Grid Operation-based Outage Maintenance Planning

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Overview

- Problem Description
- Problem instances
- Deterministic model
- Robust model
- Next Steps

What is the ROADEF challenge?

- French Operations Research & Decision Support Society (ROADEF)
- Ongoing competition
  - Google : Machine reassignment
  - Air Liquide: Inventory routing
  - Saint-Cobain : Glass manufacturing
  - Electricity Transmission Network (RTE) 2020: Maintenance scheduling
    - Due date: 2022
- Why this problem?
  - Competing objectives
  - Uncertainty
  - Discrete decisions
  - No solutions published
  - Real data

What’s the problem?

- RTE responsible for the largest high-voltage transmission system in Europe
  - 105k km miles of transmission lines in France
- Minimize service disruption during maintenance
  - 15k events/yr
- Long term planning for
  - Renewable options
  - Consumption changes
- Given the risk (in Euros) of scheduling an intervention (maintenance event), can we design short and long-term maintenance schedules?

What are the parameters?

- Planning horizon
  - Daily or weekly
  - Ex. Create a 90-day plan
- Resources
  - 9-10 skilled teams available
- Interventions
  - What items need to be scheduled?
  - 18-706 possible
- Risk assessment per Intervention
  - Source of uncertainty
  - Euros for each timestep
  - Independent
  - If we start intervention i on July 5th, how much money could we lose on July 7th under the medium risk scenario?

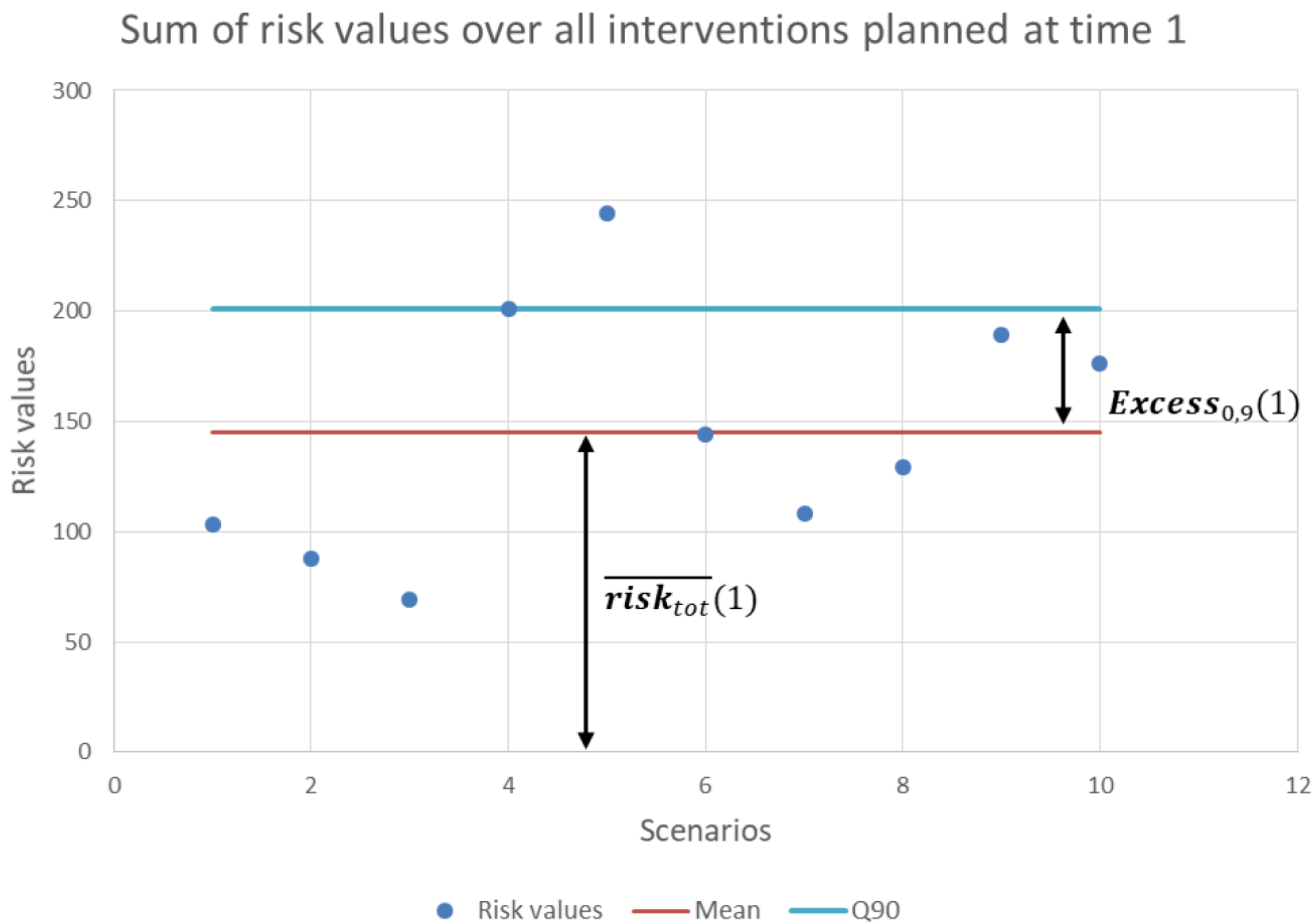
What are the constraints?

