Linear Algebra Lecture 21

1. Eigenvalues - Eigenvectors
2. $\det[A - \lambda I] = 0$
3. $TRACE = \lambda_1 + \lambda_2 + \cdots + \lambda_n$

Ax parrallel to x called Eigenvectors

$$Ax = \lambda x$$

If A is singular, $\lambda = 0$ is eigenvalue

What are x's and λ's for projection matrix

Any x in plane: $Px = x$ $\quad \lambda = 1$

Any $x \perp$ Plane: $Px = 0x$ $\quad \lambda = 0$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad Ax = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \lambda = 1$$

$$x = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad Ax = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \lambda = -1$$

Fact: sum of λ's $= a_{11} + a_{22} + \cdots + a_{nn}$

How to solve $Ax = \lambda x$

Rewrite $\underbrace{(A - \lambda I)}_{\text{SINGULAR}} x = 0$

$\det(A - \lambda I) = 0$ FIND λ First

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \quad \det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix}$$

$$= (3-\lambda)^2 - 1 = \lambda^2 - 6\lambda + 8 = (\lambda - 2)(\lambda - 4)$$

$\lambda_1 = 4 \quad \lambda_2 = 2$

$$A - 4I = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A - 2I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

If $Ax = \lambda x$ then $(A + 3I)x = \lambda x + 3x = (\lambda + 3)x$

NOT SO GREAT $A+B$ AB (NOT LINEAR)

Example

90° rotation

$$Q=\begin{bmatrix}\cos 90 & -\sin 90\\ \sin 90 & \cos 90\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

trace : $0+0=\lambda_1+\lambda_2$

det $=1=\lambda_1\lambda_2$

$$\det(Q-\lambda I)=\begin{bmatrix}-\lambda & -1\\ 1 & -\lambda\end{bmatrix}=\lambda^2+1=0$$

$\lambda_1=i \quad \lambda_2=-i$

$$A=\begin{bmatrix}3 & 1\\ 0 & 3\end{bmatrix}$$

$$\det(A-\lambda I)=\begin{bmatrix}3-\lambda & 1\\ 0 & 3-\lambda\end{bmatrix}=(3-\lambda)^2$$

$\lambda_1=3 \quad \lambda_2=3$

$$(A-\lambda I)x=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\end{bmatrix}=\begin{bmatrix}0\end{bmatrix}$$

$x_1=\begin{bmatrix}1\\ 0\end{bmatrix}$ $\quad x_2=$ NO 2^{nd} INDEP x