Supplementary Information for *Helping Friends or Influencing Foes: Electoral and Policy Effects of Campaign Finance Contributions*

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1 Relaxing the Uniform Distribution Assumption

thereby generalizing the model to include to other, weaker, distributional assumptions. We replace the Uniform distribution with the Beta distribution with parameters a and b and density

$$f(\theta; a, b) = \frac{1}{B(a,b)}\theta^{a-1}(1-\theta)^{b-1} \tag{1}$$

where $B(\cdot)$ is the beta function. It is well known that the expected value of θ under this distribution is $\frac{a}{a+b}$.

Though it is possible to derive conditions for general distributions on $[0,1]$ that support our original results, the beta distribution is easy to work with and is flexible enough to demonstrate that our results do not rely on anything peculiar to the Uniform distribution.

1.1 Beliefs

Bayesian updating from the signals in our model corresponds to the standard Beta-Binomial model. In particular, if n is the total number of signals observed and k is the number of high signals, posterior beliefs about θ are distributed Beta$[a+k, b+n-k]$ with expectation equal to $\frac{a+k}{a+b+n}$. Thus, the posterior expectation of θ for G upon observing only s_G is

$$\mathbb{E}[\theta | s_G = 0] = \frac{a}{a+b+1} \tag{2}$$

$$\mathbb{E}[\theta | s_G = 1] = \frac{a+1}{a+b+1}. \tag{3}$$

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In a pooling equilibrium, M's beliefs are the same as above upong observing s_M. In a separating equilibrium, M acts as if she observes both s_G and s_M, and her posterior beliefs have the following expectations:

$$\mathbb{E}[\theta|s_G = s_M = 0] = \frac{a}{a+b+2} \tag{4}$$

$$\mathbb{E}[\theta|s_G \neq s_M] = \frac{a+1}{a+b+2} \tag{5}$$

$$\mathbb{E}[\theta|s_G = s_M = 1] = \frac{a+2}{a+b+2}. \tag{6}$$

Noting that Beta$[1,1]$ is the Uniform distribution, we can easily verify that the expected values from the main text satisfy the above equations: $\mathbb{E}[\theta|s_G = 0] = \frac{1}{3}$, $\mathbb{E}[\theta|s_G = 1] = \frac{2}{3}$, $\mathbb{E}[\theta|s_G = s_M = 0] = \frac{1}{4}$, $\mathbb{E}[\theta|s_G \neq s_M] = \frac{2}{4} = \frac{1}{2}$, and $\mathbb{E}[\theta|s_G = s_M = 1] = \frac{3}{4}$.

1.2 Interim Expected Payoffs

We derive interim expected payoffs to G for different contibutions in a separating strategy profile in order to derive the conditions for existence of such a separating equilibrium. Recall that $\Pr[s_M = 1|s_G] = \mathbb{E}[\theta|s_G]$ and that M sets policy equal to its expectation of θ. Consider a separating strategy and belief profile profile in which $\pi < \pi^*$ induces the belief that $s_G = 0$ and $\pi \geq \pi^*$ induces the belief that $s_G = 1$. Since we are focused on group-optimal equilibrium we will only consider the interim expected payoffs for sending $\pi = 0$ versus $\pi = \pi^*$.

Let $\mu(\pi, s_G)$ denote the expected policy chosen by M from the perspective of G given s_G and a choice π. We have

$$\mu(0,0) = \frac{a}{a+b+1}\frac{a+1}{a+b+2} + \left(1 - \frac{a}{a+b+1}\right)\frac{a}{a+b+2} \tag{7}$$

$$= \frac{a}{a+b+1} \tag{8}$$

$$\mu(\pi^*,0) = \frac{a}{a+b+1}\frac{a+2}{a+b+2} + \left(1 - \frac{a}{a+b+1}\right)\frac{a+1}{a+b+2} \tag{9}$$

$$= \frac{a}{a+b+1} + \frac{1}{a+b+2} \tag{10}$$

$$\mu(0,1) = \frac{a+1}{a+b+1}\frac{a+1}{a+b+2} + \left(1 - \frac{a+1}{a+b+1}\right)\frac{a}{a+b+2} \tag{11}$$

$$= \frac{a+1}{a+b+1} - \frac{1}{a+b+2} \tag{12}$$

$$\mu(\pi^*,1) = \frac{a+1}{a+b+1}\frac{a+2}{a+b+2} + \left(1 - \frac{a+1}{a+b+1}\right)\frac{a+1}{a+b+2} \tag{13}$$

$$= \frac{a+1}{a+b+1} \tag{14}$$

Setting $a = b = 1$ for the Uniform distribution gives us the expressions from the main text: $\mu(0,0) = \frac{1}{3}$, $\mu(\pi^*,0) = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$, $\mu(0,1) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$, $\mu(\pi^*,1) = \frac{2}{3}$, so these expressions for μ are a direct generalization of those from the main text.

