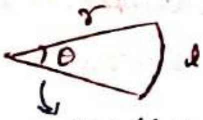
UNITS AND DIMENSIONS

Fundamental Quant:

- Luminous intensity : Candela : cd.

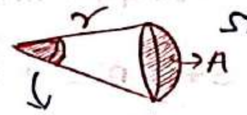
Plane Angle:



$\theta = l/r$

radian

Solid Angle:



$\Omega = \frac{A}{r^2}$

Steradian

Must know Dim. formulae:

1) $\varepsilon_0 - M^{-1}L^{-3}T^{4}A^{2}$
2) $\mu_0 - MLT^{-2}A^{-2}$
3) $G - M^{-1}L^{3}T^{-2}$
4) Gas Const., $R - ML^{2}T^{-2}\theta^{-1}$
5) Boltzmann's Const, $K_B = ML^{2}T^{-2}\theta^{-1}$
6) Planks const, $h = ML^{2}T^{-1}$
7) Rydberg's const., $R_y = L^{-1}$.
8) Magnetising field, $H = \frac{B}{\mu_0} = \frac{\mu_0 i/2r}{\mu_0} = i/r \Rightarrow AL^{-1}$

$\frac{dQ}{dT} = \sigma e A T^4$],

9) Stephen Boltzmann Const (σ) : $ML^{0}T^{-3}\theta^{-4}$.

Dimentionless Quantities:

→ All trignometric ratios

→ Angle, θ ($\sin\frac{ab}{c}$, here $[ab]=[c]$)

→ Exponential fns., e^x → x must be dim'less.

($e^{-t/Rc}$ → $[t]=[RC]$)

→ Reynold's no. ($R_e = \frac{\rho v D}{\eta}$), Dielectric const (k), Refractive index, and many more...

Principle of Homogeneity:

If $a = b + c$ then,

$$[a] = [b] = [c].$$

Conversion of Units:

amt. of physical quantity ← $q = nu$ → units (CGS/SI)

n ↓ numeric value

Ex: find how many poise (CGS unit of vis) is equal to 1 Poiseuille (SI)?

Solu) $F = 6\pi\eta r v \Rightarrow [\eta] = \frac{[F]}{[r][v]} = ML^{-1}T^{-1}$

$$n_1u_1 = n_2u_2$$

$$\Rightarrow n_1 \times (g\,cm^{-1}s^{-1}) = 1 \times (kg\,m^{-1}s^{-1})$$

$$n_1 = \frac{kg\,m^{-1}}{g\,cm^{-1}} = \frac{10^3 g \times 10^{-2} cm^{-1}}{g\,cm^{-1}} = \underline{\underline{10}}$$

Dimension in terms of other phy. quantity:

#Ex: Expression for time in terms of G, h, c is prop. to?

Solut $t \propto G^a h^b c^c \Rightarrow [t] = [G]^a[h]^b[c]^c$.

$$\Rightarrow [M^0L^0T^1] = [M^{-1}L^3T^{-2}]^a[ML^2T^{-1}]^b[LT^{-1}]^c$$

$$[M^0L^0T^1] = [M^{-a+b}L^{3a+2b+c}T^{-2a-b-c}]$$

$0 = -a+b$; $0 = 3a+2b+c$; $1 = -2a-b-c$.

$\Rightarrow a = b = 1/2$; $c = -5/2$

$\therefore t \propto G^{1/2}h^{1/2}c^{-5/2}$ (or) $\underline{\underline{t \propto \sqrt{\frac{Gh}{c^5}}}}$.

ERROR ANALYSIS

1. Absolute, Relative & Percentage Error:

(Error = True value − Mes value)

Step1: $R_{avg} = \frac{R_1 + R_2 + \cdots + R_n}{n}$ (we take R_{avg} as true value)

Step2: Absolute Error $\Delta R_1 = R_1 - R_{avg}$.

$\Delta R_2 = R_2 - R_{avg}$.

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$\Delta R_n = R_n - R_{avg}$.

Step3: Mean Abs. Error

$$\Delta R_{avg} = \frac{|\Delta R_1| + |\Delta R_2| + \cdots + |\Delta R_n|}{n}.$$

Final Result: $R_{avg} \pm \Delta R_{avg}$.

Rel Error: $\frac{\Delta R_{avg}}{R_{avg}}$.

% Error: $\frac{\Delta R_{avg}}{R_{avg}} \times 100$

2. Combination of Error:

- Sum / Diff —

$l = 4.1 \pm 0.1\,cm$, $b = 3.3 \pm 0.1\,cm$

$S = L + B = 7.4 \pm 0.2\,cm$

$S = L - B = 0.8 \pm 0.2\,cm$.

- Prod / Div —

$P = d\frac{x^a y^b}{z^c}$ $\left\{ \text{Error is } \Delta x, \Delta y \text{ and } \Delta z. \text{ Find Error in } P \right\}$

↳ $\ln p = a\ln x + b\ln y + \ln d - c\ln z$.

↳ $\frac{dp}{p} = a\frac{dx}{x} + b\frac{dy}{y} - c\frac{dz}{z} + 0$.

to find max Error in p:

$$\boxed{\frac{\Delta p}{p} = a\frac{\Delta x}{x} + b\frac{\Delta y}{y} + c\frac{\Delta z}{z}}$$ $\left\{ \frac{\Delta x}{x} \text{ is rel Error in } x \right\}$

* If, $x = \frac{l}{100 - l}$; $\frac{\Delta x}{x} = \frac{\Delta l}{l} + \frac{\Delta l}{100 - l}$.

BASIC VECTORS.

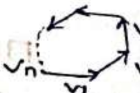
- $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

- Coplanar Vectors: $\vec{a} = \lambda\vec{b} + \mu\vec{c}$

- Addition of vectors:

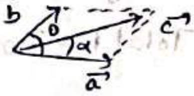
• Triangle law – $\vec{a} + \vec{b} = \vec{c}$.

• Polygon law –



$\vec{v}_1 + \vec{v}_2 + \dots \vec{v}_n = 0$

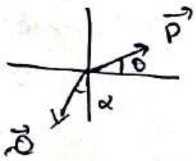
• Parallelogram law –



$$C = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

$$\tan\alpha = \frac{b\sin\theta}{a + b\cos\theta}.$$

- Resolution of vectors:

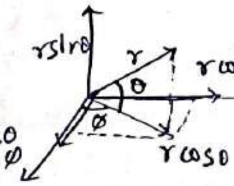


$\vec{P} = P\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}$

$\vec{Q} = Q\cos\alpha\,(-\hat{j}) + Q\sin\alpha(-\hat{i})$

— 3-D:

$\vec{r} = r\cos\theta\sin\phi\,\hat{i} + r\sin\theta\,\hat{j} + r\cos\theta\cos\phi\,\hat{k}$



- Dot Product:

– $\vec{a} \cdot \vec{b} = ab\cos\theta$

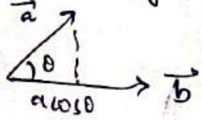
– $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

- Uses of dot product:

→ If $\vec{a} \cdot \vec{b} = 0$ then $\vec{a} \perp \vec{b}$

→ $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|ab|}$

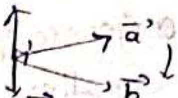
→ Finding projection of one vect on anoth



$\vec{a} \cdot \vec{b} = ab\cos\theta$

$\therefore a\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{b}$.

- Cross Product:



– $\vec{a} \times \vec{b} = \vec{c}$, $\vec{c} = ab\sin\theta\,\hat{n}$

– $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{c}$ ($\vec{a} \times \vec{b} = -\vec{b} \times \vec{c}$)

– $\vec{a} \times \vec{a} = 0$

– To find area of Δ^{le} & llgm.

– $\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

– If $\vec{A} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$; $\vec{B} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$\hat{n} = \dfrac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$ {unit vect is ⊥ to both A & B}

- Scalar Triple Prod:

$\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$ [cond. for coplanarity]

- Direction Cosines:

$l = \cos\alpha = x/|\vec{r}|$

$m = \cos\beta = y/|\vec{r}|$

$n = \cos\gamma = z/|\vec{r}|$

$l^2 + m^2 + n^2 = 1$

VERNIER CALIPER.

- Generally, each division on VS is smaller than MS.
- LC: smallest meas. that can be made.

$$LC = 1MSD - 1VSD.$$

let $nVSD = mMSD$

$$LC = MSD\left(1 - m/n\right).$$

- Reading $= MSR + (\cancel{n} \; VSR)\, LC$

MSR: reading on MS just before (0) on VS

VSR: VSR coinciding with MSD.

→ MSR + least count × VS div coinc. with MS.

- When 1MSD < 1VSD:
  - calculate using diagram {better dont use formula & think}

- Zero Error: when zero of VS donot coincide with zero of MS.

+ve zero: 0 of vs is right. Reading = Total R − Error

−ve zero: 0 of vs is left. Reading = T.R + Error

SCREW GAUGE.

- Pitch: dist. b/w 2 consecutive threads. {Made by 1 complete rotation of Circular scale}
- Least Count: $\dfrac{\text{Pitch}}{\text{no. of div. on circular scale}}$.
- Reading: MS reading + Circular Scale count. {circ. div coinc. with MS × LC}

⇒ +ve zero error: 0 of circular scale is below of MS (zero).

Reading = Total R − Error.

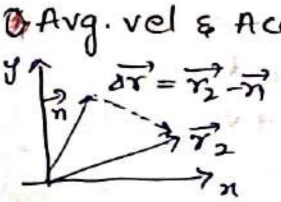
⇒ −ve zero error: 0 of c.scale is above the MS (zero).

Reading = Total R + Error.

KINEMATICS.

Dist : Actual length of the path
Displacement: shortest dist. b/w i & f.

Avg. vel & Acc :



$\vec{v}_{avg} = \frac{\vec{r_2} - \vec{r_1}}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$.

(OR)

$v = f(t)$ is given

$v_{avg} = \int_{t_1}^{t_2} v\,dt \Big/ t_2 - t_1$

- $\vec{a}_{avg} = \frac{\vec{v_2} - \vec{v_1}}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$ (or)

If $a = f(t)$ is given, $a_{av} = \frac{\int_{t_1}^{t_2} a\,dt}{t_2 - t_1}$.

Inst. vel & acc :

$x = f(t)$
$v = \frac{dx}{dt}$

$v_{ist} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$.

$a_{ist} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$.

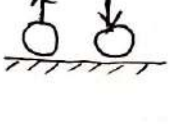
$a = \frac{dv}{dt} = \frac{v\,dv}{ds} = \frac{d^2 s}{dt^2}$

Uniform Acceleration :

→ $v = u + at$
→ $s = ut + \frac{1}{2}at^2$
→ $v^2 - u^2 = 2as$
→ $s_n = u + \frac{a}{2}(2n-1)$

here v, u, a & s are vectors, so, be careful with signs.

1-D Motion under gravity :



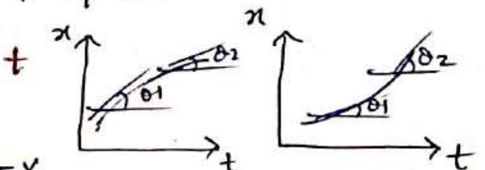
$t_{ascent} = t_{descent} = u/g$

$T = 2u/g$. & $H = u^2/2g$.

$t = \sqrt{\frac{2H}{g}}$.

1-D Graphs :

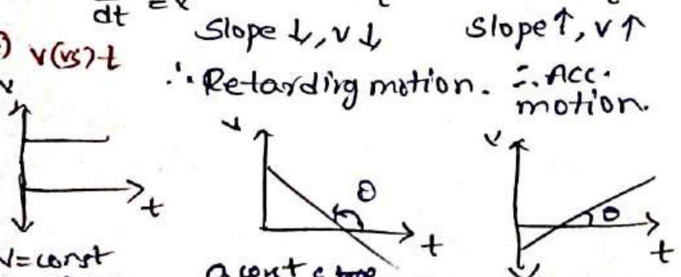
1) x(vs) t



$\frac{dx}{dt} = v$

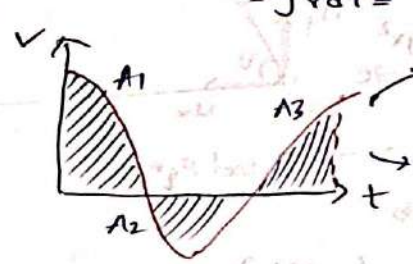
Slope ↓, v ↓ ∴ Retarding motion.
Slope ↑, v ↑ ∴ Acc. motion.

2) v(vs) t



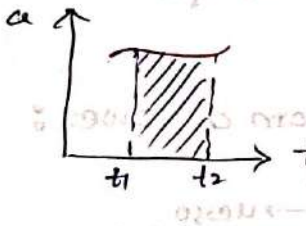
v = const, a = 0
a const & -ve
a const & +ve.

2) v (vs) t : → slope $\frac{dv}{dt} = a$.
→ area under curve $= \int v\,dt =$ displ.



→ displ: $= A_1 - A_2 + A_3$.
→ dist: $= A_1 + A_2 + A_3$

3) a (vs) t :



Area $= \Delta v = v - u$
(if area is below t-axis, take it as -ve)

4) V (vs) x :
↳ slope $\frac{dv}{dx} = a/v$.

2-D Motion :

Motion in a plane -

if $\vec{a}$ is const: {for along k axis}

$v_k = u_k + a_k t$
$s = u_k t + \frac{1}{2} a_k t^2$
$v_k^2 - u_k^2 = 2a_k s$.

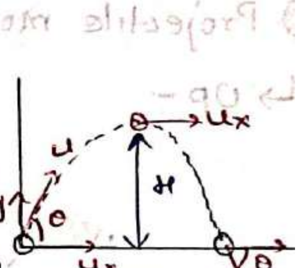
Projectile Motion - (Ground to Ground)

- standard Eq^n :

$T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

$H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$

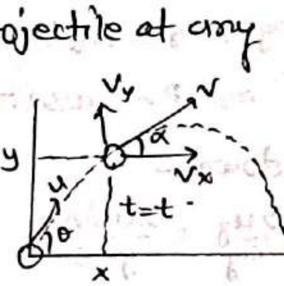
$R = u_x \cdot T = \frac{u^2 \sin 2\theta}{g}$



∴ $a_x = 0 \Rightarrow u_x = u\cos\theta =$ const.

- Analysis of projectile at any time t :

(i) $v_x = u_x$,
$v_y = u_y - gt$
$v = \sqrt{v_x^2 + v_y^2}$
$\alpha = \tan^{-1}(v_y/v_x)$.



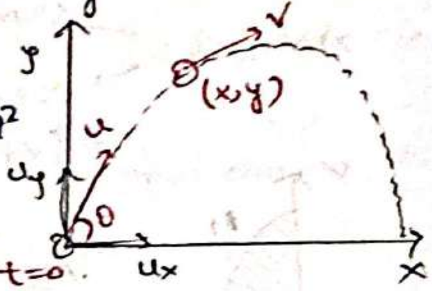
(ii) $x = u_x t$, $y = u_y t - \frac{1}{2} g t^2$

$\vec{r} = x\hat{i} + y\hat{j}$, $r = \sqrt{x^2 + y^2}$.

Eq^n of Trajectory:

using $x = u\cos\theta . t$

& $y = u\sin\theta . t - \frac{1}{2}gt^2$



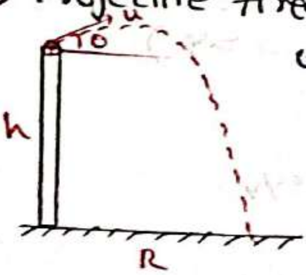
(i) $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

$(y = ax - bx^2) \longrightarrow$ Quad Eq^n.

Range, $R = a/b$.

(ii) $y = x\tan\theta\left(1 - x/R\right)$ → Range.

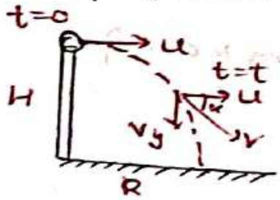
- Projectile fired from a Tower:



$a_y = -g$ & $a_x = 0$

$R = u\cos\theta . T$

— projected horizontally:



$T = \sqrt{\frac{2h}{g}}$.

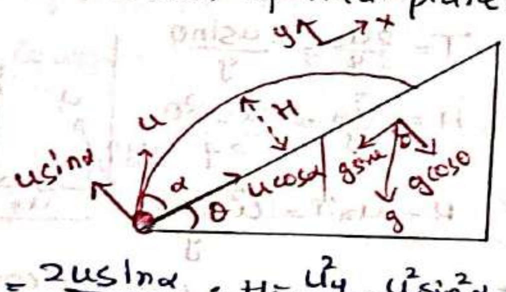
$R = u.T = u\sqrt{\frac{2h}{g}}$.

$v_y = gt \therefore \alpha = \tan^{-1}\left(\frac{v_y}{u}\right)$

& $v = \sqrt{u^2 + v_y^2}$.

- Projectile motion of incl. plane:

↳ up –

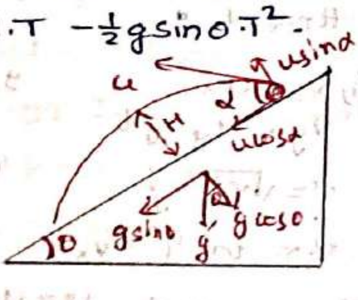


$T = \dfrac{2u_y}{a_y} = \dfrac{2u\sin\alpha}{g\cos\theta}$; $H = \dfrac{u_y^2}{2a_y} = \dfrac{u^2\sin^2\alpha}{2g\cos\theta}$.

Range, $x = u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow R = u\cos\alpha . T - \frac{1}{2}g\sin\theta . T^2$.

↳ down –



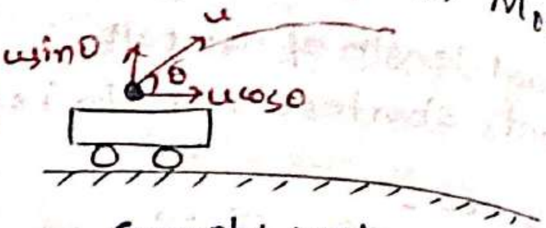
$T = \dfrac{2u_y}{a_y} = \dfrac{2u\sin\alpha}{g\cos\theta}$.

$H = \dfrac{u_y^2}{2a_y} = \dfrac{u^2\sin^2\alpha}{2g\cos\theta}$.

Range, $x = u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow R = u\cos\alpha . T + \frac{1}{2}g\sin\theta . T^2$.

- Projectile from Moving platform



Concept: wrt ground.

$u_x = u\cos\theta + v_T$

$u_y = u\sin\theta$.

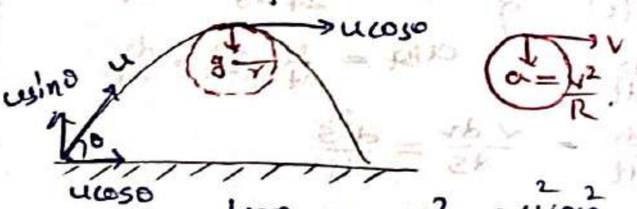
(i) $T = \dfrac{2u\sin\theta}{g}$.

(ii) $H = \dfrac{u^2\sin^2\theta}{2g}$.

(iii) $R = u_x . T = (u\cos\theta + v_T) . \dfrac{2u\sin\theta}{g}$.

↳ to find R_{max}, $\dfrac{dR}{d\theta} = 0$.

Radius of Curvature in a Projectile



here, $g = v^2/r \Rightarrow \dfrac{u^2\cos^2\theta}{r}$

- $\int F\Delta T = \Delta P = (P_f - P_i)$

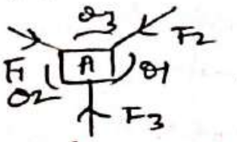
NLM

Classification of Forces:

1) EM Force } long range
2) G Force }
3) Strong Nuclear F. } short range.
3) Weak Nu. Force }

3>1>4>~~3~~2

Lami's theorem:



$$\frac{F_1}{\sin\theta_1}=\frac{F_2}{\sin\theta_2}=\frac{F_3}{\sin\theta_3}$$

{applicable for 3/more forces}

Newton's first law:

- Equilibrium $\vec{F}_{net}=0$
- (rotational) $\vec{\tau}_{net}=0$.

Newton's Second law:

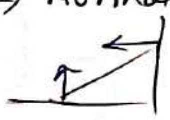
$\frac{d\vec{P}}{dt}=F=ma$ & $\Sigma\vec{F}=m\vec{a}$.

Newton's third law:

- force always exists in pairs.
- action-reaction pair {T, N & f}
- Acts on diff. objects. {F, mg}
- Same nature.

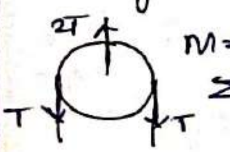
Common forces acting on Body:

1) Weight, mg: due to gravit. pull
2) Normal, N: Bodies in touch. Pushing ⊥ to contact.



3) Tension, T: pulling force.

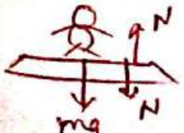
Pully - Block systems:



$M=0$
$\Sigma\vec{F}=0$

Concept of Weighing Machine:

- measures N react. acting on surface.

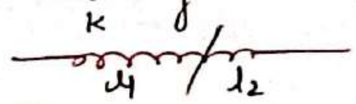


$N=mg$
Reading $= N/g = \underline{\underline{m}}$.

Spring Forces:

→ Spring balance - $T=mg$, Reading $= T/g$.

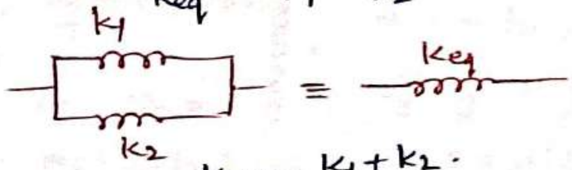
→ Cutting & Comb -



} $Kl=\underline{\underline{const}}$

$K_1=\frac{Kl}{l_1}$ & $K_2=\frac{Kl}{l_2}$

$$\frac{1}{K_{eq}}=\frac{1}{K_1}+\frac{1}{K_2}.$$



$K_{eq}=K_1+K_2$.

{spring force}:

$T=kx$ {Hooke's law}

$k \propto 1/\text{natural length}$.

Note! If we cut spring, T becomes 0 instantly.

Pseudo force:

$$\vec{F}_p=-m\vec{a}_{FOR}$$

Variable force:

$\vec{a}$, $\vec{F}$

- $F(t)$: $F(t)=m\frac{dv}{dt} \Rightarrow \int_0^t F(t)dt=\int_u^v m\,dv$
- $F(x)$: $F(x)=m\frac{v\,dv}{dx} \Rightarrow \int_0^x F(x)dx=\int_u^v mv\,dv$
- $F(v)$: $F(v)=mv\frac{dv}{dx} \Rightarrow \int_0^x dx=\int_u^v \frac{mv\,dv}{F(v)}$; $F(v)=m\frac{dv}{dt} \Rightarrow \int_0^t dt=\int_u^v \frac{m\,dv}{F(v)}$

FRICTION.

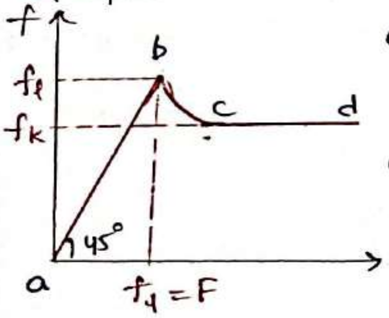
Types of Friction:

- viscous/wet friction {drag F}
- rolling friction {due to deformation of materials during rolling}
- sliding friction
  - a) Static – at rest
  - b) Limiting – max. limit
  - c) Kinetic – during movement.

$$\boxed{f = \mu_k N} \qquad \mu_k < \mu_s$$

- $f \propto N$
- independent of area of contact. $[\mu \leq 1]$
- self adjusting force.
- always acts opp. to dir^n of net vel.

Graph:



ab: static friction

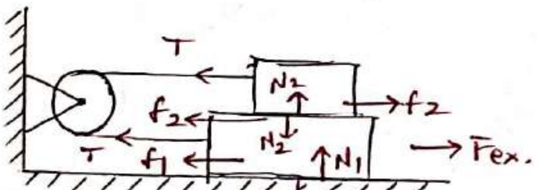
b : $f_{s\,max}$ or f_l

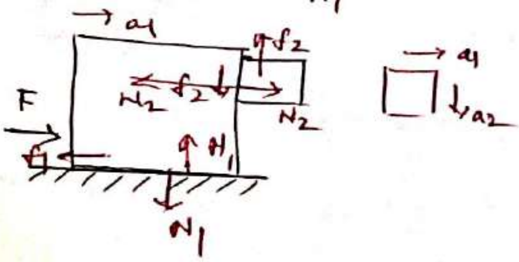
cd : kinetic friction ($f_k < f_{s\,max}$)

(Horizontally)

→ Pulling is easier than pushing

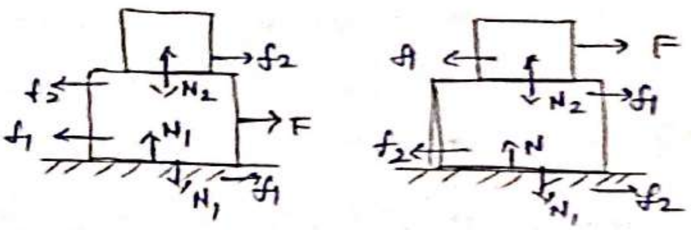
Friction in pully & block cases:





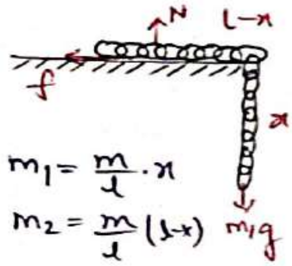
Angle of Repose: min angle for which body slips due to its own weight.

Block over Block:



they move only when $F \geq f_L$

Friction on a Chain:



$m_1 = \frac{m}{l} \cdot x$

$m_2 = \frac{m}{l}(l-x)$

at rest when,

$m_1 g = f_{max}$

$\frac{m}{l} x g = \mu \frac{m}{l}(l-x) g$

$\underline{\underline{x = \mu(l-x)}}$

WORK ENERGY & POWER.

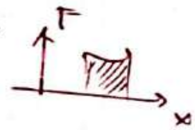
Work: transfer of $\mathcal{E}$ due to displ.

+ve: supply of $\mathcal{E}$; −ve: gain of $\mathcal{E}$.

↳ Constant: $W = \vec{F}\cdot\vec{s} = |\vec{F}||\vec{s}|\cos\theta$.

↳ Variable: $F(x)$.

$$W = \int_{x_1}^{x_2} F(x)\,dx$$



− $F(t)$.

$$a(t) = \frac{F(t)}{m} \xrightarrow{\int} v = f(t)$$
$$\Rightarrow dx = f(t)\,dt$$
$$W = \int_{t_1}^{t_2} m\,a(t)\, f(t)\,dt.$$

Energy: Ability to do work.

↳ KE: possessed by virtue of its motion.

↳ PE: " " " " position.

- Gravitational: $U = mgh$, $U = -Gm_1m_2/r$
- Spring: $U = \frac{1}{2}k(x_i^2 - x_f^2)$
- Electrostatics: $U = kq_1q_2/r$; Dipole, $U = -\vec{P}\vec{E}$
- Magnetism: $U = -\vec{M}\cdot\vec{B}$
- Elastic: $U = \frac{1}{2}\times$stress$\times$strain$\times$vol.

Work-Energy Theorem: (for rigid bodies)

$\rightarrow$ [m] (v_1) − − − − − − [m] (v_2) $\rightarrow$

		Int	Ext
if $KE_i > KE_f$ ⇒	W/D by body	+ve	−ve
if $KE_i < KE_f$ ⇒	W/D by body	−ve	+ve

KE_i + W/D by supp f − W/D by opp f = KE_f

$$\Rightarrow \boxed{KE_i + W_c + W_{nc} + W_{oth} = KE_f}$$

$$W_{net} = \Delta K = KE_f - KE_i$$

Law of $\mathcal{E}$ cons. $U_i + KE_i = U_f + KE_f$.

Conservative force $\left(F = -\frac{dU}{dx}\right)$

↳ $W = U_B - U_A$ ~ $K_f = KE_i = 0$

also, $\vec{F} = -\vec{\nabla} U$ gradient.

$$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

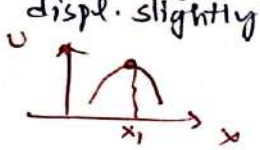
* all conservative forces are time variant.

Equilibrium: $F = 0$ (or) $\frac{dU}{dx} = 0$.

a) Stable Equi − $\boxed{U\,min}$
- SHM occurs
- $\frac{d^2U}{dx^2} > 0$

b) Neutral Equi −
- No SHM (when body slightly displ. it neither comes back nor move away) $\boxed{U\ const}$.
- $\frac{d^2U}{dx^2} = 0$

c) Unstable Equi − $\boxed{U\,max}$
- No SHM (moves away when displ. slightly).
- $\frac{d^2U}{dx^2} < 0$



Power: (Watt) W

- rate of doing W
- rate at which $\mathcal{E}$ is being transferred.

a) Avg. P:

$$\langle \vec{P} \rangle_{avg} = \frac{\text{Total W/D}}{\text{time}}$$ +ve − $\mathcal{E}$ supplied; −ve − $\mathcal{E}$ gained.

$$\Delta P = \Delta W/\Delta t.$$

b) Inst. P:

$$P_{inst} = \frac{dW}{dt} = \vec{F}\cdot\frac{d\vec{s}}{dt} = \vec{F}\cdot\vec{v} = Fv\cos\theta$$

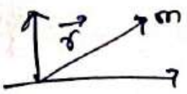
Efficiency:

$$\eta = \frac{P_{delivered}}{P_{consumed}} \times 100.$$

CENTRE OF MASS

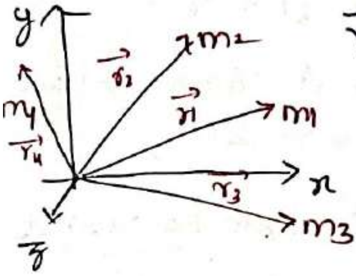
- pt. rep. of a large body.

