

# Lecture 4

## Non-competitive output markets

---

Ivan Rudik  
AEM 7510

# Roadmap

Previously we assumed output markets were competitive and found that the conditions for efficiency ( $MAC = MD$ ) still hold

But markets are often **not** competitive

Will this affect our results?

Why might it?

Now we have **two** distortions in our market, pollution and market power

# Environmental policy under monopoly

Suppose we have an industry with a **single** firm that generates a local pollutant

We will again look at both the general and specific cases of our model

Lets begin with the specific case where abatement is only possible through output reductions and  $E = \delta X$

# Environmental policy under monopoly

The monopolist's profit-maximization problem under an emission tax is:

$$\max_X \Pi(X) = P(X)X - C(X) - \tau E = P(X)X - C(X) - \tau\delta X$$

Where the monopolist now controls  $X$ : aggregate production

The FOC for the problem is:

$$P'(X)X + P(X) = C'(X) + \tau\delta$$

and the profit-maximizing output choice is given by  $X^M(\tau)$

# Environmental policy under monopoly

The FOC for the problem is:

$$P'(X)X + P(X) = C'(X) + \tau\delta$$

This illustrates the monopolists decision rule:  $MR = MC$

MR consists of two pieces:

1.  $P(X)$ : Additional revenue from increased  $X$
2.  $P'(X)X$ : decreases in the price for **inframarginal units** from increased  $X$

The monopolist now accounts for how increasing production lowers price

# Environmental policy under monopoly

How does the firm respond to the tax? Differentiate the FOC wrt  $\tau$ :

$$[P''(X^M)X^M + 2P'(X^M) - C''(X^M)] \frac{dX^M}{d\tau} = \delta$$

and rearrange:

$$\frac{dX^M}{d\tau} = \frac{\delta}{[P''(X^M)X^M + 2P'(X^M) - C''(X^M)]}$$

What's the sign on this expression?

Let's make two assumptions

# Environmental policy under monopoly

$$\frac{dX^M}{d\tau} = \frac{\delta}{[P''(X^M)X^M + 2P'(X^M) - C''(X^M)]}$$

Assume the inverse demand function  $P(X)$  is decreasing, and for all  $X > 0$ :

$$\frac{P''(X)}{P'(X)}X > -1$$

This just comes from the second-order sufficient condition for a maximum being satisfied, it means inverse demand isn't too convex:  $P''$  is bounded above

It also ensures that  $\frac{dX^M}{d\tau} < 0$

# Environmental policy under monopoly

Now lets look at the regulators problem of maximizing the benefits minus the costs of production, accounting for damages:

$$\max_X \int_0^X P(t)dt - C(X) - D(\delta X)$$

She is maximizing the consumption value minus production and environmental costs

The FOC for this problem is:

$$P(X^*) = C'(X^*) + \delta D'(\delta X^*)$$

which doesn't map as nicely to the firm's FOC as in competitive markets

# Environmental policy under monopoly

Lets re-write the regulator's problem and explicitly include the firm's optimal response to  $\tau$ :

$$W(\tau) = \int_0^{X^m(\tau)} P(t)dt - C(X^m(\tau)) - D(\delta X^m(\tau))$$

Then differentiate with respect to  $\tau$  to get:

$$[P(X^m) - C'(X^m) - \delta D'(\delta X^m)] \times \frac{dX^m}{d\tau} = 0$$

# Environmental policy under monopoly

$$[P(X^m) - C'(X^m) - \delta D'(\delta X^m)] \times \frac{dX^m}{d\tau} = 0$$

Recognize  $P(X^m) - C'(X^m) = \tau\delta - P'(X^m)X^M$  from the firm FOC

Rearranging gives us that the optimal tax rate is characterized by:

$$\tau = D'(\delta X^m) + \frac{P'(X^m)X^m}{\delta}$$

Can we achieve the first-best with this tax?

Yes! If we plug it into the firm FOC it is the same as the regulator's welfare-maximizing FOC

# Environmental policy under monopoly

$$\tau = D'(\delta X^m) + \frac{P'(X^m)X^m}{\delta}$$

What is the intuition behind this expression?

