

$$1. U = x^3 + 3xy^2 + z^2 - 3x - 36y + 2z + 26$$

$$U'_x = 3x^2 + 3y^2 - 39$$

$$U''_{xy} = 6y$$

$$U''_{xz} = 0$$

$$U''_{xx} = 6x$$

$$U'_y = 6xy - 36$$

$$U''_{yx} = 6y$$

$$U''_{yz} = 0$$

$$U''_{yy} = 6x$$

$$U'_z = 2z + 2$$

$$U'_{zx} = 0$$

$$U'_{zy} = 0$$

$$U''_{zz} = 2$$

исчезающие производные равны

$$2. \ U = \frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2$$

$$U'_x = -\frac{256}{x^2} + \frac{2x}{y}$$

$$U''_{xy} = -\frac{2x}{y^2} \quad U''_{xz} = 0 \quad U''_{xx} = -\frac{256 \cdot 2}{x^3} + \frac{2}{y}$$

$$U'_y = -\frac{x^2}{y^2} + 2\frac{y}{z}$$

$$U''_{yx} = -\frac{2x}{y^2} \quad U''_{yz} = -\frac{2y}{z^2} \quad U''_{yy} = \frac{2x^2}{y^3} + \frac{2}{z}$$

$$U'_z = -\frac{y^2}{z^2} + 2z$$

$$U''_{zx} = 0 \quad U''_{zy} = -\frac{2y}{z^2} \quad U''_{zz} = \frac{2y^2}{z^3} + 2$$

существование производных

$$3. U = x^2 + y^2 + z^2 ; \vec{C}(-9, 8, -12), M(8, -12, 9)$$

$$U_x' = 2x \quad U_y' = 2y \quad U_z' = 2z$$

$$\text{grad } U = (2x, 2y, 2z)$$

$$\text{grad } U|_{(8, -12, 9)} = (16, -24, 18)$$

$$|\vec{C}| = \sqrt{x_0^2 + y_0^2 + z_0^2} = \sqrt{81 + 64 + 144} = 17$$

$$\vec{C}_0 = \frac{\vec{C}}{|\vec{C}|} = \left(\frac{-9}{17}, \frac{8}{17}, -\frac{12}{17} \right)$$

$$\begin{aligned} U'_{\text{grad } U}|_{(8, -12, 9)} &= -\frac{9}{17} \cdot 16 - \frac{8}{17} \cdot 24 - \frac{12}{17} \cdot 18 = \\ &= \frac{-144 - 192 - 216}{17} = -\frac{552}{17} \end{aligned}$$

$$4. U = e^{x^2+y^2+z^2}, \vec{d} = (4, -13, -16), L(-16, 4, -13)$$

$$U'_x = 2xe^{x^2+y^2+z^2}$$

$$U'_y = 2ye^{x^2+y^2+z^2}$$

$$U'_z = 2ze^{x^2+y^2+z^2}$$

$$\text{grad } U \Big|_{(-16, 4, -13)} = (-32e^{441}, 8e^{441}, -26e^{441})$$

$$|\vec{d}| = \sqrt{16+16+256} = 21$$

$$\vec{d}_0 = \frac{\vec{d}}{|\vec{d}|} = \left(\frac{4}{21}, -\frac{13}{21}, -\frac{16}{21} \right)$$

$$U'_{\text{grad } U \Big|_{(-16, 4, -13)}} = -32e^{441} \cdot \frac{4}{21} - 8e^{441} \cdot \frac{13}{21} +$$

$$+ 26e^{441} \cdot \frac{16}{21} = \frac{2e^{441}}{21} (-16 \cdot 4 - 4 \cdot 13 + 13 \cdot 16) =$$

$$= \frac{8e^{441}}{21} (-16 - 13 + 52) = \frac{184e^{441}}{21}$$

$$5. U = \log_{21} (x^2 + y^2 + z^2), F(-19; 8; -4)$$

$$U'_x = \frac{2x}{(x^2 + y^2 + z^2) \ln 21}$$

$$U'_y = \frac{2y}{(x^2 + y^2 + z^2) \ln 21}$$

$$U'_z = \frac{2z}{(x^2 + y^2 + z^2) \ln 21}$$

$$\text{grad } U(-19; 8; -4) = \left(\frac{2 \cdot (-19)}{(-19)^2 + 64 + 16} \right) \ln 21 ;$$

$$\left(\frac{2 \cdot 8}{441 \ln 21}, \frac{-8}{441 \ln 21} \right) = \left(\frac{-38}{441 \ln 21}, \frac{16}{441 \ln 21}, \frac{-8}{441 \ln 21} \right)$$

$$|\text{grad } U|_F = \sqrt{\left(\frac{-38}{441 \ln 21} \right)^2 + \left(\frac{16}{441 \ln 21} \right)^2 + \left(\frac{-8}{441 \ln 21} \right)^2} =$$

$$= \sqrt{\frac{7764}{(441 \ln 21)^2}} = \frac{42}{441 \ln 21} \leftarrow \text{uechte oberein}$$

$$6. U = x^2y + \frac{1}{3}y^3 + 2x^2 + 3y^2 - 1$$

$$U'_x = 2xy + 4x \quad U'_y = x^2 + y^2 + 6y$$

$$\text{ реш } \begin{cases} 2xy + 4x = 0 \\ x^2 + y^2 + 6y = 0 \end{cases} \Rightarrow \begin{cases} x(2y + 4) = 0 \\ x^2 + y^2 + 6y = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x_1 = 0 \\ y_1 = -2 \end{cases} \Rightarrow x^2 + (-2)^2 + 6(-2) = 0 \Rightarrow$$

