1. $\int (2x^2 - 2x - 1 + \sin x - \cos x + \ln x + e^x)\,dx =$

$= \int 2x^2 dx - \int 2x dx - \int 1 dx + \int \sin x dx -$

$- \int \cos x dx + \int \ln x dx + \int e^x dx = \frac{2x^3}{3} -$

$- 2 \cdot \frac{x^2}{2} - x - \cos x - \sin x + x\ln x - x + e^x +$

$+ C = \frac{2x^3}{3} - x^2 - 2x - \cos x - \sin x + x\ln x + e^x + C$

2. $\int (2x + 6xz^2 - 5x^2y - 3\ln z)\,dx =$

$= \int 2x\,dx + \int 6xz^2 dx - \int 5x^2 y\,dx - \int 3\ln z\,dx =$

$= x^2 + 3z^2x^2 - \frac{5yx^3}{3} - 3\ln(z)\,x + C$

3. $\int_0^{\pi} 3x^2 \sin 2x\,dx = 3\int_0^{\pi} x^2 \sin 2x\,dx =$

$u = x^2 \Rightarrow du = 2x\,dx$

$dv = \sin 2x\,dx \Rightarrow v = -\frac{\cos 2x}{2}$

$= 3\left(-\frac{x^2\cos 2x}{2}\Big|_0^{\pi} - \int_0^{\pi}\left(-\frac{\cos 2x}{2}\right)2x\,dx = \right.$

$= \frac{-3\pi^2}{2} + \frac{1}{2}\int_0^{\pi} 2x\cos 2x\,dx =$

$u = x \quad du = dx$

$dv = \cos 2x\,dx \quad v = \frac{\sin 2x}{2}$

$$= -\frac{3\pi^2}{2} + \frac{x\sin 2x}{2}\Big|_0^{\pi} - \frac{1}{2}\int_0^{\pi} \sin 2x\,dx =$$

$$= -\frac{3\pi^2}{2} - \frac{1}{4}\int_0^{\pi} \sin 2x\,d2x = -\frac{3\pi^2}{2} + \frac{\cos 2x}{4}\Big|_0^{\pi} =$$

$$= -\frac{3\pi^2}{2} + \frac{1}{4} - \frac{1}{4} = -\frac{3\pi^2}{2}$$

4. $$\int \frac{1}{\sqrt{x+1}}\,dx = \int \frac{1}{\sqrt{x+1}}\,d(x+1) = \int \frac{1}{\sqrt{t}}\,dt =$$

$t = x+1$

$$= \frac{\sqrt{t}}{1/2} + C = 2\sqrt{t} + C = 2\sqrt{x+1} + C$$

1. $$\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$$

$$\lim_{n\to\infty} \frac{(n+1)^{n+1}}{((n+1)!)^2} : \frac{n^n}{(n!)^2} = \frac{(n+1)^{n+1}\cdot(n!)^2}{(n+1)^2\cdot(n!)^2\cdot n^n} =$$

$$= \frac{(n+1)^{n-1}}{n^n} = \lim_{n\to\infty} \frac{\left(\frac{n+1}{n}\right)^n}{n+1} = \lim_{n\to\infty} \frac{e}{n+1} = 0$$

ряд сходится, т.к. $0 < 1$.

2. $\sum\limits_{n=1}^{\infty} \frac{n}{2^n}$

$\lim\limits_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2}$ – ряд расходится
<1

3. $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n+\ln n}$ – монотонно убывает

$\lim\limits_{n\to\infty} \left|\frac{(-1)^n}{n+\ln n}\right| = \lim\limits_{n\to\infty} \frac{1}{n+\ln n} = \frac{1}{\infty} = 0 \Rightarrow$

$\Rightarrow$ ряд сходится

$\sum\limits_{n=1}^{\infty} \left|\frac{(-1)^n}{n+\ln n}\right| = \sum\limits_{n=1}^{\infty} \frac{1}{n+\ln n} \sim O\left(\frac{1}{n^1}\right) \Rightarrow$

$\Rightarrow$ ряд сходится условно.

4. $\sum\limits_{n=1}^{\infty} \frac{3^n}{2^n}$

$\lim\limits_{n\to\infty} \left(n\left(\frac{a_n}{a_{n+1}} - 1\right)\right) = \lim\limits_{n\to\infty} \left(n\left(\frac{3^n}{2^n} : \frac{3^{n+1}}{2^{n+1}} - 1\right)\right) =$

$= \lim\limits_{n\to\infty} \left(n\left(\frac{2}{3} - 1\right)\right) = \lim\limits_{n\to\infty} \left(-\frac{n}{3}\right) = -\infty \Rightarrow$

