Linear Algebra Lecture 24

Markov matrices

Steady state

Fourier series and projection

$$A = \begin{bmatrix} .1 & .01 & .3 \\ .2 & .99 & .3 \\ .7 & 0 & .4 \end{bmatrix}$$

① All entries ≥ 0

② Sum of column $= 1$

1. $\lambda = 1$ is an evalue

2. all other $|\lambda_i| < 1$

$$A - 1 \cdot I = \begin{bmatrix} -.9 & .01 & .3 \\ .2 & -.01 & .3 \\ .7 & 0 & -.6 \end{bmatrix}$$

All columns add to zero of $A - I$

$A - I$ is singular, because rows are dependent, because $c(1,1,1)$ is in $N(A^T)$

eigenvalues of A is the same as eigenvalues of A^T

$$\det(A - \lambda I) = 0 \xrightarrow{⑩} \det(A^T - \lambda I) = 0$$

$u_{k+1} = A u_k \quad A$ is Markov

$$\begin{bmatrix} u_{cal} \\ u_{mass} \end{bmatrix}_{t=k+1} = \begin{bmatrix} .9 & .2 \\ .1 & .8 \end{bmatrix} \begin{bmatrix} u_{cal} \\ u_{mass} \end{bmatrix}_{t=k}$$

$$\begin{bmatrix} u_{cal} \\ u_{mass} \end{bmatrix} = \begin{bmatrix} 0 \\ 1000 \end{bmatrix} \quad \begin{bmatrix} u_{cal} \\ u_{mass} \end{bmatrix}_1 = \begin{bmatrix} 200 \\ 800 \end{bmatrix}$$

$$\begin{bmatrix} .9 & .2 \\ .1 & .8 \end{bmatrix} \quad \lambda_1 = 1 \quad \lambda_2 = 0.7$$

$$\begin{bmatrix} -0.1 & 0.2 \\ 0.1 & -0.2 \end{bmatrix} \begin{bmatrix} x_1 \end{bmatrix} = 0 \quad x_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.2 & 0.2 \\ 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} x_2 \end{bmatrix} = 0 \quad x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$u_k = c_1 1^k \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 (0.7)^k \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$u_0 = \begin{bmatrix} 0 \\ 1000 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$c_1 = \frac{1000}{3} \qquad c_2 = \frac{2000}{3}$$

projections with orthonormal basis

$q_1 \cdots q_n$

$$v = x_1 q_1 + x_2 q_2 + \cdots + x_n q_n$$

$$q_1^T v = x_1 q_1^T q_1 + 0 \cdots + 0$$

$$= x_1$$

$$Qx = v \qquad x = Q^{-1} v = Q^T v$$

Fourier series

$$f(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \cdots$$

$$f^T g = \int_0^{2\pi} f(x)\, g(x)\, dx$$

$$\int_0^{2\pi} \cos x \sin x = \frac{1}{2}(\sin x)^2 \Big|_0^{2\pi} = 0$$

$$a_1 \int_0^{2\pi} (\cos x)^2 \, dx = \int_0^{2\pi} f(x) \cos x \, dx$$

$$a_1 = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos x \, dx$$