~~$x^2 + y^2 + 6y = 0 + 4 - 12$~~

$$\Rightarrow x_{2,3} = \pm \sqrt{8}$$

$$\text{ при } x=0 \Rightarrow 0 + y^2 + 6y = 0 \Rightarrow \begin{cases} y_1 = 0 \\ y_2 = -6 \end{cases}$$

получаем точки:

$$(0, 0), (0, -6), (\sqrt{8}, -2), (-\sqrt{8}, -2)$$

$$U''_{xx} = 2y + 4$$

~~$U''_{xy} = U''_{yx} = 2x$~~

$$U''_{yy} = 2y + 6$$

$$\begin{pmatrix} U_{xx}'' & U_{xy}'' \\ U_{yx}'' & U_{yy}'' \end{pmatrix} = \begin{pmatrix} 2y+4 & 2x \\ 2x & 2y+6 \end{pmatrix} \quad \Delta_1 = 2y+4$$

$$\Delta_2 = \begin{vmatrix} 2y+4 & 2x \\ 2x & 2y+6 \end{vmatrix} = (2y+4)(2y+6) - 4x^2$$

при $(0; 0) =$

$$\Delta_1 = 4, \quad \Delta_2 = 24$$

$(0, 0)$ — точка минимума

при $(0; -6) =$

$$\Delta_1 = -8, \quad \Delta_2 = (-12+4)(-12+6) = 48$$

$(0, -6)$ — точка максимума

при $(\sqrt{8}; -2) =$

$\Delta_1 = 0$ эту точку можно дальше
не считать по теории Гессе

при $(-\sqrt{8}; -2)$ — аналогично, т.к. $\Delta_1 = 0$

$$7. u = e^{-\frac{x}{2}}(x^2 + y^2)$$

$$u'_x = -\frac{e^{-\frac{x}{2}}}{2}(x^2 + y^2) + 2xe^{-\frac{x}{2}}$$

$$u'_y = 2ye^{-\frac{x}{2}}$$

$$u''_{xx} = \frac{e^{-\frac{x}{2}}}{4}(x^2 + y^2) + (-e^{\frac{x}{2}} \cdot x) + 2e^{-\frac{x}{2}} + \left(-\frac{e^{-\frac{x}{2}}}{2} \cdot 2x\right) =$$

$$= \frac{e^{-\frac{x}{2}}}{4}(x^2 + y^2) - 2x \cdot e^{-\frac{x}{2}} + 2e^{-\frac{x}{2}} \cancel{- e^{-\frac{x}{2}} \cdot 2x}$$

$$u''_{xy} = u''_{yx} = -ye^{-\frac{x}{2}}$$

$$u''_{yy} = 2e^{-\frac{x}{2}}$$

$$\begin{cases} e^{-\frac{x}{2}} \left(-\frac{x^2 + y^2}{2} + 2x \right) = 0 \\ 2ye^{-\frac{x}{2}} = 0 \end{cases} \Rightarrow e^{-\frac{x}{2}} = 0 \text{ не имеет корней}$$

$$\begin{cases} y = 0 \\ -\frac{x^2 + y^2}{2} + 2x = 0 \end{cases} \Rightarrow -\frac{x^2}{2} + 2x = 0 \Rightarrow$$

$$\Rightarrow \begin{cases} x_1 = 0 \\ -\frac{x_2}{2} + 2 = 0 \end{cases} \quad x_2 = 4$$

находим точки:

$$(0; 0), (4; 0)$$

$$\begin{pmatrix} U_{xx}'' & U_{xy}'' \\ U_{yx}'' & U_{yy}'' \end{pmatrix} = \begin{pmatrix} e^{-\frac{x}{2}} \left(\frac{x^2+y^2}{4} - 2x + 2 \right) & -ye^{-\frac{x}{2}} \\ -e^{-\frac{x}{2}}y & 2e^{-\frac{x}{2}} \end{pmatrix}$$

при $(0; 0)$:

$$\Delta_1 = 2 \quad \Delta_2 = 2 \cdot 2 - 0 = 4$$

точка $(0; 0)$ — минимум

при $(4; 0)$:

$$\Delta_1 = e^{-2} \left(\frac{16}{4} - 8 + 2 \right) = -2e^{-2}$$

$$\Delta_2 = (-2e^{-2}) \cdot 2e^{-2} - 0 = -4e^{-4}$$

точка $(4; 0)$ является седловой