Another Approach in Traffic Intersection Modeling Based on Queue Theory

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***Abstract*— The traffic control using traffic light in the intersection becomes complex since the huge growth of the vehicular in the crowded city. Traffic jam couldn't be avoided since the capacity of the road is less than the amount of the existing car in the road. Queue on the intersection couldn't be avoided in the peak hours. This kind of conventional traffic control couldn't help the problem of traffic jam in the intersection especially in the metropolitan city at peak hours. With queue theory approach, we can optimize the traffic control. But the M/M/1 model of queue system could not satisfy the intersection model with more than one road line. So we introduce the M/M/m model to overcome the problem.**

Keywords*—*System Modeling, Multiple Server Model, M/M/m Model, Traffic Control Optimization

I. Introduction

Traffic has attracted much attention from researcher to research in many ways including research of traffic model in vehicular ad hoc networks (VANETs). Road Traffic modeling has become an important issue after the IEEE 802.11p was released. Traffic intersection based on queue theory has been introduced as one of the traffic model in vehicular networks.

Traffic control using traffic light in the intersection becomes complex since the huge growth of the vehicular in the crowded city. Traffic jam couldn't be avoided since the capacity of the road is less than the amount of the existing car in the road. Queue on the intersection couldn't be avoided in the peak hours. This kind of conventional traffic control couldn't help the problem of traffic jam in the intersection especially in the metropolitan city at peak hours.

The existence of traffic control should give benefit; even though, it doesn't give much help in the very high density of the road traffic. In this project, I'm going to use queueing theory as traffic model in the intersection to improve the performance of the traffic control.

The aim of using queueing theory in improving the performance of the traffic control is to map the behavior of the vehicles in the intersection. We can assume the intersection as a system with an input and an output in queue system. As long as we have this assumption, we can model the traffic in intersection as a queue and calculate the expected delay time or waiting time in the queue in the system to predict how crowded the road.

Traffic intersection based on the queue theory has been discussed by [1] by assuming the system as single server system which is using M/M/1 as a model. I quite disagree with the author of [1]; therefore, I want to introduce another approach to model this system. The explanation of the model will be discussed in section 2 thoroughly including mathematical equation. Section 3 will discuss about traffic model for an intersection using queue theory, including comparison between existing model and my model. Section 4 will discuss about model validation. Section 5 will discuss about performance metrics and analysis. Section 6 will give the conclusion of this simulation. The last part will mention about references.

II. Queue Models

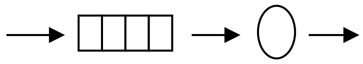
Queue Model is a system model which is represented by a queue of arriving customer in a system. There are three main components in the queue model; those are customers, queues, and servers. In queue system, customers are generated by an input source according to a statistical distribution. The customers have to join the queue in order to get a service from the server. At various times, customers are selected for service by the server using a certain queue discipline. There are 6 basic components of queueing theory based on [2]:

- Arrival process: a process that describe how customers arrive. If the customers arrive at times t_1, t_2, …,t_j, the random variables $\tau_j = t_j - t_{j-1}$ are called interarrival times. The most common arrival process is Poisson arrivals, which means the interarrival times are independent and identically distributed (IID) and are exponentially distributed.
- Service time distribution: it describes how long the service will take. It is common to assume that the service times are random variables, which are IID. The distribution most commonly used is the exponential distribution.
- Number of servers: the terminal room may have more than one server if the servers are all identical. If the servers are not identical, the identical servers should be divided into groups of identical servers with separate queues for each group. In this case each group is queueing system.
- Number of buffers (system capacity): the maximum customer which can stay in the queue line.
- Population size: the total number of potential customers who can ever come to the service center.

- Service discipline: the order in which the customers are served. The most common discipline is First Come First Served (FCFS) or usually called First In First Out (FIFO).

