

$$1. y = \frac{1}{x} + \frac{2}{x^2} - \frac{5}{x^3} + \sqrt{x} - \sqrt[3]{x} + \frac{3}{\sqrt{x}}$$

$$y' = -\frac{1}{x^2} - \frac{4}{x^3} + \frac{15}{x^4} + \frac{1}{2\sqrt{x}} - \frac{1}{3\sqrt[3]{x^2}} - \frac{3}{2\sqrt{x^3}}$$

$$2. y = x\sqrt{1+x^2}$$

$$y' = \sqrt{1+x^2} + x \cdot \frac{1}{2\sqrt{1+x^2}} \cdot (1+x^2)' =$$

$$= \sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}}$$

$$3. y = \frac{2x}{1-x^2}$$

$$y' = \frac{2(1-x^2) - 2x(-2x)}{(1-x^2)^2} = \frac{2(1-x^2) + 4x^2}{(1-x^2)^2} =$$

$$= \frac{2(1+x^2)}{(1-x^2)^2}$$

$$4. y = \sqrt{x + \sqrt{x + \sqrt{x}}}$$

$$y' = \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \left(1 + \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}} \right) \right)$$

$$5. y = (x^2 + 2)^5 (3x - x^3)^3$$

$$y' = (x^2 + 2)^5 (3x - x^3)^3 \cdot (\ln((x^2 + 2)^5 (3x - x^3)^3))' =$$

$$= \cancel{(x^2 + 2)^5 (3x - x^3)^3} \left(\frac{1}{\cancel{(x^2 + 2)^5}} + \frac{1}{\cancel{(3x - x^3)^3}} \right) \cdot \cancel{5(x^2 + 2)^4}$$

2x.

$$= (x^2 + 2)^5 (3x - x^3)^3 \left(\frac{5(x^2 + 2)^4 \cdot 2x}{(x^2 + 2)^5} + \frac{3(3x - x^3)^2 \cdot (3 - 3x^2)}{(3x - x^3)^3} \right) =$$

$$= (x^2 + 2)^5 (3x - x^3)^3 \left(\frac{10x}{(x^2 + 2)} + \frac{9 - 9x^2}{3x - x^3} \right) =$$

$$= (x^2 + 2)^5 (3x - x^3)^3 \left(\frac{10x}{x^2 + 2} + \frac{3}{x} \right)$$

$$6. y = \sqrt[x]{x}$$

$$y' = \sqrt[x]{x} (\ln(\sqrt[x]{x}))' = \sqrt[x]{x} \cdot \frac{(\sqrt[x]{x})'}{\sqrt[x]{x}} = \frac{x^{-\frac{1}{x}}}{x}$$

$$7. y = \frac{(2 - x^2)^3 (x - 1)^2}{(2x^3 - 3x)e^x}$$

$$y' = \frac{(2 - x^2)^3 (x - 1)^2}{(2x^3 - 3x)e^x} \cdot (\ln y)'$$

$$(\ln y)' = (\ln(2 - x^2)^3 + \ln(x - 1)^2 - \ln(2x^3 - 3x) - \ln e^x)' =$$

$$= \frac{3 \cdot (-2x)}{(2 - x^2)} + \frac{2}{x - 1} - \frac{6x^2 - 3}{2x^3 - 3x} - 1$$

$$y' = \frac{(2-x^2)^3 \cdot (x-1)^2}{(2x^3-3x)e^x} \left(-\frac{6x}{2-x^2} + \frac{2}{x-1} - \frac{6x^2-3}{2x^3-3x} - 1 \right)$$

$$8. \begin{cases} x = \frac{t^2}{t-1} \\ y = \frac{t}{t^2-1} \end{cases} \Rightarrow \begin{cases} x'_t = \frac{2t}{1} = 2t \\ y'_t = \frac{1}{2t} \end{cases} \Rightarrow$$

$$y'_x = \frac{y'_t}{x'_t} = \frac{1}{2t} : \frac{1}{2t} = 1$$

$$9. \operatorname{arctg} \frac{y}{x} = \ln \sqrt{x^2+y^2} \Rightarrow$$

$$\Rightarrow \frac{1}{1+\left(\frac{y}{x}\right)^2} y' = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{2\sqrt{x^2+y^2}} (2x+2y y')$$

$$\frac{y'}{1+\left(\frac{y}{x}\right)^2} = \frac{x+y y'}{x^2+y^2} \Rightarrow y' \left(\frac{1}{1+\left(\frac{y}{x}\right)^2} - \frac{y}{x^2+y^2} \right) =$$

$$= \frac{x}{x^2+y^2} \Rightarrow y' \left(\frac{x^2-y}{x^2+y^2} \right) = \frac{x}{x^2+y^2} \Rightarrow$$

$$\Rightarrow y' = \frac{x}{x^2-y}$$

$$10. y = \ln(x + \sqrt{x^2 + 1})$$

$$y' = \frac{1}{x + \sqrt{x^2 + 1}} \cdot \left(1 + \frac{2x}{2\sqrt{x^2 + 1}}\right) = \frac{\sqrt{x^2 + 1} + x}{(x + \sqrt{x^2 + 1})\sqrt{x^2 + 1}} =$$

$$= \frac{1}{\sqrt{x^2 + 1}}$$

$$11. y = x \cdot \ln(x + \sqrt{x^2 + 1}) - \sqrt{x^2 + 1}$$

$$y' = \ln(x + \sqrt{x^2 + 1}) + \frac{x}{x + \sqrt{x^2 + 1}} \left(1 + \frac{2x}{2\sqrt{x^2 + 1}}\right) -$$

$$- \frac{2x}{2\sqrt{x^2 + 1}} = \ln(x + \sqrt{x^2 + 1}) + \frac{x}{\sqrt{x^2 + 1}} - \frac{x}{\sqrt{x^2 + 1}} =$$

$$= \ln(x + \sqrt{x^2 + 1})$$

$$12. y = \arcsin(\sin x)$$

$$y' = -\frac{\cos x}{\sqrt{1 - \sin^2 x}} = -\frac{\cos x}{\sqrt{\cos^2 x}} = -\frac{\cos x}{\pm \cos x} = \pm 1$$

13. $x = ?$ $y = ?$ $P = 144$

$$\begin{cases} x + y = 144 \\ x \cdot y = S_{\max} \end{cases} \Rightarrow \begin{cases} y = 144 - x \\ S_{\max} = xy \end{cases} \Rightarrow S = x(144 - x)$$

$$S'_x = 144 - 2x = 0$$

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$$x = \underline{72}$$

Берем $x = 73$

$$S'_{73} = 144 - 2 \cdot 73 = -2$$

Берем $x = 71$

$$S'_{71} = 144 - 2 \cdot 71 = 2$$

$$y = 144 - 72 = \underline{72}$$

$$S_{\max} = 72 \cdot 72 = 5184$$

