Gurobi Workshop - Part II

Lan Peng, PhD Candidate

Department of Industrial & Systems Engineering
University at Buffalo, SUNY

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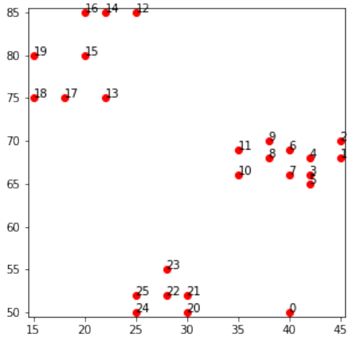
Overview

- Part I - 26th Oct.
  - Introduction and Installation
  - Manufactures Problem - Small Example of LP
  - Parallel Machine Scheduling Problem - Simple Example of IP
  - Traveling Salesman Problem - Lazy Constraints and Callback
- Part II - 2nd Nov.
  - Vehicle Routing Problem with Time Windows - Column Generation
  - Facility Location Problem - Benders Decomposition

Special thanks to Dr. Walteros. I learned the material in this workshop from his lecture.

VRPTW

Vehicle routing problem with time windows (VRPTW) can be defined as choosing routes for limited number of vehicles to serve a group of customers in the time windows. Each vehicle has a limited capacity. It starts from the depot and terminates at the depot. Each customer should be served exactly once. (From Hindawi)



Set partitioning master problem

Create and select routes

$$
\begin{aligned}
\min \quad & \sum_{r \in Y} c_r y_r \\
\text{s.t.} \quad & \sum_{r \in Y} y_r \leq V \\
& \sum_{r \in Y} a_{ir} y_r = 1, \quad \forall i \in C \\
& y_r \geq 0 \quad \text{(Lower-bound searching)}, \\
\text{or,} \quad & y_r \in \{0, 1\} \quad \text{(Early branching)}
\end{aligned}
$$

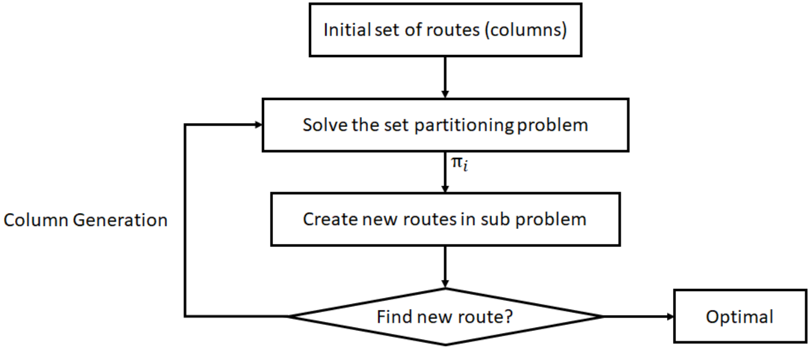
Set Y is updated in each iteration.

Pricing subproblem

Finding the time-windowed shortest routes.

$$
\begin{aligned}
\min \quad & \sum_{(i,j)\in E} (c_{ij} - \pi_i) w_{ij} \\
\text{s.t.} \quad & \sum_{j:(i,j)\in E} w_{ij} - \sum_{j:(j,i)\in E} w_{ji} = 0, \quad \forall i \in C \setminus \{0\} \\
& \sum_{j:(0,j)\in E} w_{0j} = \sum_{j:(j,0)\in E} w_{j0} = 1 \\
& \sum_{(i,j)\in E} w_{ij} d_j \leq q \\
& s_i + c_{ij} + t_i - M(1 - w_{ij}) \leq s_j, \quad \forall i,j \in C \\
& a_i \leq s_i \leq b_i, \quad \forall i \in C \\
& w_{ij} \in \{0,1\}, \quad \forall (i,j) \in E
\end{aligned}
$$

Column Generation



Facility Location Problem

Consider the following facility location problem, where m is the number of potential facilities, n is the number of customers, and $\mathbb{Y}$ is the set of feasible plans of facility location plans, where $\mathrm{y} \subset \{0,1\}^m \in \mathbb{Y}$. c_{ij} is the cost for customer i to be assigned to facility j, d_j is the cost for opening facility j. The formulation is as following

$$
\begin{aligned}
\min \quad & v = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} + \sum_{j=1}^{m} d_j y_j \\
\text{s.t.} \quad & \sum_{j=1}^{m} x_{ij} \geq 1, \quad \forall i \in \{1, 2, \dots, n\} \\
& x_{ij} \leq y_j, \quad \forall i \in \{1, 2, \dots, n\}, j\{1, 2, \dots, m\} \\
& x_{ij} \geq 0, \quad \forall i \in \{1, 2, \dots, n\}, j\{1, 2, \dots, m\} \\
& y_j \in \{0, 1\}, \quad \forall j \in \{1, 2, \dots, m\}
\end{aligned}
$$

Sub problem

If y are **fixed**, i.e., y $= \hat{y} \in \mathbb{Y}$, the rest of the formulation will become a LP model with x_{ij} as the nonnegative decision variables.

$$
\begin{aligned}
\min \quad & v = \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} + \sum_{j=1}^{m} d_j\hat{y}_j \\
\text{s.t.} \quad & \sum_{j=1}^{m} x_{ij} \geq 1, \quad \forall i \in \{1,2,\dots,n\} \\
& x_{ij} \leq \hat{y}_j, \quad \forall i \in \{1,2,\dots,n\}, j\{1,2,\dots,m\} \\
& x_{ij} \geq 0, \quad \forall i \in \{1,2,\dots,n\}, j\{1,2,\dots,m\}
\end{aligned}
$$

Dual of sub problem

If we take the dual of the subproblem, we get

$$
\begin{aligned}
\text{(Dual-Sub)} \quad v(\hat{y}) = \max \quad & \sum_{i=1}^{n}(\lambda_i - \sum_{j=1}^{m} \hat{y}_j \pi_{ij}) + \sum_{j=1}^{m} d_j \hat{y}_j \\
\text{s.t.} \quad & \lambda_i - \pi_{ij} \leq c_{ij} \quad \forall i \in \{1, 2, \ldots, n\}, j \in \{1, 2, \ldots, m\} \\
& \lambda_i \geq 0 \quad \forall i \in \{1, 2, \ldots, n\} \\
& \pi_{ij} \geq 0 \quad \forall i \in \{1, 2, \ldots, n\}, j \in \{1, 2, \ldots, m\}
\end{aligned}
$$

Remind: Weak Duality theorem

Prime Problem	Dual Problem
Optimal	Optimal
Unbounded or infeasible	Infeasible

Optimality Cuts and Feasibility Cuts

- If the dual of sub problem has an optimal solution - add optimality cut

$$\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} \geq \sum_{i=1}^{n}(\lambda_i - \sum_{j=1}^{m}\pi_{ij}\hat{y}_j)$$

- If the dual of sub problem is unbounded - add feasibility cut

$$\sum_{i=1}^{n}(\lambda_i - \sum_{j=1}^{m}\pi_{ij}\hat{y}_j) \leq 0$$

- If the dual of sub problem is infeasible - terminate