CC 4

Concept Check

Suppose that two portfolio managers who work for competing investment management houses each employ a group of security analysts to prepare the input list for the Markowitz algorithm. When all is completed, it turns out that the efficient frontier obtained by portfolio manager *A* dominates that of manager *B*. By domination we mean that *A*'s optimal risky portfolio lies northwest of *B*'s. Hence, given a choice, investors will always prefer the risky portfolio that lies on the CAL of *A*.

a. What should be made of this outcome?
b. Should it be attributed to better security analysis by *A*'s analysts?
c. Could it be that *A*'s computer program is superior?
d. If you were advising clients (and had an advance glimpse at the efficient frontiers of various managers), would you tell them to periodically switch their money around to the manager with the most northwesterly portfolio?

Asset Allocation and Security Selection

As we have seen, the theories of security selection and asset allocation are identical. Both activities call for the construction of an efficient frontier and the choice of a particular portfolio from along that frontier. The determination of the optimal combination of securities proceeds in the same manner as the analysis of the optimal combination of asset classes. Why, then, do we (and the investment community) distinguish between asset allocation and security selection?

Three factors are at work. First, as a result of greater need and ability to save (for college education, recreation, longer life in retirement and health care needs, etc.), the demand for sophisticated investment management has increased enormously. Second, the growing spectrum of financial markets and financial instruments have put sophisticated investment beyond the capacity of most amateur investors. Finally, there are strong economic returns to scale in investment management. The end result is that the size of a competitive investment company has grown with the industry, and efficiency in organization has become an important issue.

A large investment company is likely to invest both in domestic and international markets and in a broad set of asset classes, each of which requires specialized expertise. Hence, the management of each asset-class portfolio needs to be decentralized, and it becomes impossible to simultaneously optimize the entire organization's risky portfolio in one stage (although this would be prescribed as optimal on *theoretical* grounds).

The practice is therefore to optimize the security selection of each asset-class portfolio independently. At the same time, top management continually updates the asset allocation of the organization, adjusting the investment budget of each asset-class portfolio. When changed frequently in response to intensive forecasting activity, the reallocations are called *market timing*. The shortcoming of this two-step approach to portfolio construction versus the theory-based one-step optimization approach is the failure to exploit the covariance of the individual securities in one asset-class portfolio with the individual securities in the other asset classes. Only the covariance matrix of the securities within each asset-class portfolio can be used. However, this loss might be small, due to the depth of diversification of each portfolio and the extra layer of diversification at the asset allocation level.

7.5 A SPREADSHEET MODEL

Several software packages can be used to generate the efficient frontier. We will demonstrate the method using Microsoft Excel. Excel is far from the best program for this purpose and is limited in the number of assets it can handle, but working through a simple portfolio optimizer in Excel can illustrate concretely the nature of the calculations used in more sophisticated "black box" programs. You will find that even in Excel, the computation of the efficient frontier is fairly easy.

We apply the Markowitz portfolio optimization program to a practical problem of international diversification. We take the perspective of a portfolio manager serving U.S. clients, who in 2006 intended to construct for the next year an optimal risky portfolio of large stocks in the United States and six developed capital markets (Japan, Germany, the United Kingdom, France, Canada, and Australia). First we describe the input list: forecasts of risk premiums and the covariance matrix. Next, we describe Excel's Solver, and finally we show the solution to the manager's problem.

The Covariance Matrix

To capture recent risk parameters the manager compiles an array of the most recent 60 monthly (annualized) rates of return for the years 2001–2005, as well as the monthly T-bill rates for the same period.

The standard deviations of excess returns are shown in Table 7.4 (column C). They range from 14.95 percent (U.S. large stocks) to 22.7 percent (Germany). For perspective on how these parameters can change over time, standard deviations for the period 1991–2000 are also shown (column B). In addition, we present the correlation coefficient between large stocks in the six foreign markets with U.S. large stocks for the same two periods. Here we see that correlations are higher in the more recent period, consistent with the process of globalization.

The covariance matrix shown in Table 7.5 was estimated from the array of 60 returns of the seven countries using the COVARIANCE function from the dialogue box of Data Analysis in Excel's Tools menu. Due to a quirk in the Excel software, the covariance matrix is not corrected for bias; hence, each of the elements in the matrix was multiplied by 60/59 to eliminate downward bias.

