

Linear Algebra Lecture 20

① Formula for A^{-1}

② Cramers Rule for $x = A^{-1}b$

③ $|\det A|$ = volume of box

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

① $A^{-1} = \frac{1}{\det A} C^T$ ← product of $n-1$ entries

product of n entries

Check $AC^T = (\det A) I$ ✓

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} c_{11} & c_{n1} \\ c_{12} & c_{n2} \\ \vdots & \vdots \\ c_{1n} & c_{nn} \end{bmatrix} = \begin{bmatrix} \det A & 0 & \dots & 0 \\ 0 & \det A & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \det A \end{bmatrix}$$

$$a_{11}c_{n1} + a_{12}c_{n2} + \dots + a_{1n}c_{nn} = 0$$

(two same rows)

$$Ax = b$$

$$x = A^{-1}b = \frac{1}{\det A} C^T b$$

CRAMER'S RULE

$$x_1 = \frac{\det B_1}{\det A}$$

$$x_j = \frac{\det B_j}{\det A}$$

A with
column 1
replaced by
b

$$B_1 = \begin{bmatrix} | & & | \\ & n-1 & \\ | & b & | \\ & \text{columns} & \\ & \text{of } A & \end{bmatrix}$$

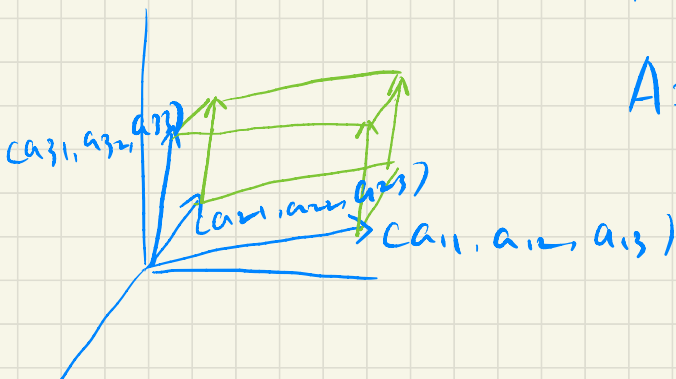
$$= b_1 C_{11} + b_2 C_{12} + \dots$$

$B_j = A$ with column j replaced by b

(3)
 $|\det A| = \text{volume of box}$

$$A = I \quad \det A = 1$$

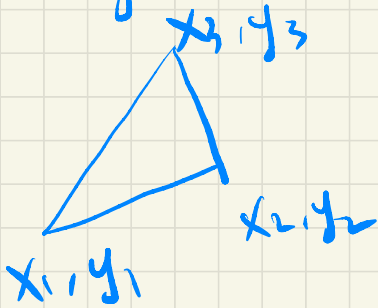
$$A = Q \quad Q^T Q = I$$



$|\det A| = \text{volume of box}$

$1v, 2v, 3av, 3bv$

triangle



$$S = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$