Name:

ID:

CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**

Problem Set 2a (12 points) **Fall 2019, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.

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1. *(6 pts) For each of the following pairs of functions $f(n)$ and $g(n)$, we have that $f(n) \in \mathcal{O}(g(n))$. Find valid constants c and n_0 in accordance with the definition of Big-O. For the sake of this assignment, both c and n_0 should be strictly less than 10. You do* ***not*** *need to formally prove that $f(n) \in \mathcal{O}(g(n))$ (that is, no induction proof or use of limits is needed).*

   (a) $f(n) = n^3 \log(n)$ and $g(n) = n^4$.

   $n^3\log(n) \leq c * n^4$
   $(n^3/n^4) * (\log(n)/n^4) \leq c$
   $(1/n) * (\log(n)/n^4) \leq c$
   $(1/2) * (\log(2)/54) \leq c$

   $n_0 \geq 2$, $c = 1$

   (b) $f(n) = n2^n$ and $g(n) = 2^{n \log_2(n)}$.

   $n2^n < c * 2^{n\log 2(n)}$
   $n2^n \leq c * n2^n$

   $n_0 \geq 1$, $c = 1$

   (c) $f(n) = 4^n$ and $g(n) = (2n)!$

   $4^n \leq c * (2n)!$
   $4^1 \leq c * (2)!$
   $4 \leq c * 2$
   $2 \leq c$

   $n_0 \geq 1$, $c = 2$

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2. *(2 pts) Let* $f(n) = 3n^3 + 6n^2 + 6000$. *So* $f(n) \in \Theta(n^3)$. *Find appropriate constants* c_1, c_2, *and* n_0 *in accordance with the definition of Big-Theta.*

$T(n) = 3n^3 + 6n^2 + 6000$
$c1n^3 \leq 3n^3 + 6n^2 + 6000 \leq c2n^3$
$c1 \leq 3 + (6/n) + (6000/n^3) \leq c2$
RS: $3 + (6/n) + (6000/n^3) \leq c2$
$3 \leq c2 \ \forall \ n$
LS: $c1 \leq 3 + (6/n) + (6000/n^3)$
$c1 \leq 3 + (6/n) + (6000/n^3)$
$3 + (6/n) + (6000/n^3) = 0$ if $n = -13.3$
so $n_0 \geq -14$
$c1 \leq 3 + (6/-14) + (6000/-14^3)$
$c1 \leq 3 + (6/-14) + (6000/-2744)$
$c1 \leq (8232/2744) - (1176/2744) - (6000/2744)$
$c1 \leq 1056/2744$
$c1 \leq 132/343$

Inequality holds for $c1 = 132/343$, $c2 = 3$, $n_0 \geq -14$

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3. *(2 pts) Consider the following algorithm. Find a suitable function $g(n)$, such that the algorithm's worst-case runtime complexity is $\Theta(g(n))$. You do **not** need to formally prove that $f(n) \in \Theta(g(n))$ (that is, no induction proof or use of limits is needed).*

```
count = 0
for(i = n; i >= 0; i = i - 1){
    for(j = i-1; j >= 0; j = j-1){
        count = count+1
    }
}
```

Cost	Time	
c_1	1	
c_2	n+1	
c_3	(n(n+1))/2	(Based on $\Sigma_{i=1}^{n} i = (n(n+1))/2$)
c_4	n	

$T(n) = c_1(1) + c_2(n+1) + c_3((n(n+1))/2) + c_4(n)$

$= (c_1 + c_2) + (c_2 + c_4)n + (c_3)((n(n+1)/2))$

$T(n) = \Theta(n)$

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4. *(2 pts) Consider the following algorithm. Find a suitable function $g(n)$, such that the algorithm's worst-case runtime complexity is $\Theta(g(n))$. You do **not** need to formally prove that $f(n) \in \Theta(g(n))$ (that is, no induction proof or use of limits is needed).*

```
count = 0
for(i = 1; i < n; i = i * 3){
    for(j = 0; j < n; j = j + 2){
        count = count + 1
    }
}
```

Cost	Time	
c_1	1	
c_2	$\log_3 n$	(maximizes i and still < n)
c_3	$(n(n+1))/2$	
c_4	n	

$T(n) = (c_1) + (c_2)(\log_3 n) + (c_3)((n(n+1))/2) + (c_4)(n)$

$T(n) = \Theta(n)$