Expected Returns

While estimation of the risk parameters (the covariance matrix) from excess returns is a simple technical matter, estimating the risk premium (the expected excess return) is a daunting task. As we discussed in Chapter 5, estimating expected returns using historical data is unreliable. Consider, for example, the negative average excess returns on U.S. large stocks over the period 2001–2005 (cell G6) and, more generally, the big differences in average returns between the 1991–2000 and 2001–2005 periods, as demonstrated in columns F and G.

In this example, we simply present the manager's forecasts of future returns as shown in column H. In Chapter 9 we will establish a framework that makes the forecasting process more explicit.

The Bordered Covariance Matrix and Portfolio Variance

The covariance matrix in Table 7.5 is bordered by the portfolio weights, as explained in Section 7.2 and Table 7.2. The values in cells A18–A24, to the left of the covariance matrix, will be selected by the optimization program. For now, we arbitrarily input 1.0 for the U.S. and zero for the others. Cells A16–I16, above the covariance matrix, must be set equal to the column of weights on the left, so that they will change in tandem as the column weights are changed by Excel's Solver. Cell A25 sums the column weights and is used to force the optimization program to set the sum of portfolio weights to 1.0.

Cells C25–I25, below the covariance matrix, are used to compute the portfolio variance for any set of weights that appears in the borders. Each cell accumulates the contribution to portfolio variance from the column above it. It uses the function SUMPRODUCT to accomplish this task. For example, row 33 shows the formula used to derive the value that appears in cell C25.

Tables 7.4, 7.5, 7.6 Spreadsheet Model for International Diversification

eXcel Please visit us at www.mcgrawhill.ca/olc/bodie

7.4 Country Index Statistics and Forecasts of Risk Premiums

Country	Standard Deviation 1991–2000	Standard Deviation 2001–2005	Correlation with the U.S. 1991–2000	Correlation with the U.S. 2001–2005	Average Excess Return 1991–2000	Average Excess Return 2001–2005	Forecast 2006
US	0.1295	0.1495	1	1	0.1108	−0.0148	0.0600
UK	0.1466	0.1493	0.64	0.83	0.0536	0.0094	0.0530
France	0.1741	0.2008	0.54	0.83	0.0837	0.0247	0.0700
Germany	0.1538	0.2270	0.53	0.85	0.0473	0.0209	0.0800
Australia	0.1808	0.1617	0.52	0.81	0.0468	0.1225	0.0580
Japan	0.2432	0.1878	0.41	0.43	−0.0177	0.0398	0.0450
Canada	0.1687	0.1727	0.72	0.79	0.0727	0.1009	0.0590

7.5 The Bordered Covariance Matrix

Portfolio Weights →

Portfolio Weights		US	UK	France	Germany	Australia	Japan	Canada
		1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0000	US	0.0224	0.0184	0.0250	0.0288	0.0195	0.0121	0.0205
0.0000	UK	0.0184	0.0223	0.0275	0.0299	0.0204	0.0124	0.0206
0.0000	France	0.0250	0.0275	0.0403	0.0438	0.0259	0.0177	0.0273
0.0000	Germany	0.0288	0.0299	0.0438	0.0515	0.0301	0.0183	0.0305
0.0000	Australia	0.0195	0.0204	0.0259	0.0301	0.0261	0.0147	0.0234
0.0000	Japan	0.0121	0.0124	0.0177	0.0183	0.0147	0.0353	0.0158
0.0000	Canada	0.0205	0.0206	0.0273	0.0305	0.0234	0.0158	0.0298
1.0000		0.0224	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0600	Mean							
0.1495	SD							
0.4013	Slope							

Cell A18 - A24		A18 is set arbitrarily to 1 while A19 to A24 are set to 0
Formula in cell	C16	=A18 ... Formula in cell I16 = A24
Formula in cell	A25	=SUM(A18:A24)
Formula in cell	C25	=C16*SUMPRODUCT($A$18:$A$24,C18:C24)
Formula in cell	D25-I25	Copied from C25 (note the absolute addresses)
Formula in cell	A26	=SUMPRODUCT($A$18:$A$24,H6:H12)
Formula in cell	A27	=SUM(C25:I25)^0.5
Formula in cell	A28	=A26/A27