A. *Single Server System*

Single server system can be represented as customers who make a queue line in the system as we can see in figure 1. A single server processes the customers one at a time. An arriving customer that find the server in idle condition will enter the service immediately; otherwise, it will enter a buffer of a queue and join at the end of the queue to wait until the server becomes idle.



B. *Little's Law*

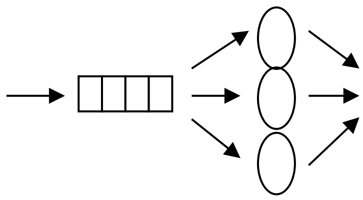
Based on [2], the theorem most commonly used in queueing theory is Little's law, which allows us to relate the mean number of jobs in any system with the mea time spent in the system as follows:

$$n = \lambda \times \tau$$

Where n is the mean number in the system, λ is the arrival rate, and τ is the mean response time. This relationship applies to all systems or parts of systems in which the number of jobs entering the system is equal to those completing service.

C. *Multiple Servers System*

Multiple servers system can be represented as customers who make a queue line in the system as we can see in figure 2. Multiple servers could process the group of customer at a time as much as the capacity of servers. An arriving customer that finds the idle condition in any servers can enter service immediately; otherwise, it enters a buffer and join at the end of the queue until the server becomes idle.



The multiple servers system is also known as M/M/m queue. The number of m is starting from 1 until the certain value of m (1,2,…,m). If m is equal to 1, then the system will become M/M/1 queue which is single server system. The shorthand notation like M/M/m is introduced by Kendall to characterize the range of queueing models.

Some relevant performance measures in the M/M/m model can be applied for traffic light system [2].

- Traffic intensity: $\rho = \lambda/(m\mu)$ (1)
- Probability that the service facility is idle (probability of 0 costumer in the system)

$$p_0 = \left[1 + \frac{(m\rho)^m}{m!(1-\rho)} + \sum_{n=1}^{m-1} \frac{(m\rho)^n}{n!}\right]^{-1} \quad (2)$$

- Probability of queueing

$$\varrho = \frac{(m\rho)^m}{m!(1-\rho)} p_0 \quad (3)$$

- The mean number of vehicle in the sytem

$$\mathrm{E}[n] = m\rho + \frac{\rho\varrho}{(1-\rho)} \quad (4)$$

- The mean response time in the system

  - $\mathrm{E}[r] = {}^{1}/_{\mu}\,(1 + \frac{\varrho}{m(1-\rho)})$ (5)

III. Traffic Model for an Intersection Using Queue Theory

The traffic model in an intersection can be represented as figure 3. We can assume each line of the intersection as 1 system. In the system, there are customers, queue, and servers. In this traffic model, a system is represented as customer that is processed by multiple servers at the same time. If we compare this model with the model introduced by [1] which is shown by figure 4, we can derive that the customers in the traffic intersection model should be processed with multiple servers. The vehicle as a customer has a probability of choosing the way in the intersection. The road can be divided into several lines. If we assume that there are 3 lines in a road and each line represents the selected path of the vehicle, then each vehicle in each line will have the same opportunity to access the intersection. Therefore, by using this assumption, we can apply this traffic intersection model by using multiple servers instead of single server model.

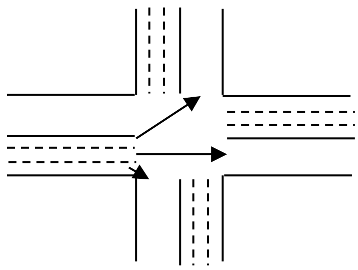


Figure 3.Multiple servers traffic intersection model

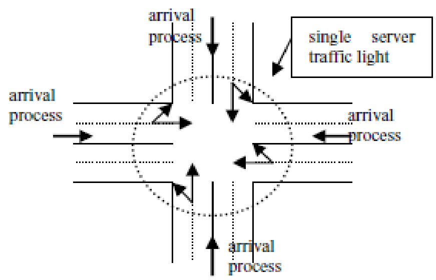


Figure 4. Single server traffic intersection model [1]

Since we are using multiple servers system, then we can apply M/M/m queue theory. The M/M/m refers to the basic components of queue theory. The first "M" stands for a "memoryless" distribution of inter-arrival times. The second "M" stands for "memoryless" distribution of "i.i.d" service times. The third "m" stands for the number of server in the system (multiple servers). The vehicle queue has FIFO (first in first out) as queue discipline.

