STA457 Assignment

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Model

Fit the following transfer function noise model $$co2_t =\alpha + \sum^s_{j} v_j \cdot gas_{t-j-b} + e_t, (1)$$ where e_t is serially correlated.

1. Use the ideas of prewhitening taught in class to identify b and s.

According the following figure, I identify that $b=?$ and $s=?$.

```
# Type R codes used to produce the figure for identifying b and s
install.packages("markdown", repos='http://cran.utstat.utoronto.ca/')
```

```
## Installing package into 'C:/Users/Isaac/Documents/R/win-library/3.1'
## (as 'lib' is unspecified)
```

```
## package 'markdown' successfully unpacked and MD5 sums checked
##
## The downloaded binary packages are in
##  C:\Users\Isaac\AppData\Local\Temp\RtmpKwlF1M\downloaded_packages
```

```
# library(markdown)
require(markdown)
```

```
## Loading required package: markdown
```

```
## Warning: package 'markdown' was built under R version 3.1.3
```

```
install.packages("forecast", repos='http://cran.utstat.utoronto.ca/')
```

```
## Installing package into 'C:/Users/Isaac/Documents/R/win-library/3.1'
## (as 'lib' is unspecified)
```

```
## Warning in download.file(url, destfile, method, mode = "wb", ...):
## downloaded length 1364370 != reported length 1364370
```

```
## package 'forecast' successfully unpacked and MD5 sums checked
##
## The downloaded binary packages are in
##  C:\Users\Isaac\AppData\Local\Temp\RtmpKwlF1M\downloaded_packages
```

```
library(forecast)
```

```
## Warning: package 'forecast' was built under R version 3.1.3
```

```
## Loading required package: zoo
```

```
## Warning: package 'zoo' was built under R version 3.1.3
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
## Loading required package: timeDate
```

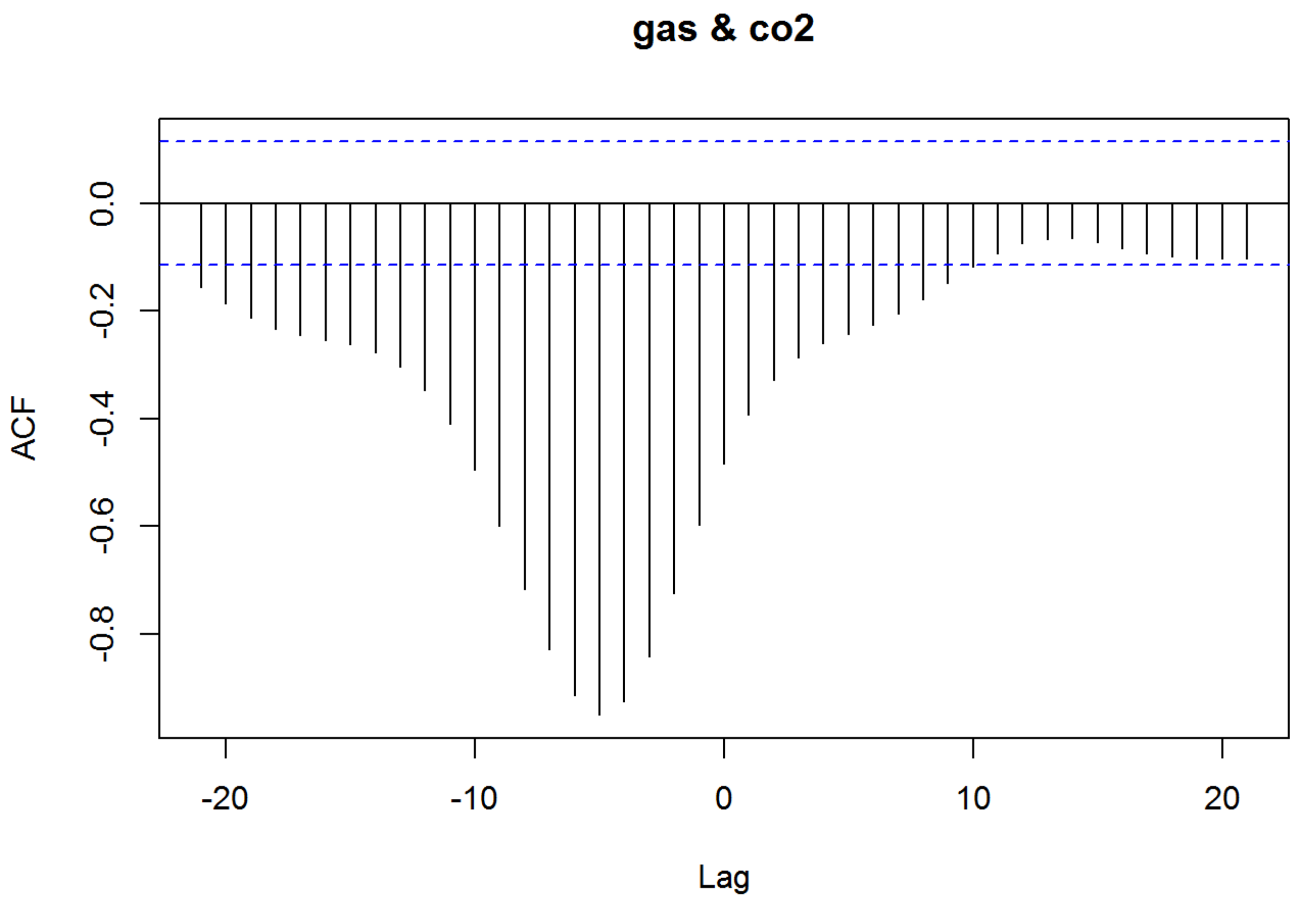
```
## Warning: package 'timeDate' was built under R version 3.1.3
```

```
## This is forecast 7.1
```

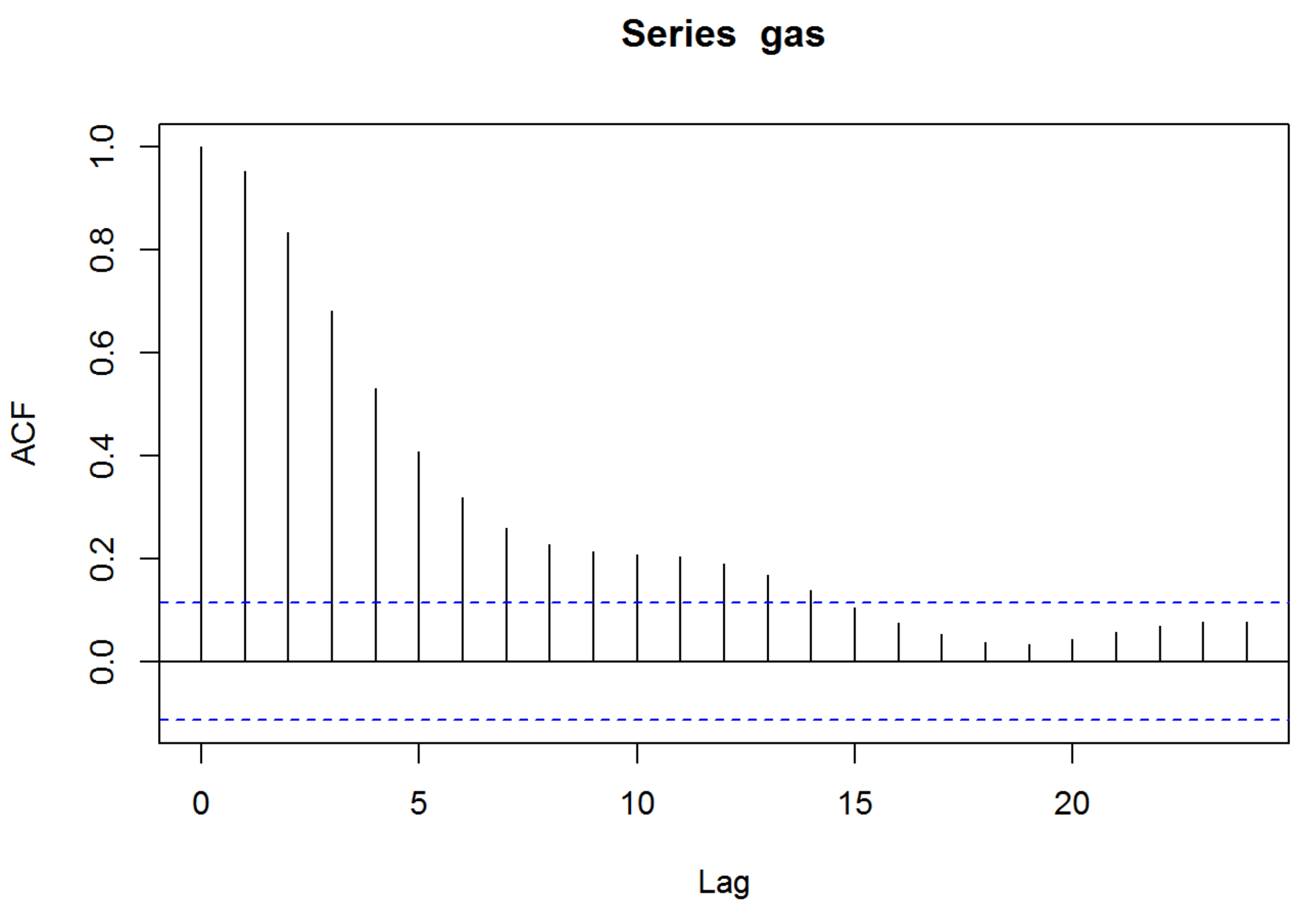
```
##
## Attaching package: 'forecast'
```

```
## The following object is masked _by_ '.GlobalEnv':
##
##     gas
```

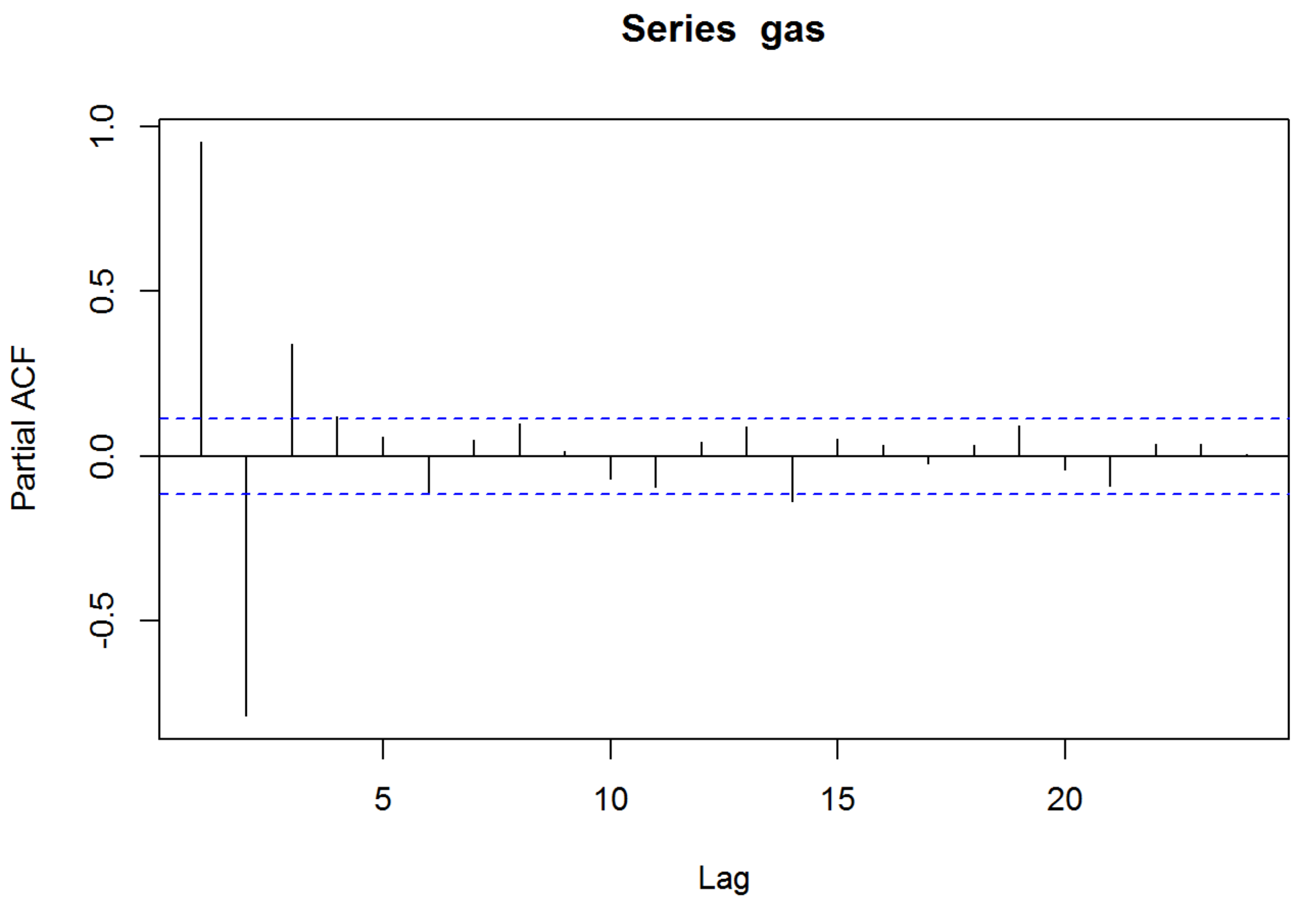
```
ccf(gas, co2)
```


gas & co2
ACF
0.0
-0.2
-0.4
-0.6
-0.8
-20
-10
0
10
20
Lag

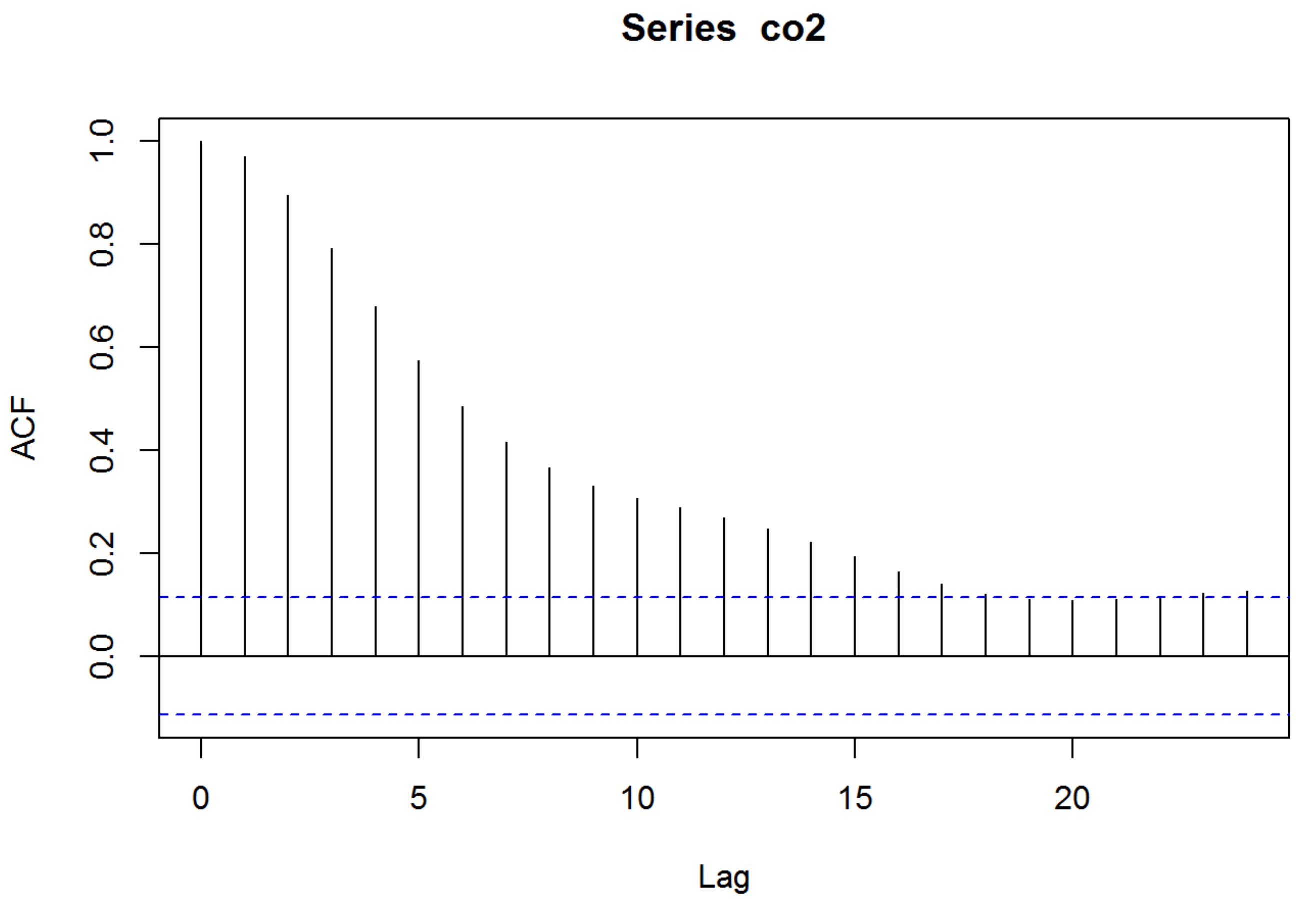
```
acf(gas)
```


Series gas
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Lag

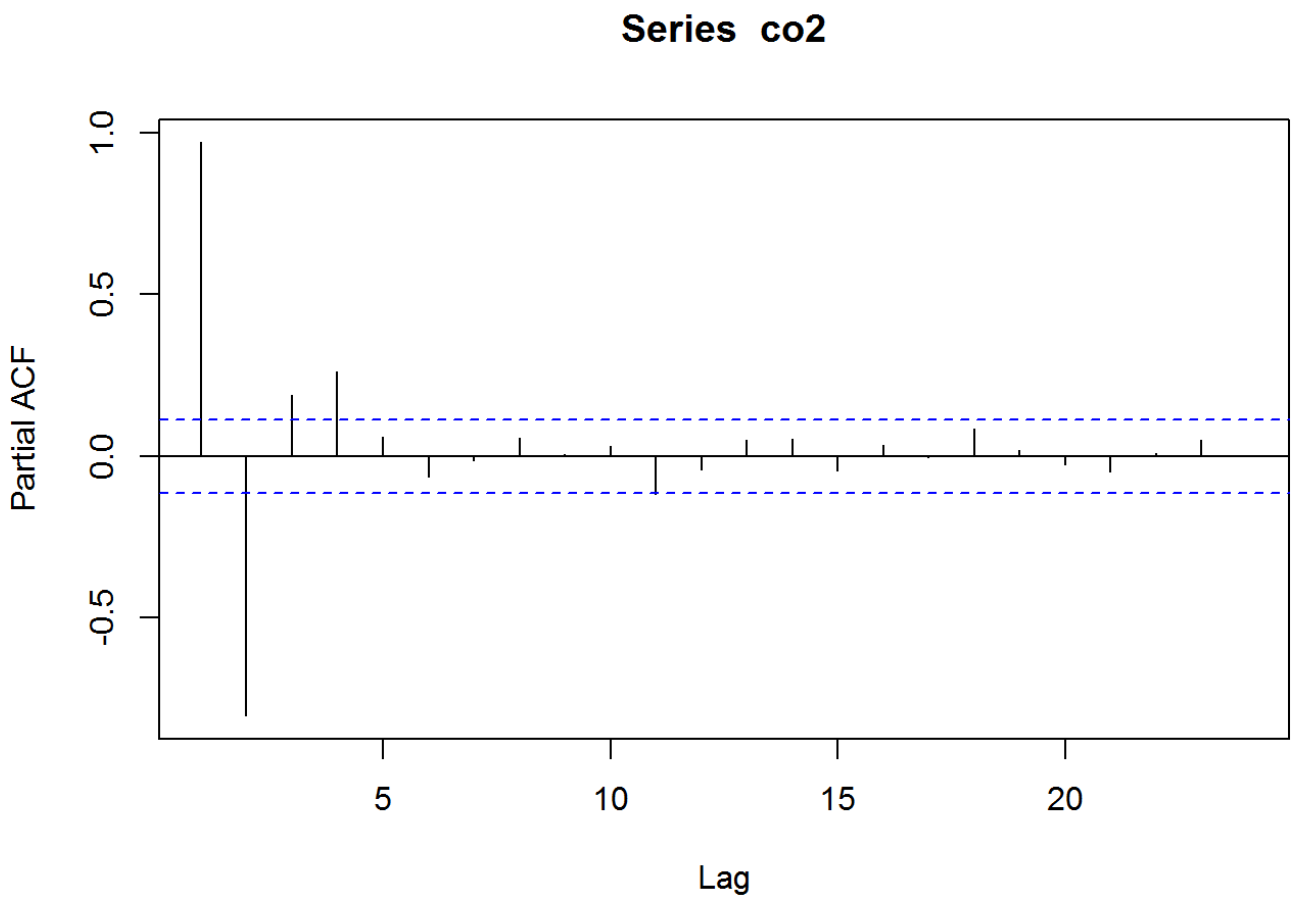
```
pacf(gas)
```




```
acf(co2)
```


Series co2
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Lag

```
pacf(co2)
```


