

Missile_1feat_fuel_isp

January 16, 2023

```
[1]: write_images = False

wirte_output_txt = False
# Specify everytime Simulation is called
# WARNING --> Set to False when running more then 10 simulations
#           (otherwise it will be super slow and might crash)
```

```
[2]: import numpy as np
```

```
[3]: from emukit.core import ContinuousParameter, ParameterSpace
from emukit.core.initial_designs import RandomDesign

import GPy
from GPy.models import GPRegression
from emukit.model_wrappers import GPyModelWrapper
from emukit.sensitivity.monte_carlo import MonteCarloSensitivity

import matplotlib.pyplot as plt
import mlai.plot as plot
```

```
[4]: %run Missile_utils.ipynb
```

```
[ ]:
```

```
[5]: simulation_output = 'range'
# We divide by 1000 to avoid dealing with too large numbers
```

```
[ ]:
```

```
[ ]:
```

We consider missiles with only 1 stage

```
[6]: basic_param_spaces = {
    'payload': [10, 2410],
    'missilediam': [0.1, 9.9],
    'rvdiam': [0.1, 9.9],
    'estrange': [100, 4900],
    'fuelmass': [500, 6000], # [500, 7000],
    'drymass': [1000, 3000],
    'Isp0': [100, 800], # [100, 800],
    'thrust0': [10000, 69000],
```

```
}
```

```
[7]: from sklearn.metrics import mean_squared_error
import math

def compute_rmse(y_actual, y_predicted):
    MSE = mean_squared_error(y_actual, y_predicted)
    RMSE = math.sqrt(MSE)

    return RMSE

def evaluate_prediction(y_actual, y_predicted):
    return compute_rmse(y_actual, y_predicted)
```

```
[ ]:
```

1 0. Only one param - m0

```
[8]: m0_param_1 = 'fuelmass'
m0_domain_param_1 = basic_param_spaces[m0_param_1]

m0_space = ParameterSpace(
    [ContinuousParameter(m0_param_1, *m0_domain_param_1),
    ])

custom_param_names = [m0_param_1]
```

```
[9]: def run_missile_sim(custom_params):
    """
    Recives in input an array of custom parameters.
    Each row represents a set of different parameters
    Each column is a different parameter (#cols = len(custom_param_names))
    """
    default_params_IRAQ = {
        'payload':500,
        'missilediam':0.88,
        'rvdiam':0,
        'estrange':600,
        'numstages':1,
        'fuelmass':[0,5600],
        'drymass':[0,1200],
        'Isp0':[0,226],
        'thrust0':[0,9177.4]
    }

    y = np.zeros((custom_params.shape[0], 1))
    for i in range(custom_params.shape[0]):
        params_to_use = default_params_IRAQ
        # Overwrite default param variables
```

```

for j in range(custom_params.shape[1]):
    param_name = custom_param_names[j]
    if param_name in ['fuelmass', 'drymass', 'Isp0', 'thrust0']:
        params_to_use[param_name][1] = custom_params[i,j]
    else:
        params_to_use[param_name] = custom_params[i, j]

    if j==0:
        print('\nNew simulation \n')
    str_to_print = param_name + ': ' + str(custom_params[i,j])
    print(str_to_print)

# Run simulation
output_path = 'results/results_' + str(i) + '.txt'
sim_output = run_one_sim(
    numstages=params_to_use["numstages"],
    fuelmass=params_to_use["fuelmass"],
    drymass=params_to_use["drymass"],
    thrust0=params_to_use["thrust0"],
    Isp0=params_to_use["Isp0"],
    payload=params_to_use["payload"],
    missilediam=params_to_use["missilediam"],
    rvdiam=params_to_use["rvdiam"],
    est_range=params_to_use["estrange"],
    output_path=output_path,
    simulation_output=simulation_output,
)

y[i, 0] = sim_output
return y

```

```

[10]: # Get true points (to build model)
wirte_output_txt = True

m0_design = RandomDesign(m0_space)
m0_x = m0_design.get_samples(3)
m0_y = run_missile_sim(m0_x)

```

New simulation

fuelmass: 1007.8840088887675

Stage 1 burnout

Velocity (km/s): 0.6978016112431643

Angle (deg h): 43.69724960661012

Range (km): 3.6927765908978962

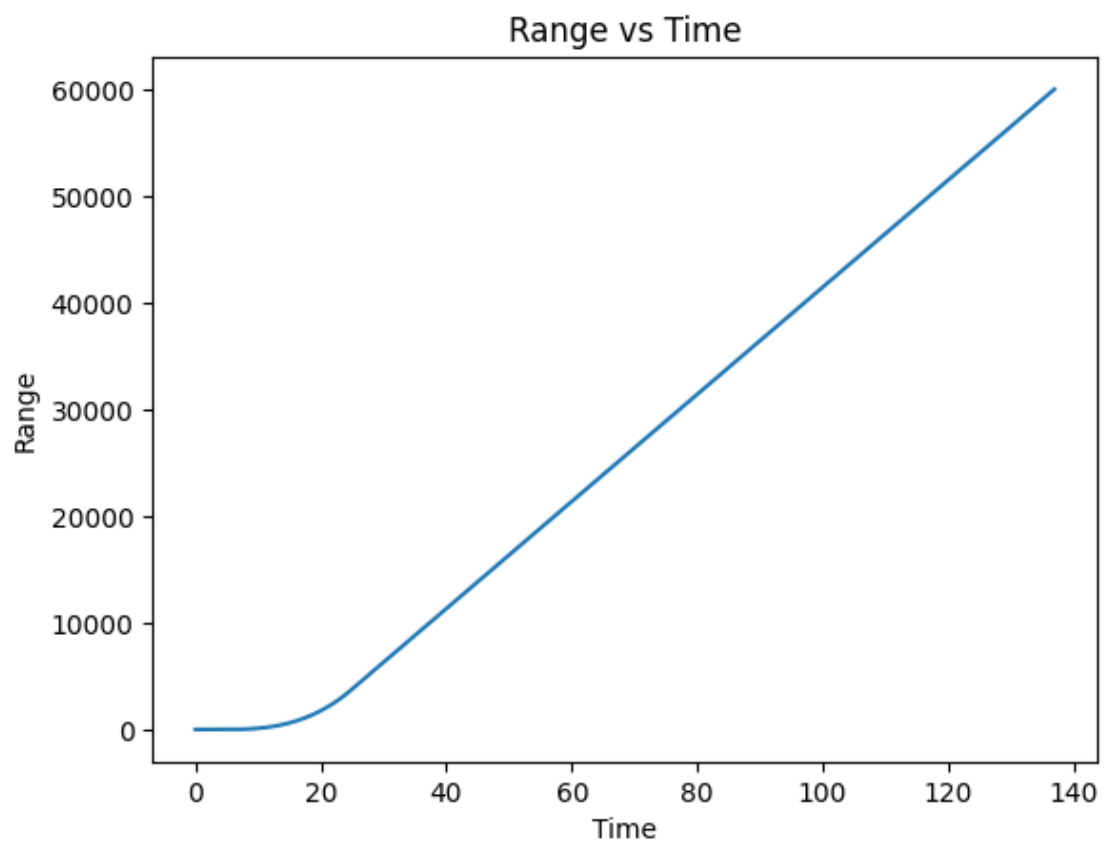
Time (sec): 24.8000000000000086

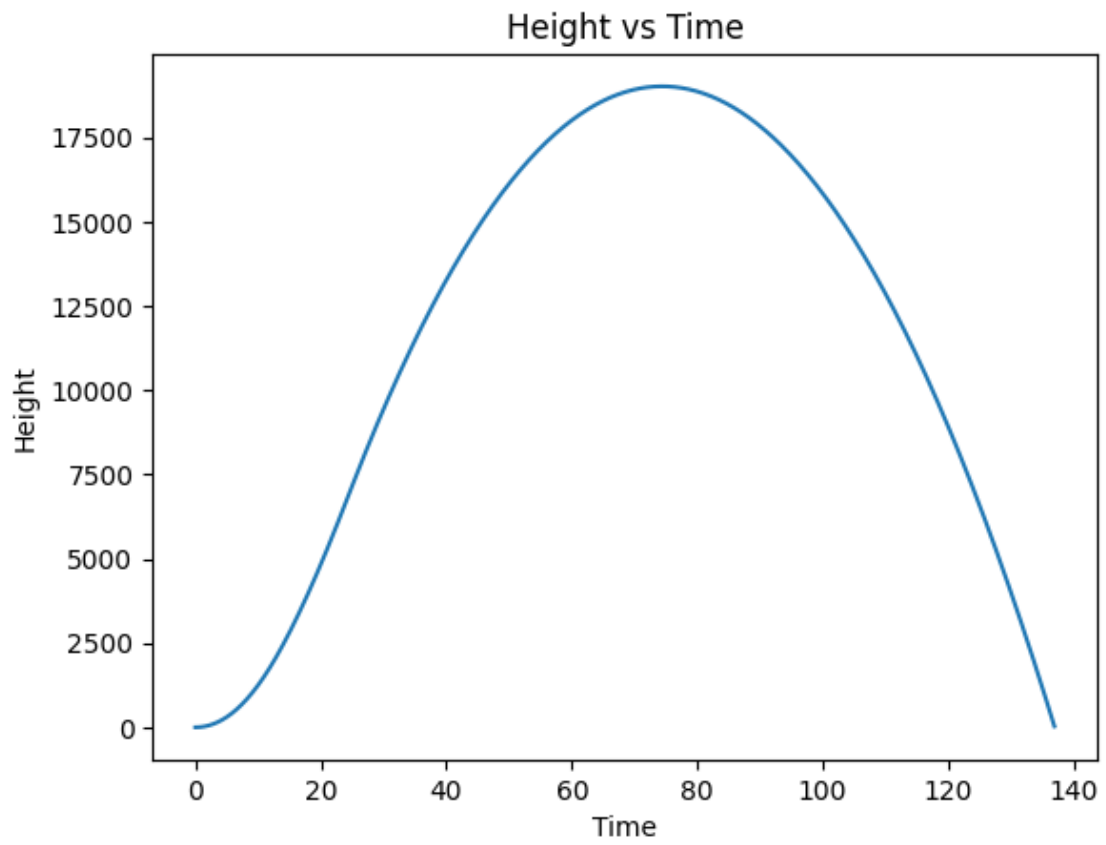
Final results:

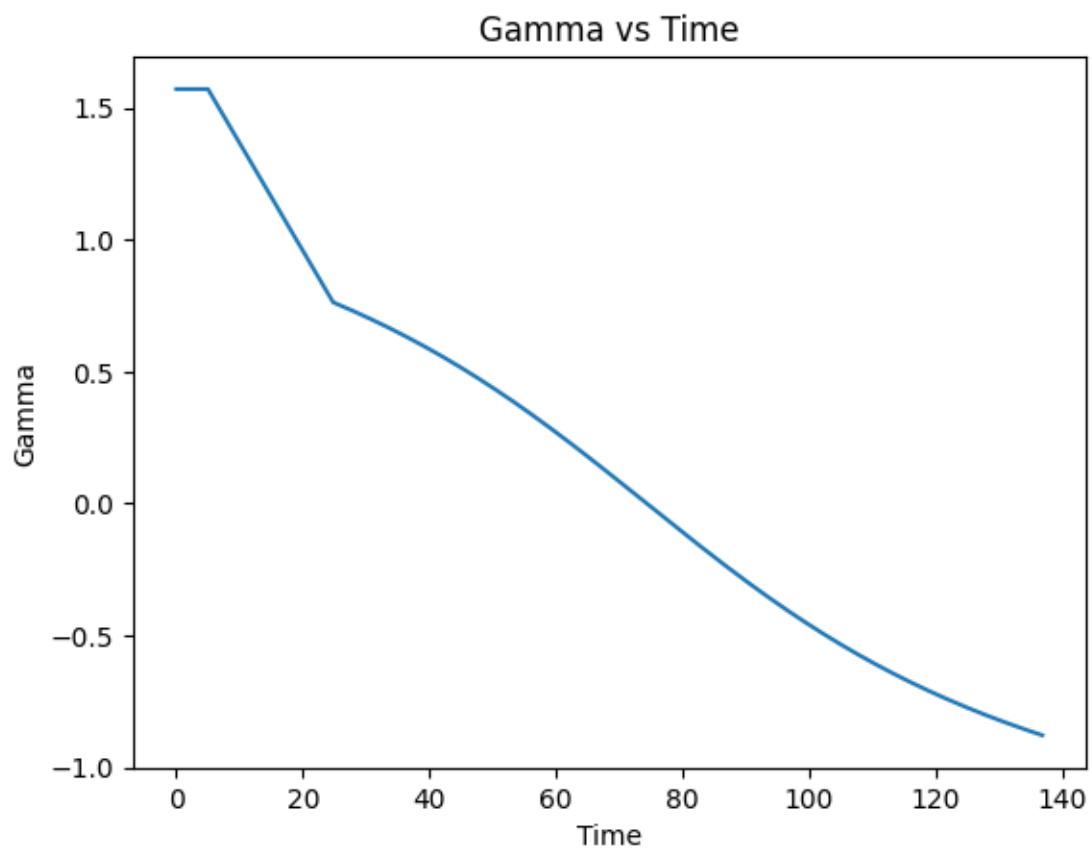
Range (km): 60.11879127124692

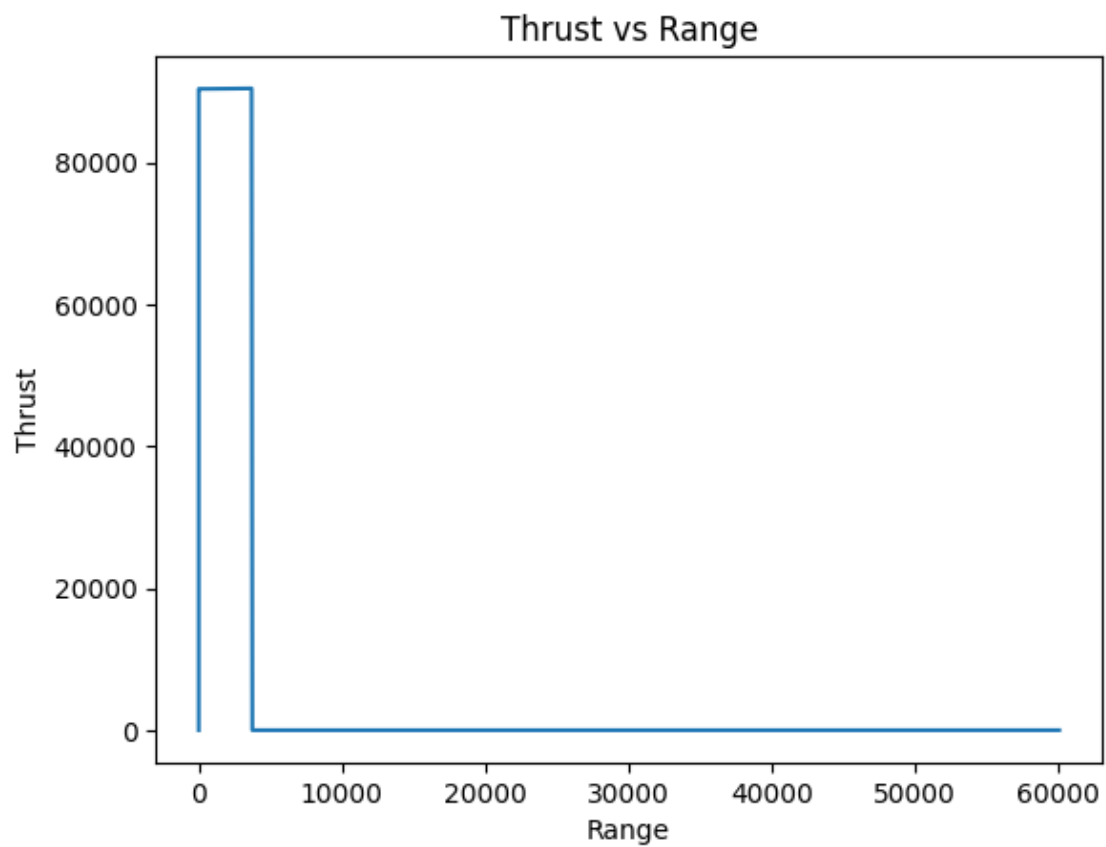
Apogee (km): 19.01539535261587

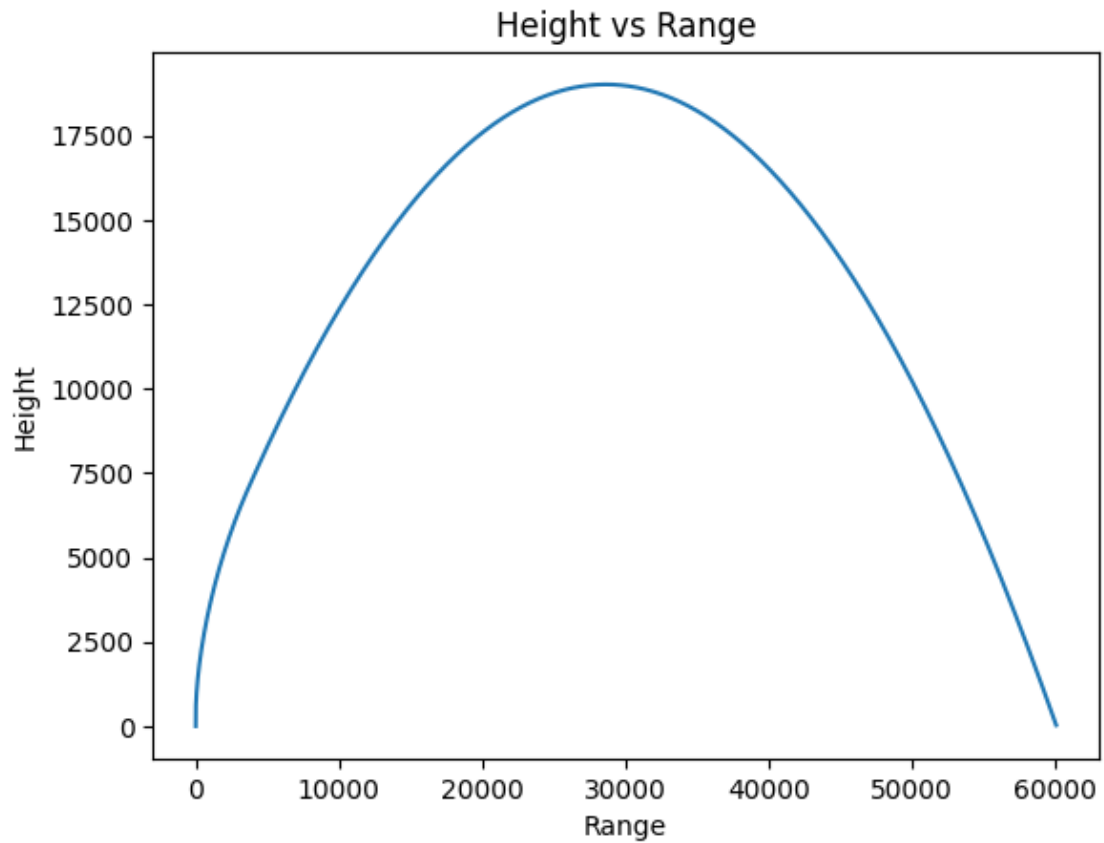
Time to target (sec): 136.9999999999965











Data written to 'results/results_0.txt'

New simulation

fuelmass: 4113.2114793652045

Stage 1 burnout

Velocity (km/s): 1.7930023621327982

Angle (deg h): 43.673294728018924

Range (km): 31.900601741935937

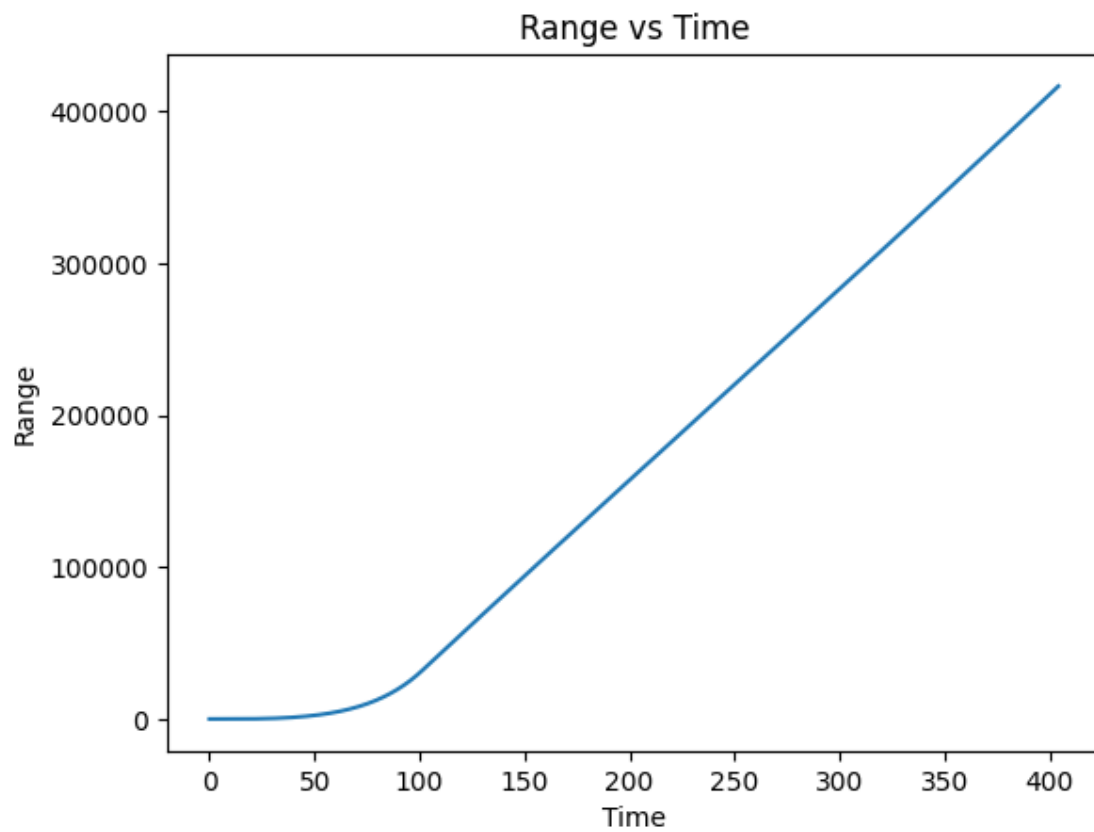
Time (sec): 101.29999999999852

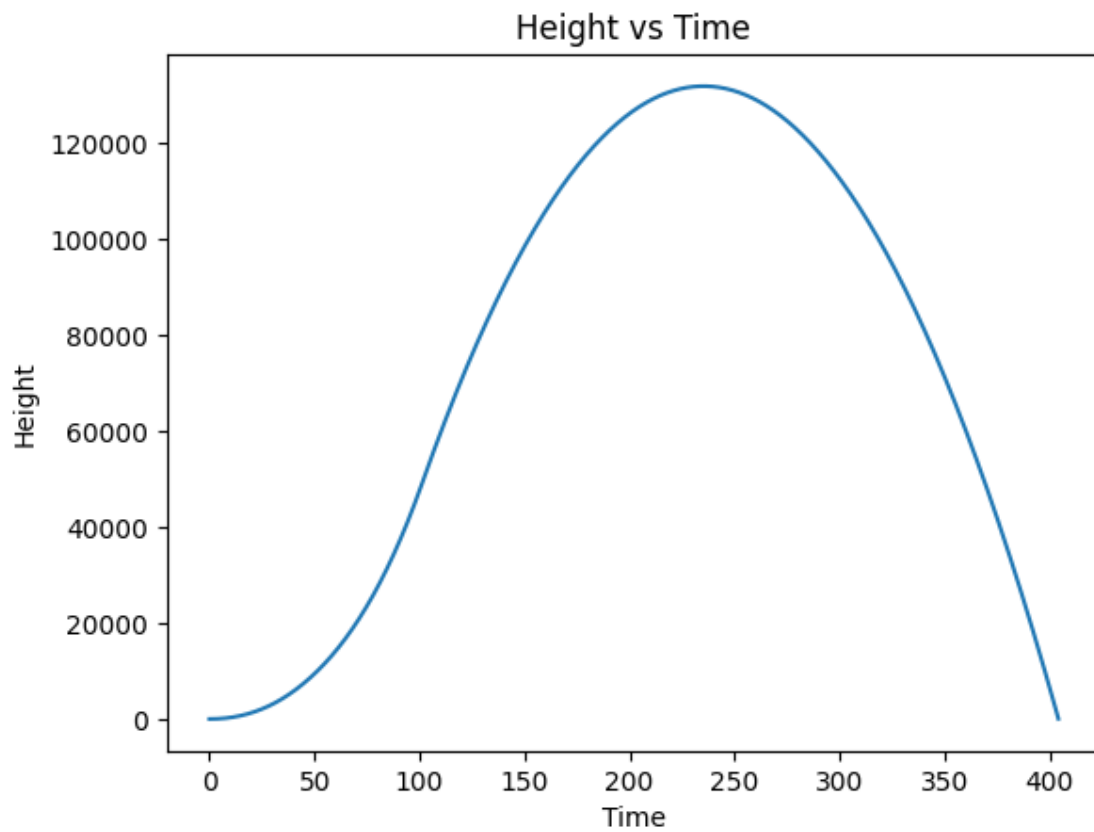
Final results:

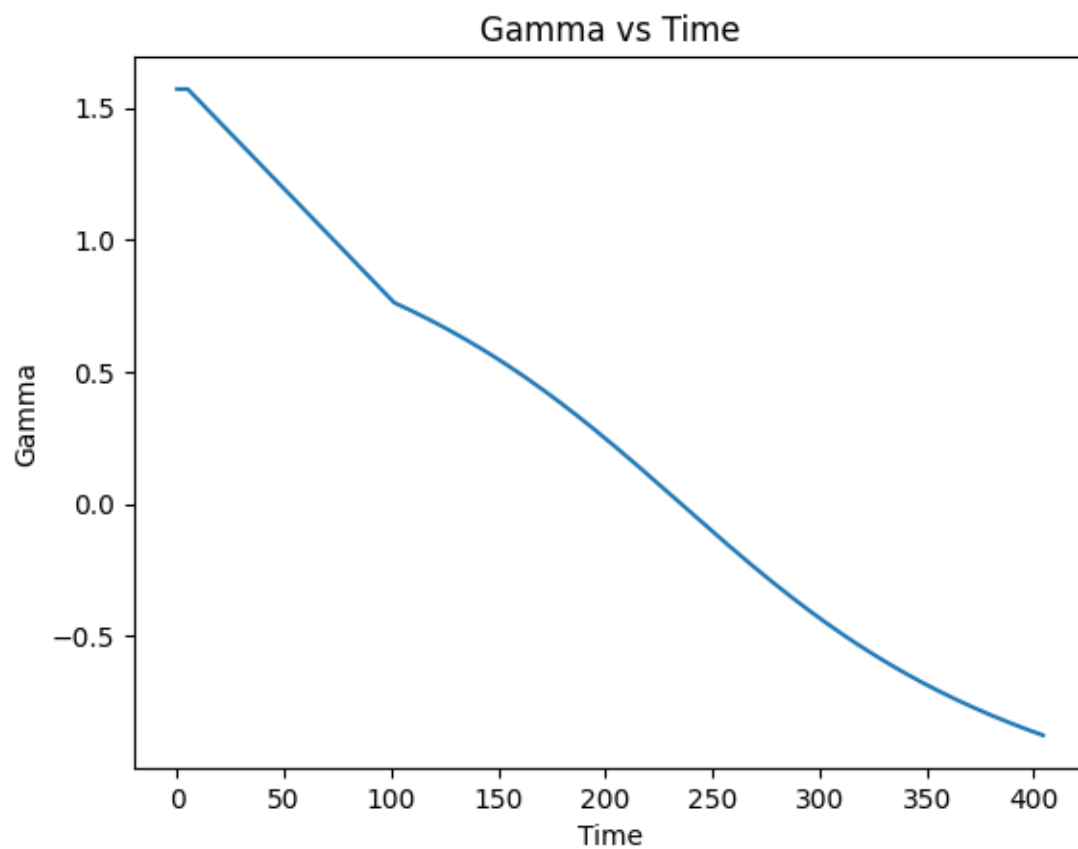
Range (km): 416.4717608333348

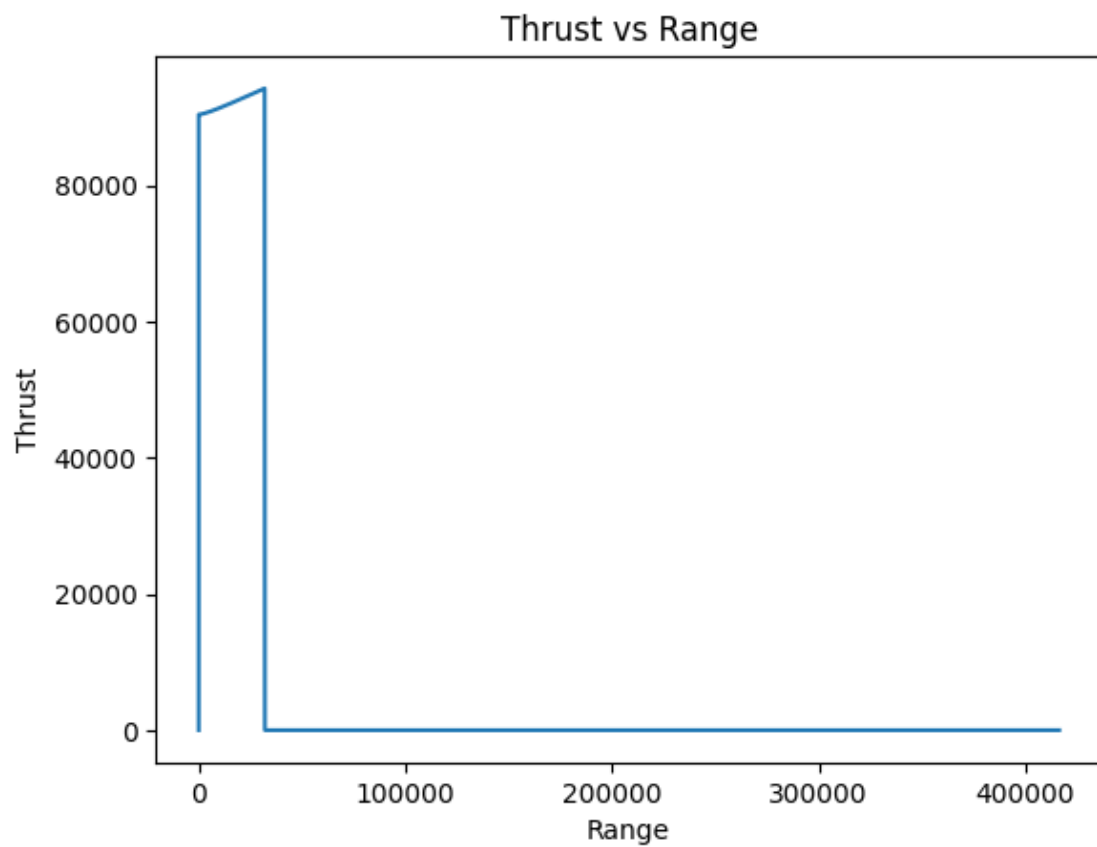
Apogee (km): 131.7242674622094

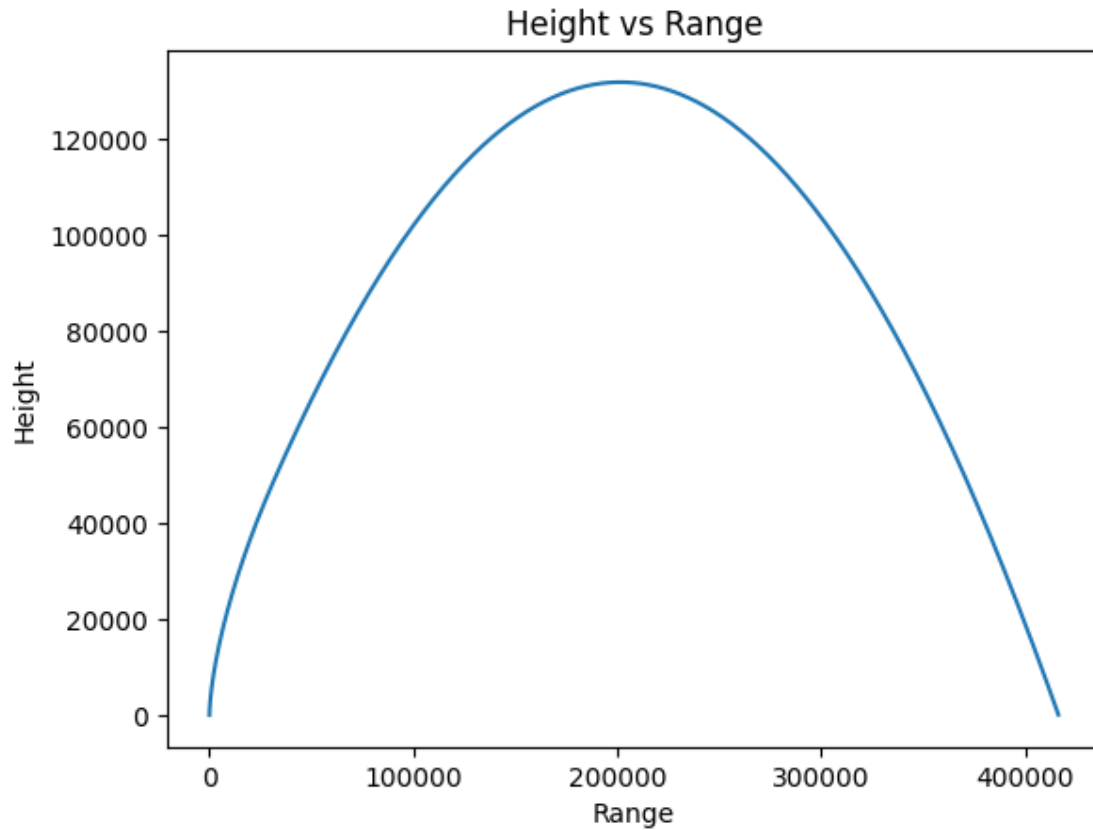
Time to target (sec): 404.40000000002345











Data written to 'results/results_1.txt'

New simulation

fuelmass: 849.9309768862413

Stage 1 burnout

Velocity (km/s): 0.6221936524437466

Angle (deg h): 43.67346391307597

Range (km): 2.758683030930767

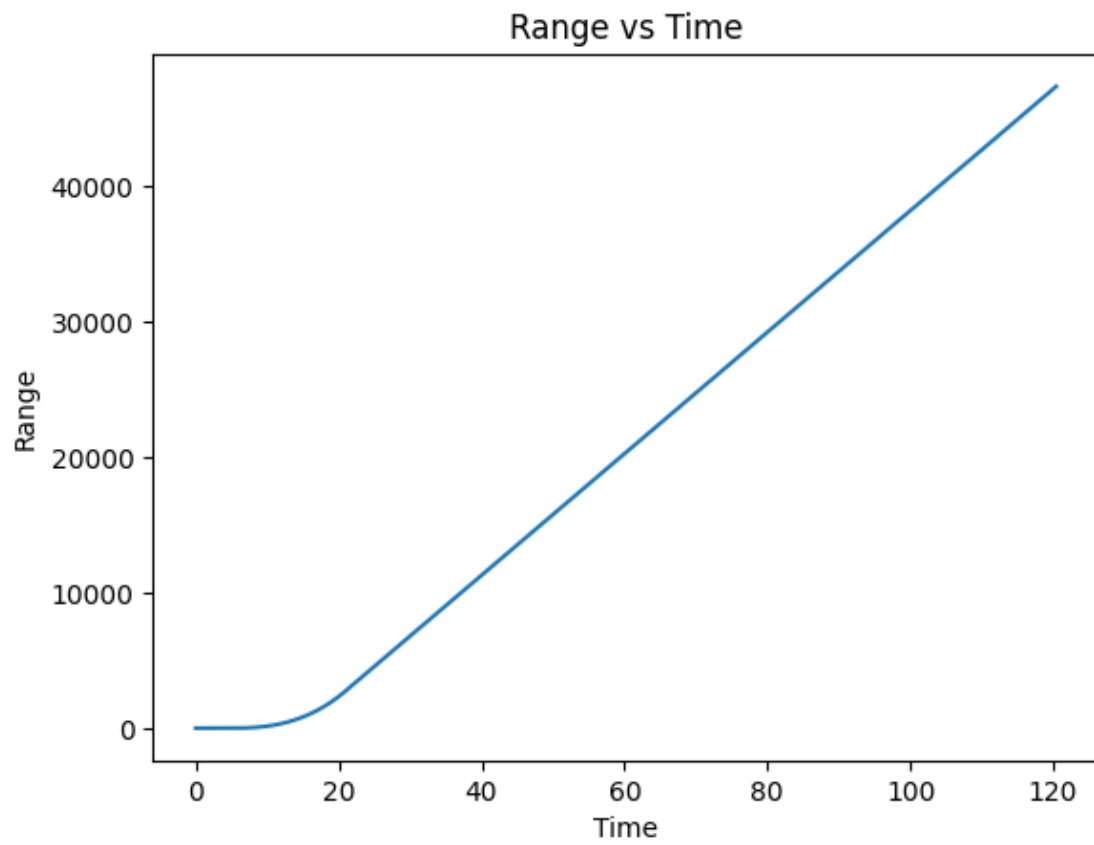
Time (sec): 21.000000000000032

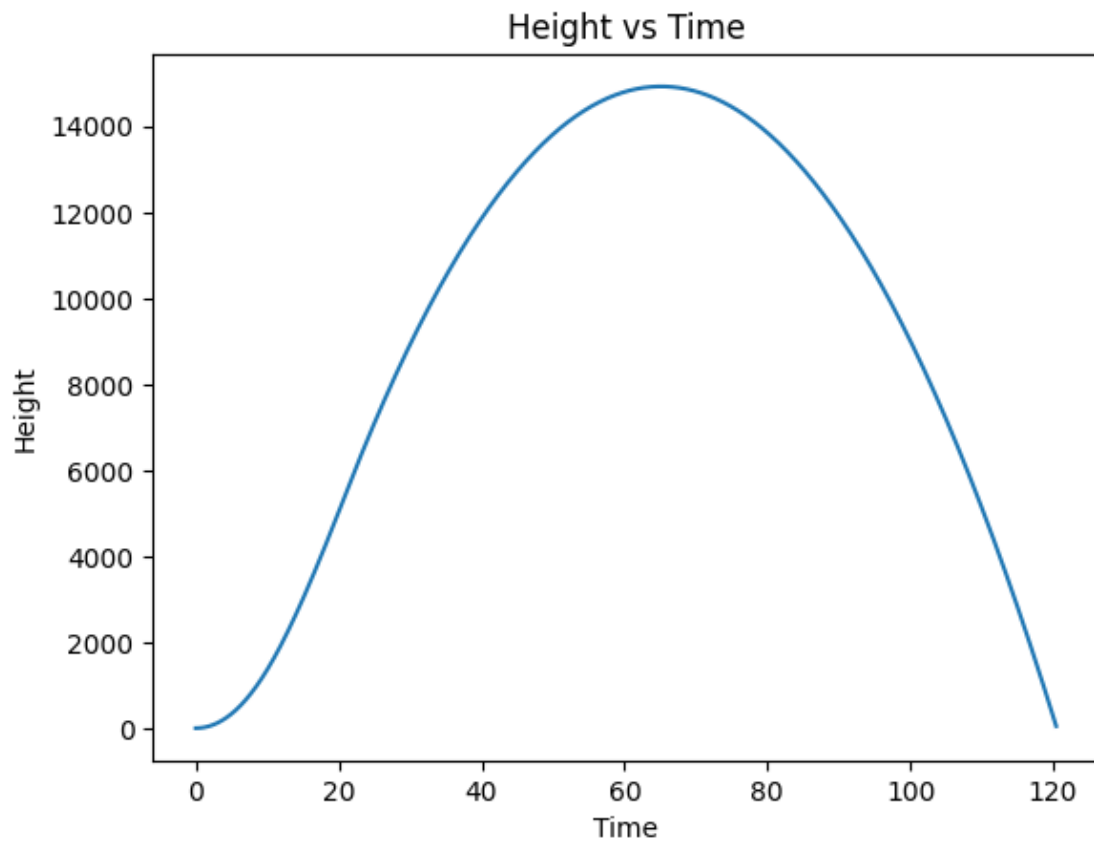
Final results:

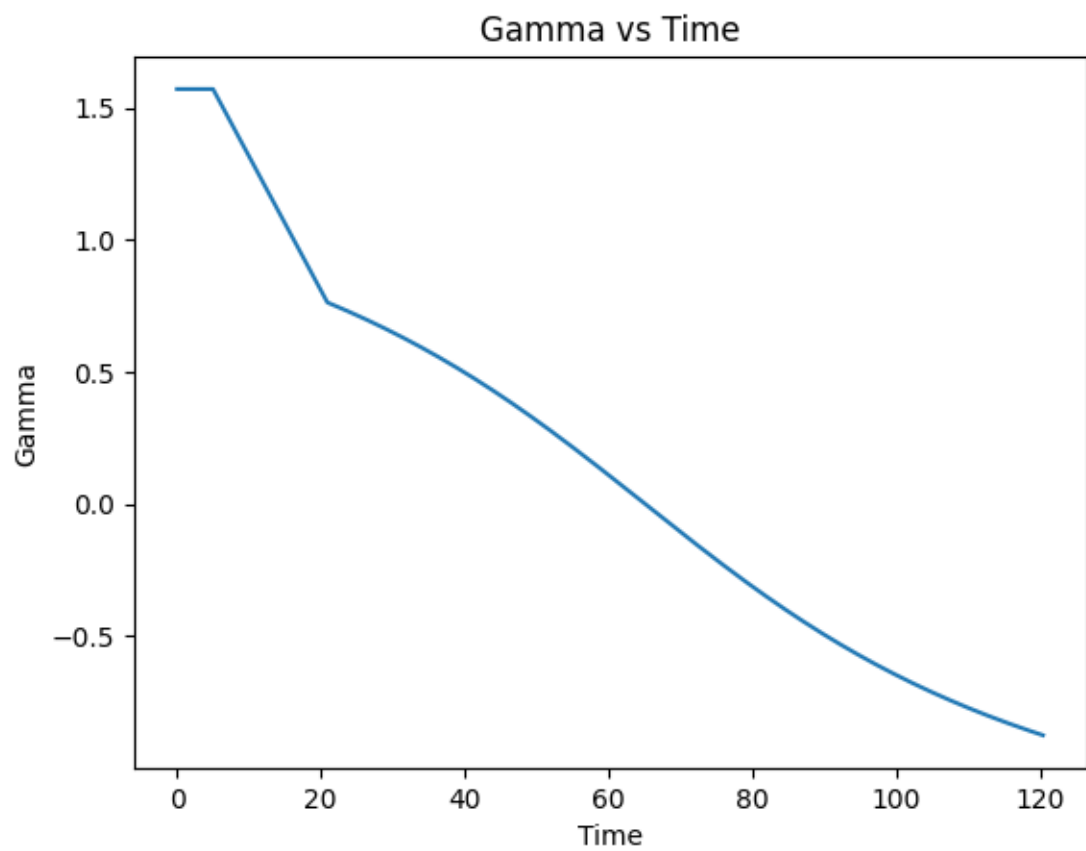
Range (km): 47.42350449217844

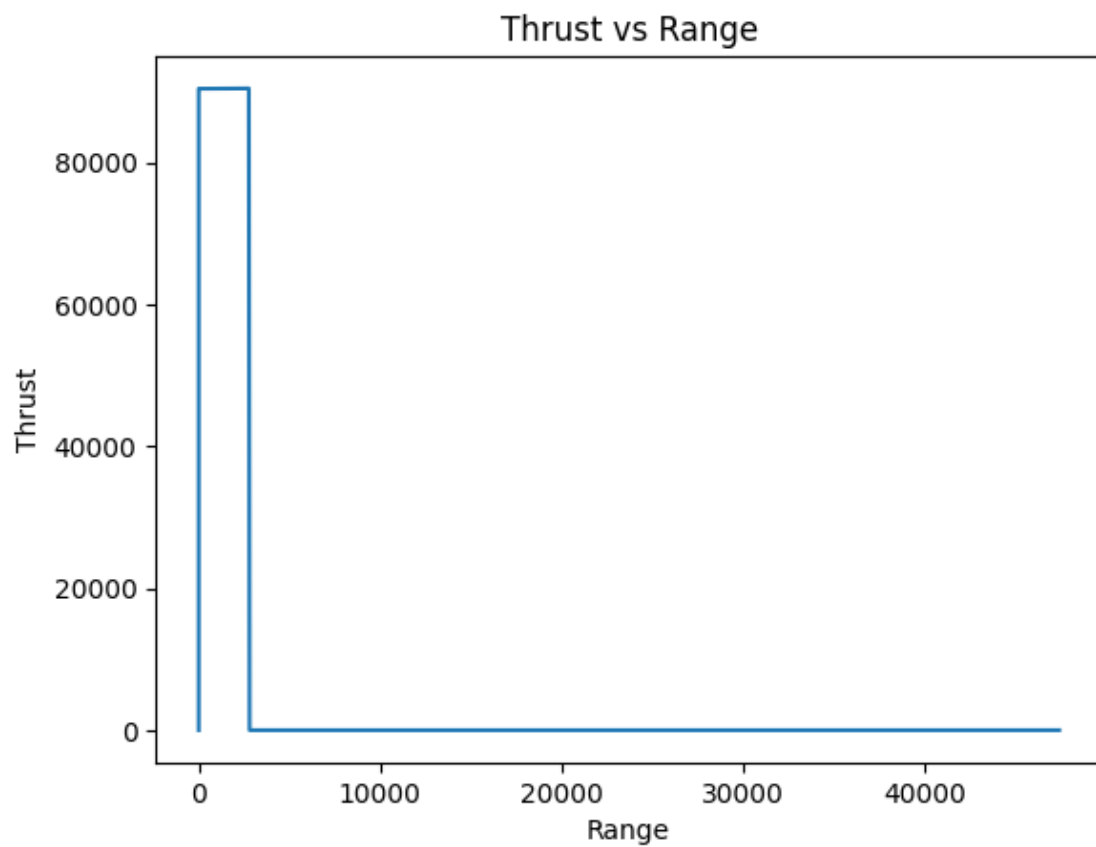
Apogee (km): 14.926926793285853

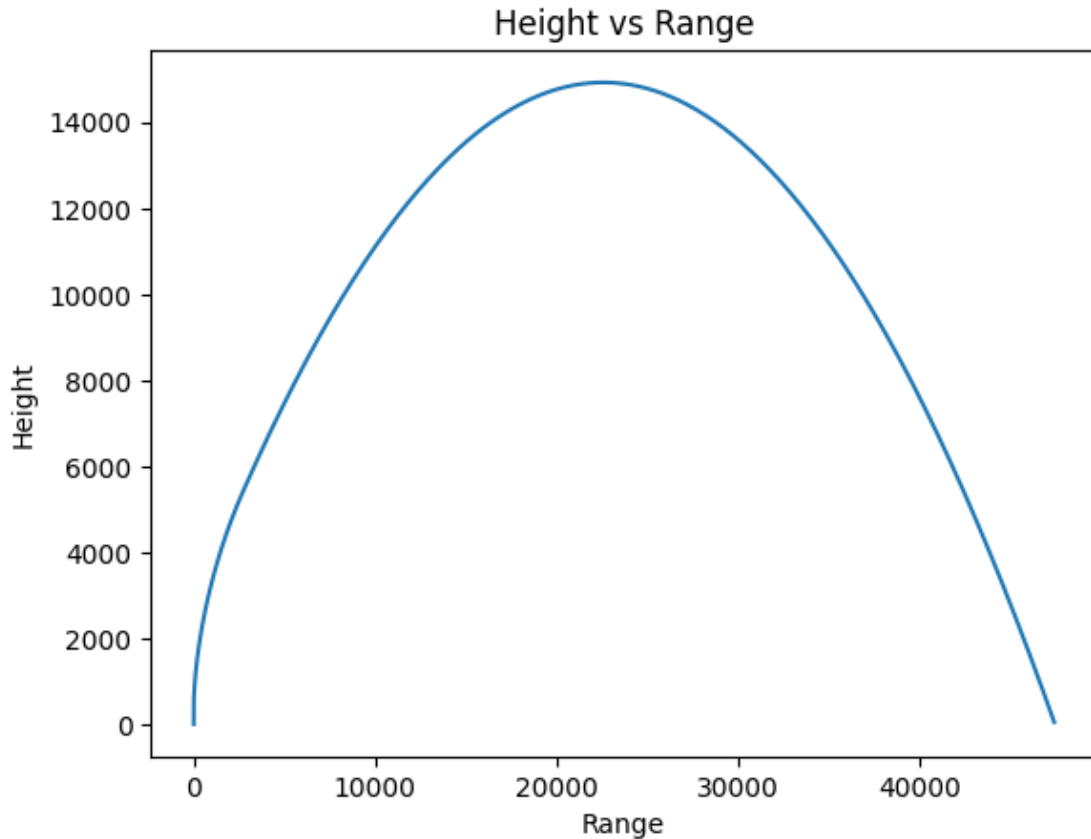
Time to target (sec): 120.49999999999743











Data written to 'results/results_2.txt'

```
[11]: # Build model
m0_var_kernel = (100)**2
m0_lengthscale = 100 # 1
m0_var_linear_kernel = (100)**2
m0_var_noise = 1e-5 # small value

#kern = GPy.kern.RBF(input_dim=1, lengthscale=100, variance =var_kernel ) # ,␣
→lengthscale=0.08, variance=20
# kern = GPy.kern.Matern32(input_dim=1)
# kern = GPy.kern.Linear(input_dim=1)

constrain_lengthscale = False

m0_rbf_kern = GPy.kern.RBF(input_dim=1, lengthscale=m0_lengthscale)
if constrain_lengthscale:
    m0_rbf_kern.lengthscale.constrain_bounded(m0_lengthscale, m0_lengthscale*1e12)

m0_kern = m0_rbf_kern + \
    GPy.kern.Linear(input_dim=1)
```

```

m0_model_gpy = GPRegression(m0_x,m0_y, kernel=m0_kern)
m0_model_gpy.kern.variance = m0_var_kernel
m0_model_gpy.likelihood.variance.fix(m0_var_noise)

display(m0_model_gpy)

```

```
<GPpy.models.gp_regression.GPRegression at 0x7ff43be81db0>
```

```

[12]: # Fit emulator
m0_model_emukit = GPpyModelWrapper(m0_model_gpy)
m0_model_emukit.optimize() # Optimize model hyperparameters

```

```
[13]: display(m0_model_gpy)
```

```
<GPpy.models.gp_regression.GPRegression at 0x7ff43be81db0>
```

```

[14]: # Get true points corresponding to param_1_x_plot (for plot)
wirte_output_txt = False

nr_points_plot = 301
m0_param_1_x_plot = np.linspace(m0_space.parameters[0].min, m0_space.parameters[0].
    ↪max, nr_points_plot)[: , None]
m0_param_1_y_plot = run_missile_sim(m0_param_1_x_plot)

```

```

[15]: # Get model prediction on param_1_x_plot
m0_mu_plot, m0_var_plot = m0_model_emukit.predict(m0_param_1_x_plot)

```

```
[ ]:
```

```

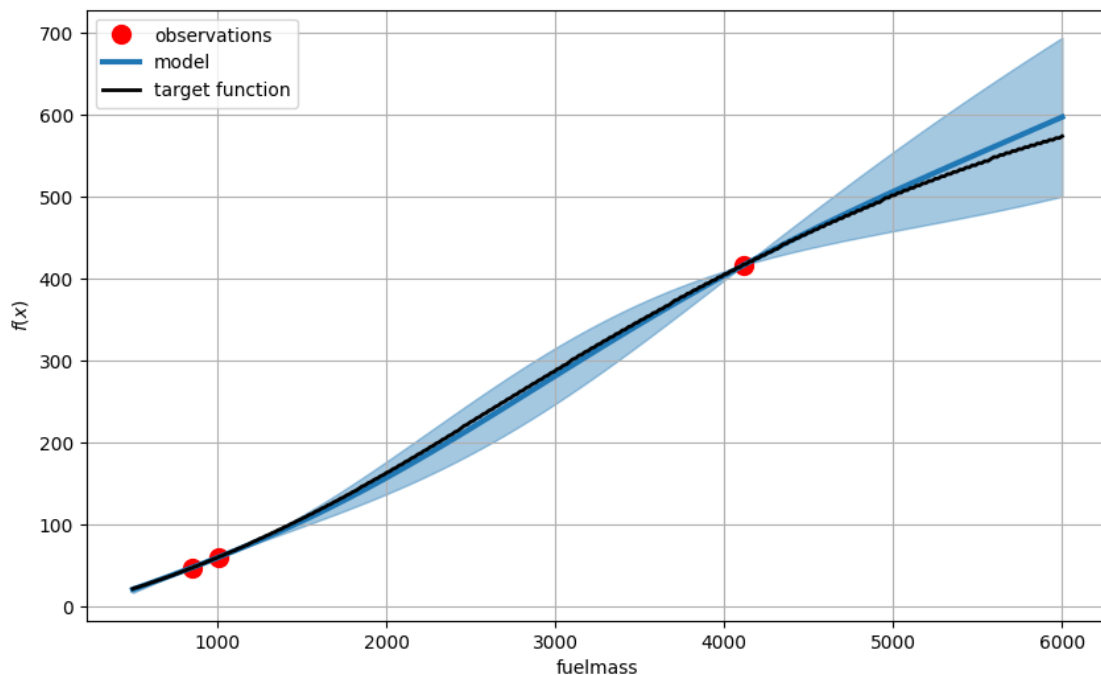
[16]: # Plot
def helper_plot_emulator_errorbars(x_plot, y_plot, mu_plot, var_plot, model_emukit):
    """Helper function for plotting the emulator fit."""
    ax.plot(model_emukit.X[:, 0], model_emukit.Y, 'ro', markersize=10,
    ↪label='observations')
    ax.plot(x_plot[:, 0], mu_plot, 'C0', label='model', linewidth=3)
    ax.plot(x_plot[:, 0], y_plot, 'k', label='target function', linewidth=2)
    # ax.fill_between(x_plot[:, index],
    #                 mu_plot[:, 0] + np.sqrt(var_plot[:, 0]),
    #                 mu_plot[:, 0] - np.sqrt(var_plot[:, 0]), color='C0', alpha=0.6)
    ax.fill_between(x_plot[:, 0],
                    mu_plot[:, 0] + 2 * np.sqrt(var_plot[:, 0]),
                    mu_plot[:, 0] - 2 * np.sqrt(var_plot[:, 0]), color='C0', alpha=0.4)
    # ax.fill_between(x_plot[:, index],
    #                 mu_plot[:, 0] + 3 * np.sqrt(var_plot[:, 0]),
    #                 mu_plot[:, 0] - 3 * np.sqrt(var_plot[:, 0]), color='C0', alpha=0.2)
    ax.legend(loc=2)
    ax.set_xlabel(custom_param_names[0])
    ax.set_ylabel('$f(x)$')
    ax.grid(True)
    #ax.set_xlim(-0.01, 1)
    #ax.set_ylim([-20, 20])

```

```
[17]: fig, ax = plt.subplots(figsize=plot.big_wide_figsize)
      helper_plot_emulator_errorbars(x_plot=m0_param_1_x_plot, y_plot=m0_param_1_y_plot,
                                     mu_plot=m0_mu_plot, var_plot=m0_var_plot,
                                     model_emukit=m0_model_emukit)

      m0_rmse = evaluate_prediction(y_actual=m0_param_1_y_plot, y_predicted=m0_mu_plot)
      print("RMSE m0 (pre experiment design loop): ", m0_rmse)
```

RMSE m0 (pre experiment design loop): 7.175915276215401



1.0.1 Experiment design loop

```
[18]: from emukit.experimental_design.experimental_design_loop import ExperimentalDesignLoop
      from emukit.experimental_design.acquisitions import IntegratedVarianceReduction, \
      ↪ ModelVariance
```

```
[19]: m0_2_model_emukit = m0_model_emukit
```

```
[20]: write_output_txt = False

      integrated_variance = IntegratedVarianceReduction(space=m0_space,
                                                         model=m0_2_model_emukit)
      m0_ed = ExperimentalDesignLoop(space=m0_space,
                                     model=m0_2_model_emukit,
                                     acquisition = integrated_variance,
                                     batch_size = 1)

      # batch size is set to one in this example as we'll collect evaluations
      # sequentially but parallel evaluations are allowed
```

```
m0_ed.run_loop(user_function=run_missile_sim, stopping_condition=5)
```

```
/Users/ilariasartori/opt/anaconda3/envs/mlphysical/lib/python3.10/site-  
packages/paramz/transformations.py:111: RuntimeWarning:overflow encountered in  
expm1
```

New simulation

fuelmass: 5336.286633499005

Stage 1 burnout

Velocity (km/s): 2.0031498796355165
Angle (deg h): 43.65441827225012
Range (km): 42.40099812276978
Time (sec): 131.39999999999682
Final results:
Range (km): 527.3657480797298
Apogee (km): 167.1947200518729
Time to target (sec): 475.4000000000396

New simulation

fuelmass: 2870.056178551053

Stage 1 burnout

Velocity (km/s): 1.4603312433832933
Angle (deg h): 43.67861432087781
Range (km): 19.80304752155
Time (sec): 70.70000000000026
Final results:
Range (km): 271.76080359339636
Apogee (km): 85.81808126129849
Time to target (sec): 312.7000000000026

New simulation

fuelmass: 5437.440730935407

Stage 1 burnout

Velocity (km/s): 2.016959034014756
Angle (deg h): 43.65110927817635
Range (km): 43.185262500067815
Time (sec): 133.89999999999668
Final results:
Range (km): 535.1532002014948
Apogee (km): 169.6705625053785

Time to target (sec): 480.6000000000408

New simulation

fuelmass: 2436.318992076523

Stage 1 burnout

Velocity (km/s): 1.3059979248887084

Angle (deg h): 43.70171652329588

Range (km): 15.559733488322564

Time (sec): 60.00000000000058

Final results:

Range (km): 216.34529712094323

Apogee (km): 68.50093463288768

Time to target (sec): 275.09999999999405

New simulation

fuelmass: 5771.536834897998

Stage 1 burnout

Velocity (km/s): 2.0607572828510845

Angle (deg h): 43.642312875155234

Range (km): 45.71206092652399

Time (sec): 142.1999999999962

Final results:

Range (km): 560.3907944209011

Apogee (km): 177.55485806745324

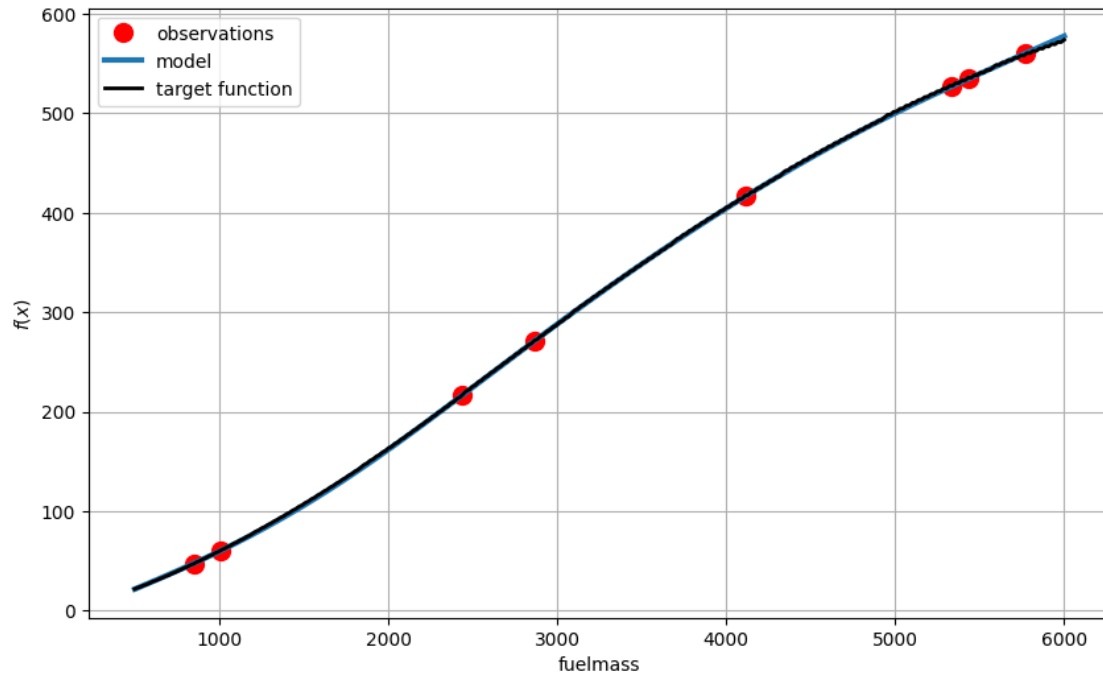
Time to target (sec): 497.60000000004464

```
[21]: m0_2_mu_plot, m0_2_var_plot = m0_2_model_emukit.predict(m0_param_1_x_plot)
```

```
[22]: fig, ax = plt.subplots(figsize=plot.big_wide_figsize)
      helper_plot_emulator_errorbars(x_plot=m0_param_1_x_plot, y_plot=m0_param_1_y_plot,
                                     mu_plot=m0_2_mu_plot, var_plot=m0_2_var_plot,
                                     model_emukit=m0_2_model_emukit)

      m0_2_rmse = evaluate_prediction(y_actual=m0_param_1_y_plot, y_predicted=m0_2_mu_plot)
      print("RMSE m0 (post experiment design loop): ", m0_2_rmse)
```

RMSE m0 (post experiment design loop): 1.0654800808660763



2 0. Only one param - m1

```
[23]: m1_param_1 = 'Isp0'
m1_domain_param_1 = basic_param_spaces[m1_param_1] # [500, 6000] # [5000, 15000]

m1_space = ParameterSpace(
    [ContinuousParameter(m1_param_1, *m1_domain_param_1),
    ])

custom_param_names = [m1_param_1]
```

```
[24]: def run_missile_sim(custom_params):
    """
    Recives in input an array of custom parameters.
    Each row represents a set of different parameters
    Each column is a different parameter (#cols = len(custom_param_names))
    """
    default_params_IRAQ = {
        'payload':500,
        'missilediam':0.88,
        'rvdiam':0,
        'estrange':600,
        'numstages':1,
        'fuelmass':[0,5600],
        'drymass':[0,1200],
        'Isp0':[0,226],
        'thrust0':[0,9177.4]
```

```

}

y = np.zeros((custom_params.shape[0], 1))
for i in range(custom_params.shape[0]):
    params_to_use = default_params_IRAQ
    # Overwrite default param variables
    for j in range(custom_params.shape[1]):
        param_name = custom_param_names[j]
        if param_name in ['fuelmass', 'drymass', 'Isp0', 'thrust0']:
            params_to_use[param_name][1] = custom_params[i,j] # OK as long as we
→are considering missiles with only 1 stage
        else:
            params_to_use[param_name] = custom_params[i, j]

        ## TEMP ## Better customise this
        if j==0:
            print('\nNew simulation \n')
            str_to_print = param_name + ': ' + str(custom_params[i,j])
            print(str_to_print)
        ##

    # Run simulation
    output_path = 'results/results_' + str(i) + '.txt' # TODO Define better
→identifier
    sim_output = run_one_sim(
        numstages=params_to_use["numstages"],
        fuelmass=params_to_use["fuelmass"],
        drymass=params_to_use["drymass"],
        thrust0=params_to_use["thrust0"],
        Isp0=params_to_use["Isp0"],
        payload=params_to_use["payload"],
        missilediam=params_to_use["missilediam"],
        rvdiam=params_to_use["rvdiam"],
        est_range=params_to_use["estrange"],
        output_path=output_path,
        simulation_output=simulation_output,
    )

    y[i, 0] = sim_output
return y

```

```

[25]: # Get true points (to build model)
wirte_output_txt = True

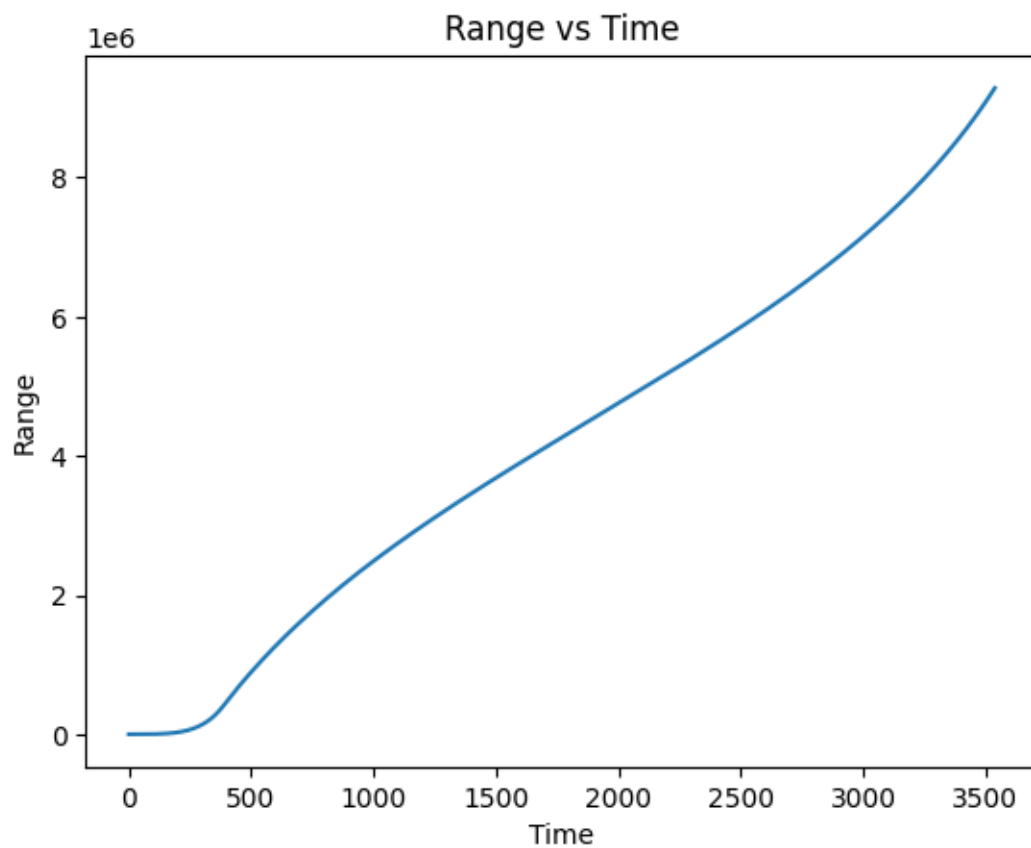
m1_design = RandomDesign(m1_space)
m1_x = m1_design.get_samples(3)
m1_y = run_missile_sim(m1_x)

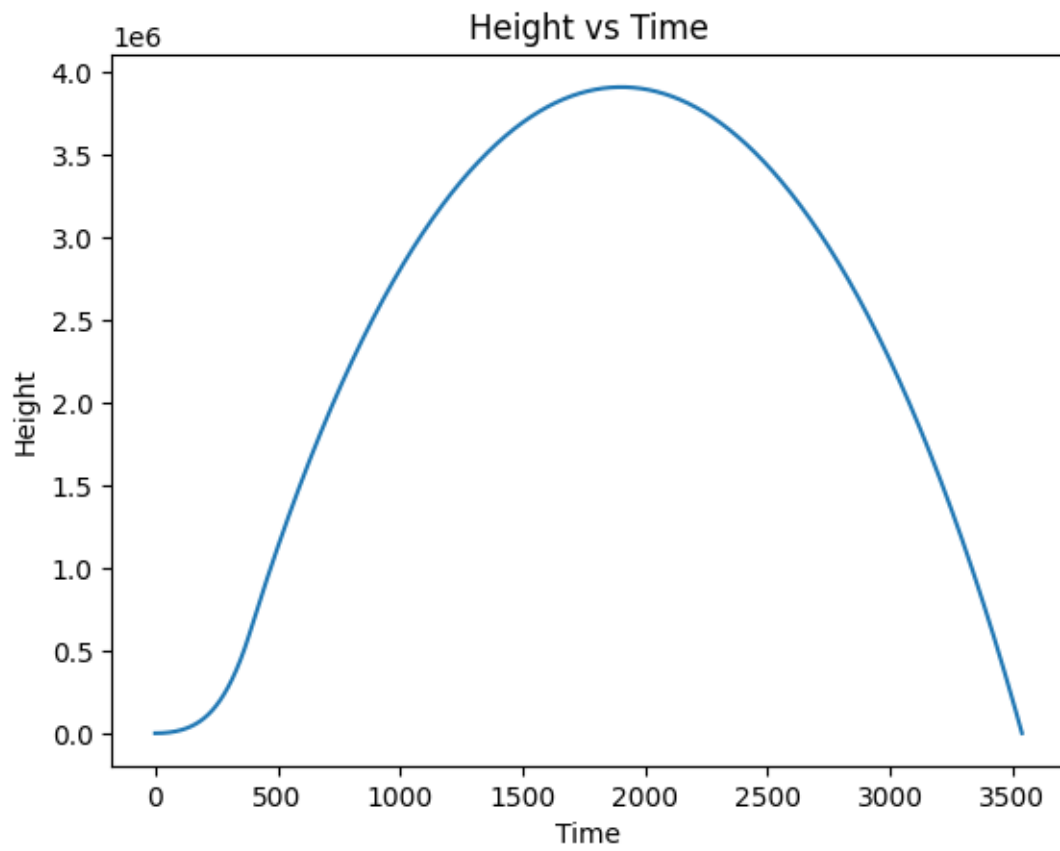
```

