

$$\begin{aligned}
 1. \int (2x^2 - 2x - 1 + \sin x - \cos x + \ln x + e^x) dx &= \\
 &= \int 2x^2 dx - \int 2x dx - \int 1 dx + \int \sin x dx - \\
 &\quad - \int \cos x dx + \int \ln x dx + \int e^x dx = \frac{2x^3}{3} - \\
 &\quad - 2 \cdot \frac{x^2}{2} - x - \cos x - \sin x + x \ln x - x + e^x + C \\
 &= \frac{2x^3}{3} - x^2 - 2x - \cos x - \sin x + x \ln x + e^x + C
 \end{aligned}$$

$$\begin{aligned}
 2. \int (2x + 6xz^2 - 5x^2y - 3\ln z) dx &= \\
 &= \int 2x dx + \int 6xz^2 dx - \int 5x^2y dx - \int 3\ln z dx = \\
 &= x^2 + 3z^2x^2 - \frac{5y x^3}{3} - 3\ln(z)x + C
 \end{aligned}$$

$$3. \int_0^{\pi} 3x^2 \sin 2x dx = 3 \int_0^{\pi} x^2 \sin 2x dx =$$

$$U = x^2 \Rightarrow dU = 2x dx$$

$$dV = \sin 2x dx \Rightarrow V = -\frac{\cos 2x}{2}$$

$$= 3 \left(-\frac{x^2 \cos 2x}{2} \right) \Big|_0^{\pi} - \int_0^{\pi} \left(-\frac{\cos 2x}{2} \right) 2x dx =$$

$$= \frac{-3\pi^2}{2} + \frac{1}{2} \int_0^{\pi} 2x \cos 2x dx =$$

$$U = x \quad dU = dx$$

$$dV = \cos 2x dx \quad V = \frac{\sin 2x}{2}$$

$$= -\frac{3\pi^2}{2} + \frac{x \sin 2x}{2} \Big|_0^\pi - \frac{1}{2} \int_0^\pi \sin 2x dx =$$

$$= -\frac{3\pi^2}{2} - \frac{1}{4} \int_0^\pi \sin 2x d(2x) = -\frac{3\pi^2}{2} + \frac{\cos 2x}{4} \Big|_0^\pi =$$

$$= -\frac{3\pi^2}{2} + \frac{1}{4} - \frac{1}{4} = -\frac{3\pi^2}{2}$$

$$4. \int \frac{1}{\sqrt{x+1}} dx = \int \frac{1}{\sqrt{x+1}} d(x+1) = \int \frac{1}{\sqrt{t}} dt =$$

$$t = x+1$$

$$= \frac{\sqrt{t}}{1/2} + C = 2\sqrt{t} + C = 2\sqrt{x+1} + C$$

$$1. \sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$$

$$\lim_{n \rightarrow \infty} \frac{(n+1)^{n+1}}{(n+1)!^2} \cdot \frac{n^n}{(n!)^2} = \frac{(n+1)^{n+1} \cdot (n!)^2}{(n+1)^2 \cdot (n!)^2 \cdot n^n} =$$

$$= \frac{(n+1)^{n+1}}{n^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{n+1}{n}\right)^n}{n+1} = \lim_{n \rightarrow \infty} \frac{e}{n+1} = 0$$

mag exognax, m.k. $a < 1$

$$2. \sum_{n=1}^{\infty} \frac{n}{2^n}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} \quad - \text{наглядно}$$

$$3. \sum_{n=1}^{\infty} \frac{(-1)^n}{n + \ln n} \quad - \text{монотонно убывает}$$

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^n}{n + \ln n} \right| = \lim_{n \rightarrow \infty} \frac{1}{n + \ln n} \doteq \frac{1}{\infty} = 0 \Rightarrow$$

\Rightarrow нагл. сходима

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n + \ln n} \right| = \sum_{n=1}^{\infty} \frac{1}{n + \ln n} \sim O\left(\frac{1}{n^1}\right) \Rightarrow$$

\Rightarrow нагл. сходима. условно.

$$4. \sum_{n=1}^{\infty} \frac{3^n}{2^n}$$

$$\lim_{n \rightarrow \infty} \left(n \left(\frac{a_n}{a_{n+1}} - 1 \right) \right) = \lim_{n \rightarrow \infty} \left(n \left(\frac{3^n}{2^n} - \frac{3^{n+1}}{2^{n+1}} - 1 \right) \right) =$$

$$= \lim_{n \rightarrow \infty} \left(n \left(\frac{2}{3} - 1 \right) \right) = \lim_{n \rightarrow \infty} \left(n \left(-\frac{1}{3} \right) \right) = -\infty \Rightarrow$$

