Linear Algebra Lecture 26

complex inner products

vector DISCRETE FOURIER

matrices FAST Transform = FFT

In $\mathbb{C}^n$ length $= z^H z = \bar{z}^T z$

(H: Hermitian)

inner product $= y^H x = \bar{y}^T x$

Symmetric $A^T = A$ no good if A is complex

For complex: $\bar{A}^T = A = \begin{bmatrix} 2 & 3+i \\ 3-i & 5 \end{bmatrix} = A^H$

perpendicular: $q_1, q_2, \cdots q_n$

unitary

$$\bar{q}_i^T q_j = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases} \qquad Q^T Q = I = Q^H Q$$

$$F_n = \begin{bmatrix} 1 & 1 & 1 & & 1 \\ 1 & w & w^2 & & w^{n-1} \\ \vdots & & & \ddots & \vdots \\ 1 & w^{n-1} & w^{2n-2} & & w^{(n-1)^2} \end{bmatrix} \qquad (F_n)_{ij} = w^{ij}$$

$i, j = 0, \cdots n-1$

$$w^n = 1 \qquad w_n = e^{i2\pi/n} = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$$

$$n = 4 \qquad w^4 = 1 \qquad w_4 = e^{i2\pi/4} = i$$

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

cols orthogonal $\quad (1, -i, -1, i)\begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix} = 0$

$$\begin{bmatrix} F_{64} \end{bmatrix} = \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} F_{32} & 0 \\ 0 & F_{32} \end{bmatrix} \begin{bmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \\ & \dots & & \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & & & & \\ & w & & & \\ & & w^2 & & \\ & & & \ddots & \\ & & & & w^{31} \end{bmatrix}$$