1. Mass moment : $m\vec{r}$



vector
dirⁿ along $\vec{r}$
kg-meter

2. COM of discrete particles:

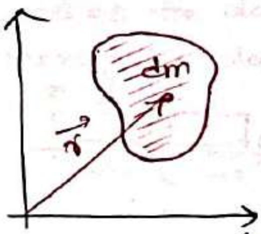


$$\vec{r}_{com} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + \cdots m_n\vec{r_n}}{m_1 + m_2 + \cdots + m_n}$$

$$= \frac{\sum_{i=1}^{n} m_i\vec{r_i}}{\sum_{i=1}^{n} m_i}$$

$$x_{com} = \frac{\sum_{i=1}^{n} m_i\vec{x_i}}{\sum_{i=1}^{n} m_i} ; \quad y_{com} = \frac{\sum_{i=1}^{n} m_i\vec{y_i}}{\sum_{i=1}^{n} m_i}.$$

3. COM of continuous body:



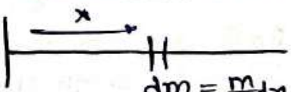
$$\vec{r}_{com} = \frac{\int dm\,\vec{r}}{\int dm} = \frac{1}{M}\int \vec{r}\,dm.$$

$$x_{com} = \frac{\int dm\,\vec{x}}{\int dm} ;$$

$$y_{com} = \frac{\int dm\,\vec{y}}{\int dm} ; \quad z_{com} = \frac{\int dm\,\vec{z}}{\int dm}.$$

4. COM of standard bodies:

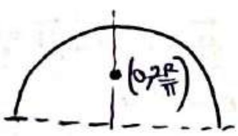
→ Uniform Rod –

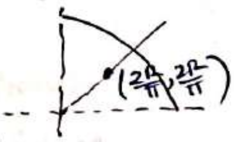


$$x_{com} = \frac{\int_0^l \frac{m}{l}dx \cdot x}{\int_0^l \frac{m}{l}dx}$$

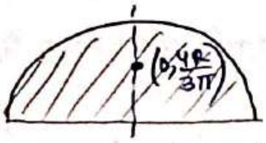
$$= \underline{l/2}$$

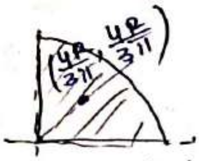
→ Semicircular ring



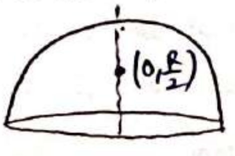


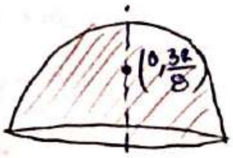
→ Semicircular disc



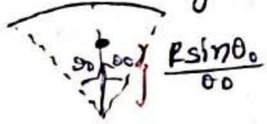


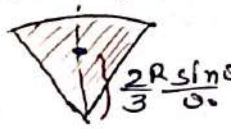
→ Hollow hemisphere → Solid



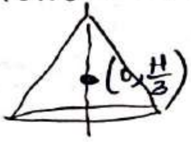


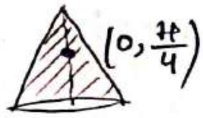
→ Arc of Ring → Sector of Disc



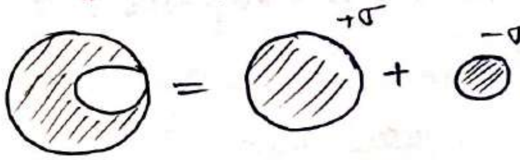


→ Hollow cone → Solid cone





5) COM of Cavity:



6) Displ. of com:

$$\vec{r}_c = \frac{m_1\vec{r_1} + m_2\vec{r_2} + \cdots}{m_1 + m_2 + \cdots}$$

$$\Delta\vec{r}_c = \frac{m_1\Delta\vec{r_1} + m_2\Delta\vec{r_2} + \cdots m_n\Delta\vec{r_n}}{m_1 + m_2 + \cdots + m_n}.$$

$\downarrow \frac{d\vec{r_c}}{dt}$

$$\vec{V}_{cm} = \frac{m_1\vec{v_1} + m_2\vec{v_2} + \cdots}{m_1 + m_2 + \cdots}$$

$\downarrow \frac{d\vec{v}_{cm}}{dt}$

$$\vec{a}_{com} = \frac{m_1\vec{a_1} + m_2\vec{a_2} + \cdots}{m_1 + m_2 + \cdots}$$

Note!

i) $\vec{F}_{net} = \frac{d\vec{P}_{net}}{dt}$

ii) $\vec{F}_{net} = 0 \Rightarrow \vec{P}_{net} = \text{const}$

$(a_{com} = 0)$ $\quad (\vec{V}_{cm} = \text{const.})$

Conserv. of Momentum & Collision.

1. Law of Cons. of linear momentum:

if $\vec{F}_{net} = \Sigma\vec{F} = 0$. $\vec{V}_{com} = const$.

$\{\vec{a}_{com} = 0;\ \frac{d\vec{v}_{com}}{dt} = 0\}$

$\vec{P}_{sys} = const. = m\vec{v}_{com}$.

$\boxed{\vec{P}_i = \vec{P}_f}$ LMCT.

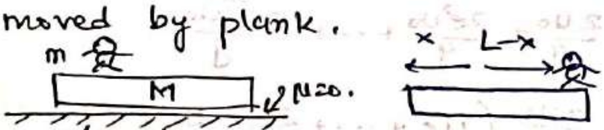
→ Internal forces can never change the momentum of system.

2. COM at rest $(\vec{F}_{net} = 0, V_{com} = 0)$

$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$0 = m\Delta\vec{r}_1 + m\Delta\vec{r}_2 + \cdots \quad \{\Delta\vec{r}_{cm} = 0\}$$

Eg! If a boy walks to the other end of the plank, find the dist. moved by plank.

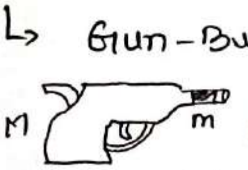


$m_1\Delta x_1 + m_2\Delta x_2 = 0$

$m(L-x) + M(-x) = 0$

$$\boxed{x_{cm} = \frac{mL}{m+M}}$$

↳ **Gun-Bullet (muzzle vel):**



Muzzle vel. is u, find recoil speed of Gun.

solut Muzzle vel: is vel of bullet wrt muzzle.

$V_{b/G} = V_b - V_G$

$\Rightarrow u = u_1 + v \Rightarrow u_1 = u - v$

$\because \vec{P}_i = 0 \Rightarrow \vec{P}_f = 0$

$\Rightarrow MV = m(u - v)$

$\therefore V = mu/m+M$ ans

3. Explosion Concept:

$\boxed{m\vec{u} = m_1\vec{v}_1 + m_2\vec{v}_2}$

(i) Explosion due to int. forces

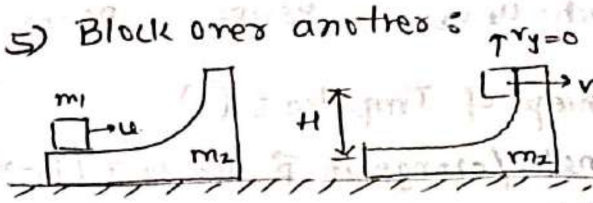
$\because \Sigma\vec{F}_{ext} = 0 \Rightarrow \vec{P}_{sys} = const$.

$\Rightarrow \vec{V}_{cm} = const$.

4) Bullet hitting block:

$\vec{P} = const.$ $KE + PE = const.$

5) Block over another:



$\vec{F}_{net}$ along horizontal $= 0 \Rightarrow \vec{P}_x = k$

$\vec{F}_{net}$ along vertical $\neq 0 \Rightarrow \vec{P}_y \neq k$

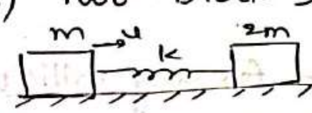
(i) $\vec{P}_x = const \Rightarrow m_1u = (m_1+m_2)V_x$

$$V_x = \frac{m_1u}{m_1+m_2}.$$

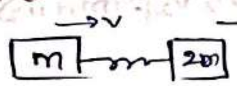
(ii) Gain in PE = loss in KE.

$$m_1gh = \frac{1}{2}m_1u^2 - \frac{1}{2}(m_1+m_2)V_x^2.$$

6) Two-Block spring sys:



#① At max comp. both have same V.



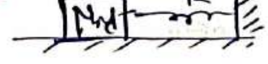
(i) $\vec{P} = const \Rightarrow mu = mV + 2mV$

$$\underline{V = u/3}$$

(ii) $TE = const. \Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}2mv^2 + \frac{1}{2}kx^2$.

$$\boxed{x = u\sqrt{\frac{2m}{3k}}}$$

#②



$\mu_{red} = \frac{m_1m_2}{m_1+m_2}$

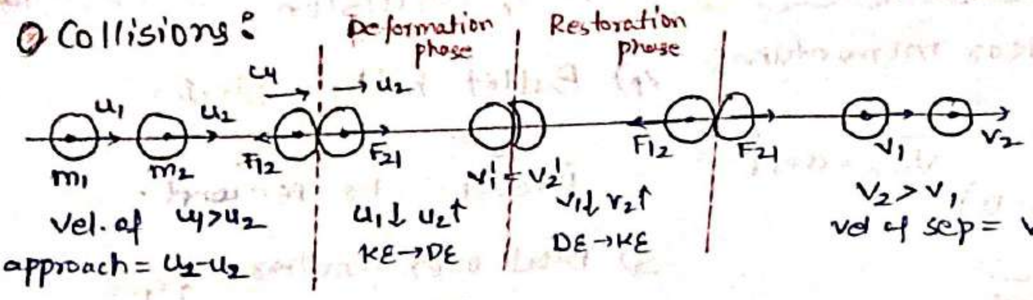
Loss in KE = gain in PE

$\Rightarrow \frac{1}{2}\mu_{red}V_{rel}^2 = \frac{1}{2}kx^2$

$\frac{1}{2}\left(\frac{m \times 2m}{3m}\right)u^2 = \frac{1}{2}kx^2$

$\therefore x = u\sqrt{2m/3k}$.

Collisions:



Concept of Impulse: (J)

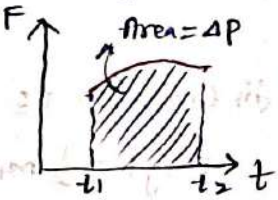
- transfer/change in $\vec{P}$ due to $F\ (t_1 \to t_2)$

$\vec{F} = \frac{d\vec{P}}{dt} \Rightarrow dp = \vec{F}dt$

$F = \text{const.}$: $\Delta p = F\Delta t$

$F = f(t)$: $J = \Delta \vec{P} = \int_{t_1}^{t_2} F\,dt$

(i) Impulsive Force:
Large force acting for short time (they bring Δp)



LCLM holds good for any collision:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 = (m_1+m_2)v'$$

coeff of restitution –

$$e = \frac{\text{vel. of sep}}{\text{vel. of app.}} \quad \{\text{along line of impact}\}$$

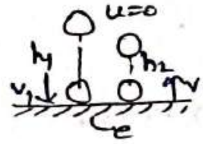
- Case I: Inelastic.
  $e = 0$; $v_1 = v_2 = v_f$; KE loss Max.
- Case II: Elastic.
  $e = 1$; $u_1 - u_2 = v_2 - v_1$; KE = const.
- Case III: Partially elastic.
  $0 < e < 1$; KE Loss.

// fractional Loss in KE $= \frac{KE_i - KE_f}{KE_i}$

// % loss $= \frac{KE_i - KE_f}{KE_i} \times 100$.

% restored in KE $= \frac{KE_f}{KE_i} \times 100$.

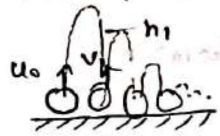
Example I:



$e = \frac{v_2}{v_1} \Rightarrow v_2 = ev_1$ $\{v_1 = \sqrt{2gh_1}\}$

$h_2 = \frac{v_2^2}{2g} = \frac{e^2v_1^2}{2g} = e^2h_1$

Example 2:



$v_1 = eu_0$
$v_2 = ev_1 = e^2u_0$

$h_1 = \frac{e^2u_0^2}{2g}$; $h_2 = \frac{e^4u_0^2}{2g}$; $h_n = \frac{e^{2n}u_0^2}{2g}$

$t_0 = \frac{2u_0}{g}$; $t_1 = \frac{2e^2u_0}{g}$ ...

T after which ball stops:

→ $t_1 + t_2 + \dots + t_\infty$

$= \frac{2u_0}{g} + \frac{2e^2u_0}{g} + \dots + \frac{2e^{2(n)}u_0}{g}$

$= \frac{2u_0}{g}\left(1 + e^2 + \dots + e^{2n} + \dots e^\infty\right)$

$= \frac{2u_0}{g}\left(\frac{1}{1-e^2}\right)$

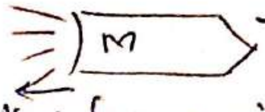
total dist. it travels:

$s = (h_0 + h_1 + h_2 + \dots + h_\infty) \times 2$

$s = \frac{2u^2}{2g}\left(1 + e^2 + e^4 + \dots\right)$

$s = \frac{u^2}{g}\left(\frac{1}{1-e^2}\right)$

Mass Variation:

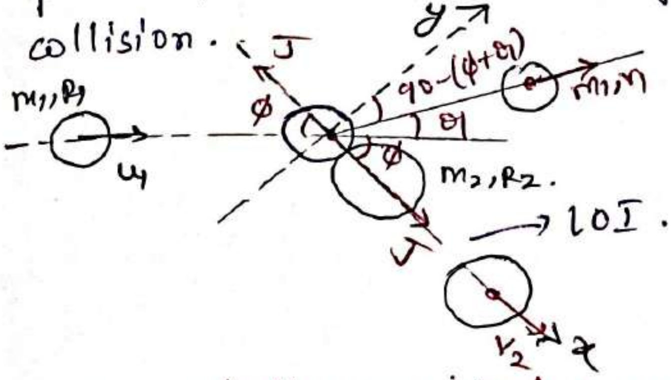


→ $F_{th} = v_{rel}\frac{dm}{dt}$.

v_{rel} ↳ mass ejection rate $= \frac{dm}{dt}$ kg/s.

Oblique collision {2-D} :

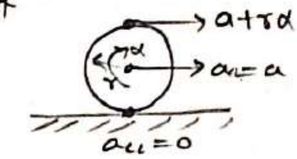
→ Line of impact: dir^n. in which p transfer takes plae during collision.



* When 2 bodies are ideal & one collides other when at rest, they both separate $\perp^{lrly}$.

* Conserve linear $\vec{p}$ along LOI & $\perp^{ar}$ to it. (y-axis)

*



for pure rolling, {$r\alpha = a$}

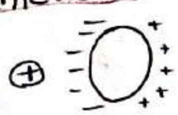
$a + r\alpha = 2a$

ELECTROSTATICS.

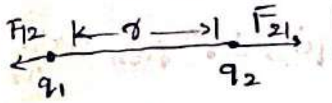
Charge: $q = ne$

- scalar; coulumb [C]; [IT]
- $e = 1.6\times10^{-19}C$
- charges of isolated systems remain unchanged.
- charge is quantised {e/2 doesn't exist}
- it is invariant from FOR.

Induction –



Coulumb's law:



$$\boxed{F = \frac{kq_1q_2}{r^2}}$$

$$K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,Nm^2/C^2 \Big\}\ \varepsilon = \varepsilon_0\varepsilon_r$$

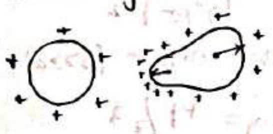
$$\varepsilon_0 = 8.85\times10^{-12}\,C^2/Nm^2.$$

$$\vec{F}_{21} = \frac{kq_1q_2}{|\vec{r}|^3}\cdot\vec{r}$$ Head on the charge at which F is to be found.

Limitations:

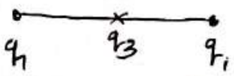
- only applicable for point charges
- charge must be at rest {Static}

Charge distribution in isolated cond:



surface charge density $\boxed{\sigma \propto 1/r}$

Equilibrium:



$q_3(+)$ → Along line stable; → ⊥ to line unstable

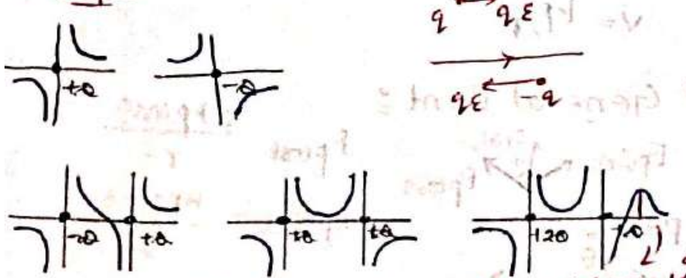
$q_3(-ve)$ → along line unstable; → ⊥ to line stable.

{large dist. from large charge}

Electric Field:

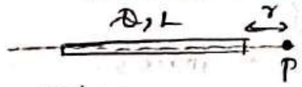
$$\boxed{\vec{E} = \frac{\vec{F}}{q_0} = \frac{KQ}{r^2}}$$

Graphs!

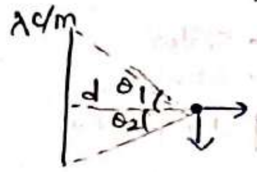


to find this, equate $\frac{dE}{dr}=0$

Special Cases:

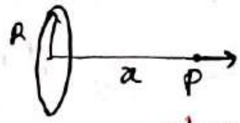


$$E_{P_{\parallel}} = \frac{KQ}{L}\left[\frac{1}{r} - \frac{1}{r+L}\right].$$

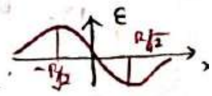


$$\varepsilon_\perp = \frac{K\lambda}{d}(\sin\theta_1 + \sin\theta_2)$$

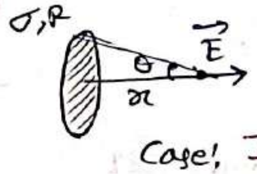
$$\varepsilon_\parallel = \frac{K\lambda}{d}(\cos\theta_2 - \cos\theta_1)$$



$$E = \frac{KQx}{(x^2+R^2)^{3/2}}$$



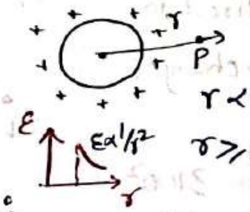
- at $x = \pm R/\sqrt{2}$, E is max.
- at $x = 0$ (centre), $E = 0$.



$$E = \frac{\sigma}{2\varepsilon_0}(1 - \cos\theta)$$

Case! If disc is very large ($x \ll R$) → for sheet $\boxed{E = \frac{\sigma}{2\varepsilon_0}}$

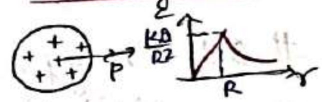
→ Metal sphere {conductor}



$r<R$, $\varepsilon = 0$

$r \geq R$, $E = \frac{KQ}{r^2}$

→ Non-conductor.



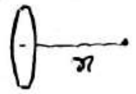
$r<R$, $E = \frac{KQr}{R^3} = \frac{\rho r}{3\varepsilon_0}$

$r \geq R$, $\varepsilon = \frac{KQ}{r^2}$

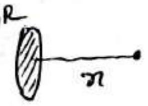
Electric Potential: (J/C = volt)

$$V = \frac{U}{q_0} = \left[\frac{(W_{ext})_{\infty\to P}}{q_0}\right] = \frac{-W_{ext\ \infty\to P}}{q_0}$$

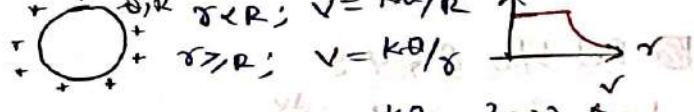
$$\boxed{V = \frac{KQ}{r}}$$ • Scalar quantity



$$V = KQ/\sqrt{x^2+R^2}.$$

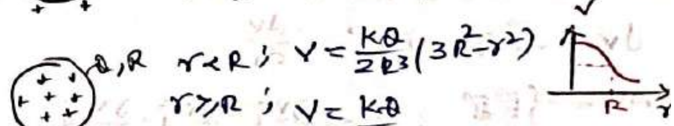


$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+x^2} - x\right)$$



$r<R$; $V = KQ/R$

$r \geq R$; $V = KQ/r$



$r<R$; $V = \frac{KQ}{2R^3}(3R^2 - r^2)$

$r \geq R$; $V = \frac{KQ}{r}$

Electrostatic P.E :

- only defined for a sys.
- stored form of Energy.
- W.D against conservative forces.

$q_1 \xrightarrow{\vec{r}} q_2$

- Scalar
- SI <<J>>
- Ref pt at ∞.

$\Delta U = U_f - U_i = kq_1q_2\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$

$$\boxed{U = \frac{kq_1q_2}{r}}$$

→ For $W_1, W_2, \ldots$

$U_{sys} = \sum_1^n U_{ij} = \sum \frac{kq_iq_j}{r_{ij}}$

total no. of terms $= \frac{n(n-1)}{2}$.

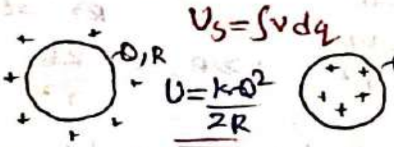
Q>> Closest approach :

$Q \overset{\leftarrow x \rightarrow}{\bullet}$ ———— $\leftarrow q$ $\frac{kQq}{r} + \frac{1}{2}mu^2 = \frac{kQq}{x}$

for such probleme, $\boxed{W_{net} = k_f - k_i}$

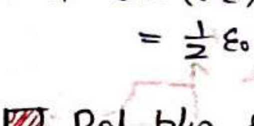
Self Energy : only defined for charge distribution (not pt-charge).

$U_s = \int v\, dq$



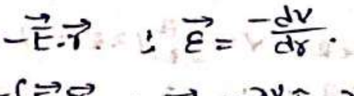
$U = \frac{kQ^2}{2R}$ $U = \frac{3kQ^2}{5R}$

Energy density :



$\frac{du}{dv}$ = energy density $= \frac{1}{2}\varepsilon_0 E^2$.

Eg.



$* \; U = (e.d) \times vol$

$= \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{2\varepsilon_0}\right)^2 \cdot a^3$.

Rel b/w E & V :

$dv = -\vec{E}\cdot\vec{r}$; $\vec{E} = \frac{-dv}{dr}$.

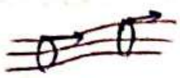
$\Delta V = -\int \vec{E}\cdot d\vec{r}$; $\vec{E} = -\frac{\partial v}{\partial x}\hat{i} - \frac{\partial v}{\partial y}\hat{j} - \frac{\partial v}{\partial z}\hat{k}$.

\# $\overset{A}{\bullet} \xrightarrow{v} \overset{B}{\bullet} \longrightarrow$ E.F

$\{V_B < V_A\}$ – on moving in the dir^n of EF, elec. potential dec & vice-versa

- charge left free moves from high V to lower V {EF moves from h.V to l.V}

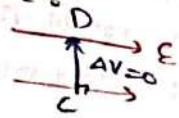
Electric Field Lines :



- cant intersect
- start from +ve (or) ∞
- crowded lines rep. strong field
- no. of lines originating / terminating on a charge is prop. to mag. of chge.

Equipotential Surface :

- always ⊥ to EF lines.



\# non-cond ∞ Sheet : $E = \frac{\sigma}{2\varepsilon_0}$

\# cond ∞ Sheet : $E = \frac{\sigma}{\varepsilon_0}$

Electric Dipole :

$-q$ ——d—— $+q$, $P = qd$, dir^n from -ve to +ve

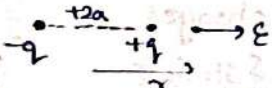
→ cubic-meter

→ 1 Debye $= 3.3 \times 10^{-30}$ c-m.

→ vector $-q$ to q.

Electrical Field

Axial point



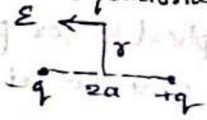
$E = \frac{2pkr}{(r^2-a^2)^2}$

*SHORT Dipole $(r >> a)$

$E_{axial} = 2pk/r^3$.

dir^n : along $\vec{P}$

Equatorial point



$E = \frac{Pk}{(r^2+a^2)^{3/2}}$

*SHORT Dipole $(r>>a)$

$E_q = kP/r^3$

dir^n : opp $\vec{P}$

Potential

Axial Point

$V = \frac{kP}{r^2 - a^2}$

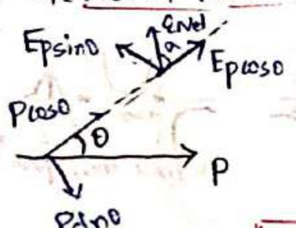
*SHORT Dipole $(r>>a)$

$V = kP/r^2$

Equatorial point.

$V = 0$

General Point :



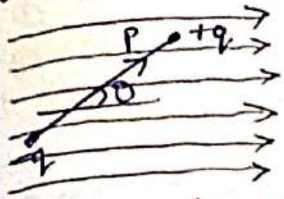
$E_{P\cos\theta} = \frac{2kP\cos\theta}{r^3}$

$E_{P\sin\theta} = \frac{kP\sin\theta}{r^3}$

$E_{Net} = \frac{kP}{r^3}\sqrt{1+3\cos^2\theta}$

$$\boxed{\tan\alpha = \frac{\tan\theta}{2}}$$

Dipole in Uniform E field :



$f = 0$

$\tau = pE\sin\theta$

$\vec{\tau} = \vec{p} \times \vec{E}$

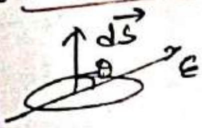
for SHM ; $T = 2\pi\sqrt{\frac{I}{pE}}$

$U = -\vec{p}\cdot\vec{E}$

$\theta = 0°$, stable

$\theta = 180°$, unstable

Electric flux :



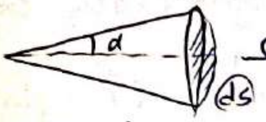
$d\phi = \vec{E}\cdot d\vec{s}$
$= E\,ds\cos\theta$

$\phi = \int \vec{E}\cdot d\vec{s} = \frac{q_{en}}{\varepsilon_0}$

Gauss law :

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$ → Charge enclosed → Elect. force is due to all the charges.

{solid angle (useful in problms)}



$\Omega = 2\pi(1 - \cos\alpha)$

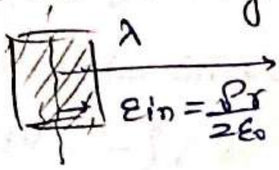
$d\Omega = ds/R^2$.

E due to long hollow cylinder :

$E_{out} = \frac{2k\lambda}{r}$

$E_{in} = 0$

E due to long solid cylinder :



$E_{out} = \frac{2k\lambda}{r}$.

$\lambda \cancel{l} = \rho\pi r^2 \cancel{l}$

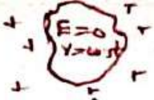
$\lambda = \rho\pi R^2$

$E_{in} = \frac{\rho r}{2\varepsilon_0}$

add :

1) PE of interaction b/w dipole $= \frac{-2P_1 P_2 k\cos\varphi}{r^3}$.

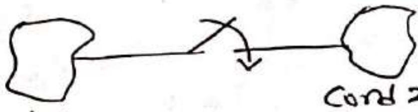
CONDUCTOR :



- charge remains on surface.
- electric field inside is zero.
- V is constant.
- field lines are ⊥ to surface.
- Connecting 2 cond : they share charge until V of both bodies equals.
- Earthing :

V of body will always be zero.

Eg :



Cond I: $V_1, Q_1 \rightarrow q_1$

Cond 2: $V_2, Q_2 \rightarrow q_2$

$Q_1 + Q_2 = q_1 + q_2$

finally $\boxed{V_1 = V_2}$

large conducting sheet :

$\rightarrow \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$.

GRAVITATION.

- Att. b/w 2 masses.

$\vec{F}_{12} = -\vec{F}_{21}$

univ. gravit. const $6.67\times10^{-11} Nm^2/kg^2$

$F \propto m_1 m_2 \Rightarrow \boxed{F = \frac{G m_1 m_2}{r^2}}$

$F \propto 1/r^2$

$\{\vec{r} = \vec{r}_2 - \vec{r}_1\}$

→ In vector form, $\vec{F} = \frac{G m_1 m_2}{r^3}\vec{r}$

Gravitational Field:

– a way of expressing influence of mass. $\boxed{E = \vec{F}/M_0}$

↳ Acc. due to gravity –

$g = \frac{F}{m} = \frac{GM_e}{R^2} = 9.8\, m/s^2$

$g = \frac{G}{R^2}\rho\frac{4}{3}\pi R^3$

⇒ Work done by Gravity:

$W_g = \int_A^B \vec{F}\cdot d\vec{s} = m\int_A^B \vec{E}\cdot d\vec{s}$.

* Gravitational potential in any gravit. field is defined as the WD by an ext. agent to bring a unit mass of 1kg from ∞ to pt.

⇒ Gravitational potential:

$V_A - V_B = \int_A^B \vec{E}\cdot d\vec{s}$

↳ Rel b/w GF & GP:

$\int_A^B dV = -\int_A^B E\cdot ds$

$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$.