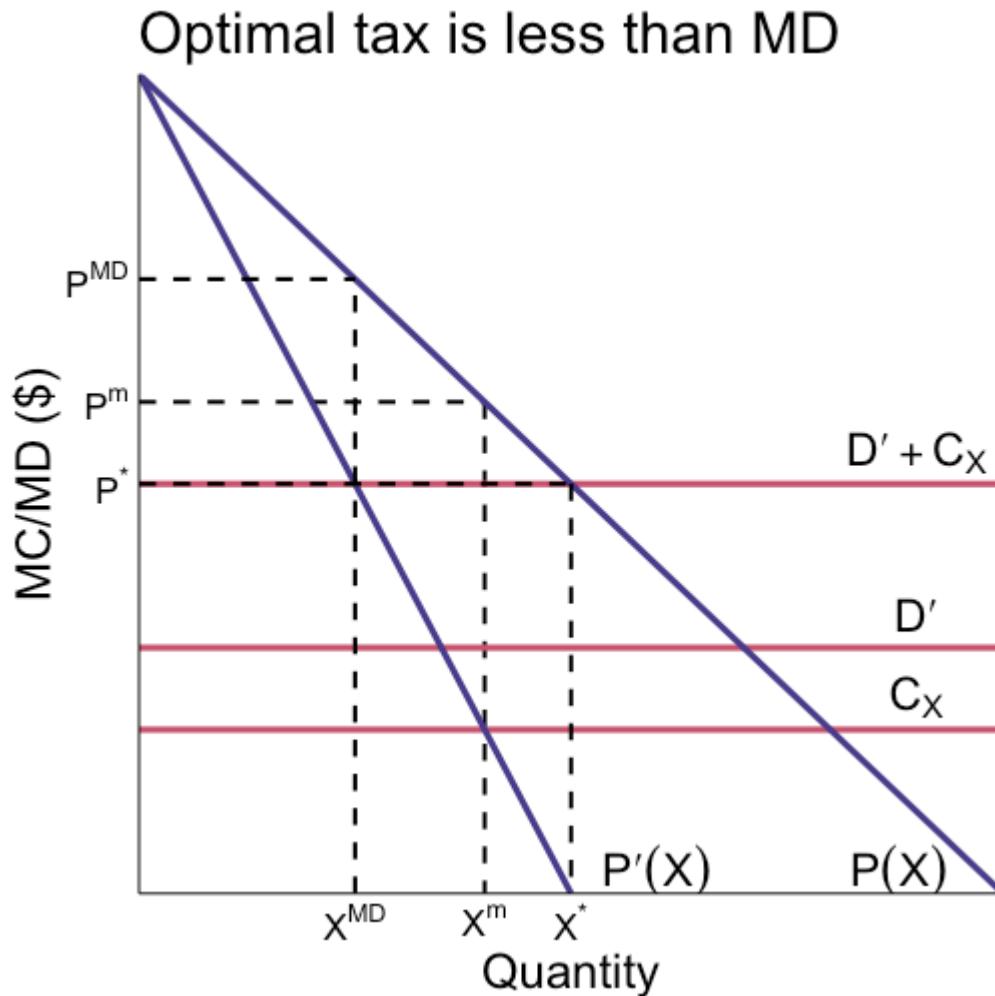
First, since  $P'(X^m) < 0, \tau < D'(\delta X^m)$

Why?

The monopolist already reduces output to exercise its market power

So we don't need as big of a tax, or may even need a **subsidy** if the monopolist was reducing output too much, to achieve the first-best

