

<u>Gameboard</u>

Maths

Maxima and Minima: Problems 1ii

Maxima and Minima: Problems 1ii



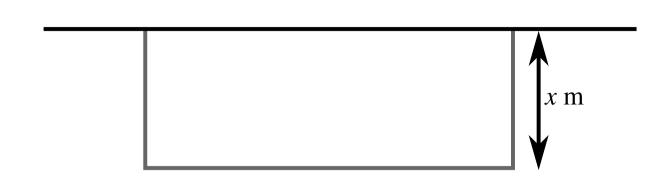


Figure 1: The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length $20\,$ metres, is used to form the remaining three sides. The width of the enclosure is x metres, and the area of the enclosure is x metres.

Part A Express as equation

Show that A can be expressed in the form $px-qx^2$, and find this expression.

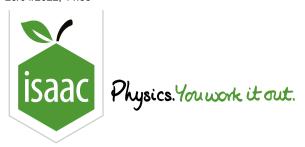
The following symbols may be useful: x

Part B Use differentiation

Use differentiation to find the maximum value of A.

The following symbols may be useful: A

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Maths

Maxima and Minima: Problems 2ii

Maxima and Minima: Problems 2ii



A curve has equation $y=3x^3-7x+rac{2}{x}$

Part A Verify stationary point

Verify the curve has a stationary point when x = 1.

Part B Nature of stationary point

Determine the nature of this stationary point.

Maximum

Minimum

Neither/inconclusive

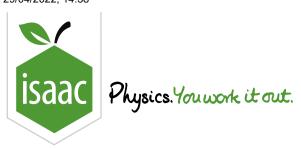
Part C Tangent to curve

The tangent to the curve at this stationary point meets the y-axis at the point Q. Find the y-coordinate of Q.

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Maths

Maxima and Minima: Problems 1i

Maxima and Minima: Problems 1i



A cuboid has an volume of exactly $8\,\mathrm{m}^3$. The base of the cuboid is a square with side length x metres. The surface area of the cuboid is $A\,\mathrm{m}^2$.

Part A Find expression for A

Show that A can be expressed in the form $ax^2 + \frac{b}{x}$, where a and b are constants, and find this expression.

The following symbols may be useful: x

Part B Find $\frac{\mathrm{d}A}{\mathrm{d}x}$

Find $\frac{\mathrm{d}A}{\mathrm{d}x}$.

The following symbols may be useful: x

Part C Find minimum

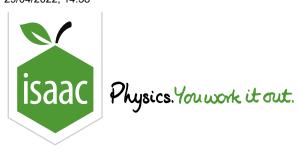
Find the value of x which gives the smallest surface area of the cuboid.

The following symbols may be useful: x

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Maths

Calculus

Differentiation

Minimising the area

Minimising the area



A rectangular cuboid has a base with sides of length a and b and a height c. Its volume V and height c are fixed. By following the steps below find expressions in terms of V and c for the values of a and b which will minimise the surface area a of the cuboid, find an expression for this minimum surface area and check that this is indeed a minimum.

Part A Volume V and surface area A

Write down the equation for the volume V of the rectangular cuboid in terms of a, b and c.

The following symbols may be useful: V, a, b, c

Write down the equation for the surface area A of the rectangular cuboid in terms of a, b and c.

The following symbols may be useful: A, a, b, c

From your equation for V deduce an expression for b in terms of V, a and c. Hence, by substitution, obtain an equation for A in terms of V, a and c.

The following symbols may be useful: A, $\,$ V, a, $\,$ c

Part B Expressions for a and b

Differentiate with respect to a the expression for A you found in Part A (since V and c are fixed you may treat them as constants). Hence find in terms of V and c an expression for the value of a for which the surface area A is minimised.

The following symbols may be useful: v, c

Find, in terms of V and c, the expression for b corresponding to this value of a.

The following symbols may be useful: v, c

Part C The minimum area

Find an expression for the minimum surface area in terms of V and c.

The following symbols may be useful: v, c

Part D Check that the area is a minimum

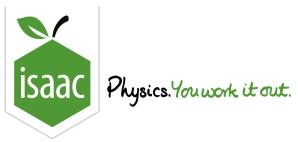
Find, at the value of a deduced in Part B, an expression in terms of V and c for the second derivative of A with respect to a; convince yourself that the value of the second derivative indicates that the value of A is a minimum at this point.

The following symbols may be useful: v, c

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Maths

Stationary Points 1ii

Stationary Points 1ii



The curve $y=x^3-kx^2+x-3$ has two stationary points.

