1. $y = \frac{1}{x} + \frac{2}{x^2} - \frac{5}{x^3} + \sqrt{x} - \sqrt[3]{x} + \frac{3}{\sqrt{x}}$

$$y' = -\frac{1}{x^2} - \frac{4}{x^3} + \frac{15}{x^4} + \frac{1}{2\sqrt{x}} - \frac{1}{3\sqrt[3]{x^2}} - \frac{3}{2\sqrt{x^3}}$$

2. $y = x\sqrt{1+x^2}$

$$y' = \sqrt{1+x^2} + x \cdot \frac{1}{2\sqrt{1+x^2}} \cdot (1+x^2)' =$$

$$= \sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}}$$

3. $y = \frac{2x}{1-x^2}$

$$y' = \frac{2(1-x^2) - 2x \cdot (-2x)}{(1-x^2)^2} = \frac{2(1-x^2) + 4x^2}{(1-x^2)^2} =$$

$$= \frac{2(1+x^2)}{(1-x^2)^2}$$

4. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

$$y' = \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \left(1 + \frac{1}{2\sqrt{x+\sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)\right)$$

5. $y=(x^2+2)^5(3x-x^3)^3$

$$y'=(x^2+2)^5(3x-x^3)^3\cdot\left(\ln\left((x^2+2)^5\cdot(3x-x^3)^3\right)\right)'=$$

$$\cancel{=(x^2+2)^5(3x-x^3)^3\cdot\left(\frac{1}{(x^2+2)^5}+\frac{1}{(3x-x^3)^3}\right)\cdot 5(x^2+2)^4}$$

$\cancel{\cdot 2x\cdot}$

$$=(x^2+2)^5(3x-x^3)^3\left(\frac{5(x^2+2)^4\cdot 2x}{(x^2+2)^5}+\frac{3(3x-x^3)^2\cdot(3-3x^2)}{(3x-x^3)^3}\right)=$$

$$=(x^2+2)^5(3x-x^3)^3\left(\frac{10x}{(x^2+2)}+\frac{9-9x^2}{3x-x^3}\right)=$$

$$=(x^2+2)^5(3x-x^3)^3\left(\frac{10x}{x^2+2}+\frac{3}{x}\right)$$

6. $y=\sqrt[x]{x}$

$$y'=\sqrt[x]{x}\left(\ln(\sqrt[x]{x})\right)'=\sqrt[x]{x}\cdot\frac{(\sqrt[x]{x})'}{\sqrt[x]{x}}=\frac{x^{-\frac{1}{x}}}{x}$$

7. $y=\dfrac{(2-x^2)^3(x-1)^2}{(2x^3-3x)e^x}$

$$y'=\frac{(2-x^2)^3(x-1)^2}{(2x^3-3x)e^x}\cdot(\ln y)'$$

$$(\ln y)'=\left(\ln(2-x^2)^3+\ln(x-1)^2-\ln(2x^3-3x)-\ln e^x\right)'=$$

$$=\frac{2\cdot(-2x)}{(2-x^2)}+\frac{2}{x-1}-\frac{6x^2-3}{2x^3-3x}-1$$

$$y' = \frac{(2-x^2)^3 \cdot (x-1)^2}{(2x^3-3x)e^x}\left(-\frac{4x}{2-x^2} + \frac{2}{x-1} - \frac{6x^2-3}{2x^3-3x} - 1\right)$$

8. $$\begin{cases} x = \dfrac{t^2}{t-1} \\ y = \dfrac{t}{t^2-1} \end{cases} \Rightarrow \begin{cases} x'_t = \dfrac{2t}{1} = 2t \\ y'_t = \dfrac{1}{2t} \end{cases} \Rightarrow$$

$$y'_x = \frac{y'_t}{x'_t} = \frac{1}{2t} : \frac{1}{2t} = 1$$

9. $$\operatorname{arctg}\frac{y}{x} = \ln\sqrt{x^2+y^2} \Rightarrow$$

$$\Rightarrow \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot y' = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{2\sqrt{x^2+y^2}}(2x+2y\cdot y')$$

$$\frac{y'}{1+\left(\frac{y}{x}\right)^2} = \frac{x+y\cdot y'}{x^2+y^2} \Rightarrow y'\left(\frac{1}{1+\left(\frac{y}{x}\right)^2} - \frac{y}{x^2+y^2}\right) =$$

$$= \frac{x}{x^2+y^2} \Rightarrow y'\left(\frac{x^2-y}{x^2+y^2}\right) = \frac{x}{x^2+y^2} \Rightarrow$$

$$\Rightarrow y' = \frac{x}{x^2-y}$$

10. $y = \ln\left(x + \sqrt{x^2+1}\right)$

$$y' = \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{2x}{2\sqrt{x^2+1}}\right) = \frac{\sqrt{x^2+1}+x}{\left(x+\sqrt{x^2+1}\right)\sqrt{x^2+1}} =$$

$$= \frac{1}{\sqrt{x^2+1}}$$

11. $y = x \cdot \ln\left(x+\sqrt{x^2+1}\right) - \sqrt{x^2+1}$

$$y' = \ln\left(x+\sqrt{x^2+1}\right) + \frac{x}{x+\sqrt{x^2+1}}\left(1 + \frac{2x}{2\sqrt{x^2+1}}\right) -$$

$$- \frac{2x}{2\sqrt{x^2+1}} = \ln\left(x+\sqrt{x^2+1}\right) + \frac{x}{\sqrt{x^2+1}} - \frac{x}{\sqrt{x^2+1}} =$$

$$= \ln\left(x+\sqrt{x^2+1}\right)$$

12. $y = \arcsin(\sin x)$

$$y' = -\frac{\cos x}{\sqrt{1-\sin^2 x}} = -\frac{\cos x}{\sqrt{\cos^2 x}} = -\frac{\cos x}{\pm|\cos x|} = \pm 1$$

13. $x = ?$ $y = ?$ $P = 144$

$$\begin{cases} x + y = 144 \\ x \cdot y = S_{max} \end{cases} \Rightarrow \begin{cases} y = 144 - x \\ S_m = xy \end{cases} \Rightarrow S = x(144 - x)$$

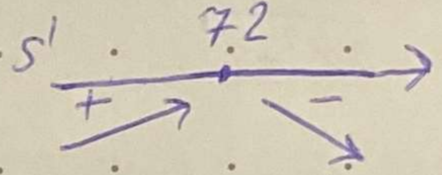
$$S'_x = 144 - 2x = 0$$

$$144 - 2x = 0$$

$$x = \underline{72}$$

берем $x = 73$

$$S'_{73} = 144 - 2 \cdot 73 = -2$$



берем $x = 71$

$$S'_{71} = 144 - 2 \cdot 71 = 2$$

$$y = 144 - 72 = \underline{72}$$

$$S_{max} = 72 \cdot 72 = 5184$$