

Binary Search Trees: AVL Tree Implementation

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Data Structures
Data Structures and Algorithms

Learning Objectives

- Implement AVL trees.
- Understand the cases required for rebalancing algorithms.

Outline

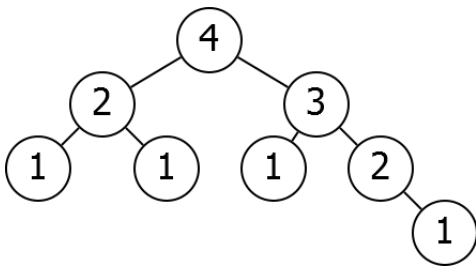
1 AVL Trees

2 Insert

3 Delete

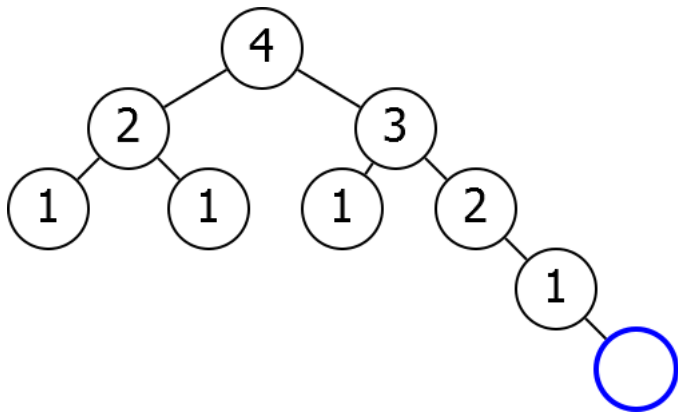
AVL Trees

Need ensure that children have nearly the same height.



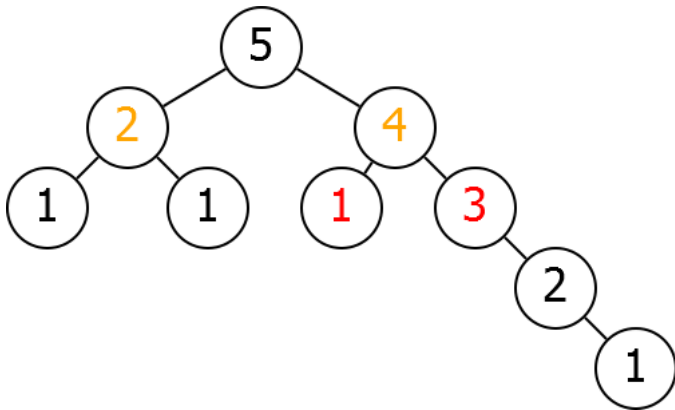
Problem

Updates to the tree can destroy this property.



Problem

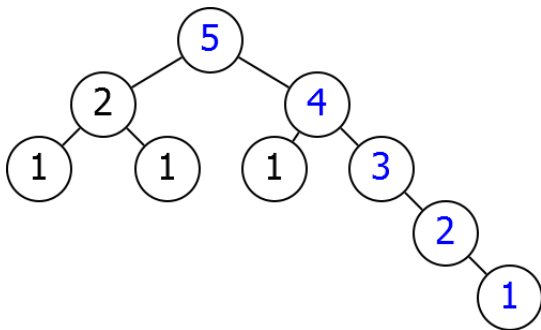
Updates to the tree can destroy this property.



Need to correct this.

Errors

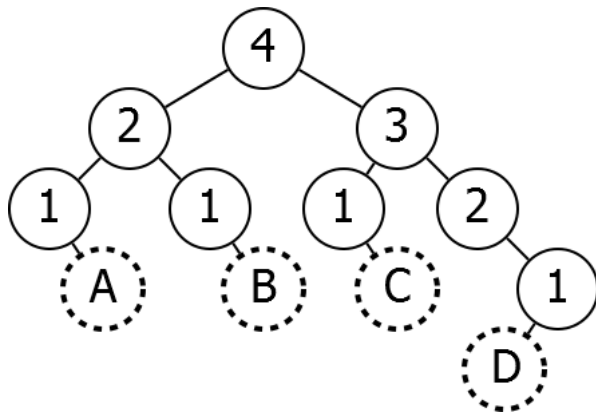
Heights stay the same except on the insertion path.



Only need to worry about this path.

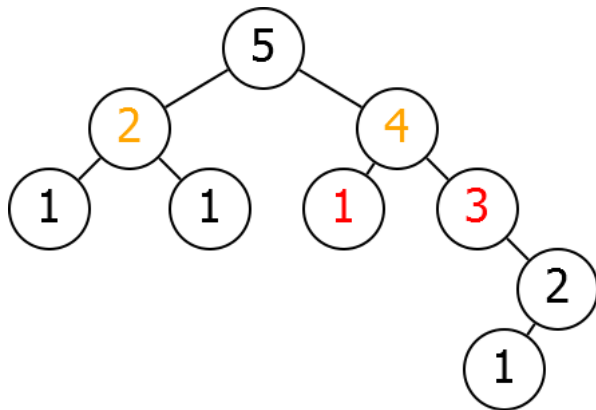
Problem

Which insertion would require the tree to be rebalanced in order to maintain the AVL property?



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Insertion

We need a new insertion algorithm that involves rebalancing the tree to maintain the AVL property.

