Linear Algebra Lecture 19

Formula for $\det A$ ($n!$ terms)

cofactor formula

Tridiagonal matrics

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & 0 \\ c & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & d \end{vmatrix} = \cancel{\begin{vmatrix} a & 0 \\ c & 0 \end{vmatrix}} + \begin{vmatrix} a & 0 \\ 0 & d \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & b \\ c & 0 \end{vmatrix} + \cancel{\begin{vmatrix} 0 & b \\ 0 & d \end{vmatrix}}$$

$$= ad - bc$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & 0 & a_{23} \\ 0 & a_{32} & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & a_{12} & 0 \\ a_{21} & 0 & 0 \\ 0 & 0 & a_{33} \end{vmatrix} + \begin{vmatrix} 0 & a_{12} & 0 \\ 0 & 0 & a_{23} \\ a_{31} & 0 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & 0 & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & 0 \\ a_{31} & 0 & 0 \end{vmatrix}$$

$$= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31}$$

$+ a_{13} a_{21} a_{32} - a_{13} a_{22} a_{31}$

Big formula

$$\det A = \sum_{n! \text{ terms}} \pm a_{1\alpha} a_{2\beta} a_{3\gamma} \cdots a_{n\omega}$$

$(\alpha, \beta, \gamma \cdots \omega) = \text{perm of } (1, 2, \cdots, n)$

Cofactors 3x3 In PARNTS

$$\det = a_{11} (a_{22} a_{33} - a_{23} a_{32})$$
$$+ a_{12} (-a_{21} a_{33} + a_{23} a_{31})$$
$$+ a_{13} (a_{21} a_{32} - a_{22} a_{31})$$

cofactor of $a_{ij} = C_{ij}$

$$(-1)^{i+j} \det \begin{pmatrix} n-1 \text{ matrix} \\ \text{with row } i \text{ erased} \\ \text{col } j \end{pmatrix}$$

Cofactor formula (along row 1)

$$\det A = a_{11} C_{11} + a_{12} C_{12} + \cdots + a_{1n} C_{1n}$$

Ex

$$A_4 = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$|A_1| = 1 \quad |A_2| = 0 \quad |A_3| = -1$

$|A_4| = 1 \cdot |A_3| - 1 \cdot |A_2| = -1$

$|A_n| = |A_{n-1}| - |A_{n-2}|$