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Railway timetable rescheduling with flexible stopping and flexible short-turning during disruptions

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ABSTRACT

Railway operations are vulnerable to unexpected disruptions that should be handled in an efficient and passenger-friendly way. To this end, we propose a timetable rescheduling model where flexible stopping (i.e. skipping stops and adding stops) and flexible short-turning (i.e. full choice of short-turn stations) are innovatively integrated with three other dispatching measures: retiming, reordering, and cancelling. The Mixed Integer Linear Programming model also ensures that each train serving a station is ensured with a platform track. To consider the rescheduling impact on passengers, the weight of each decision is estimated individually according to the time-dependent passenger demand. The objective is minimizing passenger delays. A case study is carried out for hundreds of disruption scenarios on a subnetwork of the Dutch railways. It is found that (1) applying a mix of flexible stopping and flexible short-turning results in less passenger delays; (2) shortening the recovery duration mitigates the post-disruption consequence by less delay propagation but is at the expense of more cancelled train services during the disruption; and (3) the optimal rescheduling solution is sensitive to the disruption duration, but some steady behaviour is observed when the disruption duration increases by the timetable cycle time.

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1. Introduction

In most countries, the railway plays a major role in people's daily travelling. Therefore, reliability of the operations is essential and in particular rapid responses are needed after disruptive events. To this end, efforts are made either during the planning processes to design robust timetables, or during the operations to provide high-quality rescheduled timetables. The latter are considered as real-time rescheduling problems that are further classified into disturbance management and disruption management, depending on the severities of service interruptions.

Usually, disturbance management handles relatively small delays due to, for example, extended running or dwell times of trains, by rescheduling the timetable only. On the contrary, disruption management deals with large incidents, like open-track blockages, station closures, extreme weather conditions, etc., which consists of not only timetable rescheduling, but also rolling stock and crew rescheduling (Jespersen-Groth et al., 2009). An overview of the real-time rescheduling models towards either disturbances or disruptions can be found in Cacchiani et al. (2014).

Until now, many efforts have been put on developing models and algorithms to achieve automatic disturbance management that actually has been realized in Norway since February 2014 (Lamorgese and Mannino, 2015). However, automatic

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disruption management has not been realized yet. In practice, it is still highly dependent on manual work to deal with disruptions, which usually results in rescheduling solutions of low quality and imposes much work load on the traffic controllers (Ghaemi et al., 2017b). Thus, developing models to generate disruption solutions automatically attracts increasing attention recently, which is exactly the starting point of our paper.

In this paper, a Mixed Integer Linear Programming (MILP) model is proposed to handle the railway timetable rescheduling problem during complete track blockages, by retiming, reordering, cancelling, flexible stopping, and flexible short-turning. This is the first time that flexible stopping and flexible short-turning are introduced in one rescheduling model.

Flexible stopping means that for each train the original scheduled stops could be skipped while extra stops could be added, considering that during disruptions a skipped stop could reduce the delays of passengers at their expected destinations, while an added stop could provide passengers with more alternative paths for re-routing. However, a skipped stop will cause inconvenience to some passengers who therefore need to reroute and possibly arrive with some delay at their destinations; while an added stop may increase the total travel times of many passengers. When deciding whether to skip or add stops, the negative and positive impacts on passengers are both considered while the adjusted train running times due to reduced (extra) decelerations and accelerations are explicitly taken into account.

Short-turning means that a train ends its operation at a station before the blocked tracks and the corresponding rolling stock turns at the station to be used by another train in the opposite direction. Usually, a train is short-turned at one station only. As such, the train will be cancelled instead of short-turned if the short-turn station lacks capacity. To reduce the possibility of cancelling trains due to lack of station capacity, we introduce flexible short-turning by giving each train a full choice of short-turn station candidates and the proposed model decides the optimal station and time of short-turning the train. Among all the stations that a train originally serves or passes through, the ones of which the infrastructure layouts allow short-turning are all chosen as the short-turn station candidates for the train.

Compared to most literature, our model is more complete by including realistic characteristics of the infrastructure, disruption, and passengers as much as possible. Regarding to the infrastructure, the model focuses on networks of both double-track railway lines and single-track railway lines (described at a macroscopic level), where multiple types of headways are considered to prevent operational conflicts at stations/sections. The platform tracks and pass-through tracks of a station are distinguished to make sure that each train is assigned to an appropriate track when arriving at the station. The rolling stock circulations at short-turning and terminal stations are both taken into account, and whether a station has turning facilities for the trains arriving from different directions is explicitly considered. The rolling stock circulations at the origins/destinations of trains are called OD turns. To be specific, the model ensures that the rolling stock of a train that reaches its destination turns at the station to operate an opposite train that departs from the same station as the origin. Regarding to a disruption, our model considers all disruption phases, i.e., the transition phase from the original timetable to the disruption timetable, the stable phase where the disruption timetable is performed, and the recovery phase of the disruption timetable resuming to the original timetable (Ghaemi et al., 2017b). As for the passengers, a method is proposed to determine the impacts of dispatching decisions in terms of passengers' planned paths, which are then used as the decision weights in the objective of minimizing passenger delays. The model is tested on real-life instances of a subnetwork of the Dutch railways, which demonstrates fast computations of rescheduling solutions.

The contributions of this paper are summarized as follows.

- A new rescheduling model is proposed, which includes both flexible stopping and flexible short-turning as well as retiming, reordering, and cancelling trains.
- The model deals with all phases of a disruption.
- Adjusted train running times due to saved (extra) decelerations and accelerations are explicitly considered when skipping (adding) stops.
- Station capacity is considered by ensuring that each train corresponding to passenger boarding/alighting stops at a platform track while the minimum headway times are taken into account.
- Rolling stock circulations at the short-turning and terminal stations of trains are included.
- Dispatching decisions are optimized with the objective of minimizing passenger delays.

In the following, we first give an overview of the literature on timetable rescheduling models in Section 2, followed by the mathematical modelling of the problem in Section 3. Then, the case study is given in Section 4 and finally Section 5 concludes the paper and points out directions for future research.

2. Literature review and problem challenge

In this section, we first give an overview of the publications on timetable rescheduling, particularly differentiated by the used dispatching measures. Then, the characteristics of papers relevant to disruptions, including the infrastructure modelling level, the used method, the objective, and whether considering OD turn or station capacity, are discussed and compared to the ones of this paper. In the end, the challenges of the problem considered in this paper are explained.

2.1. Literature review

During disturbances that cause service perturbations rather than dropped infrastructure capacity, local re-routing and re-timing are commonly adopted to adjust the timetable. For example, D'Ariano et al. (2008) and Corman et al. (2010) sequentially determine train routes and then the arrival and departure times of trains, while Meng and Zhou (2014) specify the routes and schedules of trains simultaneously. These papers describe infrastructure at a microscopic level, and so does Pellegrini et al. (2014), where blocking times are explicitly formulated. Recently, Pellegrini et al. (2019) propose valid inequalities that allow reformulating the model presented in Pellegrini et al. (2014) to boost computation efficiency. A detailed review regarding timetable rescheduling during disturbances and disruptions can be found in Cacchiani et al. (2014). In the following we review the literature on timetable rescheduling during disruptions.

Narayanaswami and Rangaraj (2013) establish an MILP model for a track blockage between two adjacent stations for a single-track railway with the objective of minimizing the delays of trains at the destinations. In their model, the affected trains that will run through the disrupted section during the disruption are forced to be delayed to at least after the disruption ends. Due to this delaying measure, new conflicts may rise up between these delayed trains and the trains that are originally scheduled after the time the disruption ends. Thus, binary precedence variables are introduced to allow re-ordering at stations. Meng and Zhou (2011) also deal with the disruption occurring in a single-track railway, by additionally considering uncertain disruption length and varied running times of trains. A stochastic programming model is established and embedded in a rolling horizon framework, where the measure of delaying trains is applied. Different scenarios are tested in their model with the objective of minimizing the expected secondary delays of trains.

Compared to delaying a train, cancelling usually leads to more passenger inconvenience, if the focus is on the cancelled train only. However, if the disruption is rather long or trains run with high frequencies, cancelling a train might be better than delaying it. Otherwise, more subsequent trains could be delayed, thus resulting in more passenger inconvenience across the whole network. Under these circumstances, train cancellation is necessary. Cadarso et al. (2013) propose an integrated optimization model for rescheduling both timetable and rolling stock. Two dispatching measures are applied, complete train cancellation and emergency train insertion, with the objective of minimizing operational cost, cancellations, denied passengers and service deviations. The departure/arrival times of emergency trains are pre-determined and fixed in the disruption timetable, and the departure/arrival times of planned trains are also fixed. Thus, only binary decision variables are needed to decide which planned trains should be cancelled and which emergency trains should be inserted. An extension on Cadarso et al. (2013) is made by Binder et al. (2017) who include three additional dispatching measures, partial cancellation, delaying, and global rerouting, into an ILP model with the objective of minimizing operational cost, service deviations and passenger inconveniences. This model depends on a pre-constructed rescheduling graph where delaying and rerouting arcs for planned trains are constructed to make delaying and global rerouting of trains possible. In addition, conflicting arcs are also pre-constructed to prevent any conflicts between trains. As a consequence, binary decision variables are needed in the model to decide which arcs are chosen by trains, to produce a conflict-free disruption timetable. Zhan et al. (2015) propose an MILP model for timetable rescheduling in case of complete track blockage, by including the measures of cancelling, retiming, and reordering trains. Disruption length is assumed to be known and fixed when the disruption starts. Later, they proposed another model to take uncertain disruption lengths into account, and the target case is changed to partial segment blockage (Zhan et al., 2016). In both models, they aim to minimize train delays and cancellations. As seat reservations are necessary in Chinese railways, the measure of short-turning trains is not considered in their models.

Seat reservations are not required in most urban rail transit systems and European railway systems, which makes short-turning trains widely used there during disruptions. Puong and Wilson (2008) declare that when service disruptions are less than 20 min, only holding strategies (i.e. increasing dwell times of trains) are used. However for longer disruptions, short-turning trains are usually used together with the holding strategies. The purpose is to keep the headways of both operation directions as regular as possible (Wilson et al., 1992), or to isolate the disrupted area from the whole network (Ghaemi et al., 2018a). Short-turning trains is considered by Louwerse and Huisman (2014) who propose an ILP model to deal with partial or complete blockage on a double-track railway. In their model, the capacities of short-turn stations are considered, while assuming the capacities of other stations to be infinite. By extending the model of Louwerse and Huisman (2014), Veelenturf et al. (2015) take the capacities of all stations into account, while short-turn stations are fixed to trains. This means that for each train, the last scheduled stop approaching the disrupted area is set as the only short-turn station. Instead, Ghaemi et al. (2018a) propose an MILP model where two short-turn station candidates are provided to each train. The model deals with complete track blockages and describes the infrastructure at a macroscopic level. To improve the practicability, another MILP model is proposed by Ghaemi et al. (2017a), which deals with the same problem but describes the infrastructure at a microscopic level. In both models, the objectives are minimizing train delays and cancellations. Van Aken et al. (2017a) establish an MILP model to deal with timetable adjustments for full-day multiple maintenance possessions (i.e. planned disruptions). Each train has one short-turn station to be chosen, and whether a train will be short-turned or not is decided in a preprocessing step. Further, Van Aken et al. (2017b) include short-turning decisions into the model and each train is provided with more short-turn station options. In their model, pre-processing is necessary to identify which services corresponding to a train should be cancelled in case of the train being short-turned or completely cancelled, and the short-turn durations are fixed.

In addition to the dispatching measures mentioned above, changing stopping patterns is also widely adopted when passenger inconvenience (e.g. waiting times, total travel times, etc.) are taken into account. Among the literature on

passenger-oriented timetable rescheduling, Sato et al. (2013) allow the types of trains to be changed. This means that the express trains can change to local trains, and vice versa, while the stopping pattern of each train type is fixed. Gao et al. (2016) and Altazin et al. (2017) both allow trains to skip certain stops, while Veelenturf et al. (2017) allow additional stops to be added to trains without considering the extra deceleration and acceleration times.

Most literature is operator-oriented, which usually aims to minimize train delays and cancellations, where the penalties of both measures are determined without clear rules and thus varying across papers. It has been reported in Zhan et al. (2015) and Ghaemi et al. (2017a) that the rescheduling solutions are sensitive to the cancellation and delaying penalties. In other words, different solutions can be obtained when setting different values to the cancellation and/or delaying penalties although the disruption characteristics (the disrupted section and the disruption starting/ending time) remain. For example, increasing the cancellation penalty without changing the delaying penalty may lead to less services cancelled but more services delayed. Zhan et al. (2015) conclude that the penalty choice is a trade-off to be made by railway managers: cancelling less trains by delaying more trains or the other way around; while Ghaemi et al. (2017) suggest to generate multiple solutions by varying penalties, which then will be evaluated by experts from different perspectives to decide an overall best solution.

The passenger-oriented rescheduling models for disruptions are limited, but are increasing over the last years. Cadarso et al. (2013) use a frequency-based passenger assignment model to estimate passengers' geographic travel paths with no timetable known yet, and the rescheduling model aims to accommodate the estimated passenger demand as much as possible. Veelenturf et al. (2017) use a schedule-based passenger assignment model that estimates passengers' exact travel paths with time information. They consider passenger demand in a dynamic way by iteratively adjusting the timetable by adding a stop in each iteration if it is indicated by the passenger assignment model that this stop can reduce passenger inconvenience. The decisions are limited to adding stops. Until now, Binder et al. (2017) is the only one that integrates passenger rerouting with rescheduling in one single model for disruption cases, where trains can be partially cancelled by neglecting the rolling stock connection between any two of them (i.e. no short-turning).

2.2. Summary and contributions of this paper

In Table 1, the dispatching measures used in the publications on timetable rescheduling during disruptions are summarized, as well as the ones used in this paper. Other characteristics like the infrastructure modelling level, the used method, the objective, and whether considering OD turn or station capacity are also given. The symbol '-' indicates that OD turn or station capacity is neglected in a paper, while the symbol '✓' indicates that it is considered. The publications that consider passengers in the objectives are shown in the lower part.

This paper describes the infrastructure also at a macroscopic level to allow fast computations, like most of the existing literature. The problem is formulated as an MILP model with the objective of minimizing passenger delays. A schedule-based passenger assignment is adopted to obtain passengers' planned paths, which are used to estimate passenger-dependent weights for decisions included in the objective function. The weight of each decision is individually estimated considering the affected passengers and the impact on them due to the decision. For example, the penalty of skipping a stop is calculated as the number of passengers who originally plan to board or alight from the train at the stop multiplied by an assumed delay per passenger considering that each of these passengers has to reroute. Compared to operator-oriented models, the model generates more passenger-friendly solutions that are also preferred by train operators. This is because the passenger-dependent decision weights are calculated according to passengers' planned paths estimated based on the planned timetable. In that sense, our model aims to reduce the impact on passengers' planned paths, which is, to some extent, in line with reducing the deviations from the planned timetable, while less timetable deviations help to reduce the complexity of rescheduling the rolling stock/crew further. Although passenger demand is considered in a static way, the model ensures fast computations of solutions, which satisfy the real-time requirement. Until now, only a few rescheduling models that consider passenger demand in a dynamic way (i.e. timetable-dependent passenger behaviour) in case of disruptions. These models can reflect passenger behaviour more accurately but are at the expense of more computation time (Binder et al., 2017) that is usually not acceptable in practice. Different from the existing literature, our model allows more flexibilities for the timetable rescheduling: (1) delaying, reordering, and cancelling are all allowed; (2) short-turning is considered and in a completely flexible way by giving each train a full choice of short-turn station candidates; and (3) adding and skipping stops (i.e. flexible stopping) is innovatively introduced with the deceleration and acceleration times of trains taken into account. To ensure solution feasibility in practice, the rolling stock circulations at the origins/destinations of trains (i.e. OD turns) and the capacity of each station are both considered.

2.3. Problem challenge

There are three main challenges of handling the problem considered in this paper. The first challenge is modelling flexible stopping. In the planned timetable, there are stops and non-stops only. However in a rescheduled timetable, there could be stops, non-stops, skipped stops, added stops, and also cancelled stops and cancelled non-stops due to cancellation measures. These stop types must be recognized by the model individually, as they have different impacts on station capacity, train running times and passengers. The second challenge is modelling flexible short-turning. For fixed short-turning, the station where a train can be short-turned is fixed, thus the decision is only about which opposite train should be served at the

Table 1

Comparisons between publications on railway disruption management and this paper.

Publication	Level	Method	Objective	Dispatching decisions	OD turn	Station capacity
Narayanaswami and Rangaraj (2013)	Macro	MILP	Min. train delays at the destinations	D, RO	—	—
Meng and Zhou (2011)	Macro	MILP, RH	Min. the expected secondary delays of trains under different forecasted conditions	D	—	✓
Zhan et al. (2015)	Macro	MILP	Min. train delays and cancellations	D, RO, C	—	✓
Zhan et al. (2016)	Macro	MILP, RH	Min. train deviations and cancellations	D, RO, C	—	✓
Louwense and Huisman (2014)	Macro	ILP	Min. train delays, cancellations and operation imbalance	D, RO, C, ST (one fixed short-turn station)	✓	(only short-turn stations)
Veelenturf et al. (2015)	Macro	ILP	Min. train delays and cancellations	D, RO, C, RR, ST (one fixed short-turn station)	✓	✓
Ghaemi et al. (2018a)	Macro	MILP	Min. train delays and cancellations	D, C, ST (two short-turn station candidates)	—	✓
Ghaemi et al. (2017a)	Micro	MILP	Min. train delays and cancellations	D, RO, C, ST (two short-turn station candidates)	—	✓
Cadarso et al. (2013)	Macro	MILP, PA	Min. operational cost, cancellations, denied passengers and service deviations	C, E	✓	—
Binder et al. (2017)	Macro	ILP	Min. operational cost, service deviations and passenger inconveniences	D, RO, C, RR, E	—	✓
Veelenturf et al. (2017)	Macro	ILP, PA	Min. operational costs and passenger delays	A	✓	—
This paper	Macro	MILP, PA	Min. passenger delays	D, RO, C, A, S, FST (full choice of short-turn station candidates)	✓	✓

Method: MILP, ILP, Rolling Horizon (RH), Stochastic Programming (SP), Passenger Assignment (PA). Dispatching decisions: Delaying (D), Reordering (RO), Cancelling (C), Emergency Train Insertion (E), Rerouting (RR), Short-turning (ST), Adding Stop (A), Skipping Stop (S), Flexible Short-turning (FST).

station; whereas for flexible short-turning, two decisions are needed, which are where to short-turn a train and which opposite train to be served at the station. If a train is short-turned, its following services that were originally scheduled after the short-turn station must be cancelled. In the literature, this is usually based on a preprocessing step where the cancelled services of a train are included in a set as input. Without pre-processing, the model has to decide which services of a train should be cancelled if the train is short-turned. The third challenge is modelling station capacity under flexible stopping and flexible short-turning, because whether a train needs a platform track at a station can be different than planned. For example, it is unnecessary to assign a platform track to a train that passes through a station. However, a platform track must be assigned: if the train has an additional stop at the station where passengers will board or alight from the train, or if the train is short-turned at a station where passengers will alight from the train. To handle these challenges, a new rescheduling model is proposed.

3. Timetable rescheduling model

Timetable rescheduling during complete track blockages adjusts the routes and time-distance train paths to fit the reduced infrastructure capacity without any conflicts between trains. In this section, it is formulated as an MILP model based on an event-activity network that is explained first. Next, the constraints for cancelling, delaying, reordering, flexible stopping, flexible short-turning, rolling stock circulations at terminal stations, and station capacity are introduced successively. Finally, the objective is given including the passenger-dependent weight for each decision considering the affected passengers and the impact on them due to the decision.