- Interventions must start at the beginning of a period
- Interventions cannot be interrupted
- All interventions must be executed exactly once
- Interventions must be completed no later than the end of the horizon
- Resources consumed must be within upper and lower bounds
  - Resources are consumed while the intervention is active- not just start
- Some interventions cannot be conducted at the same time

How do we evaluate the solution?

- Time cutoff
  - 15-90-minute time limit
- Mean risk cost: $\frac{1}{T}\sum_{t\in T}\overline{risk^t}$
  - $\overline{risk^t}$: Given decisions, average risk at time t
- Expected excess: $\frac{1}{T}\sum_{t\in T} E\ xcess(t)$
  - Q_τ^t: τ-quantile of risk at time t
  - $Excess(t) = max(0, Q_\tau^t - \overline{risk^t})$
    - Positive difference between quantile τ and mean cost
  - Mean cost may not capture extremely high-risk situations
  - Intended to quantify variability of scenarios
- Weighted objective

$$obj(\tau) = \alpha\times\frac{1}{T}\sum_{t\in T}\overline{risk^t} + (1-\alpha)\times\frac{1}{T}\sum_{t\in T} E\ xcess(t)$$



Sum of risk values over all interventions planned at time 1
Risk values
Scenarios
300
250
200
150
100
50
0
0
2
4
6
8
10
12
$\mathbf{Excess}_{0,9}(1)$
$\overline{\mathbf{risk}_{tot}}(1)$
Risk values
Mean
Q90

How do we select problem instances?

- 8 of 15 instances used
  - 11 with more than one scenario
- Prefer instances instances with more complexity
  - Largest number of interventions
  - Longest planning horizon
  - Largest number of scenarios
- $\alpha = .5$
- 95th percentile for τ

Instance	Horizon	# Interventions	Mean # Scenarios	# Exclusions	# Resources
A_08	17	18	645.6	4	9
A_05	182	180	120	87	9
A_02	90	89	120	32	9
A_11	53	54	640	4	9
A_14	53	108	160	22	10
A_10	53	108	6	40	9
A_09	17	18	6	10	4
A_07	17	36	6	3	9

Plan on iteration

- Multiple factors
  - Multiple objectives
  - Uncertainty
  - Time limit
- Many potential solutions:
  - Expected Risk Assessment + Deterministic model
    - Average risk assessments for each time step and schedule with MILP solver
    - Manage expected risk component of objective
  - Robust optimization
    - Use techniques for managing variability to address Expected Excess
    - Manage Excess risk component of objective
  - Approximate dynamic programming
  - Local search heuristics
  - Hybrid solutions
  - Tuning
    - Weights
    - Solver settings
    - Reformulations

Deterministic MIP with Estimates

Why start here?

- ”Quick” prototype
    1. Point estimates for risk for each time step and intervention
    2. Formulate IP
    3. Leverage solver
    4. Hope
- Expected value of risk is one of the two objectives
- Iterative modeling + incremental complexity
- Reusable
    - Sequential multiple objective modeling
    - Store feasible solutions for neighborhood search
    - Warm start

Formulation: Notation

- Sets
  - T: Planning horizon
  - I : Intervention
  - C: Resources
  - $Exclusions = A \in I, B \in B, a \in T, b \in T$: If intervention A started at time a, then intervention B started at b must not be active
- Parameters
  - $u_t^c \forall t \in T, c \in C$: Max resource consumption for resources c at t
  - $l_t^c \forall t \in T, c \in C$: Min consumption for resource c at t
  - r_{ib}^{ct}: Amount of resource c consumed at time t given resource i started at time b
    - Preprocess zero workloads
  - $\Delta_{i,t}$ Duration given intervention i starts at time t
  - $risk_{i,b}^{s,t}$: Risk assessment at time t for starting i at time b under scenario s
    - $meanRisk_{i,b}^{t}$
- Decision variables
  - x_t^i Intervention i started at time t
    - Preprocess invalid start times

Formulation

$$\min \sum_{i \in I} \sum_{t \in T} \sum_{\substack{b \in \{start_i: \\ t \in [start_i, start_i + \Delta_{i, start_i}]\}}} x_b^i \, meanRisk_{i,b}^t$$

$$\sum_{t \in T} x_t^i = 1 \quad \forall i \in I$$

$$\sum_{i \in I} \sum_{\substack{b \in \{start_i: \\ t \in [start_i, start_i + \Delta_{i, start_i}]\}}} x_b^i \, r_{ib}^{ct} \le u_t^c \quad \forall c, \forall t$$

$$\sum_{i \in I} \sum_{\substack{b \in \{start_i: \\ t \in [start_i, start_i + \Delta_{i, start_i}]\}}} x_b^i \, r_{ib}^{ct} \ge l_t^c \quad \forall c, \forall t$$

$$x_a^A \le 1 - x_b^B \quad \forall Exclusions$$

$$x_t^i \in \{0,1\} \quad \forall t \in T, i \in I$$

Solution Quality

Instance	Runtime (sec)	Expected Excess	Mean Risk	Objective
A_05	9.73	151	1169	660
A_02	0.75	12749	3231	7990
A_08	0	162	1331	746
A_07	.02	27	4536	2281
A_09	0	21	3181	1601
A_10	.14	88	5962	3024
A_11	.03	141	896	518
A_14	.37	630	4265	2448

15 minute time limit not required

How can we improve?