Series co2
Partial ACF
1.0
0.5
0.0
-0.5
5
10
15
20
Lag

```r
ar5model = arima(gas, order = c(5,0,0), include.mean = FALSE)
ar5model # Prints the AR(5) coefficients for gas
```

```
##
## Call:
## arima(x = gas, order = c(5, 0, 0), include.mean = FALSE)
##
## Coefficients:
##          ar1      ar2     ar3     ar4     ar5
##       1.9203  -1.2031  0.1670  0.0122  0.0566
## s.e.  0.0578   0.1258  0.1438  0.1256  0.0576
##
## sigma^2 estimated as 0.03466:  log likelihood = 75.24,  aic = -138.47
```

```r
f_co2 = c(1,-ar5model$coef[1:5]) #create a filter to transform co2
f_co2
```

```
##                     ar1         ar2         ar3         ar4         ar5
##  1.00000000 -1.92034209  1.20314165 -0.16697269 -0.01218442 -0.05655448
```

```r
co2_filtered = filter(co2, f_co2, method=c("convolution"),sides=1)
co2_filtered
```

```
## Time Series:
## Start = 1
## End = 296
## Frequency = 1
##   [1]       NA       NA       NA       NA       NA 2.293056 2.361373
##   [8] 2.490920 2.437077 2.336302 2.551674 2.770718 2.655379 2.998178
##  [15] 2.644243 3.114019 2.687565 2.278355 2.589678 2.459934 2.550960
##  [22] 2.374546 1.898340 2.416549 2.286400 2.379599 2.108803 2.285740
##  [29] 2.006803 1.917283 2.129573 2.313097 2.167391 2.213526 2.183095
##  [36] 2.264208 2.620804 2.699132 2.561353 2.739134 2.367876 2.240876
##  [43] 2.444288 2.261832 1.693939 1.689366 2.290446 1.759320 2.626398
##  [50] 2.272340 2.475925 2.250410 2.008443 2.279295 1.984654 2.044451
##  [57] 2.834964 2.935714 2.623505 2.441039 2.490025 2.077457 2.301144
##  [64] 2.251177 2.618632 2.259241 2.270186 2.191252 2.329246 2.240323
##  [71] 2.464841 2.582938 2.564342 2.883570 3.122761 2.934676 3.069936
##  [78] 2.525839 2.980571 2.339710 2.485406 2.787380 2.472571 2.988276
##  [85] 3.386415 2.413155 2.921267 2.614466 2.663080 2.644457 2.698779
##  [92] 2.355856 2.246309 2.094896 2.026920 2.441473 2.517974 1.877542
##  [99] 2.203644 2.523651 2.882600 2.529155 2.371488 2.711109 2.545555
## [106] 2.555439 2.599890 2.640451 2.496530 2.575089 2.045862 2.699159
## [113] 2.568035 2.982614 3.158312 3.304102 2.825955 3.118259 2.811357
## [120] 2.402528 2.794055 2.464508 2.807436 2.497945 2.565265 2.462933
## [127] 2.512682 2.344787 2.564450 2.142539 2.326868 2.121154 1.933724
## [134] 2.296465 1.765774 2.517964 2.158893 2.493527 2.419876 2.425601
## [141] 2.479160 2.279883 2.508078 2.149181 2.482543 2.455000 2.220397
## [148] 2.708672 2.914420 2.764699 2.489534 2.659978 2.393837 2.543046
## [155] 2.385652 2.540652 2.602204 2.361713 2.082471 2.462904 2.387923
## [162] 2.580003 2.103253 2.370564 2.040281 2.110185 2.288685 2.595925
## [169] 2.600358 2.748897 2.765442 2.712221 2.702087 3.291560 2.332369
## [176] 2.494789 1.619559 2.843426 2.210887 2.882939 2.535096 2.760678
## [183] 2.582772 2.721168 2.845056 2.617878 2.437306 2.150308 2.556070
## [190] 2.498809 2.869138 2.361081 2.006983 2.145590 2.549379 2.139796
## [197] 2.412185 2.473074 3.829715 2.031265 3.879105 3.343223 2.564709
## [204] 2.622858 3.378868 2.913836 2.665565 2.704269 2.553649 1.922569
## [211] 2.650335 2.210287 2.641495 2.398978 2.619081 2.637192 3.304273
## [218] 2.297301 3.049047 2.467647 2.806808 2.371399 2.560423 2.665775
## [225] 2.552665 2.618633 2.534864 2.793084 2.955247 2.585426 2.926274
## [232] 2.478675 2.214384 2.074974 3.108832 1.964384 3.365955 3.042141
## [239] 2.687879 2.590149 2.858820 2.961963 2.820141 2.873469 2.714302
## [246] 2.664814 2.383555 2.675729 2.728133 2.277005 2.966759 2.644940
## [253] 2.471582 2.238526 2.225117 2.084571 2.487601 2.102625 2.652562
## [260] 2.444395 1.710161 2.105860 2.509957 2.542003 4.183719 2.446630
## [267] 2.156691 1.539275 1.956622 2.778171 2.794980 2.891742 2.490128
## [274] 2.448495 2.894433 2.532085 2.625039 2.477138 2.684086 2.344247
## [281] 2.247608 2.704666 2.198612 2.604936 2.485374 2.772693 3.366665
## [288] 3.082191 2.326216 3.229276 2.789761 2.479490 2.918109 2.512941
## [295] 2.685316 2.744596
```

```
co2_prew = co2_filtered[6:296] # Transformed co2
co2_prew
```

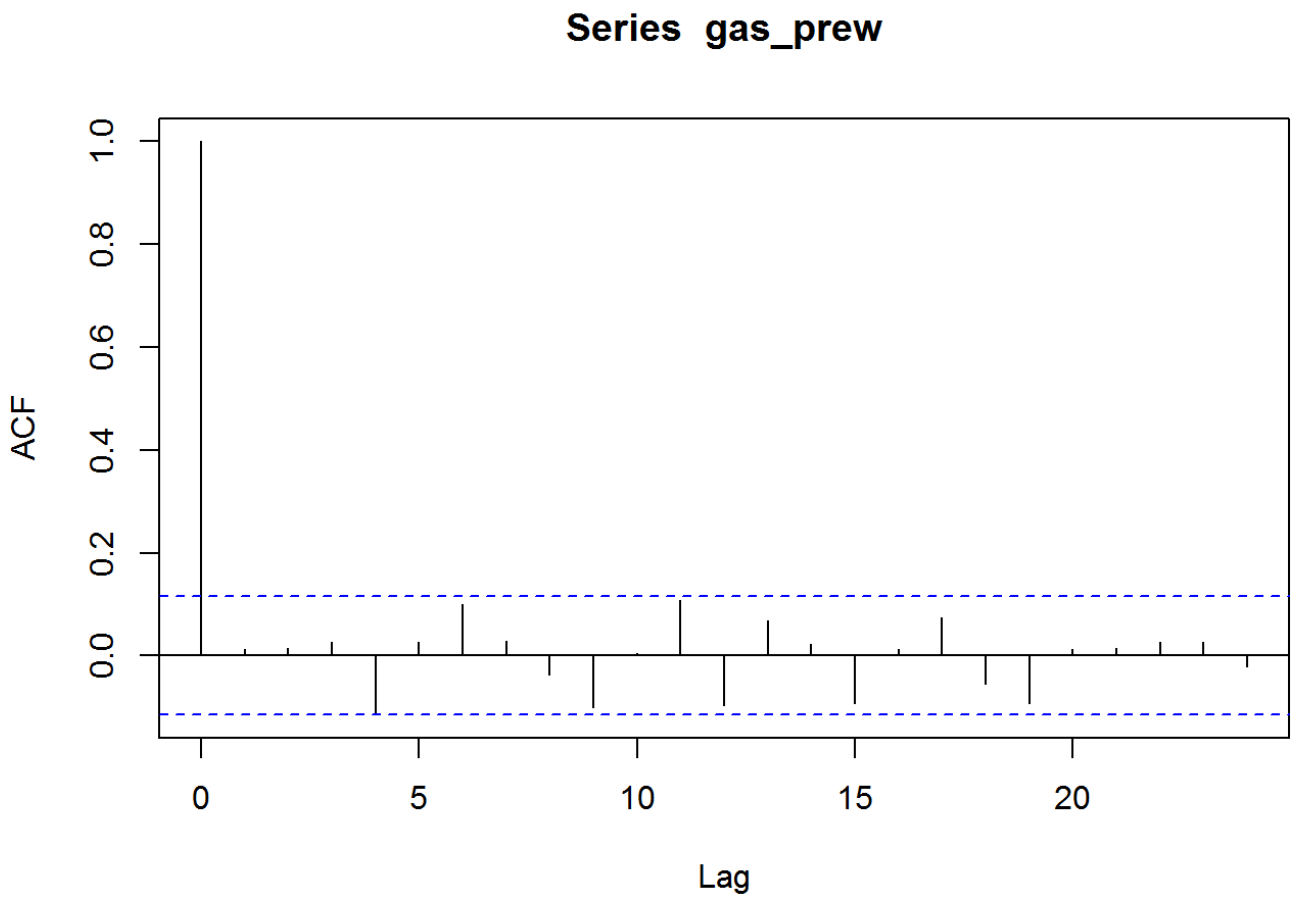
```
##   [1] 2.293056 2.361373 2.490920 2.437077 2.336302 2.551674 2.770718
##   [8] 2.655379 2.998178 2.644243 3.114019 2.687565 2.278355 2.589678
##  [15] 2.459934 2.550960 2.374546 1.898340 2.416549 2.286400 2.379599
##  [22] 2.108803 2.285740 2.006803 1.917283 2.129573 2.313097 2.167391
##  [29] 2.213526 2.183095 2.264208 2.620804 2.699132 2.561353 2.739134
##  [36] 2.367876 2.240876 2.444288 2.261832 1.693939 1.689366 2.290446
##  [43] 1.759320 2.626398 2.272340 2.475925 2.250410 2.008443 2.279295
##  [50] 1.984654 2.044451 2.834964 2.935714 2.623505 2.441039 2.490025
##  [57] 2.077457 2.301144 2.251177 2.618632 2.259241 2.270186 2.191252
##  [64] 2.329246 2.240323 2.464841 2.582938 2.564342 2.883570 3.122761
##  [71] 2.934676 3.069936 2.525839 2.980571 2.339710 2.485406 2.787380
##  [78] 2.472571 2.988276 3.386415 2.413155 2.921267 2.614466 2.663080
##  [85] 2.644457 2.698779 2.355856 2.246309 2.094896 2.026920 2.441473
##  [92] 2.517974 1.877542 2.203644 2.523651 2.882600 2.529155 2.371488
##  [99] 2.711109 2.545555 2.555439 2.599890 2.640451 2.496530 2.575089
## [106] 2.045862 2.699159 2.568035 2.982614 3.158312 3.304102 2.825955
## [113] 3.118259 2.811357 2.402528 2.794055 2.464508 2.807436 2.497945
## [120] 2.565265 2.462933 2.512682 2.344787 2.564450 2.142539 2.326868
## [127] 2.121154 1.933724 2.296465 1.765774 2.517964 2.158893 2.493527
## [134] 2.419876 2.425601 2.479160 2.279883 2.508078 2.149181 2.482543
## [141] 2.455000 2.220397 2.708672 2.914420 2.764699 2.489534 2.659978
## [148] 2.393837 2.543046 2.385652 2.540652 2.602204 2.361713 2.082471
## [155] 2.462904 2.387923 2.580003 2.103253 2.370564 2.040281 2.110185
## [162] 2.288685 2.595925 2.600358 2.748897 2.765442 2.712221 2.702087
## [169] 3.291560 2.332369 2.494789 1.619559 2.843426 2.210887 2.882939
## [176] 2.535096 2.760678 2.582772 2.721168 2.845056 2.617878 2.437306
## [183] 2.150308 2.556070 2.498809 2.869138 2.361081 2.006983 2.145590
## [190] 2.549379 2.139796 2.412185 2.473074 3.829715 2.031265 3.879105
## [197] 3.343223 2.564709 2.622858 3.378868 2.913836 2.665565 2.704269
## [204] 2.553649 1.922569 2.650335 2.210287 2.641495 2.398978 2.619081
## [211] 2.637192 3.304273 2.297301 3.049047 2.467647 2.806808 2.371399
## [218] 2.560423 2.665775 2.552665 2.618633 2.534864 2.793084 2.955247
## [225] 2.585426 2.926274 2.478675 2.214384 2.074974 3.108832 1.964384
## [232] 3.365955 3.042141 2.687879 2.590149 2.858820 2.961963 2.820141
## [239] 2.873469 2.714302 2.664814 2.383555 2.675729 2.728133 2.277005
## [246] 2.966759 2.644940 2.471582 2.238526 2.225117 2.084571 2.487601
## [253] 2.102625 2.652562 2.444395 1.710161 2.105860 2.509957 2.542003
## [260] 4.183719 2.446630 2.156691 1.539275 1.956622 2.778171 2.794980
## [267] 2.891742 2.490128 2.448495 2.894433 2.532085 2.625039 2.477138
## [274] 2.684086 2.344247 2.247608 2.704666 2.198612 2.604936 2.485374
## [281] 2.772693 3.366665 3.082191 2.326216 3.229276 2.789761 2.479490
## [288] 2.918109 2.512941 2.685316 2.744596
```

```
gas_prew = ar5model$residuals[6:296] # Transformed gas
gas_prew
```

