



# A Tree-based CP-ABE Scheme with Hidden Policy Supporting Secure Data Sharing in Cloud Computing

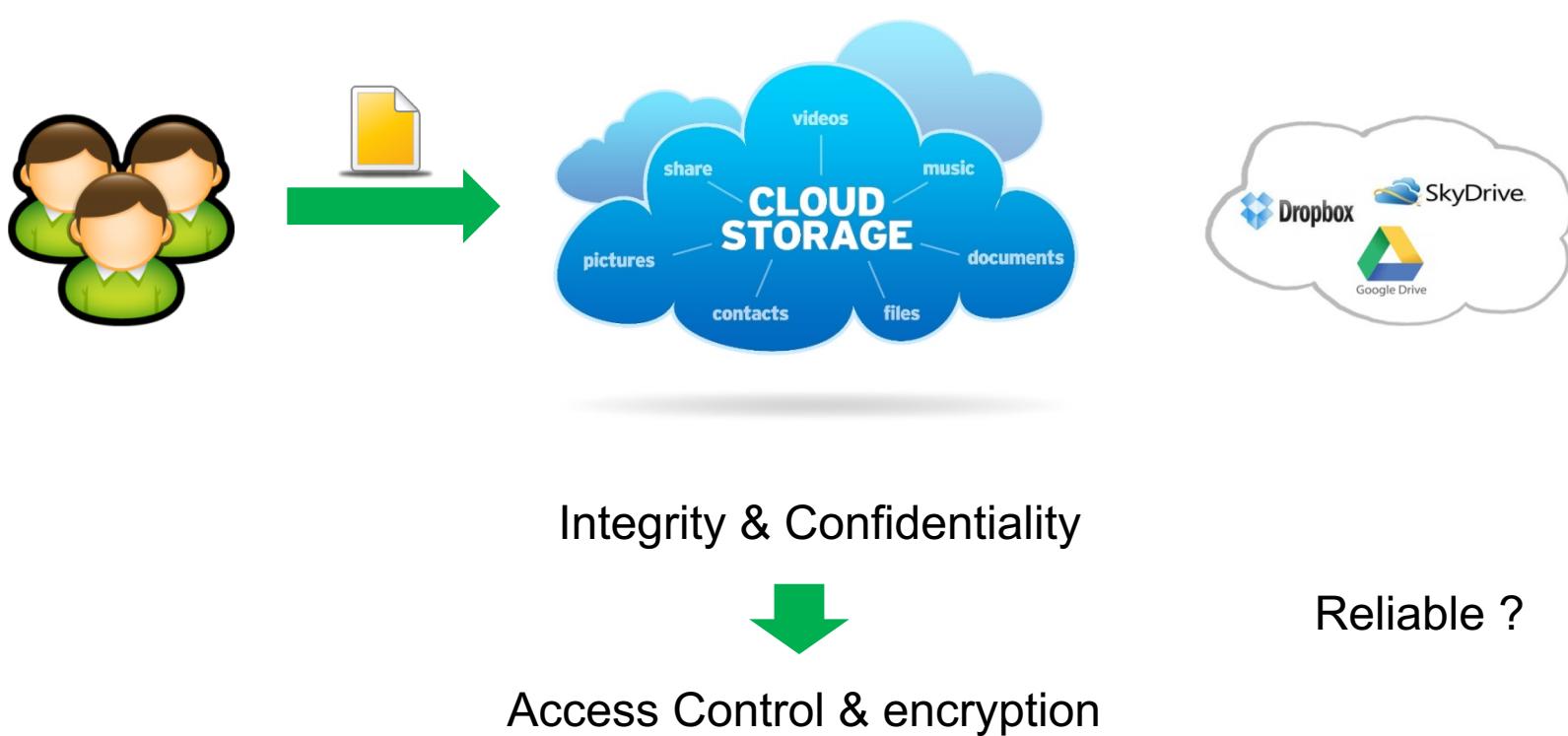


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13 Dec, 2013