⇒ Gravitational P.E:

$U = -\frac{G m_1 m_2}{r}$.

→ Gravitational P:

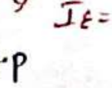
$V = -\frac{Gm}{r}$.

Interaction Energy:

m P

$IE = m V_P$ (potential at pt P)

eg. M,R



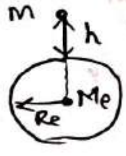
$I_E = m \cdot \frac{-Gm}{\sqrt{R^2+x^2}}$.

for all the standard fig. replace $k \to G$ & $Q \to m$ with (-ve) sign from Electrostatics.

Self-Energy: (E req. of its existence)

- SE of ○ = $-GM^2/2R$.
- SE of ● = $-3GM^2/5R$.

Variation of g with height:



$F_g = \frac{GM_e m}{(R+h)^2}$.

$\boxed{g' = \frac{g}{(1+h/R)^2}}$

if $h/R << 1 \Rightarrow h << R$

$g' \approx g(1 - 2h/R)$ $\quad \frac{\Delta g}{g}\times100 = \frac{-2h}{R}\times100$

Variation of g with Depth:

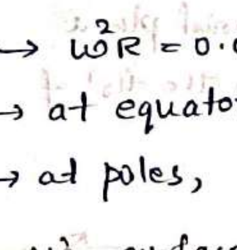
$F_g = \frac{GM'm}{r^2}$

$M' = \frac{Mr^3}{R_e^3}$

$\boxed{g' = g(1 - d/R)}$

$\frac{\Delta g}{g}\times100 = -d/R\times100$.

Variation of g with rotation:



$r = R\cos\theta$.

$mg = N + m\omega^2 r\cos\theta$.

$N = mg - m\omega^2 R\cos^2\theta$

$\boxed{g_{eff} = g_0 - \omega^2 R\cos^2\theta}$

→ $\omega^2 R = 0.034$ rad/s {for earth}

→ at equator, $g_{eff} = g_0 - \omega^2 R$, $\theta = 0°$

→ at poles, $g_{eff} = g_0$, $\theta = 90°$

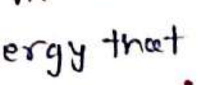
- At earth's surface:

$h << R$, $\Delta U \approx mgh$

$\Delta U = U_A - U_B$

$= -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$ $\quad \Delta U = \frac{mgh}{1+h/R}$.

$= \frac{GMm}{R^2}\left(\frac{1}{1+h/R}\right)$.



- Binding Energy: Energy that bonds the particles of a system (-ve) {BE = |E|

Escape Velocity: min. vel so that particle escapes planet's gravitational pull.

$U_i + K_i = K_f + U_f$

$\left\{ -\frac{GMm}{R} + \frac{1}{2}mv_e^2 = 0 + 0 \right\}$

$$\boxed{v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}} \approx 11.2 \text{ km/s for Earth.}$$

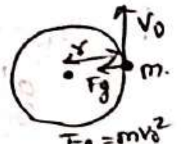
Motion of Satellites:

- Orbital Vel $v_0 = \sqrt{\frac{GM}{r}}$

  $\left\{ F = \frac{mv^2}{r} \Rightarrow \frac{GMm}{r^2} = \frac{mv_0^2}{r} \right\}$

- $T = \frac{2\pi r}{v_0} = 2\pi\sqrt{\frac{r^3}{GM}}$.

  $T^2 \propto r^3$ & $\omega = \sqrt{\frac{GM}{r^3}}$.



- Energy of a satellite:

  $K = \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$.

  $U = -\frac{GMm}{r}$, $T.E = K + U = -\frac{GMm}{2r}$.

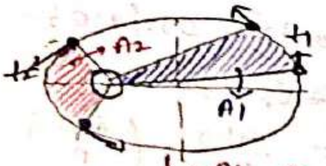
  $\therefore K = \left|\frac{U}{2}\right| = |T.E|$

- Binding Energy, $\boxed{BE = -TE} = \frac{GMm}{2r}$.

  ↳ E req. to be given so that "m" escapes

- Weightlessness $(N = 0)$

- Geostationary Satellite:
  - $T = 24$ hrs
  - Must lie on equitorial plane
  - Must revolve along the dir^n of earth.



↳ $\frac{A_1}{t_1} = \frac{A_2}{t_2}$

① Elliptical path {on L, τ, U+K}:

(1) As 'm' revolves, F always passes through c. Thus τ about c is zero.

$\because \tau = 0 \Rightarrow L = \text{const}$.

(or) $mvr\sin\theta = \text{const}$.

(2) $U + K = \text{const}$.

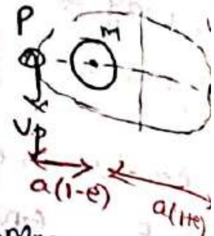
$\Rightarrow -\frac{GMm}{r} + \frac{1}{2}mv^2 = \text{const}$

② Speed at Perigee/Apogee:

$L_P = L_A$

$\Rightarrow mv_P a(1-e) = mv_A a(1+e)$ — ①

$U_P + K_P = U_A + K_A$.



$\Rightarrow -\frac{GMm}{a(1-e)} + \frac{1}{2}mv_P^2 = -\frac{GMm}{a(1+e)} + \frac{1}{2}mv_A^2$. — ②

Solving ① & ②

$$v_P = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)} \; ; \; v_A = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)}$$

Note! TE of sys. is const. i.e., $-\frac{GMm}{2a}$.

③ Projected Vel vs Path:

If!

a) $v = 0$; path 1; st. line
b) $0 < v < v_0$; ②; Elliptical
c) $v = v_0$; ③; circular
d) $v_0 < v < v_e$; ④; Elliptical
e) $v = v_e$; ⑤; Parabola
f) $v > v_e$; ⑥; Hyperbola.

Kepler's Laws:

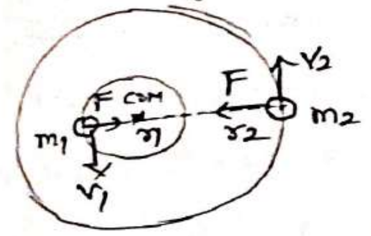
1) Law of Orbits — planet revolves around Sun in eliptical orbit; sun being one of the foci

2) Law of Areas — Line joining planet & sun sweeps equal area in equal time

$\boxed{\frac{dA}{dt} = \frac{L}{2m}} = \text{const}$ {Ang p. is conserved}

3) Law of Periods — $\boxed{T^2 \propto a^3}$

Binary Star System :



- 2 stars move about their COM (c) called "BARYCENTRE".

$$r_1 = \frac{m_2}{m_1+m_2} r \; ; \; r_2 = \frac{m_1}{m_1+m_2} r \quad \{r_1 + r_2 = r\}$$

$$F_g = \frac{G m_1 m_2}{r^2} = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

$T = const$; $\omega_1 = \omega_2$

$$\boxed{\frac{v_1}{r_1} = \frac{v_2}{r_2}}$$

↳ Energy consideration -

$$KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \; ; \; U = -\frac{G m_1 m_2}{r} \qquad TE = -KE = U/2$$

$$KE = \frac{F_g r_1}{2} + \frac{F_g r_2}{2} = \frac{G m_1 m_2}{2r} \qquad TE = -\frac{G m_1 m_2}{2r}$$

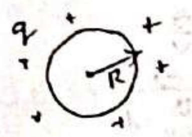
CAPACITORS.

- stores Energy in form of E Field and it releases instantaneously.

○ capacitance (C): cap. to hold the Q.

$\boxed{C = \frac{Q}{V}}$ {unit: farad}

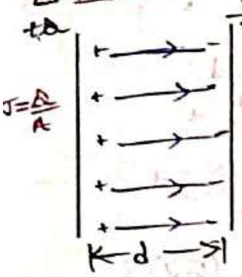
• capacitance of cond. is very small. Hence, measured in μ/nF.



$E = \frac{KQ^2}{2R}$ $(R \to \infty)$

$C = \frac{Q}{Kq/R} = 4\pi\varepsilon_0 R$.

Parallel plate Capacitor:



$\sigma = \frac{Q}{A}$

$V = E \cdot d$

$C = \frac{Q}{\text{P.d b/w plates}}$

$C = \frac{Q}{V} = \frac{Q}{E \cdot d}$

$\left(E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \right)$

$\boxed{C = \frac{\varepsilon_0 A}{d}}$

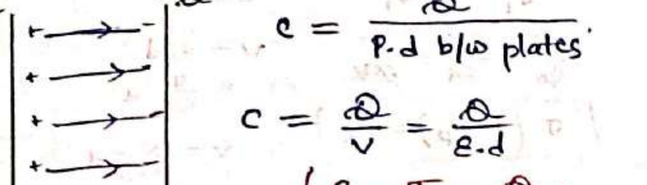
Energy in a Capacitor:

* E. d × vol. → $\frac{1}{2}\varepsilon_0 E^2$

$E = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{A\varepsilon_0}\right)^2 \times Ad$

$= \frac{Q^2 d}{2\varepsilon_0 A} = \frac{Q^2}{2C} = \frac{C^2V^2}{2C} = \frac{CV^2}{2}$.

Charge / E stored / W battery / Heat:



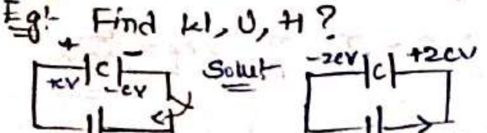
→ $q = CV$

→ $W_{battery} = q_{flown} \times V = (CV)V = CV^2$

→ $U = \frac{CV^2}{2}$

→ Heat generated $= W_b - (U_f - U_i)$

Eg: Find W, U, H?

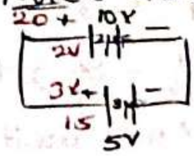


• $W_b = 2V(3CV) = 6CV^2$

• $U_i = \frac{1}{2}CV^2$; $U_f = \frac{1}{2}C(2V)^2 = 2CV^2$

$\Delta U = \quad$ ... ... $= 6CV^2 - 3/2CV^2 = 9/2 CV^2$

* Note: Charge is conserved b/w two connected plates.

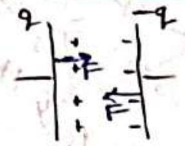


$3Q = 5V$

$V = 7V$

{For problems; refer NV sir notes}

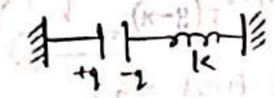
Force b/w plates:



$F = q \times E_q = q \times \frac{q}{2A\varepsilon_0}$.

$\Rightarrow F = q^2/2A\varepsilon_0$:

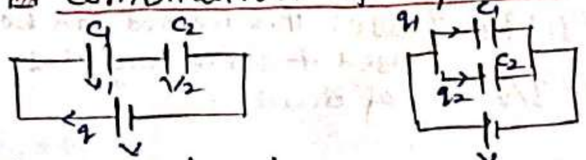
Eg:



$Kx = \frac{q^2}{2A\varepsilon_0}$

↓ spring elongation.

Combination of capacitors:



(i) $C_{eq} = \frac{1}{C_1} + \frac{1}{C_2}$ (ii) $C_{eq} = C_1 + C_2$

(ii) $q = C_{eq} V$ (ii) $q_1 = C_1 V$, $q_2 = C_2 V$.

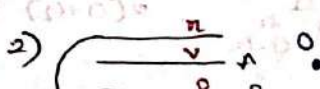
(iii) $V_1 : V_2 = \frac{1}{C_1} : \frac{1}{C_2}$

Note:

→ n identical capacitor in series, $C_{eq} = C/n$;

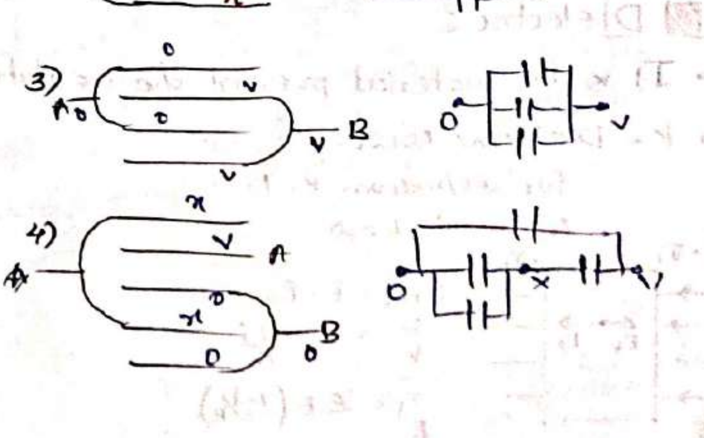
→ if in parallel, $C_{eq} = nC$.

Plates (connected):

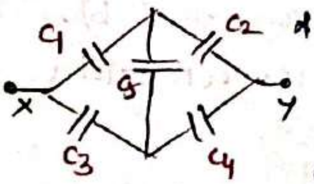


n cap in series → 2n plates.

→ (n+1) plates!



Wheatstone Bridge (Cap.)



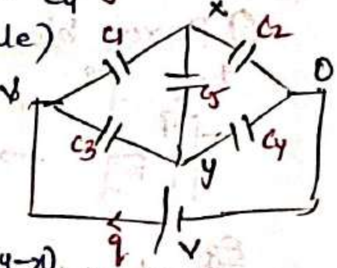
If $C_1 C_4 = C_2 C_3$

↳ $\frac{C_1}{C_3} = \frac{C_2}{C_4}$; then,

cancel / short circuit C_5

⟹ Unbalanced $\{ \frac{C_1}{C_3} \neq \frac{C_2}{C_4} \}$

(point potential + Jn. rule)



At x: $C_1(x-V) + C_5(x-y) + C_2(x-0) = 0$ — ①

At y: $C_3(y-V) + C_4(y-0) + C_5(y-x) = 0$ — ②

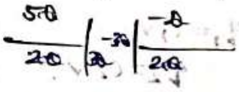
ⓐ Solve ① & ② to find x & y.

ⓑ Then we can find q_1, q_2, q_3, q_4 & q_5

ⓒ $q = q_1 + q_3$ #Note: This method can be used to solve any kind of circuit.

ⓓ $C_{eq} = q/V$

Charge on plates:



outside charge = $\frac{\text{total}}{2}$

$V = \frac{3Q}{A\varepsilon_0} d$

$3Q = CV$

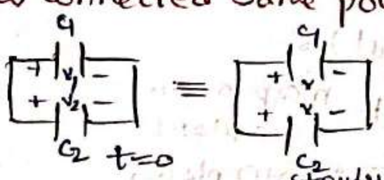
$\boxed{Q = CV}$

↓

charge on inner surface of higher pot. plate.

Charge Sharing & Heat generated:

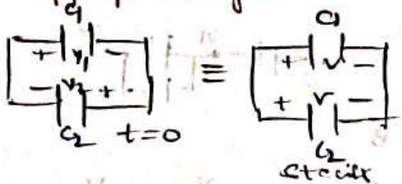
(a) Connected Same polarity –



$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$H = \frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$

(b) Opp. polarity –



$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$

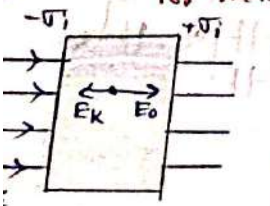
$H = \frac{C_1C_2(V_1+V_2)^2}{2(C_1+C_2)}$

Dielectric:

- It is the material present b/w the plates.
- K = Dielectric const.

for air/vaccum K = 1.

for metal K = ∞



$E_{net} = E - E_K$

$\frac{E}{K} = E - \frac{\sigma_i}{\varepsilon_0}$

$\sigma_i = \varepsilon_0 E(1 - 1/K)$

↓

Induced charge d, $\sigma_i = Q_i/A$

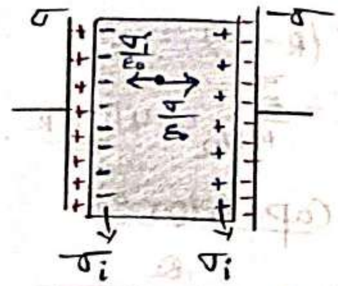
1) To support the cap. plates.

2) Capacitance of the sys. inc.

3) Some has better K than air.

{When we put E.F on a dielectric, str. increases at certain Electric field, it breaks off called Dielectric Strength of ...

Slab in Cap:



$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A\varepsilon_0 K} \cdot d}$

$\boxed{C = \frac{\varepsilon_0 K A}{d}}$

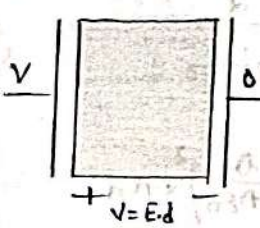
$\frac{\sigma}{\varepsilon_0} - \frac{\sigma_i}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0 K}$

$\boxed{\sigma(1 - 1/K) = \sigma_i}$

$V = E \cdot d = \frac{\sigma}{\varepsilon_0 K} d$

$V = \frac{Q}{A\varepsilon_0 K} d$

Break down Voltage (BV):



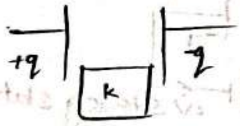
$V = E \cdot d$

$\boxed{B.V = D.S \times d}$ → Material, Dielec. strth., Dimension.

- Voltage at which dielectric breaks off.

Insertion of Dielectric:

1) At const. charge –



→ q is const.

→ C: $C_i = \frac{\varepsilon_0 A}{d}$; $C_f = KC$

C → CK

⇒ $V = \frac{Q}{C}$; $V_i = \frac{Q}{\varepsilon_0 A/d}$; $V_f = \frac{Q}{\varepsilon_0 AK/d}$

V → V/K

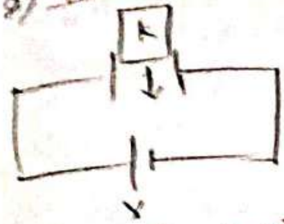
⟹ $\mathcal{E}_i = \frac{Q_0^2}{2C}$; $\mathcal{E}_f = \frac{Q_0^2}{2KC}$ $\mathcal{E} \to \mathcal{E}/K$

{dec in $\mathcal{E}$ is due to Ext forces}

⇒ W.D by Ext : $\frac{Q^2}{2C}\left(\frac{1}{K} - 1\right)$

⟹ E.F : $E_i = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$; $E_f = \frac{Q}{A\varepsilon_0 K}$

a) At const. Potential :



$V = \text{const.}$
$C \to KC$
$q \to Kq$
$E = \text{const} \; (E = V/d)$

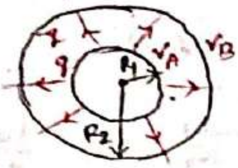
$\{U = \frac{1}{2}CV^2\}$ $U \to KU$

charge flown through battery $\to KCV - CV = (K-1)CV$

W.D by battery $\to$ charge flown $\times V = (K-1)CV^2$.

W.D by ext. agent $= -\frac{1}{2}(K-1)CV^2$.

Spherical Capacitor :

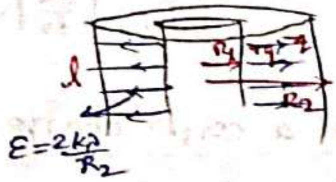


$V = V_A - V_B$
$V = \frac{Kq}{R_1} - \frac{Kq}{R_2}$.

$C = Q/V$

$C = \frac{Q}{\frac{Kq}{R_1} - \frac{Kq}{R_2}} = \frac{4\pi\varepsilon_0 R_1 R_2}{(R_2 - R_1)}$.

Cylindrical Capacitor:



$E = \frac{2K\lambda}{R_2}$

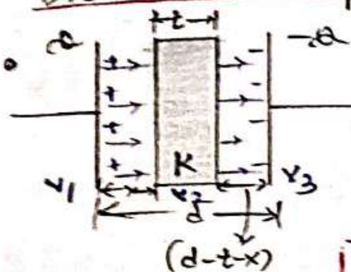
$C = \frac{Q}{V}$ $\quad \Big| \; C = \frac{2\pi\varepsilon_0 l}{\ln(R_2/R_1)}$

$V_1 - V_f = \int E \cdot dr$

$V = \int_{R_1}^{R_2} \frac{2K\lambda}{r}\, dr$

$V = 2K\lambda \ln(R_2/R_1)$

Different Combinations of Dielectric in Capacitor :

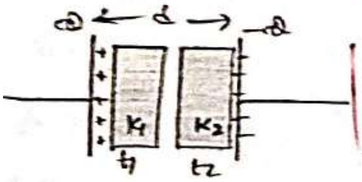


$V = V_1 + V_2 + V_3$

$\frac{Q}{C_{eq}} = \frac{Q}{A\varepsilon_0}x + \frac{Q}{A\varepsilon_0 K}\cdot t + \frac{Q}{A\varepsilon_0}(d-t-x)$

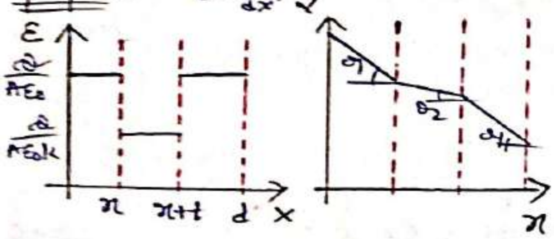
$= \frac{t}{\varepsilon_0 A K} + \frac{d}{A\varepsilon_0} - \frac{t}{A\varepsilon_0}$.

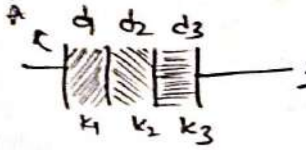
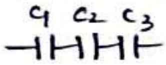
$$\boxed{C_{eq^m} = \frac{\varepsilon_0 A}{d - t + t/k}}$$



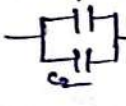
$$\boxed{C_{eq^m} = \frac{\varepsilon_0 A}{d - \Sigma t_i + \Sigma t_i/k_i}}$$

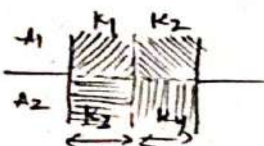
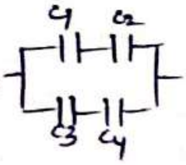
Graphs : $E = -\frac{dV}{dx}$.

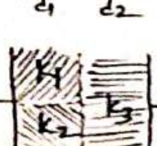
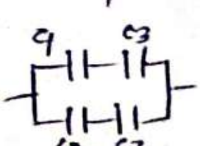


$\Rightarrow$  $\equiv$ 

$C_1 = \frac{\varepsilon_0 A k_1}{d_1}$.

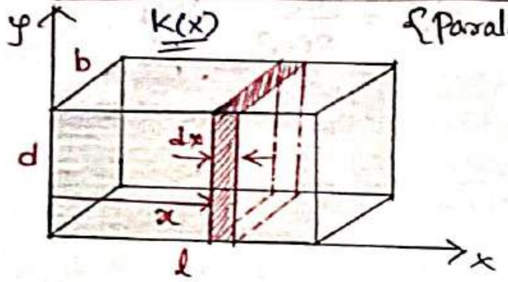
$\Rightarrow$  $=$  $C_1 = \frac{\varepsilon_0 A_1 k_1}{d}$

$\Rightarrow$  $=$ 

$\Rightarrow$  $=$ 

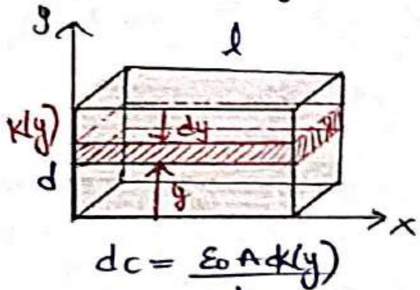
Variable Dielectric Constant (K):

{Parallel}



$dC = \frac{K(x)\, b\, dx\, \varepsilon_0}{d}$

$\Rightarrow C = \frac{b\varepsilon_0}{d}\int_0^l K(x)\, dx$

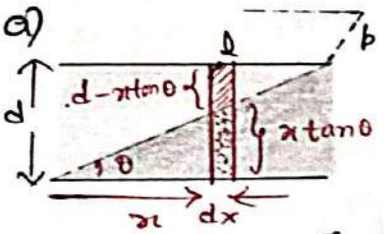


$dC = \frac{\varepsilon_0 A\, K(y)}{dy}$

$\frac{1}{C} = \int \frac{1}{dC} = \frac{1}{A\varepsilon_0}\int_0^d \frac{dy}{K(y)}$.

Capacitance with variable dim:

a)



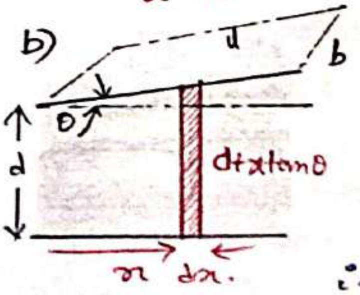
$dC_1 = \frac{K_1 b\, \varepsilon_0 dx}{d - x\tan\theta}$

$dC_2 = \frac{K_2\, b\, dx\, \varepsilon_0}{x\tan\theta}$.

$dC_{eq} = \frac{dC_1 \times dC_2}{dC_1 + dC_2}$

$$\boxed{C_{eq} = \int_0^l dC_{eq}}$$

b)

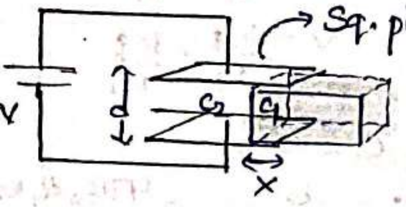


$dC = \frac{b\, dx\, \varepsilon_0}{d + x\tan\theta}$.

$\therefore \boxed{C = b\varepsilon_0 \int_0^l \frac{dx}{d + x\tan\theta}}$.

F on dielectric at a const. potential:

- At const. voltage, insertion of dielectric force is also const.



$\curvearrowright$ Sq. plate (side l).

$$F = -\frac{du}{dx}$$

$$U = \frac{1}{2} C_{eq} V$$

$$\frac{du}{dx} = \frac{V^2}{2} \frac{dC_{eq}}{dx}$$

$$C_{eq} = C_1 + C_2$$

$$= \frac{\varepsilon_0 k(x) l}{d} + \frac{\varepsilon_0 l (l-x)}{d}$$

$$= \frac{\varepsilon_0 l}{d} (l + x(k-1))$$

$$\Rightarrow \frac{dC_{eq}}{dx} = \frac{\varepsilon_0 l (k-1)}{d}$$

$$\boxed{F = \left| \frac{du}{dx} \right| = \frac{\varepsilon_0 l (k-1) V^2}{2d}}$$

→ The F becomes zero once it completely enters both the plates then due to inertia, the dielectric oscillates b/w plates to & fro.

F on dielectric at a const. charge:

$$U = \frac{Q_0^2}{2C}$$

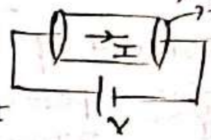
$$\frac{du}{dx} = \frac{Q_0}{2C_{eq}^2} \cdot \frac{dC_{eq}}{dx}$$

Current Electricity.

- $q_{flown} = \int_{t_1}^{t_2} i(t)\,dt$
- $v_d = \frac{dx}{dt} \simeq \frac{eE\tau}{m}$ — relaxation time. (avg. time elapsed b/w 2 collisions)

$i = \frac{dq}{dt}$; $dq = neA\,dx$

$dq = neA(v_d)dt$



- $\boxed{i = neAv_d}$ n: no. of free $\bar{e}$/unit vol.
- Mobility, $\mu = \frac{v_d}{E} = \frac{eE\tau/m}{E} = \frac{e\tau}{m}$. ($m^2/Vs$)
- Current density : $J = I/A = nev_d$.

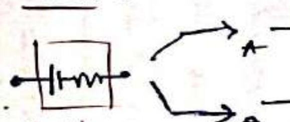
$\Rightarrow I = \vec{J}\cdot\vec{A}$

$\boxed{E = \rho J}$ — Resistivity.

* $R = R_0(1+\alpha(T-T_0))$ — α: temp. coeff
* $\rho = \rho_0(1+\alpha\Delta T)$

- Resistance : $\left(R = \frac{m}{ne^2\tau}\frac{l}{A}\right)$, ohm ($\Omega$)
  - $R \propto l/A$ — ρ: resistivity, $\sigma = 1/\rho$ — conductivity.
  - $R \propto$ Temp
  - for semicond, $T \propto 1/R$

- Cell :



Cell.

$V_{AB} = \mathcal{E} - Ir$ WD by batt (Disch.)

$V_{AB} = -\mathcal{E} + Ir$ WD on batt (Charg)

$\Rightarrow$ Combination of cell:

series — $\mathcal{E}_1, r_1$; $\mathcal{E}_2, r_2$; $\mathcal{E}_3, r_3$ $\Rightarrow$ $\mathcal{E}_{eq}, r_{eq}$

$\mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2 - \mathcal{E}_3$

$r_{eq} = r_1 + r_2 + r_3$

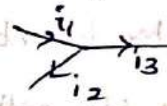
Parallel — $\mathcal{E}_1, r_1$; $\mathcal{E}_2, r_2$ $\Rightarrow$ $\mathcal{E}_{eq}, r_{eq}$

$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$. $\qquad \mathcal{E}_{eq} = \dfrac{\frac{\mathcal{E}_1}{r_1} - \frac{\mathcal{E}_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$

{here we took $-|\!|-$ ($\mathcal{E}$) as $+\mathcal{E}$ }

- Kirchoff's low :

1) KCL ($\Sigma i_n = 0$). $\frac{V_1}{R_1} - \frac{V_2}{R_2} - \frac{V_3}{R_3} = 0$



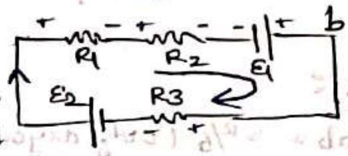
$i_1 - i_2 - i_3 = 0$.

