# Linear Programming: Introduction

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## Learning Objectives

- See an example of the type of problem solved by linear programming.

# Factory

You are running a widget factory and trying to optimize your production procedures to save money.

# Machines vs. Workers

Can use combination of machines and workers.

- Have only 100 machines.
- Unlimited workers.
- Each machine requires 2 workers to operate.

# Production

- Each machine makes 600 widgets a day.
- Each worker makes 200 widgets a day.

# Limited Demand

Total demand for only $100,000$ widgets a day.

# Algebra

Let $W$ be the number of workers and $M$ the number of machines.

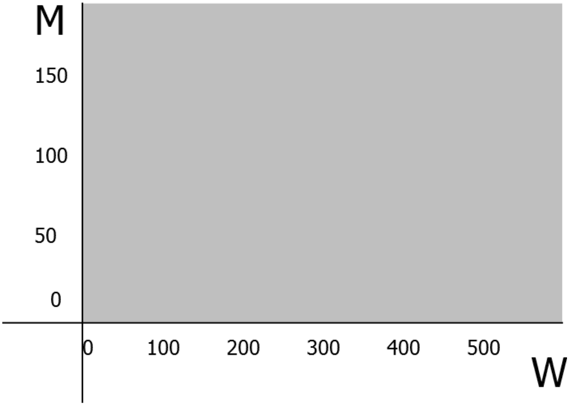
Constraints:

- $W \geq 0$.
- $100 \geq M \geq 0$.
- $W \geq 2M$. W-2M are unoccupied workers
- $100,000 \geq 200(W - 2M) + 600M$.

# Graph

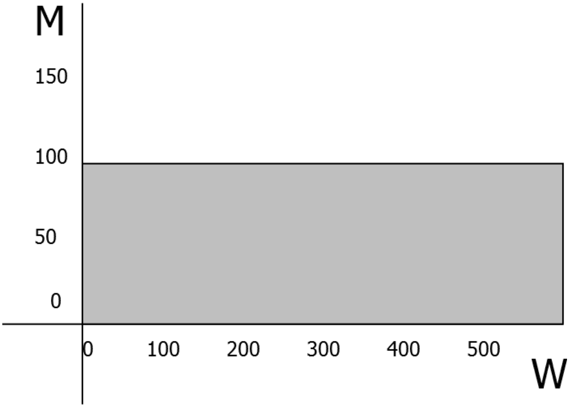
$$M, W \geq 0$$

Plane of possible values of M and W which satisfy these constraints



# Graph

$M \leq 100$



# Graph

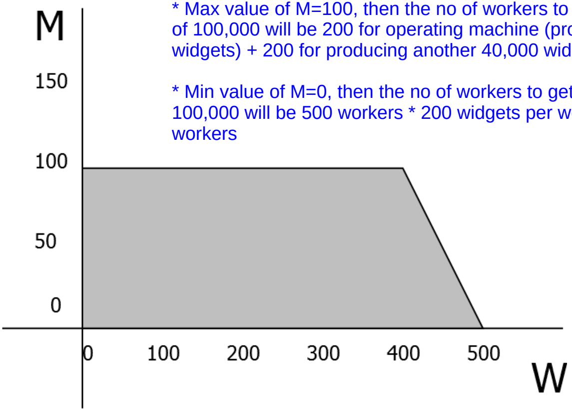
$$M + W \leq 500$$

When we look at our constraint based on the total demand.

