Lecture 7

Hedonics: Property value models

Ivan Rudik
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Roadmap

- What can we use to infer the demand for environmental goods?
- What do housing prices tell us?
- When do changes in house prices give us welfare measures

Environmental quantity changes

Last time we learned that we cannot directly compute the welfare consequences of a change in environmental quantities

There are no markets for the environmental goods

We don't observe the ordinary inverse demand curve

We can't compute CS, EV, CV, etc!

Revealed preference approaches

One way to circumvent this problem is to look at private goods that interact with the environmental good

If there are changes in the environmental good, holding everything else fixed, that should be reflected in *some way* in changes in the price of the related private good

This change in price can tell us something about how people value the change in the environmental good

Revealed preference approaches

There is no market for orcas

Suppose there's a massive decline in orcas off the Washington coast, what happens?

We will likely see demand for sightseeing tours go down (MB of these tours went down!)

This drops the price of tours

A non-market good had an effect on a market price

What does this price change mean?

Hedonics: Property value models

Common market goods to use for revealed preference valuation are **properties**

When people buy a home they are purchasing a bundle of goods:

- Rooms
- Bathrooms
- School quality
- **Environmental quality**

Homes located in pristine areas are likely to be more valuable than identical homes located near toxic facilities

Hedonics: Property value models

Real estate is virtually ideal for measuring environmental changes

Real estate markets are often competitive and thick

Property purchases are large and consequential: buyers and sellers are likely to be well-informed

It is uncontroversial that property values should reflect local attributes

The hedonic model

Property value approaches are often called **hedonics** because they rely on the hedonic model

Suppose that we have some quality-differentiated good (i.e. a home)

This good is characterized by a set of J property characteristics x

- parcel size, school quality, bedrooms, etc

It is also characterized by an environmental good q

The hedonic model

The price of a house is determined by a **hedonic price function** $P(x, q)$

P maps characteristics of the house and local environment to the market value of the home

For a particular house k its price is $p_k = P(x_k, q_k)$

P arises in equilibrium from the interaction of all buyers in sellers in the market

Here we will assume the supply of houses is fixed in the short run so the price function arises from buyer behavior

The hedonic model: consumer's choice problem

Households buy a single property given their budget constraint and their preferences

Here we will assume that households are effectively just choosing (x, q) instead of a specific house with the following objective:

$$\max_{x,q,z} U(x, q, z; s) \quad s.t. \quad y = z + P(x, q)$$

- z is the numeraire good (spending on other private goods)
- y is income
- s is the set of the household's characteristics like family size

Unrealistic pieces of the model

One unrealistic part of this model is that we are assuming household characteristics are continuous

Many housing characteristics are discrete (bedrooms, bathrooms, etc)

Another is that you just can't purchase some sets of x (i.e. a huge lot in downtown manhattan with a farm)

We won't touch on this in class but there is a **discrete choice** literature that works to alleviate these issues

Choosing q

Another thing to note: the consumer *chooses* q where as before it was fixed

The idea is that mobile households can move to get their desired level of the environmental good

We are thus also implicitly assuming q varies across space so that households can sort into areas they prefer

- q is really picking up **local** environmental goods

What is $P(x, q)$

In the model we are thinking of $P(x, q)$ as the annual **rental rate**, not the purchase price

This allows us to mesh more cleanly with annual income and view the problem as static rather than dynamic

This clearly works well for renting households

For homeowners we are basically assuming they rent from themselves every year

The hedonic model: consumer's choice problem

$$\max_{x,q,z} U(x,q,z;s) \quad s.t. \quad y = z + P(x,q)$$

The FOCs for this problem are:

$$\begin{aligned}
\frac{\partial U}{\partial x_j} =& \lambda \frac{\partial P}{\partial x_j} \quad j = 1, \ldots, J \\
\frac{\partial U}{\partial q} =& \lambda \frac{\partial P}{\partial q} \\
\frac{\partial U}{\partial z} =& \lambda
\end{aligned}$$

Next, combine the last two FOCs

The hedonic model: consumer's choice problem

$$\begin{aligned}\frac{\partial U}{\partial q} &= \lambda \frac{\partial P}{\partial q} \\ \frac{\partial U}{\partial z} &= \lambda\end{aligned}$$

gives us that

$$\frac{\partial P}{\partial q} = \frac{\partial U}{\partial q} \Bigg/ \frac{\partial U}{\partial z}$$

At a utility-maximizing choice, a household equates their MRS between q and z and the marginal implicit cost of q

The hedonic model: consumer's choice problem

$$\frac{\partial P}{\partial q} = \frac{\partial U}{\partial q} \bigg/ \frac{\partial U}{\partial z}$$

Recall that z is the numeraire good so we can think of it in terms of dollars

This means that $\frac{\partial U}{\partial q} \big/ \frac{\partial U}{\partial z}$ is the WTP for q, the reduction in income needed to compensate for an additional unit of q

Knowledge of the hedonic price function P is enough to tell us about household WTP for q!