# Graphical intuition for the optimal tax



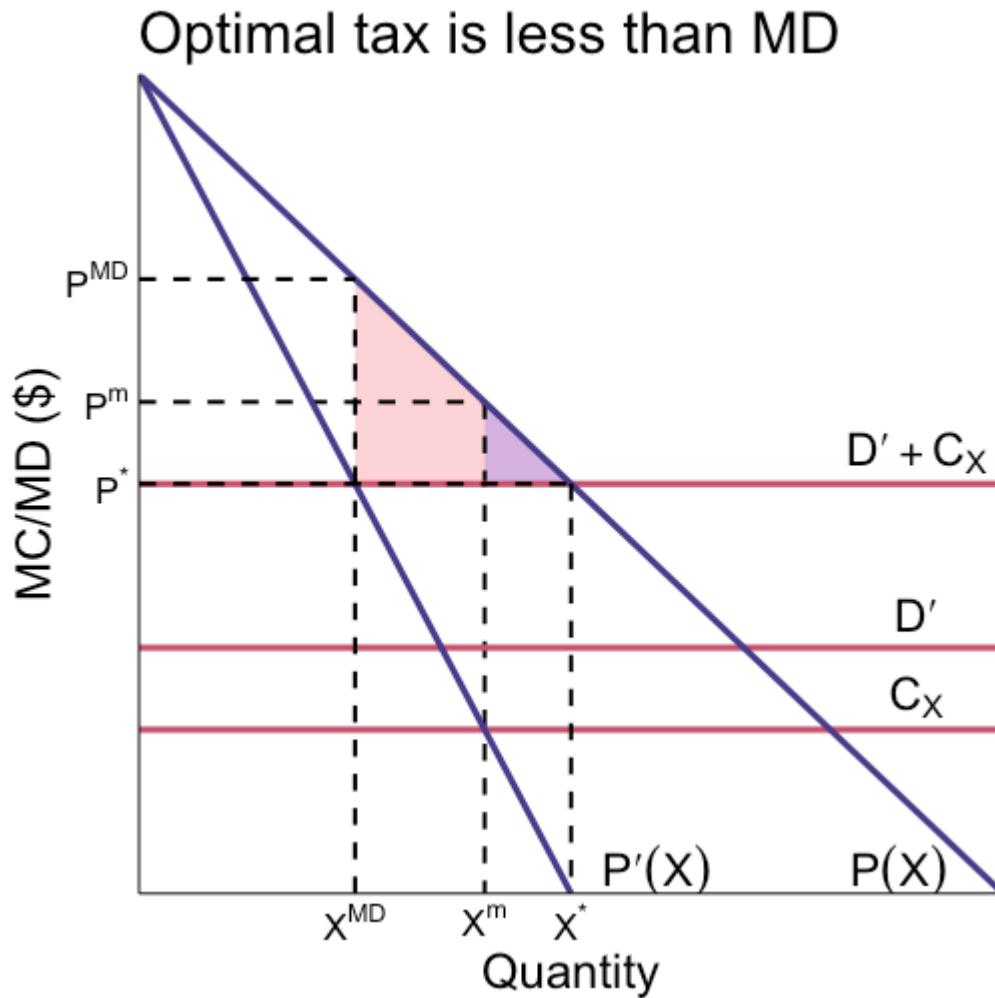
Suppose MD and MC are constant for simplicity and we are in the specific case

