Differentiating Sums and Differences 1

A Level P P P Further A P P P

Part A Differentiate $ax^3 + \frac{b}{x} + c$

Differentiate $ax^3 + \frac{b}{x} + c$ with respect to x (a, b and c are constants).

The following symbols may be useful: `a, b, c, x`

Part B Differentiate $(2m+3)(m-1)$

Differentiate $(2m+3)(m-1)$ with respect to m.

The following symbols may be useful: `m`

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Error in power dissipation

A Level C C C Further A P P P

The power P dissipated in a resistance R when there is a voltage V applied across it is given by

$$P = \frac{1}{R}V^2.$$

The resistance $R = 2.00\,\Omega$ and is known very precisely, whereas the voltage $V = 10.4 \pm 0.4\,\mathrm{V}$. Find the value of P and estimate its associated error ΔP given that there is a random measurement error ΔV of $0.4\,\mathrm{V}$ in V. You are asked to take two approaches to calculating the error in P which give very similar answers.

Part A The value of P

Calculate the value of P.

Part B ΔP by differentiation

One estimate of the error ΔP in P resulting from the error in V is given by

$$\Delta P = \frac{\mathrm{d}P}{\mathrm{d}V}\Delta V$$

where $\frac{\mathrm{d}P}{\mathrm{d}V}$ is evaluated at $V = 10.4\,\mathrm{V}$ and $\Delta V = 0.4\,\mathrm{V}$. Find ΔP using this method. Give your answer to 2 significant figures.

Part C ΔP by averaging differences

Another estimate of ΔP can be obtained in the following way.

Evaluate P_+, the value of the power using $V + \Delta V = 10.4 + 0.4\,\mathrm{V}$ and P_-, the value using $V - \Delta V = 10.4 - 0.4\,\mathrm{V}$.

Now find your second estimate for ΔP which is given by $\frac{P_+ - P_-}{2}$. (This is the average of the difference between P_+ and P_0 and between P_0 and P_-, where P_0 is the value of P when $V = 10.4\,\mathrm{V}$.) Give your answer to 2 significant figures.

Physics. You work it out.

Differentiating powers 5

A Level P P P Further A P P P

The line $y = 2x + 3$ is a tangent to the curve $y = ax^2$ (where a is a constant) at a certain value of x. Find the value of x and deduce the value of a.

Find the value of x at which the line $y = 2x + 3$ is a tangent to the curve $y = ax^2$.

The following symbols may be useful: x

Find the value of a in the equation for the curve $y = ax^2$, given that $y = 2x + 3$ is a tangent to the curve at a certain point.

The following symbols may be useful: a

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Differentiating Sums and Differences 3

A Level Further A

C C C P P P

Part A Velocity if $s = ut + bt^2$

A particle is moving in one dimension. Its displacement s at time t is given by $s = ut + bt^2$, where u and b are constants. The velocity v of the particle at time t is given by the rate of change of displacement with time, i.e. $v = \frac{\mathrm{d}s}{\mathrm{d}t}$.

Find an expression for the velocity.

The following symbols may be useful: `b, t, u, v`

Part B Acceleration if $s = ut + bt^2$

A particle is moving in one dimension. Its displacement s at time t is given by $s = ut + bt^2$, where u and b are constants. The acceleration a of the particle at time t is given by the rate of change of velocity with time.

Find an expression for the acceleration.

The following symbols may be useful: `a, b, t, u`

Part C Velocity if $x = \alpha t + \beta t^3$

The displacement of a body at time t is given by $x = \alpha t + \beta t^3$ where $\alpha = 4\,\mathrm{m\,s^{-1}}$ and $\beta = 5\,\mathrm{m\,s^{-3}}$. Use the fact that the velocity is the rate of change of displacement to find the velocity of the body at $t = 2\,\mathrm{s}$.

Find the velocity of the body at $t = 2\,\mathrm{s}$.

Part D Acceleration if $x = \alpha t + \beta t^3$

The displacement of a body at time t is given by $x = \alpha t + \beta t^3$ where $\alpha = 4\,\mathrm{m\,s^{-1}}$ and $\beta = 5\,\mathrm{m\,s^{-3}}$. Use the fact that the acceleration is the rate of change of velocity to find the acceleration of the body at $t = 2\,\mathrm{s}$.

Find the acceleration of the body at $t = 2\,\mathrm{s}$.

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Differentiating powers 6

A Level Further A

A quadratic function has the form $y = a + bx + cx^2$ where a, b and c are constants. It has a stationary point at $(2, 2)$ and, at $x = 1$, the tangent to the curve has a gradient of -2. Find the values of a, b and c. (In practice, with the information given, you will need to find b and c before you can find a.)

Part A The value of b

Find the value of b.

The following symbols may be useful: b

Part B The value of c

Find the value of c.

The following symbols may be useful: c

Part C The value of a

Find the value of a.

The following symbols may be useful: a

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Stationary Points 1

A Level P P P Further A P P P

Part A Number of stationary points of $y = 2x^3 - 24x - 5$

Find the position and nature of the stationary points of the function $y = 2x^3 - 24x - 5$.

How many stationary points are there?

- 3
- 1
- 0
- 4
- 2

Part B First stationary point of $y = 2x^3 - 24x - 5$

Find the position and nature of the stationary points of the function $y = 2x^3 - 24x - 5$.

Find x_1, the x coordinate of the stationary point with the lowest value of x.

The following symbols may be useful: `x_1`, `y_1`

Find y_1, the y coordinate of the stationary point (x_1, y_1).

The following symbols may be useful: `x_1`, `y_1`

What is the nature of this stationary point?

- ○ Maximum
- ○ Minimum

Part C Second stationary point of $y = 2x^3 - 24x - 5$

Find the position and nature of the stationary points of the function $y = 2x^3 - 24x - 5$.

Find x_2, the x coordinate of the stationary point with the second lowest value of x.

The following symbols may be useful: `x_2`, `y_2`

Find y_2, the y coordinate of the stationary point (x_2, y_2).

The following symbols may be useful: `x_2`, `y_2`

What is the nature of this stationary point?

- ○ Minimum
- ○ Maximum

Part D Number of stationary points of $y = 2x^3 - 5x^2 + 4x + 6$

Find the position and nature of the stationary points of the function $y = 2x^3 - 5x^2 + 4x + 6$.

How many stationary points are there?

- 4
- 1
- 2
- 3
- 0

Part E First stationary point of $y = 2x^3 - 5x^2 + 4x + 6$

Find the position and nature of the stationary points of the function $y = 2x^3 - 5x^2 + 4x + 6$.

Find x_1, the x coordinate of the stationary point with the lowest value of x.

The following symbols may be useful: `x_1`, `y_1`

Find y_1, the y coordinate of the stationary point (x_1, y_1). (Give your answer in the form of an improper fraction.)

The following symbols may be useful: `x_1`, `y_1`

What is the nature of this stationary point?

- Maximum
- Minimum

Part F **Second stationary point of $y = 2x^3 - 5x^2 + 4x + 6$**

Find the position and nature of the stationary points of the function $y = 2x^3 - 5x^2 + 4x + 6$.

Find x_2, the x coordinate of the stationary point with the second lowest value of x.

The following symbols may be useful: `x_2`, `y_2`

Find y_2, the y coordinate of the stationary point (x_2, y_2).

The following symbols may be useful: `x_2`, `y_2`

What is the nature of this stationary point?

- Maximum
- Minimum

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Stationary Points 3

A Level C C C Further A P P P

Part A Find the maximum height of a projectile

A particle is fired upwards into the air with a initial speed w and moves subsequently under the influence of gravity with an acceleration g downwards, such that its height h at time t is given by $h = wt - \frac{1}{2}gt^2$, where w and g are constants. Find an expression for its maximum height above its initial position.

The following symbols may be useful: `g, h, w`

Part B Examine the potential energy of two molecules

The potential energy of two molecules separated by a distance r is given by

$$U = U_0 \left(\left(\frac{a}{r}\right)^{12} - 2\left(\frac{a}{r}\right)^{6} \right)$$

where U_0 and a are positive constants. The equilibrium separation of the two molecules occurs when the potential energy is a minimum; find expressions for the equilibrium separation and the value of the potential energy at this separation.

(a) Find an expression for the equilibrium separation of the molecules.

The following symbols may be useful: `U, U_0, a, r`

(b) Find an expression for the potential energy when the molecules are at their equilibrium separation.

The following symbols may be useful: `U, U_0, a, r`

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