The hedonic model: bid functions

Now let's dive deeper by looking at some reference utility level $\bar{u}$:

$$U(x, q, z; s) = \bar{u}$$

Next we will define something called a **bid function** $b(x, q, y, s, \bar{u})$ where:

$$U(x, q, y - b(x, q, y, s, \bar{u}); s) = \bar{u}$$

The bid function b is the maximum amount the household is willing to pay for:

- A house with characteristics x, q
- Given income y and household characteristics s
- Holding utility fixed

The hedonic model: bid functions

$$U(x, q, z; s) = \bar{u}$$

We can also invert this to solve for z:[1]

$$z = U^{-1}(x, q, \bar{u}, s)$$

Income, the bid function and z are related by:

$$b(x, q, y, s, \bar{u}); s) = y - U^{-1}(x, q, \bar{u}, s)$$

Now we have everything we need to derive a marginal WTP function for q

[1] We can do this because U is monotonically increasing in z

The hedonic model: deriving MWTP

$$U(x, q, y - b(x, q, y, s, \bar{u}); s) = \bar{u}$$

Differentiate with respect to q to get:

$$\frac{\partial U}{\partial q} - \frac{\partial U}{\partial z}\frac{\partial b}{\partial q} = 0$$

We can then rearrange to get:

$$\frac{\partial b}{\partial q} = \frac{\partial U}{\partial q} \bigg/ \frac{\partial U}{\partial z}$$

The hedonic model: deriving MWTP

Recall that the bid function is separable in income: $b(x, q, y, s, \bar{u}); s) = y - U^{-1}(x, q, \bar{u}, s)$

This lets us re-write $\frac{\partial b}{\partial q}$ as:

$$\pi^q(x, q, s, \bar{u}) = \frac{\partial b}{\partial q} = \frac{\partial U}{\partial q} \bigg/ \frac{\partial U}{\partial z}$$

Conditional on x, this defines our **compensated inverse demand function** for q!

The hedonic model: deriving MWTP

$$\pi^q(x, q, s, \bar{u}) = \frac{\partial b}{\partial q} = \frac{\partial U}{\partial q} \bigg/ \frac{\partial U}{\partial z}$$

By assuming q is a part of a larger bundle of house characteristics, we are able to characterize its demand through its relationship to the housing market

We can then use the bid function (which maps into prices) to understand the marginal WTP for q

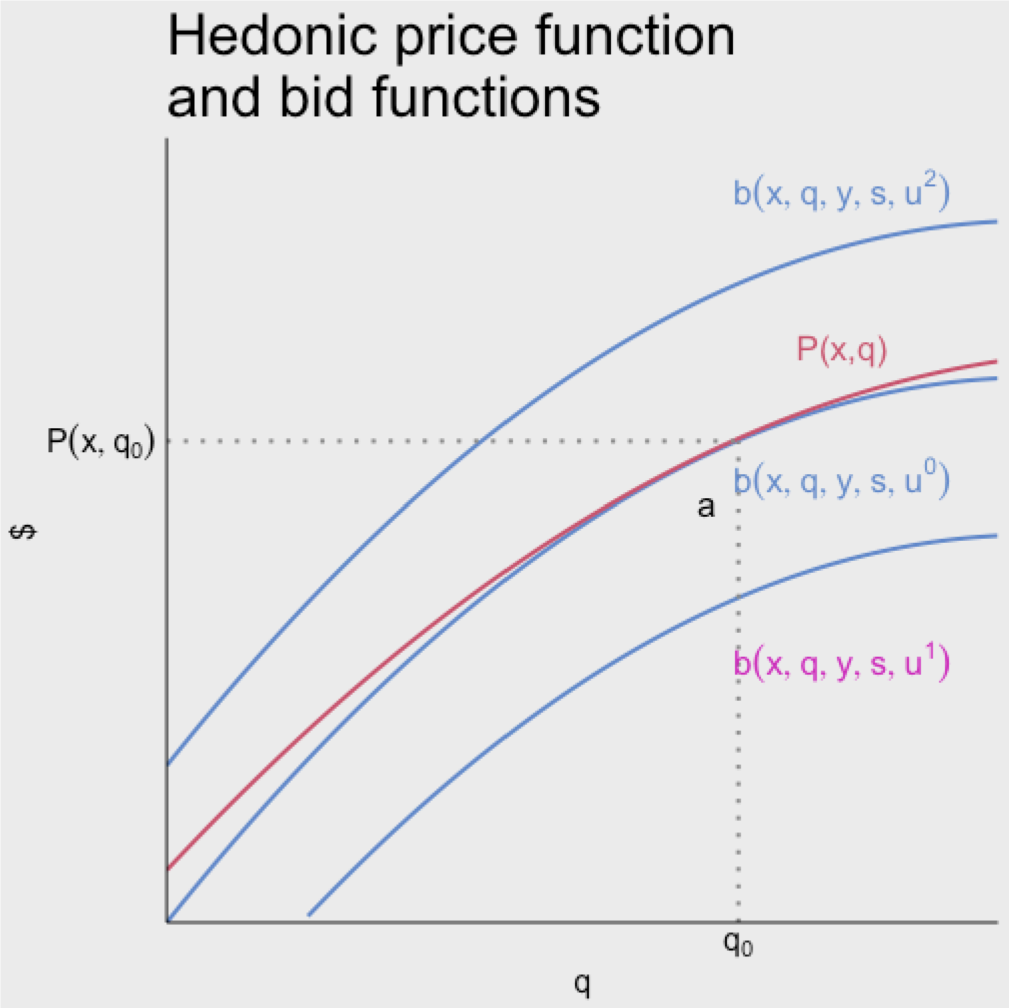
Our ultimate empirical goal is to estimate $\pi^q(x, q, s, \bar{u})$