3.1. Event-activity network

Each departure/arrival of a train is formulated as a departure/arrival event e with corresponding information: original scheduled time o_e , station st_e , train line tl_e , train number tr_e , and operation direction dr_e . A train line indicates the origin, the

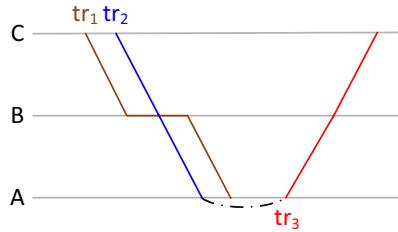


Fig. 1. A timetable with three trains and three stations located on double-track railway lines.

destination, all intermediate stops between the origin and the destination, and the operation frequency (e.g. every 30 min). For the train that passes through a station, we divide the pass-through action into two events: pass-through departure and pass-through arrival. The benefit of formulating the pass-through action this way is twofold. First, it enables the modelling of the case that a train does not stop at a station but could be short-turned. Second, it makes the modelling of additional stops of a train possible.

Each activity a is a directed arc from one event to another event, i.e., from $\text{tail}(a)$ to $\text{head}(a)$. The type of activities include running activities, dwell activities, pass-through activities, short-turn activities, OD turn activities and headway activities (arrival headway, departure headway, arrival-departure headway, or departure-arrival headway).

- A running activity a_{run} is defined from a departure event to an arrival event with both events belonging to the same train but occurring at two adjacent stations. The departure event occurs at the upstream station relative to the station where the arrival event occurs.
- A dwell (pass-through) activity $a_{\text{dwell}} (a_{\text{pass}})$ is defined from an arrival event to a departure event that belongs to the same train, occurs at the same station, and with the departure event occurring later (at the same time as the arrival event).
- An OD turn activity a_{odturn} describes a turn of a train at its destination where the rolling stock continues to operate an opposite train from the same train line.
- A short-turn activity a_{turn} is defined from an arrival event to a departure event that occurs at the same station but operates in the opposite direction. Both events are with the same train line but different trains.

In general, different train lines may use the same or different rolling stock types, and intercity and local lines use different rolling stock types. Two local lines may use the same rolling stock type but for rolling stock circulations we prefer to keep the rolling stock units on the same train line so that they stay in the same circulations, rather than ending up in complete different areas corresponding to different train line routes. This eases the recovery after the disruption. Therefore, we only allow a short-turn activity to be created between two events from the same train line. More details about creating short-turn activities can be found in [Section 3.2.4](#).

- An arrival-departure headway activity $a_{\text{head}}^{\text{ar,de}}$ is defined from an arrival event to a departure event that occurs at the same station, but belongs to a different train operating in the opposite direction.
- An arrival (departure) headway activity $a_{\text{head}}^{\text{ar}} (a_{\text{head}}^{\text{de}})$ is defined from an arrival (departure) event to another arrival (departure) event that occurs at the same station, operates in the same direction, but belongs to a different train.
- A departure-arrival headway activity $a_{\text{head}}^{\text{de,ar}}$ is defined from a departure event to an arrival event that occurs at the same station but belongs to a different train.

Arrival-departure headway activities are needed for trains operating on single-track railway lines. This is because any two adjacent stations located on single-track railway lines are linked by one track only, which makes it necessary to keep a headway between the arrival of a train and the departure of another train that will enter the open-track section where the arriving train comes from. The area between two adjacent stations is an open-track section. On double-track railway lines, any two adjacent stations are linked by two tracks where each is used by trains operating in the same direction. Arrival (departure) headway activities are needed for following trains operating in either single-track railway lines or double-track railway lines. Departure-arrival headway activities are needed for trains that use the same track at a station, and also both departure-arrival and arrival-departure headway activities are used for trains with crossing routes at stations.

Example: Fig. 1 shows a timetable where train tr_1 runs from station C to station A with a stop at station B, train tr_2 runs from station C to station A directly, and train tr_3 runs from station A to station C directly. It is assumed that stations A, B and C are located on double-track railway lines, tr_1 continues its operation further after station A, and tr_2 ends its operation at station A and the corresponding rolling stock turns to be used in the operation of tr_3 . The event-activity network formulation of this example is presented in Fig. 2, where headway activities are always pairwise created between two events considering that the order between them may change. A rescheduling solution to the event-activity network of Fig. 2 is shown in Fig. 3, where a stop is added to tr_2 at station B by delaying the departure of tr_2 at this station. The activities that are not valid in the rescheduling solution are all coloured in grey. For example, if there is no short-turning, the short-turn activity is invalid. Also between two events, one headway activity must be invalid, with the other one being valid, if these two events are

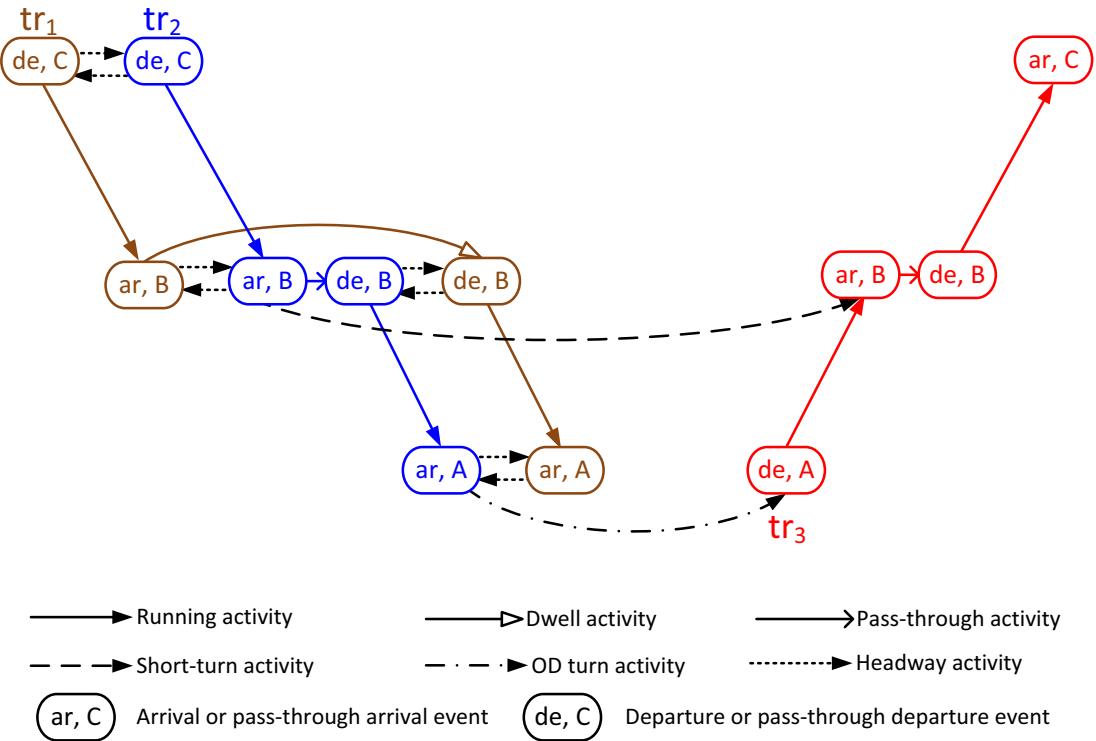


Fig. 2. Event-activity network formulation of Fig. 1.

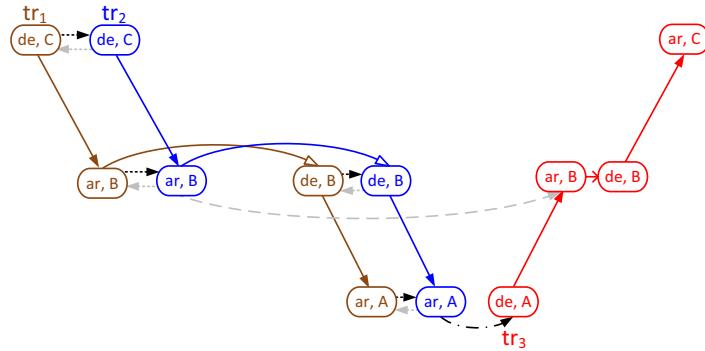


Fig. 3. A rescheduling solution to Fig. 1 by adding stop, reordering and delaying.

not cancelled. Otherwise, both headway activities between them are invalid. A valid activity in a rescheduling solution must satisfy the conditions that (1) the head of the activity occurs no earlier than the tail of the activity and the time difference between them respects the required duration, (2) both the head and tail events are not cancelled, and (3) the activity must be active if it is a short-turn/OD turn activity.

The notation used for sets and parameters are given in Appendix A. The decision variables used in the proposed model are described in Table 2.

The proposed model is based on three assumptions. First, it is assumed that the end time of a disruption is given at the moment the disruption starts and will not change. Although uncertain disruption duration is not considered in our model, it can be handled by embedding the proposed model in a rolling horizon framework like Meng and Zhou (2011) and Zhan et al. (2016) did. In the case study, we investigate how the proposed model reacts to different disruption durations. The second assumption is that trains that have already entered the blocked tracks when the disruption starts have passed the blocked points already, and thus can run as planned. The third assumption is that trapped trains that cannot turn at a station before the blocked tracks will dwell at the last possible station until the disruption ends. The passengers may be evacuated using bus services to stations with running trains or stay in the train, depending on the disruption length. Our model considers that the passengers in a trapped train are delayed at least for the dwelling duration.

Table 2
Decision variables.

Notation	Description	Domain
x_e	The rescheduled time of event e	$x_e \geq 0$
d_e	The delay of event e	$d_e \geq 0$
c_e	The binary variable deciding whether event e is cancelled. If yes, $c_e = 1$.	$c_e \in \{0, 1\}$
p_e	The binary variable deciding whether train tr_e needs a platform track, $e \in E_{ar}$. If yes, $p_e = 1$.	$p_e \in \{0, 1\}$
$q_{e,e'}$	The binary variable deciding whether event e occurs before e' . If yes, $q_{e,e'} = 1$.	$q_{e,e'} \in \{0, 1\}$
$n_{e,e'}$	The binary variable deciding whether train tr_e is occupying a track at station st_e when another train $tr_{e'}$ arrives, $e, e' \in E_{ar}, st_e = st_{e'}, tr_e \neq tr_{e'}$. If yes, $n_{e,e'} = 1$.	$n_{e,e'} \in \{0, 1\}$
$n_{e,e'}^p$	The binary variable deciding whether train tr_e is occupying a platform track at station st_e when another train $tr_{e'}$ arrives, $e, e' \in E_{ar}, st_e = st_{e'}, tr_e \neq tr_{e'}$. If yes, $n_{e,e'}^p = 1$.	$n_{e,e'}^p \in \{0, 1\}$
s_a	The binary variable deciding whether a scheduled stop $a \in A_{dwell}$ is skipped or whether an extra stop is added to $a \in A_{pass}$. If $a \in A_{dwell}$, then $s_a = 1$ indicates a is skipped. If $a \in A_{pass}$, then $s_a = 0$ indicates a is added with a stop.	$s_a \in \{0, 1\}$
m_a	The binary variable deciding whether a short-turn or an OD turn activity $a \in A_{turn} \cup A_{odturn}$ is selected. If yes, $m_a = 1$.	$m_a \in \{0, 1\}$
y_e	The binary variable deciding whether station st_e is chosen as the short-turn station for train tr_e , $e \in E_{ar}^{turn} \cup E_{de}^{turn}$. If yes, $y_e = 1$.	$y_e \in \{0, 1\}$

3.2. Constraints

In the following, the constraints for cancelling, delaying, reordering, flexible stopping, flexible short-turning, rolling stock circulations at terminal stations, and station capacity are introduced successively.

3.2.1. Constraints for cancelling and delaying trains

For an event e , the relation between its rescheduled time x_e , the cancelling decision c_e and the delaying decision d_e is formulated by

$$M_1 c_e \leq x_e - o_e \leq M_1, \quad e \in E_{ar} \cup E_{de}, \quad (1)$$

$$x_e - o_e = d_e + M_1 c_e, \quad e \in E_{ar} \cup E_{de}, \quad (2)$$

$$d_e \geq 0, \quad e \in E_{ar} \cup E_{de}, \quad (3)$$

$$d_e \leq D, \quad e \in (E_{ar} \cup E_{de}) \setminus E^{NMdelay}, \quad (4)$$

where o_e is the original scheduled time of e , E_{ar} (E_{de}) is the set of arrival (departure) events, and $E^{NMdelay}$ is the set of events that do not have an upper limit on their delays. The events in $E^{NMdelay}$ correspond to the trains that are originally scheduled to run through the disrupted section during the disruption but have already departed from the origins before the disruption starts. Thus, these trains can only be short-turned or delayed, but not cancelled. In case these trains are unable to be short-turned due to insufficient station/rolling stock capacity, they have to be delayed at least to the end of the disruption. Considering these situations, no upper limit is imposed on the delays of the events corresponding to these trains. Here, we use $E^{NMdelay}$ to contain such events that do not have an upper limit on their delays. Suppose events e and e' are the departure events of train tr at the origin station and the entry station of the disrupted section, respectively. If event e originally occurs before the disruption starting time t_{start} while event e' originally occurs after t_{start} , then all departure and arrival events that correspond to train tr and originally occur after t_{start} belong to the set $E^{NMdelay}$. Constraint (1) means that for each event e , the rescheduled time x_e is not allowed to occur earlier than the original scheduled time o_e , and it should be removed after the end of the day if it is cancelled (i.e. $c_e = 1$). As we use minute as the unit for any x_e or o_e , M_1 is set to 1440 (i.e. one day has 1440 min). Thus according to (1), the rescheduled time of a cancelled event is the original scheduled time plus 1440. For a cancelled event, its delay d_e is equal to 0, while for a non-cancelled event, its delay is equal to the time difference between the rescheduled time and the original scheduled time (2). Constraint (3) means that event delay is non-negative. Constraint (4) means that an event that does not belong to $E^{NMdelay}$ is allowed to be delayed by at most D minutes, considering that it is not preferred to delay a train too much. Imagine that a train arrives at a station on time but departs from the station much later than planned, then a track of the station will be occupied by the train for a rather long time, which is not good for station capacity utilization. Besides, we consider a cyclic planned timetable to be rescheduled, thus delaying a train longer than the cycle time does not make much sense, since another train with the same origin, destination and stopping patterns will operate later. Considering these, the parameter of maximum allowed delay per

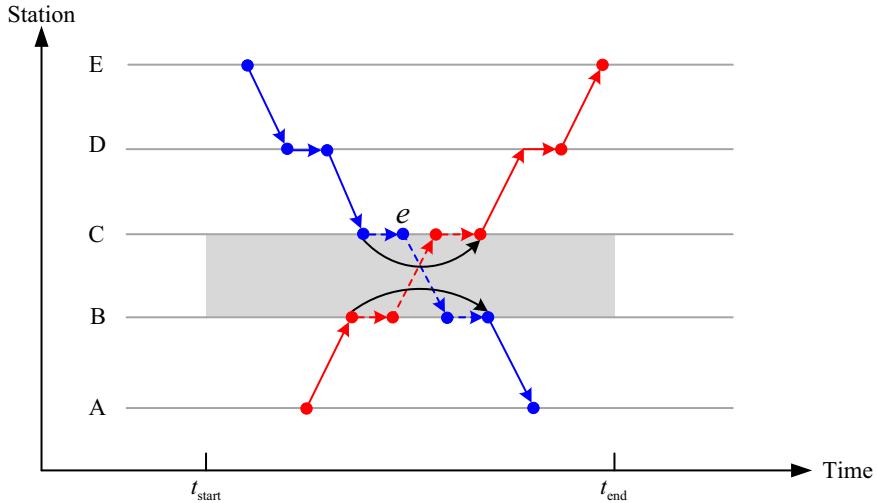


Fig. 4. An example used for explaining the entry/exit station of the disrupted section for a train. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

event, D , is used. This parameter is also adopted by some rescheduling models for disruptions, like [Zhan et al. \(2015\)](#) and [Veelenturf et al. \(2015\)](#).

During the disruption period, a departure event that is originally scheduled to occur at the entry station of the disrupted section is either cancelled or delayed at least to the end of the disruption,

$$x_e \geq t_{\text{end}}(1 - c_e), \quad e \in E_{\text{de}}, st_e = st_{\text{en}}^{dr_e}, t_{\text{start}} \leq o_e < t_{\text{end}}, \quad (5)$$

where $st_{\text{en}}^{dr_e}$ represents the entry station of the disrupted section considering the operation direction of the train corresponding to e (i.e. dr_e). An example of explaining the entry/exit station of a disrupted section is shown in Fig. 4 where the section between stations B and C is completely blocked from t_{start} to t_{end} , and a short-turning occurs between the blue and red trains at either station B or C. For the blue train that operates in downstream direction, the entry (exit) station of the disrupted section is station C (station B); whereas for the red train that operates in upstream direction, the entry (exit) station of the disrupted section is station B (station C). The dashed lines indicate the cancelled services due to the short-turnings. Here, the departure event of the blue train at station C is the event e of which the original scheduled time o_e satisfies $t_{\text{start}} \leq o_e < t_{\text{end}}$, thus it must be cancelled or delayed after t_{end} according to (5). In this case, e is cancelled (i.e. $c_e = 1$). Imagine that e is not cancelled (i.e. $c_e = 0$), then (1) both short-turnings indicated in Fig. 4 will not occur; and (2) the blue train will depart from station C later than t_{end} . This means that event e will be delayed by at least $t_{\text{end}} - o_e$ minutes, thus the blue train will occupy a track of station C for $t_{\text{end}} - o_e$ minutes at least. However due to the upper limit on delay D , the train will not occupy the track for longer than D minutes. An exception could be that a train is dwelling at the entry station of the disrupted section when the disruption starts but the infrastructure layout of the station is unable for short-turning; thus the train has to remain at the station until the disruption ends and the waiting time can be longer than D minutes. This is defined in the set E^{NMdelay} .

An event that originally occurs before the disruption start t_{start} cannot be cancelled, and should run as planned:

$$c_e = 0, \quad e \in E_{\text{ar}} \cup E_{\text{de}}, o_e < t_{\text{start}}, \quad (6)$$

$$x_e - o_e = 0, \quad e \in E_{\text{ar}} \cup E_{\text{de}}, o_e < t_{\text{start}}. \quad (7)$$

A departure event that originally occurs after $t_{\text{end}} + R$ cannot be cancelled and should run as planned:

$$c_e = 0, \quad e \in E_{\text{de}}, o_e \geq t_{\text{end}} + R, \quad (8)$$

$$x_e - o_e = 0, \quad e \in E_{\text{de}}, o_e \geq t_{\text{end}} + R. \quad (9)$$

Here, R represents the required time length for the disruption timetable resuming to the original timetable after the disruption ends. Setting R helps to avoid the disruption affecting the timetable for the entire day, which is also adopted by [Veelenturf et al. \(2015\)](#). For an arrival event, if its corresponding departure event in the running activity originally occurs after $t_{\text{end}} + R$, this arrival event cannot be cancelled and should run as planned:

$$c_{e'} = 0, \quad (e, e') \in A_{\text{run}}, o_e \geq t_{\text{end}} + R, \quad (10)$$

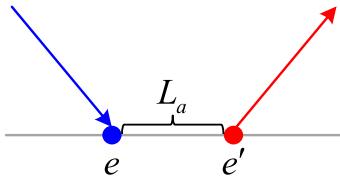


Fig. 5. Arrival-departure headway on single-track railway lines to ensure at most one train running in an open-track section: $a = (e, e') \in A_{\text{head}}^{\text{ar}, \text{de}}$.

$$x_{e'} - o_{e'} = 0, \quad (e, e') \in A_{\text{run}}, o_e \geq t_{\text{end}} + R. \quad (11)$$

Note that for an arrival event that originally occurs after $t_{\text{end}} + R$, its corresponding departure event in the running activity could originally occur before $t_{\text{end}} + R$ and possibly be cancelled/delayed, which makes this arrival cancelled or unable to run as planned. Constraints (8)–(11) require a disruption to be fully recovered after $t_{\text{end}} + R$. This might be impossible if R is set to a very small value like 0, thus resulting in infeasible solution. Considering that an event that originally occurs during the disruption period could be delayed by at most D minutes, it is better to set R at least larger than D to avoid infeasibility.

