Linear Algebra Lecture 18

1. Determinants $\det A = |A|$

2. Properties 1. 2. 3. 4-10 ± signs.

① $\det I = 1$

② Exchange rows reserve sign of det

det permutation = -1 or 1
odd or even

$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

3a $$\begin{vmatrix} ta & tb \\ c & d \end{vmatrix} = t \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

3b $$\begin{vmatrix} a+a' & b+b' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}$$

Linear for each row

④ 2 equal rows $\longrightarrow$ det = 0

Exchange those rows $\longrightarrow$ same matrix

$\longrightarrow$ det shouldn't change $\xrightarrow{②}$

det should multiple -1 $\longrightarrow$ det $=0$

5 Substract $l\times$ row$_i$ from row$_k$, DET Doesn't change

$$\begin{vmatrix} a & b \\ c-la & d-lb \end{vmatrix} \overset{3b}{=} \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a & b \\ -la & -lb \end{vmatrix}$$

$$\overset{3a}{=} \begin{vmatrix} a & b \\ c & d \end{vmatrix} - l \begin{vmatrix} a & b \\ a & b \end{vmatrix} \overset{4}{=} \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

6 Row of Zeros $\longrightarrow$ det $A=0$

$$5\begin{vmatrix} 0 & 0 \\ c & d \end{vmatrix} \overset{3a}{=} \begin{vmatrix} 5\cdot 0 & 5\cdot 0 \\ c & d \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ c & d \end{vmatrix}$$

7 $U = \begin{bmatrix} d_1 & & * \\ & d_2 & \\ 0 & & \ddots & \\ & & & d_n \end{bmatrix}$ $\det U = (d_1)(d_2)\cdots(d_n)$

$$U \xrightarrow{5} \begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ 0 & & \ddots & \\ & & & d_n \end{bmatrix}$$

$$\det U \xrightarrow{3a} d_n \cdots d_n d_1 \begin{vmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{vmatrix} = (d_1)(d_2)\cdots(d_n)$$

⑧ $\det A = 0$ when A is singular

$\det A \neq 0$ when A is invertible

⑨ $\det AB = (\det A)(\det B)$

$\det A^{-1}A = \det I = 1 = (\det A)(\det A^{-1})$

$\Rightarrow \det A^{-1} = 1/\det A$

$\det A^2 = (\det A)^2$

$\det 2A = 2^n \det A$

⑩ $\det A^T = \det A \quad (A = LU)$

$|A^T| = |U^T L^T| = |U^T||L^T| = |L||U|$

cause L and U are both diagonal