NATURE AND SCOPE OF INVESTMENT AND PORTFOLIO MANAGEMENT

NATURE

An investment is the current commitment of money for a period of time to derive future payments that will compensate the investor for time the funds are committed, the expected rate of inflation and the uncertainty of the future payments. The investments include securities (shares, bonds and other securities) and assets (e.g., real estate).

A portfolio is a combined holding of more than one stock, bond, money market instrument, commodity, collectible, or real estate investment. An individual investor might have a portfolio that includes several of these investments.

Portfolio management on the other hand is a process of encompassing many activities of investment in assets and securities. The portfolio management includes the planning, supervision, timing, in the selection of securities to meet investor's objectives. It is the process of selecting a list of securities that will provide the investor with a maximum yield constant with the risk he wishes to assume. Investors invest their funds in a portfolio expecting to get a good return consistent with the risk that he has to bear. The return realized from the portfolio has to be measured and the performance of the portfolio has to be evaluated. Portfolio management will comprise all the processes involved in the creation and maintenance of an investment portfolio. It deals specially with security analysis, portfolio analysis, portfolio selection, portfolio revision and portfolio evaluation. Portfolio management makes use of analytical techniques of analysis and conceptual theories regarding rational allocation of funds.

SCOPE

Portfolio management is a continuous process. It is a dynamic activity. The following are the basic operations of a portfolio management.

i) Identification of the investor's objective, constraints and preferences
ii) Implementation of the strategies in tune with investment objectives.
iii) Monitoring the performance of portfolio by incorporating the latest market conditions.
iv) Making revision in the portfolio.
v) Making an evaluation of portfolio income (comparison with targets and achievement).

Objectives

Safety of the investment

The first important objective of a portfolio, no matter who owns it, is to ensure that the investment is absolutely safe. Other considerations like income, growth, etc., only come into the picture after the safety of the investment is ensured.
Investment safety or minimization of risks is one of the important objectives of portfolio management. There are many types of risks, which are associated with investments. Relatively low risk investment gives correspondingly lower returns. You can try and minimize the overall risk or bring it to an acceptable level by developing a balanced and efficient portfolio. A good portfolio of growth stocks satisfies the entire objectives outline above.

Stable Current Return

Once investment safety is guaranteed, the portfolio should yield a steady current income. The current returns should at least match the opportunity cost of the funds of the investor. The returns relate to current income by way of interest or dividends, not capital gains.

Marketability

A good portfolio consists of investment, which can be marketed without difficulty. If there are too many unlisted or inactive shares in your portfolio, you will face problems in encashing them, and switching from one investment to another. It is desirable to invest in companies listed on major stock exchanges, which are actively traded.

Tax Planning
Since taxation is an important variable in total planning, a good portfolio should enable its owner to enjoy a favorable tax shelter. The portfolio should be developed considering not only income tax, but capital gains tax, and gift tax, as well. What a good portfolio aims at is tax planning, not tax evasion or tax avoidance.

Appreciation in the value of capital
A good portfolio should appreciate in value in order to protect the investor from any erosion in purchasing power due to inflation. In other words, a balanced portfolio must consist of certain investments, which tend to appreciate in real value after adjusting for inflation.

Liquidity
The portfolio should ensure that there are enough funds available at short notice to take care of the investor's liquidity requirements. It is desirable to keep a line of credit from a bank for use in case it becomes necessary to participate in right issues, or for any other personal needs.

EQUITY MARKETS

Legally, holders of a corporation's common stock or equity have an ownership stake in the issuing firm that reflects the percentages of the corporation's stock they hold. Specifically corporate stockholders have the rights to a share in the issuing firm's profits after the payment of interest to bond holders and taxes.

Common stock

Common stock is the fundamental ownership claim in a public corporation. There are many characteristics of common stock that differentiate it from other types of financial securities (e.g. bonds, mortgages preferred stock). These include discretionary dividend payment, residual claim status, limited liability and voting rights.

Dividends
While common stock holders can potentially receive unlimited dividends payment if the firm highly profitable, they have no special or guaranteed dividends rights. Rather that payment and the size of dividends are determined by the board of directors or of the issuing firm (who are elected by the common stockholders). Further unlike interest payments on debt a corporation does not default if it misses a dividend payment to common stockholders. Thus common stockholders have no legal recourse to use if dividends era not received, even if a company is highly profitable and chooses to use these profit to reinvest in new projects and firm growth.

Residual claim
In the event of liquidation common stockholders have the lowest priority in terms of any cash distribution.
Limited liability
No matter what financial difficulties the issuing corporation encounters, neither it nor its creditors can seek repayment from the firm common stockholders. This implies that common stockholders, losses are limited to the original amount of their investment.
Voting rights
Fundamental privilege assigned to common stock is voting rights while common stockholders do not exercise control over the firm daily activities these activates these activities are overseen by managers hired to acts the best interests of the firm's common over the firm's activities indirectly through the election of the board of directors.

Straight voting: The vote on the board of directors occur one director at a time. Thus, the number of votes eligible for each director is the number of shares outstanding. Straight voting results in a situation in which, an owner of over half the voting shares can elect the entire board of directors.
Cumulative voting: All directors up for election are voted on at the same time. The number of votes assigned to each stockholder equals the number of shares held multiple by the number of directors to be elected. Cumulative voting permits minority stockholders to have some real say in the election of the board of directors. Since less than a majority of the votes can affect the outcome.
Dual-class firms: Two classes of common stock are outstanding with differential voting right assigned to each class.

Preferred stock
Preferred stock is a hybrid security that has characteristic of both a bond and a common stock. Preferred stock is similar to common stock in that it represents an ownership interest in the issuing firm, but a like a bond it pays fixed periodic (dividends) payment. Preferred stock is senior to common stock but junior to bond.

Buying and Selling of Equities

Primary markets
These are markets in which corporation raise funds through new issues of stocks. The new securities are sold to initial investors (suppliers of funds) in exchange for funds (money) that the issuer (user of funds) needs. As illustrated in the figure below most primary markets transaction go though investment banks, who serve as the intermediary between the issuing corporation (funds users) and ultimate investors (fund supplier) in securities.

Stock market Transaction

These first-time issues are also referred to as initial public offerings (IPOs) Alternatives, a primary market sale may be seasoned offering in which the firm already has shares of the stock trading in the secondary markets.
Also like the primary sales of corporate bond issues corporate stocks may initially be issued through either a public sale (where the stock issue is offered to the general investing public) or a placement.

Preemptive rights
A right of existing stockholders in which new shares must be offered of existing holders first in such a way that they can maintain their proportional ownership in the corporation.
A "right offering" generally allows existing stockholders to purchase shares at a price slightly below the market price. Stockholders can then exercise their rights (buying the allocated shares in the new shares in the new stock) or sell them. The result can be a low cost distribution method of new share a firms (i.e. the issuing firm avoids the expense of an underwritten offering).

Secondary markets
Secondary stock markets are markets which stock once issued are traded – that is bought and sold by investors. The New York stock Exchange (NYSE) and the National Association of Securities Dealers Automated Quotation (NASDAQ) system are well – known examples secondary markets in the stocks. Regionally we have the NSE, JSE etc Public sources of capital must be registered with the securities and exchange commission

(SEC) or capital markets authority (CMA) in the country. They can be traded in the secondary markets like the LSE, NSE etc.

Raising capital from the public (public borrowing)

- Many regulations govern public debt and equity issues in all countries
- Certainly, this regulations increase the cost of borrowing from the public but they provide protection for investments which increases their value
- Requirements for public borrowing are:
  - ✓ Filling a registration statement (prospectus) with the SEC/CMA. This statement indicates:
    - *a)* General information about the firm and detailed financial data
    - *b)* A description of the security being issued
    - *c)* Agreement between the investment bank that acts as an underwriter, who originates and distributes the issue and the issuing firm
    - *d)* The composition of the underwriting syndicate – *(a group of investment banks and commercial banks that sell the issue)*

Once the registration statement is approved the underwriter is free to start selling the securities

The underwriter must perform due diligence *(investigating and disclosing any information that is relevant to investors and providing an audit of the accounting numbers by a CPA firm)*, otherwise they are held liable if the security performs poorly.

Underwriting Process

1. *Origination*

This involves advising the issuing firm, about the *type of security* to issue, the *timing* of the issue and the *pricing* of the issue. It also involves working with the firm to develop its prospectus and syndicating (the lead underwriter performs all these tasks)

2. *Distribution*

Distribution involves selling the issue. Distribution is generally carried but by a syndicate of banks formed by the underwriter.
These banks are listed in the prospectus along with how much of the issue they have agreed to sell. After the SEC/CMA approves the prospectus, the names of the syndicated banks appear on a ***tombstone*** (a public announcement of a new issue of the financial instrument through the financial press).

3. *Risk Bearing*

The underwriter buys the securities from the issuing firm and sells them to the public.
The underwriter is bound to sell the securities at the agreed price and not higher (so the upper margin is limited) if the securities do poorly the underwriter will be stuck with issue that will be sold at a discounted price.

4. *Certification*

Underwriters have to certify the quality of an issue. Investors heavily depend on them for this role.
If an underwriter misprices an issue, their future business will be damaged and may even be sued.

Types of underwriting arrangements

1. *Firm commitment Vs. Best-Efforts Offering*
   - *a. Firm commitment*

The underwriter agrees to buy the whole offering from the firm at a set price and to offer it to the public at a slightly higher price. The underwriter bears the risk of not selling the issue and the firm's proceeds are guaranteed.

   - *b. Best efforts offering*

Underwriter and the firm fix the **price** and the **minimum** and **maximum** of the shares to be sold
The underwriter makes the best effort to sell, investors' deposit payments to the underwriter's escrow account. If the underwriter has not sold the **minimum** number of the security units after a specific time (usually 90 days) the offer is withdrawn and the money refunded to the investors.

2. *Negotiated Vs competitive offering*
    a. *Negotiated offering*

The firm negotiates the underwriting agreement with the underwriter.

    b. *Competitive offering*

The firm specifies the underwriting agreement and puts it out for bidding.

3. *Shelf offerings*

The SEC/CMA may allow firms to register all the securities it plans to issue within 2 years. After which the firm can make offerings of any amount at any time without further notice to the SEC/ CMA. When the need arises, the firm simply puts the securities for bidding (takes the securities off the shelf).

4. *Rights offering*

Rights entitle existing shareholders to buy new shares in the firm at a discounted price.
Right offerings can be made without underwriters or with them on a standby basis to take up unexercised rights and exercise them by paying the subscription price to the firm in exchange for new shares (a subscription price is the price that the right holders must pay for the stock).

RISK AND RETURN
Measurement of rate of return and risk

RETURN
In measuring return we may consider **historical** and **expected** returns

Historical returns
Refers to income received on an investment plus any change in market price of the investment usually expressed as parentage of the beginning market of price of the investment. The period during which one holds an investment is called a ***Holding Period*** the return from the investment during this holding period expressed as an annual percentage is referred to as the ***Holding Period Yield (HPY)*** or ***annual return***

Illustration: Consider an investor that buys a security for 100 Ksh that would pay of 7 Ksh at the end of the year and whose market price at the end of the year would be 106 Ksh. What would be the HPY to the investor? The HPY from this investment comes from two sources; income plus price appreciation (or depreciation)

i.e. $HPY = \frac{7 + (106 - 100)}{100} = 13\%$

Thus, for common stock we can define one period return as:

$$HPY = \frac{D_t + (P_t - P_{t-1})}{P_{t-1}}$$

Where;

HPY = Holding Period Yield (Annual return)
t = Past future period of time
D_t = Cash dividend at time t
P_t = Stocks price at time t
P_{t-1} = Stocks price at time t minus 1

Computing Mean Historical Returns
Given a set of annual rates of return (HPYs) for an individual investment, there are two main summary measures of return performance, the arithmetic mean and the geometric mean

Arithmetic mean
To find the arithmetic mean, the sum of HPYs is divided by the number of years

$$AM = \sum HPY/n$$

Where: $\sum$HPY = the sum of annual holding period yields,
n = number of years

Illustration
Find the arithmetic mean given $HPY_1 = 0.14$; $HPY_2 = 0.12$; $HPY_3 = -0.08$; $HPY_4 = 0.25$; $HPY_5 = 0.02$

$AM = [0.14 + 0.12 + -0.08 + 0.25 + 0.02]/5 = 0.09$

Geometric Return
This is used to calculate the average compound rate of growth that has actually occurred over multiple periods. It reflects the compound rate of growth overtime. This is computed as.

$$= \sqrt[n]{[(1 + R1)\,(1 + R2) \ldots\ldots\ldots..(1 + Rn)]} - 1$$

Illustration

Calculate the Geometric Return using the figures in the above illustration

$$\sqrt[5]{[(1 + 0.14)(1 + 0.12)(1 - 0.08)(1 + 0.25)(1 + 0.02)]} - 1$$

$$\sqrt[5]{[(1.14)(1.12)(0.92)(1.25)(1.02)]} - 1 = 0.0841 \approx 8.41\%$$

Other measures of return are:

Cumulative Wealth Index

Age in total Returns reflect change in the level of wealth. To measure wealth level you must measure cumulative effect of returns over time given some stated initial amount usually Ksh1. The Cumulative Wealth Index measure the cumulative effects of total Returns measured as;

$$CWI_n = WI_0 (1+R_1)(1+R_2) \ldots\ldots\ldots\ldots (1+R_n)$$

Where:

CWI_n = Cumulative Wealth Index at the end of n years

WI_0 = Beginning Index value which is typically Ksh.1

R_i = Total Returns for years 1, 2…………………n

Illustration

Using the returns in the illustration above calculate cumulative wealth index at the end of 5 years assuming a beginning value of Ksh.1

$$CWI_n = WI_0 (1+R_1)(1+R_2) \ldots\ldots\ldots (1+R_5)$$

$$= 1(1 + 0.14)(1 + 0.12)(1 - 0.08)(1 + 0.25)(1 + 0.02)$$

$$= 1.498$$

Ksh1 invested in year 1 will be Ksh. 1.5 at the end of year 5.

Returns Relative

In this method negative returns cannot be used and hence the ending price is compared with the beginning price.

$RR = [C + P_E] / P_B$

Where: C = Cash payment or receipts during the period

P_B = Beginning Price

P_E = Ending Price

OR

$RR = 1 + \text{Total Returns } \%$

Note: the above is referred to as ***Nominal** or **Money Returns*** to make it real returns. It will be adjusted for inflation as follows:

$$\text{Real Return} = \frac{1 + \text{Nominal Returns}}{1 + \text{Inflation Rate}} - 1$$

Suppose in Geometric mean example that, inflation rate is 6% what would be the Real Returns:

$$\text{Real Return} = \frac{1+0.224}{1+0.06} - 1 = 0.1547 = 15.47\%$$

Expected returns
Refers to the weighted average of the possible returns, with the weights being the probabilities of occurrence

Illustration 1: Assume that the economy has 4 equally likely states of outcomes Depression Recession Normal and Boom. This means that each of these states of outcomes have a 0.25 chance of occurrence. Kenya Airways expect that the following returns would be made in the states of Outcomes respectively -20 %10% 30% and 50%.

The expected returns ($\bar{R}$) for the Kenya Airways will be:

$\bar{R}$ = *(0.25× -20) + (0.25× 10) + (0.25× 30+ (0.25× 50) =17.5%. Alternatively;*

$$\bar{R} = \frac{-20+10+30+50}{4} = 17.5$$

This Illustration reveals that expected returns are given by the formula:

$$\bar{R} = \sum_{i=1}^{n} (Ri)(Pi)$$

Where; R_i = Return for the i^{th} possibility (outcome)
P_i = Probability of that return occurring
n = Total number of possibilities (outcomes)

Illustration 2: If the weights for the possibilities (states of outcomes) were to change to 0.1, 0.2, 0.6 and 0.1 respectively for Kenya Airways what be the expected return?

$$\bar{R} = (-20 \times 0.1) + (10 \times 0.2) + (30 \times 0.6) + (50 \times 0.1) = 23\%$$

RISK
Refers to variability of actual returns from those expected. Consider two individuals A and B, A invests in one year Treasury bill to pay 8% and B invests in common shares at 30 Ksh. A is guaranteed by the government to receive a return of 8% at the end of the year. B is not guaranteed of any divided at the end of the year and the price of the share may be lower than 30 Ksh more than 30 Ksh. For individual A, the expected return (8 %) will be received *(expected return = actual return)*.For individual B the expected return may not equal the actual return. Therefore A is said to have invested in a risk less (risk free) asset while B has invested in a risky asset. The greater the variability of actual returns from expected returns the riskier the security said to be.

Measuring Risk
Risk has been defined as the variability of actual return from those that are expected. Variability means spread or dispersion of outcomes. The conventional measure of dispersion is the standard deviation. The greater the standard deviation of returns, the greater the variability of returns, and thus, the greater the risk of an investment Mathematically standard deviation (δ) can be expressed as:

$$\delta = \sqrt{\sum_{i=1}^{n} (Ri - \bar{R})^2 (Pi)}$$

Illustration: Using data from Illustration 1 of KQ, calculate its standard deviation

$$\bar{R} = 17.5\% = 0.175$$

R = -20% (-0.2), 10% (0.1), 30% (0.3), and 50% (0.5) repressively

P_i	R_i	$\overline{R}$	$R_i - \overline{R}$	$(R_i - \overline{R})^2$	$(R_i - \overline{R})^2 (P_{i)}$
0.25	-0.2	0.175	-0.375	0.140625	0.03515625
0.25	0.1	0.175	-0.075	0.005625	0.00140625
0.25	0.3	0.175	0.125	0.015625	0.00390625
0.25	0.5	0.175	0.325	0.105625	0.02640625
$\sum$					0.066875

(δ^2)Variance = 0.066875

$\delta = \sqrt{0.066875}$

= <u>0.2586</u>

Similarly consider Safaricom operating the in same possible states of outcomes but with the following expected return respectively, 5%, 20%, -12% and 9%. What is the standard deviation? Expected returns for SF are:

$$\overline{R} = \frac{5+20-12+9}{4} = 5.5\%(0.055)$$

Standard deviation is calculated as;

P_i	R_i	$\overline{R}$	$R_i - \overline{R}$	$(R_i - \overline{R})^2$	$(R_i - \overline{R})^2 (P_{i)}$
0.25	0.050	0.055	-0.005	0.000025	0.00000625
0.25	0.20	0.055	0.145	0.021025	0.00525625
0.25	-0.120	0.055	-0.175	0.030625	0.00765625
0.25	0090	0.055	0.035	0.001225	0.00030625
$\sum$					0.013225

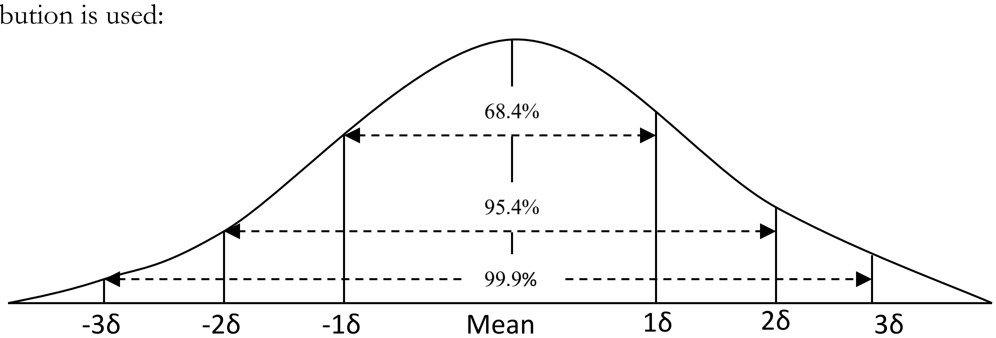
(δ^2) Variance = 0.013225

$\delta = \sqrt{0.013225}$

= <u>0.115</u>

Normal distribution

In order to better under and the application of standard derivation in measuring risk, the concept of the normal distribution is used:



For Kenya Airways we are sure that 68% of all the observation will lie between 0.175 – 0.2586 and 0.175 + 0.2586 i.e. $(\overline{X} \pm 1\delta)$. Where, $\overline{X} = \overline{R}(mean)$ (0.175 – 0.2586) is the lower limit while (0.175 + 0.2164) is the upper limit. i.e. the returns of Kenya Airways are expected to lie between: – 0.0836 to 0.4336 (i.e. – 8.4% to 43.4%) if one is 68% confident. Similarly, at 68% confidence level, Safaricom is return are expected to lie between $0.055 \pm 1\delta$. This translates to 0.055 – 0.115 to 0.055 + 0.115 = - 0.06 to 0.17 (i.e. – 6% to 17%)

***Note**: The higher the standard deviation (spread) the riskier the venture. In the preceding example, Kenya Airway is riskier (since return vary from – 8.4% to 43.4%) while Safaricom return only vary from – 6% to 17%.*

***Work out the spread for 95.4 and 99.9% confidence level for KQ & SF** *(check: at 95.4%, -0.3422 to 0.6922 for KQ and -0.265 to 0.375 for SF; at 99.9%,-0.6008 to 0.9508 for KQ and -0.29 to 0.4 for SF)*

PORTFOLIO THEORY

Portfolio Theory proposed by Harry Markowitz (1960) was the first formal attempt of quantifying the risk of a portfolio and to develop a methodology of determining optimal portfolio. The means of controlling portfolio risk is called diversification whereby investment is made in a variety of assets so that exposure to any particular type of asset is limited. Markowitz model is based on the following assumptions regarding investor behavior.

i) *Investors consider each investment alternative as being presented by a probability distribution of expected returns over some holding period*

ii) *Investors maximize one period expected utility and their utility curves demonstrate diminishing marginal utility of wealth*

iii) *Investors estimate the risk of the portfolio on the basis of the variability of expected returns*

iv) *Investors base their decisions solely on expected return and risk, so their utility curves are a function of expected returns and the expected variance (or standard deviation) of returns only*

v) *For a given level of risk, investors prefer higher return to lower returns. Similarly for a given level; of expected returns, investors prefer less risk to more risk.*

Portfolio Expected Returns and Risk

Portfolio Return

The expected return of a portfolio is simply a weighted average of the expected return of the securities comprising that portfolio. The weights are equal to the proportion of total funds invested in each security (the weights must sum to 100%)

Illustration: Assume an investor, invests 25% of his/her funds in Kenya Airways and 75% in Safaricom what are the returns of the portfolio?

