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Theme 4. Camera Views

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Bachelor's Degree in Video Game Design and Development

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Outline

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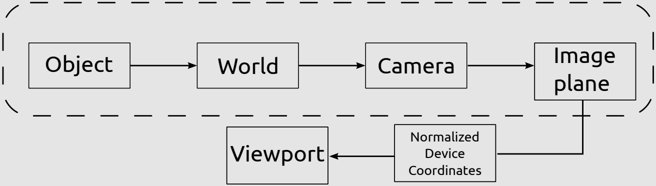
Outline

Usual graphics pipeline



Transformations involved:

- Object to world: Affine transformation defined by Att + trans.
- World to camera: Affine transformation defined by Att + trans.
- Camera to image plane: Defined by $\rightarrow$ subject of the day
- Image plane to NDC: Out of scope but affordable
- NDC to viewport: Needed to accommodate any kind of output

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Outline

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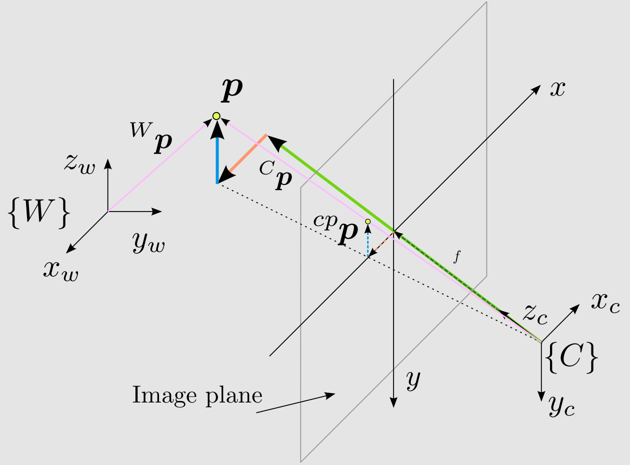
Simple Camera Model

What is the representation of $\boldsymbol{p}$ on the camera plane?



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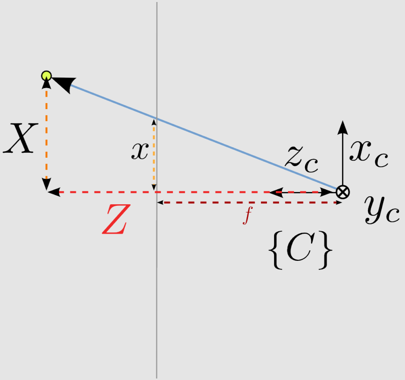

$$^{C}\boldsymbol{p} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$^{cp}\boldsymbol{p} = \begin{pmatrix} x \\ y \end{pmatrix}$$

f is the focal length: distance between the image plane and the camera

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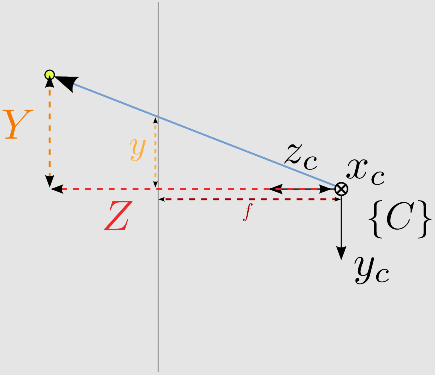

$$^{C}\boldsymbol{p} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

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Simple Camera Model

What is the representation of $\boldsymbol{p}$ on the camera plane?