Any two events that constitute the same running activity are either cancelled or kept simultaneously:

$$c_{e'} - c_e = 0, \quad (e, e') \in A_{\text{run}}. \quad (12)$$

Any two events that constitute the same station activity are either cancelled or kept simultaneously, if neither of these two events corresponds to a short-turn activity:

$$c_{e'} - c_e = 0, \quad (e, e') \in A_{\text{station}}, e \notin E_{\text{ar}}^{\text{turn}}, e' \notin E_{\text{de}}^{\text{turn}}. \quad (13)$$

A station activity can either be a dwell activity or a pass-through activity. Here, $E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{\text{turn}}$) is the set of the tails (heads) of all short-turn activities contained in A_{turn} . The tail of $a \in A_{\text{turn}}$ must be an arrival event, and the head of $a \in A_{\text{turn}}$ must be a departure event.

Instead of requiring the running time on an open-track section to respect the minimum running time, we constrain it at least to respect the original scheduled running time, in order to keep disruption timetable robustness by the original scheduled time supplements:

$$x_{e'} - x_e \geq o_{e'} - o_e, \quad a = (e, e') \in A_{\text{run}}. \quad (14)$$

During disruptions, small service perturbations could also happen, of which the resulting delays are expected to be mitigated by the time supplements kept in the disruption timetable.

To prevent overlong running in an open-track section, the following constraint is given,

$$x_{e'} - x_e \leq (1 + \lambda)(o_{e'} - o_e), \quad a = (e, e') \in A_{\text{run}}, \quad (15)$$

where λ represents the maximum percentage allowed to the running time extension. During disruptions, it happens that a train runs with a slower speed than usual if the station which it is approaching to lacks capacity to receive it. As a result, longer time is needed for the running. However, a train cannot run too slow, thus a maximum percentage allowed to the running time extension, λ , is required here.

3.2.2. Constraints for reordering trains

Minimum arrival/departure headways are required between trains running in the same directions:

$$x_{e'} - x_e \geq L_a + M_2(q_{e,e'} - 1), \quad a = (e, e') \in A_{\text{head}}^{\text{ar}} \cup A_{\text{head}}^{\text{de}}, \quad (16)$$

$$q_{e,e'} + q_{e',e} = 1, \quad (e, e') \in A_{\text{head}}^{\text{ar}} \cup A_{\text{head}}^{\text{de}}, \quad (17)$$

where the order between events e and e' is described by the binary decision variable $q_{e,e'}$ with value 1 indicating that e occurs before e' . Here, L_a represents the minimum duration of activity a , and M_2 is set to two times of M_1 .

Train overtaking on an open-track section is forbidden:

$$q_{e_1,e'_1} - q_{e_2,e'_2} = 0, \quad (e_1, e'_1) \in A_{\text{head}}^{\text{de}}, (e_2, e'_2) \in A_{\text{head}}^{\text{ar}}, (e_1, e_2) \in A_{\text{run}}, (e'_1, e'_2) \in A_{\text{run}}. \quad (18)$$

For the stations located on single-track railway lines, minimum headway should be respected between the arrival of a train and the departure of another train that occurs at the same station but operates in opposite direction. An example of such an arrival-departure headway is shown in Fig. 5.

The constraints for ensuring arrival-departure headways are:

$$x_{e'} - x_e \geq L_a + M_2(q_{e,e'} - 1), \quad a = (e, e') \in A_{\text{head}}^{\text{ar}, \text{de}}, r_{st_e} = 1, \quad (19)$$

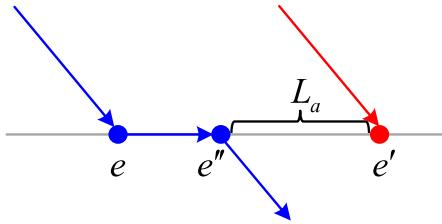
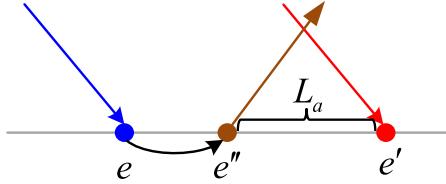
Situation 1:**Situation 2:**

Fig. 6. Two situations where departure-arrival headways are required in a station that has only two tracks and is located on a double-track railway line: $a = (e'', e') \in A_{\text{head}}^{\text{de,ar}}$.

Table 3

The stop type of activity $a = (e, e') \in A_{\text{dwell}}$ in the disruption timetable according to c_e , $c_{e'}$ and s_a .

c_e	$c_{e'}$	s_a	Stop type
0	0	0	Stop
0	0	1	Skipped stop
1	0	0	Cancelled stop
0	1	0	Cancelled stop
1	1	0	Cancelled stop

$$q_{e,e'} + q_{e',e} = 1, \quad (e, e') \in A_{\text{head}}^{\text{ar,de}}, r_{st_e} = 1, \quad (20)$$

where r_{st_e} is a binary parameter with value 1 indicating that station st_e is located on single-track railway lines; and 0 otherwise.

For a station that is located on double-track railway lines and has two tracks, each track of the station is used for trains coming from the same direction. Thus, a minimum headway is required between the departure of a train and the arrival of another train that uses the same track at the station. Likewise, such a headway is needed when a train turns at the station and departs towards the other track. In Fig. 6, two situations that require such departure-arrival headways are shown.

The constraints for ensuring headways shown in Fig. 6 are:

$$x_{e'} - x_{e''} \geq L_a + M_2(q_{e,e'} - 1 - c_e - c_{e'} - c_{e''}), \quad a = (e'', e') \in A_{\text{head}}^{\text{de,ar}}, (e, e'') \in A_{\text{station}}, N_{st_e} = 2, \quad (21)$$

$$x_{e'} - x_{e''} \geq L_a + M_2(q_{e,e'} - 1 - c_e - c_{e'} - (1 - m_{a'})), \quad a = (e'', e') \in A_{\text{head}}^{\text{de,ar}}, a' = (e, e'') \in A_{\text{turn}} \cup A_{\text{odturn}}, N_{st_e} = 2, \quad (22)$$

where N_{st_e} represents the number of tracks in the corresponding station of event e , and $m_{a'}$ is a binary decision variable with value 1 indicating that a short-turn/OD turn activity a' is active, and L_a is the minimum headway duration. Constraints (21) and (22) are for situation 1 and situation 2, respectively. In (21), the time difference between events e' and e'' , $x_{e'} - x_{e''}$, does not need to respect the minimum headway L_a , if e , e' or e'' is cancelled, or all of them are kept but e occurs after e' (i.e. $q_{e,e'} = 0$). Also in (22), $x_{e'} - x_{e''}$ does not need to respect the minimum headway L_a , if e or e' is cancelled, both of them are kept but the short-turn/OD turn activity a' relevant to e is not active (i.e. $m_{a'} = 0$), or both e and e' are kept and a' is active but e occurs after e' . The details of short-turn/OD turn activities can be found in Sections 3.2.4 and 3.2.5.

3.2.3. Constraints for flexible stopping

Recall that flexible stopping means that scheduled stops can be skipped, and extra stops can be added. To realize flexible stopping, a binary variable s_a is introduced. For a scheduled stop $a \in A_{\text{dwell}}$, $s_a = 1$ indicates that the stop is skipped. For a scheduled non-stop $a \in A_{\text{pass}}$, $s_a = 0$ indicates that a stop is added. It happens that a scheduled stop or non-stop is cancelled, which means that the stop type of $a \in A_{\text{dwell}}$ ($a \in A_{\text{pass}}$) in the disruption timetable is also relevant to cancellation decision. Table 3 (Table 4) shows the values of decision variables c_e , $c_{e'}$ and s_a that indicate the corresponding stop type of $a \in A_{\text{dwell}}$ ($a \in A_{\text{pass}}$) in the disruption timetable.

The constraints deciding the values of c_e , $c_{e'}$ and s_a are:

$$s_a \leq 1 - c_e, \quad a = (e, e') \in A_{\text{dwell}}, \quad (23)$$

$$s_a \leq 1 - c_{e'}, \quad a = (e, e') \in A_{\text{dwell}}, \quad (24)$$

Table 4

The stop type of activity $a = (e, e') \in A_{\text{pass}}$ in the disruption timetable according to c_e , $c_{e'}$ and s_a .

c_e	$c_{e'}$	s_a	Stop type
0	0	0	Extra stop
0	0	1	Non-stop
1	0	1	Cancelled non-stop
0	1	1	Cancelled non-stop
1	1	1	Cancelled non-stop

$$s_a \geq c_e, \quad a = (e, e') \in A_{\text{pass}}, \quad (25)$$

$$s_a \geq c_{e'}, \quad a = (e, e') \in A_{\text{pass}}, \quad (26)$$

$$x_{e'} - x_e \geq L_a(1 - s_a - c_e - c_{e'}) - M_1 c_e, \quad a = (e, e') \in A_{\text{station}}, \quad (27)$$

$$x_{e'} - x_e \leq M_2(1 - s_a + c_e + c_{e'}), \quad a = (e, e') \in A_{\text{station}}, \quad (28)$$

where $A_{\text{station}} = A_{\text{dwell}} \cup A_{\text{pass}}$. For $a = (e, e') \in A_{\text{dwell}}$, (23) and (24) constrain s_a to be 0, if either e or e' is cancelled. For $a = (e, e') \in A_{\text{pass}}$, (25) and (26) constrain s_a to be 1, if either e or e' is cancelled. According to Tables 3 and 4, a true stop in the disruption timetable can only be one of which the corresponding c_e , $c_{e'}$ and s_a are all equal to 0. Constraint (27) requires a true stop to satisfy the minimum dwell duration. Besides, a skipped stop or a non-stop in the disruption timetable can only be one of which the corresponding c_e and $c_{e'}$ are both equal to 0, and $s_a = 1$. Constraints (27) and (28) together ensure that the duration of a skipped stop or a non-stop is 0. For a cancelled stop (non-stop) that either the corresponding c_e or $c_{e'}$ is equal to 1, (27) and (28) also remain feasible.

When changing the stopping pattern of a train, an adjusted train running time due to saved/extraneous acceleration and deceleration should be considered. Fig. 7 enumerates all cases of the planned stopping patterns relevant to a train run $a' = (e', e'') \in A_{\text{run}}$, as well as the composition of the planned minimum running time $L_{a'}$ in each case. Here, $\Delta_{a'}^{\text{acce}}$ and $\Delta_{a'}^{\text{dece}}$ represent the acceleration time and the deceleration time needed for a' , respectively; while $\tau_{a'}$ represents the pure running time of a' . Note that time supplement is not included in either $L_{a'}$ or $\tau_{a'}$, and it is always satisfied that $L_{a'} \geq \tau_{a'}$. In Fig. 7, case 1 means that a train stops at two adjacent stations; case 2 means that a train stops at a station after which it reaches the destination where the corresponding rolling stock turns to operate the opposite train (i.e. OD turn); case 3 means that a train starts from the origin and stops at the next adjacent station; case 4 means that a train passes through a station but stops at the next adjacent station; case 5 means that a train passes through a station after which it reaches the destination; case 6 means that a train stops at a station but passes through the next adjacent station; case 7 means that a train starts from the origin and passes through the next adjacent station; and case 8 means that a train passes through two adjacent stations.

In the rescheduled timetable, the minimum running time of a train between two adjacent stations may become longer or shorter than planned, if either of the stopping patterns at these two stations changes. Considering this, the following three constraints are established for cases 1–3 of Fig. 7, respectively, each requiring that the minimum running time dependent on the changed stopping patterns is respected:

$$x_{e''} - x_{e'} \geq L_{a'} - \Delta_{a'}^{\text{acce}} s_a - \Delta_{a'}^{\text{dece}} s_{a''}, \quad a = (e, e') \in A_{\text{dwell}}, a' = (e', e'') \in A_{\text{run}}, a'' = (e'', e''') \in A_{\text{dwell}},$$

$$x_{e''} - x_{e'} \geq L_{a'} - \Delta_{a'}^{\text{acce}} s_a, \quad a = (e, e') \in A_{\text{dwell}}, a' = (e', e'') \in A_{\text{run}}, a'' = (e'', e''') \in A_{\text{odturn}}^{\text{plan}},$$

$$x_{e''} - x_{e'} \geq L_{a'} - \Delta_{a'}^{\text{dece}} s_{a''}, \quad a = (e, e') \in A_{\text{odturn}}^{\text{plan}}, a' = (e', e'') \in A_{\text{run}}, a'' = (e'', e''') \in A_{\text{dwell}},$$

where $A_{\text{odturn}}^{\text{plan}}$ is the set of all planned turnings of rolling stock at terminal stations. Recall that for $a \in A_{\text{dwell}}$, $s_a = 1$ indicates that a is skipped. According to (14), we have $x_{e''} - x_{e'} \geq o_{e''} - o_{e'}$ with $a' = (e', e'') \in A_{\text{run}}$ and $o_{e''} - o_{e'}$ representing the original scheduled running time that includes time supplement. It is always satisfied that $o_{e''} - o_{e'} \geq L_{a'}$, which makes $x_{e''} - x_{e'} \geq L_{a'}$ always respected. As such, the three constraints shown above are always satisfied, because $s_a, s_{a''} \in \{0, 1\}$ thus either $-\Delta_{a'}^{\text{acce}} s_a$ or $-\Delta_{a'}^{\text{dece}} s_{a''}$ must be non-positive. Considering this, these three constraints are not included in the proposed model.

For cases 4–8 of Fig. 7, the following constraints are established, respectively. Each of them requires that the minimum running time depending on the changed stopping pattern is respected. Note that these constraints take effect only if a stop is skipped or a passage becomes an added stop.

$$x_{e''} - x_{e'} \geq L_{a'} + \Delta_{a'}^{\text{acce}} (1 - s_a) - \Delta_{a'}^{\text{dece}} s_{a''}, \quad a = (e, e') \in A_{\text{pass}}, a' = (e', e'') \in A_{\text{run}}, a'' = (e'', e''') \in A_{\text{dwell}}, \quad (29)$$

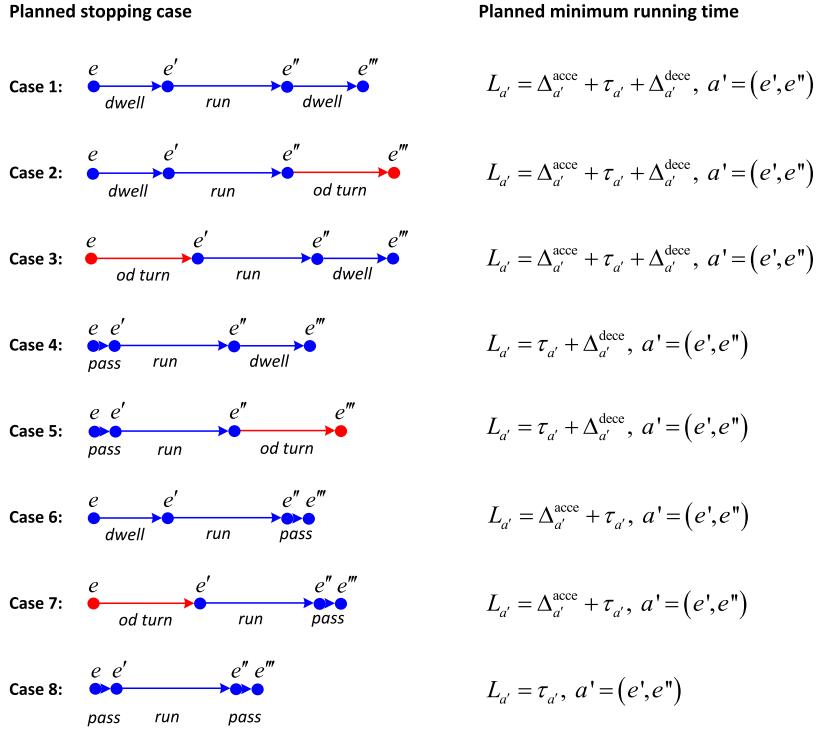


Fig. 7. Planned stopping patterns of a train at two adjacent stations and the minimum running time between these two stations under different cases where $e, e'' \in E_{\text{ar}}, e', e''' \in E_{\text{de}}, (e', e'') \in A_{\text{run}}$.

$$x_{e''} - x_{e'} \geq L_{a'} + \Delta_{a'}^{\text{acce}}(1 - s_a), \quad a = (e, e') \in A_{\text{pass}}, a' = (e', e'') \in A_{\text{run}}, a'' = (e'', e''') \in A_{\text{odturn}}^{\text{plan}}, \quad (30)$$

$$x_{e''} - x_{e'} \geq L_{a'} - \Delta_{a'}^{\text{acce}}s_a + \Delta_{a'}^{\text{dece}}(1 - s_{a''}), \quad a = (e, e') \in A_{\text{dwell}}, a' = (e', e'') \in A_{\text{run}}, a'' = (e'', e''') \in A_{\text{pass}}, \quad (31)$$

$$x_{e''} - x_{e'} \geq L_{a'} + \Delta_{a'}^{\text{dece}}(1 - s_{a''}), \quad a = (e, e') \in A_{\text{odturn}}^{\text{plan}}, a' = (e', e'') \in A_{\text{run}}, a'' = (e'', e''') \in A_{\text{pass}}, \quad (32)$$

$$x_{e''} - x_{e'} \geq L_{a'} + \Delta_{a'}^{\text{acce}}(1 - s_a) + \Delta_{a'}^{\text{dece}}(1 - s_{a''}), \quad a = (e, e') \in A_{\text{pass}}, a' = (e', e'') \in A_{\text{run}}, a'' = (e'', e''') \in A_{\text{pass}}, \quad (33)$$

Recall that for $a \in A_{\text{pass}}, s_a = 0$ indicates that a is added with a stop; while for $a \in A_{\text{dwell}}, s_a = 1$ indicates that a is skipped.

3.2.4. Constraints for flexible short-turning

Recall that flexible short-turning means that a train is provided with a full choice of short-turn station candidates and the proposed model decides at which station the train will be short-turned and which short-turn activity at the station will be selected. In the following, we first explain how to generate the set of all possible short-turn activities A_{turn} by [Algorithm 1](#), and then introduce the constraints that decide which short-turn activities of A_{turn} can be selected in the rescheduled timetable.

One input of [Algorithm 1](#), $ST_{\text{turn}}^{tl, dr}$, contains the short-turn station candidates for the trains serving train line tl and operating in direction dr . Note that each station contained in $ST_{\text{turn}}^{tl, dr}$ is the upstream/same station compared to st_{en}^{dr} where $dr \in \{\text{up}, \text{down}\}$. Recall that st_{en}^{dr} is the entry station of the disrupted section for the trains operating in direction dr . In [Fig. 4](#), suppose the infrastructure layouts of stations A, B, C, D and E all allow short-turning trains. Then, for the downstream blue train, its short-turn station candidates includes station C that is the entry station of the disrupted section for this train, and also station D that is the upstream station compared to station C. For the upstream red train, station B is the only short-turn station candidate. Another input of [Algorithm 1](#) is L_{turn} that contains the minimum short-turn duration required in each station. The set TL_{dis} includes all train lines that can be affected by the disruption, which is also input to [Algorithm 1](#). A train line of which the planned operation covers the disrupted section is a train line that could be affected by the disruption.

Algorithm 1: Constructing the set of all possible short-turn activities A_{turn} .

Input: $ST_{\text{turn}}^{tl,dr}, E_{\text{de}}, E_{\text{ar}}, t_{\text{start}}, t_{\text{end}}, R, D, L_{\text{turn}}, TL_{\text{dis}}$

Output: A_{turn}

- 1 $A_{\text{turn}}^{tl,dr} = \emptyset;$
- 2 **foreach** $st \in ST_{\text{turn}}^{tl,dr}$ **do**
- 3 Define $E_{\text{de},st}^{\text{dis},tl,dr} = \{e' \in E_{\text{de}} | tl_{e'} = tl, dr_{e'} = dr, st_{e'} = st, t_{\text{start}} \leq o_{e'} < t_{\text{end}} + R\};$
- 4 Define $E_{\text{ar}}^{\text{tn}} = \{e \in E_{\text{ar}} | (e, e') \in A_{\text{station}}, e' \in E_{\text{de},st}^{\text{dis},tl,dr}\};$
- 5 Define $E_{\text{de}}^{\text{tn}} = \{e'' \in E_{\text{de}} | tl_{e''} = tl, dr_{e''} \neq dr, st_{e''} = st, t_{\text{start}} \leq o_{e''} < t_{\text{end}} + R\};$
- 6 **foreach** $e \in E_{\text{ar}}^{\text{tn}}$ **do**
- 7 **foreach** $e'' \in E_{\text{de}}^{\text{tn}}$ **do**
- 8 **if** $o_{e''} + D - o_e \geq L_{\text{turn}}^{\text{st}}$ **then**
- 9 $a_{\text{turn}} = (e, e'');$
- 10 $A_{\text{turn}}^{tl,dr} = A_{\text{turn}}^{tl,dr} \cup \{a_{\text{turn}}\};$

11 $A_{\text{turn}} = \bigcup_{\substack{tl \in TL_{\text{dis}}, \\ dr \in \{\text{up}, \text{down}\}}} A_{\text{turn}}^{tl,dr};$

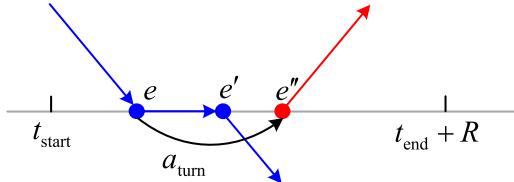


Fig. 8. Construct a_{turn} from $e \in E_{\text{ar}}$ to $e'' \in E_{\text{de}}$ where $(e, e') \in A_{\text{station}}$, e' and e'' both originally occur after t_{start} but before $t_{\text{end}} + R$, and e'' could occur $L_{\text{turn}}^{\text{st}}$ minutes later than e after (without) being delayed.

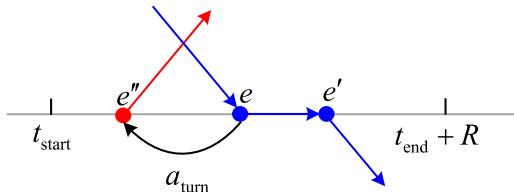


Fig. 9. Construct a_{turn} from $e \in E_{\text{ar}}$ to $e'' \in E_{\text{de}}$ where $(e, e') \in A_{\text{station}}$, e' and e'' both originally occur after t_{start} but before $t_{\text{end}} + R$, and e'' could occur $L_{\text{turn}}^{\text{st}}$ minutes later than e after being delayed.

In [Algorithm 1](#), we first initialize $A_{\text{turn}}^{tl,dr}$ as an empty set (line 1). $A_{\text{turn}}^{tl,dr}$ contains all possible short-turn activities for trains that serve train line tl and operate in direction dr . Then, we iterate over each station st contained in $ST_{\text{turn}}^{tl,dr}$ (line 2) to define the set $E_{\text{de},st}^{\text{dis},tl,dr}$ that contains the departure events corresponding to the trains that serve train line tl , operate in direction dr , and originally occur at station st after the disruption starts but before the disruption end time plus the recovery time (line 3). For example in [Fig. 8](#) or [Fig. 9](#), $e' \in E_{\text{de},st}^{\text{dis},tl,dr}$ is a departure event of which the original scheduled time $o_{e'}$ is within the time period $[t_{\text{start}}, t_{\text{end}} + R]$ and is approaching to the disrupted area. Such a departure event could be cancelled, thus its corresponding arrival event e in the station activity could be short-turned, which is included in the set $E_{\text{ar}}^{\text{tn}}$ (line 4). Also, we define the set $E_{\text{de}}^{\text{tn}}$ to contain the departure events that could be served by the arrival events in $E_{\text{ar}}^{\text{tn}}$ for short-turning (line 5), for example the departure event e'' in [Fig. 8](#) or [Fig. 9](#). Between any $e \in E_{\text{ar}}^{\text{tn}}$ and $e'' \in E_{\text{de}}^{\text{tn}}$, a short-turn activity a_{turn} is constructed only if the required minimum short-turn duration could be respected in the rescheduled timetable (lines 6–9). Here, $o_{e''} + D$ is the largest rescheduled time which departure event e'' could occur at, and o_e is the smallest rescheduled time which arrival event e could occur at. $L_{\text{turn}}^{\text{st}}$ refers to the minimum short-turn duration required at station st . For example in [Fig. 9](#), although e'' originally occurs before e , it could be delayed to occur later than e to make the minimum short-turn duration respected. Thus, a short-turn activity a_{turn} can be constructed from e to e'' . Any constructed a_{turn} for a train serving train line tl and operating in direction dr is added to the set $A_{\text{turn}}^{tl,dr}$ (line 10), and the set A_{turn} is constructed by including all $A_{\text{turn}}^{tl,dr}$ with $tl \in TL_{\text{dis}}, dr \in \{\text{up}, \text{down}\}$ (line 11).

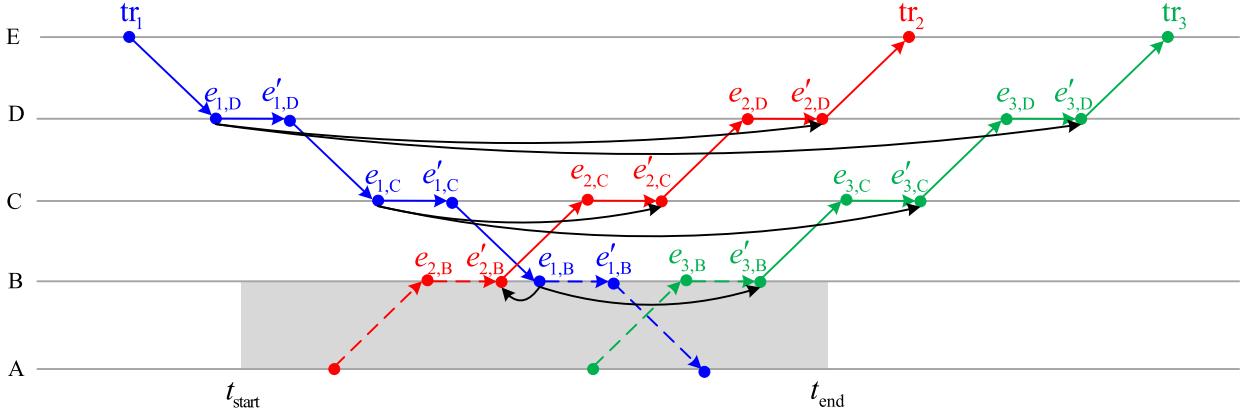


Fig. 10. The possible short-turn activities (black arcs) at each short-turn station candidate of the blue train. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

A train may have multiple short-turn activities at each short-turn station candidate. An example is given in Fig. 10 where the tracks between stations A and B are completely blocked from t_{start} to t_{end} and a blue train tr_1 has two short-turn activities at each of stations B, C, D and E. Among all of these short-turn activities, at most one can be selected for the blue train. This means that for each train, the proposed model decides (1) at which station the train will be short-turned; and (2) which short-turn activity at the station is selected for the train. Considering these, we introduce a binary variable y_e with value 1 indicating that station st_e is chosen as the short-turn station for train tr_e ; and a binary variable m_a with value 1 indicating that a short-turn activity a is selected. Besides, we construct the set $E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{\text{turn}}$) that contains all arrival (departure) events that are the tails (heads) of activities included in A_{turn} . For example in Fig. 10, $e_{1,B}, e_{1,C}, e_{1,D} \in E_{\text{ar}}^{\text{turn}}$, $e'_{2,B}, e'_{2,C}, e'_{2,D}, e'_{3,B}, e'_{3,C}, e'_{3,D} \in E_{\text{de}}^{\text{turn}}$.

Suppose the blue train in Fig. 10 is short-turned at station B, then $e_{1,B}$ must be kept with $e'_{1,B}$ being cancelled (i.e. $c_{e_{1,B}} = 0, c_{e'_{1,B}} = 1$). In that sense, the operation consistency between events $e_{1,B}$ and $e'_{1,B}$ is broken. Thus, deciding where to short-turn a train is to decide where to break the operation consistency. The operation consistency can be broken at most one station at each side of the disrupted section for a train. Considering these, the following constraints are established:

$$c_e \leq c_{e'}, \quad e \in E_{\text{ar}}^{\text{turn}}, (e, e') \in A_{\text{station}}, \quad (34)$$

$$c_{e'} \leq c_e + y_e, \quad e \in E_{\text{ar}}^{\text{turn}}, (e, e') \in A_{\text{station}}, \quad (35)$$

$$c_{e'} \geq y_e, \quad e \in E_{\text{ar}}^{\text{turn}}, (e, e') \in A_{\text{station}}, \quad (36)$$

$$c_{e'} \leq c_e, \quad e' \in E_{\text{de}}^{\text{turn}}, (e, e') \in A_{\text{station}}, \quad (37)$$

$$c_e \leq c_{e'} + y_{e'}, \quad e' \in E_{\text{de}}^{\text{turn}}, (e, e') \in A_{\text{station}}, \quad (38)$$

$$c_e \geq y_{e'}, \quad e' \in E_{\text{de}}^{\text{turn}}, (e, e') \in A_{\text{station}}, \quad (39)$$

$$\sum_{e: tr_e = tr} y_e = c_{e'}, \quad tr \in TR_{\text{turn}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, tr_{e'} = tr, st_{e'} = s_{\text{en}}^{dr_{e'}}, \quad (40)$$

$$\sum_{e': tr_{e'} = tr} y_{e'} = c_e, \quad tr \in TR_{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, e \in E_{\text{ar}}, tr_e = tr, st_e = s_{\text{ex}}^{dr_e}, \quad (41)$$

where TR_{turn} is the set of trains that correspond to the events in $E_{\text{ar}}^{\text{turn}} \cup E_{\text{de}}^{\text{turn}}$, and $s_{\text{en}}^{dr_e}$ ($s_{\text{ex}}^{dr_e}$) represents the entry (exit) station of the disrupted section considering the operation direction of event e . Constraints (34) and (35) mean that if station st_e is not chosen as the short-turn station for train tr_e (i.e. $y_e = 0$), then the operation consistency between events e and e' is kept by requiring them to be cancelled or kept simultaneously. Constraint (36) means that if station st_e is chosen as the

short-turn station for train tr_e (i.e. $y_e = 1$), then the operation consistency between events e and e' are broken by forcing event e' to be cancelled (i.e. $c_{e'} = 1$) while event e can be either cancelled or kept according to (34). Constraints (37) and (38) mean that if station $st_{e'}$ is not chosen as the short-turn station for train $tr_{e'}$ (i.e. $y_{e'} = 0$), then the operation consistency between events e and e' are kept by requiring them to be cancelled or kept simultaneously. Constraint (39) means that if station $st_{e'}$ is chosen as the short-turn station for train $tr_{e'}$ (i.e. $y_{e'} = 1$), then the operation consistency between events e and e' are broken by forcing event e to be cancelled (i.e. $c_e = 1$) while event e' can be either cancelled or kept according to (37). Constraints (40) and (41) mean that if the operation of a train in the disrupted section is cancelled, then at each side of the disrupted section, one station is chosen for the train as the short-turn station.

At the short-turn station, at most one short-turn activity will be selected for the train, which is formulated as

$$\sum_{a \in A_{\text{turn}}, \text{tail}(a)=e} m_a = c_{e'} - c_e, \quad e \in E_{\text{ar}}^{\text{turn}}, (e, e') \in A_{\text{station}}, \quad (42)$$

$$\sum_{a \in A_{\text{turn}}, \text{head}(a)=e'} m_a = c_e - c_{e'}, \quad e' \in E_{\text{de}}^{\text{turn}}, (e, e') \in A_{\text{station}}, \quad (43)$$

$$M_1 c_e + 2D(1 - m_a) + x_{e'} - x_e \geq m_a L_a, \quad a = (e, e') \in A_{\text{turn}}. \quad (44)$$

Constraint (42) means that if event $e \in E_{\text{ar}}^{\text{turn}}$ is kept (i.e. $c_e = 0$) while its corresponding departure event e' in the station activity is cancelled (i.e. $c_{e'} = 1$), one and only one of the short-turn activities corresponding to e will be selected. If e and e' are both cancelled, no short-turn activities will be selected for e . Constraint (43) means that if event $e' \in E_{\text{de}}^{\text{turn}}$ is kept (i.e. $c_{e'} = 0$) while its corresponding arrival event e in the station activity is cancelled (i.e. $c_e = 1$), one and only one of the short-turn activities corresponding to e' will be selected. If e and e' are both cancelled, no short-turn activities will be selected for e' . Constraint (44) means that if a short-turn activity $a \in A_{\text{turn}}$ is selected (i.e. $m_a = 1$), it has to respect the minimum short-turn duration L_a . For a short-turn activity a that is not selected (i.e. $m_a = 0$), (44) also remains feasible.

To summarize, whether a train will be short-turned or not depends on (42)–(44), while (34)–(41) together help to decide the values of the variables that are in the right sides of (42) and (43).

3.2.5. Constraints for rolling stock circulations at terminal stations

When a train arrives at the destination, the corresponding rolling stock turns at the station to operate an opposite train that departs from the station as the origin. We call this an OD turn and take it into account in the proposed model. In the following, we first explain how to generate the set of all possible OD turn activities A_{odturn} by [Algorithm 2](#), and then introduce the constraints that decide which OD turn activities of A_{odturn} can be selected in the rescheduled timetable.

[Algorithm 2](#) construct the set of OD turn activities: A_{odturn} . We first initialize A_{odturn} as an empty set (line 1). Then, we select any arrival (departure) event that corresponds to a planned OD turn activity but the corresponding departure (arrival) event in this activity originally occurs after t_{start} but before $t_{\text{end}} + R$ (making the planned OD turn possibly impossible in the disruption timetable). These selected arrivals (departures) constitute the set $E_{\text{ar}}^{\text{odturn}}$ ($E_{\text{de}}^{\text{odturn}}$) (lines 2–3). Between any $e \in E_{\text{ar}}^{\text{odturn}}$ and $e' \in E_{\text{de}}^{\text{odturn}}$, an OD turn activity a_{odturn} is created, if e and e' correspond to the same train line, occur at the same station, and the possible largest occurrence time of e' (i.e. $o_{e'} + D$) could be $L_{\text{odturn}}^{st_e}$ minutes later than the possible smallest occurrence time of e (i.e. o_e) (lines 4–7). $L_{\text{odturn}}^{st_e}$ refers to the minimum OD turn duration required at the corresponding station. In line 8, a created a_{odturn} is added to the set A_{odturn} .

Algorithm 2: Constructing the set of all possible OD turn activities A_{odturn} .

Input: $A_{\text{odturn}}^{\text{plan}}$, E_{ar} , E_{de} , t_{start} , t_{end} , R , D , L_{odturn}
Output: A_{odturn}

- 1 $A_{\text{odturn}} = \emptyset$;
- 2 Define $E_{\text{ar}}^{\text{odturn}} = \left\{ e \in E_{\text{ar}} \mid (e, e') \in A_{\text{odturn}}^{\text{plan}}, t_{\text{start}} \leq o_{e'} < t_{\text{end}} + R \right\}$;
- 3 Define $E_{\text{de}}^{\text{odturn}} = \left\{ e' \in E_{\text{de}} \mid (e, e') \in A_{\text{odturn}}^{\text{plan}}, t_{\text{start}} \leq o_e < t_{\text{end}} + R \right\}$;
- 4 **foreach** $e \in E_{\text{ar}}^{\text{odturn}}$ **do**
- 5 **foreach** $e' \in E_{\text{de}}^{\text{odturn}}$ **do**
- 6 **if** $tl_{e'} = tl_e$, $st_{e'} = st_e$ and $o'_e + D - o_e \geq L_{\text{odturn}}^{st_e}$ **then**
- 7 $a_{\text{odturn}} = (e, e')$;
- 8 $A_{\text{odturn}} = A_{\text{odturn}} \cup \{a_{\text{odturn}}\}$;

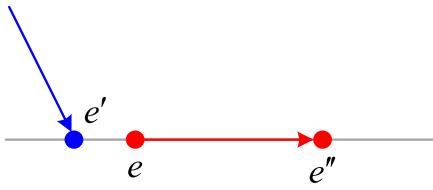


Fig. 11. If $e' \in E_{ar}$ occurs before $e \in E_{ar}$, then $n_{e,e'} = 0$, where (e, e'') is a station activity, a short-turn activity or an OD turn activity.

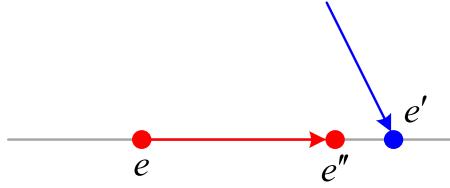
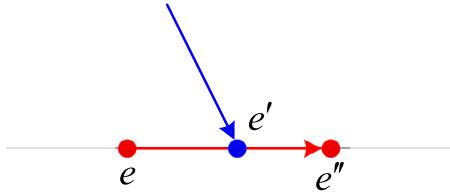


Fig. 12. If $e' \in E_{ar}$ occurs after $e'' \in E_{de}$, then $n_{e,e'} = 0$, where (e, e'') is a station activity, a short-turn activity or an OD turn activity.



Based on the constructed E_{ar}^{odturn} , E_{de}^{odturn} and A_{odturn} , we establish the constraints for rolling stock circulations at terminal stations:

$$\sum_{a \in A_{odturn}, tail(a)=e} m_a = 1 - c_e, \quad e \in E_{ar}^{odturn}, \quad (45)$$

$$\sum_{a \in A_{odturn}, head(a)=e'} m_a = 1 - c_{e'}, \quad e' \in E_{de}^{odturn}, \quad (46)$$

$$M_1 c_e + 2D(1 - m_a) + x_{e'} - x_e \geq m_a L_a, \quad a = (e, e') \in A_{odturn}. \quad (47)$$

Constraint (45) means that if an arrival event $e \in E_{ar}^{odturn}$ is not cancelled (i.e. $c_e = 0$), then one and only one of the OD turn activities corresponding to e will be selected. Otherwise, no OD turn activities corresponding to e will be selected. Constraint (46) means that if a departure event $e' \in E_{de}^{odturn}$ is not cancelled (i.e. $c_{e'} = 0$), then one and only one of the OD turn activities corresponding to e' will be selected. Otherwise, no OD turn activities corresponding to e' will be selected. For an selected OD turn activity, the minimum turn duration must be respected (47).

3.2.6. Constraints for station capacities

It is necessary to ensure each arrival train to be assigned with a track to dwell or pass through within a station. In other words, there should be at least one station track available for each arrival train. Hence, for each arrival event $e' \in E_{ar}$, we need to ensure that when e' occurs, the number of trains currently occupying the tracks at station $st_{e'}$ is smaller than the total number of tracks within this station. Here, we introduce a binary variable $n_{e,e'}$ for any two events $e, e' \in E_{ar}$ that $st_e = st_{e'}, tr_e \neq tr_{e'}$, which indicates whether train tr_e is currently occupying a track at station st_e when another train $tr_{e'}$ is approaching to the same station. If yes, $n_{e,e'} = 1$. In addition, we use N_{st} to represent the total number of tracks at station st . Thus, ensuring $e' \in E_{ar}$ to be assigned with a station track is actually to make sure that $\sum_{e \in E_{ar}, st_e = st_{e'}, tr_e \neq tr_{e'}} n_{e,e'}$ is smaller than N_{st} .

Figs. 11–13 show all cases where the values of $n_{e,e'}$ could be. When e' occurs before e or after e'' (see Fig. 11 or Fig. 12), $n_{e,e'} = 0$. When e' occurs after e but before e'' (see Fig. 13), $n_{e,e'} = 1$. According to the enumerated cases, we establish the following constraints to determine the value of $n_{e,e'}$:

$$n_{e,e'} \leq q_{e,e'}, \quad e, e' \in E_{ar}, st_e = st_{e'}, tr_e \neq tr_{e'}, \quad (48)$$

$$x_{e'} - x_{e''} \geq M_2(q_{e,e'} - n_{e,e'} - 1), \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, (e, e'') \in A_{\text{station}} \cup A_{\text{turn}} \cup A_{\text{odturn}}, \quad (49)$$

$$x_{e'} - x_{e''} \leq M_2(q_{e,e'} - n_{e,e'}), \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, (e, e'') \in A_{\text{station}} \cup A_{\text{turn}} \cup A_{\text{odturn}}, \quad (50)$$

where $q_{e,e'}$ is a binary variable indicating whether or not e occurs before e' . If yes, $q_{e,e'} = 1$. Otherwise, $q_{e,e'} = 0$.

If e' occurs before e (i.e. $q_{e,e'} = 0$), $n_{e,e'}$ is forced to be 0 in (48). If e' occurs after e (i.e. $q_{e,e'} = 1$), the value of $n_{e,e'}$ is further dependent on the occurrence times of e' and e'' . This means that if $q_{e,e'} = 1$ and $x_{e'} - x_{e''} \geq 0$, $n_{e,e'}$ is forced to be 0 by (49) and (50), and if $q_{e,e'} = 1$ and $x_{e'} - x_{e''} \leq 0$, $n_{e,e'}$ is forced to be 1 by (49) and (50).

However, it is possible that either e or e' is cancelled, thus $n_{e,e'}$ should be 0:

$$n_{e,e'} \leq 1 - c_e, \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, \quad (51)$$

$$n_{e,e'} \leq 1 - c_{e'}, \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}. \quad (52)$$

Considering $e \in E_{\text{ar}}$ may correspond to one station activity only but not any short-turn or OD turn activities (i.e. $e \notin E_{\text{ar}}^{\text{turn}} \cup E_{\text{ar}}^{\text{odturn}}$), $n_{e,e'}$ should be 0, if the corresponding departure event of e in the station activity is cancelled (53). Considering $e \in E_{\text{ar}}$ may correspond to one station activity and at least one short-turn activity (i.e. $e \in E_{\text{ar}}^{\text{turn}}$), $n_{e,e'}$ should be 0, if the corresponding departure event of e in the station activity is cancelled and none of the short-turn activities relevant to e is selected (54). Moreover, $e \in E_{\text{ar}}$ could correspond to OD turn activities only, if e is a destination arrival. In such a case, $n_{e,e'}$ should be 0, if none of the OD turn activities relevant to e is selected which actually equals to e is cancelled, according to (45). Thus, the value of $n_{e,e'}$ in this case can be reflected well by (51).

$$n_{e,e'} \leq 1 - c_{e''}, \quad e \in E_{\text{ar}} \setminus (E_{\text{ar}}^{\text{turn}} \cup E_{\text{ar}}^{\text{odturn}}), e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, (e, e'') \in A_{\text{station}}, \quad (53)$$

$$n_{e,e'} \leq 1 - (c_{e''} - \sum_{a \in A_{\text{turn}}, \text{tail}(a)=e} m_a), \quad e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, (e, e'') \in A_{\text{station}}. \quad (54)$$

Considering these cancellation situations, (49) and (50) are changed to (55)–(58) where $L_{a'}$ is the minimum headway required between the departure of a train and the arrival of another train in case they are assigned to the same track at the corresponding station.

$$x_{e'} - (x_{e''} + L_{a'}) \geq M_2(q_{e,e'} - n_{e,e'} - 1 - c_e - c_{e'} - c_{e''}), \quad e, e' \in E_{\text{ar}}, (e, e'') \in A_{\text{station}}, a' = (e'', e') \in A_{\text{head}}^{\text{de.ar}}, \quad (55)$$

$$x_{e'} - (x_{e''} + L_{a'}) \leq M_2(q_{e,e'} - n_{e,e'} + c_e + c_{e'} + c_{e''}), \quad e, e' \in E_{\text{ar}}, (e, e'') \in A_{\text{station}}, a' = (e'', e') \in A_{\text{head}}^{\text{de.ar}}, \quad (56)$$

$$x_{e'} - (x_{e''} + L_{a'}) \geq M_2(q_{e,e'} - n_{e,e'} - 1 - c_e - c_{e'} - (1 - m_a)), \quad e, e' \in E_{\text{ar}}, a = (e, e'') \in A_{\text{turn}} \cup A_{\text{odturn}}, a' = (e'', e') \in A_{\text{head}}^{\text{de.ar}} \quad (57)$$

$$x_{e'} - (x_{e''} + L_{a'}) \leq M_2(q_{e,e'} - n_{e,e'} + c_e + c_{e'} + 1 - m_a), \quad e, e' \in E_{\text{ar}}, a = (e, e'') \in A_{\text{turn}} \cup A_{\text{odturn}}, a' = (e'', e') \in A_{\text{head}}^{\text{de.ar}}. \quad (58)$$

To summarize, (48) and (51)–(58) together decide the value of $n_{e,e'}$, $\forall e, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}$. In these constraints, the element $q_{e,e'}$ that indicates the sequence of e and e' is necessary. Recall that constraints (16) and (17) decide the value of $q_{e,e'}$ but only for such e and e' that correspond to the same operation directions. Thus, additional constraint is needed for determining the $q_{e,e'}$ that e and e' correspond to different operation directions:

$$M_2(q_{e,e'} - 1) \leq x_{e'} - x_e \leq M_2 q_{e,e'}, \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, dr_e \neq dr_{e'}, \quad (59)$$

$$q_{e,e'} + q_{e',e} = 1, \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, dr_e \neq dr_{e'}. \quad (60)$$

Based on $n_{e,e'}$, we ensure that each arrival train has at least one station track to dwell or pass through by

$$\sum_e n_{e,e'} \leq N_{st} - 1, \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, st = st_{e'}, tr_e \neq tr_{e'}, \quad (61)$$

where N_{st} represents the total number of tracks at station st . Here, N_{st} is the sum of N_{st}^p and N_{st}^{th} that refer to the number of platform tracks and the number of pass-through tracks at station st , respectively. To ensure that each arrival that corresponds to passenger boarding/alighting to be assigned with a platform track, additional constraints need to be added, which are based on two kinds of decision variables. One is the binary variable p_e , for all $e \in E_{\text{ar}}$ with value 1 indicating that e needs a platform track. The other one is the binary variable $n_{e,e'}^p$ for any two events $e, e' \in E_{\text{ar}}$ such that $st_e = st_{e'}, tr_e \neq tr_{e'}$. $n_{e,e'}^p = 1$

indicates that train tr_e is occupying a platform track at station st_e at the moment that another train $tr_{e'}$ arrives at the same station. In the following, how to decide the values of p_e and $n_{e,e'}^p$ are explained successively.

If event $e \in E_{\text{ar}}$ is cancelled, it does not need a platform track:

$$p_e \leq 1 - c_e, \quad e \in E_{\text{ar}}. \quad (62)$$

Otherwise, e needs a platform track, if e corresponds to a true stop (i.e. $s_a = 0, c_e = 0, c_{e'} = 0, a = (e, e') \in A_{\text{station}}$, see Tables 3 and 4):

$$p_e \geq 1 - s_a - c_e - c_{e'}, \quad e \in E_{\text{ar}}, a = (e, e') \in A_{\text{station}}, \quad (63)$$

or e corresponds to an selected short-turning/OD turning (i.e. $\sum_{a \in A_{\text{turn}} \cup A_{\text{odturn}}, \text{tail}(a)=e} m_a = 1$):

$$p_e \geq \sum_{a \in A_{\text{turn}} \cup A_{\text{odturn}}, \text{tail}(a)=e} m_a, \quad e \in E_{\text{ar}}^{\text{turn}} \cup E_{\text{ar}}^{\text{odturn}}. \quad (64)$$

For arrival event e that is not relevant to any short-turn/OD turn activities, no platform track is needed by e , if e corresponds to a skipped stop or a non-stop (i.e. $c_e = 0, c_{e'} = 0, s_a = 1$, see Tables 3 and 4):

$$p_e \leq 1 - s_a, \quad e \in E_{\text{ar}} \setminus (E_{\text{ar}}^{\text{turn}} \cup E_{\text{ar}}^{\text{odturn}}), a = (e, e') \in A_{\text{station}}. \quad (65)$$

For an arrival event e that is relevant to short-turn activities, no platform track is needed by e , if e does not correspond to a true stop and no short-turn activities relevant to e are selected:

$$p_e \leq 1 - s_a + \sum_{a \in A_{\text{turn}}, \text{tail}(a)=e} m_a, \quad e \in E_{\text{ar}}^{\text{turn}}, a = (e, e') \in A_{\text{station}}. \quad (66)$$

Based on p_e and $n_{e,e'}$, we can determine $n_{e,e'}^p$, of which the value 1 indicating that train tr_e is occupying a platform track at the moment that another train $tr_{e'}$ arrives at the same station. $n_{e,e'}^p = 1$ happens only if $n_{e,e'} = 1$ and $p_e = 1$, which is formulated by

$$n_{e,e'}^p \leq n_{e,e'}, \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, \quad (67)$$

$$n_{e,e'}^p \leq p_e, \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, \quad (68)$$

$$n_{e,e'}^p \geq n_{e,e'} + p_e - 1, \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}. \quad (69)$$

When $e' \in E_{\text{ar}}$ needs a platform track (i.e. $p_{e'} = 1$), the constraint below ensures that e' is assigned with a platform track:

$$\sum_e n_{e,e'}^p \leq (N_{\text{st}} - 1)(1 - p_{e'}) + p_{e'}(N_{\text{st}}^p - 1), \quad e, e' \in E_{\text{ar}}, st_e = st_{e'}, st = st_{e'}, tr_e \neq tr_{e'}. \quad (70)$$

3.3. Objective

The proposed model is based on constraints (1)–(48) and (51)–(70), with the objective

$$\min \sum_{e \in E_{\text{ar}}} w_e^{\text{delay}} d_e + \sum_{e \in E_{\text{ar}}} w_e^{\text{cancel}} c_e + \sum_{a \in A_{\text{dwell}}} w_a^{\text{skip}} s_a - \sum_{a \in A_{\text{pass}}} w_a^{\text{add}} (1 - s_a). \quad (71)$$

This objective considers the potential impacts of different dispatching measures on passengers, which include the impacts of delaying and cancelling trains, the negative impact of skipping stops, and the positive impact of adding stops. Although the positive impact of skipping stops and the negative impact of adding stops are not directly included in the objective, they are actually accounted for by the first term of the objective. For example, a skipped stop can help a passenger who should have been delayed to arrive on time (i.e. zero delay) or to be delayed less, while an added stop can delay a passenger who should have arrived on time. The weight of each decision variable is passenger-dependent, which considers the influenced passengers and the impact on these passengers. Each weight is individually estimated, based on the data of the passengers' planned paths that are obtained by the schedule-based passenger assignment model of Zhu and Goverde (2018). In the following, we first introduce how to obtain passengers' planned paths and then elaborate how they will be used to estimate passenger-dependent weights.

The schedule-based passenger assignment model proposed by Zhu and Goverde (2018) is able to estimate passenger path choices when given a timetable and passenger information regarding the origins, the destinations and the arrival times at the origins. In Zhu and Goverde (2018), the path with the shortest generalized travel time is chosen for each passenger. Generalized travel time is the weighted travel time considering passenger's preferences on waiting time, in-vehicle time, transfer time and the number of transfers. A path is constituted by a series of time-ordered departure and arrival events corresponding to the trains that the passenger wishes to take. The departure (arrival) event that corresponds to the boarding

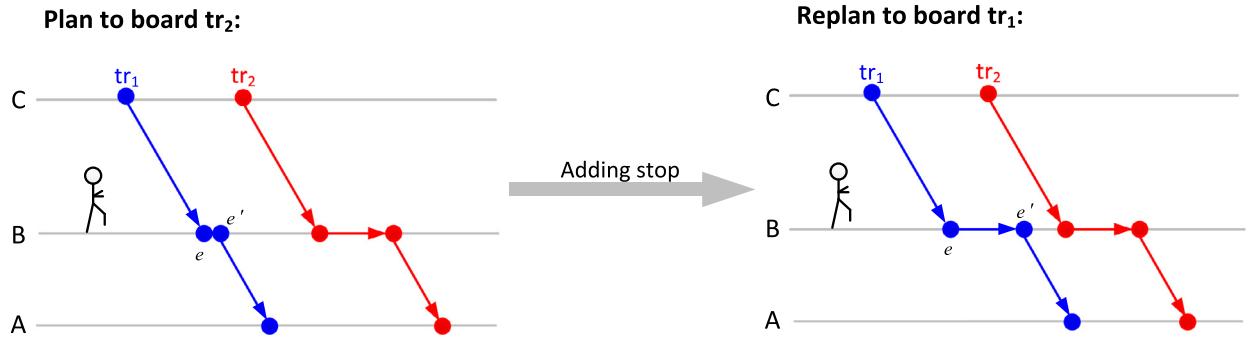


Fig. 14. Illustration of earlier boarding when adding a stop to $a = (e, e') \in A_{\text{pass}}$.

(alighting) of the passenger is indicated in the path. This means that from the path of a passenger, it is able to tell when and where the passenger will board or alight from which train. Also, the events that are in the path of a passenger but do not correspond the boarding/alighting of the passenger indicate that when the passenger will pass through which station by which train. In this paper, we input the planned timetable to the schedule-based passenger assignment model to obtain the planned path of each passenger, namely the path that a passenger wishes to take on normal days. If a disruption occurs, the planned path of a passenger could be influenced due to different dispatching decisions applied. For example, if a train skips a stop, then the passengers who plan to board or alight from the train at the stop will be affected, and thus have to reroute and arrive possibly with delay at their destinations. In that sense, the weight of a decision is constituted by two parts: (1) the affected passengers, and (2) the passenger delays due to the decision. How to estimate these two parts for each decision is elaborated in the following.

- When delaying an arrival event $e \in E_{\text{ar}}$, the affected passengers include (1) the passengers z_e^{alight} who plan to alight from train tr_e at station st_e ; and (2) the passengers z_e^{pass} who plan to pass through station st_e in train tr_e . The delay to each of these passengers is d_e minutes, where d_e is a decision variable representing the delay of event e . The weight of delaying an arrival event is:

$$w_e^{\text{delay}} = z_e^{\text{alight}} + z_e^{\text{pass}}, \quad e \in E_{\text{ar}}.$$

The resulting passenger delays due to delaying event $e \in E_{\text{ar}}$ is $w_e^{\text{delay}} d_e$.

- When cancelling an arrival event $e \in E_{\text{ar}}$, the affected passengers include: (1) the passengers z_e^{alight} who plan to alight from train tr_e at station st_e ; and (2) the passengers z_e^{pass} who plan to pass through station st_e in train tr_e . Because of the cancellation, these passengers cannot stick to their planned paths but have to reroute, and the delay due to the rerouting is assumed as α minutes for each of them. Thus, the weight of cancelling an arrival event is:

$$w_e^{\text{cancel}} = \alpha(z_e^{\text{alight}} + z_e^{\text{pass}}), \quad e \in E_{\text{ar}},$$

which represents the resulting passenger delays when e is cancelled (i.e. $c_e = 1$).

- When skipping a stop $a = (e, e') \in A_{\text{dwell}}$ where $e \in E_{\text{ar}}$, $e' \in E_{\text{de}}$, the affected passengers include (1) the passengers z_e^{alight} who plan to alight from train tr_e at station st_e ; and (2) the passengers $z_{e'}^{\text{board}}$ who plan to board train $tr_{e'}$ at station $st_{e'}$. Here, tr_e and $tr_{e'}$ must be the same train and st_e and $st_{e'}$ must be the same station, since $(e, e') \in A_{\text{dwell}}$. Because of the skipped stop, these passengers cannot stick to their planned paths but have to reroute, and the delay due to the rerouting is assumed as β minutes for each of them. Thus, the negative impact of skipping a stop is:

$$w_a^{\text{skip}} = \beta(z_e^{\text{alight}} + z_{e'}^{\text{board}}), \quad a = (e, e') \in A_{\text{dwell}},$$

which represents the resulting passenger delays when $a \in A_{\text{dwell}}$ is skipped (i.e. $s_a = 1$).

- When adding a stop to $a = (e, e') \in A_{\text{pass}}$ where $e \in E_{\text{ar}}$, $e' \in E_{\text{de}}$, passengers can benefit from the added stop by earlier boarding or earlier alighting. Fig. 14 shows an example of a passenger boarding earlier due to an added stop. In Fig. 14, a passenger who arrives at station B earlier than time e' and plans to board train tr_2 that departs later than tr_1 , may board train tr_1 instead if a stop is added to tr_1 at the station. We calculate z_a^{board} as the number of passengers who can benefit from such earlier boarding due to an added stop to $a \in A_{\text{pass}}$. Fig. 15 shows an example of a passenger alighting earlier due to an added stop. In Fig. 15, a passenger plans to pass through station B by train tr_1 and then transfer to train tr_2 at station C to reach his/her destination station B. However, if train tr_1 is added with a stop at station B, this passenger will alight from tr_1 at station B. We calculate z_a^{off} as the number of such passengers who can benefit from earlier alighting due to an added stop to $a \in A_{\text{pass}}$. The saved time for each passenger who benefits from earlier boarding/alighting is assumed as γ minutes. Thus, the positive impact of adding a stop is

$$w_a^{\text{add}} = \gamma(z_a^{\text{board}} + z_a^{\text{off}}), \quad a \in A_{\text{pass}},$$

which represents the resulting passenger saved times when $a \in A_{\text{pass}}$ is added with a stop (i.e. $s_a = 0$).

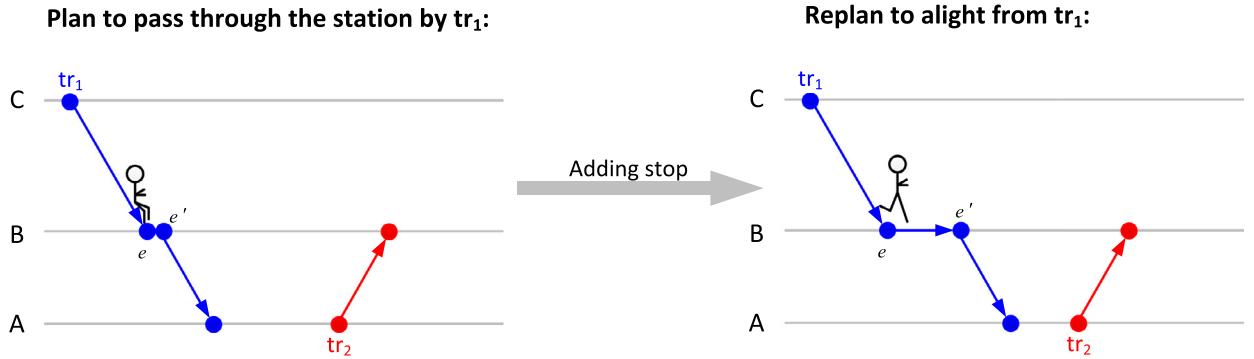


Fig. 15. Illustration of earlier alighting when adding a stop to $a = (e, e') \in A_{\text{pass}}$.

Note that the values of α , β and γ depend on many factors, which makes it difficult to estimate them. For example, α could be affected by the disruption locations, the disruption durations, the travel times of re-routing paths, the frequencies of external alternatives (e.g. shuttle buses), etc. Therefore, we have to make some assumptions here to simplify the estimation of these values in our case study. The delay of a passenger whose planned path is cancelled and the delay of a passenger whose planned boarding/alighting is skipped, are both assumed to be the disruption length (i.e. $\alpha = \beta = t_{\text{end}} - t_{\text{start}}$). In the case study, we applied the rescheduling model in a limited network that does not contain the areas that are far beyond the disruption sections, while adding stops could lead to delay propagation to the considered network beyond and further increase the passenger inconvenience there. Thus, we only assume 1 min earliness (i.e. $\gamma = 1$) to each passenger who can benefit from adding stops, in order to offset the underestimation of the negative effects of adding stops.

The formulas of weights and the passenger groups and parameters that are necessary to determine the weights can be found in Appendix B.

4. Case study

In this section, two experiments are carried out. In the first experiment, the proposed model is performed for 408 disruption scenarios, in order to explore (1) the effect of the recovery duration setting; and (2) the effect of the setting for the maximum allowed delay per event, when applying flexible stopping or applying flexible short-turning. In this experiment, the disruption duration is fixed for each scenario. However in the second experiment, different disruption durations are tested to investigate the influence of the disruption duration on the optimal rescheduling solution. In the end, the computation efficiency of the proposed model is analysed. All computational scenarios were solved to optimality (with a gap less than 0.00001%) by using the optimization software GUROBI release 7.0.1 on a desktop with Intel Xeon CPU E5-1620 v3 at 3.50 GHz and 16 GB RAM.

The network considered is shown in Fig. 16 where six train lines operate every 30 min in either upstream or downstream direction. The black arrows indicate the upstream direction, which is the clockwise direction starting from Roermond (Rm) and back to itself; while the downstream direction is the anticlockwise one. The rolling stock circulations are only taken into account for the trains of which the terminal stations are located in the considered network. Such terminal stations are indicated in Fig. 16, as well as the type of each train line.

The schematic track layout in the considered network is shown in Fig. 17 where stations Tg, Rv and Sm are located on single-track railway lines while the others are located on double-track railway lines. Due to the infrastructure layouts, some stations are unable for short-turning the trains that operate in a specific direction or even both directions: (1) stations Hze, Hmbv, Hmh and Hmbh are unable for short-turning the trains operating in both directions; (2) stations Mz and Gp are unable for short-turning the trains operating in upstream direction; while the others are able for short-turning the trains operating in both directions.

The parameter settings are detailed here: the minimum short-turn or OD turn duration is 300 s; the minimum arrival/departure headway on open-track section is 180 s; the minimum headway of following trains at a station is 180 s; and the minimum dwell time at a station is 30 s. The maximum percentage allowed to running time extensions is set to 67%, considering that added stops increase the running times. Note that excessive running time extensions are unlikely to happen when no stops are added to a train, because that will lead to train delays that are not preferred by the model. The acceleration (deceleration) time needed for a train that serves train line 800, 1900 or 3500 is set to 62 s (34 s); the acceleration (deceleration) time needed for a train that serves train line 6400 or 9600 is set to 39 s (28 s); and the acceleration (deceleration) time needed for a train that serves train line 32200 is set to 74 s (26 s).

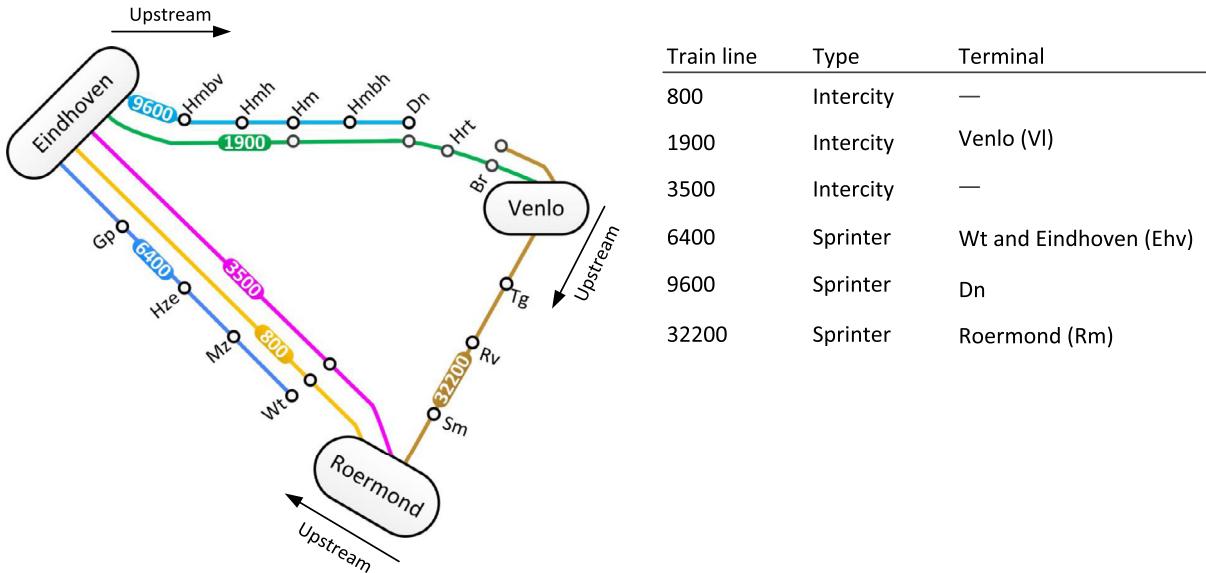


Fig. 16. The train lines operating in the considered network.

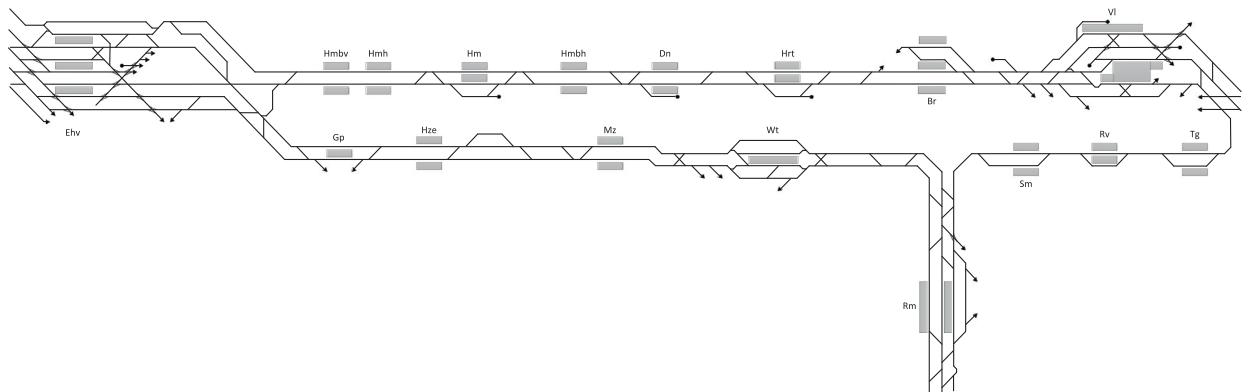


Fig. 17. The schematic track layout in the considered network.

4.1. Experiment 1: disruptions with fixed duration

In this experiment, we construct 408 disruption scenarios that differ in (1) the disrupted section; (2) whether flexible stopping is applied; (3) whether flexible short-turning is applied; and (4) the maximum allowed delay per event. The characteristics of these disruptions scenarios are shown in Table 5.

In each scenario, the disrupted section is assumed with a complete track blockage that starts at 8:00 and ends at 11:00, thus the passengers who start travelling during the period of 8:00–11:00 can be affected by the disruption. The passengers who start travelling before 8:00 or after 11:00 could also be affected. For example, a passenger who boards a train at 7:45 could be forced to alight from the train if the train is short-turned at a station during the disruption, or a passenger who plans to board a train at 11:15 may be unable to board the train as planned considering that the train could be delayed during the recovery period. Thus in each scenario, the passengers who start travelling during the period of 7:00–12:00 are all considered by using their planned paths to estimate the passenger-dependent weight of each decision in the objective. In other words, we consider the rescheduling impact on these passengers. The method of estimating passenger-dependent weights based on passengers' planned paths is introduced in Section 3.3. Recall that the planned path of a passenger is a series of time-ordered departure/arrival events that correspond to the train(s) the passenger plans to take. The dynamic passenger assignment proposed by Zhu and Goverde (2018) is adopted to estimate the planned path of each passenger, which uses the planned timetable and the information of each passenger (i.e. the origin, the destination, the time he/she arrives at the origin) as the input. This passenger information is obtained from a full-day OD matrix of the whole Dutch railways by applying the hourly distribution considering the time period concerned. The used full-day OD matrix and the hourly distribution are the same ones as adopted in Ghaemi et al. (2018b).

Table 5

Disruption scenarios for experiment 1.

Scenario No.	Disrupted section	Stopping	Short-turning	Maximum allowed delay per event [min]
1-6	Rm - Wt	Fixed	Fixed	5,10,15,20,25 and 30
7-12	Rm - Wt	Flexible	Fixed	5,10,15,20,25 and 30
13-18	Rm - Wt	Fixed	Flexible	5,10,15,20,25 and 30
19-24	Rm - Wt	Flexible	Flexible	5,10,15,20,25 and 30
25-30	Wt - Mz	Fixed	Fixed	5,10,15,20,25 and 30
31-36	Wt - Mz	Flexible	Fixed	5,10,15,20,25 and 30
37-42	Wt - Mz	Fixed	Flexible	5,10,15,20,25 and 30
43-48	Wt - Mz	Flexible	Flexible	5,10,15,20,25 and 30
:	:	:	:	:
385-390	Sm - Rm	Fixed	Fixed	5,10,15,20,25 and 30
391-396	Sm - Rm	Flexible	Fixed	5,10,15,20,25 and 30
397-402	Sm - Rm	Fixed	Flexible	5,10,15,20,25 and 30
403-408	Sm - Rm	Flexible	Flexible	5,10,15,20,25 and 30

For each scenario, the rescheduled timetable is generated and the analysis of the result is described as follows.

4.1.1. The effect of recovery duration

To avoid the disruption affecting the timetable for the whole day, we set the recovery duration R to ensure that trains run as planned again after R minutes of the disruption ending time. In each disruption scenario, R is set with the same value as the maximum allowed delay per event.

The value of R affects solution feasibility. When it is set to 5 or 10 min in the scenarios where section Rm-Wt or Dn-Hrt is disrupted, no solutions can be found unless increasing R to 15 min. In other scenarios where the disrupted section is neither Rm-Wt nor Dn-Hrt, optimal solutions can be obtained even though R is set to 5 min. This indicates that the location of the disruption affects the required recovery duration. Thus a proper setting of R is necessary. Recall that we set R to the same value as the maximum allowed delay per event in each scenario. This setting of R ensures optimal solutions for 392 of the 408 scenarios.

The value of R affects the number of cancelled train services and the total train delay. When it is set to a smaller value, more train services are cancelled but with less train delays. This indicates that shortening the recovery duration aggravates the consequence during the disruption but mitigates the post-disruption consequence due to less delay propagation. In that sense, the value of R defines the trade-off between the consequence during the disruption and the post-disruption consequence. This setting deserves attention particularly when focusing on a large-scale network where a longer recovery duration may worsen the problem of delay propagation across the network. Optimizing this parameter is out of the scope for this paper, but is interesting to be investigated in future research.

4.1.2. The effect of maximum allowed delay per event, flexible stopping or flexible short-turning

To explore the effect of the maximum allowed delay per event, flexible stopping or flexible short-turning, three performance indicators are used: the total number of cancelled train services, total train arrival delay, and total passenger delay. The total number of cancelled train services is the number of the train services that are cancelled. A train service represents the running of a train between two adjacent stations. Total train arrival delay is the sum of arrival delays of the train services that are not cancelled. Total passenger delay refers to the objective value. These three indicators are calculated for the rescheduled timetable of each disruption scenario. The minimal, average, and maximal values of these indicators over the scenarios that have the same settings about stopping, short-turning and maximum allowed delay per event are calculated and shown in Table 6.

Setting the maximum allowed delay with a larger value results in less cancelled train services and less passenger delays, but sometimes more train delays. This is because when more train services are kept instead of cancelled, more conflicts could emerge between them, which are resolved at the expense of introducing more train delays.

When applying flexible stopping or flexible short-turning, the average value of total number of cancelled services decreases, while the corresponding maximal or minimal value remains. This is because applying flexible stopping or flexible short-turning helps to reduce the number of cancelled train services in most scenarios, but not in a few scenarios where the disruption occurs in the area where only one train line operates (e.g. section Tg-Rv) or the disrupted section is Hmbv-Hmh or Hmh-Hm and the maximum allowed delay per event is set to 5 min. This indicates that flexible stopping or flexible short-turning is more likely to bring benefits in the situations that (1) the operation frequency is relatively high; and (2) a train departure/arrival is allowed to be delayed for a relatively long time. This is because such a situation provides a wider search space for the flexible dispatching measures to explore.

When applying flexible stopping and flexible short-turning, the average total passenger delay is the smallest, compared to the one when either or neither of flexible stopping and flexible short-turning is applied. To have a deeper insight on the impact of these two measures, Table 7 shows the details of the rescheduled timetables by applying flexible stopping and flexible short-turning with the maximum allowed delay per event set to 30 min.

Table 6

Results on total number of cancelled services, total train arrival delay, and total passenger delay in experiment 1.

Stopping	Short-turning	Maximum allowed delay per event	Total number of cancelled train services			Total train arrival delay [min]			Total passenger delay (i.e. the objective value) [min]		
			Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Fixed	Fixed	5	12	60	138	0	90	367	72,321	456,361	1,141,471
		10	12	58	138	0	114	367	72,321	439,018	1,141,471
		15	12	56	135	0	145	402	72,321	424,651	1,080,987
		20	12	51	130	0	241	999	72,321	398,095	1,025,639
		25	10	49	130	0	277	1,075	72,321	389,929	1,022,740
		30	10	49	122	0	274	671	72,321	386,815	1,022,740
Flexible	Fixed	5	12	59	138	0	107	530	72,321	452,857	1,141,471
		10	12	57	138	0	147	530	72,321	435,240	1,141,471
		15	12	54	135	0	211	658	72,321	411,151	1,021,524
		20	12	50	130	0	251	775	72,321	390,513	1,008,233
		25	10	49	130	0	283	816	72,321	385,841	1,004,4318
		30	10	48	122	0	295	655	72,321	382,415	989,612
Fixed	Flexible	5	12	59	138	0	90	367	72,321	4,446,90	1141471
		10	12	56	138	0	127	367	72,321	427,536	1,137,755
		15	12	53	135	0	157	431	72,321	408,345	1,051,481
		20	12	50	130	0	244	1,010	72,321	390,583	1,017,767
		25	10	48	130	6	291	1,134	72,321	383,980	1,004,484
		30	10	48	122	6	287	749	72,321	380,866	1,004,484
Flexible	Flexible	5	12	58	138	0	108	530	72,321	441,066	1,141,471
		10	12	55	138	0	158	530	72,321	423,981	1,137,755
		15	12	51	135	0	219	721	72,321	398,397	1,004,790
		20	12	49	130	0	251	775	72,321	386,729	1,000,334
		25	10	48	130	6	297	875	72,321	379,970	984,648
		30	10	47	122	6	297	720	72,321	376,544	971,251

Table 7

The results of applying flexible stopping and flexible short-turning with the maximum allowed delay per event set to 30 min.

Disrupted section	# Skipped stops	# Added stops	Total number of cancelled train services	Total train arrival delay [min]	Total passenger delay [min]
Rm-Wt	6	12	20	240	128,175
Wt-Mz	1	5	48	308	195,580
Mz-Hze	1	5	95	504	528,350
Hze-Gp	0	3	95	465	513,271
Gp-Ehv	0	6	122	633	694,103
Ehv-Hmbv	5	1	76	704	971,251
Hmbv-Hmh	6	1	70	720	958,273
Hmh-Hm	7	1	70	510	966,616
Hm-Hmbh	6	0	44	22	383,259
Hmbh-Dn	6	0	44	278	398,090
Dn-Hrt	1	0	12	69	98,366
Hrt-Br	0	1	10	96	77,066
Br-Vl	1	1	20	330	86,132
Vl-Tg	3	0	24	58	140,020
Tg-Rv	3	0	12	58	75,220
Rv-Sm	0	0	12	52	72,321
Sm-Rm	0	0	20	6	115,154

Table 7 indicates that when Rm-Wt is disrupted, the number of added stops is the largest. The disruption timetable for this case is shown in Fig. 18, where the dotted (dashed) lines represent the planned services that are delayed (cancelled) in the disruption timetable; the solid lines represent the services that are scheduled in the disruption timetable; and each red triangle (circle) represents an added (skipped) stop. Stops are added to five trains from line IC3500 (in pink color) at station Mz. These trains additionally stop at station Mz to wait for the trains from line SPR6400 (in blue color) to leave from station Wt, which can only provide platform tracks for two trains at the same time. To respect the minimum short-turn duration, six trains from line SPR6400 (in blue color) depart from station Wt with delays, and these delays continue to station Gp. As such, six trains from line IC800 (in yellow color) and one train from line IC3500 (in pink color) have to be delayed at station Gp to respect the minimum departure headway. These trains are all added with stops at station Gp.

Table 7 indicates that when Gp-Ehv is disrupted, the number of cancelled train services is the largest. The disruption timetable for this case is shown in Fig. 19. Although only section Gp-Ehv is disrupted, lots of train services between stations Wt and Gp are cancelled. Recall that due to the infrastructure layouts, stations Gp and Mz are unable for short-turning the trains operating in upstream direction and station Hze is unable for short-turning the trains operating in both directions.

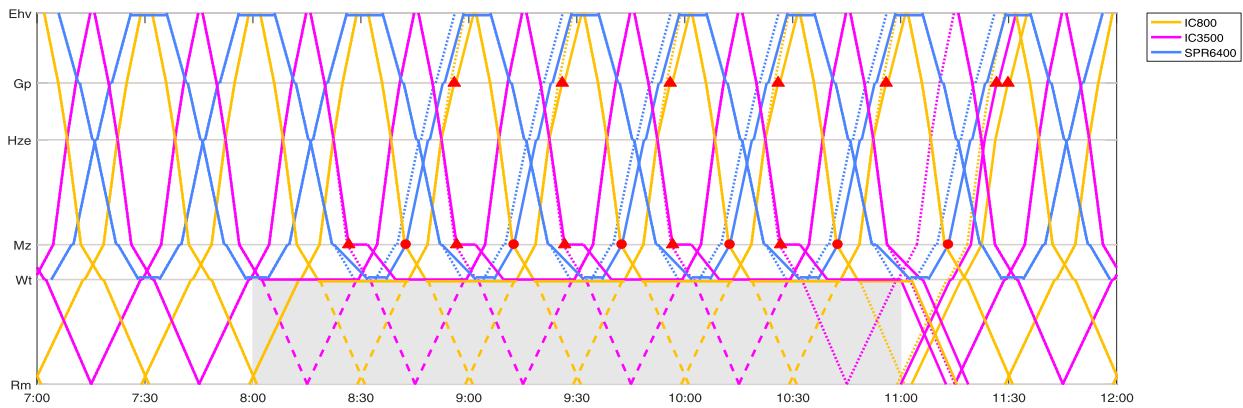


Fig. 18. The disruption timetable with flexible stopping and flexible short-turning and maximum allowed delay per event set to 30 min for disrupted section Rm-Wt. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

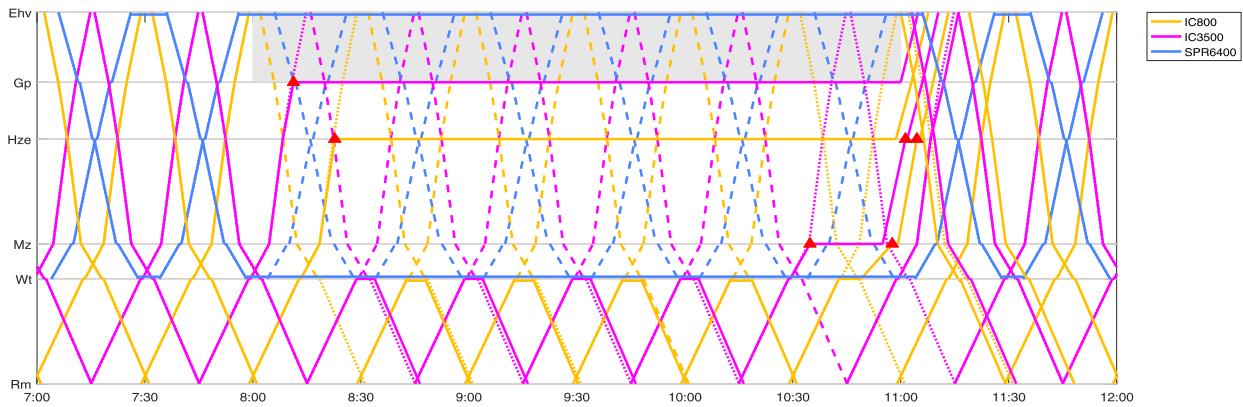


Fig. 19. The disruption timetable with flexible stopping and flexible short-turning and maximum allowed delay per event set to 30 min for disrupted section Gp-Ehv. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

As such, an upstream train from line IC3500 (in pink color) cannot be short-turned at station Gp to serve the opposite operation, thus can only dwell at station Gp until the disruption ends. This is why lots of train services from line IC3500 (in pink color) are cancelled. The same reason explains the cancelled train services from line IC800 (in yellow color) or SPR6400 (in blue color). When a train has to stop at a station where it originally passes through, the required extra acceleration/deceleration time may cause an infeasible solution, if the resulting extension on the train running times is larger than allowed. For example, the upstream train from line IC3500 (in pink color) has to stop at station Gp, and thus extra deceleration time should be added to the train when running from station Hze to station Gp. The required deceleration time is 34 s that accounts for 22.7% extension on the scheduled running time of 150 s for the train from station Hze to station Gp. Recall that our model avoids overlong running in an open-track section by constraint (15) where a maximum percentage λ allowed to a running time extension is imposed. If λ is set to 20%, it would cause infeasibility of the adjusted timetable in the example. However in our case study, no infeasible rescheduling solutions were found due to adding stops, because we set λ to 67%, which is large enough to avoid the infeasibility caused by adding stops (to short station distances) in the disruption scenarios considered. We can take a relatively high percentage because when no stops are added to a train, excessive running time extensions are unlikely to happen, because that will lead to train delays that are not preferred by the model.

Table 7 indicates that when Ehv-Hmbv is disrupted, the total passenger delay is the largest. The disruption timetable for this case is shown in Fig. 20. Compared to Fig. 19, less train services are cancelled in Fig. 20; however the services that are cancelled here correspond to more passenger demand, thus cancelling them results in more passenger delays. Here, five short-turnings of the IC1900 occur at station Hm, while one short-turning occurs at an earlier station Dn. This early short-turning is due to the required recovery duration, which is 30 min in this case. If this short-turning does not occur at station Dn but at station Hm instead, more delays will happen to trains, which cannot be completely absorbed within the recovery duration.

From experiment 1 it is concluded that (1) although shortening the recovery duration is at the expense of more train services being cancelled, it can mitigate the problem of delay propagation; (2) better solutions can be found when setting the maximum allowed delay per event to a larger value; and (3) applying flexible stopping and flexible short-turning helps

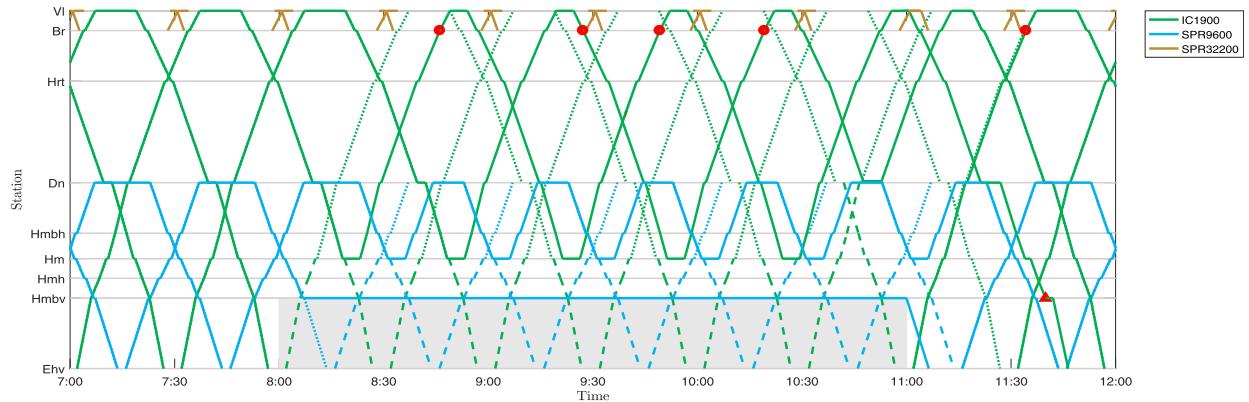


Fig. 20. The disruption timetable with flexible stopping and flexible short-turning and setting allowed delay per event set to 30 min for disrupted section Ehv-Hmbv.

to reduce the number of cancelled train services and the total passenger delay, especially when the disruption occurs in the area where the train operation frequency is relatively high.

4.2. Experiment 2: disruptions with different durations

To explore the influence of the disruption duration on the optimal rescheduling solution, we construct 85 scenarios that differ in (1) the disrupted section and (2) the disruption ending time. These scenarios are shown in [Table 8](#), where the maximum allowed delay per event D and the required recovery duration R are both set to 30 min.

For each scenario, the rescheduled timetable is generated and three performance indicators are calculated: the total passenger delay (i.e. the objective value), the number of cancelled train services, and the total train arrival delay, of which the values are shown in [Figs. 21–23](#), respectively. The y-axis represents the indicator value, the x-axis represents the disrupted section, and the legend indicates the disruption ending time corresponding to each point.

[Fig. 21](#) indicates that in each disrupted section, the total passenger delay increases gradually with the extension of disruption duration. From [Figs. 22](#) and [23](#), we found that similar patterns exist among disruption ends with an interval of 30 min apart, indicated with lines of the same colors. The green lines correspond to the scenarios where the disruption ends at 10:00, 10:30 or 11:00, while the pink lines correspond to the scenarios where the disruption ends at 10:15 or 10:45. Recall that in our case study, the trains from each train line operate every 30 min in each direction. This reveals that the optimal rescheduling solution is sensitive to the disruption duration, but keeps some regularities if the disruption duration extends periodically. In the following, an example is given to show how the optimal rescheduling solution changes when the disruption duration extends with different time lengths.

In [Fig. 24](#), three rescheduled timetables are shown for the scenarios where the disruptions all start at 8:00, but end at 10:30, 10:45 and 11:00, respectively. The disruption ending time is highlighted on the bottom of each timetable and three dotted black rectangles are used to highlight the parts that are different in these timetables. When the disruption ends at 10:30, a train from line IC3500 (in pink color) is delayed at station Wt; while when the disruption ends at 10:45, the train is suggested to short-turn at station Wt where a following train from line IC3500 (in pink color) is delayed instead.

Table 8
Disruption scenarios for experiment 2.

Scenario No.	Disrupted section	Start time	End time	Stopping	Short-turning	D [min]	R [min]
1	Rm - Wt	8:00	10:00	Flexible	Flexible	30	30
2	Rm - Wt	8:00	10:15	Flexible	Flexible	30	30
3	Rm - Wt	8:00	10:30	Flexible	Flexible	30	30
4	Rm - Wt	8:00	10:45	Flexible	Flexible	30	30
5	Rm - Wt	8:00	11:00	Flexible	Flexible	30	30
6	Wt - Mz	8:00	10:00	Flexible	Flexible	30	30
7	Wt - Mz	8:00	10:15	Flexible	Flexible	30	30
8	Wt - Mz	8:00	10:30	Flexible	Flexible	30	30
9	Wt - Mz	8:00	10:45	Flexible	Flexible	30	30
10	Wt - Mz	8:00	11:00	Flexible	Flexible	30	30
:	:	:	:	:	:	:	:
81	Sm - Rm	8:00	10:00	Flexible	Flexible	30	30
82	Sm - Rm	8:00	10:15	Flexible	Flexible	30	30
83	Sm - Rm	8:00	10:30	Flexible	Flexible	30	30
84	Sm - Rm	8:00	10:45	Flexible	Flexible	30	30
85	Sm - Rm	8:00	11:00	Flexible	Flexible	30	30

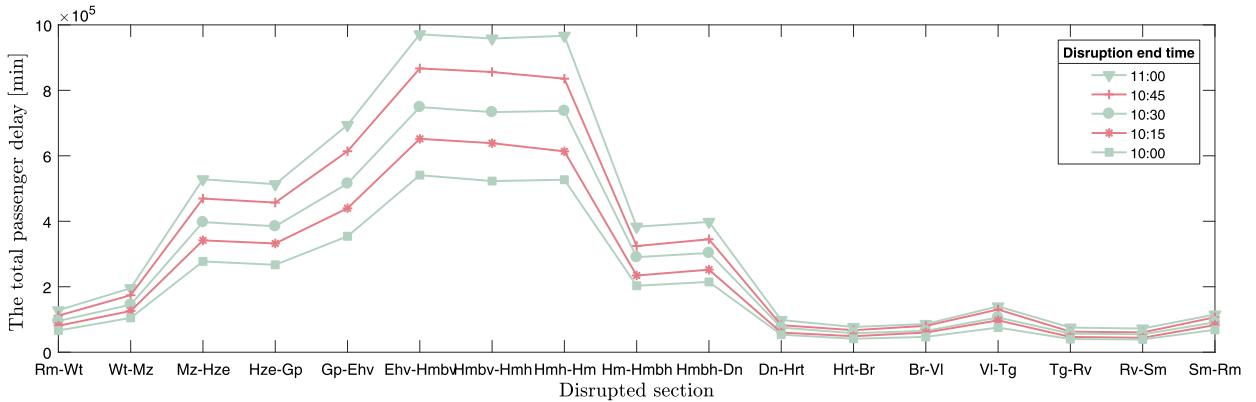


Fig. 21. The total passenger delay in each scenario of Table 8.

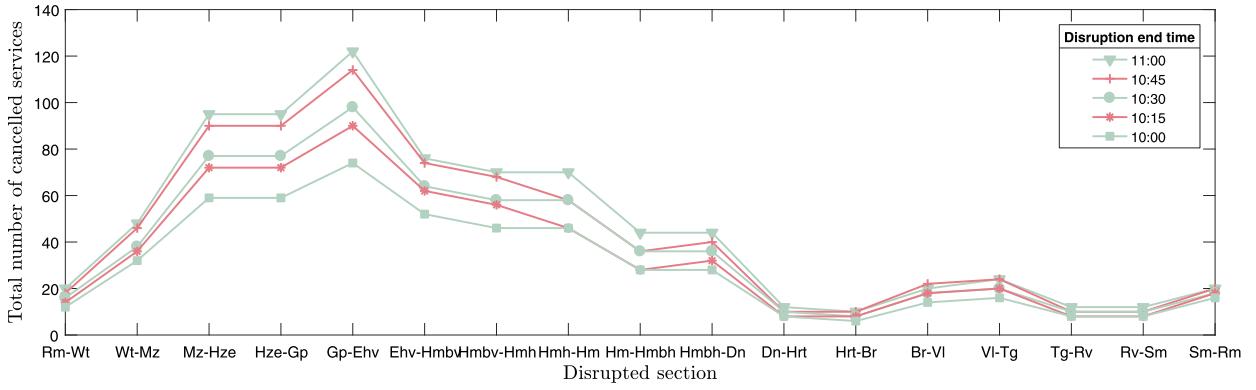


Fig. 22. The number of cancelled train services in each scenario of Table 8. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

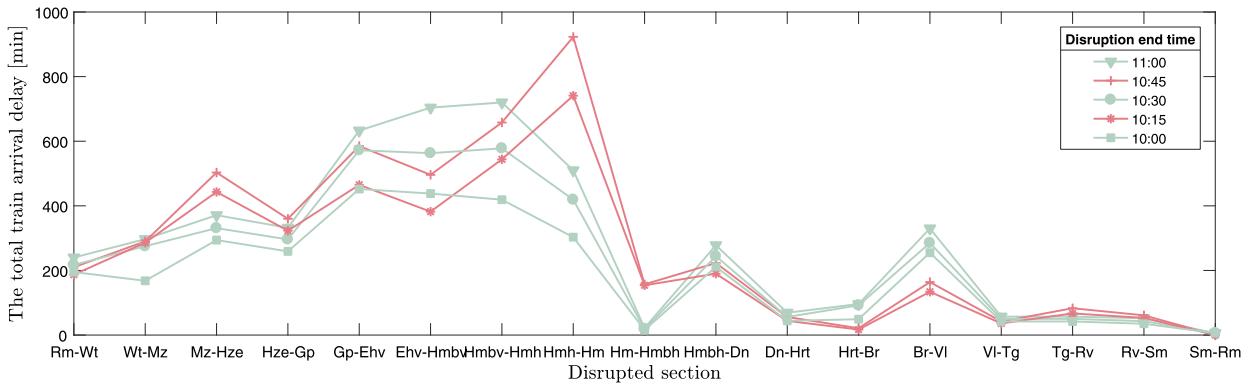


Fig. 23. The total train arrival delay in each scenario of Table 8. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

When the disruption ends at 11:00, more delays are introduced to trains, but the short-turning patterns and the stopping patterns both look similar to the case where the disruption ends at 10:30. This indicates that the optimal rescheduling solution is sensitive to the disruption ending time that affects the decisions of short-turning or delaying the last trains that approach the disrupted section before the disruption ends and these decisions will further affect the stopping patterns of trains during the recovery period. However, some regularities can be kept in the rescheduled timetables corresponding to the periodic pattern.

In real life, the disruption ending time is uncertain, which means that the first predicted ending time may be extended to another new one that could also be extended further (Zilko et al., 2016). Under these circumstances, the rescheduled timetable has to be updated every time a new disruption ending time is renewed. A direct solution to this problem is to apply the model at the time when the disruption ending time is renewed, where the current time (the renewed ending time) is regarded as the disruption starting (ending) time and the train arrivals/departures that have already been realized

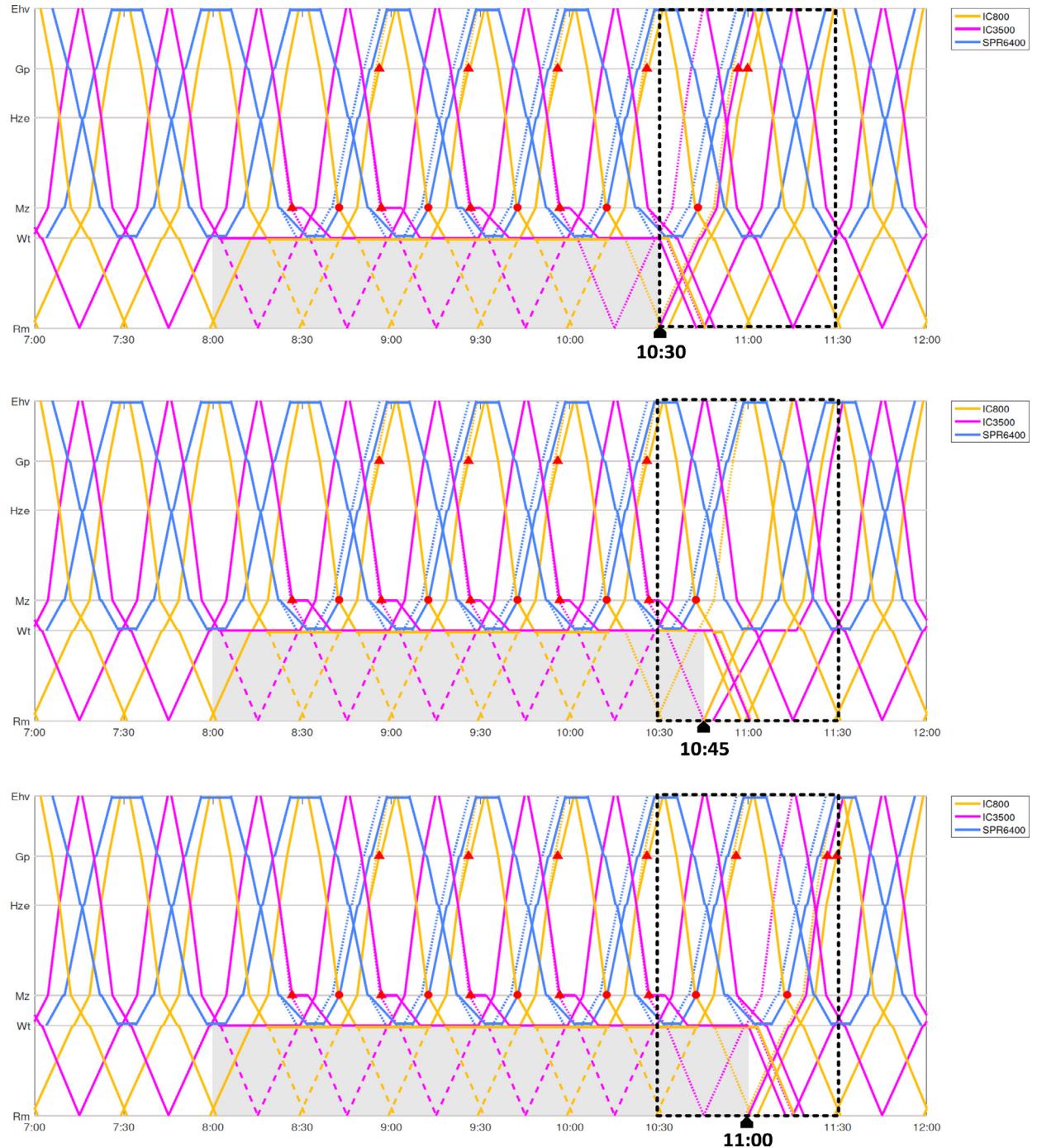


Fig. 24. The rescheduled timetables of three scenarios that only differ in the disruption ending times. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

are respected with the previous rescheduled timetable as the reference. In this way, a rescheduled timetable can be obtained for the extended disruption. Including the uncertainty of disruption duration during the rescheduling helps to generate a robust solution. This is out of scope of this paper but is interesting to be investigated further.

