

---

# 2016–2017

# MATHCOUNTS®

## School Handbook

---

Contains 250 creative math problems  
that meet the NCTM Grades 6-8 Standards.



2017 MATHCOUNTS  
National Competition Sponsor

**National Sponsors:**

Raytheon Company  
Northrop Grumman Foundation  
U.S. Department of Defense  
National Society of Professional Engineers  
CNA Foundation  
Phillips 66  
Texas Instruments Incorporated  
3Mgives  
Art of Problem Solving  
NextThought

**Executive Sponsor:**  
General Motors Foundation

**Official Sponsors:**

Bentley Systems Incorporated  
Brookhill Institute of Mathematics  
The National Council of Examiners for  
Engineering and Surveying

**Founding Sponsors:**

National Society of Professional Engineers  
National Council of Teachers of Mathematics  
CNA Foundation



*Change the Equation has recognized MATHCOUNTS  
as having one of the nation's most effective STEM  
learning programs, listing the Math Video Challenge as  
an Accomplished Program in STEMworks.*



*The National Association of Secondary School  
Principals has placed all three MATHCOUNTS  
programs on the NASSP Advisory List of National  
Contests and Activities for 2016-2017.*

# **HOW TO USE THIS SCHOOL HANDBOOK**

## **If You're a New Coach**



Welcome! We're so glad you're a coach this year.  
Check out the **Guide for New Coaches**  
starting on the next page.

.....

## **If You're a Returning Coach**



Welcome back! Thank you for coaching again.  
Get the **2016-2017 Handbook Materials**  
starting on page 8.

# **GUIDE FOR NEW COACHES**

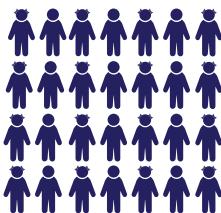
**Welcome to the MATHCOUNTS® Competition Series!** Thank you so much for serving as a coach this year. Your work truly does make a difference in the lives of the students you mentor. We've created this Guide for New Coaches to help you get acquainted with the Competition Series and understand your role as a coach in this program.

If you have questions at any point during the program year, please feel free to contact the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.

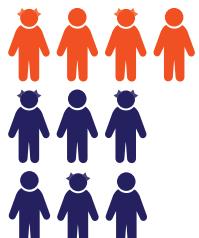
## ***The MATHCOUNTS Competition Series in a Nutshell***

The **MATHCOUNTS Competition Series** is a national program that provides students the opportunity to compete in live, in-person math contests against and alongside their peers. Created in 1983, it is the longest-running MATHCOUNTS program and is open to all sixth-, seventh- and eighth-grade students.

**HOW DOES IT WORK?** The Competition Series has 4 levels of competition—school, chapter, state and national. Here's what a typical program year looks like.



**Schools register in the fall and work with students during the year. Coaches administer the School Competition, usually in January.** Any number of students from your school can participate in your team meetings and compete in the School Competition. MATHCOUNTS provides the School Competition to coaches in November. Many coaches use this to determine which student(s) will advance to the Chapter Competition.



**Between 1 and 10 students from each school advance to the local Chapter Competition, which takes place in February.** Each school can send a team of 4 students plus up to 6 individual competitors. All chapter competitors—whether they are team members or individuals—participate in the individual rounds of the competition; then just the 4 team members participate in the team round. Schools also can opt to send just a few individual competitors, rather than forming a full team. Over 500 Chapter Competitions take place across the country.



**Top students from each Chapter Competition advance to their State Competition, which takes place in March.** Your school's registration fees cover your students as far as they get in the Competition Series. If your students make it to one of the 56 State Competitions, no additional fees are required.



**Top 4 individual competitors from each State Competition receive an all-expenses-paid trip to the National Competition, which takes place in May.** These 224 students combine to form 4-person state teams, while also competing individually for the title of National Champion.

**WHAT DOES THE TEST LOOK LIKE?** Every MATHCOUNTS competition consists of 4 rounds—Sprint, Target, Team and Countdown Round. Altogether the rounds are designed to take about 3 hours to complete. Here's what each round looks like.



### Sprint Round

40 minutes  
30 problems total  
no calculators used  
focus on speed and accuracy



### Target Round

Approx. 30 minutes  
8 problems total  
calculators used  
focus on problem-solving and mathematical reasoning

*The problems are given to students in 4 pairs. Students have 6 minutes to complete each pair.*



### Team Round

20 minutes  
10 problems total  
calculators used  
focus on problem-solving and collaboration

*Only the 4 students on a school's team can take this round officially.*



### Countdown Round

Maximum of 45 seconds per problem  
no calculators used  
focus on speed and accuracy

*Students with highest scores on Sprint and Target Rounds compete head-to-head. This round is optional at the school, chapter and state level.*

**HOW DO I GET MY STUDENTS READY FOR THESE COMPETITIONS?** What specifically you do to prepare your students will depend on your schedule as well as your students' schedules and needs. But in general, working through lots of different MATHCOUNTS problems and completing practice competitions is the best way to prepare to compete. Each year MATHCOUNTS provides the *School Handbook* to all coaches, plus lots of additional free resources online.

The next sections of this Guide for New Coaches will explain the layout of the *MATHCOUNTS School Handbook* and other resources, plus give you tips on structuring your team meetings and preparation schedule.

## ***The Role of the Competition Coach***

Your role as the coach is such an important one, but that doesn't mean you need to know everything, be a math expert or treat coaching like a full-time job. Every MATHCOUNTS coach has a different coaching style and you'll find the style that works best for you and your students. But in general **every good MATHCOUNTS coach must do the following**.

- Schedule and run an adequate number of practices for participating students.
- Help motivate and encourage students throughout the program year.
- Select the 1-10 student(s) who will represent the school at the Chapter Competition in February.
- Take students to the Chapter Competition or make arrangements with parents and volunteers to get them there.



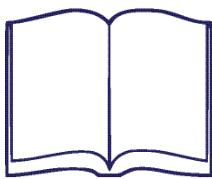
***Looking for tools to help you become a top-notch coach? Check out our videos at the Coach Section of the MATHCOUNTS website!***

You don't need to know how to solve every MATHCOUNTS problem to be an effective coach. In fact, many coaches have told us that they themselves improved in mathematics through coaching. Chances are, you'll learn with and alongside your students throughout the program year.

You don't need to spend your own money to be an effective coach. You can prepare your students using solely the free resources and this handbook. We give coaches numerous detailed resources and recognition materials so you can guide your Mathletes® to success even if you're new to teaching, coaching or competition math, and even if you use only the free resources MATHCOUNTS provides all competition coaches.

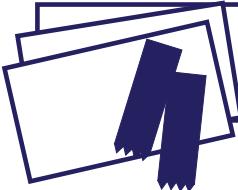
## Making the Most of Your Resources

As the coach of a registered competition school, you already have received what we at MATHCOUNTS call the **School Competition Kit**. Your kit includes the following materials for coaches.



### 2016-2017 MATHCOUNTS School Handbook

The most important resource included in the School Competition Kit. Includes 250 problems.



### Student Recognition Ribbons and Certificates

10 participation certificates and 1 ribbon for each registered chapter competitor.

You'll also get access to electronic resources. The following resources are available to coaches online at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches). This section of the MATHCOUNTS website is restricted to coaches and you already should have received an email with login instructions. *If you have not received this email, please contact us at info@mathcounts.org to make sure we have your correct email address.*

#### Official 2017 MATHCOUNTS School Competition

Released in November 2016  
Includes all 4 test rounds and the answer key

#### 2016 MATHCOUNTS School, Chapter + State Competitions

Released in mid-April 2016  
Each level includes all 4 test rounds and the answer key

#### MATHCOUNTS Problem of the Week

Released each Monday  
Each multi-step problem relates to a timely event

You can use the **2017 MATHCOUNTS School Competition** to choose the students who will represent your school at the Chapter Competition. Sometimes coaches already know which students will attend the Chapter Competition. If you do not need the School Competition to determine your chapter competitors, then we recommend using it as an additional practice resource for your students.

The **2016-2017 MATHCOUNTS School Handbook** will be your primary resource for the Competition Series this year. It is designed to help your students prepare for each of the 4 rounds of the test, plus build critical thinking and problem-solving skills. This section of the Guide for New Coaches will focus on how to use this resource effectively for your team.

**WHAT'S IN THE HANDBOOK?** There is a lot included in the *School Handbook*, and you can find a full table of contents on pg. 8 of this book, but below are the sections that you'll use the most when coaching your students.

- **Handbook Problems:** 250 math problems divided into Warm-Ups, Workouts and Stretches. These problems in-



**Check out our online coach resource videos:**  
**Making the Most of Your Coaching Resources**  
**How to Use the Handbook**

- crease in difficulty as the students progress through the book. (pg. 13)
- **Solutions to Handbook Problems:** complete step-by-step explanations for how each problem can be solved. These detailed explanations are only available to registered coaches. (pg. 59)
  - **Answers to Handbook Problems:** key available to the general public. Your students can access this key, but not the full solutions to the problems. (pg. 51)
  - **Problem Index + Common Core State Standards Mapping:** catalog of all handbook problems organized by topic, difficulty rating and mapping to Common Core State Standards. (pg. 55)

There are 3 types of handbook problems to prepare students for each of the rounds of the competition. You'll want to have your students practice all of these types of problems.

<b>Warm-Ups</b>	<b>Workouts</b>	<b>Stretches</b>
14 Warm-Ups in handbook 10 questions per Warm-Up no calculators used  	8 Workouts in handbook 10 questions per Workout calculators used  	3 Stretches in handbook Number of questions and use of calculators vary by Stretch  <i>Each Stretch covers a particular math topic that could be covered in any round. These help prepare students for all 4 rounds.</i>
<i>Warm-Ups prepare students particularly for the Sprint and Countdown Rounds.</i>   	<i>Workouts prepare students particularly for the Target and Team Rounds.</i>   	   

**IS THERE A SCHEDULE I SHOULD FOLLOW FOR THE YEAR?** On average coaches meet with their students for an hour once a week at the beginning of the year, and more often as the competitions approach. Practice sessions may be held before school, during lunch, after school, on weekends or at other times, co-ordinating with your school's schedule and avoiding conflicts with other activities.

Designing a schedule for your practices will help ensure you're able to cover more problems and prepare your students for competitions. We've designed the *School Handbook* with this in mind. Below is a suggested schedule for the program year that mixes in Warm-Ups, Workouts and Stretches from the *School Handbook*, plus free practice competitions from last year. This schedule allows your students to tackle more difficult problems as the School and Chapter Competition approach.

<b>Mid-August – September 2016</b> Warm-Ups 1, 2 + 3 Workouts 1 + 2	<b>October 2016</b> Warm-Ups 4, 5 + 6 Workout 3 Fractions Stretch	<b>November 2016</b> Warm-Ups 7 + 8 Workouts 4 + 5 Angles and Arcs Stretch	<b>December 2016</b> Warm-Ups 9, 10 + 11 Workout 6 Bases Stretch
<b>January 2017</b> Warm-Ups 12, 13 + 14 Workouts 7 + 8 <i>2017 MATHCOUNTS School Competition</i> <i>Select chapter competitors (optional at this time)</i>		<b>February 2017</b> Practice Competition: 2016 School Competition Practice Competition: 2016 Chapter Competition <i>Select chapter competitors (required by this time)</i> <i>2017 MATHCOUNTS Chapter Competition</i>	

You'll notice that in January or February you'll need to select the 1-10 student(s) who will represent your school at the Chapter Competition. This must be done before the start of your local Chapter Competition. You'll submit the names of your chapter competitors either online at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches) or directly to your local Chapter Coordinator.

It's possible you and your students will meet more frequently than once a week and need additional resources. If that happens, don't worry! You and your Mathletes can work together using the **Interactive MATHCOUNTS Platform**, powered by NextThought. This free online platform contains numerous *MATHCOUNTS School Handbooks* and past competitions, not to mention lots of features that make it easy for students to collaborate with each other and track their progress. You and your Mathletes can sign up for free at [mathcounts.nextthought.com](http://mathcounts.nextthought.com).

And remember, just because you and your students will meet once a week doesn't mean your students can only prepare for MATHCOUNTS one day per week. Many coaches assign "homework" during the week so they can keep their students engaged in problem solving outside of team practices. Here's one example of what a 2-week span of practices in the middle of the program year could look like.



**Check out the  
Interactive  
MATHCOUNTS Platform  
to get even more  
handbook problems  
and past  
competitions!**

Monday	Tuesday	Wednesday (Weekly Team Practice)	Thursday	Friday
-Students continue to work individually on Workout 4, due Wednesday	-Students continue to work on Workout 4 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 4 -Coach gives Warm-Up 7 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 5 as individual work, due Wednesday	-Students continue to work individually on Workout 5
-Students continue to work individually on Workout 5, due Wednesday	-Students continue to work on Workout 5 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 5 -Coach gives Warm-Up 8 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 6 as group work, due Wednesday	-Students work together on Workout 6 using online Interactive Platform

**WHAT SHOULD MY TEAM PRACTICES LOOK LIKE?** Obviously every school, coach and group of students is different, and after a few practices you'll likely find out what works and what doesn't for your students. Here are some suggestions from veteran coaches about what makes for a productive practice.

- Encourage discussion of the problems so that students learn from each other
- Encourage a variety of methods for solving problems
- Have students write math problems for each other to solve
- Use the **Problem of the Week** (posted online every Monday)
- Practice working in groups to develop teamwork (and to prepare for the Team Round)
- Practice oral presentations to reinforce understanding

On the following page is a sample agenda for a 1-hour practice session. There are many ways you can structure math team meetings and you will likely come up with an agenda that works better for you and your group. It also is probably a good idea to vary the structure of your meetings as the program year progresses.

## MATHCOUNTS Team Practice Sample Agenda – 1 Hour

### *Review Problem of the Week (20 minutes)*

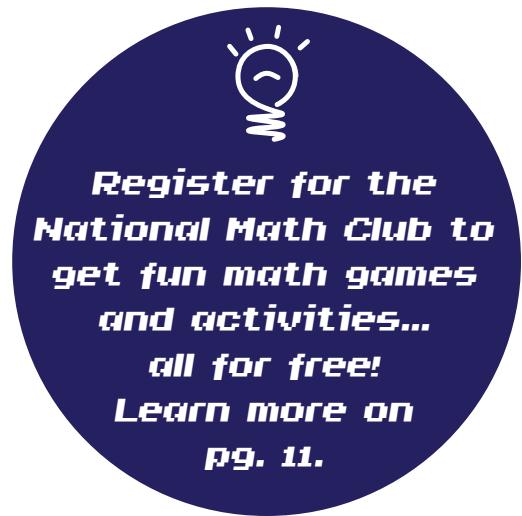
- Have 1 student come to the board to show how s/he solved the first part of the problem.
- Discuss as a group other strategies to solve the problem (and help if student answers incorrectly).
- Have students divide into groups of 4 to discuss the solutions to the remaining parts of the problem.
- Have 2 groups share answers and explain their solutions.

### *Timed Practice with Warm-Up (15 minutes)*

- Have students put away all calculators and have one student pass out Warm-Ups (face-down).
- Give students 12 minutes to complete as much of the Warm-Up as they can.
- After 12 minutes is up, have students hold up pencils and stop working.

### *Play Game to Review Warm-Up Answers (25 minutes)*

- Have students divide into 5 groups (size will depend on number of students in meeting).
- Choose a group at random to start and then rotate clockwise to give each group a turn to answer a question. When it is a group's turn, ask the group one question from the Warm-Up.
- Have the group members consult their completed Warm-Ups and work with each other for a maximum of 45 seconds to choose the group's official answer.
- Award 2 points for a correct answer on questions 1-3, 3 points for questions 4-7 and 5 points for questions 8-10. The group gets 0 points if they answer incorrectly or do not answer in 45 seconds.
- Have all students check their Warm-Up answers as they play.
- Go over solutions to select Warm-Up problems that many students on the team got wrong.



**OK I'M READY TO START. HOW DO I GET STUDENTS TO JOIN?** Here are some tips given to us from successful competition coaches and club leaders for getting students involved in the program at the beginning of the year.

- Ask Mathletes who have participated in the past to talk to other students about participating.
- Ask teachers, parent volunteers and counselors to help you recruit.
- Reach parents through school newsletters, PTA meetings or Back-to-School-Night presentations.
- Advertise around your school by:
  1. posting intriguing math questions (specific to your school) and referring students to the first meeting for answers.
  2. designing a bulletin board or display case with your MATHCOUNTS poster (included in your School Competition Kit) and/or photos and awards from past years.
  3. attending meetings of other extracurricular clubs (such as honor society) so you can invite their members to participate.
  4. adding information about the MATHCOUNTS team to your school's website.
  5. making a presentation at the first pep rally or student assembly.

**Good luck in the competition! If you have any questions during the year, please contact the MATHCOUNTS national office at (703) 299-9006 or [info@mathcounts.org](mailto:info@mathcounts.org).**

.....

***Coach Resources: [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches)***

# 2016-2017 HANDBOOK MATERIALS

**Thank you for being a coach in the MATHCOUNTS Competition Series this year!**

We hope participating in the program is meaningful and enriching for you and your Mathletes.

Don't forget to log in at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches) for additional resources!