Traffic arrival and service times at a given intersection are considered as independent random variables, with known distributions. Here we are using Poisson distribution due to the random nature of traffic arrival. [4] also explain that Poisson distribution usually makes a good fit for the memoryless nature of the exponential distribution.

In this model, the arriving vehicles are modeled by a Poisson process with mean arrival rate λ (vehicles per unit of time). Service time is defined as the time used to cross the intersection by individual vehicle in each line during the green light of the traffic light. Total service time (T_s) is defined as total of the service time from all vehicles from starting of green light until starting of red light.

$$T_s^j = t_{out1}^j + t_{out2}^j + \cdots + t_{out(f_{-1})}^j + t_{outf}^j \tag{9}$$

$$T_s = T_s^1 + T_s^2 + \cdots + T_s^m \tag{10}$$

Where t_{out1}^j is the departure time (service time) for the first vehicles leaving with j (j = 1, 2, …, m) denotes the j-th-line of the road. T_s^j is total of service time for each vehicle of each line. T_s is a total service time for all lines (servers) in the road. The service time (t_{out1}^j) can be obtained by calculating the departure time (t_d) and arrival time (t_a) where $t_{out1}^j = \mathrm{t_{d1}} - \mathrm{t_{a1}}$.

We define the interval time for red light in the intersection as 60 seconds (T_{green}) and green light as 20 seconds (T_{red}). This assumption is used by considering the environment that we used which is 4-way intersection. There are two events that occur in this model: arrival event and departure event. The arrival event can happen in both conditions which are red light condition and green light condition. While the departure event, it only occurs in green light condition.

The interchange from green light into red light, or vice versa, can be shown in figure 5. In figure 5-a, the vehicle may come after the green light change into red light. The ΔT is the excess time, so we can define the T_{red} = 60 – ΔT seconds and T_{green} = 20 seconds. T_x is the previous event which this event may be arrival event or departure event.

For the case 2, which is shown in figure 5-b, the vehicle can't depart after the green light turns to red light. Therefore T_d is defined as the starting time of next green light and ΔT = 0 or there is no excess time after interchange of the light. At the end of green light, T_{green} become 20 s and T_{red} start count down from 60 s.

The last possible case can be seen in figure 5-c which the vehicle can arrive after the red light turns into green light. Since the departure time T_d from the previous event is set into the starting green time, so there is no excess time ΔT. The vehicle will start depart right after the light turns into green light.

IV. Validation of Model

Before we analyze the result of simulation, we should validate the outcome of simulation with the mathematical analysis. To obtain the outcome for validation purpose, we have to bypass the red light condition so the traffic will always in green light condition.