$\overline{R}$ for KQ = 17.5% and $\overline{R}$ for SF = 5.5%

Proportion of funds in KQ = 25% and Proportion of funds in SF = 75%

Returns

If the proportions are exchanged the returns will be: (17.5 × 0.25) + (5.5 × 0.75) = 8.5% Thus the general formula for the expected return on a portfolio $\overline{R}_p$ is as follows:

$$\overline{R}_p = \sum_{j=1}^{n} W_j \overline{R}_j$$

Where; W_j = Proportion or weight of total funds invested in security j

R_j = Expected return for security j

n= Total number of different securities in the portfolio

Problem 1

Suppose we are planning to invest in the following Assets X and Y and the probability and Returns on the 2 assets are as follows:

Probability	Returns (X) %	Returns (Y) %
0.25	10	-6
0.25	8	23
0.25	20	5
0.25	15	15

Calculate the expected Returns of the portfolio to be invested in X and Y as follows:-

i) 25% in X and 75% in Y

ii) 100% in X and 0 in Y

iii) 50% in X and 50% in Y

Solution:

Problem 2
Suppose the expected returns on asset are ER1= 10%; ER2 = 15%; ER3 = 20%; ER4 = 16%.
If the investor invested in these 4 assets equally, what are the returns of the Portfolio?
Solution:

Portfolio Risk and the Importance of Covariance
When a portfolio of assets is involved, we **CANNOT** take the weighted average of the individual security's standard derivation since this will ignore the relationship (or covariance) between the returns of the securities. Covariance is a statistical measure of the degree to which two variables (e.g. securities returns) move together. A positive value means that on average, they move in the same direction. A negative value means that or average they move in the opposite direction and a zero covariance means that the two variables show no tendency to vary together in either a positive or negative linear fashion. Covariance of two assets is given by the following expression

$$\text{Cov}_{A,B} = \sum_{t=1}^{n}[(R_A - \overline{R_A})(R_B - \overline{R_B})]Pi$$

Consider the KQ and SF example

State of economy	*Pi*	*Return on KQ*	*Deviations on KQ (KQ_D)*	*Return on SF*	*Deviation on SF (SF_D)*	*($KQ_D \times SF_D$)*	*($KQ_D \times SF_D$) (Pi)*
Depression	0.25	-20	-0.375	5	- 0.005	0.001875	0.00046875
Recession	0.25	10	- 0.075	20	0.145	0.010875	-0.00271875
Normal	0.25	30	0.125	-12	- 0.175	0.021875	-0.00546875
Boom	0.25	50	0.325	9	0.035	0.011375	0.00284375
$\sum$						-0.0195	-0.004875

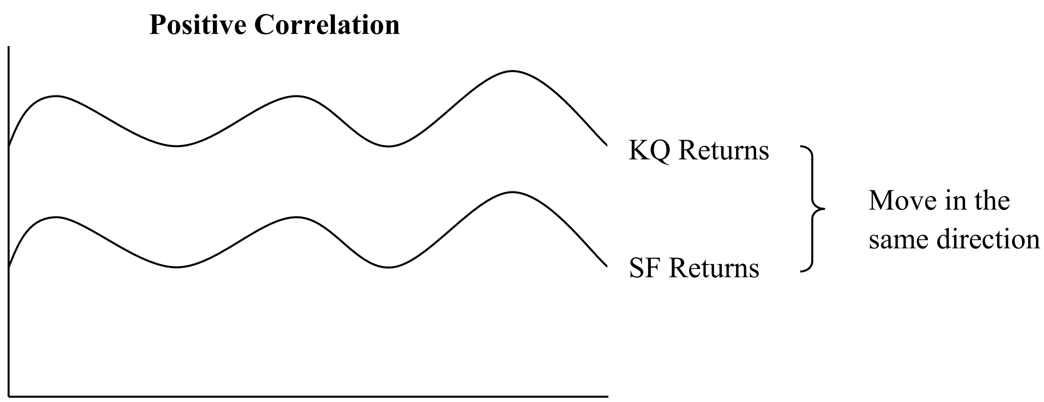
* - 0.195/4 (outcomes) = 0.004875
$\text{Cov}_{KQ, SF}$ = - 0.004875

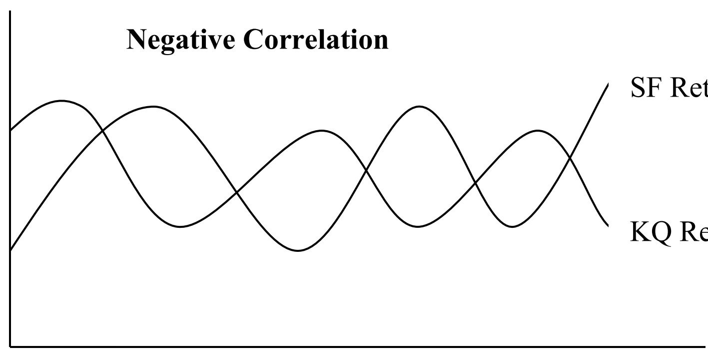
We use the correlation coefficient (**r**) in interpreting the covariance. Correlation coefficient is a standardized statistical measure of the linear relationship between two variables. It ranges from – 1.0 (perfect negative correlation) through 0 (no correlation), to + 1.0 (perfect positive correlation).

Correlation coefficient (**r**) of KQ and SF $\mathbf{r}_{KQ,SF}$, will be given by covariance of KQ and SF divided by their respective standard deviations i.e.

$$r_{KQ,SF} = \frac{\text{cov}_{KQ,SF}}{\delta_{KQ}.\delta_{SF}} = \frac{-0.004875}{0.2586 \times 0.115} = -0.163926157$$

This indicates that there is a weak negative correlation between KQ is returns and SF is returns





Measuring Portfolio Variance

1. *A two Asset Portfolio*

The variance of a two asset portfolio $(\sigma^2{}_P)$ is given by the following expression

$$\sigma_P^2 = X^2\sigma_A^2 + (1-X)^2\sigma_B^2 + 2X(1-X)Cov_{A,B}$$

$$\sigma_p = \left[X^2\sigma^2{}_A + (1-X)^2\sigma^2{}_B + 2X(1-X)Cov_{A,B}\right]^{1/2}$$

Where; σ_P^2 = Portfolio variance

σ_p = Portfolio standard deviation

X = Proportion of funds invested in security A

σ_A^2 = Variance of security A

σ_B^2 = Variance of security B

$Cov_{A,B}$ = Covariance of returns on security A and B, also given by: $Cov_{A,B} = \sigma_B.\sigma_B.r_{A,B}$

Where; σ_A = Standard deviation of security A

σ_B = Standard deviation of security B

$r_{A,B}$ = Correlation coefficient of A and B

Note*: The square root of portfolio variance gives portfolio Standard deviation (risk)*

Illustration: What is the portfolio risk of a portfolio comprising KQ and SF assuming 75% and 25% investment respectively?

$\sigma_{KQ} = O.2586$, $\sigma_{SF} = 0.115$, $Cov_{KQ,SF} = -0.004875$ and X = 0.75

$$\sigma_P^2 = (0.75)^2(0.2586)^2 + (0.25)^2(0.115)^2 + 2(0.75)(0.25)(-0.004875)$$

= 0.0376166025 + 0.000826562 – 0.001828125

= 0.036615039

$$\sigma = \sqrt{0.036615039} = 0.191350567$$

What is the spread of outcomes at 68% confidence level?

Recall R_p = 14.5%

$\sigma_P \approx 0.19$

@ 68% = $\pm 1\sigma$ from the mean $(\overline{R})$

∴ Spread = 14.5 ± 0.19
= 0.145 – 0.19 to 0.145 + 0.19
= - 0.045 to 0.335

Returns vary from – 4.5% to 33.5%

****Insight***
How does this spread of the portfolio compare with the individual spreads of KQ and SF?

Problem
Suppose you want to invest in two securities X and Y in the following proportions and returns are as follows:

Probability	Returns (X) %	Returns (Y) %
0.2	11	-3
0.2	9	15
0.2	25	2
0.2	7	20
0.2	-2	6

Calculate the portfolio return and risk if the proportions invested in X and Y are:

X	Y
100%	0%
75%	25%
50%	50%

Solution:

Minimum Variance Portfolio

Most investors and portfolio managers invest in 2 broad categories, namely bonds and stocks. The general equation for the weight of the first security to achieve minimum variance (in a two asset portfolio is given by:

$$W_{b\,(min)} = \frac{\sigma_s^2 - \sigma_b\sigma_s rbs}{\sigma_b^2 + \sigma_s^2 - 2\sigma_b\sigma_s rbs} = \frac{\sigma_s^2 - Cov_{bs}}{\sigma_b^2 + \sigma_s^2 - 2Cov_{bs}}$$

Illustration

Suppose the expected return (ER) of bond is 8% ER of stock is 15% σ_{bond} = 10% σ_{stock} = 20%. Find the ER of portfolio (ER_P) and standard deviation of portfolio consisting of bonds and stocks.

$$\boldsymbol{ER_P} = (W_b \times 8) + (W_s \times 15)$$

$$= 8W_b + 15W_s$$

$$\sigma_P^2 = W_b^2 \times 10^2 + W_s^2 20^2 + 2 \times W_b \times W_s \times 10 \times 20 \times r_{b,s}$$

$$= 100 W_b^2 + 400\ W_s^2 + 400 W_b W_s \times r_{bs}$$

Find the minimum variance portfolio for correlation of -1, 0 and 0.5

i) Portfolio weights at r_{bs} = -1

$$W_{b\,(min)} = \frac{20^2 - (10 \times 20 \times -1)}{10^2 + 20^2 - (2 \times 10 \times 20 \times -1)} = \frac{600}{900} = 0.67$$

$W_s = 1 - W_b = 1 - 0.67 = 0.33$

ii) Portfolio weights at r_{bs} = 0

$$W_{b\,(min)} = \frac{20^2 - (10 \times 20 \times 0)}{10^2 + 20^2 - (2 \times 10 \times 20 \times 0)} = \frac{400}{500} = 0.8$$

$W_s = 1 - W_b = 1 - 0.8 = 0.2$

iii) Portfolio weights at r_{bs} = 0.5

$$W_{b\,(min)} = \frac{20^2 - (10 \times 20 \times 0.5)}{10^2 + 20^2 - (2 \times 10 \times 20 \times 0.5)} = \frac{300}{300} = 1$$

$W_s = 1 - W_b = 1-1 = 0$

i) ER_P and minimum portfolio variance at r_{bs} = -1

$ER_P = (0.67 \times 8) + (0.33 \times 15) = 10.31$

$$\sigma_p^2 = 100 \times 0.67^2 + 400 \times 0.33^2 + 400 \times 0.67 \times 0.33 \times -1 = 0.01$$

ii) ER_P and minimum portfolio variance at r_{bs} = 0

$ER_P = (0.8 \times 8) + (0.2 \times 15) = 9.4$

$$\sigma_p^2 = (100 \times 0.8^2) + (400 \times 0.2^2) + (400 \times 0.8 \times 0.2 \times 0) = 80$$

iii) ER_P and minimum portfolio variance at r_{bs} = 0.5

$ER_P = (1 \times 8) + (0 \times 15) = 8$

$$\sigma_p^2 = (100 \times 1^2) + (400 \times 0^2) + (400 \times 0.8 \times 0.2 \times 0.5)$$

$=132$

The summary is given in the table below

r_{bs}	**-1**	**0**	**0.5**
$W_{b(min)}$	0.67	0.8	1
W_s	0.33	0.2	0
ER_P	10.31	9.4	8
σ_p^2	0.01	80	132

2. Multi Assets Portfolio

We will limit ourselves to a 3 asset portfolio due by the intensity of the calculations involved. The general formula for portfolio variance is given by:

$$\sigma^2{}_p = \sum_{j=1}^{m} \sum_{k=1}^{m} W_j W_k \sigma_{jk}$$

Where; m is the total number of different securities

W_j, *Proportion* of funds invested in security j

W_k, Proportion of funds invested in security k

δ_{jk}, Covariance of returns security j & k

Double summation $(\sum\sum)$ means we sum across all elements within a square matrix (m by m)

Illustration: An investment portfolio consists of share of company N, P and Q in the respective proportions 20, 70 and 10%. The following are the characteristics of shares of company N, P and Q.

Company	Portfolio proportion	$\overline{R}_j$	δ_i	r_{jk} (correlation of *j & k*)
N	0.2	20%	6%	NN= +1 NP = + 0.7 NQ = + 0.4
P	0.7	30%	10%	PN = +0.7 PP = +1 PQ =+ 0.8
Q	0.1	12%	2%	QQ = +1 QP = + 0.8 QN = + 0.4

Calculate the portfolio risk and portfolio return

$$\sigma_p = [X^2\sigma^2{}_N + Y^2\sigma_P^2 + Z^2\sigma_Q^2 + 2XY\sigma_N\sigma_P r_{N,P} + 2XZ\sigma_N\sigma_Q r_{NQ} + 2YZ\sigma_P\sigma_Q r_{PQ}]^{1/2}$$

Where X, Y and Z are proportion of funds invested in security N, P & Q respectively.
And, *r* is the correlation sign

$\sigma_p =$ $[(0.2^2 \times 6^2) + (0.7^2 \times 10^2) + (0.1^2 \times 2^2) + (2 \times 0.2 \times 0.7 \times 6 \times 10 \times 0.7) + (2 \times 0.2 \times 0.1 \times 6 \times 2 \times 0.4) + (2 \times 0.7 \times 0.1 \times 10 \times 2 \times 0.8)]^{0.5}$

$= [1.44 + 49 + 0.04 + 11.76 + 0.192 + 2.24]^{0.5}$

$= [64.672]^{0.5}$

$$\sigma_P = \sqrt{64.672} = 8.041893\ \%$$

$$\overline{R}_P = (0.2 \times 20) + (0.7 \times 30) + (0.1 \times 12) = 26.2\%$$

Diversification and Portfolio Risk

The idea diversification is to spread the risk across a number of assets or investment (i.e. don't put all your eggs in one basket)

If one diversifies by investing in several assets irrespective of their covariance (or correlation). He /she is said to have naïve diversification. Meaningful diversification combines securities in a way that will reduce risk. Benefits of diversification in the form of risk reduction occur as long as the securities are not perfectly positively correlated (+1)

Illustration:

Suppose an investor wishes to construct a portfolio which consists of placing 60% of his funds in firm E and the rest in firm F having the following possible return and risk. $E(r_j)$ = 30%, $F(r_j)$ =10%, $E(\sigma) = 12\%$, $F(\sigma) = 3\%$.

Find the expected returns on the portfolio and the portfolio risk assume ***r*** of (a) + 1, (b) + 0.5, (c) 0, (d) -1

$$\overline{R}_P = (0.6 \times 30) + (0.4 \times 10) = 19$$

(a) $\sigma_p = [0.6 \times 12 + 0.4 \times 3 + 2(0.6 \times 0.4 \times 12 \times 3 \times 1)]^{1/2} = 8.4\%$

(b) $\sigma_p = [0.6 \times 12 + 0.4 \times 3 + 2(0.6 \times 0.4 \times 12 \times 3 \times 0.5)]^{1/2} = 7.87\%$

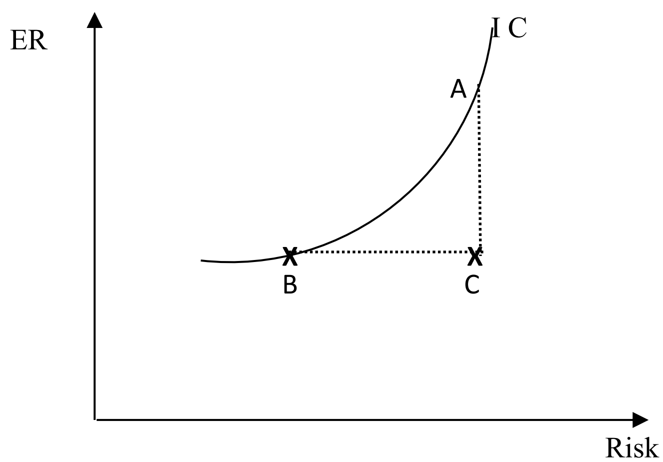
(c) $\sigma_p = [0.6 \times 12 + 0.4 \times 3 + 2(0.6 \times 0.4 \times 12 \times 3 \times 0)]^{1/2} = 7.299\%$

(d) $\sigma_p = [0.6 \times 12 + 0.4 \times 3 + 2(0.6 \times 0.4 \times 12 \times 3 \times -1)]^{1/2} = 6\%$

Note that the risk reduction potential as correlation moves away from + 1 to – 1, Maximum risk reduction is attained when the securities are perfectly negatively correlated (-1)

Indifference Curves (ICs)

When choosing the portfolio, the investor would wish to maximize expected returns (ER) and minimize the risk. The indifference curve for an investor represents the mixture of risk and returns which will be acceptable in terms of an investment. Any point along the indifference curves, the risk returns relationship is the same for the investor and will be indifferent to whether the investment is at point A or B as shown below:



Indifference curves are curved because of the diminishing returns provided as the quantities of risk or return become disproportionate in the mixture towards the extremes of one curve. A will be preferable to C because it offers a higher ER for the same level of risk. A is said to dominate C. B on the other hand will be preferable to C because it offers the same ER at lower risk compared to C. Whether an investor would choose A or B will depend on their attitude to risk. The Markowitz Model of investment analysis seeks to measure investor's utility U as function of risk and expected return $U = F(\bar{R}\sigma)$.

Where: $\bar{R} = Investor's\ E(R)\ from\ the\ investment$

$\sigma = Std.Deviation\ \ or\ Risk\ of\ the\ investment$.

The risk-return utility function of an investor can be exhibited using the following formula:

U = E(R) – ½ A ×Variance

Where:

U is a given level of utility/happiness

A is the level of risk tolerance/averseness

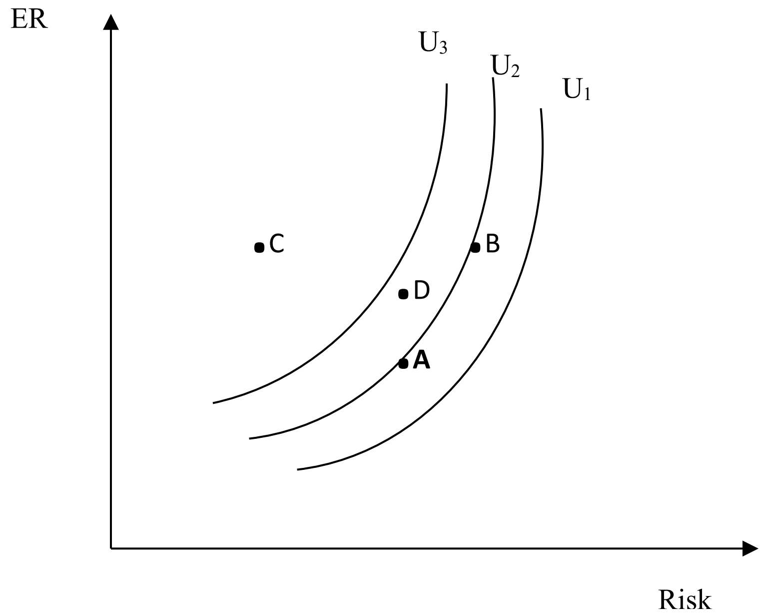
We can make some interesting observations from this function:

Utility increases as expected returns increase

Utility decreases when variance or risk increases

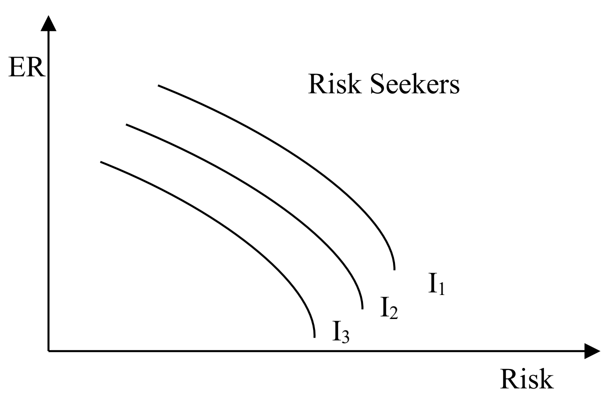
Utility reduces as risk-aversion (A) increases

As the risk aversion increases, an investor demands more return for every unit of increase in risk. When the risk increases, the investor demands more return based on his utility function, thereby keeping the level of utility the same. This concept can be explained with the help of indifference curve. An indifference curve presents the risk-return requirements of an investor at a certain level of utility. The following graph shows three indifference curves for the same investor:

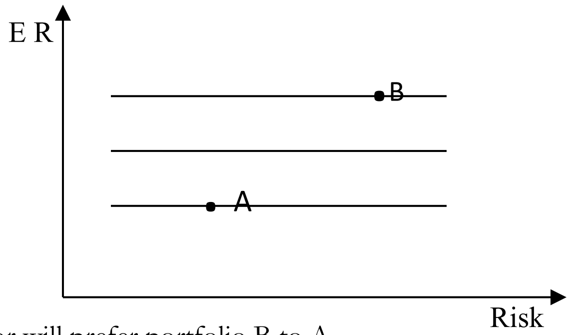


The further the indifference curves goes to the left, the greater will be the value of utility to the investor because these portfolios provides higher ER for the same level of risk or lower risk for the same level of ER. Portfolio's to the left of the curves are preferred because those below are Mean Variance inefficient and those on or above the utility curve are Mean Variance Efficient. Mean Variance Efficient Portfolio's are those which give maximum return for a given level of risk or have the minimum risk for a given level of return.

Markowitz assumption was mainly on risk averse investors. However investors can be risk seeking or risk neutral. Investors who are risk seekers will choose portfolio with higher risks hence suggesting that they will have negative sloping indifference curve which is concave.



The investor will choose I_1which is in the North East indifference curve. The risk averse investor does not want to take a fair gamble while risk seeking investor will take the fair gamble and the risk neutral investor does not care whether the gamble is taken i.e. The risk is unimportant to the risk neutral investors in horizontal lines as shown below:



The investor will prefer portfolio B to A.

The Efficient Frontier

The Efficient Set Theory states that an investor would choose his/her portfolio from set of portfolio's that:

i) Offers maximum ER for varying levels of risks.