## **What's in This Year's Handbook**

Highlighted Resources .....	9
<b><i>the best materials and tools for coaches and Mathletes!</i></b>	
Critical 2016–2017 Dates .....	10
Other MATHCOUNTS Programs .....	11
<b><i>the National Math Club and Math Video Challenge</i></b>	
School Registration Form for the National Math Club .....	12
 <b>This Year's Handbook Problems</b> ..... 13	
<b><i>250 problems designed to boost math + problem-solving skills</i></b>	
Official Rules + Procedures .....	39
<b><i>all of the ins-and-outs + dos-and-don'ts of competing</i></b>	
Registration .....	39
Eligibility Requirements .....	40
Levels of Competition .....	42
Competition Components .....	43
Scoring .....	44
Results Distribution .....	45
Additional Rules .....	45
Forms of Answers .....	47
Vocabulary and Formulas .....	48
<b>Answers to Handbook Problems</b> ..... 51	
<b><i>available to the general public...including your students</i></b>	
Problem Index + Common Core State Standards Mapping .....	55
<b><i>all 250 problems are categorized + mapped to the CCSS</i></b>	
<b>Solutions to Handbook Problems</b> ..... 59	
<b><i>step-by-step explanations (just for coaches) of how each problem can be solved</i></b>	
Additional Students Registration Form .....	79

# HIGHLIGHTED RESOURCES

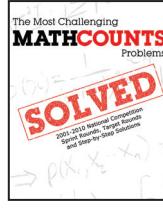
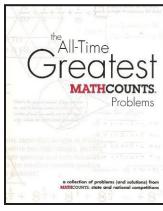
Also access resources at  
[www.mathcounts.org/coaches!](http://www.mathcounts.org/coaches)

 Great for  
Coaches

 Great for  
Mathletes

 Advanced  
Level Book

 Free  
Resource

<p><b>OPLET</b>            Online database of over 13,000 problems and over 5,000 step-by-step solutions. Create personalized quizzes, flash cards, worksheets and more!  <b>Save \$25 when you buy your subscription by Oct. 17, 2016</b>  <i>Renewers:</i> use code RENEW17  <i>First-Time Subscribers:</i> use code NEW17</p>	<p><b>Practice Competitions for MATHCOUNTS, Vol. I &amp; II</b>              Practice books written by repeat national-level coach Josh Frost. Each volume includes 4 complete mock-competitions plus solutions.</p>	<p><b>Most Challenging MATHCOUNTS Problems Solved</b>              Advanced level practice book with 10 years of national-level Sprint Rounds, plus detailed step-by-step solutions to each problem.</p>
<p> <a href="http://www.mathcounts.org/myoplet">www.mathcounts.org/myoplet</a></p> <p><b>All Time Greatest MATHCOUNTS Problems</b>              A collection of some of the most creative, interesting and challenging MATHCOUNTS competition problems.</p> <p>  </p> <p><a href="http://www.mathcounts.org/store">www.mathcounts.org/store</a></p>	<p>  <a href="http://www.mathcounts.org/store">www.mathcounts.org/store</a></p> <p><b>Interactive MATHCOUNTS Platform</b>                Online platform of past and current handbook and competition problems. Interactive features make collaboration easy and fun!</p> <p>    <a href="http://mathcounts.nexthought.com">mathcounts.nexthought.com</a></p>	<p><b>MATHCOUNTS Trainer App</b>              Train your Mathletes with this fun app, featuring real-time leaderboards and lots of past MATHCOUNTS problems.</p> <p>   <a href="http://aops.com/mathcounts_trainer">aops.com/mathcounts_trainer</a> or download at the App Store</p>
<p><b>Past Competitions</b>              Last year's School, Chapter and State competitions are free online! Other years' competitions can be purchased.</p> <p>  </p> <p><a href="http://www.mathcounts.org/pastcompetitions">www.mathcounts.org/pastcompetitions</a>  <a href="http://www.mathcounts.org/store">www.mathcounts.org/store</a></p>	<p><b>Problem of the Week</b>            A new, multi-step problem every week! Each problem focuses on a particular set of math skills and coincides with a timely event, holiday or season. Get the problem at the beginning of the week and the step-by-step solution the following week.</p> <p>    <a href="http://www.mathcounts.org/potw">www.mathcounts.org/potw</a></p>	<p><b>MATHCOUNTS Minis</b>            A fun monthly video series featuring Richard Rusczyk from Art of Problem Solving. Each video looks at a particular math skill and walks through how to solve different MATHCOUNTS problems using creative problem-solving strategies.</p> <p>    <a href="http://www.mathcounts.org/minis">www.mathcounts.org/minis</a></p>

# Critical 2016-2017 Dates

## 2016



Aug. 15 –  
Dec. 16

Submit your school's registration to participate in the Competition Series and receive this year's School Competition Kit, which includes a hard copy of the 2016-2017 *MATHCOUNTS School Handbook*. Kits are shipped on an ongoing basis between mid-August and December 31.

The fastest way to register is online at [www.mathcounts.org/compreg](http://www.mathcounts.org/compreg). You also can download the MATHCOUNTS Competition Series Registration form and mail or email it with payment to:

MATHCOUNTS Foundation – Competition Series Registrations  
1420 King Street, Alexandria, VA 22314  
*Email:* reg@mathcounts.org

To add students to your school's registration, log in at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches) to access the Dashboard. **Questions?** Call the MATHCOUNTS national office at (703) 299-9006 or email us at info@mathcounts.org.



Nov. 1

The 2017 School Competition will be available online. All registered coaches can log in at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches) to download the competition.



Nov. 18  
(postmark)

**Deadline to register for the Competition Series at reduced registration rates** (\$90 for a team and \$25 for each individual). After November 18, registration rates will be \$100 for a team and \$30 for each individual.



Dec. 16  
(postmark)

### Competition Series Registration Deadline

In some circumstances, late registrations might be accepted at the discretion of MATHCOUNTS and the local coordinator. *Late fees will apply. Register on-time to ensure your students' participation.*

## 2017



Early Jan.

If you have not been contacted with details about your upcoming competition, call your local or state coordinator. Coordinator contact information is available at [www.mathcounts.org/findmycoordinator](http://www.mathcounts.org/findmycoordinator).



Late Jan.

If you have not received your School Competition Kit, contact the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.



Feb. 1-28

### Chapter Competitions



March 1-31

### State Competitions



May 14-15

### 2017 Raytheon MATHCOUNTS National Competition in Orlando, FL

# **Other MATHCOUNTS Programs**

MATHCOUNTS was founded in 1983 as a way to provide new avenues of engagement in math for middle school students. MATHCOUNTS began solely as a competition, but has grown to include 3 unique but complementary programs: the **MATHCOUNTS Competition Series**, the **National Math Club** and the **Math Video Challenge**. Your school can participate in all 3 MATHCOUNTS programs!



The **National Math Club** is a free enrichment program that provides teachers and club leaders with resources to run a math club. The materials provided through the National Math Club are designed to engage students of all ability levels—not just the top students—and are a great supplement for classroom teaching. This program emphasizes collaboration and provides students with an enjoyable, pressure-free atmosphere in which they can learn math at their own pace.

Active clubs also can earn rewards by having a minimum number of club members participate (based on school/organization/group size). **There is no cost to sign up for the National Math Club**, and registration is open to schools, organizations and groups that consist of at least 4 students in 6th, 7th and/or 8th grade and have regular in-person meetings. More information can be found at [www.mathcounts.org/club](http://www.mathcounts.org/club), and the 2016-2017 School Registration Form is included on the next page.



The **Math Video Challenge** is an innovative program that challenges students to work in teams of 4 to create a video explaining the solution to a MATHCOUNTS handbook problem and demonstrating its real-world application. This project-based activity builds math, communication and collaboration skills.

Students post their videos to the contest website, where the general public votes for the best videos. The 100 videos with the most votes advance to judging rounds, in which 20 semifinalists and, later, 4 finalists are selected. This year's finalists will present their videos to the students competing at the 2017 Raytheon MATHCOUNTS National Competition, and the 224 Mathletes will vote to determine the winner. Members of the winning team receive college scholarships. **Registration is completely free** and open to all 6th, 7th and 8th grade students. More information can be found at [videochallenge.mathcounts.org](http://videochallenge.mathcounts.org).

! The fastest way to register  
your school is online at  
[www.mathcounts.org/clubreg!](http://www.mathcounts.org/clubreg)



## 2016-2017 SCHOOL REGISTRATION FORM

This registration form is for U.S. middle schools only. To register a non-school group (such as a Girl Scout Troop, Boys and Girls Club Chapter or math circle) for the National Math Club, please go to [www.mathcounts.org/club](http://www.mathcounts.org/club) to review eligibility requirements and register.

### Step 1: Fill in your school's name and confirm eligibility to participate.

\*required information

**U.S. school with students in 6th, 7th and/or 8th grade**

School Name\* \_\_\_\_\_

**There can be multiple clubs at the same U.S. school, as long as each club has a different club leader.**

By signing below I, the club leader, affirm that the school named above is a U.S. school with students in sixth-, seventh- and/or eighth-grade and is therefore eligible to participate in the National Math Club. I affirm that I have permission to register the school above for this program and I understand that MATHCOUNTS can cancel my membership at any time if it is determined that my group is ineligible.

**Club Leader Signature\*** \_\_\_\_\_

### Step 2: Provide your information so we can send you materials and set up your online access.

\*required information

Club Leader Name\* \_\_\_\_\_ Club Leader Phone \_\_\_\_\_

Club Leader Email Address\* \_\_\_\_\_

Club Leader Alternate Email Address \_\_\_\_\_

Club Address\* \_\_\_\_\_

City, State ZIP\* \_\_\_\_\_

Estimated total number of participating students in club (minimum 4 students)\*: \_\_\_\_\_

My school previously participated in the National Math Club.

School Type (please check one)\*:  Public  Charter  Private  Homeschool  Virtual

#### Department of Defense or State Department schools, please provide additional information below.

Clubs located outside of the U.S. states or territories are not eligible to participate in the National Math Club unless they are in schools affiliated with the U.S. Department of Defense or State Department.

My school is sponsored by (please check one):  U.S. Department of Defense (DoDDS)  
 U.S. State Department

Country \_\_\_\_\_

### Step 3: Almost done... just turn in your form.

Mail or email a scanned copy of this completed form to:  
**MATHCOUNTS Foundation**, 1420 King Street, Alexandria, VA 22314  
Email: [reg@mathcounts.org](mailto:reg@mathcounts.org)

**Questions?**  
Please call the national office at  
**703-299-9006**



# Warm-Up 1

1. \_\_\_\_\_ years A time capsule was sealed in 1940 and will be opened on the same date in 2017. How long will the capsule remain sealed?

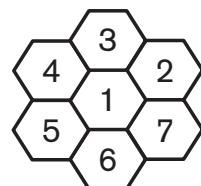
2. \_\_\_\_\_ large paper clips



The length of 5 small paper clips is equal to the length of 2 large paper clips. The length of 8 small paper clips is equivalent to the length of how many large paper clips? Express your answer as a mixed number.

3. \_\_\_\_\_ paths

In the figure shown, if no cell may be visited more than once and not every cell must be visited, how many paths start in cell 1 and end in cell 7?



4. \_\_\_\_\_

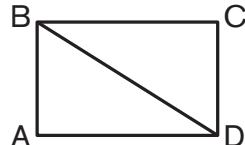
What is the value of  $4 \div \frac{2}{3} - 5$ ?

5. \_\_\_\_\_

What is the absolute difference between  $\frac{1}{2}$  and  $\frac{1}{3}$ ? Express your answer as a common fraction.

6. \_\_\_\_\_ cm

If the perimeter of rectangle ABCD is 34 cm and AB = 5 cm, what is the perimeter of  $\triangle ABD$ ?



7. \_\_\_\_\_

Harvey has a fair eight-sided die that has a different number from 1 to 8 on each side. If he rolls this die twice, what is the probability that the second number rolled is greater than or equal to the first number? Express your answer as a common fraction.



8. \_\_\_\_\_

In scientific notation, what is the product of  $1.2 \times 10^3$  and  $1.4 \times 10^2$ ? Express your answer to two significant figures.

9. \_\_\_\_\_

When the integer  $n$  is squared, the result is less than 150. What is the sum of all possible values of  $n$ ?

10. \$ \_\_\_\_\_

Minnie paid a one-time registration fee of \$30 for dance lessons. Additionally, she paid \$20 per lesson. If she took seven lessons, how much did she pay altogether?





## Warm-Up 2

11. \_\_\_\_\_ pieces How many pieces that are exactly 5 inches long can Sue cut from a string that is 7 feet long?

12. \_\_\_\_\_ units How many units away from the origin is the point  $(-3, -4)$ ?

13. \_\_\_\_\_ tables A hexagonal table that is by itself seats 6 people, one person at each side. A row of hexagonal tables is created by pushing together a certain number of hexagonal tables so that a side of one table meets a side of the next table, in the way shown here. If 50 people can sit at the row of tables that was created, how many tables are in the row?



14. \_\_\_\_\_ coins In Oregon, which has no sales tax, Gloria bought three notebooks for \$1.57 each. If she paid \$5.00, what is the least number of U.S. coins that she could get in change?

15. \_\_\_\_\_ g  The mass of a fluorine atom is  $3.16 \times 10^{-23}$  g. What is the mass of 1,000,000,000 fluorine atoms? Express your answer in scientific notation to three significant figures.

16. \_\_\_\_\_ meals A restaurant offers a dinner special in which diners can choose any one of 3 appetizers, any one of 4 entrées, any two different side dishes out of 5 and any one of 6 desserts. How many different meals are possible?

17. \_\_\_\_\_ minutes It takes 15 machines 15 minutes to make 500 raviolis. The machines produce raviolis at a steady rate. How long would it take 75 of these machines to make 6000 raviolis?

18. \_\_\_\_\_ Four chips are distinctly labeled with the digits 2, 3, 1 and 7, one chip for each digit. Two chips are drawn at random without replacement and placed in the order in which they are drawn, from left to right, to form a two-digit number. What is the probability that the two-digit number is a prime number? Express your answer as a common fraction.



19. \_\_\_\_\_ The letters A, B, ..., Z are equally spaced in order on a number line, with A at 0 and Z at 25. What is the average of the two numbers that are 4 units from the letter M?

20. \_\_\_\_\_ units On a coordinate grid, point B is located 8 units below and 2 units to the left of A(0, 6). What is the length of segment AB? Express your answer in simplest radical form.



# Warm-Up 3

21. \_\_\_\_\_ What is the average of the integers from 13 to 31, inclusive?

22. \_\_\_\_\_ socks Kelly has 6 identical white socks and 5 identical black socks in a drawer. If she selects without looking, how many socks must she take from the drawer to be assured of a matching pair?



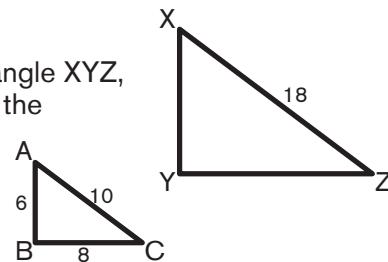
23. \_\_\_\_\_ For what value of  $m$  does  $\frac{3}{m} = \frac{27}{72}$ ?

24. \_\_\_\_\_ chairs A room has 23 rows of 27 chairs each. How many chairs are in the room?

25. \_\_\_\_\_ Esme is thinking of two integers. One integer is 4 times the other, and their sum is 18 more than 3 times the smaller integer. What is the smaller of the integers Esme is thinking of?



26. \_\_\_\_\_ Right triangle ABC has side lengths 6, 8 and 10 units. Right triangle XYZ, with hypotenuse of length 18 units, is similar to  $\triangle ABC$ . What is the ratio of the area of  $\triangle ABC$  to the area of  $\triangle XYZ$ ? Express your answer as a common fraction.



27. \_\_\_\_\_ A number is randomly selected from the integers 1 through 25, inclusive. What is the probability that the number chosen is divisible by 2, 3, 4 or 5? Express your answer as a common fraction.

28. \_\_\_\_\_ sequences Using only 1s and 2s, in how many different ordered sequences can Siddarth write a sum that equals 5? For example, a sum of 3 can be written 1 + 2, 2 + 1 or 1 + 1 + 1.

29. \_\_\_\_\_ miles Kevin takes a bus from home to school. The bus travels 8 miles west, then turns and travels 8 miles north, then turns and travels 7 miles west to the school. If the bus were able to travel directly from Kevin's house to the school, along a straight path, how much shorter would the trip be?



30. \_\_\_\_\_ What is the result when Ellen starts with the integer 123,456 and performs the following sequence of operations: subtract 6, divide by 10, subtract 5, divide by 10, subtract 4, divide by 10, subtract 3, divide by 10, subtract 2, divide by 10?



# Workout 1

31. NZ\$

When one New Zealand dollar, NZ\$1.00, was worth US\$0.77, how much was US\$1.00 worth in New Zealand dollars?



32. \_\_\_\_\_ pennies

A jar contains some number of pennies. When pennies are removed 2, 3, 4, 5, 6 or 8 at a time, one penny is left over. There are no pennies left over when they are removed 7 at a time. What is the least number of pennies that could be in the jar?

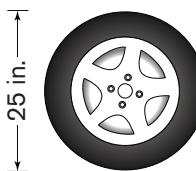
33. \_\_\_\_\_ lemmings

The population of lemmings on an island in Norway varies drastically. In a base year, the population was  $n$  lemmings. The next year, it tripled. The third year, the number dropped by 3000 from the second year. The fourth year, the population was  $\frac{1}{2}$  that of the third year. The fifth year, it increased by 1300 to a total of 1450 lemmings. What is the value of  $n$ ?

34. \_\_\_\_\_ integers

How many distinct positive four-digit integers can be formed using the digits 1, 2, 3 and 4 each once, such that no adjacent digits differ by more than 2?

35. \_\_\_\_\_ rotations



The tires on a certain car are 25 inches in diameter. If the car is moving at a constant speed of 65 miles per hour, how many rotations per second is the front left tire making? Express your answer as a decimal to the nearest tenth. (1 mile = 5280 feet)

36. \_\_\_\_\_ week

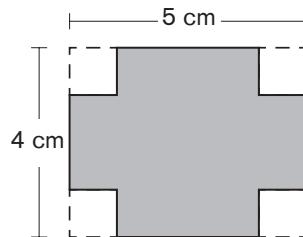
Nish trains to run a half marathon, a distance of 13.1 miles. Her training starts in week 1 with two 2-mile runs and one 4-mile run. Each week thereafter, the distances of her runs increase by 10% over the previous week's distances; therefore, she runs 8.8 miles in week 2. In which week of training does she first exceed 13.1 miles for the week?

37. \_\_\_\_\_

Kylie writes a 1 after A, a 2 after B, and so on, writing a single digit in counting order after each of the first nine letters of the alphabet. When she reaches 10, she writes the 1 after J and the 0 after K. When she reaches the Z, the 1 from 18 follows it; then, she cycles back to the start of the alphabet, and the 8 follows the A. After what letter does Kylie write the 6 in 26?

38. \_\_\_\_\_  $\text{cm}^2$ 

A dodecagon is formed when a 1-cm by 1-cm square is removed from each corner of a 4-cm by 5-cm rectangle as shown. What is the area of the dodecagon?



39. \_\_\_\_\_

Jesse added all but one of the first ten positive integers together. The sum was a perfect square. Which one of the first ten positive integers did Jesse not include?

40. \_\_\_\_\_ purple marbles

A bag contains 20 purple and 40 green marbles. How many purple marbles need to be added so that  $\frac{5}{12}$  of the marbles will be green?



# Workout 2

41. \_\_\_\_\_ feet A cube of cheese has edge length 1 inch and weighs 0.6 ounce. What is the edge length, in feet, of a cube of the same cheese that weighs 64.8 pounds? (1 pound = 16 ounces)

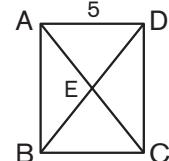
42. \_\_\_\_\_ integers How many positive integers from 1 to 100, inclusive have an even number of positive divisors?

