Linear Algebra Lecture 30

- LINEAR TRANSFORMATIONS

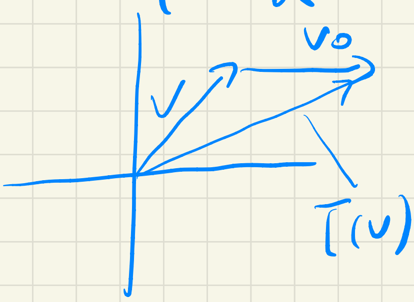
without coordinates : no matrix

with coordinates : MATRIX

Example 1: Projection

$: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

Example 2: Shift whole plane by v_0



Not Linear Transformation (Counter Example)

$$T(v+w) = T(v) + T(w)$$

$$T(cv) = cT(v) \qquad c \in \mathbb{R}$$

$$T(cv+dw) = cT(v) + dT(w)$$

Start : $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

Example : $T(v) = A \underset{\text{input in } \mathbb{R}^3}{v}$ $\xrightarrow{} 2 \times 3$ matrix

$\uparrow$ output in $\mathbb{R}^2$

Information needed to know $T(v)$ for all inputs

$T(v_1), T(v_2), \dots, T(v_n)$ for any $v_1, \dots, v_n$

basis

$$\text{Every } v = c_1 v_1 + \dots + c_n v_n$$

Construct matrix A that represents lin transformation $T: \mathbb{R}^n \to \mathbb{R}^m$

Choose basis $v_1, \dots, v_n$ for inputs $\mathbb{R}^n$

choose basis $w_1, \dots, w_m$ for output $\in \mathbb{R}^m$

1st column of A: $T(v_1) = a_{11}w_1 + a_{21}w_2 + \cdots + a_{n1}w_n$

2nd column of A: $T(v_2) = a_{12}w_1 + a_{22}w_2 + \cdots + a_{n2}w_n$

$$A\begin{pmatrix}\text{input} \\ \text{columns}\end{pmatrix} = \begin{pmatrix}\text{output} \\ \text{columns}\end{pmatrix}$$

$c_1 + c_2x + c_3x^2$ basis: $1, x, x^2$

$c_2 + 2c_3x$ basis: $1, x$

$T = \frac{d}{dx}$

linear!

$$A\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} c_2 \\ 2c_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$