— sum of v.d across all elements in a closed loop = 0

— sum of incoming I = outgoing I.

— in series comb. I is equal & in || comb. V is equal.

2) KVL ($\Sigma V_n = 0$) In a loop.

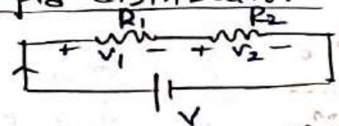


In loop abcda

$-iR_1 - iR_2 + \mathcal{E}_1 - iR_3 + \mathcal{E}_2 = 0$.

- Circuit Analysis :

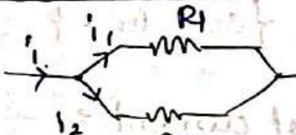
* p.d distribution



$V_1 = \frac{VR_1}{R_1+R_2}$, $V_2 = \frac{VR_2}{R_1+R_2}$

$\because$ i is same, $V \propto R$

* i distribution

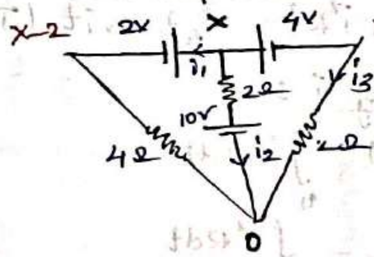


$\because$ V is same, $i \propto 1/R$

$i_1 = \frac{iR_2}{R_1+R_2}$

$i_2 = \frac{iR_1}{R_1+R_2}$.

* Point potential method :



$i_1 + i_2 + i_3 = 0$

$\frac{x-2}{4} + \frac{x-10}{2} + \frac{x-4}{2} = 0$

- Combination of resistors :

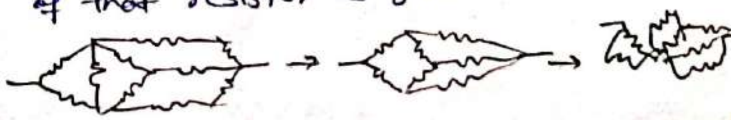
$R_{eq} = R_1 + R_2$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$.

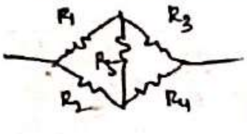
* pts. with same potential can be joined.

- Equipotential reduction :

• if V of 2 pts are same & if any resistor present b/w them, then i of that resistor is zero.



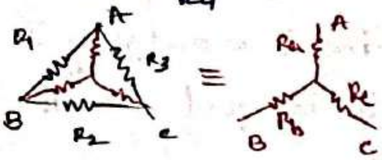
Wheatstone Bridge:



Where, if $\frac{R_1}{R_2} = \frac{R_3}{R_4}$, then no current passes through $\underline{R_5}$.

Q>> What if Wheatstone bridge is unbalanced?

if $\frac{R_1}{R_2} \neq \frac{R_3}{R_4}$, we use Delta-Star method.



$R_a = \frac{R_1 R_3}{R_1+R_2+R_3}$

$R_b = \frac{R_1 R_2}{R_1+R_2+R_3}$

$R_c = \frac{R_3 R_2}{R_1+R_2+R_3}$.

• Replace this (λ) in place of (Δ).

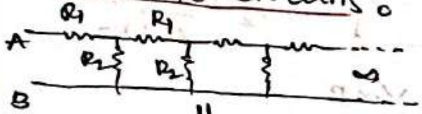
Cube Resistors:

$R_{eq,ab} = 5R/6$ (body diagonal)

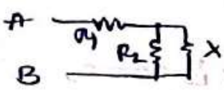
$R_{eq,ac} = 7R/12$ (edge)

$R_{eq,ad} = 3R/4$ (face diagonal)

Infinite Circuits:



⇓



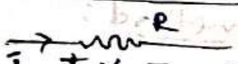
$\frac{R_2 x}{R_2+x} + R_1 = x$

{solve for x}

Thermal effect of current:

{Joules heating effect}

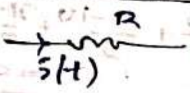
↳ Const. Current –



$P = i^2 R = \frac{v^2}{R} = vi$ (Watt)

$H = i^2 Rt = \frac{v^2}{R} t = vit$ (Joules)

↳ Time varying i:–



$H = \int_{t_1}^{t_2} i^2 R\,dt$.

$P_{av} = \frac{\int i^2 R\,dt}{\int dt}$

Ex: $i(t) = i_0 \sin\omega t$

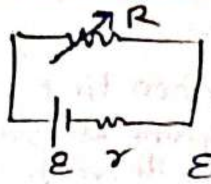
Power:

$dW = dq\,(V_f - V_i)$

$\frac{dW}{dt} = P = \underline{i^2 R}$ & Heat $= \underline{i^2 Rt}$

Efficiency $= \frac{P}{P+P_{loss}} \times 100$

Max Power transfer theorem



Condition: $R = r$ for max P transfer.

External R = Int R.

Concept of Power Rating:

Bulb.

Specifications

220V, 50W

↳ Rated Power.

Rated Voltage

(i) Means bulb will consume 50W if 220V is across it.

(ii) $R_{bulb} = \frac{V^2}{P} = \frac{220^2}{50} = 968\,\Omega$

(iii) If V > 220V is across bulb, it will fuse.

(iv) More power ⇒ more bright.

GALVANOMETER AMMETER VOLTMETER METER BRIDGE POTENTIOMETER

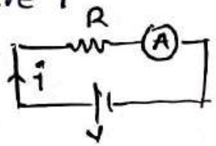
1) Galvanometer: Measures i, but has very less cap. to show.

* Range of Galvanometer of order (10μA to 10mA)

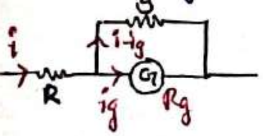
$i_g \rightarrow$ for full deflection

$V_g = i_g G \Rightarrow \boxed{G = V_g / i_g}$

2) Ammeter: to measure i



- connected in Series.
- has very low R.



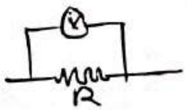
- i_g is the max current that can pass through G for full deflection.
- $S << R_g$ (S: Shunt)

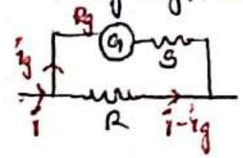
$i_g R_g = (i - i_g) S$

$i = i_g \left(\frac{R_g + S}{S} \right)$

3) Voltmeter: to measure V.diff.

- connected in Parallel
- very high R.



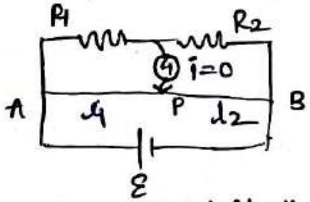


- S connected in series with G

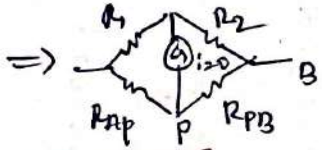
$V = i_g (R_g + S)$

4) Meter Bridge:

- to measure R of unk. resistor (R_2)



$\Rightarrow$



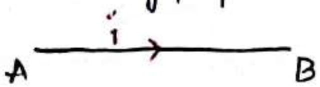
- P is null deflection point.

$\boxed{\frac{R_1}{R_2} = \frac{R_{AP}}{R_{BP}}}$

$\Rightarrow \boxed{\frac{R_1}{R_2} = \frac{l_1}{l_2}}$

5) Potentiometer: to calculate EMF of battery.

- the potentiometer is based upon the working principle that when a const i is passed through a wire of uniform cross section, the potential drop across any portion of the wire is directly prop. to the length of that portion.

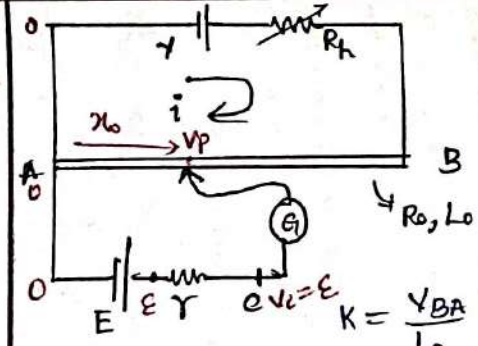


$V_A - V_B = iR$

$\Delta V = i \times \frac{\rho l}{A}$

$\frac{\Delta V}{l} = \frac{i\rho}{A} = \text{const}$

$\hookrightarrow$ potential gradient (K).



$\boxed{V_c = V_p}$

$\mathcal{E} = K x_0$

$\boxed{\mathcal{E} = \frac{V R_0}{L_0 (R_h + R_0)} x_0}$

K across AB:

$K = \frac{V_{BA}}{L_0}$

$= \frac{1}{L_0} \times i R_0$

$= \frac{1}{L_0} \times \frac{V R_0}{R_h + R_0}$

$V_{PA} = K \cdot x_0$

$V_P - V_A = K x_0$

$\boxed{V_P = K x_0}$ also $V_{PA} = V_{CA}$

Magnetism.

Magnetic Force:

$\vec{F} = q(\vec{v} \times \vec{B})$

$F = qvB\sin\theta$.

→ It is a no-work force.
→ No change in speed or KE.

$\frac{mv^2}{R} = qvB$

$R = \frac{mv}{qB}$ or $\frac{\sqrt{2mKE}}{qB}$.

$\omega = \frac{v}{R} = qB/m$.

$T = \frac{2\pi}{\omega}$.

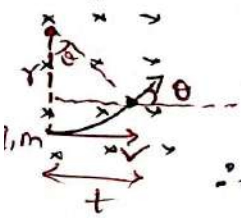
~~If not complete cycle~~:

1) $r < t$, $T = \frac{\pi m}{qB}$

2) $T = \frac{2\pi m}{qB} \times \frac{\theta}{2\pi} = \frac{\theta m}{qB}$

Duration of T inside B.

~~Angle of Deviation~~:



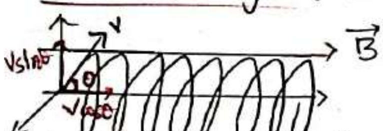
$r > t$

$\sin\theta = t/r$

$= tqB/mv$

$\therefore \theta = \sin^{-1}\left(\frac{qBt}{mv}\right)$.

V at an angle: {Helical}



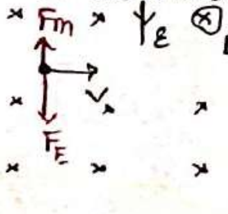
(i) $R = \frac{mv_\perp}{Bq} = \frac{mv\sin\theta}{Bq}$

(ii) $T = \frac{2\pi}{\omega}$ & $\omega = v/R$

(iii) Pitch, $P = v_{\parallel} \times T = v\cos\theta \cdot T$.

Special Case: $E \perp B \perp V$

in this situation, possibility of q moving undeviated.

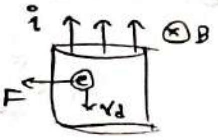


Cond^n.

$F_E = F_M$.

$\Rightarrow qE = qvB$ $\boxed{v = \frac{E}{B}}$

Force on i carrying wire:

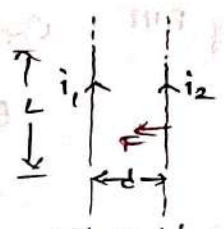


$\vec{F} = i\int(d\vec{l} \times \vec{B})$. ↳ Closed loop: $F = 0$

* $i = neAv_d$.
* $J = nev_d$.

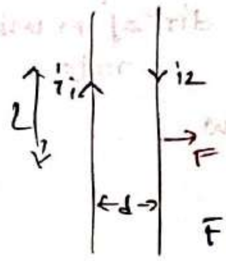
* Parallel wire:

1) Parallel i



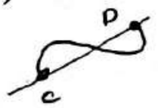
$F = \frac{\mu_0 i_1 i_2 L}{2\pi d}$

2) Antiparallel i



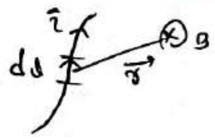
$F = \frac{\mu_0 i_1 i_2 L}{2\pi d}$.

3) Curved:



$F = i(\vec{CD} \times \vec{B})$

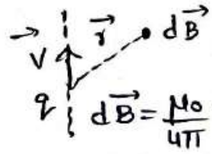
Biot-Savart's law:



$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i(d\vec{l} \times \vec{r})}{r^3}$.

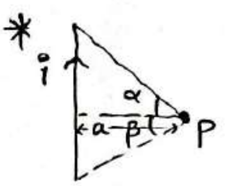
$\frac{\mu_0}{4\pi} = 10^{-7}$

• $d\vec{l}$ is along i



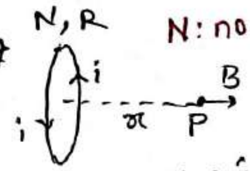
$d\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v} \times \vec{r})}{r^3}$.

*



$B_P = \frac{\mu_0 i}{4\pi a}(\sin\alpha + \sin\beta)$

*



N: no. of turns.

$B_P = \frac{\mu_0 N i R^2}{2(R^2+x^2)^{3/2}}$.

*

$B = \frac{\mu_0 i \alpha}{4\pi r}$.

* Solenoid:

$\vec{B} = \mu_0 n i$

* Toroid:

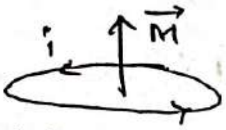
$\vec{B} = \frac{\mu_0 N i}{2\pi r}$.

Ampere Circuital Law :

Note! Outward i is +ve.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0(i_{en})$$

Magnetic moment :



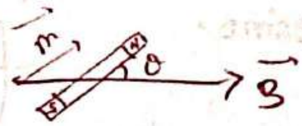
n: no of turns.
A: loop area.

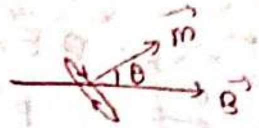
$\vec{M}=ni\vec{A}$. dir^n of M using RHT rule.

$\vec{m}=\left(\frac{Q}{2m}\right)I\omega$.

Torque on loop in B :

$$\vec{\tau}=\vec{m}\times\vec{B}.$$





PE of loop in B :

$$U=-\vec{m}\cdot\vec{B}.$$

Stable : $\theta=0^\circ \Rightarrow U_{min}=-MB$

Unstable : $\theta=180^\circ \Rightarrow U_{max}=MB$

Force on loop in B :
(Non-uniform B)

$F = M\frac{dB}{dx}$ [use variation of B is small.]

Mag. moment of loop.

ELECTRO MAGNETIC INDUCTION

- $\phi = \int \vec{B}\cdot d\vec{A}$ – no. of field lines crossing given area
- SI unit (Wb) $\rightarrow Tm^2$
- if B is uniform, $\phi = \vec{B}\cdot\vec{A} = |\vec{B}||\vec{A}|\cos\theta$.

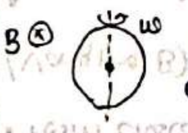
- Faraday's law:
- $\mathcal{E}mf = \left|\frac{d\phi}{dt}\right|$; $\langle \mathcal{E} \rangle = \left|\frac{\Delta\phi}{\Delta t}\right|$

Induced # if N turns, $\mathcal{E} = N\left|\frac{d\phi}{dt}\right|$.

→ Charge flown in time t:

$\boxed{Q = \frac{\Delta\phi}{R}}$, R → Resistance.

Eg:- Find the inst. Emf, If ring is rotated about with ang v = ω rad/s.

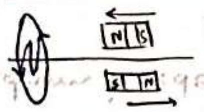


$\phi = BA\cos\omega t$

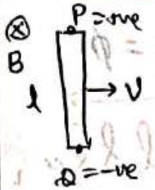
$d\phi = -BA\omega\sin\omega t$

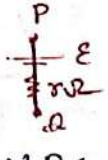
$|\mathcal{E}mf| = BA\omega\sin\omega t$.

- Lenz's law: dirⁿ. of i will such that its effect will oppose the $\Delta\phi$.
- Based on law of cons. of Energy.

a)  $\otimes \vec{B}\uparrow$ b) 

Motional Emf:

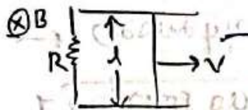


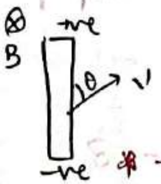
$V_p - V_q = \mathcal{E} = Bl v$ ≡ 

- applicable only when v, B & l are mutually ⊥ᵉʳ.

Note: If any 2 are ∥, $\mathcal{E} = 0$.

Q>> Find the induced i in the circuit?

 →  $i = Blv/R$.

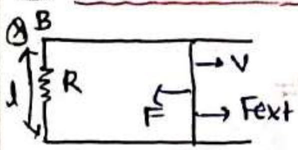


$\mathcal{E} = Bl$ (component of $v \perp$ to l)

$= Blv\sin\theta$.

* +ve polarity is towards dirⁿ of $\vec{v}\times\vec{B}$.

Force and Power analysis:



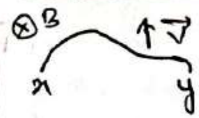
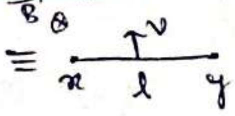
wkt, $F = Bil$, $\mathcal{E}mf = Blv$

$i = Bvl/R$

$F = Bl(Bvl/R) = \frac{B^2l^2v}{R}$

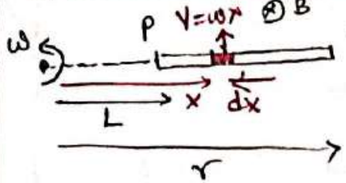
$P_{ext} = F_{ext}\cdot v = \frac{B^2l^2v^2}{R} = P_{loss}$.

Random Shape:

 ≡ 

$\mathcal{E} = Blv$

EMF due to rotation:

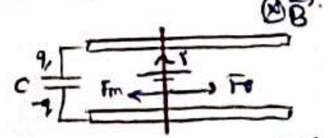


$d\mathcal{E} = B\,dx\cdot\omega x$

$\mathcal{E} = B\omega\int_L^r x\,dx$

$\mathcal{E} = \frac{B\omega}{2}(r^2 - L^2)$

Parallel Rail-Track problems:



$F_m = ilB = \frac{Blv}{R}\times lB$.

i) $\frac{dv}{dt} = \frac{F_0 - F_m}{m}$

$v = \frac{F_0R}{B^2l^2}\left(1 - e^{-\frac{B^2l^2t}{mR}}\right)$

At $t \to \infty$ (steady state)

Terminal vel, $v_t = \frac{F_0R}{B^2l^2}$.

$P = F_0 v_t$ = power across R.

$q = C\mathcal{E} \rightarrow q = CBlv$

$\Rightarrow i = \frac{dq}{dt} = CBla$

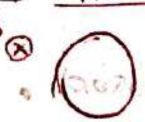
$F_0 - F_m = ma$

$\Rightarrow F_0 - (CBla)lB = ma$

$\therefore a = \frac{F_0}{m + B^2l^2C}$

→ a is const.

Induced E. Field : (const. B)



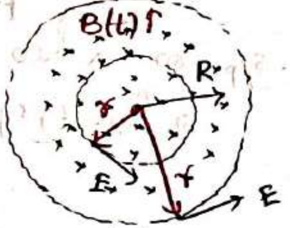
$\oint \vec{E}\cdot d\vec{l} = \frac{-d\phi}{dt}$.

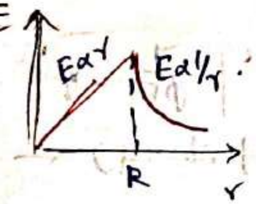
- produced by charging mag. field.
- non electrostatic, non-conservative.
- can't define a voltage for it.
- Form closed loops {No source & sink}

(Varying B) {cylindrical region}

for $r<R$, $E = \frac{r}{2}\frac{dB}{dt}$.

for $r \geq R$, $E = \frac{R^2}{2r}\frac{dB}{dt}$

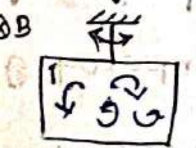




(i) E is in closed loop.

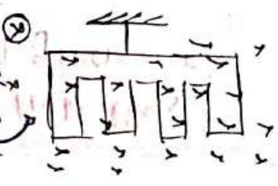
(ii) Here, it's in non-conservative in nature.

Eddy Currents :

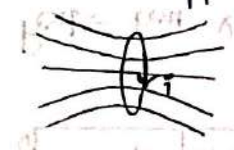


random i loops generated due to motion.

(ii) Due to this, heat is dissipated.



Self induction : a property of coil by which it opposes the change in itself.



(a) For any coil carrying, I

$\phi \propto I$ (Self flux linkage due to own I)

$\Rightarrow \phi = Li$

Inductance : Henry (H) $\left\{\frac{Web}{Amp}\right\}$

(b) If I varies, Emf induced

$\mathcal{E} = \frac{-LdI}{dt}$ (or) $\mathcal{E} = \left|\frac{LdI}{dt}\right|$.

↳ polarity of $\mathcal{E}$ can be found by Lenz's law.

L depends on,

→ Geometrical construction of coil

→ Material properties

for Solenoid,

Area $= \pi r^2$

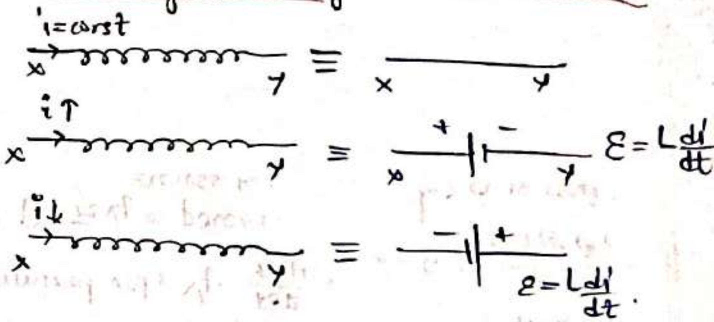
$\phi = NBA \Rightarrow \phi = N \times \frac{\mu_0 N i}{l} A$

$\Rightarrow \phi = \left(\frac{\mu_0 N^2 A}{l}\right) i$ $\therefore$ $\boxed{L = \frac{\mu_0 N^2 A}{l}}$

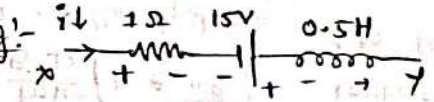
Note! If any medium inside, $L = \frac{\mu_0 \mu_r N^2 A}{l}$.

$B = \mu_0 \mu_r n i$ {mag. flux density}

Battery polarity induced in ind :



Eg :-

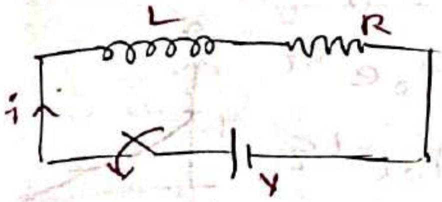


$i = 5A$

$V_x - V_y = 5 - 15 - 5 = -15V$. $\frac{di}{dt} = 10\ A/s$

LR - Circuits.

→ **Growth of Current:**

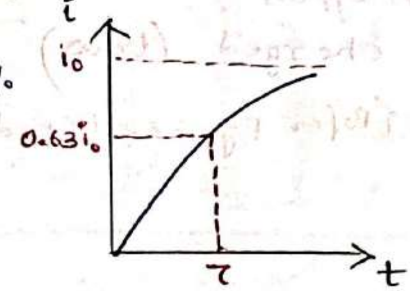


at $t=0,\ i=0$

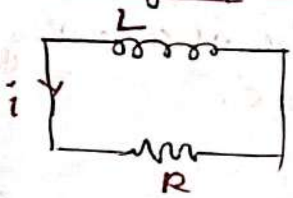
at $t=t$, $\boxed{i = i_0\left(1-e^{-t/\tau}\right)}$

$\{ i_0 = \mathcal{E}/R\ ;\ \tau = L/R \}$

$1\tau \rightarrow 63\%$



→ **Decay of Current:**

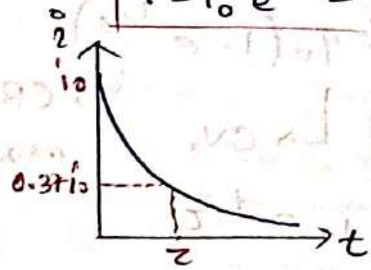


at $t=0,\ i=i_0$

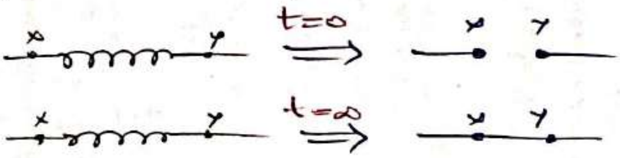
at $t=t$,

$\boxed{i = i_0\, e^{-t/\tau}}$

{ 63% decay in 1τ }



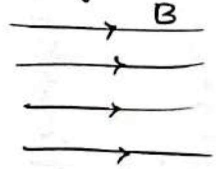
⇒ Behaviour of L at $t=0$ & $t=\infty$:



⇒ Energy stored in Inductor:

$\mathcal{E}$ (or) $U = \frac{1}{2}Li^2$

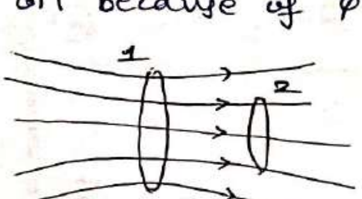
⇒ Magnetic Energy density:



$= \frac{B^2}{2\mu_0}$ $\mathcal{E}$ per unit vol.

Mutual Inductance.

→ Property of pair of coils due to which a change in current 1 coil is opposed by EMF induced in other coil because of ϕ linkage.



$\phi_{22} = L_2 i_2$

$\phi_{11} = L_1 i_1$

$\phi_{21} \rightarrow$ flux on 2 due to 1

$\phi_{21} \propto i_1$

$\boxed{\phi_{21} = M_{21} i_1}$

Mutual ind. of ② wrt ①.

- $\frac{d\phi_{21}}{dt} = M_{21}\frac{di_1}{dt}$

$\mathcal{E}mf = -M_{21}\frac{di_1}{dt}$

Reciprocality: $\boxed{M_{21} = M_{12}}$

$0 \le K \le 1$

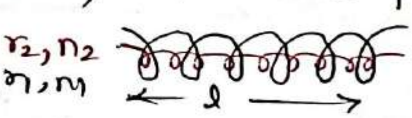
$M = K\sqrt{L_1 L_2}$ { K: coeff. of coupling }

Eg:

$\underline{K=0}$ $\quad$ $\underline{K=1}$.

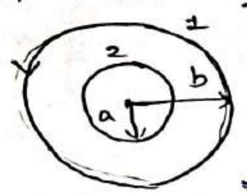
Examples:

1) Find the M.I of following setup?



$M_{21} = \mu_0 n_1 n_2 l \pi r_2^2$

2) Find M?



$a \ll b$

$\phi_2 = \frac{\mu_0 i}{2b} \times \pi a^2$

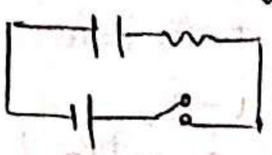
$\Rightarrow \phi_2 = \left(\frac{\mu_0 \pi a^2}{2b}\right) i$

$\therefore M = \frac{\mu_0 \pi a^2}{2b}$

PTL

R-C. Circuit.

→ **Charging:**



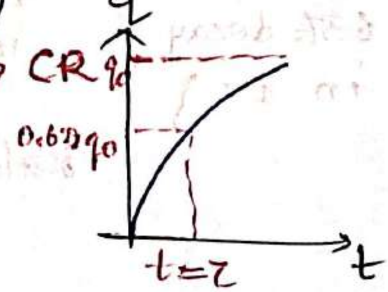
Circuit consisting a R & C.

$q = q_0(1 - e^{-t/\tau})$

$\tau \to CR$

$q_0 \to CV_0$

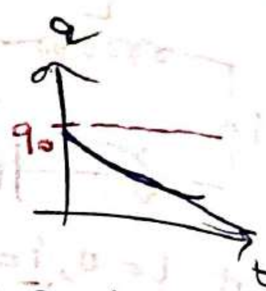
$i = i_0 e^{-t/\tau}$.



→ **Discharging:**

$q_t = q_0 e^{-t/\tau}$.

$i = -i_0 e^{-t/\tau}$.



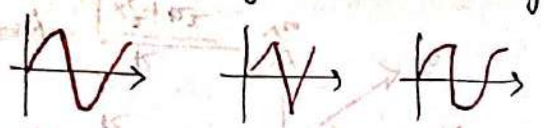
cap. offers 0 Resistance when uncharged $(t=0)$

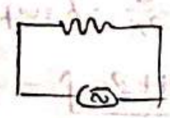
offers $\infty\ \Omega$ when fully charged $(t=\infty)$

{Refer pg. 232 DC Pandey}

Alternating Current.

• Dir^n. of i changes alternatively.





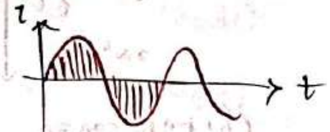
⇒ Avg. Value of a Fn:

$\langle \sin(\omega t+\varphi)\rangle = 0$; $\langle \sin^2(\omega t+\varphi)\rangle = 1/2$.

$\langle \cos(\omega t+\varphi)\rangle = 0$; $\langle \cos^2(\omega t+\varphi)\rangle = 1/2$.

I_{avg} :

$$\bar{I}_{avg} = \frac{\int_{t_1}^{t_2} i(t)\,dt}{t_2 - t_1}$$



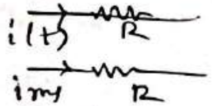
for $i = i_0\sin\omega t$

$i_{avg} = 0$.

⇒ RMS value : $y_{rms} = \sqrt{\langle y^2\rangle}$.

