Flows in Networks: Slow Example

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Advanced Algorithms and Complexity
Data Structures and Algorithms

Learning Objectives

- Understand why the Ford-Fulkerson algorithm is sometimes slow.
- Provide an example of a potential slow execution of this algorithm.

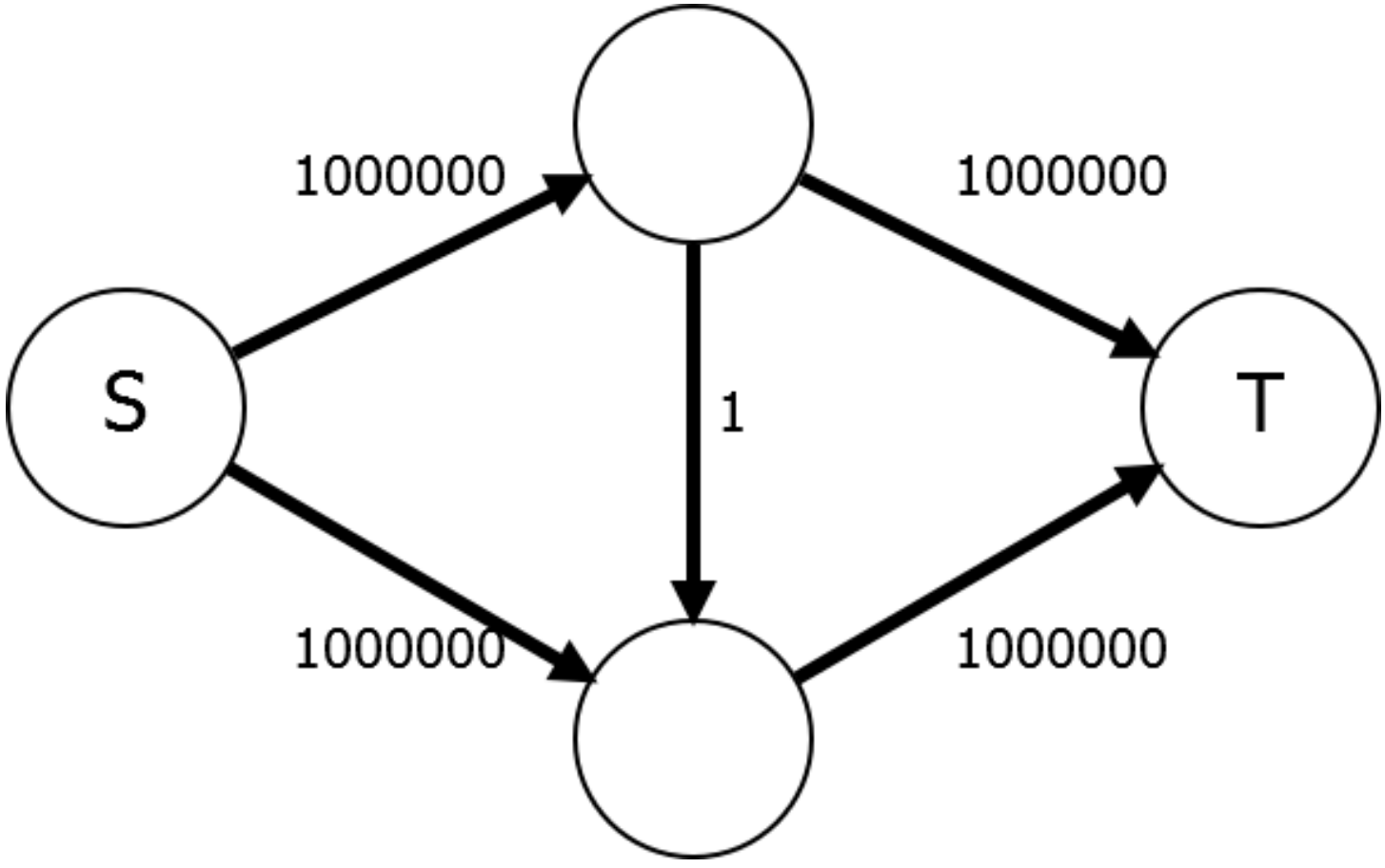
Last Time

Ford-Fulkerson Algorithm for Maxflow:

- Runtime $O(|E||f|)$.
- Potentially bad if $|f|$ is large.
- Does this problem ever actually happen?

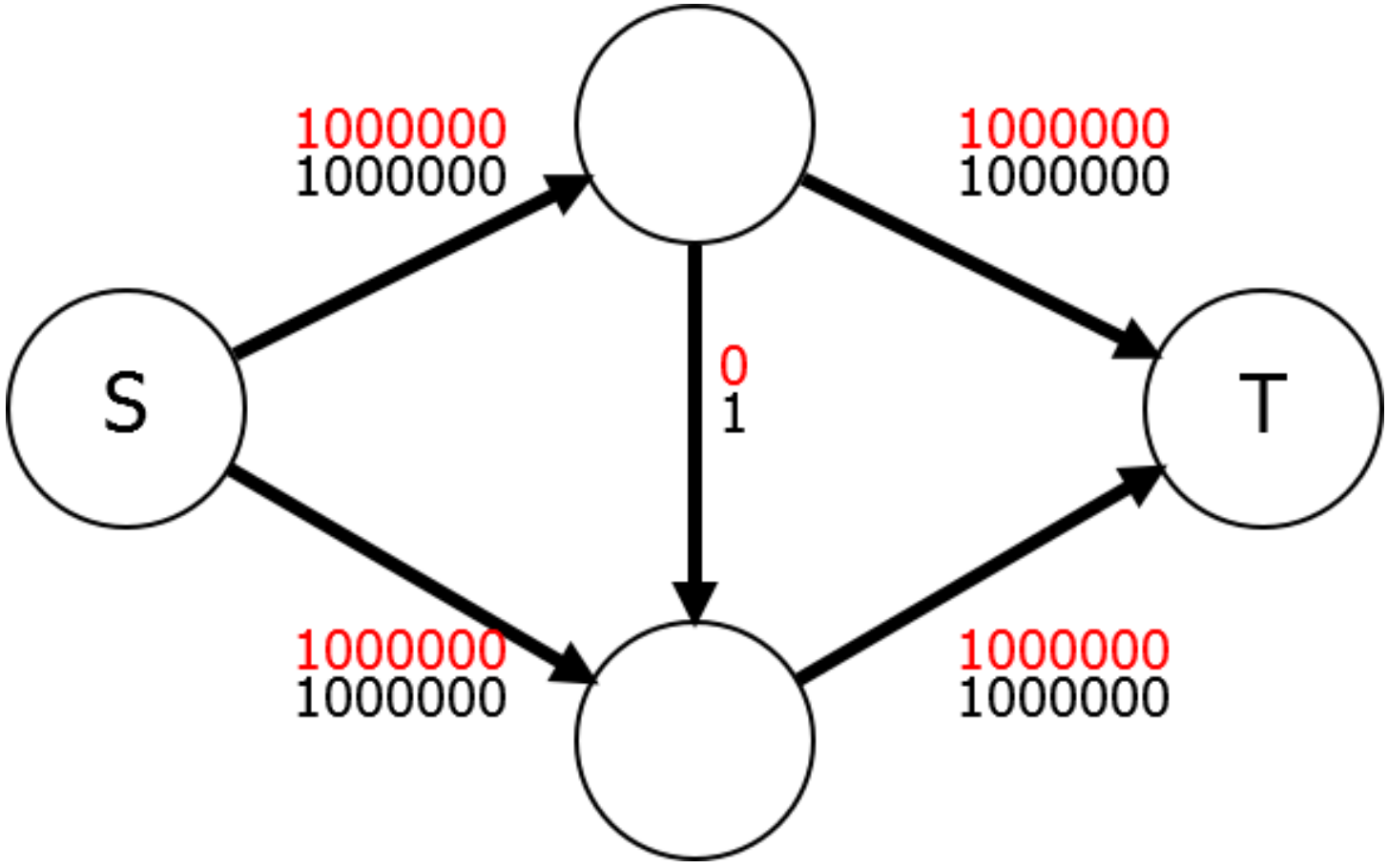
Example

Consider the following example:

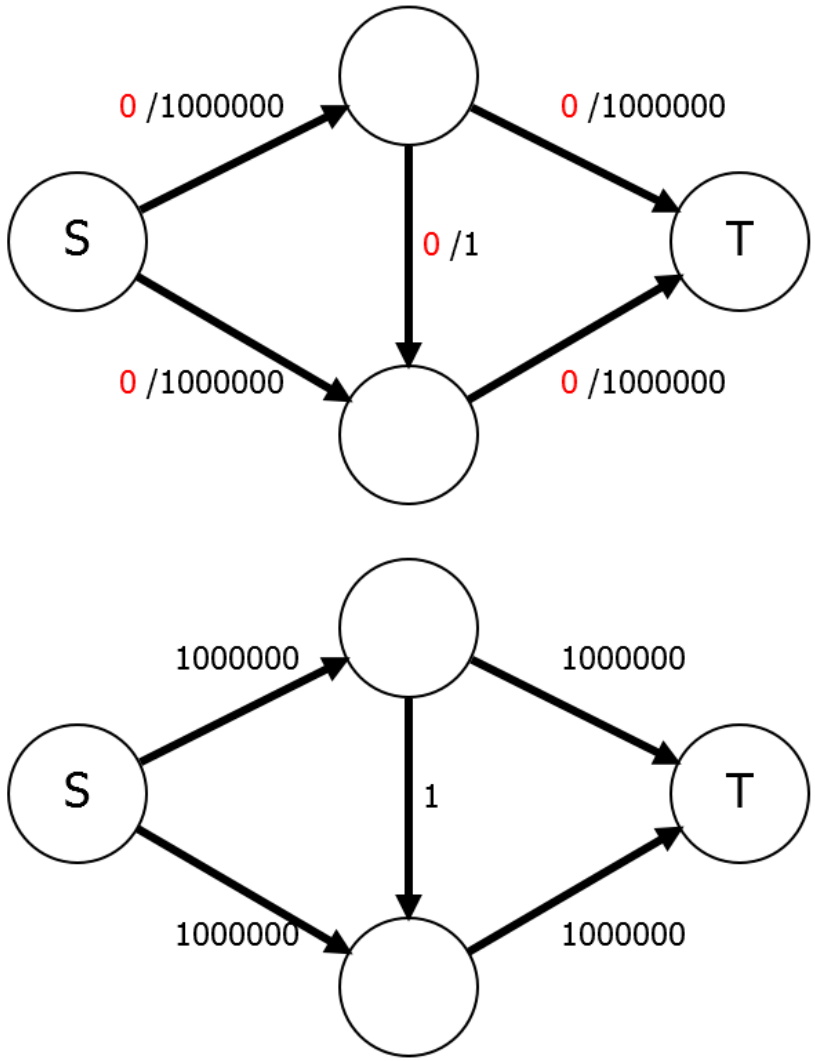


Flow

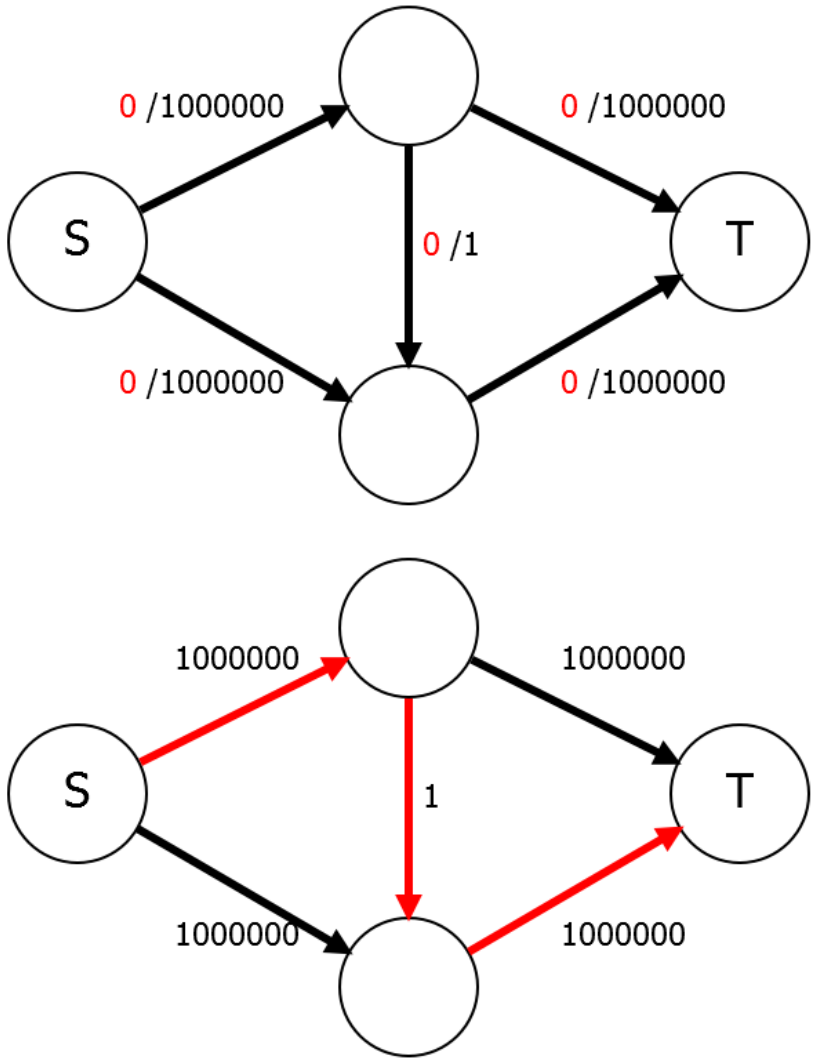
Has following maxflow ($|f| = 2000000$):



