Finding Roots

A Level

The polynomial $f(x)$ is given by:

$$f(x) = x^3 + 6x^2 + x - 4$$

Part A Factorisation

Show that $(x+1)$ is a factor of $f(x)$ and enter the quotient obtained when $f(x)$ is divided by $(x+1)$.

The following symbols may be useful: `f, x`

Part B Find the roots

Find the exact roots of $f(x)$.

Give the expression for the value for the lowest, most negative root to check your answer.

The following symbols may be useful: `f, x`

Part C Sketch the graph of $f(x)$

Sketch the curve $f(x) = x^3 + 6x^2 + x - 4$ and find the coordinates of intersection with the axes.

Give the coordinate of intersection with the y-axis. Please give your answer in the form "y=", "asymptote", or "none" if there is no intersection.

Part D Logarithmic equation

Write the equation

$$2\log_2(x+3)+\log_2 x-\log_2(4x+2)=1$$

in the form $g(x)=0$, where $g(x)$ is a polynomial function of x.

Give the polynomial expression of $g(x)$.

The following symbols may be useful: `g, x`

Part E Sketch $\log_2 x$

Sketch the curve $h(x)=\log_2 x$ and find the coordinates of intersection with the axes.

Give the coordinate of intersection with the y-axis. Please give your answer in the form "y=", "asymptote" or "none" if there is no intersection.

Part F Find the root

Explain why the equation

$$2\log_2(x+3)+\log_2 x-\log_2(4x+2)=1$$

only has one real root.

State its value.

The following symbols may be useful: `x`

Adapted with permission from UCLES, A Level, June 2007, Paper 4722, Question 9.

Circles and Geometry

A Level

A circle has equation

$$(x-5)^2+(y+2)^2=25$$

Part A Centre

(i) Find the x coordinate of the centre C of the circle.

The following symbols may be useful: x

(ii) Find the y coordinate of C.

The following symbols may be useful: y

Part B Diameter

Find the diameter d of the circle.

The following symbols may be useful: d

Part C Line through C and P

Find the equation of the line passing through the centre C and the point $P(7, 2)$.

The following symbols may be useful: `x, y`

Part D Length of CP and position of P

(i) Calculate the exact length of CP.

(ii) Using the previous result, determine which **one** of these statements is correct.

- P lies outside the circle.
- P lies inside the circle.
- P lies on the circumference of the circle.

Part E Relation to the line $y = 2x$

Which **one** of the choices below best describes how the circle is related to the line $y = 2x$?

The line and circle do not meet.

The line meets the circle once tangentially.

The line intersects the circle at two distinct points.

(ii) If the line and circle do meet, give the x-coordinate of point of intersection with the largest x coordinate. If they do not meet, give the largest positive value of a for which the line $y = ax$ meets the circle.

The following symbols may be useful: a

Adapted with permission from UCLES, A Level, June 2012, Paper 4721, Question 10.

Gameboard:
STEM SMART Single Maths 14 - Pure Revision (non-calculus)

Exponentials and Logs

A Level

C C C

Part A Sketching

Consider the curve $y = 6 \times 5^x$, sketch it and find the value of the y intercept of the curve.

What is the value of the y intercept of the curve?

The following symbols may be useful: y

Part B Find x-coordinate

The point P on the curve $y = 9^x$ has y-coordinate equal to 150. Use logarithms to find the x-coordinate of P.

Give the x-coordinate of P to 3 significant figures.

Part C New x-coordinate

The curves $y = 6 \times 5^x$ and $y = 9^x$ intersect at the point Q. Find the exact value of the x-coordinate at point Q, giving any logarithms in base three.

Give the exact value of the x-coordinate at point Q, giving any logarithms in base three $(\log_3)$.
When typing $\log_3$ into the answer box, use the syntax: $\log$(number, base), i.e. $\log_3 2 = \log(2, 3)$.

The following symbols may be useful: `log(), x`

Gameboard:

STEM SMART Single Maths 14 - Pure Revision (non-calculus)

Applying Trigonometry

A Level

C C C

A rower attempts to row across a river from a place A where the banks of the river are straight and parallel. They wish to reach a point B which is directly opposite to A on the other bank (i.e. on a line perpendicular to the bank at point A).