$\sin(\omega t+\varphi)_{rms} = 1/\sqrt{2}$

$\cos(\omega t+\varphi)_{rms} = 1/\sqrt{2}$



$$I_{rms} : \sqrt{\frac{\int_{t_1}^{t_2} i^2(t)\,dt}{t_2 - t_1}}$$

// in same time, same heat is desipated.

for $i = i_0\sin\omega t$

$i_{rs} = i_0/\sqrt{2}$.

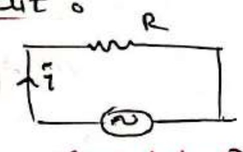
Power factor, $\cos\varphi = R/Z$

① Pure Resistive Circuit :

$V = iR$

$i = V/R = \frac{V_0\sin(\omega t)}{R}$

$i = i_0\sin(\omega t)$

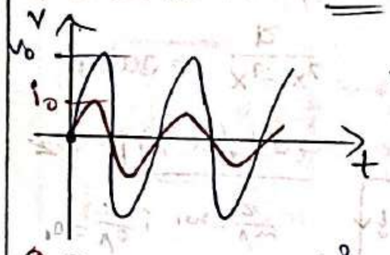


$i_0 = V_0/R$; $V_{rs} = V_0/\sqrt{2}$

$i_{rs} = i_0/\sqrt{2}$; $i_{rs} = V_{rs}/R$.

• i & v in phase

• R doesn't depend on ω



$V = V_0\sin(\omega t)$

$i = i_0\sin(\omega t)$.

Power : $\langle P\rangle = i_{rs}^2 R = \left(\frac{V_{rs}}{R}\right)^2 R = \frac{V_{rs}^2}{R}$.

• 220 V is the rms value of voltage in In.

• $f = 50\,Hz$ • $\langle p\rangle$ is max here.

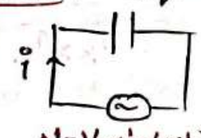
• $\omega = 2\pi f = 100\pi$

② Pure Capacitive Circuit : c, q

$q = CV = CV_0\sin(\omega t)$

$i = \frac{dq}{dt} = CV_0\omega\cos(\omega t)$

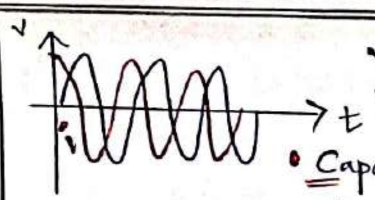
$i = i_0\sin(\omega t + \pi/2)$ $i_0 = CV_0\omega$



$i_0 = \frac{V_0}{1/\omega C}$

$\frac{V_0}{X_C} \to 1/\omega C$ {unit Ω} $i_0 = \frac{V_0}{X_C}$

Capacitive reactance.



$V = V_0\sin(\omega t)$

$i = i_0\sin(\omega t + \pi/2)$

• Capacitive circuit Current leads voltage by phase $\pi/2$.

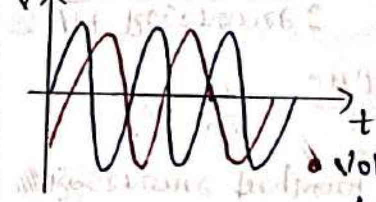
Power ⇒ $\langle P\rangle = 0$

③ Pure Inductive Circuit :

$i = i_0\sin(\omega t - \pi/2)$

$i_0 = \frac{V_0}{X_L}$; $i_{rs} = \frac{V_{rs}}{X_L}$

$X_L = \omega L$ {Inductive reactance} Ω



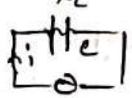
$V = V_0\sin(\omega t)$

$i = i_0\sin(\omega t - \pi/2)$

• Voltage leads current by phase $\pi/2$

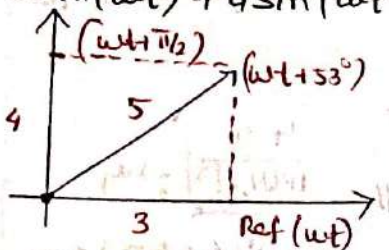
Power ⇒ $\langle P\rangle = 0$

Impedance : $\boxed{i_0 = \frac{V_0}{Z}}$ → Impedance.

$Z = R$	$Z = X_C = \frac{1}{\omega C}$	$Z = X_L = \omega L$
$i_0 = \frac{V_0}{R}$	$i_0 = \frac{V_0}{X_C}$	$i_0 = V_0/X_L$
		

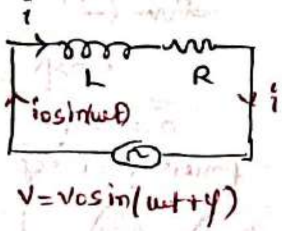
Eg. Draw phasor diagram for

$3\sin(\omega t) + 4\sin(\omega t + \pi/2) = ?$



$= 5\sin(\omega t + 53^\circ)$.

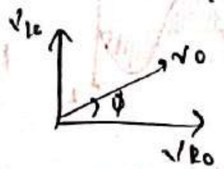
$\Rightarrow$ L-R circuit :



$i_0 = \frac{V_0}{Z}$; $i_{rms} = \frac{V_{rms}}{Z}$

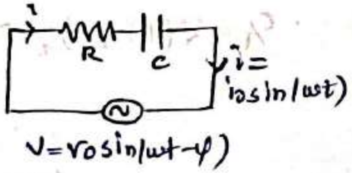
$Z = \sqrt{X_L^2 + R^2}$

$\tan\phi = \frac{X_L}{R}$.



P.F: $\cos\phi = \frac{R}{\sqrt{X_L^2 + R^2}}$

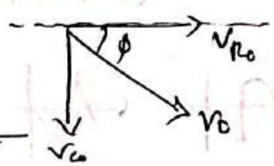
$\Rightarrow$ R-C circuit :



$i_0 = \frac{V_0}{Z}$; $i_{rms} = \frac{V_{rms}}{Z}$

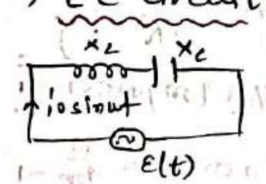
$Z = \sqrt{X_C^2 + R^2}$

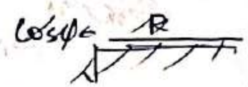
$\tan\phi = \frac{X_C}{R}$.



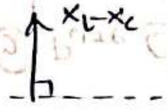
P.F: $\cos\phi = \frac{R}{\sqrt{X_C^2 + R^2}}$

$\Rightarrow$ L-C circuit :





- $X_L > X_C$

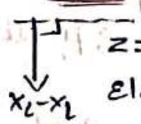


$Z = X_L - X_C$

$\mathcal{E}$ leads i by $\pi/2$

$\mathcal{E} = \mathcal{E}_0\sin(\omega t + \pi/2)$
$\hookrightarrow i_0 Z$

- $X_L < X_C$

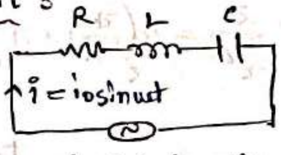


$Z = X_C - X_L$

$\mathcal{E}$ lags i by $\pi/2$

$\mathcal{E} = \mathcal{E}_0\sin(\omega t - \frac{\pi}{2})$

$\Rightarrow$ L-C-R circuit :



$i_0 = \frac{V_0}{Z}$; $i_{rms} = \frac{V_{rms}}{Z}$

$Z = \sqrt{(X_L - X_C)^2 + R^2}$

$\tan\theta = \frac{X_L - X_C}{R}$.

(a) If $X_L > X_C$ (Inductive) $\Rightarrow \mathcal{E}$ leads by ϕ

(b) If $X_L < X_C$ (capacitive) $\Rightarrow \mathcal{E}$ lags by ϕ.

(a) $V_L = V_L \sin(\omega t + \pi/2)$
$\hookrightarrow V_L = i_0 X_L$

$V_C = V_C \sin(\omega t - \pi/2)$.
$\hookrightarrow V_C = i_0 X_C$

P.F. $\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

$\langle power \rangle = i_{rms} V_{rms} \cos\phi$ $\hookrightarrow$ Power factor.

$\cos\phi = \frac{R}{Z}$ $\langle P_c \rangle = i_{rms}^2 R$

$\Rightarrow$ Resonance in LCR circuit :

$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{(X_L - X_C)^2 + R^2}}$. $\rightarrow$ & $Z = \sigma$

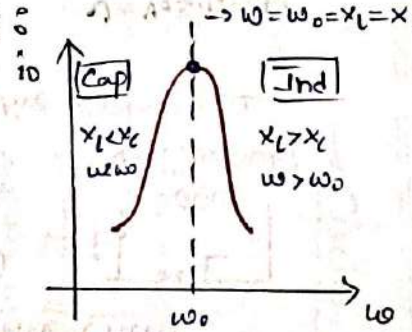
i_0 is max $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow \omega_R = \frac{1}{\sqrt{LC}}$.

Resonance frequency :

$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

At Resonance :

1) i_0 is max
2) $X_L = X_C$
3) $Z = R$
4) $\cos\phi = 1$
5) $\omega_0 = 1/\sqrt{LC}$



Current Variation with ω: (series LCR)

$$i_{rms} = \frac{E_{rms}}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

(a) For ω_1 & ω_2
P_{av} is $\frac{1}{2}$ of P_{max}.

(b) Bandwidth
$\Delta\omega = \omega_2-\omega_1 = R/L$

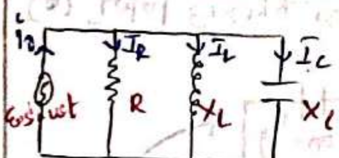
(c) Quality factor

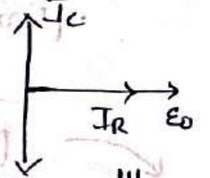
$$Q = \frac{\omega_R}{\Delta\omega} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

(d) If $\omega < \omega_R \Rightarrow X_C > X_L$ (i leads)

(e) If $\omega > \omega_R \Rightarrow X_C < X_L$ (i lags).

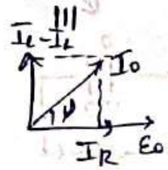
RLC (parallel):





(a) $\tan\phi = \frac{I_C - I_L}{I_R}$

$$= \frac{\frac{E_0}{X_C}-\frac{E_0}{X_L}}{E_0/R}$$



(b) $I_0 = \sqrt{I_R^2+(I_C-I_L)^2}$. $\Rightarrow \frac{1}{Z} = \sqrt{\frac{1}{R^2}+\left(\frac{1}{X_C}+\frac{1}{X_L}\right)^2}$

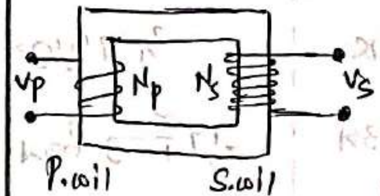
$$\Rightarrow \frac{E_0}{Z} = \sqrt{\left(\frac{E_0}{R}\right)^2+\left(\frac{E_0}{X_C}-\frac{E_0}{X_L}\right)^2}$$

(c) $\omega_R = 1/\sqrt{LC}$

(d) At ω_R, Z is max
$\Rightarrow i$ is min.

Transformer:

- Used to change Alternating Voltage from one value to another using principle of Mutual Induction.



$V_P = -N_P \frac{d\phi}{dt}$.

$V_S = -N_S \frac{d\phi}{dt}$.

$\boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P}}$

$N_S > N_P$: Step-up transformer

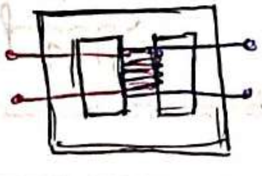
$N_S < N_P$: step-down transformer.

$$P = Vi \Rightarrow \boxed{V_P i_P = V_S i_S}$$

Few problems:

1) Cause – Due to poor design & air gaps in the core, all the flux due to primary doesn't pass through secondary.

Rectification – By winding the P & S coil one over the another.



2) Cause – Resistance of Windings causes heat loss.

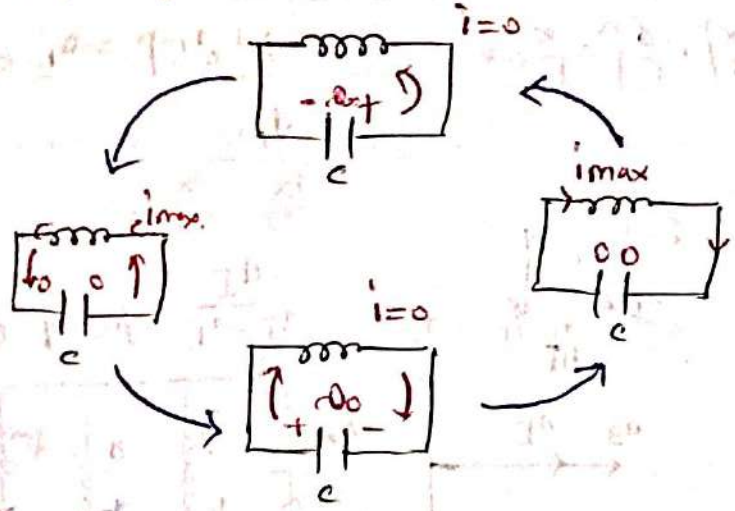
Rectification – In high i, low V windings, these are minimal by using thick wire.

3) Eddy Currents – induced Eddy currents causes heating.
– Laminated core reduces latter i.

4) Hysteresis – Alternating magnetization of core causes hysteresis loss.

Rectification – It is kept min. by using a magnetic material having low hysteresis loss.

LC Oscillations.



(a) Total Energy is const.

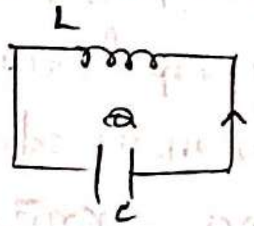
$$\frac{q_0^2}{2C} = \frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{1}{2}Li_0^2.$$

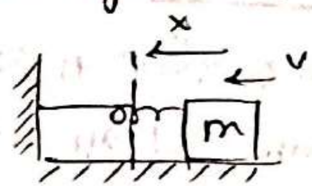
(b) $\omega = \frac{1}{\sqrt{LC}}$, $T = 2\pi\sqrt{LC}$.

(c) General Equation:

$$q = q_0 \sin(\omega t + \varphi),\ i = i_0 \cos(\omega t + \varphi).$$

⇒ LC vs Spring block.





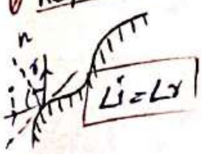
LC	Spring block
Q	x
i	vel
$1/C$	k
L	m
Elect. $E = Q^2/2C$	$PE = \frac{1}{2}kx^2$
Mag. $E = \frac{1}{2}Li^2$	$KE = \frac{1}{2}mv^2$.
$Q, i, L, 1/C$	x, v, k, m
$i = \omega\sqrt{Q_0^2 - Q}$	$v = \omega\sqrt{A^2 - x^2}$
$i_{max} = \omega Q_0$	$V_{max} = \omega A$
$\omega = \frac{1}{\sqrt{LC}}$	$\omega = \sqrt{\frac{k}{m}}$.

Geometrical Optics.

PTL

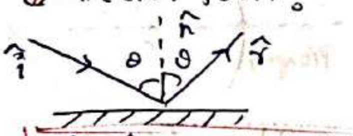
- Rectilinear propagation of light
- Light is an EM Wave $\lambda : 3600$ to $7800 \mathring{A}$
- Speed of light in vacuum is
  * $c = 3\times10^{8}$ m/s & in other med $v = \underline{\underline{4m}}$
- $v = f\lambda$
- particle of light, Photon $\rightarrow$ $P = h/\lambda$; $E = hf = hc/\lambda$.

Reflection :



$\boxed{\angle i = \angle r}$

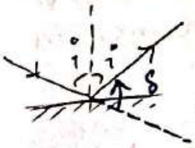
Vector form :



$$\boxed{\hat{r} = \hat{i} - 2(\hat{i}\cdot\hat{n})\hat{n}}$$

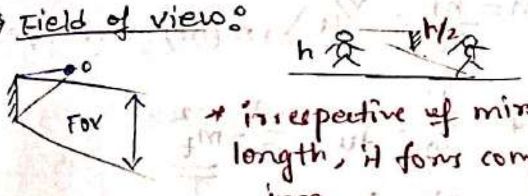
- Incident, R & N ray lies on same plane. $\boxed{[\hat{i}\,\hat{n}\,\hat{r}] = 0}$.

Angle of Deviation : (δ)



- $\delta = \pi - 2i$
- Here, it is anti-clockwise rotation.

→ Field of view :



* irrespective of mirror length, it forms complete img.

Effect of Motion :

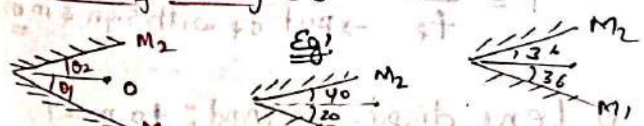
$\vec{V}_{OM} = -\vec{V}_{IM}$

$\vec{v}_0 - \vec{v}_m = -(\vec{v}_I - \vec{v}_m)$

$\boxed{\vec{V}_I = 2\vec{V}_m - \vec{V}_0}$

* If obj rotated by θ → Img $-\theta$.
* If mirror rotated by θ → Img 2θ.

→ No. of images :



M_1	M_2
θ_1	θ_2
$\theta_2 + (\theta_1+\theta_2)$	$\theta_1 + (\theta_1+\theta_2)$

$= 360 \cdot (\theta_1+\theta_2)$

Eg:

M_1	M_2
① 20	40 ②
③ 100	80 ④
⑤ 140	160 ⑥

$140+160+60 = 360$ $\therefore 6+1 = 5$

M_1	M_2
① 36	36 ②
③ 108	108 ④
⑤ 180	180 ✗

$180+180+72 > 360$ $\therefore 5-1 = 4$

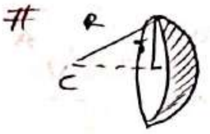
then steps -1 = images.
{if >360 then ignore 1 step}

* if $\frac{360}{\theta}$ is even, $n = \frac{360}{\theta} - 1$
* if $\frac{360}{\theta}$ is odd,

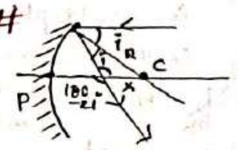
Sign Conventions :

- All are measured from pole
- incident ray dir^n +ve.

Spherical Mirror :



r : radius of aperture.
2r : ape. size.
C : centre of curvature.
R : radius of curvature.



by sine law

$$\frac{R}{\sin(180-2i)} = \frac{x}{\sin i}$$

$$\boxed{x = \frac{R}{2\cos i}}$$

$FP = R - \frac{R}{2}\sec\theta$ CP = R

Note!
If rays were Paraxial (close to axis) then θ is very small.
$\Rightarrow$ F is focus & $\boxed{FP = \frac{R}{2}}$

Mirror Formulae :

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

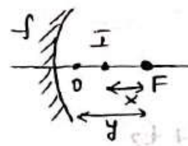
Magnification :

Lateral − $m = \frac{h_i}{h_o} = \frac{-v}{u}$

longitudinal − $m = \frac{l \text{ of img.}}{l \text{ of obj.}}$

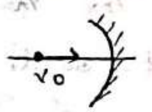
- $\frac{dv}{du} = -m^2$ # areal mag : $= -m^4$

Newton's Formula :

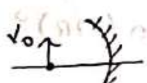


$$f = \sqrt{xy}$$

Img velocity w.r.t mirror :



$\overline{V}_{i\parallel} = -m^2 \overline{V}_{o\parallel}$



$\overline{V}_{i\perp} = \overline{V}_{o\perp} \times m$

$\overline{V}_o = \frac{dh_o}{dt}$; $\overline{V}_i = \frac{dh_i}{dt}$

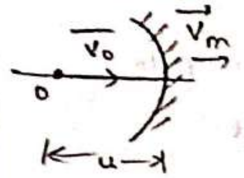
$V_{i\perp} = m v_{o\perp} + h_o\, du/dt$

$V_{o\parallel} = -m^2 \overline{v}_{o\parallel}$

$V_I = -m^2 v_{o\parallel}\hat{i} + (m\overline{v}_{o\perp} + h_o\, du/dt)\hat{j}$

$V \propto \frac{\mu_1}{\mu_2}$ {in a med} $\mu_1 \rightarrow$ obs, $\mu_2 \rightarrow$ obj

In general :-



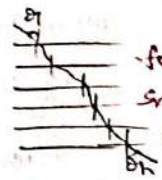
$$v_{I/m} = -\left(\frac{v}{u}\right)^2 v_{o/M}$$

If $v_{M}=0$, $v_I = -m^2 v_o$.

Refraction :

• $\mu = \frac{\text{vel. of light in vacuum}}{\text{vel. of light in med}} = c/v$

• Snell's law -

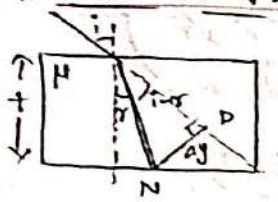


for multiple ref. snells law can be applied for 1st & last.

> $\mu_i \sin i = \mu_r \sin r$

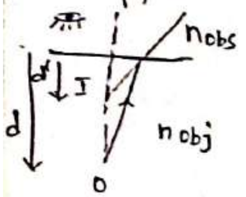
> rel ref indx $\mu = \frac{\mu_2}{\mu_1}$ * $n\sin\theta = \text{const}$.

Lateral shift :



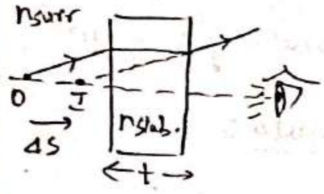
$$\Delta y = \frac{t\sin(i-r)}{\cos r}$$

Apparent Depth :



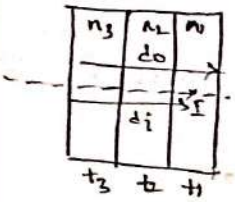
$$h_{app} = \frac{h_{real}}{n_{obj}/n_{obs}}$$

Apparant Shift :



$$\Delta S = t\left(1 - \frac{n_{surr}}{n_{med}}\right)$$

↳ parallel slabs.

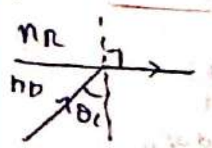


$d_o = t_1 + t_2 + t_3$

$d_i = \frac{t_1}{n_1/n_o} + \frac{t_2}{n_2/n_o} + \frac{t_3}{n_3/n_o}$

normal shift $= d_o - d_i$

Total internal reflection (TIR) :

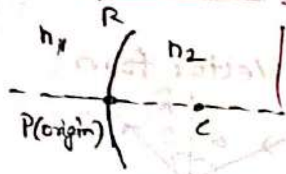


$$\theta_c = \sin^{-1}\left(\frac{n_R}{n_D}\right).$$

#

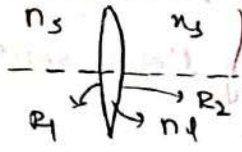
Optical Fibre : used to transfer EM-waves with min. Attenuation.
↳ loss.

Spherical Surfaces :



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Thin lenses :



$$\frac{1}{f} = \left(\frac{n_l}{n_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

(1) $n_l > n_s$: Convex f: +ve, concave f: -ve

(2) $n_l < n_s$: Convex f: -ve, concave f: +ve

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

• $m = \frac{h_i}{h_o} = v/u$ → m(-ve): real & inverted, → m(+ve): virtual & erect

• $m_l = \frac{d_i}{d_o} = \frac{dv}{du} = \frac{v^2}{u^2} = m_t^2$

$v^{-1} - u^{-1} = f^{-1} \Rightarrow \frac{-1}{v^2}dx - \frac{1}{u^2}du = 0$

Lens type :

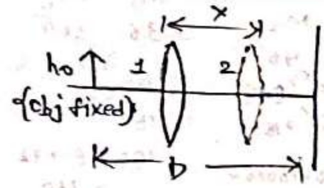
Convex : fat at centre (converging)

Concave : flat at edge (diverging)

Optical Power :

$P = \frac{1}{f_l}$ → unit : dioptre → put f_l with sign & in m

Lens displ. method : to meas f.



{screen fixed}

$$f = \frac{D^2 - x^2}{4D}$$

$h_o = \sqrt{h_1 h_2}$

h_1 : img height when lens is at pos. 1

h_2 : img h at pos. 2

θ Vel. of Img :

a) Along p. axis.

$v^{-1} - u^{-1} = f^{-1}$

$\frac{dv}{dt} = \frac{v^2}{u^2}\left(\frac{du}{dt}\right)$

$\boxed{v_i = \frac{v^2}{u^2} v_o}$

b) $\perp$ to p. axis.

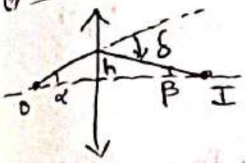
$\frac{h_i}{h_o} = m$

$h_i = m h_o$

$\frac{dh_i}{dt} = m \frac{dh_o}{dt}$

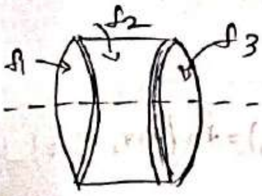
$\boxed{v_i = m v_o}$

θ Deviation by lenses :



$\delta = h/f$.

Combination of Lenses :

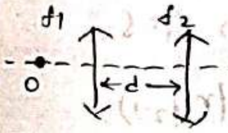


$P = P_1 + P_2 + P_3$

$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$.

- put with sign's
- they must be in contact
- f_1, f_2, f_3 are individual f's wrt surr.

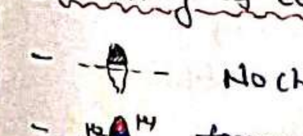
» a) if lenses are far ?



$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

Δ only when they mention.

Silvering of Lens :



$P = 2P_L + P_m$

$\frac{-1}{f_{eq}} = \frac{2}{f_L} - \frac{1}{f_m}$.

{behaves just as a mirror}

θ Cutting of Lens :

- No change,
- forms 3 diff. images.
- $f = 2f_0$, $P = P_0/2$

{DD) (ID D(I (I(I} → all arang. have same focal length.

Dispersion : VIBGYOR {$n_R < n_V$} $\lambda \uparrow$

– ang. splitting of white light into {VIBGYOR}

– diff colours (λ) will have slightly diff. ref. index of a medium.

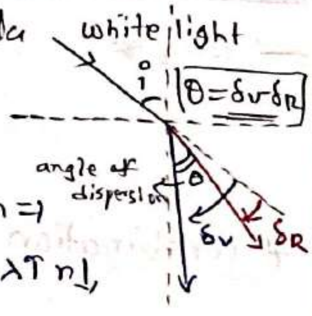
↳ Cauchy's formula

$n = f(\lambda)$

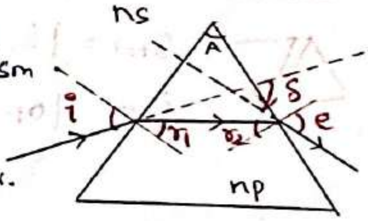
$n = a + b/\lambda^2$

Vaccum : $a = 1, b = 0, n = 1$

Other med : $b = +ve$; $\lambda \uparrow n \downarrow$,



#PRISM :



A : Angle of prism

$A = r_1 + r_2$

δ : angle of dev.

$\delta = i + e - A$.

for δ_{min} : $i = e$; $r_1 = r_2 = r$

$\frac{n_p}{n_s} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin(A/2)}$.

$i = \frac{\delta_{min} + A}{2}$.

⟹ Thin Prism (A ~ small)

take $\sin\theta \simeq \theta$

A < 6°

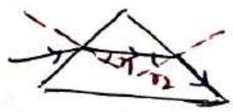
$\boxed{\delta = \left(\frac{n_p}{n_s} - 1\right) A = (\mu_y - 1) A}$

↳ Dispersion ⟹ $\theta = \delta_V - \delta_R$

$\theta = (\mu_V - \mu_R) A$

also, $n_y = \frac{n_V + n_R}{2}$ {use only when n_y not given}

θ For no emergence : {TIR}



$r_2 > \theta_c = \sin^{-1}(1/\mu)$

$\boxed{\mu > \operatorname{cosec}(A/2)}$

$\boxed{i < \sin^{-1}(\mu \sin(A - \theta_c))}$.

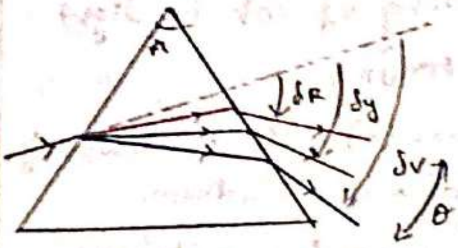
θ For grazing emergence :

$r_2 = e = \theta_c$ {90°}

$\boxed{\mu = \operatorname{cosec}(A/2)}$

$\boxed{i = \sin^{-1}(\mu \sin(A - \theta_c))}$.

Dispersive Power (ω) :



$\delta_R = A(n_R - 1)$; $\delta_y = A(n_y - 1)$; $\delta_v = A(n_v - 1)$

{n_R, n_y, n_v are the R.I of med for Red, Yellow & Vi

(Cauchy's eqⁿ , $n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots$)

(a) θ (Dispersion angle) $= \delta_v - \delta_R = A(n_v - n_R)$

(b) ω (Dispersion Power) $= \frac{\delta_v - \delta_R}{\delta_y} = \frac{n_v - n_R}{n_y - 1}$.

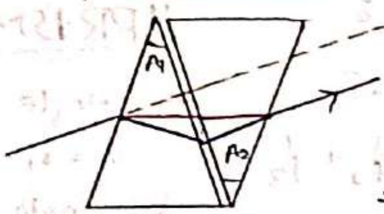
Combination of Prism's :

$\delta_{net} = \delta_1 + \delta_2$

$\delta_{net} = |\delta_1 - \delta_2|$

$\theta_{net} = |\theta_1 - \theta_2|$.