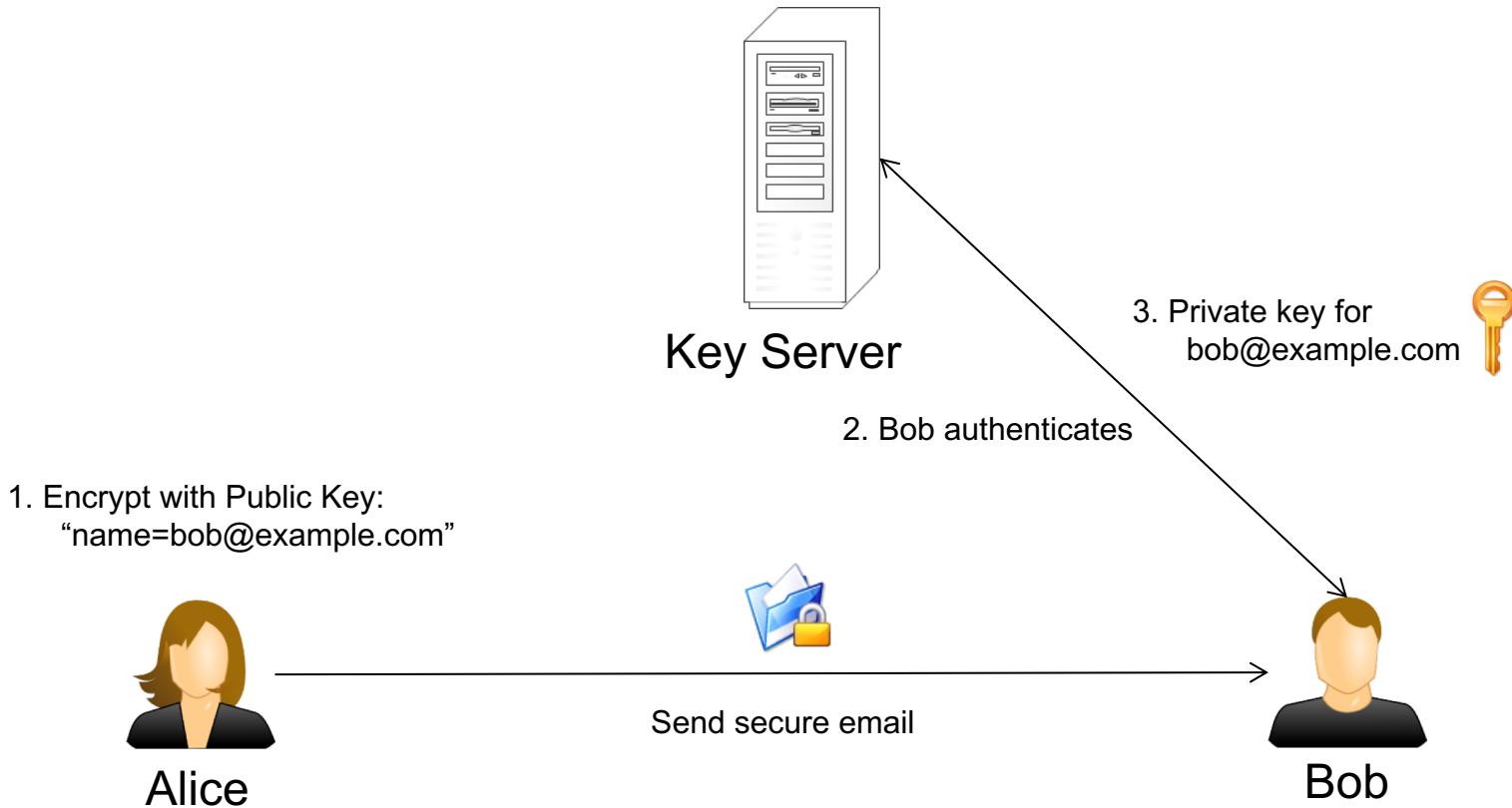
# Background

- The importance of data self-protection capability



# Background

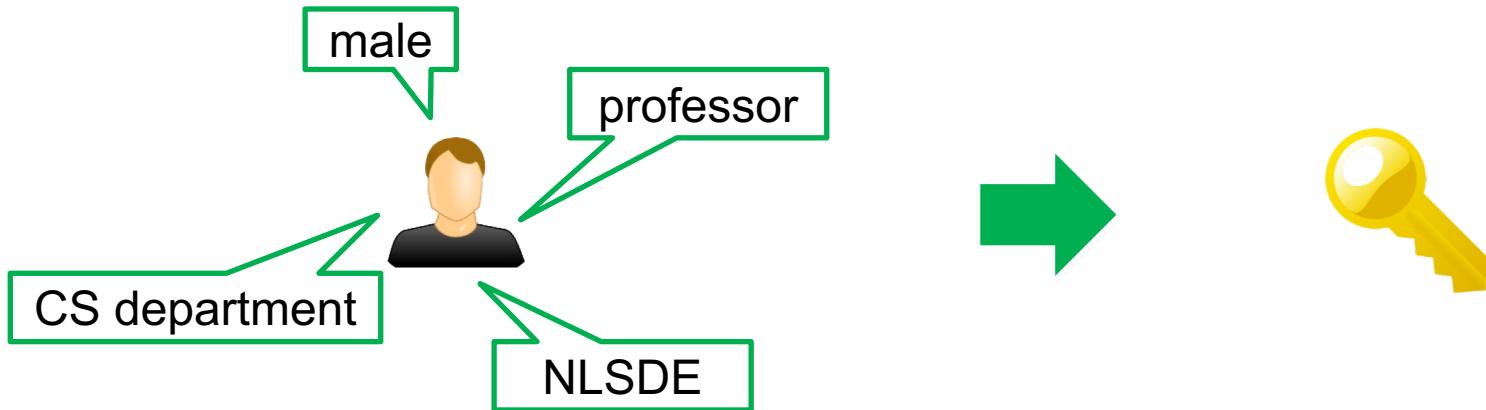
## ➤ Identity Based Encryption



# Background

## ➤ Attribute Based Encryption