- May not use all information available
  - Point estimate
- Does not address excess objective
    - $Excess(t) = max(0, Q_\tau^t - \overline{risk^t})$
  - Measure of variability
- Try robust optimization
  - Mathematical programming formulation which identifies solutions under the worst case scenarios

Robust Optimization

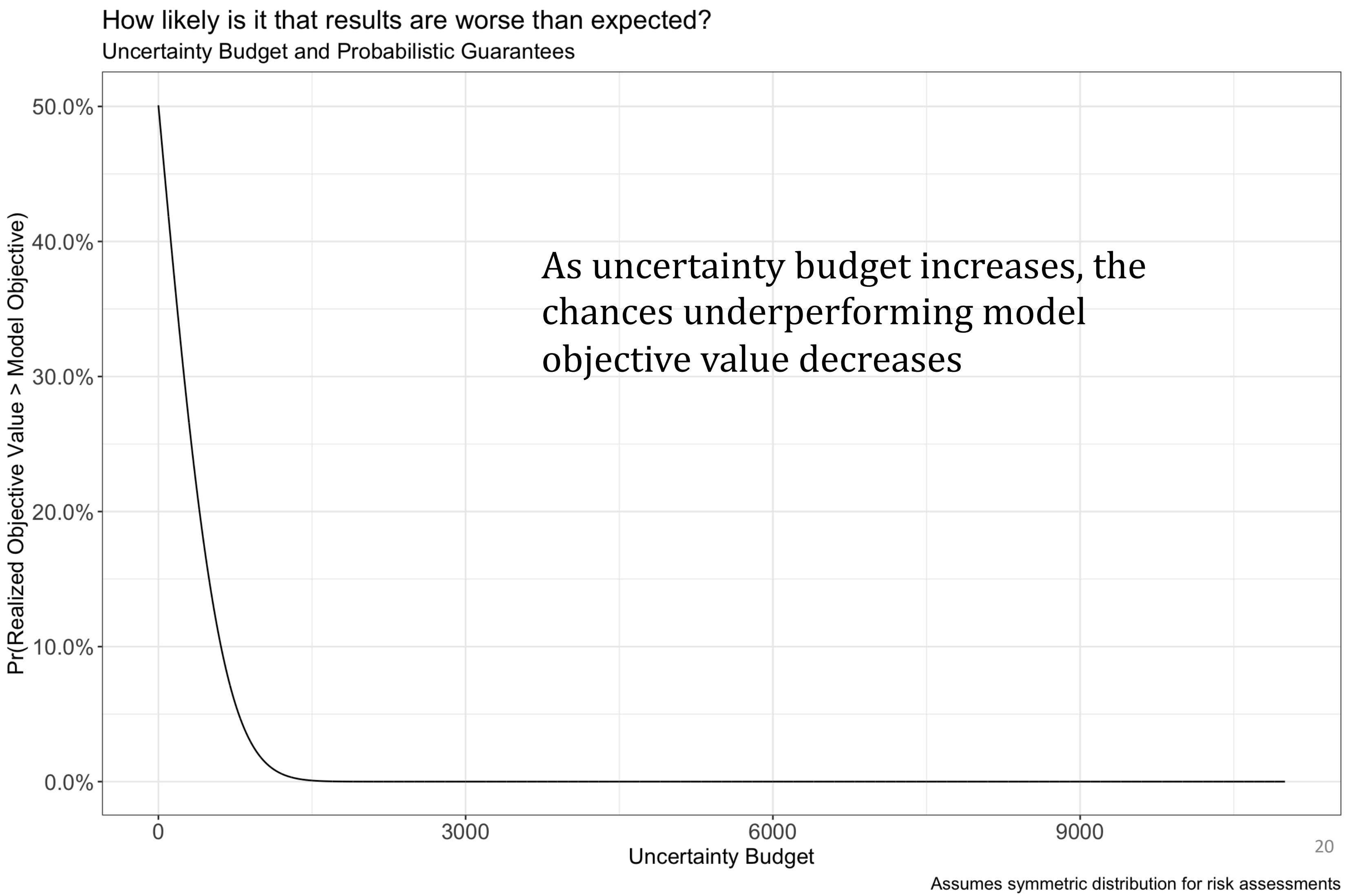
What does it mean to be robust?

- Each deterministic program has a robust formulation counterpart that can be formulated
  - Counterparts search for best solution under worst case scenario
  - How bad should your worst case be?
- Soyster Framework
  - Assumes all uncertain parameters can take their worst values simultaneously
  - Offers highest protection, but is the most conservative
- Ben-tal and Nemirovski Framework
  - Allows tradeoff between performance and conservatism
  - Nonlinear robust counterpart
- Bertsimas Framework
  - Allow tradeoff between performance and conservatism via uncertainty budget Γ
    - $\mathrm{Pr}(\, Obj.Underperforming) = \mathrm{f}(\Gamma, \mathrm{numUncertainParameters})$
    - Increasing parameter increases protection
  - Avoid nonlinear optimization