4.3. Computation efficiency analysis

Among the 408 scenarios of experiment 1 (see Table 5), only 7 take more than 15 s (but less than 80 s). These scenarios are the ones where both flexible stopping and flexible short-turning are applied and the maximum event per delay is set

to 30 min. Fig. 25 illustrates the impact of dispatching measures and parameter settings on computation time, based on the results of the scenarios in Table 5 where the disruptions have the same duration (3 h). Here, each circle represents the average computation time over the scenarios that only differ in the disrupted sections. The average computation time grows with the increase of maximum allowed delay per event. With the same setting of maximum allowed delay per event, longer computation times are needed when more flexible dispatching measures are applied. This is because more binary variables about stopping (short-turning) decisions are needed when applying flexible stopping (short-turning), thus increasing the computation complexity. Compared to applying flexible short-turning, more binary variables are needed when applying flexible stopping, which is why the average computation times due to flexible stopping and fixed short-turning are longer than the ones due to fixed stopping and flexible short-turning.

Fig. 26 shows the impact of disruption duration on computation time, based on the results of the scenarios in Table 8. Recall that in these scenarios, flexible stopping and flexible short-turning are both applied, and the maximum event per

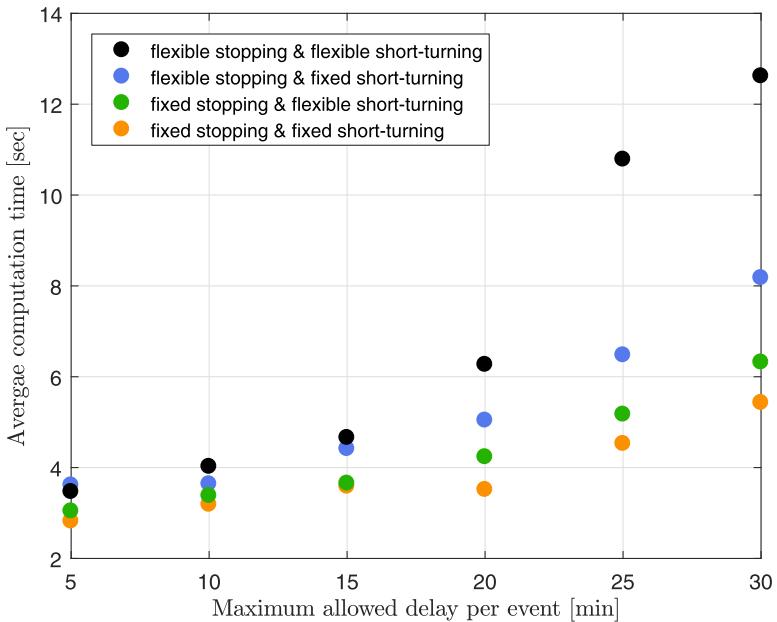


Fig. 25. The average computation times over the scenarios that use the same dispatching measures and the same setting of maximum allowed delay per event.

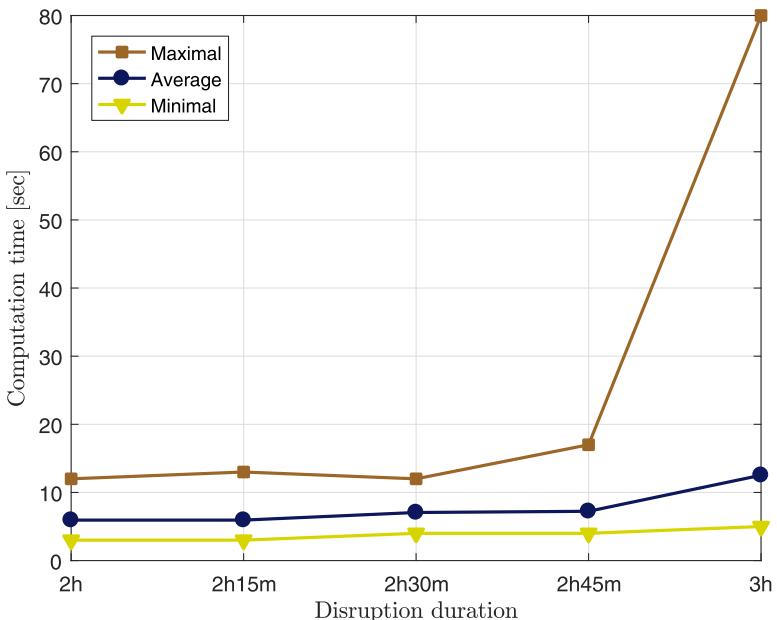


Fig. 26. The maximal, average, and minimal computation times over the scenarios that have the same disruption durations.

delay and the required recovery duration are both set to 30 min. In Fig. 26, each square, circle and triangle represent the maximal, average, and minimal computation time over the scenarios that only differ in the disrupted sections, respectively. The average and minimal computation time both grow gradually with the extension of disruption duration; whereas a steep growth is observed for the maximal computation time when the disruption duration extends from 2 h and 45 min to 3 h. Nevertheless, the maximal value of the computation time is 80 s, which is still acceptable in practice.

In the Dutch railways, the average disruption duration was 2 h and 40 min over the time period from January, 2011 to September 14, 2018 (data source: www.rijdendetreinen.nl). Once a disruption occurs, it is expected to be handled as soon as possible. For an up to three-hour disruption, the proposed model is able to generate an optimal rescheduling solution in an average of 13 s approximately, and thus can be applied for real time dispatching.

5. Conclusions and future directions

In this paper, an MILP model is proposed for rescheduling a timetable during railway disruptions, where flexible stopping and flexible short-turning are innovatively integrated with delaying, cancelling and reordering. The deceleration and acceleration times are considered when changing the stopping patterns, and each train that corresponds to passenger boarding/alighting at a station is ensured with a platform track. To make the disruption timetable passenger-friendly, each decision in the objective is assigned with an individual weight that is estimated from time-dependent passenger demand. In the case study, hundreds of disruption scenarios are established on a subnetwork of the Dutch railways. By the proposed model, the optimal rescheduling solutions to these scenarios were generated mostly within 13 s, and the worst case cost no longer than 80 s. The results indicate that flexible stopping and flexible short-turning are more likely to work in the situations where the operation frequency is relatively high and trains are allowed to be delayed with a relatively long time, because such situations provide a wider search space for the flexible dispatching measures to explore. It is found that applying flexible stopping and flexible short-turning results in less passenger delays, compared to applying either or neither of them. Moreover, shortening the recovery duration is good for mitigating the post-disruption consequence by less delay propagation, but is at the expense of more cancelled train services during the disruption. It will be interesting to explore how to make the trade-off between the consequence during the disruption and the post-disruption consequence, particularly when focusing on a large-scale network where a longer recovery duration may worsen the problem of delay propagation across the network. Also, it is found that the optimal rescheduling solution is sensitive to the disruption duration, but keeps some regularities when the disruption duration extends periodically.

In this paper, passenger demand is handled in a static way. In other words, the dynamic interaction between passengers and the disruption timetable is neglected. To consider such dynamic interaction, one way is to embed the timetable rescheduling model and the passenger assignment model in an iterative framework where in each iteration a disruption timetable is generated and the resulting passenger inconvenience is evaluated and then included to the rescheduling model in the next iteration as feedback from the passengers. Another way is to consider passenger reactions towards the disruption timetable during the rescheduling process (i.e. integrating passenger routing and timetable rescheduling in one single model). We will further explore both ways by considering the trade-off between the solution quality and the computational efficiency in future work. In real life, the duration of a disruption is uncertain, thus another future direction is extending the model to deal with uncertain disruption duration.

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Appendix A. Sets and parameters

Table 9
Sets.

Notation	Description
E_{de}	Set of departure and pass-through departure events
E_{ar}	Set of arrival and pass-through arrival events
E_{ar}^{turn}	The subset of E_{ar} , which includes all tails of activities in A_{turn} : $E_{ar}^{turn} = \bigcup_{a \in A_{turn}} \{tail(a)\}$
E_{de}^{turn}	The subset of E_{de} , which includes all heads of activities in A_{turn} : $E_{de}^{turn} = \bigcup_{a \in A_{turn}} \{head(a)\}$
E_{ar}^{odturn}	The subset of E_{ar} , which includes all tails of activities in A_{odturn} : $E_{ar}^{odturn} = \bigcup_{a \in A_{odturn}} \{tail(a)\}$
E_{de}^{odturn}	The subset of E_{de} , which includes all heads of activities in A_{odturn} : $E_{de}^{odturn} = \bigcup_{a \in A_{odturn}} \{head(a)\}$
$E^{N\Delta delay}$	Set of events that are not given the upper limit on their delays
TL	Set of train lines
TL_{dis}	Set of train lines that are affected by the disruption: $TL_{dis} \subseteq TL$
TR_{turn}	Set of trains that correspond to the events contained in $E_{ar}^{turn} \cup E_{de}^{turn}$

(continued on next page)

Table 9 (continued)

Notation	Description
A_{run}	Set of running activities
A_{dwell}	Set of dwell activities
A_{pass}	Set of pass-through activities
A_{station}	Set of station activities: $A_{\text{station}} = A_{\text{dwell}} \cup A_{\text{pass}}$
$A_{\text{arr}}^{\text{head}}$	Set of arrival headway activities for following trains
$A_{\text{head}}^{\text{de}}$	Set of departure headway activities for following trains
$A_{\text{arr}, \text{de}}^{\text{head}}$	Set of arrival-departure headway activities for crossing trains and trains operating on single-track railways
$A_{\text{de}, \text{ar}}^{\text{head}}$	Set of departure-arrival headway activities for crossing trains and trains using the same track at a station
$A_{\text{turn}}^{tl, dr}$	Set of short-turn activities for trains serving train line tl and operating in direction dr , $A_{\text{turn}}^{tl, dr} \subset A_{\text{turn}}$
A_{turn}	Set of short-turn activities
A_{odturn}	Set of OD turn activities
$A_{\text{odturn}}^{\text{plan}}$	Set of planned OD turn activities: $A_{\text{odturn}}^{\text{plan}} \subset A_{\text{odturn}}$
$S_{\text{turn}}^{tl, dr}$	The activities in $A_{\text{odturn}}^{\text{plan}}$ are the planned turnings of rolling stock at terminal stations
$ST_{\text{turn}}^{tl, dr}$	Set of short-turn station candidates for the trains serving train line tl and operating in direction dr
L_{turn}	Each station contained in $ST_{\text{turn}}^{tl, dr}$ must be the upstream/same station compared to st_{en}^{dr}
L_{turn}	Set of minimum short-turn times at stations
L_{odturn}	Set of minimum OD turn times at stations
$E_{\text{de}, st}^{\text{dis}, tl, dr}$	A local set used in Algorithm 1 , which contains the departure events of the trains that serve train line tl , operate in direction dr , and occur at station st after t_{start} but before $t_{\text{end}} + R$
$E_{\text{ar}}^{\text{tn}}$	A local set used in Algorithm 1 , which contains the corresponding arrival events of $E_{\text{de}, st}^{\text{dis}, tl, dr}$ in station activities
$E_{\text{de}}^{\text{tn}}$	A local set used in Algorithm 1 , which contains the departure events that could be served by the arrival events of $E_{\text{ar}}^{\text{tn}}$ for short-turning

Table 10
Parameters.

Notation	Description
e_{de}	Departure or pass-through departure event: $e_{\text{de}} \in E_{\text{de}}$
e_{ar}	Arrival or pass-through arrival event: $e_{\text{ar}} \in E_{\text{ar}}$
o_e	The original time of event e
st_e	The corresponding station of event e
tr_e	The corresponding train of event e
tl_e	The corresponding train line of event e
dr_e	The operation direction of event e , which is either <i>upstream</i> or <i>downstream</i> : $dr_e \in \{\text{up}, \text{down}\}$
r_{st}	A binary parameter indicating whether or not station st is located on single-track railway lines. If yes, $r_{\text{st}} = 1$.
a_{run}	Running activity: $a_{\text{run}} \in A_{\text{run}}$ $a_{\text{run}} = (e, e')$, $e \in E_{\text{de}}, e' \in E_{\text{ar}}, tr_e = tr_{e'}, dr_e = dr_{e'}, st_e$ is upstream neighbouring to $st_{e'}$
a_{dwell}	Dwell activity: $a_{\text{dwell}} \in A_{\text{dwell}}$ $a_{\text{dwell}} = (e, e')$, $e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e = tr_{e'}, dr_e = dr_{e'}, st_e = st_{e'}, o_e < o_{e'}$
a_{pass}	Pass-through activity: $a_{\text{pass}} \in A_{\text{pass}}$ $a_{\text{pass}} = (e, e')$, $e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e = tr_{e'}, dr_e = dr_{e'}, st_e = st_{e'}, o_e = o_{e'}$
$a_{\text{head}}^{\text{ar}}$	Arrival headway activity: $a_{\text{head}}^{\text{ar}} \in A_{\text{head}}^{\text{ar}}$ $a_{\text{head}}^{\text{ar}} = (e, e')$, $e, e' \in E_{\text{ar}}, tr_e \neq tr_{e'}, dr_e = dr_{e'}, st_e = st_{e'}$
$a_{\text{head}}^{\text{de}}$	Departure headway activity: $a_{\text{head}}^{\text{de}} \in A_{\text{head}}^{\text{de}}$ $a_{\text{head}}^{\text{de}} = (e, e')$, $e, e' \in E_{\text{de}}, tr_e \neq tr_{e'}, dr_e = dr_{e'}, st_e = st_{e'}$
$a_{\text{head}}^{\text{ar}, \text{de}}$	Arrival-departure headway activity: $a_{\text{head}}^{\text{ar}, \text{de}} \in A_{\text{head}}^{\text{ar}, \text{de}}$ $a_{\text{head}}^{\text{ar}, \text{de}} = (e, e')$, $e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e \neq tr_{e'}, dr_e \neq dr_{e'}, st_e = st_{e'}$
$a_{\text{head}}^{\text{de}, \text{ar}}$	Departure-arrival headway activity: $a_{\text{head}}^{\text{de}, \text{ar}} \in A_{\text{head}}^{\text{de}, \text{ar}}$ $a_{\text{head}}^{\text{de}, \text{ar}} = (e, e')$, $e \in E_{\text{de}}, e' \in E_{\text{ar}}, tr_e \neq tr_{e'}, st_e = st_{e'}$
a_{turn}	Short-turn activity: $a_{\text{turn}} \in A_{\text{turn}}$ $a_{\text{turn}} = (e, e')$, $e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e \neq tr_{e'}, dr_e \neq dr_{e'}, st_e = st_{e'}, tl_e = tl_{e'}, st_e \in ST_{\text{turn}}^{tl, dr_e}$
a_{odturn}	OD turn activity that refers to the rolling stock of one train turning at a terminal station to operate an opposite train from the same train line: $a_{\text{odturn}} \in A_{\text{odturn}}$ $a_{\text{odturn}} = (e, e')$, $e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e \neq tr_{e'}, dr_e \neq dr_{e'}, st_e = st_{e'}, tl_e = tl_{e'}$
st_{en}^{dr}	The entry station of the disrupted section for trains operating in direction $dr \in \{\text{up}, \text{down}\}$
st_{ex}^{dr}	The exit station of the disrupted section for trains operating in direction $dr \in \{\text{up}, \text{down}\}$
$\text{tail}(a)$	The tail of activity a , which is the event that a starts from
$\text{head}(a)$	The head of activity a , which is the event that a points to
t_{start}	The start time of the disruption
t_{end}	The end time of the disruption

(continued on next page)

Table 10 (continued)

Notation	Description
L_a	The minimum duration of activity a
I_{st}^{turn}	The minimum short-turn time at station st
λ	The maximum percentage allowed to running time extension
D	The maximum allowed delay per event
R	The required recovery duration after the disruption end time.
M_1	A positive large number that is set to 1440
M_2	A positive large number that is set to twice of M_1 : $M_2 = 2M_1$
N_{st}^p	The number of platform tracks at station st
N_{st}^{th}	The number of pass-through tracks at station st
N_{st}	The number of tracks at station st : $N_{st} = N_{st}^p + N_{st}^{\text{th}}$
τ_a	Pure running time that does not include acceleration time, deceleration time, and time supplement: $a \in A_{\text{run}}$
Δ_a^{acce}	Acceleration time needed for a train run: $a \in A_{\text{run}}$
Δ_a^{dece}	Deceleration time needed for a train run: $a \in A_{\text{run}}$

Appendix B. Weights of decisions, and the passenger groups and parameters used for estimating the weights

Table 11
The descriptions of weights.

Weight	Description
w_e^{delay}	The weight of delaying an arrival event e
w_e^{cancel}	The weight of cancelling an arrival event e
w_a^{skip}	The negative impact of skipping a stop
w_a^{add}	The positive impact of adding a stop

Table 12
The passenger groups and parameters used for estimating the weights.

Symbol	Description
z_e^{alight}	The number of passengers who plan to alight from train tr_e at station st_e , $e \in E_{\text{ar}}$
z_e^{pass}	The number of passengers who plan to pass through station st_e in train tr_e , $e \in E_{\text{ar}}$
z_e^{board}	The number of passengers who plan to board train tr_e at station st_e , $e \in E_{\text{de}}$
z_a^{Board}	The number of passengers who may benefit from earlier boarding, $a = (e, e') \in A_{\text{pass}}$
z_a^{Eoff}	It is calculated as the number of passengers who arrive at station st_e before time o_e and plan to board another train that departs later than train $tr_{e'}$, $(e, e') \in A_{\text{pass}}$
α	The number of passengers who may benefit from earlier alighting, $a = (e, e') \in A_{\text{pass}}$
β	It is calculated as the number of passengers who plan to pass through station st_e by train tr_e while their destinations are $st_{e'}$, $(e, e') \in A_{\text{pass}}$
γ	The delay of a passenger whose planned path is unavailable due to partial/complete train cancellation
	The delay of a passenger whose planned alighting/boarding is impossible due to a skipped stop
	The saved time of a passenger who has earlier alighting/boarding option compared to his/her planned path due to an added stop

Appendix C. Standard abbreviations of stations

Table 13
The standard abbreviation of each station considered in the case study.

Station	Abbreviation	Station	Abbreviation
Blerick	Br	Horst-Sevenum	Hrt
Deurne	Dn	Maarheeze	Mz
Eindhoven	Ehv	Reuver	Rv
Geldrop	Gp	Roermond	Rm
Heeze	Hze	Swalmen	Sm
Helmond	Hm	Tegelen	Tl
Helmond Brandevoort	Hmbv	Venlo	Vi
Helmond Brouwhuis	Hmbh	Weert	Wt
Helmondct Hout	Hmh		

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