1. $\int \frac{2x+3}{(x-2)(x+5)}dx = \int \left(\frac{A}{x-2} + \frac{B}{x+5}\right)dx =$

$= \int \frac{A(x+5)+B(x-2)}{(x-2)(x+5)}dx = \int \frac{x(A+B)+5A-2B}{(x-2)(x+5)}dx$

$\begin{cases} A+B=2 \\ 5A-2B=3 \end{cases} \Rightarrow \begin{cases} A=2-B \\ 5(2-B)-2B=3 \end{cases} \Rightarrow 10-7B=3 \Rightarrow$

$\Rightarrow B=1 \;,\; A=1 \Rightarrow$

$\Rightarrow \int \frac{x(A+B)+5A-2B}{(x-2)(x+5}dx = \int \left(\frac{1}{x-2} + \frac{1}{x+5}\right)dx =$

$= \ln(x-2) + \ln(x+5) + C$

2. $\int e^{2x}\cos 3x\,dx =$

$u = e^{2x} \Rightarrow du = 2e^{2x}dx$

$dV = \cos 3x\,dx \qquad V = \frac{\sin 3x}{3}$

$= \frac{e^{2x}\sin 3x}{3} - \int \frac{\sin 3x}{3} \cdot 2e^{2x}dx =$

$u = e^{2x} \qquad du = 2e^{2x}dx$

$dV = \sin 3x\,dx \qquad V = -\frac{\cos 3x}{3}$

$$= \frac{e^{2x}\sin 3x}{3} - \frac{2}{3}\left(-\frac{e^{2x}\cos 3x}{3} + \frac{2}{3}\int e^{2x}\cos 3x\,dx\right)$$

$$\int e^{2x}\cos 3x\,dx = \frac{e^{2x}\sin 3x}{3} + \frac{2e^{2x}\cos 3x}{9} - \frac{4}{9}\int e^{2x}\cos 3x\,dx$$

$$\frac{13}{9}\int e^{2x}\cos 3x\,dx = \frac{e^{2x}\sin 3x}{3} + \frac{2e^{2x}\cos 3x}{9}$$

$$\int e^{2x}\cos 3x\,dx = \frac{\cancel{3}e^{2x}(3\sin 3x + 2e^{2x}\cos 3x)}{13}$$

3. $$\int_0^{\ln 2} x\cdot e^{-x}\,dx = -xe^{-x}\Big|_0^{\ln 2} + \int_0^{\ln 2} e^{-x}\,dx =$$

$U = x \qquad dU = dx$

$dV = e^{-x}\,dx \qquad V = -e^{-x}$

$$= -\frac{\ln 2}{2} - \int_0^{\ln 2} e^{-x}\,d(-x) = -\frac{\ln 2}{2} - e^{-x}\Big|_0^{\ln 2} =$$

$$= -\frac{\ln 2}{2} - \frac{1}{2} + 1 = \frac{1-\ln 2}{2}$$

4. $y = e^x \quad [-\pi; \pi]$

ряд Фурье:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^x dx = \frac{1}{2\pi} e^x \Big|_{-\pi}^{\pi} = \frac{e^{\pi} - e^{-\pi}}{2\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cos nx\, dx$$

$$U = e^x \qquad dU = e^x dx$$

$$dV = \cos nx\, dx \qquad V = \frac{1}{n}\sin nx$$

$$a_n = \frac{1}{\pi}\left(\frac{e^x \overset{=0}{\sin nx}}{n}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{1}{n}\sin nx \cdot e^x dx \right)$$

$$U = e^x \qquad dU = e^x dx$$

$$dV = \sin nx\, dx \qquad V = -\frac{1}{n}\cos nx$$

$$a_n = \frac{1}{\pi}\left(-\frac{1}{n}\left(-\frac{e^x \overset{=-1}{\cos nx}}{n}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\left(-\frac{1}{n}\cos nx\, e^x\right)dx\right)\right) =$$

$$a_n = \frac{1}{\pi}\left(-\frac{1}{n}\left(\frac{+e^{\pi}}{n} - \frac{e^{-\pi}}{n} + \frac{1}{n}\int_{-\pi}^{\pi}\cos nx\, e^x dx\right)\right)$$

