

Waiting Time Problem

Assignment 05



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Our Problem

Waiting Time Problem

A long queue is observed at the coffee shop of the Faculty during rush hours. Propose a model to reduce the waiting time at the coffee shop.



A Usual Day at the Coffee Shop

- Small scale coffee shop.
- Shop is operated by two employers.
- The customer base includes the university crowd, including students, lecturers, and the non-academic staff.



Data Collection Process

Employers - Short Interview

Observational Data

Customers - Questionnaire

● Summary of Data Collected from Employers

- ❖ Rush hours : 7.30 - 8.00 , 9.00 - 9.30 , 12.00 - 1.00 , 4.00 - 4.30
- ❖ Busiest days : “No rush on Fridays and Saturdays”
- ❖ Top-selling food items : Nescafe, coffee, eggrolls, plain-tea, egg rotti
- ❖ The average length of the queue during a rush hour : no answer
- ❖ The average waiting time of an order during a rush hour : 1 - 2 minutes.
However, for items such as egg rotti, the wait time may extend to 4 to 5 minutes, **leading to longer queues.**



Data Collection Process

- **Data Collected from the Customers (SUMMARY)**
 - ❖ Rush hours : 7.30 - 8.00 , 9.00 - 9.30 , 12.00 - 1.00 , 3.00 - 4.00
 - ❖ Busiest days : Monday, Tuesday and Wednesday
 - ❖ Top-selling food items : Nescafe, coffee, eggrolls, plain-tea, ice-cream
 - ❖ The average length of the queue during a rush hour : 10 students
 - ❖ The average waiting time of an order during a rush hour : 5 to 7 minutes.



Model Development : **Key Variables**

- ❖ Customer Arrival rate
- ❖ Service Rate
- ❖ Order Complexity : Off the shelf, pre-prepared food
- ❖ Size of Order
- ❖ Number of Servers
- ❖ Waiting Area Design
- ❖ Cashier Time



Model Development

- ❖ Queueing theory is the area in mathematics to model queue related problems.
- ❖ Using queueing theory we can determine queueing systems.
- ❖ An example of a standard queueing system is $(M/M/C):(FCFS/\infty/\infty)$.

Where,

M: Markovian (or Poisson) arrivals or departures distribution (or equivalently exponential interarrival or service time distribution)

C : Number of servers

FCFS : Queue discipline is first come first serve

Inf: Limit of the customers in the system is infinity

Inf : Size of the source from which customers arrive is infinite.



Assumptions

- ❖ The queue has infinite capacity and follows a first-come-first-serve (FCFS) discipline.
- ❖ There is no limit to number of customers that can be catered by the coffee shop.



Adding another server parallel the 1st server

- ❖ The arrival and service process are both poisson distributed.
- ❖ There are two identical servers that serve customers in parallel.
- ❖ The queue has infinite capacity and follows a first-come-first-serve (FCFS) discipline.

So we can model this using M/M/2 (FCFS; inf; inf)

$$\text{Average number of customer in system} = \frac{\lambda^3}{\mu(4\mu^2 - \lambda^2)} + \frac{\lambda}{2\mu}$$

$$\text{Average wating time in the system} = \frac{\lambda^2}{\mu(4\mu^2 - \lambda^2)} + \frac{1}{\mu}$$

λ = Average rate of service

μ = Average rate of customer



Using Queuing Theory

- ❖ We expected arrival and service processes both to be poisson distributed.
- ❖ Hence both the interarrival distribution and service time distribution are exponential distributions.(independent in time)

So we can model this using M/M/1 (FCFS; inf; inf)

$$\text{Average number of customer in system} = \frac{\lambda}{\mu - \lambda}$$

$$\begin{aligned}\lambda &= \text{Average rate of service} \\ \mu &= \text{Average rate of customer}\end{aligned}$$

$$\text{Average waiting time in the system} = \frac{1}{\mu - \lambda}$$



Data Overview

- 86 Total data points
- On average, customers arrived every 21.8 seconds or every 0.363 minutes. Thus, the interarrival rate is 2.75 customers per minute.
- Service completion took an average of 41.1 seconds or 0.685 minutes. Thus, the service rate is 1.46 customers per minute.



Results

We used the M/M/s model for queueing analysis, but we faced some difficulties with fitting the real time data to the model.

- ❖ Our initial expectation was that the model would closely align with observed data, based on our assumptions
- ❖ However with a closer examination and modeling, it became evident that the actual data didn't fit as expected.
- We used a **chi-square goodness of fit test** to see whether the interarrival times and the service times follow an exponential distribution.
- **For interarrival times,**
 - Chi-square value - 28.21
- **For service times,**
 - Chi-square value - 14.57
- One possible recommendation is to get more data in order to reduce the effect of outliers and reinvestigate the chi-square goodness of fitness.



Our Suggestions



Adding another service counter

- ❖ Even though we cannot model using queueing theory, it is obvious that the waiting time will drop substantially if we add another counter.
- ❖ We identified that there is sufficient space to open another counter.
- ❖ On the other hand, If we hire extra staff to open the counter, we have to take into consideration the cost for hiring.



Additional Staff

- ❖ Having sufficient number of staff members during rush hours could minimize waiting times.
- ❖ Employing a student from the faculty during peak hours not only enhances efficiency but also provides an opportunity for the student to earn through part-time work.



Queue Line Barrier Implementation

- ❖ Queue line barriers will guide customers to stay in a line. As one of the main issues for waiting time, students making orders without a queue can be stated.
- ❖ This would violate the first come first serve basis which we have mentioned in the previous slide.



Efficient Equipment and Technology

- ❖ Allowing customers to scan QR codes of items with their smartphones and pay through the app and use POS system would increase efficiency of the transaction and also will minimize the waiting time.



Online Pre-Ordering App

- ❖ Use a platform for online ordering.
- ❖ This method would let the customers to track the preparation of their order which would save their time, in fact they can pick up the order when it's ready.



Conclusion

- ❖ In conclusion, considering all the suggestions, the best option is to open another counter and having two servers.
- ❖ Since the new science canteen was opened yesterday the queue at the coffee shop seemed to be very short, actually there was no queue.



References

1. Winston, W. L. (1991). Operations Research: Applications and Algorithms (2nd ed.). PWS-Kent Publishing.
2. Taha, H.A. (2018) Operations Research: An Introduction, Global Edition. Pearson Higher Ed.