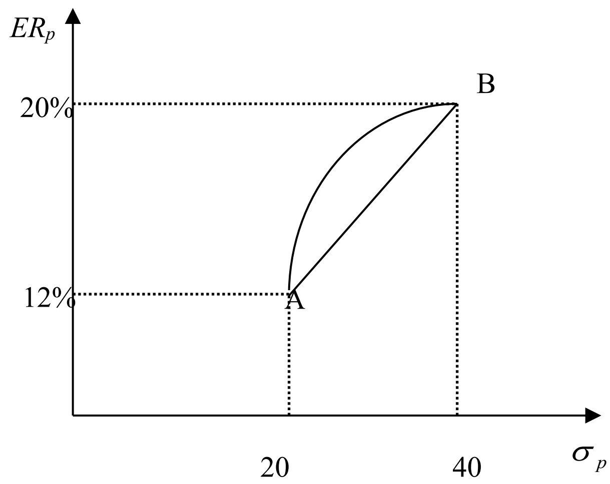
ii) Offers minimum Risk for varying levels of ER.

The set of portfolio's meeting these 2 conditions is known as efficient set also known as efficient frontier. The Feasible Set also known as Opportunity Set is the set from which the efficient set can be identified.

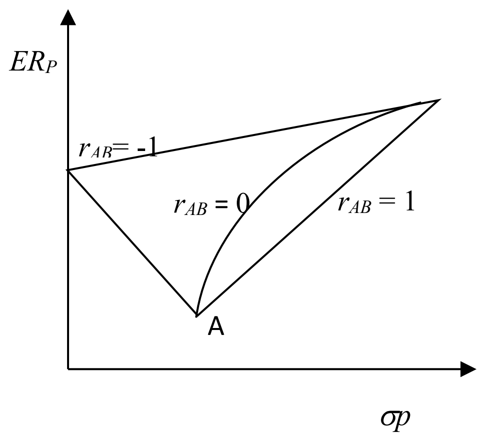
Case 1: Two Securities

Suppose an investor is evaluating 2 securities A and B with ER_A = 12% and ER_B = 20% while $\sigma_A = 20 \text{ and } \sigma_B = 40$. Correlation coefficient between the two securities $r_{A,B}$ = -0.20. These securities can be combined as follows:

Portfolio	W_A	W_B	ER_P	σ_P
1	1	0	12	20
2	0.9	0.1	12.8	17.64
3	0.8	0.2	13.6	16.39
4	0.75	0.25	14	16.27
5	0.7	0.3	14.4	16.52
6	0.5	0.5	16	20.49
7	0.3	0.7	17.6	27.44
8	0.25	0.75	18	29.41
9	0.2	0.8	18.4	31.45
10	0.1	0.9	19.2	35.65
11	0	1	20	40



The feasible frontier for different degree's correlations e.g. -1, 0 and 1 can be illustrated as follows:



When r_{AB} = 1 Diversification does not reduce risk while when r_{AB} = -1 diversification can reduce maximum risk. This relationship can be mathematically determined as follows:

$$\sigma_p^2 = W_A^2 \times \sigma_A^2 + W_B^2 \times \sigma_B^2 + 2 \times W_A \times W_B \times r_{AB} \times \sigma_A \times \sigma_B$$

Since $W_B = (1 - W_A)$

$$\sigma_p^2 = W_A^2 \times \sigma_A^2 + (1 - W_A)^2 \times \sigma_B^2 + 2 \times W_A \times (1 - W_A) \times r_{AB} \times \sigma_A \times \sigma_B$$

$$\sigma_p^2 = W_A^2 \sigma_A^2 + \sigma_B^2 - 2W_A \sigma_B - W_A^2 \sigma_B^2 + 2W_A r_{AB} \sigma_A \sigma_B - 2W_A^2 r_{AB} \sigma_A \sigma_B$$

If r_{AB} = -1 Then the expression can be rearranged and expressed as:

$$= W_A^2 [\sigma_A^2 + 2\sigma_A \sigma_B + \sigma_B^2] - 2W_A [\sigma_A \sigma_B + \sigma_B] + \sigma_B^2$$

$$= W_A^2[\sigma_A + \sigma_B]^2 - 2W_A\sigma_B[\sigma_A + \sigma_B] + \sigma_B^2$$

$$= [W_A(\sigma_A + \sigma_B) - \sigma_B]^2$$

If it can be set to zero:

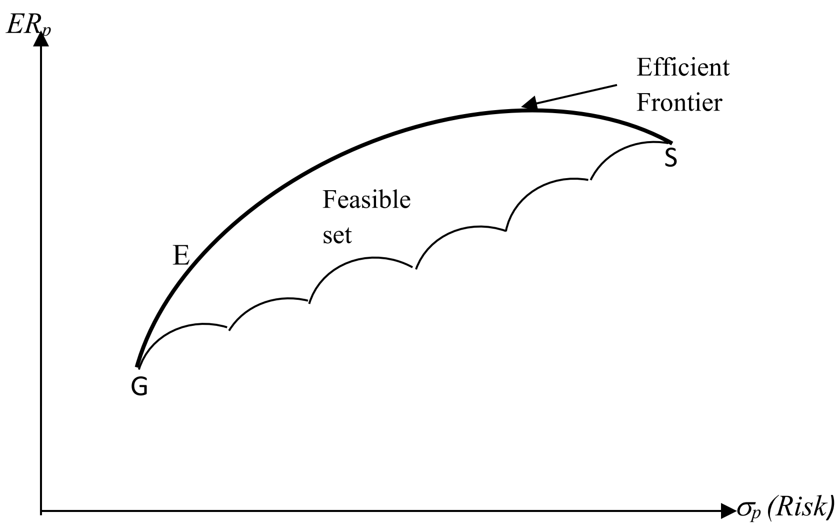
$$W_A = \frac{\sigma_B}{\sigma_A + \sigma_B} \text{ and } W_B = \frac{\sigma_A}{\sigma_A + \sigma_B}$$

Case 2: Efficient Frontier for n Securities

In a 2 security case, a curved line separates all possible portfolios. In a multi-security case, the collection of all possible portfolios is represented by a broken egg shape region referred to as the feasible region. What matters to the investor is the North-West boundary of the feasible region represented by the thick dark line referred to as Efficient Frontier

Efficient frontier contains all the efficient portfolios. A portfolio is efficient if and only if there is no alternative with:

i) The same ER and lower portfolio risk
ii) The same portfolio Risk and a higher ER
iii) A higher ER and a lower portfolio risk. This is as shown below:-



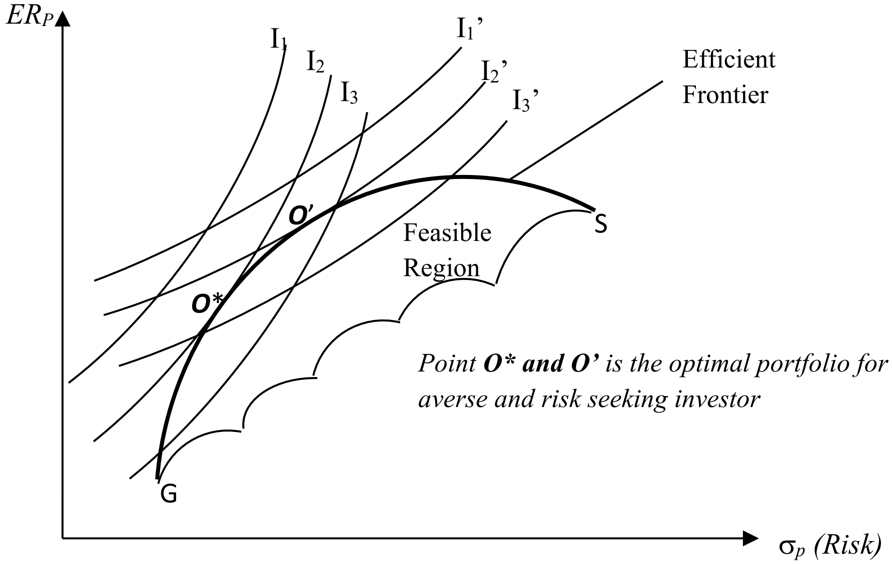
All possible portfolios that could be formed from any securities lie either on or within the boundary of the feasible set. Only those portfolios lying on the North-West boundary between point E and S are referred to as efficient which the risk averse investors find it optimal.

Assignment

Why is the feasible region broken egg shaped?

Selection of optimal Portfolio

The investor selects optimal portfolio by plotting his/her indifference curve on the graph as the efficient set and then choose the portfolio on the furthest North-West indifference curve. This portfolio will correspond to the point at which an indifference curve is just tangent to the efficient frontier as shown below.



*Point **O* and O’** is the optimal portfolio for the risk averse and risk seeking investor*

ASSETS PRICING THEORIES

CAPITAL MARKET THEORY

Capital Market Theory extends portfolio theory and develops a model for pricing all risky assets. Because capital market theory builds on the Markowitz portfolio model, it requires the same assumptions, along with some additional ones

Assumptions

i) *All investors are Markowitz efficient investors who want to target points on the efficient frontier. The exact location on the efficient frontier and, therefore, the specific portfolio selected, will depend on the individual investor's risk-return utility function.*
ii) *Investors can borrow or lend any amount of money at the risk-free rate of return (RFR).*
iii) *All investors have homogeneous expectations; that is, they estimate identical probability distributions for future rates of return.*
iv) *All investors have the same one-period time horizon such as one month, six months, or one year. A difference in the time horizon would require investors to derive risk measures and risk-free assets that are consistent with their investment horizons.*
v) *All investments are infinitely divisible, which means that it is possible to buy or sell fractional shares of any asset or portfolio.*
vi) *There are no taxes or transaction costs involved in buying or selling assets. This is a reasonable assumption in many instances. Relaxing this assumption modifies the results, but it does not change the basic thrust.*
vii) *There is no inflation or any change in interest rates, or inflation is fully anticipated. This is a reasonable initial assumption, and it can be modified.*
viii) *Capital markets are in equilibrium. This means that we begin with all investments properly priced in line with their risk levels.*

Development of Capital Market Theory

The major factor that allowed portfolio theory to develop into capital market theory is the concept of a risk-free asset. Such an asset would have ***zero variance*** and ***zero correlation*** with all other risky assets and would provide the ***risk-free rate of return (RFR).*** It would lie on the vertical axis of a portfolio graph. This assumption allows us to derive a generalized theory of capital asset pricing under conditions of uncertainty from the Markowitz portfolio theory. This achievement is generally attributed to William Sharpe, for which he received the Nobel Prize, but Lintner and Mossin derived similar theories independently.

We have defined a ***risky asset*** as one from which future returns are uncertain and we have measured this uncertainty by the variance, or standard deviation of expected returns. Because the expected return on a risk-free asset is entirely certain, the standard deviation of its expected return is zero ($\sigma_{RF} = 0$). The rate of return earned on such an asset should be the risk-free rate of return (RFR) which, should equal the expected long-run growth rate of the economy with an adjustment for short-run liquidity. We are then interested in what happens when we introduce this risk-free asset into the risky world of the Markowitz portfolio model.

Covariance with a Risk-Free Asset

Recall that the covariance between two sets of returns is

$$Cov_{ij} = \sum_{i=1}^{n} [R_i - E(R_i)][R_j - E(R_j)]/n$$

Because the returns for the risk-free asset are certain, σRF = 0, which means that $R_i = E(R_i)$ during all periods. Thus, R_i - $E(R_i)$ will also equal zero, and the product of this expression with any other expression will equal zero. Consequently, the covariance of the risk-free asset with any risky asset or portfolio of assets will always equal zero. Similarly, the correlation between any risky asset *i* and the risk-free asset, RF, would be zero because it is equal to:

$$r_{RF,i} = Cov_{RF,i} / \sigma_{RF}\sigma_j$$

Combining a Risk-Free Asset with a Risky Portfolio

Expected Return

Like the expected return for a portfolio of two risky assets, the expected rate of return for a portfolio that includes a risk-free asset is the weighted average of the two returns:

$$E(R_{port}) = W_{RF}(RFR) + (1 - W_{RF})E(R_i)$$

W_{RF} = the proportion of the portfolio invested in the risk free asset

$E(R_i)$ = the expected rate of return on risky Portfolio *i*

Standard Deviation

In the previous chapter the expected variance for a two-asset portfolio was given as:

$$\sigma_{port}^2 = W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_AW_Br_{AB}\sigma_A\sigma_B$$

Substituting the risk-free asset for Security A, and the risky asset portfolio for Security B, this formula would become:

$$E(\sigma_{port}^2) = W_{RF}^2\sigma_{RF}^2 + (1 - W_{RF})^2\sigma_i^2 + 2W_{RF}(1 - W_{RF})r_{RF,i}\sigma_{RF}\sigma_i$$

We know that the variance of the risk-free asset is zero i.e. σ_{RF}^2 =0. Because the correlation between the risk-free asset and any risky asset, *i,* is also zero, the factor $r_{RF,i}$ in the equation above also equals zero. Therefore, any component of the variance formula that has either of these terms will equal zero. When you make these adjustments, the formula becomes:

$$E(\sigma_{port}^2) = (1 - W_{RF})^2\sigma_i^2$$

The standard deviation is:

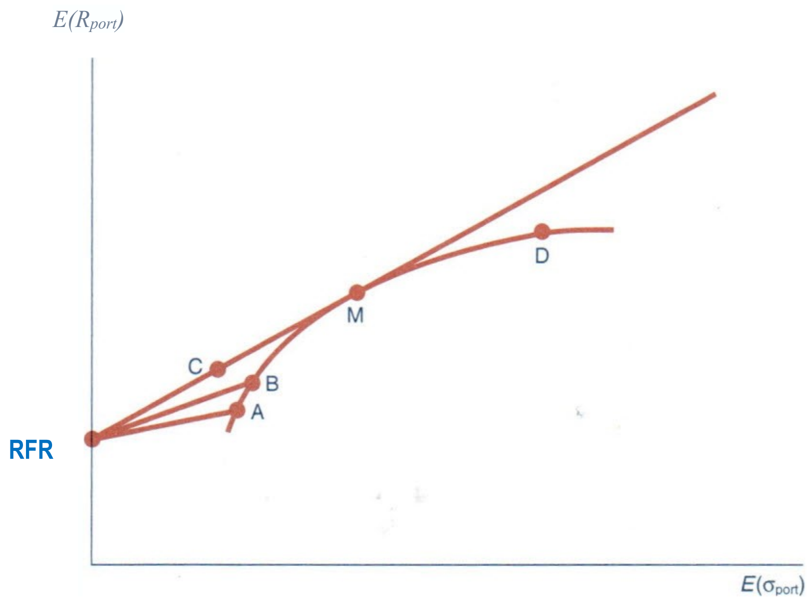
$$E(\sigma_{port}) = \sqrt{(1 - W_{RF})^2\sigma_i^2}$$

$$= (1 - W_{RF})\sigma_i$$

Therefore, the standard deviation of a portfolio that combines the risk-free asset with risky assets is *the linear proportion of the standard deviation of the risky asset portfolio.*

The figure below shows the portfolio possibilities when a risk-free asset is combined with alternative risky portfolios on the Markowitz efficient frontier, one can attain any point along the straight line RFR-A by investing some portion of their portfolio in the risk-free asset W_{RF} and the remainder (1 - W_{RF}) in the risky asset portfolio at Point A on the efficient frontier. This set of portfolio possibilities dominates all the risky asset portfolios on the efficient frontier below Point A because some portfolio along Line RFR-A has equal variance with a higher rate of return than the portfolio on the original efficient frontier. Likewise, you can attain any point along the Line RFR-B by investing in some combination of the risk-free asset and the risky asset portfolio at Point B. Again, these potential combinations dominate all portfolio possibilities on the original efficient frontier below Point B (including Line RFR-A).

One can draw further lines from the RFR to the efficient frontier at higher and higher points until they reach the point where the line is tangent to the frontier, which occurs in at Point M. The set of portfolio possibilities along Line RFR-M dominates all portfolios below Point M. For example, one could attain a risk and return combination between the RFR and Point M (Point C) by investing one-half of his portfolio in the risk-free asset (that is, lending money at the RFR) and the other half in the risky portfolio at Point M.



Risk Return Possibilities with leverage

An investor may want to attain a higher expected return than is available at Point M in exchange for accepting higher risk. One alternative would be to invest in one of the risky asset portfolios on the efficient frontier beyond Point M such as the portfolio at Point D. A second alternative is to add *leverage* to the portfolio by *borrowing* money at the risk-free rate and investing the proceeds in the risky asset portfolio at Point M.

If one borrows an amount equal to 50 percent of his original wealth at the risk-free rate, W_{RF} will not be a positive fraction, but rather a negative 50 percent (W_{RF} = -0.50). The effect on the expected return for the portfolio is:

$$E(R_{port}) = W_{RF}(RFR) + (1 - W_{RF})\,E(R_M)$$
$$= -0.50(RFR) + [1 - (-0.50)]\,E(R_M)$$
$$= -0.50(RFR) + 1.50\,E(R_M)$$

The return will increase in a *linear* fashion along the Line RFR-M because the gross return increases by 50 percent, but you must pay interest at the RFR on the money borrowed.

Example

Assume that $E(RFR) = 0.06$ and $E(R_M) = 0.12$. The return on your leveraged portfolio would be:

$$E(R_{port}) = -0.50(0.06) + 1.5(0.12)$$
$$= -0.03 + 0.18$$
$$= 0.15$$

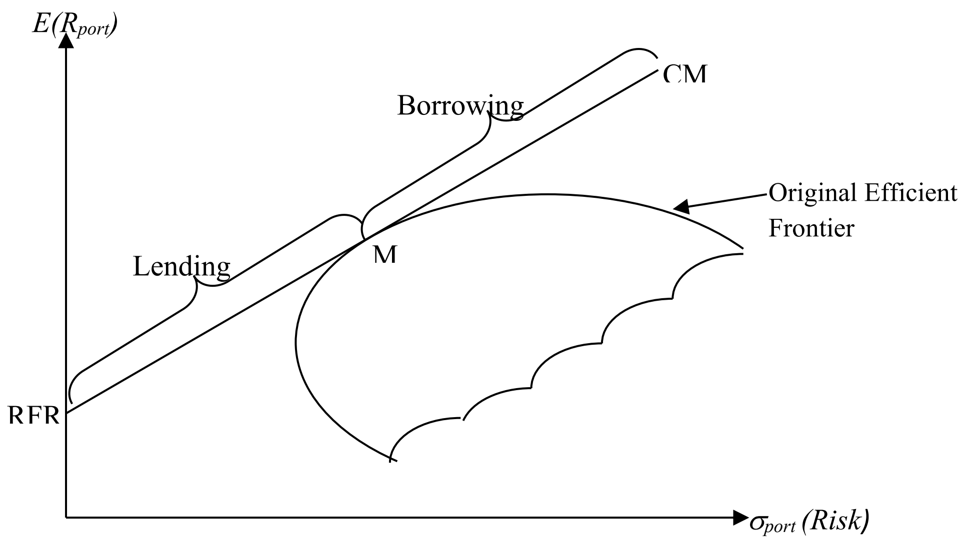
The effect on the standard deviation of the leveraged portfolio is similar:

$$E(\sigma_{port}) = (1 - W_{RF})\sigma_M$$
$$= [1 - (1 - 0.50)]\sigma_M$$

$= 1.50\sigma_M$

Where, σ_M = the standard deviation of the M portfolio

Capital Market Line (CML)
Since both return and risk increase in a linear fashion along the original *Line RFR-M,* and this extension dominates everything below the line on the original efficient frontier, you have a new efficient frontier: the straight line from the *RFR* tangent to Point M. This line is referred to as the ***Capital Market Line (CML)*** and is shown in the Figure below.



When two assets are perfectly correlated, the set of portfolio possibilities falls along a straight line. Therefore, because the CML is a straight line, it implies that all the portfolios on the CML are perfectly positively correlated. Since all the portfolios on the CML combine the risky asset Portfolio M and the risk-free asset, you either invest part of your portfolio in the risk-free asset and the rest in the risky asset portfolio M, or you borrow at the risk-free rate and invest these funds in the risky asset portfolio. In either case, all the variability comes from the risky asset M portfolio. The only difference between the alternative portfolios on the CML is the magnitude of the variability, which is caused by the proportion of the risky asset portfolio in the total portfolio.

The Market Portfolio
Given that Portfolio M lies at the point of tangency, it has the highest portfolio possibility line, and everybody will want to invest in Portfolio M and borrow or lend to be somewhere on the CML. This portfolio must, therefore, include *all risky assets*. Because the market is in equilibrium, it is also necessary that all assets are included in this portfolio in *proportion to their market value*. This portfolio that includes all risky assets is referred to as the ***market portfolio***.

Because the market portfolio contains all risky assets, it is a ***completely diversified portfolio,*** which means that all the risk unique to individual assets in the portfolio is diversified away. Specifically, the unique risk of any asset is offset by the unique variability of the other assets in the portfolio. This unique (diversifiable) risk is also referred to as ***unsystematic risk***. This implies that only ***systematic risk*** (undiversifiable) risk remains in the market portfolio.

Total Risk

The total risk on all risky assets can be divided in to two: Systematic risk and Unsystematic risk.

Systematic Risk

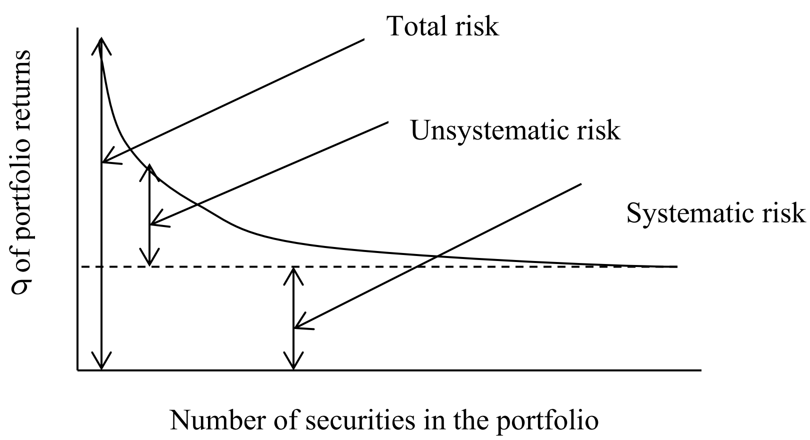
This is the variability of return on risky assets associated with changes in return and the whole market such as changes in the nation's economy, tax rules change, world energy (oil) changes etc. These risks are unavoidable or non – diversifiable

Unsystematic Risk

This is the variability of return on risky assets not explained by general market movements. They are risks unique to a particular company or industry and are independent of the political, economic and other factors affecting all securities. Examples include breakthrough in technology making an existing product obsolete. These risks are avoidable or diversifiable. For most stocks, unsystematic risk accounts for approximately 50% of the stock total risk.