$X^m, P^m$  is the unregulated monopoly allocation

$X^{MD}, P^{MD}$  is the outcome if we set  $\tau = D'(E^*)$

Since  $X^{MD} < X^m < X^*$  this clearly reduced welfare

# Graphical intuition for the optimal tax

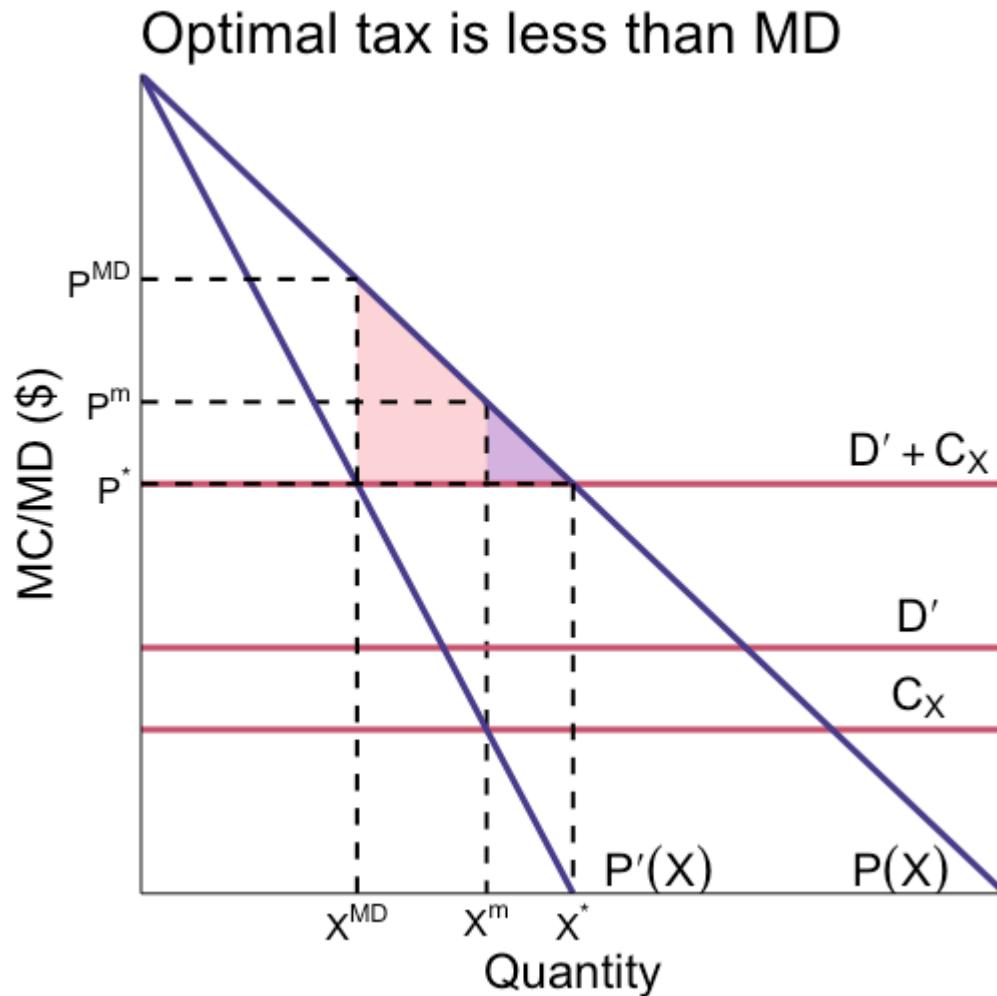


The purple/blue area is the original DWL under monopoly

The red + purple/blue area is the DWL if we tax at marginal damage, this **worsened welfare**

What's the optimal tax that gets us  $E^*$ ?

# Graphical intuition for the optimal tax



We want to shift the marginal cost of the firm so that it intersects MR at  $X^*$ , this vertical distance is the optimal tax

MR at  $X^*$  is 0, so we want to shift marginal cost down to zero

Our tax is then:  $0 - C_X$ : we actually **subsidize output**

# Environmental policy under monopoly

In the specific case we could achieve the first-best with just an emission tax and solve two externalities at once

Does this hold more generally?

The general monopoly problem is:

$$\max_{X,E} \Pi(X, E) = P(X)X - C(X, E) - \tau E$$

# Environmental policy under monopoly

The FOCs are

$$P'(X)X + P(X) = C_X(X, E) \quad - C_E(X, E) = \tau$$

The first is just  $MR = MC$  of production

The second is the  $MAC = \text{tax for emissions}$

Recall the regulators solution will look like:

$$P(X^*) = C_X(X^*, E^*) \quad - C_E(X^*, E^*) = D'(E^*)$$

Can the regulator use an emission tax alone to achieve the efficient outcome?

# Environmental policy under monopoly

The regulator's problem is:

$$W(\tau) = \int_0^{X^m(\tau)} P(t)dt - C(X^m(\tau), E^m(\tau)) - D(E^m(\tau))$$

The FOC of this problem is:

$$[P(X^M) - C_X(X^m, E^m)] \frac{dX^m}{d\tau} - [C_E(X^m, E^m) + D'(E^m)] \frac{dE^m}{d\tau} = 0$$

To get the **second-best** tax rate, substitute in the conditions from the firm FOCs:

$$P'(X)X + P(X) = C_X(X, E) \quad - C_E(X, E) = \tau$$

# Environmental policy under monopoly

This gives us that:

$$\tau = D'(E^m) + P'(X^m)X^m \frac{dX^m/d\tau}{dE^m/d\tau}$$

What is the second term?

