

Linear Algebra Lecture 18

1. Determinants $\det A = |A|$

2. Properties 1-2-3, 4-10 \pm signs

① $\det I = 1$

② Exchange rows reverse sign of det

$\det \text{ permutation} = -1 \text{ or } 1$

Odd or even

$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

③a $\begin{vmatrix} ta & tb \\ c & d \end{vmatrix} = t \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

③b $\begin{vmatrix} a+ta' & b+tb' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} ta' & tb' \\ c & d \end{vmatrix}$

Linear for each row

④ 2 equal rows $\rightarrow \det = 0$

Exchange those rows \rightarrow same matrix

$\rightarrow \det$ shouldn't change $\xrightarrow{(2)}$

det should multiple $-1 \rightarrow \det = 0$

⑤ Subtract $L \times \text{row}_i$ from row_k , DET
Doesn't change

$$\begin{vmatrix} a & b \\ c-La & d-Lb \end{vmatrix} \stackrel{\textcircled{3b}}{=} \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} -La & -Lb \\ c & d \end{vmatrix}$$

$$\stackrel{\textcircled{3a}}{=} \begin{vmatrix} a & b \\ c & d \end{vmatrix} - L \begin{vmatrix} a & b \\ a & b \end{vmatrix} \stackrel{\textcircled{4}}{=} \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

⑥ Row of Zeros $\rightarrow \det A = 0$

$$5 \begin{vmatrix} 0 & 0 \\ c & d \end{vmatrix} \stackrel{\textcircled{3a}}{=} \begin{vmatrix} 5 \cdot 0 & 5 \cdot 0 \\ c & d \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ c & d \end{vmatrix}$$

$$\textcircled{7} U = \begin{bmatrix} d_1 & * & \\ & d_2 & \\ 0 & & \ddots & d_n \end{bmatrix} \det U = (d_1)(d_2) \dots (d_n)$$

$$U \xrightarrow{\textcircled{8}} \begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ 0 & & \ddots & d_n \end{bmatrix}$$

$$\det U \stackrel{\textcircled{3a}}{\rightarrow} d_n \dots d_n d_1 \begin{vmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{vmatrix} = (d_1)(d_2) \dots (d_n)$$

⑧ $\det A = 0$ when A is singular
 $\det A \neq 0$ when A is invertible

⑨ $\det AB = (\det A)(\det B)$

$$\det A^T A = \det I = 1 = (\det A)(\det A^T)$$

$$\Rightarrow \det A^T = 1 / \det A$$

$$\det A^2 = (\det A)^2$$

$$\det 2A = 2^n \det A$$

⑩ $\det A^T = \det A$ ($A = LU$)

$$|A^T| = |U^T L^T| = |U^T| |L^T| = |L| |U|$$

cause L and U are both diagonal