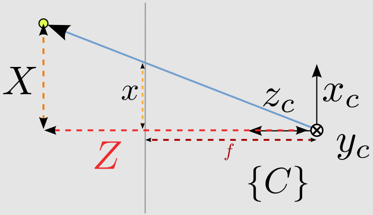
Thales Theorem about similar triangles establishes

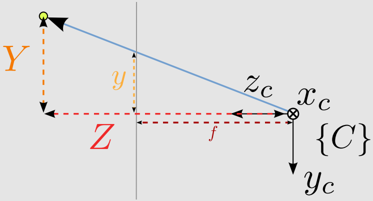
$$\frac{X}{Z} = \frac{x}{f}$$

$$\frac{Y}{Z} = \frac{y}{f}$$

which implies that

$${}^{c}\boldsymbol{p} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{X}{Z} f \\ \\ \frac{Y}{Z} f \end{pmatrix}$$





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Outline

Perspective Transformation

Perspective Transformation

$$^{cp}\boldsymbol{p} = \begin{pmatrix} \dfrac{X}{Z} f \\ \\ \dfrac{Y}{Z} f \end{pmatrix}$$

is a transformation, known as Perspective Transformation, with the properties

- It Is a mapping from 3-dimensional space to 2-dimensional space.
- Straight lines in the world are projected to straight lines in the camera plane.
- Parallel lines in the world are translated to lines that intersect at a vanishing point.

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Perspective Transformation

Perspective Transformation, continuation

- Conics (circles, ellipses, parabolas and hyperbolas) are translated to other conics.
- The transformation does not preserve angles between lines.
- The mapping in general has not a unique inverse: any point

$$^{c}\boldsymbol{p} = \begin{pmatrix} \lambda X \\ \lambda Y \\ \lambda Z \end{pmatrix}, \forall \lambda$$

is mapped to the same point $^{cp}\boldsymbol{p}$ on the camera

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Outline

Homework
Camera Views

Exercise 1

A triangle is represented by three points in space. Of these 3 points, two of them

$$
{}^{f1}\boldsymbol{p}_1 = (0,\ -0.5977,\ 1.2817)^\intercal \ \mathrm{m}
$$
$$
{}^{f1}\boldsymbol{p}_2 = (-1,\ 1.0261,\ 2.8191)^\intercal \ \mathrm{m}
$$

are known in frame $\{F_1\}$. The coordinates of the third point are known in the frame $\{F_2\}$ as

$$
{}^{f2}\boldsymbol{p}_3 = (0,\ -0.2724,\ -1.7821)^\intercal \ \mathrm{m}
$$

Knowing that:

- The origin of $\{F_1\}$ is at coordinates ${}^{w}\boldsymbol{o}_{f1} = (0,\ 4,\ 1)^\intercal\ \mathrm{m}$ with respect a world frame and that its orientation is achieved by rotating the world frame 30 deg about the world x axis.

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Homework
Camera Views

Exercise 1, continuation

- The origin of $\{F_2\}$ is at coordinates ${}^{w}\boldsymbol{o}_{f2} = (1, 7, 4)^{\mathsf{T}}$ m with respect the world frame and the orientation of $\{F_2\}$ is achieved by rotating the world frame -25 deg about the world x axis.

If a camera with focal length $f = 1/55\,\mathrm{m}$, with origin at ${}^{w}\boldsymbol{o}_c = (6, 3, 0)^{\mathsf{T}}$ m, and which orientation is achieved by consecutively apply the next rotations to the world reference system

1. A rotation defined by the euler angles $(\psi = \pi/2, \theta = 0, \phi = -\pi/2)\,\mathrm{rad}$.
2. Followed by a rotation of $-\pi/20\,\mathrm{rad}$ about the z axis
3. Followed by a rotation of $0.3\,\mathrm{rad}$ about the y axis.

Exercise 1, continuation

Calculate:

- The transformation needed to go from frames $\{F_1\}$ and $\{F_2\}$ to the world frame
- The transformation needed to go from world frame to the camera frame
- The position of points $\boldsymbol{p}_1$, $\boldsymbol{p}_2$ and $\boldsymbol{p}_3$ projected on the camera plane

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Exercise 2

The coordinates of a point $\boldsymbol{p}$, expressed in the reference frame of a camera are, ${}^{c}\boldsymbol{p} = (\beta, 3, 10)^{\intercal}$. If the coordinates, of this point in the image plane are given by ${}^{p}\boldsymbol{p} = (0.0625, 0.0075)^{\intercal}$, find the value of β and the focal distance.