Possible Execution



Possible Execution



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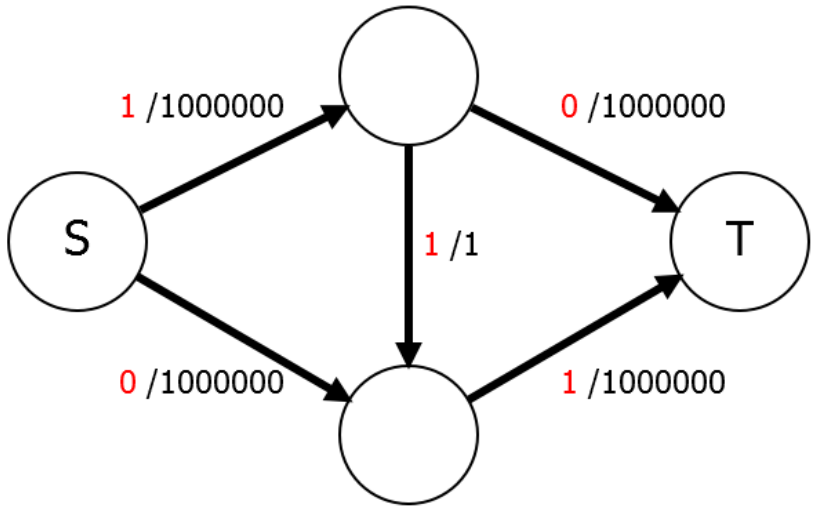
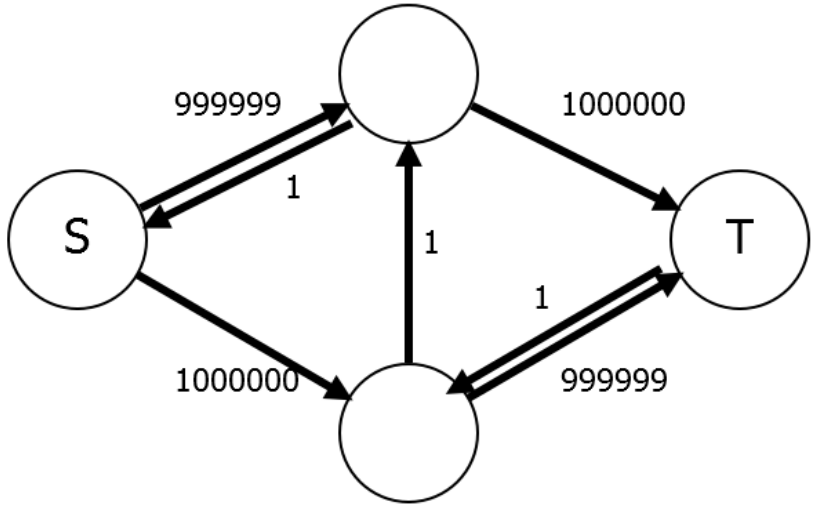
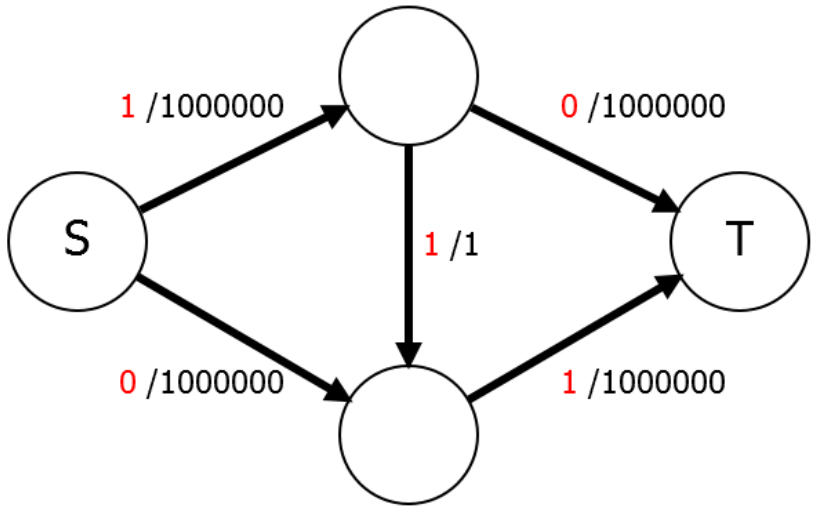
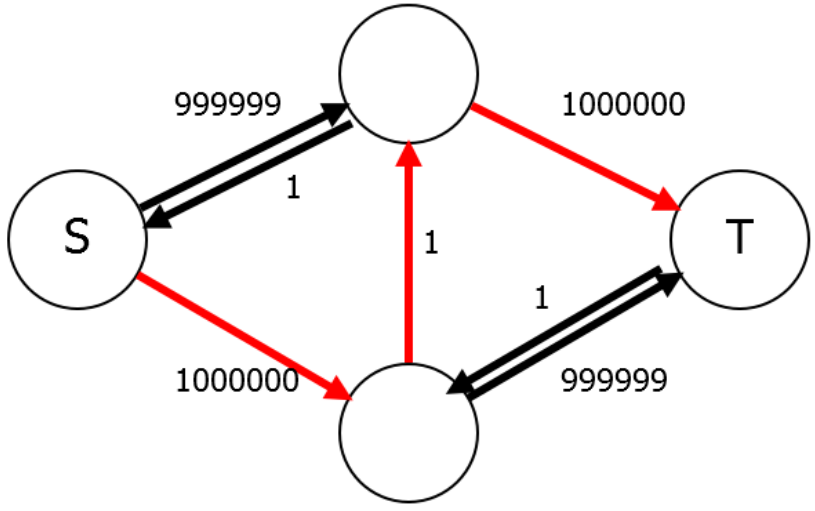




Possible Execution





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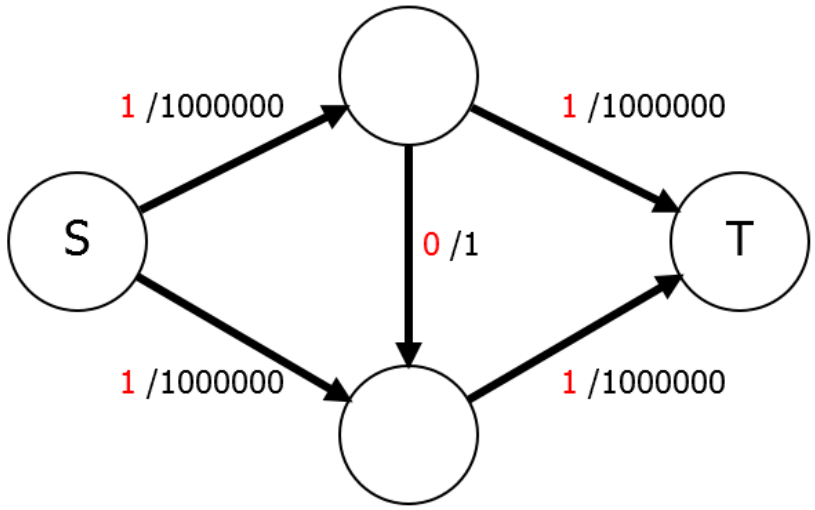

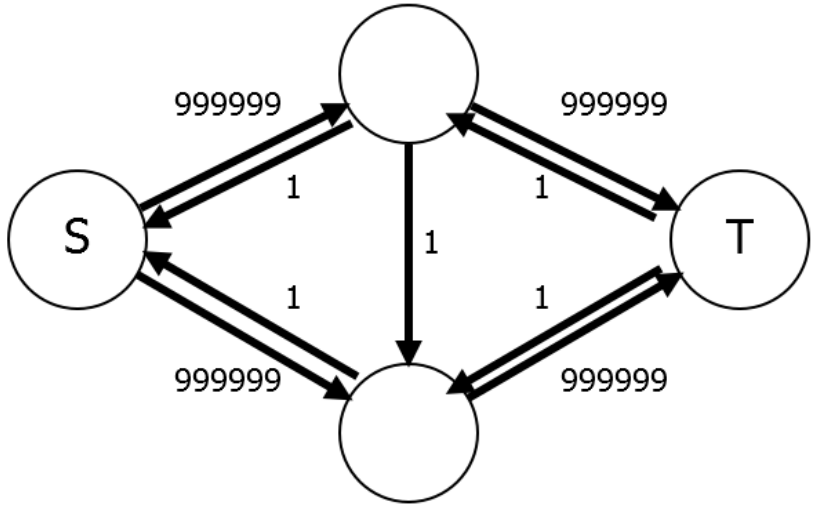

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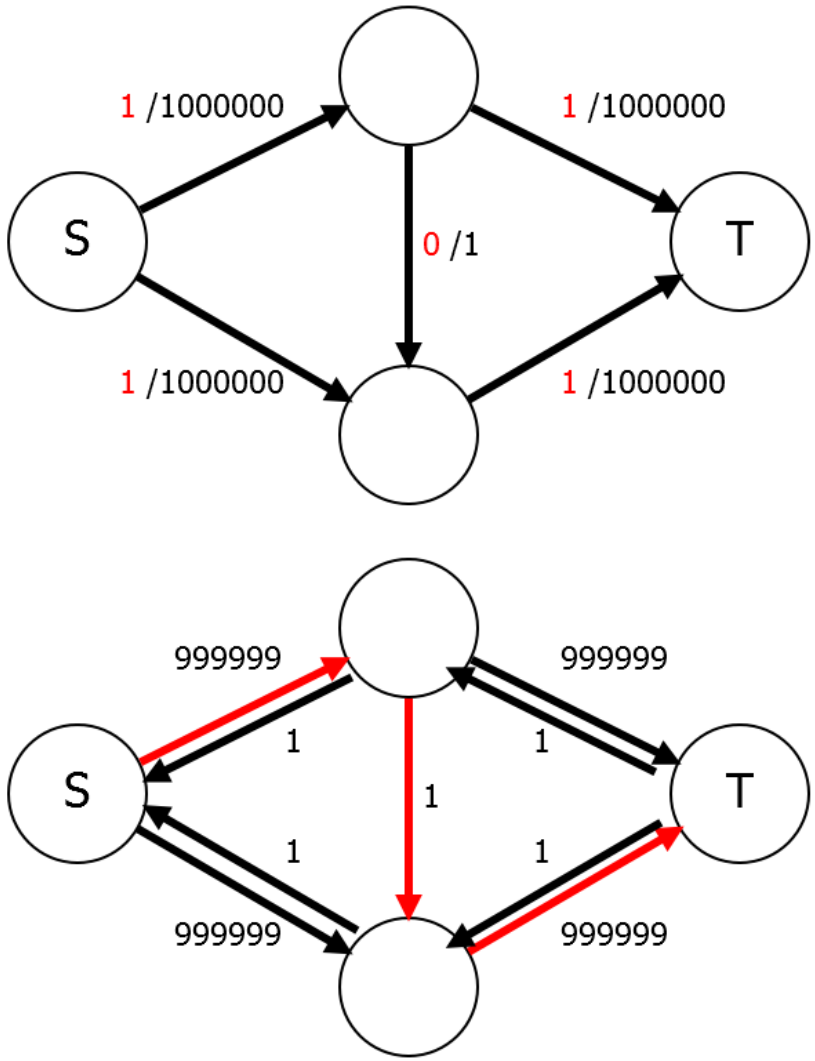

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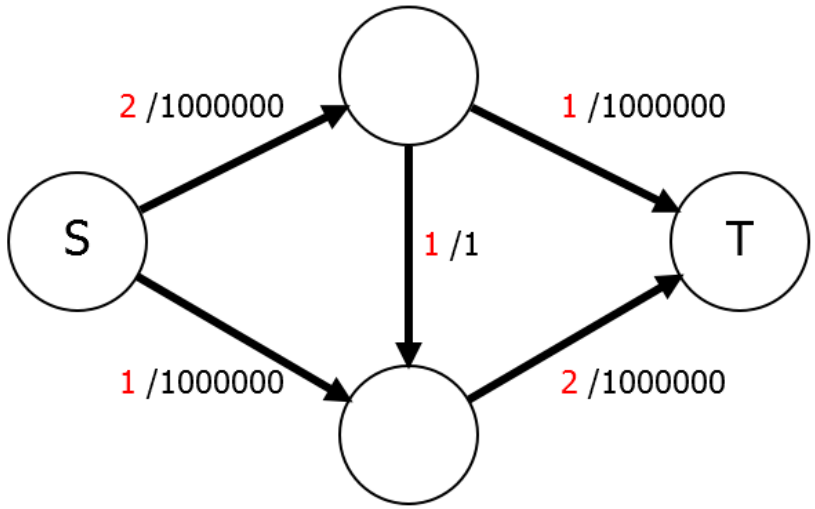

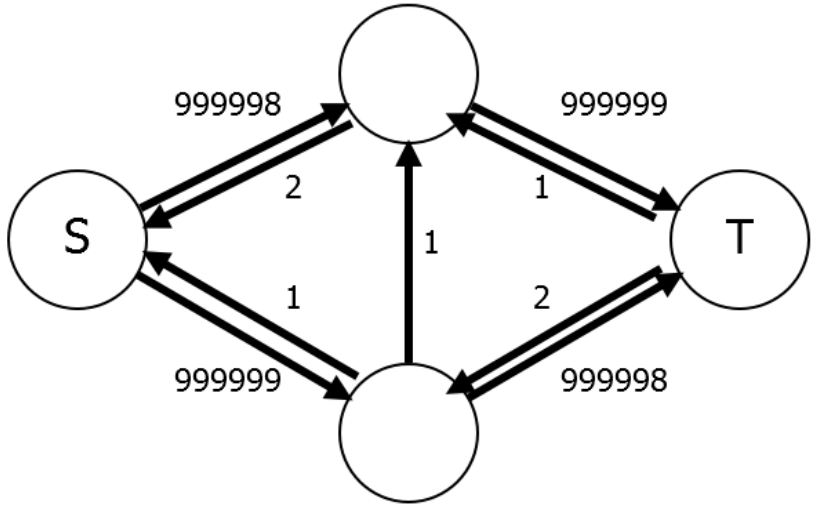

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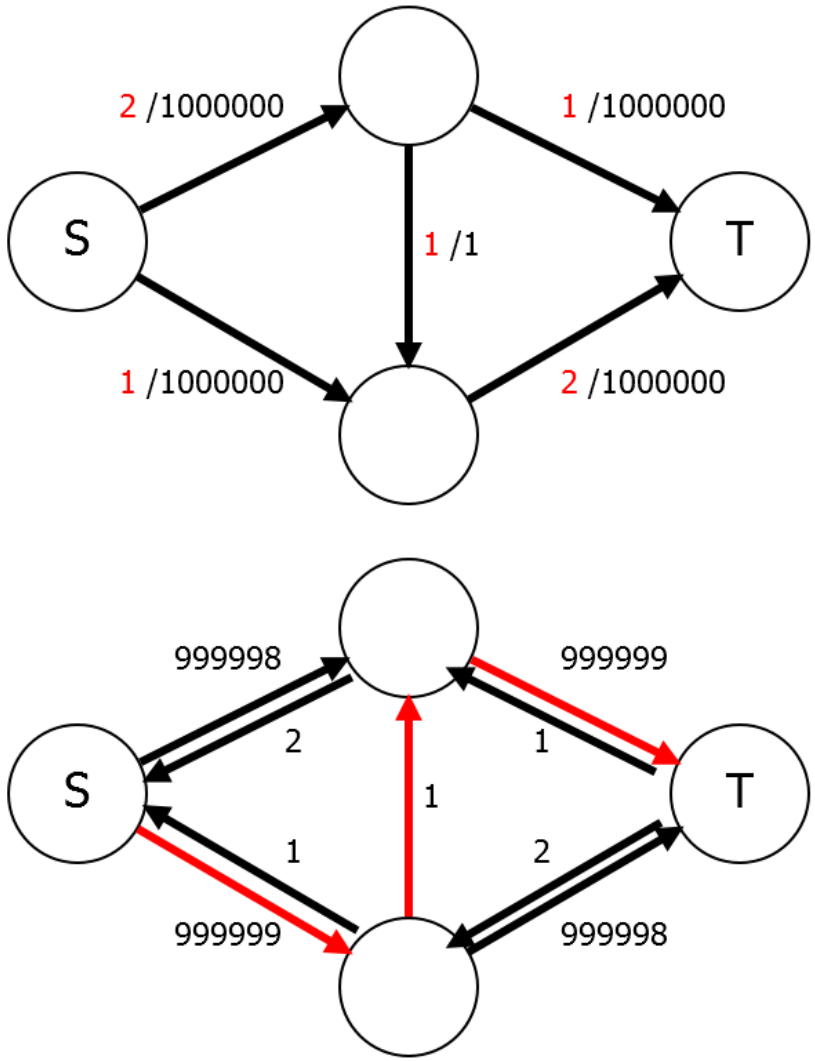

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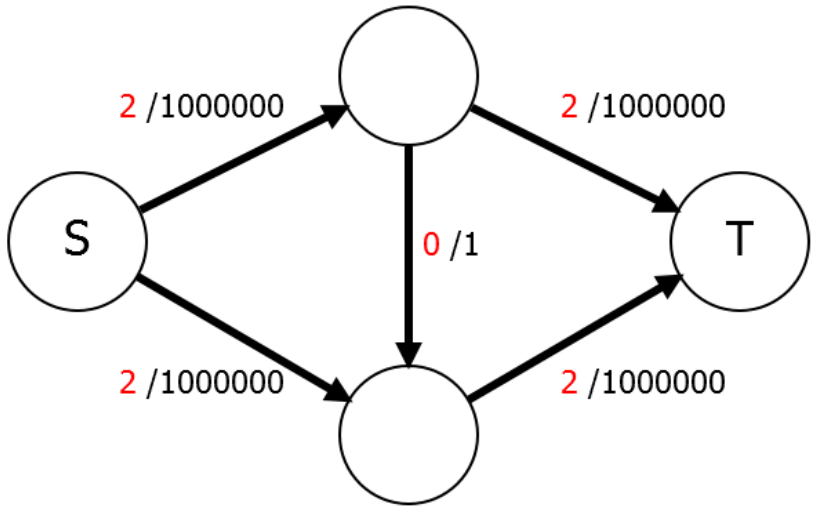

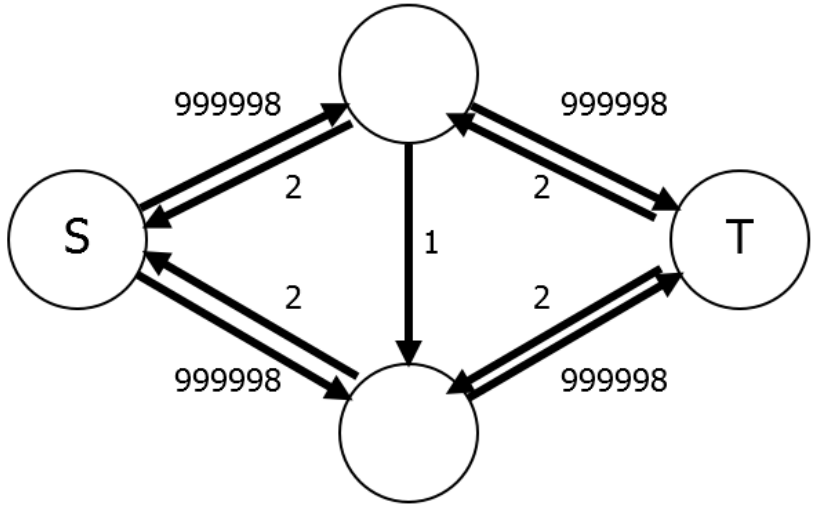

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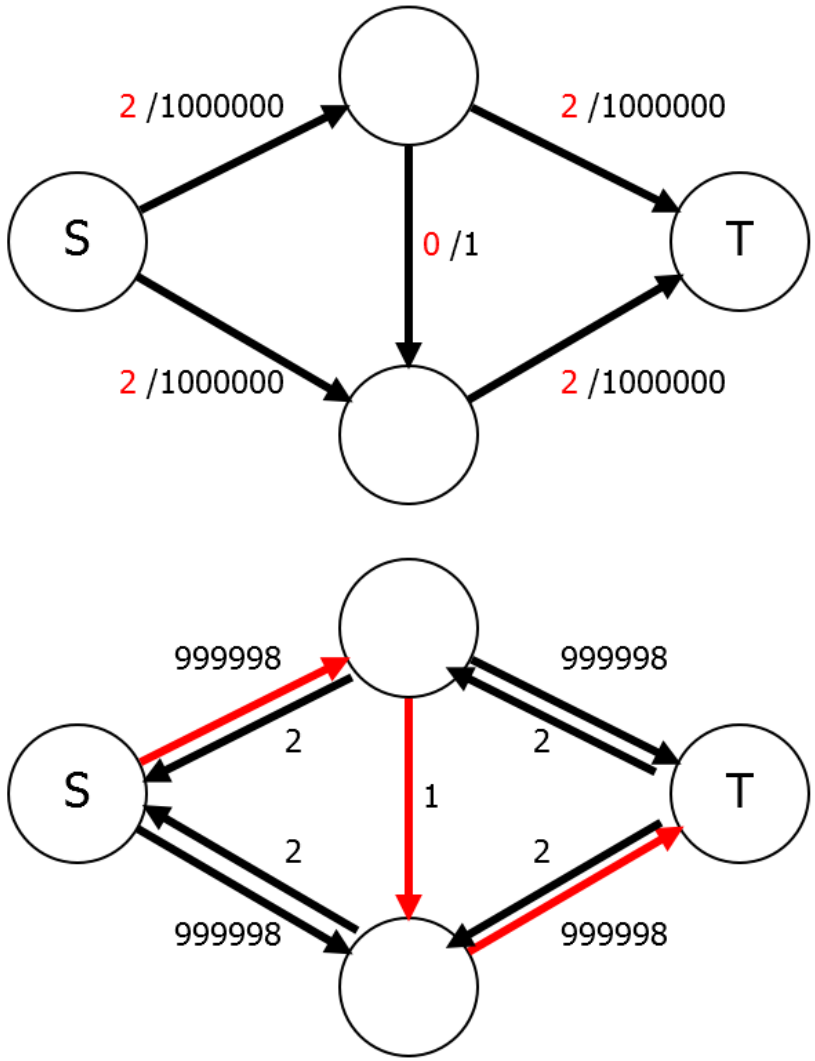

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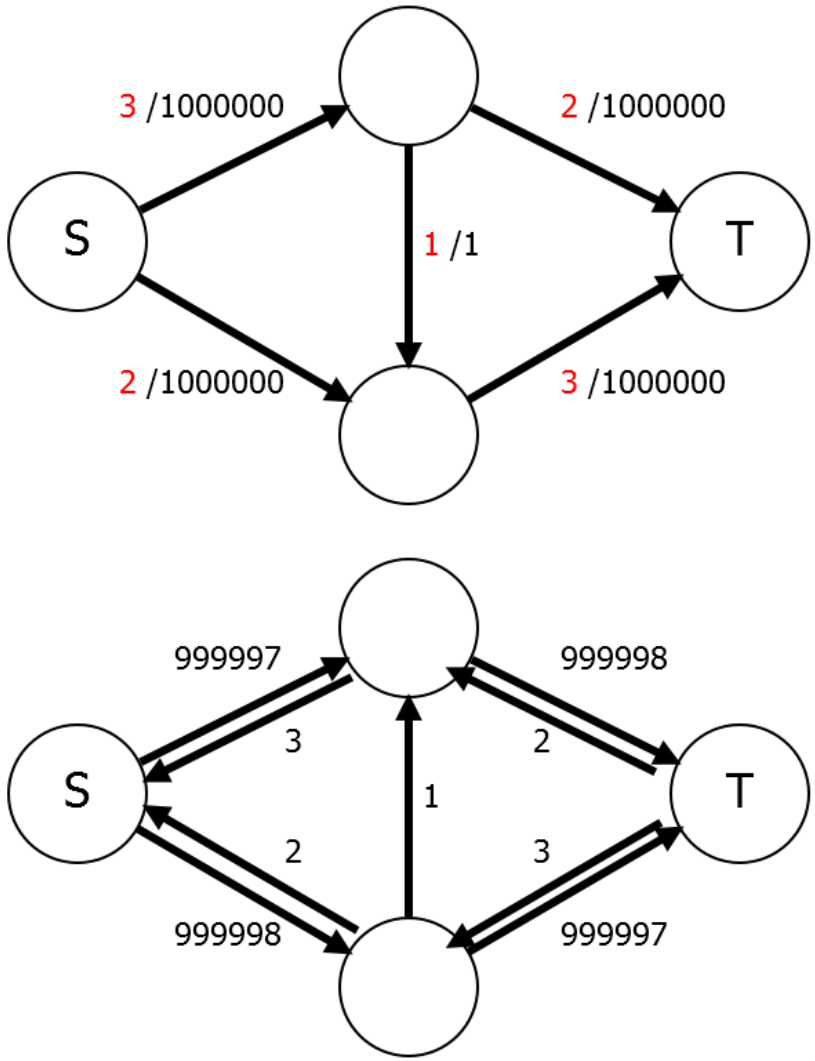

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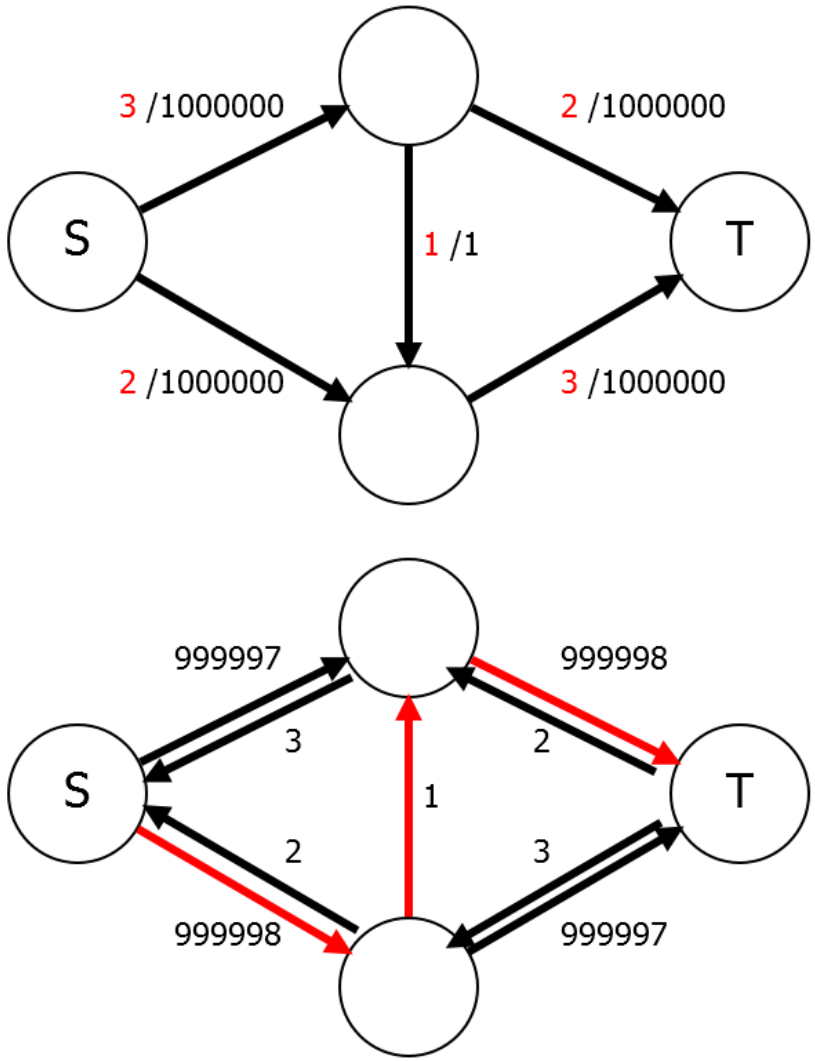

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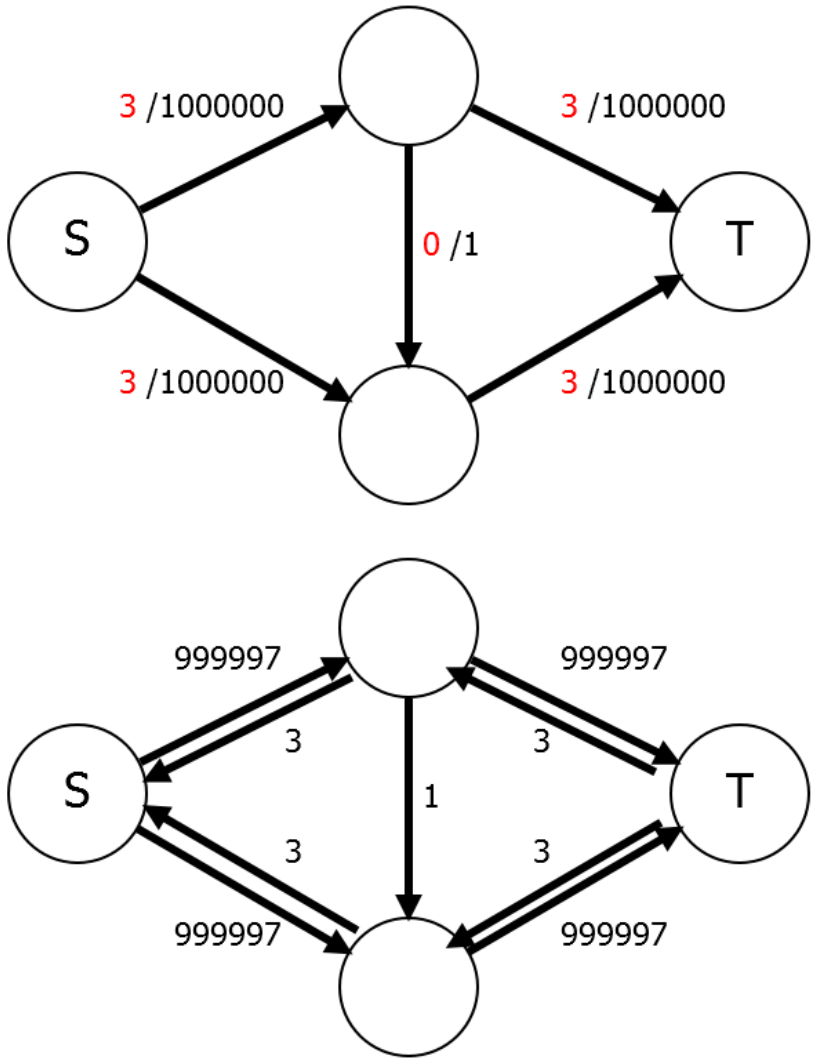

Problem

How many iterations of Ford-Fulkerson will be required to compute a maximum flow, if it keeps choosing augmenting paths in this way?

Time

- Each step adds only one unit of flow (limited by middle edge).
- Need 2000000 total units.
- Ford-Fulkerson requires 2000000 iterations.

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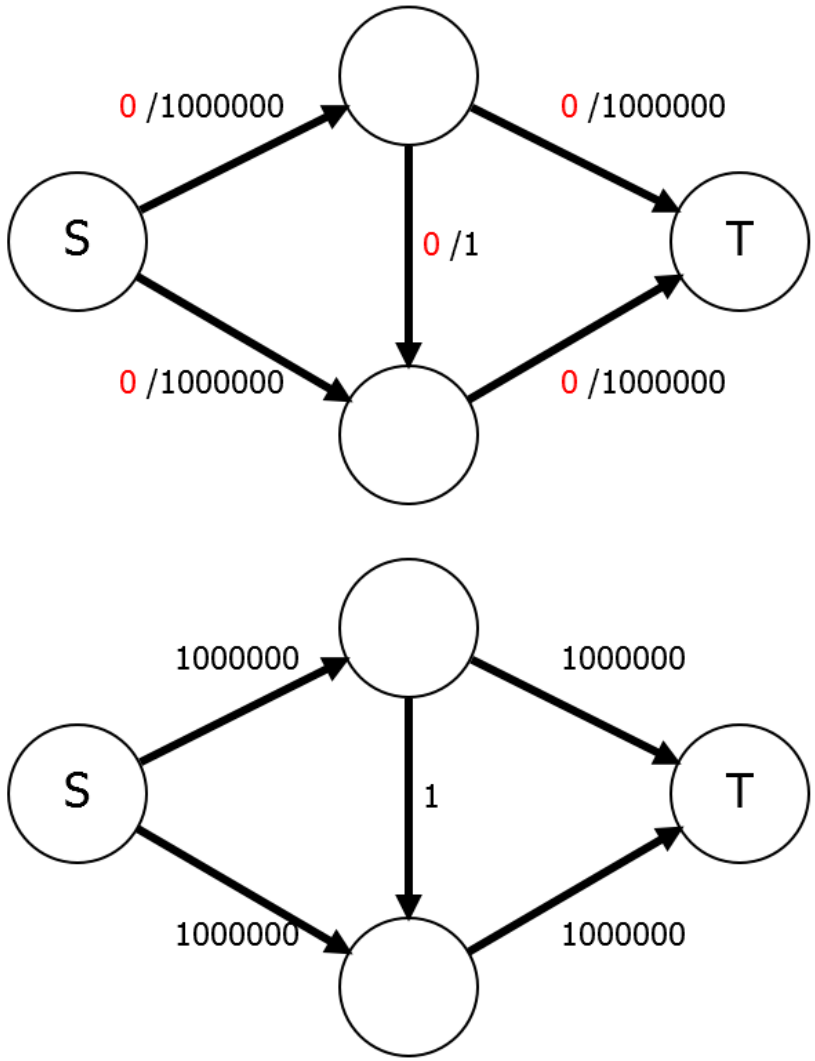
Doesn't need to be this bad.

Another Possible Execution



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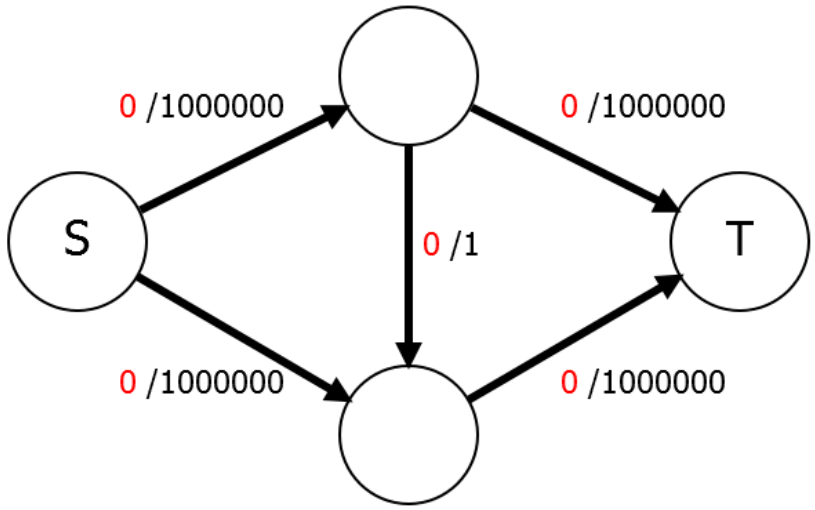
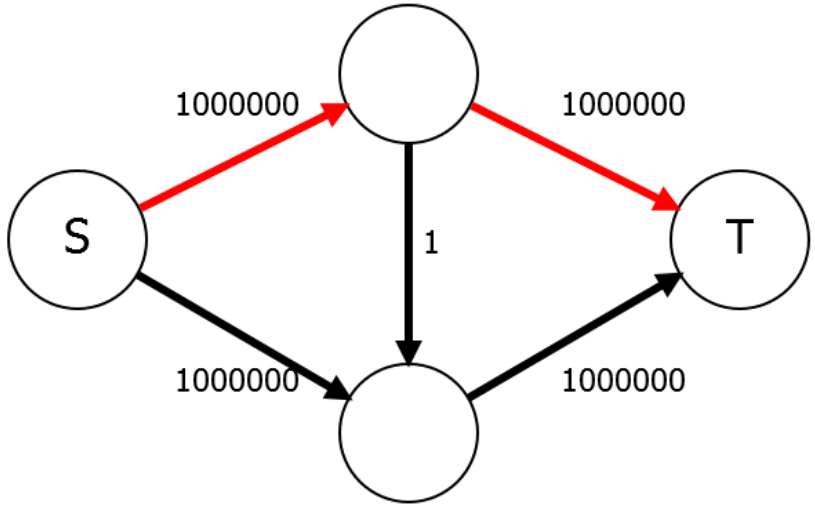
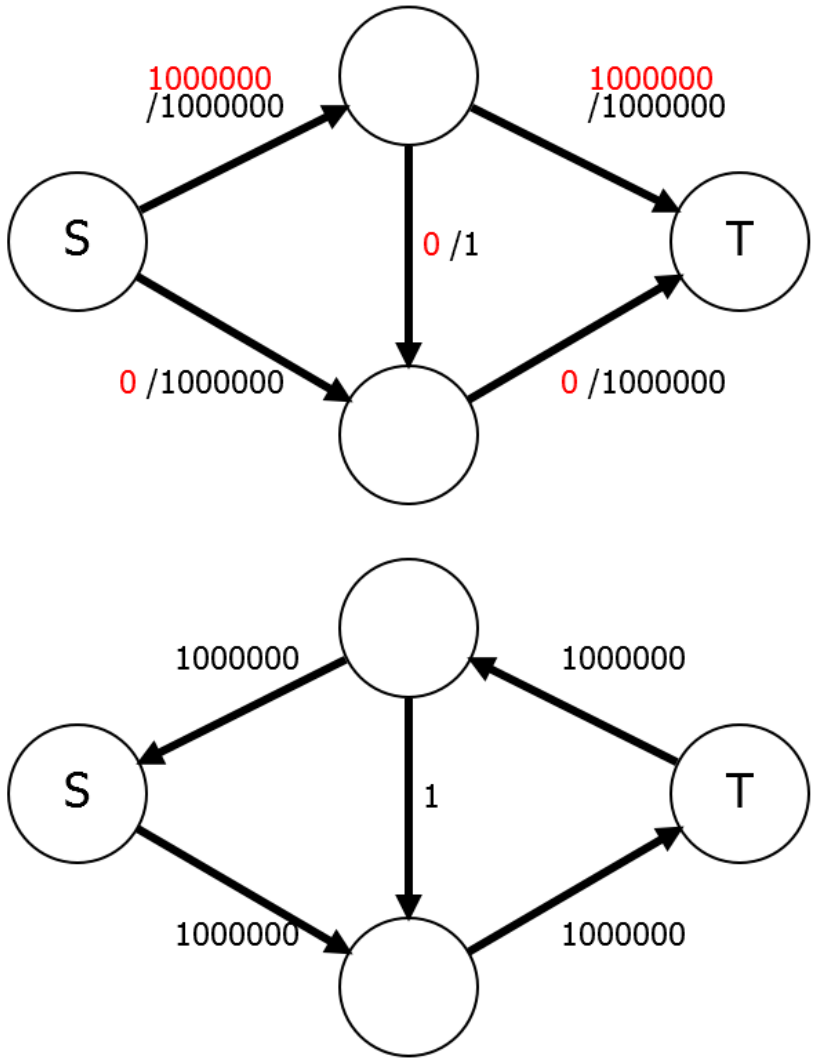




Routing the maximum of 1000000 units from S to T

Another Possible Execution



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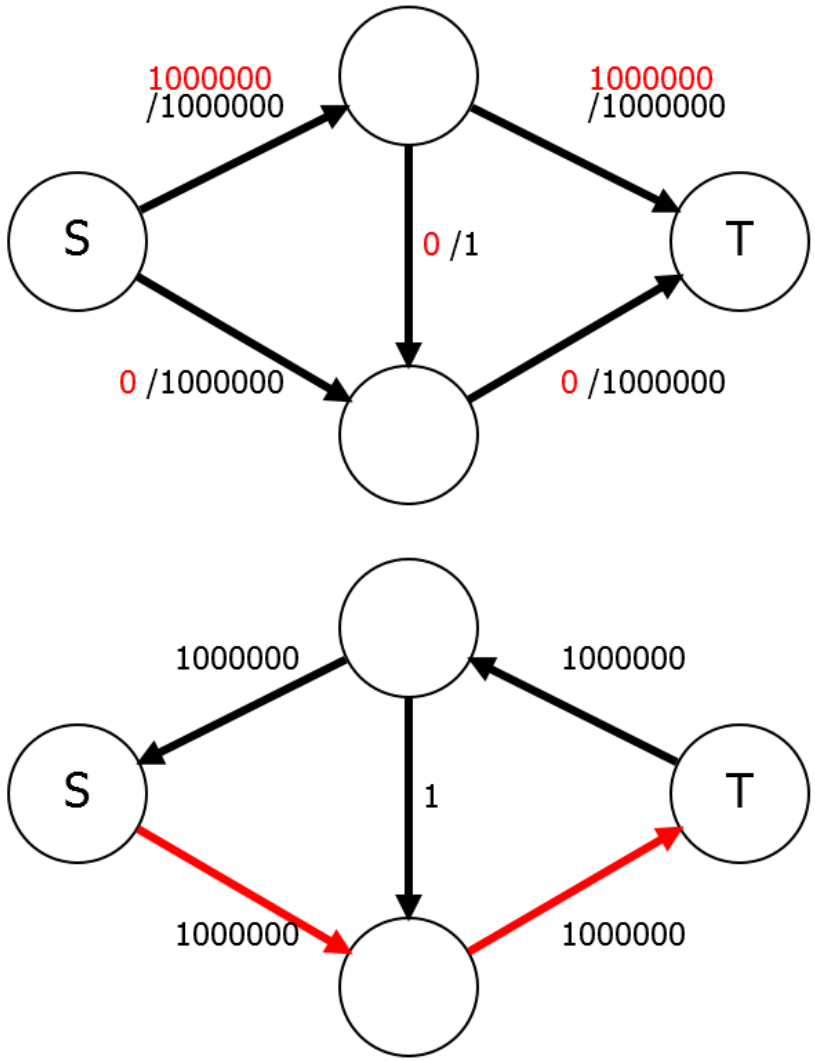

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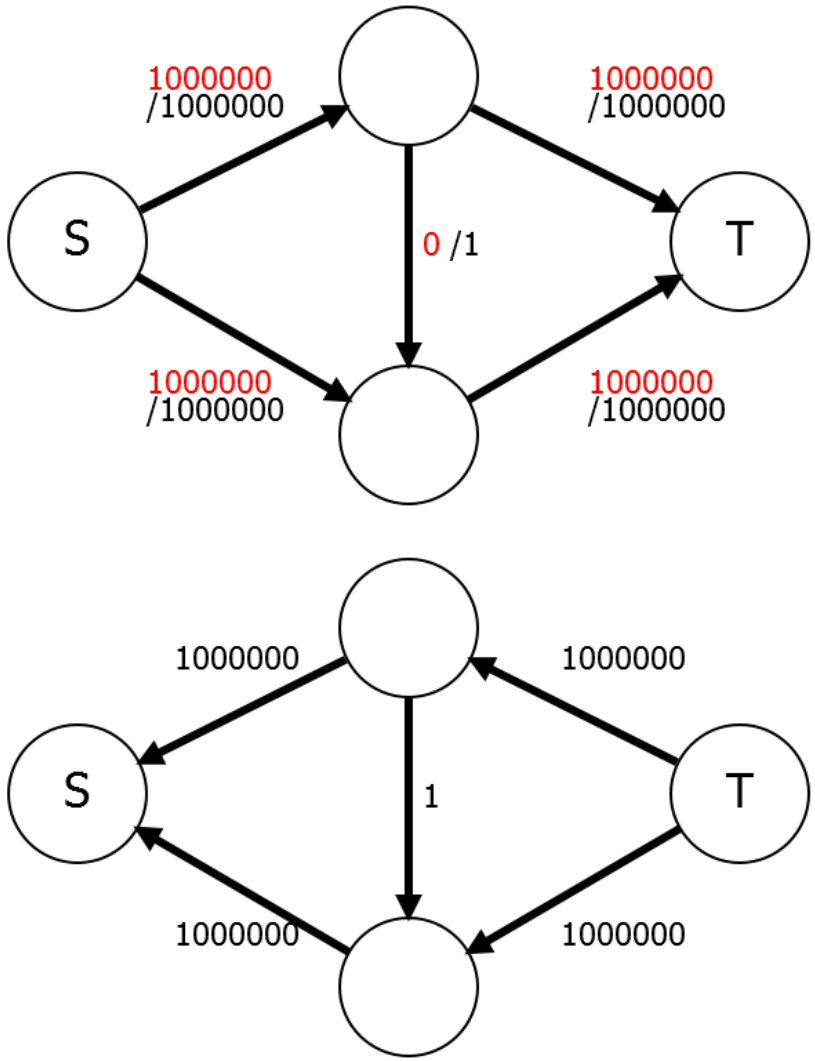

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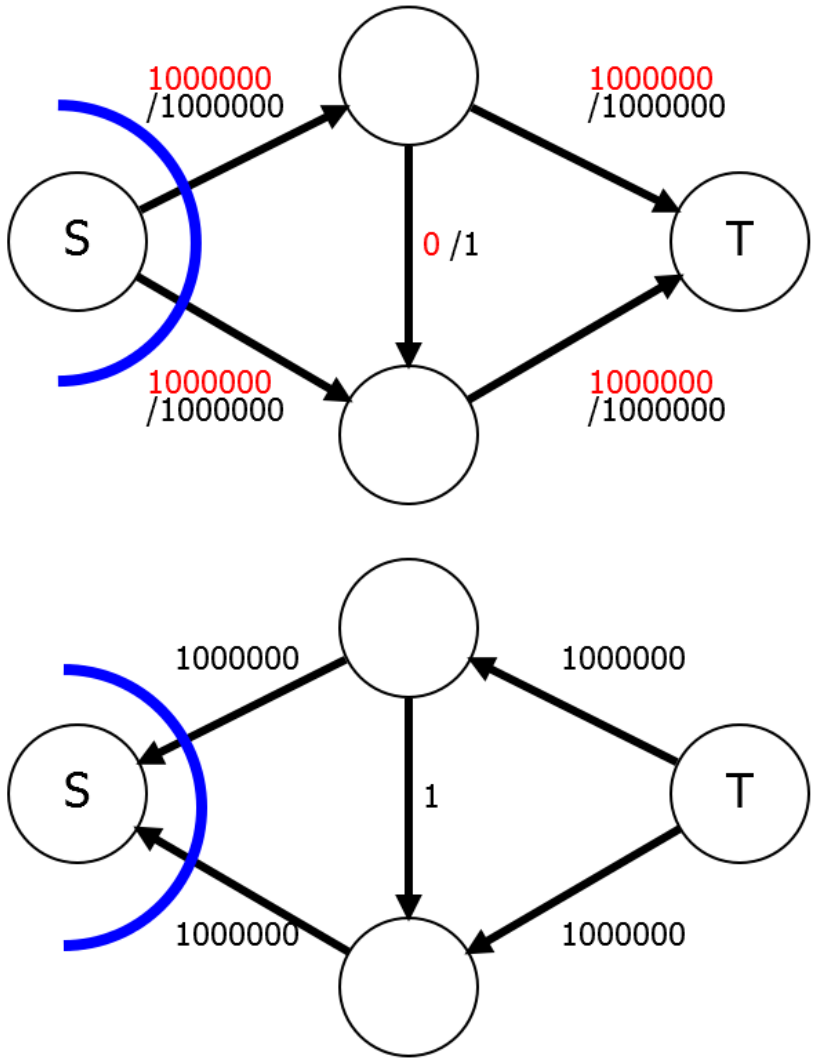

No path from S to T left

Next Time

We'll find a way to ensure that we never have this type of problem.