$$a_n = \frac{1}{\pi}\left(\frac{e^{-\pi} - e^{\pi}}{n^2} - \frac{1}{n^2}\int_{-\pi}^{\pi} e^x \cos nx\, dx\right)$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cos nx\,dx = \frac{1}{\pi}\left(\frac{e^{-\pi}-e^{\pi}}{n^2} - \frac{1}{n^2}\int_{-\pi}^{\pi} e^x \cos nx\,dx\right)$$

$$\frac{n^2+1}{n^2}\int_{-\pi}^{\pi} e^x \cos nx\,dx = \frac{e^{-\pi}-e^{\pi}}{n^2}$$

$$\int_{-\pi}^{\pi} e^x \cos nx\,dx = \frac{e^{-\pi}-e^{\pi}}{n^2+1}$$

$$a_n = \frac{1}{\pi}\left(\frac{e^{-\pi}-e^{\pi}}{n^2} - \frac{1}{n^2}\left(\frac{e^{-\pi}-e^{\pi}}{n^2+1}\right)\right) =$$

$$= \left(\frac{e^{-\pi}-e^{\pi}}{\pi n^2}\right)\left(1 - \frac{1}{n^2+1}\right)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \sin nx\,dx = \frac{1}{\pi}\left(-\frac{e^x \cos nx}{n}\Big|_{-\pi}^{\pi} + \frac{1}{n}\int_{-\pi}^{\pi} e^x \cos nx\,dx\right)$$

$u = e^x \qquad du = e^x\,dx$

$dv = \sin nx\,dx \qquad v = -\frac{1}{n}\cos nx\,dx$

$$b_n = \frac{1}{\pi}\left(\frac{e^{\pi}-e^{-\pi}}{n} + \frac{1}{n}\int_{-\pi}^{\pi} e^x \cos nx\,dx\right)$$

$u = e^x \qquad du = e^x\,dx$

$dv = \cos nx\,dx \qquad v = \frac{1}{n}\sin nx\,dx$

$$b_n = \frac{1}{\pi}\left(\frac{e^{\pi}-e^{-\pi}}{n} + \frac{1}{n}\left(\underbrace{\frac{e^x \sin nx}{n}\Big|_{-\pi}^{\pi}}_{=0} - \frac{1}{n}\int_{-\pi}^{\pi} e^x \sin nx\,dx\right)\right)$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} e^{x}\sin nx\,dx = \frac{1}{\pi}\left(\frac{e^{\pi}-e^{-\pi}}{n} - \frac{1}{n^2}\int_{-\pi}^{\pi} e^{x}\sin nx\,dx\right)$$

$$\frac{n^2+1}{n^2}\int_{-\pi}^{\pi}\sin nx\, e^{x}\,dx = \frac{e^{\pi}-e^{-\pi}}{n}$$

$$\int_{-\pi}^{\pi} e^{x}\sin nx\,dx = \frac{(e^{\pi}-e^{-\pi})\cdot n}{n^2+1}$$

$$b_n = \frac{1}{\pi}\left(\frac{e^{\pi}-e^{-\pi}}{n} - \frac{1}{n^2}\left(\frac{n(e^{\pi}-e^{-\pi})}{n^2+1}\right)\right)$$

$$b_n = \frac{e^{\pi}-e^{-\pi}}{\pi\cdot n}\left(1-\frac{1}{n^2+1}\right)$$

$$f(x) = e^{x} = \frac{e^{\pi}-e^{-\pi}}{2\pi} + \sum_{n=1}^{\infty}\left(\left(\frac{e^{-\pi}-e^{\pi}}{\pi n^2}\right)\left(1-\frac{1}{n^2+1}\right)\cos nx + \cancel{\cos nx} + \frac{e^{\pi}-e^{-\pi}}{\pi n}\left(1-\frac{1}{n^2+1}\right)\sin nx\right)$$

ряд Маклорена:

$$e^{x} = e^{0} + \frac{e^{0}}{0!}x^{1} + \frac{e^{0}}{2!}x^{2} + \frac{e^{0}}{3!}x^{3} + \frac{e^{0}}{4!}x^{4} + \ldots =$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots \frac{x^n}{n!} = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

График построил, но он не сходится https://www.desmos.com/calculator/vzarr4mp7c