The relationship of total, systematic and unsystematic

** Total risk = systematic risk (non diversifiable or unavoidable) + unsystematic risk diversifiable or avoidable*



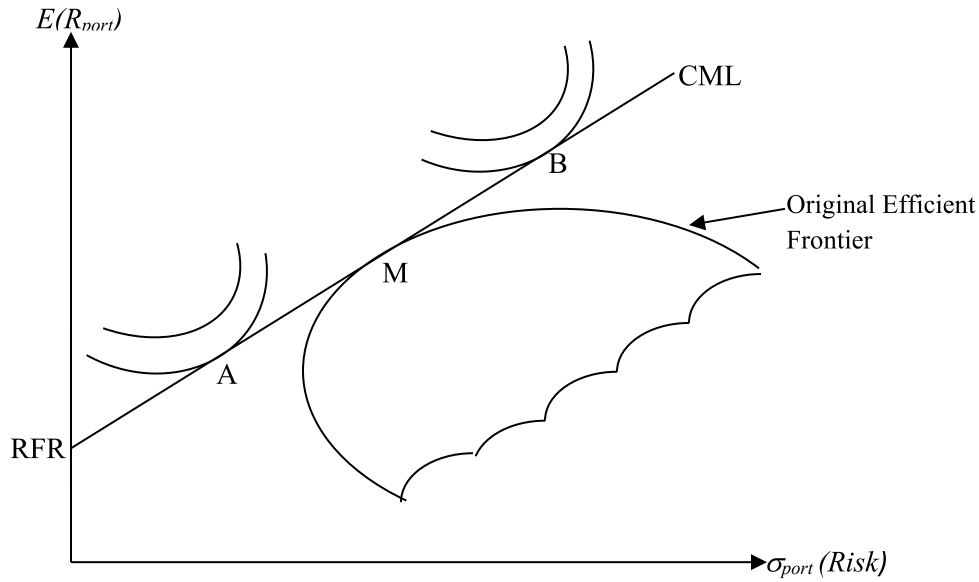
Diversification and CML

Since all portfolios on the CML are perfectly positively correlated, this means that all portfolios on the CML are perfectly correlated with the completely diversified market portfolio M. This implies measure of complete diversification.' Specifically, a completely diversified portfolio would have a correlation with the market portfolio of + 1.00. This is logical because complete diversification means the elimination of all the unsystematic or unique risk. Once you have eliminated all unsystematic risk, only systematic risk is left, which cannot be diversified away. Therefore, completely diversified portfolios would correlate perfectly with the market portfolio because it has only systematic risk.

CML and the Separation Theorem

The CML leads all investors to invest in the same risky asset portfolio, the M portfolio. Individual investors should only differ regarding their position on the CML, which depends on their risk preferences

In turn, how they get to a point on the CML is based on their *financing decisions.* If you are relatively risk averse, you will lend some part of your portfolio at the RFR by buying some risk-free securities and investing the remainder in the market portfolio of risky assets. For example, you might invest in the portfolio combination at Point A in the figure below.



In contrast, if you prefer more risk, you might borrow funds at the RFR and invest everything (all of your capital plus what you borrowed) in the market portfolio, building the portfolio at Point B. This financing decision provides more risk but greater returns than the market portfolio. This division of the investment decision from the financing decision was called ***separation theorem*** by Tobin. Specifically to be somewhere on the CML efficient frontier, you initially decide to invest in the market portfolio M, which means that you will be on the CML. This is your *investment* decision. Subsequently, based on your risk preferences, you make a separate *financing* decision either to borrow or to lend to attain your preferred point on the CML.

Measurement of risk for the CML

The relevant measure for risky assets is *their covariance with the M portfolio,* which is referred their systematic risk. The importance of this covariance is apparent from two points view.

First, the Markowitz portfolio model, revealed that when adding a security to a portfolio *its average covariance with all other assets in the portfolio* is the relevant factor for consideration. The Capital Market Theory has shown that *the only relevant portfolio is the M portfolio.* Together, these two findings mean that the only important consideration for any individual risky asset is its average covariance with all the risky assets in the M portfolio or simply, *the asset's covariance with the market portfolio.* This covariance, then, the relevant risk measure for an individual risky asset.

Second, because all individual risky assets are a part of the M portfolio, one can describe their rates of return in relation to the returns for the M portfolio using the following linear model:

$$R_{it} = a_i + b_i R_{mt} + \varepsilon$$

Where,

R_{it} = return for asset *i* during period *t*

$a_,$ = constant term for asset *i*

$b,$ = slope coefficient for asset *i*

R_{Mt} = return for the M portfolio during period *t*

ε = random error term

The variance of returns for a risky asset could be described as

$$\begin{aligned} Var(R_{it}) &= Var(a_i + b_i R_{Mt} + \varepsilon) \\ &= Var\ (a_i) + Var\ (b_i R_{Mt}) + Var\ (\varepsilon) \\ &= 0 + Var\ (b_i R_{Mt}) + Var\ (\varepsilon) \end{aligned}$$

Note that $Var\ (b_i R_{Mt})$ is the variance of return for an asset related to the variance of the market return, or the *systematic variance or risk*. Also, $Var\ (\varepsilon)$ is the residual variance of return for the individual asset that is not related to the market portfolio. This residual variance is the variability that we have referred to as the unsystematic or *unique risk or variance* because it arises from the unique features of the asset. Therefore:

Var (R$_{it}$) = Systematic Variance + Unsystematic Variance

Assignment

i) Discuss leverage and its effect of on CML

ii) Given the CML, discuss and justify the relevant measure of risk for an individual security

iii) Explain why the set of points between the risk free asset and a portfolio on the Markowitz efficient frontier is a straight line

THE CAPITAL ASSET PRICING MODEL (CAPM)

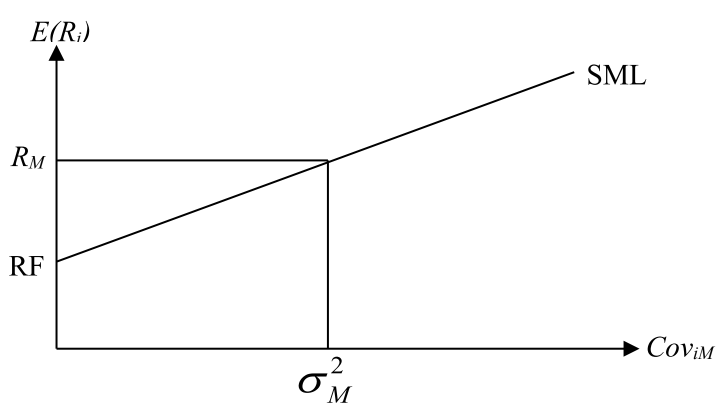
CAPM was developed in 1964 by William Sharp building on the insights provided by Harry Markowitz's portfolio model.
CAPM was developed in a hypothetical world with the following assumptions

i) *Investors are risk averse individuals who maximize the expected utility of their one period of wealth*
ii) *There exist risk free assets such that investor can lend and borrow at the risk free rate*
iii) *All assets are marketable and perfectly divisible*
iv) *Assets markets are frictionless and information is costless and simultaneously available to all investors. Frictionless means that there are or market imperfections such a as taxes is restrictions on short selling*
v) *Asset returns have a normal distribution*
vi) *Investors have homogenous expectations about asset returns.*

The model further assumes (perhaps more importantly) that, there exists two types of investment opportunities. The first is the risk free security whose return over the holding period is known with certainty e.g. T bills. The second is the market portfolio of common stocks e.g. the S&P 500 Index. Since one cannot hold a more diversified portfolio than the market portfolio, it represents the limits to attainable diversification. Thus market portfolio risk is unavoidable or systematic. The existence of this risk-free asset resulted in the derivation of a capital market line (CML) that became the relevant efficient frontier. Because all investors want to be on the CML, an asset's covariance with the market portfolio of risky assets emerged as the relevant risk measure.

Now that we understand this relevant measure of risk, we can proceed to use it to determine an appropriate expected rate of return on a risky asset. This step takes us into the Capital Asset Pricing Model (CAPM), which is a model that indicates what should be the expected or required rates of return on risky assets. This transition is important because it helps one to value an asset by providing an appropriate discount rate to use in any valuation model. Alternatively, if you have already estimated the rate of return that you think you will earn on an investment, you can compare this *estimated* rate of return to the *required* rate of return implied by the CAPM and determine whether the asset is undervalued, overvalued, or properly valued.

The Security Market Line (SML)
To estimate the expected or required rates of return on risky assets, or whether the asset is undervalued, overvalued, or properly valued, we need to develop the ***Security Market Line (SML)*** that visually represents the relationship between risk and the expected or the required rate of return on an asset. The equation of this SML, together with estimates for the return on a risk-free asset and on the market portfolio, can generate expected or required rates of return for any asset based on its systematic risk. Given that the relevant risk measure for an individual risky asset is its covariance with the market portfolio *(Cov_{iM})*. Therefore, we can draw the risk-return relationship with the systematic covariance variable *(Cov_{iM})* as the risk measure as shown in figure below.



The return for the market portfolio *(R_M)* should be consistent with its own risk, which is the covariance of the market with itself. If you recall the formula for covariance, you see that the covariance of any asset with itself is its variance, $Cov_{i,i} = \sigma_i^2$ In turn, the covariance of the market with itself is the variance of the market rate of return $Cov_{m,m} = \sigma_M^2$ Therefore, the equation for the risk-return line in the figure above

$$E(R_i) = RFR + \frac{R_M - RFR}{\sigma_M^2}(Cov_{i,M})$$

Rearranging,

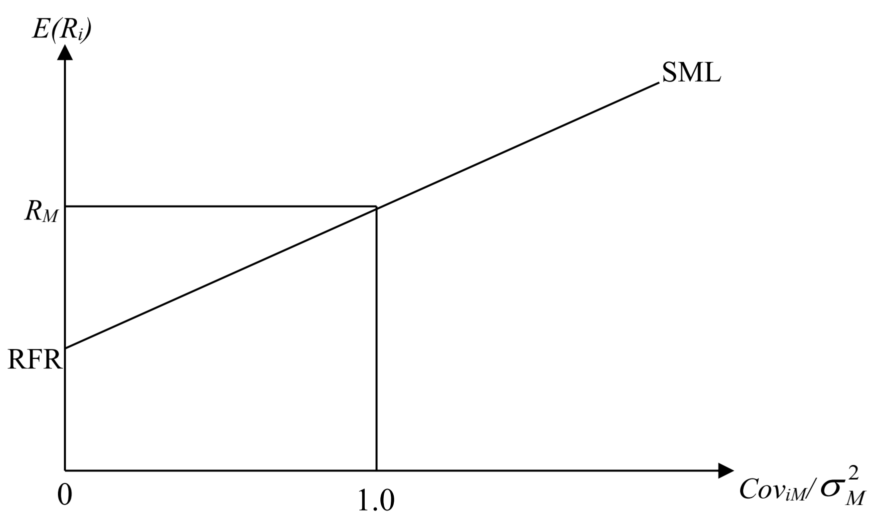
$$E(R_i) = RFR + \frac{Cov_{i,M}}{\sigma_M^2}(R_M - RFR)$$

Defining $\frac{Cov_{i,M}}{\sigma_M^2}$ as beta, (β), this equation can be stated:

$$E(R_i) = RFR + \beta(R_M - RFR)$$

Beta can be viewed as a *standardized* measure of systematic risk. Specifically, we already know that the covariance of any asset *i* with the market portfolio ($Cov_{i,m}$) is the relevant measure of risk. Beta is a standardized measure of risk because it relates this covariance to the variance of the market portfolio. As a result, the market portfolio has a beta of 1. Therefore, if the β_i for an asset is above 1.0, the asset has higher normalized systematic risk than the market, which means that it is more volatile than the overall market portfolio.

Given this standardized measure of systematic risk, the SML graph can be expressed as shown in below. This is the same graph as in above, except there is a different measure of risk. Specifically, the graph in Figure below replaces the covariance of an asset's returns with the market portfolio as the risk measure with the standardized measure of systematic risk (beta), which is the covariance of an asset with the market portfolio divided by the variance of the market portfolio.



Determining the Expected Rate of Return for a Risky Asset
The above graph tells us that the expected rate of return for a risky asset is determined by the RFR plus a risk premium for the individual asset. In turn, the risk premium is determined by the systematic risk of the asset (β), and the prevailing market risk premium (R_M - RFR). To demonstrate how you would compute the expected or required rates of return, consider the following example stocks assuming you have already computed betas:

Stock	Beta
A	0.80
B	1.00
C	1.50
D	1.30
E	-0.50

Assume that we expect the economy's RFR to be 5 percent (0.05) and the return on the market portfolio (R_M) to be 10 percent (0.10). This implies a market risk premium of 5 percent (0.05). With these inputs, the SML equation would yield the following expected (required) rates of return for these five stocks:

$$E(R_i) = RFR + \beta(R_M - RFR)$$

$E(R_A)$ = 0.05 + 0.80 (0.10 - 0.05)
= 0.09 = 9 %

$E(R_B)$ = 0.05 + 1.00 (0.10 - 0.05)
= 0.10 =10%

$E(R_C)$ = 0.05 + 1.50 (0.10 - 0.05)
= 0.125 = 12.5%

$E(R_D)$ = 0.05 + (1.30)(0.10 - 0.05)
= 0.115 = 11.5%

$E(R_E)$ = 0.05 + (-0.50)(0.10 - 0.05)
= 0.05 - 0.025
= 0.025 = 2.5%

As stated, these are the expected (required) rates of return that these stocks should provide based on their systematic risks and the prevailing SML

Insight
What does each of the illustration reveal about the relationship between stock beta and expected returns?

Identifying Undervalued and Overvalued Assets
Given that we can now compute the rate of return one should expect or require for specific risky asset using the SML, we can compare this *required* rate of return to the asset's *estimated* rate of return over a specific investment horizon to determine whether it would be an appropriate investment. To make this comparison, we need an independent estimate of the return outlook for the security based on either fundamental or technical analysis techniques. Using the previous example for the five assets, assume that based on extensive fundamental analysis, the analysts provide the price and dividend outlook as shown in the table below:

Stock *(1)*	*Current Price (P_i)* *(2)*	*Expected Price (P_i+1)* *(3)*	*Expected Dividend (P_i+ 1)* *(4)*	*Estimated Future Rate of Return (Percent)* *(5)*
A	25	27	0.50	10.0
B	40	42	0.50	6.2
C	33	39	1.00	21.2
D	64	65	1.10	3.3
E	50	54	-	8.0

Column*(5)*={ *[*Column*(3)*- Column*(2)]*+ Column*(4)*}/ Column*(2)*

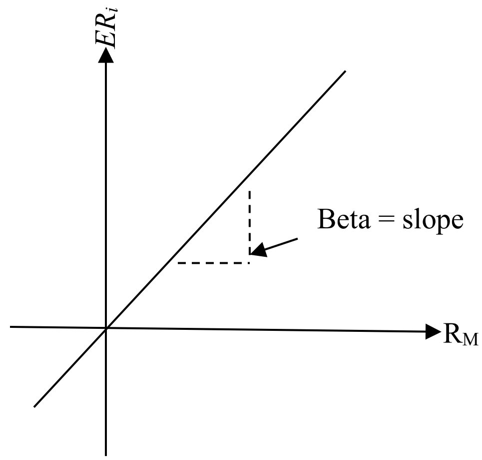
The independent estimate of return given in column 5 of the table above is then compared with the expected rate of return of the assets obtained using the SML (column 4 and 3 respectively) in the table below.

Stock *(1)*	*Beta* *(2)*	*Required Return E(Ri)* *(3)*	*Estimated Return* *(4)*	*Estimated Return Minus E(Ri)* *(5)*	*Evaluation* *(6)*
A	0.80	9	10.0	1	Undervalued
B	1.00	10.0	6.2	-3.8	Overvalued
C	1.50	12.5	21.2	8.7	Undervalued
D	1.30	11.5	3.3	-8.2	Overvalued
E	-0.50	2.5	8.0	5.5	Undervalued

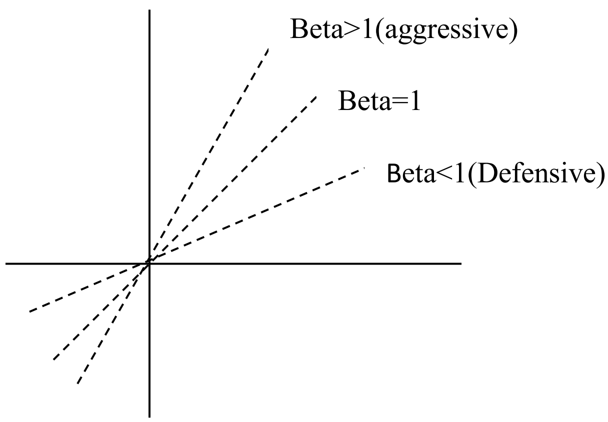
Column 5 presents the difference between the estimated returns and the expected return which is sometimes referred to as *alpha* or its *excess returns.*
When alpha is positive the asset is undervalued and if its negative the asset is overvalued as presented in column 6. If the alpha is zero, the stock is on the SML and is properly valued in line with its systematic risk

Insight
What action would you recommend for each of the above stocks?

Calculating Systematic Risk: The Characteristic
The systematic risk input for an individual asset is derived from a regression referred to as the asset's characteristic line with the market portfolio. The characteristic line is the regression line of best fit through a scatter plot of return for the individual risky asset and for the market portfolio of risky assets over designated past period.



The slope of the characteristic line is referred to as Beta. It measures the sensitivity of a stock return to changes in the returns of the market portfolio. The beta of a portfolio is simply a weighted average of the individual betas in the portfolio (therefore beta is the measure of the systematic risk). A beta of 1 means that excess return for the stock vary proportionately with excess return for market portfolio (Beta >1 more proportionately and Beta <1 less proportionately)



If we assume financial markets are efficient and investors as a whole are efficiently diversified, unsystematic risk becomes irrelevant and the major risk associated with a stock because it's systematic risk. The greater the beta of a stock the greater the relevant risk of that stock and the greater the required rate of return. If we assume unsystematic risk has been diversified away, the required rate of return for stock j is: $E(R_i) = RFR + \beta(R_M - RFR)$ Put another way, the required rate of return on a stock equals risk free rate plus a risk premium. And the risk premium is a function of expected market return less the risk free rate and the beta coefficient.

Problem

1. *Assume the risk free rate is 10% and the expected return on the market portfolio is 15%. Market analysts return expectations for four stocks are listed below:*

Stock	*Expected return*	*Beta*
Safaricom	*17.0*	*1.3*
Rift valley Railways	*14.5*	*0.8*
Lake Victoria Fish	*15.5*	*1.1*
Oserian Flowers	*18.0*	*1.7*

(a) If the analysts' expectations are correct, which stocks (if any) are overvalued and which (if any) are undervalued?
(b) If the risk free rate were to suddenly rise to 12% and the expected return on the market portfolio to 16%. Which stocks (if any) would be overvalued and which (if any) are undervalued? (Assume the market analysts' return expectations remain the same for the four stocks)

2. *Suppose T bills have an 8% expected return, and the expected return on market portfolio is 13%, and the beta of Kenya Airways is 1.3.*
   i) *What is the required rate of return for KQ will be*

   ii) *If beta for KQ = 1.0*

   iii) *If beta for KQ = 0.7*

CAPM provides us the means by which to estimate the required rate of return on a security. This return can then be used of the discount rate in a divided valuation model. The intrinsic value of a stock can be expressed as the present value of the expected future dividends.

$$V = \sum_{t=1}^{\infty} \frac{D_t}{(1+K_e)^t}$$

Where; D_t = Expected dividend in period t

K_e = Required rate of return for the stock

$\sum$= Sum of present value of future dividends from period 1 to infinity

Suppose we wished to determine to value of stock of KQ and that the perceptual divided growth model was appropriate i.e.

$$V = \frac{D_t}{K_e - g}$$

Where g = Expected annual growth rate in dividend per share.

Further assume KQ is expected annual growth rate in dividend per share is 10% and the dividends expected at the end of period 1= 2 Ksh. What is the value of KQ's stock using KQ's required rate of return assuming 14.5% expected return?

$$V=\frac{D_1}{K_e-g}=\frac{2}{0.145-0.1}=44.44Ksh$$

Problem

Suppose the economy enters a period of stable growth characterized by low inflation and low interest rates and variables for the company change as follows: R_f *from 8% to 7%;* $\overline{R}_m$ *from 13% to 11%,* β *from 1.3 to 1.2 and g from 10 to 9%. What is the value of the new stock?*

$\overline{R}_{KQ}=0.7=(0.11-0.07)1.2=11.8\%$ Using this rate as K_e the value of the stock is:

ARBITRAGE PRICING THEORY (APT)
APT was developed in the early 1970s by Stephen Ross as an alternative model to CAPM which is reasonable initiative requires only limited assumptions and allows for multiple risk factors. The development of APT was in response to some criticisms leveled against CAPM some of these included the assumption that investors have quadratic utility functions, some tests of the CAPM indicated that the beta coefficients for individual securities are not stable, some studies have also supported a positive linear relationship between rates of return and systematic risk for portfolios of stock. Yet still the usefulness of the model was criticized because of its dependence on a market portfolio of risky assets, which may not be available. APT made only three major assumptions:

i) *Capital markets are perfectly competitive.*
ii) *Investors always prefer more wealth to less wealth with certainty.*
iii) *The stochastic process generating asset returns can be represented as a K factor model*

Equally important, the following major assumptions are *not* required:

i) *Quadratic utility function,*
ii) *Normally distributed security returns, and*
iii) *A market portfolio that contains all risky assets and is mean-variance efficient*

As noted, the theory assumes that the stochastic process generating asset returns can be represented as a *K* factor model of the form.

$$R_i = E_i + b_{i1}\delta_1 + b_{i2}\delta_2 + \ldots.. + b_{ik}\delta_k + \varepsilon_i$$

Where:

R_i = return on asset *i* during a specified time period
E_i = expected return for asset *i*
b_{ik} = reaction in asset *i*'s returns to movements in a common factor
δ_k = a common factor with a zero mean that influences the returns on all assets
ε_i = a unique effect on asset *i*'s return that, by assumption, is completely diversifiable in large portfolios and has a mean of zero
N = number of assets

As indicated, the δ_k terms are the *multiple* factors expected to have an impact on the returns of *all* assets. Examples of such factors might include inflation, growth in GDP, major political upheavals, or changes in interest rates. The APT contends there are many such factors, in contrast to the CAPM, where it is contended that the only relevant variable is the covariance of the asset with the market portfolio, that is, it's beta coefficient.

