Error analysis on our formula for Brazilian test

see Figure 11 from Brazilian paper

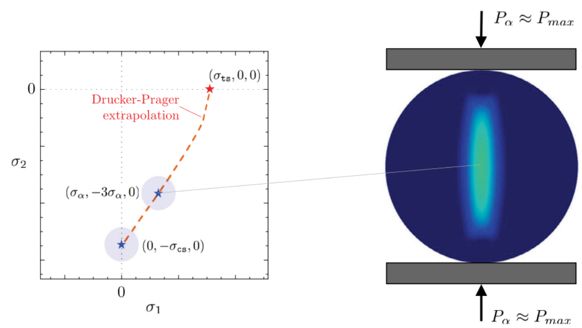


Fig. 11. Schematic illustrating that the formula (21) corresponds to the extrapolation of the strength point $\mathcal{F} = (\mathrm{diag}(\sigma_\alpha, -3\sigma_\alpha, 0)) = 0$, with $\sigma_\alpha = P_{max}/(\pi RH)$ obtained from a Brazilian test carried out with flat ($R_p = +\infty$) platens, and the point of uniaxial compressive strength $\mathcal{F} = (\mathrm{diag}(0, -\sigma_{cs}, 0)) = 0$ to the point of uniaxial tensile strength $\mathcal{F} = (\mathrm{diag}(\sigma_{ts}, 0, 0)) = 0$ in the space of principal stresses $(\sigma_1, \sigma_2, 0)$. The point $\boldsymbol{\sigma} = \mathrm{diag}(\sigma_\alpha, -3\sigma_\alpha, 0)$ corresponds to the violation of the strength surface of the material at the center of the specimen.

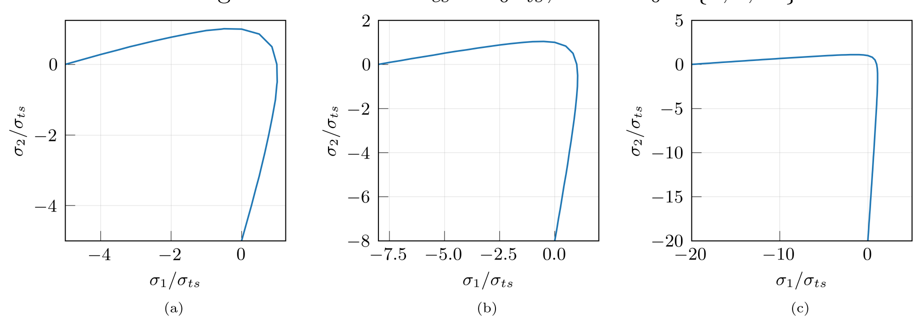
In Brazilian tests, assume a material with the Drucker-Prager strength surface, if we perturb compressive strength σ_{cs} by $5, 10\%$, how much will the measured tensile strength σ_{ts} be changed?

Verify with our formula

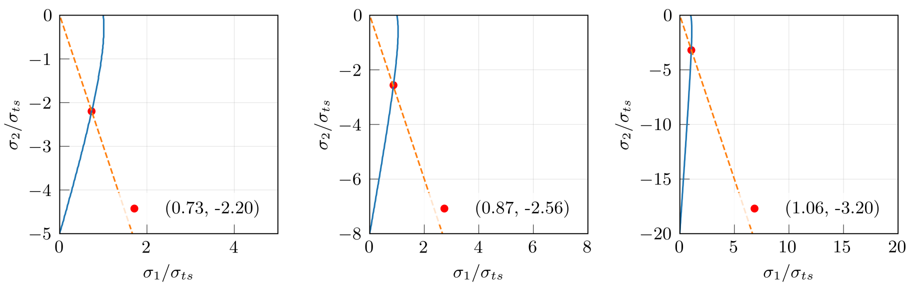
The Drucker-Prager strength surface reads

$$\mathcal{F}(\sigma) = \sqrt{J_2} + \frac{\sigma_{cs} - \sigma_{ts}}{\sqrt{3}(\sigma_{cs} + \sigma_{ts})} I_1 - \frac{2\sigma_{cs}\sigma_{ts}}{\sqrt{3}(\sigma_{cs} + \sigma_{ts})} = 0.$$

Consider the strength surface with $\sigma_{cs} = c_0 \sigma_{ts}$, where $c_0 \in \{5, 8, 20\}$ as follows



If the crack initiate at the center of the disc, we have $\sigma_2 = -3\sigma_1$:



(a) (b) (c)

Following equation (21):

$$\sigma_{ts} = f(\sigma_\alpha, \sigma_{cs})\sigma_\alpha,$$

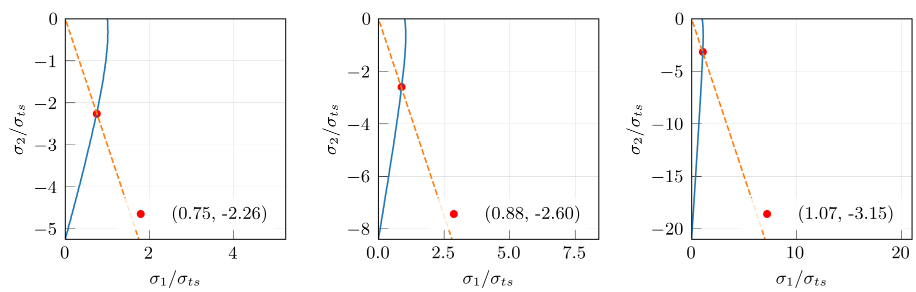
where

$$f(\sigma_\alpha, \sigma_{cs}) = \frac{(\sqrt{13} - 2)\sigma_{cs}/\sigma_\alpha}{2\sigma_{cs}/\sigma_\alpha - \sqrt{13} - 2}.$$

Verify it using $c_0 \in \{5, 8, 20\}$, $\sigma_\alpha \in \{0.73, 0.87, 1.06\}$, getting $\overline{\sigma_{ts}}/\sigma_{ts} = \{0.99, 1.00, 1.00\}$.