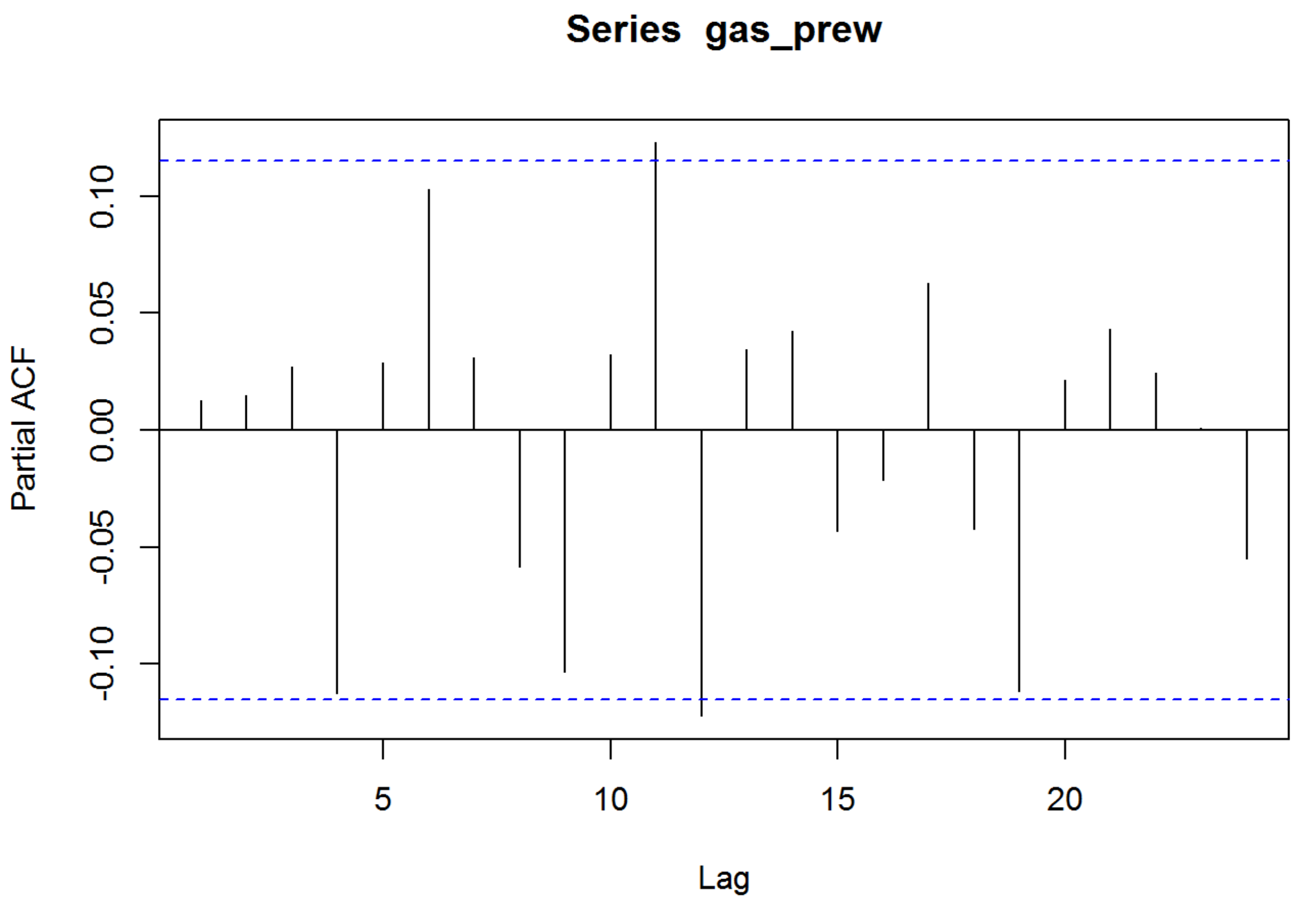
```
##   [1]  0.1090207185  0.0041284040 -0.0831702665 -0.0839824622 -0.1086327136
##   [6] -0.1782034754 -0.1939216742 -0.0936596813  0.0326823894  0.1007561295
##  [11]  0.1408868888 -0.1475544346  0.0560878715  0.1248764369  0.1966676381
##  [16]  0.1255763325 -0.0766957593  0.0768156259  0.1136083763  0.1501304022
##  [21]  0.2393652488  0.3286985322 -0.1381828417  0.0016778825  0.1618815727
##  [26]  0.1525239146  0.0466098046 -0.0279763373 -0.1349652521 -0.1274742379
##  [31] -0.0287258333  0.0434139798  0.0620303306  0.1590939075  0.0469769440
##  [36]  0.0831648205  0.2786540025  1.0025467552 -0.7849590627  0.2986995131
##  [41] -0.0245757614 -0.0413933683  0.1130025022  0.0740210553  0.1829946144
```

```
##  [46]  0.2774271441 -0.0221098810 -0.0062543639 -0.1301129312 -0.9242118655
##  [51]  0.8263823907 -0.0170417565  0.0676453760  0.2373564586 -0.1054802109
##  [56] -0.0379133704  0.0657433936  0.0852093614  0.1334935145  0.1437442104
##  [61] -0.0074146607 -0.0514736582 -0.0772416652 -0.0294212332 -0.1148131212
##  [66] -0.2613718350 -0.4155840737  0.0843231765 -0.1113608510 -0.0670565250
##  [71]  0.1721415622  0.0366453224 -0.0946103728 -0.1050804081 -0.1565486356
##  [76] -0.1814968152 -0.3343865471  0.2070876151 -0.0429363104 -0.0563668503
##  [81] -0.0091674464 -0.0411178889  0.0265147695  0.1855202654  0.2481851083
##  [86]  0.3335185714 -0.3553933206  0.1193247600  0.2522171202  0.1547388396
##  [91]  0.0135921004 -0.1112346827 -0.2423780121 -0.2252730247  0.3006080499
##  [96]  0.0387926861 -0.0312282922 -0.1419381911 -0.1053249826  0.0197755726
## [101]  0.0720516744  0.1671620489  0.0429439190 -0.1142694204 -0.2077344773
## [106] -0.2512129466 -0.2617747989 -0.6320761898  0.4551235737 -0.2822125208
## [111]  0.0023631537  0.2875882010 -0.3147075356 -0.0509464334  0.0295271220
## [116] -0.0405777198  0.0004909993  0.0867895034  0.1459846065 -0.1239382600
## [121]  0.0672563429  0.1705371763  0.1982811622  0.1481860941  0.1816870293
## [126]  0.0992388110  0.0607129252  0.0159328783  0.0222051670 -0.2176077999
## [131]  0.1345935639  0.1601330330 -0.0304805348  0.0138945090  0.0671591845
## [136]  0.0858690242  0.0182498441 -0.0211333008 -0.0692911022 -0.1724451936
## [141] -0.2013623196  0.0363670489  0.1342508367  0.0459719291 -0.0335370145
## [146] -0.0700579039  0.0301055372  0.0584867819  0.0985379615  0.1342330808
## [151] -0.0895137229  0.0394968688  0.0794889391  0.0475745182  0.0939178919
## [156]  0.2222018503  0.2151288616 -0.1366237853 -0.0395405866 -0.3421946504
## [161]  0.0505105256 -0.0461258698 -0.2245520260 -0.1353449501 -0.0243309151
## [166]  0.0494387731  0.2592212803  0.1831187397 -0.0468867258 -0.1860486577
## [171] -0.0126551239 -0.1018335663 -0.0530010075 -0.0906451695 -0.1419572887
## [176] -0.0229304972  0.0102800575  0.1341830029  0.1726903745 -0.0705258566
## [181] -0.2121724587  0.0481329615  0.1450106447  0.2112886384  0.1512652218
## [186] -0.0391711456 -0.0096702342  0.1157633411 -0.0177455732 -0.3190859256
## [191] -0.3057149766 -0.0168976359 -0.5075281539 -0.0479484899  0.0269717931
## [196] -0.2017273028 -0.1142578292 -0.0051996459 -0.0219988430  0.1689331797
## [201]  0.2713319144 -0.1143349037 -0.0976531120  0.0399780608  0.0332650978
## [206] -0.0390547914 -0.1500206817 -0.1530965776 -0.1623025970  0.0002100134
## [211]  0.0134159295  0.0727653982  0.0372124223 -0.1611875497 -0.0477562194
## [216] -0.0396667459 -0.0307964992 -0.0798777991 -0.1210850981 -0.2114471876
## [221] -0.1076568361  0.1120286224  0.2866822949  0.1175786104 -0.2389954581
## [226]  0.0167705036 -0.0840159933 -0.3541077944  0.0246399650 -0.1609608182
## [231] -0.1310502272 -0.1017057153 -0.0601336695 -0.1251673469 -0.1736960312
## [236]  0.0629001279  0.2614049672 -0.2238601050 -0.0723017402  0.0752897806
## [241] -0.1497463206 -0.1027056268  0.0471549788  0.2272603646  0.1666063759
## [246]  0.1388831978  0.0506594863 -0.1995468110  0.0634218520  0.1017694437
## [251]  0.2318970062  0.2407113875  0.0064898049 -0.1060201889 -0.2962012825
## [256] -0.2274378262  0.5437325106  0.2347419275  0.1208001414 -0.3309733152
## [261] -0.0763531723 -0.0543912016 -0.0822532797 -0.2118520702  0.0864983093
## [266] -0.1325488218 -0.0648498634  0.0338733408  0.0921837409 -0.0762385041
## [271] -0.0167557975  0.0390814464  0.0694848853  0.0630032055  0.0691451175
## [276] -0.0562411424 -0.2434007522 -0.1974069978  0.1321494489  0.0227840007
## [281] -0.0043403465  0.0344036209  0.0943306360 -0.0329843986  0.0370464170
## [286] -0.0154968517 -0.0187363368  0.0379885438 -0.0466047577 -0.1042137070
## [291]  0.0693979999
```

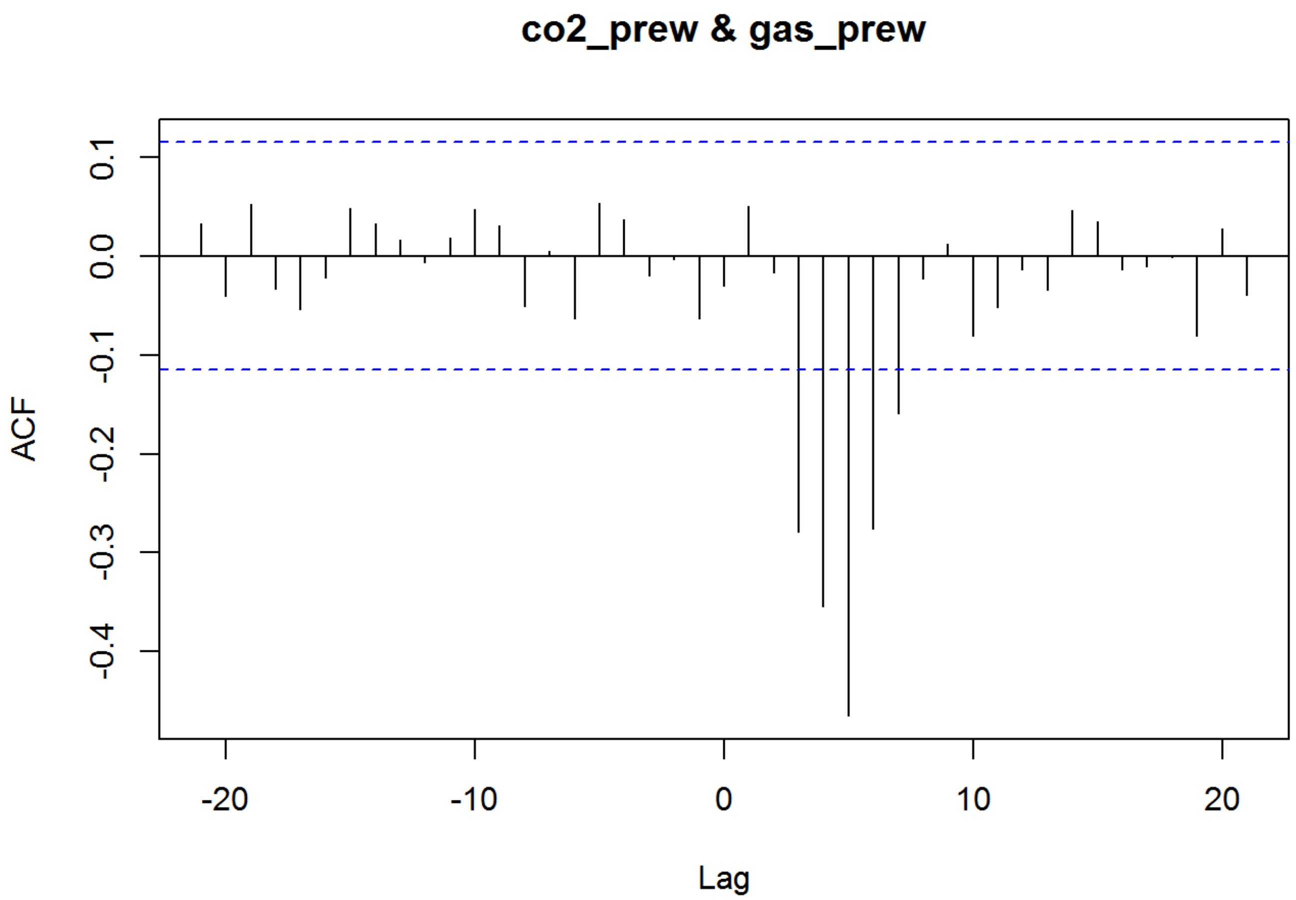
```
acf(gas_prew)
```




```
pacf(gas_prew)
```



```r
CCF = ccf(co2_prew, gas_prew) #retrieve cross-correlations
```



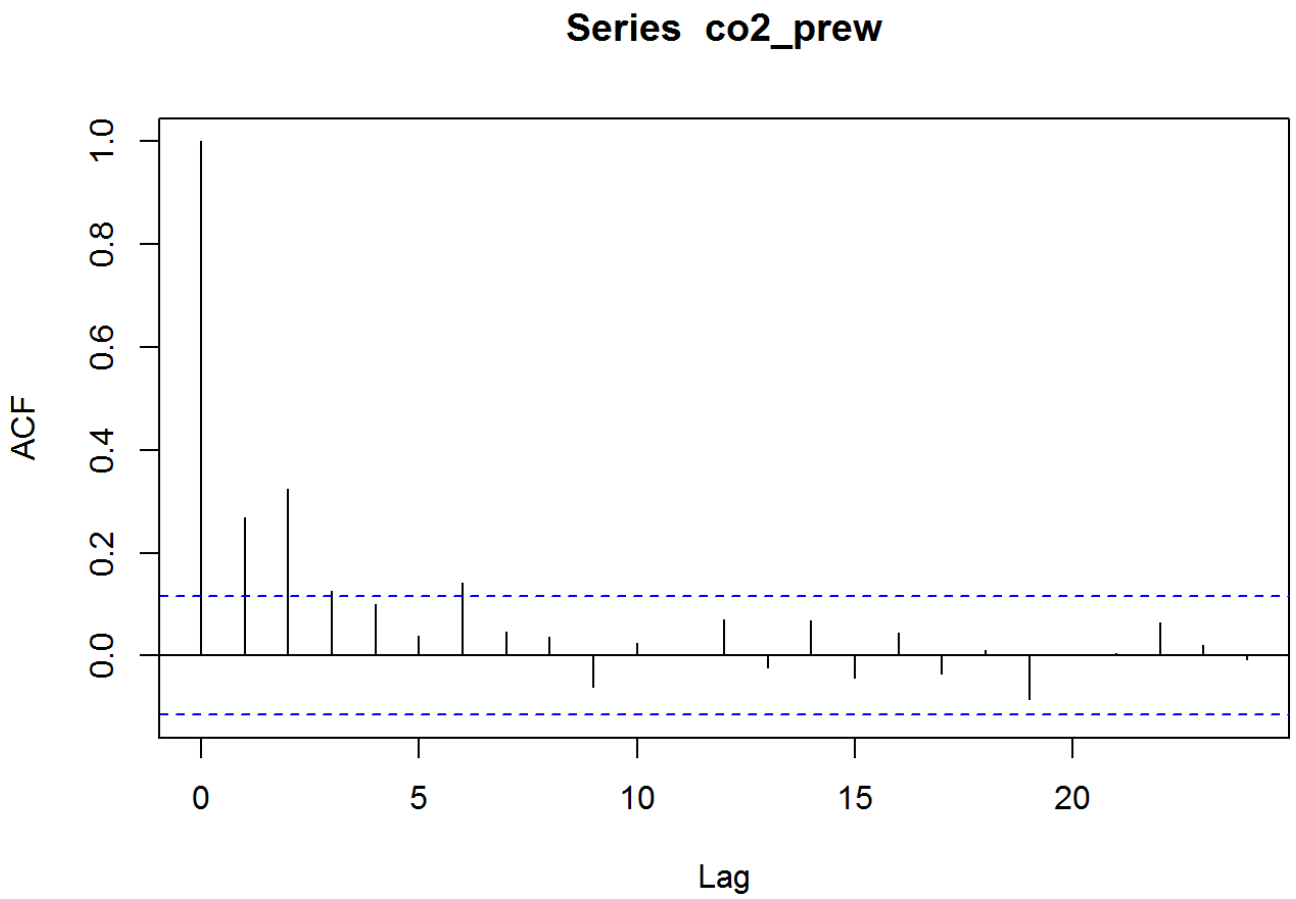

```r
CCF
```

```
##
## Autocorrelations of series 'X', by lag
##
##    -21    -20    -19    -18    -17    -16    -15    -14    -13    -12
##  0.033 -0.041  0.052 -0.034 -0.054 -0.022  0.048  0.033  0.016 -0.006
##    -11    -10     -9     -8     -7     -6     -5     -4     -3     -2
##  0.019  0.047  0.031 -0.051  0.005 -0.064  0.053  0.037 -0.019 -0.004
##     -1      0      1      2      3      4      5      6      7      8
## -0.063 -0.031  0.050 -0.017 -0.279 -0.355 -0.465 -0.276 -0.159 -0.023
##      9     10     11     12     13     14     15     16     17     18
##  0.012 -0.081 -0.051 -0.014 -0.035  0.046  0.035 -0.014 -0.011 -0.002
##     19     20     21
## -0.081  0.028 -0.039
```

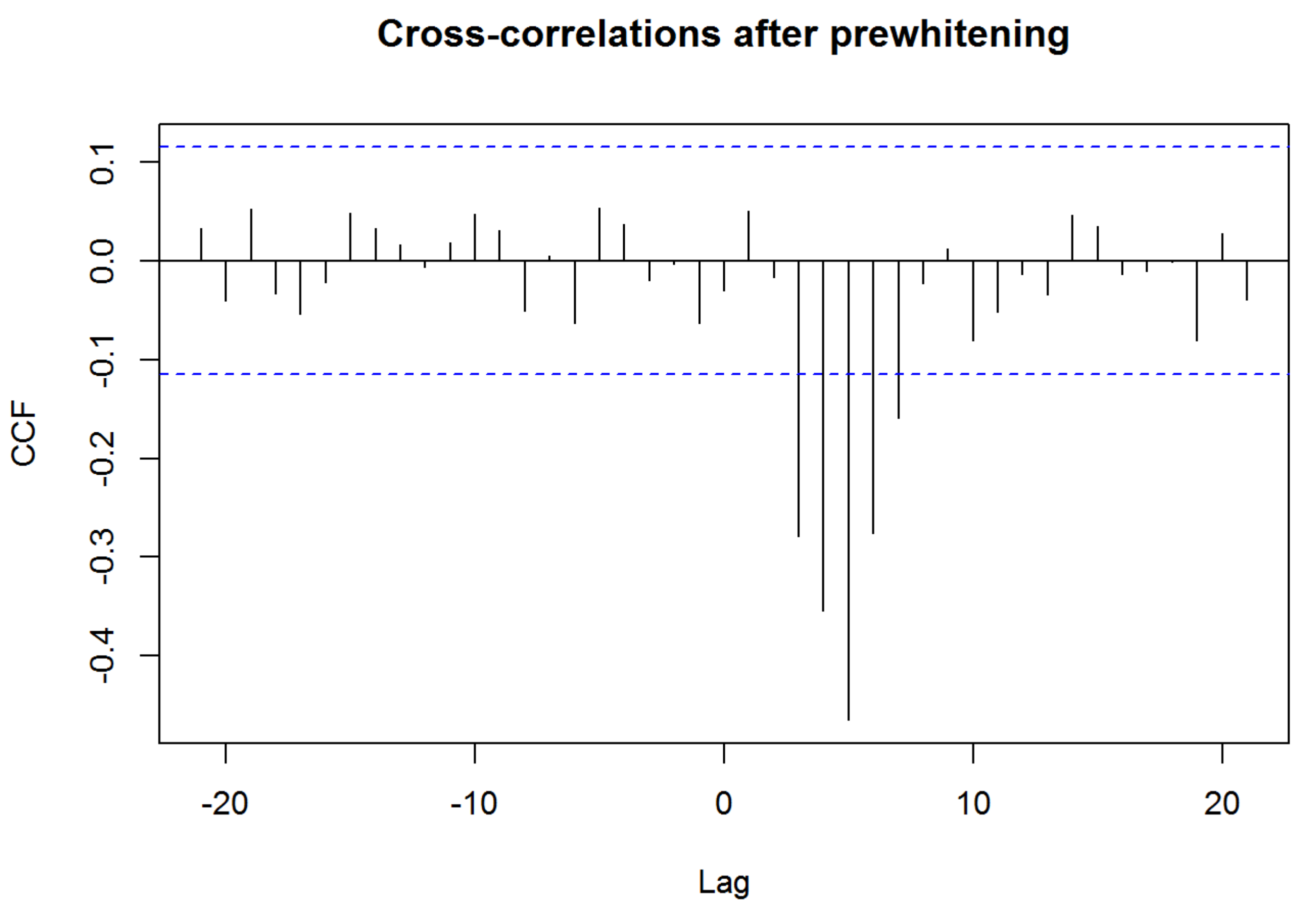
```r
vk = (sd(co2_prew)/sd(gas_prew))*CCF$acf # impulse response function
vk
```

```
## , , 1
##
##                [,1]
##  [1,]  0.062833717
##  [2,] -0.078255040
##  [3,]  0.100329919
##  [4,] -0.064428787
##  [5,] -0.104006915
##  [6,] -0.041340992
##  [7,]  0.091911876
##  [8,]  0.062346138
##  [9,]  0.031495922
## [10,] -0.012341757
## [11,]  0.035643819
## [12,]  0.089829141
## [13,]  0.059760643
## [14,] -0.097937123
## [15,]  0.009703520
## [16,] -0.121993125
## [17,]  0.102508151
## [18,]  0.070994317
## [19,] -0.037329530
## [20,] -0.006880635
## [21,] -0.121252162
## [22,] -0.058859089
## [23,]  0.096187920
## [24,] -0.033082812
## [25,] -0.535409457
## [26,] -0.680115942
## [27,] -0.891629032
## [28,] -0.529383614
## [29,] -0.304350174
## [30,] -0.044173839
## [31,]  0.022902659
## [32,] -0.154943668
## [33,] -0.098693730
## [34,] -0.026191128
## [35,] -0.066204971
## [36,]  0.088790262
## [37,]  0.066493047
## [38,] -0.026973540
## [39,] -0.020697075
## [40,] -0.003302980
## [41,] -0.154777233
## [42,]  0.053053561
## [43,] -0.075687688
```

```
ACF = acf(co2_prew)
```




```
plot(CCF, ylab = "CCF",main = "Cross-correlations after prewhitening")
```



```
#According to CCF plot, I identify that s = 4, b = 3
```

2. Fit Eqn. (1) based on your preliminary identification and R arima function

Type the mathematical form of your model here # We choose ARIMA(3,1,2) since it has the lowest aic. Also, its ccf shows that all of its values are within 95% CI. Therefore, its error terms behave like white noise. # Fit for the error terms \[(1-03796*B-0.6830*B^2+0.5802*B^3)*(1-B)*e_{t} = (1+0.2184*B-0.6421*B^2)*j_{t} \] \[(1-1.3796*B-0.3034*B^2+1.2632*B^3-0.5802*B^4)e_{t} = j_{t}+0.2184*j_{t-1}-0.6421*j_{t-2} \] \[e_t = 1.3796*e_{t-1}+0.3034*e_{t-2}-1.2632*e_{t-3}+0.5802*e_{t-4}+j_{t}+0.2184*j_{t-1}-0.6421*j_{t-2} \] # Therefore, our final equation is \[co2_t = -.5249*gas_{t-3}-0.6202*gas_{t-4}-0.8556*gas_{t-5}-0.4746*gas_{t-6}-0.3414*gas_{t-7}+1.3796*e_{t-1}+0.3034*e_{t-2}-1.2632*e_{t-3}+0.5802*e_{t-4}+j_{t}+0.2184*j_{t-1}-0.6421*j_{t-2} \] where \(j_{t}\) ~ N(0,0.05645)

```
# Lagging the gas

gas_lag3 = c(NA, NA, NA, gas[1:293])
gas_lag4 = c(NA, NA, NA,NA, gas[1:292])
gas_lag5 = c(NA, NA, NA,NA,NA, gas[1:291])
gas_lag6 = c(NA, NA, NA,NA,NA,NA, gas[1:290])
gas_lag7 = c(NA, NA, NA,NA,NA,NA,NA, gas[1:289])
gas_lag_master = cbind(gas_lag3,gas_lag4,gas_lag5,gas_lag6,gas_lag7)
colnames(gas_lag_master) = paste("gas_lag",3:7, sep = "")

gas_lag_master
```

```
##        gas_lag3 gas_lag4 gas_lag5 gas_lag6 gas_lag7
##   [1,]       NA       NA       NA       NA       NA
##   [2,]       NA       NA       NA       NA       NA
##   [3,]       NA       NA       NA       NA       NA
##   [4,]   -0.109       NA       NA       NA       NA
##   [5,]    0.000   -0.109       NA       NA       NA
##   [6,]    0.178    0.000   -0.109       NA       NA
##   [7,]    0.339    0.178    0.000   -0.109       NA
##   [8,]    0.373    0.339    0.178    0.000   -0.109
##   [9,]    0.441    0.373    0.339    0.178    0.000
##  [10,]    0.461    0.441    0.373    0.339    0.178
##  [11,]    0.348    0.461    0.441    0.373    0.339
##  [12,]    0.127    0.348    0.461    0.441    0.373
##  [13,]   -0.180    0.127    0.348    0.461    0.441
##  [14,]   -0.588   -0.180    0.127    0.348    0.461
##  [15,]   -1.055   -0.588   -0.180    0.127    0.348
##  [16,]   -1.421   -1.055   -0.588   -0.180    0.127
##  [17,]   -1.520   -1.421   -1.055   -0.588   -0.180
##  [18,]   -1.302   -1.520   -1.421   -1.055   -0.588
##  [19,]   -0.814   -1.302   -1.520   -1.421   -1.055
##  [20,]   -0.475   -0.814   -1.302   -1.520   -1.421
##  [21,]   -0.193   -0.475   -0.814   -1.302   -1.520
##  [22,]    0.088   -0.193   -0.475   -0.814   -1.302
##  [23,]    0.435    0.088   -0.193   -0.475   -0.814
##  [24,]    0.771    0.435    0.088   -0.193   -0.475
##  [25,]    0.866    0.771    0.435    0.088   -0.193
##  [26,]    0.875    0.866    0.771    0.435    0.088
##  [27,]    0.891    0.875    0.866    0.771    0.435
##  [28,]    0.987    0.891    0.875    0.866    0.771
##  [29,]    1.263    0.987    0.891    0.875    0.866
```

```
## [30,]  1.775  1.263  0.987  0.891  0.875
## [31,]  1.976  1.775  1.263  0.987  0.891
## [32,]  1.934  1.976  1.775  1.263  0.987
## [33,]  1.866  1.934  1.976  1.775  1.263
## [34,]  1.832  1.866  1.934  1.976  1.775
## [35,]  1.767  1.832  1.866  1.934  1.976
## [36,]  1.608  1.767  1.832  1.866  1.934
## [37,]  1.265  1.608  1.767  1.832  1.866
## [38,]  0.790  1.265  1.608  1.767  1.832
## [39,]  0.360  0.790  1.265  1.608  1.767
## [40,]  0.115  0.360  0.790  1.265  1.608
## [41,]  0.088  0.115  0.360  0.790  1.265
## [42,]  0.331  0.088  0.115  0.360  0.790
## [43,]  0.645  0.331  0.088  0.115  0.360
## [44,]  0.960  0.645  0.331  0.088  0.115
## [45,]  1.409  0.960  0.645  0.331  0.088
## [46,]  2.670  1.409  0.960  0.645  0.331
## [47,]  2.834  2.670  1.409  0.960  0.645
## [48,]  2.812  2.834  2.670  1.409  0.960
## [49,]  2.483  2.812  2.834  2.670  1.409
## [50,]  1.929  2.483  2.812  2.834  2.670
## [51,]  1.485  1.929  2.483  2.812  2.834
## [52,]  1.214  1.485  1.929  2.483  2.812
## [53,]  1.239  1.214  1.485  1.929  2.483
## [54,]  1.608  1.239  1.214  1.485  1.929
## [55,]  1.905  1.608  1.239  1.214  1.485
## [56,]  2.023  1.905  1.608  1.239  1.214
## [57,]  1.815  2.023  1.905  1.608  1.239
## [58,]  0.535  1.815  2.023  1.905  1.608
## [59,]  0.122  0.535  1.815  2.023  1.905
## [60,]  0.009  0.122  0.535  1.815  2.023
## [61,]  0.164  0.009  0.122  0.535  1.815
## [62,]  0.671  0.164  0.009  0.122  0.535
## [63,]  1.019  0.671  0.164  0.009  0.122
## [64,]  1.146  1.019  0.671  0.164  0.009
## [65,]  1.155  1.146  1.019  0.671  0.164
## [66,]  1.112  1.155  1.146  1.019  0.671
## [67,]  1.121  1.112  1.155  1.146  1.019
## [68,]  1.223  1.121  1.112  1.155  1.146
## [69,]  1.257  1.223  1.121  1.112  1.155
## [70,]  1.157  1.257  1.223  1.121  1.112
## [71,]  0.913  1.157  1.257  1.223  1.121
## [72,]  0.620  0.913  1.157  1.257  1.223
## [73,]  0.255  0.620  0.913  1.157  1.257
## [74,] -0.280  0.255  0.620  0.913  1.157
## [75,] -1.080 -0.280  0.255  0.620  0.913
## [76,] -1.551 -1.080 -0.280  0.255  0.620
## [77,] -1.799 -1.551 -1.080 -0.280  0.255
## [78,] -1.825 -1.799 -1.551 -1.080 -0.280
## [79,] -1.456 -1.825 -1.799 -1.551 -1.080
## [80,] -0.944 -1.456 -1.825 -1.799 -1.551
## [81,] -0.570 -0.944 -1.456 -1.825 -1.799
## [82,] -0.431 -0.570 -0.944 -1.456 -1.825
## [83,] -0.577 -0.431 -0.570 -0.944 -1.456
## [84,] -0.960 -0.577 -0.431 -0.570 -0.944
## [85,] -1.616 -0.960 -0.577 -0.431 -0.570
## [86,] -1.875 -1.616 -0.960 -0.577 -0.431
```

```
##  [87,]   -1.891   -1.875   -1.616   -0.960   -0.577
##  [88,]   -1.746   -1.891   -1.875   -1.616   -0.960
##  [89,]   -1.474   -1.746   -1.891   -1.875   -1.616
##  [90,]   -1.201   -1.474   -1.746   -1.891   -1.875
##  [91,]   -0.927   -1.201   -1.474   -1.746   -1.891
##  [92,]   -0.524   -0.927   -1.201   -1.474   -1.746
##  [93,]    0.040   -0.524   -0.927   -1.201   -1.474
##  [94,]    0.788    0.040   -0.524   -0.927   -1.201
##  [95,]    0.943    0.788    0.040   -0.524   -0.927
##  [96,]    0.930    0.943    0.788    0.040   -0.524
##  [97,]    1.006    0.930    0.943    0.788    0.040
##  [98,]    1.137    1.006    0.930    0.943    0.788
##  [99,]    1.198    1.137    1.006    0.930    0.943
## [100,]    1.054    1.198    1.137    1.006    0.930
## [101,]    0.595    1.054    1.198    1.137    1.006
## [102,]   -0.080    0.595    1.054    1.198    1.137
## [103,]   -0.314   -0.080    0.595    1.054    1.198
## [104,]   -0.288   -0.314   -0.080    0.595    1.054
## [105,]   -0.153   -0.288   -0.314   -0.080    0.595
## [106,]   -0.109   -0.153   -0.288   -0.314   -0.080
## [107,]   -0.187   -0.109   -0.153   -0.288   -0.314
## [108,]   -0.255   -0.187   -0.109   -0.153   -0.288
## [109,]   -0.229   -0.255   -0.187   -0.109   -0.153
## [110,]   -0.007   -0.229   -0.255   -0.187   -0.109
## [111,]    0.254   -0.007   -0.229   -0.255   -0.187
## [112,]    0.330    0.254   -0.007   -0.229   -0.255
## [113,]    0.102    0.330    0.254   -0.007   -0.229
## [114,]   -0.423    0.102    0.330    0.254   -0.007
## [115,]   -1.139   -0.423    0.102    0.330    0.254
## [116,]   -2.275   -1.139   -0.423    0.102    0.330
## [117,]   -2.594   -2.275   -1.139   -0.423    0.102
## [118,]   -2.716   -2.594   -2.275   -1.139   -0.423
## [119,]   -2.510   -2.716   -2.594   -2.275   -1.139
## [120,]   -1.790   -2.510   -2.716   -2.594   -2.275
## [121,]   -1.346   -1.790   -2.510   -2.716   -2.594
## [122,]   -1.081   -1.346   -1.790   -2.510   -2.716
## [123,]   -0.910   -1.081   -1.346   -1.790   -2.510
## [124,]   -0.876   -0.910   -1.081   -1.346   -1.790
## [125,]   -0.885   -0.876   -0.910   -1.081   -1.346
## [126,]   -0.800   -0.885   -0.876   -0.910   -1.081
## [127,]   -0.544   -0.800   -0.885   -0.876   -0.910
## [128,]   -0.416   -0.544   -0.800   -0.885   -0.876
## [129,]   -0.271   -0.416   -0.544   -0.800   -0.885
## [130,]    0.000   -0.271   -0.416   -0.544   -0.800
## [131,]    0.403    0.000   -0.271   -0.416   -0.544
## [132,]    0.841    0.403    0.000   -0.271   -0.416
## [133,]    1.285    0.841    0.403    0.000   -0.271
## [134,]    1.607    1.285    0.841    0.403    0.000
## [135,]    1.746    1.607    1.285    0.841    0.403
## [136,]    1.683    1.746    1.607    1.285    0.841
## [137,]    1.485    1.683    1.746    1.607    1.285
## [138,]    0.993    1.485    1.683    1.746    1.607
## [139,]    0.648    0.993    1.485    1.683    1.746
## [140,]    0.577    0.648    0.993    1.485    1.683
## [141,]    0.577    0.577    0.648    0.993    1.485
## [142,]    0.632    0.577    0.577    0.648    0.993
## [143,]    0.747    0.632    0.577    0.577    0.648
```

```
## [143,]    0.747    0.632    0.577    0.577    0.648
## [144,]    0.900    0.747    0.632    0.577    0.577
## [145,]    0.993    0.900    0.747    0.632    0.577
## [146,]    0.968    0.993    0.900    0.747    0.632
## [147,]    0.790    0.968    0.993    0.900    0.747
## [148,]    0.399    0.790    0.968    0.993    0.900
## [149,]   -0.161    0.399    0.790    0.968    0.993
## [150,]   -0.553   -0.161    0.399    0.790    0.968
## [151,]   -0.603   -0.553   -0.161    0.399    0.790
## [152,]   -0.424   -0.603   -0.553   -0.161    0.399
## [153,]   -0.194   -0.424   -0.603   -0.553   -0.161
## [154,]   -0.049   -0.194   -0.424   -0.603   -0.553
## [155,]    0.060   -0.049   -0.194   -0.424   -0.603
## [156,]    0.161    0.060   -0.049   -0.194   -0.424
## [157,]    0.301    0.161    0.060   -0.049   -0.194
## [158,]    0.517    0.301    0.161    0.060   -0.049
## [159,]    0.566    0.517    0.301    0.161    0.060
## [160,]    0.560    0.566    0.517    0.301    0.161
## [161,]    0.573    0.560    0.566    0.517    0.301
## [162,]    0.592    0.573    0.560    0.566    0.517
## [163,]    0.671    0.592    0.573    0.560    0.566
## [164,]    0.933    0.671    0.592    0.573    0.560
## [165,]    1.337    0.933    0.671    0.592    0.573
## [166,]    1.460    1.337    0.933    0.671    0.592
## [167,]    1.353    1.460    1.337    0.933    0.671
## [168,]    0.772    1.353    1.460    1.337    0.933
## [169,]    0.218    0.772    1.353    1.460    1.337
## [170,]   -0.237    0.218    0.772    1.353    1.460
## [171,]   -0.714   -0.237    0.218    0.772    1.353
## [172,]   -1.099   -0.714   -0.237    0.218    0.772
## [173,]   -1.269   -1.099   -0.714   -0.237    0.218
## [174,]   -1.175   -1.269   -1.099   -0.714   -0.237
## [175,]   -0.676   -1.175   -1.269   -1.099   -0.714
## [176,]    0.033   -0.676   -1.175   -1.269   -1.099
## [177,]    0.556    0.033   -0.676   -1.175   -1.269
## [178,]    0.643    0.556    0.033   -0.676   -1.175
## [179,]    0.484    0.643    0.556    0.033   -0.676
## [180,]    0.109    0.484    0.643    0.556    0.033
## [181,]   -0.310    0.109    0.484    0.643    0.556
## [182,]   -0.697   -0.310    0.109    0.484    0.643
## [183,]   -1.047   -0.697   -0.310    0.109    0.484
## [184,]   -1.218   -1.047   -0.697   -0.310    0.109
## [185,]   -1.183   -1.218   -1.047   -0.697   -0.310
## [186,]   -0.873   -1.183   -1.218   -1.047   -0.697
## [187,]   -0.336   -0.873   -1.183   -1.218   -1.047
## [188,]    0.063   -0.336   -0.873   -1.183   -1.218
## [189,]    0.084    0.063   -0.336   -0.873   -1.183
## [190,]    0.000    0.084    0.063   -0.336   -0.873
## [191,]    0.001    0.000    0.084    0.063   -0.336
## [192,]    0.209    0.001    0.000    0.084    0.063
## [193,]    0.556    0.209    0.001    0.000    0.084
## [194,]    0.782    0.556    0.209    0.001    0.000
## [195,]    0.858    0.782    0.556    0.209    0.001
## [196,]    0.918    0.858    0.782    0.556    0.209
## [197,]    0.862    0.918    0.858    0.782    0.556
## [198,]    0.416    0.862    0.918    0.858    0.782
## [199,]   -0.336    0.416    0.862    0.918    0.858
```

```
## [200,]   -0.959   -0.336    0.416    0.862    0.918
## [201,]   -1.813   -0.959   -0.336    0.416    0.862
## [202,]   -2.378   -1.813   -0.959   -0.336    0.416
## [203,]   -2.499   -2.378   -1.813   -0.959   -0.336
## [204,]   -2.473   -2.499   -2.378   -1.813   -0.959
## [205,]   -2.330   -2.473   -2.499   -2.378   -1.813
## [206,]   -2.053   -2.330   -2.473   -2.499   -2.378
## [207,]   -1.739   -2.053   -2.330   -2.473   -2.499
## [208,]   -1.261   -1.739   -2.053   -2.330   -2.473
## [209,]   -0.569   -1.261   -1.739   -2.053   -2.330
## [210,]   -0.137   -0.569   -1.261   -1.739   -2.053
## [211,]   -0.024   -0.137   -0.569   -1.261   -1.739
## [212,]   -0.050   -0.024   -0.137   -0.569   -1.261
## [213,]   -0.135   -0.050   -0.024   -0.137   -0.569
## [214,]   -0.276   -0.135   -0.050   -0.024   -0.137
## [215,]   -0.534   -0.276   -0.135   -0.050   -0.024
## [216,]   -0.871   -0.534   -0.276   -0.135   -0.050
## [217,]   -1.243   -0.871   -0.534   -0.276   -0.135
## [218,]   -1.439   -1.243   -0.871   -0.534   -0.276
## [219,]   -1.422   -1.439   -1.243   -0.871   -0.534
## [220,]   -1.175   -1.422   -1.439   -1.243   -0.871
## [221,]   -0.813   -1.175   -1.422   -1.439   -1.243
## [222,]   -0.634   -0.813   -1.175   -1.422   -1.439
## [223,]   -0.582   -0.634   -0.813   -1.175   -1.422
## [224,]   -0.625   -0.582   -0.634   -0.813   -1.175
## [225,]   -0.713   -0.625   -0.582   -0.634   -0.813
## [226,]   -0.848   -0.713   -0.625   -0.582   -0.634
## [227,]   -1.039   -0.848   -0.713   -0.625   -0.582
## [228,]   -1.346   -1.039   -0.848   -0.713   -0.625
## [229,]   -1.628   -1.346   -1.039   -0.848   -0.713
## [230,]   -1.619   -1.628   -1.346   -1.039   -0.848
## [231,]   -1.149   -1.619   -1.628   -1.346   -1.039
## [232,]   -0.488   -1.149   -1.619   -1.628   -1.346
## [233,]   -0.160   -0.488   -1.149   -1.619   -1.628
## [234,]   -0.007   -0.160   -0.488   -1.149   -1.619
## [235,]   -0.092   -0.007   -0.160   -0.488   -1.149
## [236,]   -0.620   -0.092   -0.007   -0.160   -0.488
## [237,]   -1.086   -0.620   -0.092   -0.007   -0.160
## [238,]   -1.525   -1.086   -0.620   -0.092   -0.007
## [239,]   -1.858   -1.525   -1.086   -0.620   -0.092
## [240,]   -2.029   -1.858   -1.525   -1.086   -0.620
## [241,]   -2.024   -2.029   -1.858   -1.525   -1.086
## [242,]   -1.961   -2.024   -2.029   -1.858   -1.525
## [243,]   -1.952   -1.961   -2.024   -2.029   -1.858
## [244,]   -1.794   -1.952   -1.961   -2.024   -2.029
## [245,]   -1.302   -1.794   -1.952   -1.961   -2.024
## [246,]   -1.030   -1.302   -1.794   -1.952   -1.961
## [247,]   -0.918   -1.030   -1.302   -1.794   -1.952
## [248,]   -0.798   -0.918   -1.030   -1.302   -1.794
## [249,]   -0.867   -0.798   -0.918   -1.030   -1.302
## [250,]   -1.047   -0.867   -0.798   -0.918   -1.030
## [251,]   -1.123   -1.047   -0.867   -0.798   -0.918
## [252,]   -0.876   -1.123   -1.047   -0.867   -0.798
## [253,]   -0.395   -0.876   -1.123   -1.047   -0.867
## [254,]    0.185   -0.395   -0.876   -1.123   -1.047
## [255,]    0.662    0.185   -0.395   -0.876   -1.123
## [256,]    0.709    0.662    0.185   -0.395   -0.876
```

```
## [257,]   0.605   0.709   0.662   0.185  -0.395
## [258,]   0.501   0.605   0.709   0.662   0.185
## [259,]   0.603   0.501   0.605   0.709   0.662
## [260,]   0.943   0.603   0.501   0.605   0.709
## [261,]   1.223   0.943   0.603   0.501   0.605
## [262,]   1.249   1.223   0.943   0.603   0.501
## [263,]   0.824   1.249   1.223   0.943   0.603
## [264,]   0.102   0.824   1.249   1.223   0.943
## [265,]   0.025   0.102   0.824   1.249   1.223
## [266,]   0.382   0.025   0.102   0.824   1.249
## [267,]   0.922   0.382   0.025   0.102   0.824
## [268,]   1.032   0.922   0.382   0.025   0.102
## [269,]   0.866   1.032   0.922   0.382   0.025
## [270,]   0.527   0.866   1.032   0.922   0.382
## [271,]   0.093   0.527   0.866   1.032   0.922
## [272,]  -0.458   0.093   0.527   0.866   1.032
## [273,]  -0.748  -0.458   0.093   0.527   0.866
## [274,]  -0.947  -0.748  -0.458   0.093   0.527
## [275,]  -1.029  -0.947  -0.748  -0.458   0.093
## [276,]  -0.928  -1.029  -0.947  -0.748  -0.458
## [277,]  -0.645  -0.928  -1.029  -0.947  -0.748
## [278,]  -0.424  -0.645  -0.928  -1.029  -0.947
## [279,]  -0.276  -0.424  -0.645  -0.928  -1.029
## [280,]  -0.158  -0.276  -0.424  -0.645  -0.928
## [281,]  -0.033  -0.158  -0.276  -0.424  -0.645
## [282,]   0.102  -0.033  -0.158  -0.276  -0.424
## [283,]   0.251   0.102  -0.033  -0.158  -0.276
## [284,]   0.280   0.251   0.102  -0.033  -0.158
## [285,]   0.000   0.280   0.251   0.102  -0.033
## [286,]  -0.493   0.000   0.280   0.251   0.102
## [287,]  -0.759  -0.493   0.000   0.280   0.251
## [288,]  -0.824  -0.759  -0.493   0.000   0.280
## [289,]  -0.740  -0.824  -0.759  -0.493   0.000
## [290,]  -0.528  -0.740  -0.824  -0.759  -0.493
## [291,]  -0.204  -0.528  -0.740  -0.824  -0.759
## [292,]   0.034  -0.204  -0.528  -0.740  -0.824
## [293,]   0.204   0.034  -0.204  -0.528  -0.740
## [294,]   0.253   0.204   0.034  -0.204  -0.528
## [295,]   0.195   0.253   0.204   0.034  -0.204
## [296,]   0.131   0.195   0.253   0.204   0.034
```

```r
# Need to choose optimal model with lowest abs(AIC)

# Run for loop to find the model with optimal p,d,q

aic_c = c()
p_c = c()
d_c = c()
q_c = c()
for (p in 1:5){
  for (d in 0:1){
    for (q in 1:5){
      p_c = c(p_c, p)
      d_c = c(d_c, d)
      q_c = c(q_c, q)
      fit_reg = arima(co2, xreg = gas_lag_master, order = c(p,d,q))
      print(fit_reg)
      print(c(p,d,q))
      aic_c = c(aic_c, fit_reg$aic)
    }
  }
}
```

```
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ma1  intercept  gas_lag3  gas_lag4  gas_lag5  gas_lag6
##       0.9420  0.4835    53.5348   -0.4941   -0.6279   -0.8272   -0.4748
## s.e.  0.0241  0.0410     0.3729    0.0805    0.0890    0.0891    0.0890
##       gas_lag7
##        -0.3422
## s.e.    0.0806
##
## sigma^2 estimated as 0.06593:  log likelihood = -18.75,  aic = 55.5
## [1] 1 0 1
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ma1     ma2  intercept  gas_lag3  gas_lag4  gas_lag5
##       0.8988  0.6244  0.3273    53.4597   -0.5257   -0.6008   -0.8580
## s.e.  0.0318  0.0582  0.0523     0.2690    0.0773    0.0793    0.0789
##       gas_lag6  gas_lag7
##        -0.4610   -0.3557
## s.e.    0.0791    0.0773
##
## sigma^2 estimated as 0.05884:  log likelihood = -2.38,  aic = 24.76
## [1] 1 0 2
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ma1     ma2     ma3  intercept  gas_lag3  gas_lag4
```

```
##          ar1     ma1     ma2     ma3  intercept  gas_lag3  gas_lag4
##       0.8505  0.6804  0.4807  0.2036    53.4228   -0.5379   -0.6235
## s.e.  0.0411  0.0618  0.0682  0.0672     0.2186    0.0763    0.0810
##       gas_lag5  gas_lag6  gas_lag7
##        -0.8405   -0.4814   -0.3554
## s.e.    0.0807    0.0808    0.0764
##
## sigma^2 estimated as 0.05714:  log likelihood = 1.85,  aic = 18.31
## [1] 1 0 3
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ma1     ma2     ma3     ma4  intercept  gas_lag3
##       0.7540  0.8142  0.6902  0.4329  0.1867    53.3935   -0.5540
## s.e.  0.0679  0.0860  0.1152  0.1159  0.0772     0.1748    0.0758
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6387   -0.8613   -0.4765   -0.3696
## s.e.    0.0801    0.0800    0.0798    0.0764
##
## sigma^2 estimated as 0.05601:  log likelihood = 4.69,  aic = 14.63
## [1] 1 0 4
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ma1     ma2     ma3     ma4      ma5  intercept  gas_lag3
##       0.8363  0.7175  0.5405  0.2721  0.0454  -0.1016    53.4117   -0.5405
## s.e.  0.0855  0.1111  0.1599  0.1698  0.1421   0.0919     0.2082    0.0770
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6319    -0.848   -0.4721   -0.3633
## s.e.    0.0805     0.081    0.0801    0.0764
##
## sigma^2 estimated as 0.05576:  log likelihood = 5.31,  aic = 15.38
## [1] 1 0 5
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ma1  gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##       0.4421  0.1655   -0.4953   -0.6172   -0.8378   -0.4655   -0.3194
## s.e.  0.0789  0.0781    0.0765    0.0800    0.0799    0.0800    0.0766
##
## sigma^2 estimated as 0.06258:  log likelihood = -9.25,  aic = 34.5
## [1] 1 1 1
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ma1     ma2  gas_lag3  gas_lag4  gas_lag5  gas_lag6
##       0.2158  0.3962  0.2276   -0.5088   -0.6046   -0.8445   -0.4606
## s.e.  0.1399  0.1355  0.0749    0.0759    0.0784    0.0781    0.0782
##       gas_lag7
##        -0.3305
```

```
##       -0.3305
## s.e.    0.0758
##
## sigma^2 estimated as 0.06096:  log likelihood = -5.51,  aic = 29.01
## [1] 1 1 2
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ma1     ma2     ma3  gas_lag3  gas_lag4  gas_lag5
##       0.0021  0.6142  0.3847  0.1220   -0.5102   -0.6102   -0.8359
## s.e.  0.2417  0.2344  0.1414  0.0929    0.0756    0.0791    0.0792
##       gas_lag6  gas_lag7
##        -0.4661   -0.3288
## s.e.    0.0789    0.0755
##
## sigma^2 estimated as 0.06069:  log likelihood = -4.89,  aic = 29.79
## [1] 1 1 3
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ma1      ma2      ma3      ma4  gas_lag3  gas_lag4
##       0.8220  -0.2902  -0.1841  -0.2684  -0.2031   -0.5311   -0.6202
## s.e.  0.0516   0.0677   0.0629   0.0549   0.0659    0.0758    0.0810
##       gas_lag5  gas_lag6  gas_lag7
##        -0.8372   -0.4798   -0.3505
## s.e.    0.0807    0.0808    0.0760
##
## sigma^2 estimated as 0.05749:  log likelihood = 2.47,  aic = 17.05
## [1] 1 1 4
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ma1      ma2      ma3      ma4      ma5  gas_lag3
##       0.6836  -0.1159  -0.0850  -0.2355  -0.2464  -0.2021   -0.5409
## s.e.  0.1087   0.1194   0.0781   0.0596   0.0638   0.0763    0.0753
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6317   -0.8541   -0.4709   -0.3620
## s.e.    0.0799    0.0798    0.0797    0.0755
##
## sigma^2 estimated as 0.05614:  log likelihood = 5.8,  aic = 12.4
## [1] 1 1 5
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2     ma1  intercept  gas_lag3  gas_lag4  gas_lag5
##       1.5004  -0.5910  0.0586    53.3849   -0.5491   -0.6415   -0.8568
## s.e.  0.0727   0.0761  0.0810     0.1654    0.0783    0.0805    0.0805
##       gas_lag6  gas_lag7
##        -0.4835   -0.3591
```

```
## s.e.    0.0803    0.0776
##
## sigma^2 estimated as 0.05812:  log likelihood = -0.56,  aic = 21.12
## [1] 2 0 1
```

```
## Warning in arima(co2, xreg = gas_lag_master, order = c(p, d, q)): possible
## convergence problem: optim gave code = 1
```

```
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2     ma1      ma2  intercept  gas_lag3  gas_lag4
##       1.4898  -0.5782  0.0813  -0.0055    53.3937   -0.5475   -0.6374
## s.e.  0.1100   0.1057  0.1159   0.0997     0.1727    0.0785    0.0805
##       gas_lag5  gas_lag6  gas_lag7
##        -0.8556   -0.4866   -0.3524
## s.e.    0.0803    0.0801    0.0777
##
## sigma^2 estimated as 0.0582:  log likelihood = -0.77,  aic = 23.53
## [1] 2 0 2
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2     ma1     ma2     ma3  intercept  gas_lag3
##       1.1479  -0.2900  0.4012  0.3452  0.1427    53.3998   -0.5505
## s.e.  0.1754   0.1627  0.1710  0.1178  0.0933     0.1844    0.0768
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6346   -0.8517   -0.4834   -0.3622
## s.e.    0.0811    0.0809    0.0805    0.0767
##
## sigma^2 estimated as 0.05665:  log likelihood = 3.09,  aic = 17.81
## [1] 2 0 3
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2     ma1     ma2     ma3     ma4  intercept  gas_lag3
##       0.4797  0.2445  1.0814  0.8629  0.5588  0.2430    53.4019   -0.5484
## s.e.  0.3414  0.3078  0.3332  0.2236  0.1738  0.0859     0.1866    0.0760
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6372   -0.8529   -0.4770   -0.3654
## s.e.    0.0801    0.0810    0.0801    0.0766
##
## sigma^2 estimated as 0.05587:  log likelihood = 5.05,  aic = 15.91
## [1] 2 0 4
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2     ma1     ma2      ma3      ma4      ma5
```

```
##       1.3604  -0.4023  0.1934  0.1258  -0.0779  -0.1739  -0.1975
## s.e.  0.2670   0.2304  0.2643  0.1884   0.1516   0.0966   0.0687
##       intercept  gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##         53.4772   -0.5315   -0.6264   -0.8466   -0.4665   -0.3594
## s.e.     0.2947    0.0758    0.0800    0.0800    0.0798    0.0755
##
## sigma^2 estimated as 0.05549:  log likelihood = 5.95,  aic = 16.09
## [1] 2 0 5
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2      ma1  gas_lag3  gas_lag4  gas_lag5  gas_lag6
##       1.3432  -0.5871  -0.7391   -0.5198   -0.6269   -0.8499   -0.4784
## s.e.  0.0876   0.0538   0.0945    0.0747    0.0784    0.0785    0.0783
##       gas_lag7
##        -0.3386
## s.e.    0.0746
##
## sigma^2 estimated as 0.05776:  log likelihood = 2.11,  aic = 13.78
## [1] 2 1 1
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2      ma1     ma2  gas_lag3  gas_lag4  gas_lag5
##       1.3684  -0.6682  -0.7999  0.1512   -0.5204   -0.6278   -0.8518
## s.e.  0.0901   0.0673   0.1116  0.0849    0.0740    0.0792    0.0789
##       gas_lag6  gas_lag7
##        -0.4774   -0.3405
## s.e.    0.0789    0.0739
##
## sigma^2 estimated as 0.0572:  log likelihood = 3.5,  aic = 13.01
## [1] 2 1 2
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2      ma1     ma2      ma3  gas_lag3  gas_lag4
##       1.3017  -0.6000  -0.7382  0.1598  -0.0816   -0.5252   -0.6217
## s.e.  0.1132   0.1022   0.1231  0.0962   0.0840    0.0743    0.0796
##       gas_lag5  gas_lag6  gas_lag7
##        -0.8510   -0.4747   -0.3468
## s.e.    0.0788    0.0791    0.0743
##
## sigma^2 estimated as 0.05703:  log likelihood = 3.92,  aic = 14.16
## [1] 2 1 3
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2      ma1      ma2      ma3      ma4  gas_lag3
```

```
##       1.1467  -0.3197  -0.5959  -0.0250  -0.1859  -0.1370   -0.5410
## s.e.  0.1738   0.1579   0.1698   0.0983   0.0686   0.0927    0.0758
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6305   -0.8474   -0.4803   -0.3553
## s.e.    0.0809    0.0806    0.0804    0.0758
##
## sigma^2 estimated as 0.05684:  log likelihood = 4.04,  aic = 15.92
## [1] 2 1 4
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2     ma1      ma2      ma3      ma4      ma5  gas_lag3
##       0.5001  0.1678  0.0622  -0.1524  -0.2682  -0.2929  -0.2381   -0.5384
## s.e.  0.3215  0.2864  0.3143   0.1392   0.0817   0.0991   0.0855    0.0753
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6315   -0.8492    -0.472   -0.3596
## s.e.    0.0800    0.0805     0.080    0.0757
##
## sigma^2 estimated as 0.05606:  log likelihood = 5.99,  aic = 14.02
## [1] 2 1 5
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ma1  intercept  gas_lag3  gas_lag4
##       0.5899  0.8547  -0.6273  0.9359    53.3716   -0.5646   -0.6407
## s.e.  0.0589  0.0554   0.0526  0.0441     0.1490    0.0768    0.0800
##       gas_lag5  gas_lag6  gas_lag7
##         -0.868   -0.4803   -0.3681
## s.e.     0.080    0.0799    0.0763
##
## sigma^2 estimated as 0.05738:  log likelihood = 1.24,  aic = 19.53
## [1] 3 0 1
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ma1     ma2  intercept  gas_lag3
##       0.5141  0.8477  -0.5415  1.0596  0.1337    53.3870   -0.5495
## s.e.  0.0832  0.0597   0.0808  0.0928  0.0843     0.1705    0.0770
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6338   -0.8613   -0.4803   -0.3580
## s.e.    0.0780    0.0780    0.0778    0.0764
##
## sigma^2 estimated as 0.05687:  log likelihood = 2.5,  aic = 19.01
## [1] 3 0 2
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ma1     ma2     ma3  intercept  gas_lag3
##       0.4041  0.8509  -0.4589  1.1606  0.3464  0.1293    53.3940   -0.5530
```

```
## s.e.  0.1197  0.0528   0.1085  0.1219  0.1708  0.0862     0.1797    0.0768
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6248   -0.8653   -0.4747   -0.3633
## s.e.    0.0786    0.0781    0.0781    0.0763
##
## sigma^2 estimated as 0.05647:  log likelihood = 3.52,  aic = 18.97
## [1] 3 0 3
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2     ar3     ma1     ma2     ma3     ma4  intercept
##       1.4036  -1.1181  0.5664  0.1483  0.7734  0.3757  0.1867    53.4381
## s.e.  0.2412   0.4522  0.2617  0.2532  0.1557  0.0985  0.1025     0.2301
##       gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.5472   -0.6174   -0.8399   -0.4602   -0.3758
## s.e.    0.0773    0.0824    0.0816    0.0797    0.0787
##
## sigma^2 estimated as 0.05556:  log likelihood = 5.74,  aic = 16.52
## [1] 3 0 4
```

```
## Warning in arima(co2, xreg = gas_lag_master, order = c(p, d, q)): possible
## convergence problem: optim gave code = 1
```

```
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2     ar3     ma1     ma2      ma3      ma4      ma5
##       1.4236  -0.5943  0.1211  0.1317  0.2194  -0.0221  -0.1264  -0.1729
## s.e.  0.3356   0.5145  0.2791  0.3332  0.3047   0.2189   0.1575   0.0945
##       intercept  gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##         53.4823   -0.5306   -0.6239   -0.8473   -0.4636   -0.3605
## s.e.     0.2965    0.0757    0.0801    0.0798    0.0799    0.0756
##
## sigma^2 estimated as 0.05546:  log likelihood = 6.02,  aic = 17.96
## [1] 3 0 5
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2      ar3      ma1  gas_lag3  gas_lag4  gas_lag5
##       1.1194  -0.3477  -0.1477  -0.5575   -0.5224   -0.6245   -0.8523
## s.e.  0.1698   0.1440   0.0779   0.1673    0.0739    0.0795    0.0789
##       gas_lag6  gas_lag7
##        -0.4756   -0.3439
## s.e.    0.0790    0.0741
##
## sigma^2 estimated as 0.05709:  log likelihood = 3.78,  aic = 12.44
## [1] 3 1 1
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
```

```
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ma1      ma2  gas_lag3  gas_lag4
##       0.3796  0.6830  -0.5802  0.2184  -0.6421   -0.5249   -0.6202
## s.e.  0.0947  0.0741   0.0557  0.1054   0.1013    0.0733    0.0767
##       gas_lag5  gas_lag6  gas_lag7
##        -0.8556   -0.4746   -0.3414
## s.e.    0.0764    0.0765    0.0732
##
## sigma^2 estimated as 0.05645:  log likelihood = 5.33,  aic = 11.35
## [1] 3 1 2
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1    ar2      ar3     ma1      ma2     ma3  gas_lag3  gas_lag4
##       0.3891  0.659  -0.6238  0.1891  -0.6006  0.0820   -0.5253   -0.6207
## s.e.  0.0955  0.076   0.0727  0.1151   0.1135  0.0923    0.0731    0.0774
##       gas_lag5  gas_lag6  gas_lag7
##        -0.8564   -0.4738   -0.3431
## s.e.    0.0770    0.0771    0.0731
##
## sigma^2 estimated as 0.05631:  log likelihood = 5.69,  aic = 12.61
## [1] 3 1 3
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ma1      ma2     ma3     ma4  gas_lag3
##       0.3892  0.6589  -0.6238  0.1890  -0.6005  0.0821  0.0001   -0.5253
## s.e.  0.1317  0.0766   0.1067  0.1422   0.1396  0.1215  0.1037    0.0733
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6208   -0.8564   -0.4737   -0.3430
## s.e.    0.0778    0.0770    0.0772    0.0736
##
## sigma^2 estimated as 0.05631:  log likelihood = 5.69,  aic = 14.61
## [1] 3 1 4
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ma1      ma2      ma3      ma4      ma5
##       0.0658  0.6042  -0.3481  0.5136  -0.3455  -0.0289  -0.2341  -0.1999
## s.e.  0.1701  0.1465   0.1701  0.1679   0.2010   0.2058   0.1396   0.0944
##       gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.5257   -0.6287   -0.8335   -0.4824   -0.3468
## s.e.    0.0736    0.0785    0.0786    0.0782    0.0735
##
## sigma^2 estimated as 0.05579:  log likelihood = 6.96,  aic = 14.09
## [1] 3 1 5
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
```

```
##
## Coefficients:
##          ar1     ar2      ar3     ar4     ma1  intercept  gas_lag3
##       0.6813  0.7274  -0.6752  0.1238  0.9082    53.3968   -0.5432
## s.e.  0.0820  0.0988   0.0603  0.0650  0.0600     0.1865    0.0771
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6283   -0.8589   -0.4789   -0.3549
## s.e.    0.0776    0.0774    0.0771    0.0762
##
## sigma^2 estimated as 0.05666:  log likelihood = 3,  aic = 17.99
## [1] 4 0 1
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ar4     ma1      ma2  intercept
##       1.2885  0.3572  -1.1923  0.5165  0.2956  -0.5707    53.5163
## s.e.  0.1412  0.1185   0.1377  0.0853  0.1498   0.1417     0.3422
##       gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.5300   -0.6226   -0.8569   -0.4755   -0.3448
## s.e.    0.0745    0.0770    0.0768    0.0767    0.0743
##
## sigma^2 estimated as 0.05564:  log likelihood = 5.52,  aic = 14.97
## [1] 4 0 2
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ar4     ma1      ma2     ma3  intercept
##       1.2507  0.3430  -1.1727  0.5387  0.3147  -0.4711  0.0951    53.5031
## s.e.  0.2135  0.1335   0.1908  0.1161  0.2230   0.2547  0.1044     0.3309
##       gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.5300   -0.6213   -0.8571   -0.4749   -0.3491
## s.e.    0.0745    0.0779    0.0774    0.0774    0.0748
##
## sigma^2 estimated as 0.05547:  log likelihood = 5.95,  aic = 16.09
## [1] 4 0 3
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ar4     ma1     ma2     ma3     ma4
##       0.4876  0.4386  -0.5147  0.3683  1.0801  0.6428  0.7419  0.2797
## s.e.  0.2285  0.1749   0.1951  0.1743  0.2290  0.2492  0.1692  0.0912
##       intercept  gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##         53.4340   -0.5359   -0.6299   -0.8347   -0.4754    -0.359
## s.e.     0.2316    0.0748    0.0789    0.0785    0.0787     0.075
##
## sigma^2 estimated as 0.05485:  log likelihood = 7.57,  aic = 14.86
## [1] 4 0 4
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
```

```
## Coefficients:
##          ar1     ar2      ar3     ar4     ma1     ma2     ma3     ma4
##       0.4066  0.3873  -0.4633  0.4065  1.1623  0.8242  0.9166  0.4129
## s.e.  0.3238  0.2347   0.2596  0.2355  0.3281  0.4336  0.3453  0.2614
##          ma5  intercept  gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##       0.0671    53.4307   -0.5384   -0.6277   -0.8393   -0.4704   -0.3623
## s.e.  0.1321     0.2269    0.0751    0.0792    0.0792    0.0795    0.0758
##
## sigma^2 estimated as 0.05481:  log likelihood = 7.68,  aic = 16.65
## [1] 4 0 5
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2      ar3     ar4      ma1  gas_lag3  gas_lag4
##       1.2055  -0.3910  -0.1696  0.0432  -0.6414   -0.5250   -0.6216
## s.e.  0.2287   0.1586   0.0930  0.0952   0.2199    0.0742    0.0795
##       gas_lag5  gas_lag6  gas_lag7
##        -0.8535   -0.4739   -0.3457
## s.e.    0.0787    0.0790    0.0742
##
## sigma^2 estimated as 0.05705:  log likelihood = 3.89,  aic = 14.23
## [1] 4 1 1
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3      ar4     ma1      ma2  gas_lag3
##       0.2649  0.7068  -0.5378  -0.0723  0.3138  -0.5745   -0.5259
## s.e.  0.1712  0.0764   0.0775   0.0846  0.1634   0.1392    0.0732
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6200   -0.8564   -0.4733   -0.3437
## s.e.    0.0774    0.0770    0.0771    0.0732
##
## sigma^2 estimated as 0.05631:  log likelihood = 5.68,  aic = 12.64
## [1] 4 1 2
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ar4      ma1      ma2     ma3  gas_lag3
##       1.2871  0.2904  -1.2707  0.5248  -0.7355  -0.7461  0.7453   -0.5259
## s.e.  0.1145  0.0989   0.0989  0.0784   0.1107   0.0827  0.0901    0.0714
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6244   -0.8535   -0.4648   -0.3215
## s.e.    0.0769    0.0768    0.0769    0.0712
##
## sigma^2 estimated as 0.05563:  log likelihood = 7.31,  aic = 11.39
## [1] 4 1 3
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
```

```
##          ar1     ar2      ar3     ar4     ma1      ma2     ma3      ma4
##       0.5185  0.6072  -0.7081  0.0836  0.0595  -0.6218  0.1583  -0.0160
## s.e.  2.0084  0.7860   1.3205  1.2587  2.0073   0.3980  1.2014   0.2188
##       gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.5255   -0.6203   -0.8562   -0.4736   -0.3435
## s.e.    0.0733    0.0779    0.0770    0.0772    0.0739
##
## sigma^2 estimated as 0.05631:  log likelihood = 5.7,  aic = 16.61
## [1] 4 1 4
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ar4     ma1      ma2     ma3      ma4
##       0.4445  0.4485  -0.5011  0.3334  0.1261  -0.4196  0.0940  -0.4380
## s.e.  0.2410  0.1766   0.1984  0.1826  0.2403   0.1146  0.1964   0.0997
##           ma5  gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##       -0.2729   -0.5311   -0.6278   -0.8330   -0.4747   -0.3549
## s.e.   0.0898    0.0743    0.0788    0.0785    0.0786    0.0746
##
## sigma^2 estimated as 0.05518:  log likelihood = 8.22,  aic = 13.57
## [1] 4 1 5
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ar4     ar5     ma1  intercept  gas_lag3
##       0.6383  0.8483  -0.7720  0.0446  0.1292  0.9384    53.4390   -0.5382
## s.e.  0.0771  0.1046   0.0761  0.0756  0.0655  0.0488     0.2402    0.0758
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6149   -0.8588   -0.4718   -0.3580
## s.e.    0.0780    0.0773    0.0775    0.0754
##
## sigma^2 estimated as 0.05588:  log likelihood = 4.97,  aic = 16.07
## [1] 5 0 1
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ar4     ar5     ma1      ma2  intercept
##       1.1106  0.5363  -1.1284  0.3597  0.0790  0.4554  -0.4443    53.5090
## s.e.  0.2864  0.2394   0.1964  0.2139  0.0862  0.2837   0.2524     0.3346
##       gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.5313   -0.6211   -0.8575   -0.4741   -0.3490
## s.e.    0.0745    0.0779    0.0773    0.0774    0.0746
##
## sigma^2 estimated as 0.05548:  log likelihood = 5.92,  aic = 16.16
## [1] 5 0 2
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2      ar3     ar4      ar5     ma1     ma2     ma3
```

```
##       1.1532  -0.2859  -0.6887  1.0674  -0.4373  0.3910  0.3522  0.8113
## s.e.  0.1113   0.1554   0.1075  0.1305   0.0925  0.0929  0.0956  0.0885
##       intercept  gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##         53.3983   -0.5646   -0.6191   -0.8435   -0.4710   -0.3803
## s.e.     0.1831    0.0766    0.0797    0.0795    0.0796    0.0766
##
## sigma^2 estimated as 0.05486:  log likelihood = 7.51,  aic = 14.97
## [1] 5 0 3
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2      ar3     ar4      ar5     ma1     ma2     ma3
##       0.9915  -0.1715  -0.6337  0.9422  -0.3352  0.5755  0.4675  0.8764
## s.e.  0.2016   0.2299   0.1105  0.2107   0.1604  0.2063  0.1282  0.1085
##          ma4  intercept  gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##       0.1389    53.4137   -0.5580   -0.6186   -0.8355   -0.4711   -0.3781
## s.e.  0.1284     0.2025    0.0768    0.0788    0.0790    0.0784    0.0766
##
## sigma^2 estimated as 0.05463:  log likelihood = 8.1,  aic = 15.81
## [1] 5 0 4
```

```
## Warning in arima(co2, xreg = gas_lag_master, order = c(p, d, q)): possible
## convergence problem: optim gave code = 1
```

```
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2      ar3     ar4      ar5     ma1     ma2     ma3
##       0.9653  -0.1622  -0.6276  0.9330  -0.3229  0.6013  0.5017  0.9022
## s.e.  0.2793   0.2389   0.1172  0.2208   0.1859  0.2807  0.2808  0.2191
##          ma4     ma5  intercept  gas_lag3  gas_lag4  gas_lag5  gas_lag6
##       0.1689  0.0175    53.4117   -0.5581   -0.6173   -0.8361   -0.4706
## s.e.  0.2560  0.1272     0.2035    0.0769    0.0795    0.0791    0.0784
##       gas_lag7
##        -0.3786
## s.e.    0.0768
##
## sigma^2 estimated as 0.05462:  log likelihood = 8.11,  aic = 17.79
## [1] 5 0 5
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##           ar1     ar2      ar3      ar4      ar5     ma1  gas_lag3
##       -0.3206  0.4900  -0.1899  -0.2300  -0.1532  0.9084   -0.5149
## s.e.   0.0823  0.0708   0.0658   0.0632   0.0598  0.0611    0.0726
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6214   -0.8422   -0.4800   -0.3446
## s.e.    0.0778    0.0772    0.0773    0.0730
##
## sigma^2 estimated as 0.0564:  log likelihood = 5.46,  aic = 13.09
## [1] 5 1 1
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3      ar4      ar5    ma1      ma2  gas_lag3
##       0.2120  0.6780  -0.4962  -0.0844  -0.0284  0.366  -0.5205   -0.5241
## s.e.  0.2727  0.1296   0.1724   0.0990   0.0990  0.268   0.2563    0.0734
##       gas_lag4  gas_lag5  gas_lag6  gas_lag7
##        -0.6219   -0.8558   -0.4746   -0.3422
## s.e.    0.0777    0.0771    0.0772    0.0732
##
## sigma^2 estimated as 0.0563:  log likelihood = 5.72,  aic = 14.55
## [1] 5 1 2
```

```
## Warning in arima(co2, xreg = gas_lag_master, order = c(p, d, q)): possible
## convergence problem: optim gave code = 1
```

```
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2     ar3      ar4      ar5     ma1      ma2      ma3
##      -0.6892  0.8807  0.1595  -0.5570  -0.1191  1.2770  -0.2014  -0.5091
## s.e.   0.2294  0.2266  0.1490   0.1274   0.0870  0.2278   0.3179   0.2142
##      gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##       -0.5269   -0.6281   -0.8451   -0.4791   -0.3440
## s.e.    0.0726    0.0771    0.0772    0.0772    0.0729
##
## sigma^2 estimated as 0.05597:  log likelihood = 6.49,  aic = 15.02
## [1] 5 1 3
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##         ar1      ar2      ar3     ar4      ar5     ma1     ma2     ma3
##      0.3071  -0.2404  -0.1976  0.6011  -0.5428  0.2803  0.3771  0.2111
## s.e.  0.1032   0.0787   0.0768  0.0743   0.0564  0.1125  0.1111  0.1132
##          ma4  gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##      -0.6330   -0.5203   -0.6271   -0.8562   -0.4671   -0.3434
## s.e.   0.1067    0.0736    0.0776    0.0770    0.0772    0.0732
##
## sigma^2 estimated as 0.05448:  log likelihood = 8.65,  aic = 12.7
## [1] 5 1 4
##
## Call:
## arima(x = co2, order = c(p, d, q), xreg = gas_lag_master)
##
## Coefficients:
##         ar1      ar2      ar3     ar4      ar5      ma1      ma2     ma3
##      0.9988  -0.2106  -0.6371  0.9545  -0.3628  -0.4294  -0.0699  0.4297
## s.e.  0.1901   0.2118   0.1102  0.1930   0.1507   0.1964   0.1417  0.1003
##          ma4      ma5  gas_lag3  gas_lag4  gas_lag5  gas_lag6  gas_lag7
##      -0.7327  -0.1149   -0.5526   -0.6148   -0.8341   -0.4698   -0.3732
## s.e.   0.1201   0.1261    0.0759    0.0787    0.0788    0.0783    0.0759
##
## sigma^2 estimated as 0.05487:  log likelihood = 8.9,  aic = 14.21
## [1] 5 1 5
```

```
aic_c
```

```
##  [1] 55.49929 24.76413 18.30549 14.62617 15.38100 34.50020 29.01373
##  [8] 29.78511 17.05403 12.39720 21.11766 23.53402 17.81044 15.90578
## [15] 16.09057 13.78203 13.00936 14.15833 15.92333 14.01569 19.52612
## [22] 19.00749 18.96676 16.51508 17.95719 12.43723 11.34509 12.61001
## [29] 14.61001 14.08512 17.99405 14.96506 16.09080 14.85784 16.64564
## [36] 14.22952 12.63885 11.38565 16.60836 13.56897 16.06659 16.16463
## [43] 14.97290 15.80683 17.78692 13.08623 14.55499 15.02179 12.69700
## [50] 14.20963
```

```r
master_data = cbind(p_c,d_c,q_c,aic_c)
which.min(master_data[,4]) #row location of min aic
```

```
## [1] 27
```