A. Validation of M/M/1 Model

From the simulation by using validation model with

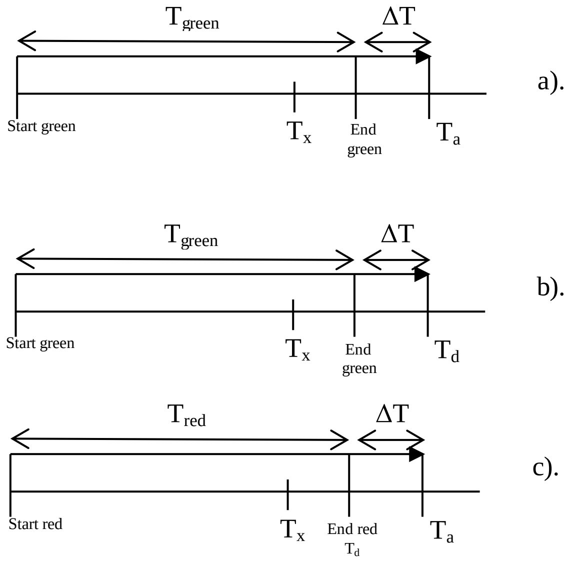


Figure 5, Traffic light interchange model

bypass the red light using $\lambda = \frac{1}{3}$ and $\mu = \frac{1}{2}$, we obtain this result of M/M/1 model.

Mean number of job in system = 2.03552 job

Mean time spent in the system = 6.11845 sec

We were using the equation of [1] to calculate the mean response time (E[r]) and mean number of job in the system (E[n]).

$$\mathrm{E}[r] = \frac{1/\mu}{1-\rho} = \frac{1/\mu}{1-\lambda/\mu} = \frac{1}{\mu - \lambda}$$

$$\mathrm{E}[r] = \frac{1}{\frac{1}{2} - \frac{1}{3}} = 6$$

$$\mathrm{E[n]} = \lambda \times \mathrm{E[r]} = \frac{1}{3} \times 6 = 2$$

From the mathematical calculation, we obtain E[r] = 6 seconds and E[n] = 2 jobs. These values are similar with the value obtained in simulation.

B. Validation of M/M/3 Model

From the simulation by using validation model with bypass the red light using $\lambda = \frac{1}{3}$ and $\mu = \frac{1}{2}$, we obtain this result of M/M/3 model.

Mean number of job in system = 0.514498 job

Mean time spent in the system = 1.5484 sec

We were using the equation of [1] to calculate the mean response time (E[r]) and mean number of job in the system (E[n]).

$$\rho = \lambda/(m\mu) = 0.2222$$

$$p_0 = \left[1 + \frac{(m\rho)^m}{m!\,(1-\rho)} + \sum_{n=1}^{m-1} \frac{(m\rho)^n}{n!}\right]^{-1} = 0.512195$$

$$\varrho = \frac{(m\rho)^m}{m!\,(1-\rho)} p_0 = 0.03252$$

$$\mathrm{E}[n] = m\rho + \frac{\rho\varrho}{(1-\rho)} = 0.675958$$

$$\mathrm{E}[r] = {}^{1}/_{\mu}\left(1 + \frac{\varrho}{m(1-\rho)}\right) = 2.027875$$

From the mathematical calculation, we obtain E[r] = 0.675958 seconds and E[n] = 2.027875 jobs. The result of mathematical calculation and simulation are close enough, so we can say the model of M/M/3 is valid enough.

V. Performance Metrics and Analysis

A. Performance Metrics

In this project, there are 2 performance metrics that we used to compare the existing system with our system. Those 2 performance metrics are:

- Average number of vehicle in the system
- Average time for a vehicle spends a time to wait in the system

In order to achieve the good result, I'm going to vary the value arrival rate (λ) from λ = 1/3, 1/6, 1/9, 1/12, and 1/15 which each arrival rate represent how busy the traffic of the intersection. The service rate is constant with $\mu = \frac{1}{2}$. We observe this simulation for 1 day or 86400 s. Green light interval (T_{green}) = 20 s and red light interval (T_{red}) = 60 s.

B. Performance Analysis

There are two outcomes from this simulation which can be a comparison from M/M/1 and M/M/3 models. We compared the mean response time (E[r]) and mean number of jobs in system (E[n]) for each model.

From figure 6, we can see the graph of mean number of jobs in the system for M/M/3 model. By using the λ from 1/3 and $\mu = 1/3$, we can see the graph is decreasing predictively.