They start to row towards B, keeping the boat aligned in a direction parallel to $\underline{\boldsymbol{j}}$, but discover that there is a current flowing in a direction $\underline{\boldsymbol{i}}$ parallel to the banks, such that their resultant travel is along a vector $\underline{\boldsymbol{v}} = \underline{\boldsymbol{i}} + 4\underline{\boldsymbol{j}}$.

Part A Speed of the rower

Find the magnitude of vector $\underline{\boldsymbol{v}}$.

The following symbols may be useful: v

Part B Angle between $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{i}}$

Find the angle between vectors $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{i}}$. Give your answer to no more than 3 sig figs.

Part C Direction of travel

If they are to arrive at B, but can adjust their rowing speed to take the same time as before to cross to the other bank, in what direction should they actually row? Give your answer as a vector in terms of the unit vectors i and j.

The following symbols may be useful: `i, j, v_row`

What is the angle between $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{j}}$? Give your answer to no more than 3 sig figs.

Part D A tower in the distance

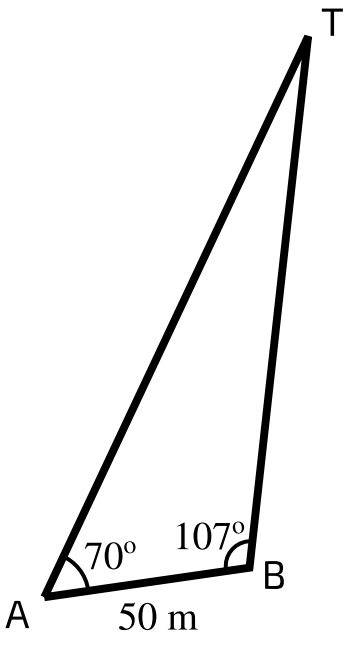

Figure 1: The tower, and points A and B.

Some walkers see a tower, T, in the distance and want to know how far away it is. They take a bearing from point A and walk for $50\,\mathrm{m}$ in a straight line before taking another bearing from point B. They find that the angle TAB is $70\,^\circ$ and angle TBA is $107\,^\circ$ (see **Figure 1**).

Find the distance of the tower from A. Give you answer to three significant figures.

Part E Distance from C

They continue walking in the same direction for another $100\,\mathrm{m}$ to a point C, so that AC is $150\,\mathrm{m}$. What is the distance of the tower from C? Give your answer to three significant figures.

Part F Shortest distance from A to C

Find the shortest distance of the walkers from the tower as they walk from A to C. Give your answer to three significant figures.

Part G Area swept out

D is the point on AC such that TD is the shortest distance of the walkers from the tower.

Find the area of ground represented by the triangle ATD. Give your answer in km^2 and to 3 significant figures.

Adapted with permission from SAW 2017 and UCLES, A Level, January 2009, Paper 4722, Question 5.

Gameboard:

STEM SMART Single Maths 14 - Pure Revision (non-calculus)

Expansions and Algebra

A Level

C C C

Part A Indices and surds

Simplify $2x^{\frac{2}{3}} \times 3x^{-1}$.

The following symbols may be useful: x

Part B Indices

Express $2^{40} \times 4^{30}$ in the form 2^n.

The following symbols may be useful: n

Part C Simplifying expressions

Express $\frac{26}{4-\sqrt{3}}$ in the form $a + b\sqrt{3}$.

The following symbols may be useful: a, b

Part D Binomial expansions

Find the first four terms in the expansion, in ascending powers of x, of

$$(1+3x)^8 \ .$$

The following symbols may be useful: x

Part E Summing binomial expansions

Show that, if terms involving x^4 and higher powers of x may be ignored,

$$(1+3x)^8 + (1-3x)^8 = a + bx^2$$

Enter $a + bx^2$, substituting in the values for a and b.

The following symbols may be useful: x

Part F Estimating

Use the equation from the previous part $(1+3x)^8 + (1-3x)^8 = a + bx^2$ to solve this question.

Find the value of $1.000\,003^8 + 0.999\,997^8$ correct to 12 decimal places.

Adapted with permission from UCLES, A Level, January 2008, Paper 4722, Question 10, and June 2002, Paper 2, Question 5.