The interim expected payoffs $U(\pi,s_G)$ for sending $\pi=0$ and $\pi=\pi^*$ for each type are

$$U(\pi,s_G)=(\mu(\pi,s_G)-1)V(\pi)+1. \tag{15}$$

Thus, an interest group of type s_G should choose $\pi=\pi^*$ rather than $\pi=0$ if

$$\frac{V(\pi^*)}{V(0)}\leq\frac{\mu(0,s_G)-1}{\mu(\pi^*,s_G)-1}. \tag{16}$$

Inequality 16 holds for the bad type if

$$\frac{V(\pi^*)}{V(0)}\leq\frac{\frac{a}{a+b+1}-1}{\frac{a}{a+b+1}+\frac{1}{a+b+2}-1}=\frac{ab+a+b^2+3b+2}{ab+b^2+2b+1}. \tag{17}$$

Inequality 16 holds for the good type if

$$\frac{V(\pi^*)}{V(0)}\leq\frac{\frac{a+1}{a+b+1}-\frac{1}{a+b+2}-1}{\frac{a+1}{a+b+1}-1}=\frac{ab+a+b^2+3b+1}{ab+b^2+2b}. \tag{18}$$

Note that

$$\frac{ab+a+b^2+3b+1}{ab+b^2+2b}-\frac{ab+a+b^2+3b+2}{ab+b^2+2b+1}=>0 \tag{19}$$

for $a>0$ and $b>0$, which implies the single crossing condition from the main text: if the bad type has a weak incentive to send the high contribution then the good type has a strict incentive to do so. Proposition 3 states our result.

Proposition 3. *If*

$$\beta\geq\frac{ab+a+b^2+3b+2}{ab+b^2+2b+1}$$

then there is a separating equilibrium. Otherwise, any equilibrium is pooling.

Proof. Given the single-crossing condition from above, existence when

$$\beta\geq\frac{ab+a+b^2+3b+2}{ab+b^2+2b+1}$$

follows from the Intermediate Value Theorem. Non-existence when $\beta<\frac{ab+a+b^2+3b+2}{ab+b^2+2b+1}$ follows from the face that $V(\pi)/V(0)$ is increasing in π, which implies that the bad type would deviate to the high contribution for any π. ■

2 Relaxing the Assumption of a Fixed Budget

In this section we relax the assumption that G must distribute its entire budget across the moderate and ally candidates. To do so, we must introduce a modified model of campaign contributions. As before, there is a moderate candidate M and an ally candidate A. The policy space is $X=[0,1]$

and the state of nature θ is uniform on $[0,1]$ as in the original model. The game proceeds as follows: The interest group receives a noisy signal $s_G \in \{0,1\}$, with $\Pr[s_G = 1|\theta] = \theta$. The interest groups then chooses a contribution schedule (d_M, d_A) where $d_i \geq 0$ for $i \in \{M, A\}$ and d_M and d_A represent contributions to the moderate and ally candidate, respectively. G's budget constraint is given by $d_M + d_A \leq 1$. The probability that M wins the elections determined by $V(d_M, d_A)$ where $\frac{\partial V(d_M,d_A)}{\partial d_M} > 0$, $\frac{\partial V(d_M,d_A)}{\partial d_A} < 0$, and $\frac{\partial^2 V(d_M,d_A)}{\partial {d_A}^2} > 0$ for all pairs of contributions. Finally, the winner of the election $j \in \{M, A\}$ receives a signal $s_j \in \{0,1\}$ with $\Pr[s_j = 1|\theta] = \theta$ and chooses a policy $x \in X$.

The candidates' utility functions are the same as in the original model:

$$u_A(x) = x, \tag{20}$$

$$u_M(x, \theta) = -(x-\theta)^2. \tag{21}$$

The interest group's utility function reflects state-independent policy preferences as before, but now includes a cost of donations:

$$u_G(x, d_M, d_a) = x - c * (d_M + d_A) \tag{22}$$

where $c \geq 0$ is the marginal cost of contributions.

Our main objective is to show that a separating equilibrium of the type described in Proposition 1 exists for some parameters and that separating equilibria vanish when the electoral effects of contributions are too small. Thus, we will do away with the requirement of selecting group-optimal equilibria and ignore questions of uniqueness for the purposes of this exercise.

The candidates' policy choices follow from the arguments in the main text. Furthermore, since the costs of contributions do not differ by type, the arguments from the main text imply that expected payoffs satisfy a single-crossing condition with respect to changes in V. Thus, existence of a separating equilibrium hinges on whether or not there exists a good type contribution schedule that makes the bad type indifferent between its separating equilibrium payoff and its payoff from imitating the good type. First, we characterize the bad type's optimal contribution in a separating equilibrium.