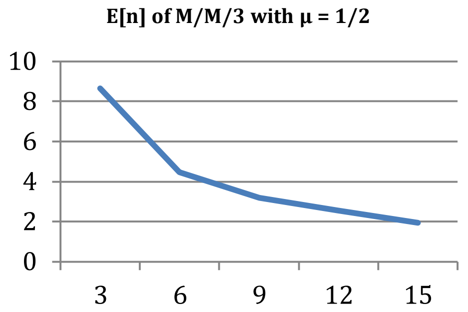


Figure 6, E[n] of M/M/1 model

Compared to figure 7, the graph of M/M/1 model is not well shaped. The M/M/1 model isn't decreasing predictively because the waiting time in red condition makes this system in un-steady state condition. It means the M/M/1 model can't be applied in the intersection with more than 1 line of the road.

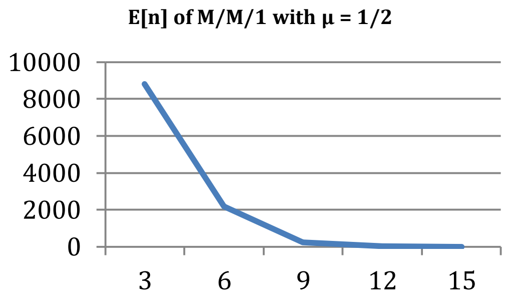


Figure 7, E[n] of M/M/3 model

The mean response time (E[r]) of each model can be seen in figure 8 and figure 9. The big difference can be seen in both figure, where in M/M/3 the line decreasing predictively. But in M/M/1, the result of mean response time is not good enough. This is happen because the system is not in steady state yet at the first simulation. From this result we can conclude that with more than 1 line of the road, M/M/1 approach can't be applied. Therefore we can use M/M/m approach instead of M/M/1 approach for more than 1 line of the road. In figure 9, the graph is not decreasing because in very small arrival rate, the response time should be close to constant because there will be no more queue in the system. Since this system is applied in traffic intersection, there will always be queue in red light condition. Therefore the mean response time is constant, but still close to constant.

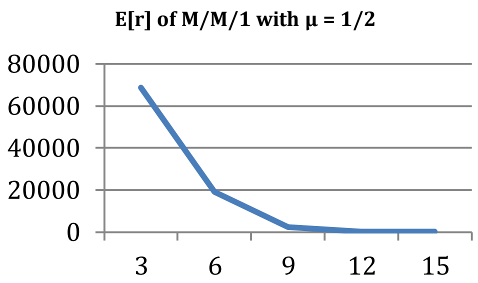


Figure 8, E[r] of M/M/1 model

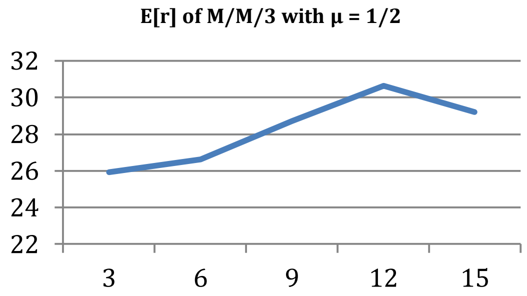


Figure 9, E[r] of M/M/3 model

VI. CONCLUSION

The traffic control using traffic light in the intersection becomes complex since the huge growth of the vehicular in the crowded city. Traffic jam couldn't be avoided since the capacity of the road is less than the amount of the existing car in the road. Queue on the intersection couldn't be avoided in the peak hours. This kind of conventional traffic control couldn't help the problem of traffic jam in the intersection especially in the metropolitan city at peak hours. The existence of traffic control should give benefit; even though, it doesn't give much help in the very high density of the road traffic.

With queue theory approach, we can optimize the traffic control. But the M/M/1 model of queue system could not satisfy the intersection model with more than one road line. So we introduce the M/M/m model to overcome the problem. The result of the simulation is quite satisfied. By using mathematical analysis we have validated our model. Finally we can say this M/M/m model can be used for more than one road line of traffic intersection model.

VII. REFERENCES

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