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Woodland Survey

A Level

P P P

A conservationist wishes to collect data on the number of trees in a woodland nature reserve that have been parasitised by mistletoe. The reserve consists of birch, aspen and oak trees.

Part A Conducting a census

Why would it be impractical to conduct a census to collect data from every tree in the woodland?

- The act of collecting data from a tree would destroy it.
- The population would need to be classified into distinct strata.
- Collecting this data might spread the mistletoe
- It would be very time-consuming and expensive.
- A sampling frame would be needed.

Part B Type of sample

The conservationist wishes to determine if any particular species of tree are more likely to be parasitised by mistletoe. To do so, they decide to survey the first 10 birches, 10 aspen and 10 oak trees that they come across in the reserve.

What type of sample are they collecting?

Part C Disadvantage of sampling

What is a disadvantage of the type of sampling described in Part B?

- It is very time-consuming and expensive.
- A sampling frame is needed in order to select the sample.
- This non-random sampling could introduce bias.

Part D Species data type

What type of data is the species of a tree?

Part E Number of trees data type

What type of data is the number of trees that have been parasitised by mistletoe?

- Discrete qualitative data.
- Discrete quantitative data.
- Continuous quantitative data.
- Continuous qualitative data.

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Researcher Survey

A research institute wishes to collect data from its employees on the methods by which they travel to work. The institute has an alphabetised list of its 450 employees.

Part A Systematic sample

Describe how they could take a systematic sample of size 30.

Assign numbers from 1 to ______ to the employees on the alphabetised list. Calculate $450 \div$ ______ $=$ ______. Generate a random number between 1 and ______. Select the employee corresponding to that number and then select every ______ th employee on the list after that.

Items:

30 15 450

Part B Alternative method

One researcher suggests that it would be much easier to collect a sample by speaking to the first 30 employees in the canteen.

What type of sampling would this be?

Give one disadvantage of this type of sampling.

- This method requires the use of a sampling frame.
- This method is likely to introduce bias towards employees who use the canteen.
- This method would be time-consuming to carry out.
- Increasing the size of the sample can be very expensive.
- The sample is unlikely to be representative of the different groups among the employees.

Part C Stratified sample

The research institute also wishes to gather data from its research staff as to which new pieces of lab equipment will be required over the next year. The research staff within the institute consist of 40 geneticists, 25 ecologists and 55 epidemiologists. It is thought that the different types of staff will have different requirements for lab equipment.

Describe how the institute could could collect a stratified sample of size 20 from its research staff.

There are a total of 120 research staff. The sample is ____ $\div 120 =$ ____ of the research staff.

The institute should survey $40 \times$ ____ $\approx$ ____ geneticists, $25 \times$ ____ $\approx$ ____ ecologists and $55 \times$ ____ $\approx$ ____ epidemiologists. The staff should be selected randomly from each group, by generating random numbers from 1 to ____ for geneticists, 1 to ____ for ecologists and 1 to ____ for epidemiologists and selecting the corresponding members of staff, discarding and replacing any duplicate numbers within each group.

Items:

40 55 20 $\frac{1}{6}$ 4 $\frac{11}{24}$ 6 5 7 25 10 $\frac{1}{3}$ $\frac{5}{24}$ 9

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Permutations and Combinations 1

A Level

P P P

This question is about the number of possible orders when rearranging the letters of the word NEVER.

Part A Distinct permutations

The five letters of the word NEVER are arranged in random order in a straight line.

How many different orders of the letters are possible?

Part B Orders with adjacent Es

The five letters of the word NEVER are arranged in random order in a straight line.

In how many of the possible orders are the two E's next to each other?

Part C Orders with one E in the first two letters

The five letters of the word NEVER are arranged in random order in a straight line.

Find the probability that the first two letters in the order include exactly one letter E.

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Permutations and Combinations 3

A Level

P P P

This question is about the number of possible orders of 7 students sitting on a bench.

Part A Number of permutations

A group of 7 students sit in a random order on a bench. Find the number of orders in which they can sit.

Part B Permutations where two students must be adjacent

A group of 7 students sit in a random order on a bench. The 7 students include Tom and Jerry. Find the probability that Tom and Jerry sit next to each other.

Part C Permutations where no boys are adjacent

A group of 7 students sit in random order on a bench. The students consist of 3 girls and 4 boys. Find the probability that no two boys sit next to each other.

Part D Permutations where all girls are adjacent

A group of 7 students sit in a random order on a bench. The students consist of 3 girls and 4 boys. Find the probability that all three girls sit next to each other.

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Permutations and Combinations 4

A Level

This question is about students taking numbered cards out of a bag to make 4-digit numbers, and finding the probabilities of the results they might get.

Part A How many 4-digit numbers can be made?

