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Contextual Areas

Online Passenger Flow Control in Metro Lines

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Abstract. Crowd management during peak commuting hours is a key challenge facing oversaturated metro systems worldwide, which results in serious safety concerns and uneven service experience for commuters on different origin-destination (o-d) pairs. This paper develops real-time passenger flow control policies to manage the inflow of crowds at each station, to optimize the total load carried or revenue earned (efficiency), and to ensure that adequate service is provided to passengers on each o-d pair (fairness), as much as possible. For given train capacity, we use Blackwell's approachability theorem and Fenchel duality to characterize the attainable service level of each o-d pair. We use these insights to develop online policies for crowd control problems. Numerical experiments on a set of transit data from Beijing show that our approach performs well compared with existing benchmarks in the literature.

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Keywords: metro system oversaturation • passenger flow control • online algorithm • fairness

1. Introduction

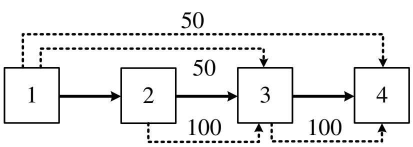
The metro system is becoming a critical part of public transport systems in many megacities because of its inherent advantages in high reliability, large capacity, and low environmental footprint. A significant portion of trips are, however, generated during peak hours (Tirachini et al. 2013, Muñoz et al. 2018, Renken et al. 2018, Bansal et al. 2019). This brings about serious adverse effects on metro system operations and management. On one hand, the accumulation of stranded passengers may increase the risk of potential stampede accidents (Xu et al. 2016). On the other hand, the service experience at different stations can vary considerably without crowd control (Shi et al. 2018), and passengers in downstream stations often cannot board a fully packed train coming from upstream stations.

To mitigate these adverse effects, we develop passenger flow control policies to improve the efficiency and service experience in an oversaturated metro line. By proper admission of passenger flow on each origin-destination (o-d) pair, this approach would enable more passengers to board a train (efficiency) and ensure equitable service level (fairness) for each o-d pair. In this paper, we borrow the *fill rate*[1] concept from the supply chain management area to quantify the service level, which measures the proportion of satisfied demand for each o-d pair.

To make the discussion concrete, we use an example in Figure 1 to illustrate the logic of passenger flow control. Note that the train cannot accommodate all passengers; therefore, different flow control policies may result in differentiated service experiences at each o-d pair. We compare the performance of three typical policies (i.e., first come, first serve (FCFS) policy; revenue maximization policy (RMP); fairness-oriented policy (FOP)). The FCFS policy greedily accepts passengers during the train journey and does not reserve capacity for downstream stations. The RMP maximizes the total number of passengers carried without considering the fairness issue, whereas the FOP maximizes the minimum fill rate (50% in this example) to all o-d pairs and optimizes the total load carried simultaneously. Table 1 shows the performances of the three policies.

Note that the total load improvement in the example comes from exploiting the reusable nature of train capacity (e.g., capacity occupied by upstream passengers can be released and reused by downstream passengers). It

Figure 1. There Is a Train with Capacity 100 Running from Station 1 to 4 to Serve Passengers with Trip $1 \to 3, 1 \to 4, 2 \to 3$, and $3 \to 4$



Note. The number of passengers with trip $1 \to 3$ and $1 \to 4$ is set as 50, and the number of passengers with trip $2 \to 3$ and $3 \to 4$ is set as 100.

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shows the impact of passenger flow control under fixed and known demand settings, but often, the demand information is revealed in an online fashion and known only upon the train's arrival at a station. We study passenger flow control policies under stochastic demand in an online setting in this paper. More concretely, upon the arrival of the train at each station i, the decision maker (DM) learns the realized demands originating from that station and decides an admission policy for the realized demand, based on the capacity available at that point in time. Our aim is to design the flow control process that can exploit the reusable nature of train capacity so that the number of boarding passengers or total revenue (efficiency) can be maximized. Meanwhile, the policy must ensure a certain fill rate delivered to each o-d pair.

Our problem is a stochastic dynamic program (DP) with fill rate constraints on the control policies. This problem is related to the online resource allocation literature in the sense that the DM needs to sequentially allocate seats (resources) to passengers during a finite planning horizon. Some early works addressed this *reusable resource* allocation problem in a (quantity-based) revenue management setting and proposed approximation algorithms with theoretical performance guarantee (e.g., Rusmevichientong et al. 2020, Baek and Ma 2022, Gong et al. 2022, Goyal et al. 2022). However, as far as we know, the service-level issue is often ignored in the aforementioned revenue management works. Our problem is technically more challenging as there is no good technique to deal with the fill rate constraints in the DP recursion. On the other hand, we note that the resource allocation problem with fill rate constraints has been considered in the supply chain management setting (e.g., Zhong et al. 2018, Lyu et al. 2019, Jiang et al. 2022), in which the resources are allocated simultaneously after knowing all the demand realization. However, the problem of reusable resource allocation over multiple stages, with service-level consideration, is new to this literature.

The remainder of this paper is organized as follows. Section 2 presents the model formulation and technical analysis. Section 3 provides the numerical experiments, and Section 4 concludes the paper. All the proofs are relegated to Online Appendix A. We provide more detailed algorithmic design and a case study using a set of transit data from Beijing in Online Appendices B and C, respectively. All the data and source code are available in Online Appendix D.

