Physics. You work it out.

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Complex Numbers: $re^{\mathrm{i}\theta}$ 3ii

Further A

P P P

Part A Expression for $z_1 z_2$

Given that $z_1 = 2e^{\frac{1}{6}\pi \mathrm{i}}$ and $z_2 = 3e^{\frac{1}{4}\pi \mathrm{i}}$, express $z_1 z_2$ in the form $re^{\mathrm{i}\theta}$.

$r > 0$ and $0 \leqslant \theta < 2\pi$.

The following symbols may be useful: `e`, `i`, `pi`

Part B Expression for $\frac{z_1}{z_2}$

Express $\frac{z_1}{z_2}$ in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$.

The following symbols may be useful: `e`, `i`, `pi`

Part C Expression for w^{-5}

Given that $w = 2\left(\cos\frac{1}{8}\pi + \mathrm{i}\sin\frac{1}{8}\pi\right)$, express w^{-5} in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $0 \leqslant \theta < 2\pi$

.

The following symbols may be useful: `cos()`, `i`, `pi`, `sin()`

Adapted with permission from UCLES, A Level, June 2006, Paper 4727, Question 2.

Complex Numbers: $r\mathrm{e}^{\mathrm{i}\theta}$ 1i

Further A

P P P

Part A $z^6 = 1$

Solve the equation $z^6 = 1$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $0 \leq \theta < 2\pi$ and $0 < r$.

Write your answer in terms of k where $k = 0, 1, 2, 3, 4, 5$.

The following symbols may be useful: `e`, `i`, `k`, `pi`

Part B Argand diagram

Sketch an argand diagram showing the solutions to $z^6 = 1$.

When you have made your sketch, answer this question to see an example sketch: Which of the four sketches in **Figure 1** is most accurate?

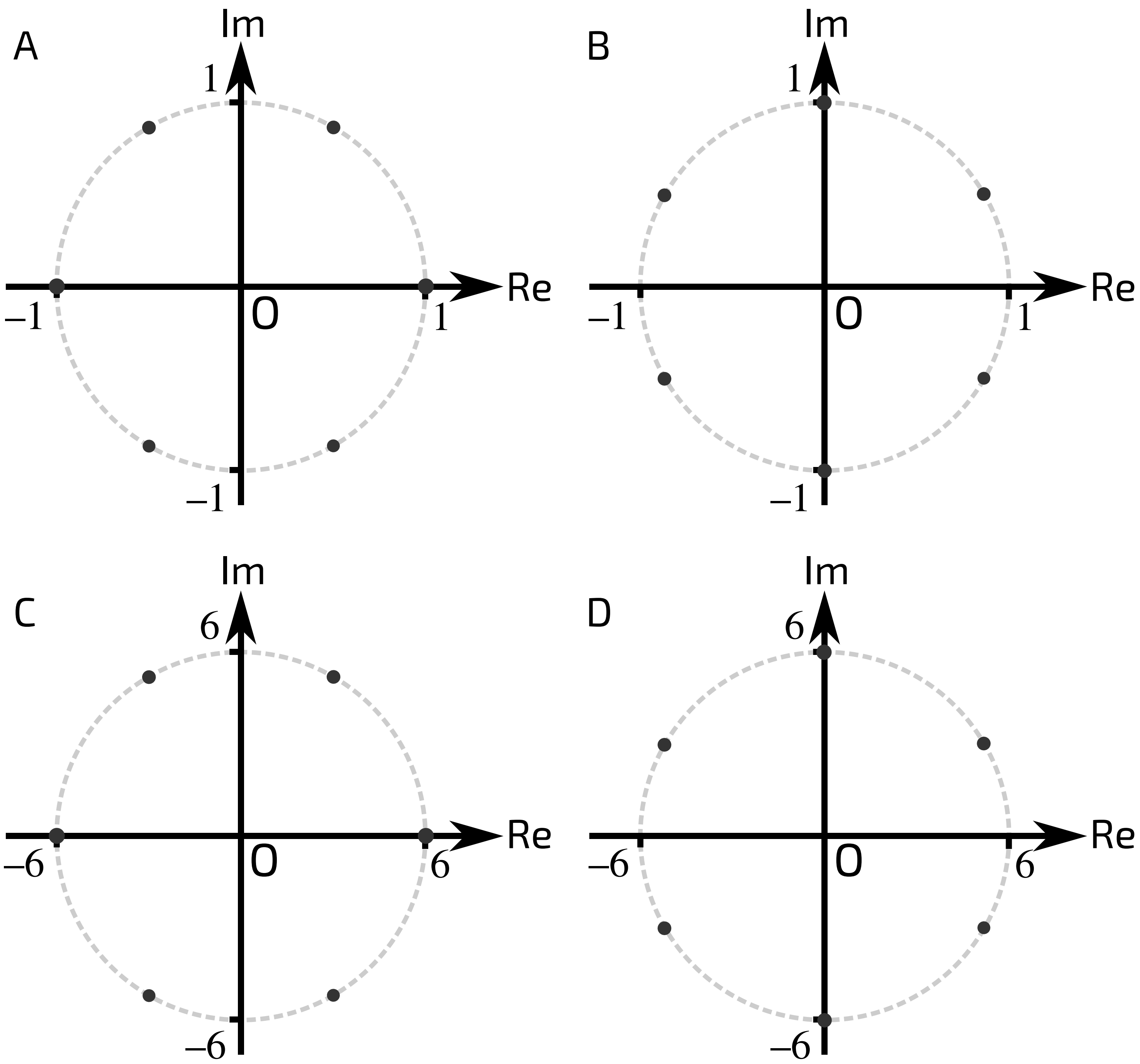

Figure 1: Four Argand diagram sketches.

- ○ Sketch A
- ○ Sketch B
- ○ Sketch C
- ○ Sketch D

Part C $(1+\mathrm{i})^6$

Evaluate $(1+\mathrm{i})^6$.

Give your answer in the form $x+\mathrm{i}y$.

The following symbols may be useful: `i`

Part D $z^6+8\mathrm{i}=0$

Hence, or otherwise, solve the equation $z^6+8\mathrm{i}=0$, giving your answers in the form $r\mathrm{e}^{\pi\mathrm{i}(a+bk)}$ where $k=0,1,2,3,4,5$.

$r>0$ and $0\leq \arg z<2\pi$.

Find r.

Find a.

Find b.

Adapted with permission from UCLES, A Level, June 2014, Paper 4727, Question 3.

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STEM SMART Double Maths 31 - Complex Exponentials & Hyperbolic Functions

Complex Numbers: De Moivre 3ii

Further A

P P P

Part A $\cos 5\theta$

Use de Moivre's theorem to show that $\cos 5\theta \equiv \mathrm{f}(\cos\theta)$.

What is $\mathrm{f}(\cos\theta)$?

The following symbols may be useful: `cos()`, `theta`

Part B Quartic roots

Hence find the roots of $16x^4 - 20x^2 + 5 = 0$ in the form $\cos\alpha$ where $0 \leqslant \alpha \leqslant \pi$.

Give the solutions x_i in order of increasing value of α.

State x_1.

The following symbols may be useful: `cos()`, `pi`

State x_2.

The following symbols may be useful: `cos()`, `pi`

State x_3.

The following symbols may be useful: `cos()`, `pi`

State x_4.

The following symbols may be useful: `cos()`, `pi`

Part C $\cos\frac{1}{10}\pi$

Hence find the exact value of $\cos\frac{1}{10}\pi$.

Adapted with permission from UCLES, A Level, June 2013 , Paper 4727, Question 8.

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Complex Numbers: De Moivre 1i

Further A

P P P

The series C and S are defined for $0 < \theta < \pi$ by

$$\begin{aligned} C &= 1 + \cos\theta + \cos 2\theta + \cos 3\theta + \cos 4\theta + \cos 5\theta, \\ S &= \quad\;\; \sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta + \sin 5\theta. \end{aligned}$$

Part A $C + \mathrm{i}S$

Write $C + \mathrm{i}S$ in terms of exponentials.

The following symbols may be useful: `e`, `i`, `theta`

Part B Expression for C

Deduce that C can be written as a product of trigonometric functions of the form $\sin a\theta \cos b\theta \operatorname{cosec} c\theta$ where a, b and c are rational numbers. Write down that expression for C.

The following symbols may be useful: `cos()`, `cosec()`, `sin()`, `theta`

Part C Expression for S

Write down a corresponding expression for S as a product of trigonometric functions.

The following symbols may be useful: `cosec()`, `sin()`, `theta`

Part D Solving $C = S$

Hence find the values of θ, in the range $0 < \theta < \pi$, for which $C = S$.

Write your answers, θ_i, in increasing order.

What is θ_1?

The following symbols may be useful: `pi`

What is θ_2?

The following symbols may be useful: `pi`

What is θ_3?

The following symbols may be useful: `pi`

What is θ_4?

The following symbols may be useful: `pi`

What is θ_5?

The following symbols may be useful: `pi`

Adapted with permission from UCLES, A Level, June 2006, Paper 4727, Question 7.

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Complex Numbers: De Moivre 5i

Further A

P P P

Part A $\sin^6\theta$

By expressing $\sin\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, show that

$$\sin^6\theta \equiv f(\cos 6\theta, \cos 4\theta, \cos 2\theta).$$

What is $f(\cos 6\theta, \cos 4\theta, \cos 2\theta)$?

The following symbols may be useful: `cos()`, `theta`

Part B $\cos^6\theta$.

Replace θ by $\left(\frac{1}{2}\pi - \theta\right)$ in the identity in part A to obtain a similar identity for $\cos^6\theta$ of the form

$$\cos^6\theta = g(\cos 6\theta, \cos 4\theta, \cos 2\theta).$$

What is $g(\cos 6\theta, \cos 4\theta, \cos 2\theta)$?

The following symbols may be useful: `cos()`, `theta`

Part C Value of an integral

Hence find the exact value of

$$\int_0^{\frac{1}{4}\pi} \left(\sin^6\theta - \cos^6\theta\right)\ d\theta.$$

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STEM SMART Double Maths 31 - Complex Exponentials & Hyperbolic Functions

Hyperbolic Functions: Manipulations 1ii

Further A

P P P

Part A $\cosh x \cosh y - \sinh x \sinh y$

Using the definitions of hyperbolic functions in terms of exponentials, prove that

$$\cosh x \cosh y - \sinh x \sinh y = \cosh(f(x,y)).$$

What is $f(x,y)$?

The following symbols may be useful: x, y

Part B Solving for y

Given that $\cosh x \cosh y = 9$ and $\sinh x \sinh y = 8$, write an expression for y in terms of x.

The following symbols may be useful: x, y

Part C Possible values of x and y

Hence find the values of x and y which satisfy the equations given in part B, giving the answers in logarithmic form.

What are the values of x? Write your answer using the $\pm$ symbol.

The following symbols may be useful: `ln()`, `log()`

What are the values of y? Write your answer using the $\pm$ symbol.

The following symbols may be useful: `ln()`, `log()`

Adapted with permission from UCLES, A Level, June 2007 , Paper 4726, Question 7.

Gameboard:

STEM SMART Double Maths 31 - Complex Exponentials & Hyperbolic Functions

Hyperbolic Functions: Manipulations 3i

Further A
P P P

Part A Defining $\tanh y$

Write an expression for $\tanh y$ in terms of e^{y} and e^{-y}.

The following symbols may be useful: `e, y`

Part B Log form of $\operatorname{artanh} x$

Given that $y = \operatorname{artanh} x$, where $-1 < x < 1$, write an expression for y as a logarithm in terms of x.

The following symbols may be useful: `ln(), log(), x`

Part C Solve $3\cosh x = 4\sinh x$

Find the exact solution of the equation $3\cosh x = 4\sinh x$, giving the answer in terms of a logarithm.

The following symbols may be useful: `ln(), log()`

Part D Solve $\text{artanh}\, x + \ln(1-x) = \ln\left(\frac{4}{5}\right)$

Solve the equation

$$\text{artanh}\, x + \ln(1-x) = \ln\left(\frac{4}{5}\right).$$

You may wish to use the $\pm$ symbol.

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STEM SMART Double Maths 31 - Complex Exponentials & Hyperbolic Functions

Hyperbolic Functions: Differentiation 2ii

Further A

P P P

The equation of a curve is $y = \cosh x - 2\sinh 2x$.

Part A $\frac{\mathrm{d}y}{\mathrm{d}x}$

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of hyperbolic functions.

The following symbols may be useful: `cosech()`, `cosh()`, `coth()`, `sech()`, `sinh()`, `tanh()`

Part B Turning points

Hence, explain why the curve has no turning points.

Drag six of the items to the right-hand column, and order them correctly to make an example proof.

Available items

- For the function in this question, the equation for the x coordinate of a turning point is: $\sinh x - 4\cosh 2x = 0$.
- This is a quadratic equation in $\cosh x$.
- The determinant is > 0, therefore the equation has no real roots.
- The determinant is < 0, therefore the equation has no real roots.
- For the function in this question, the equation for the x coordinate of a turning point is: $\sinh x + 4\cosh 2x = 0$
- This equation rearranges to $8\sinh^2 x - \sinh x + 4 = 0$.
- $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ for all x, so we have no turning points. QED
- We begin by stating that at a turning point, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.
- We begin by stating that at a turning point, $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$.
- This is a quadratic equation in $\sinh x$.
- $\frac{\mathrm{d}y}{\mathrm{d}x} \neq 0$ for all x, so we have no turning points. QED
- This equation rearranges to $8\sinh^2 x - \sinh x - 4 = 0$.

Adapted with permission from UCLES, A Level, June 2018, Paper 4726, Question 3.

Gameboard:

STEM SMART Double Maths 31 - Complex Exponentials & Hyperbolic Functions

Hyperbolic functions: Integration 1ii

Further A

P P P

Part A Definition of $\cosh x$

Using the definition of $\cosh x$ in terms of e^x and e^{-x}, write $\cosh 2x$ in terms of $\cosh^2 x$.

Give your answer in the form $\cosh 2x = f(\cosh^2 x)$

The following symbols may be useful: `cosech()`, `cosh()`, `coth()`, `sech()`, `sinh()`, `tanh()`, `x`

Part B $\int_0^1 \cosh^2 3x \; \mathrm{d}x$

Find

$$\int_0^1 \cosh^2 3x \; \mathrm{d}x,$$

giving your answer in the form $A + B\sinh C$, where A, B and C are constants to be found.

The following symbols may be useful: `cosech()`, `cosh()`, `coth()`, `sech()`, `sinh()`, `tanh()`

Adapted with permission from UCLES, A Level, June 2018 , Paper 4726, Question 2.

Gameboard:

STEM SMART Double Maths 31 - Complex Exponentials & Hyperbolic Functions

Hyperbolic functions: Integration 2i

Further A

P P P

By first completing the square, find

$$\int_0^1 \frac{1}{\sqrt{x^2+4x+8}}\,\mathrm{d}x$$

giving your answer in an exact form.

The following symbols may be useful: `arccosech()`, `arccosh()`, `arccoth()`, `arcsech()`, `arcsinh()`, `arctanh()`, `ln()`, `log()`

Adapted with permission from UCLES, A Level, January 2013 , Paper 4726, Question 6.