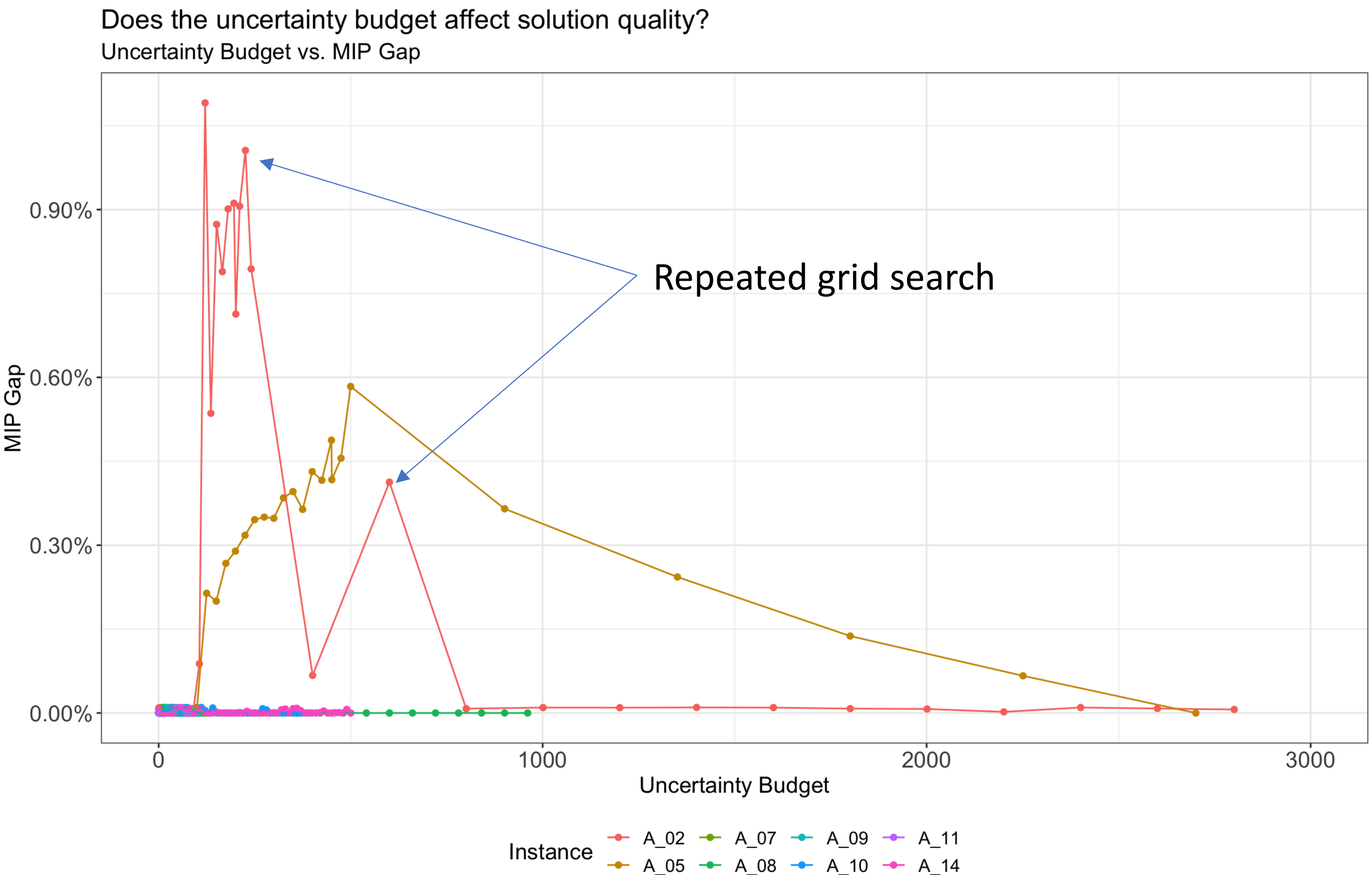
How does it work?

1. Assume symmetric distribution about each risk assessment
   - Provide central nominal value (ex. mean of risk on day t)
   - Provide window size (ex. standard deviation of risk on day t)
2. Reformulate MILP with uncertainty budget parameter Γ
   - $\Pr(\,Obj.Underperforming) = \mathrm{f}(\Gamma, \mathrm{numUncertainParameters})$
     - Bertsimas and Sim Theorem
   - $\Gamma \in [0, numUncertainParameters]$
   - $\Gamma = 0 \Longrightarrow$ deterministic
   - $\Gamma = numUncertainParameters \Longrightarrow$ most conservative total risk estimate
3. Select Γ
   - Domain expertise
     - “We accept at most a 40% chance that results could be worse than projected.”
   - Sensitivity analysis w/ objective(s)
   - Out-of-sample and in-sample backtesting
4. Implement MIP using Gurobi and 15 minute limit

What's the relationship between the uncertainty budget and Pr(underperforming)?

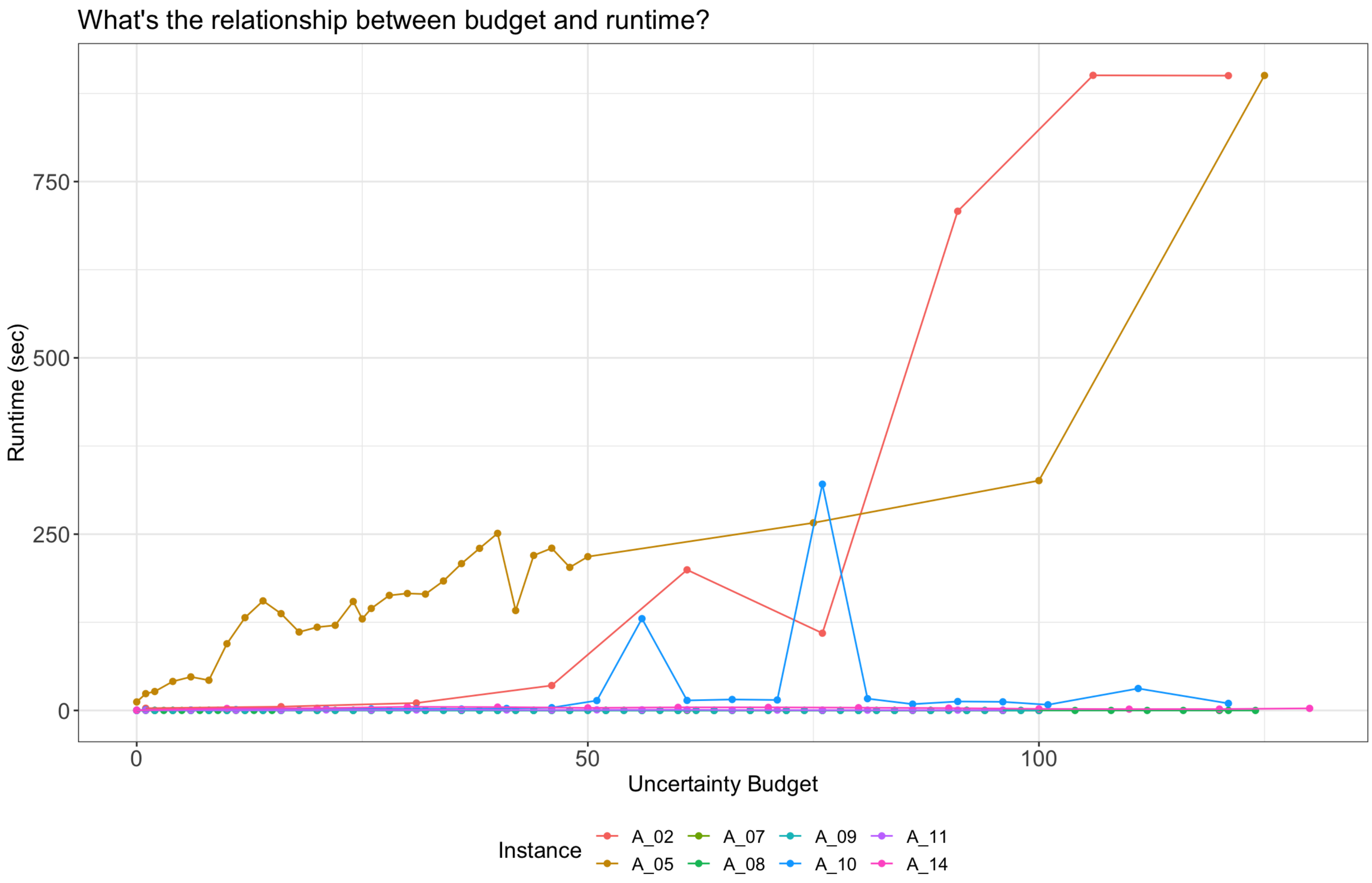


Budget Parameter vs. MIP Gap



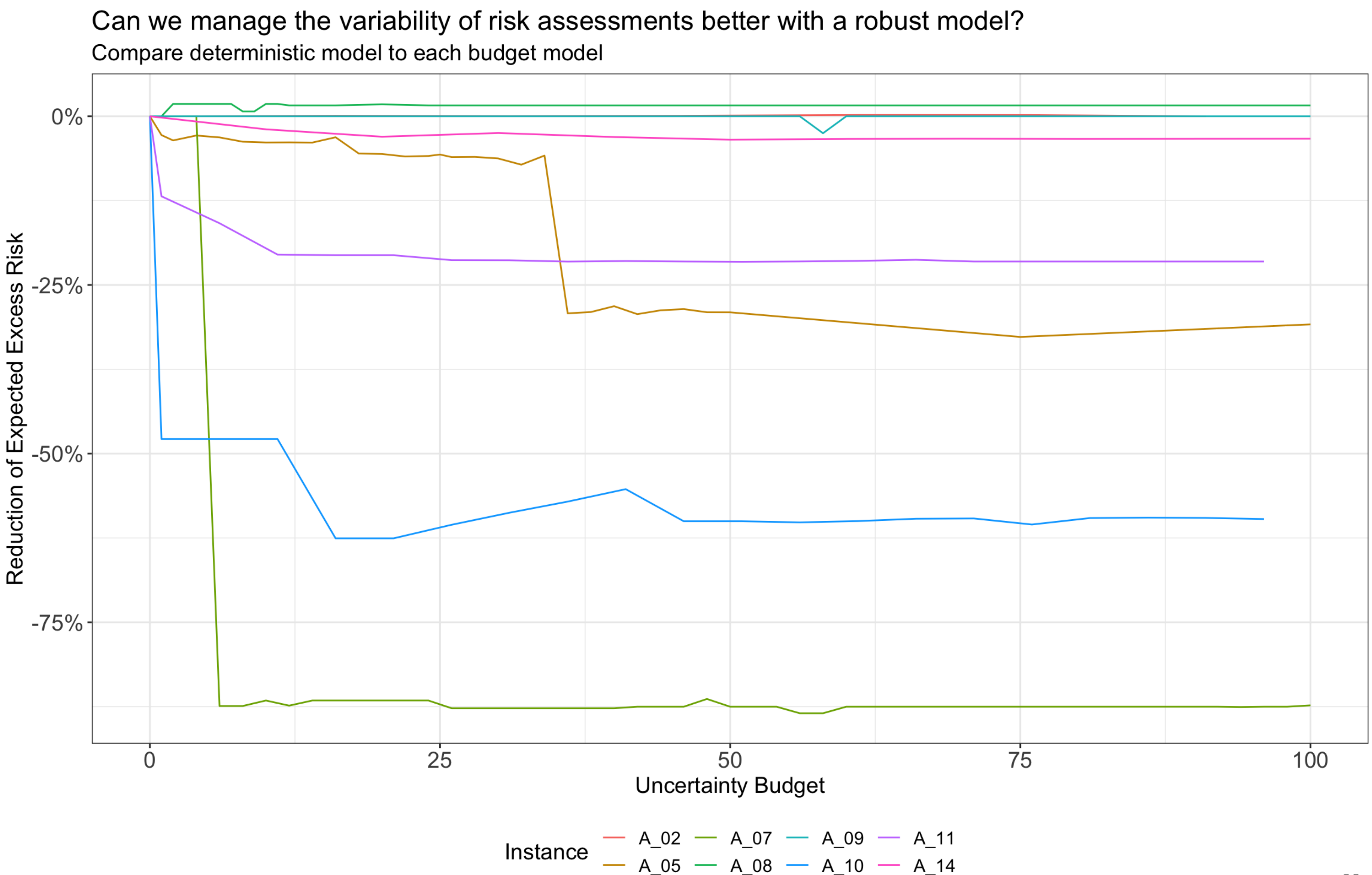
Increased uncertainty budget increases size of model.