43. \_\_\_\_\_ What is the coordinate of the point on a number line that is  $\frac{2}{3}$  of the way from  $-1.3$  to  $3\frac{1}{8}$ ? Express your answer as a decimal to the nearest hundredth.

44. \_\_\_\_\_ calories

A stick of butter is a  $1\frac{1}{2}$ -inch by  $1\frac{1}{2}$ -inch by  $3\frac{1}{4}$ -inch rectangular prism and contains 800 calories. How many calories are in a pre-formed "pat" of butter measuring 1 inch by 1 inch by  $\frac{3}{8}$  inch? Express your answer to the nearest whole number.

45. \_\_\_\_\_ degrees In rectangle ABCD, shown here,  $AD = BC = 5$  units. Diagonals AC and BD, each of length 10 units, intersect at E. What is the degree measure of  $\angle AEB$ ?



46. \_\_\_\_\_ What is the value of  $n$  if  $4! + 5! = n!3!$ ?

47. \_\_\_\_\_

In a standard set of dominoes, a face of each domino has a line through the center, with 0 to 6 dots on each side of the line. Each possible combination of dots is used exactly once, one combination per domino. What is the probability that a randomly selected domino will have the same number of dots on both sides of the line? Express your answer as a common fraction.

48. \_\_\_\_\_ % Paige cuts a square out of a circular pizza. The corners of the square lie on the circumference of the pizza. To the nearest whole number, what percent of the pizza is left when Paige removes the square?



49. \_\_\_\_\_ zaggles In exchange for 5 ziggles and 4 zoggles, Jefferson gets 30 zaggles. In exchange for 2 ziggles and 3 zoggles, Monroe gets 19 zaggles. How many zaggles should Carter expect to get in exchange for 1 ziggle and 1 zoggle?

50. \_\_\_\_\_ feet When leaned against a vertical structure, a straight ladder can be used safely if its top is no more than 4 feet above the base of the structure for every foot that the bottom of the ladder is away from the base. How high can a 22-foot ladder safely reach up a vertical structure? Express your answer as a decimal to the nearest tenth.



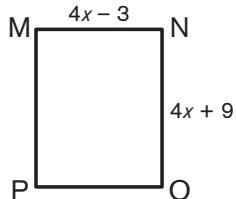
## Warm-Up 4

51. \_\_\_\_\_ Jamie's scores on the first five tests in his algebra class were 81, 75, 86, 98 and 92. After three more tests the median of his test scores was 88. What is the greatest possible value for the lowest score on these three tests?

52. \_\_\_\_\_ Consider the set of all possible two-digit numbers that can be created using an unlimited supply of 1s, 3s, 7s and 9s. What is the greatest absolute difference between any two primes in this set?

53. \_\_\_\_\_ The mean of  $a$  and  $b$  is 8. The mean of  $b$  and  $c$  is 16. The mean of  $a$  and  $c$  is 14. What is the value of  $a + b + c$ ?

54. \_\_\_\_\_ units Rectangle MNOP has length  $4x + 9$  and width  $4x - 3$ . What is the absolute difference between the length and width of rectangle MNOP?



55. \_\_\_\_\_ The faces of a cube are randomly and independently painted either red or blue with equal likelihood. What is the probability that the cube has all blue faces? Express your answer as a common fraction.

56. \_\_\_\_\_ If  $(\textcircled{n}) = n^2 - n$ , what is the value of  $(\textcircled{5})$ ?

57. \_\_\_\_\_ Kavon has a fair eight-sided die with each side having a different one of the digits 1 through 8. He rolls the die twice and writes down, in order, the results to form a two-digit number. What is the probability that his two-digit number is prime? Express your answer as a common fraction.

58. \_\_\_\_\_ What is the value of  $2x^2 + 3y^2 - 4x + 2y - 17$  when  $x = 3$  and  $y = -2$ ?

59. \_\_\_\_\_ Colorado used to issue license plates with the format of two letters (excluding Q) followed by four digits from 0 through 9. Later the state switched to a format of three letters (excluding Q) followed by three digits. What is the ratio of the number of possible old-style plates to the number of possible new-style ones? Express your answer as a common fraction.



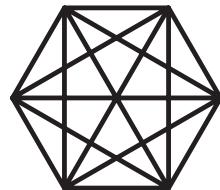
60. \_\_\_\_\_ What is  $40\% \times \frac{2}{3} \times 24 \div 0.8$ ?



# Warm-Up 5

61. \_\_\_\_\_ What is the value of  $(2 \times 6^3 + 6^2) - 7 \times 6^2$ ?

62. \_\_\_\_\_ triangles The figure shown consists of a regular hexagon and all of its diagonals. How many triangles in the figure have at least two congruent sides?



63. \_\_\_\_\_ ways Fido has to climb five stairs. If he steps on at least three of the five stairs, but never climbs more than three stairs in one step, in how many possible ways can Fido climb the stairs?

64. \_\_\_\_\_ The quotient  $\frac{x^2(x^2)^3}{x^2}$  can be expressed as  $x^y$ . What is the integer value of  $y$ ?

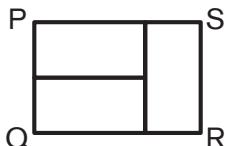
65. \$ \_\_\_\_\_ A grocery store is required to charge customers an 8% sales tax on certain items. However, some purchases at the store, such as food products, are not subject to sales tax. During a certain month, the store sold \$400,000 worth of groceries, not including the sales tax. If the store also collected \$10,000 in sales tax that month, then what was the total amount (in dollars) of the store's sales that month that were not subject to sales tax?

66. \_\_\_\_\_ ways A school of 100 fish swims in the ocean and comes to a very wide horizontal pipe. The fish have three choices to get to the food on the other side: swim above the pipe, through the pipe or below the pipe. If we do not consider the fish individually, in how many ways can the entire school of fish be partitioned into three groups with each group choosing a different one of the three options and with at least one fish in each group?

67. \_\_\_\_\_ miles If Reid is traveling at a speed of 44 ft/s, how many miles will he travel in an hour given that 1 mile = 5280 feet?

68. \_\_\_\_\_ ways In how many different ways can the letters of CHAIRS be arranged?

69. \_\_\_\_\_ cm The figure shows rectangle PQRS composed of three congruent rectangles. If the area of PQRS is  $1536 \text{ cm}^2$ , what is its perimeter?



70. \_\_\_\_\_ What is the value of the following expression?

$$2^{0^{1^2}} - 2^{2^{2^{1^0}}}$$



# Warm-Up 6

71. \$ \_\_\_\_\_ Amy's favorite lotion costs \$3.00 for 4 fluid ounces. At that same rate, what would she expect to pay for a quart of lotion? (1 quart = 32 fluid ounces)

72. \_\_\_\_\_
- |            |               |
|------------|---------------|
| Multiply → |               |
| ↓          | 3    2    6   |
|            | 4    1    4   |
|            | 12    2    24 |
- 
- |            |         |
|------------|---------|
| Multiply → |         |
| ↓          | 18    a |
|            | b    4  |
|            | 5184    |

In the first grid, numbers were multiplied vertically and horizontally until a value was found for the shaded box. For instance,  $3 \times 2 = 6$  and  $4 \times 1 = 4$  were the results from the first two rows, and then  $6 \times 4 = 24$  in the third column. The partially completed second grid follows the same rules, and both  $a$  and  $b$  are positive integers. What is the least possible value of  $a + b$ ?

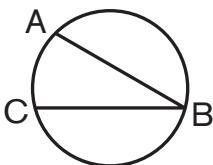
73. \_\_\_\_\_ The consecutive counting numbers are written in a triangular table, as shown, with one more number in each successive row. What is the sum of the numbers in the row that contains 25?

1		
2	3	
4	5	6
7	8	9
		10

74. \_\_\_\_\_ In the arithmetic sequence  $12, w, x, y, z, 47$ , what is the value of  $y$ ?

75. \_\_\_\_\_  $\text{cm}^2$  The width of a rectangle is one-third of its length. If the perimeter of the rectangle is 136 cm, what is its area?

76. \_\_\_\_\_ permutations How many three-letter permutations can be made using letters from ALASKA?

77. \_\_\_\_\_ inches
- 
- The circle shown has a diameter of 12 inches,  $m\angle ABC = 30$  degrees and  $AB = BC$ . What is the length of minor arc AC? Express your answer in terms of  $\pi$ .

78. \_\_\_\_\_ If  $A$  represents a digit in the equation  $0.0A = \sqrt{0.0049}$ , what is the value of  $A$ ?

79. \_\_\_\_\_ The absolute difference between two numbers is 6, and the absolute difference between their squares is 24. What is the product of the two numbers?

80. \_\_\_\_\_ If  $a \heartsuit b$  is defined as  $a^2 - 2b^2$ . What is  $5 \heartsuit (4 \heartsuit 3)$ ?



# Workout 3

81. \_\_\_\_\_ The sum of eleven consecutive integers is 11. What is the least of these eleven integers?

82. \_\_\_\_\_ The mean of  $x$  and  $y$  is 12 and the mean of  $y$  and 12 is  $\frac{z}{2}$ . What is the mean of  $x$  and  $z$ ?

83. \_\_\_\_\_ cm Squares A and B have at least one point in common. The area of square A is  $225 \text{ cm}^2$  and the area of square B is  $16 \text{ cm}^2$ . What is the maximum distance between the centers of the squares? Express your answer as a common fraction in simplest radical form.

84. \_\_\_\_\_ In the number sequence 3, 5, 2, ..., after the first two terms, the  $n$ th term is defined as  $a_n = a_{n-1} - a_{n-2}$ . For example,  $a_3 = a_2 - a_1 = 5 - 3 = 2$ . What is the sum of the first 200 terms of this sequence?

85. \_\_\_\_\_ What is the sum of the positive integer factors of 2017?

86. \_\_\_\_\_ feet Oberon and Lance sit directly opposite each other at a large round table. Arthur sits at the same table, 20 feet from Oberon and 21 feet from Lance. What is the diameter of the table?

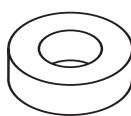
87. \$ \_\_\_\_\_ The cost of 1 binder with photos of celebrities on the cover plus the cost of 8 regular binders is a total of \$32.60. The cost of 1 binder with photos of celebrities on the cover plus the cost of 12 regular binders is a total of \$46.00. How much more does it cost to buy a celebrity binder than a regular binder?

88. \_\_\_\_\_ widgets Mr. Jones makes 3% commission on his sales of widgets. At a different company, Mr. Smith makes 5% commission selling the same widgets at the same price. Mr. Smith sold 500 fewer widgets than Mr. Jones, and they both earned the same commission. How many widgets did Mr. Smith sell?

89. \_\_\_\_\_ The table of values shows the relationship between  $x$  and  $y$ , which can be modeled with the equation  $y = ax^b$ , for integers  $a$  and  $b$ . What is the value of  $a + b$ ?

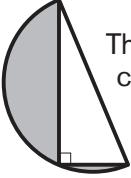
$x$	2	3	5	6
$y$	8	27	125	216

90. \_\_\_\_\_ in<sup>2</sup> Donny needs to find the surface area of a dozen donuts so he can make enough glaze. He doesn't know how to calculate the exact surface area of a donut, so he makes an approximation based on a roll of 1-inch-wide masking tape, which has the same volume as the donut. The outer radius of the tape is 2.5 inches, and the inner radius is 1.5 inches. What surface area did Donny estimate for a dozen donuts? Express your answer in terms of  $\pi$ .





# Workout 4

91. \_\_\_\_\_ combinations On Saturday, three different football games are televised at noon and four different games are televised at 8 p.m. On Sunday, five different games are televised at noon. If Amanda watches one Saturday game at noon and another at 8 p.m. and one game at noon on Sunday, how many different combinations of games can she watch?
92. \_\_\_\_\_ If  $a = 12$ ,  $b = 4$ ,  $c = 5$  and  $x = \frac{1}{2}$ , then what is the value of  $\frac{\left(\frac{abc}{x}\right) - (6b^2 - 4)}{0.5}$ ?
93. \_\_\_\_\_ hours At noon, Randy's family left the Texas-Oklahoma border, traveling north on I-35. At noon, Marco's family left their home in Minnesota, 1029 miles from the Texas-Oklahoma border, traveling south on I-35. If Randy's family is traveling 45 mi/h and Marco's family is traveling 53 mi/h, how many hours will it take for the two families to pass each other? Express your answer as a decimal to the nearest tenth.
94. \_\_\_\_\_ What is the 41st digit after the decimal point in the decimal expansion of  $\frac{1}{27}$ ?
95. \_\_\_\_\_ For the integers 15, 17, 11, 13,  $x$ ,  $y$ , the mode, median and mean form an increasing arithmetic sequence, in that order. If  $x \leq y$ , what is the greatest possible value of  $y$ ?
96. \_\_\_\_\_ units<sup>2</sup>  The hypotenuse of a 5-12-13 right triangle is the diameter of a semicircle containing the right angle vertex, as shown. What is the total area of the shaded regions? Express your answer to the nearest whole number.
97. \_\_\_\_\_ ways A group of 12 tourists will split up for two tours. A tour guide will lead one group on a hike. Another tour guide will lead the other group on a safari. If at least one tourist goes with each guide, in how many different ways can the tourists split up for the two tour groups?
98. \_\_\_\_\_ intersections What is the maximum number of distinct intersections of 30 different coplanar circles?
99. \_\_\_\_\_ % A sphere, a cone and a cylinder all have the same height and radius. The sphere and cone are completely filled with water. If the amount of water in the cylinder is the same as the total of the amounts in the sphere and cone, what percent of the cylinder's volume is filled?
100. \_\_\_\_\_ What is the sum of all the prime numbers less than 500 with only 3s and 4s as digits?

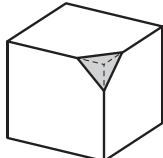


# Warm-Up 7

101. \_\_\_\_\_ marbles Mac has 25 marbles, of which 20% are red. Thayer has 20 marbles, of which 75% are not red. What is the absolute difference between the numbers of red marbles they have?

102. \_\_\_\_\_ If  $f(x) = x^2 - 2$  and  $g(x) = 2x + 4$ , what is the value of  $f(g(-3))$ ?

103. \_\_\_\_\_



In the figure shown, a triangular pyramid has been cut off the corner of the cube so that an equilateral triangle face is formed. If each corner of the cube is cut off in this manner, what is the maximum sum of the number of faces, edges and vertices on the new polyhedron?

104. \$ \_\_\_\_\_ Four oranges cost a total of 90 cents. At this rate, what is the cost of 3 dozen oranges?

105. \_\_\_\_\_

When fully matured, a grape contains 80% water. After the drying process, called dehydration, the resulting raisin is only 20% water. What fraction of the original water in the grape remains after dehydration? Express your answer as a common fraction.

106. \_\_\_\_\_

The proper factors of a positive integer are all of the distinct positive integer factors of the number except the number itself. An abundant number is a positive integer whose proper factors sum to a value greater than the number. Which abundant number less than 50 has the greatest proper factor sum?

107. \_\_\_\_\_

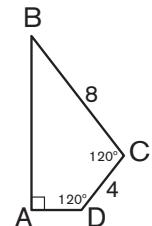
If  $(AAA)^3 = A6,926,0A7$ , what digit does A represent?

108. \_\_\_\_\_ liters

A Vermont syrup maker has 100 liters of a mixture that is  $\frac{1}{4}$  maple syrup and  $\frac{3}{4}$  base. She wants to add enough maple syrup to bring the ratio of maple syrup to base up to 1:1. If she has to evaporate 90% of the maple sap to get the maple syrup to add to that mixture, how many liters of maple sap does she need to start with?

109. \_\_\_\_\_ units<sup>2</sup>

In quadrilateral ABCD,  $m\angle C = m\angle D = 120$  degrees,  $m\angle A = 90$  degrees,  $BC = 8$ ,  $CD = 4$ . What is the area of ABCD? Express your answer in simplest radical form.



110. \_\_\_\_\_ arrangements

Zzyzx Road is in California near Nevada. How many five-letter arrangements of the letters in the English alphabet follow Zzyzx alphabetically?



# Warm-Up 8

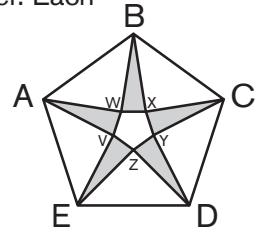
111. \_\_\_\_\_ If  $\frac{x}{y} = 10$ ,  $x = 3z$  and  $z = 20$ , what is the value of  $y$ ?

112. \_\_\_\_\_ Cora has five balls—two red, two blue and one yellow—which are indistinguishable except for their color. She has two containers—one red and one green. If the balls are randomly distributed between the two containers, what is the probability that the two red balls will be alone in the red container? Express your answer as a common fraction.

113. \_\_\_\_\_ units<sup>2</sup> What is the area of a circle that has diameter AB with endpoints A(-2, 4) and B(10, 2)? Express your answer in terms of  $\pi$ .

114. \_\_\_\_\_ sets An arithmetic sequence of integers has 20 as the first term and 56 as the last term. How many different sets of integers form such a sequence?

115. \_\_\_\_\_ degrees In the figure, regular pentagons ABCDE and VWXYZ have the same center. Each side of pentagon ABCDE is the hypotenuse of an isosceles right triangle. In each right triangle, the vertex opposite the hypotenuse is a vertex of pentagon VWXYZ. Each side of the smaller regular pentagon VWXYZ is also the base of one of the shaded acute isosceles triangles. What is the degree measure of the vertex angle of each shaded triangle?

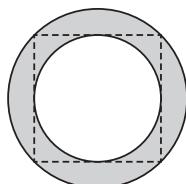


116. \_\_\_\_\_ A 12-foot by 12-foot square bathroom needs to be tiled with 1-foot square tiles. Two of the tiles are the wrong color. If the tiles are placed randomly, what is the probability that the two wrong-colored tiles share an edge? Express your answer as a common fraction.

117. \_\_\_\_\_ What digit is in the units place in the product  $3^{17} \times 7^{23}$ ?

118. \_\_\_\_\_ What is the geometric mean of 14 and 126?

119. \_\_\_\_\_ in<sup>2</sup> The figure shows a square inscribed in a circle of radius 12 inches, and another circle inscribed within that square. What is the area of the shaded region between the two circles? Express your answer in terms of  $\pi$ .



120. \_\_\_\_\_ base 3 What is the value of the sum  $321_5 + 321_4$  when written in base 3?



