Notes for Classic IP & CO Paper List

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November 8, 2020

November 8, 2020

To My Beloved Motherland China

Contents

Chapter 1

The Traveling-Salesman Problem and Minimum Spanning Tree

Michael Held and Richard M. Karp

***Operations Research*, 1969**

1.1 The Traveling-Salesman Problem and a Related Spanning-Tree Problem

Definition 1.1.1 (1-tree)**.** In graph $G = (V, E)$, where $V = \{1, 2, \cdots, n\}$, a 1-tree consists of a tree on the vertex set $\{2, 3, \cdots, n\}$, together with two distinct edges at vertex 1.

Thus, a 1-tree has a single cycle, this cycle contains vertex 1 and vertex 1 always has degree 2. A minimal weighted 1-tree can be found by constructing a minimum spanning tree on the vertex set $\{2, 3, \cdots, n\}$, and then adjoining two edges of lowest weight at vertex 1.
Also notice that every tour is a 1-tree, and a 1-tree is a tour iff each of its vertices has degree 2. If a minimum-weight 1-tree is a tour, it is the solution of the TSP.

Example. An example of 1-tree can be found in figure 1.1, solid arcs are minimum spanning tree of $\{2, 3, \cdots, n\}$ and two dashed arcs links the MST to vertex 1 with minimal cost.

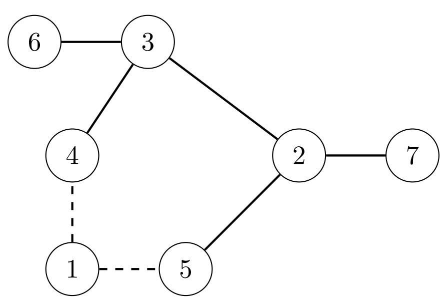

Figure 1.1: 1-tree

Lemma 1.1. *Let $\pi = (\pi_1, \pi_2, \cdots, \pi_n)$ be a real n-vector. If C^* is a minimum-weight tour with respect to the edge weights c_{ij}, then it is also a minimum-weight tour C' with respect to the edge weight $c_{ij} + \pi_i + \pi_j$.*

Proof. For tour C, the weight is $C = \sum_{(i,j) \in C} c_{ij}$. Therefore $C' - C^* = 2 \sum_{i=1}^{n} \pi_i$, which is a constant. $\square$