Perturb σ_{cs} by 5%

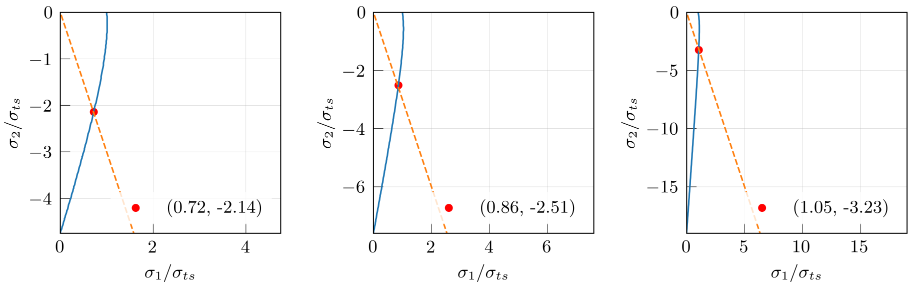
- σ_{cs} +5%



(a) (b) (c)

Verify it using $c_0 \in \{5, 8, 20\}$, $\sigma_\alpha \in \{0.75, 0.88, 1.07\}$, getting $\overline{\sigma_{ts}}/\sigma_{ts} = \{1.039, 1.021, 1.0105\}$.

- σ_{cs} -5%

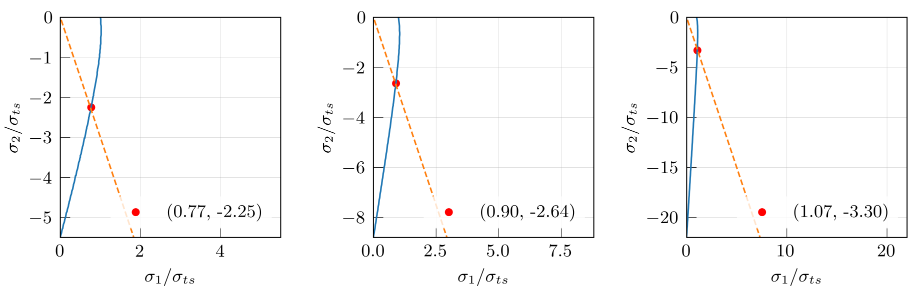


(a) (b) (c)

Verify it using $c_0 \in \{5, 8, 20\}$, $\sigma_\alpha \in \{0.72, 0.86, 1.05\}$, getting $\overline{\sigma_{ts}}/\sigma_{ts} = \{0.969, 0.988, 0.988\}$.

Perturb σ_{cs} by 10%

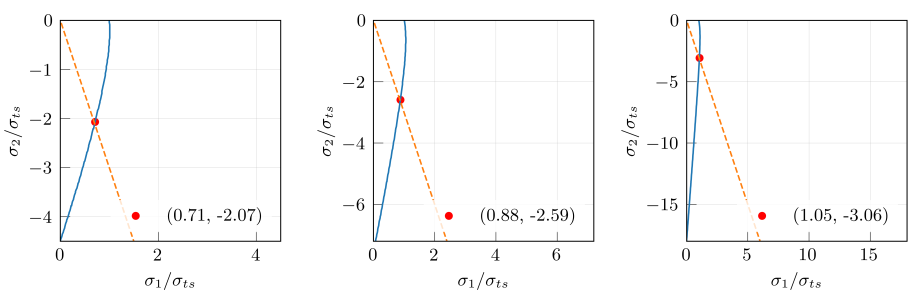
- σ_{cs} +10%



(a) (b) (c)

Verify it using $c_0 \in \{5, 8, 20\}$, $\sigma_\alpha \in \{0.77, 0.90, 1.07\}$, getting $\overline{\sigma_{ts}}/\sigma_{ts} = \{1.088, 1.055, 1.0105\}$.

- σ_{cs} -10%



(a) (b) (c)

Verify it using $c_0 \in \{5, 8, 20\}$, $\sigma_\alpha \in \{0.71, 0.88, 1.05\}$, getting $\overline{\sigma_{ts}}/\sigma_{ts} = \{0.947, 1.021, 0.988\}$.

Summary

Let's define the error $e = |\overline{\sigma_{ts}} - \sigma_{ts}|/\sigma_{ts}$.

σ_{cs}/σ_{ts}	5	8	20
$\sigma_{cs} + 5\%$	0.039	0.021	0.0105
$\sigma_{cs} - 5\%$	0.031	0.012	0.012
$\sigma_{cs} + 10\%$	0.088	0.055	0.0105
$\sigma_{cs} - 10\%$	0.053	0.021	0.012