# Workout 5

121. \_\_\_\_\_

A set of six different positive integers has a median and mean of 6. If the largest number in the set is 12, what is the largest possible sum for the three largest numbers?

122. \_\_\_\_\_ minutes per mile

Nathan ran 2.5 miles at a pace of 7 minutes 36 seconds per mile. If he wishes to complete the entire 5-mile run at an average pace of 7 minutes 24 seconds per mile, what should his pace be for the next 2.5 miles? Express your answer as a decimal to the nearest tenth.

123. \_\_\_\_\_

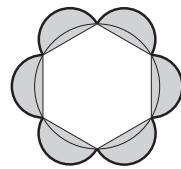
The graphs of  $y = x^2 - 3x + 3$  and  $4x - 12y = -19$  intersect in two points. What is the sum of the  $x$ -coordinates of those points? Express your answer as a common fraction.

124. \_\_\_\_\_ cm

In right triangle ABC,  $m\angle B = 30$  degrees and BC = 9 cm. If D is on side BC so that segment AD bisects acute  $\angle A$ , what is DC?

125. \_\_\_\_\_ units

A regular hexagon is inscribed in a circle of radius 1 unit. On each side of the hexagon, a semicircle is constructed with the side of the hexagon as a diameter, as shown. What is the perimeter of the figure formed by these semicircles? Express your answer as a decimal to the nearest tenth.



126. \_\_\_\_\_ integers

For how many three-digit positive integers is the sum of the digits of the integer equal to 9?

127. \_\_\_\_\_

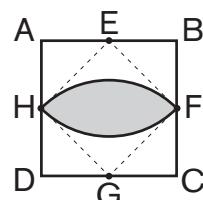
A toy manufacturer produces blue yo-yos and red yo-yos simultaneously at the same rate. During production, yo-yos of each color exit the assembly line in random order. What is the probability that the next four yo-yos that exit are all the same color? Express your answer as a common fraction.

128. \_\_\_\_\_ cm

Ginger wants to make bubble tea in a cylinder-shaped cup with inside measurements of diameter 6 cm and height 12 cm. After she places 48 identical spherical tapioca bubbles into her empty cup, exactly  $100\pi$  mL of liquid will fill the cup right to the top. Given that 1 mL = 1  $\text{cm}^3$ , what is the radius of each tapioca bubble? Express your answer as a decimal to the nearest tenth.

129. \_\_\_\_\_  $\text{m}^2$ 

Square ABCD, shown here, has side length 2 meters, and E, F, G and H are midpoints of the sides. The curved lines are arcs of circles with centers at E and G. What is the area of the shaded region? Express your answer as a decimal to the nearest hundredth.



130. \_\_\_\_\_

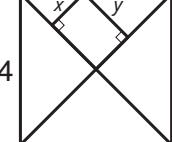
Tony chooses a positive integer  $k$ . After his friends make the following five statements, Tony says that exactly two of them are correct. What is the least possible value of  $k$ ?

- a. Bruce guesses that  $k$  is a multiple of 15.
- b. Steven guesses that  $k$  is a multiple of 18.
- c. Thor guesses that  $k$  is a multiple of 20.
- d. Clint guesses that  $k$  is a multiple of 28.
- e. Natasha guesses that  $k$  is a multiple of 60.



# Workout 6

131. \_\_\_\_\_ Caynan wrote a sequence of consecutive integers beginning with  $-37$ . If the sum of the integers he wrote is  $200$ , what is the greatest integer in the sequence Caynan wrote?

132. \_\_\_\_\_  In the figure, the segments of lengths  $x$  and  $y$  lie on perpendiculars to the diagonals of a square of side length  $4$ . The sum  $x + y$  can be written in the form  $\sqrt{z}$ . What is the value of  $z$ ?

133. \_\_\_\_\_ A box contains  $26$  slips of paper, each showing a different letter of the alphabet. If two slips of paper are drawn from the box at the same time, what is the probability that both letters appear in the word ALGEBRA? Express your answer as a common fraction.

134. \$ \_\_\_\_\_ In Swimmington, where Maxwell lives, the charge for water usage is  $0.15$  cent per gallon. Maxwell has a cylindrical pool of height  $4.5$  feet and diameter  $24$  feet. What is the cost for Maxwell to fill his pool so that the water surface is  $3$  inches below the top of the pool, given that  $1$  gallon =  $231$  in $^3$ ? Express your answer to the nearest whole number.

135. \_\_\_\_\_ base 8 What is  $10111010_2$  when written in base  $8$ ?

136. \_\_\_\_\_ spheres Solid metal spheres with diameter  $\frac{1}{6}$  inch are dropped into a rectangular prism tank, where they sink to the bottom. The tank is  $10$  inches wide by  $15$  inches long by  $8$  inches deep, and the water level is currently  $3$  inches. How many spheres does it take to raise the water level  $1$  inch? Express your answer to the nearest hundred.

137. \_\_\_\_\_ % Recently, the manufacturer changed how Leon's favorite pens are sold. The price of a box of pens has been reduced by  $10\%$ , and there are now  $25\%$  fewer pens per box. What is the percent change in the cost per pen?

138. \_\_\_\_\_ What is the greatest integer  $n$  such that  $n!$  has  $n$  digits?

139. \_\_\_\_\_ Adam has a triangle with vertices labeled  $1$  through  $3$ . Jayvon has an octagon with vertices labeled  $1$  through  $8$ . Each boy starts at position  $1$  and counts consecutive vertices on his polygon, continuing in the same direction, until he has reached  $120$  and is back at the vertex labeled  $1$ . Percy did the same activity with his polygon, and he also finished at the vertex labeled  $1$ . If Percy's polygon is not a triangle or an octagon, what is the sum of all the possible numbers of sides his polygon might have?

140. \_\_\_\_\_ On Mars a day is called a sol. Mars has a  $668$ -sol year with a  $7$ -sol week. If a regular Martian year has  $95$  weeks, and a leap year is one week longer, what fraction of the years are leap years? Express your answer as a common fraction.



# Warm-Up 9

141. \_\_\_\_\_

Two positive integers have a sum of 11 and a product of 24. What is the absolute difference between those two numbers?

142. \_\_\_\_\_

Becca and Varun are walking side by side at the same constant speed. Becca steps onto a moving walkway and continues to walk at the same speed, while Varun walks alongside, maintaining his speed. When Becca reaches the end, Varun has covered only two-fifths of the length of the walkway. What is the ratio of the walkway's rate to Becca and Varun's walking speed? Express your answer as a common fraction.

143. \_\_\_\_\_ %

Two sides of a regular pentagon are doubled and a new pentagon is formed. By what percent is the perimeter increased?

144. \_\_\_\_\_ quarters

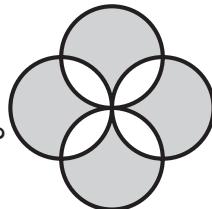
Joe has some nickels, dimes and quarters. He has 37 coins in all, with 4 more nickels than dimes and 2 more quarters than nickels. How many quarters does Joe have?

145. \_\_\_\_\_ numbers

A *repeating* integer is one in which a sequence of digits occurs two or more times to make the entire number. The 4-digit number 4242 is a repeating integer. How many numbers are six-digit repeating integers?

146. \_\_\_\_\_  $\text{cm}^2$ 

Lucky draws a four-leaf clover by shading portions of four overlapping circles of radius 2 cm as shown. What is the area of the shaded regions?

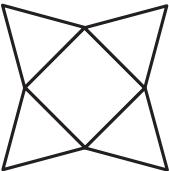


147. \_\_\_\_\_ dates

Eve's cousin, Fin, lives in a different country. According to a postcard Eve got, Fin plans to visit the U.S., but Eve can't tell the exact date of Fin's visit, because of the way the date is written. Eve doesn't know if the date format used in Fin's country is M/D or D/M, where M and D are different and represent the two-digit month and two-digit day, respectively. For how many dates in the year would both interpretations of the date written on the postcard result in a valid date?

148. \_\_\_\_\_

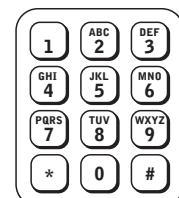
The net of a square pyramid, shown here, is a square with an equilateral triangle on each of its sides. The side length of the square can be expressed as  $6x - 6$  or  $2x + 14$ , for the same value of  $x$ . When the net is folded to form a square pyramid, its surface area can be expressed in simplest radical form as  $a^2(\sqrt{b} + c)$ . What is the value of  $a + b + c$ ?



149. \_\_\_\_\_

What single digit does D represent when  $2D \times D51 = 807D$ ?

150. \_\_\_\_\_ extensions



The numbers 2 through 9 on a telephone keypad, like the one shown, are associated with the letters of the alphabet. Each person in a particular office is assigned a phone extension based on the first three letters of his or her last name. For instance, John DOE has the extension 363, and Marrella JOHnson has the extension 564. How many unique three-digit extensions can be assigned using the digits 2 through 9?



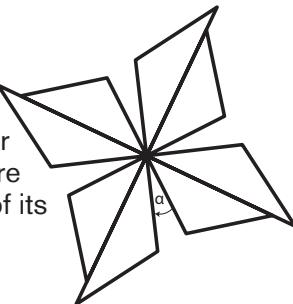
# Warm-Up 10

151. \_\_\_\_\_ inches Keaton wants to build a rectangular prism with volume  $2016 \text{ in}^3$  so that the length of each edge is a whole number of inches. What is the least possible sum of the three dimensions of the prism he builds?

152. \_\_\_\_\_ Cody's ZIP code is a five-digit number whose digits are all different. In this number, there are two pairs of adjacent digits in which the digits differ by 1. There is a pair of adjacent digits in which one digit is 4 times the other. There is a pair of adjacent digits whose sum is 10, as well as a pair of non-adjacent digits whose sum is 10. The sum of all five digits is a multiple of 10. If the leftmost digit of the number is 7, what is Cody's ZIP code?

153. \_\_\_\_\_ The product of the 3-digit number ABC and its reverse, CBA, is 140,209. If A, B and C each represent a different digit, what is the value of  $A + B + C$ ?

154. \_\_\_\_\_ degrees A fan design has four pairs of similar isosceles triangles that create four blades, as shown. In each pair of triangles, the base of the smaller triangle is a segment of the base of the larger triangle, and the measure of the vertex angle of each triangle is twice the sum of the measures of its base angles. What is the degree measure of the angle labeled  $\alpha$ ?



155. \_\_\_\_\_ teams  Twelve couples participate in a fitness retreat. One strength-building exercise requires participants to form teams of three so that the two people who make up a couple are not on the same team. How many different teams of three can be formed in this manner?

156. \_\_\_\_\_ What common fraction is equivalent to  $0.\overline{327}$ ?

157. \_\_\_\_\_ Parallel lines  $l$ ,  $m$  and  $n$  are in a plane with line  $m$  a distance of 1 cm from each of the lines  $l$  and  $n$ . Line  $l$  is tangent to a circle that has radius 3 cm. Lines  $m$  and  $n$  intersect the circle, and the four points of intersection are connected to form a trapezoid. If the area of the trapezoid is expressed in the form  $\sqrt{a} + \sqrt{b}$   $\text{cm}^2$ , what is the value of the product  $ab$ ?

158. \_\_\_\_\_ What is the value of the expression  $\frac{2017^2 + 11(2017) - 42}{2014}$ ?

159. \_\_\_\_\_ What is the greatest integral value of  $n$  for which  $32!$  has  $2^n$  as a factor?

160. \_\_\_\_\_ combinations  Storm places coins, having a total value of at least \$1.00, in a bag. The coins may include pennies, nickels, dimes and quarters, but no more than three of any single denomination. How many different combinations of coins can Storm place in the bag?



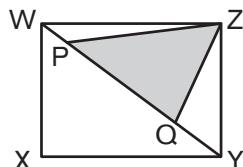
# Warm-Up 11

161.

Three students each flip three fair coins. What is the probability that all three students get the same number of tails? Express your answer as a common fraction.

162. \_\_\_\_\_ years

Twins Taylor and Tyler were born on 05/02/07. This date is referred to as a *sum date* because the sum of the month and day is equal to the two-digit year:  $05 + 02 = 07$ . How many years in the 21st century will have a sum date in each month during that year?

163. \_\_\_\_\_  $\text{cm}^2$ 

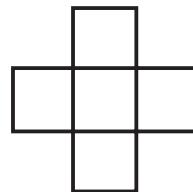
The area of rectangle WXYZ is  $90 \text{ cm}^2$ . P and Q are points on diagonal WY such that  $3(WP + QY) = 2PQ$ . What is the area of triangle PQZ?

164. \_\_\_\_\_

There are 50 equally spaced points marked on a circle. Sara numbers them clockwise from 1 to 50. Starting at 1, she then draws congruent, connected segments between points that are 8 spaces apart, moving clockwise from the end of the previous segment. For example, the first segment is drawn from 1 to 9, and the second from 9 to 17. What is the sum of the numbers on the endpoints of the 23rd segment drawn?

165. \_\_\_\_\_ colorings

The five squares of the diagram shown are to be colored orange, yellow, green, blue and indigo, with exactly one color per square. Two colorings are the same if one is just a rotation of the other (but not if the diagram must be flipped over). How many distinct colorings are there?



166. \_\_\_\_\_

What is the greatest four-digit palindrome that is divisible by 7 and 8?

167. \_\_\_\_\_ players

Tryouts were held for three positions on the school basketball team: center, guard and forward. There were 4 players who tried out for center, 10 for guard and 10 for forward. These numbers include 1 player who tried out for center and guard, 3 who tried out for center and forward, and 4 who tried out for guard and forward. All six of these counts include 1 player who tried out for all three positions. If 17 players, in all, tried out for the team, how many players tried out only for guard?

168. \_\_\_\_\_ cm

When ice melts and becomes water, its volume decreases by 8%. A cylindrical block of ice completely fills a container with a height of 10 cm and a radius of 4 cm. When all of the ice melts, what will be the height of the water in the cylinder? Express your answer as a decimal to the nearest tenth.

169. \_\_\_\_\_ values

A triangle exists with side lengths  $2x$ ,  $3x + 7$  and  $6x - 5$  for how many integer values of  $x$ ?

170. \_\_\_\_\_

If  $x$  and  $y$  are integers, such that  $x > y$ ,  $(x + y)^2 = 9$  and  $x^2 + y^2 = 29$ , what is the smallest possible value for  $x$ ?



# Workout 7

171. \_\_\_\_\_ in<sup>2</sup> A circular pizza is cut along four diameters into eight identical sectors. If the total perimeter of each sector is 10 inches, what is the area of the whole pizza? Express your answer as a decimal to the nearest tenth.

172. \_\_\_\_\_ minutes Because of a traffic jam, Alana's 18-mile commute to work took 4 minutes longer than usual, and her average speed was decreased by 9 mi/h. How many minutes did it take her to get to work that day?

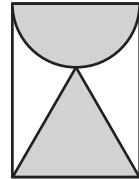
173. \_\_\_\_\_ g<sub>n</sub> The standard gravitational acceleration of an object near Earth's surface is  $g_n \approx 32 \text{ ft/s}^2$ . Kingda Ka is a roller coaster at the Six Flags amusement park in Jackson, New Jersey. It accelerates from a stop to 128 mi/h in 3.5 seconds. What is the acceleration of the roller coaster as a multiple of  $g_n$ ? Express your answer as a decimal to the nearest tenth.

174. \_\_\_\_\_ % To the nearest whole percent, what percent of all positive integers are not multiples of 2, 3, 4, 5 or 6?

175. \_\_\_\_\_ Seventy-five bingo balls, each with a different positive integer from 1 through 75, are placed in a cage. A random ball is selected, its number is announced, and the ball is returned to the cage. This process occurs a total of 20 times. What is the probability that at least one ball is selected more than once? Express your answer as a decimal to the nearest hundredth.

176. \_\_\_\_\_ Tina and Tricia play on a softball team. Tina has 8 hits out of 20 times at bat, and Tricia has 6 hits out of 16 times at bat. Based on their past performance, what is the probability that both girls will get a hit the next time they bat? Express your answer as a common fraction.

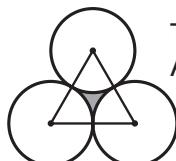
177. \_\_\_\_\_ % To create a new flag design, Howard paints a semicircle and an equilateral triangle inscribed in a rectangle as shown. What percent of the flag does the painted area cover? Express your answer to the nearest whole number.



178. \_\_\_\_\_ The inhabitants of the planet Rundia run footraces similar to those run on Earth. However, Rundians measure the distances that they run in *bars*, where one bar measures four-fifths of a meter, and they measure time in *ticks*, where there are 100 ticks in one minute. Rundian sprinter Sejes Wesno can run a 100-bar race in 11.53 ticks. The great Earth sprinter Usain Bolt ran a 100-meter race in 9.58 seconds. If Wesno is  $k$  times as fast as Bolt, what is the value of  $k$ ? Express your answer as a decimal to the nearest hundredth.

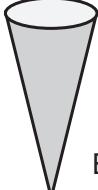
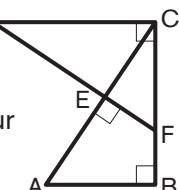
179. \_\_\_\_\_ Jason's line has a slope of  $-\frac{1}{3}$  and contains the point  $(5, -2)$ . Amisha's line is perpendicular to Jason's and passes through the point  $(4, 1)$ . If the intersection of these lines is  $(x, y)$ , what is the value of  $x + y$ ? Express the answer as a common fraction.

180. \_\_\_\_\_ units<sup>2</sup> Three circles, each having radius 4 units, are externally tangent to each other. A triangle joins the centers of the circles. What is the area of the shaded region within the triangle but outside the circles? Express your answer as a decimal to the nearest tenth.





