Linear Algebra Lecture 25

Symmetric matrices

Eigenvalues / Eigenvectors

Start: Positive Definite Matrices

$A = A^T$

① The eigenvalues are real

② The eigenvectors are PERPENDICULAR

usual case : $A = S \Lambda S^{-1}$

when $A = A^T$ $\quad A = Q \Lambda Q^{-1} = Q \Lambda Q^T$

Why real eigenvalues? $\quad \overline{a+ib} = a - ib$

$Ax = \lambda x \xRightarrow{\text{always}} \bar{A}\bar{x} = \bar{\lambda}\bar{x} \Longrightarrow A\bar{x} = \bar{\lambda}\bar{x}$

$A\bar{x} = \bar{\lambda}\bar{x} \Rightarrow \bar{x}^T A^T = \bar{x}^T \bar{\lambda} \Longrightarrow \bar{x}^T A = \bar{x}^T \bar{\lambda}$

$\Downarrow$

$Ax = \lambda x \qquad\qquad \bar{x}^T A x = \bar{x}^T \bar{\lambda} x$

$\Downarrow$

$\bar{x}^T A x = \lambda \bar{x}^T x \qquad \lambda \bar{x}^T x = \bar{\lambda} \bar{x}^T x \Rightarrow \lambda = \bar{\lambda}$

λ is REAL

$\bar{x}^T x = \bar{x}_1 x_1 + \bar{x}_2 x_2 + \cdots$

$\bar{x}_1 x_1 = (a - ib)(a + ib) = a^2 + b^2$

$A = A^T$

$A = Q \Lambda Q^T$

$$= \begin{bmatrix} | & | & \\ q_1 & q_2 & \cdots \\ | & | & \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{bmatrix} \begin{bmatrix} q_1^T \\ q_2^T \\ \vdots \end{bmatrix} = \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T + \cdots$$

Every symm matrix is a comb of perp

Projection matrix

signs of pivots same as signs of λ's

\# positive pivots = \# positive λ's

product of pivots = product of λ's because they all equal to det

Positive definite symmetric matrix

all eigenvalues are positive

all pivots are positive

all subdeterminents are positive