Remedial Mathematics for Undergraduate Students in Nanoscience and nanotechnology

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Índice general

1. Complex Numbers

1.1. Definitions and Notation of Complex Numbers

1.1.1. A bit of history

Luca Pacioli published in 1494 the book “Summa de arithmetica, geometria, proportioni et proportionalita”, in this text the general cubic equation $ax^3 + bx^2 + d = 0$ was proposed and it was believed that such an equation was impossible to solve, however, in the following decades and centuries some mathematicians began to find some partial solutions of the general cubic equation.

Some mathematicians like Niccolò Tartaglia and Scipione del Ferro solved the cubic equation in the 16th century, but partially. In that same century, Gerolamo Cardano published an algebraic method to analytically solve any cubic equation, Cardano solved the cubic equation, but, only the real ones and he was the first to introduce complex numbers $a + \sqrt{-b}$.

Rafael Bombelli was an mathematician and engineer is central figure in the understanding of imaginary numbers. He was the one who finally managed to address the problem with imaginary numbers. In his 1572 book, L'Algebra, Bombelli solved equations using the method of del Ferro/Tartaglia. He introduced the rhetoric that preceded the representative symbols $+\mathbf{i}$ and $-\mathbf{i}$ and described how they both worked because they were necessary for his calculations. Bombelli introduces a notation for $\sqrt{-1}$, and calls it “pí u di meno”.

René Descartes associated imaginary numbers with geometric impossibility. Euler, first of all, introduced the notation $i = \sqrt{-1}$, and visualized complex numbers as points with rectangular coordinate while Gauss introduced the term complex number and Cauchy constructed the set of complex numbers in 1847.

Complex numbers are used in many fields of engineering and science, including:

- Electronics.
- Electromagnetism.

- To simplify the unknown roots if roots are not real for quadratic equations.
- Computer science engineering.
- Civil engineering.
- Control systems.
- Quantum mechanics, $H\Psi = i\hbar\,\partial\,\Psi/\,\partial\,t$.

1.1.2. Introduction

A complex number is an element of a number system that extends the real numbers with a specific element denoted i, called the imaginary unit and satisfying the equation

$$i^2 = -1 \tag{1.1}$$

or equivalently

$$\sqrt{-1} = i. \tag{1.2}$$

Every complex number can be expressed in the form $a + bi$, where a and b are real numbers. Because no real number satisfies the above equation, i was called an imaginary number by René descartes. For the complex number $a + bi$, a is called the real part and b is called the imaginary part. The set of complex numbers is denoted by either of the symbols $\mathbf{C}$ or $\mathbb{C}$. Despite the historical nomenclature "imaginary", complex numbers are regarded in the mathematical sciences as just as "real" as the real numbers and are fundamental in many aspects of the scientific description of the natural world. Complex numbers can be expressed in three different ways: binomial, rectangular and exponencial form.
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1.2. Rectangular or binomial form of complex numbers and the complex plane

It is customary to express any real number by means of the letter z. The rectangular form of a complex number is a sum of two terms: the number's real part and the number's imaginary part multiplied by i.

$$z = a + bi \tag{1.3}$$

It is possible also plot a complex number given in rectangular form in the complex plane (Fig. 1.1). The real and imaginary parts determine the real and imaginary coordinates of the number.

Recommended Lectures

[1] Budden, F. J. *Complex numbers and their applications.* Longmans, 2010.

[2] Capilla, R. F., and Giménez, I. *Variable compleja y ecuaciones diferenciales.* Editorial Reverté, 1995.

[3] Carlomán, A. G. *Didáctica del número complejo.* Universidad de Oviedo. Servicio de Publicaciones, 1994.

[4] Carr, V. *Complex Numbers Made Simple.* Butterworth-Heinemann, 1996.

[5] Levinson, N., and Redheffer, R. M. *Curso de variable compleja.* Editorial Reverté, 2010.

[6] Malacrida, S. *Introducción a los números complejos.* Simone Malacrida, 2023.

[7] Orive, A. L. *Curso básico de variable compleja.* UNAM. Facultad de Ciencias, 2007.

[8] Restrepo, G. *Funciones de una variable compleja.* Universidad del Valle, 2003.

[9] Saff, E., and Snider, A. D. *Fundamentals of Complex Analysis with Applications to Engineering and Science.* Prentice Hall, 2003.