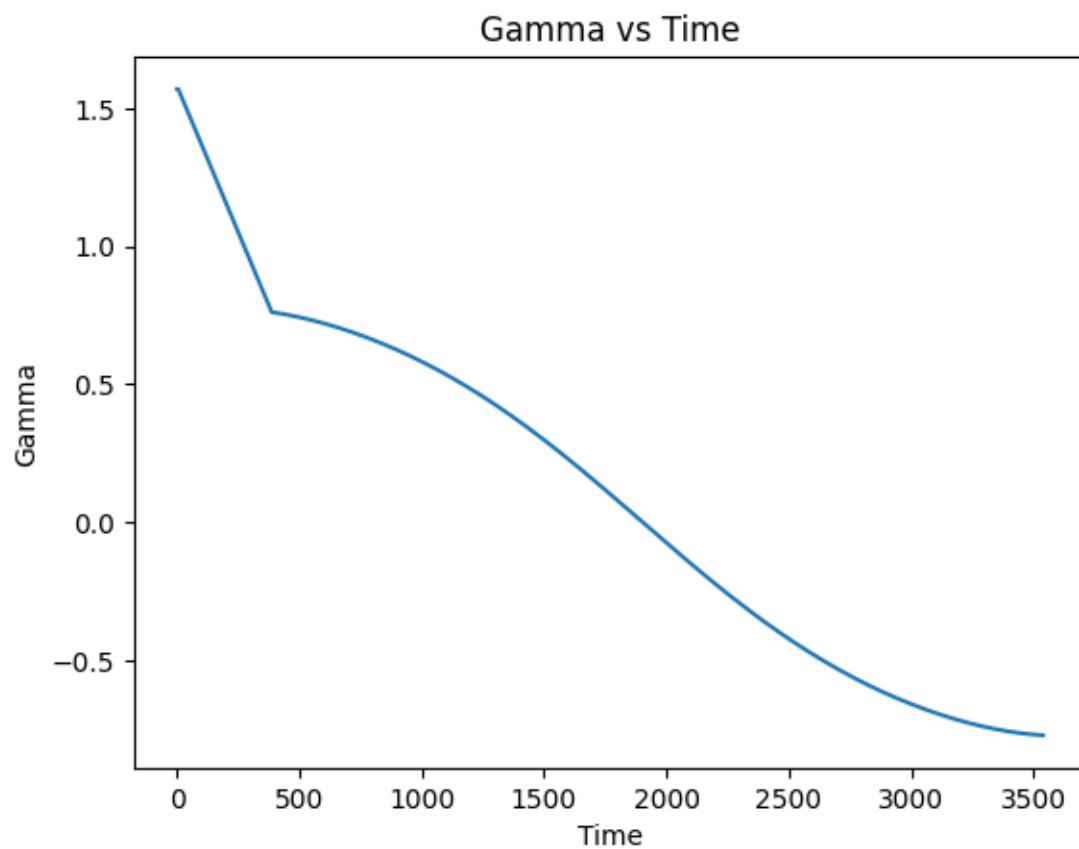
New simulation

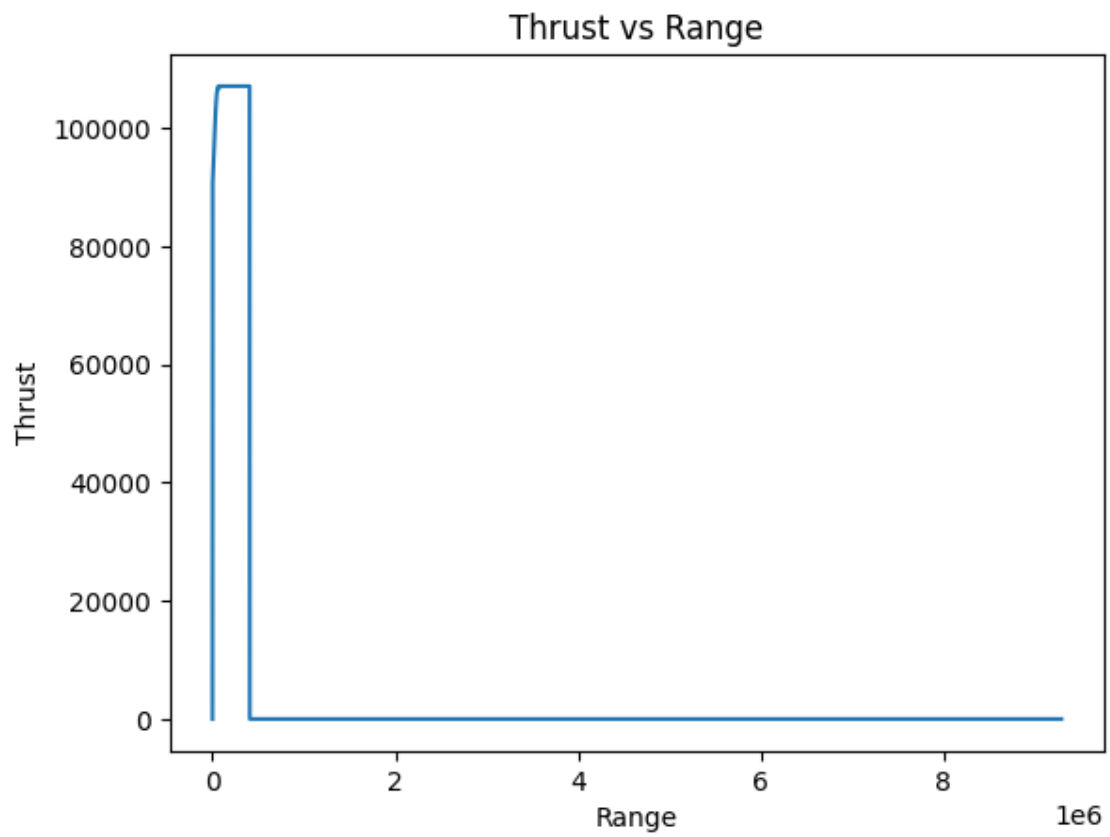
Isp0: 632.4850588045783

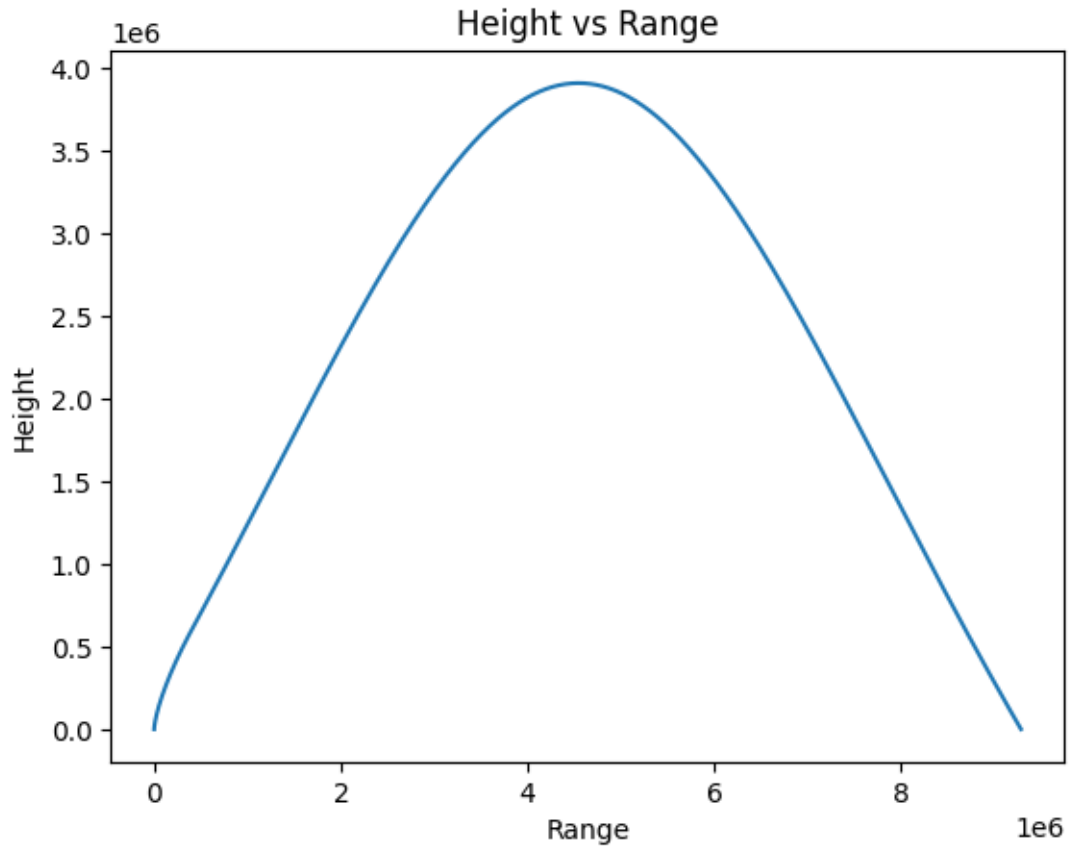
Stage 1 burnout
Velocity (km/s): 6.951986958952771
Angle (deg h): 43.65479851742443
Range (km): 406.2582827315758
Time (sec): 386.00000000001927
Final results:
Range (km): 9287.467919626113
Apogee (km): 3911.6038081153692
Time to target (sec): 3538.899999997877











Data written to 'results/results_0.txt'

New simulation

Isp0: 261.652194637604

Stage 1 burnout

Velocity (km/s): 2.4293213942317604

Angle (deg h): 43.653675344881215

Range (km): 60.73458609084893

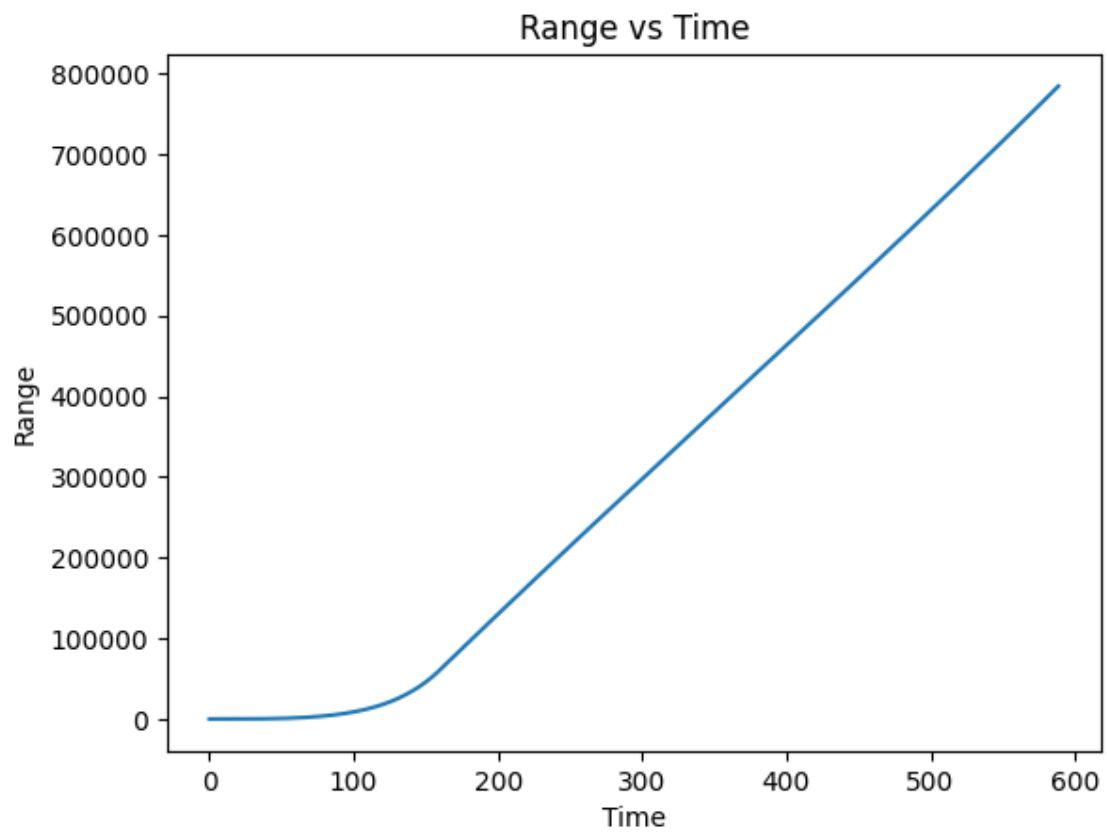
Time (sec): 159.6999999999952

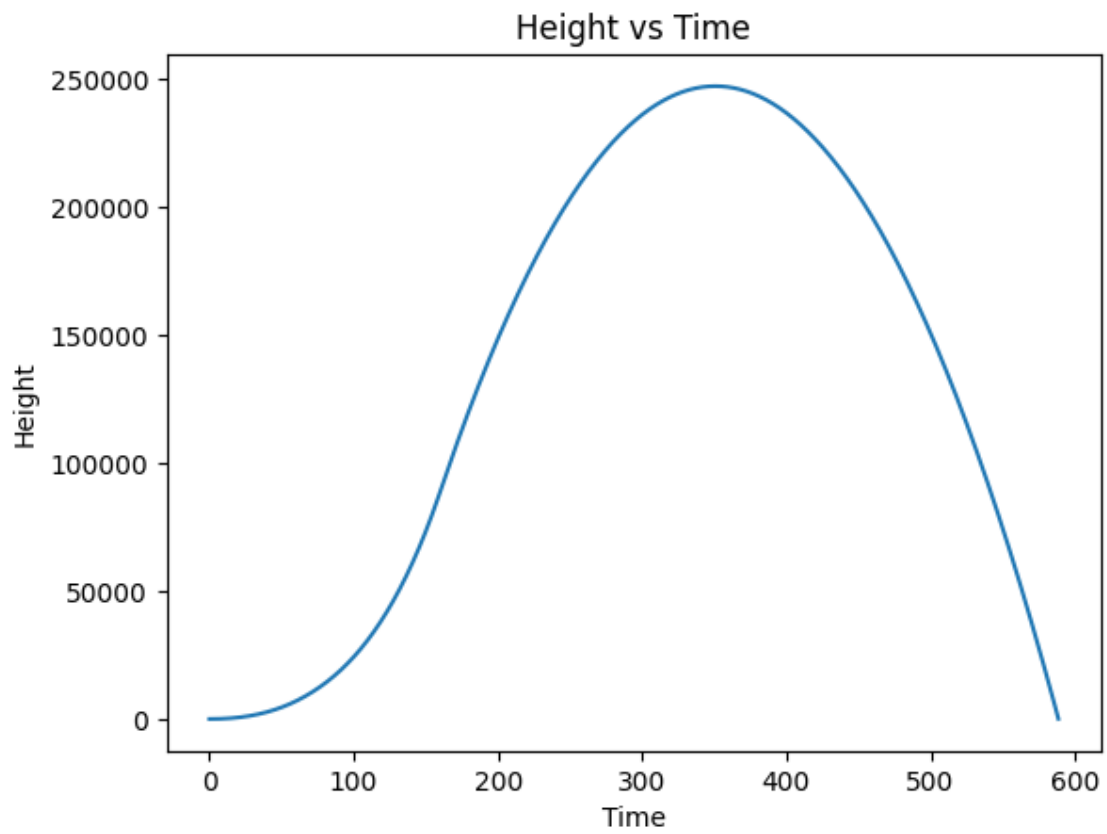
Final results:

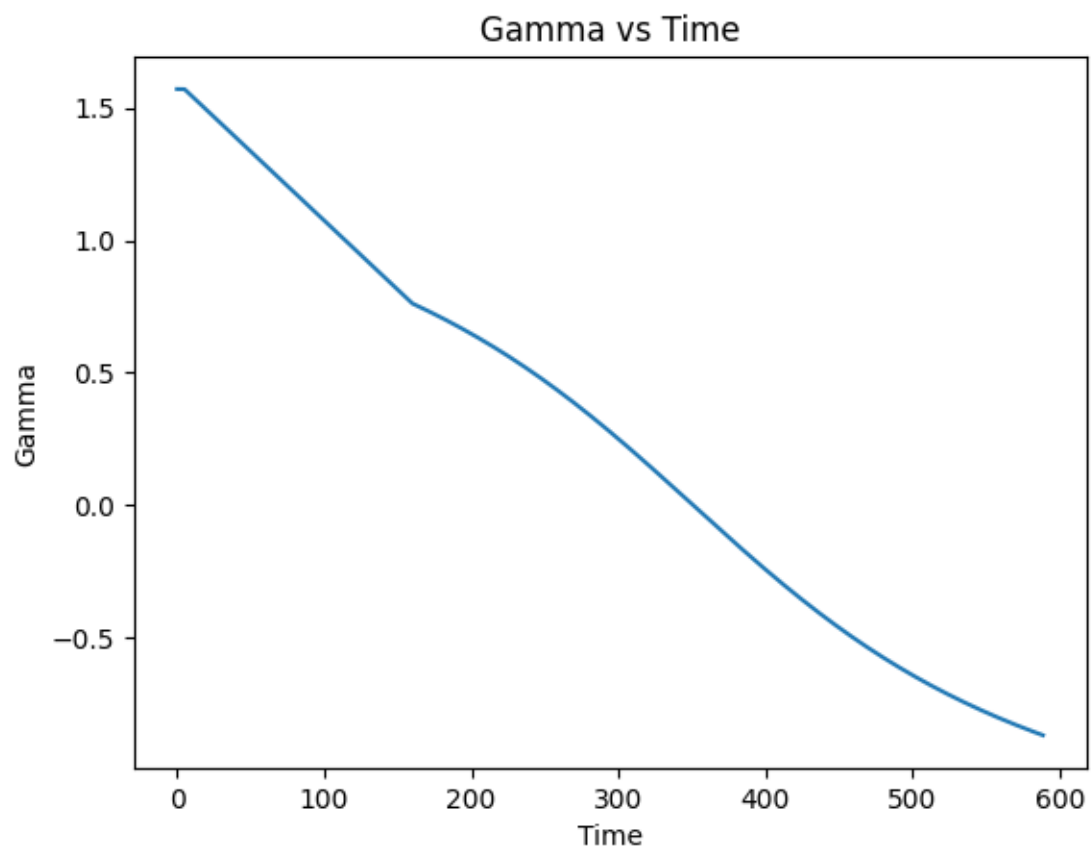
Range (km): 784.223400078734

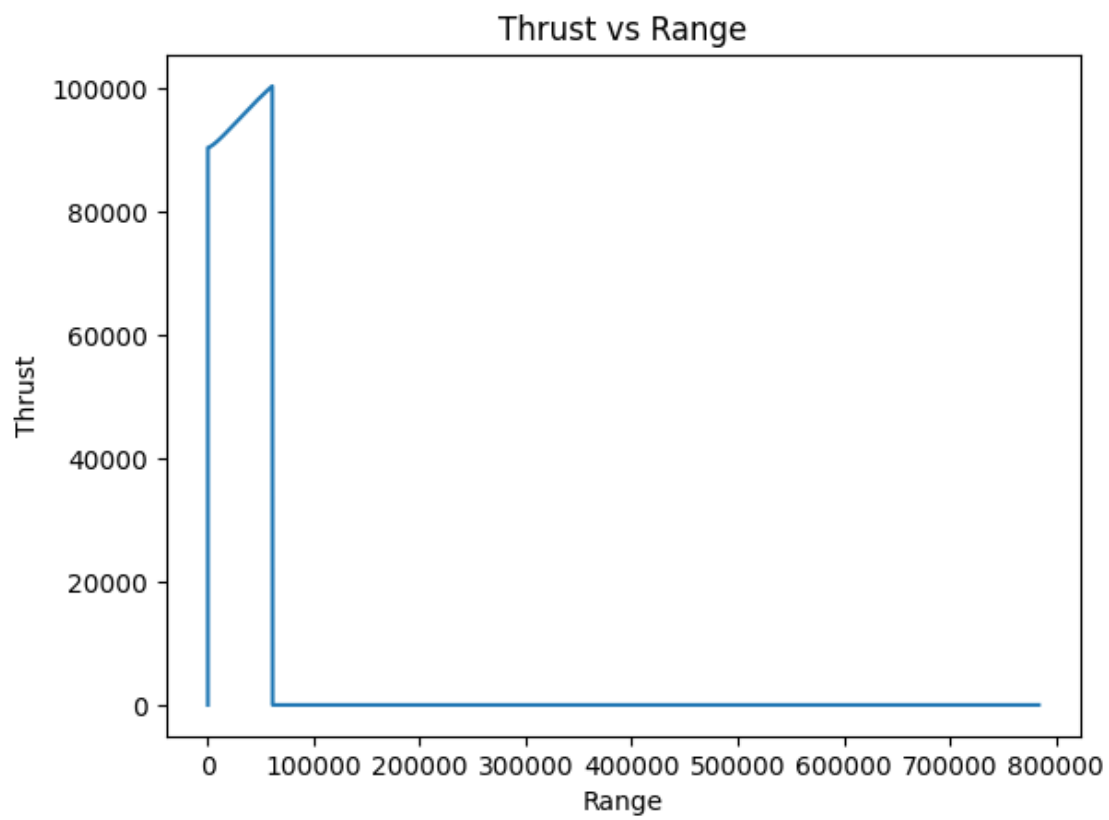
Apogee (km): 247.42768830021498

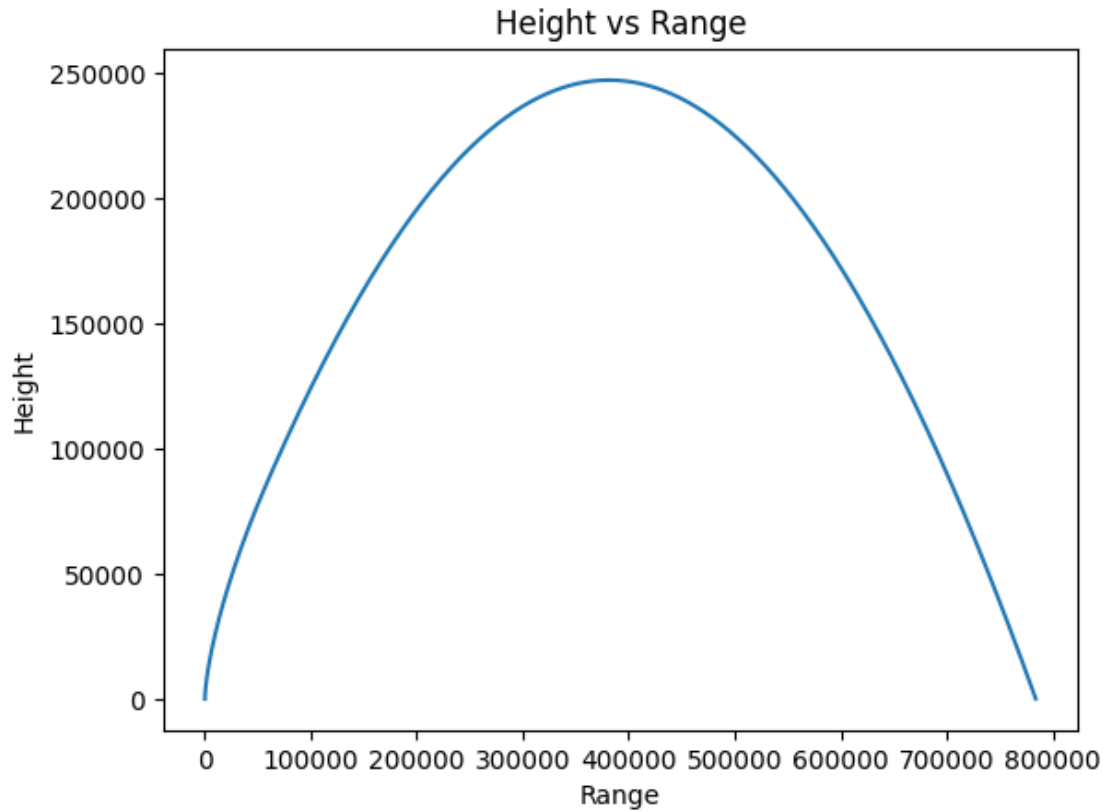
Time to target (sec): 588.6000000000653











Data written to 'results/results_1.txt'

New simulation

Isp0: 220.8452126160107

Stage 1 burnout

Velocity (km/s): 1.98477240573621

Angle (deg h): 43.65288429473307

Range (km): 42.32475216111083

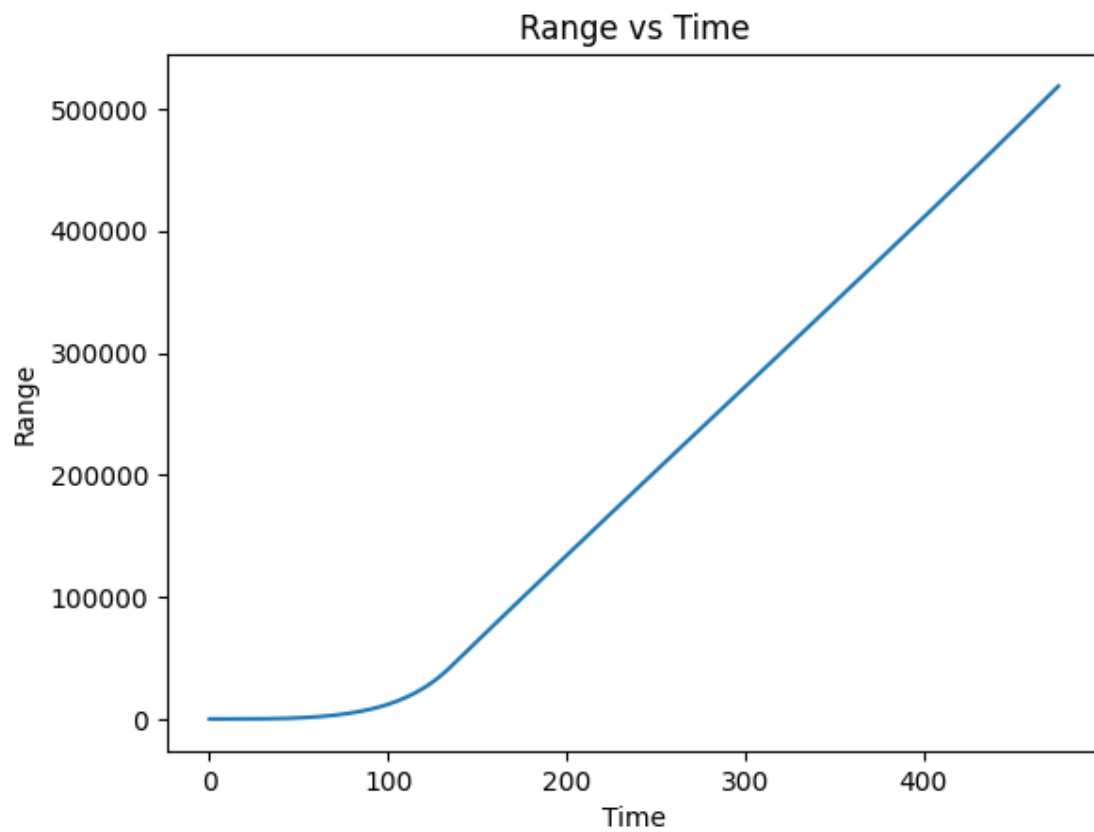
Time (sec): 134.79999999999663

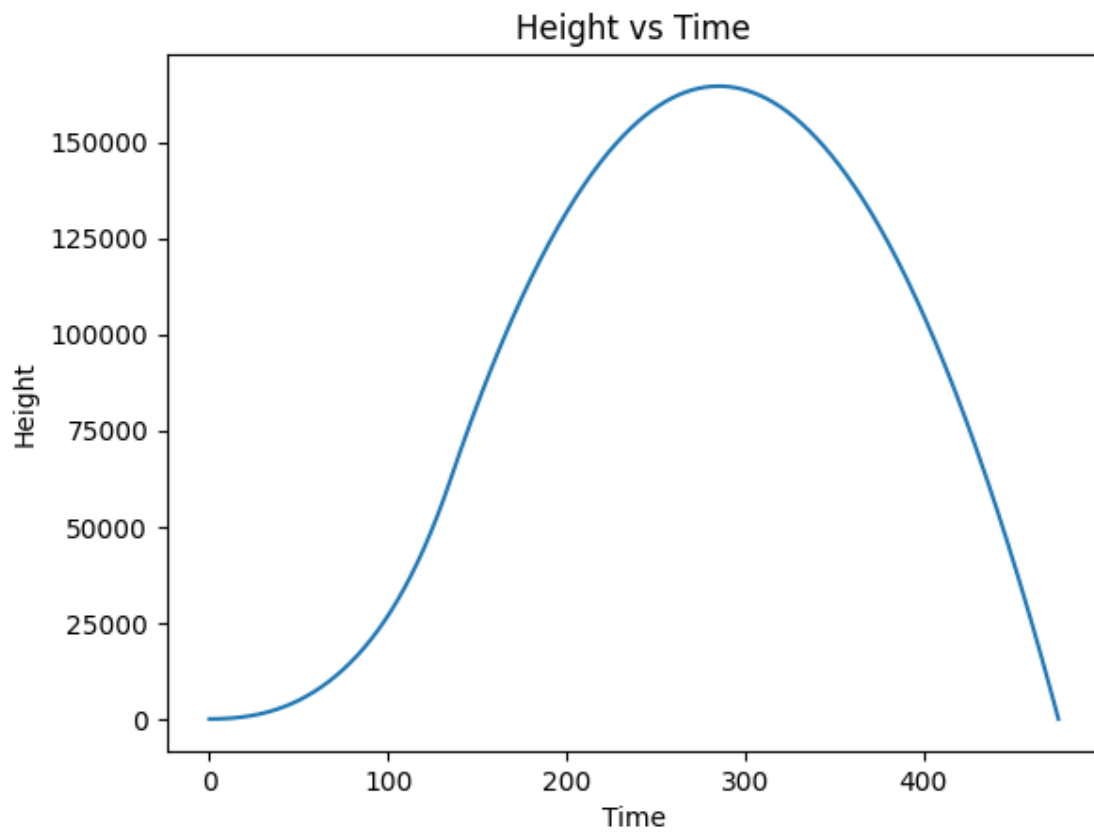
Final results:

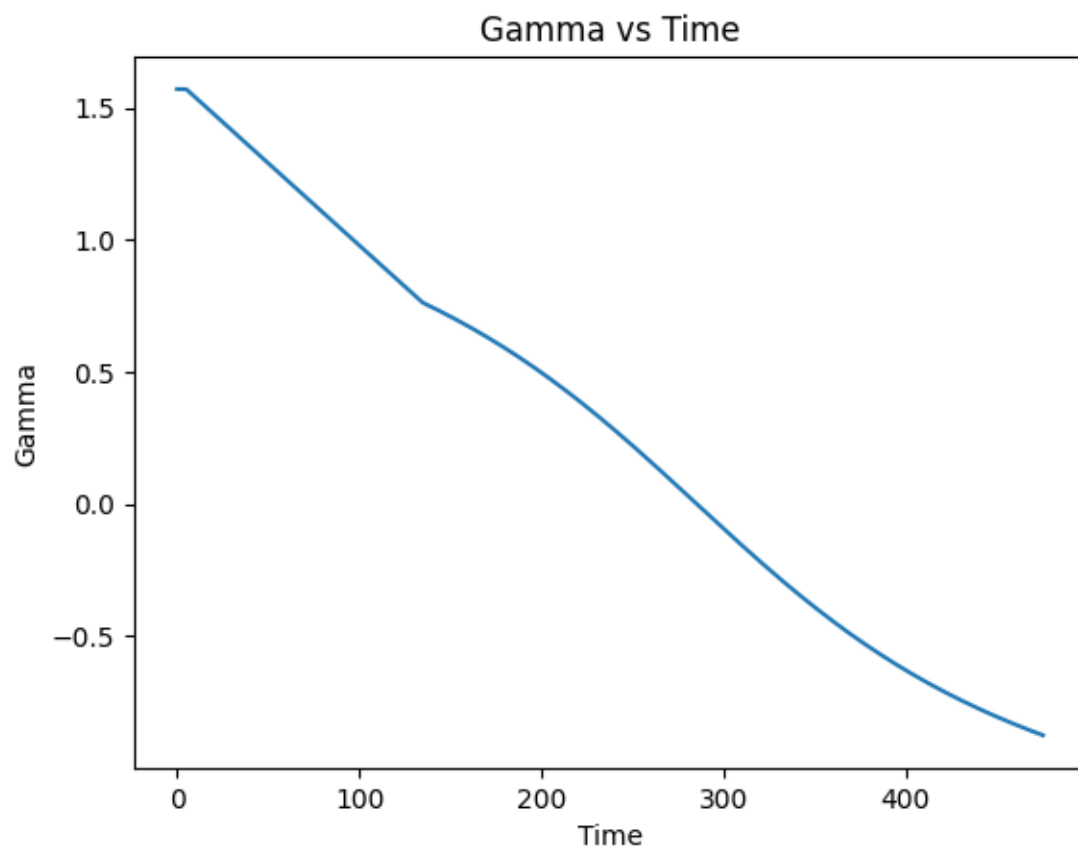
Range (km): 518.6145973012602

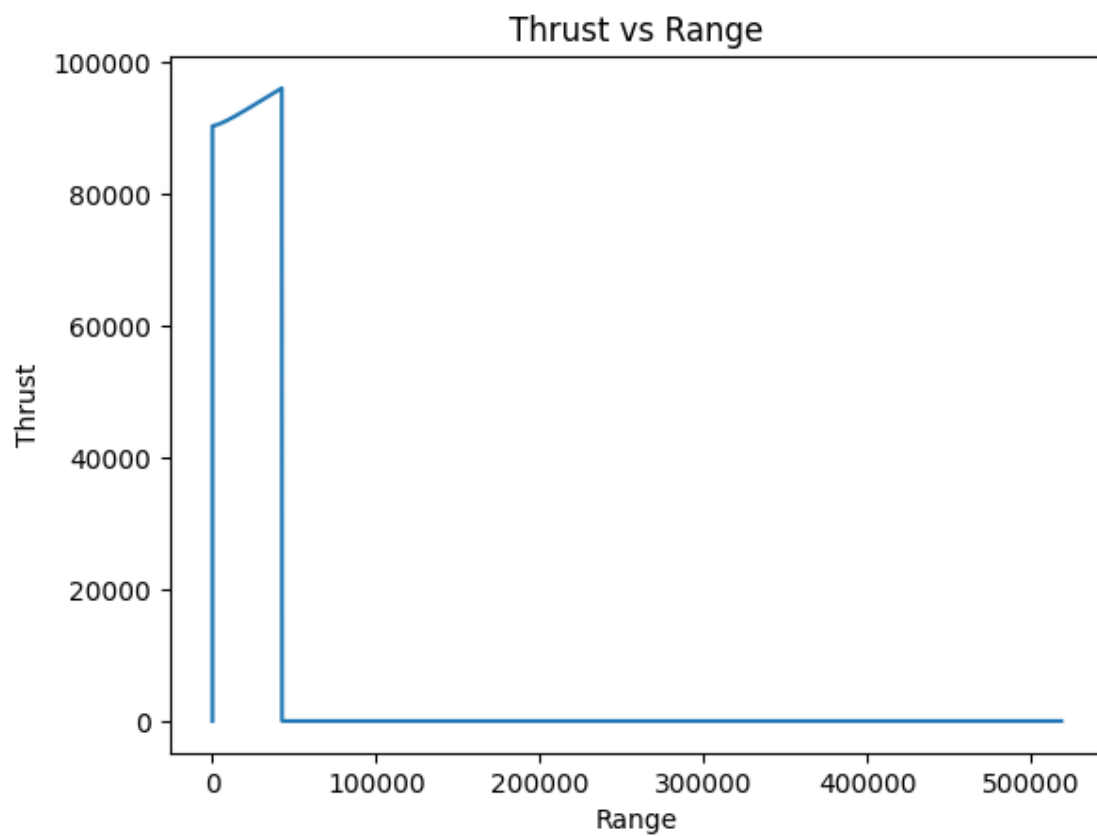
Apogee (km): 164.50094646411802

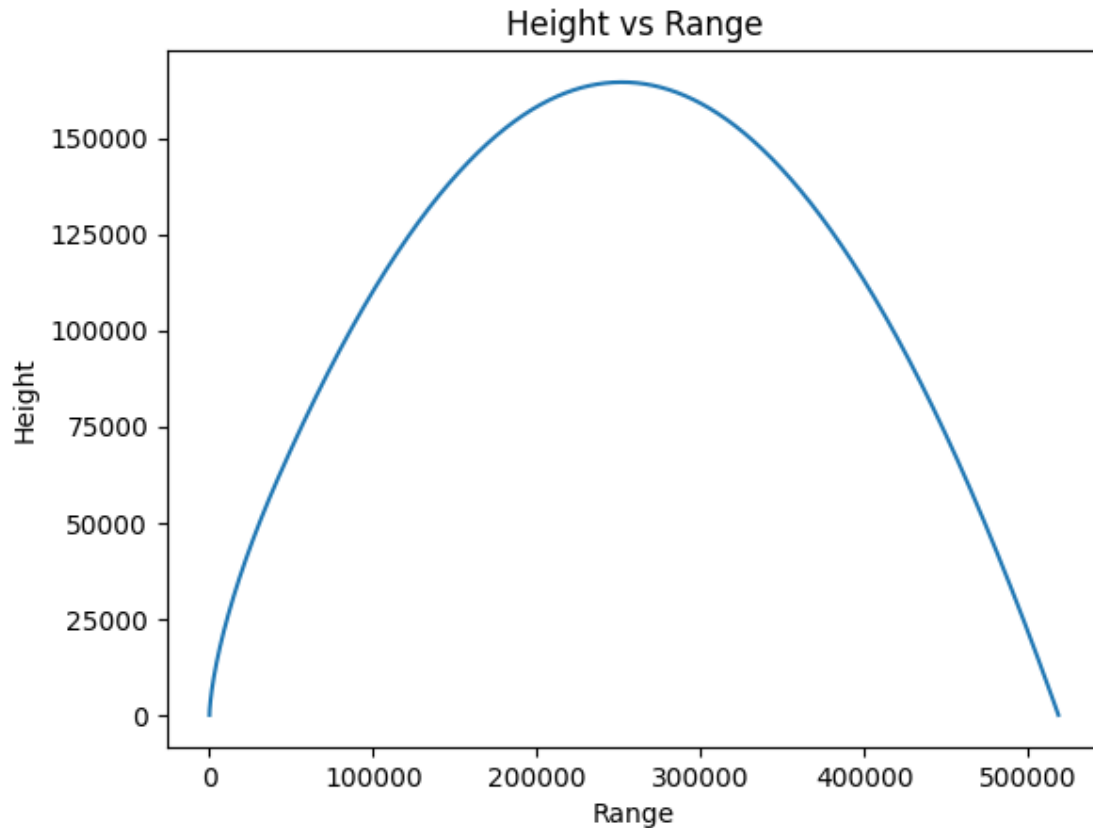
Time to target (sec): 475.60000000003964











Data written to 'results/results_2.txt'

```
[26]: # Build model
m1_var_kernel = (100)**2
m1_lengthscale = 100 # 1
m1_var_linear_kernel = (100)**2
m1_var_noise = 1e-5 # small value

constrain_lengthscale = True

#kern = GPy.kern.RBF(input_dim=1, lengthscale=100, variance = var_kernel ) # ,
↳ lengthscale=0.08, variance=20
# kern = GPy.kern.Matern32(input_dim=1)
# kern = GPy.kern.Linear(input_dim=1)
m1_rbf_kern = GPy.kern.RBF(input_dim=1, lengthscale=m1_lengthscale)
if constrain_lengthscale:
    m1_rbf_kern.lengthscale.constrain_bounded(m1_lengthscale, m1_lengthscale*1e12)

m1_kern = m1_rbf_kern + \
    GPy.kern.Linear(input_dim=1)
# m1_kern = m1_rbf_kern

m1_model_gpy = GPyRegression(m1_x,m1_y, kernel=m1_kern)
m1_model_gpy.kern.variance = m1_var_kernel
```

```
m1_model_gpy.likelihood.variance.fix(m1_var_noise)
display(m1_model_gpy)
```

reconstraining parameters rbf.lengthscale

<GPpy.models.gp_regression.GPRegression at 0x7ff428841810>

```
[27]: # Fit emulator
m1_model_emukit = GPpyModelWrapper(m1_model_gpy)
m1_model_emukit.optimize() # Optimize model hyperparameters
```

```
[28]: display(m1_model_gpy)
```

<GPpy.models.gp_regression.GPRegression at 0x7ff428841810>

```
[29]: # Get true points corresponding to param_1_x_plot (for plot)
wirte_output_txt = False

nr_points_plot = 301
m1_param_1_x_plot = np.linspace(m1_space.parameters[0].min, m1_space.parameters[0].
    ↪max, nr_points_plot)[: , None]
m1_param_1_y_plot = run_missile_sim(m1_param_1_x_plot)
```

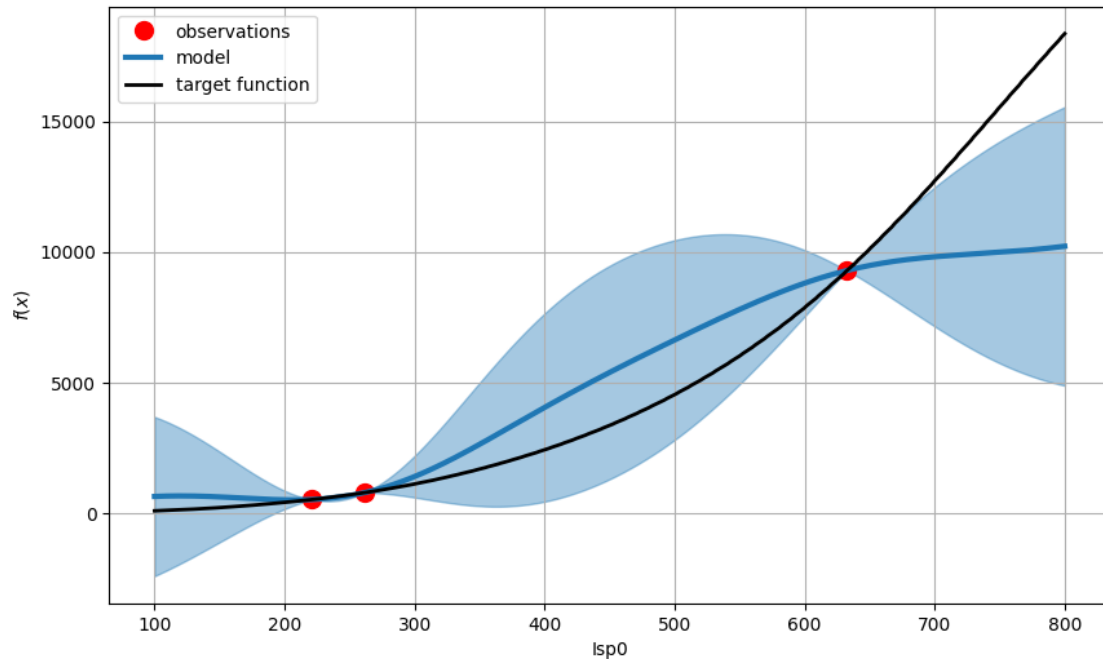
```
[30]: # Get model prediction on param_1_x_plot
m1_mu_plot, m1_var_plot = m1_model_emukit.predict(m1_param_1_x_plot)
```

```
[ ]:
```

```
[31]: fig, ax = plt.subplots(figsize=plot.big_wide_figsize)
helper_plot_emulator_errorbars(x_plot=m1_param_1_x_plot, y_plot=m1_param_1_y_plot,
                                mu_plot=m1_mu_plot, var_plot=m1_var_plot,
                                model_emukit=m1_model_emukit)

m1_rmse = evaluate_prediction(y_actual=m1_param_1_y_plot, y_predicted=m1_mu_plot)
print("RMSE m1 (pre experiment design loop): ", m1_rmse)
```

RMSE m1 (pre experiment design loop): 2495.4252546081



[]:

2.0.1 Experiment design loop

```
[32]: m1_2_model_emukit = m1_model_emukit
```

```
[33]: write_output_txt = False

integrated_variance = IntegratedVarianceReduction(space=m1_space,
                                                    model=m1_2_model_emukit)
m1_ed = ExperimentalDesignLoop(space=m1_space,
                               model=m1_2_model_emukit,
                               acquisition = integrated_variance,
                               batch_size = 1)

# batch size is set to one in this example as we'll collect evaluations
# sequentially but parallel evaluations are allowed
m1_ed.run_loop(user_function=run_missile_sim, stopping_condition=5)
```

```
/Users/ilariasartori/opt/anaconda3/envs/mlphysical/lib/python3.10/site-
packages/paramz/transformations.py:111: RuntimeWarning:overflow encountered in
expm1
```

New simulation

Isp0: 453.8115854906686

Stage 1 burnout

Velocity (km/s): 4.757773066937232

Angle (deg h): 43.65308698446824
Range (km): 200.62970617155497
Time (sec): 276.89999999999446
Final results:
Range (km): 3435.23616105572
Apogee (km): 1124.8530801502245
Time to target (sec): 1409.0999999999814

New simulation

Isp0: 747.8325958719347

Stage 1 burnout

Velocity (km/s): 8.39211805728013
Angle (deg h): 43.65426424435811
Range (km): 572.8125394418782
Time (sec): 456.4000000000353
Final results:
Range (km): 15428.140370527834
Apogee (km): 10983.199112700311
Time to target (sec): 9072.500000015474

New simulation

Isp0: 459.0419390861636

Stage 1 burnout

Velocity (km/s): 4.822031152282082
Angle (deg h): 43.65163411011696
Range (km): 205.6886754418494
Time (sec): 280.0999999999952
Final results:
Range (km): 3548.962160892317
Apogee (km): 1165.8277895746091
Time to target (sec): 1442.2999999997837

New simulation

Isp0: 136.0755293944842

Stage 1 burnout

Velocity (km/s): 1.1243932391878897
Angle (deg h): 43.63489310225416
Range (km): 15.337397515705034
Time (sec): 83.0999999999955

Final results:
Range (km): 166.75202528661995
Apogee (km): 54.302467772155566
Time to target (sec): 270.7999999999931

New simulation

Isp0: 664.1460850222866

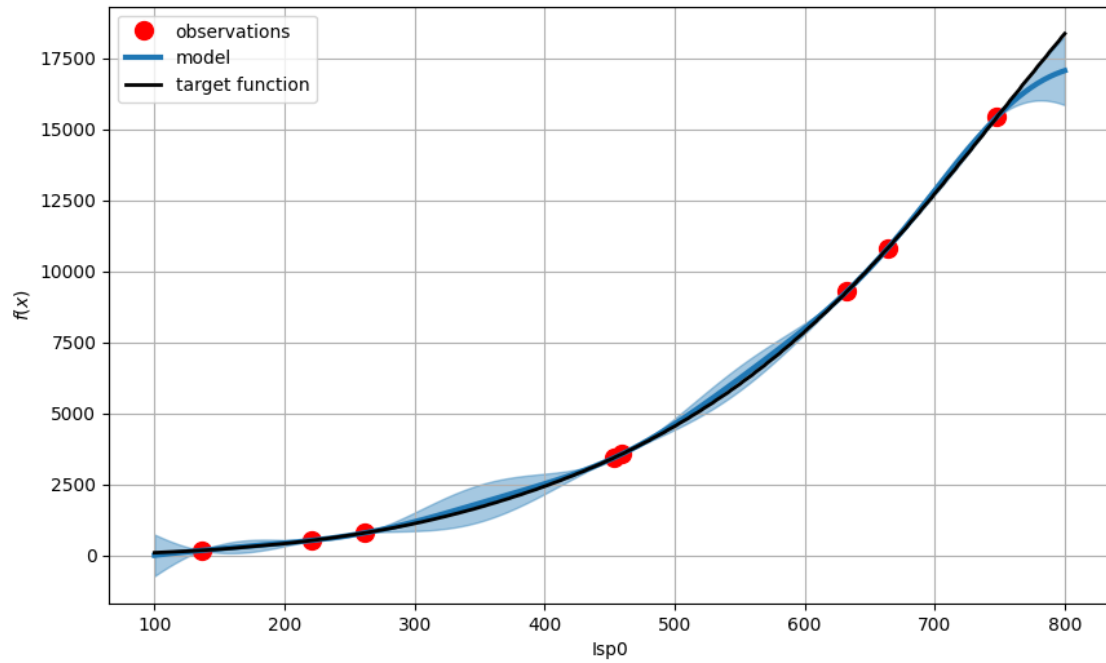
Stage 1 burnout
Velocity (km/s): 7.34343675835312
Angle (deg h): 43.657354331140674
Range (km): 449.3726221339817
Time (sec): 405.30000000002366
Final results:
Range (km): 10819.07080280896
Apogee (km): 5024.52619565074
Time to target (sec): 4380.199999998404

```
[34]: m1_2_mu_plot, m1_2_var_plot = m1_2_model_emukit.predict(m1_param_1_x_plot)
```

```
[35]: fig, ax = plt.subplots(figsize=plot.big_wide_figsize)
      helper_plot_emulator_errorbars(x_plot=m1_param_1_x_plot, y_plot=m1_param_1_y_plot,
                                     mu_plot=m1_2_mu_plot, var_plot=m1_2_var_plot,
                                     model_emukit=m1_2_model_emukit)

      m1_2_rmse = evaluate_prediction(y_actual=m1_param_1_y_plot, y_predicted=m1_2_mu_plot)
      print("RMSE m1 (post experiment design loop): ", m1_2_rmse)
```

RMSE m1 (post experiment design loop): 202.81859322929324



[]:

[]: