

```
1 import components.naturalnumber.NaturalNumber;
5
6 /**
7  * Program with implementation of {@code NaturalNumber} secondary
  operation
8  * {@code root} implemented as static method.
9  *
10 * @author Isaac Frank
11 *
12 */
13 public final class NaturalNumberRoot {
14
15     /**
16      * Private constructor so this utility class cannot be
  instantiated.
17      */
18     private NaturalNumberRoot() {
19     }
20
21     /**
22      * Updates {@code n} to the {@code r}-th root of its incoming
  value.
23      *
24      * @param n
25      *      the number whose root to compute
26      * @param r
27      *      root
28      * @updates n
29      * @requires  $r \geq 2$ 
30      * @ensures  $n^r \leq \#n < (n + 1)^r$ 
31      */
32     public static void root(NaturalNumber n, int r) {
33         assert n != null : "Violation of: n is not null";
34         assert r >= 2 : "Violation of: r >= 2";
35
36         // initial bounds for the interval
37         NaturalNumber lowEnough = new NaturalNumber2(0);
38         n.increment();
39         NaturalNumber tooHigh = new NaturalNumber2(n);
40         n.decrement();
41
42         // finding the average of lowEnough and tooHigh to halve
  the interval
43         NaturalNumber two = new NaturalNumber2(2);
```

```

44     NaturalNumber midInterval = new NaturalNumber2();
45     midInterval.add(lowEnough);
46     midInterval.add(tooHigh);
47     midInterval.divide(two);
48     NaturalNumber guess = power(midInterval, r);
49
50     midInterval.increment();
51     // iterating through until midInterval = the rth root of n
52     while (guess.compareTo(n) > 0
53         || power(midInterval, r).compareTo(n) <= 0) {
54         midInterval.decrement();
55
56         // halving the interval
57         if (guess.compareTo(n) <= 0) {
58             lowEnough.transferFrom(midInterval);
59         } else {
60             tooHigh.transferFrom(midInterval);
61         }
62
63         // resetting midInterval
64         midInterval = new NaturalNumber2(tooHigh);
65         midInterval.add(lowEnough);
66         midInterval.divide(two);
67         guess = power(midInterval, r);
68         midInterval.increment();
69     }
70
71     // changing n to be the rth root of #n
72     midInterval.decrement();
73     n.transferFrom(midInterval);
74 }
75
76 /**
77  * Returns a NaturalNumber, the power of {@code n} to the
78  * {@code p}-th
79  * power.
80  *
81  * @param n
82  *         the base of the power
83  * @param p
84  *         the exponent of the power
85  * @return n raised to the p power
86  * @requires p >= 0
87  * @ensures n = #n and power = n ^ p

```

```
87     */
88     public static NaturalNumber power(NaturalNumber n, int p) {
89
90         NaturalNumber ans = new NaturalNumber2(1);
91
92         // recursive call if p > 1
93         if (p > 1) {
94             ans = power(n, p / 2);
95             ans.multiply(power(n, p / 2));
96         }
97
98         if (p % 2 != 0) {
99             ans.multiply(n);
100         }
101         return ans;
102     }
103
104     /**
105     * Main method.
106     *
107     * @param args
108     *         the command line arguments
109     */
110     public static void main(String[] args) {
111         SimpleWriter out = new SimpleWriter1L();
112
113         final String[] numbers = { "0", "1", "13", "1024",
114 "189943527", "0",
115 "1", "13", "4096", "189943527", "0", "1", "13",
116 "1024",
117 "189943527", "82", "82", "82", "82", "82", "9",
118 "27", "81",
119 "243", "143489073", "2147483647", "2147483648",
120 "9223372036854775807", "9223372036854775808",
121 "618970019642690137449562111",
122 "162259276829213363391578010288127",
123 "170141183460469231731687303715884105727" };
124         final int[] roots = { 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 15,
15, 15, 15, 15,
2, 3, 4, 5, 15, 2, 3, 4, 5, 15, 2, 2, 3, 3, 4, 5,
6 };
125         final String[] results = { "0", "1", "3", "32", "13782",
"0", "1", "2",
"16", "574", "0", "1", "1", "1", "3", "9", "4",
```

```
125     "3", "2", "1",
126         "3", "3", "3", "3", "3", "46340", "46340",
127         "2097151", "2097152",
128         "4987896", "2767208", "2353973" };
129
130     for (int i = 0; i < numbers.length; i++) {
131         NaturalNumber n = new NaturalNumber2(numbers[i]);
132         NaturalNumber r = new NaturalNumber2(results[i]);
133         root(n, roots[i]);
134         if (n.equals(r)) {
135             out.println("Test " + (i + 1) + " passed: root(" +
136                 numbers[i]
137                 + ", " + roots[i] + ") = " + results[i]);
138         } else {
139             out.println("*** Test " + (i + 1) + " failed:
140                 root("
141                 + numbers[i] + ", " + roots[i] + ")
142                 expected <"
143                 + results[i] + "> but was <" + n + ">");
144         }
145     }
146     out.close();
147 }
```