Ballistic missile range estimation and maximisation

Tomas Paulik, Mohd Sadiq,
Ilaria Sartori, Vilde Schulerud Bøe

L48 – Machine Learning and the Physical World

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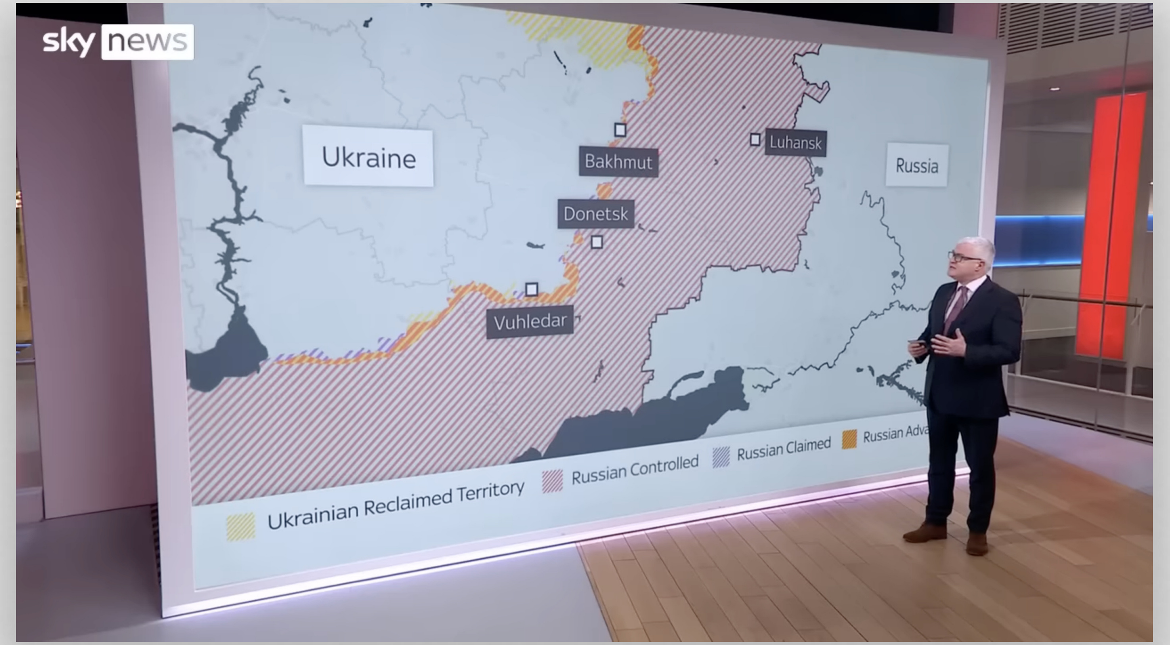
A post-diplomatic solution



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Project overview

Key project deliverables:

- Range estimation and maximisation
- Low-fidelity and high-fidelity simulator
- Sensitivity analysis
- Multi-fidelity analysis
- Conclusions

The high-fidelity simulator

$$\frac{dV}{dt} = \frac{T}{m}cos\eta - \frac{C_d \rho V^2 A}{2m} - gsin\gamma$$

The rate of change of velocity (acceleration)

$$\frac{d\psi}{dt} = \frac{V cos\gamma}{R_e + h}$$

The rate of change of positional angle with respect to the earth's centre

$$\frac{d\gamma}{dt} = \frac{d\psi}{dt} + \frac{T}{Vm}sin\eta - \frac{g}{V}cos\gamma$$

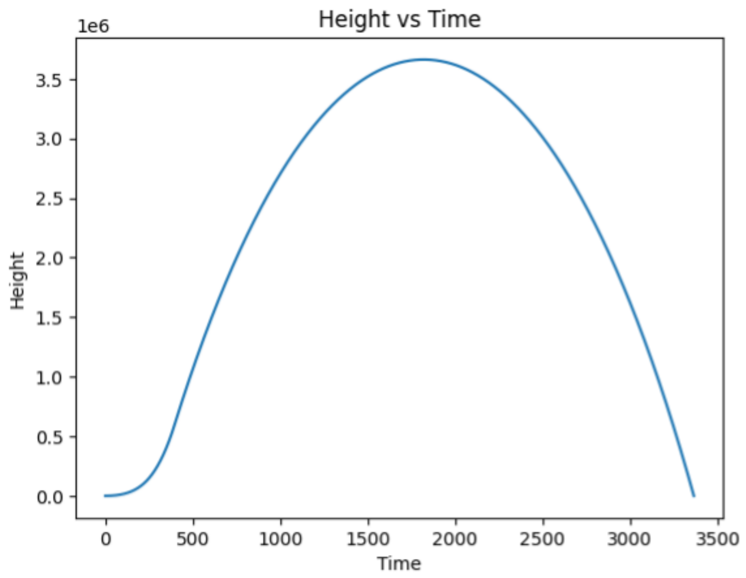
The rate of change of angle between the missile and the vertical axis