1) Achromatic Combination {dev. without disp } (No colour)

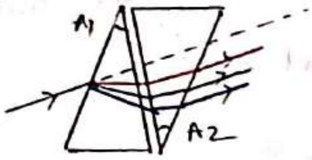


$\theta_{net} = 0$

$\Rightarrow \theta_1 = \theta_2$

$\Rightarrow A_1(n_{v_1} - n_{R_1}) = A_2(n_{v_2} - n_{R_2})$.

2) Direct vision comb. {disp without dev}



Here, final emergent yellow is parallel to incident white light.

So, $\delta_{net} = 0 \Rightarrow \delta_1 = \delta_2$.

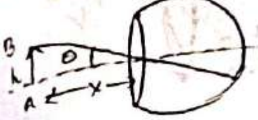
$A_1(n_{y_1} - 1) = A_2(n_{y_2} - 1)$.

Resolution limit : $\frac{1.22\lambda}{d}$.

{$\sin\theta = \theta = \lambda/d$}

Optical Instruments.

1. Human Eye :



$v = 2.5\,cm$ (fixed)

$u, f \rightarrow$ changeable.

θ: visual angle. $\boxed{\theta = \frac{h}{x}}$

→ Near point :
- least dist. of distinct vision.
- $D = 25\,cm$ (for normal eye)

→ Far point :
- stress-free $\{\infty\}$

2. EYE Defects :

↳ Myopia (Nearsightedness) :
- far objects not clear.
- ray's converge before retina.
- * Correction – Concave lens.
  - makes img of obj at ∞.

↳ Hypermetropia (Farsightedness) :
- near objects not clear
- ray's converge after retina.
- * Correction – Convex lens.
  - makes img of obj at 25cm.

↳ Presbyopia :
- old age, weak eye muscles
- f of eye doesn't adjust.
- both myo & hypermetro.
- Near pt. > 25,
  Far pt. $< \infty$.
- * Correction – Bifocal lens.

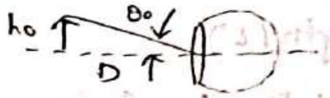
↳ Astigmatism :
- Distorted eye (Not spherical)
- R of eye isn't same in orthogonal dirn.
  - * correction – Cylindrical lens.

Magnification of opt. inst :

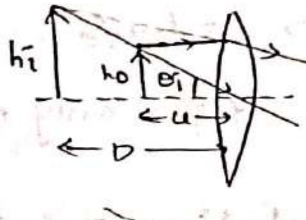
$$m = \frac{\theta}{\theta_0} = \frac{\text{vis. angle with opt. inst}}{\text{max. vis. angle with eye.}}$$

$\boxed{m > 1}$

3. Simple Microscope :



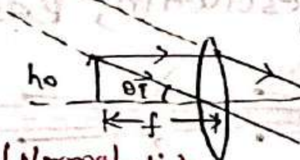
Ang. size of obj, $\theta_0 = \frac{h_0}{D}$.



(a) Img. at near pt; D

$\theta_i = \frac{h_0}{u}$, $u = \frac{Df}{D+f}$

$m = \frac{\theta_i}{\theta_0} = 1 + \frac{D}{f}$.



{Normal adj}

(b) Img. at far pt, ∞

$\theta_i = h_0/f$

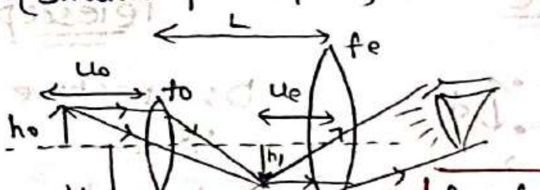
$m = \frac{\theta_i}{\theta_0} = \frac{D}{f}$.

* put only magnitudes.

4. Compound Microscope :
- 2 convex lenses (f = +ve)
- small focal lengths

2 lenses → Objective lens (near obj) ; Eye piece. (near eye)

{smaller aperture & |F|}



$\boxed{fe > fo}$

$m = m_0 \times m_e$

Img. at D	Img. at ∞
$m = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$	$m = \frac{v_0}{u_0}\frac{D}{f_e}$.

Generally, $f_0 << L$, $f_e << L$

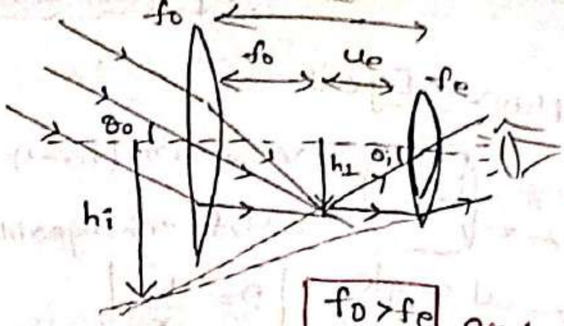
$m = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$ $\quad m = \frac{L}{f_0}\frac{D}{f_e}$.

Tube length (L):

(a) If img at D, $L = v_0 + u_e$

(b) If img at ∞, $L = v_0 + f_e$.

5. Refracting Telescope :
(Astronomical)



$\theta_0 = h_1/f_0$, $\theta_i = h_1/u_e$, $u_e = \frac{Df_e}{D+f_e}$.

Img. at D

$m = \frac{\theta_i'}{\theta_0} = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$

Img. at ∞

$\theta_i = h_1/f_e$

$\therefore m = \frac{\theta_i'}{\theta_0} = \frac{f_0}{f_e}$.

$\boxed{f_0 > f_e}$ and aperture of objective is large to capture more light.

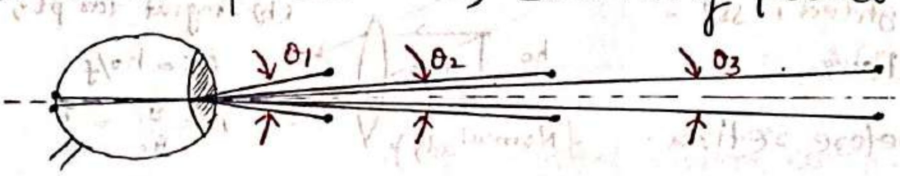
Tube Length (L)

(a) If img at D, $L = f_0 + u_e$,

(b) If img at ∞, $L = f_0 + f_e$.

* put only magnitudes.

6. Limit of resolution, Resolving power :



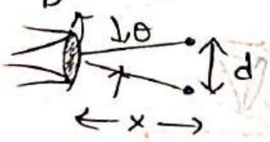
Limit of resolution tells the ability of objective lens to resolve two objects distinctively.

→ There will be min. value of θ for which two objects are just resolved in image.

that θ is called "Limit of Resolution"

& 1/θ is Resolving power.

Eye



LOR, $\theta = \frac{1.22\lambda}{D}$.

$RP = \frac{1}{\theta}$. $d = \theta x$.

Telescope.

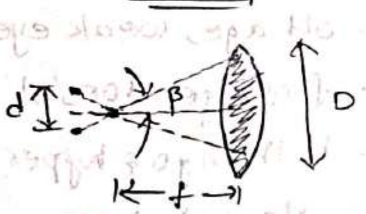
D: Aperture of obj. lens.

Two stars will be just resolved if,

$\theta = \frac{1.22\lambda}{D}$, $RP = \frac{D}{1.22\lambda}$

If dist. of star is x, separation b/w them, $d = x\theta$

Microscope.



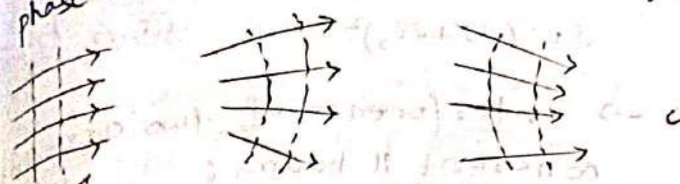
$d_{min} = \frac{1.22\lambda}{2\mu\sin\beta}$

$RP = \frac{1}{d_{min}} = \frac{2\mu\sin\beta}{\lambda}$

• μ: ref. index of med. b/w lens & obj.

• Numerical Aperture (NA) $= \mu\sin\beta$

→ **Wave front** : Locus of all the pts. that are vibrating in the same phase. At 90° with propagation of light

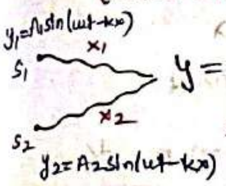


Hyugen's Principles :

- Every point on primary wave front acts as a fresh source of light emit. disturbances known as wavelets
- These wavelets travel with speed of light in air.
- The new position of wavelet is given by the geometrical envelop to these sec. wavelets

Laws of reflection & refraction can be proved using this.

A_{net} & I_{net} in interference :
(coherent source, same ω)



$y = y_1 + y_2$

* phase diff,

$\Delta\phi = k(x_2 - x_1) = k\Delta x$

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda}\Delta x}$$

* $I \propto A^2$

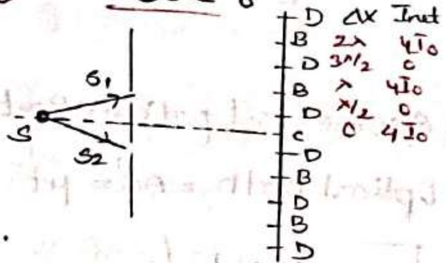
- $A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\phi}$
- $I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$
- $A_{max} = |A_1 + A_2|$; $A_{min} = |A_1 - A_2|$
  ↳ $\Delta\phi = 2n\pi$ ↳ $\Delta\phi = (2n+1)\pi$.
- $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$; $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$
- $I_{max} = (A_1 + A_2)^2$; $I_{min} = (A_1 - A_2)^2$

Interference :

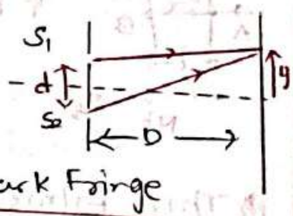
Constructive	Destructive
→ $\Delta\phi = 2n\pi$	$\Delta\phi = (2n+1)\pi$
→ $2n\pi = \frac{2\pi}{\lambda}\Delta x$	$(2n+1)\pi = \frac{2\pi}{\lambda}\Delta x$
$\Delta x = n\lambda$	$\Delta x = (2n+1)\lambda/2$
→ $A_{net} = A_1 + A_2$	$A_{net} = A_1 - A_2$
→ $I_{net} = (\sqrt{I_1} + \sqrt{I_2})^2$	$I_{net} = (\sqrt{I_1} - \sqrt{I_2})^2$

Coherent Sources :
- const. or no ϕ
- same f
- same or almost same λ.

YDSE :



$D >> d >> \lambda$



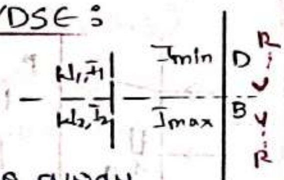
Path diff at y, $\Delta x = \frac{yd}{D}$

Bright Fringe	Dark Fringe
$\Delta x = n\lambda$	$\Delta x = (2n-1)\lambda/2$
$\frac{y_n d}{D} = n\lambda$	$\frac{y_n d}{D} = (2n-1)\lambda/2$
$y_n = \frac{n\lambda D}{d}$	$y_n = (2n-1)\frac{\lambda D}{2d}$.

↳ Fringe width & ang fringe width:
- dist. b/w a successive bright or dark fringe.

(a) $\beta = \frac{\lambda D}{d}$ (b) $\beta_\theta = \lambda/d$

White light in YDSE :



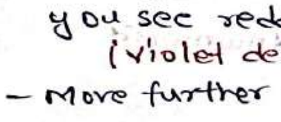
- Central bright f will be white colour.
- as you move a little away you see reddish colour. (violet destructive interference)
- Move further away, it's blue colour.

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$
$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$\beta_V < \ldots \beta_Y < \ldots \beta_R$.
$\lambda = 430nm$ $\lambda = 750nm$

$$\boxed{I \propto \text{slit width} \propto A^2}$$

Geometry of Fringes :

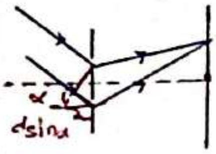


- In vertical slits, the fringes are Hyperbolic
- In horizontal slits, the fringes are semicircular in shape.

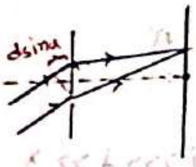
if the whole setup is submerged into med of (μ) then ;

$$\boxed{\lambda = \frac{\lambda_{vacm}}{\mu}} \quad \boxed{\Delta x = \mu(\Delta x)_{vacm}}$$

Diff cases in YDSE :

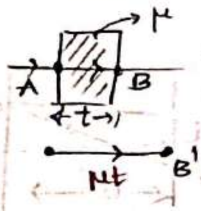


Total path diff,
$$\Delta x = d\sin\theta + \frac{yd}{D}$$



Total path diff,
$$\Delta x = \frac{yd}{D} - d\sin\theta$$

Slabs :

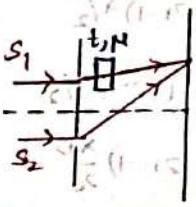


Geometrical path = AB = t

Optical path = AB' = μt

$$\boxed{\Delta x = (\mu - 1)t}$$

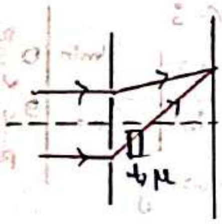
Thin Films in YDSE :



The optical path of upper ray inc. by $(\mu-1)t$.

$$\boxed{\Delta x = \frac{yd}{D} - (\mu-1)t}$$

for n^{th} maxima on upper side, $\Delta x = n\lambda$ the fringes shift upward by, $\frac{(\mu-1)tD}{d}$.

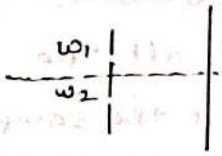


The optical path of lower ray inc. by $(\mu-1)t$.

$$\boxed{\Delta x = \frac{yd}{D} + (\mu-1)t}$$

the fringes shift downword by, $\frac{(\mu-1)tD}{d}$.

→ Effect of changing slit width :



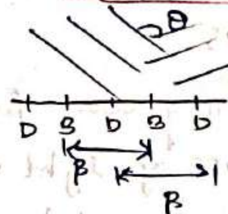
Contrast ratio:
$$r_c = \left[\frac{I_{max}}{I_{min}}\right]$$

$I_D : (\sqrt{I_1} - \sqrt{I_2})^2$

$I_B : (\sqrt{I_1} + \sqrt{I_2})^2$

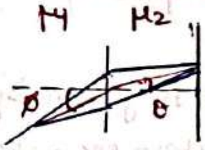
$$r_c = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$$

→ Interference of two convergent coherent ‖ beams :

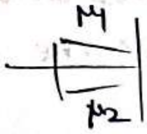


$$\beta = \frac{\lambda}{\theta}$$

Different μ :

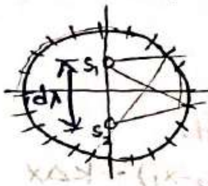


$$\Delta x = \mu_1 d\sin\phi + \mu_2(d\sin\theta)$$



$$\Delta x = \mu_1(S_1P) - \mu_2(S_2P)$$

Q>> No of fringes :



$d\sin\theta = n\lambda$

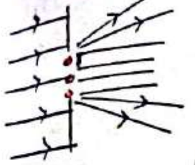
$\sin\theta = n\lambda/d$

$\therefore \frac{n\lambda}{d} \le 1 \Rightarrow \underline{n \le d/\lambda}$

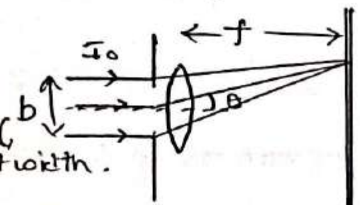
▣ **Diffraction:** Bending of light round the sharp corners and spreading into the regions of geometrical shadow.

- If slit width is large; diffraction effect is negligible. But if small, effect is significant.

1. Fresnel diffraction: (near field dif.)
   - source & slit are at finite distances.
   - no lens used.
2. Fraunhofer diffraction:
   - source & slit are at ∞ distances.
   - plane wave front
   - lens {source & screen are at focal planes}



Intensity in dif:



$$I(\theta) = \frac{I_0 \sin^2\beta}{\beta^2}, \quad \beta = \frac{\pi b \sin\theta}{\lambda}.$$

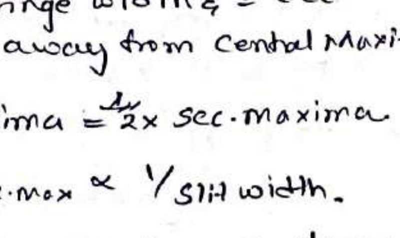
for central maxima,
$\theta = 0$ and $I = I_0$ $\quad \boxed{d\sin\theta = n\lambda}$ min.

Minima,
$\beta = n\pi$ $\rightarrow \sin\theta = \frac{n\lambda}{b}$

$\therefore n\pi = \frac{\pi b \sin\theta}{\lambda}$ $\rightarrow \boxed{\theta = \frac{n\lambda}{b}}$

Fringe pattern:

a) for $\theta = 0$, $I = I_0$
b) n^{th} minima,
$\sin\theta = \frac{n\lambda}{b}$ $n \neq 0$

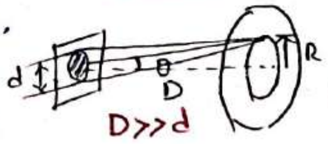


(c) unlike YDSE, here both fringe width & I dec. as you move away from central maxi.

- Central maxima = $2\times$ sec. maxima
- Width$_{sec.max} \propto$ 1/slit width.

Diffraction by circular aperture:

First minima on screen,
$\theta = 1.22\lambda/d$.

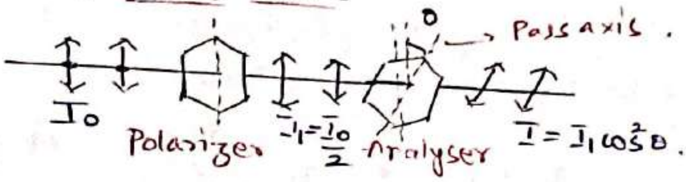


(a) R of first dark fringe or radius of central bright fringe,
$$\boxed{R = \theta D = \frac{1.22\lambda D}{d}}$$

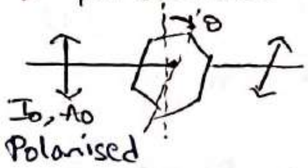
(b) If light is converged using convex lens at the screen placed at focal plane of lens, $R = \theta f = 1.22\lambda f$.

Polarisation of light.
↳ Electric field oscillating in one plane.

Malus law:



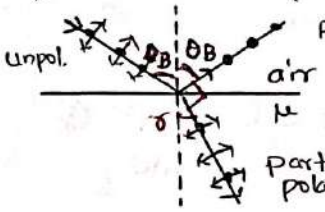
Generalised



$A = A_0\cos\theta$
$I = I_0\cos^2\theta$.

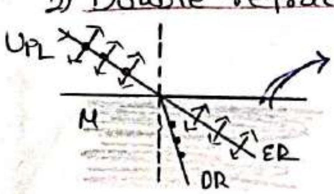
▣ **Methods of polarisation of light:**

1) Reflection (Brewster's law):



→ Brewster's angle (θ_B)
$\boxed{\tan\theta_B = \mu}$

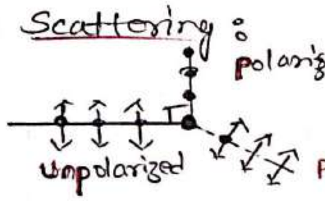
2) Double refraction:



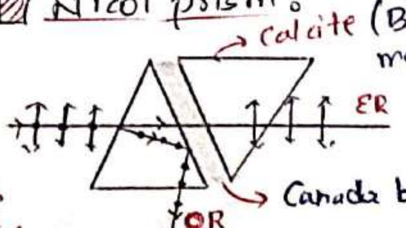
→ Birefringent material (Calcite, quartz)
OR: ordinary ray (polarized normal to plane)
ER: Extraordinary ray (polarised in plane)

3) Dichroism:
- in this, due to horizontal vibration, it gradually dec's the UPL to PL by restriction.

4) Scattering:



▣ **Nicol prism:** uses double refr.

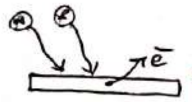


* MODERN PHYSICS. *

⇒ 1. Photoelectric Effect.

- When EM radiations of suitable λ are incident on a metalic surface, e^-s are emitted. (Photo electrons)
- i generated in above process (Photo electric current).
- Energy of photon $E = hf$
- Momentum of photon $p = h/\lambda$
- Photons :- rest mass = 0
  - Neutral particle
  - Work's on principle of "all or none"

- Work function (ϕ): min. E req. to eject weakly bonded e^- from surface.



$$\begin{cases} KE \le h\nu - \phi \\ KE_{max} = h\nu - \phi \end{cases}$$

$$\phi = hf_{th} = \frac{hc}{\lambda_{th}} \text{ [max wavelength]}$$

f_{th} ← min freq. for PE effect.

* Incident energy $\boxed{h\nu = \phi + \frac{1}{2}mv^2_{max}}$

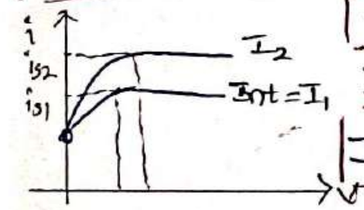
* for PEE: $\nu \geq \nu_{th}$ & $\lambda \leq \lambda_{th}$ ||

- Effect of intensity variation PEE:
  $I \propto$ photon flux (no. of ph/sec m²)
  As $I\uparrow$, $i\uparrow$
  as $KE_{max} = h\nu - \phi$
  As freq ↑ ⇒ KE of e^- ejected ↑

- Temp: it has no effect on PEE.
  - as PEE is a quantum phenomenon so it is independent from thermionic emission.

- Time lag: No time lag occurs. (b/w absorp. of photon & emission of e^-)

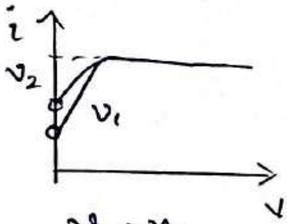
- Potential vs photocurrent:



$\boxed{I_2 > I_1}$

$I \propto i_{saturation}$

$$\boxed{I = \left(\frac{n}{At}\right) h\nu}$$



$\nu_2 > \nu_1$

- Stopping potential:
  - if we reverse the potential, it starts repelling e^-s, at one value no e^- would pass by; called "stopping potential".

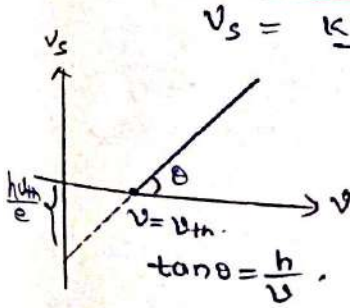
$$\boxed{eV_s = KE_{max}}$$

$$V_s = \frac{KE_{max}}{e} \Rightarrow V_s = \frac{h\nu}{e} - \frac{\phi}{e}$$

$$\Downarrow$$

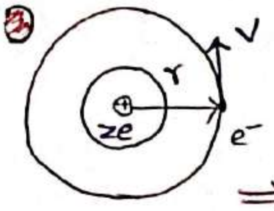
$$\boxed{eV_s = h(\nu - \nu_{th})}$$

Einstein's PEE eqn
" NOBLE Prize wala "



$\tan\theta = \frac{h}{\nu}$.

Atomic Physics.

③ 

$$\frac{Kq_1q_2}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{mv_n^2}{r_n} = \frac{KZe^2}{r_n^2} \quad -①$$

③ $L = mvr$ (ang momentum)

$$\Rightarrow mvr = \frac{nh}{2\pi} \quad -②$$

From ① & ②:

$$\Rightarrow r_n = \frac{h^2}{4\pi^2 kme^2}\left(\frac{n^2}{Z}\right) = 0.529\left(\frac{n^2}{Z}\right)$$

$$\Rightarrow v_n = \frac{2\pi ke^2}{h}\left(\frac{Z}{n}\right) = 2.18\times10^6\left(\frac{Z}{n}\right)$$

$$\Rightarrow \omega = \frac{v_n}{r_n} \propto Z^2 n^{-3} m^1 e^4$$

$$\Rightarrow T = \frac{2\pi}{\omega} \propto Z^{-2} n^3 m^{-1} e^{-4}$$

$$\Rightarrow KE = \frac{1}{2}mv_n^2 = \frac{2\pi^2 k^2 e^4 m}{h^2}\left(\frac{Z^2}{n^2}\right)$$

$$* \; KE = 13.6\frac{Z^2}{m^2}eV \qquad \boxed{KE \propto m} \; \boxed{KE \propto e^4}$$

$$\Rightarrow PE = \frac{Kq_1q_2}{r^2} = \frac{K(Ze)(e)}{r^2} = \frac{-4\pi^2k^2e^4m}{h^2}\left(\frac{Z^2}{n^2}\right) = -27.2\left(\frac{Z^2}{n^2}\right)$$

$$\Rightarrow TE = KE + PE = -13.6\left(Z^2/n^2\right) \qquad KE : PE : TE = 1 : -2 : -1$$

$$\Rightarrow L \text{ (ang momentum)} = mvr = \frac{nh}{2\pi}.$$

$$\Rightarrow i_{avg} = \frac{dq}{dt} = \frac{e}{T} \propto Z^2 n^{-3} m^{-1} e^5$$

$$\Rightarrow B = \frac{\mu_0 i}{2\pi r} \propto Z^3 n^{-5} m^2 e^7$$

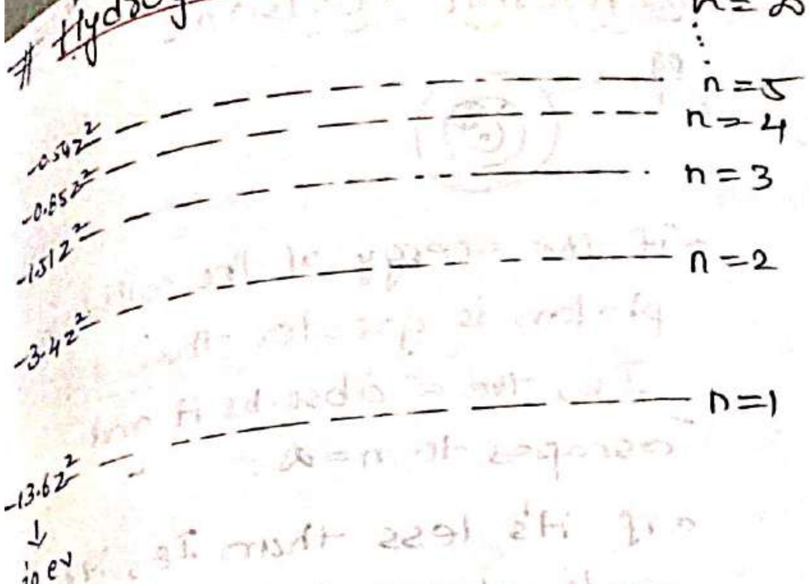
$$\Rightarrow M = i \times \pi r^2 \propto Z^0 n^1 m^{-1} e^1$$

* alpha particle vs proton:

m_p = mass of proton

$$\left\{\begin{array}{l} 4m_p = m_\alpha \\ 2q_p = q_\alpha \end{array}\right\}$$

Hydrogen : (for single e^-)



- $E_n = -13.6 \times \frac{z^2}{n^2}$ eV
- Excitation Energy : it's the E req. for e^- to jump from lower orbit to one step higher.

* $E_m - E_n = 13.6 z^2 \left(\frac{1}{n^2} - \frac{1}{m^2}\right)$ eV.

- Ionisation Energy :

$$13.6(z^2/n^2) \text{ eV}$$

=> to give Energy we need potential.

$$\boxed{I.P\,(V) = \frac{IE}{e}}$$

$$\frac{hc}{\lambda} = 13.6 z^2 \left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

⊙ λ of Emitted radiation :

$$\frac{hc}{\lambda} = E_{n_2} - E_{n_1}$$

$$\boxed{\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

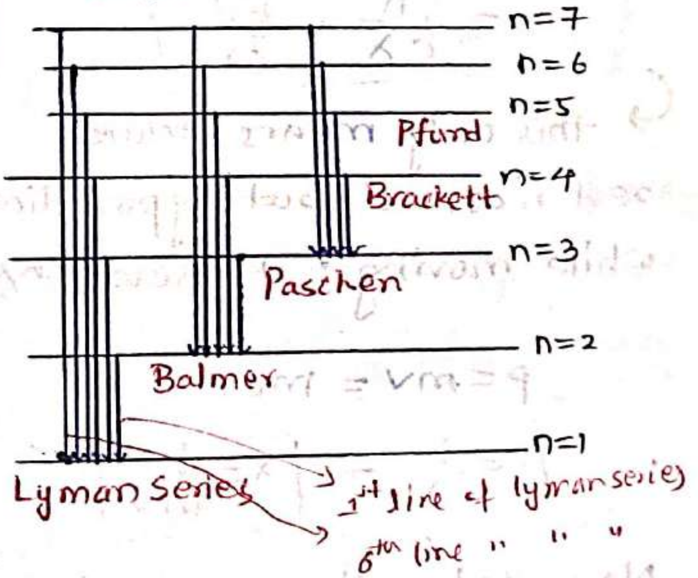
→ $R = 1.097 \times 10^7$

* $\lambda = \frac{12430}{\Delta E \text{ (in eV)}}$ Å $= \frac{1243}{\Delta E}$ nm

* $\Delta E = \frac{12430}{\lambda\,(Å)}$ eV

$\Delta E = h\nu$

⊙ Hydrogen Spectrum :



⊙ No. of spectral lines :

- possible no. of photon energies emitted due to de excitation of e^- from $n=n_2$ to $n=1$ state

$$= {}^nC_2 = \frac{n(n-1)}{2}$$

⇒ Wave

⇒ **Wave particle duality:**

Particle Nature	Wave nature.
Treated as photon	As EM waves
$E = pc$ (mom.)	$E = hc/\lambda$

$\therefore pc = \frac{hc}{\lambda}$

$$\boxed{p = h/\lambda}$$

Photon momentum

Debroglie's hypothesis:

Acc. to einstein $E = mc^2$

$\because E = \frac{hc}{\lambda} = h\nu \; \{ m = E/c^2$

$$\boxed{m = \frac{h}{c\lambda} = \frac{h\nu}{c^2}}$$

↳ this only means actual rest mass = 0 but hypothetically while moving; it shows $h/\lambda c$.