Idea

AVLInsert(k, R)

Insert(k, R)

$N \leftarrow \text{Find}(k, R)$

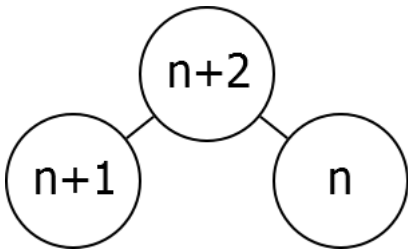
Rebalance(N)

Rebalancing

If

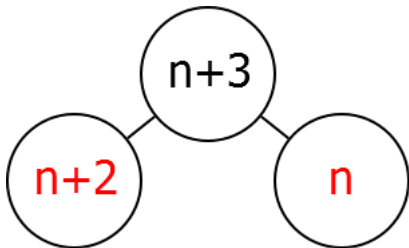
$$|N.\text{Left.Height} - N.\text{Right.Height}| \leq 1$$

fine.



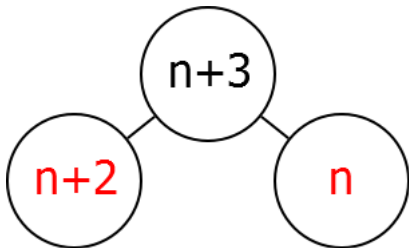
Problem

Difficulty if heights differ by more.



Problem

Difficulty if heights differ by more.



Never more than 2.

Code

Rebalance(*N*)

```
P ← N.Parent
if N.Left.Height > N.Right.Height + 1:
    RebalanceRight(N)
if N.Right.Height > N.Left.Height + 1:
    RebalanceLeft(N)
AdjustHeight(N)
if P ≠ null:
    Rebalance(P)
```

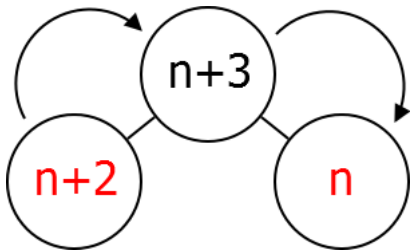

Adjust Height

AdjustHeight(N)

$$N.\text{Height} \leftarrow 1 + \max(\begin{array}{l} N.\text{Left}.\text{Height}, \\ N.\text{Right}.\text{Height} \end{array})$$

Rebalancing

If left subtree too heavy, rotate right:

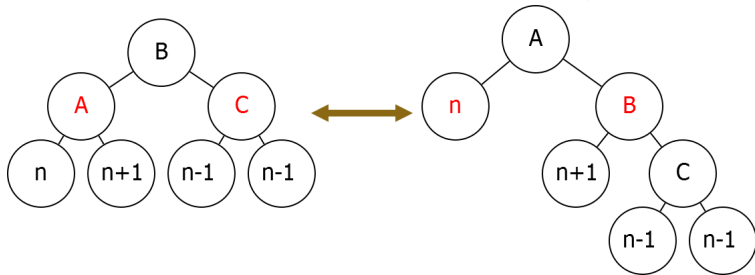


Bad Case

Doesn't work in this case.

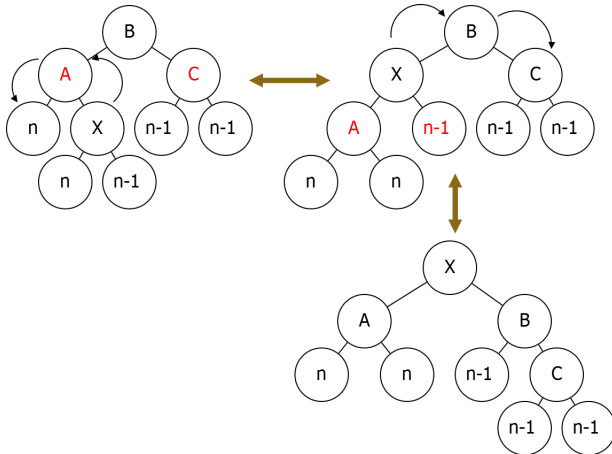
Notice this special case. Rotate right creates unbalance when the left node of the node to be rotated right has a heavier right node.

In this case right node of A has weight $n+1$ as compared to its left node of weight n



Fix

Must rotate left first.



Rebalance

RebalanceRight(N)

$M \leftarrow N.\text{Left}$

if $M.\text{Right}.\text{Height} > M.\text{Left}.\text{Height}$:

 RotateLeft(M)

RotateRight(N)

AdjustHeight on affected nodes

Outline

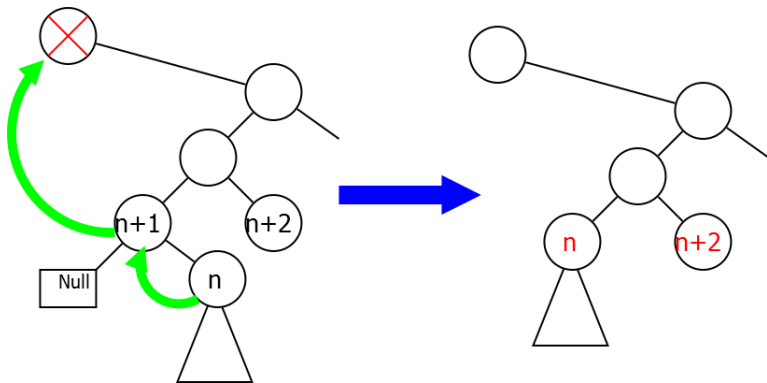
① AVL Trees

② Insert

③ Delete

Delete

Deletions can also change balance.



New Delete

AVLDelete(N)

Delete(N)

$M \leftarrow$ Left child of node replacing N

Rebalance(M)

Conclusion

Summary

AVL trees can implement all of the basic operations in $O(\log(n))$ time per operation.