$$\frac{dm}{dt} = \frac{T}{g_0 I_{sp}}$$

The rate of change of mass of the missile

$$\frac{dh}{dt} = V sin\gamma$$

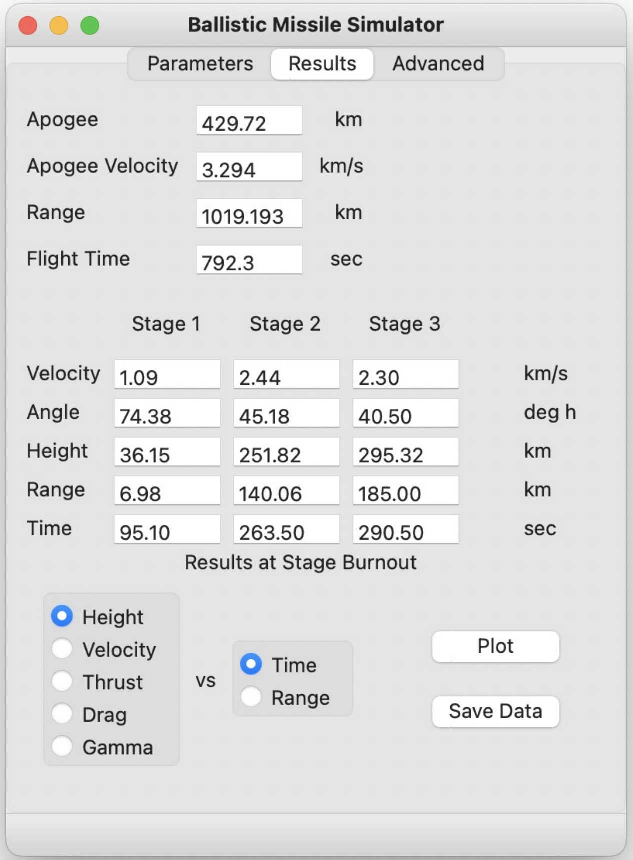
The rate of change of height of the missile

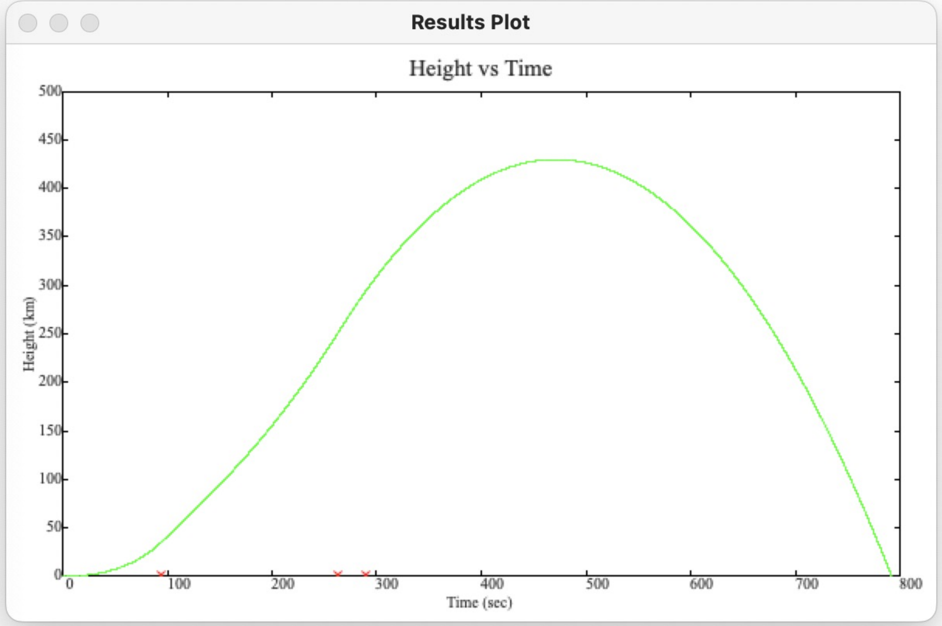


An example output of the simulator, plotted with matplotlib.

Source: David C. Wright, Lisbeth Gronlund. Depressed trajectory SLBMs: A Technical Evaluation and Arms Control Possibilities. *Science & Global Security*, 3(1-2):101–159, 1992.

Attributes and features





Attributes and features

- **payload (float)** – The extra mass attached to the missile (in kg).
- **missilediam (float)** – The diameter of the cross-sectional area of the missile (in meters).
- **rvdiam (float)** – The diameter of the cross-sectional area of the missile, after it has burnt all its fuel and exhausted through all its stages.
- **numstages (int)** – The number of missile stages.
- **fuelmass (List[float])** – The collection of fuel masses at different stages (in kg) burnt during the trajectory.
- **drymass (List[float])** – The weight of the missile stages (in kg) without the fuel.
- **Isp0 (List[float])** – Specific impulses of the missile at different stages (in seconds).
- **thrust0 (List[float])** – The force generated by the missile by burning of the fuel at different stages (in Newtons).

The emulator

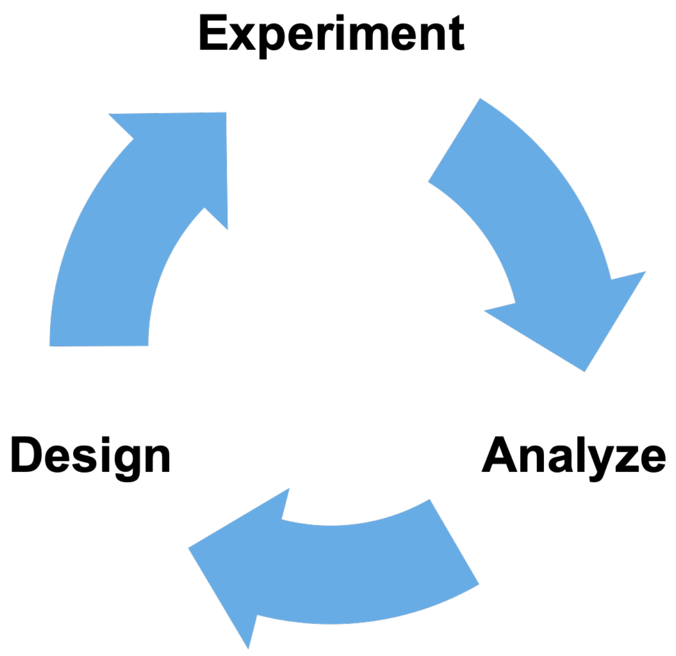
- **Define feature space**

  Two conditions where the simulator fails or doesn't return a value:

  - **Simulation takes too long** [>20,000 seconds]
  - **Missile is too heavy to lift off** [$T / m < g * \sin(\gamma)$]

- **Run experiments**

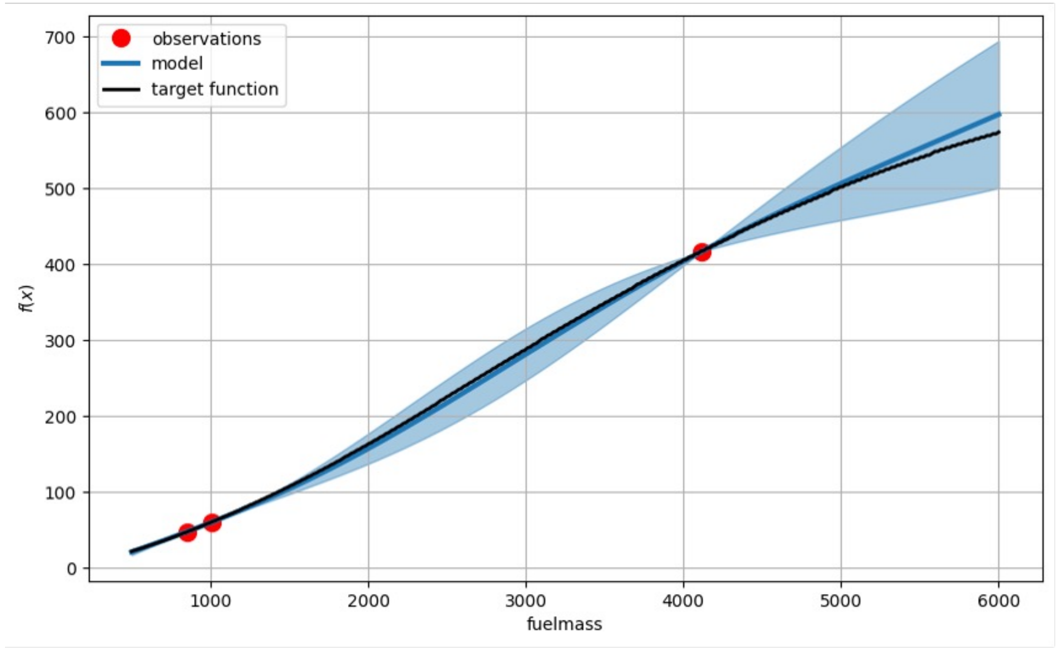
  1. **Range estimation**
  2. **Range maximisation**
  3. **Sensitivity analysis**



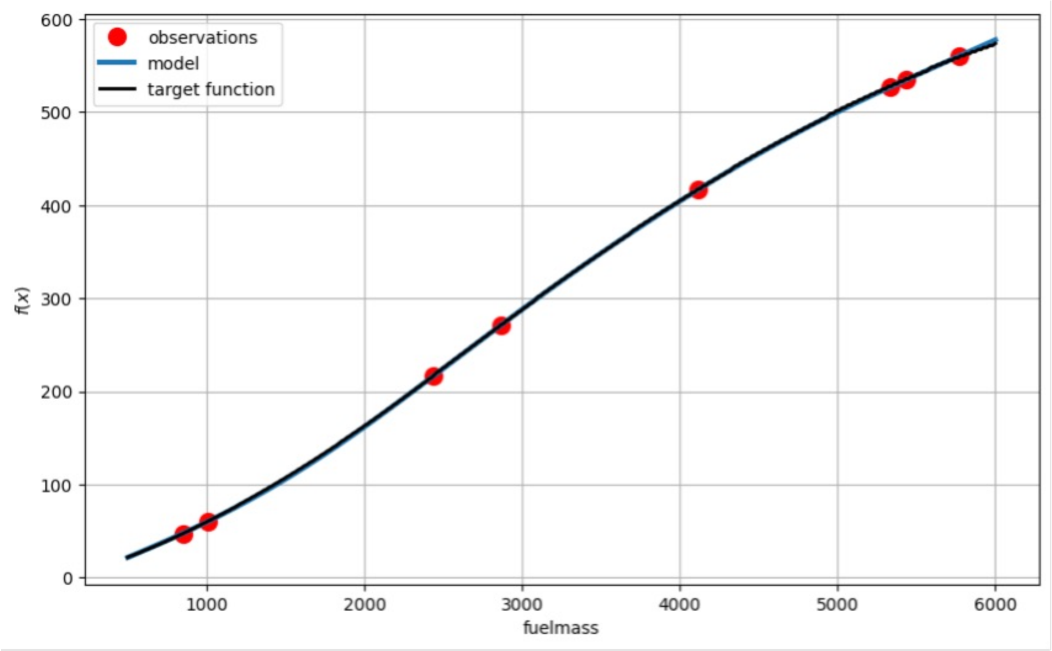
Range estimation

One-feature model

Features	fuelmass
Model	**Mean**: 0 **Kernel**: RBF + Linear
Acquisition function	Integrated variance reduction (IVR)



The initial state, with 3 data points randomly selected from the input domain.