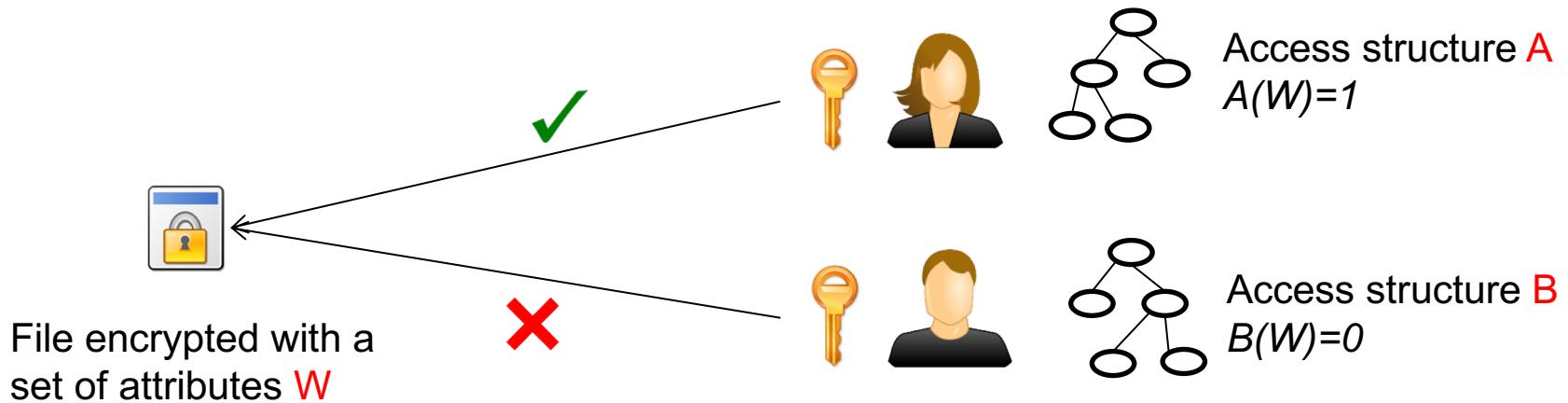
- ✓ An extend scheme of Identity based Encryption
- ✓ Utilization of attribute information for Encryption/Decryption
- ✓ Dynamically control the user group of the encrypted data