Given these common factors, the β_{ik} terms determine how each asset reacts to this common factor. To extend the earlier example, although all assets may be affected by growth GDP, the effects will differ across assets. For example, stocks of cyclical firms that produce autos, steel, or heavy machinery will have larger b_{ik} terms for this common factor than non cyclical firms, such as grocery chains.

Similar to the CAPM model, it is assumed that the unique effects (ε_i) are independent and will be diversified away in a large portfolio. The APT assumes that, in equilibrium, return on a zero-investment, zero-systematic-risk portfolio is zero when the unique effects are diversified away. This assumption and some theory from linear algebra imply that expected return on any asset *i* $[E(R_i)]$ can be expressed as:

$$E(R_i) = \lambda_0 + \lambda_{i1}b_1 + \lambda_{i2}b_2 + \ldots.. + \lambda_{ik}\beta_k$$

Where:

λ_0 = the expected return on an asset with zero systematic risk where $\lambda_0 = E_0$
λ_1 = the risk premium related to each of the common factors-for example, the risk premium related to interest rate risk ($\lambda_1 = E_i = E_0$)

b_i = the pricing relationship between the risk premium and asset i - that is, how responsive asset i is to this common factor K

Illustration

Consider the following example of two stocks and a two-factor model:

λ_1 = changes in the rate of inflation. The risk premium related to this factor is 1 percent for every 1 percent change in the rate ($\lambda_1 = 0.01$)

λ_2 = percent growth in real GDP. The average risk premium related to this factor is 2 percent for every 1 percent change in the rate ($\lambda_2 = 0.02$)

λ_0 = the rate of return on a zero-systematic-risk asset (zero beta: $b_{0j} = 0$) is 3 percent ($\lambda_0 = 0.03$)

The two assets *(X, Y)* have the following response coefficients to these factors:

b_{x1} = the response of asset X to changes in the rate of inflation is 0.50 ($b_{x1} = 0.50$). This is not very responsive to changes in the rate of inflation

b_{y1} = the response of asset Y to changes in the rate of inflation is 2.00 ($b_{y1} = 2.00$)

b_{x2} = the response of asset X to changes in the growth rate of real GDP is 1.50 ($b_{x2} = 1.50$)

b_{y2} = the response of asset Y to changes in the growth rate of real GDP is 1.75 ($b_{y2} = 1.75$)

These response coefficients indicate that if these are the major factors influencing as returns, asset Y is a higher-risk asset, and therefore its expected (required) return should greater, as shown below:

$$E(R_i) = \lambda_0 + \lambda_{i1}b_1 + \lambda_{i2}b_2$$

$$= 0.03 + (0.01)b_{i1} + (0.02)b_{i2}$$

Therefore:

$$E(R_x) = 0.03 + (0.01)(0.50) + (0.02)(1.50)$$
$$= 0.065 = 6.5\%$$

$$E(R_y) = 0.03 + (0.01)(2.00) + (0.02)(1.75)$$
$$= 0.085 = 8.5\%$$

If the prices of the assets do not reflect these returns, we would expect investors to enter into arbitrage arrangements whereby they would sell overpriced assets short and use the proceeds to purchase the underpriced assets until the relevant prices were corrected. Given these linear relationships, it should be possible to find an asset or a combination of assets with equal risk to the mispriced asset, yet a higher return.

VALUATION THEORY

COMPANY ANALYSIS AND VALUATION OF COMMON STOCK

The approach to company analysis and stock valuation may be accomplished using two different kinds analysis. These are fundamental analysis and technical analysis.

Fundamental analysis, involves making investment decisions based on the examination of the economy, an industry, and company variables that lead to an estimate of value for an investment, which is then compared to the prevailing market price of the investment.

In contrast to the efficient market hypothesis or fundamental analysis, technical analysis involves the examination of past market data such as prices and the volume of trading, which leads to an estimate of future price trends and, therefore, an investment decision.

FUNDAMENTAL ANALYSIS

As pointed out, the general approach in fundamental analysis is to examine the economy, industry and company variables in estimating the value of an investment. Because of the complexity and importance of valuing common stock, various techniques for accomplishing fundamental analysis have been devised over time. These techniques fall into one of two general approaches:

i) The *discounted cash-flow valuation techniques,* where the value of the stock is estimated based upon the present value of some measure of cash flow, including dividends, operating cash flow, and free cash flow; and

ii) The *relative valuation techniques,* where the value of a stock is estimated based upon its current price relative to variables considered to be significant to valuation, such as earnings, cash flow, book value, or sales.

An important point is that *both of these approaches and all of these valuation techniques have several common factors.*

i) All of them are significantly affected by the investor's *required rate of return* on the stock because this rate becomes the discount rate or is a major component of the discount rate.

ii) All of them are affected by *the estimated growth rate of the variable* used in the valuation technique-for example, dividends, earnings, cash

Why and when to use discounted cash-flow valuation techniques

These discounted cash-flow valuation techniques are obvious choices for valuation because they are the epitome of how we describe value-that is, the present value of expect flows. The major difference between the alternative techniques is how one specific cash flow-that is, the measure of cash flow used.

The cleanest and most straightforward measure of cash flow is *dividends* because are clearly cash flows that go directly to the investor, which implies that you should use the *cost of equity* as the discount rate. However, this dividend technique is difficult to apply to firms that do not pay dividends during periods of high growth, or that currently pay very limited dividends because they have high rate of return investment alternatives available. On the other hand, an advantage is that the reduced form of the dividend discount model (DDM) is very useful when discussing valuation for a stable, mature entity where the assumption of relatively constant growth for the long term is appropriate.

The second specification of cash flow is the *operating cash flow,* which is generally described as cash flows after direct costs (cost of goods and S, G & A expenses) and before any payments to capital suppliers. Because we are dealing with the cash flows available for all capital suppliers, the discount rate employed is the firm's *weighted average cost of capital* (WACC). This is a very useful model when comparing firms with diverse capital structures because you determine the value of the total firm and then subtract the value of the firm's debt obligations to arrive at a value for the firm's equity.

The third cash-flow measure is *free cash flow to equity,* which is a measure of cash flows available to the equity holder after payments to debt holders and after allowing for expenditures to maintain the firm's asset base. Because these are cash flows available to equity owners, the appropriate discount rate is the firm's *cost of equity*.

A potential difficulty with these cash-flow techniques is that they are very dependent on the two significant inputs:

i) The growth rates of cash flows (both the *rate* of growth and the *duration* of growth), and

ii) The estimate of the discount rate. As we will show in several instances, a small change in either of these values can have a significant impact on the estimated value. This is the problem when using any theoretical model: Everyone knows and uses the same model, but it is the *inputs* that are critical-GIGO: garbage in, garbage out!

Why and when to use relative valuation techniques

An advantage of the relative valuation techniques is that they provide information about how the market is *currently* valuing stock at several levels-that is, the aggregate market, alternative industries, and individual stocks within industries. However the relative valuation approach provides this information on current valuation, but it does not provide guidance on whether these current valuations are appropriate-that is, *all* valuations at a point in time could be too high or too low. The relative valuation techniques are appropriate to consider under two conditions:

i) You have a good set of comparable entities-that is, either comparable industries or companies that are similar in terms of industry, size, and, it is hoped, risk.

ii) The aggregate market is not at a valuation extreme-that is, it is not either seriously undervalued or overvalued.

Discounted cash-flow valuation techniques

We have three discounted cash-flow valuation techniques:

i) Present Value of Dividends (DDM)

ii) Present Value of Operating Cash Flow

iii) Present Value of Free Cash Flow

All of these valuation techniques are based on the basic valuation model, which asserts that the value of an asset is the present value of its expected future cash flows as follows:

$$V_j = \sum_{t=1}^{n} \frac{CF_t}{(1+k)^t}$$

Where:

V_j = Value of stock j

n = Life of the asset

CF_t = Cash flow in period t

k = The discount rate that is equal to the investors' required rate of return for asset j, which is determined by the uncertainty (risk) of the stock's cash flows

Present Value of Dividends (DDM)

The **dividend discount model** assumes that the value of a share of common stock is the present value of all future dividends as follows.

$$V_j = \sum_{t=1}^{n} \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \ldots\ldots + \frac{D_\infty}{(1+k)^\infty}$$

$$V_j = \sum_{t=1}^{n} \frac{D_t}{(1+k)^t}$$

Where:

V_j = Value of stock j
D_t = Dividend during period t
k = Required rate of return on stock j

One-Year Holding Period

Assume an investor wants to buy the stock, hold it for one year, and then sell it. To determine the value of the stock-that is, how much the investor should be willing to pay for it-using the DDM, we must estimate the dividend to be received during the period, the expected sale price at the end of the holding period, and the investor's required rate of return.

Illustration

Assume that ABC Company paid dividends at Ksh 1 per share last year. Dividends for the current year are projected to grow by 10% analysts expect the share price of the company to close the year at Ksh 22. If your required rate of return is 14%. Estimate the value of the ABC stock

Dividend payable at the end of the year will be 1 × 0.10 = 1.10

$$V_1 = \frac{1.10}{(1+0.14)} + \frac{22.00}{(1+0.14)} = \frac{1.10}{(1.14)} + \frac{22.00}{(1.14)} = 0.96 + 19.30 = Ksh20.26$$

Multi-Year Holding Period

If you anticipate holding the stock for several years and then selling it, the valuation estimate is harder. You must forecast several future dividend payments and estimate the sale price of the stock several years in the future. The exact estimate of the future dividends depends on two projections. The first is your outlook for earnings growth because earnings are the dividends. The second projection is the firm's dividend policy.

Illustration:

Assume the expected holding period is three years, and you estimate the following dividend payments at the end of each year***:***

Year 1	$1.10/share
Year 2	$1.20/share
Year 3	$1.35/share

If analysts project the share price to be Ksh 34 at the end of the third year, and your required rate of return is 14 percent estimate the value of the stock.

$$V = \frac{1.10}{(1+0.14)} + \frac{1.20}{(1+0.14)} + \frac{1.35}{(1+0.14)} + \frac{34}{(1+0.14)}$$

$$V = \frac{1.10}{(1.14)} + \frac{1.20}{(1.14)^2} + \frac{1.35}{(1.14)^3} + \frac{34}{(1.14)^3} = 0.96 + 0.92 + 0.91 + 22.95 = Ksh25.74$$

Infinite Period Model
We can extend the multi-period model to the infinite period dividend discount model, which assumes investors estimate future dividend payments for an infinite number of periods. We must make some simplifying assumptions about this future stream of dividends to make the task viable. The easiest assumption is that *the future dividend stream will grow at a constant rate for an infinite period.*

This model is generalized as follows:

$$V_j = \frac{D_0(1+g)}{(1+k)} + \frac{D_0(1+g)^2}{(1+k)^2} + \frac{D_0(1+g)^3}{(1+k)^3} + \ldots\ldots + \frac{D_0(1+g)^n}{(1+k)^n}$$

Where:

V_j = the value of stock j
D_0 = the dividend payment in the current period
g = the constant growth rate of dividends
k = the required rate of return on stock j
n = the number of periods, which we assume to be infinite

This infinite period constant growth rate model can be simplified to the following expression:

$$V_j = \frac{D_1}{k-g}$$

Illustration:
Consider the example of a stock with a current dividend of $1 a share, which is expected to grow at 9% forever. If your expected rate of return is 13 %, the estimated value of return will be:

g = 0.09
k = 0.13
D_1= 1 × 0.09 = 1.09

$$V_j = \frac{1.09}{0.13-0.09} = \frac{1.09}{0.04} = Ksh27.25$$

What happens to the value of the stock if k increases for example from13% to 15%

$$V_j = \frac{1.09}{0.15-0.09} = \frac{1.09}{0.06} = Ksh18.17$$

What happens to the value of the stock if g decreases for example from 9% to 11%?

$$V_j = \frac{1.09}{0.13 - 0.11} = \frac{1.09}{0.02} = Ksh54.50$$

Valuation with Temporary Supernormal Growth

To determine the value of a temporary supernormal growth company, you must combine the previous models. In analyzing the initial years of exceptional growth, you examine each year individually. If the company is expected to have two or three stages of supernormal growth, you must examine each year during these stages of growth. When the firm's growth rate stabilizes at a rate below the required rate of return, you can compute the remaining value of the firm assuming constant growth using the DDM and discount this lump-sum constant growth value back to the present.

Illustration

The XYZ Company has a current dividend *(D_0)* of Ksh 2 a share. The following are the expected annual growth rates for dividends.

Dividend Year	Growth Rate
1-3	25
4-6	20
7-9	15
10 onwards	9

The required rate of return for the stock (the company's cost of equity) is 14 percent. Therefore, the value equation becomes:

$$V_i = \frac{2(1.25)}{(1.14)} + \frac{2(1.25)^2}{(1.14)^2} + \frac{2(1.25)^3}{(1.14)^3}$$

$$+ \frac{2(1.25)^3(1.20)}{(1.14)^4} + \frac{2(1.25)^3(1.20)^2}{(1.14)^5} + \frac{2(1.25)^3(1.20)^3}{(1.14)^6}$$

$$+ \frac{2(1.25)^3(1.20)^3(1.15)}{(1.14)^7} + \frac{2(1.25)^3(1.20)^3(1.15)^2}{(1.14)^8} + \frac{2(1.25)^3(1.20)^3(1.15)^3}{(1.14)^9}$$

$$+ \frac{\dfrac{2(1.25)^3(1.20)^3(1.15)^3(1.09)}{0.14 - 0.09}}{(1.14)^9} = Ksh94.355$$

Value of dividend stream for Year 10 and all future dividends (that is, \$11.21/(0.14 - 0.09) = \$224.20). The discount factor is the ninth-year factor because the valuation of the remaining stream is made at the end of Year 9 to reflect the dividend in Year 10 and all future dividends.

NB: *The growth rate of dividends may be estimated by the retention rate × return on equity (RR × ROE)*

Present Value of Operating Cash Flow

In this model, you derive the value of the total firm because you discount total operating cash flows prior to the payment of interest to the debt holders. Also, because you discount the total firm's operating cash flow, you use the firm's weighted average cost of capital (WACC) as your discount rate. Therefore, once you estimate the value of the total firm, you subtract the value of debt, assuming your goal is to estimate the value of the firm's equity. The total value of the firm is equal to:

$$V_j = \sum_{t=1}^{n} \frac{OCF_t}{(1+WACC_j)^t}$$

Where:

V_j = Value of firm j

n = Number of periods assumed to be infinite

OCF_t = The firm operating cash flow in period t

$WACC_j$ = Firm j's weighted average cost of capital

If you are dealing with a mature firm whereby its operating cash flows have reached a stage of stable growth, you can adapt the infinite period constant growth DDM model as follows:

$$V_j = \frac{OCF_1}{WACC_j - g_{OCF}}$$

Where:

OCF_1= Operating cash flow in period 1 equal to $OCF_0\,(1+ g_{OCF})$

g_{OCF} = Long-term constant growth rate of operating cash flow

NB: *OCF = EBIT (1- Tax Rate) + Depreciation expense – Capital spending - Δ in working capital- Δ in other assets*

$WACC = W_E k + W_D i$

Where:

W_E = *proportion of equity in total capital*

k = *the after tax cost of equity*

W_D = *proportion of debt in total capital*

i = *the after tax cost of debt*

$g = (RR)\,(ROIC)$

Where:

RR= *the average retention rate*

ROIC = EBIT (1 – Tax Rate)/Total Capital

Alternatively, assuming that the firm is expected to experience several different rates of growth for OCF, these estimates can be divided into three or four stages, as demonstrated with the temporary supernormal dividend growth model. Similar to the dividend model, the analyst must estimate the rate of growth and the duration of growth for each of these periods of supernormal growth. A point to note is that after determining the value of the total firm Vj you must subtract the value of all no equity items, including accounts payable, total interest-bearing debt, deferred taxes, and preferred stock, to arrive at the estimated value of the firm's equity.

Present Value of Free Cash Flow to Equity
The third discounted cash-flow technique deals with "free" cash flows to equity, which would be derived *after* operating cash flows, have been adjusted for debt payments (interest and principle) and after deducting capital expenditures necessary to maintain the firm's asset base. Also, these cash flows precede dividend payments to the common stockholder. Such cash flows are referred to as "free" because they are what is left after meeting all obligations to other capital suppliers (debt and preferred stock) and after providing the funds needed to maintain the firm's asset base. Since, because these are cash flows available to equity owners, the discount rate used is the firm's cost of equity *(k)* rather than the firm's WACC.

$$V_j = \sum_{t=1}^{n} \frac{FCFE_t}{(1+k_j)^t}$$

Where:

V_J = value of the stock of firm j
n = number of periods assumed to be infinite
$FCFE_t$ = the firm's free cash flow in period t.

NB: FCFE=Net Income + Depreciation Expense –Capital Expenditures - Δin Working Capital – Principal Debt Repayments + New Debt Issues
Again, how an analyst would implement this general model depends upon the firm's position in its life cycle. That is, if the firm is expected to experience stable growth, analysts can use the infinite growth model. In contrast, if the firm is expected to experience a period of temporary supernormal growth, analysts should use the multistage growth model similar to the process used with dividends and for operating cash flow i.e.

$$V = \frac{FCFE_1}{k - g_{FCFE}}$$

Where:

g_{FCFE} = Expected constant growth rate of free cash flow to equity for the firm
We have three discounted cash-flow valuation techniques:
Relative Valuation Techniques
In contrast to the various discounted cash-flow techniques that attempt to estimate a specific value for a stock based on its estimated growth rates and its discount rate, the relative valuation techniques implicitly contend that it is possible to determine the value of an economic entity (i.e., the market, an industry, or a company) by comparing it to similar entities on the basis of several relative ratios that compare its stock price to relevant variables that affect a stock's value, such as earnings, cash flow, book value, and sales.

We have four relative valuation techniques:
i) Price/Earnings Ratio (P/E)
ii) Price/Cash Flow Ratio (P/CF)
iii) Price/Book Value Ratio (P/BV)
iv) Price/Sales Ratio (P/S)

Price/Earnings Ratio (P/E)
Under this model, investors can estimate value by determining how many dollars they are willing to pay for a dollar of expected earnings (typically represented by the estimated earnings during the following 12-month period). For example, if investors are willing to pay 10 times expected earnings, they would value a stock they expect to earn $2 a share during the following year at $20. You can compute the prevailing price/earnings (P/E) ratio**,** as follows:

$$\text{Price/ Earning Ratio} = \frac{\text{Current Market Price}}{\text{Expected 12} - \text{Month Earning}}$$

The infinite period dividend discount model can also be used to determine the value of the P/E ratio as follows:

$$V_i = \frac{D_1}{k-g}$$

Replacing V with P on the left hand of the equation gives:

$$P_i = \frac{D_1}{k-g}$$

Dividing by E_1 (expected earnings during the next 12 months) gives:

$$P_i / E_1 = \frac{D_1 / E_1}{k-g}$$

Consider the following estimates for example:

$D/E = 0.50$, $k = 0.12$, $g = 0.09$, $E_0 = \$2.00$

$$P/E = \frac{0.50}{0.12 - 0.09} = \frac{0.50}{0.03} = 16.7$$

Given current earnings (E_0) of \$2.00 and a g of 9 percent, you would expect E_1 to be \$2.18. Therefore, you would estimate the value (price) of the stock as:

V = 16.7 × 2.18
= \$36.41

As before, you would compare this estimated value of the stock to its current market price to decide whether you should invest in it.

Price/Cash Flow Ratio (P/CF)

The growth in popularity of this relative valuation technique can be traced to concern over the propensity of some firms to manipulate earnings per share, whereas cash flow values are generally less prone to manipulation. The price to cash flow ratio is computed as follows:

$$P/CF_j = \frac{P_t}{CF_{t+1}}$$

Where:

P/CF_j = the price/cash flow ratio for firm j
P_t = the price of the stock in period t
CF_{t+1} = the expected cash flow per share for firm j

The specific cash flow measure used is typically EBITDA

Price/Book Value Ratio (P/BV)

The price/book value (P/BV) ratio has been widely used for many years by analysts in the banking industry as a measure of relative value. This ratio gained in popularity and credibility as a relative valuation technique for all types of firms based upon a study by Fama and French that indicated significant inverse relationship between P/BV ratios and excess rates of return for a cross section of stocks. The P/BV ratio is specified as follows:

$$P/BV_j = \frac{P_t}{BV_{t+1}}$$

Where:

P/B_j = the price/book value ratio for firm j

P_t = the price of the stock in period t

BV_{t+1} = the estimated end-of-year book value per share for firm j

As with other relative valuation ratios, it is important to match the current price with the estimated book value that is expected to prevail at the end of the year. One can derive an estimate based upon the historical growth rate for the series or use the growth rate implied by the sustainable growth formula: g = ROE × Retention Rate

Price/Sales Ratio (P/S)

Advocates consider this ratio meaningful and useful for two reasons First, they believe that strong and consistent sales growth is a requirement for a growth company. Although they note the importance of an above-average profit margin, they consider tend that *the growth process must begin with sales.* Second, given all the data in the balance sheet and income statement, sales information is subject to less manipulation than any other data item. The specific P/S ratio is:

$$P/S_j = \frac{P_t}{S_{t+1}}$$

Where:

P/S_j = the price to sales ratio for firm j

P_t = the price of the stock in period t

S_{t+1} = the expected sales per share for firm j

Again, it is important to match the current stock price with the firm's expected sales per share, which may be difficult to derive for a large cross section of stocks. Two caveats are relevant to the price to sales ratio. First, it is important to recognize that this particular relative valuation ratio varies dramatically by industry. The reason for this difference is related to the second consideration, the profit margin on sales. Therefore, your relative valuation analysis using the *P/S* ratio should be between firms in the same or similar industries.