Lemma 1. *There exists $\tilde{d} \in [0,1]$ such that the bad type's optimal contribution in a separating equilibrium is $(d_M, d_A) = (0, \tilde{d})$.*

Proof. Assume that the contribution $(0,d)$ induces $\hat{s} = 0$. Then the expected utility for such a contribution is

$$V(0,d)\frac{1}{3} + (1 - V(0,d)) - cd = 1 - \frac{2}{3}V(0,d) - cd. \tag{23}$$

Since G should not donate to the Moderate absent any signaling motivation, the optimal contribution for the low type in a separating equilibrium is then $(0, \tilde{d})$ where $\tilde{d}$ solves

$$\arg\max_{d} 1 - \frac{2}{3}V(0,d) - cd. \tag{24}$$

The first order condition is

$$-\frac{2}{3}\frac{\partial V(0,d)}{\partial d} = c \tag{25}$$

and the second order condition for a maximum is

$$-\frac{2}{3}\frac{\partial^2 V(0,d)}{\partial d^2} < 0. \tag{26}$$

The second order condition is satisfied by our assumption that $\frac{\partial^2 V(d_M,d_A)}{\partial {d_A}^2} > 0$. Thus, if $-\frac{2}{3}\frac{\partial V}{\partial d_A}(0,0) < c$ then we can set $\tilde{d} = 0$, if $-\frac{2}{3}\frac{\partial V}{\partial d_A}(0,1) > c$ then we have $\tilde{d} = 1$, and otherwise (by the Intermediate Value Theorem) there exists some $\tilde{d} \in (0,1)$ satisfying the first order condition. In each of these cases, $(0,\tilde{d})$ is a best response for the low type with no signaling motivation. ■

We will separately analyze the case where the bad type's budget constraint is binding (i.e. $\tilde{d} = 1$ in the above Lemma) and the case where it is not (i.e. $\tilde{d} < 1$). If $\tilde{d} = 1$, the results of the model are identical to Proposition 1 in the model with no costs.

Lemma 2. *Suppose $\tilde{d} = 1$. Then there is a separating equilibrium if and only if $\frac{V(1,0)}{V(0,1)} \geq \frac{8}{5}$.*

Proof. If the low type weakly prefers to give $(0,1)$ and reveal itself as a low type rather than give $(1,0)$ and have M believe it to be a high type, then by continuity there is some $\hat{d} \in [0,1]$ such that the low type is indifferent between giving $(0,1)$ and $(\hat{d},0)$ when the former induces the belief that $s_G = 0$ and the latter induces the belief that $s_G = 1$. Since the low type's expected utility is monotone with respect to changes in V, there can be no seaparating equilibrium if the low type would be willing to deviate to $(1,0)$.

Since G's expected payoffs satisfy a single crossing condition with respect to changes in s_G and V, this yields a separating equilibrium. Since the contributions schedules $(0,1)$ and $(1,0)$ have the same cost, the parameter c drops out of this comparison and the relevant calculation is identical to that in the model with no costs. Thus, the same computations that are in the main text imply that there is a separating equilibrium if and only if $\frac{V(1,0)}{V(0,1)} \geq \frac{8}{5}$. ■

Lemma 2 is intuitive: when altering the model to include costs does not change the preferred contribution of the bad type, the results of the costly contributions model is essentially the same as the results in the baseline model with a fixed budget. When costs are substantial enough to cause the bad type to donate only part of its budget to the Ally when there is no signaling motivation, the results change in one key way: the bad type's utility from deviating from a separating equilibrium can be reduced by changes in V *or* by increased costs of contributions.

Lemma 3. *Suppose $\tilde{d} < 1$. Then there is a separating equilibrium if and only if $V(1,0) \geq \frac{8}{5}V(0,\tilde{d}) - \frac{12}{5}c(1-\tilde{d})$.*

Proof. As before, if the bad type weakly prefers its separating payoff over deviating to $(1,0)$ then there is some $\hat{d} \in [0,1]$ that makes the bad type indifferent between deviating $(\hat{d},0)$ or receiving its separating equilibrium payoff. By the single-crossing condition, this point is an equilibrium. Thus, there is a separating equilibrium if and only if

$$-\frac{2}{3}V(0,\tilde{d}) - c\tilde{d} \geq -\frac{5}{12}V(1,0) - c \tag{27}$$

$$\Rightarrow V(1,0) \geq \frac{8}{5}V(0,\tilde{d}) - \frac{12}{5}c(1-\tilde{d}). \tag{28}$$

for any $\tilde{d} \in [0,1)$. ■

The equilibrium conditions in Lemma 3 suggest that equilibrium existence depends on changes in V as well as the costs of contributions. Comparisons between the costly contributions model and the baseline model with no costs are made in Corollary 1.

Corollary 1.

1. *As $\tilde{d} \to 1$ the conditions for existence of a separating equilibrium converge to the baseline model with no costs of contributions*

2. *As $c \to 0$ from above, the conditions for existence of a separating equilibrium converge monotonically to the conditions from the baseline model with no costs.*

3. *For $c \in (0,1)$ and $\tilde{d} < 1$, separating equilibria can be supported for lower values of $\frac{V(1,0)}{V(0,1)}$ than in the baseline model.*

The picture painted by this analysis is that including costs of contributions and relaxing the fixed budget assumption generates results with similar intuition to the baseline model. In closing, we also note that we have assumed that all contributions to M or A strictly increase the probability that the recipient wins the election: if the electoral effects of contributions are zero in any version of this model, there would be no separating equilibrium since changes in contributions would affect the expected utilities of bad types and good types in exactly the same way.