# Background

## ➤ Key Policy Attribute based Encryption

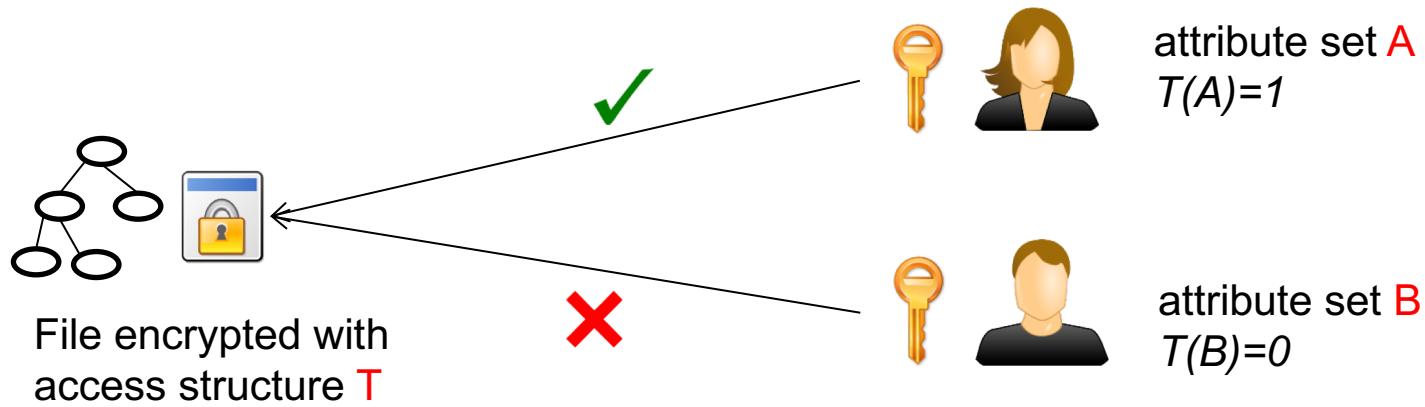
(Goyal et al., 2006)





# Background

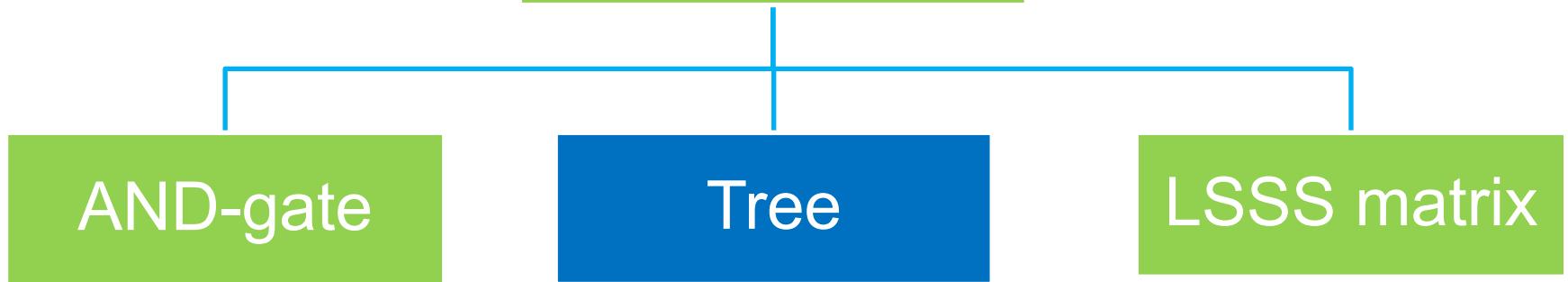
- Ciphertext Policy Attribute based Encryption (Bethencourt et al., 2007)



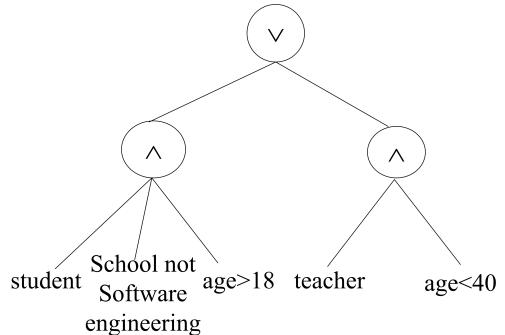


# Background

## CP-ABE Access Structure



$$A_1 = (1 \wedge 2 \wedge 3 \wedge 4)$$



$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$$

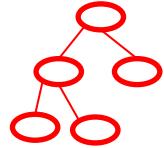
$$A_3 = (1 \wedge 2 \wedge 3) \vee (1 \wedge 4)$$

$$A_2 = (\textit{student} \wedge \textit{school not SE} \wedge \textit{age} > 18) \vee (\textit{teacher} \wedge \textit{age} < 40)$$



# Background

- Original CP-ABE scheme
  - ✓ the access structure is embedded in the ciphertext
  - ✓ someone obtains the ciphertext can see the content of the access structure
- Privacy disclose
  - ✓ Full exposure of data's access policy will disclose sensitive information of the decryption or encryption party.