Part A Differentiate

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: k, x

Part B Find k

Given that there is a stationary point when x=1, find the value of k.

The following symbols may be useful: \ensuremath{k}

Part C Differentiate twice

Find $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

The following symbols may be useful: \boldsymbol{x}

Hence determine whether the stationary point is a minimum or a maximum.

Minimum

Maximum

Part D Find coordinate

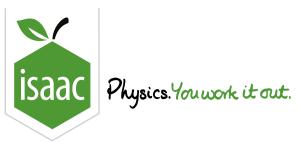
Find the x-coordinate of the other stationary point.

The following symbols may be useful: x

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Maths

Stationary Points 1i

Stationary Points 1i



Part A Find stationary points

Find the coordinates of the stationary points on the curve $y=2x^3-3x^2-12x-7$. Enter the x
and y coordinates of the stationary point with the largest x coordinate.

Enter the x coordinate:

The following symbols may be useful: \times

Enter the y coordinate:

The following symbols may be useful: y

Part B Nature of stationary points

Determine whether each stationary point is a minimum or maximum point. Identify the nature of the stationary point whose coordinates you have entered in Part A.

() Maximum

Minimum

Part C Expand and simplify

Expand and simplify $(x+1)^2(2x-7)$.

The following symbols may be useful: x

Part D Sketch

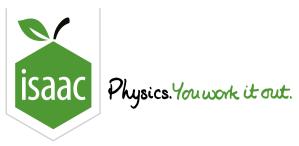
Hence sketch the curve $y=2x^3-3x^2-12x-7$, indicating the coordinates of all stationary points and intercepts with the axes. In order to check your answer, give the value of the intercept with the y-axis.

The following symbols may be useful: y

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Maths

Stationary Points 2i

Stationary Points 2i



Part A Find minimum

Find the coordinates of the minimum point of the curve $y=(x+2)(x^2-3x+5)$. Enter the x and y coordinates below.

Enter the *x*-coordinate:

The following symbols may be useful: \times

Enter the y-coordinate:

The following symbols may be useful: y

Part B Finding nature of stationary point

How did you know that the stationary point in part A was a minimum point?

At this point, $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ is positive.

- At this point, $\frac{\mathrm{d}y}{\mathrm{d}x}$ is zero.
- At this point, $\frac{d^2y}{dx^2}$ is negative.

Part C Calculate discriminant

Calculate the discriminant of x^2-3x+5 . Enter the exact value.

Part D Explain

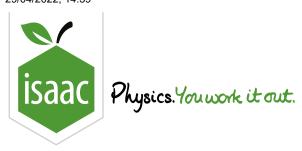
Explain why $(x+2)(x^2-3x+5)$ is always positive whenever x>-2.

Easier question?

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Maths

Stationary Points 2ii

Stationary Points 2ii



Part A Find coordinate

Find the coordinates of the stationary points on the curve $y=x^3-3x^2+4$. Enter the x and y coordinates of the stationary point with the greatest x coordinate.

Enter the *x*-coordinate:

The following symbols may be useful: \times

Enter the y-coordinate:

The following symbols may be useful: y

Part B Stationary point

Determine whether the stationary point whose coordinates you entered is a maximum point or a minimum point.

Minimum

- () Maximum
- () Inconclusive

${\bf Part \ C} \qquad {\bf Range \ of} \ x$

For which range of values of x does $x^3 - 3x^2 + 4$ decrease as x increases?

What form does your answer take? Choose from the list below, where a and b are constants and a < b, and then find a and/or b.

- $\bigcirc x < a$
- $x \leq a$
- x > a
- $x \geq a$
 - a < x < b
- $a \le x \le b$
- x < a or x > b
- $x \le a \text{ or } x \ge b$

Write down the value of a.

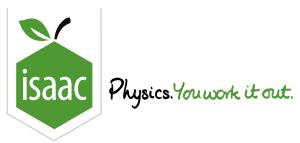
Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

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Maths

Stationary Points 4ii

Stationary Points 4ii



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Part A	4 FI	nd c	oordi	nates

Find the coordinates of the stationary point on the curve $y=x^4+32x$. Enter the x and y coordinates below.

Enter *x* coordinate:

The following symbols may be useful: x

Enter y coordinate:

The following symbols may be useful: y

Part B Maxima or Minima

Determine whether this stationary point is a maximum or a minimum.

Minimum

() Maximum

${\bf Part \ C} \qquad {\bf Range \ of} \ x$

For what range of values of x does x^4+32x increase as x increases? Give your answer in the form of an inequality.

The following symbols may be useful: <, <=, >, >=, \times