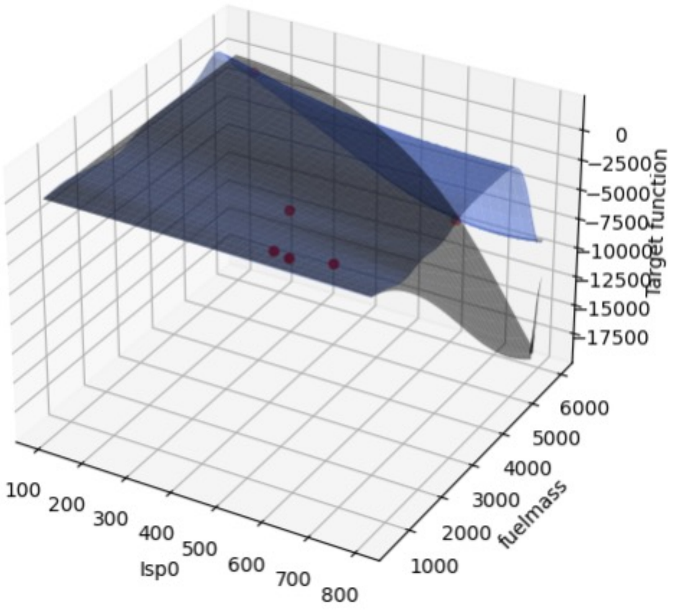


The final state, after 5 iterations of the experimental design loop.

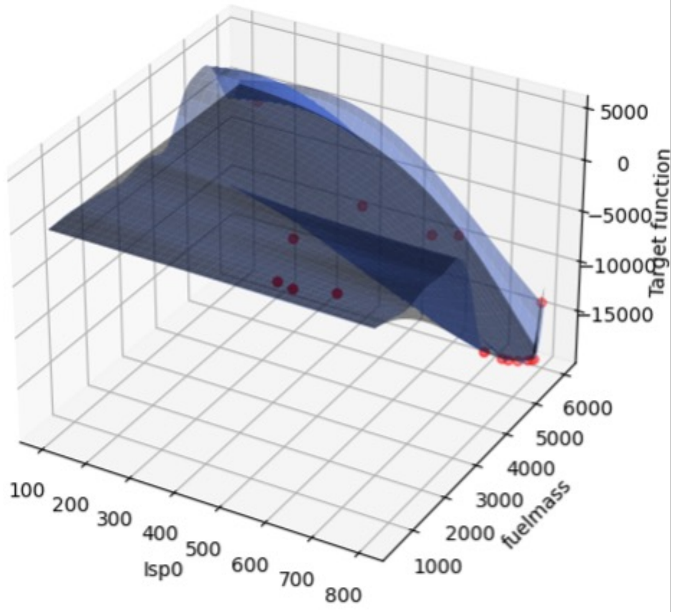
*In **black** we have the target function (i.e., the range), in **blue** the model mean prediction (±2 standard deviation), and in **red** the points where the simulations have been run.*

Range maximisation
Two-feature model

Features	fuelmass + Isp0
Model	**Mean**: 0 **Kernel**: RBF * RBF + Linear
Acquisition function	Expected Improvement



The initial state, with 6 data points randomly selected from the input domain.

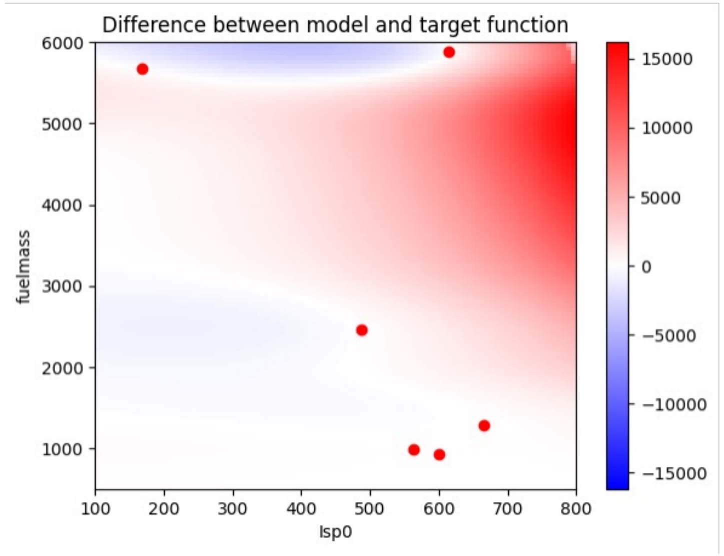


The final state, after 10 iterations of the Bayesian optimisation loop.