A bag contains 9 discs numbered $1, 2, 3, 4, 5, 6, 7, 8, 9$. Andrea chooses 4 discs at random, without replacement, and places them in a row.

How many different 4-digit numbers can be made?

Part B How many odd 4-digit numbers?

A bag contains 9 discs numbered $1, 2, 3, 4, 5, 6, 7, 8, 9$. Andrea chooses 4 discs at random, without replacement, and places them in a row.

How many different **odd** 4-digit numbers can be made?

Part C Numbers with 3 odd digits

A bag contains 9 discs numbered $1, 2, 3, 4, 5, 6, 7, 8, 9$. Martin chooses 4 discs at random, without replacement.

Find the probability that the 4 digits include at least 3 odd digits.

Part D Digits that add up to 28

A bag contains 9 discs numbered $1, 2, 3, 4, 5, 6, 7, 8, 9$. Martin chooses 4 discs at random, without replacement.

Find the probability that the 4 digits add up to 28.

Used with permission from UCLES, A level, January 2012, Paper 4732, Question 9

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Home Gameboard Maths Statistics Hypothesis testing 3.1

Hypothesis testing 3.1

A Level

P P P

An experiment has two possible outcomes, A and B. An experimenter investigates the probabilities with which the two outcomes occur by carrying out the experiment a large number of times; X, the number of times the experimenter gets outcome A, is recorded. Answer the following questions.

Part A Outcomes equally likely

In a previous set of measurements the two outcomes of the experiment (A and B) have been found to be equally likely. An experimenter wishes to test, at the 5% significance level, the hypothesis that the two outcomes are not equally likely. The experiment is carried out 50 times and X, the number of times the experimenter gets outcome A, is recorded.

Find the lower bound X_l of the acceptance region.

Find the upper bound X_u of the acceptance region.

Part B Slight difference in probability of outcomes

In a previous set of measurements the two outcomes of the experiment (A and B) have been found to be equally likely. The experiment is carried out 50 times and the experimenter gets outcome A 34 times i.e. $X = 34$. By answering the following three questions use the data to test, at the 1% significance level, the hypothesis that the outcomes are not equally likely.

Find the probability P_p, assuming the null hypothesis, that $X = 34$. Give your answer to 2 s.f.

Find the probability P_c, assuming the null hypothesis, that $X \geq 34$. Give your answer to 2 s.f.

You were asked to use the data above to test, at the 1% significance level, the hypothesis that the outcomes are not equally likely. Using the appropriate answer from earlier is there sufficient evidence to reject, at this level, the null hypothesis that the outcomes are equally likely? Which of the following arguments is correct in this regard?

- $P_c > 0.005$ so there is not sufficient evidence to reject the null hypothesis
- $P_p < 0.01$ so reject the null hypothesis
- $P_p < 0.005$ so reject the null hypothesis
- $P_p > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_c < 0.005$ so reject the null hypothesis
- $P_c < 0.01$ so reject the null hypothesis
- $P_p > 0.005$ so there is not sufficient evidence to reject the null hypothesis
- $P_c > 0.01$ so there is not sufficient evidence to reject the null hypothesis

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Part C Probability of A greater than that of B

In a previous set of measurements the two outcomes of the experiment (A and B) have been found to be equally likely. As described in Part B the experiment is carried out 50 times and the experimenter gets outcome A 34 times i.e. $X = 34$. Test, at the 1% significance level, the hypothesis that the probability of getting the outcome A is greater than that of getting the outcome B.

Is there sufficient evidence to reject, at this level, the null hypothesis that the outcomes are equally likely? Which of the following arguments is correct in this regard?

- $P_c > 0.005$ so there is not sufficient evidence to reject the null hypothesis
- $P_p > 0.005$ so there is not sufficient evidence to reject the null hypothesis
- $P_p > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_c < 0.005$ so reject the null hypothesis
- $P_p < 0.005$ so reject the null hypothesis
- $P_c > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_p < 0.01$ so reject the null hypothesis
- $P_c < 0.01$ so reject the null hypothesis

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Hypothesis testing 3.2

A Level

P P P

An experimenter uses a piece of test equipment to make measurements of samples she has manufactured. There is a 10% probability that the equipment will fail when she makes a measurement. She makes some changes to the equipment which she hopes will increase its reliability. Answer the following questions about how she will test whether her changes have increased the reliability.

Part A Carrying out 30 measurements

The experimenter would like to test the hypothesis, at the 2% level of significance, that her changes have increased the reliability of her equipment. She decides to make 30 measurements and there are no failures.

Assuming that the failure rate is unchanged find the probability that there are no failures. Give your answer to 2 s.f.

Deduce what the experimenter can conclude about the changes she has made.

- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; therefore the changes have not improved the reliability of the test equipment.
- Given the original failure rate she would have expected about 3 failures, so there is sufficient evidence to support her hypothesis
- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; there is insufficient evidence to indicate that the changes have improved the reliability of the test equipment.
- There is sufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- The changes have not improved the reliability of the test equipment.