# Workout 8

181. \_\_\_\_\_ units<sup>2</sup> In square WXYZ, point V is the midpoint of side YZ, and the area of  $\triangle XYV$  is  $\frac{4}{5}$  unit<sup>2</sup>. What is the area of square WXYZ? Express your answer as a common fraction.
182. \_\_\_\_\_ tokens Ali gives Stan a closed box that contains at least one of each token worth 5, 11 or 19 points. Ali says that the tokens have a combined value of 56 points. How many tokens are in the box?
183. \_\_\_\_\_ Xera and Yeta use this method to decide who will sit in the front passenger seat of the car. Xera throws a standard six-sided die, after which Yeta picks a card from a standard deck of 52 cards, with replacement. They continue to take turns die-throwing and card-picking until either Xera wins by rolling a four or Yeta wins by picking a card with a four. What is the probability that Yeta wins? Express your answer as a common fraction.
184. \_\_\_\_\_ %  When a cone's height is decreased by a factor of four, to maintain the same volume, the radius must be increased by a factor of two, or 100%. When the cone's height is decreased by a factor of three, by what percent must the radius be increased to maintain the same volume? Express your answer to the nearest whole number. 
185. \_\_\_\_\_ integers Given the expression  $\frac{n^2 - 9}{n^2 - 4}$ , for how many positive integers  $n$  from 1 to 2016, inclusive, is the GCF of the numerator and denominator greater than 1?
186. \_\_\_\_\_ values The mean, the median and the mean of all modes of the integers 7, 4, 5, 6, 5,  $x$  are equal. How many possible values are there for  $x$ ?
187. \_\_\_\_\_ A box contains 15 slips of paper, each bearing a different natural number from 1 to 15, inclusive. If three of these slips are randomly drawn, one at a time, without replacement, what is the probability that three consecutive numbers are drawn in increasing order? Express your answer as a common fraction.
188. \_\_\_\_\_ inches A rectangle has a diagonal of length 8 inches and an area of 26 square inches. What is its perimeter? Express your answer in simplest radical form.
189. \_\_\_\_\_ units  In pentagon ABCDE, with right angles ABC, BCD and AEF, as shown, AE = 14, DE = 18 and EF = 8. What is the length of side BC? Express your answer in simplest radical form.
190. \_\_\_\_\_ hours Pump P can fill a water tank in 12 hours, and Pump Q can fill the same tank in 15 hours. The two pumps started filling the tank at the same time and worked together until the tank was 60% full. At that point, Pump P was turned off, and Pump Q continued to fill the tank until it was completely full. How many hours did it take to completely fill the tank?



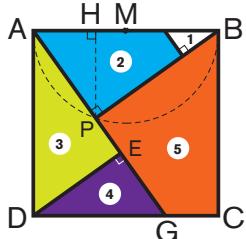
# Warm-Up 12

191. \_\_\_\_\_ minutes The digital clock shown can display up to four digits to represent the hour and the minute. For how many minutes in a 12-hour period does the digit 0 appear on the clock?

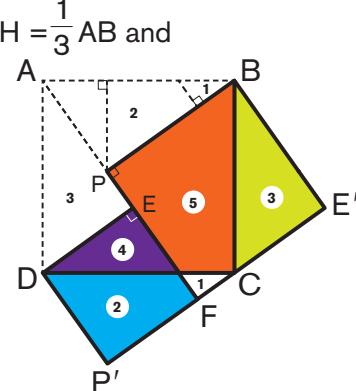


192. \_\_\_\_\_ What is the absolute difference between  $1.\overline{18}$  and  $2.\overline{36}$ ? Express your answer as a common fraction.

193. \_\_\_\_\_ inches A square is dissected into five pieces as shown on the left, with  $AH = \frac{1}{3}AB$  and  $AM = MP = MB$ . The pieces are shaded and numbered. The pieces are rearranged so as to form two squares, one with twice the area of the other. The rearrangement of the pieces is shown on the right, overlaid on top of the original square.



If square  $DEFP'$  in the second picture has a side length of 1 inch, then what is the side length of the original square? Express your answer in simplest radical form.

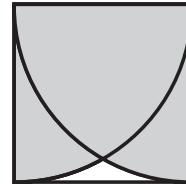


194. \_\_\_\_\_ arrangements How many arrangements of all eight letters in TRESPASS do not have S as the final letter?

195. \_\_\_\_\_ Four vertices of a regular octagon are chosen at random. What is the probability that a square can be made by connecting the vertices? Express your answer as a common fraction.

196. \_\_\_\_\_ mi/h A private jet made a trip from Denver to Los Angeles in 3 hours, flying against a steady headwind. On the return trip the wind speed doubled and became a tailwind. The return trip took only 2.5 hours. If the plane's speed on the return trip was 450 mi/h, what was the speed of the original headwind?

197. \_\_\_\_\_ In the square shown, the side lengths are 6 cm, and the intersecting arcs are quarter-circles. The area of the shaded region, expressed in simplest radical form in terms of  $\pi$ , is  $a\pi + b\sqrt{c}$  cm<sup>2</sup>. What is the value of  $a + b + c$ ?



198. \_\_\_\_\_ prisms A rectangular prism with length  $\geq$  width  $\geq$  height has positive integer dimensions and a volume of 60 units<sup>3</sup>. How many different prisms are there that meet these conditions?

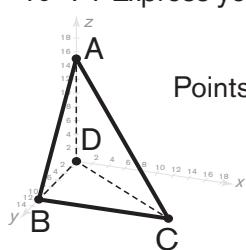
199. \_\_\_\_\_ solutions
- 
- The numbers 1, 2, 3, 4, 5 and 6 are to be placed in this figure, one number per circle, so that the sums of the numbers on each side of the triangle are the same. How many distinct solutions are there, not including rotations and reflections?

200. \_\_\_\_\_ minutes Xena runs halfway across a field in 1 minute. The next fourth of the field takes her  $\frac{2}{3}$  of a minute to cross, the next eighth takes  $\frac{4}{9}$  of a minute, and so on, with each half of the previous distance taking  $\frac{2}{3}$  of the previous time. How many minutes does it take Xena to cross the field?



# Warm-Up 13

201. \_\_\_\_\_ Given  $a - b = 3$  and  $a^2 + b^2 = 65$ , what is the value of  $a^3 - b^3$ ?
202. \_\_\_\_\_ integers How many positive three-digit integers have one digit equal to the average of the other two digits?
203. \_\_\_\_\_ marbles A bag contains only red marbles and green marbles, two of which are to be drawn without replacement. There are at least two marbles of each color in the bag. If the probability of both marbles being red is half the probability of both marbles being green, then what is the minimum possible number of marbles in the bag?
204. \_\_\_\_\_  $M(a, b)$  is the midpoint of the longest side of the triangle bounded by the lines  $x + 2y = 8$ ,  $5x + 2y = 48$  and  $x - 2y = 0$ . What is the value of  $a + b$ ? Express the answer as a common fraction.
205. \_\_\_\_\_ A math club has 16 members. The coach wants to select three boys and three girls to represent their school at a tournament. There are six times as many ways to choose the girls as there are ways to choose the boys. What is the ratio of girls to boys in the club? Express your answer as a common fraction.
206. \_\_\_\_\_ The number 3638 has a digital sum of  $3 + 6 + 3 + 8 = 20$  and a digital product of  $3 \times 6 \times 3 \times 8 = 432$ . What is the absolute difference between the least and greatest four-digit numbers that each have a digital sum of 20 and a digital product of 432?
207. \_\_\_\_\_ Jebediah has two coins in his pocket. One is a fair coin, while the other has heads on both sides. He pulls one coin out at random and flips it three times. If the coin lands heads all three times, what is the probability that it is the fair coin? Express your answer as a common fraction.
208. \_\_\_\_\_ people What is the minimum number of people that must be in a room to ensure that there are three people who, when considered pairwise, all know each other or three people who, when considered pairwise, all do not know each other?
209. \_\_\_\_\_ % Tire pressure is directly proportional to temperature on a temperature scale where zero degrees is absolute zero. Given that temperatures in degrees Celsius ( $C$ ) and degrees Fahrenheit ( $F$ ) are related by the formula  $F = \frac{9}{5}C + 32$ , and absolute zero is  $-273.15^\circ\text{C}$ , by what percent does tire pressure decrease when the temperature drops from  $80^\circ\text{F}$  to  $40^\circ\text{F}$ ? Express your answer to the nearest whole number.
210. \_\_\_\_\_ units Points A(0, 0, 15), B(0, 12, 0), C(16, 12, 0) and D(0, 0, 0) determine the vertices of a tetrahedron as shown. What is the shortest distance from B to the face ADC? Express your answer as a common fraction.





# Warm-Up 14

211. \_\_\_\_\_ chords Eight points on a circle are labeled. How many chords can be drawn connecting any two of these points?
212. \_\_\_\_\_ Tad draws three cards at random, without replacement, from a deck of ten cards numbered 1 through 10. What is the probability that no two of the cards drawn have numbers that differ by 1? Express your answer as a common fraction.
213. \_\_\_\_\_ people Twelve people have sheared  $\frac{1}{3}$  of a field of pine trees in 7 days. How many more people need to be added to the crew to shear the rest of the trees in the field in the next 6 days?
214. \_\_\_\_\_ If three distinct integer lattice points are randomly selected from the interior of the circle defined by  $x^2 + y^2 = 8$ , what is the probability that they are the vertices of a triangle? Express your answer as a common fraction.
215. \_\_\_\_\_ numbers For how many four-digit numbers is the sum of the digits equal to the product of the digits?
216. \_\_\_\_\_ cubes A cube has one red, one green, one yellow and three blue faces. How many distinct cubes satisfying this description are possible?
217. \_\_\_\_\_ units In right triangle PQR, shown here, M is on PQ such that  $PM = MR$ . If  $PQ = 12$  units and  $QR = 9$  units, what is the value of  $MQ$ ? Express your answer as a common fraction.
218. \_\_\_\_\_ Karla writes down six different prime numbers in increasing order. She notices that the product of the first three prime numbers she has written is equal to the sum of the last three prime numbers she has written. What is the least possible value of the last prime number Karla wrote?
219. \_\_\_\_\_ Circle A has center  $(0, 0)$  with radius 4. Circle B has center  $(40, 40)$  with radius 6. The radius of circle B increases at twice the rate as the radius of circle A increases. When the circles are externally tangent at  $P(x, y)$ , which is located between the centers of circles A and B, what is the value of  $x$ ? Express your answer as a fraction in simplest radical form.
220. \_\_\_\_\_ ways   
Figure 1  
Figure 1 shows a  $2 \times 1$  gray rectangle and a  $6 \times 4$  white rectangle with the midpoints of two sides labeled A and B. If the larger rectangle cannot be rotated, in how many ways can 12 of the gray rectangles be arranged inside the white rectangle so that none crosses a segment from A to B? One such arrangement is shown in Figure 2, but not in Figure 3.  
YES   
Figure 2  
NO   
Figure 3



# Fractions Stretch

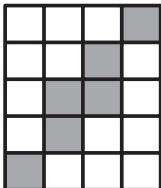
Solve the following problems. Express any non-integer answer as a common fraction.

221. \_\_\_\_\_ What fraction of 100 is 25?

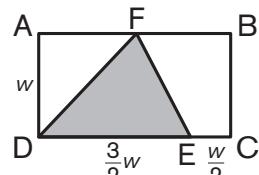
222. \_\_\_\_\_ What fraction of  $\frac{3}{8}$  is  $\frac{9}{16}$ ?

223. \_\_\_\_\_ What is the value of  $\sqrt{\frac{3}{11} \div \frac{11}{12}}$ ?

224. \_\_\_\_\_ What fractional part of this grid of 20 unit squares is shaded?



225. \_\_\_\_\_ What fraction of the area of rectangle ABCD is the area of inscribed triangle DEF?



226. \_\_\_\_\_ On a number line, what common fraction is  $\frac{3}{4}$  of the way from  $\frac{1}{2}$  to  $\frac{3}{4}$ ?

227. \_\_\_\_\_ What is the reciprocal of  $\frac{1}{2 + \frac{1}{3}}$ ?

228. \_\_\_\_\_ What common fraction is equal to 0. $\overline{75}$ ?

229. \_\_\_\_\_ If  $\frac{1}{\frac{1}{n} + \frac{1}{3}} + \frac{1}{\frac{1}{3} + \frac{1}{n}} = \frac{5}{12}$ , what is the value of  $n$ ?

230. \_\_\_\_\_ If  $\frac{2x}{x-3} - 2 = \frac{4}{x+2}$ , what is the value of  $x$ ?

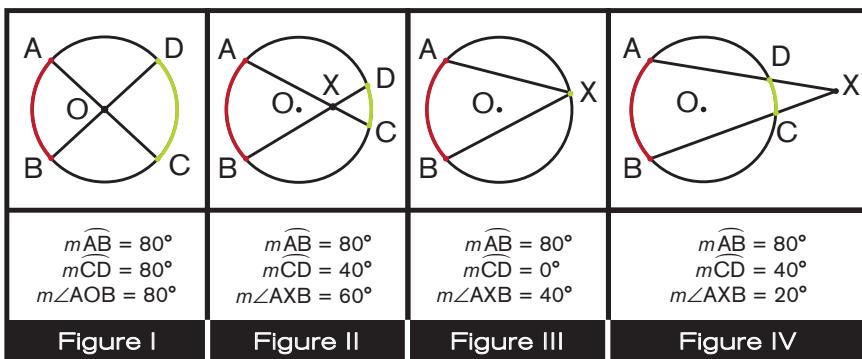


# Angles and Arcs Stretch

<b>S<sub>E</sub>CANT</b>	a line that intersects the circle at two points
<b>C<sub>H</sub>ORD</b>	a line segment whose endpoints are two points on the circle
<b>T<sub>A</sub>NGET</b>	a coplanar line that intersects the circle at a single point of tangency
<b>C<sub>E</sub>NTRAL A<sub>N</sub>GLE</b>	an angle with its vertex at the center of the circle
<b>I<sub>N</sub>SCRIBED A<sub>N</sub>GLE</b>	an angle with its vertex on the circle and whose sides are chords of the circle
<b>M<sub>A</sub>JOR A<sub>R</sub>C</b>	an arc of the circle with measure greater than or equal to 180°
<b>M<sub>I</sub>NOR A<sub>R</sub>C</b>	an arc of the circle with measure less than 180°

## ANGLE AND ARC MEASURES

In the figures below, observe how the degree measure of  $\angle AXB$  decreases as the distance between the vertex of the angle and the center of the circle increases.



- In Figure I, angles AOB and COD are central angles of circle O that intercept arcs AB and CD, respectively. The degree measure of a central angle and the arc it intercepts are equal.

$$m\angle AOB = m\widehat{AB} \text{ and } m\angle COD = m\widehat{CD}$$

- In Figure II, vertical angles AXB and CXD, formed by the intersection of chords AC and BD inside circle O, intercept arcs AB and CD, respectively. The degree measure of vertical angles formed by two chords intersecting inside a circle is half the sum of the measures of their intercepted arcs.

$$m\angle AXB = m\angle CXD = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$$

- In Figure III,  $\angle AXB$  is inscribed in circle O. The degree measure of an inscribed angle is half the measure of the intercepted arc.

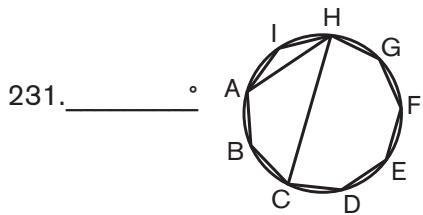
$$m\angle AXB = \frac{1}{2}m\widehat{AB}$$

- In Figure IV,  $\angle AXB$ , formed by the intersection of two secants at point X outside of circle O, intercepts arcs AB and CD. The degree measure of an angle formed by two secants, two tangents or a secant and a tangent is half the difference of the measures of its intercepted arcs.

$$m\angle AXB = \frac{1}{2}(m\widehat{AB} - m\widehat{CD})$$

It may appear that there are four different formulas for calculating the four types of angles. But in each case, the measure of the angle in question is, essentially, the average of the measures of the intercepted arcs. In Figure IV, note that, with respect to  $\angle AXB$ ,  $\widehat{AB}$  appears concave, while  $\widehat{CD}$  appears convex. So the measure of  $\angle AXB$  can be thought of as the average of 80° and -40°.

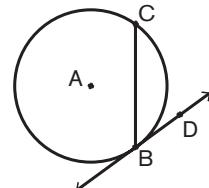
Solve the following problems by using what you've learned about angles and arcs. Express any non-integer value as a decimal to the nearest tenth.



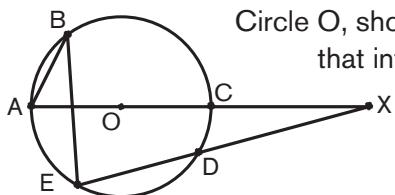
231. \_\_\_\_\_ ° Regular nonagon ABCDEFGHI is inscribed in a circle, as shown.

What is  $m\angle AHC$ ?

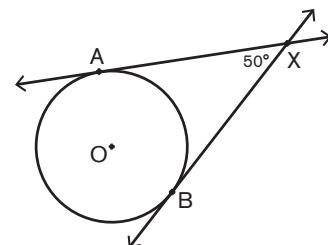
232. \_\_\_\_\_ ° In circle A, shown here,  $\overleftrightarrow{BD}$  is tangent to the circle at B, and major  $\widehat{BC}$  has measure  $230^\circ$ . What is  $m\angle CBD$ ?



233. \_\_\_\_\_ ° Circle O, shown here with chords AB and BE, has secants AC and DE that intersect at X. If  $m\angle ABE = 35^\circ$  and  $m\angle AXE = 15^\circ$ , what is the measure of  $\widehat{CD}$ ?

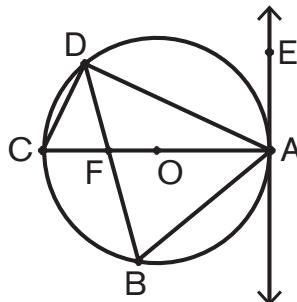


234. \_\_\_\_\_ ° In this figure, lines AX and BX are tangent to circle O at A and B, respectively. If  $m\angle AXB = 50^\circ$ , what is the measure of major  $\widehat{AB}$ ?



*Use the figure at the right for questions 235 through 238.*

235. \_\_\_\_\_ ° What is  $m\angle ABD$ ?



$\overleftrightarrow{AE}$  is tangent to circle O

$\overleftrightarrow{AE} \perp \overline{AC}$

$m\angle BDC = 40^\circ$

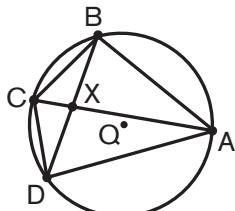
$m\widehat{AD} = 125^\circ$

236. \_\_\_\_\_ ° What is  $m\widehat{AB}$ ?

237. \_\_\_\_\_ ° What is  $m\angle BAE$ ?

238. \_\_\_\_\_ ° What is  $m\angle CFD$ ?

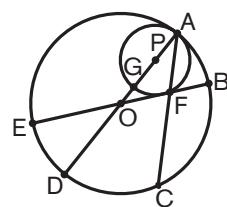
239. \_\_\_\_\_ °



- Quadrilateral ABCD is inscribed in circle Q, as shown, with diagonals intersecting at X. If  $m\widehat{AB} = 110^\circ$ ,  $m\widehat{BC} = 60^\circ$  and  $AB = BD$ , what is  $m\angle CXD$ ?