7.6 The Efficient Frontier

Cell to store constraint on risk premium: 0.0400

	B	Min Var	D	E	F	G	Optimum	I	J	K	L
Mean		0.0383	0.0400	0.0450	0.0500	0.0550	0.0564	0.0575	0.0600	0.0700	0.0800
SD	0.1	0.1132	0.1135	0.1168	0.1238	0.1340	0.1374	0.1401	0.1466	0.1771	0.2119
Slope		0.3386	0.3525	0.3853	0.4037	0.4104	0.4107	0.4106	0.4092	0.3953	0.3774
US		0.6112	0.6195	0.6446	0.6696	0.6947	0.7018	0.7073	0.7198	0.7699	0.8201
UK		0.8778	0.8083	0.5992	0.3900	0.1809	0.1214	0.0758	−0.0283	−0.4465	−0.8648
France		−0.2140	−0.2029	−0.1693	−0.1357	−0.1021	−0.0926	−0.0852	−0.0685	−0.0014	0.0658
Germany		−0.5097	−0.4610	−0.3144	−0.1679	−0.0213	0.0205	0.0524	0.1253	0.4185	0.7117
Australia		0.0695	0.0748	0.0907	0.1067	0.1226	0.1271	0.1306	0.1385	0.1704	0.2023
Japan		0.2055	0.1987	0.1781	0.1575	0.1369	0.1311	0.1266	0.1164	0.0752	0.0341
Canada		−0.0402	−0.0374	−0.0288	−0.0203	−0.0118	−0.0093	−0.0075	−0.0032	0.0139	0.0309
CAL*	0.0411	0.0465	0.0466	0.0480	0.0509	0.0550	0.0564	0.0575	0.0602	0.0727	0.0871

*Risk premium on CAL = SD * slope of optimal risky portfolio

Finally, the short column A26–A28 below the bordered covariance matrix presents portfolio statistics computed from the bordered covariance matrix. First is the portfolio risk premium in cell A26, with formula shown in row 35, which multiplies the column of portfolio weights by the column of forecasts (H6–H12) from Table 7.4. Next is the portfolio standard deviation in cell A27. The variance is given by the sum of cells C25–I25 below the bordered covariance matrix. Cell A27 takes the square root of this sum to produce the standard deviation. The last statistic is the portfolio Sharpe ratio, cell A28, which is the slope of the CAL (capital allocation line) that runs through the portfolio constructed using the column weights (the value in cell A28 equals cell A26 divided by cell A27). The optimal risky portfolio is the one that maximizes the Sharpe ratio.

Using the Excel Solver

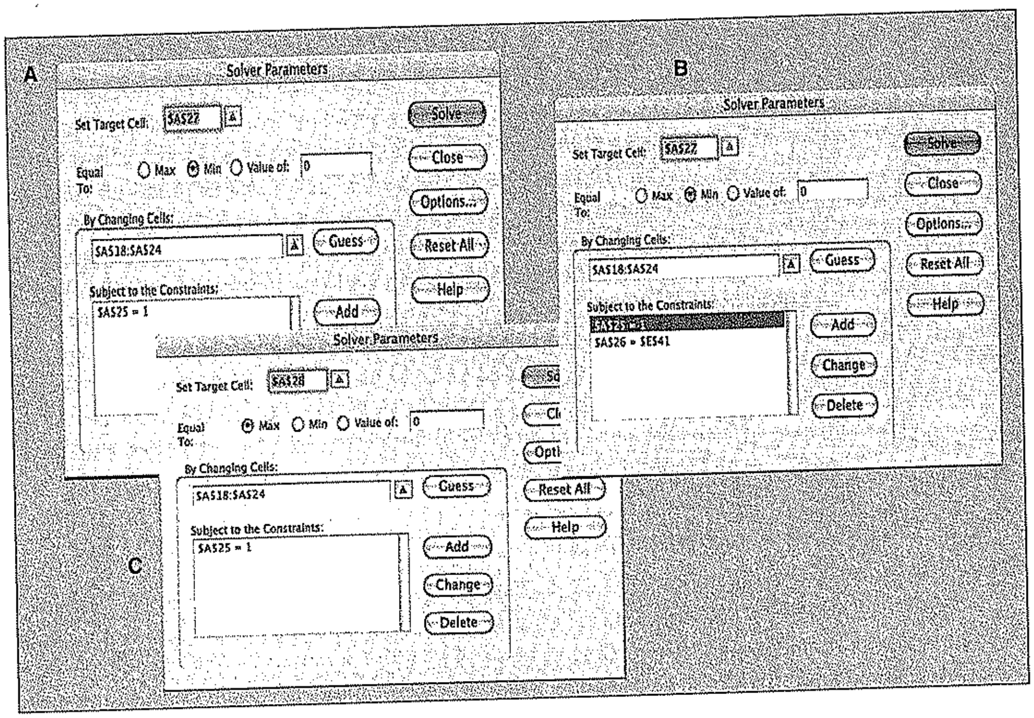
Excel's Solver is a user-friendly, but quite powerful, optimizer. It has three parts: (1) an objective function, (2) decision variables, and (3) constraints. Figure 7.14 shows three pictures of the Solver. For the current discussion we refer to picture A.