$p = mv = mc$

$p = \frac{h}{\lambda} \Rightarrow \boxed{\lambda = \frac{h}{mv}}$

Now, debroglie approached it oppositely.

$m \rightarrow v$ when particle is moving, it behaves as a wave of wavelength λ

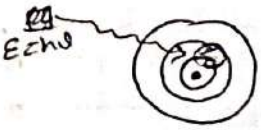
$\lambda = \frac{h}{p}$; $p = mv$; $KE = \frac{1}{2}mv^2$

1) $$\boxed{\lambda = \frac{h}{\sqrt{2mKE}}}$$

for a charge "q", potential "v"

2) $$\boxed{\lambda = \frac{h}{\sqrt{2mqv}}} \; \{KE = qv\}$$

Atomic Collision:



- if the energy of the colliding photon is greater than the IE, the e^- absorbs it and escapes to $n = \infty$.
- if it's less than IE, it won't absorb, instead stay there itself.
- if $E = 10.2$, e^- absorbs & excites to 1st ex. state.

{ An e^- to absorb a photon, the energy of the photon has to be either greater than IE or equal to any exact Excitation energies. }

KEi: $(H) \rightarrow$ (m) rest: (H) (m)

$mv = 2mv_f$

$KE_f = \frac{KE_i}{2}$

Perfectly Elastic → $\Delta KE = 0$.

" Inelastic → $\Delta KE_{max} = KE_i/2$.

* In classical mech → any value b/w them
  ↳ loss in E $[0, \Delta KE_{max}]$
* In quantum mech
  ↳ loss in E $\{0, 10.2, 12.09, ...\}$ only sp. values.

for an electron

$$\boxed{\lambda = \sqrt{\frac{150}{V(\text{volt})}}}$$

for a gas molecule in random position,

$KE = 3/2\, kT$ (translational KE)

$$\boxed{\lambda = \frac{h}{\sqrt{3mkT}}}$$

Boltzmann's const.

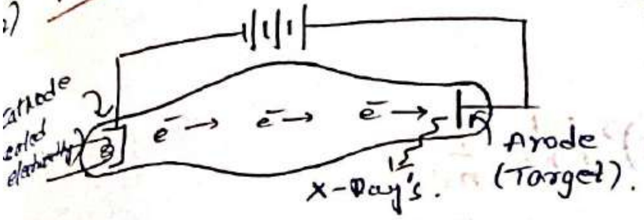
X-Rays.

1) X-Ray's :

Soft X-Ray's - 1A° - 100A° - Low Energy.

Hard X-Ray's - 0.1A° - 1A° - High Energy.

2) Production of X-Ray's :

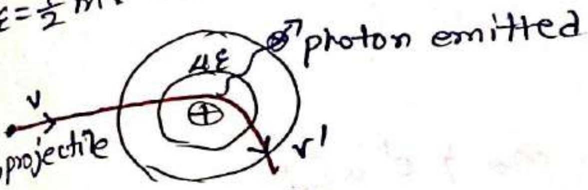


X-Ray's are produced by incidence of accelerated $\bar{e}$ (cathode rays) on target material.

I Continuous X-Rays :

→ Mechanism

$KE = \frac{1}{2}mv^2 = eV$



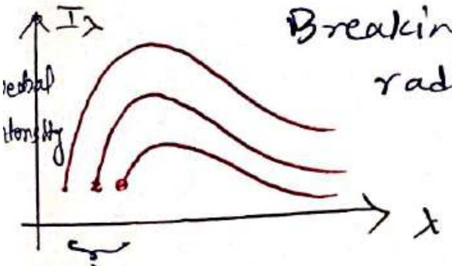
photon emitted :

$\Delta E = h\nu = hc/\lambda = K_i - K_f$

$\Delta KE_{max} = \frac{hc}{\lambda_{min}} = eV \Rightarrow \lambda_{min} = \frac{12431}{V} A°$.

• photon ejected due to sudden change in acc. of Nu.

Spectrum :



Breaking radiation

Cutoff λ / min λ :

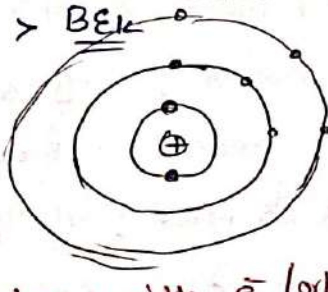
$\lambda_c = \frac{12431}{V} A°$

$\boxed{\lambda_{min} = \frac{12431}{V} A°}$

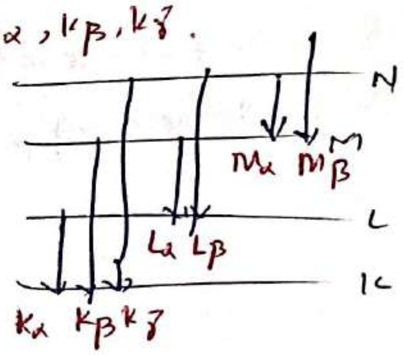
II Characteristic X-Rays :

→ Mechanism → BE associated with K.

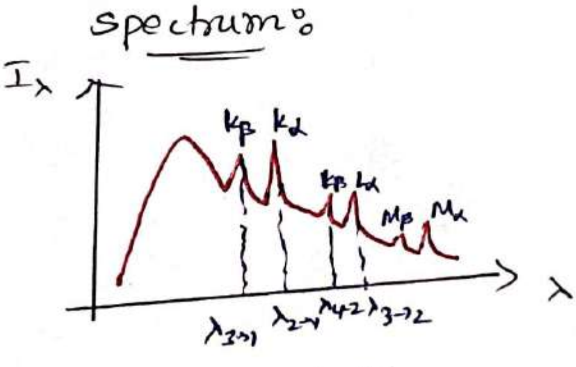
$K = \frac{1}{2}mv^2 = eV > BE_K$



• The $\bar{e}$ collides with $\bar{e}$ (orbiting), both eject creating vacancy, then $\bar{e}$ from L, M, N shell occupy producing $K_\alpha, K_\beta, K_\gamma$.



Spectrum :



$eV > BE$

Mosley's law :

If $V_e > BE$ then self x-ray is generated.

λ of charact - x-ray is given by $\frac{1}{\lambda} = R(Z-\sigma)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Modified Rydberg's formulae

Screening const.

$Z_{eff} = Z - \sigma$

Eg: for K_α X-Ray ; $\sigma = 1$.

* freq. of rad $\vartheta = c/\lambda = Rc(Z-\sigma)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\sqrt{\vartheta} = b(Z-\sigma)$; $b = \sqrt{Rc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$.

1) Electron : 9.1×10^{-31} kg

2) Proton : 1836 M_E

3) Neutron : 1839 M_E

4) Positron : Antiparticle of e^- ; mass = m_e ; e^+ or $\left({}_{+1}\beta^0\right)$.

5) Antiproton : Antiparticle of proton ; charge = $-e$; $m = m_p$; $\bar{p}$

6) Antineutron : No charge ; $m = m_n$; if N & A.N spin in same dirn their mag.m will be in opp dirn

7) Neutrino : Rest mass & Charge = 0 but have E & P.

8) Alpha particle : Charge = $2q_p$ & $m = 4m_p$.

EM WAVES.

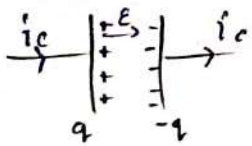
- Transverse Waves $(P = h/\lambda)$
- Carry Energy and Momentum
- Electric and Mag. field Energy

$u_B = B^2/2\mu_0$, $u_E = \frac{1}{2}\varepsilon_0 E^2$.

1) Radiowaves few mm - m
2) Microwave 10^{-1}m - 10^{-4}m
3) Infrared 10^{-4}m - 7.8×10^{-7}m
4) Visible light 7.8×10^{-7}m - 4.2×10^{-9}m
5) Ultra violet 4.2×10^{-9}m - 10^{-9}m
6) X-rays 10^{-9}m - 10^{-12}m
7) γ-rays $< 10^{-12}$m

Displacement Current: (i_d) due to t varying E. field.

$\boxed{i_d = \varepsilon_0 \frac{d\phi_E}{dt}}$, $\phi_E \to$ Electric flux.



$\longrightarrow$ $\phi = \frac{q}{\varepsilon_0}$

$q = \phi \cdot \varepsilon_0$

$\frac{dq}{dt} = \frac{d\phi}{dt}\cdot\varepsilon_0$

(a) If q changes, E changes:

$i_d = \varepsilon_0 \frac{d(EA)}{dt}$, $E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$.

$i_d = \varepsilon_0 A \times \frac{1}{A\varepsilon_0}\frac{dq}{dt} = dq/dt$.

∴ In cap. for time varying current conduction current is same as i_d.

(b) b/w plates $i_d \neq 0$, $i_c = 0$
outside plates $i_c \neq 0$, $i_d = 0$

(c) i_d is uniform across plate cross-section.

Ampere's law: $\nearrow i_c + \varepsilon_0 \frac{d\phi_E}{dt}$.

$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enclosed}$

Maxwell's Equations:

1) Gauss law in Electrostatics

$\phi_E = \oint \vec{E}\cdot d\vec{S} = q_{en}/\varepsilon_0$

2) Gauss law in Magnetism.

$\phi_m = \oint \vec{B}\cdot d\vec{S} = 0$.

{because of closed loop lines}

3) Faraday's law of emi,

$\int \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt}$.

4) Maxwells Ampere law:

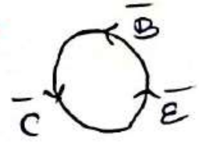
$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_c + \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$.

Nature of EM Waves:

$E_{rms} = \frac{E_0}{\sqrt{2}}$.

$\vec{E} = E_0 \sin(\omega t - kx)\hat{j}$ $\to$ dirn of prop of wave (EM)

$\vec{B} = B_0 \sin(\omega t - kx)\hat{k}$



- $E_0 = cB_0$

- $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ in free space.

$c_{med.} = \frac{1}{\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$.

- $I = \frac{1}{2}\varepsilon_0 E_0^2 c = \varepsilon_0 E_{rms}^2 c$.

{in above I due to Electric field = I due to Mag. field = I/2}

FLUIDS.

→ Liquids and Gases together called Fluids i.e., that can flow.

→ can't withstand any tangential sheer.

- **Density**: $\rho = \frac{dm}{dv}$ rel density $= \frac{\rho_{sub}}{\rho_{H_2O}(4°C)}$.

* S.I unit of P: (N/m^2 pascal)

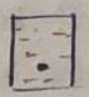
$dF_{\perp} = PdA$

$F = PA$

$[ML^{-1}T^{-2}]$

$1atm = 1.013 \times 10^5$ pa.

- **P. variation with depth:**



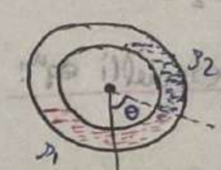
$\Delta P = h\rho g$ ↳ $P = P_0 + \rho g h$

→ Abs P. → Gauge P → p. at (h↑)

- **Hydrostatics:** Ideal fluids are incompressible.

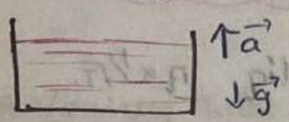
→ Non viscous

→ A liq. is always known by its density.

* 

$\tan\theta = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$.

→ **Vertically acc:**



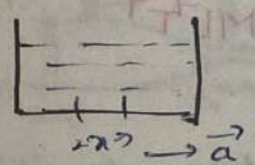
$\frac{dP}{dh} = \rho|\vec{a_y}|$.

- **Force on a wall:**

$F = \frac{\rho g b h^2}{2}$

$\tau = \frac{\rho g b h^3}{6}$

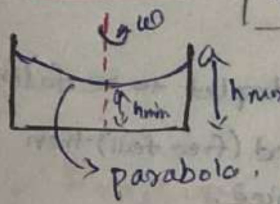
→ **Horizontal acc:**



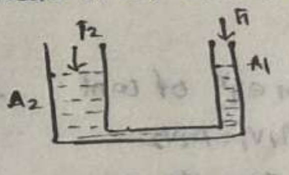
$\frac{dP}{dx} = \rho a_x$

Pascal's law: Pressure exerted to any enclosed fluid is transmited equally and undiminished to all parts of the fluid in all directions.

→ **Rotation:** $\boxed{h_{max} - h_{min} = \frac{\omega^2 x^2}{2g}}$



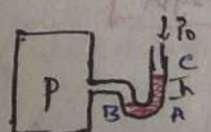
$\boxed{h_i = \frac{h_{max} + h_{min}}{2}}$



$\boxed{\frac{F_2}{A_2} = \frac{F_1}{A_1}}$

- **P. measuring instruments:**

1) Manometer –



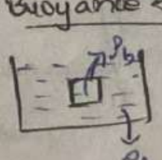
$\boxed{P - P_0 = h\rho g}$.

↳ P. of gas = P at B = P at A = P at C + $h\rho g$ = $P_0 + h\rho g$.

Archimedes' Principle: when a body is partially or fully dipped into a fluid at rest, the fluid exerts an upward force of buoyancy equal to the weight of the displaced liq.

→ [loss in wt = wt of fluid displaced]

* Buoyance force –



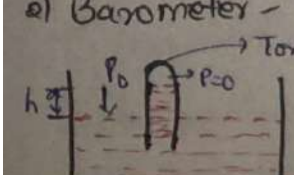
$\rho_b > \rho_l$ {body will sink}

$\rho_L > \rho_b$ {Float partially}

$\rho_L = \rho_b$ {eq. in H_2O}.

$\boxed{F_b = W_{LD} = \rho_L V_{BI} g}$

2) Barometer –



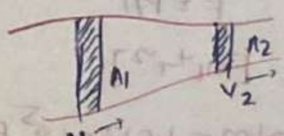
$\boxed{\rho_H g h = P_0}$

● Critical velocity: The largest velocity that allows a steady flow.

↳ steady / Turbulent flow depends on ρ, v & η & diam. of tube.

Raynold's no: $N = \frac{\rho v D}{\eta}$ $N < 2000$ } steady 2k-3k } unstable $N > 3k$] Turbulent

● Eq^n. of continuity:



$A_1 v_1 = A_2 v_2$.

■ Bernoulli Equation:

$P + \rho g h + \frac{1}{2}\rho v^2 = \text{const}$.

● Viscosity:

$F = -\eta A \frac{dv}{dz}$

- depends on nature of fluid.
- $\eta_{liq} > \eta_{gas}$.
- for liq $\rho \propto \eta$.
- for gas $\rho \propto 1/\eta$
- Immisible impurities ($\eta \downarrow$)
- missible impurites ($\eta \uparrow$)

S.I unit: Poiseuille.

CGS unit: Poise

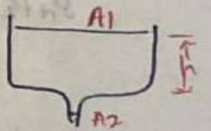
1 poise = 0.1 poiseulle.

liq $\eta \propto 1/\sqrt{T}$.

gas $\eta \propto \sqrt{T}$.

$ML^{-1}T^{-1}$

* Appl. of Bernoulli eq^n:



$P_a + \frac{1}{2}\rho v_1^2 + \rho g h = P_0 + \frac{1}{2}\rho v_2^2$.

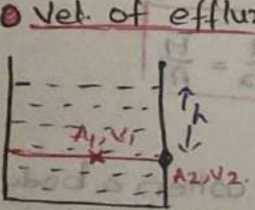
$A_1 v_1 = A_2 v_2$.

$\frac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 + \rho g h = \frac{1}{2}\rho v_2^2$.

↳ $\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] v_2^2 = 2gh$.

$A_2 << A_1 \longrightarrow v_2 = \sqrt{2gh}$

● Vel. of efflux:



from eq^n of cont.

$A_1 v_1 = A_2 v_2$

$\frac{A_1}{A_2} = \frac{v_2}{v_1}$ —①

By Bernoulli:

$\cancel{P_0} + \rho g h + \frac{1}{2}\rho v_1^2 = \cancel{P_0} + \rho g(0) + \frac{1}{2}\rho v_2^2$

$\boxed{gh + \frac{v_1^2}{2} = \frac{v_2^2}{2}}$

→ by ①

but for $A_2 << A_1 \Rightarrow v_1 << v_2$

$gh = \frac{v_2^2}{2} \Rightarrow \boxed{v_2 = \sqrt{2gh}}$

■ Toricelli's theorem:

→ take/assume the water droplet to be falling from upper layer to ground (free fall) then calculate the speed at orifice.

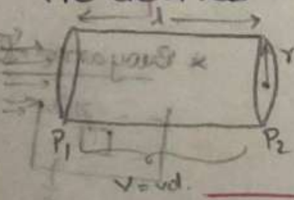
{OR}

$2gh + v_1^2 = v_2^2$

↳ $\boxed{v_2 = \sqrt{v_1^2 + 2gh}}$

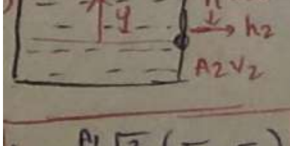
● Poiseuille's Equation:

He derived a formula for the rate of flow of viscous fluid through a cylindrical tube.



$\boxed{\frac{V}{t} = \frac{\pi(\Delta P) r^4}{8\eta l}}$

● Time taken to Empty:



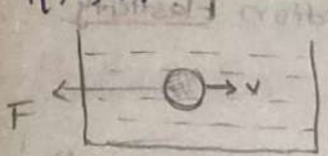
$A_1 v_1 = A_2 v_2$

$A_1\left(-\frac{dy}{dt}\right) = A_2\sqrt{2gy}$

$-\left(\frac{A_1}{A_2\sqrt{2g}}\right)\int_{h_1}^{h_2}\frac{1}{\sqrt{y}}\,dy = \int_0^t dt$

$t = \frac{A_1}{A_2}\sqrt{\frac{2}{g}}\left(\sqrt{h_2} - \sqrt{h_1}\right)$

Stoke's law: gives rel. b/w viscous force, η, r, v.



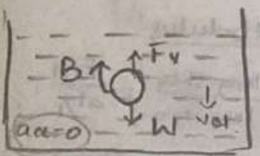
$$\boxed{F = 6\pi\eta r v}$$

Critical Velocity: The max. velocity at steady flow after which goes turbulent.

$$\boxed{v \propto \frac{\eta}{\rho D}} \quad N = \frac{\rho v D}{\eta}$$

Reynolds no.

Terminal Velocity: When a mass is dropped into a fluid, W acts downward and B upwards initially it acc. ↓ but then due to F_{visus} it gets fallen into zero acc. The velocity from then is Terminal vel.

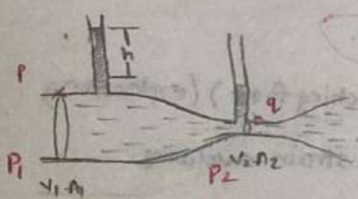


$$\boxed{v_0 = \frac{2r^2(\rho - \sigma)g}{9\eta}}$$

ρ = density of body

σ = density of liquid.

Ventury Tube:



first apply eqⁿ of cont. at P, q
then Bernoulli
then equate $P_1 - P_2 = \rho g h$

This allows us to know the rate of flow of liquid past a cross section.

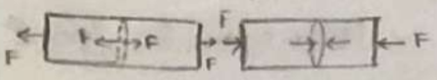
- ✓ Pressure
- ✓ Pascals Law
- ✓ Buoyancy.
- ✓ Surface E & Tension
- ✓ Capillary rise
- ✓ Viscosity
- ✓ Stoke's law
- ✓ Terminal velocity
- ✓ Streamline flow
- ✓ Eqⁿ of continuity
- ✓ Bernoulli's theorem.

Mechanical Properties of Matter.

- The property to restore original shape/oppose deformation Elasticity.

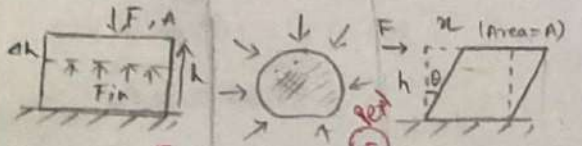
- External force – Deformation force
- Internal force – Reformation force.

Stress: $\frac{\text{Rest } F}{\text{Area}}$



Tensile stress   Comprex. stress.

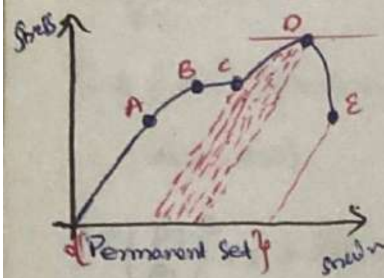
Strain: rel. deformation.



N. stress = $\frac{F}{A}$ Vol stress = $\frac{F}{V}$ Shear Stress = F/A

" strain = $\Delta h/h$ " strain = $\frac{\Delta V}{V}$ " Strain = $\tan\theta = x/h$.

Stress-Strain Curve:



A → Hooke's limit

B → Elastic limit / Yield pt

D → Ultimate stress.

E → Breaking pt. / fracture pt.

Ductile: large deformation takes place b/w the elastic limit & the fracture pt.

Brittle: Breaks soon after elastic limit.

Bulk Modulus of Gases:

$$B = \frac{-dP}{dV} \cdot V$$

isochoric $V = \text{const}$
$dV = 0$
$B \to$ not defined

Isothermal $PV = \text{const}$.
$PdV + VdP = 0$
$\frac{dP}{dV} = -P/V$
$B = -\left(\frac{-P}{V}\right)V$
$B = P$

isobaric $P = \text{const}$
$dP = 0$
$B = 0$

Polytropic $PV^a = \text{const}$.
$B = aP$

adiabatic $PV^\gamma = \text{const}$ $\boxed{B = \gamma P}$

Hooke's law: For small deformations stress ∝ strain.

$\boxed{E = \frac{\text{stress}}{\text{Strain}}}$

Modulus of Elasticity.

$\{E_{solid} > E_{liq} > E_{gas}\}$

Types of Elasticity Modulus:

1) Young's Modulus: $Y = \frac{\text{long. Stress}}{\text{long. Strain}} = \frac{F/A}{\Delta l/l}$.

2) Bulk Modulus: $B = \frac{\text{Vol. Stress}}{\text{Vol. Strain}} = \frac{P_{ext}}{\Delta V/V}$.

incompressability $C = 1/B$.

3) Shear Modulus: $\eta = \frac{\text{Shear Stress}}{\text{Shear strain}} = \frac{F/A}{x/h}$.

Potential Energy:

$$U = \frac{1}{2}\left(AY\frac{l}{L}\right)l.$$

$= \frac{1}{2}$(max. stretching force) (extension)

$= \frac{1}{2} \times$ stress × strain × volume.

$$\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}.$$

Elastic Potential Energy:

→ $U = \frac{1}{2}kx^2$

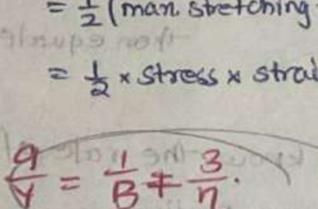
$= \frac{1}{2}\frac{YA}{l}x^2$

$= \frac{1}{2}YAl\left(x/l\right)^2$ $\frac{\text{Stress}}{\text{Strain}} = Y$

$= \frac{1}{2}Y$(vol.)(strain)

$= \frac{1}{2}y$ (vol)(strain)(stress)

Poisson's Ratio:



$\sigma = -\frac{dR/R}{dl/l}$.

#

⟹ Thermodynamics.

- **Temp**: Degree of hotness/coldness.
- **Heat**: The amt. of energy transferred without any Mech. work.

→ Thermometer –

$$\frac{\text{Temp.} - \text{M.pt}}{\text{B.pt} - \text{M.pt}} = \text{const}$$

Q: + heat supplied to the sys., – heat drawn from sys.

W: + W done by gas, – W done on gas.

ΔU: + with inc. temp (↑), – with dec. temp (↓)

→ 1st law: Conservation of Energy

for a gaseous system.

$$Q_{in} \longrightarrow \boxed{\Delta U} \longrightarrow W, \quad \downarrow Q_{out}$$

$$\boxed{dQ = du + dW}$$

↳ $Q = \Delta U + W$

- Work done: $\boxed{W = \int P dv = P\Delta V}$
- Heat: $\boxed{Q = nC\Delta T}$
- ΔU: $\Delta U = nC_v\Delta T$ $\quad \boxed{U = \frac{f}{2}nRT}$ $\quad \boxed{C = \frac{Q}{\Delta T}}$

$$PV = nRT \Rightarrow Pdv + vdP = nRdT$$

↳ $ndT = \frac{Pdv + vdP}{R}$

$$\frac{ndT}{Pdv} = \frac{Pdv + vdP}{(Pdv)R}$$

$$\Rightarrow \frac{vdP}{Pdv} = k \Rightarrow Vdp = kPdv.$$

$$\Rightarrow \ln p = k \ln v.$$

$$P = v^k \text{ (or) } \boxed{PV^x = \text{const}}$$

$$\boxed{W = \frac{nR\Delta T}{1-x}} \qquad \boxed{C_v = \frac{fR}{2}}$$

$$\boxed{C = C_v + \frac{R}{1-x}}$$

$$\boxed{C_p - C_v = nR}$$

$$\frac{C_p}{C_v} = \gamma \Rightarrow C_v = \frac{R}{\gamma - 1} \quad C_p = \frac{\gamma R}{\gamma - 1}$$

$$U = nC_vT = \frac{nRT}{\gamma - 1} = \frac{PV}{\gamma - 1}$$

① Polytropic

C is constant.

⟹ $\boxed{Q = nC\Delta T}$

→ Isochoric (C_v) → $\boxed{x = \infty}$

→ Isobaric (C_p) → $\boxed{x = 0}$

→ Isothermal (C_v) → $\boxed{x = 1}$

→ Adiabatic (C_v)

② Non Polytropic

c is variable

⟹ $dQ = ncdT$

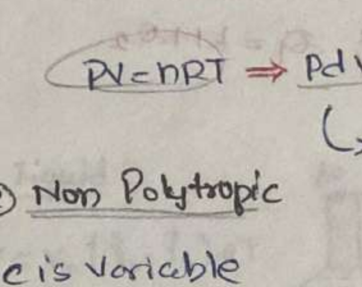
$$\boxed{Q = \int ncdT}$$

⟹ Isochoric: vol const.

$W = 0$

$\Delta U = nC_v\Delta T = \frac{f}{2}nR\Delta T$

$C_v = \frac{f}{2}R$



⟹ Isobaric: P const.

$Q = nC_p\Delta T$

$\Delta U = nC_v\Delta T = \frac{f}{2}nR\Delta T$

$W = P\Delta V = nR\Delta T$.

⟹ Isothermal: T const.

$P_1V_1 = P_2V_2$ $\qquad \Delta U = 0$, $Q = W$

$$W = nRT \ln\frac{V_2}{V_1} = nRT \ln\frac{P_1}{P_2}$$

molar heat cap: $C = \frac{\Delta Q}{n\Delta T} \Rightarrow C = \infty$.

⟹ Adiabatic:

$Q = 0$

$W = -\Delta U = -nC_v\Delta T$.

PV^γ, $TV^{\gamma-1}$, $P^{1-\gamma}T^\gamma$

↳ constants.

$$W = \frac{nR\Delta T}{1-\gamma}$$

	f	C_v	C_p	γ
Mono	3	$\frac{3}{2}R$	$\frac{5}{2}R$	5/3
di	5	$\frac{5}{2}R$	$\frac{7}{2}R$	7/5
Poly	6	3R	4R	4/3

$\gamma_{mono} > \gamma_{di} > \gamma_{poly}$ {in adiabatic}

2nd law of Thermod:

All work can be converted to heat but not all heat " to work.

- **Calusius:** It's impossible to transfer heat from a body at low temp. to a body at high temp. without help of any ext. agents

- **Heat Engine:**

$$\underset{T_1}{\text{Source}} \xrightarrow{Q_1} \boxed{\text{Engine}} \xrightarrow{Q_2} \underset{T_2}{\text{Sink}}, \quad \text{Engine} \Downarrow W$$

$T_1 > T_2$ $\Longrightarrow \frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ &

$W = Q_1 - Q_2$ $\quad$ & $Q = W + Q_2$.

$$\text{Efficiency}(\eta) = \frac{W}{\text{Heat}} \times 100.$$

$$\eta = \frac{W}{Q_1} \times 100 = \frac{Q_1 - Q_2}{Q_1} \times 100.$$

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100.$$

$$\eta = 1 - T_2/T_1$$

→ When 2 diff gases taken;

$$f_{mix} = \frac{n_1 f_1 + n_2 f_2 + \cdots}{n_1 + n_2 + \cdots}$$

$$\gamma_{mix} = 1 + \frac{2}{f_{mix}} = \frac{C_{p_{mix}}}{C_{v_{mix}}}$$

$$C_{v_{mix}} = \frac{n_1 C_{v_1} + n_2 C_{v_2} + \cdots}{n_1 + n_2 + \cdots}$$

$$C_{p_{mix}} = \frac{n_1 C_{p_1} + n_2 C_{p_2} + \cdots}{n_1 + n_2 + \cdots}$$

$$* \quad T_{mix} = \frac{n_1 C_{v_1} T_1 + n_2 C_{v_2} T_2}{n_1 C_{v_1} + n_2 C_{v_2}}.$$

- **Coeff. of performance (COP)**

$$COP_{cooling} = \frac{|Q_{cold}|}{W}$$

$$COP_{heating} = \frac{Q_{hot}}{Q_{hot} - Q_{cold}}$$

$$COP_{cooling} = \frac{Q_{cold}}{Q_{hot} - Q_{cold}} = \frac{t_{cold}}{t_{hot} - t_{cold}}$$

Thermal Expansion.