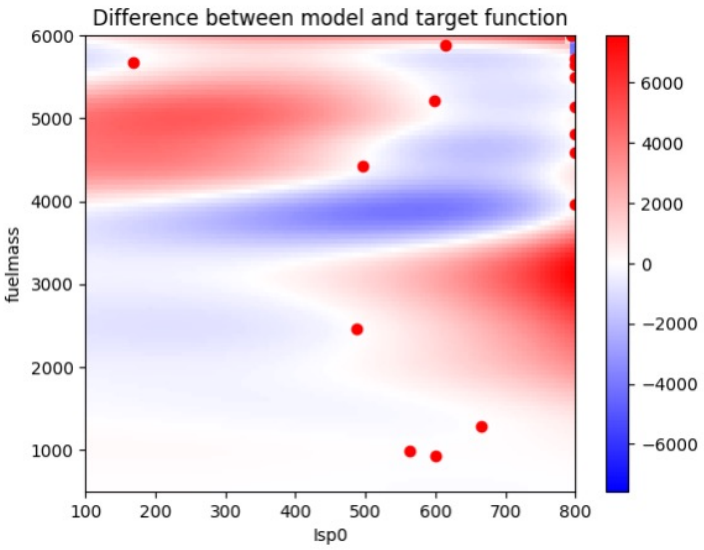
*In **black** we have the target function (i.e., minus the range), in **blue** the model mean prediction, and in **red** the points where the simulations have been run.*

Range maximisation
Two-feature model

Features	fuelmass + Isp0
Model	**Mean**: 0 **Kernel**: RBF * RBF + Linear
Acquisition function	Expected Improvement



The initial state, with 6 data points randomly selected from the input domain.



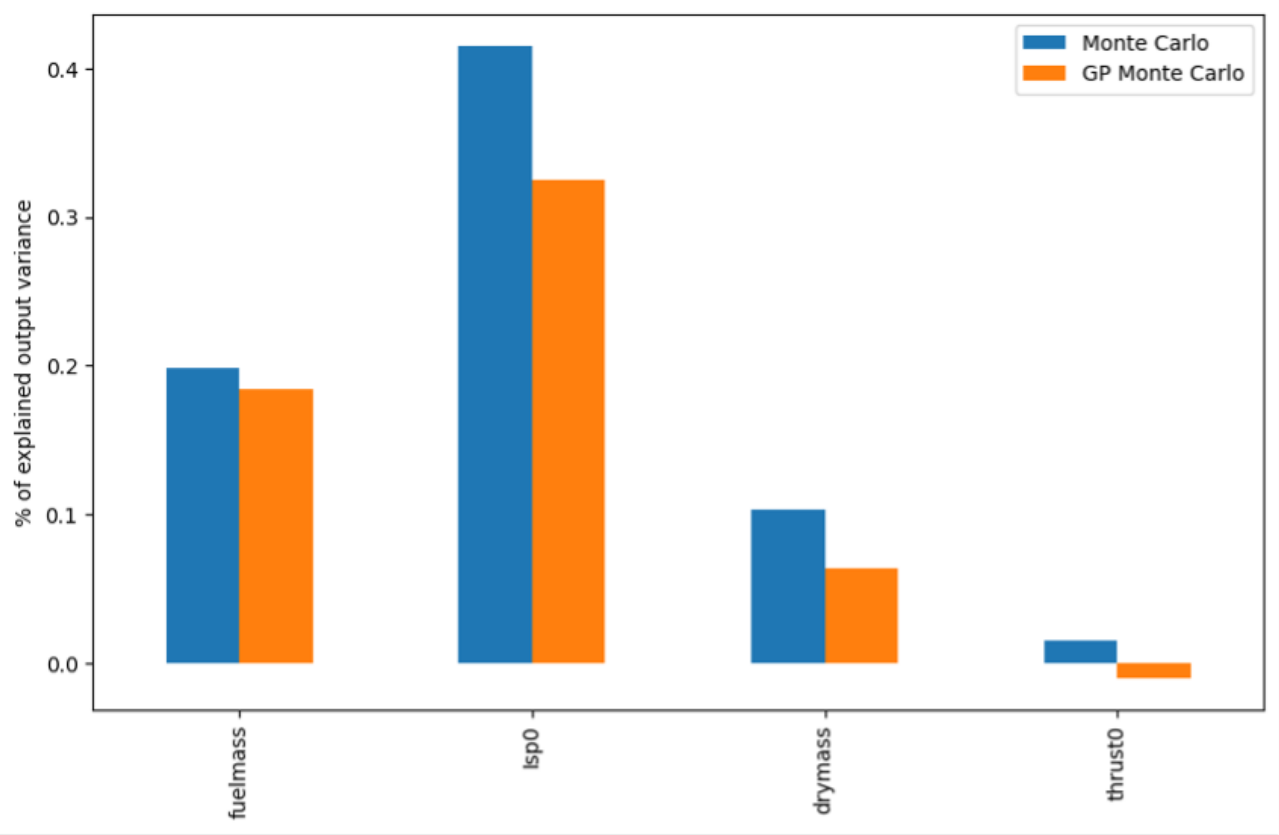
The final state, after 10 iterations of the Bayesian optimisation loop.

Heatmap of the difference between the model prediction and the target function.