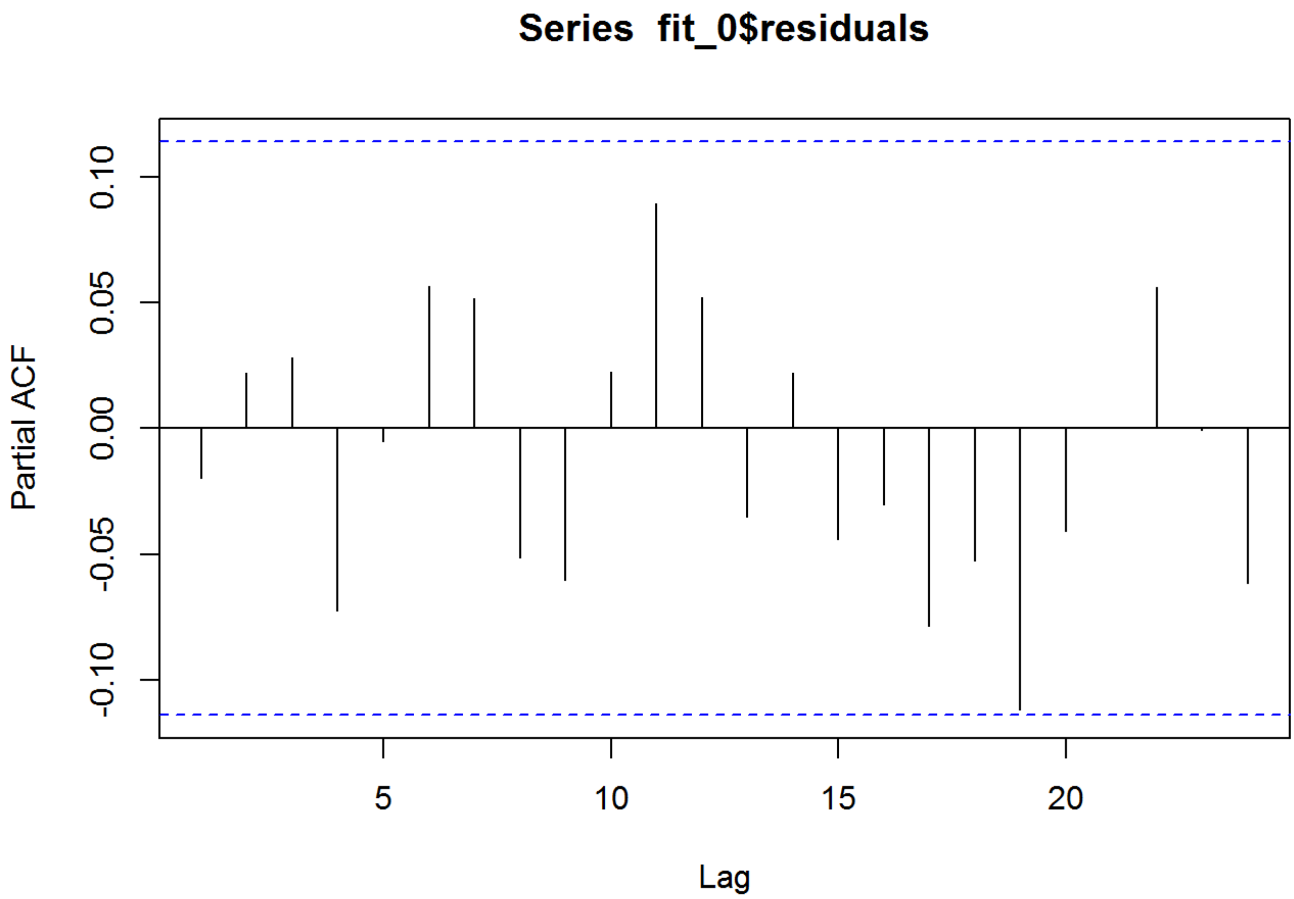
```r
master_data[27,] # Get the row with the p, d, and q values related to min aic
```

```
##      p_c      d_c      q_c    aic_c
##  3.00000  1.00000  2.00000 11.34509
```

```r
fit_0 = arima(co2, xreg = gas_lag_master, order = c(3,1,2))
fit_0 #aic is 11.34
```

```
##
## Call:
## arima(x = co2, order = c(3, 1, 2), xreg = gas_lag_master)
##
## Coefficients:
##          ar1     ar2      ar3     ma1      ma2  gas_lag3  gas_lag4
##       0.3796  0.6830  -0.5802  0.2184  -0.6421   -0.5249   -0.6202
## s.e.  0.0947  0.0741   0.0557  0.1054   0.1013    0.0733    0.0767
##       gas_lag5  gas_lag6  gas_lag7
##        -0.8556   -0.4746   -0.3414
## s.e.    0.0764    0.0765    0.0732
##
## sigma^2 estimated as 0.05645:  log likelihood = 5.33,  aic = 11.35
```