VALUATION OF PREFERENCE STOCK

The owner of a preferred stock receives a promise to pay a stated dividend, for an infinite period. Preferred stock is perpetuity because it has no maturity. As was true with a bond, stated payments are made on specified dates although the issuer of this stock does not have the same legal obligation to pay investors as do issuers of bonds. Payments are made only after the firm meets its bond interest payments. Because this reduced legal obligation increases the uncertainty of returns, investors should require a higher rate of return on a firm's preferred stock than on its bonds. Because preferred stock is perpetuity, its value is simply the stated annual dividend divided by the required rate of return on preferred stock (k_p) as follows:

$$V = \frac{Dividend}{k_p}$$

Illustration:

Assume a preferred stock has a \$100 par value and a dividend of \$8 a year. Your required rate of return on this stock is 9 percent. Therefore, the value of this preferred stock to you is:

$$V = \frac{8}{0.09} = \$88.89$$

If the current market price is \$95 you would decide against a purchase, whereas if it is \$80, you would buy the stock. Also, given the market price of preferred stock, you can derive its promised yield. Assuming a current market price of \$85, the promised yield would be:

$$k_p = \frac{Dividend}{\text{Pr}ice} = \frac{\$8}{\$85.00} = 0.0941$$

Problem

KBC company preferred stock has a par value of KSh100and a Ksh 9 dividend rate. You require an 11% rate of return on this stock.

i) What is the maximum price you would pay for it

ii) Would you buy it at Ksh 96?

Solution:

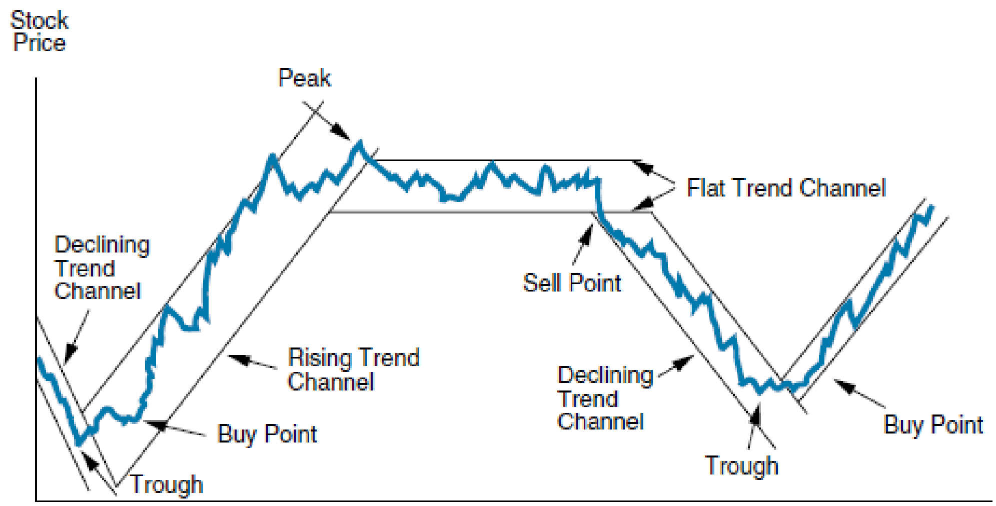
TECHNICAL ANALYSIS

Technical analysis involves the examination of past market data such as prices and the volume of trading, which leads to an estimate of future price trends and, therefore, an investment decision. Several assumptions lead to this view of price movements:

i) *The market value of any good or service is determined solely by the interaction of supply and demand.*
ii) *Supply and demand are governed by numerous rational and irrational factors. Included in these factors are those economic variables relied on by the fundamental analyst as well as opinions, moods, and guesses. The market weighs all these factors continually and automatically.*
iii) *Disregarding minor fluctuations, the prices for individual securities and the overall value of the market tend to move in trends, which persist for appreciable lengths of time.*
iv) *Prevailing trends change in reaction to shifts in supply and demand relationships. These shifts, no matter why they occur, can be detected sooner or later in the action of the market itself.*

Technical trading rules and indicators

The graph shows a peak and trough, along with a rising trend channel, a flat trend channel, a declining trend channel, and indications of when a technical analyst would ideally want to trade.



The graph begins with the end of a declining (bear) market that finishes in a ***trough*** followed by an upward trend that breaks through the ***declining trend channel***. Confirmation that the trend has reversed would be a buy signal. The technical analyst would buy stocks that showed this pattern. The analyst would then look for the development of a ***rising trend channel.*** As long as the stock price stayed in this rising channel, the technician would hold the stock(s). Ideally, you want to sell at the peak of the cycle, but you cannot identify a peak until after the trend changes.

If the stock (or the market) begins trading in a flat pattern, it will necessarily break out of its rising trend channel. At this point, some technical analysts would sell, but most would hold to see if the stock experiences a period of consolidation and then breaks out of the flat trend channel on the upside and begins rising again. Alternatively, if the stock were to break out of the channel on the downside, the technician would take this as a sell signal and would expect a declining trend channel. The next buy signal would come after the trough when the price breaks out of the declining channel and establishes a rising trend.

There are several technical trading rules and a range of interpretations for each of them. They are discussed below

Contrary-Opinion Rules

Many technical analysts rely on technical trading rules that assume that the majority of investors are wrong as the market approaches peaks and troughs. Therefore, these technicians try to determine when the majority of investors is either strongly bullish or bearish and then trade in the opposite direction. They tend to watch the following:

Mutual Fund Cash Positions- A high cash position is also a bullish indicator because of potential buying power. Irrespective of the reason for the increase in cash balances, technicians believe these cash funds will eventually be invested and will cause stock prices to increase

Credit balances of brokerage accounts- A buildup of credit balances is an increase in buying power and a bullish signal

Investment Advisory Opinions- Many technicians believe that if a large proportion of investment advisory services are bearish, this signals the approach of a market trough and the onset of a bull market.

Follow the Smart Money

Some technical analysts have created a set of indicators that they expect to indicate the behavior of smart, sophisticated investors and create rules to follow them.

The Confidence Index- Published by Barron's, the Confidence Index is the ratio of *Barron's* average yield on 10 top-grade corporate bonds divided by the yield on the Dow Jones average age of 40 bonds. Technicians believe that a decrease in this index in a bullish indicator because investors are willing to invest in lower-quality bonds

T-Bill–Eurodollar Yield Spread- An alternative measure of investor attitude or confidence on a global basis is the spread between T-bill yields and Eurodollar rates. It is reasoned that, at times of international crisis, this spread widens as money flows to safe haven U.S. T-bills, which causes a decline in this ratio. The stock market typically experiences a trough shortly thereafter.

Debit Balances in Brokerage Accounts (Margin Debt)- Debit balances in brokerage accounts represent borrowing (margin debt) by knowledgeable investors from their brokers. This is taken to imply buying attitude and is considered a bullish sign, while a decline in debit balances would indicate selling by these sophisticated investors and would be a bearish indicator.

Other indicators

Stocks above Their 200-Day Moving Average- The market is considered to be ***overbought*** and subject to a negative correction when more than 80 percent of the stocks are trading above their 200-day moving average. In contrast, if less than 20 percent of the stocks are selling above their 200-day moving average, the market is considered to be ***oversold***, which means investors should expect a positive correction.

Stock Price and Volume Techniques

Dow Theory

Dow described stock prices as moving in trends analogous to the movement of water. He postulated three types of price movements over time:

i) Major trends that are like tides in the ocean
ii) Intermediate trends that resemble waves
iii) Short-run movements that are like ripples

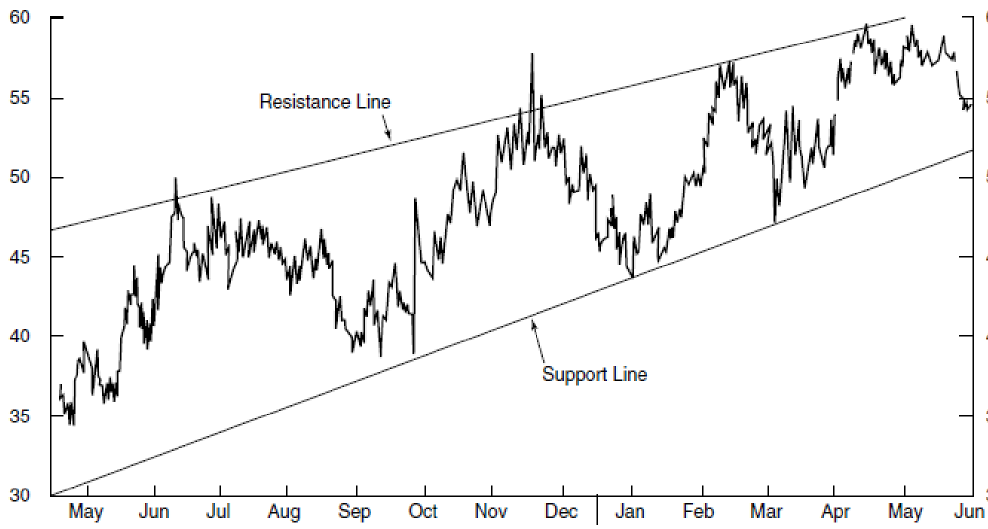
Followers of the Dow Theory attempt to detect the direction of the major price trend (tide), recognizing that intermediate movements (waves) may occasionally move in the opposite direction. They recognize that a major market advance does not go straight up but, rather, includes small price declines as some investors decide to take profits.

Importance of Volume A technician looks for a price increase on heavy volume relative to the stock's normal trading volume as an indication of bullish activity. Conversely, a price decline with heavy volume is bearish.

Support and Resistance Levels- A ***support level*** is the price range at which the technician would expect a substantial increase in the demand for a stock. Generally, a support level will develop after a stock has enjoyed a meaningful price increase and the stock experiences profit taking. Technicians reason that, at some price below the recent peak, other investors who did not buy during the first price increase and have been waiting for a small reversal to get into the stock. When the price reaches this support price, demand surges and price and volume begin to increase again.

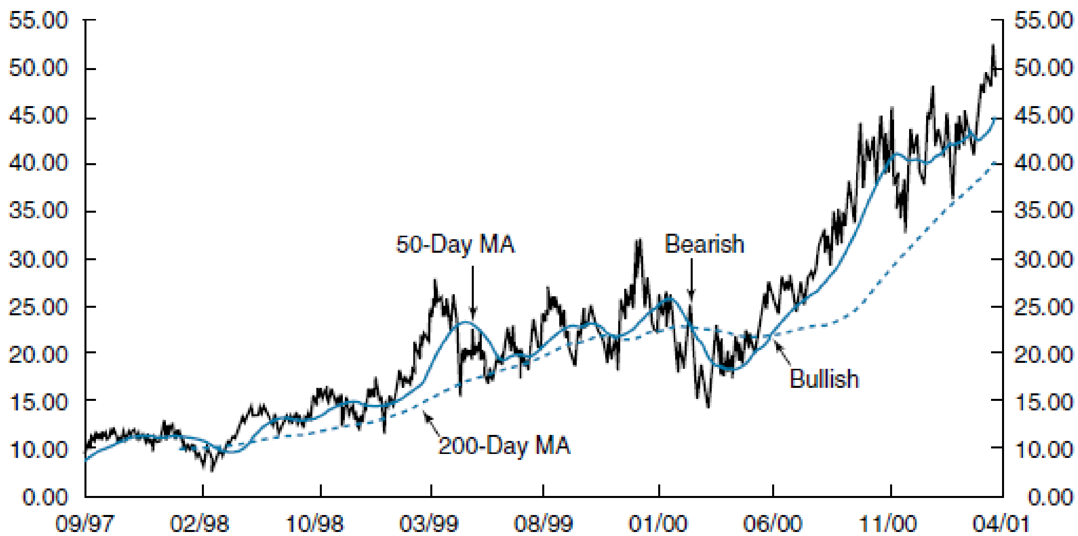
A ***resistance level*** is the price range at which the technician would expect an increase in the supply of stock and a price reversal. A resistance level develops after a steady decline from a higher price level- that is, the decline in price leads some investors who acquired the stock at a higher price to look for an opportunity to sell it near their breakeven points. Therefore, the supply of stock owned by these investors is overhanging the market. When the price rebounds to the target price set by these investors, this overhanging supply of stock comes to the market and there is a price decline on heavy volume.

The graph below illustrates support and resistance lines.



Moving-Average Lines- MA lines are meant to reflect the overall trend for the price series, with the shorter MA series (50-day) reflecting shorter trends as opposed to the longer MA series (200-day) which reflect longer trend. Two comparisons involving the MA series are considered important. The first comparison is the specific prices to the shorter run 50-day MA series. If the overall price trend of a stock or the market has been down, the moving-average price line generally would lie above current prices. If prices reverse and break through the moving-average line ***from below*** accompanied by heavy trading volume, most technicians would consider this a strong ***positive*** change and speculate that this breakthrough signals a reversal of the declining trend. In contrast, if the price of a stock had been rising, the moving average line would also be rising, but it would be below current prices. If current prices broke through the moving-average line ***from above*** accompanied by heavy trading volume, this would be considered a bearish pattern that would signal a reversal of the long-run rising trend.

The second comparison is between the 50- and 200-day MA lines. Specifically, when these two lines cross, it signals a change in the overall trend. Specifically, if the 50-day MA line crosses the 200-day MA line from below on good volume, this would be a bullish indicator (buy signal) because it signals a reversal in trend from negative to positive. In contrast, when the 50- day line crosses the 200-day line from above, it signals a change to a negative trend and would be a sell signal. The figure below illustrates the 50 and 200 MA for a stock from September 1997 to April. 2001



Relative Strength- Technicians believe that once a trend begins, it will continue until some major event causes a change in direction. They believe this is also true of relative performance. If an individual stock or an industry group is outperforming the market, technicians believe it will continue to do so. Therefore, technicians compute weekly or monthly relative-strength (RS) ratios for individual stocks and industry groups. The ratio is equal to the price of a stock or an industry index relative to the value for some stock market series such as the S&P 500.

Bar Charting - Technicians use charts that show daily, weekly, or monthly time series of stock prices. For a given interval, the technical analyst plots the high and low prices and connects the two points vertically to form a bar. Typically, he or she will also draw a small horizontal line across this vertical bar to indicate the closing price. Finally, almost all bar charts include the volume of trading at the bottom of the chart so that the technical analyst can relate the price and volume movements.

EFFICIENT MARKET HYPOTHESIS (EMH)

In 1965, Eugene Fama published his dissertation arguing for random walk hypothesis in stock prices. According to Fama (1965) an efficient market is a market where there are large numbers of rational profit maximizers actively competing, with each trying to predict future market values or individual securities and where important current information is almost freely available to all participants. In an efficient market, on the average, competition will cause the full effects of new information on intrinsic values to be reflected "instantaneously" in actual prices. A market in which prices always "fully reflect" all available information is called "efficient". The efficient market hypothesis is based on three common forms; the weak form efficiency, the ***semi-strong form efficiency***, and the ***strong form efficiency***.

The weak form efficiency

In this form of efficiency, share prices exhibit no serial dependencies, meaning there are no "patterns" to asset price. This implies that future price movements are determined entirely by information that is contained in the price series hence prices must follow a random walk. In this form of efficiency, current prices are determined solely by a technical analysis of past prices. Technical analysts study historical price movements by looking for cyclical patterns or trends likely to repeat themselves. Their research ensures that significant movements in current prices relative to their history become widely and quickly known to investors as a basis for immediate trading decisions. Current prices will then move accordingly. Technical analysis technique or chartist technique will not be able to consistently produce excess returns though some forms of fundamental analysis may provide excess returns.

It does not require that the prices remain at or near equilibrium but only that market participation will not be able to systematically profit from the market "inefficiencies". In a weak-form market, fundamentalists who make investment decisions on the expectations of individual firms should therefore be able to "out-guess" technical analyst and profit to the extent that such information is not assimilated into past prices (A phenomenon particularly applicable to companies whose financial securities are infrequently traded). In summary, a weak form efficiency market has its prices reflect all the information contained in historical returns.

Semi- strong form of efficiency
In this form of efficiency, current prices not only reflect price history but all public information, and this is where fundamental analysis comes into play. Unlike technical analyst, fundamentalist study a company and its business based on historical records, plus its current and future performance (Profitability, dividends, investment potential, managerial expertise etc) relative to its competitive position, the state of the economy and the global factors. This form of efficiency implies that share prices adjust to publicly available new information very rapidly and in an unbiased fashion, such that no excess returns can be earned by trading on that information. In a semi- strong market prices reflect all publicly available information i.e past earnings, earnings forecast etc.

Strong form efficiency
In this form of efficiency, share prices reflect all information, public and private including insider knowledge and no one can earn excess returns. If there are legal barriers to private information becoming public as with insider trading laws, strong form of efficiency becomes impossible except in cases where the laws are universally ignored. To test the strong form of efficiency, a market must exist that consists of investors that cannot consistently earn excess returns over a long period of time. In the presence of strong form efficiency, the market price of any financial security should represent its intrinsic (true) value based on anticipated returns and their degree of risk. In a strong form efficient market, prices reflect even that which is not publicly available e.g. insider trading.

MUTUAL FUNDS AND REITs VALUATION

MUTUAL FUNDs

Are financial intermediaries that pool the financial resources of investors and invest the resources in diversified portfolios of assets.

Types of Mutual Funds.

i) ***Equity Funds***: funds consisting of common and preferred stock securities.
ii) ***Bond Funds:*** *funds consisting of fixed income capital market and debt securities with a maturity of over one year.*
iii) ***Hybrid Funds:*** *funds consisting of both stock and bond securities.*
iv) ***Money Markets Mutual Funds:*** *funds consisting of money securities with an original* maturity of less than one year.

Regulations require that mutual fund manager specify the investment objectives of their funds in a prospectus available to all investors. This includes the list of all securities that the fund holds. This allows investors to invest in funds that match their risk complexion: - equity funds are high-risk, high-return funds for example.

Investor Return from Mutual Fund Ownership

Return from investing in mutual fund shares can be viewed from three dimensions of the portfolio of mutual fund assets

i) Dividends / income from the asset
ii) Capital gains (from sale of the assets at higher selling price than the buying price).
iii) Capital appreciation in the underlying values of existing assets (which adds to the values of mutual funds shares)

For capital appreciations, mutual fund assets are normally marked to the market daily i.e. computing the daily market value of the fund total asset portfolio then dividing this amount by the number of mutual funds shares outstanding to get the current value of each mutual fund Share. This is called the ***Net Asset Value (NAV)*** of the fund. This is the price that investors obtain when they sell back shares to the fund or the price they pay to buy new shares of the fund that day (less any fees that the fund may charge).

Types of Mutual Funds Ownership

i) ***Open-Ended Mutual Fund***: A fund for which the supply of shares is not fixed but can increase or decrease daily with purchase and redemption of share.
ii) ***Closed-Ended Mutual Funds/Investment Companies***: this are specialized investment companies that have a fixed supply of outstanding shares but invest in securities and assets of other firms.

Calculating the Net Asset Value (NAV)

Example 1: Calculating NAV of an Open-End Mutual Fund.

Suppose today a mutual fund contains 1000 shares of Kenyan Airways currently trading at 40.50 Ksh., 2000 shares of Shell Oil currently trading at 86.50 Ksh. and 4500 shares of Kenyan Breweries currently trading at 42.25Ksh. And the mutual fund has 15000 outstanding shares held by investors. What is the NAV of the mutual fund today?

$$NAV = \frac{Total\ Value\ of\ \ Assets\ Under\ Management}{Number\ of\ \ Mutual\ Fund\ Shares\ Outs\tan ding}$$

$$NAV = \frac{(1000 \times 40.50) + (2000 \times 86.50) + (4500 \times 42.25)}{15000} = 26.905\ Ksh.$$

If the share prices change to the following tomorrow: 35Ksh. for the Kenyan Airways 100Ksh. for Shell Oil and 45Ksh. for Kenyan Breweries, the mutual fund NAV will be:

$$NAV = \frac{(1000\times 35)+(2000\times 100)+(4500\times 45)}{15000} = 29.1667\text{ Ksh.}$$

Example 2: Calculating NAV of an Open-End Mutual Fund When the Number of Shares Changes
Suppose 10 new investors buy 100 shares each at today NAV of 26.908. The fund manager would have 26,908 new funds to invest. Assume the manager decides to buy additional Kenyan Airways shares at today's market price i.e. 26,908/40.50. The fund would acquire 664 additional shares of Kenyan Airways and the new portfolio will be 1664 in Kenyan Airways, 2000 in Shell Oil and 4500 in Kenyan Breweries. Tomorrow's NAV will be:

$$NAV = \frac{(1664\times 35)+(2000\times 100)+(4500\times 45)}{16000} = 28.796\ Ksh.$$

Example 3: Calculating the Market Value of Closed End-Mutual Fund Shares
Suppose a Kenyan Investment Company has 10,000 shares outstanding and due to a high demand for its shares, they are trading at 50Ksh per share. If the firm has a portfolio of 1000 Kenyan Airways shares, 2000 Shell Oil shares and 4500 Kenyan Breweries shares all retailing at today's price (40.50, 86.50 & 42.25 respectively). The market value balance sheet is shown below:

Assets
Markets value of asset portfolio…………………………………………. 403,625Ksh
(1000×40.50) + (2000×86.50) + (4500×42.25)
Premium ((50×10000) – 403.625)) ………………………………………. 96,375Ksh
500,000Ksh.
Liabilities
Market value of closed-ended fund shares (50×10000) …………...……500,000Ksh.

The fund's shares are trading at a premium of 9.64 Ksh (96,375/10000) per share.

Suppose there was a drop in demand and the fund's share are now trading at 37Ksh. per share while the market value of the securities remains constant. The market value balance sheet of this fund will be:

Assets
Markets value of asset portfolio…………………………………………...403,625Ksh
(1000×40.50) + (2000×86.50) + (4500×42.25)
Discount ((403.625 - (37×10000)) ….…………………………………..... (33,625Ksh)
370,000Ksh.
Liabilities
Market value of closed-ended fund shares (37×10,000) ………..……...…..370,000Ksh.

The fund's shares are trading at a premium of 3.36 Ksh (33,625/10000) per share.

Mutual Fund Costs
Mutual funds charge shareholders a price or fee for the services they provide (i.e. management of a diversified portfolio of financial securities). Investors incur two types of fees; ***Sales Loads and Fund Operating Expenses.***

Load Funds: - A mutual fund with an upfront sales or commission charge to the investors is referred to as a load fund. Mutual funds who market shares through broker will have sales or commission charge.
No Load Funds: - A mutual fund that does not charge upfront sales or commission charges to investors is referred to as a no load fund. Mutual funds that sell their shares directly to investors will often not charge sales or commission charges.

Funds operating expenses are annual fees charges to cover all fund level expenses expressed as a percent of funds' assets e.g. Management fees charged to meet operating costs.