How long does it take to find a solution?



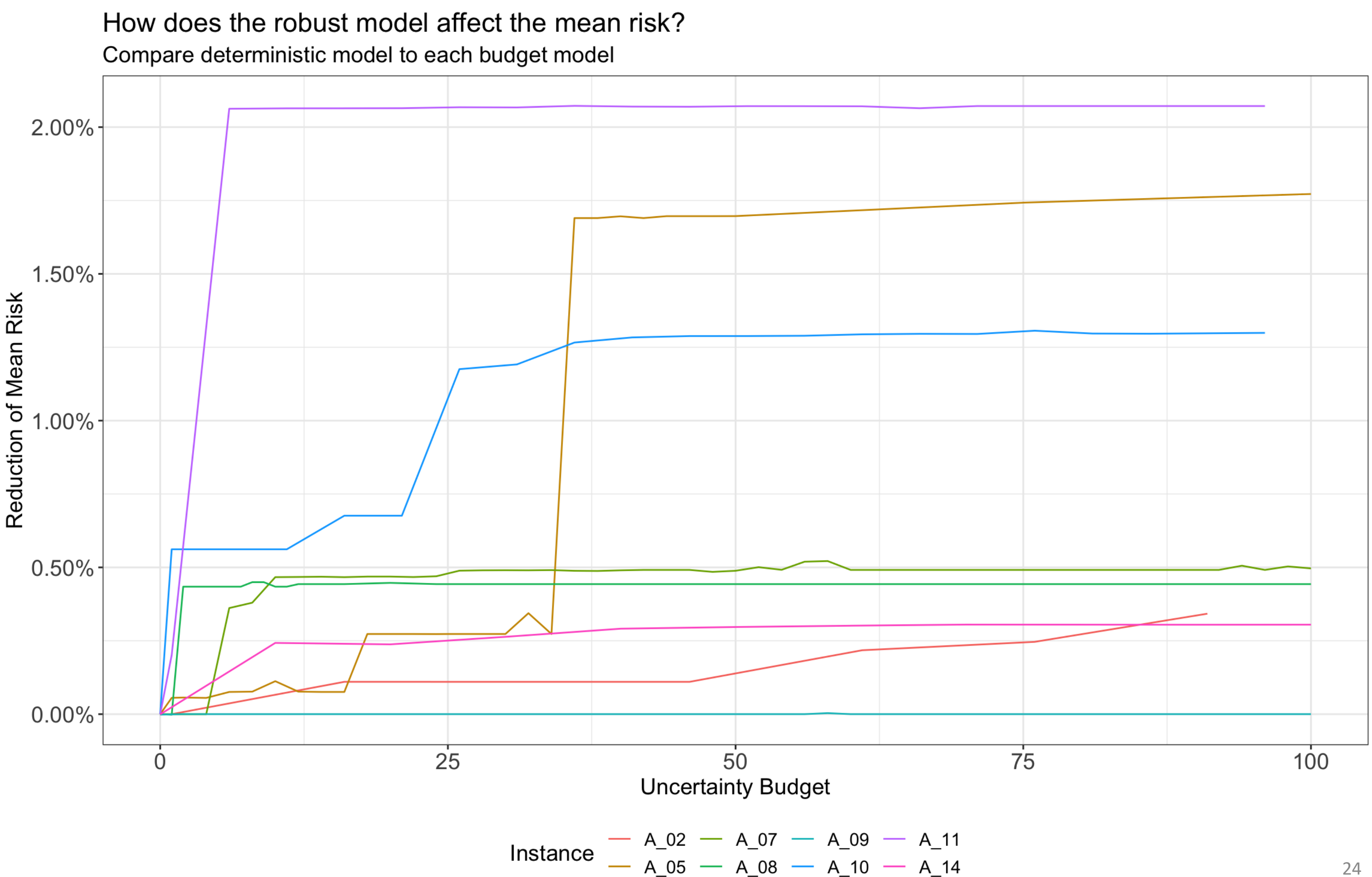
15 minute time limit

Can we manage risk assessment variance?



