1. $u = 3 - 8x + 6y \qquad x^2 + y^2 = 36$

$$L(\lambda_1, x, y) = 3 - 8x + 6y + \lambda_1 (x^2 + y^2 - 36)$$

$$\begin{cases} L'_x = -8 + \lambda_1 \cdot 2x = 0 \\ L'_y = 6 + \lambda_1 \cdot 2 \cdot y = 0 \\ L'_{\lambda_1} = x^2 + y^2 - 36 = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{4}{\lambda_1} \\ y = -\frac{3}{\lambda_1} \\ \frac{16}{\lambda_1^2} + \frac{9}{\lambda_1^2} = 36 \end{cases} \Rightarrow$$

$$\begin{cases} x = \frac{4}{\lambda_1} \\ y = -\frac{3}{\lambda_1} \\ \lambda_1 = \sqrt{\frac{25}{36}} = \pm\frac{5}{6} \end{cases} \Rightarrow \begin{matrix} \left(\frac{5}{6}, \frac{24}{5}, -\frac{18}{5}\right) \\ \left(-\frac{5}{6}, -\frac{24}{5}; \frac{18}{5}\right) \end{matrix}$$

$$L''_{xx} = 2\lambda_1 \qquad L''_{yy} = 2\lambda_1 \qquad L''_{\lambda_1\lambda_1} = 0$$

$$L''_{x \cdot y} = 0 \qquad L''_{x\lambda_1} = 2x \qquad L''_{y\lambda_1} = 2y$$

$$\begin{pmatrix} L''_{\lambda_1\lambda_1} & L''_{\lambda_1 x} & L''_{\lambda_1 y} \\ L''_{x\lambda_1} & L''_{xx} & L''_{xy} \\ L''_{y\lambda_1} & L''_{yx} & L''_{yy} \end{pmatrix} = \begin{pmatrix} 0 & 2x & 2y \\ 2x & 2\lambda_1 & 0 \\ 2y & 0 & 2\lambda_1 \end{pmatrix}$$

$$\begin{vmatrix} 0 & 2x & 2y \\ 2x & 2\lambda_1 & 0 \\ 2y & 0 & 2\lambda_1 \end{vmatrix} = 0 \cdot \begin{vmatrix} 2\lambda_1 & 0 \\ 0 & 2\lambda_1 \end{vmatrix} - 2x \begin{vmatrix} 2x & 0 \\ 2y & 2\lambda_1 \end{vmatrix} +$$

$$+ 2y \begin{vmatrix} 2x & 2\lambda_1 \\ 2y & 0 \end{vmatrix} = -2x(2x \cdot 2\lambda_1 - 0 \cdot 2y) +$$

$$+ 2y(2x \cdot 0 - 2\lambda_1 \cdot 2y) = -8x^2\lambda_1 + (-8y^2\lambda_1) =$$

$$= -8\lambda_1(x^2+y^2) = -8\lambda_1 \cdot 36 = -288\lambda_1 \Rightarrow$$

$\Rightarrow$ точка $\left(\frac{5}{6}; \frac{24}{5}; -\frac{18}{5}\right)$ – минимум

точка $\left(-\frac{5}{6}; -\frac{24}{5}; \frac{18}{5}\right)$ – максимум.

2. $U = 2x^2 + 12xy + 32y^2 + 15$ $\quad x^2 + 16y^2 = 64$

$$L(\lambda_1, x, y) = 2x^2 + 12xy + 32y^2 + 15 + \lambda_1(x^2 + 16y^2 - 64)$$

$$\begin{cases} L'_{x} = 4x + 12y + 2\lambda_1 x = 0 \\ L'_{y} = 12x + 64y + 32\lambda_1 y = 0 \\ L'_{\lambda_1} = x^2 + 16y^2 - 64 = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} y = -\dfrac{2x + \lambda_1 x}{6} = -\dfrac{x(2+\lambda_1)}{6} \\ x = -\dfrac{16y(2+\lambda_1)}{6} = -\dfrac{8y(2+\lambda_1)}{3} \\ x^2 + \cancel{y}16y^2 - 64 = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} y = -\dfrac{x(2+\lambda_1)}{6} \\ x = \dfrac{8(2+\lambda_1)^2 \cdot x}{18} \\ x^2 + 16y^2 - 64 = 0 \end{cases} \Rightarrow \begin{cases} y = -\dfrac{x(2+\lambda_1)}{6} \\ (2+\lambda_1)^2 = \dfrac{9}{4} \\ x^2 + 16y^2 - 64 = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 2 - \lambda_{1_1} = \dfrac{3}{2} \\ 2 - \lambda_{1_2} = -\dfrac{3}{2} \end{cases} \Rightarrow \begin{cases} \lambda_{1_1} = \dfrac{1}{2} \\ \lambda_{1_2} = \dfrac{7}{2} \end{cases} \Rightarrow$$

для $\lambda_1 = \frac{1}{2}$:

$$\cancel{y}\quad y = -\frac{x\left(2+\frac{1}{2}\right)}{6} = -\frac{5x}{12} \Rightarrow$$

$$\Rightarrow x^2 + 16 \cdot \frac{25x^2}{144} - 64 = 0$$

$$x^2 + \frac{25x^2}{9} - 64 = 0$$

$$\frac{34x^2}{9} = 64 \Rightarrow x^2 = \frac{288}{17} \Rightarrow x_1 = \sqrt{\frac{288}{17}} \Rightarrow$$

$$x_2 = -\sqrt{\frac{288}{17}}$$

$$\cancel{y_1 =} \Rightarrow x_1 = 12\sqrt{\frac{2}{17}} \quad x_2 = -12\sqrt{\frac{2}{17}}$$