*In **red** we mark the points where the simulations have been run.*

Sensitivity analysis
Four-feature model

Features	fuelmass + Isp0 + drymass + thurst0
Model	**Mean**: 0 **Kernel**: RBF + Linear
Acquisition function	Integrated variance reduction (IVR)



First-order Sobol indices

Multi-fidelity analysis – high and low fidelity simulators

The high-fidelity simulator		The low-fidelity simulator
$\frac{dV}{dt} = \frac{T}{m}cos\eta - \frac{C_d \rho V^2 A}{2m} - gsin\gamma$	The rate of change of velocity (acceleration)	$\frac{dV}{dt} = \frac{T}{m} - g.sin\gamma$
$\frac{d\psi}{dt} = \frac{V cos\gamma}{R_e + h}$	The rate of change of positional angle with respect to the earth's centre	$\frac{d\psi}{dt} = \frac{V cos\gamma}{R_e + h}$
$\frac{d\gamma}{dt} = \frac{d\psi}{dt} + \frac{T}{Vm}sin\eta - \frac{g}{V}cos\gamma$	The rate of change of angle between the missile and the vertical axis	$\frac{d\gamma}{dt} = \frac{d\psi}{dt} - \frac{g.cos\gamma}{V}$
$\frac{dm}{dt} = \frac{T}{g_0 I_{sp}}$	The rate of change of mass of the missile	$\frac{dm}{dt} = k$
$\frac{dh}{dt} = V sin\gamma$	The rate of change of height of the missile	$\frac{dh}{dt} = V sin\gamma$

High-fidelity simulator – additional conditions

$$T_{n+1} = T_n * (-0.4339.h_{norm}^3 + 0.6233.h_{norm}^2 - 0.01.h_{norm} + 1.004)$$

$$h_{norm} = \frac{h_n}{160934}$$

$$\rho = \begin{cases} \rho_0.e^{-\frac{h}{8420}} & if\ h\ <\ 19200 \\ \rho_0.(0.857003 + \frac{h}{57947})^{-13.201} & if\ h\ > 19200\ and\ h\ <\ 47000 \\ 0 & if\ h\ \geq\ 47000 \end{cases}$$

$$C_d = \begin{cases} 0.15 & if\ mach\ >\ 5 \\ -0.03125.mach + 0.30625 & if\ mach\ >\ 1.8\ and\ mach\ \leq\ 5 \\ -0.25.mach + 0.7 & if\ mach\ >\ 1.2\ and\ mach\ \leq\ 1.8 \\ 0.625.mach - 0.35 & if\ mach\ >\ 0.8\ and\ mach\ \leq\ 1.2 \\ 0.15 & if\ mach\ \leq\ 0.8 \end{cases}$$

$$mach = \frac{V}{\sqrt{1.4 * 287 * temperature(in\ K)}}$$

$$g = g_0 . \frac{h^2}{(R_e + h)^2}$$

$$temperature(in\ {}^0C) = \begin{cases} 15.04 - 0.00649.h & if\ h\ \leq 11000 \\ -56.46 & if\ 11000\ <\ h\ \leq 25000 \\ -131.21 + 0.00299.h & if\ t\ >\ 25000 \end{cases}$$

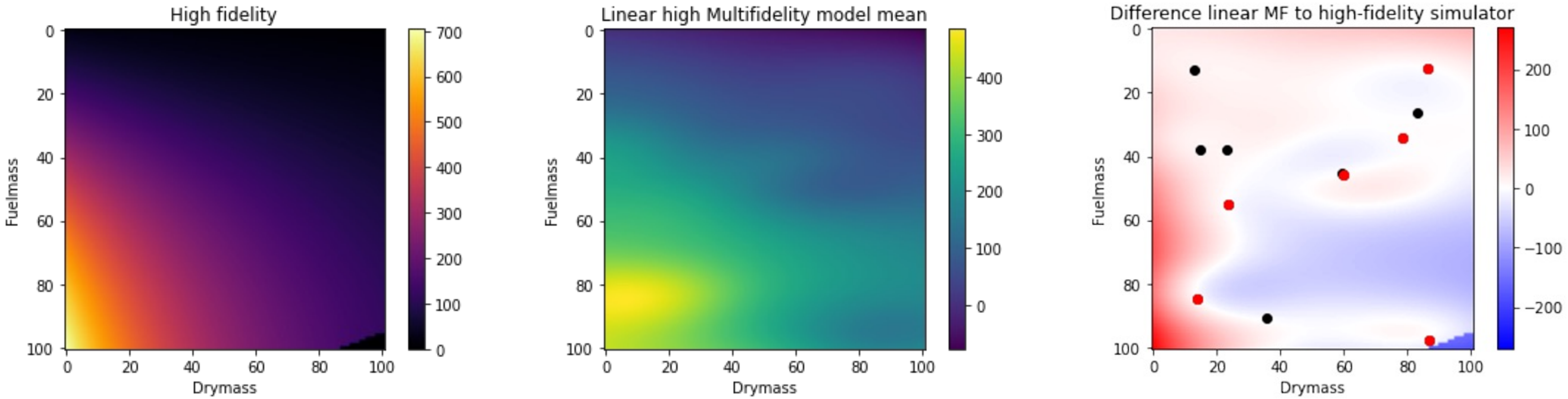
Low-fidelity parameters

- **Rate of Change of Mass (float)** – The rate at which fuel is consumed by the engine (in kg/s).
- **Exhaust Velocity (float)** – The rate at which exhaust gases are pushed out of the missile (in m/s).
- **Payload (float)** – The extra mass attached to the missile (in kg).
- **Dry mass (float)** – The weight of the missile body (in kg).
- **Fuel mass (float)** – The weight of fuel added onto the missile (in kg).
- **Diameter (float)** – The diameter of the missile (in m).