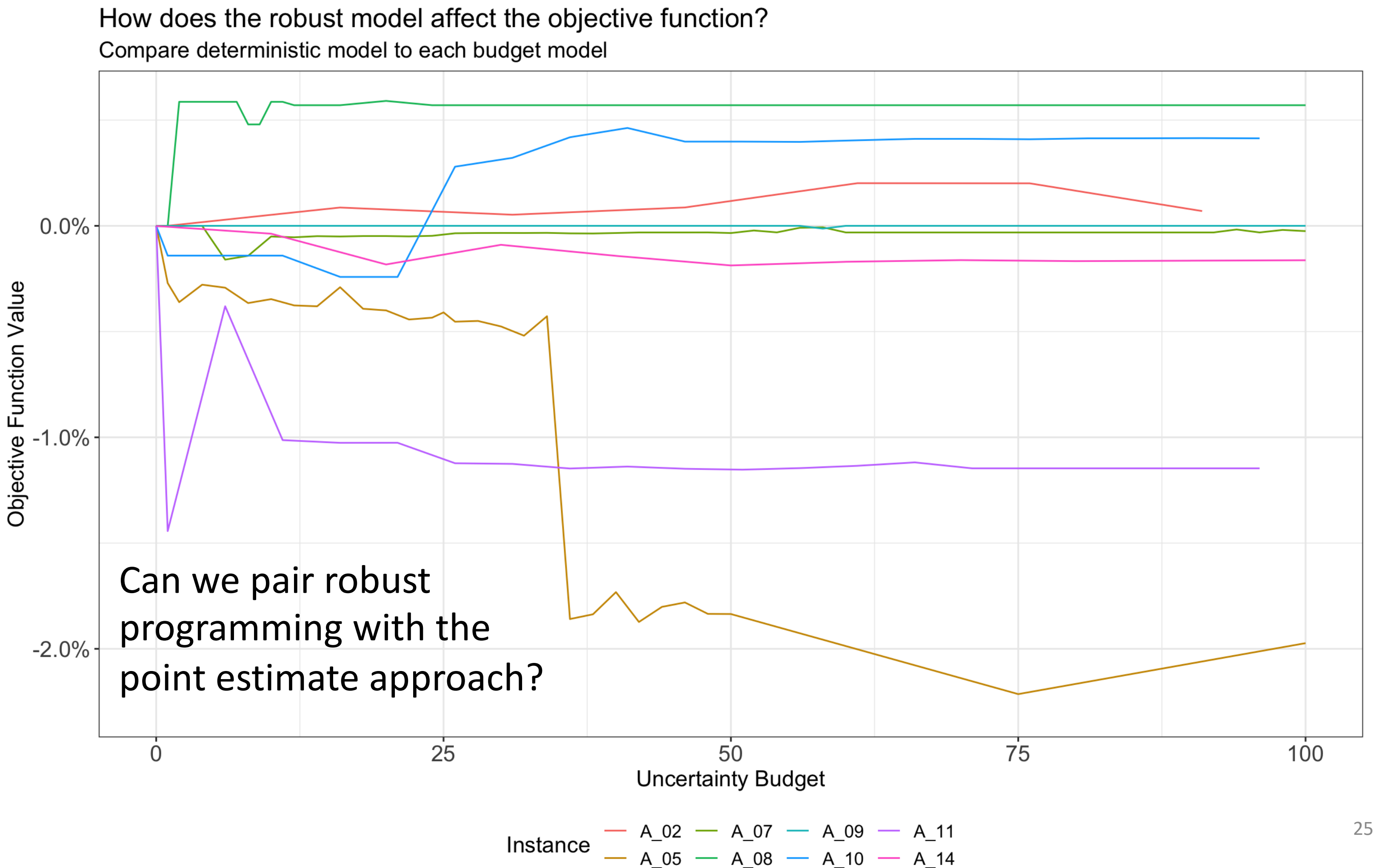
Negative indicates robust model successfully controlled variability

Does the robust model affect average risk?