240. \_\_\_\_\_ °

- Circle P is internally tangent to circle O at A, as shown.  $\overline{AC}$  and  $\overline{BE}$  intersect at F, which is also the point of tangency between  $\overline{BE}$  and circle P.  $\overline{AD}$  and  $\overline{BE}$  are diameters of circle O, and  $\overline{AG}$  is a diameter of circle P. If  $m\widehat{CD} = 50^\circ$ , what is the measure of minor  $\widehat{BC}$ ?





# Bases Stretch

The **base 10** number system, the number system we are most familiar with, uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Numerals with these digits in the ones, tens, hundreds and higher places express specific numerical quantities. In base 10, the number 245, for example, is composed of 2 hundreds, 4 tens and 5 ones. That is,  $2(10^2) + 4(10^1) + 5(10^0) = 200 + 40 + 5 = 245$ .

A **base  $b$**  number system uses the digits 0, 1, ...,  $b - 1$ . Numerical quantities are expressed with these digits in the  $b^0$ ,  $b^1$ ,  $b^2$  and higher places. In base  $b$ , if  $b \geq 6$ , the numeral  $245_b$  represents the number  $2(b^2) + 4(b^1) + 5(b^0)$ . In base 8, for example,  $245_8 = 2(8^2) + 4(8^1) + 5(8^0) = 2(64) + 4(8) + 5(1) = 128 + 32 + 5 = 165$ .

Bases greater than 10 use letters to represent the digits greater than 9. For example, the 12 digits used in base 12 are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B. The numeral 10 in base 12 has 1 twelve and 0 ones. That is,  $10_{12} = 1(12^1) + 0(12^0) = 1(12) + 0(1) = 12 + 0 = 12$ .

## Practice Problems

What is the representation of each of the following in base 10?

241. \_\_\_\_\_  $24_9$

242. \_\_\_\_\_  $24_8$

243. \_\_\_\_\_  $24_7$

What is the representation of 24 in each of the following bases?

244. \_\_\_\_\_ base 9

245. \_\_\_\_\_ base 8

246. \_\_\_\_\_ base 7

## Now try these.

247. \_\_\_\_\_ What is the representation of 4991 in base 12?

248. \_\_\_\_\_ What is the representation of  $3BB_{12}$  in base 6?

249. \_\_\_\_\_ If  $523_b = 262$ , what is the value of  $b$ ?

250. \_\_\_\_\_ If  $441_b = n^2$  and  $351_b = (n - 2)^2$ , for some  $b < 10$ , what is the value of  $n$ ?



## OFFICIAL RULES + PROCEDURES

The following rules and procedures govern all MATHCOUNTS competitions. The MATHCOUNTS Foundation reserves the right to alter these rules and procedures at any time. **Coaches are responsible for being familiar with the rules and procedures outlined in this handbook.** Coaches should bring any difficulty in procedures or in student conduct to the immediate attention of the appropriate chapter, state or national official. Students violating any rules may be subject to immediate disqualification.

Any questions regarding the MATHCOUNTS Competition Series Official Rules + Procedures articulated in this handbook should be addressed to the MATHCOUNTS national office at (703) 299-9006 or [info@mathcounts.org](mailto:info@mathcounts.org).

## REGISTRATION

**The fastest and easiest way to register for the MATHCOUNTS Competition Series is online at [www.mathcounts.org/compreg](http://www.mathcounts.org/compreg).**

For your school to participate in the MATHCOUNTS Competition Series, a school representative is required to complete a registration form and pay the registration fees. A school representative can be a teacher, administrator or parent volunteer who has received expressed permission from his/her child's school administration to register. By completing the Competition Series Registration Form, the coach attests to the school administration's permission to register students for MATHCOUNTS.

School representatives can register online at [www.mathcounts.org/compreg](http://www.mathcounts.org/compreg) or download the Competition Series Registration Form and mail or email a scanned copy of it to the MATHCOUNTS national office. Refer to the Critical 2016-2017 Dates on pg. 10 of this handbook for contact information.

**WHAT REGISTRATION COVERS:** Registration in the Competition Series entitles a school to:

- 1) send 1-10 student(s)—depending on number registered—to the Chapter Competition. *Students can advance beyond the chapter level, but this is determined by their performance at the competition.*
- 2) receive the School Competition Kit, which includes the 2016-2017 MATHCOUNTS School Handbook, one recognition ribbon for each registered student, 10 student participation certificates and a catalog of additional coaching materials. *Mailings of School Competition Kits will occur on a rolling basis through December 31, 2016.*
- 3) receive online access to the 2017 School Competition, along with electronic versions of other competition materials at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches). *Coaches will receive an email notification no later than November 1, 2016 when the 2017 School Competition is available online.*

Your state or chapter coordinator will be notified of your registration, and then you will be informed of the date and location of your Chapter Competition. **If you have not been contacted by mid-January with competition details, it is your responsibility to contact your local coordinator** to confirm that your registration has been properly routed and that your school's participation is expected. Coordinator contact information is available at [www.mathcounts.org/findmycoordinator](http://www.mathcounts.org/findmycoordinator).

**DEADLINES:** The sooner your Registration Form is received, the sooner you will receive your preparation materials. To guarantee your school's participation, submit your registration by one of the following deadlines:

<i>Early Bird Discount Deadline:</i> November 18, 2016	Online registrations: submitted by 11:59 PST Emailed forms: received by 11:59 PST Mailed forms: postmarked by November 18, 2016
<i>Regular Registration Deadline:</i> December 16, 2016*	Online registrations: submitted by 11:59 PST Emailed forms: received by 11:59 PST Mailed forms: postmarked by December 16, 2016

\*Late Registrations may be accepted at the discretion of the MATHCOUNTS national office and your local coordinators, but are not guaranteed. If a school's late registration is accepted, an additional \$20 processing fee will be assessed.

**REGISTRATION FEES:** The cost of your school's registration depends on when your registration is postmarked/mailed/submitted online. The cost of your school's registration covers the students for the entire Competition Series; there are no additional registration fees to compete at the state or national level. Title I schools (as affirmed by a school's administration) receive a 50% discount off the total cost of their registration.

Number of Registered Students	Registration Postmarked by <b>11/18/2016</b>	Postmarked between	Postmarked after
		<b>11/18/2016 and 12/16/2016</b>	<b>12/16/2016 (+ Late Fee)</b>
1 individual	\$25	\$30	\$50
2 ind.	\$50	\$60	\$80
3 ind.	\$75	\$90	\$110
1 team of 4	\$90	\$100	\$120
1 tm. + 1 ind.	\$115	\$130	\$150
1 tm. + 2 ind.	\$140	\$160	\$180
1 tm. + 3 ind.	\$165	\$190	\$210
1 tm. + 4 ind.	\$190	\$220	\$240
1 tm. + 5 ind.	\$215	\$250	\$270
1 tm. + 6 ind.	\$240	\$280	\$300

## ELIGIBILITY REQUIREMENTS

Eligibility requirements for the MATHCOUNTS Competition Series are different from other MATHCOUNTS programs. Eligibility for the National Math Club or the Math Video Challenge does not guarantee eligibility for the Competition Series.

### WHO IS ELIGIBLE:

- U.S. students enrolled in the 6th, 7th or 8th grade can participate in MATHCOUNTS competitions.
- Schools that are the students' official school of record can register.
- Any type of school, of any size, can register—public, private, religious, charter, virtual or homeschooled—but virtual and homeschooled must fill out additional forms to participate (see pgs. 41-42).
- Schools in 50 U.S. states, District of Columbia, Guam, Puerto Rico and Virgin Islands can register.
- Overseas schools that are affiliated with the U.S. Departments of Defense and State can register.

**WHO IS NOT ELIGIBLE:**

- Students who are not full-time 6th, 7th or 8th graders cannot participate, even if they are taking middle school math classes.
- Academic centers, tutoring centers or enrichment programs that do not function as students' official school of record cannot register. *If it is unclear whether your educational institution is considered a school, please contact your local Department of Education for specific criteria governing your state.*
- Schools located outside of the U.S. states and territories listed on the previous page cannot register.
- Overseas schools not affiliated with the U.S. Departments of Defense or State cannot register.

**NUMBER OF STUDENTS ALLOWED:** A school can register a maximum of one team of four students and six individuals; these 1-10 student(s) will represent the school at the Chapter Competition. Any number of students can participate at the school level. Prior to the Chapter Competition, coaches must notify their chapter coordinator of which students will be team members and which students will compete as individuals.

**NUMBER OF YEARS ALLOWED:** Participation in MATHCOUNTS competitions is limited to 3 years for each student, but there is no limit to the number of years a student may participate in school-based coaching.

**WHAT TEAM REGISTRATION MEANS:** Members of a school team will participate in the Target, Sprint and Team Rounds. Members of a school team also will be eligible to qualify for the Countdown Round (where conducted). Team members will be eligible for team awards, individual awards and progression to the state and national levels based on their individual and/or team performance. It is recommended that your strongest four Mathletes form your school team. Teams of fewer than four will be allowed to compete; however, the team score will be computed by dividing the sum of the team members' scores by four (see pg. 45), meaning, teams of fewer than four students will be at a disadvantage. *Only one team (of up to four students) per school is eligible to compete.*

**WHAT INDIVIDUAL REGISTRATION MEANS:** Students registered as individuals will participate in the Target and Sprint Rounds, but not the Team Round. Individuals will be eligible to qualify for the Countdown Round (where conducted). Individuals also will be eligible for individual awards and progression to the state and national levels. A student registered as an "individual" may not help his/her school's team advance to the next level of competition. *Up to six students may be registered in addition to or in lieu of a school team.*

**HOW STUDENTS ENROLLED PART-TIME AT TWO SCHOOLS PARTICIPATE:** *A student may compete only for his/her official school of record.* A student's school of record is the student's base or main school. A student taking limited course work at a second school or educational center may not register or compete for that second school or center, even if the student is not competing for his/her school of record. MATHCOUNTS registration is not determined by where a student takes his or her math course. If there is any doubt about a student's school of record, the chapter or state coordinator must be contacted for a decision before registering.

**HOW SMALL SCHOOLS PARTICIPATE:** MATHCOUNTS does not distinguish between the sizes of schools for Competition Series registration and competition purposes. Every "brick-and-mortar" school will have the same registration allowance of up to one team of four students and/or up to six individuals. A school's participants may not combine with any other school's participants to form a team when registering or competing.

**HOW HOMESCHOOLS PARTICIPATE:** Homeschools and/or homeschool groups in compliance with the homeschool laws of the state in which they are located are eligible to participate in MATHCOUNTS competitions in accordance with all other rules. Homeschool coaches must complete the 2016-2017 Homeschool + Virtual School Participation Form, verifying that students from the homeschool or homeschool group are in the 6th, 7th or 8th grade and that each homeschool complies with applicable state laws. Forms can be downloaded at [www.mathcounts.org/competition](http://www.mathcounts.org/competition) and must be submitted to the MATHCOUNTS national office in order for registrations to be processed.

**HOW VIRTUAL SCHOOLS PARTICIPATE:** Virtual schools that want to register must contact the MATHCOUNTS national office by December 9, 2016 for specific registration details. Any student registering as a virtual school student must compete in the MATHCOUNTS Chapter Competition assigned according to the student's home address. Additionally, virtual school coaches must complete the 2016-2017 Homeschool + Virtual School Participation Form, verifying that the students from the virtual school are in the 6th, 7th or 8th grade and that the virtual school complies with applicable state laws. Forms must be submitted to the national office in order for registrations to be processed; forms can be downloaded at [www.mathcounts.org/competition](http://www.mathcounts.org/competition).

**WHAT IS DONE FOR SUBSTITUTIONS OF STUDENTS:** Coaches determine which students will represent the school at the Chapter Competition. Coaches cannot substitute team members for the State Competition unless a student voluntarily releases his/her position on the school team. Additional requirements and documentation for substitutions (such as requiring parental release or requiring the substitution request be submitted in writing) are at the discretion of the State Coordinator. A student being added to a team need not be a student who was registered for the Chapter Competition as an individual. Coaches cannot make substitutions for students progressing to the State Competition as individuals. At all levels of competition, student substitutions are not permitted after on-site competition registration has been completed.

**WHAT IS DONE FOR RELIGIOUS OBSERVANCES:** A student who is unable to attend a competition due to religious observances may take the written portion of the competition up to one week in advance of the scheduled competition. In addition, all competitors from that student's school must take the Sprint and Target Rounds at the same earlier time. If the student who is unable to attend the competition due to a religious observance: (1) is a member of the school team, then the team must take the Team Round at the same earlier time; (2) is not part of the school team, then the team has the option of taking the Team Round during this advance testing or on the regularly scheduled day of the competition with the other school teams. The coordinator must be made aware of the team's decision before the advance testing takes place. *Advance testing will be done at the discretion of the chapter and state coordinators. If advance testing is deemed possible, it will be conducted under proctored conditions.* Students who qualify for an official Countdown Round but are unable to attend will automatically forfeit one place standing.

**WHAT IS DONE FOR STUDENTS WITH SPECIAL NEEDS:** Reasonable accommodations may be made to allow students with special needs to participate. However, many accommodations that are employed in a classroom or teaching environment cannot be implemented in the competition setting. Accommodations that are not permissible include, but are not limited to: granting a student extra time during any of the competition rounds or allowing a student to use a calculator for the Sprint or Countdown Rounds. *A request for accommodation of special needs must be directed to chapter or state coordinators in writing at least three weeks in advance of the Chapter or State Competition.* This written request should thoroughly explain a student's special need, as well as what the desired accommodation would entail. In conjunction with the MATHCOUNTS Foundation, coordinators will review the needs of the student and determine if any accommodations will be made. In making final determinations, the feasibility of accommodating these needs at the National Competition will be taken into consideration.

## LEVELS OF COMPETITION

There are four levels in the MATHCOUNTS Competition Series: school, chapter (local), state and national. Competition questions are written for 6th, 7th and 8th graders. The competitions can be quite challenging, particularly for students who have not been coached using MATHCOUNTS materials. All competition materials are prepared by the national office.

**SCHOOL COMPETITIONS (TYPICALLY HELD IN JANUARY 2017):** After several months of coaching, schools registered for the Competition Series should administer the 2017 School Competition to all interested

students. The School Competition should be an aid to the coach in determining competitors for the Chapter Competition. *Selection of team and individual competitors is entirely at the discretion of the coach and does not need to be based solely on School Competition scores.* School Competition materials are sent to the coach of a school, and it may be used by the teachers and students only in association with that school's programs and activities. The current year's School Competition questions must remain confidential and may not be used in outside activities, such as tutoring sessions or enrichment programs with students from other schools. For updates or edits, please check [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches) before administering the School Competition.

It is important that the coach look upon coaching sessions during the academic year as opportunities to develop better math skills in all students, not just in those students who will be competing. Therefore, it is suggested that the coach postpone selection of competitors until just prior to the Chapter Competition.

**CHAPTER COMPETITIONS (HELD FROM FEB. 1–28, 2017):** The Chapter Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be conducted. The chapter and state coordinators determine the date and location of the Chapter Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The winning team will advance to the State Competition. Additionally, the two highest-ranking competitors not on the winning team (who may be registered as individuals or as members of a team) will advance to the State Competition. This is a minimum of six advancing Mathletes (assuming the winning team has four members). Additional teams and/or individuals also may progress at the discretion of the state coordinator, but the policy for progression must be consistent for all chapters within a state.

**STATE COMPETITIONS (HELD FROM MAR. 1–31, 2017):** The State Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be included. The state coordinator determines the date and location of the State Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The four highest-ranked Mathletes and the coach of the winning team from each State Competition will receive an all-expenses-paid trip to the National Competition.

**2017 RAYTHEON MATHCOUNTS NATIONAL COMPETITION (HELD MAY 14–15 IN ORLANDO, FL):** The National Competition consists of the Sprint, Target, Team and Countdown Rounds (conducted officially). Expenses of the state team and coach to travel to the National Competition will be paid by MATHCOUNTS. The national program does not make provisions for the attendance of additional students or coaches. All national competitors will receive a plaque and other items in recognition of their achievements. Winning teams and individuals also will receive medals, trophies and college scholarships.

## COMPETITION COMPONENTS

The four rounds of a MATHCOUNTS competition, each described below, are designed to be completed in approximately three hours:

**TARGET ROUND** (approximately 30 minutes): In this round eight problems are presented to competitors in four pairs (six minutes per pair). The multi-step problems featured in this round engage Mathletes in mathematical reasoning and problem-solving processes. *Problems assume the use of calculators.*

**SPRINT ROUND** (40 minutes): Consisting of 30 problems, this round tests accuracy, with the time period allowing only the most capable students to complete all of the problems. *Calculators are not permitted.*

**TEAM ROUND** (20 minutes): In this round, interaction among team members is permitted and encouraged as they work together to solve 10 problems. *Problems assume the use of calculators.*

Note: The order in which the written rounds (Target, Sprint and Team) are administered is at the discretion of the competition coordinator.

**COUNTDOWN ROUND:** A fast-paced oral competition for top-scoring individuals (based on scores on the Target and Sprint Rounds), this round allows pairs of Mathletes to compete against each other and the clock to solve problems. Calculators are not permitted.

At Chapter and State Competitions, a Countdown Round (1) may be conducted officially, (2) may be conducted unofficially (for fun) or (3) may be omitted. However, the use of an official Countdown Round must be consistent for all chapters within a state. In other words, *all* chapters within a state must use the round officially in order for *any* chapter within a state to use it officially. All students, whether registered as part of a school team or as individual competitors, are eligible to qualify for the Countdown Round.

An official Countdown Round determines an individual's final overall rank in the competition. If a Countdown Round is used officially, the official procedures as established by the MATHCOUNTS Foundation must be followed, as described below.\*

- The top 25% of students, up to a maximum of 10, are selected to compete. These students are chosen based on their Individual Scores.
- The two lowest-ranked students are paired; a question is read and projected, and students are given 45 seconds to solve the problem. A student may buzz in at any time, and if s/he answers correctly, a point is scored. If a student answers incorrectly, the other student has the remainder of the 45 seconds to answer.
- Three total questions are read to the pair of students, one question at a time, and the student who scores the higher number of points (not necessarily 2 out of 3) progresses to the next round and challenges the next-higher-ranked student.
- If students are tied in their matchup after three questions (at 1-1 or 0-0), questions should continue to be read until one is successfully answered. The first student who answers an additional question correctly progresses to the next round.
- This procedure continues until the 4th-ranked Mathlete and his/her opponent compete. For the final four matchups, the first student to correctly answer three questions advances.
- The Countdown Round proceeds until a 1st place individual is identified. More details about Countdown Round procedures are included in the 2017 School Competition.