$\Rightarrow -\infty < 1$ – ряд расходится

5. $f(x) = \ln(16x^2)$ $\quad a = 1$

$f^{0}(\cancel{x}) = \ln(16x^2)^{\cancel{x}} = \ln 16$

$f'(x) = \frac{1}{16x^2} \cdot 32x = \frac{2}{x} = \frac{2}{1} = 2$

$f''(x) = -\frac{2}{x^2} = -2$

$f'''(x) = +\frac{4}{x^3} = 4$

$f^{IV}(x) = -\frac{12}{x^4} = -12$

$$f(x) = \frac{\ln 16 (x-1)^0}{0!} + \frac{2(x-1)^1}{1!} - \frac{2(x-1)^2}{2!} +$$

$$+ \frac{4(x-1)^3}{3!} - \frac{12(x-1)^4}{4!} + \ldots =$$

$$= \ln 16 + 2\sum (-1)^{n-1} \frac{(x-1)^n}{n!}$$

6. $f(x)=x^2$, $[-2;2]$

$$a_0=\frac{1}{2\pi}\int_{-2}^{2} f(x)dx=\frac{1}{2\pi}\int_{-2}^{2} x^2dx=\frac{1}{2\pi}\cdot\frac{x^3}{3}\Big|_{-2}^{2}=$$

$$=\frac{4}{3\pi}-\left(-\frac{4}{3\pi}\right)=\frac{8}{3\pi}$$

$$a_n=\frac{1}{\pi}\int_{-2}^{2} f(x)\cos nx\,dx=\frac{1}{\pi}\int_{-2}^{2} x^2\cos nx\,dx$$

$u=x^2 \Rightarrow du=2x\,dx$

$dv=\cos nx\,dx \Rightarrow v=\frac{1}{n}\sin nx$

$$a_n=\frac{1}{\pi}\cdot x^2\cdot\frac{1}{n}\sin nx\Big|_{-2}^{2}-\frac{1}{\pi}\int_{-2}^{2}\frac{1}{n}\sin nx\cdot 2x\,dx=$$

$$=\frac{x^2}{\pi n}\cdot\sin nx\Big|_{-2}^{2}-\frac{2}{\pi n}\int_{-2}^{2} x\sin nx\,dx$$

$$\cancel{=\frac{x^2}{\pi n}\sin nx\Big|_{-2}^{2}-}$$

$u=x \Rightarrow du=dx$

$dv=\sin nx\,dx \Rightarrow v=-\frac{1}{n}\cos nx$

$$a_n=\frac{x^2}{\pi n}\sin nx\Big|_{-2}^{2}-\frac{2}{\pi n}\left(-\frac{x}{n}\cos nx\Big|_{-2}^{2}-\int_{-2}^{2}\left(-\frac{1}{n}\cos nx\,dx\right)\right)=$$

$$=\frac{x^2}{\pi n}\sin nx\Big|_{-2}^{2}+\frac{2x}{\pi n^2}\cos nx\Big|_{-2}^{2}+\frac{-2}{\pi n^2}\int_{-2}^{2}\cos nx\,dx=$$

$$= \frac{x^2}{\pi n}\sin nx\Big|_{-2}^{2} + \frac{2x}{\pi n^2}\cos nx\Big|_{-2}^{2} - \frac{2}{\pi n^2}\cdot\frac{1}{n}\sin nx\Big|_{-2}^{2} =$$

$$= \frac{4\sin 2n}{\pi n} - \frac{4\sin(-2n)}{\pi n} + \frac{4\cos 2n}{\pi n^2} - \frac{(-4)\cdot\cos(-4n)}{\pi n^2} -$$

$$-\left(\frac{2\sin 2n}{\pi n^3} - \frac{2\sin(-2n)}{\pi n^3}\right) = \frac{8\sin 2n}{\pi n} +$$

$$+ \frac{8\cos 2n}{\pi n^2} - \frac{4\sin 2n}{\pi n^3}$$

$$b_n = \frac{1}{\pi}\int_{-2}^{2} x^2 \sin nx\,dx = \frac{1}{\pi}\left(-\frac{x^2\cos nx}{n}\Big|_{-2}^{2} + \frac{2}{n}\int_{-2}^{2} x\cos nx\,dx\right)$$

$u = x^2 \quad du = 2x\,dx$

$dV = \cancel{\sin}\, nx\,dx \quad V = -\frac{1}{n}\cos nx$

$u = x \quad du = dx$

$dV = \cos nx\,dx \quad V = \frac{1}{n}\sin nx$

$$b_n = \frac{1}{\pi}\left(-\frac{4\cos 2n}{n} + \frac{4\cos 2n}{n} + \frac{2}{n}\left(\frac{x\sin nx}{n}\Big|_{-2}^{2} -\right.\right.$$

$$\left. - \frac{1}{n}\int_{-2}^{2}\sin nx\,dx\right) = \frac{1}{\pi}\left(\frac{2}{n}\left(\frac{2\sin 2}{n} - \frac{2\sin 2n}{n} - \right.\right.$$

$$- \frac{1}{n^2}\left(\sin 2n + \sin 2n\right) = -\frac{4\sin 2n}{\pi n^3}$$

$$f(x) = x^2 = \frac{8}{3\pi} + \sum_{n=1}^{\infty}\left(\left(\frac{8\sin 2n}{\pi n} + \frac{8\cos 2n}{\pi n^2} - \frac{4\sin 2n}{\pi n^3}\right)\cos nx +\right.$$
$$\left. + \left(\frac{-4\sin 2n}{\pi n^3}\right)\sin nx\right)$$

График похож, но не полностью https://www.desmos.com/calculator/wsxmrndoqn