Isaac Abboudi

DamConstruction Write Up

May 3 2023

The cost of a brute force algorithm for this problem would be O(n!) since we would need to evaluate every possible ordering of the dams.

The key insight is using memoization to avoid doing computations extra times. The table is a 2D array memo[i][j] where i represents the index of the last dam built and j represents the index of the second-to-last dam built. This allows us to keep track of the minimum cost of building a series of dams up to the ith dam with the jth dam as the second-to-last dam. We compute the cost of evaluating a dam at position i as the product of the difference in height between the ith dam and the jth dam and the distance between the ith dam and the end of the river. Then update the memoization table with the minimum cost of evaluating the ith dam based on the minimum costs of evaluating the previous dams. Finally, iterate through the last row of the memoization table to find the minimum overall cost.

The optimal substructure with overlapping subproblems in this problem is that the minimum cost of building a series of dams up to the ith dam with the jth dam, as the second-to-last dam can be computed from the minimum costs of evaluating the previous dams. This is because the cost of bulding a dam at position i depends on the cost of evaluating the previous dams, which depends on the costs of evaluating the dams before them. So we can break down the problem into smaller subproblems of finding the minimum cost of building a series of dams up to a certain position with a certain dam as the second-to-last dam, and use memoization to store the results of these subproblems for later use.

The recursive formulation of the problem is as follows:

memo[i][j] = min(memo[k][j] + cost(i, j, k)) for k in 0 to i-1

where cost(i, j, k) = (yArray[i] - yArray[j]) * (riverEnd - yArray[i]) + yArray[i] - yArray[k]

The recurrence relation computes the minimum cost of building the first i+1 dams with the last dam built at position j by considering all possible positions k of the previous dam. The cost of building the dam at position i+1 with the last dam built at position k is computed using the cost function. The overall minimum cost can be obtained by computing min(minCost(n-1, j)) for all j.