\*Rules for the Countdown Round change for the National Competition.

An unofficial Countdown Round does not determine an individual's final overall rank in the competition, but is done for practice or for fun. The official procedures do not have to be followed. Chapters and states choosing not to conduct the round officially must determine individual winners solely on the basis of students' scores in the Target and Sprint Rounds of the competition.

## SCORING

**MATHCOUNTS Competition Series scores do not conform to traditional grading scales.  
Coaches and students should view an Individual Score of  
23 (out of a possible 46) as highly commendable.**

**INDIVIDUAL SCORE:** calculated by taking the sum of the number of Sprint Round questions answered correctly and twice the number of Target Round questions answered correctly. There are 30 questions in the Sprint Round and eight questions in the Target Round, so the maximum possible Individual Score is  $30 + 2(8) = 46$ . If used officially, the Countdown Round yields final individual standings.

**TEAM SCORE:** calculated by dividing the sum of the team members' Individual Scores by four (even if the team has fewer than four members) and adding twice the number of Team Round questions answered correctly. The highest possible Individual Score is 46. Four students may compete on a team, and there are 10 questions in the Team Round. Therefore, the maximum possible Team Score is  $((46 + 46 + 46 + 46) \div 4) + 2(10) = 66$ .

**TIEBREAKING ALGORITHM:** used to determine team and individual ranks and to determine which individuals qualify for the Countdown Round. In general, questions in the Target, Sprint and Team Rounds increase in difficulty so that the most difficult questions occur near the end of each round. In a comparison of questions to break ties, generally those who correctly answer the more difficult questions receive the higher rank. The guidelines provided below are very general; competition officials receive more detailed procedures.

- *Ties between individuals:* the student with the higher Sprint Round score will receive the higher rank. If a tie remains after this comparison, specific groups of questions from the Target and Sprint Rounds are compared.
- *Ties between teams:* the team with the higher Team Round score, and then the higher sum of the team members' Sprint Round scores, receives the higher rank. If a tie remains after these comparisons, specific questions from the Team Round will be compared.

## RESULTS DISTRIBUTION

Coaches should expect to receive the scores of their students, as well as a list of the top 25% of students and top 40% of teams, from their competition coordinators. In addition, single copies of the blank competition materials and answer keys may be distributed to coaches after all competitions at that level nationwide have been completed. Before distributing blank competition materials and answer keys, coordinators must wait for verification from the national office that all such competitions have been completed. Both the problems and answers from Chapter and State Competitions will be posted on the MATHCOUNTS website following the completion of all competitions at that level nationwide, replacing the previous year's posted tests.

Student competition papers and answers will not be viewed by or distributed to coaches, parents, students or other individuals. Students' competition papers become the confidential property of MATHCOUNTS.

## ADDITIONAL RULES

**All answers must be legible.**

**Pencils and paper** will be provided for Mathletes by competition organizers. However, students may bring their own pencils, pens and erasers if they wish. They may not use their own scratch paper or graph paper.

**Use of notes or other reference materials** (including dictionaries and translation dictionaries) is prohibited.

**Specific instructions stated in a given problem** take precedence over any general rule or procedure.

**Communication with coaches is prohibited during rounds but is permitted during breaks.** All communication between guests and Mathletes is prohibited during competition rounds. Communication between teammates is permitted only during the Team Round.

**Calculators are not permitted in the Sprint and Countdown Rounds, but they are permitted in the Target, Team and Tiebreaker (if needed) Rounds.** When calculators are permitted, students may use any calculator (including programmable and graphing calculators) that does not contain a QWERTY (typewriter-like) keypad. Calculators that have the ability to enter letters of the alphabet but do not have a keypad in a standard typewriter arrangement are acceptable. Smart phones, laptops, tablets, iPods®, personal

digital assistants (PDAs) and any other “smart” devices are not considered to be calculators and may not be used during competitions. Students may not use calculators to exchange information with another person or device during the competition.

**Coaches are responsible for ensuring their students use acceptable calculators, and students are responsible for providing their own calculators.** Coordinators are not responsible for providing Mathletes with calculators or batteries before or during MATHCOUNTS competitions. Coaches are strongly advised to bring backup calculators and spare batteries to the competition for their team members in case of a malfunctioning calculator or weak or dead batteries. Neither the MATHCOUNTS Foundation nor coordinators shall be responsible for the consequences of a calculator’s malfunctioning.

**Pagers, cell phones, tablets, iPods® and other MP3 players should not be brought into the competition room.** Failure to comply could result in dismissal from the competition.

**Should there be a rule violation or suspicion of irregularities, the MATHCOUNTS coordinator or competition official has the obligation and authority to exercise his/her judgment regarding the situation and take appropriate action, which might include disqualification of the suspected student(s) from the competition.**

# FORMS OF ANSWERS

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

**Units of measurement are not required in answers, but they must be correct if given.** When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

**All answers must be expressed in simplest form.** A “common fraction” is to be considered a fraction in the form  $\pm \frac{a}{b}$ , where  $a$  and  $b$  are natural numbers and  $\text{GCF}(a, b) = 1$ . In some cases the term “common fraction” is to be considered a fraction in the form  $\frac{A}{B}$ , where  $A$  and  $B$  are algebraic expressions and  $A$  and  $B$  do not have a common factor. A simplified “mixed number” (“mixed numeral,” “mixed fraction”) is to be considered a fraction in the form  $\pm N\frac{a}{b}$ , where  $N$ ,  $a$  and  $b$  are natural numbers,  $a < b$  and  $\text{GCF}(a, b) = 1$ . Examples:

Problem: What is $8 \div 12$ expressed as a common fraction?	Answer: $\frac{2}{3}$	Unacceptable: $\frac{4}{6}$
Problem: What is $12 \div 8$ expressed as a common fraction?	Answer: $\frac{3}{2}$	Unacceptable: $\frac{12}{8}, 1\frac{1}{2}$
Problem: What is the sum of the lengths of the radius and the circumference of a circle of diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of $\pi$ ?	Answer: $\frac{1+2\pi}{8}$	
Problem: What is $20 \div 12$ expressed as a mixed number?	Answer: $1\frac{2}{3}$	Unacceptable: $1\frac{8}{12}, \frac{5}{3}$

**Ratios should be expressed as simplified common fractions** unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}, \frac{3}{\pi}, \frac{4-\pi}{6}$	Unacceptable: $3\frac{1}{2}, \frac{4}{3}, 3.5, 2:1$
--	---

**Radicals must be simplified.** A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form?	Answer: $5\sqrt{3}$	Unacceptable: $\sqrt{75}$
--	---------------------	---------------------------

**Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., “How many dollars...,” “How much will it cost...,” “What is the amount of interest...”) should be expressed in the form (\$)*a.bc*, where *a* is an integer and *b* and *c* are digits.** The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* are both zero, in which case they both may be omitted. Answers in the form (\$)*a.bc* should be rounded to the nearest cent, unless otherwise specified. Examples:

Acceptable Forms: 2.35, 0.38, .38, 5.00, 5	Unacceptable: 4.9, 8.0
--	------------------------

**Do not make approximations for numbers** (e.g.,  $\pi$ ,  $\frac{2}{3}$ ,  $5\sqrt{3}$ ) in the data given or in solutions unless the problem says to do so.

**Do not do any intermediate rounding** (other than the “rounding” a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

**Scientific notation** should be expressed in the form  $a \times 10^n$  where  $a$  is a decimal,  $1 \leq |a| < 10$ , and  $n$  is an integer. Examples:

Problem: What is 6895 expressed in scientific notation?	Answer: $6.895 \times 10^3$
Problem: What is 40,000 expressed in scientific notation?	Answer: $4 \times 10^4$ or $4.0 \times 10^4$

**An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form.** Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

**The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.**

## VOCABULARY AND FORMULAS

The following list is representative of terminology used in the problems but should not be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference	decimal	infinite series
absolute value	degree measure	inscribe
acute angle	denominator	integer
additive inverse ( <i>opposite</i> )	diagonal of a polygon	interior angle of a polygon
adjacent angles	diagonal of a polyhedron	interquartile range
algorithm	diameter	intersection
alternate exterior angles	difference	inverse variation
alternate interior angles	digit	irrational number
altitude ( <i>height</i> )	digit-sum	isosceles
apex	direct variation	kite
area	dividend	lateral edge
arithmetic mean	divisible	lateral surface area
arithmetic sequence	divisor	lattice point(s)
base 10	dodecagon	LCM
binary	dodecahedron	linear equation
bisect	domain of a function	mean
box-and-whisker plot	edge	median of a set of data
center	endpoint	median of a triangle
chord	equation	midpoint
circle	equiangular	mixed number
circumference	equidistant	mode(s) of a set of data
circumscribe	equilateral	multiple
coefficient	evaluate	multiplicative inverse ( <i>reciprocal</i> )
collinear	expected value	natural number
combination	exponent	nonagon
common denominator	expression	numerator
common divisor	exterior angle of a polygon	obtuse angle
common factor	factor	octagon
common fraction	factorial	octahedron
common multiple	finite	ordered pair
complementary angles	formula	origin
composite number	frequency distribution	palindrome
compound interest	frustum	parallel
concentric	function	parallelogram
cone	GCF	Pascal's Triangle
congruent	geometric mean	pentagon
convex	geometric sequence	percent increase/decrease
coordinate plane/system	height ( <i>altitude</i> )	perimeter
coordinates of a point	hemisphere	permutation
coplanar	heptagon	perpendicular
corresponding angles	hexagon	planar
counting numbers	hypotenuse	polygon
counting principle	image(s) of a point(s) <i>(under a transformation)</i>	polyhedron
cube	improper fraction	polynomial
cylinder	inequality	prime factorization
decagon		prime number

principal square root	revolution	supplementary angles
prism	rhombus	system of equations/inequalities
probability	right angle	tangent figures
product	right circular cone	tangent line
proper divisor	right circular cylinder	term
proper factor	right polyhedron	terminating decimal
proper fraction	right triangle	tetrahedron
proportion	rotation	total surface area
pyramid	scalene triangle	transformation
Pythagorean Triple	scientific notation	translation
quadrant	sector	trapezoid
quadrilateral	segment of a circle	triangle
quotient	segment of a line	triangular numbers
radius	semicircle	trisect
random	semiperimeter	twin primes
range of a data set	sequence	union
range of a function	set	unit fraction
rate	significant digits	variable
ratio	similar figures	vertex
rational number	simple interest	vertical angles
ray	slope	volume
real number	slope-intercept form	whole number
reciprocal ( <i>multiplicative inverse</i> )	solution set	<i>x</i> -axis
rectangle	space diagonal	<i>x</i> -coordinate
reflection	sphere	<i>x</i> -intercept
regular polygon	square	<i>y</i> -axis
relatively prime	square root	<i>y</i> -coordinate
remainder	stem-and-leaf plot	<i>y</i> -intercept
repeating decimal	sum	

The list of formulas below is representative of those needed to solve MATHCOUNTS problems but should not be viewed as the only formulas that may be used. Many other formulas that are useful in problem solving should be discovered and derived by Mathletes.

## CIRCUMFERENCE

$$\text{Circle} \quad C = 2 \times \pi \times r = \pi \times d$$

## AREA

$$\text{Circle} \quad A = \pi \times r^2$$

$$\text{Square} \quad A = s^2$$

$$\text{Rectangle} \quad A = l \times w = b \times h$$

$$\text{Parallelogram} \quad A = b \times h$$

$$\text{Trapezoid} \quad A = \frac{1}{2}(b_1 + b_2) \times h$$

$$\text{Rhombus} \quad A = \frac{1}{2} \times d_1 \times d_2$$

$$\text{Triangle} \quad A = \frac{1}{2} \times b \times h$$

$$\text{Triangle} \quad A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Equilateral triangle} \quad A = \frac{s^2\sqrt{3}}{4}$$

## SURFACE AREA AND VOLUME

$$\text{Sphere} \quad SA = 4 \times \pi \times r^2$$

$$\text{Sphere} \quad V = \frac{4}{3} \times \pi \times r^3$$

$$\text{Rectangular prism} \quad V = l \times w \times h$$

$$\text{Circular cylinder} \quad V = \pi \times r^2 \times h$$

$$\text{Circular cone} \quad V = \frac{1}{3} \times \pi \times r^2 \times h$$

$$\text{Pyramid} \quad V = \frac{1}{3} \times B \times h$$

$$\text{Pythagorean Theorem} \quad c^2 = a^2 + b^2$$

$$\text{Counting/Combinations} \quad {}_nC_r = \frac{n!}{r!(n-r)!}$$



# ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.

4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.

6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

## Warm-Up 1

Answer	Difficulty			
1. 77	(1)	6. 30	(4)	
2. $3\frac{1}{5}$	(2)	7. $\frac{9}{16}$	(4)	
3. 11	(2)	8. $1.7 \times 10^5$	(2)	
4. 1	(2)	9. 0	(3)	
5. $\frac{1}{6}$	(2)	10. 170 or 170.00	(2)	

## Workout 1

Answer	Difficulty		
31. 1.30	(3)	36. 7	(4)
32. 721	(4)	37. Q	(3)
33. 1100	(4)	38. 16	(2)
34. 12	(4)	39. 6	(3)
35. 14.6	(4)	40. 36	(4)

## Warm-Up 2

Answer	Difficulty			
11. 16	(2)	16. 720	(3)	
12. 5	(3)	17. 36	(5)	
13. 12	(3)	18. $\frac{7}{12}$	(4)	
14. 5	(3)	19. 12	(3)	
15. $3.16 \times 10^{-14}$	(4)	20. $2\sqrt{17}$	(4)	

## Workout 2

Answer	Difficulty		
41. 1*	(3)	46. 4	(4)
42. 90	(3)	47. $\frac{1}{4}$	(4)
43. 1.65	(3)	48. 36	(4)
44. 41	(1)	49. 7	(4)
45. 120	(4)	50. 21.3	(4)

## Warm-Up 3

Answer	Difficulty			
21. 22	(2)	26. $\frac{25}{81}$	(4)	
22. 3	(2)	27. $\frac{18}{25}$	(4)	
23. 8	(2)	28. 8	(4)	
24. 621	(1)	29. 6	(4)	
25. 9	(4)	30. 1	(2)	

## Warm-Up 4

Answer	Difficulty		
51. 88	(4)	56. 380	(3)
52. 86	(3)	57. $\frac{15}{64}$	(4)
53. 38	(4)	58. -3	(2)
54. 12	(2)	59. $\frac{2}{5}$	(4)
55. $\frac{1}{64}$	(3)	60. 8	(3)

\* The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.

## Warm-Up 5

Answer	Difficulty			
61. 216	(2)	66. 4851	(5)	
62. 38	(4)	67. 30	(3)	
63. 11	(4)	68. 720	(3)	
64. 6	(3)	69. 160	(5)	
65. 275,000 or 275,000.00	(4)	70. -3	(4)	

## Workout 4

Answer	Difficulty			
91. 60	(3)	96. 36	(4)	
92. 776	(3)	97. 4094	(4)	
93. 10.5	(4)	98. 870	(4)	
94. 3	(2)	99. 100	(4)	
95. 35	(5)	100. 922	(3)	

## Warm-Up 6

Answer	Difficulty			
71. 24 or 24.00	(2)	76. 34	(4)	
72. 17	(3)	77. $2\pi$	(5)	
73. 175	(3)	78. 7	(3)	
74. 33	(4)	79. -5	(4)	
75. 867	(4)	80. 17	(4)	

## Warm-Up 7

Answer	Difficulty			
101. 0	(2)	106. 48	(4)	
102. 2	(3)	107. 3	(4)	
103. 74	(5)	108. 500	(4)	
104. 8.10	(2)	109. $14\sqrt{3}$	(5)	
105. $1/16$	(4)	110. 678	(4)	

## Workout 3

Answer	Difficulty			
81. -4	(3)	86. 29	(4)	
82. 18	(3)	87. 2.45	(4)	
83. $\frac{19\sqrt{2}}{2}$	(4)	88. 750	(4)	
84. 8	(2)	89. 4	(4)	
85. 2018	(2)	90. $192\pi$	(4)	

## Warm-Up 8

Answer	Difficulty			
111. 6	(2)	116. $1/39$	(5)	
112. $1/32$	(4)	117. 9	(4)	
113. $37\pi$	(4)	118. 42	(4)	
114. 9	(4)	119. $72\pi$	(5)	
115. 18	(5)	120. 12022	(4)	

## **Workout 5**

<b>Answer</b>	<b>Difficulty</b>		
121. 29	(4)	126. 45	(3)
122. 7.2	(4)	127. $\frac{1}{8}$	(3)
123. $\frac{10}{3}$	(5)	128. 0.5	(5)
124. 3	(4)	129. 1.14	(6)
125. 9.4	(5)	130. 90	(3)

## **Warm-Up 10**

<b>Answer</b>	<b>Difficulty</b>		
151. 38	(4)	156. $\frac{18}{55}$	(3)
152. 78230	(4)	157. 40	(6)
153. 14	(4)	158. 2031	(4)
154. 30	(5)	159. 31	(4)
155. 1760	(5)	160. 32	(4)

## **Workout 6**

<b>Answer</b>	<b>Difficulty</b>		
131. 42	(3)	136. 61,900	(4)
132. 8	(5)	137. 20	(5)
133. $\frac{3}{65}$	(4)	138. 24	(4)
134. 22 or 22.00	(5)	139. 346	(3)
135. 272	(3)	140. $\frac{3}{7}$	(3)

## **Warm-Up 11**

<b>Answer</b>	<b>Difficulty</b>		
161. $\frac{7}{64}$	(4)	166. 8008	(4)
162. 18	(3)	167. 6	(3)
163. 27	(5)	168. 9.2	(4)
164. 62	(3)	169. 9	(5)
165. 30	(4)	170. 2	(5)

## **Warm-Up 9**

<b>Answer</b>	<b>Difficulty</b>		
141. 5	(2)	146. 32	(6)
142. $\frac{3}{2}$	(4)	147. 132	(3)
143. 40	(3)	148. 28	(5)
144. 15	(3)	149. 3	(3)
145. 981	(4)	150. 512	(3)

## **Workout 7**

<b>Answer</b>	<b>Difficulty</b>		
171. 40.5	(4)	176. $\frac{3}{20}$	(3)
172. 24	(5)	177. 60	(5)
173. 1.7	(3)	178. 1.11	(4)
174. 27	(5)	179. $\frac{9}{5}$	(5)
175. 0.94	(4)	180. 2.6	(4)

