Assignment 2
Deadline to hand in: Nov. 7th in class

Q. 1 The following data are obtained from $n = 10$ patients with aggressive chronic hepatitis. Please note that it is a part of a larger set of data reported by Plomteux (1980, Clin. Chem. 26, 1897-1899). The measurements are made on the level of three liver enzymes: aspartate aminotransferase, alanine aminotransferase, and glutamate dehydrogenase.

Patient	X_1	X_2	X_3
1	31	63	4
2	32	56	6
3	50	59	9
4	56	72	7
5	39	87	9
6	46	95	8
7	29	57	5
8	40	50	3
9	29	44	4
10	24	42	3

(a) Compute the maximum likelihood estimates for the mean vector $\underset{\sim}{\mu}$ and the covariance matrix Σ.

(b) Compute the unbiased estimates of the covariance matrix Σ.

(c) Compute the generalized sample variance.

(d) Compute the total sample variance.

Q. 2 Let the data matrix for a random matrix for a random sample of size $n = 3$ from a bivariate normal population be

$$X = \begin{bmatrix} 6 & 9 \\ 10 & 6 \\ 8 & 3 \end{bmatrix}$$

Verify that T^2 remains unchanged if each observation $\underset{\sim}{x}_j, j = 1, 2, 3$ is replaced by $C\underset{\sim}{x}_j$, where

$$C = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

Q. 3 A number of patients with bronchus cancer were treated with ascorbate and compared with matched control patients who received no ascorbate (Cameron & Pauling, 1978). Download the data("BronchusCancer.txt") from the Blackboard. The explanation of variables is as follows:

$y_1 = $ patient (with ascorbate): survival time (days) from date of first hospital admission.
$x_1 = $ matched control (without ascorbate): survival time (days) from date of first hospital admission.
$y_2 = $ patient: survival time (days) from date of untreatability
$x_2 = $ matched control: survival time (days) from date of untreatability.

(a) Compare y_1 and y_2 with x_1 and x_2, respectively, using a paired T^2 test with $\alpha = 0.05$. Clearly sate your null hypothesis.

(b) Perform the following analyses as a follow up to your hypothesis test in part (a):

    i. Sketch 95% confidence region. Confirm it using built-in R function, "Ellipse.R".

    ii. Find 95% univariate intervals for each mean difference.

    iii. Find 95% simultaneous T^2 intervals for each mean difference.

    iv. Find 95% Bonferroni component intervals.

    v. Find the linear combination that gives the largest value of T^2.

(c) Using all the information in part (b), would you want to receive the treatment if you have the same type of cancer? Explain.

Q. 4 Four psychological tests were given to 32 men and 32 women.

Download the data("PsyTests.txt") from the Blackboard. The variables are:

$y_1 = $ pictorial inconsistencies $y_3 = $ tool recognition
$y_2 = $ paper form board $y_4 = $ vocabulary

(a) Assuming that the population matrices $\Sigma_1 = \Sigma_2$. Obtain the pooled estimate of the common variance-covariance matrix.

(b) Perform Bartlett's test of null hypothesis, $H_0 : \Sigma_1 = \Sigma_2$ against the alternative $H_a : \Sigma_1 \neq \Sigma_2$. State your conclusion.

(c) Using the pooled covariance matrix, compute Hotelling's T^2. Test the hypothesis that the mean vectors of men and women are same using T^2. State your conclusion.

(d) Construct the profile plot of two mean vectors (specify gender on the plot).

(e) Carry out profile analysis.