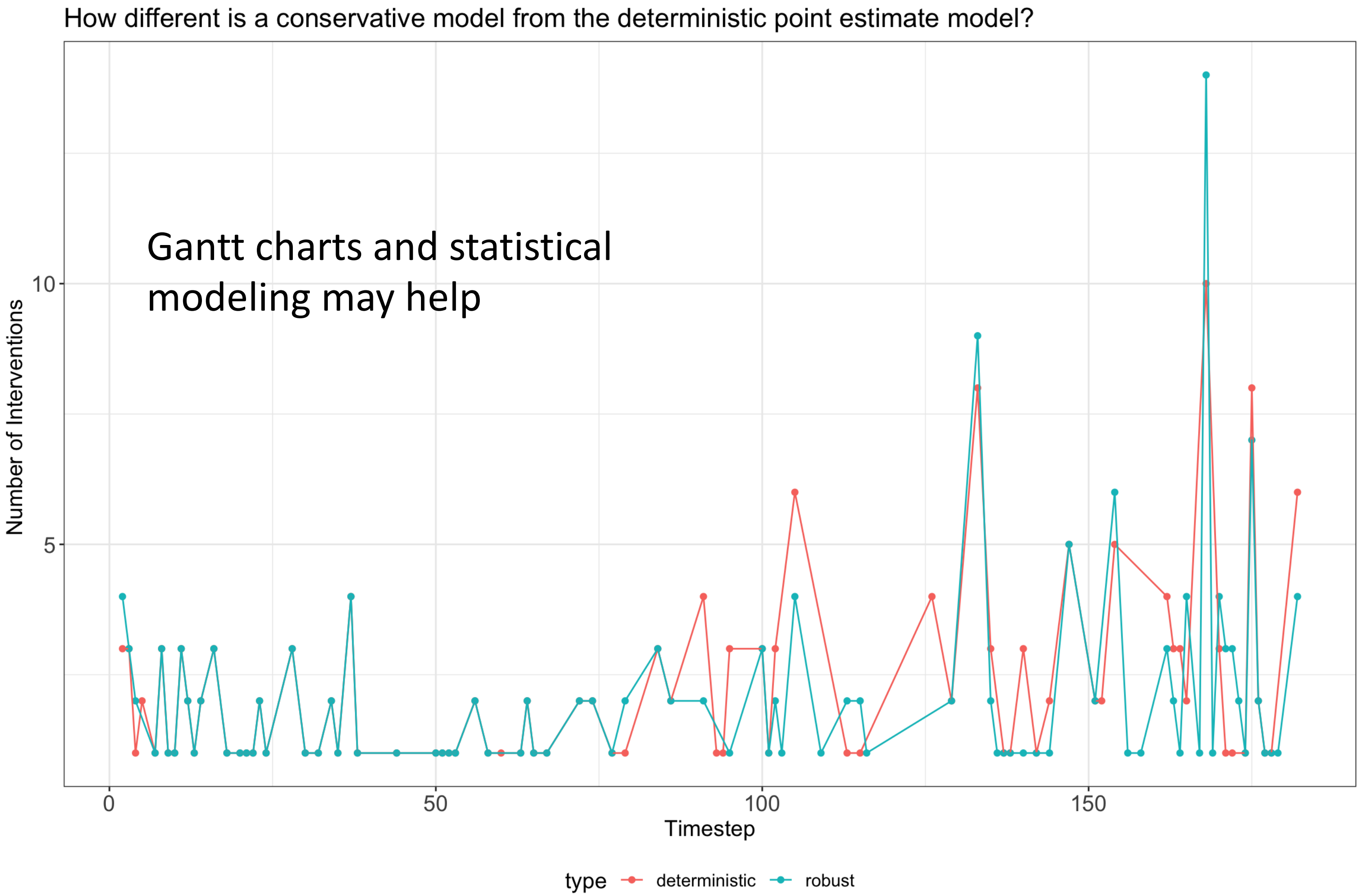


Negative indicates robust model improved mean risk component of objective

How does the robust model affect the objective function?



What do robust decisions look like?



How can we improve?

- Advantages
  - Leverages scenario information
  - Probabilistic behavior rather than worst case
    - “I’m ok with 45% chance of underperforming.”
  - Relative slow complexity growth with number of scenarios
  - Avoid nonlinear IP (Bertsimas and Sim)
    - Mean-risk formulation associated with portfolios and FE
    - Ben-Tal and Nemirovski nonlinear Robust IP counterpart
  - Leverage off-the-shelf solvers
  - Extends to multi-stage decision making
- Disadvantages
  - Proxy objective for excess
  - Still not addressing both objectives simultaneously
  - (Expensive?) Parameter tuning

Next Steps

- Apply model to remaining 3 examples
- Statistical modeling
  - Multiple objectives + Multiple problem dimensions + Parameter tuning
- Reformulation
  - Aggregated vs. disaggregated constraints
  - Decomposition?
  - Exploit binary variables in robust formulation
  - Symmetry?
- Heuristics
  - Local search
  - Hybrid algorithms
- Backtesting
  - In-sample and out-of-sample
  - Distribution of objective and components

Conclusion

- Created models which quickly and effectively produce schedules
  - Deterministic MILP + Point Estimate: <10s solutions
  - Robust MILP: Manages variability in <15min
  - Composable pieces for a larger solution
    - Stage B awaits…
- Conducted sensitivity analysis of key parameters
- Roadmap for model evolution

References

- Bertsimas, Dimitris, and Melvyn Sim. "The price of robustness." *Operations research* 52, no. 1 (2004): 35-53.
- Li, Zukui, and Marianthi G. Ierapetritou. "Robust optimization for process scheduling under uncertainty." *Industrial & Engineering Chemistry Research* 47, no. 12 (2008): 4148-4157.

What's the relationship between the uncertainty budget Γ and the probability of underperforming?

$$P\left(\sum_m \tilde{a}_{lm} x_m > p_l\right) \le \frac{1}{2^n}\left\{(1-\mu)\sum_{l=\lfloor v \rfloor}^{n} \binom{n}{l} + \mu \sum_{l=\lfloor v \rfloor+1}^{n} \binom{n}{l}\right\} \le$$

$$(1-\mu)C(n,\lfloor v \rfloor) + \sum_{k=\lfloor v \rfloor+1}^{n} C(n,k)$$

where

$$n = |M_l|,\, v = \frac{\Gamma_l + n}{2},\, \mu = v - \lfloor v \rfloor$$

$$C(n,k) = \begin{cases} \frac{1}{2^n} & (\text{if } k=0 \ \text{ or } k=n) \\ \frac{1}{\sqrt{2\pi}}\sqrt{\frac{n}{(n-k)k}} & \\ \exp\left\{n \log\left[\frac{n}{2(n-k)}\right] + k\log\left(\frac{n-k}{k}\right)\right\} & (\text{otherwise}) \end{cases}$$