It represents how a change in emissions caused by the tax  $\tau$  changes how the monopolist exercises market power

$P'(X^m)X^m$  is the market power term in MR,  $\frac{dX^m/d\tau}{dE^m/d\tau}$  is the output response to a tax-induced change in emissions

# Environmental policy under monopoly

The sign of this term depends on  $\frac{dX^m/d\tau}{dE^m/d\tau}$ , so let's sign the two components

We do this by differentiating the firm FOCs wrt  $\tau$  and first solving for  $dX^m/d\tau$ :

$$P'(X)X + P(X) = C_X(X, E) \quad -C_E(X, E) = \tau$$

to give us that (after some algebra and dropping function arguments):

$$\frac{dX^m}{d\tau} = \frac{-C_{XE}}{C_{EE} [P''X + 2P' - C_{EE} \{C_{XX}C_{EE} - C_{XE}^2\}]} < 0$$

# Environmental policy under monopoly

$$\frac{dX^m}{d\tau} = \frac{-C_{XE}}{C_{EE} [P''X + 2P' - C_{EE} \{C_{XX}C_{EE} - C_{XE}^2\}]} < 0$$

We assumed  $-C_{XE} > 0$ , strict convexity ensures  $C_{EE}, C_{XX}C_{EE} - C_{XE}^2 > 0$ , and our most recent assumption that demand is not too convex ensures  $P''X + 2P' < 0$

The numerator is positive, the denominator is negative: output declines as the tax increases

# Environmental policy under monopoly

For emissions we differentiate the firm FOCs wrt  $\tau$  solve for  $\frac{dE^m}{d\tau}$ :

$$\frac{dE^m}{d\tau} = \frac{-1}{C_{EE}} + \frac{C_{XE}^2}{C_{EE}^2 [P''X + 2P' - C_{EE} \{C_{XX}C_{EE} - C_{XE}^2\}]} < 0$$

As before, we assumed:

- $-C_{XE} > 0$
- Strict convexity ensures  $C_{EE}, C_{XX}C_{EE} - C_{XE}^2 > 0$
- Demand is not too convex assumption ensures  $P''X + 2P' < 0$

So both output and emissions decline in the tax rate

# Environmental policy under monopoly

Can we use the tax alone to achieve the first-best?

Substitute in the optimal tax expression  $\tau$  into the firm emission FOC and see if it reduces to the conditions for the first-best outcome:

$$-C_E(X, E) = D'(E^m) + P'(X^m)X^m \frac{dX^m/d\tau}{dE^m/d\tau}$$

The last term doesn't equal zero so this FOC, given the second-best tax, cannot be equal to the first-best condition

**The regulator cannot achieve the first-best in the general case with a tax alone**

# Environmental policy under monopoly

What ended up being the difference between the specific and general case?

In the specific case the monopolist only had one degree of freedom, output, and the regulator had one instrument, the tax

With one instrument and one degree of freedom the regulator can incentivize the firm to select a specific value of output AND emissions

# Environmental policy under monopoly

In the general case the monopolist has two degrees of freedom: it can choose output and emissions separately

The regulator still has only one instrument

You cannot use one instrument to pin down two values (similar to solving one equation for two unknowns)

When you have fewer instruments than market distortions you are in a  
**second-best world**

# Environmental policy under monopoly

Here's some intuition:

Suppose  $|P'(X)|$  is large so demand is very inelastic  $\rightarrow$  small changes in quantity lead to big changes in price

In this case, the market power distortion is a big problem

$$(P(X^M) \ggg C_X(X^M, E^M))$$

If we tax equal to marginal damage, we will make the market power distortion worse

The second term in the tax expression reduces the tax to account for these concerns

# Environmental policy under monopoly

We can achieve the first-best if we have two instruments to address two distortions

- Use an output subsidy to incentivize the monopolist to produce the efficient level
- Use an emission tax to get the efficient level of emissions

The monopolist's problem is:

$$\Pi(X, E) = [P(X) + \xi]X - C(X, E) - \tau E$$

# Environmental policy under monopoly

The FOCs are:

$$P(X) + \xi + P'(X)X = C_X(X, E) \quad - C_E(X, E) = \tau$$

If we set:

$$\xi = -P'(X^*)X^* \quad \tau = D'(E^*)$$

the firm FOCs reduce to the regulator's efficiency conditions

This is a special case of the *Tinbergen Rule* that says you need as many instruments as distortions to achieve the first-best