File encrypted with access structure  $T$



# Related Work

## ➤ CP-ABE Schemes

### ✓ BSW07

- The initial structure of CP-ABE
- Security:
  - General group model rather than the standard numerical theoretical assumptions
- Expressive ability : Tree Structure
  - AND, OR and threshold operations
  - “bag of bits” for express policies containing  $<$ ,  $\leq$ ,  $>$ ,  $\geq$

### ✓ CN07

- Security
  - CPA security under DBDH assumption
- Expressive ability : AND-gate Structure
  - AND, NOT opeartions

### ✓ Waters08

- Expressive ability : LSSS matrix
- Improvement of Lewko et al
  - supported any monotone access formula



# Related Work

## ➤ CP-ABE Schemes

### ✓ BCP-ABE

- Security
  - CPA security under DBDH assumption
- Expressive ability : Tree Structure
  - AND, OR and threshold operations
- Improvement of Liang et al
  - Shorten the system's public key
  - Shorten the user's private key
  - Shorten the length of the ciphertext

### ✓ ITHJ09

- Security
  - CPA security under DBDH assumption
- Expressive ability : Tree Structure
  - AND, OR and threshold operations
- the costs of key generation, encryption and decryption are lower than BSW07



# Related Work

## ➤ CP-ABE Schemes with Hidden Policy

- ✓ NY008(Nishide et al, 2008)
  - Security
    - CPA security under DBDH assumption and D-Linear assumption
  - Access Structure:
    - Only support And-gate structure
- ✓ LDL12(Lai et al, 2012)
  - Security
    - Fully secure in the standard model using the dual system encryption methodology .
  - Access Structure
    - LSSS Matrix structure
  - Partial hidden policy
- ✓ XGRDY12(Xiaohui et al, 2012)
  - Security
    - CPA security under DBDH assumption in the standard model.
  - Access Structure
    - And-gate access structure



# Related Work

## ➤ Conclusion

- ✓ CP-ABE scheme with flexible policy expression ability will have broad application prospects.
- ✓ XGRDY12 and LDL12 support the access structure hidden
  - And-gate access structure:
    - the expressive ability of policy is limited.
  - LSSS matrix structure:
    - It's hard to construct the LSSS matrix
    - No normal method of construction.
- ✓ The research of tree-based access structure CP-ABE scheme with hidden policy.



# Our Contribution

## Theorem:

Element's orthogonal property in subgroup of composite order bilinear groups

CP-ABE

Introduces some random elements into the policy key component.

ITHJ09 Scheme

CP-ABE-HP



# CP-ABE-HP specific scheme

## ➤ Initialize

- ✓ Generate public parameter  $pk$  and master key  $mk$ 
  - Generate the bilinear groups  $G$  and a bilinear map  $e: G \times G \rightarrow G_T$ 
    - $G$  and  $G_T$  are the cyclic groups of order  $N=pr$  ( $p$  and  $r$  are distinct primes)
    - $G_p$  and  $G_r$  be the subgroup of the  $G$  with order  $p$  and  $r$  respectively.
    - Also  $g_p$  and  $g_r$  are the generator of  $G_p$  and  $G_r$  respectively.
  - Generate the attribute set  $U = \{a_1, a_2, \dots, a_n\}$ , and random elements  $\alpha, t_1, t_2, \dots, t_n \in \mathbb{Z}_p^*$  and  $R_0, R_1, R_2, \dots, R_n \in G_r$
  - Compute the pk elements
$$x = g_p \cdot R_0, y = e(g_p, g_p)^\alpha, T_j = g_p^{t_j} \cdot R_j \quad (1 \leq j \leq n)$$
  - The public key is  $pk = (e, x, y, T_j \quad (1 \leq j \leq n))$ , and the master key is  $mk = (\alpha, t_j \quad (1 \leq j \leq n))$ .
- ✓ Give  $pk$  to the encryption party.