The top panel of the Solver lets you choose a target cell for the "objective function," that is, the variable you are trying to optimize. In picture A, the target cell is A27, the portfolio standard deviation. Below the target cell, you can choose whether your objective is to maximize, minimize, or set your objective function equal to a value that you specify. Here we choose to minimize the portfolio standard deviation.

The next panel contains the decision variables. These are cells that the Solver can change in order to optimize the objective function in the target cell. Here, we input cells A18–A24, the portfolio weights that we select to minimize portfolio volatility.

The bottom panel of the Solver can include any number of constraints. One constraint that must always appear in portfolio optimization is the "feasibility constraint," namely, that portfolio weights sum to 1.0. When we bring up the constraint dialogue box, we specify that cell A25 (the sum of weights) be set equal to 1.0.

Figure 7.14
Solver dialogue box.



Finding the Minimum Variance Portfolio

It is helpful to begin by identifying the global minimum variance portfolio (G). This provides the starting point of the efficient part of the frontier. Once we input the target cell, the decision variable cells, and the feasibility constraint, as in picture A, we can punch "solve" and the Solver returns portfolio G. We copy the portfolio statistics and weights to our output Table 7.6. Column C in Table 7.6 shows that the lowest standard deviation (SD) that can be achieved with our input list is 11.32 percent. Notice that the SD of portfolio G is considerably lower than even the lowest SD of the individual indices. From the risk premium of portfolio G (3.83 percent) we begin building the efficient frontier with ever-larger risk premiums.

Charting the Efficient Frontier of Risky Portfolios

We determine the desired risk premiums (points on the efficient frontier) that we wish to use to construct the graph of the efficient frontier. It is good practice to choose more points in the neighborhood of portfolio G because the frontier has the greatest curvature in that region. It is sufficient to choose for the highest point the highest risk premium from the input list (here, 8 percent for Germany). You can produce the entire efficient frontier in minutes following this procedure.

1. Input to the Solver a constraint that says: cell A26 (the portfolio risk premium) must equal the value in cell E41. The Solver at this point is shown in picture B of Figure 7.14. Cell E41 will be used to change the required risk premium and thus generate different points along the frontier.
2. For each additional point on the frontier, you input a different desired risk premium into cell E41, and ask the Solver to solve again.
3. Every time the Solver gives you a solution to the request in (2), copy the results into Table 7.6, which tabulates the collection of points along the efficient frontier. For the next step, change cell E41 and repeat from step 2.

Finding the Optimal Risky Portfolio on the Efficient Frontier

Now that we have an efficient frontier, we look for the portfolio with the highest Sharpe ratio (i.e., reward-to-volatility ratio). This is the efficient frontier portfolio that is tangent to the CAL. To find it, we just need to make two changes to the Solver. First, change the target cell from cell A27 to cell A28, the Sharpe ratio of the portfolio, and request that the value in this cell be maximized. Next, eliminate the constraint on the risk premium that may be left over from the last time you used the Solver. At this point the Solver looks like picture C in Figure 7.14.

The Solver now yields the optimal risky portfolio. Copy the statistics for the optimal portfolio and its weights to your Table 7.6. In order to get a clean graph, place the column of the optimal portfolio in Table 7.6 so that the risk premiums of all portfolios in the table are steadily increasing from the risk premium of portfolio G (3.83 percent) all the way up to 8 percent.

The efficient frontier is graphed using the data in cells C45–I45 (the horizontal or x-axis is portfolio standard deviation) and C44–I44 (the vertical or y-axis is portfolio risk premium). The resulting graph appears in Figure 7.15.