2. Model and Analysis

2.1. Model Formulation

We consider a single directional metro line with N stations that are consecutively indexed as $1, \ldots, N$. A train with capacity S stops at each station to ferry passengers to the downstream destinations. Passenger flows in o-d pairs (i, j) are random (denoted $\tilde{d}_{i,j}$) with mean $\bar{d}_{i,j}$ and cumulative distribution function $F_{i,j}(\cdot)$, respectively. We use boldface letters to denote vectors throughout this paper (e.g., the o-d demand vector is denoted as $\tilde{\boldsymbol{d}} := (\tilde{d}_{1,2}, \ldots, \tilde{d}_{N-1,N})$). Furthermore, we let $\tilde{\boldsymbol{d}}_{i,\bullet}$ denote $(\tilde{d}_{i,i+1}, \ldots, \tilde{d}_{i,N})$, the demand revealed at station i. The o-d demand information is revealed sequentially during the journey of the train, and the DM makes irrevocable admission decisions upon arrival at each station i after learning the realized demands originating from that station.

The flow control policies could be divided into different classes according to whether the destination information of the o-d pair (i, j) is known or not at the origin station i. We focus here on the *differentiated setting*, where the destination of all passengers originating from station i is known when the train stops at station i. The DM determines a differentiated acceptance rate $x_{i,j}(\tilde{\boldsymbol{d}})$ for each o-d pair (i, j), so that the policy attains desired individual fill rate performance while maximizing the number of passengers carried.[2]

Let π denote an online (sequential) control policy for this problem and Π denote the set of all feasible flow control policies. We use $x^{\pi}_{i,j}(\tilde{\boldsymbol{d}})$ to denote the proportion

Table 1. Performance of Different Policies in an Illustrative Example

	Number of boarding passengers (fill rate)									
Policy type	$1 \to 3$		$1 \to 4$		$2 \to 3$		$3 \to 4$		Total load	Minimal fill rate
FCFS	50	(1.0)	50	(1.0)	0	(0.0)	50	(0.5)	150	0.0
RMP	0	(0.0)	0	(0.0)	100	(1.0)	100	(1.0)	200	0.0
FOP	25	(0.5)	25	(0.5)	50	(0.5)	75	(0.75)	175	0.5

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of the demand in o-d pair (i, j) that is accepted at station i under policy π. Furthermore, we use the notation $\boldsymbol{x}$ to represent the decision when the policy π is not prespecified. We proceed next to describe the model formulation.

2.1.1. Feasibility Constraint. The flow control decisions at station i must satisfy the train capacity constraint and carry no more than the realized demand. We construct the solution $\boldsymbol{x}_i(\tilde{\boldsymbol{d}})$ in stages by first deciding $\boldsymbol{x}_{1,\bullet}$, based on $\boldsymbol{d}_{1,\bullet}$, followed by $\boldsymbol{x}_{2,\bullet}$ given $\boldsymbol{x}_{1,\bullet}$ and $\boldsymbol{d}_{2,\bullet}$ and so on. More formally, the feasible region of flow control decisions at station i can be formulated as

$$\mathcal{X}_i(\boldsymbol{x}_{m,\bullet} : m < i) := \left\{ \boldsymbol{x}_{i,\bullet} \in \mathbb{R}_+^{|N-i|} \;\middle|\; \begin{array}{l} \sum_{j=i+1}^{N} \tilde{d}_{i,j} x_{i,j}(\tilde{\boldsymbol{d}}) \le S - \sum_{m=1}^{i-1} \sum_{n=i+1}^{N} \tilde{d}_{m,n} x_{m,n}(\tilde{\boldsymbol{d}}), \\ \hfill \forall \tilde{\boldsymbol{d}} \in \Omega \\ 0 \le x_{i,j}(\tilde{\boldsymbol{d}}) \le 1, \;\; \forall i < j \le N, \tilde{\boldsymbol{d}} \in \Omega \end{array} \right\}, \tag{1}$$

where Ω represents the potentially infinite support set for passenger demand $\tilde{\boldsymbol{d}}$. The first set of constraints ensures that, under any demand realization $\tilde{\boldsymbol{d}}$, the number of accepted passengers at station i cannot exceed the reserved train capacity. Furthermore, we say that $\boldsymbol{x}$ is a *feasible solution* if $\boldsymbol{x}_{i,\bullet} \in \mathcal{X}_i(\boldsymbol{x}_{m,\bullet} : m < i)$ for all $i = 1, \ldots, N$. We write $\boldsymbol{x} \in \mathcal{X}(\tilde{\boldsymbol{d}})$ to denote that $\boldsymbol{x}$ is feasible in short.

2.1.2. Objective Function. The DM aims at designing the passenger flow control policies during the train journey in such a way that the expected number of accepted passengers is maximized. We can formulate the online flow control problem as the following multistage stochastic DP model and find the solution $\boldsymbol{x}$ to optimize

$$\mathbf{E}_{\tilde{d}_{1,\bullet}} \left[\max_{\boldsymbol{x}_{1,\bullet} \in \mathcal{X}_1} \sum_{j=2}^{N} \tilde{d}_{1,j} x_{1,j}(\tilde{\boldsymbol{d}}) + \mathbf{E}_{\tilde{d}_{2,\bullet}} \left[\max_{\boldsymbol{x}_{2,\bullet} \in \mathcal{X}_2(\boldsymbol{x}_{1,\bullet})} \sum_{j=3}^{N} \tilde{d}_{2,j} x_{2,j}(\tilde{\boldsymbol{d}}) + \cdots + \mathbf{E}_{\tilde{d}_{N-1,\bullet}} \left[\max_{\boldsymbol{x}_{N-1,\bullet} \in \mathcal{X}_{N-1}(\boldsymbol{x}_{j,\bullet} : j < N-1)} \tilde{d}_{N-1,N} x_{N-1,N}(\tilde{\boldsymbol{d}}) \right] \right] \right].$$

