# Linear Algebra Lecture 27

## Positive Definite Matrix (Tests)

Tests for Minumum ($x^TAx > 0$)

Ellipsoids in $R^n$

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

① $\lambda_1 > 0 \quad \lambda_2 > 0$

② $a > 0$, $ac - b^2 > 0$

③ pivots $a > 0 \quad \dfrac{ac-b^2}{a} > 0$

④ $x^TAx > 0$ (*)

## Examples

$$A = \begin{bmatrix} 2 & 6 \\ 6 & 18 \end{bmatrix} \rightarrow$$

Pivots : 2

positive semidefinite

$\lambda_1 = 0 \quad \lambda_2 = 20$

$$[x_1, x_2] \begin{bmatrix} 2 & 6 \\ 6 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [x_1 \ x_2] \begin{bmatrix} 2x_1 + 6x_2 \\ 6x_1 + 20x_2 \end{bmatrix}$$

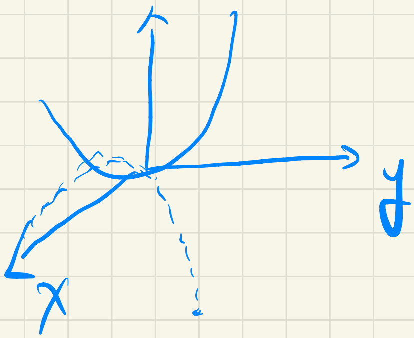
$= 2x_1^2 + 12x_1x_2 + 20x_2^2$ (quadratic form)

$ax^2 + 2bxy + cy^2$

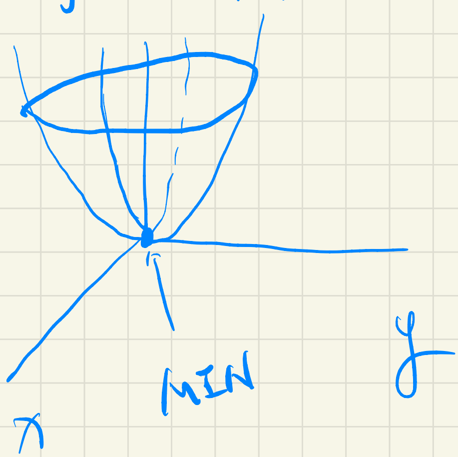
# Graphs of $f(x,y) = x^TAx = ax^2 + 2bxy + y^2$

$= 2x_1^2 + 12x_1x_2 + 7x_2^2$

$$\begin{bmatrix} 2 & 6 \\ 6 & 7 \end{bmatrix}$$




$f(x,y) = 2x^2 + 12x_1x_2 + 20x_2^2$




1st DERIV = 0

Calculus: MIN $\sim \frac{d^2u}{dx^2} > 0$

MIN ~ MATRIX of 2nd DERIVS is POSITIVE

2st derivs decides the min or max

$$f(x,y) = 2x_1^2 + 12x_1x_2 + 20x_2^2$$

$$= 2(x_1+3x_2)^2 + 2x_2^2$$

$$A = \begin{bmatrix} 2 & 6 \\ 6 & 20 \end{bmatrix} \longrightarrow U = \begin{bmatrix} 2 & 6 \\ 0 & 2 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

## 3x3 example

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

det 2, 3, 4

pivots 2, $\frac{3}{2}$, $\frac{4}{3}$

eigenvalues: $2-\sqrt{2}$, 2, $2+\sqrt{2}$

$$x^T A x = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3 > 0$$