$\Rightarrow -\infty < 1$ - нагл. расходится

$$5. \quad f(x) = \ln(16x^2) \quad a=1$$

$$f'(x) = \ln(16x^2)^2 = \ln 16$$

$$f'(x) = \frac{1}{16x^2} \cdot 32x = \frac{2}{x} = \frac{2}{1} = 2$$

$$f''(x) = -\frac{2}{x^2} = -2$$

$$f'''(x) = +\frac{4}{x^3} = 4$$

$$f^{(iv)}(x) = -\frac{12}{x^4} = -12$$

$$\begin{aligned} f(x) &= \frac{\ln 16(x-1)^0}{0!} + \frac{2(x-1)^1}{1!} - \frac{2(x-1)^2}{2!} + \\ &+ \frac{4(x-1)^3}{3!} - \frac{12(x-1)^4}{4!} + \dots = \\ &= \ln 16 + 2 \sum (-1)^{n-1} \frac{(x-1)^n}{n!} \end{aligned}$$

$$6. f(x) = x^2 ; [-2, 2]$$

$$a_0 = \frac{1}{2\pi} \int_{-2}^2 f(x) dx = \frac{1}{2\pi} \int_{-2}^2 x^2 dx = \frac{1}{2\pi} \left. \frac{x^3}{3} \right|_{-2}^2 = \frac{16}{3\pi}$$

$$= \frac{4}{3\pi} - \left(-\frac{4}{3\pi} \right) = \frac{8}{3\pi}$$

$$a_n = \frac{1}{\pi} \int_{-2}^2 f(x) \cos nx dx = \frac{1}{\pi} \int_{-2}^2 x^2 \cos nx dx$$

$$U = x^2 \Rightarrow dU = 2x dx$$

$$dV = \cos nx dx \Rightarrow V = \frac{1}{n} \sin nx$$

$$a_n = \frac{1}{\pi} \cdot x^2 \cdot \frac{1}{n} \sin nx \Big|_{-2}^2 - \frac{1}{\pi} \int_{-2}^2 \frac{1}{n} \sin nx \cdot 2x dx =$$

$$= \frac{x^2}{\pi n} \cdot \sin nx \Big|_{-2}^2 - \frac{2}{\pi n} \int_{-2}^2 x \sin nx dx =$$

~~$$= \frac{x^2}{\pi n} \sin nx \Big|_{-2}^2$$~~

$$U = x \Rightarrow dU = dx$$

$$dV = \sin nx dx \Rightarrow V = -\frac{1}{n} \cos nx$$

$$a_n = \frac{x^2}{\pi n} \sin nx \Big|_{-2}^2 - \frac{2}{\pi n} \left(-\frac{x}{n} \cos nx \Big|_{-2}^2 - \int_{-2}^2 \left(-\frac{1}{n} \cos nx \right) dx \right) =$$

$$= \frac{x^2}{\pi n} \sin nx \Big|_{-2}^2 + \frac{2x}{\pi n^2} \cos nx \Big|_{-2}^2 + \frac{-2}{\pi n^2} \int_{-2}^2 \cos nx dx =$$

$$\begin{aligned}
 &= \frac{x^2}{\pi n} \sin nx \Big|_{-2}^2 + \frac{2x}{\pi n^2} \cos nx \Big|_{-2}^2 - \frac{2}{\pi n^2} \cdot \frac{1}{n} \sin nx \Big|_{-2}^2 = \\
 &= \frac{4 \sin 2n}{\pi n} - \frac{4 \sin(-2n)}{\pi n} + \frac{4 \cos 2n}{\pi n^2} - \frac{(-4) \cos(-4n)}{\pi n^2} = \\
 &= \left(\frac{2 \sin 2n}{\pi n^3} - \frac{2 \sin(-2n)}{\pi n^3} \right) + \frac{8 \sin 2n}{\pi n} + \\
 &\quad + \frac{8 \cos 2n}{\pi n^2} - \frac{4 \sin 2n}{\pi n^3}
 \end{aligned}$$

Попробовал построить график разложения Фурье, но похоже, что сделал задание не верно – функция не приближается к x^2 - <https://www.desmos.com/calculator/95dygzxwxm>