For ease of exposition, we reformulate the model as $\mathbf{E}_{\tilde{\boldsymbol{d}}}\left[\max_{\boldsymbol{x} \in \mathcal{X}(\tilde{\boldsymbol{d}})} \sum_{1 \le i < j \le N} \tilde{d}_{i,j} x_{i,j}(\tilde{\boldsymbol{d}})\right]$, when the context is clear that this is a multistage stochastic DP problem. More generally, if $r_{i,j}$ denotes the revenue earned by serving one passenger in the o-d pair (i, j), then the revenue optimization problem can be cast as

$$\mathbf{E}_{\tilde{\boldsymbol{d}}}\left[\max_{\boldsymbol{x} \in \mathcal{X}(\tilde{\boldsymbol{d}})} \sum_{1 \le i < j \le N} r_{i,j} \tilde{d}_{i,j} x_{i,j}(\tilde{\boldsymbol{d}}) \right]. \tag{2}$$

2.1.3. Fill Rate Requirement. Note that the DP model (2) only considers revenue maximization under feasibility constraint $\boldsymbol{x} \in \mathcal{X}(\tilde{\boldsymbol{d}})$ and does not incorporate fairness consideration. Next, we introduce a set of fill rate constraints into DP model (2) to address the fairness concern. Let $\boldsymbol{\beta} = (\beta_{1,2}, \ldots, \beta_{N-1,N})$ denote the ex ante fill rate target for every o-d pair (i, j), which quantifies the expected proportion of demand that is satisfied immediately. The flow control decisions must ensure that the fill rate delivered to each o-d pair (i, j) is at least $\beta_{i,j}$ for the sake of fairness:

$$\mathbf{E}[\tilde{d}_{i,j} x_{i,j}(\tilde{\boldsymbol{d}})] \ge \beta_{i,j} \mathbf{E}[\tilde{d}_{i,j}], \;\; \forall 1 \le i < j \le N. \tag{3}$$

We focus on the fill rate constraints for ease of exposition. More generally, we can incorporate other types of side constraints into the model. For instance, with revenue target Z, we have

$$\mathbf{E}\left[\sum_{1 \le i < j \le N} r_{i,j} \tilde{d}_{i,j} x_{i,j}(\tilde{\boldsymbol{d}})\right] \ge Z.$$

Combining (1), (2), and (3) gives rise to the *online passenger flow control with fairness* (OPFCwF) problem studied in this paper. To solve this problem, we will first focus on the fill rate attainability problem; given any $\boldsymbol{\beta}'$, can we find a (sequential) policy to deliver the stipulated fill rate target $\boldsymbol{\beta}'$ to the passengers? In Section 2.2, we characterize the attainability of any fill rate targets $\boldsymbol{\beta}'$ and construct an adaptive flow control policy to attain the targets. Next, we extend the fill rate attainability result to solve the revenue maximization problem with fill rate constraint in Section 2.3. We set a revenue target Z on the revenue objective and reformulate (OPFCwF) as an attainability problem with both fill rate target and revenue target. By fixing the fill rate target and using bisection search for the largest attainable revenue target as input, we can apply our attainability framework to maximize the total revenue and satisfy the fill rate requirement.[3]

2.2. Attainability of Fill Rate Target

In this subsection, we focus on the fill rate requirement and characterize the attainability of any fill rate targets by explicitly constructing an adaptive flow control policy $\pi \in \Pi$ (or $\boldsymbol{x}^{\pi}(\tilde{\boldsymbol{d}}) \in \mathcal{X}(\tilde{\boldsymbol{d}})$). The necessary and sufficient conditions for attainable fill rate targets are described in Theorem 1.

Theorem 1. *The fill rate target $\boldsymbol{\beta}'$ in the passenger flow control problem can be attained if and only if*

$$\max_{\boldsymbol{\theta} \ge \boldsymbol{0}} \left\{ \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \theta_{i,j} \beta'_{i,j} \bar{d}_{i,j} - \max_{\pi \in \Pi} \mathbf{E}\left[\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \theta_{i,j} \tilde{d}_{i,j} x^{\pi}_{i,j}(\tilde{\boldsymbol{d}}) \right] \right\} \le 0. \tag{4}$$

Note that $\boldsymbol{\theta}$ is given prior to the determination of the optimal dynamic policy π. This represents the revenue contribution of each o-d pair in a related revenue optimization problem. This result generalizes all equivalent results known in the supply chain setting (e.g., Zhong et al. 2018, Lyu et al. 2019) to the multistage setting with reusable resources.

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2.2.1. Necessary Conditions. To see that (4) gives a set of necessary conditions for each o-d pair (i, j), consider a feasible adaptive policy $\mathcal{A} \in \Pi$ that attains the fill rate target. In other words, we have

$$\beta'_{i,j}\bar{d}_{i,j} - \mathbf{E}[\tilde{d}_{i,j}x^{\mathcal{A}}_{i,j}(\tilde{\boldsymbol{d}})] \le 0, \quad \forall 1 \le i < j \le N. \tag{5}$$

By taking a linear combination with nonnegative vector $\boldsymbol{\theta} \ge \mathbf{0}$ among all the o-d pairs in Equation (5), the conditions (4) in Theorem 1 hold clearly.

2.2.2. Sufficient Conditions. Note that the fill rate Constraint (3) defined the "expected" service level in a single-period stochastic setting, but it is straightforward to recast it as a sample average problem using a long sequence of demand samples $\{\tilde{\boldsymbol{d}}(t)\}_{t=1}^{T} \sim \tilde{\boldsymbol{d}}$, where sample size $T \to \infty$. We first construct a *debt-associated adaptive* (DAA) policy iteratively over the demand samples and then, convert it back into a randomized policy to attain the fill rate target in (OPFCwF).

Let $R_{i,j}(t) := \tilde{d}_{i,j}(t)[\beta'_{i,j} - x_{i,j}(t)]$ denote the *debt* for o-d pair (i, j) at sampling epoch t, say the gap between actual satisfied demand $d_{i,j}(t)x_{i,j}(t)$ and those required by fill rate target.[4] The average debt $\hat{\alpha}_{i,j}(t+1)$ for o-d pair (i, j) at the beginning of sampling epoch $(t+1)$, which is accumulated from sampling epoch 1 to t, is simply

$$\hat{\alpha}_{i,j}(t+1) := \frac{1}{t}\sum_{s=1}^{t} R_{i,j}(s). \tag{6}$$

We construct the allocation policy for the sampling epoch $t+1$ by prioritizing o-d pairs with positive average debt so that the fill rate targets can be satisfied in the long run. Intuitively, when $\hat{\alpha}_{i,j}(t+1) > 0$, the previous control decisions over the first t epochs have not achieved the required fill rate targets $\beta'_{i,j}$. As a result, the flow control decision at epoch $(t+1)$ shall give a higher priority to o-d pair (i, j) with a larger positive debt $\hat{\alpha}_{i,j}(t+1)$. Otherwise, when $\hat{\alpha}_{i,j}(t+1) \le 0$, the previous flow control decisions have already met the fill rate target for o-d pair (i, j). Hence, the control decisions at epoch $(t+1)$ do not need to prioritize these passengers.

We let $\alpha_{i,j}(t+1) \leftarrow \max\{\hat{\alpha}_{i,j}(t+1), 0\}$ so that passengers from o-d pair (i, j) with $\hat{\alpha}_{i,j}(t+1) < 0$ could also be served whenever there is remaining capacity after serving o-d pairs with positive average debts.[5] Let $f_a(\boldsymbol{\alpha}(t+1))$ denote the optimal value of the revenue maximization problem in the sampling epoch $(t+1)$:

$$f_a(\boldsymbol{\alpha}(t+1)) = \mathbf{E}_{\tilde{\boldsymbol{d}}(t+1)}\left[\max_{x \in \mathcal{X}(\tilde{\boldsymbol{d}}(t+1))} \sum_{1 \le i < j \le N} \alpha_{i,j}(t+1)\tilde{d}_{i,j}(t+1)x_{i,j}(t+1)\right]. \tag{7}$$

Clearly, the DP formulation in Equation (7) is essentially identical to the revenue optimization model (2). In this way, our DAA policy decomposes the fill rate attainability problem into a series of standard DP problems without the service-level constraints. Therefore, the value of $f_a(\boldsymbol{\alpha}(t+1))$ can be efficiently obtained as long as there exists an efficient algorithm for the revenue optimization Problem (2).[6] To prove the sufficiency of conditions (4), we first develop the following lemma to show that $f_a(\boldsymbol{\alpha}(t+1))$ is convex with respect to (w.r.t.) debt vector $\boldsymbol{\alpha}(t+1)$.

Lemma 1. *For any sample epoch $(t+1)$ and any average debt vector $\boldsymbol{\alpha}(t+1)$, the optimal value of DP formulation $f_a(\boldsymbol{\alpha}(t+1))$ is a convex function w.r.t. debt vector $\boldsymbol{\alpha}(t+1)$.*

Next, we exploit the convexity of $f_a(\boldsymbol{\alpha}(t+1))$ to derive its Fenchel dual and prove the sufficiency of conditions (4). Let $u_{i,j}(t+1) = \beta'_{i,j}\bar{d}_{i,j}$ denote the expected required numbers of accepted passengers in o-d pair (i, j) to ensure a fill rate target $\beta'_{i,j}$ at epoch $(t+1)$; then, the Fenchel dual of $f_a(\boldsymbol{\alpha}(t+1))$ is denoted as

$$f_a^*(\boldsymbol{u}(t+1)) = \max_{\boldsymbol{\alpha}(t+1) \ge \mathbf{0}}\left[\textstyle\sum_{1 \le i < j \le N}\alpha_{i,j}(t+1)u_{i,j}(t+1) - f_a(\boldsymbol{\alpha}(t+1))\right].$$

By the Fenchel–Young inequality, we have

$$f_a(\boldsymbol{\alpha}(t+1)) + f_a^*(\boldsymbol{u}(t+1)) \ge \textstyle\sum_{1 \le i < j \le N}\alpha_{i,j}(t+1)u_{i,j}(t+1).$$

Let $\boldsymbol{x}^*(t+1)$ denote the optimal solution to Problem (7). By the definition of $f_a(\boldsymbol{\alpha}(t+1))$, it is straightforward to obtain the following inequality:

$$\begin{aligned} f_a^*(\boldsymbol{u}(t+1)) &\ge \sum_{1 \le i < j \le N} \alpha_{i,j}(t+1)u_{i,j}(t+1) - f_a(\boldsymbol{\alpha}(t+1)) \\ &= \sum_{1 \le i < j \le N} \alpha_{i,j}(t+1)\left(\beta'_{i,j}\bar{d}_{i,j} - \mathbf{E}[\tilde{d}_{i,j}(t+1)x^*_{i,j}(t+1)]\right) \\ &= \sum_{1 \le i < j \le N} \alpha_{i,j}(t+1)\mathbf{E}[R_{i,j}(t+1)]. \end{aligned}$$

Note that the conditions (4) imply that $f_a^*(\boldsymbol{u}(t+1)) \le 0$, which means that

$$\sum_{1 \le i < j \le N} \alpha_{i,j}(t+1)\mathbf{E}[R_{i,j}(t+1)] \le 0. \tag{8}$$

Finally, following Blackwell's approachability theorem (Blackwell 1956), our DAA policy is shown to be a Blackwell strategy that can move the average debt vector $\hat{\boldsymbol{\alpha}}(T)$, reaching the closed convex set $\mathcal{D} := \{\boldsymbol{z} = [z_{1,2}, z_{1,3}, \ldots, z_{N-1,N}] : z_{i,j} \le 0, \ \forall 1 \le i < j \le N\}$ in $\mathbb{R}^{\frac{N(N-1)}{2}}$. Therefore, we claim that the fill rate requirements can be satisfied in the long term if conditions (4) hold. This finishes the sufficiency part. We provide the detailed proof in Online Appendix A.2.

Note that the DAA policy is implemented iteratively to solve a sample average problem. We exploit the independence between debt vector $\boldsymbol{\alpha}(t+1)$ and demand scenario $\tilde{\boldsymbol{d}}(t+1)$ to construct a randomized policy (cf. Algorithm 1) to attain the fill rate target in (OPFCwF).

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Algorithm 1 (Randomized Passenger Flow Control Policy)

1. **Input:** Train capacity S, distribution of o-d pair demand $\{\tilde{\boldsymbol{d}}\}$, fill rate targets $\boldsymbol{\beta}'$, and sample size T.
2. Generate independent and identically distributed (i.i.d.) demand samples $\tilde{d}_{i,j}(1), \ldots, \tilde{d}_{i,j}(T)$ according to demand distribution of each o-d pair (i, j).
3. Solve the revenue maximization Problem (7) iteratively over the demand samples $\tilde{\boldsymbol{d}}(1), \ldots, \tilde{\boldsymbol{d}}(T)$ based on the DAA policy, and collect a sequence of average debt vectors $\{\boldsymbol{\alpha}(t)\}_{t=1}^{T}$.
4. Sample a period index $\bar{t}$ uniformly at random from $\{1, \ldots, T\}$, and let $\bar{\boldsymbol{\alpha}} = \boldsymbol{\alpha}(\bar{t})$.
5. Solve the revenue optimization Problem (7) with coefficient $\bar{\boldsymbol{\alpha}}$, and return the optimal flow control decision $\boldsymbol{x}^{\tilde{G}}(\tilde{\boldsymbol{d}})$.
6. **Output:** The solution oracle $\boldsymbol{x}^{\tilde{G}}(\cdot)$.

Let $\mathrm{dist}(\hat{\boldsymbol{\alpha}}(T), \mathcal{D})$ denote the Euclidean distance from the average debt vector $\hat{\boldsymbol{\alpha}}(T)$ to the target set $\mathcal{D}$. Cesa-Bianchi and Lugosi (2006) showed that the nonasymptotic convergence rate for the Blackwell strategy is given by $\mathrm{dist}(\hat{\boldsymbol{\alpha}}(T), \mathcal{D}) \le \sqrt{4W/T}$, where T and W denote the sample size and the number of o-d pairs, respectively. In this way, we can state the necessary and sufficient condition for the attainable fill rate targets in (OPFCwF).

Theorem 2. *Suppose conditions (4) hold in (OPFCwF), and let $\epsilon > 0$. The flow control decision $\boldsymbol{x}^{\tilde{G}}(\tilde{\boldsymbol{d}})$, obtained by performing Algorithm 1 with $T = \lceil 4W/\epsilon^2 \rceil$, satisfies that $\mathbf{E}_{\bar{t},\tilde{\boldsymbol{d}}}[x^{\tilde{G}}_{i,j}(\tilde{\boldsymbol{d}})\tilde{d}_{i,j}] \ge \beta'_{i,j}\mathbf{E}[\tilde{d}_{i,j}] - \epsilon$, $\forall 1 \le i < j \le N$, and $\boldsymbol{x}^{\tilde{G}}(\tilde{\boldsymbol{d}}) \in \mathcal{X}(\tilde{\boldsymbol{d}})$ for any $\tilde{\boldsymbol{d}} \in \Omega$.*

Theorem 2 implies that our randomized policy provides a certificate for the attainability of any fill rate target $\boldsymbol{\beta}'$. If the target $\boldsymbol{\beta}'$ is not attained by our randomized policy (i.e., the Euclidean distance $\mathrm{dist}(\hat{\boldsymbol{\alpha}}(T), \mathcal{D})$ does not converge to zero as the sample size T increases), it means that the fill rate target cannot be attained by any flow control policy.

2.3. Revenue Maximization with Fill Rate Constraint

In this section, we extend our attainability framework to maximize the revenue objective (2) as well as satisfy the fill rate constraint (3). We exploit the equivalence between feasibility and optimization in classical optimization theory and use the fact that we can now check for service fill rate feasibility in the DP problem to demonstrate how we can incorporate the revenue objective into our framework. To this end, we construct a revenue target Z on the objective function to ensure that the attained revenue is at least Z. In this way, we can reformulate (OPFCwF) as a refined attainability problem with $1 + \frac{N(N-1)}{2}$ targets:

$$\frac{1}{M}\mathbf{E}\left[\sum_{1\le i<j\le N} r_{i,j}\tilde{d}_{i,j}x_{i,j}(\tilde{\boldsymbol{d}})\right] \ge \frac{Z}{M} \quad \text{and}$$

$$\mathbf{E}[\tilde{d}_{i,j}x_{i,j}(\tilde{\boldsymbol{d}})] \ge \beta_{\mathrm{i,j}}\mathbf{E}[\tilde{d}_{i,j}], \quad \forall 1 \le i < j \le N, \tag{9}$$

where M is introduced to normalize the revenue target and fill rate target to the same scale. Note that the revenue target is also defined in a single-period stochastic setting. We recast the stochastic attainability Problem (9) as a sample average problem by generating a sequence of demand samples $\{\tilde{\boldsymbol{d}}(t)\}_{t=1}^{T} \sim \tilde{\boldsymbol{d}}$, where sample size $T \to \infty$.

We refine the DAA policy by incorporating the revenue target and implementing it iteratively over the demand samples. More concretely, we introduce an additional dimension to the debt vector to trace the gap between the attained revenue and the revenue target at each sample epoch. This lifts the average debt vector $\boldsymbol{\alpha}(\cdot)$ from $\mathbb{R}_{+}^{N(N-1)/2}$ to $\mathbb{R}_{+}^{1+N(N-1)/2}$. Next, we incorporate the revenue component into the DP formulation (7). This gives rise to the *refined DAA policy*. Similar to Algorithm 1, we construct a (refined) randomized policy to ensure a revenue maximization solution given any feasible fill rate constraints. The randomized policy developed to solve the attainability Problem (9) is summarized in Algorithm 2 in Online Appendix B.1. The details are relegated to Online Appendix B.

Following Theorem 2, we demonstrate that the refined DAA policy can ensure that the average debt vector is able to hit the corresponding target set in $\mathbb{R}^{1+N(N-1)/2}$. Therefore, we claim that both revenue target Z and fill rate targets $\boldsymbol{\beta}$ in (9) can be satisfied under the randomized policy as long as the targets $(Z, \boldsymbol{\beta})$ are attainable.

Theorem 3. *Suppose the targets $(Z, \boldsymbol{\beta})$ are attainable in the attainability problem (9), and let $\epsilon > 0$. The flow control decision $\boldsymbol{x}^{\tilde{G}}(\tilde{\boldsymbol{d}})$, obtained by performing Algorithm 2 in Online Appendix B.1 with $T = \lceil 4(W+1)/\epsilon^2 \rceil$, satisfies that $\mathbf{E}_{\bar{t},\tilde{\boldsymbol{d}}}\left[\sum_{1\le i<j\le N} r_{i,j}\tilde{d}_{i,j}x^{\tilde{G}}_{i,j}(\tilde{\boldsymbol{d}})\right] \ge Z - M\epsilon$, $\mathbf{E}_{\bar{t},\tilde{\boldsymbol{d}}}[x^{\tilde{G}}_{i,j}(\tilde{\boldsymbol{d}})\tilde{d}_{i,j}] \ge \beta_{i,j}\mathbf{E}[\tilde{d}_{i,j}] - \epsilon$, $\forall 1 \le i < j \le N$, and $\boldsymbol{x}^{\tilde{G}}(\tilde{\boldsymbol{d}}) \in \mathcal{X}(\tilde{\boldsymbol{d}})$ for any $\tilde{\boldsymbol{d}} \in \Omega$.*

Note that the refined randomized policy provides a certificate for the attainability of both revenue target and fill rate target. To ensure the attained revenue is maximal for a given set of fill rate target $\boldsymbol{\beta}$, we can set the largest revenue target (denoted by Z^{max}) as the input of Algorithm 2 in Online Appendix B.1. If the revenue target is not attainable, we need to reduce the revenue target; otherwise, we can improve the revenue target for a higher revenue. In this way, we can obtain the largest revenue target Z^{max} via a bisection search procedure (cf. Algorithm 3 in Online Appendix B.2).

Theorem 3 implies that our refined randomized flow control policy can maximize the revenue objective (2) as well as satisfy the fill rate requirement (3) if we set $(Z^{max}, \boldsymbol{\beta})$ as the target. Interestingly, it is straightforward to show that the attained fill rate, denoted by $\boldsymbol{\beta}'$, is Pareto optimal[7] because we cannot improve the attained revenue by modifying the target. Note that $Z^{max} = \sum_{1\le i<j\le N} r_{i,j}\beta'_{i,j}\mathbf{E}[\tilde{d}_{i,j}]$ as the objective function in (2) is

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linear in the fill rate. Therefore, if we set $\boldsymbol{\beta}'$ as the fill rate target and perform Algorithm 1 to solve (OPFCwF), we can also maximize the revenue objective (2) as well as satisfy the fill rate requirement (3). To this end, we show that solving (OPFCwF) is equivalent to solving an attainability problem with an appropriate $\boldsymbol{\beta}'$.

Corollary 1. *Denote $\boldsymbol{\beta}'$ as the set of attained fill rates by implementing Algorithm 2 in Online Appendix B.1 with $(Z^{max}, \boldsymbol{\beta})$ as input. Then, the flow control decision $\boldsymbol{x}^{\tilde{G}}(\tilde{\boldsymbol{d}})$, obtained by performing Algorithm 1 with $\boldsymbol{\beta}'$ as the fill rate target input and $T = \lceil 4W/\epsilon^2 \rceil$ $(\epsilon > 0)$ as the sample size, satisfies that $\mathbf{E}_{\bar{t},\tilde{\boldsymbol{d}}}[x^{\tilde{G}}_{i,j}(\tilde{\boldsymbol{d}})\tilde{d}_{i,j}] \geq \beta'_{i,j}\mathbf{E}[\tilde{d}_{i,j}] - \epsilon,\ \forall 1 \leq i < j \leq N$, and $\boldsymbol{x}^{\tilde{G}}(\tilde{\boldsymbol{d}}) \in \mathcal{X}(\tilde{\boldsymbol{d}})$ for any $\tilde{\boldsymbol{d}} \in \Omega$. Furthermore, we have $\mathbf{E}_{\bar{t},\tilde{\boldsymbol{d}}}[\sum_{1\leq i<j\leq N} r_{i,j} x^{\tilde{G}}_{i,j}(\tilde{\boldsymbol{d}})\tilde{d}_{i,j}] \geq Z^{max} - \sum_{1\leq i<j\leq N} r_{i,j}\epsilon$.*

3. Numerical Experiment

We consider the example in Figure 1 with different capacity levels (from 100 to 120) to illustrate the numerical performance of our DAA policy. We assume that the demand of each o-d pair follows a (truncated) normal distribution $d_{i,j} \sim \max\{0; \text{Normal}(\bar{d}_{i,j}, (\bar{d}_{i,j}/3)^2)\}$, where the demand mean $\bar{d}_{i,j}$ is represented along each link (i, j). We generate two sets of i.i.d. o-d demands samples (i.e., training data and test data), each with sample size $T =$ 5,000. We first solve an offline optimization problem with training data to obtain the fill rate targets as the inputs of our DAA policy.

We consider two version of the flow control problems: (i) RMP and (ii) FOP. In the RMP (FOP) case, we set $\beta_{i,j} = 0$ $(\beta_{i,j} = \beta_{\max})$ for each pair (i, j) to solve the offline optimization problem. Here, $\beta_{\max}$ denotes the maximum of the minimal fill rates that can be served based on the demand samples and train capacity. We use the attained fill rate $\boldsymbol{\beta}' = \boldsymbol{\beta}^{RMP}$ $(\boldsymbol{\beta}^{FOP})$ to guide the online passenger flow control with the test data. Note that both $\boldsymbol{\beta}^{RMP}$ and $\boldsymbol{\beta}^{FOP}$ are Pareto optimal fill rate targets, and the associated revenue attained is optimal for the respective fill rate targets.

We compare the performance of our DAA policies with three benchmarks. (1) The hindsight optimal solution is obtained by the classic sample average approximation method using the realized demand information in the test data. It serves as an upper bound for the optimal DP solution. (2) The responsive policy in Zhong et al. (2018) is similar to our DAA policy, except that the debt vectors are generated after knowing all the demand information at each sample (perfect information). (3) The FCFS policy greedily accepts passengers without reserving capacity for downstream stations. We measure the performance of different policies in terms of (i) objective value (i.e., average number of passengers served), (ii) Gini coefficient[8] of fill rate delivered to each o-d pair, and (iii) Euclidean distance from the attained fill rate vector to the targeted fill rate. For ease of comparison, we set the attained fill rate from the hindsight optimal solution as the same target for the three policies in each setting.

As shown in Table 2, the gap between the hindsight optimal solution and the DAA policy is consistently no more than 2.71% in all the cases. The performance gaps between the hindsight optimal solution and DAA policy in terms of Gini coefficient and Euclidean distance are also negligible. The Euclidean distances under the responsive policy by Zhong et al. (2018) are much higher than ours in both RMP and FOP cases. Therefore, it is necessary to obtain the "debt" in the right way (to account for the cost-to-go function) to solve (OPFCwF). Our DAA policy also improves upon the FCFS policy in terms of both efficiency and fairness considerations. We plot the performance of different policies (under capacity level 110) in Figure 2. Our DAA policy clearly dominates both the policy by Zhong et al. (2018) and the FCFS policy.

In Online Appendix C, we extend this approach to deal with a train system across 23 stations and stretching over 27.5 kilometers in Beijing. We compare our

Table 2. Performance of Different Policies for the Illustrative Example

	RMP			FOP		
Capacity and policy	Objective	Gini coefficient	Euclidean distance	Objective	Gini coefficient	Euclidean distance
100						
Hindsight optimal	185.254	0.293	0.062	173.245	0.079	0.001
DAA	182.021	0.282	0.018	169.537	0.082	0.028
Zhong et al. (2018)	175.796	0.401	0.392	166.441	0.188	0.241
FCFS	152.786	0.288	0.627	—	—	—
110						
Hindsight optimal	199.011	0.255	0.007	189.438	0.077	0.001
DAA	194.649	0.262	0.028	184.737	0.080	0.036
Zhong et al. (2018)	190.716	0.307	0.175	180.716	0.141	0.176
FCFS	169.526	0.257	0.618	—	—	—
120						
Hindsight optimal	211.487	0.236	0.026	204.975	0.073	0.000
DAA	205.755	0.247	0.040	199.079	0.076	0.046
Zhong et al. (2018)	202.379	0.256	0.089	194.124	0.110	0.139
FCFS	186.322	0.223	0.587	—	—	—

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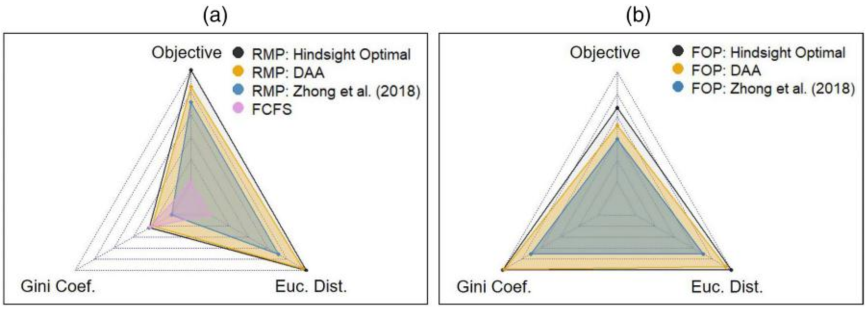
Figure 2. (Color online) Performance of Different Policies Under Capacity = 110



Notes. Along each spoke, better performance is farther from the central vertex. The best performance (among seven policies) is indicated at the peripheral vertex. (a) RMP. (b) FOP.

approach (for differentiated and undifferentiated) with the more common FCFS policy. Instead of solving the DP exactly in DAA, we implement an approximate DP version of the control policy to obtain the algorithm heuristic debt-associated adaptive (H-DAA) that can deal with a larger train system. Again, the H-DAA policies perform exceedingly well, especially for the differentiated case, compared with the FCFS policy. For details, we refer the readers to the online appendices.

4. Concluding Remarks

In this paper, we propose a novel class of passenger flow control policies to deal with the oversaturation problem in public transit systems, to maximize the total load carried, and to ensure service fairness in high-volume metro lines. Using Blackwell's approachability theorem and Fenchel duality, we streamline the analysis to obtain an online approach to solve the DP problem with the service-level side constraints. The numerical results are promising, and we hope this analytical framework can be adapted for other related operations research and transportation problems.

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Endnotes

[1] This kind of service-level measurement is also called *type 2 service level*, and our fairness consideration requires that each o-d pair can be guaranteed with a stipulated fill rate.

[2] In the *undifferentiated setting*, the problem is the same as before, except that the destination of the outgoing passenger is unknown when the train arrives at each station i. In this setting, the DM can only determine a common acceptance rate $x_i(\tilde{d})$ for all the passengers from this station. Furthermore, we assume that passengers in different o-d pairs arrive randomly and uniformly at each station i, with each o-d pair (i, j) receiving the same acceptance rate $x_i(\tilde{d})$ in expectation.

[3] We remark that we solve the optimization problem (OPFCwF) by translating it as a feasibility/attainability checking problem. This solution scheme is in the same line with the classic result that checking the feasibility of a linear system of inequalities and optimization for linear programming are equivalent. The key challenge to be addressed in this work is solving the attainability problem for any given fill rate targets, which is much harder than checking the feasibility of a linear system of inequalities.

[4] The notion of debt used here is motivated by the work in Zhong et al. (2018) and Lyu et al. (2019). However, its incorporation into a DP framework is the main contribution of this technical note. This allows us to extend the technique beyond the single-stage setting in the earlier literature to consider more challenging multistage problems.

[5] If there are multiple optimal solutions, we make the tiebreaking arbitrarily: for example, based on the optimization solver. As shown in the proof of Theorem 1, the modification of $\hat{\alpha}_{i,j}(t+1)$ does not affect our analysis.

[6] We assume that an optimization oracle can be used to solve the revenue optimization Problem (2) for any revenue coefficient $\boldsymbol{r}$. For ease of numerical computation, we apply the Python package MSPPy developed by Ding et al. (2019) to solve the DP Problem (7). Note that MSPPy can deal with moderate size multistage linear stochastic programs based on the celebrated stochastic dual dynamic programming method (cf. Pereira and Pinto 1991).

[7] A fill rate target $\boldsymbol{\beta}'$ lies on the Pareto frontier if there does not exist any other feasible $\boldsymbol{\beta}''$ such that $\beta''_{i,j} \geq \beta'_{i,j}$, $\forall 1 \leq i < j \leq N$ (i.e., the target $\boldsymbol{\beta}'$ is Pareto optimal).

[8] The Gini index can be represented as $\sum_{i<j}\sum_{m<n} |\beta_{i,j} - \beta_{m,n}|/(2W^2\bar{\beta})$, where W and $\bar{\beta}$ denote the number of o-d pairs and the average fill rate attained, respectively. Clearly, a Gini coefficient of zero indicates identical fill rate delivered to all o-d pairs, whereas a large value of the Gini coefficient implies unfair transport service. Note that it was also used to quantify the demand imbalance in public transport (Hörcher and Graham 2020).

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