Calculation of Mutual Fund Costs
Suppose an investor purchases fund shares with a 6% load and a total funds expense ratio of 2% and the investor wishes to hold the shares for 12 years. The annual total cost to the shareholder for this fund is: 6/12 (annualized load) % + 2% = 2.5%. The 6% load is a one off expense it is therefore annualized i.e. divided by the investors holding period.

Calculation of Number of Outstanding Shares in a Money Market Mutual Fund (MMMF)
Suppose investors rates fall and the market values of asset held be a given MMMF increase from 10000 to 11000Ksh. The market values balance sheet for the mutual fund before and after the drop in interest is.

a) Before:

Assets
Mutual value of MMMF assets……………………………………………… 10,000Ksh.

Liabilities
Market value of MMMF fund shares (10000 Shares×1Ksh) ……………..........10,000Ksh.

b) After

Assets
Mutual value of MMMF assets……………………………………………… 11,000Ksh.

Liabilities
Market value of MMMF fund shares (11000 Shares×1Ksh)………..…………. 11,000Ksh.

The fall in interest rates results in 1000 (11000 – 10000) new equity types shares that are held by investors in the MMMF i.e. an increase of 1000 shares of 1Ksh each. (It is customary to specify in the shares in MMMF in terms of the units of currently e.g. 1$, 1€, 1£, 1¥, 1Kshetc.)

REAL ESTATE INVESTMENT TRUSTS (REITs)
What are Real Estate Investment Trusts?
Operating companies that own, develop and manage commercial real estate
Chartered as a corporation or business trust
Elective choice under tax code creates pass-through of income
Revenue must primarily come from real estate investments
Required to distribute at least 90 percent of their taxable income
Taxation of income is passed through to shareholder level

What Makes a REIT Different?
Asset Requirements
75 percent of assets must be invested in:
- Equity ownership of real property
- Mortgages
- Other REIT shares
- Government securities and cash

Not more than 5% of the value of the assets may consist of the securities of any one issuer if the securities are not includable under the 75% test
A REIT may not hold more than 10% of the outstanding voting securities of any one issuer if those securities are not includable under the 75% test
Not more than 20% of its assets can consist of stocks in taxable REITs subsidiaries

Income Requirements
At least 95% of the entity's gross income must be derived from dividends, interest, rents or gains from the sale certain assets.
75 percent of revenue must come from:
- Rents from real property
- Mortgage interest
- Gains from sales of real property
- Income attributable to investments in other REITs

Large REITs are actively-managed, vertically integrated firms providing commercial real estate goods and services for their "customers" (tenants & users of space).
Vertical integration:
- Land acquisition/holding
- Development
- Ownership
  - Financial capital provision
  - Asset (portfolio) management
- Operation
  - Asset management (franchise value, synergy)
  - Property management
- Tenant services

Characteristics of REITs
Exempt from corporate income tax. Major REIT constraints required to maintain tax exempt status:

Five or Fewer Rule A REIT cannot be a closely held corporation. No five or fewer individuals (and certain trusts) may own more than 50% of the REIT's stock, and there must be at least 100 different shareholders. [Ownership Test] *"Rule"*.

Real Estate Pure Play 75% or more of the REITs total assets must be real estate, mortgages, cash, or federal government securities, and 75% or more of the REIT's yearly gross income must be derived directly or indirectly from real property (including mortgages, partnerships and other REITs).[Asset Test] *"Play"*.

Passive Investment Entity RequirementREITs must derive their income from primarily passive sources like rents and mortgage interest, as distinct from short-term trading or sale of property assets. They cannot use their tax status to shield non-real-estate income from corporate taxation. A REIT is subject to a tax of 100% on net income from "prohibited transactions", such as the sale or other disposition of property held primarily for sale in the ordinary course of its trade or business. However, if the REIT sells property it has held for at least 4 years and the aggregate adjusted basis of the property sold does not exceed 10% of the aggregate basis of all assets of the REIT as of the beginning of the year, then no prohibited transaction is deemed to have occurred. [Income Test] *"Requirement"*.

Earnings Payout Requirement 90% or more of the REIT's annual taxable income must be distributed to shareholders as dividends each year. (Shareholders will then pay ordinary income tax on the earnings in their personal taxes.) [Distribution Test] *"Requirement"*.

UMBRELLA PARTNERSHIP REIT (UPREIT)

*An UPREIT is a REIT that owns a controlling interest in a limited partnership that ownsthe real estate,*as opposed to a traditional structure in which the REIT directly owns the real estate. This structure provides a tax-deferred mechanism through which realestate developers and other real estate owners could transfer their properties to the REITform of ownership. Since the transfer is an exchange of one partnership interest for another,it is not a taxable event. These partnership interests, known as operating partnership units,or OP units, are generally convertible into shares of the REIT, offering voting rights anddividend payments matching those of the REIT shares.

Tax Treatment

One area of importance in accounting for REITs is the treatment of depreciation for financial reporting and the determination of taxable income, For example, *a REIT may use an accelerated method of depreciation in its determination of taxable income, but when determining income available for dividends it is required to use a 40-year asset life.*

*The use of inconsistent methods of income calculation sometimes results in shareholders receiving dividends in excess of the REIT's calculated taxable income.*However, *to the extent that the distribution represents a return on investment, these dividends will be taxed as ordinary income.*

BecauseREITs do not generally pay corporate taxes, the majority of REIT dividends continue to be taxed as ordinary income at prevailing tax rates. Any additional amounts distributed, such as those representing depreciation, will be considered a return of original capital and thus will simply reduce the shareholder's tax basis.

TYPES OF RIETs

The three principal types of publicly traded real estate trusts are *equity trusts, mortgage trusts,* and *hybrid trusts.*There are also REITs that are not listed on an exchange or traded
Overthe counter, which are generally called "private" REITs.

The difference between assets held by the equity trust and those held by the mortgage trust is fairly obvious. *The equity trust acquires property interests, while the mortgage trust purchases mortgage obligations and thus becomes a creditor* with mortgage liens given priority to equity holders.

Over time, more heterogeneous investment policies have developed whlch combine the advantages of both type of trusts to suit specific investment objectives.Such combinations are called *hybrid trusts.*

REIT EXPANSION AND GROWTH

Because of the requirement that 90 percent of earnings be paid out as dividends, REITs have limited opportunities to retain earnings or cash flow to acquire additional real estate assets. Stated another way, REITs have very little free cash flow. Consequently, most REITs must plan for expansion by reserving the right to issue additional stock at some future time.

This is referred to as a *secondary, or follow-on, stock offering*to raise more equity capital, which may in turn be used to acquire additional real estate assets. Analysts may view eventual issuance of these shares as a potential source of dilution of future earnings. The general tendency in the industry is to evaluate the use of funds from follow-on offerings to determine if they will generate an increase in cash flow that more than offsets the dilution. This is referred to as an ***accretive transaction***. This is particularly important when looking at the period just after additional shares are issued and before additional cashflow is realized from the newly acquired assets. Furthermore, any interim problems with developing, leasing, managing, and renovating the new real estate assets could require timeto correct and thus serve as a potential drag on earnings. The dilution of earnings from issuing additional shares might also have a depressing effect on the stock price of the REIT,through the impact on the dividend. Redevelopment primarily relates to remodeling ofspace to meet changing tenant needs.

Potential avenues for expansion or growth of REITs include:

Growing income from existing properties- The most obvious method for growing income in an existing portfolio by increasing occupancy by renting more space. The second is by raising rents. Obviously, the two are intrinsically related and both are dependent on the supply and demand conditions in the market. Redevelopment offers a third alternative

Crowing income through acquisitions- There are two methods of growing the portfolio through acquisitions. are (I) purchasing properties with cash at positive spreads specially when the REITs trading above its NAV is and (2) swapping shares in the REIT or operating partnership units for interests in properties, taking advantage of tax benefits.

Growing income through development-RElTs may also choose to grow their income through development of properties. Risk is generally higher than in redevelopment or acquisition, but can be mitigated. For example,the risks associated with build-to-suit development of properties subject to long-term net leases with quality credit tenants are considerably lower than those associated with speculative development.

Growing income through provision of services- REITs may also derive a portion of their income from provision of services to related and unrelated third parties.

REIT ANALYSIS

Importance of Funds From Operations(FFO) in REIT Analysis

FFO stands for funds from operations, which most analysts consider the REIT equivalent of earnings in industrial stocks. FFO is used by analysts and investors as a measure of the cash flow available to the REIT for distributions (dividends) to shareholders. Most investors are familiar with the use of earnings per share in this capacity. However, for REITs, earnings are not the best measure of cash flow, largely due to the element of depreciation.

Because REITs own real estate assets that are subject to large depreciation allowances, one should be aware of the difference between REIT earnings per share (EPS) and funds from operations (FFO) per share. The distinction between the two can be best made with a simple example:

	REIT Income Statement	REIT FFO
Rent	$100	$100
-Operating expenses	40	40
Net operating income	60	60
- Depreciation	40	-
+Gains on sale of property	20	-
Net income	40	-
Cash flow	-	60
EPS	4	-
FFO per share	-	6

Assuming that the REIT above has 10 shares of stock outstanding, its earnings per share (EPS) would be reported as $4.00 per share. However, its funds from operations (FFO) per share would be $6.00. Generally Accepted Accounting Practices (GAAP) provide for depreciation of assets over time as their useful life is expended. Depreciation is assumed to occur in a predictable fashion and the time periods and rates of depreciation for different types of assets are well established. Most people are familiar with the concept and logic of depreciation based on their experiences with automobiles and other durable goods. As these goods get older, their mechanical parts break down and function less efficiently, decreasing their value. Real estate values tend to rise and fall over time based more on market conditions than physical conditions, although physical conditions can and do play a role in value. The result is that GAAP earnings calculations that use historical cost depreciation do not provide an accurate or meaningful picture of REIT financial performance.
This problem was recognized is defined as:

> *Funds from operations means net income (computed in accordance with generally accepted accounting principles), excluding gains (or losses) from sales of property, plus depreciation and amortization, and after adjustments for unconsolidated partnerships and joint ventures.*
> *Adjustments for unconsolidated partnerships and joint ventures is calculated to reflect funds from operations on the same basis.*

Many analysts and investors have gone beyond the FFO to look at adjusted funds from operations (AFFO), funds available for distribution (FAD), or cash available for distribution (CAD). AFFO. FAD, and CAD are largely interchangeable, with different analysts using the term they prefer. *The major difference between FFO and these supplementals relates to the issue of capital improvements, particularly ongoing capital improvements.* To understand the difference, consider a multifamily apartment building. There are several major expenditures, such as painting and replacement of carpets, that have to be made on a recurring basis. For example, carpeting may be replaced every five years, and painting redone every three years. Accounting policies vary from REIT to REIT on how to handle these expenses. The most conservative treatment is to classify these as expenses, counting against" the current year's income. Others choose to classify them as capital improvements, capitalizing them on the balance sheet and amortizing them over time. In the latter case, the amount spent for capital expenditures will not affect FFO because amortization is added back to EPS when calculating FFO, Thus, although either treatment is valid, the variation causes difficulty in comparing income and expense figures across REITs.

Financial Analysis of an Equity REIT

Table1: Financial Statement MA REIT

A. Operating Statement Summary	
Net revenue	$70,000,000
Less:	
Operating expenses	30,000,000
Depreciation and amortization	15,000,000
General and administrative expenses	4,000,000
Management expense	1,000,000
Income from operations	$20,000,000
Less:	
Interest expense	6,400,000
Net income (loss)	$13,600,000
Net income (loss) per share	$ 2.72

B. Balance Sheet Summary

Assets			Liabilities	
Cash		500,000	Short term	2,000,000
Rents receivable		1,500,000	Mortgage debt	80,000,000
Properties @ cost	$300,000,000		Total	82,000,000
Less: Acc. depr.	130,000,000			
Properties-net		170,000,000	Shareholders' equity	90,000,000
Net assets		$172,000,000	Total liabilities and equity	$172,000,000

Table 2: Summary indicators of financial performance of MA REIT

I. General Summary:

Properties: 5 million sq. ft.	Mortgage debt: $80,000,000
Original cost: $300 million	Avg. interest 8%, 10-yr. maturity
Depreciated cost: $170 million	Number of common shares: 5 million

II. Profit Summary:	$Amount	Per Share
Earnings per share *(EPS)*[1]	13,6000,000	$2.72
Income from operations plus depreciation and amortization *(NOI per share)*[2]	35,000,000	$7.00
Funds from operations *(FFO per share)*[3]	28,600,000	$5.72

III. Other Important Financial Data:	
Market price per share of common stock	$75.00
Dividend per share	$4.00
Shareholder recovery of capital *(ROC per share)*[4]	$1.28
Cash retention per share (*CRPS)*[5]	$1.72
Earnings yield[6]	3.63%
FFOyield[7]	7.62%
Dividend yield[8]	5.33%
Current earnings multiple[9]	27.6 times
Current FFO muitiple[10]	13.1 times
Net assets per share *(NAPS)*[11]	$34.00
Equity or book value per share *(BVPS)*[12]	$18.00

V. Explanation and Calculations:

[1]*EPS:* Net income $13,600,000/5,000,000 shares outstanding
[2]*NOI:* Income from operations plus depreciation and amortization ($20,000,000 + $15,000,000)/5,000,000 shares outstanding
[3]*FFO:* Net Income + Depreciation & Amortization ($13,600,000 + $15,000,000)/5,000,000 shares outstanding= 5.72
[4]*ROC:* Dividend pershare - EPS = $4.00 - $2.72 = $1.28
[5]*CRPS:* FFO - Dividend per share $5.72 - $4.00 = $1.72
[6]*EPS*/Market Price per share = $2.72/$75 = 3.62%
[7]*FFO*/Market Price per share =$5.72/$75 = 7.62%
[8]Dividend per share/Market price per share = $4.00/$75 = 5.33
[9]Current price per share/EPS = $75/$2.72 = 27.6times
[10]Current price per share/FFO = $75/$5.72 = 13.1 times
[11]*NAPS:* Net assets $172,000,000/5,000,000
[12]*BVPS*: (Assets - Liabilities)/shares = $90,000,000/5,000,000

The preceding is an analysis of an equity REIT that a prospective investor or shareholder might make. The financial statement for MA REIT is provided below in table 1. MA owns and manages approximately five million square feet of real estate property office. The cost basis for these assets is $300 million: the REIT has made or assumed mortgages totaling $80 million as part of financing its asset acquisitions. MA stock is currently trading at $75 per share, making its current market value worth $375 million. Referring to table 1, we see that MA earned $13.600.000 in net income or $2.72 per share during the past year. However additional data in table 2 (financial performance indicators) show that other interesting and important relationships must be understood.

We can see that the FFO per share for MA was $5.72 during the past year versus earnings per share (EPS) of $2.72. The difference in this simplified example is due to the $ 15 million depreciation allowance.

One REIT regulation previously detailed indicates that 90 percent of net income must be paid out as dividends. Therefore, another very important relationship shown in section III is the dividend payment per share. In our example, the payment of $4.00 per share meets the 90 percent requirement, but this amount is also greater than the earnings per share; thus, MA paid dividends ofS4.00 per share even though EPS was only $2.72. This can occur because FFO, or cashflow per share, was $5.72, which exceeded earnings per share. Indeed, MA could have paid dividends of$5.72 per share even though it was required to pay only 90 percent of $2.72, or only $2.45 per share. By paying a $4.00 dividend, MA met the 90 percent of earnings requirement and retained cash of $ 1.72 per share for operations and acquisitions of new assets.

The difference between REIT earnings and dividends has a very important effect on the taxes that shareholders pay. Tax regulations provide that even though investors in MA receive $4.00 per share, only $2.72 of earnings are reported as a taxable dividend. The remaining $1.28 is treated as recovery of capital (ROC) and serves to reduce the cost basis of the stock acquired by the investor. For example, if a share of MA stock was purchased for $75 prior to the dividend declaration date, the investor would reduce the investment basis of the stock by $1.28, from $75.00 to $73.72. When the stock is eventually sold, the investor would then calculate any gain or loss based on the sale price less $73.72, or the reduced basis of the stock. If the stock has been owned for one year or more and results in a gain, it would be taxed at the prevailing capital gains tax rate. This also means that if there is a difference between ordinary and capital gains tax rates, the investor saves taxes in the amount of $I.28 times the difference in the two tax rates. Consequently, this treatment allows investors to receive a portion of the dividend ($1.28) "tax free" until the stock is sold or the REIT is liquidated. At that point if the investor has owned stock long enough to qualify for capital gains treatment and capital gains tax rates are lower than tax on ordinary income, the investor will also save taxes. When REITs report operating losses, none of the losses can be passed through to investors. Instead, losses must be carried forward to offset income in income in future periods.

With respect to capital gains from the sale of property. RE1Ts may either

i) *Retain the gain and defer its distribution to shareholders, in which case the gain is taxed at the corporate tax rate or*
ii) *Distribute the gain as a dividend to shareholder*

In the latter case, the REIT is not taxed on the distributed gain; however, the REIT is required to designate such dividends as a capital gain distribution to shareholders who must recognize it as a capital gain in their individual taxes. Capital losses cannot be passed through to individual investors but must be carried forward by the REIT and offset against any future capital gains.
Also important in section III of table 2 is cash flow retention, or the difference between FFO per share and dividends per share, which amounts to $1.72. MA may have retained this amount as a cash reserve or to acquire properties during the past year, as pointed out, MA could have paid this amount as a dividend and been taxed at ordinary income rates. However, because it was not paid currently, the cash flow retention is converted eventually into a capital gain if the price of MA stock responds favorably to management's decision to retain and invest these funds instead of paying dividends. Unlike corporations that may choose not to pay any dividends and retain all earnings for future expansion, MA must pay at least 90 percent of $2.72, or $2.45 per share. In other words, MA has far less discretion than corporations with respect to paying a minimum dividend- a major difference between REITs and corporate entities, which affects REIT dividend reinvestment and expansion policy in very important ways.

POTFOLIO PERFORMNCE EVALUATION

There are two major requirements of a portfolio manager:

i) The ability to derive above-average returns for a given risk class

ii) The ability to diversify the portfolio completely to eliminate all unsystematic risk, relative to the portfolio's benchmark

In terms of return, the first requirement is obvious, but the need to consider risk in this context was generally not apparent before the 1960s, when work in portfolio theory showed its significance. In modern theory, superior risk-adjusted returns can be derived through either superior timing or superior security selection.

An equity portfolio manager who can do a superior job of predicting the peaks or troughs of the equity market can adjust the portfolio's composition to anticipate market trends, holding a completely diversified portfolio of ***high-beta stocks*** through ***rising markets*** and favoring ***low beta stocks*** and money market instruments during **declining markets**. Bigger gains in rising markets and smaller losses in declining markets give the portfolio manager above-average risk adjusted returns.

A fixed-income portfolio manager with superior timing ability changes the portfolio's duration in anticipation of interest rate changes by ***increasing the duration*** of the portfolio in anticipation of ***failing interest rates*** and ***reducing the duration*** of the portfolio when ***rates are expected to rise***. If properly executed, this bond portfolio management strategy likewise provides superior risk-adjusted returns.

As ***an alternative strategy***, a portfolio manager and his or her analysts may try consistently ***to select undervalued stocks or bonds for a given risk class***. Even without superior market timing, such a portfolio would likely experience above-average risk-adjusted returns.

The ***second factor*** to consider in evaluating a portfolio manager is the ***ability to diversify completely***. On average the market rewards investors only for bearing systematic (market) risk. Unsystematic risk is not considered when determining required returns because it can be eliminated in a diversified market portfolio. Because they can expect no reward for bearing this uncertainty, investors often want their portfolios completely diversified, which means they want the portfolio manager to eliminate most or all unsystematic risk. The level of diversification can be judged on the basis of the correlation between the portfolio returns and the returns for a market portfolio or some other benchmark index. A completely diversified portfolio is perfectly correlated with the fully diversified benchmark portfolio.

These two requirements of a portfolio manager are important because some portfolio evaluation techniques take into account one requirement but not the other. Other techniques implicitly consider both factors but do not differentiate between them

COMPOSITE PORTFOLIO PERFORMANCE MEASURES

Peer Group Comparison

Before examining measures of portfolio performance that adjust an investor's return for the level of investment risk, we first consider the concept of a peer group comparison. This method, which Kritzman describes as the most common manner of evaluating portfolio managers, collects the returns produced by a representative universe of investors over a specific period of time and displays them in a simple boxplot format.

To aid the comparison, the universe is typically divided into percentiles, which indicate the relative ranking of a given investor. For instance, a portfolio manager that produced a one-year return of 12.4 percent would be in the 10th percentile if only nine other portfolios in a universe of 100 produced a higher return. Although these comparisons can get quite detailed, it is common for the boxplot graphic to include the maximum and minimum returns, as well as the returns falling at the 25th, 50th (i.e., the median), and 75th percentiles.

There are several potential **problems with the peer group comparison method** of evaluating an investor's performance.

First, and foremost, the ***boxplots do not make any explicit adjustment for the risk level of the portfolios in the universe***. In fact, investment risk is only implicitly considered to the extent that all the portfolios in the universe have essentially the same level of volatility. This is not likely to be the case for any sizable peer group, particularly if the universe mixes portfolios with different investment styles.

A second, related point is that ***it is almost impossible to form a truly comparable peer group that is large enough to make the percentile rankings valid and meaningful.***

Finally, ***by focusing on nothing more than relative returns***, such a comparison **loses sight of whether *the investor*** in question or any in the universe, for that matter, ***has accomplished his individual objectives and satisfied his investment constraints***

Treynor Portfolio Performance Measure

Treynor developed the first composite measure of portfolio performance that included risk. ***He postulated two components of risk***: (1) risk produced by general ***market fluctuations*** and (2) risk resulting from ***unique fluctuations*** in the portfolio securities.

To identify risk due to ***market fluctuations***, he introduced the ***characteristic line***, which defines the relationship between the rates of return for a portfolio over time and the rates of return for an appropriate market portfolio, as was discussed earlier. He noted that the characteristic line's slope measures the relative volatility of the portfolio's returns in relation to returns for the aggregate market. This slope is the portfolio's beta coefficient. A higher slope (beta) characterizes a portfolio that is more sensitive to market returns and that has greater market risk.

