Lecture 1

Theory of externalities

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The welfare-theoretic basis

Before diving into environmental econ we need some definitions

The normative criterion we use to judge the desirability of economic outcomes is called **Pareto optimality**

The welfare-theoretic basis

> An economic outcome is said to be **Pareto optimal** if a reallocation of resources cannot make at least one person better off without making another person worse off

Is there a way to make everyone weakly better off? If not, then we have a Pareto optimal allocation of resources

This does not make any judgments on whose well-being matters more, sidesteps equity and fairness and focuses on efficiency

Big draw back of Pareto optimality is that some seemingly undesirable outcomes can be Pareto optimal

The welfare-theoretic basis

Pareto optimal is central to the first and second welfare theorems of economics:

First Welfare Theorem: If markets are perfectly competitive and complete, then a decentralized price system will deliver a Pareto optimal allocation

Second Welfare Theorem: If markets are perfectly competitive and complete, then any Pareto optimal allocation can be supported by a decentralized price system and lump sum taxes and transfers

The welfare-theoretic basis

The welfare theorems' appeal is pretty clear: **if the conditions are met we can have the largest economic pie (efficiency) simply by letting the free market function**

If we perceive there are inequities in outcomes under the free market, we can simply (lump sum) transfer income to adjust the distribution and still have efficiency

Unfortunately the conditions required by the welfare theorems are not usually met, especially in environmental economics

Why?

Externalities

Most environmental goods have no market or price

There is no price for noise, air pollution, water pollution, biodiversity, etc

Why?

Generally because property rights are not well-defined: does a factory have the right to pollute or do you have the right to clean air?

There are not complete markets!

Externalities

A key concept in environmental economics is externalities

> An **externality** exists when agent A's utility or production function depends directly on real variables chosen by another agent B, without an offer of compensation or other attention given to the effect on A's well-being.

A critical aspect of this is that it must be a *real* variable:

Pollution, noise, etc

Not changes in wages or prices of regular market goods

Public goods

Another concept closely related to externalities is public goods

> A **public good** is a good that is non-rival and non-excludable. The good can be simultaenously enjoyed by several people (non-rivalry), and none of these people can be prevented from enjoying the good (non-excludability).

Non-excludability causes problems for the welfare theorems: how are decentralized price systems suppose to work if anyone can enjoy the good whenever they want without having to pay?

This is the **free rider problem**

Our beginning model

Lets start building our model of the economy

Suppose there is:

- Two people
- A dirty good
- A clean good
- Labor as the only factor (input) of production

Our beginning model

Define each person i's utility as $U_i(x_i, z_i, E)$ for $i = 1, 2$

x_i is consumption of the dirty good,

z_i is consumption of the clean good,

and E is the level of pollution emissions

Our beginning model

Production of x causes the emissions E

We can define production of x as: $x = f(l_x, E)$ where $f_l > 0$ and $f_E > 0$

Emissions are defined as an *input*, so if you want less emissions you will produce less because $f_E > 0$

We could equivalently define emissions as a joint product but the input setup is cleaner.

Our beginning model

Production of the clean good z only uses labor: $z = g(l_z)$

Labor can be measured in terms of hours and there is only so many hours that can be worked l such that $l_x + l_z = l$

This is all we need for the model, now we just need to set it up, and define two terms

Consumption trade offs

A key piece of economics is the **marginal rate of substitution** (MRS)

The MRS tells us how an individual trades off consumption of two different goods and is defined as:

$$MRS_1^{xz} \equiv \frac{\partial U_1}{\partial x_1} \bigg/ \frac{\partial U_1}{\partial z_1} \equiv dz_1/dx_1$$

It tells us how many units of z person 1 is willing to give up to get 1 more unit of x

It is just the slope of an indifference curve

Production trade offs

Another key piece of economics is the **marginal rate of transformation** (MRT)

The MRT tells us how a firm trades off production of two different goods and is defined as:

$$MRT_1^{xz} \equiv \frac{\partial g}{\partial l_z} \bigg/ \frac{\partial f}{\partial l_x} \equiv MRT \equiv dz/dx$$

It tells us how many units of z the firm has to give up to produce 1 more unit of x

It is just the slope of the production possibility frontier

Pareto optimality

First lets derive the conditions for a Pareto optimal allocation of resources

How do we do this?

By just following the definition of Pareto optimality:

We find the allocations of consumption, labor, and pollution, that maximizes one person's utility while making the other person no worse off than some (arbitrary) benchmark

While also satisfying technology (production) and endowment (labor) constraints

Pareto optimality

The problem is given by:

$$\max_{x_1,x_2,z_1,z_2,l_x,l_z,E} U_1(x_1,z_1,E) \quad \text{subject to:}$$

$$U_2(x_2,z_2,E) \geq \bar{u}_2 \tag{1}$$

$$f(l_x,E) = x_1 + x_2 \tag{2}$$

$$g(l_z) = z_1 + z_2 \tag{3}$$

$$l = l_x + l_z \tag{4}$$

We are maximizing person 1's utility subject to (1) keeping person 2s utility at least some level, (2,3) production constraints, (4) total labor allocation

Now let's look at the Lagrangian

Pareto optimality

The Lagrangian is given by:

$$\begin{aligned}\mathcal{L} = \max_{x_1,x_2,z_1,z_2,l_x,l_z,E} U_1(x_1, z_1, E) \\ &+ \lambda_u[U_2(x_2, z_2, E) - \bar{u}_2] \\ &+ \lambda_x[f(l_x, E) - x_1 - x_2] \\ &+ \lambda_z[g(l_z) - z_1 - z_2] \\ &+ \lambda_l[l - l_x - l_z]\end{aligned}$$

Now let's look at the FOCs

Pareto optimality: Problem

$$\max_{x_1,x_2,z_1,z_2,l_x,l_z,E} U_1(x_1, z_1, E)$$
$$+ \lambda_u[U_2(x_2, z_2, E) - \bar{u}_2]$$
$$+ \lambda_x[f(l_x, E) - x_1 - x_2]$$
$$+ \lambda_z[g(l_z) - z_1 - z_2]$$
$$+ \lambda_l[l - l_x - l_z]$$

Pareto optimality: Consumption FOCs

$$\frac{\partial U_1}{\partial x_1} = \lambda_x, \qquad \frac{\partial U_1}{\partial z_1} = \lambda_z$$

and

$$\lambda_u \frac{\partial U_2}{\partial x_2} = \lambda_x, \qquad \lambda_u \frac{\partial U_2}{\partial z_2} = \lambda_z$$

Marginal utility equals the shadow price of the good

Pareto optimality: Labor FOCs

$$\lambda_x \frac{\partial f}{\partial l_x} = \lambda_l, \qquad \lambda_z \frac{\partial g}{\partial l_z} = \lambda_l$$

The marginal product of labor equals the shadow price of labor

Pareto optimality: Emissions FOC

$$-\left[\frac{\partial U_1}{\partial E} + \lambda_u \frac{\partial U_2}{\partial E}\right] = \lambda_x \frac{\partial f}{\partial E}$$

The marginal utility cost of emissions equals the marginal product of emissions

Consumption efficiency

The consumption FOCs give us efficiency in consumption:

$$MRS_1^{xz} \equiv \frac{\partial U_1}{\partial x_1} \bigg/ \frac{\partial U_1}{\partial z_1} = \lambda_x / \lambda_z = \frac{\partial U_2}{\partial x_2} \bigg/ \frac{\partial U_2}{\partial z_2} \equiv MRS_2^{xz}$$

Efficiency in consumption requires that the marginal rate of substitution (MRS) between individuals is equal (i.e. the slopes of their indifference curves are equal)

Exchange efficiency

The consumption and labor supply FOCs give us

$$MRS_i^{xz} \equiv \frac{\partial U_i}{\partial x_i} \Bigg/ \frac{\partial U_i}{\partial z_i} = \lambda_x / \lambda_z = \frac{\partial g}{\partial l_z} \Bigg/ \frac{\partial f}{\partial l_x} \equiv MRT^{xz}$$

MRSs must equal the marginal rate of transformation (MRT)

The slope of the indifference curves must equal the slope of the production possibility frontier

Emissions efficiency

Substitute in the consumption FOCs to obtain a new expression for emissions efficiency:

$$MRS_1^{Ex} + MRS_2^{Ex} \equiv -\frac{\partial U_1}{\partial E} \Big/ \frac{\partial U_1}{\partial x_1} - \frac{\partial U_2}{\partial E} \Big/ \frac{\partial U_2}{\partial x_2} = \frac{\partial f}{\partial E}$$

The marginal product of emissions must equal the **sum** of the marginal rates of substitution of between x and E across both individuals

General Intuition

We can understand (prove) why these conditions need to be met by considering cases when they are not met

For any case where the three efficiency conditions are not met, we can show there exists a possible **Pareto improvement** $\rightarrow$ the conditions must be met for Pareto optimality

> A Pareto improvement is a reallocation of resources that makes at least one person better off without making anyone else worse off

Consumption Efficiency

Suppose Ann's MRS is 2, and Bob's MRS is 4, the MRSs tell us that:

- Ann is willing to give up 2 units of z for 1 more unit of x
- Bob is willing to give up 4 units of z for 1 more unit of x

If Bob gives Ann 3 units of z in exchange for 1 unit of x, **both are better off**

Bob was willing to give up 4 units, Ann was willing to accept 2 units $\rightarrow$ both are 1 unit of z better off

Exchange Efficiency

Suppose MRS^{xz} is 1 and MRT^{xz} is 3, the MRS and MRT tell us that:

- The consumer is willing to give up 1 unit of x for 1 unit of z
- If we move labor from producing x to z we can get 3 units of z for 1 unit of x

If we give up 1 unit of x, we get 3 units of z $\rightarrow$ the consumer gets 2 more units of z than they needed to be better off

Emissions Efficiency

Suppose $\text{MRS}_1^{Ex} + \text{MRS}_2^{Ex} = 3$ and $\frac{\partial f}{\partial E} = 4$, these conditions tell us that:

- The total cost to both consumers of 1 more unit of E in terms of units of x is 3
- 1 more unit of E gets us 4 more units of x

If we increase E by 1 unit, the benefits (4) outweigh the costs (3), we can make the consumers better off

Competitive markets

We now know the necessary conditions for a Pareto allocation, which serves as a useful baseline

Now we can ask whether free markets can achieve the Pareto optimal outcome

In the free market:

- Consumers take the price of goods as given
- Firms take the price of inputs as given

Competitive markets

Let p_x and p_z be the prices of x and z

Let w be the wage paid to labor

Let y_i be income for person i

Each person maximizes their utility

Each firm maximizes profit

Competitive markets

The utility maximization problem is:

$$\max_{x_i, z_i} U_i(x_i, z_i, E) \quad \text{subject to: } y_i = p_x x_i + p_z z_i$$

The Lagrangian is

$$\max_{x_i, z_i} U_i(x_i, z_i, E) + \lambda_i [y_i - p_x x_i - p_z z_i]$$

with first-order conditions:

$$\frac{\partial U_i}{\partial x_i} = \lambda_i p_x, \qquad \frac{\partial U_i}{\partial z_i} = \lambda_i p_z$$

Competitive markets

The profit maximization problems for firms producing z and x are:

$$\max_{l_z} p_z g(l_z) - wl_z, \qquad \max_{l_x, E} p_x f(l_x, E) - wl_x$$

This gives us the FOCs for the firm producing x:

$$p_x \frac{\partial f}{\partial l_x} = w \qquad p_x \frac{\partial f}{\partial E} = 0$$

and the firm producing z

$$p_z \frac{\partial g}{\partial l_z} = w$$

Competitive markets: Consumption

The consumption first-order conditions:

$$\frac{\partial U_i}{\partial x_i} = \lambda_i p_x, \qquad \frac{\partial U_i}{\partial z_i} = \lambda_i p_z$$

give us:

$$\frac{\partial U_1}{\partial x_1} \Bigg/ \frac{\partial U_1}{\partial z_1} = p_x/p_z = \frac{\partial U_2}{\partial x_2} \Bigg/ \frac{\partial U_2}{\partial z_2}$$

The equal MRS condition for efficiency in consumption is met

Competitive markets: Exchange

Combining the consumption and labor FOCs gives us that:

$$\frac{\partial U_i}{\partial x_i} \Bigg/ \frac{\partial U_i}{\partial z_i} = p_x/p_z = \frac{\partial g}{\partial l_z} \Bigg/ \frac{\partial f}{\partial l_x}$$

The MRS=MRT exchange efficiency condition is met

Competitive markets: Emissions

The efficiency condition for emissions:

$$-\frac{\partial U_1}{\partial E}\Big/\frac{\partial U_1}{\partial x_1}-\frac{\partial U_2}{\partial E}\Big/\frac{\partial U_2}{\partial x_2}=\frac{\partial f}{\partial E}$$

is not met!

The free market provides no incentive for the firm to treat E as scarce or to account for its impact on consumers: the firm faces a price on pollution of 0

Market intervention

Much of environmental economics is about designing policy to correct market failures

The intellectual starting point comes from Pigou: that externalities can be corrected by imposing a fee on emissions

Suppose a firm now has to pay a fee τ^* per unit of emissions, can we achieve the optimal outcome?

Emissions taxation

The firm's problem is now:

$$\max_{l_x, E} p_x f(l_x, E) - w l_x - \tau^* E$$

with first-order conditions:

$$p_x \frac{\partial f}{\partial l_x} = w, \qquad p_x \frac{\partial f}{\partial E} = \tau^*$$

Is there a fee that can satisfy the emissions efficiency condition achieve the Pareto optimal outcome?

Emissions taxation: Conditions

Recall the firm's FOC is:

$$p_x \frac{\partial f}{\partial E} = \tau^*$$

and the emissions efficiency condition is:

$$-\frac{\partial U_1}{\partial E} \bigg/ \frac{\partial U_1}{\partial x_1} - \frac{\partial U_2}{\partial E} \bigg/ \frac{\partial U_2}{\partial x_2} = \frac{\partial f}{\partial E}$$

Emissions taxation: Solution

Notice that if you multiply the right hand side of the emissions efficiency condition by p_x, it is equal to the left hand side of the firm FOC

The Pareto optimal tax is thus:

$$\tau^* = -p_x \left[\frac{\partial U_1}{\partial E} \Bigg/ \frac{\partial U_1}{\partial x_1} + \frac{\partial U_2}{\partial E} \Bigg/ \frac{\partial U_2}{\partial x_2} \right]$$

This tax can thus make the firm's profit-maximizing condition consistent with Pareto optimality

Emissions taxation

$$\tau^* = -p_x \left[\frac{\partial U_1}{\partial E} \Big/ \frac{\partial U_1}{\partial x_1} + \frac{\partial U_2}{\partial E} \Big/ \frac{\partial U_2}{\partial x_2} \right]$$

There are two parts to the intuition for the tax:

1. $-\left[\frac{\partial U_1}{\partial E} \Big/ \frac{\partial U_1}{\partial x_1} + \frac{\partial U_2}{\partial E} \Big/ \frac{\partial U_2}{\partial x_2} \right]$ tells us how much total x we need in order to compensate both consumers for an additional unit of E and keep their utilities constant
2. p_x tells us the dollar value of this much x

The tax is the marginal utility cost of emissions in dollar terms

Emissions taxation

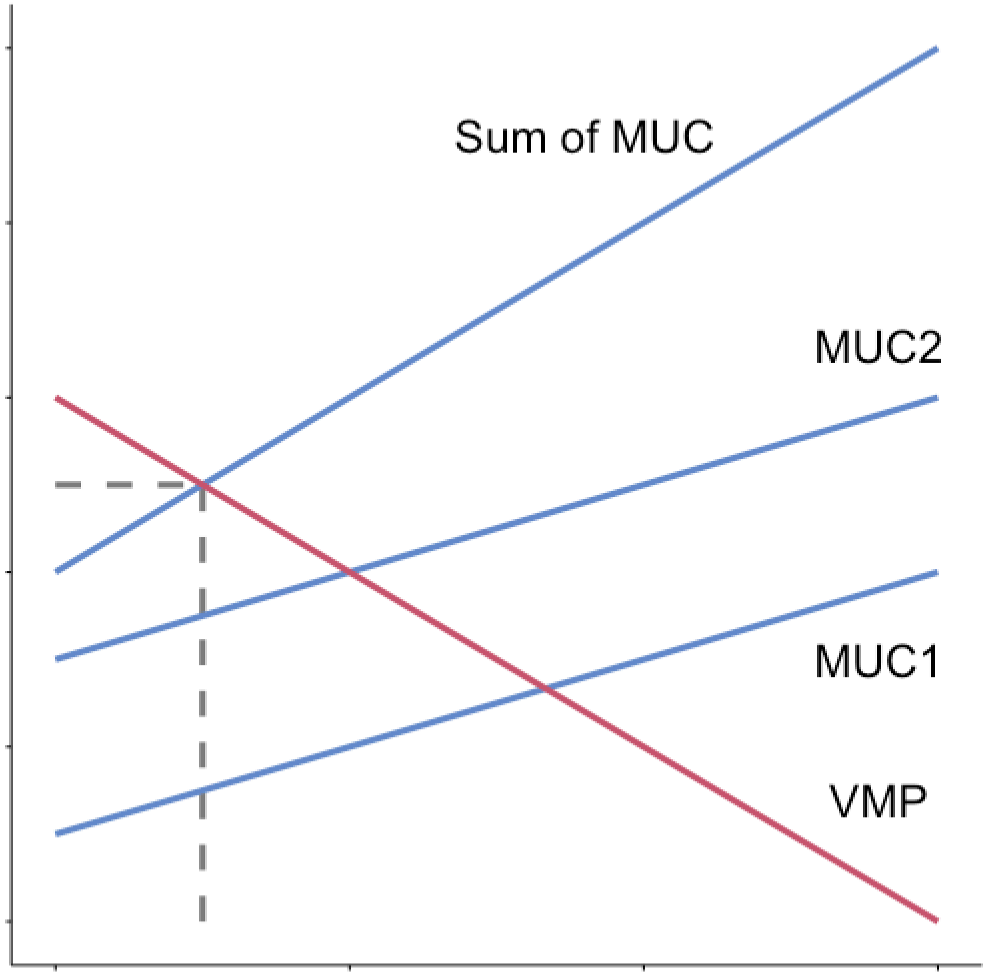
$$\tau^* = -p_x \left[\frac{\partial U_1}{\partial E} \Big/ \frac{\partial U_1}{\partial x_1} + \frac{\partial U_2}{\partial E} \Big/ \frac{\partial U_2}{\partial x_2} \right]$$

The tax depends on marginal utility of consumption, this implies two things:

1. The tax depends on the distribution of income
2. The tax depends on the distribution of endowments in general (e.g. income, labor)

Changes in income or factor endowments will therefore change the level of the Pareto optimal tax!

Graphical emissions taxation



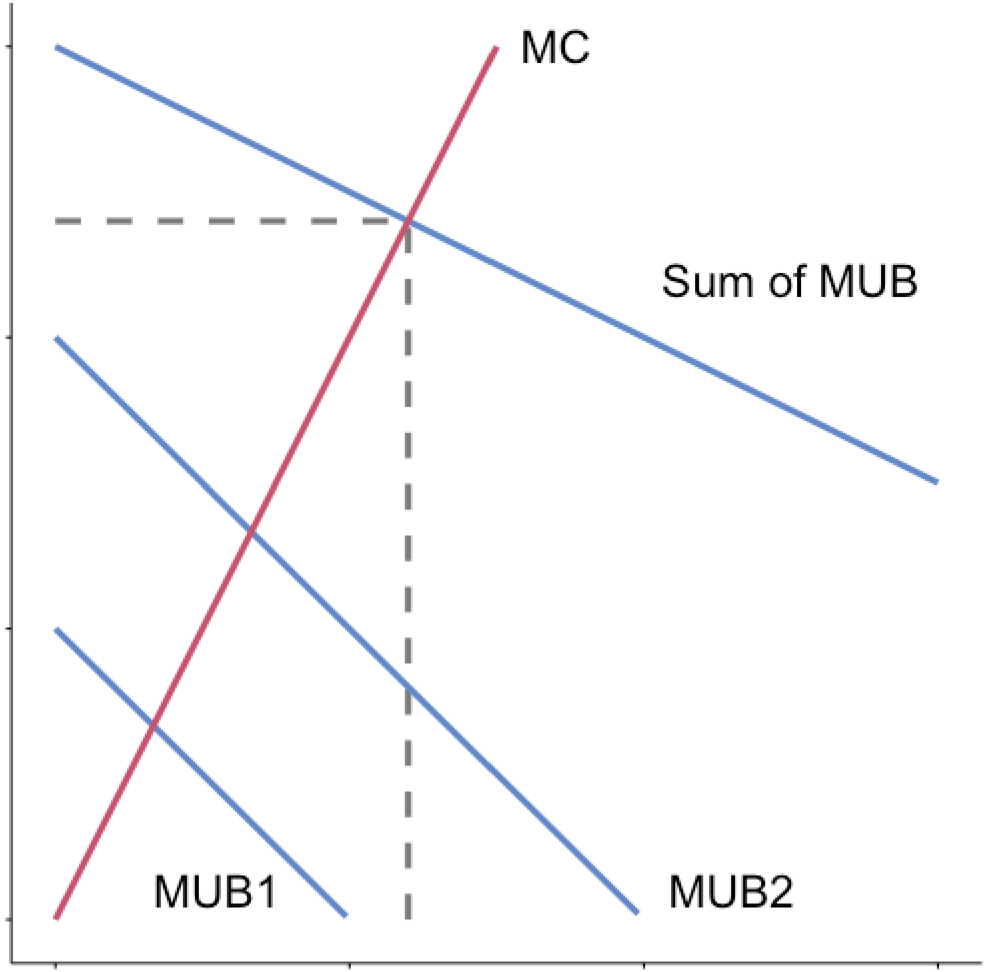
$-p_x \left[\frac{\partial U_i}{\partial E} \bigg/ \frac{\partial U_i}{\partial x_i} \right]$ is the individual marginal utility cost of emissions

$p_x \frac{\partial f}{\partial E}$ is the value of marginal product of emissions (VMP)

At a Pareto optimum the sum of MUCs must be equal to the VMP

MC of emissions must equal MB

Graphical emissions taxation: abatement



Let abatement A^* be how much we reduce emissions below baseline: $A^* = \bar{E} - E^*$

$-p_x \left[\frac{\partial U_i}{\partial E} \middle/ \frac{\partial U_i}{\partial x_i} \right]$ is the individual marginal utility benefit

$p_x \frac{\partial f}{\partial E}$ is the marginal cost of abatement