# CP-ABE-HP specific scheme

## ➤ Encryption

- ✓ Select a random element  $s \in Z_p^*$  and compute  
 $c_0 = x^s \cdot R'_0, c_1 = m \cdot y^s = m \cdot e(g_p, g_p)^{\alpha s}$
- ✓ Set the value of the root node of  $\tau$  to be  $s$
- ✓ Mark all child nodes as un-assigned, and mark the root node assigned
- ✓ Recursively, for each un-assigned non-leaf node, do the following:
  - If its child nodes are un-assigned, the secret  $s$  is divided using  $(t,n)$ -Shamir secret sharing technique. The relation of  $n$  and  $t$  is:
    - if the symbol is  $\oplus$ , then  $1 < t < n$ ;
    - if the symbol is AND, then  $t = n$ ;
    - if the symbol is OR, then  $t = 1$ .
  - To each child node a share secret  $s_i = f(i)$  is assigned ,  $f(x) = \sum_{j=0}^{t-1} b_j x^j$
  - Mark this node assigned.
- ✓ For each leaf attribute  
 $\forall a_{j,i} \in \tau, c_{j,i} = T_j^{s_i} \cdot R'_j$
- ✓ Return the ciphertext:  $c_\tau = (c_0, c_1, \forall a_{j,i} \in \tau : [i, c_{j,i}])$ .



# CP-ABE-HP specific scheme

## ➤ Secret key generation

- ✓ Verify the basic attribute
- ✓ Generate the secret key  $sk_{w^*}$  corresponds to  $w^*$ 
  - Select a random value  $r \in \mathbb{Z}_p^*$ ,  $d_0 = g^{\alpha-r}$ .
  - For each attribute  $a_j$  in  $w$ , compute  $d_j = g^{rt_j^{-1}}$
- ✓ Send key back to the user

## ➤ Decryption

- ✓ For every attribute element  $a_j \in w'$ , computing:

$$m = \frac{c_1}{e(c_0, d_0) \cdot \prod_{a_j \in w'} e(c_{j,i}, d_j)^{l_i(0)}}$$

- ✓  $l_i(0)$  is a Lagrange coefficient.



# CP-ABE-HP specific scheme

## ➤ Correctness Proof:

$$\begin{aligned} m' &= \frac{c_1}{e(c_0, d_0) \cdot \prod_{a_j \in w} e(c_{j,i}, d_j)^{l_i(0)}} \\ &= \frac{m \cdot e(g_p, g_p)^{\alpha s}}{e(g_p^s, g_p^{\alpha-r}) \cdot e(R_0^s \cdot R_0^{'}, g_p^{\alpha-r})} \cdot \frac{1}{\prod_{a_j \in w} (e(g_p^{t_j s_i}, g_p^{r t_j^{-1}})^{l_i(0)} \cdot e(R_j^{s_i} \cdot R_j^{'}, g_p^{r t_j^{-1}})^{l_i(0)})} \\ &= \frac{m \cdot e(g_p, g_p)^{\alpha s}}{e(g_p^s, g_p^{\alpha-r}) \cdot e(g_p, g_p)^{\sum r s_i l_i(0)}} \\ &= \frac{m \cdot e(g_p, g_p)^{\alpha s}}{e(g_p^s, g_p^{\alpha-r}) \cdot e(g_p, g_p)^{rs}} \\ &= m \end{aligned}$$



# CP-ABE-HP Performance analysis

Table Comparison of our scheme with other schemes in computing cost

Scheme	Access Structure	Hidden Policy	Encrypt	Decrypt
CN07	And-gate	N	$(n+1)G + 2G_t$	$(n+1)C_e + (n+1)G_t$
Emura09	And-gate	N	$(n+1)G + 2G_t$	$2C_e + 2G_t$
Xiao12	And-gate	Y	$(n+3)G + 2G_t$	$2C_e + 2G_t$
BSW07	Tree	N	$(2 A_c +1)G + 2G_t$	$2 A_u C_e + (2 S +2)G_t$
ITHJ09	Tree	N	$( A_c +1)G + 2G_t$	$( w +1)C_e + ( w +1)G_t$
CP-ABE-HP	Tree	Y	$2( A_c +1)G + 2G_t$	$( w +1)C_e + ( w +1)G_t$



# CP-ABE-HP Security Model

## ➤ IND-sAtt-CPA game

- ✓ Init Phase.
  - The adversary chooses a challenge access tree  $\tau^*$  and gives it to the challenger.
- ✓ Setup Phase.
  - The challenger runs Setup algorithm to generate  $(PK, MK)$  and gives the public key  $PK$  to adversary  $A$ .
- ✓ Phase 1.
  - Adversary  $A$  makes a secret key request to the key generation oracle for any attribute sets. The challenger runs Key-Generation  $(MK, S)$  algorithm to generate a private key.
- ✓ Challenge Phase.
  - Adversary  $A$  sends to the challenger two equal length messages  $m_0, m_1$ . The challenger picks a random bit  $b \in \{0, 1\}$  and returns  $c_b = \text{Encrypt}(m_b, \tau^*, PK)$ .
- ✓ Phase 2.
  - Adversary  $A$  can continue querying key generation oracle with the same restriction as in Phase 1.
- ✓ Guess Phase.
  - Adversary  $A$  outputs a guess  $b' \in \{0, 1\}$ .