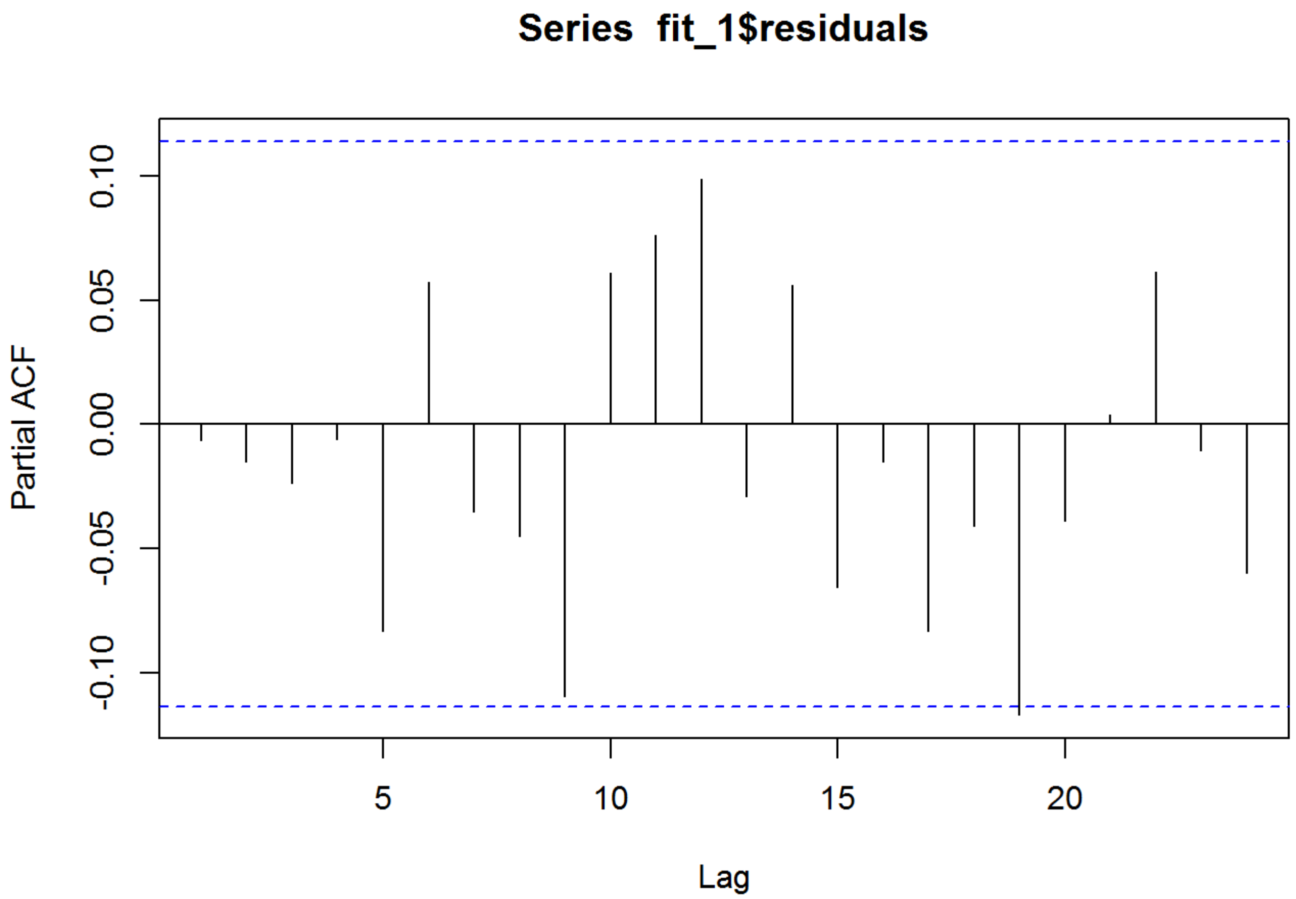
```r
pacf(fit_0$residuals, na.action = na.pass)
```




```r
fit_1 = auto.arima(co2, xreg = gas_lag_master)
fit_1 #aic is 14.4
```

```
## Series: co2
## ARIMA(4,1,0)
##
## Coefficients:
##          ar1      ar2      ar3      ar4  gas_lag3  gas_lag4  gas_lag5
##       0.5764  -0.0132  -0.1782  -0.1052   -0.5158   -0.6168   -0.8424
## s.e.  0.0587   0.0672   0.0674   0.0593    0.0739    0.0797    0.0789
##       gas_lag6  gas_lag7
##        -0.4762   -0.3496
## s.e.    0.0792    0.0742
##
## sigma^2 estimated as 0.0577:  log likelihood=2.8
## AIC=14.4   AICc=15.18   BIC=51.27
```

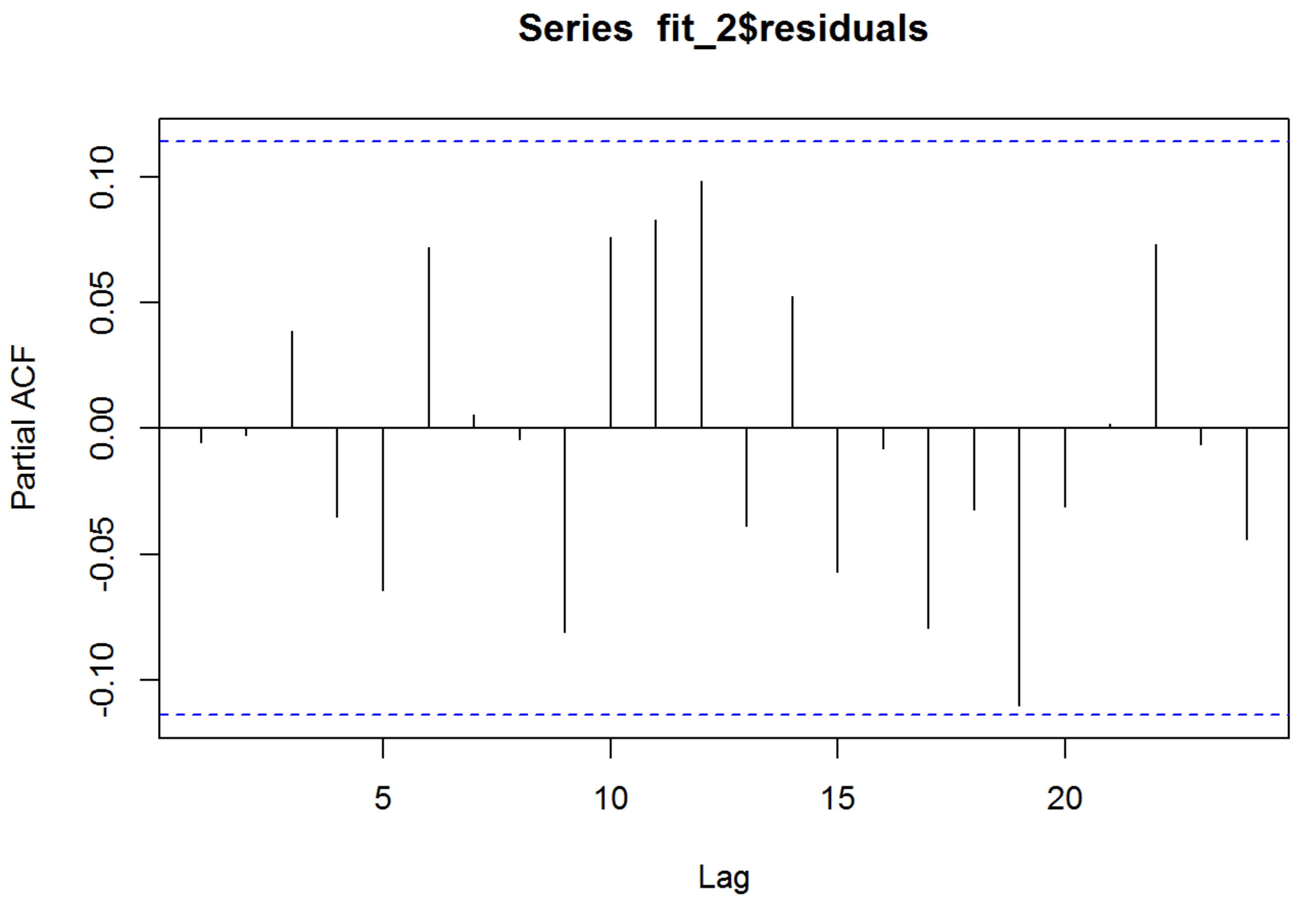
```r
pacf(fit_1$residuals, na.action = na.pass)
```




```
fit_2 = arima(co2, xreg = gas_lag_master, order = c(4,0,0))
fit_2 # aic is 14.68
```

```
##
## Call:
## arima(x = co2, order = c(4, 0, 0), xreg = gas_lag_master)
##
## Coefficients:
##          ar1      ar2      ar3     ar4  intercept  gas_lag3  gas_lag4
##       1.5619  -0.5963  -0.1987  0.1773    53.4475   -0.5363   -0.6165
## s.e.  0.0580   0.1083   0.1089  0.0615     0.2494    0.0764    0.0794
##       gas_lag5  gas_lag6  gas_lag7
##        -0.8567   -0.4725   -0.3585
## s.e.    0.0787    0.0789    0.0759
##
## sigma^2 estimated as 0.05641:  log likelihood = 3.66,  aic = 14.68
```

```
pacf(fit_2$residuals, na.action = na.pass)
```




```
fit_3 = arima(co2, xreg = gas_lag_master, order = c(7,1,4), include.mean = FALSE)
# testing of various p,d,q
```

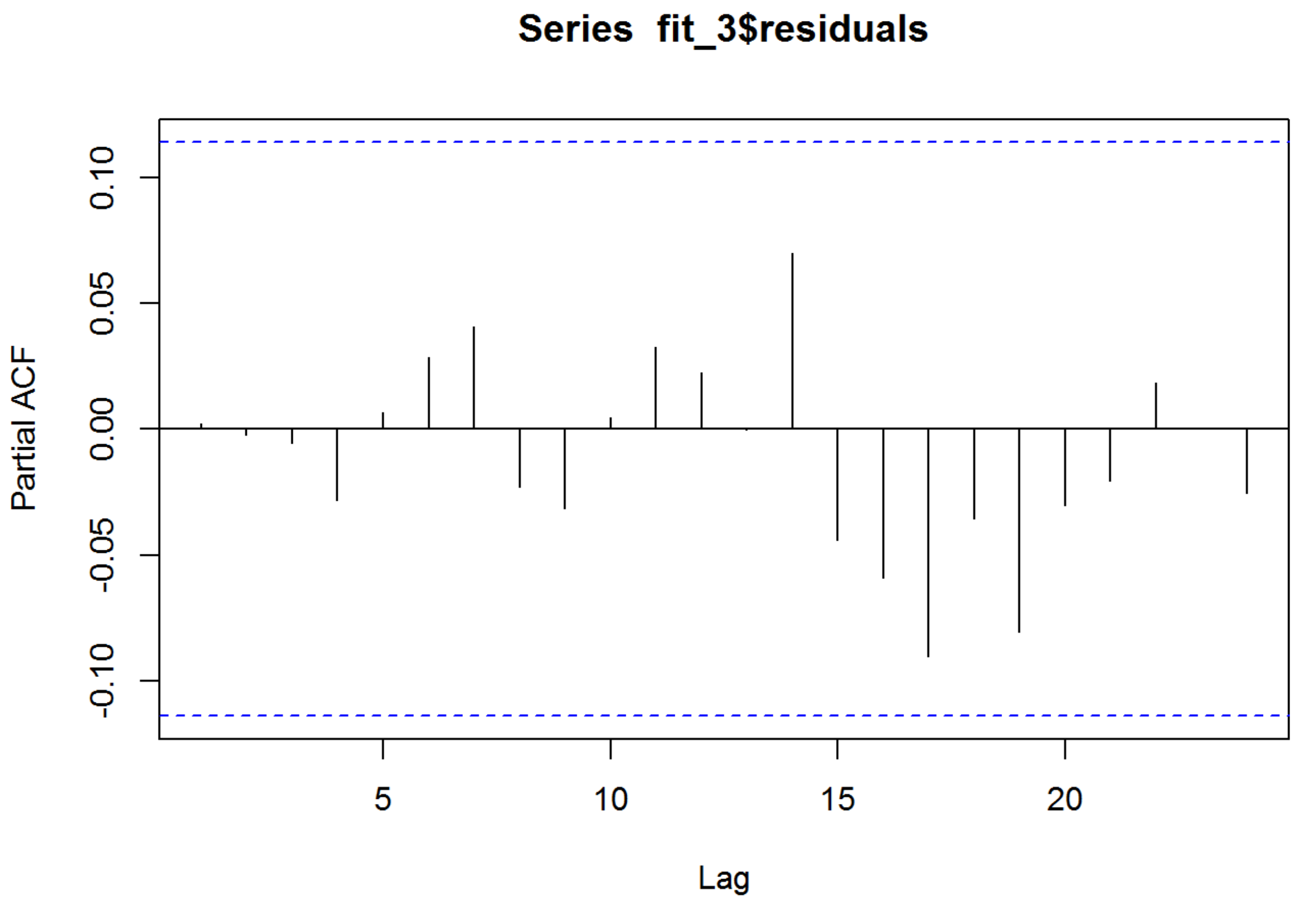
```
## Warning in log(s2): NaNs produced
```

```
## Warning in log(s2): NaNs produced
```

```
fit_3 # aic is 11.55
```

```
##
## Call:
## arima(x = co2, order = c(7, 1, 4), xreg = gas_lag_master, include.mean = FALSE)
##
## Coefficients:
##          ar1      ar2      ar3     ar4      ar5      ar6      ar7      ma1
##       0.7930  -0.3842  -0.7644  0.7373  -0.3482  -0.0471  -0.1131  -0.2299
## s.e.  0.2893   0.1535   0.1346  0.3150   0.1631   0.0857   0.0977   0.2905
##          ma2     ma3      ma4  gas_lag3  gas_lag4  gas_lag5  gas_lag6
##       0.2429  0.7537  -0.3977   -0.5249   -0.6140   -0.8456   -0.4871
## s.e.  0.0990  0.1172   0.2815    0.0750    0.0782    0.0782    0.0798
##       gas_lag7
##        -0.3486
## s.e.    0.0734
##
## sigma^2 estimated as 0.05384:  log likelihood = 11.22,  aic = 11.55
```

```
pacf(fit_3$residuals, na.action = na.pass) # Seems to have best PACF graph so far
```




```
aic_list = list(ARIMA312 = fit_0$aic, ARIMA410 = fit_1$aic, ARIMA400 = fit_2$aic, A
RIMA714 = fit_3$aic)
aic_list
```

```
## $ARIMA312
## [1] 11.34509
##
## $ARIMA410
## [1] 14.40271
##
## $ARIMA400
## [1] 14.68467
##
## $ARIMA714
## [1] 11.55232
```

```
# Choose model fit_0 since it has the lowest aic of 11.34


# Show R codes that estimate your model and the corresponding estimation result
```

3. Checking model adequacy of your fitted model.

The analysis of the model adequacy of the fitted model is as follows:

- e.g. My model pass the residual correlation check using ?? because the corresponding p-value is ??
- e.g. The cross correlation between xx and xx .....

```
# Show your R codes and results for checking model adequacy

install.packages("TSA", repos='http://cran.utstat.utoronto.ca/')
```

```
## Installing package into 'C:/Users/Isaac/Documents/R/win-library/3.1'
## (as 'lib' is unspecified)
```

```
## package 'TSA' successfully unpacked and MD5 sums checked
##
## The downloaded binary packages are in
##  C:\Users\Isaac\AppData\Local\Temp\RtmpKwlF1M\downloaded_packages
```

```
library("TSA")
```

```
## Warning: package 'TSA' was built under R version 3.1.3
```

```
## Loading required package: leaps
```

```
## Warning: package 'leaps' was built under R version 3.1.3
```

```
## Loading required package: locfit
```

```
## Warning: package 'locfit' was built under R version 3.1.3
```

```
## locfit 1.5-9.1    2013-03-22
```

```
## Loading required package: mgcv
```

```
## Loading required package: nlme
```

```
##
## Attaching package: 'nlme'
```

```
## The following object is masked from 'package:forecast':
##
##     getResponse
```

```
## This is mgcv 1.8-3. For overview type 'help("mgcv-package")'.
```

```
## Loading required package: tseries
```

```
## Warning: package 'tseries' was built under R version 3.1.3
```

```
##
## Attaching package: 'TSA'
```

```
## The following objects are masked from 'package:forecast':
##
##     fitted.Arima, plot.Arima
```

```
## The following objects are masked from 'package:timeDate':
##
##     kurtosis, skewness
```

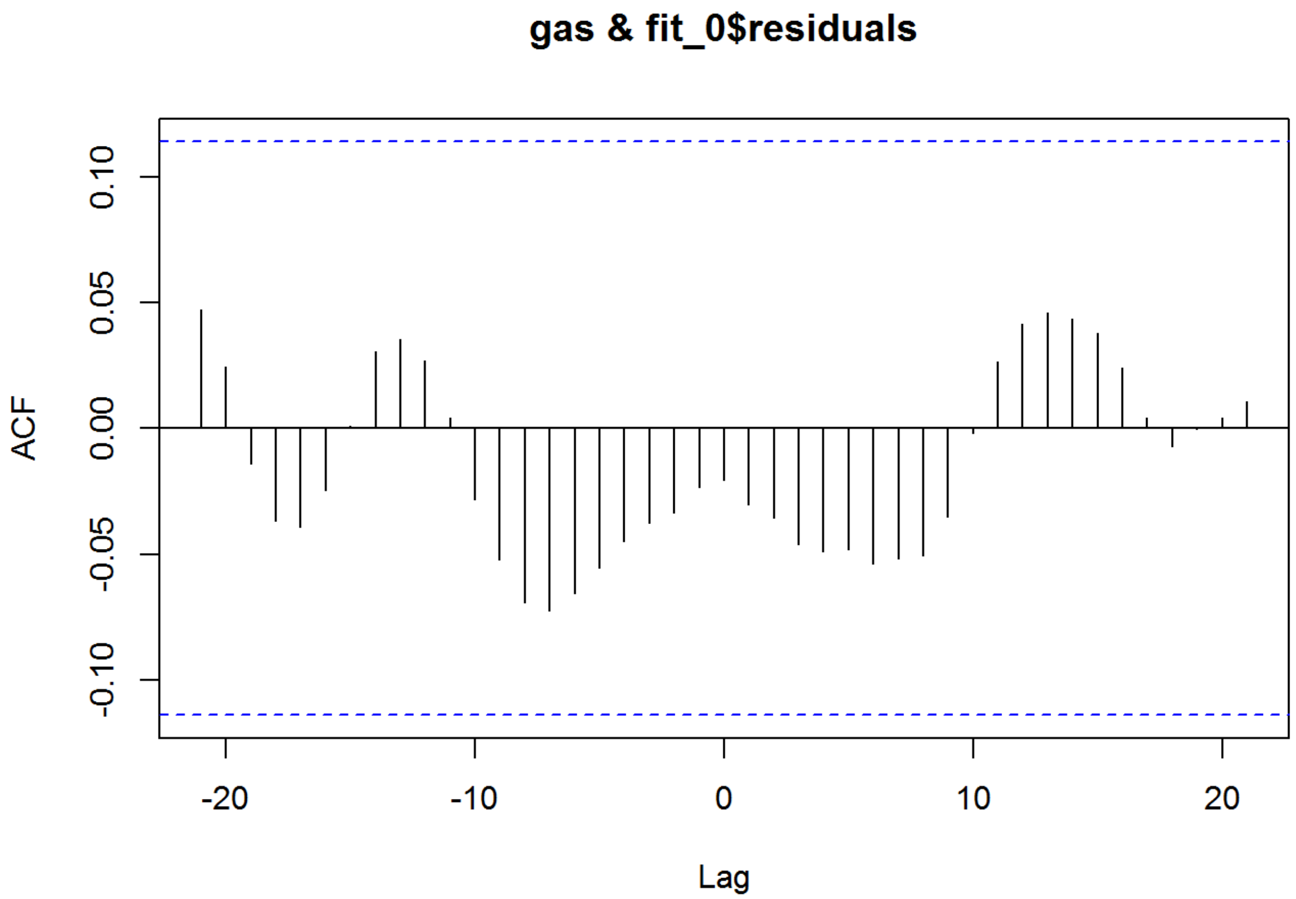
```
## The following objects are masked from 'package:stats':
##
##     acf, arima
```

```
## The following object is masked from 'package:utils':
##
##     tar
```

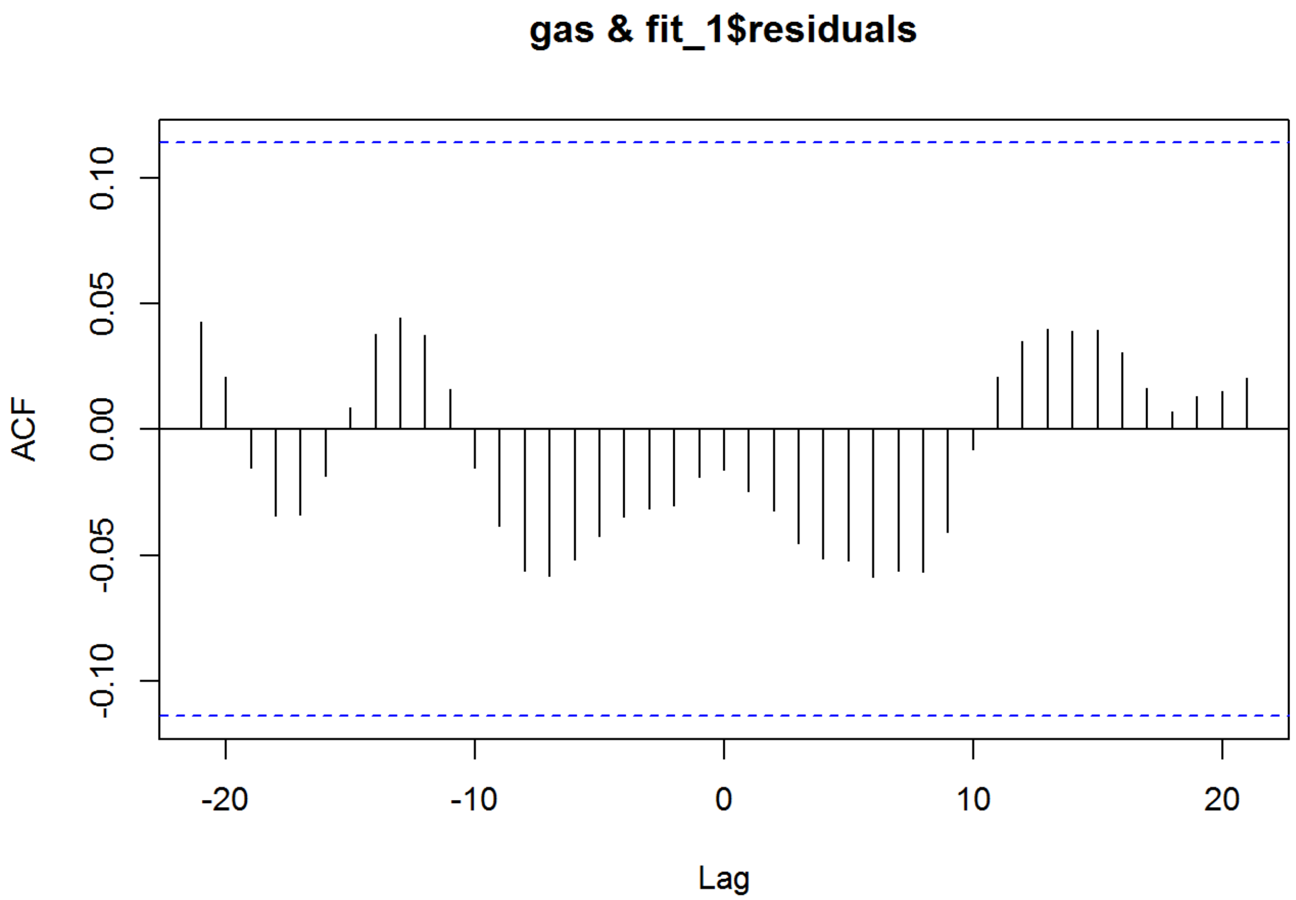
```
p_value_list = list(ARIMA_3_1_2 = LB.test(fit_0, type = c("Ljung-Box"))$p.value, A
RIMA_4_1_0 = LB.test(fit_1, type = c("Ljung-Box"))$p.value, ARIMA_4_0_0 = LB.test(
fit_2, type = c("Ljung-Box"))$p.value,  ARIMA_7_1_4 = LB.test(fit_3, type = c("Lju
ng-Box"))$p.value)
p_value_list
```

```
## $ARIMA_3_1_2
## [1] 0.3776446
##
## $ARIMA_4_1_0
## [1] 0.07792931
##
## $ARIMA_4_0_0
## [1] 0.1986914
##
## $ARIMA_7_1_4
## [1] 0.1640317
```

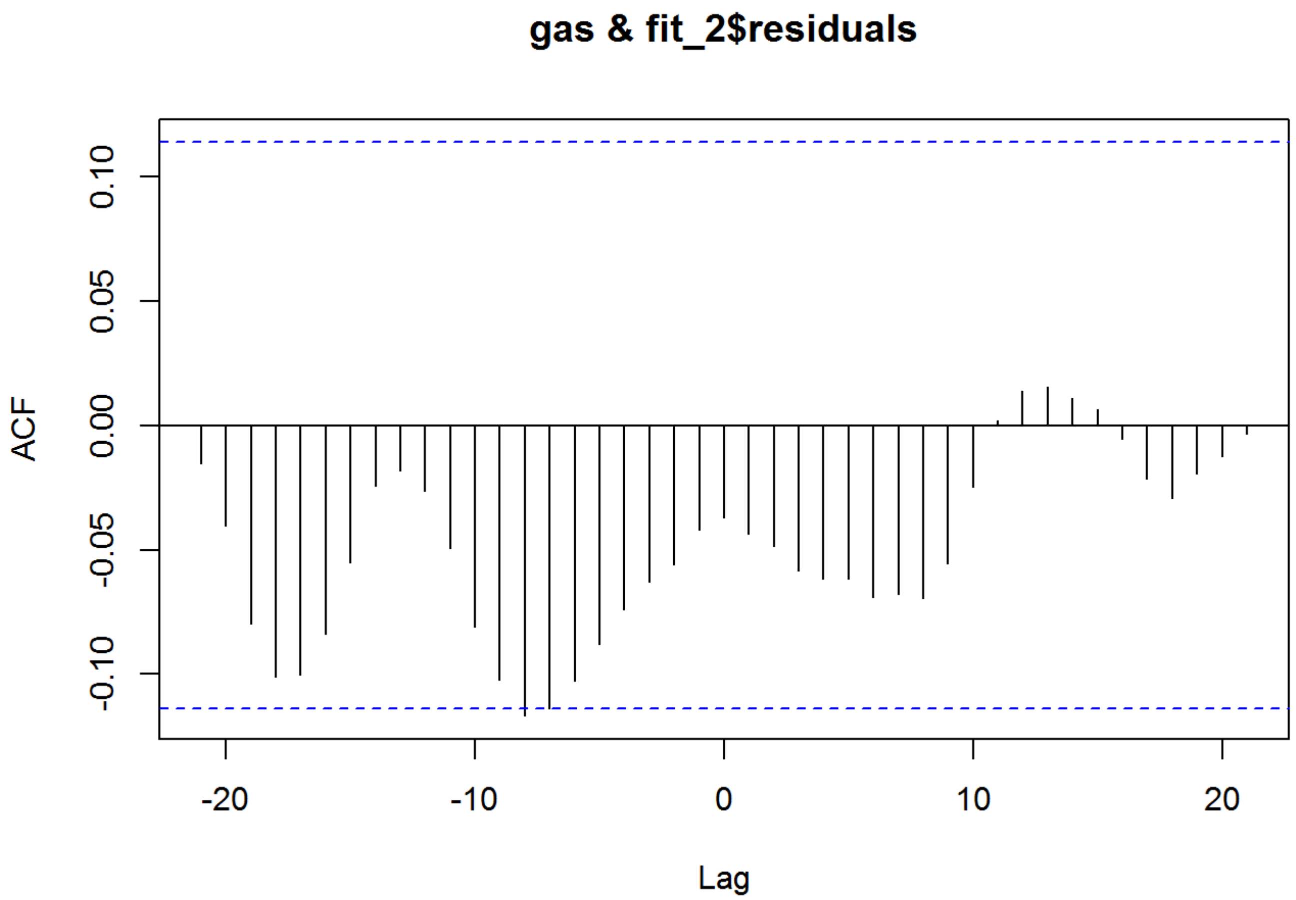
```
# All of my models pass the residual correlation check using Ljung-Box test because
all of the corresponding p-values are greater than 0.05
# The cross correlation between gas and residuals of ARIMA(3,1,2) model shows that
all of the values are within
# the 95% CI, or upper and lower threshold

ccf(gas, fit_0$residuals, na.action = na.pass)
```



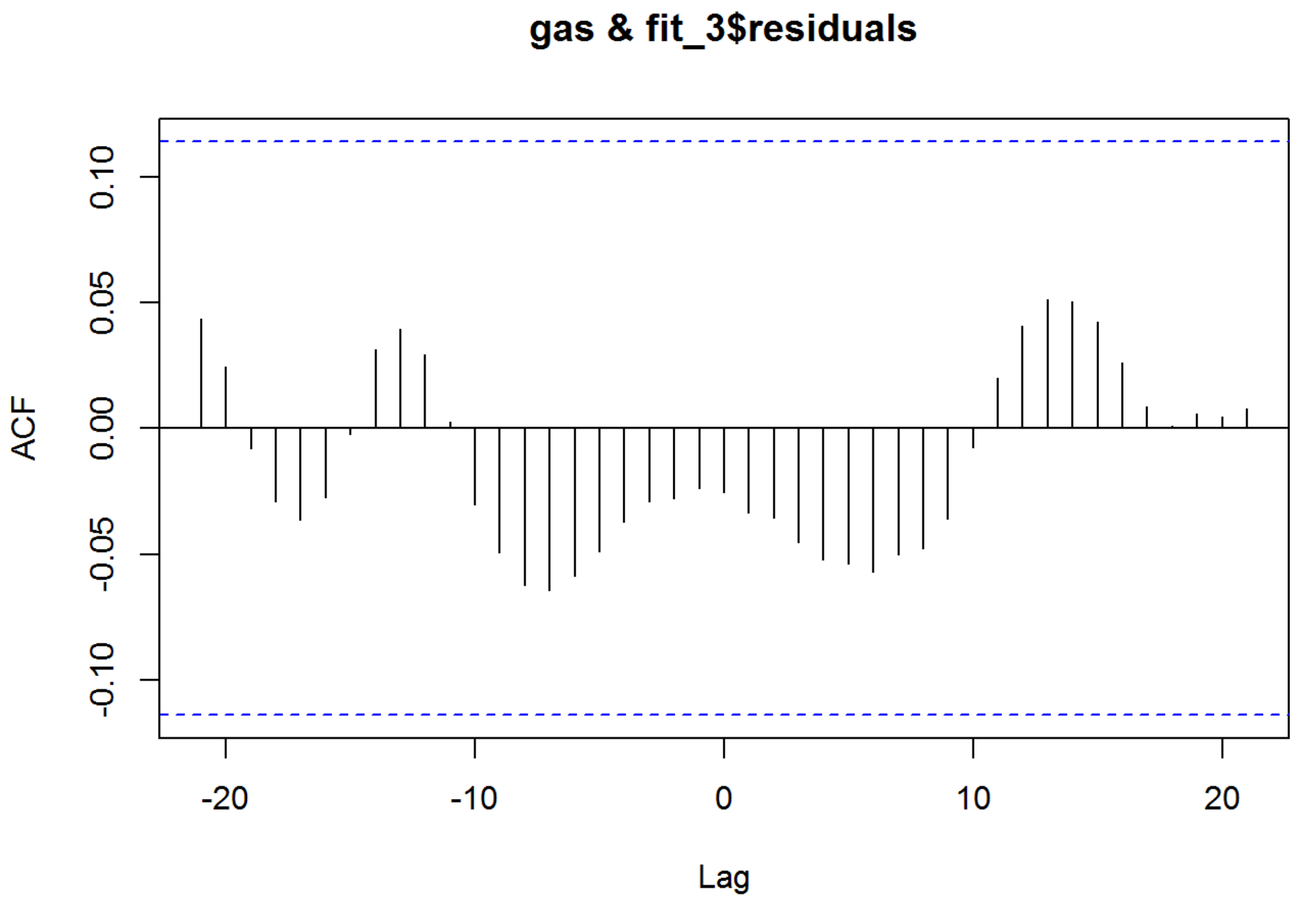

```
ccf(gas, fit_1$residuals, na.action = na.pass)
```



```r
ccf(gas, fit_2$residuals, na.action = na.pass)
```




```r
ccf(gas, fit_3$residuals, na.action = na.pass)
```




```
# From the graph of ccf's, we can see that fit_2 (ARIMA(4,0,0)) has the least optio
mal CCF graph.Therefore, we need to choose the most optimal model from:
# fit_0, ARIMA(3,1,2)
# fit_1, ARIMA(4,1,0)
# fit_3, ARIMA(7,1,4)
# From the above 3 models, fit_0, ARIMA(3,1,2), has the least aic, therefore
# it is reasonable to choose it as the most optimal model
# Also, the ccf(gas, fit_0$residuals) graph shows that residuals behave like white
noise and thus have no spikes outside of 95% CI. Beautiful graph indeed.
```