Deviations from the characteristic line indicate unique returns for the portfolio relative to the market. These differences arise from the returns on individual stocks in the portfolio. In a completely diversified portfolio, these unique returns for individual stocks should cancel out. As the correlation of the portfolio with the market increases, unique risk declines and diversification improves. Because Treynor was not concerned about this aspect of portfolio performance, he gave no further consideration to the diversification measure.

Treynor was interested in a measure of performance that would apply to all investors regardless of their risk preferences. Building on developments in capital market theory, he introduced a risk-free asset that could be combined with different portfolios to form a straight portfolio possibility line. He showed that rational, risk-averse investors would always prefer portfolio possibility lines with larger slopes because such high-slope lines would place investors on higher indifference curves. The slope of this portfolio possibility line (designated T) is equal to:

$$T = \frac{\bar{R}_i - \overline{RFR}}{\beta_i}$$

Where:

$\bar{R}_i$=*the average rate of return for portfolio "i" during a specified time period*
$\overline{RFR}$=*the average rate of return on a risk-free investment during the same time period*
β_i=*the slope of the fund's characteristic line during that time period (this indicates the portfolio's relative volatility)*

As noted, *a larger T value indicates a larger slope and a better portfolio for all investors (regardless of their risk preferences)*. Because the numerator of this ratio ($\bar{R}_i - \overline{RFR}$) is the risk premium and the denominator is a measure of risk***, the total expression indicates the portfolio's risk premium return per unit of risk.*** All risk-averse investors would prefer to maximize this value. Note that the risk variable beta measures systematic risk and tells us nothing about the diversification of the portfolio. It implicitly assumes a completely diversified portfolio, which means that systematic risk is the relevant risk measure.

Comparing a portfolio's T value to a similar measure for the market portfolio indicates whether the portfolio would plot above the SML. Calculate the T value for the aggregate market as follows:

$$T = \frac{\bar{R}_M - \overline{RFR}}{\beta_M}$$

In this expression, β_M equals 1.0 (the market's beta) and indicates the slope of the SML. Therefore, a portfolio with a higher T value than the market portfolio plots above the SML, indicating superior risk-adjusted performance.

Demonstration of Comparative Treynor Measures

To understand how to use and interpret this measure of performance, suppose that during the most recent 10-year period, the average annual total rate of return (including dividends) on an aggregate market portfolio, such as the S&P 500, was 14 percent ($\bar{R}_M$=0.14) and the average nominal rate of return on government T-bills was 8 percent ($\overline{RFR}$=0.08). Assume that, as administrator of a large pension fund that has been divided among three money managers during the past 10 years, you must decide whether to renew your investment management contracts with all three managers. To do this, you must measure how they have performed.

Assume you are given the following results:

Investment manager	Average annual rate of return	Beta
W	0.12	0.90
X	0.16	1.05
Y	0.18	1.20

You can compute T values for the market portfolio and for each of the individual portfolio managers as follows:

$$T_M = \frac{0.14 - 0.08}{1.00} = 0.060$$

$$T_W = \frac{0.12 - 0.08}{0.90} = 0.044$$

$$T_X = \frac{0.16 - 0.08}{1.05} = 0.076$$

$$T_Y = \frac{0.18 - 0.08}{1.20} = 0.083$$

These results indicate that Investment Manager W not only ranked the lowest of the three managers but did not perform as well as the aggregate market. In contrast, both X and Y beat the market portfolio, and Manager Y performed somewhat better than Manager X. In terms of the SML, both of their portfolios plotted above the line,

A portfolio with a negative beta and an average rate of return above the risk-free rate of return would likewise have a negative T value. In this case, however, it indicates exemplary performance. As an example, assume that Portfolio Manager G invested heavily in gold mining stocks during a period of great political and economic uncertainty. Because gold often has a negative correlation with most stocks, this portfolio's beta could be negative. Assume that our gold portfolio G had a beta of –0.20 and yet experienced an average rate of return of 10 percent. The T value for this portfolio would then be:

$$T_G = \frac{0.10 - 0.08}{-0.20} = -0.100$$

Although the T value is –0.100, if you plotted these results on a graph, it would indicate a position substantially above the SML.

Because negative betas can yield T values that give confusing results, it is preferable either to plot the portfolio on an SML graph or to compute the expected return for this portfolio using the SML equation and then compare this expected return to the actual return. This comparison will reveal whether the actual return was above or below expectations. In the preceding example for Portfolio G, the expected return would be:

$E(R_G) = RFR + \beta_i(R_m - RFR)$
$= 0.08 + (-0.20)(0.06)$
$= 0.08 - 0.012$
$= 0.068$

Comparing this expected (required) rate of return of 6.8 percent to the actual return of 10 percent shows that Portfolio Manager G has done a superior job.

Sharpe Portfolio Performance Measure
Sharpe likewise conceived of a composite measure to evaluate the performance of mutual funds. *The measure followed closely his earlier work on the capital asset pricing model (CAPM), dealing specifically with the capital market line (CML).* The Sharpe measure of portfolio performance (designated S) is stated as follows:

$$S = \frac{\bar{R}_i - \overline{RFR}}{\sigma_i}$$

Where:
$\bar{R}_i$ = *the average rate of return for portfolio i during a specified time period*
$\overline{RFR}$ = *the average rate of return on a risk-free investment during the same time period*
σ_i = *the standard deviation of the rate of return for portfolio iduring the time period*

This composite measure of portfolio performance clearly is similar to the Treynor measure; however, it seeks to measure the total risk of the portfolio by including the standard deviation of returns rather than considering only the systematic risk summarized by beta. Because the numerator is the portfolio's risk premium, ***this measure indicates the risk premium return earned per unit of total risk.*** In terms of capital market theory, this portfolio performance measure uses total risk to compare portfolios to the CML, whereas the Treynor measure examines portfolio performance in relation to the SML. Finally, notice that in practice the standard deviation can be calculated using either total portfolio returns or portfolio returns in excess of the risk-free rate.

Demonstration of Comparative Sharpe Measures
The following examples use the Sharpe measure of performance. Again, assume that $\bar{R}_M$=0.14, $\overline{RFR}$=0.08. Suppose you are told that the standard deviation of the annual rate of return for the market portfolio over the past 10 years was 20 percent (σ_M=0.20). Now you want to examine the performance of the following portfolios:

Assume you are given the following results:

Investment manager	Average annual rate of return	Standard deviation of returns
D	0.13	0.18
E	0.17	0.22
F	0.16	0.23

The Sharpe measures for these portfolios are as follows:

$$S_M = \frac{0.14 - 0.08}{0.20} = 0.300$$

$$S_D = \frac{0.13 - 0.08}{0.18} = 0.278$$

$$S_E = \frac{0.17 - 0.08}{0.22} = 0.409$$

$$S_F = \frac{0.16 - 0.08}{0.23} = 0.348$$

The D portfolio had the lowest risk premium return per unit of total risk, failing even to perform as well as the aggregate market portfolio. In contrast, Portfolios E and F performed better than the aggregate market: Portfolio E did better than Portfolio F.

Given the market portfolio results during this period, it is possible to draw the CML. If we plot the results for Portfolios D, E, and F on this graph, we see that Portfolio D plots below the line, whereas the E and F portfolios are above the line, indicating superior risk-adjusted performance.

Treynor versus Sharpe Measure

The **Sharpe** portfolio performance measure ***uses the standard deviation of returns as the measure of total risk***, whereas the **Treynor** performance measure ***uses beta (systematic risk***). The **Sharpe** measure, therefore, ***evaluates the portfolio manager on the basis of both rate of return performance and diversification.***

For a completely diversified portfolio, one without any unsystematic risk, the two measures give identical rankings because the total variance of the completely diversified portfolio is its systematic variance. *Alternatively, a poorly diversified portfolio could have a high ranking on the basis of the Treynor performance measure but a much lower ranking on the basis of the Sharpe performance measure.*Any difference in rank would come directly from a difference in diversification.

Therefore, these two performance measures provide complementary yet different information, and both measures should be used. If you are dealing with a group of well-diversified portfolios, as many mutual funds are, the two measures provide similar rankings.

A disadvantage of the Treynor and Sharpe measures is that they produce relative, but not absolute, rankings of portfolio performance. That is, the Sharpe measures for Portfolios E and F show that both generated risk-adjusted returns above the market. Further, E's risk-adjusted performance measure is larger than F's. What we cannot say with certainty, however, is whether any of these differences are statistically significant.

Modigliani risk-adjusted performance

Also, known as M^2, M2, Modigliani–Modigliani measure or RAP is a measure of the risk-adjusted returns of some investment portfolio. It measures the returns of the portfolio, adjusted for the risk of the portfolio relative to that of some benchmark (e.g., the market). It is derived from the widely-used Sharpe ratio, but it has the significant advantage of being in units of percent return (as opposed to the Sharpe ratio – an abstract, dimensionless ratio of limited utility to most investors), which makes it dramatically more intuitive to interpret. It is given as:

$$M^2 = S \times \sigma_B + RFR$$

Where:

S is the Sharpe ratio,

σ_B is the standard deviation of the excess returns for some benchmark portfolio against which you are comparing the portfolio in question (often, the benchmark portfolio is the market), and

RFR is the average risk-free rate for the period in question

Jensen Portfolio Performance Measure

The Jensen measure is similar to the measures already discussed because it is based on the capital asset pricing model (CAPM). All versions of the CAPM calculate the expected one-period return on any security or portfolio by the following expression:

$$E(R_j) = RFR + \beta_j[E(R_M) - RFR]$$

Where:

$E(R_j)$ = the expected return on security or portfolio j

RFR = the one-period risk-free interest rate

β_j = the systematic risk (beta) for security or portfolio j

$E(R_M)$ = the expected return on the market portfolio of risky assets

The expected return and the risk-free return vary for different periods. Consequently, *we are concerned with the time series of expected rates of return for security or portfolio j.* Moreover, assuming the asset pricing model is empirically valid, you can express the above equation in terms of realized rates of return as follows:

$$R_{jt} = RFR_t + \beta_j[R_{mt} - RFR_t] + e_{jt}$$

This equation states that the realized rate of return on a security or portfolio during a given time period should be a linear function of the risk-free rate of return during the period, plus a risk premium that depends on the systematic risk of the security or portfolio during the period plus a random error term (e_{jt}).

Subtracting the risk-free return from both sides, we have

$$R_{jt} - RFR_t = \beta_j [R_{mt} - RFR_t] + e_{jt}$$

This shows that *the risk premium earned on the jth portfolio is equal to βj times a market risk premium plus a random error term.* In this form, ***an intercept for the regression is not expected if all assets and portfolios were in equilibrium.***

Alternatively, superior portfolio managers who forecast market turns or consistently select undervalued securities earn higher risk premiums than those implied by this model. *Specifically, superior portfolio managers* ***have consistently positive random error terms*** *because the actual returns for their portfolios consistently exceed the expected returns implied by this model.* ***To detect and measure this superior performance, you must allow for an intercept (a non-zero constant) that measures any positive or negative difference from the model.*** Consistent positive differences cause a positive intercept, whereas consistent negative differences (inferior performance) cause a negative intercept. With an intercept, or non-zero constant, the earlier equation becomes:

$$R_{jt} - RFR_t = \alpha_j + \beta_j [R_{mt} - RFR_t] + e_{jt}$$

In the equation above, ***α_j value indicates whether the portfolio manager is superior or inferior in market timing and/or stock selection.*** A superior manager has a significant positive α(or"alpha") value because of the consistent positive residuals. In contrast, an inferior manager's returns consistently fall short of expectations based on the CAPM model giving consistently negative residuals. In such a case, α is a significant negative value.

The performance of a portfolio manager with no forecasting ability but not clearly inferior equals that of a naive buy-and-hold policy. In the equation, because the rate of return on such a portfolio typically matches the returns you expect, the residual returns generally are randomly positive and negative. *This gives a constant term that is not significantly different from zero,* indicating that the portfolio manager basically matched the market on a risk-adjusted basis. Therefore, the α represents how much of the rate of return on the portfolio is attributable to the manager's ability to derive above-average returns adjusted for risk. Superior risk-adjusted returns indicate that the manager is good at either predicting market turns, or selecting undervalued issues for the portfolio, or both.

Applying the Jensen Measure
The Jensen measure of performance requires using a different RFR for each time interval during the sample period. For example, to examine the performance of a fund manager over a 10-year period using yearly intervals, *you must examine the fund's annual returns less the return on risk-free assets for each year and relate this to the annual return on the market portfolio less the same risk-free rate.* This contrasts with the Treynor and Sharpe composite measures, which examine the average returns for the total period for all variables (the portfolio, the market, and the risk-free asset).

Also, like the Treynor measure, the Jensen measure does not directly consider the portfolio manager's ability to diversify because it calculates risk premiums in terms of systematic risk. As noted earlier, to evaluate the performance of a group of well-diversified portfolios such as mutual funds, this is likely to be a reasonable assumption. Jensen's analysis of mutual fund performance showed that complete diversification was a fairly reasonable assumption because the funds typically correlated with the market at rates above 0.90.

Jensen Measure and Multifactor Models
The Jensen composite measure of performance has several advantages over the Treynor and Sharpe.
First, it is ***easier to interpret*** in that an alpha value of 0.02 indicates that the manager generated a return of 2 percent per period more than what was expected given the portfolio's risk level.
Second, because it is estimated from a regression equation, ***it is possible to make statements about the statistical significance of the manager's skill level,*** or the difference in skill levels between two different managers.
A **third** advantage of the Jensen performance measure is that it is ***flexible enough to allow for alternative models of risk and expected return than the CAPM***. Specifically, risk-adjusted performance (i.e., a) can be computed relative to any of the multifactor models as follows:

$$R_{jt} - RFR_t = \alpha_j + \beta_j [b_{j1}F_{1t} + b_{j2}F_{2t} + \ldots\ldots\ldots + b_{jk}F_{kt}] + e_{jt}$$

Where:
F_{kt} represents the Period *t* return to the kth common risk factor. Notice that the Sharpe measure, which focuses on total risk, ignores the specific form of the return-generating process altogether, while the Treynor measure requires a single measure of systematic risk

Information Ratio

Closely related to the statistics just presented is another widely used performance measure: the information ratio. Also known as an **appraisal ratio**, ***this statistic measures a portfolio's average return in excess of that of a comparison or benchmark portfolio divided by the standard deviation of this excess return.*** Formally, the information ratio *(IR)* is calculated as:

$$IR_j = \frac{\bar{R}_j - \bar{R}_b}{\sigma_{ER}} = \frac{\overline{ER_j}}{\sigma_{ER}}$$

Where:

IR_j *=the information ratio for portfolio j*
R_j *=the average return for portfolio j during the specified time period*
R_b*=the average return for the benchmark portfolio during the period*
σ_{ER}*=the standard deviation of the excess return during the period*
$\overline{ER_j}$*=the mean excess return during the period*

To interpret *IR*, notice that *the mean excess return in the* ***numerator represents*** *the investor's ability to use her talent and information to generate a portfolio return that differs from that of the benchmark against which her performance is being measured* (e.g., NASI*). Conversely, the* ***denominator measures*** *the amount of residual (unsystematic) risk that the investor incurred in pursuit of those excess returns*. The coefficient σ_{ER} is sometimes called the ***tracking error*** of the investor's portfolio and it is a "cost" of active management in the sense that fluctuations in the periodic ER_j values represent random noise beyond an investor's control that could hurt performance. *Thus, the IR can be viewed as* ***a benefit-to-cost ratio*** *that assesses the quality of the investor's information deflated by unsystematic risk generated by the investment process.*

Goodwin has noted that the *Sharpe ratio is a special case of the IR where the risk-free asset is the benchmark portfolio, despite the fact that this interpretation violates the spirit of a statistic that should have a value of zero for any passively managed portfolio*. More importantly, he also showed that if excess portfolio returns are estimated with historical data using the same single-factor regression equation used to compute Jensen's alpha, the *IR* simplifies to

$$IR_j = \frac{\alpha_j}{\sigma_e}$$

Where:

σ_e *=the standard error of the regression*

Finally, he showed that one way an information ratio based on periodic returns measured T times per year could be annualized is as follows:

$$Annualized\ IR = \frac{(T)\alpha_j}{\sqrt{T}\sigma_e} = \sqrt{T}(\mathrm{IR})$$

For instance, an investor that generated a quarterly ratio of 0.25 would have an annualized *IR* of 0.50. Some researchers have argued that reasonable information ratio levels should range from 0.50 to 1.00, with an investor having an *IR* of 0.50 being good and one with an IR of 1.00 being exceptional.

Fama's Breakdown of Performance

Following the work of Treynor, Sharpe, and Jensen, ***Fama suggested a somewhat finer breakdown of performance.*** The basic premise for Fama's technique is that overall performance of a portfolio, which is its return in excess of the risk-free rate, can be decomposed into measures of risk-taking and security selection skill. That is:

Overall Performance =Excess Return =(Portfolio Risk) +(Selectivity)

Further, *if there is a difference between the risk level specified by the investor and the actual risk level adopted by the portfolio manager* (in cases where these are separate people), this calculation can be further refined to:

Overall Performance =[(Investor's Risk) +(Manager's Risk)] +(Selectivity)

Notice that *the selectivity component represents the portion of the portfolio's actual return beyond that available to an unmanaged portfolio with identical systematic risk. Thus, this selectivity measure is used to assess the manager's investment prowess.*

As with the preceding performance statistics, Fama's evaluation model assumes that returns to managed portfolios can be compared to those of naively selected portfolios with similar risk levels. The technique is based on the *ex-ante* market line summarizing the equilibrium relationship between expected return and risk for Portfolio j:

$$E(\hat{R}_j) = RFR + \left[\frac{E(\hat{R}_M) - RFR}{\sigma(\hat{R}_M)}\right]\frac{\text{Cov}(\hat{R}_j, \hat{R}_M)}{\sigma(\hat{R}_M)}$$

Cov(R_j,R_M) is the covariance between the returns for Portfolio j and the return on a single market portfolio. ***This equation indicates that the expected return on Portfolio jis the riskless rate of interest, RFR, plus a risk premium that is $[E(\hat{R}_M - RFR)/\sigma(\hat{R}_M)$ called the market price per unit of risk, times the risk of Asset j, which is [Cov($\hat{R}_j$, $\hat{R}_M]/\sigma(\hat{R}_M)$***

If a portfolio manager believes that the market is not completely efficient and that she can make better judgments than the market, then an *ex-post* version of this market line can provide a benchmark for the manager's performance. ***Given that the risk variable, Cov(R_j,R_M)/σ(R_M), can be denoted β_x, the ex post market line is as follows:***

$$R_x = RFR + \left(\frac{R_M - RFR}{\sigma(R_M)}\right)\beta_x$$

This *ex-post* market line provides the benchmark used to evaluate managed portfolios in a sequence of more complex measures.

Evaluating Selectivity

Formally, you can measure the return due to gross selectivity as follows:

Gross Selectivity =R_a– $R_x(\beta_a)$

Where:

R_a=*the actual return on the portfolio being evaluated*

$R_x(\beta_a)$ =*the return on the combination of the riskless asset and the market portfolio M that has risk β_x equal to β_a, the risk of the portfolio being evaluated*

Selectivity measures the vertical distance between the actual return and the *ex-post* market line and is quite similar to Treynor's measure. As already noted, you can examine overall performance in terms of selectivity and the returns from assuming risk as follows:

Overall Performance = Selectivity + Risk

$[R_a - RFR] = [R_a - R_x(\beta_a)] + [R_x(\beta a) - RFR]$

Overall performance is the total return above the risk-free return and includes the return that should have been received for accepting the portfolio risk (β_a). This *expected return for accepting risk (β_a) is equal to [$R_x(\beta_a)$ – RFR]. Any excess over this expected return is due to selectivity.*

Evaluating Diversification

The selectivity component in can also be broken down into two parts. If a portfolio manager attempts to select undervalued stocks and in the process gives up some diversification, it is possible to measure the added return necessary to justify this diversification decision. *The portfolio's **gross selectivity** is made up of **net selectivity** plus **diversification** as follows:*

Gross Selectivity = Net Selectivity + Diversification

$$R_a - R_x(\beta_a) = \text{Net Selectivity} + [R_x(\sigma(R_a)) - R_x(\beta_a)]$$

$$\text{Net Selectivity} = R_a - R_x(\beta_a) - [R_x(\sigma(R_a)) + R_x(\beta_a)]$$
$$= R_a - R_x(\sigma(R_a))$$

Where:

$R_x(\sigma(R_a))$ = *the return on the combination of the riskless asset and the market portfolio that has return dispersion equivalent to that of the portfolio being evaluated*

Evaluating Risk

Assuming the investor has a target level of risk for the portfolio equal to β_T, the portion of overall performance due to risk (the total return above the risk-free return) can be assessed as follows:

Risk = Manager's Risk + Investor's Risk

$$[R_x(\beta_a) - RFR] = [R_x(\beta_a) - R_x(\beta_T)] + [R_x(\beta_T) - RFR]$$

Where:

$R_x(\beta_T)$ = *the return on the naively selected portfolio with the target level of market risk* (β_T)

If the portfolio risk is equal to the target risk ($\beta_a = \beta_T$), *then the manager's risk does not exist. If there is a difference between* β_a *and* β_T, *then the manager's risk is the return she must earn due to the decision to accept risk* (β_a), *which is different from the risk desired by the investor* (β_T). The investor's risk is the return expected because the investor stipulated some positive level of risk. This evaluation is possible only if the client has specified a desired level of market risk, which is typical of pension funds and profit-sharing plans. Generally, it is not possible to compute this measure for ex-post evaluations because the desired risk level is typically not available

Factors affecting performance measures

Data inputs- should cover several years to ensure a fully market cycle has been captured
Incompleteness of the market portfolio- Benchmark error
Beta could differ from that computed using the true market portfolio.

Required Characteristics of Benchmarks

i) ***Unambiguous-*** The names and weights of securities comprising the benchmark are clearly delineated.
ii) ***Investable-*** The option is available to forgo active management and simply hold the benchmark.
iii) ***Measurable-*** It is possible to calculate the return on the benchmark on a reasonably frequent basis.
iv) ***Appropriate-*** The benchmark is consistent with the manager's investment style or biases.
v) ***Reflective of current investment opinions-*** The manager has current investment knowledge (be it positive, negative, or neutral) of the securities that make up the benchmark.
vi) ***Specified in advance-*** The benchmark is constructed prior to the start of an evaluation period.

If a benchmark does not possess all of these properties, it is considered flawed as an effective management tool.

Appendix
Example of Peer Group Comparison Boxplot