→ Linear: $\Delta l = l\alpha\Delta T$

→ Aerial: $\Delta A = A\beta\Delta T$.

→ Volume: $\Delta V = V\gamma\Delta T$.

$\alpha : \beta : \gamma = 1:2:3$

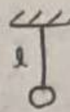
● Variation of Density:

$\rho = m/V$

$\rho = \rho_0 (1+\gamma\Delta T)^{-1}$

$\rho = \rho_0 (1-\gamma(T-T_0))$

● Pendulum:

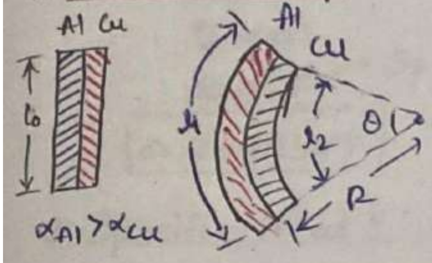


$T = 2\pi\sqrt{l/g}$.

$T_{New} = 2\pi\sqrt{\dfrac{l(1+\alpha\Delta T)}{g}}$

$\dfrac{\Delta T}{t} = \frac{1}{2}\alpha\Delta T$.

Bimetallic Strip:



$\alpha_{Al} > \alpha_{Cu}$

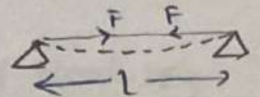
$l_1 = l_0(1+\alpha_{Al}\Delta T)$

$l_2 = l_0(1+\alpha_{Cu}\Delta T)$

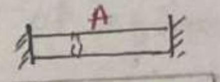
$\boxed{\theta = \dfrac{l_1}{R+t_1} = \dfrac{l_2}{R-t_2}}$

T_g (graduation temp): Clock giving correct time.

Tension b/w (in) a clamped wire:



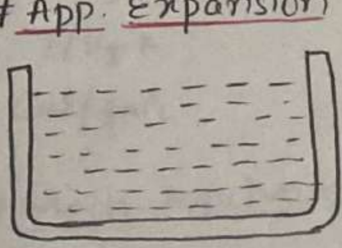
$\boxed{F = YA\alpha\Delta T}$



$\boxed{N = YA\alpha\Delta T}$

$Y = \dfrac{mgl}{A\Delta l}$

App. Expansion of a Liquid:

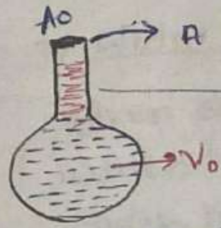


$\Delta V = V_0(\gamma_L - \gamma_S)\Delta T$

↓

liq overflown

$\gamma_{liq} > \gamma_{solid}$



→ $A(1+\beta\Delta T)$

→ $\Delta V = V_0(\gamma_L - \gamma_{gl})\Delta T$

∴ Height of liq raised $\boxed{h = \dfrac{\Delta V}{A_0}}$

$\dfrac{dQ}{dt} = \dfrac{KA}{L}(T_1 - T_2)$.

K: thermal conductivity

A: Cross-section area

L: length

T_1: Higher temp.

T_2: Lower temp.

Calorimetry.

→ Process of Measuring of heat enchanges b/w substances.

Heat → Subs. → Rise in temp. (ΔT)

ΔQ → Change in phase — Melting. / Vapourisation. / Sublimation.

If temp. inc. by ΔT:

$Q \propto \Delta T$

$Q \propto m$ — Sp. heat of substance.

$\boxed{\Delta Q = m s \Delta T}$

Specific Heat: Amt. of Heat req. to raise the temp. of unit mass of subs. by $1°C$.

$\Delta Q = m s \Delta T$

Units:
- J/kg°C
- J/kg-K
- Cal/gm°C

s is low for metals.

s is high for non metals.

Mech. equivalent of Heat:

1 cal of heat ≡ 4.187 J of work.

$\boxed{J = 4.187 \text{ joule/cal}}$

Phase Transformation: {Solid ↔ liquid}

Solid → Thermal agitation of particles → energy of random motion of particles

↑ Heat

at some temp T_m.

- Force due to collisions ≃ Cohesive force b/w particles of solid.

afterwards, the supplied heat will start breaking bonds with mol. & subs. start melting (T_m is const. till its melt comp)

T_m → m.pt

Heat Capacity: Amt. of Heat req. to raise the temp of a given body by $1°C$.

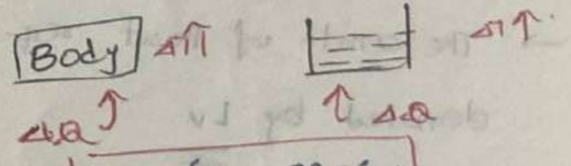
$C = ms$.

Units:

J/°C (or) J/K (or) Cal/°C

Thermal cap. of a body = mass × sp. heat.

Water eq. of a subs: For a given subs. the amt. of H_2O which has same heat cap which is that of the subs. then this amt. of H_2O is termed as →



$\boxed{m_B s_B = m_w s_w}$

→ $m_w = \dfrac{m_B s_B}{s_w}$.

Latent heat of Fusion!

The amt. of heat req. to melt a subs. of unit mass.

→ It is denoted by L_f

Thus, heat req. to melt a solid of mass m can be given as $\boxed{\Delta Q = mL_f}$

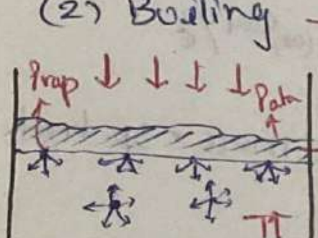
Units! Joule/kg. Cal/gm.

Phase Transformation: (liq → vap).

Vapourisation of liq takes place in 2 ways:

(1) Evaporation – at all temp's.
(2) Boiling – only at B.pt. temp.

} two takes place simultaneously.



→ Thin layer of vapor.

P_{vap} → Vap. p. of liq. surface.

$P_{vap} < P_{atm}$

As, temp inc. rate of evaporation also inc's at temp $T = T_0$ (Boiling Pt) $P_{vap} = P_{atm}$ → at Boiling.

Latent Heat of Vapourisation!

→ The amt. of heat req. to vap. a liq of unit mass; denoted by L_v

→ The heat req. to vapourise a liq of mass m is $\boxed{\Delta Q = mL_v}$

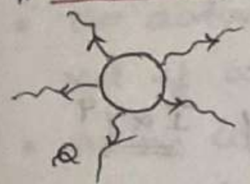
Units! Joule/kg Cal/gm

Radiation: form of Heat transfer.

- No requirement of medium (vaccum).
- EM waves.
- Amt. of rad depends on Temp, Material.

→ Each body emits radiation at all temp.
→ " " absorbs " " " "

Emissive Power (ε):

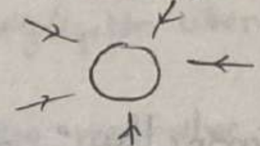


$Q \propto A$ (surface area)
$Q \propto t$

$\boxed{Q = EAT}$
depends on material

- ε depends on Material, λ.
- $E = \frac{Q}{At} = \frac{J}{Sm^2} = \frac{W}{m^2}$
- $\boxed{\varepsilon \propto T^4}$

Absorptive Power (a):



$a = \frac{\text{heat abs. by body}}{\text{that of incident}}$

- a depends on Material, λ.

$0 \le a \le 1$
ideal perfect reflector — ideal perfect absorber.

Black Body: Which absorbs all radiations falling on it at any temp. at any incident angle.
→ zero reflect
→ " transmitance.
- can be black but not always black.

[OR]

Which emit maximum radiation at a given temp. as compared to other bodies.

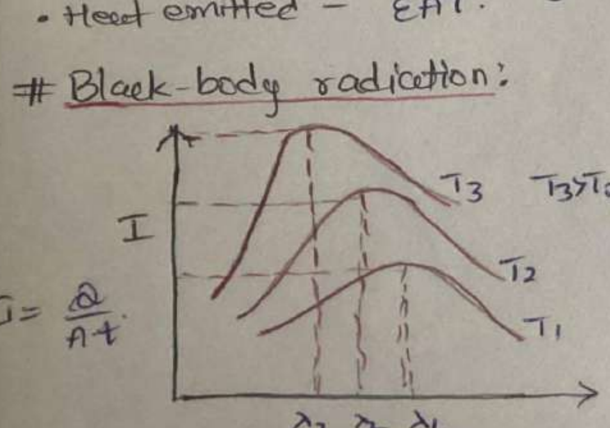
Kirchoff's law: Good absorbers are Good emittors.

$\boxed{E \propto a}$ so, also $E_\lambda \propto a_\lambda$

- Heat absorbed – $a(Q)$
- Heat emitted – EAT.

$\frac{\varepsilon_1}{a_1} = \text{const} = \varepsilon_{\text{black body}}$.

Black-body radiation:



λ_{max} → wavelength at max. intensity.

- As T ↑, λ_{max} displaces towards shorter value.

$\boxed{\lambda_1 T_1 = \lambda_2 T_2 = \lambda_3 T_3}$ wein's displ. law.

- $\lambda_{max} \propto 1/T$

↳ $\lambda_{max} T = b$ {wein's const}
↳ 2.898×10^{-3} m k.

for black body only!!

② Area under the graph → Total intensity at that temp. $\propto T^4$

Higher temp → more radiations.

③ $\frac{Q}{A \times T} \propto T^4 \propto I$.

Stephan's - Boltzmann's Law: for a perfectly black body $I \propto T^4$.

$$\frac{Q}{A \times t} \propto T^4 \Rightarrow \boxed{\frac{Q}{At} = \sigma T^4} \quad \{\text{stephan's law}\} \quad \| \quad \sigma = 5.67 \times 10^{-8}\, W/m^2\text{-}K^4$$

$$\frac{Q}{t} = \sigma A T^4$$

$$\boxed{\text{Power} = \sigma A T^4}$$

(radiated)

• But for any other body,

$$P = e\sigma A T^4$$

$e \rightarrow$ emissivity $(0 \le e \le 1)$

For a Black Body $\left.\begin{matrix} e=1 \\ a=1 \end{matrix}\right\} \boxed{e=a}$

For a body when given outer temp!

T_0 (T, Black Body) T_0 T_0 T_0

$$\left.\begin{matrix} P_{radiated} = e\sigma A T^4 \\ P_{absorbed} = e\sigma A T_0^4 \end{matrix}\right\} \text{Net P. radiated} = e\sigma A (T^4 - T_0^4)$$

Kinetic Theory of Gases.

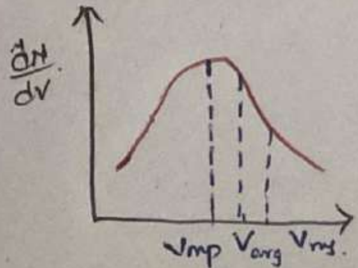
→ Gas occupies the vol. of the container.

→ A gas behaves as an ideal gas at high temp. & low p.

Assumption of KTG:

- All the molecules are identical and indistinguishable
- The actual vol. of mol's are negligible when compared to vol. of container.
- All the collisions are elastic and the molecules obey newton's laws.

Maxwell's Speed distribution:



$$V_{mp} = \sqrt{\frac{2RT}{M_0}}$$

$$V_{avg} = \sqrt{\frac{8RT}{\pi M_0}}$$

$$V_{rms} = \sqrt{\frac{3RT}{M_0}}$$

- $TKE = \frac{3}{2} PV = \frac{3}{2} nRT.$
- $KE = \frac{2}{3} K.T = \frac{1}{2} m\bar{v}^2.$
- $KE_{avg} = \frac{3}{2} K.T.$

Mean free path:

$\lambda \propto 1/P$ $\quad \lambda \propto 1/(N/V)$

→ Law of equipartition of Energy –

TE of a gas molecule is equally distributed among all of its DOF and energy associated with each DOF is $\frac{1}{2}KT$.

- $U = \frac{f}{2} nRT.$

⇒ Mean free path:

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P} \Rightarrow \lambda \propto T/P$$

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n} \Rightarrow \lambda \propto 1/n.$$

EARTH's MAGNETISM.

Naming:

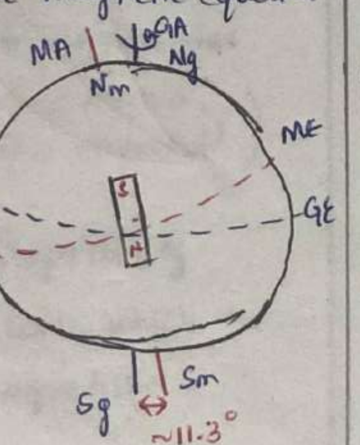
GA - Geographical Axis.

MA - Magnetic axis.

N_g, S_g - Geographical North, South.

N_m, S_m - Geomagnetic North, South.

GE, ME - Geographic Magnetic Equator.



Geographic and Mag. meridian:

(a) Geographical Meridian -

A vertical plane at any place on earth which passes through GA.

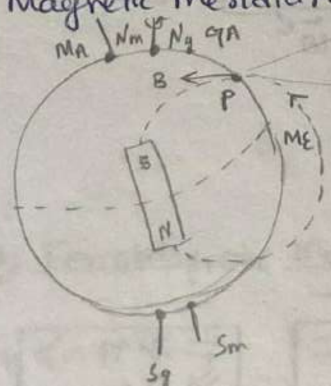
(b) Magnetic Meridian - A vertical plane on earth that passes through GA. MA.

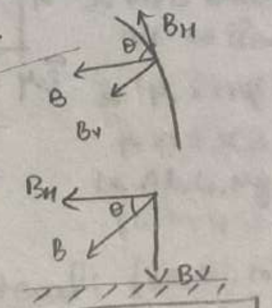
[OR]

Vertical plane that contains Magnetic field lines.

7. **Elements of Mag. field -**

(a) Angle of Dip: Angle which mag. field makes with horizontal in Magnetic Meridian.

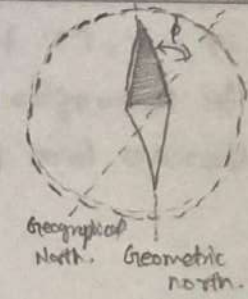




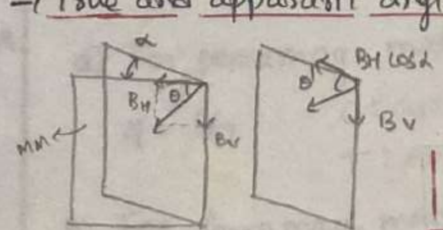
$$\boxed{\tan\theta = \frac{B_V}{B_H}}$$

(b) Angle of Declination (ϕ) - ϕ is the angle b/w Geographical meridian and Mag. meridian at a point.

Compass in horizontal plane always gets aligned along $B_H = B\cos\theta$ & is in mag. meridian



⇒ **True and apparent angle of Dip:**

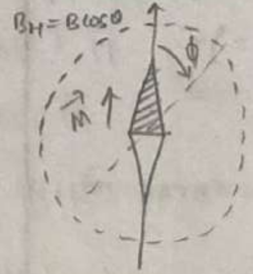


$$\tan\theta' = \frac{B_V}{B_H\cos\alpha}$$

$$\boxed{\tan\theta' = \frac{\tan\theta}{\cos\alpha}}$$

θ' is called apparent angle of Dip.

⇒ **Oscillation of Compass Needle:**



$\vec{m}$: Magnetic moment

$\tau = mB_H\sin\phi$

$I\alpha = mB_H\sin\phi$

$\alpha = \frac{mB_H}{I}\phi$ { ϕ is small

$$\boxed{T = 2\pi\sqrt{\frac{I}{mB_H}}}$$

EARTH's MAGNETISM.

MAGNETIC MATERIALS.

1. $\vec{H}, \vec{I}, \vec{B}_{net}, \chi$

a) $\vec{H}$, Magnetic field.

For ext. mag field $\vec{B}_{ext}$, $\vec{H}$ is defined.

$\vec{B}_{ext} = \mu_0 \vec{H}$, unit of $\vec{H}$ is A/m.

b) $\vec{I}$, Intensity of Magnetisation.

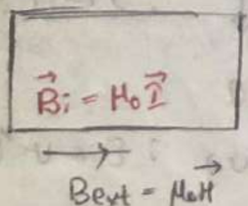
Total mag. mom. of material per unit vol. (It tells how much a material is magnetised)

$\vec{I} = \vec{m}/V$

c) χ, magnetic Susceptibility.

Tells abt. the ease with which the material can be magnetised.

$\chi = I/H$



$\vec{B}_{net} = \vec{B}_{ext} + \vec{B}_i$

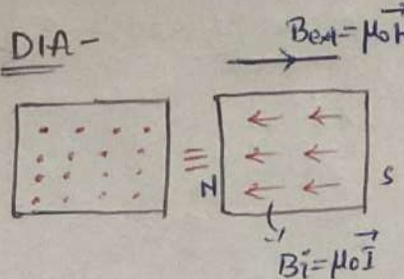
$\Rightarrow \mu_0 \mu_r \vec{H} = \mu_0 \vec{H} + \mu_0 \vec{I}$.

$\Rightarrow \mu_r \vec{H} = \vec{H} + \chi \vec{H}$.

$\therefore \boxed{\mu_r = 1 + \chi}$

2. Diamagnetic & Paramagnetic Material:

DIA -



i) Weakly mg.

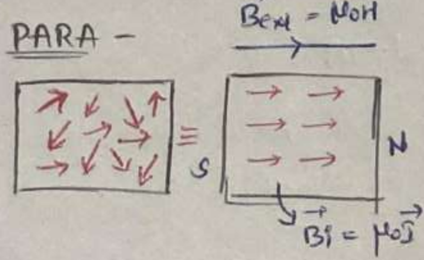
ii) Material repels $\vec{B}_{ext}$.

iii) $\mu_r = 1 + \chi$

$\mu_r < 1$ & χ is -ve.

iv) Graphite, Bismuth.

PARA -



i) Weakly mag.

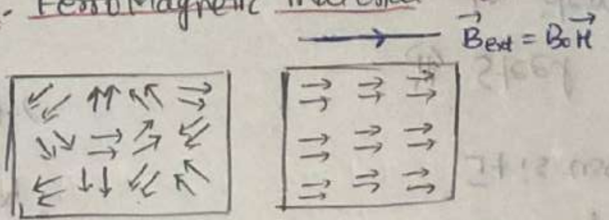
ii) Gets weakly att. to $\vec{B}_{ext}$.

iii) $\mu_r = 1 + \chi$.

$\mu_r > 1$ & χ is +ve.

iv) Al, Li, Mg!

3. FerroMagnetic Material: (Fe, Ni, Co)



$\vec{B}_{net} = \vec{B}_{ext} + \vec{B}_i$

Here, $B_i \gg B_{ext}$

$\Rightarrow I \gg H$ $\therefore \chi \gg 1$.

4. Curie's law: If $T\uparrow$, due to thermal agitation, the alignment of dipoles gets disturbed and overall $I\downarrow$ for a given H.

$\because I \downarrow \Rightarrow \chi \downarrow$.

a) For paramag. material

$\chi = C/T$ C: curie const. T: Abs. temp°.

b) For ferromag. material -

on heating it to T_c, it changes to paramagnetic. On further $\uparrow T$

$\chi = \frac{C}{T - T_c}$ T_c: curie temp°.

for paramag. material, $\chi \propto 1/T$

5. Magnetic Hysteresis :

(i) $O \to A$: on $\uparrow H$, $I\uparrow$ and gets saturated for $H = H_0$

(ii) $A \to B$: on $\downarrow H$, to zero, still I is retained.

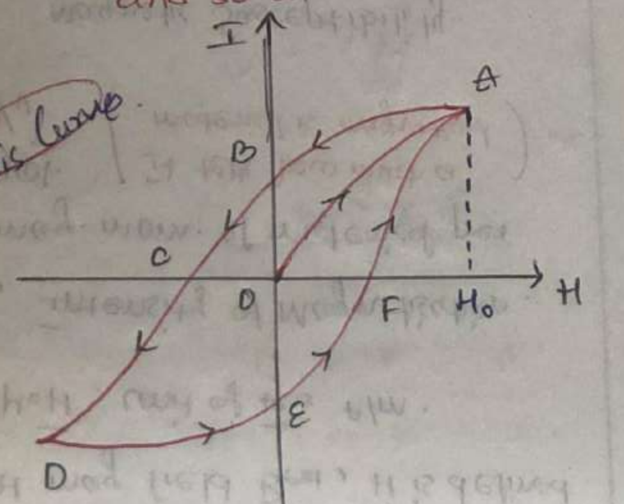
OB is called "RETENTIVITY".

(iii) $B \to C$: $\uparrow H$ in reverse dirⁿ, $I = 0$ at C.

OC is called "COERCIVITY".

(iv) $C \to D$: Further inc $\uparrow H$ & I again gets saturated at D.

and so on. .....



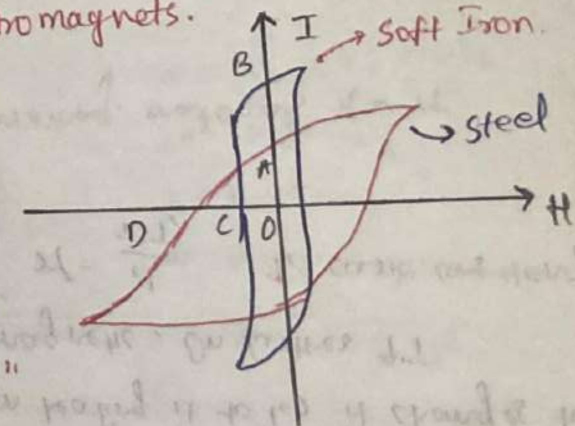
Note: Area of loop ABCDEFA shows thermal energy or heat produced / vol in 1 cyc.

6. Hysteresis Curve: Soft iron vs steel.

(i) Soft Iron gets easily magnetised & looses almost all mag. easily. (low coercivity).

It is used for "Electromagnets."

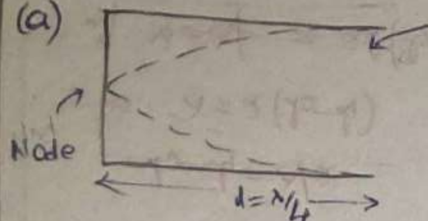
(ii) Steel is difficult to demagnetize. (OD = coercivity) it is used to make "Permanent Magnets."



ORGAN PIPES.

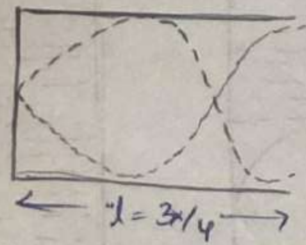
1. Closed Organ Pipe: (Stationary Wave)

(a)



$\lambda = 4l$, $\boxed{f_0 = \frac{v}{4l}}$ → Fund. freq or 1^{st} Harmonic

(b)

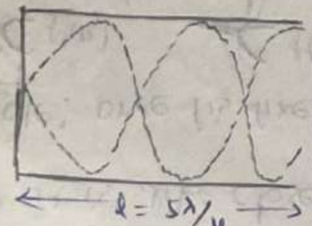


$\lambda = 4l/3$

$\boxed{f_3 = \frac{3v}{4l}}$

3^{rd} Harmonic (or) 1^{st} Overtone.

(c)



$\lambda = 4l/5$

$f_5 = \frac{5v}{4l}$

5^{th} Harmonic (or) 2^{nd} Overtone.

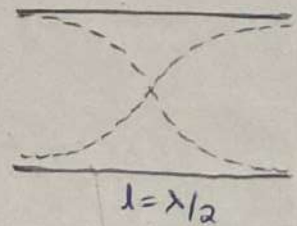
n^{th} overtone: $f_n = (2n+1) f_0$

$v = \sqrt{\frac{\gamma RT}{m}}$ (or) $\sqrt{\frac{\gamma P}{\rho}}$ $\gamma = 4/lr$

• If the frequency of Tuning fork matches with odd multiple of fundamental frequency, Resonance occurs.

2. Open Organ Pipe: (Stationary Waves)

(a)

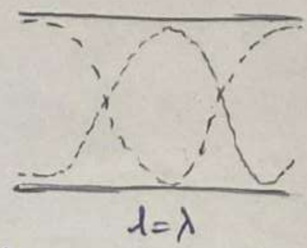


$\lambda = 2l$

$\boxed{f_0 = \frac{v}{2l}}$

fundamental freq. (OR) 1^{st} Harmonic

(b)

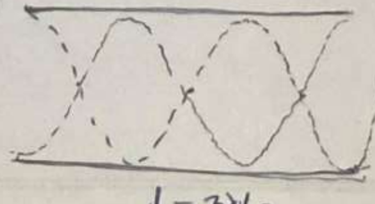


$\lambda = l$

$f_2 = \frac{2v}{2l}$

2^{nd} Harmonic (OR) 1^{st} overtone.

(c)



$\lambda = 2l/3$.

$f_3 = \frac{3v}{2l}$

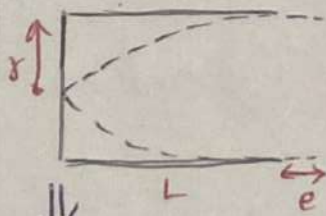
3^{rd} Harmonic (OR) 2^{nd} overtone.

n^{th} overtone: $f_n = (n+1) f_0$.

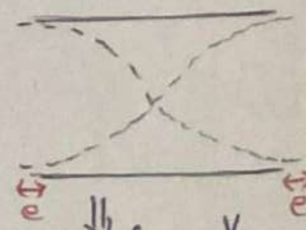
• If frequency of tuning fork matches with integer multiple of fundamental frequency, Resonance Occurs.

3. End Correction: (e)

At open side, antidote is formed a little outside.



⇓ $f_0 = \frac{v}{4(L+e)}$



⇓ $f_0 = \frac{v}{2(L+2e)}$.

$\boxed{e = 0.6r}$

r: pipe radius,

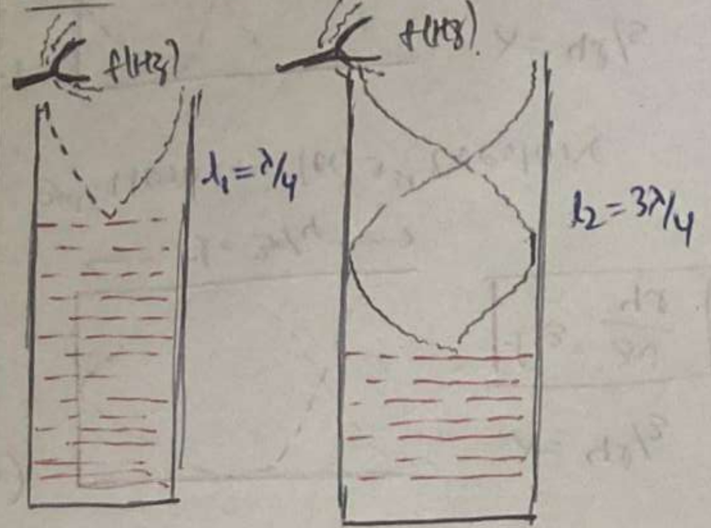
L: pipe true length.

4. Resonance Tube :

→ Uesd to find Speed of Sound in Air.

→ It is like closed organ pipe.

Note: once f is fixed, λ is also fixed.



Diff b/w any 2 consecutive Resonance / Harmonic is $\lambda/2$.

$$l_2 - l_1 = \lambda/2$$

$$\Rightarrow \lambda = 2(l_2 - l_1)$$

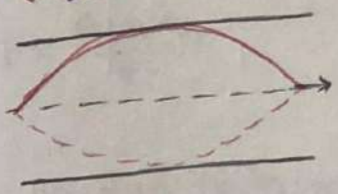
$$\therefore v = \lambda f = 2f(l_2 - l_1)$$

also, end correction isn't possible due to "difference" {cancel hojayega}.

IMPORTANT TOPICS (Short).

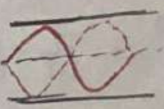
1. Waves :

→ Organ Pipes –



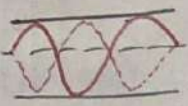
$\frac{\lambda}{2} = l$; $f = \frac{v}{2l}$ Fundamental Freq.

$f \propto 1/l$



$\frac{2\lambda}{2} = l$

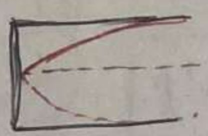
$f = \frac{2v}{2l}$



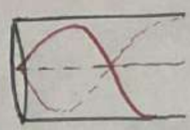
$\frac{3\lambda}{2} = l$

$f = \frac{3v}{2l}$.

Closed at one end :

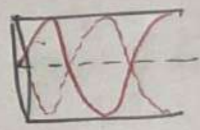


$\frac{\lambda}{4} = l$ $f = \frac{v}{4l}$



$\frac{3\lambda}{4} = l$

$f = \frac{3v}{4l}$



$\frac{5\lambda}{4} = l$

$f = \frac{5v}{4l}$.

→ Doppler's Effect – The apparent change in the frequency of a wave caused by rel. motion b/w the source of the wave & the observer.

$$\boxed{f = f_0 \left(\frac{v - v_0}{v - v_s}\right)}$$

v_s : vel of source

v_0 : " of obs.

v : vel of sound in that medium.

f_0 : real freq.

f : app. freq

All velocities in f dirn are +ve