Part B Carrying out more measurements

The experimenter realises that she needs more evidence to test, at the 2% level of significance, whether or not her changes have increased the reliability of her equipment. She decides to make more measurements. Assuming that she still gets no failures, how many measurements will she have to make in total to provide support for her hypothesis at the 2% level of significance?

Part C Carrying out 60 measurements

The experimenter makes 60 measurements in all and gets one failure.

Assuming that the failure rate is unchanged find the probability that she will get one failure or no failures. Give your answer to 2 s.f.

Hence deduce what the experimenter can conclude about the changes she has made.

- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is insufficient evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- There is sufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- The number of failures is less than the 6 expected so the null hypothesis can be rejected; the reliability of the test equipment has therefore improved.
- There is sufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is insufficient evidence to support the hypothesis that the changes have improved the reliability of the test equipment

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Hypothesis testing 3.3

A Level

P P P

According to one theory the probability that a particular result will be obtained in an experiment is 0.6 whereas according to another theory the probability is 0.7. The experiment is carried out 100 times and the particular result is obtained 76 times.

Part A Testing $p = 0.6$

Consider the first theory which suggests that $p = 0.6$; assume the null hypothesis is that $p = 0.6$ and the alternative hypothesis is that $p > 0.6$. Find the critical region for the test (in this case if X is the number of times the particular result is obtained the critical region has one part with $X \geq X_h$). If the theory is to be tested at the 2% level of confidence, deduce X_h.

Part B Testing $p = 0.7$

Now consider the second theory which suggests that $p = 0.7$; assume the null hypothesis is that $p = 0.7$ and the alternative hypothesis is that $p \neq 0.7$. Find the critical region for the test (in this case if X is the number of times the particular result is obtained the critical region has two parts $X \leq X_l$ and $X \geq X_h$). If the theory is to be tested at the 2% level of confidence, deduce X_l and X_h.

Find X_l.

Find X_h

Part C Conclusions from the tests

What can be concluded at the 2% level from the tests?

- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; the hypothesis that $p = 0.7$ is correct.
- The hypotheses that $p > 0.6$ and that $p = 0.7$ are correct.
- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; there is sufficient evidence to reject the hypothesis that $p = 0.7$ and to support the theory that $p \neq 0.7$.
- There is insufficient evidence to reject the hypothesis that $p = 0.6$; there is insufficient evidence to reject the hypothesis that $p = 0.7$.
- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; there is insufficient evidence to reject the hypothesis that $p = 0.7$.
- There is insufficient evidence to reject the hypothesis that $p = 0.6$; there is sufficient evidence to reject the hypothesis that $p = 0.7$ and to support the theory that $p \neq 0.7$.

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Expectation and Variance

Further A

A discrete random variable X has the following probability distribution:

x	0	1	2	3	4
$\mathrm{P}(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	k	$2k$	$\frac{1}{3} - k$

Part A Find k

Find the value of k.

The following symbols may be useful: `k`

Part B Expectation of X

Find $\mathrm{E}(X)$.

The following symbols may be useful: `E, X`

Part C Variance of X

Find $\mathrm{Var}(X)$.

Part D Expectation of Y

Another discrete random variable Y is defined as $Y = 2X - 3$.

Find $\mathrm{E}(Y)$.

The following symbols may be useful: `E`, `Y`

Part E Variance of Y

Find $\mathrm{Var}(Y)$.

Part F Probability that $X > Y$

Find P$(X > Y)$.

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Combining Random Variables

Further A

C C C

R is a discrete random variable. S is another discrete random variable and is defined by $S = \frac{3R+2}{4}$. It is given that $\mathrm{E}(S) = -2$ and $\mathrm{Var}(S) = 9$.

Part A Expectation of R

Find $\mathrm{E}(R)$.

The following symbols may be useful: `E`, `R`

Part B Variance of R

Find $\mathrm{Var}(R)$.

Part C Expectation of S and T

The random variable T is such that $\mathrm{E}(T) = 5$ and $\mathrm{Var}(T) = 21$.

Find $\mathrm{E}(3S - 2T)$.

The following symbols may be useful: `E`, `S`, `T`

Part D Variance of S and T

Find $\mathrm{Var}(3S - 2T)$.

Part E Expectation of R and T

Find $\mathrm{E}(R_1 + R_2 + R_3 + \frac{T}{3})$, where R_1, R_2 and R_3 indicate independent readings of R.

Part F Variance of R and T

Find $\mathrm{Var}(R_1 + R_2 + R_3 + \frac{T}{3})$, where R_1, R_2 and R_3 indicate independent readings of R.

Part G Variance of $3R$ and T

Find $\mathrm{Var}(3R + \frac{T}{3})$.

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