Consider this plot consider

* Max value of M=100, then the no of workers to get to widget demand of 100,000 will be 200 for operating machine (produces 60,000 widgets) + 200 for producing another 40,000 widgets

* Min value of M=0, then the no of workers to get to widget demand of 100,000 will be 500 workers * 200 widgets per worker = 100, 000 workers

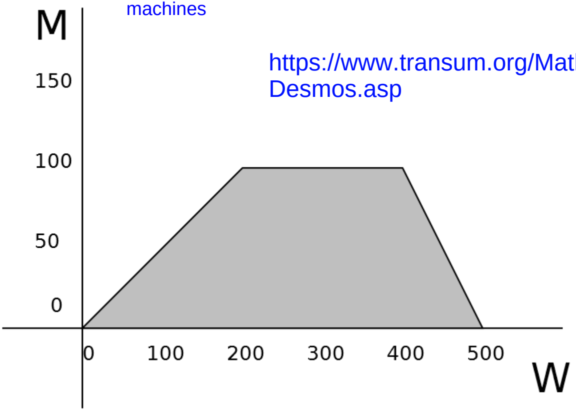


# Graph

Diagram of possible configurations:

Final constraint of Workers needs to be atleast twice that of number of machines

https://www.transum.org/Maths/Activity/Graph/Desmos.asp



# Profits

Profits are determined as follows:

- Each widget earns you \$1.
- Each worker costs you \$100/day.

Total profits (in dollars per day):

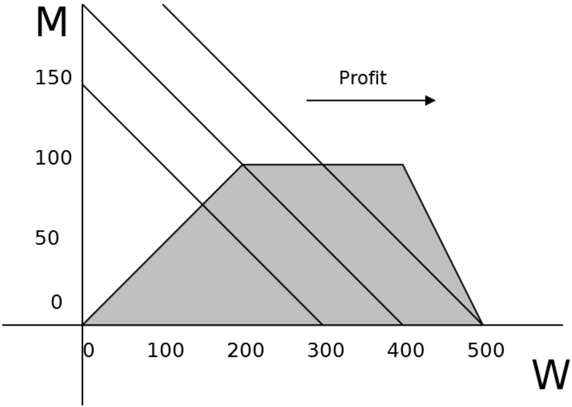
$$200(W-2M)+600M-100W = 100W+200M.$$

= 100W + 200M

# Graph

Profit mapped on graph:

These lines are lines of equal profits

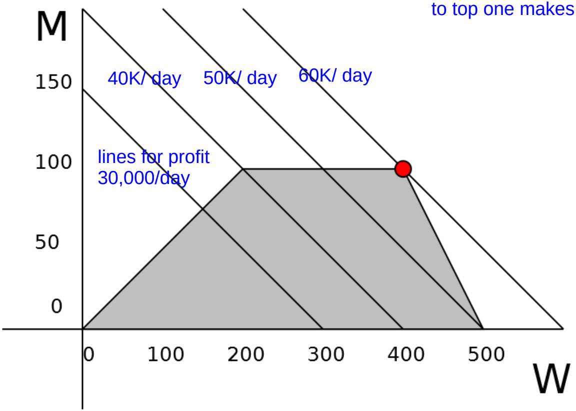


# Optimum

Best: $M = 100, W = 400$ [NB: A corner]

Profit $= \$60,000/\text{day}$.

If we go from left to right or from bottom to top one makes more profit



# Proof of Optimality

$$
\begin{array}{rll}
100 \cdot [ & 001 \cdot M + 000 \cdot W & \leq 100] \\
+0.5 \cdot [ & 200 \cdot M + 200 \cdot W & \leq 100,000] \\
\hline
 & 200 \cdot M + 100 \cdot W & \leq 60,000.
\end{array}
$$

M < 100

200W + 200M < 100,000

Here they are adding 100 times first constraint + 0.5 times second constraint

60K thus is max profit

# Summary

Maximized:

$$200M + 100W$$

Linear function

subject to constraints:

Linear Programming is nothing but minimizing/ maximing a linear function of variables subject to a bunch of linear inequality constraints

$$\begin{aligned} 0M + 1W &\geq 0 \\ 1M + 0W &\geq 0 \\ -1M + 0W &\geq -100 \\ -2M + 1W &\geq 0 \\ -1M - 1W &\geq -500 \end{aligned}$$

Linear Inequalities