Bid functions and housing prices

The red line is the hedonic price function

The blue lines are a single household's bid functions at different reference utility levels where $u_1 > u_0 > u_2$

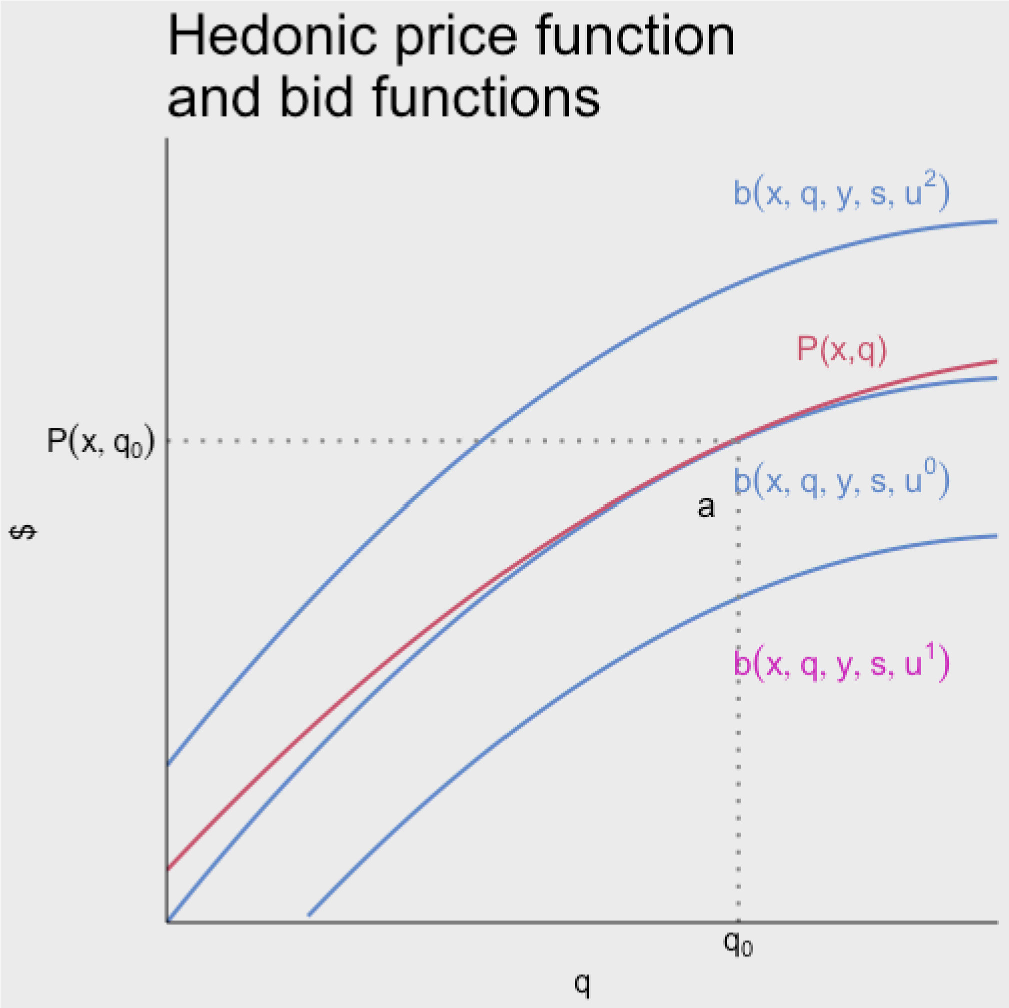
Lower bids imply higher utility because same level of q can be achieved with higher z