## Workout 8

Answer	Difficulty		
181. $\frac{16}{5}$	(2)	186. 2	(4)
182. 6	(2)	187. $\frac{1}{210}$	(4)
183. $\frac{5}{18}$	(5)	188. $4\sqrt{29}$	(6)
184. 73	(5)	189. $6\sqrt{13}$	(6)
185. 806	(5)	190. 10	(4)

## Fractions Stretch

Answer	Difficulty		
221. $\frac{1}{4}$	(1)	226. $\frac{11}{16}$	(4)
222. $\frac{3}{2}$	(2)	227. $\frac{7}{3}$	(3)
223. $\frac{6}{11}$	(3)	228. $\frac{34}{45}$	(4)
224. $\frac{3}{10}$	(1)	229. 2	(5)
225. $\frac{3}{8}$	(4)	230. -12	(4)

## Warm-Up 12

Answer	Difficulty		
191. 225	(3)	196. 25	(5)
192. $\frac{391}{330}$	(4)	197. 18	(7)
193. $\sqrt{3}$	(5)	198. 10	(3)
194. 4200	(4)	199. 4	(4)
195. $\frac{1}{35}$	(4)	200. 3	(5)

## Angles and Arcs Stretch

Answer	Difficulty		
231. 40	(3)	236. 100	(3)
232. 65	(3)	237. 130	(4)
233. 40	(4)	238. 77.5	(4)
234. 230	(4)	239. 80	(4)
235. 62.5	(3)	240. 90	(5)

## Warm-Up 13

Answer	Difficulty		
201. 279	(5)	206. 6777	(5)
202. 121	(5)	207. $\frac{1}{9}$	(5)
203. 7	(4)	208. 6	(6)
204. $\frac{15}{2}$	(5)	209. 7	(4)
205. $\frac{5}{3}$	(5)	210. $\frac{48}{5}$	(6)

## Warm-Up 14

Answer	Difficulty		
211. 28	(3)	216. 5	(5)
212. $\frac{7}{15}$	(5)	217. $\frac{21}{8}$	(5)
213. 16	(4)	218. $\frac{47}{3}$	(5)
214. $\frac{124}{133}$	(7)	219. $\frac{\sqrt{2}+40}{3}$	(5)
215. 12	(5)	or $\frac{40+\sqrt{2}}{3}$	
		220. 169	(5)

## Bases Stretch

Answer	Difficulty		
241. 22	(2)	246. 33	(3)
242. 20	(2)	247. 2A7B	(4)
243. 18	(2)	248. 2355	(5)
244. 26	(3)	249. 7	(5)
245. 30	(3)	250. 19	(6)

# **MATHCOUNTS Problems Mapped to Common Core State Standards (CCSS)**

Forty-three states, the District of Columbia, four territories and the Department of Defense Education Activity (DoDEA) have adopted the Common Core State Standards (CCSS). As such, MATHCOUNTS considers it beneficial for teachers to see the connections between the 2016-2017 *MATHCOUNTS School Handbook* problems and the CCSS. MATHCOUNTS not only has identified a general topic and assigned a difficulty level for each problem but also has provided a CCSS code in the Problem Index (pages 56-57). A complete list of the Common Core State Standards can be found at [www.corestandards.org](http://www.corestandards.org).

The CCSS for mathematics cover K-8 and high school courses. MATHCOUNTS problems are written to align with the NCTM Standards for Grades 6-8. As one would expect, there is great overlap between the two sets of standards. MATHCOUNTS also recognizes that in many school districts, algebra and geometry are taught in middle school, so some MATHCOUNTS problems also require skills taught in those courses.

In referring to the CCSS, the Problem Index code for each of the Standards for Mathematical Content for grades K-8 begins with the grade level. For the Standards for Mathematical Content for high school courses (such as algebra or geometry), each code begins with a letter to indicate the course name. The second part of each code indicates the domain within the grade level or course. Finally, the number of the individual standard within that domain follows. Here are two examples:

- 6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6
- G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry

Some math concepts utilized in MATHCOUNTS problems are not specifically mentioned in the CCSS. Two examples are the Fundamental Counting Principle (FCP) and special right triangles. In cases like these, if a related standard could be identified, a code for that standard was used. For example, problems using the FCP were coded 7.SP.8, S-CP.8 or S-CP.9 depending on the context of the problem; SP → Statistics and Probability (the domain), S → Statistics and Probability (the course) and CP → Conditional Probability and the Rules of Probability. Problems based on special right triangles were given the code G-SRT.5 or G-SRT.6, explained above.

There are some MATHCOUNTS problems that either are based on math concepts outside the scope of the CCSS or based on concepts in the standards for grades K-5 but are obviously more difficult than a grade K-5 problem. When appropriate, these problems were given the code SMP for Standards for Mathematical Practice. The CCSS include the Standards for Mathematical Practice along with the Standards for Mathematical Content. The SMPs are (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure and (8) Look for and express regularity in repeated reasoning.

# PROBLEM INDEX

It is difficult to categorize many of the problems in the *MATHCOUNTS School Handbook*. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code **9 (3) 7.SP.3** refers to problem 9 with difficulty rating 3 mapped to CCSS 7.SP.3. For an explanation of the difficulty ratings refer to page 51. For an explanation of the CCSS codes refer to page 55.

SOLID GEOMETRY	90 (4) 8.G.9	GENERAL MATH	1 (1) 4.OA.3	COORDINATE GEOMETRY	12 (3) 8.G.8
	99 (4) 8.G.9		4 (2) 5.NF.7		20 (4) 8.G.8
	103 (5) 7.G.3		8 (2) 8.EE.4		113 (4) 8.G.8
	128 (5) GGMD.3		10 (2) 4.MD.2		179 (5) 8.EE.6
	134 (5) 8.G.9		14 (3) 4.MD.2		210 (6) 8.G.8
	136 (4) GGMD.3		24 (1) 4.OA.3		219 (5) GSRT.6
	151 (4) 7.G.6		43 (3) 5.NF.1		30 (2) 4.OA.3
	168 (4) 8.G.9		44 (1) 5.NF.6		32 (4) 6.NS.4
	184 (5) 8.G.9		54 (2) 7.EE.1		42 (3) 4.OA.4
	198 (3) 4.OA.4		61 (2) 6.EE.1		46 (4) SMP
MEASUREMENT	11 (2) 4.MD.2	PLANE GEOMETRY	6 (4) 8.G.7	NUMBER THEORY	52 (3) 4.OA.4
	15 (4) 8.EE.4		38 (2) 7.G.6		72 (3) 6.EE.2
	29 (4) 8.G.7		48 (4) 7.G.4		78 (3) SMP
	35 (4) 7.G.4		77 (5) GC.5		81 (3) SMP
	45 (4) GCO.10		96 (4) 7.G.4		85 (2) 4.OA.4
	50 (4) 8.G.7		109 (5) 7.G.6		100 (3) 4.OA.4
	69 (5) 6.G.1		115 (5) GSRT.6		106 (4) 4.OA.4
	75 (4) 6.G.1		124 (4) GSRT.6		107 (4) SMP
	83 (4) 8.G.8		129 (6) GC.5		117 (4) SMP
	86 (4) 8.G.8		132 (5) GSRT.6		120 (4) SMP
	119 (5) 7.G.4		143 (3) 8.G.4		135 (3) SMP
	125 (5) GC.2		148 (5) 7.G.6		138 (4) SMP
	146 (6) 7.G.4		154 (5) GSRT.5		139 (3) 4.OA.4
	173 (3) 6.RP.3		157 (6) 8.G.7		149 (3) SMP
			163 (5) 7.G.6		153 (4) SMP
			169 (5) GCO.10		156 (3) SMP
LOGIC	3 (2) SMP	ANGLES AND ARCS	171 (4) GC.5	BASES	159 (4) 6.NS.4
	22 (2) SMP		180 (4) GC.5		166 (4) 6.NS.4
	37 (3) SMP		181 (2) 7.G.6		174 (5) 6.NS.4
	63 (4) SCP.9		189 (6) GSRT.3		185 (5) ASSE.2
	98 (4) FLE.2		193 (5) 8.G.7		206 (5) SMP
	130 (3) 6.NS.4		197 (7) GC.5		215 (5) SMP
	167 (3) SMP		217 (5) 8.G.7		218 (5) 4.OA.4
	199 (4) SMP		Angles and Arcs Stretch <sup>1</sup>		Bases Stretch <sup>2</sup>

<sup>1</sup> CCSS 7.G.5

<sup>2</sup> CCSS 8.EE.1

ALGEBRAIC EXPRESSIONS & EQUATIONS	25	(4)	6.EE.2	PROBABILITY, COUNTING & COMBINATORICS	7	(4)	SCP.9	SEQUENCES, SERIES & PATTERNS	13	(3)	SMP
	33	(4)	A-CED.1		16	(3)	SCP.9		73	(3)	SMP
	49	(4)	A-CED.2		18	(4)	SCP.9		74	(4)	FIF.3
	53	(4)	8.EE.8		27	(4)	SCP.6		84	(2)	FIF.3
	56	(3)	6.EE.2		28	(4)	SMP		94	(2)	8.NS.1
	58	(2)	6.EE.2		34	(4)	SMP		114	(4)	FLE.2
	64	(3)	8.EE.1		47	(4)	SCP.9		131	(3)	FBF.2
	79	(4)	A-REI.5		55	(3)	7.SP.8		164	(3)	SMP
	80	(4)	6.EE.2		57	(4)	SCP.9		200	(5)	FBF.2
	87	(4)	8.EE.8		59	(4)	SCP.9				
	88	(4)	8.EE.8		66	(5)	SCP.9				
	89	(4)	8.EE.1		68	(3)	SCP.9				
	92	(3)	6.EE.2		76	(4)	SCP.9		2	(2)	6.RP.3
	102	(3)	F-IF.2		91	(3)	SCP.9		17	(5)	6.RP.3
	111	(2)	8.EE.8		97	(4)	SCP.9		23	(2)	4.NF.2
	123	(5)	A-REI.7		110	(4)	SCP.9		26	(4)	8.G.4
	141	(2)	8.EE.8		112	(4)	SCP.9		31	(3)	4.MD.2
	144	(3)	8.EE.8		116	(5)	SCP.9		40	(4)	6.RP.3
	158	(4)	A-SSE.2		126	(3)	SMP		41	(3)	6.RP.3
	170	(5)	A-SSE.2		127	(3)	7.SP.8		67	(3)	6.RP.3
	172	(5)	A-CED.2		133	(4)	7.SP.8		71	(2)	6.RP.3
	188	(6)	G-SRT.3		145	(4)	SMP		93	(4)	6.RP.3
	196	(5)	G-C.5		147	(3)	SMP		104	(2)	6.RP.3
	201	(5)	S-CP.9		150	(3)	SCP.9		105	(4)	7.RP.2
	204	(5)	S-CP.9		155	(5)	SMP		108	(4)	7.RP.2
					160	(4)	SMP		122	(4)	S.RP.3
					161	(4)	SCP.9		140	(3)	7.RP.1
					162	(3)	SMP		142	(4)	7.RP.1
					165	(4)	SMP		178	(4)	6.RP.3
					175	(4)	7.SP.8		190	(4)	7.RP.1
					176	(3)	7.SP.8		209	(4)	6.EE.2
					183	(5)	7.SP.8		213	(4)	6.RP.3
					187	(4)	SCP.9				
					194	(4)	SCP.9				
					195	(4)	SCP.9				
					202	(5)	SCP.9				
					203	(4)	7.SP.8				
					205	(5)	SCP.9				
					207	(5)	7.SP.8				
					211	(3)	SMP				
					212	(5)	7.SP.8				
					214	(7)	SMP				
					220	(5)	SMP				
STATISTICS	21	(2)	6.SP.5	PROPORTIONAL REASONING				PERCENTS & FRACTIONS			
	51	(4)	6.SP.5								
	82	(3)	ACED.2								
	95	(5)	6.SP.5								
	118	(4)	SMP								
	121	(4)	6.SP.5								
	186	(4)	6.SP.5								
PROBLEM SOLVING (Misc.)	9	(3)	6.EE.1	PERCENTS & FRACTIONS				Fractions Stretch <sup>3</sup>			
	19	(3)	SMP								
	39	(3)	6.EE.1								
	62	(4)	SMP								
	152	(4)	SMP								
	182	(2)	SMP								
	191	(3)	SMP								
	208	(6)	SMP								
	216	(5)	SMP								

<sup>3</sup> CCSS 6.RP.3





## 2016–2017 ADDITIONAL STUDENTS REGISTRATION FORM

### Step 1: Tell us about your school so we can find your original registration (please print legibly).

Coach Name _____	School Name _____
Coach Email Address _____	
School Address _____	City, State ZIP _____

### Step 2: Tell us how many students you are adding to your school's registration. Following the instructions below.

Please circle the number of additional students you will enter in the Chapter Competition and the associated cost below (depending on the date your registration is postmarked). The cost is \$30 per student added, whether that student will be part of a team or will compete as an individual. The cost of adding students to a previous registration is not eligible for an Early Bird rate.

# of Students You Are Adding	1	2	3	4	5	6	7	8	9
Regular Rate (postmarked by Dec. 16, 2016)	\$30	\$60	\$90	\$120	\$150	\$180	\$210	\$240	\$270
Late Registration (postmarked after Dec. 16, 2016)	\$50	\$80	\$110	\$140	\$170	\$200	\$230	\$260	\$290

My school qualifies for the **50% Title I discount**, so the Amount Due in Step 4 will be half the amount I circled above. Principal signature required below to verify Title I eligibility.

Principal Name \_\_\_\_\_ Principal Signature \_\_\_\_\_

### Step 3: Tell us what your school's FINAL registration should be (including all changes/additions).

Total # of Registered Students	1 (1 individual)	2 (2 ind)	3 (3 ind)	4 (1 team)	5 (1 tm, 1 ind)	6 (1 tm, 2 ind)	7 (1 tm, 3 ind)	8 (1 tm, 4 ind)	9 (1 tm, 5 ind)	10 (1 tm, 6 ind)
--------------------------------	---------------------	--------------	--------------	---------------	--------------------	--------------------	--------------------	--------------------	--------------------	---------------------

### Step 4: Almost done... just fill in payment information and turn in your form!

Amount Due \$ \_\_\_\_\_  Check (payable to MATHCOUNTS Foundation)  Money Order  Purchase Order # \_\_\_\_\_ (must include copy of P.O.)  
Credit card payments cannot be accepted with paper registrations. To pay with a credit card, please register online at [www.mathcounts.org/compreg](http://www.mathcounts.org/compreg).

**Questions?**  
Please call the national office at  
**703-299-9006**.

**IMPORTANT!** By submitting this form you (1) agree to adhere to the rules of the MATHCOUNTS Competition Series; (2) attest you have the school administration's permission to register students for this program under this school's name; and (3) affirm the above named school is a U.S. school eligible for this program and is not an academic or enrichment center. The coach will receive an emailed confirmation and receipt once this additional students registration has been processed.



# **ACKNOWLEDGMENTS**

The MATHCOUNTS Foundation wishes to acknowledge the hard work and dedication of those volunteers instrumental in the development of this handbook: the question writers who develop the questions for the handbook and competitions, the judges who review the competition materials and serve as arbiters at the National Competition and the proofreaders who edit the questions selected for inclusion in the handbook and/or competitions.

## **2015-2016 QUESTION WRITING COMMITTEE**

Chair: Edward Early (STE 92), *Austin, TX*  
Susan Holtzapple, *Los Altos, CA*  
Rich Morrow, *Naalehu, HI*

Nina Otterson, *Lakeville, CT*  
Cody Patterson (STE 94, NAT 95), *Tucson, AZ*  
Kris Warloe, *Corvallis, OR*

## **2016-2017 NATIONAL JUDGES**

Sam Baethge, *San Marcos, TX*  
Flavia Colonna, *George Mason University, Fairfax, VA*  
Barbara Currier, *Addison, TX*  
Peter Kohn, *James Madison University, Harrisonburg, VA*  
Dave Sundin (STE 84), *San Mateo, CA*

## **2016-2017 NATIONAL REVIEWERS**

William Aldridge, *Springfield, VA*  
Erica Arrington, *North Chelmsford, MA*  
Alyssa Briery (STE 04, 05, 06), *Denton, TX*  
Lars Christensen (STE 89), *St. Paul, MN*  
Dan Cory (NAT 84, 85), *Seattle, WA*  
Roslyn Denny, *Valencia, CA*  
Joyce Glatzer, *Woodland Park, NJ*  
Dennis Hass *Newport News, VA*

Helga Huntley (STE 91), *Newark, DE*  
Chris Jeuell, *Kirkland, WA*  
David Kung (STE 85, NAT 86), *Lexington, MD*  
Stanley Levinson, P.E., *Lynchburg, VA*  
Howard Ludwig, *Ocoee, FL*  
Tina Pateracki, *Mt. Pleasant, SC*  
Randy Rogers (NAT 85), *Davenport, IA*  
Craig Volden (NAT 84), *Earlysville, VA*

**Special Thanks to:** Jerrold Grossman, *Rochester, MI*  
Jane Lataille, *Los Alamos, NM*  
Leon Manelis, *Orlando, FL*

The **Solutions** to the problems were written by Kent Findell, Diamond Middle School, *Lexington, MA*.

**MathType** software for handbook development was contributed by **Design Science Inc.**, [www.dessci.com](http://www.dessci.com), *Long Beach, CA*.

## **MATHCOUNTS FOUNDATION**

**Editor and Contributing Author:** Kera Johnson, *Manager of Education*

**Content Editor:** Cara Norton, *Senior Education Coordinator*

**Author of Introduction & Program Information:** Amanda Naar, *Communications Manager*

**Executive Director:** Louis DiGioia

**Honorary Chairman:** Thomas A. Kennedy, *Chairman and CEO of Raytheon Company*

## **©2016 MATHCOUNTS Foundation**

1420 King Street, Alexandria, VA 22314

703-299-9006 ♦ [www.mathcounts.org](http://www.mathcounts.org) ♦ [info@mathcounts.org](mailto:info@mathcounts.org)

Unauthorized reproduction of the contents of this publication is a violation of applicable laws.  
Materials may be duplicated for use by U.S. schools.

MATHCOUNTS® and Mathlete® are registered trademarks of the MATHCOUNTS Foundation.

The MATHCOUNTS Foundation makes its products and services available on a nondiscriminatory basis. MATHCOUNTS does not discriminate on the basis of race, religion, color, creed, gender, sexual orientation, physical disability or ethnic origin.