Multi-fidelity analysis

Linear model

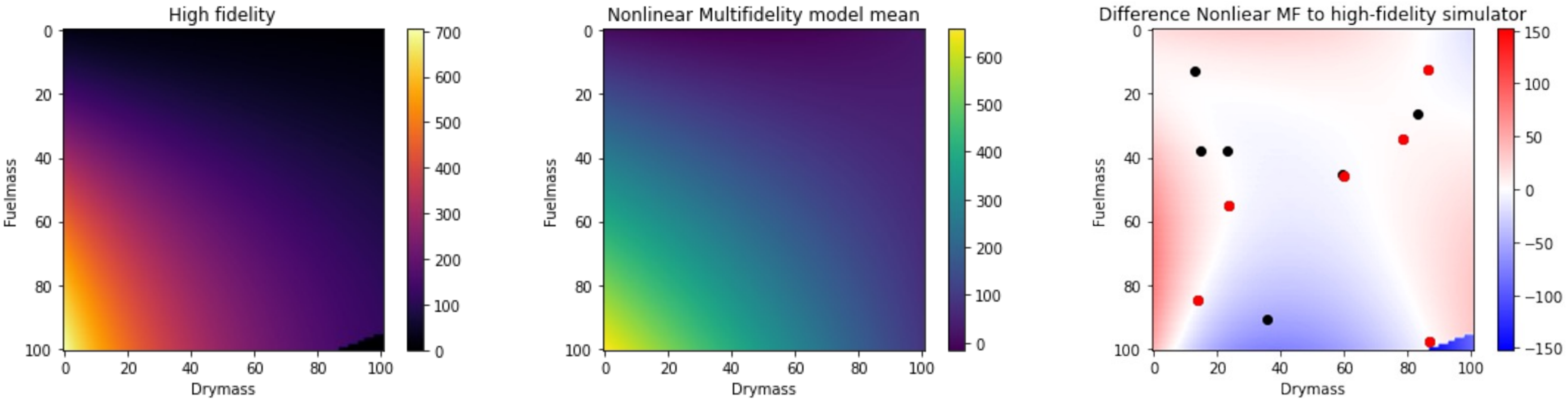
Features	fuelmass + drymass
Model	**Mean**: 0 **Kernel**: RBF * RBF + Linear