# CP-ABE-HP Security Proof

## ➤ DBDH assumption

- ✓ No probabilistic polynomial-time algorithm  $\beta$  can distinguish the tuple  $(g, g^a, g^b, g^c, e(g, g)^{abc})$  from the tuple  $(g, g^a, g^b, g^c, e(g, g)^z)$  with more than a negligible advantage.

## ➤ Methodology

- ✓ Suppose that the IND-sAtt-CPA game can be won by an adversary  $A$  with a non-negligible advantage  $\varepsilon$ .
- ✓ From the attack ability of adversary  $A$ , we will build a simulator  $\beta$ , which has the ability to solve the DBDH assumption problem with advantage  $\varepsilon/2$ .
- ✓ According to the DBDH assumption: there are no effective polynomial algorithms which can solve the DBDH assumption problem with non-negligible advantage.
- ✓ The adversary also cannot win the IND-sAtt-CPA game with the above advantage  $\varepsilon$ .

## ➤ Conclusion:

- ✓ the adversary having no advantage to break through CP-ABE-HP system



# CP-ABE-HP Security Proof

## ➤ Proof

### ✓ Init Phase.

- The adversary chooses a challenge access  $\tau^*$  and sends it to the challenger.

### ✓ Setup Phase.

- The challenger selects a random element  $x' \in Z_p$  and sets  $\alpha = ab + x'$ , then calculates  $pk$ .

$$x = g_p \cdot R_0$$

$$y = e(g_p, g_p)^\alpha = e(g_p, g_p)^{ab} e(g_p, g_p)^{x'}$$

$$\forall a_j \in U : T_j = \begin{cases} g_p^{b/t_j} \cdot R_j, & a_j \notin \tau^* \\ g_p^{t_j} \cdot R_j, & a_j \in \tau^* \end{cases}, (1 \leq j \leq n)$$

### ✓ Phase 1

- The adversary sends a user private key query request to the challenger by the following attributes set:

$$w_j = \{a_j \mid a_j \in \Omega\}, (a_j \notin \tau^*)$$

- For each query request of the adversary, the challenger selects random element  $r' \in_R Z_p$  and sets  $r = ab + br'$  and calculates private key:

$$d_0 = g_p^{\alpha-(ab+r'b)} = g_p^{x'-r'b} = g_p^{x'} (g_p^b)^{-r'} \quad d_j = g_p^{rt_j/b} = (g_p^a)^{t_j} g_p^{r't_j}, (a_j \notin \tau^*)$$



# CP-ABE-HP Security Proof

## ➤ Proof

### ✓ Challenge Phase.

- The adversary submits two plaintext messages  $m_0, m_1$  to the challenger.
- The challenger selects a random plaintext message  $m_b$  from the two messages, where  $b \in_R \{0,1\}$ . Encrypt the message as follows:

$$c_0 = g_p^c \cdot R_0^c \cdot R_0'$$

$$c_1 = m_b e(g_p, g_p)^{abc} e(g_p^c, g_p^{x'})$$

- Use the Shamir Secret Sharing scheme over the access tree.

### ✓ Phase 2.

- The adversary continues to send the secret key requests to the challenger with the same restriction as in Phase 1.

### ✓ Guess Phase.

- The adversary outputs a guess  $b' \in \{0, 1\}$ .
- If  $b' = b$ , the challenger can guess that  $u = 0$ ,  $Z_u = e(g_p, g_p)^{abc}$ .