The Optimal CAL

It is instructive to superimpose on the graph of the efficient frontier in Figure 7.15 the CAL that identifies the optimal risky portfolio. This CAL has a slope equal to the Sharpe ratio of the optimal risky portfolio. Therefore, we add at the bottom of Table 7.6 a row with entries obtained by multiplying the SD of each column's portfolio by the Sharpe ratio of the optimal risky portfolio

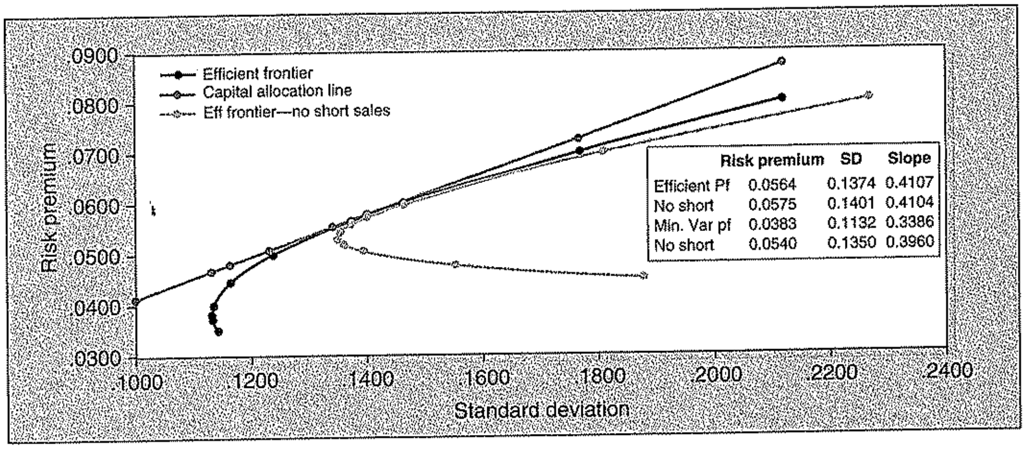


Figure 7.15
Efficient frontier and CAL for country stock indices.

from cell H46. This results in the risk premium for each portfolio along the CAL efficient frontier. We now add a series to the graph with the standard deviations in B45–I45 as the x-axis and cells B54–I54 as the y-axis. You can see this CAL in Figure 7.15.

The Optimal Risky Portfolio and the Short-Sales Constraint

With the input list used by the portfolio manager, the optimal risky portfolio calls for significant short positions in the stocks of France and Canada (see column H of Table 7.6). In many cases the portfolio manager is prohibited from taking short positions. If so, we need to amend the program to preclude short sales.

To accomplish this task, we repeat the exercise, but with one change. We add to the Solver the following constraint: each element in the column of portfolio weights, A18–A24, must be greater than or equal to zero. You should try to produce the short-sale constrained efficient frontier in your own spreadsheet. The graph of the constrained frontier is also shown in Figure 7.15.

SUMMARY

1. The expected return of a portfolio is the weighted average of the component assets' expected returns with the investment proportions as weights.
2. The variance of a portfolio is the weighted sum of the elements of the covariance matrix with the product of the investment proportions as the weight. Thus, the variance of each asset is weighted by the square of its investment proportion. Each covariance of any pair of assets appears twice in the covariance matrix, and thus the portfolio variance includes twice each covariance weighted by the product of the investment proportions in each of two assets.
3. Even if the covariances are positive, the portfolio standard deviation is less than the weighted average of the component standard deviations, as long as the assets are not perfectly positively correlated. Thus, portfolio diversification is of value as long as assets are less than perfectly correlated.
4. The greater an asset's *covariance* with the other assets in the portfolio, the more it contributes to portfolio variance. An asset that is perfectly negatively correlated with a portfolio can serve as a perfect hedge. The perfect hedge asset can reduce the portfolio variance to zero.
5. The efficient frontier is the graphical representation of a set of portfolios that maximize expected return for each level of portfolio risk. Rational investors will choose a portfolio on the efficient frontier.
6. A portfolio manager identifies the efficient frontier by first establishing estimates for the asset expected returns and the covariance matrix. This input list then is fed into an optimization program that reports as outputs the investment proportions, expected returns, and standard deviations of the portfolios on the efficient frontier.
7. In general, portfolio managers will arrive at different efficient portfolios due to a difference in methods and