Bid functions and housing prices

Optimal choice is where the household's bid function is tangent to the hedonic price schedule: a

This gives us an observed consumption level q_0, observed price $P(x, q_0)$, and realized utility u^0

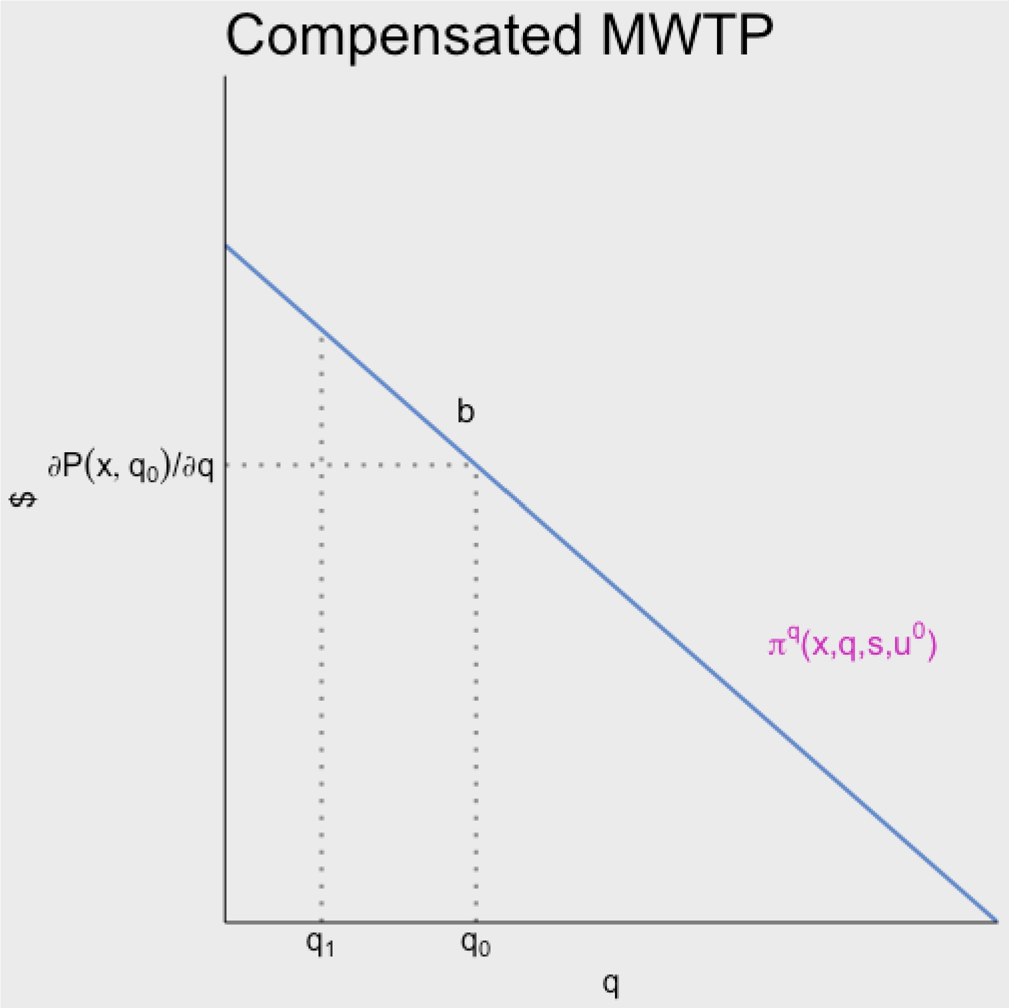


Compensated MWTP

This plot shows the corresponding compensated MWTP curve associated with $b(x, q, y, s, u^0)$

It is the slope of the bid function as q changes

We observe b if we can estimate $P(x, q)$ and its derivative



Compensated MWTP

We can estimate $P(x, q)$ using home sales prices and home attributes data

The slope of $P(x, q)$ is then equal to the MWTP for q

This gives us the consumers inverse demand for q

$$\frac{\partial P(x, q_0)}{\partial q} = \pi^q(x, q_0, s, u^0)$$