$$y_1 = -\frac{5}{12} \cdot 12\sqrt{\frac{2}{17}} = -5\sqrt{\frac{2}{17}}$$

$$y_2 = 5\sqrt{\frac{2}{17}} \Rightarrow \text{две точки: } \left(\frac{1}{2}, 12\sqrt{\frac{2}{17}}, -5\sqrt{\frac{2}{17}}\right)$$

$$\left(\frac{1}{2}, -12\sqrt{\frac{2}{17}}, 5\sqrt{\frac{2}{17}}\right)$$

для $\lambda_2 = \frac{7}{2}$:

$$y = -\frac{x\left(2+\frac{7}{2}\right)}{6} = -\frac{11x}{12} \Rightarrow$$

$$\Rightarrow x^2 + 16 \cdot \frac{121x^2}{144} - 64 = 0$$

$$x^2 + \frac{121x^2}{9} = 64 \Rightarrow \frac{130x^2}{9} = 64$$

$$x_1 = 12\sqrt{\frac{2}{65}}\ ;\quad x_2 = -12\sqrt{\frac{2}{65}}$$

$$y_1 = -\frac{11}{12} \cdot 12\sqrt{\frac{2}{65}} = -11\sqrt{\frac{2}{65}}$$

$$y_2 = 11\sqrt{\frac{2}{65}} \Rightarrow \left(\frac{7}{2}; -12\sqrt{\frac{2}{65}}; 11\sqrt{\frac{2}{65}}\right)$$

$$\left(\frac{7}{2}; 12\sqrt{\frac{2}{65}}; -11\sqrt{\frac{2}{65}}\right)$$

$$L''_{xx} = 4 + 2\lambda_1 \qquad L''_{yy} = 64 + 32\lambda_1$$

$$L''_{\lambda_1\lambda_1} = 0 \qquad L''_{xy} = 12$$

$$L''_{x\lambda_1} = 2x \qquad L''_{y\lambda_1} = 32y$$

$$\begin{pmatrix} L''_{\lambda_1\lambda_1} & L''_{\lambda_1 x} & L''_{\lambda_1 y} \\ L''_{x\lambda_1} & L''_{xx} & L''_{xy} \\ L''_{y\lambda_1} & L''_{yx} & L''_{yy} \end{pmatrix} = \begin{pmatrix} 0 & 2x & 32y \\ 2x & 4+2\lambda_1 & 12 \\ 32y & 12 & 64+32\lambda_1 \end{pmatrix}$$

$$\begin{vmatrix} 0 & 2x & 32y \\ 2x & 4+2\lambda_1 & 12 \\ 32y & 12 & 64+32\lambda_1 \end{vmatrix} = 0 - 2x \begin{vmatrix} 2x & 12 \\ 32y & 64+32\lambda_1 \end{vmatrix} +$$

$$+ 32y \begin{vmatrix} 2x & 4+2\lambda_1 \\ 32y & 12 \end{vmatrix} = -2x\left(2x(64+32\lambda_1) - 12 \cdot 32y\right) +$$

$$+ 32y\left(2x \cdot 12 - 32y(4+2\lambda_1)\right) = -128x^2(2+\lambda_1) +$$

$+768xy + 768xy - 2048y^2(2+\lambda_1) =$

$= -128x^2(2+\lambda_1) + 1536xy - 2048y^2(2+\lambda_1) =$

$= -128(2+\lambda_1)(x^2+16y^2) + 1536xy =$

$= -8192(2+\lambda_1) + 1536xy$

при $\left(\frac{1}{2}; 12\sqrt{\frac{2}{17}}; -5\sqrt{\frac{2}{17}}\right)$:

$\Delta = -8192\left(2+\frac{1}{2}\right) + 1536 \cdot 12\sqrt{\frac{2}{12}} \cdot \left(-5\sqrt{\frac{2}{17}}\right) < 0 \Rightarrow$

$\Rightarrow$ точка минимум

при $\left(\frac{1}{2}; -12\sqrt{\frac{2}{17}}; 5\sqrt{\frac{2}{17}}\right)$:

$\Delta = -8192\left(2+\frac{1}{2}\right) + 1536 \cdot \left(-12\sqrt{\frac{2}{17}}\right) \cdot \left(5\sqrt{\frac{2}{17}}\right) < 0 \Rightarrow$

$\Rightarrow$ точка минимум

при: $\left(\frac{7}{2}; -12\sqrt{\frac{2}{65}}; 11\sqrt{\frac{2}{65}}\right)$ и $\left(\frac{7}{2}; 12\sqrt{\frac{2}{65}}; -11\sqrt{\frac{2}{65}}\right)$

$\Delta < 0 \Rightarrow$ обе точки минимум

3. $\begin{cases} x^2 - y^2 + 3xy^3 - 2x^2y^2 + 2x - 3y - 5 = 0 \\ 3y^3 - 2x^2 + 2x^3y - 5x^2y^2 + 5 = 0 \end{cases}$

$x^2 - y^2 + 3xy^3 - 2x^2y^2 + 2x - 3y - 5 - 3y^3 + 2x^2 -$

$- 2x^3y + 5x^2y^2 - 5 = 0$

$u'_x = 2x + 3y^3 - 4xy^2 + 2 + 4x - 6x^2y + 10xy^2 =$

$= 6x + 3y^3 + 6xy^2 - 6x^2y + 2$

$u'_y = -2y + 9xy^2 - 4x^2y - 3 - 9y^2 - 2x^3 + 10x^2y =$

$= -2y + 9xy^2 + 6x^2y - 9y^2 - 2x^3 - 3$

$3x^2 - y^2 + 3xy^3 + 3x^2y^2 + 2x - 3y - 10 - 3y^3 - 2x^3y = 0$