$$\Pr[b' = b | Z_u = e(g_p, g_p)^{abc}] = 1/2 + \varepsilon$$

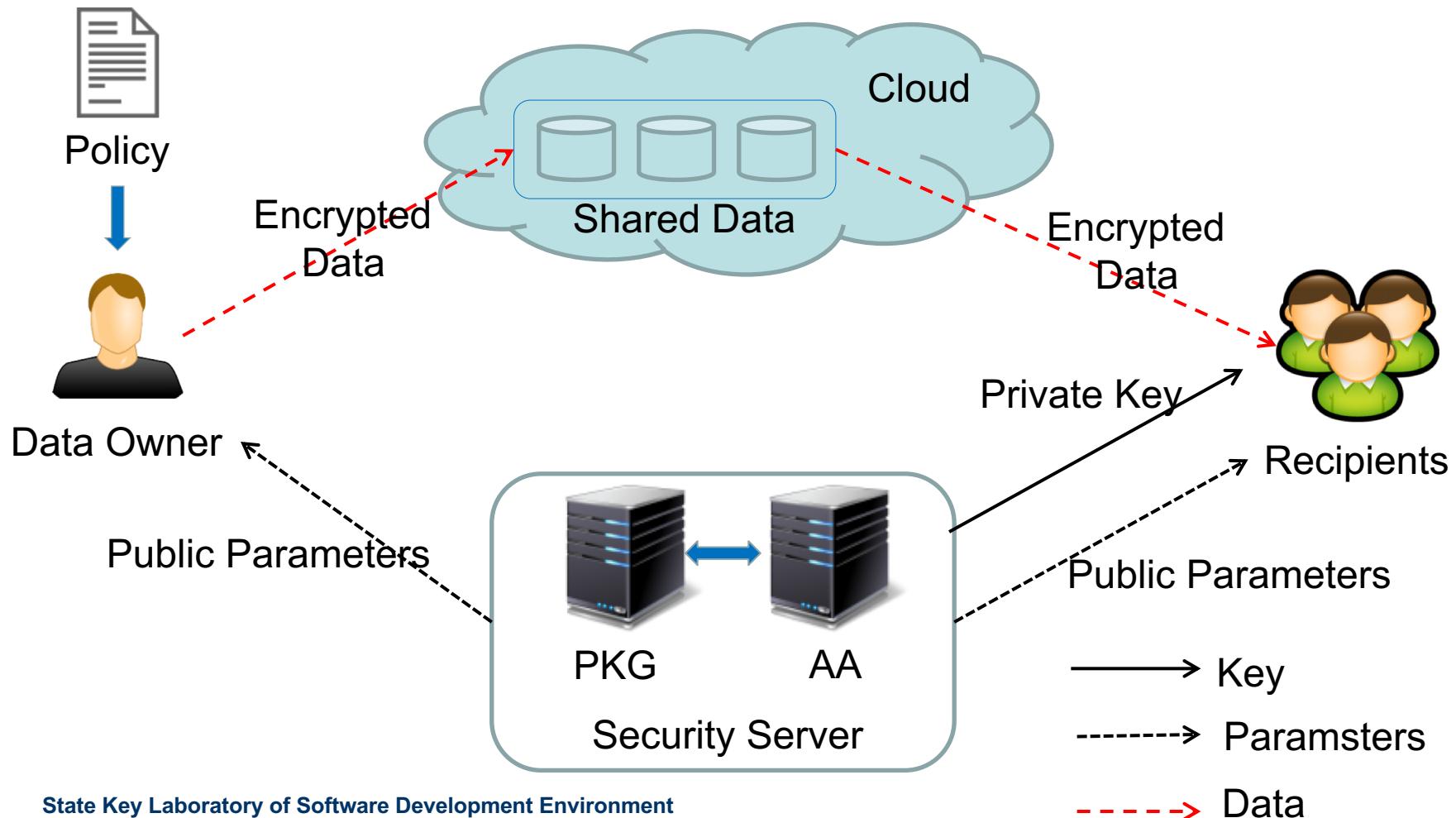
- Otherwise, the challenger guesses that  $u = 1$ ,  $Z_u = e(g_p, g_p)^\theta$ .

$$\Pr[b' \neq b | Z_u = e(g_p, g_p)^\theta] = 1/2$$

$$\left. \begin{aligned} & \frac{1}{2} \Pr[u' = u | u = 0] \\ & + \frac{1}{2} \Pr[u' = u | u = 1] - \frac{1}{2} = \frac{\varepsilon}{2} \end{aligned} \right\}$$

# An implementation framework

The implementation framework of CP-ABE-HP scheme for outsourced data sharing





# Conclusion

- Propose an CP-ABE-HP scheme
  - ✓ Introduces the tree-based CP-ABE scheme with hidden policy
    - Keep the security and efficiency properties of the CP-ABE scheme
    - Preserve the privacy in the access policy
- Future work
  - ✓ Implement an efficient CP-ABE-HP mechanism
  - ✓ Apply it to some specific cloud storage environments



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Thank you!  
Q & A

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