Multi-fidelity analysis

Non-linear model

Features	fuelmass + drymass
Model	**Mean**: 0 **Base kernel**: Linear



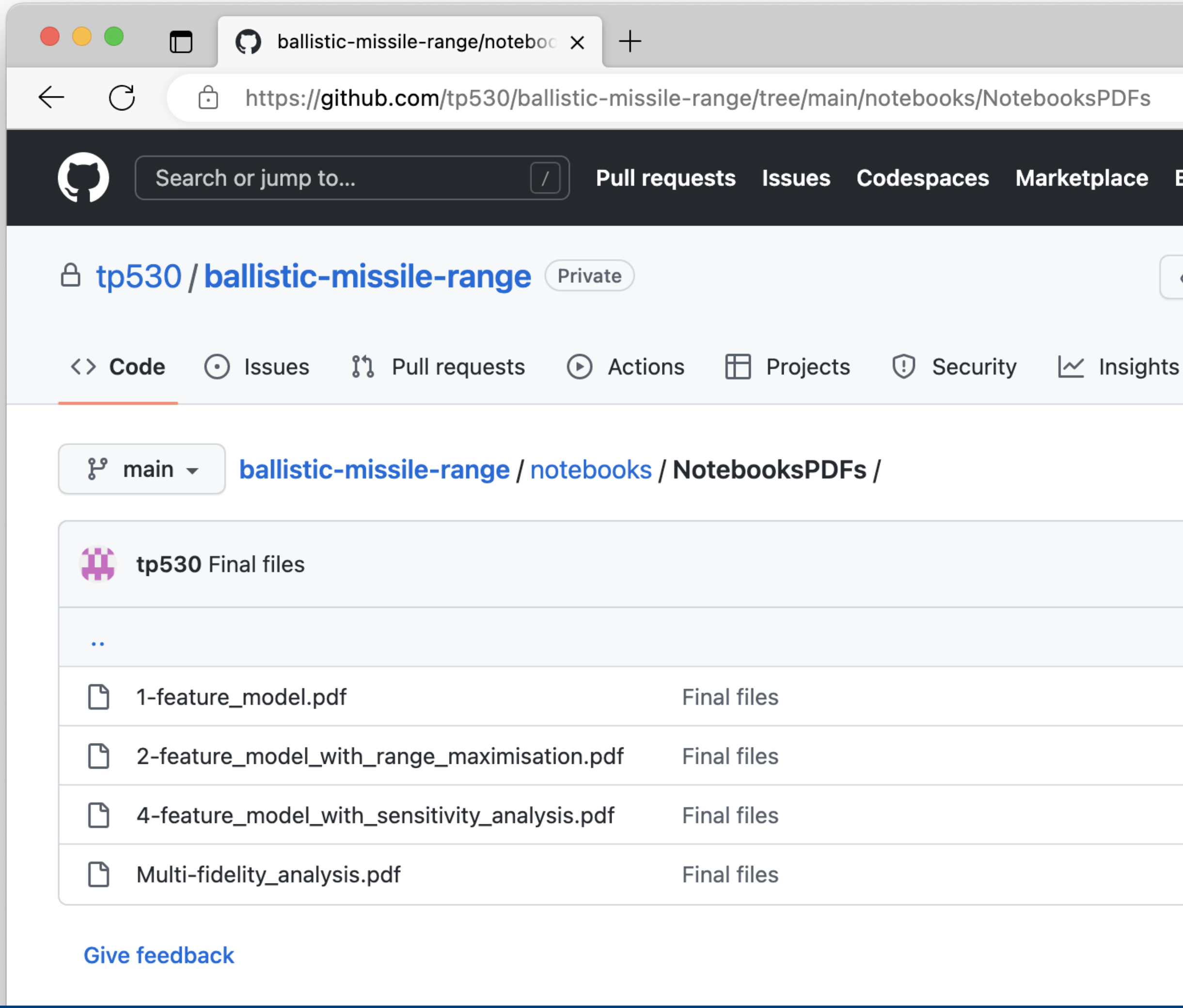
Conclusions

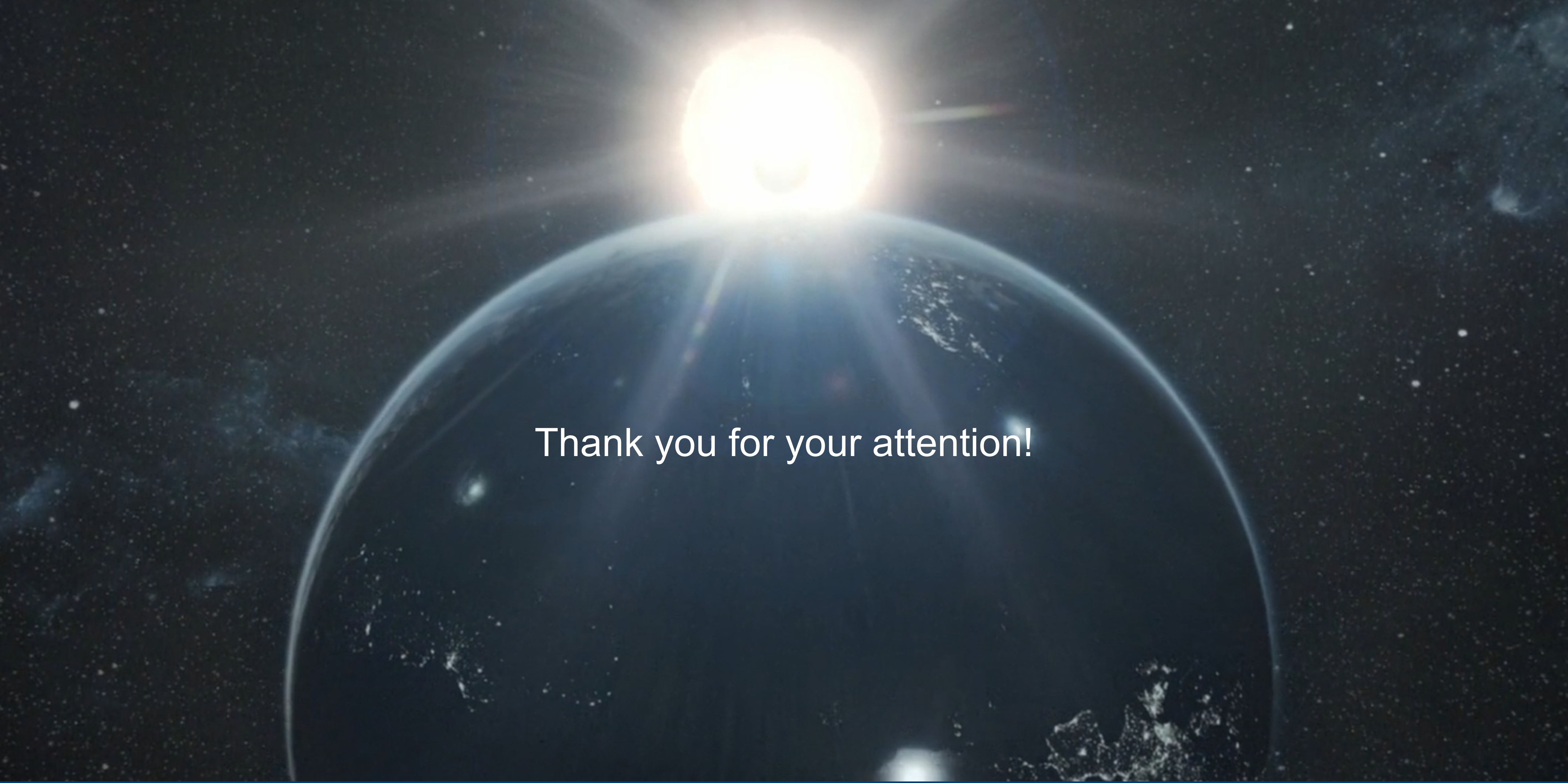
- We have demonstrated the power of the used simulation-emulation framework, and how GPs can be utilised to deliver meaningful results.
- We have identified the challenges that arise when dealing with a high number of features in a system.
- It would be interesting to explore whether neural networks could be used as a substitution for the GPs.
- Overall, our project highlights the potential of machine learning techniques in the field of data analysis and the importance of advanced fine-tuning of models when dealing with a large number of features.

Additional resources are available on GitHub

https://github.com/tp530/ballistic-missile-range

- The original (remastered) simulator with the GUI
- All the Python notebooks used in this project
- PDF files with exported **Python notebook outputs**.
- This presentation in a PDF format.





Thank you for your attention!