Decomposition of Graphs: Representing Graphs

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Graph Algorithms
Data Structures and Algorithms

Learning Objectives

- Provide ways in which a graph can be represented on a computer.
- Understand the distinction between dense and sparse graphs and how it affects algorithm efficiency.

Outline

1. Graph Representations
2. Density and Runtimes

Last Time

Graphs consist of:

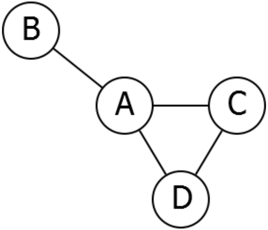
- Vertices (or nodes).
- Edges connecting pairs of vertices.

Representing Graphs

To compute things about graphs we first need to represent them.
There are many ways to do this.

Edge List

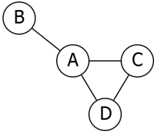
List of all edges:



Edges: (A, B), (A, C), (A, D), (C, D)

Adjacency Matrix

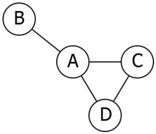
Matrix. Entries 1 if there is an edge, 0 if there is not.



$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 1 & 1 \\ B & 1 & 0 & 0 & 0 \\ C & 1 & 0 & 0 & 1 \\ D & 1 & 0 & 1 & 0 \end{array}$$

Adjacency List

For each vertex, a list of adjacent vertices.



A adjacent to B, C, D

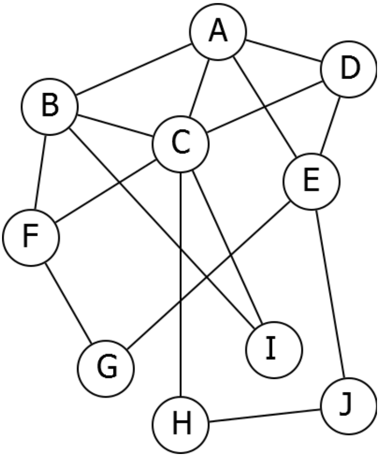
B adjacent to A

C adjacent to A, D

D adjacent to A, C

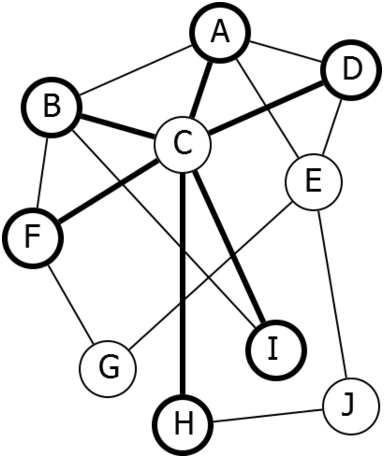
Problem

What are the neighbors of C?



Solution

$A, B, D, F, H, I.$



Summary

Different operations are faster in different representations.

|E| represents the size of the edge list
(deg) represents the number of neighbors for a particular node

Op.	Is Edge?	List Edge	List Nbrs.
Adj. Matrix	$\Theta(1)$	$\Theta(\lvert V\rvert^2)$	$\Theta(\lvert V\rvert)$
Edge List	$\Theta(\lvert E\rvert)$	$\Theta(\lvert E\rvert)$	$\Theta(\lvert E\rvert)$
Adj. List	$\Theta(\mathrm{deg})$	$\Theta(\lvert E\rvert)$	$\Theta(\mathrm{deg})$

For many problems, want adjacency list.

Outline

1. Graph Representations
2. Density and Runtimes

Algorithm Runtimes

Graph algorithm runtimes depend on $|V|$ and $|E|$.

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Graph algorithm runtimes depend on $|V|$ and $|E|$.
For example, $O(|V| + |E|)$ (linear time), $O(|V||E|)$, $O(|V|^{3/2})$, $O(|V| \log(|V|) + |E|)$.

Density

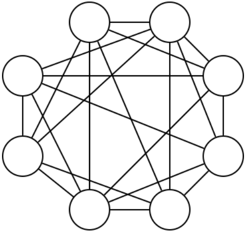
Which is faster, $O(|V|^{3/2})$ or $O(|E|)$?

Density

Which is faster, $O(|V|^{3/2})$ or $O(|E|)$?
Depends on graph! Depends on the density, namely how many edges you have in terms of the number of vertices.

Dense Graphs

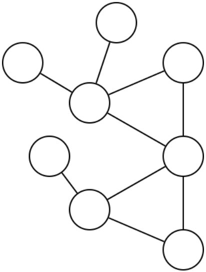
In dense graphs, $|E| \approx |V|^2$.



A large fraction of pairs of vertices are connected by edges.

Sparse Graphs

In sparse graphs, $|E| \approx |V|$.



Each vertex has only a few edges.

Next Time

Algorithms for exploring graphs.