Stationary Points 2ii

A Level

P P P

Part A Find coordinate

Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 + 4$. Enter the x and y coordinates of the stationary point with the greatest x coordinate.

Enter the x-coordinate:

The following symbols may be useful: x

Enter the y-coordinate:

The following symbols may be useful: y

Part B Stationary point

Determine whether the stationary point whose coordinates you entered is a maximum point or a minimum point.

- Inconclusive
- Minimum
- Maximum

Part C Range of x

For which range of values of x does $x^3 - 3x^2 + 4$ decrease as x increases?

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $x < a$
- $x \leq a$
- $x > a$
- $x \geq a$
- $a < x < b$
- $a \leq x \leq b$
- $x < a$ or $x > b$
- $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

Used with permission from UCLES, A level, January 2006, Paper 4721, Question 6

Powers using Chain Rule 3

A Level Further A

P P P P P P

Part A Stationary point of $y = (2 - 3x)^4 + 4$

Find the coordinates and nature of the stationary point of the function $y = (2 - 3x)^4 + 4$.

Find the x coordinate of the stationary point of the function $y = (2 - 3x)^4 + 4$.

The following symbols may be useful: x

Find the y coordinate of the stationary point of the function $y = (2 - 3x)^4 + 4$.

The following symbols may be useful: y

By considering the behaviour of the function when x is very large and positive and also when it is very large and negative deduce the nature of the stationary point of the function $y = (2 - 3x)^4 + 4$.

- Minimum
- Maximum

Part B Stationary points of $q = 4(2p-1)^3 - 3(2p-1)^4$

Consider the function $q = 4(2p-1)^3 - 3(2p-1)^4$.

Find the stationary points of the function. How many are there? The stationary point with the lowest value of p is at (p_1, q_1) and the stationary point with the second lowest value of p is at (p_2, q_2). Find the values of p and q at (p_1, q_1) and (p_2, q_2).

How many stationary points are there?

- 1
- 0
- 3
- 2
- 4

Find p_1, the p coordinate of the stationary point with the lowest value of p.

The following symbols may be useful: `p_1, q_1`

Find p_2, the p coordinate of the stationary point with the second lowest value of p.

The following symbols may be useful: `p_2, q_2`

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Maxima and Minima: Problems 2ii

A Level
P P P

A curve has equation $y = 3x^3 - 7x + \frac{2}{x}$

Part A Verify stationary point

Verify the curve has a stationary point when $x = 1$.

More practice questions?

Part B Nature of stationary point

Determine the nature of this stationary point.

- Neither/inconclusive
- Maximum
- Minimum

Part C Tangent to curve

The tangent to the curve at this stationary point meets the y-axis at the point Q. Find the y-coordinate of Q.

Used with permission from UCLES, A Level, June 2014, Paper 4721, Question 8.

Minimising the area

A Level Further A

C C C P P P

A rectangular cuboid has a base with sides of length a and b and a height c. Its volume V and height c are fixed. By following the steps below find expressions in terms of V and c for the values of a and b which will minimise the surface area A of the cuboid, find an expression for this minimum surface area and check that this is indeed a minimum.

Part A Volume V and surface area A

Write down the equation for the volume V of the rectangular cuboid in terms of a, b and c.

The following symbols may be useful: `V, a, b, c`

Write down the equation for the area A of the rectangular cuboid in terms of a, b and c.

The following symbols may be useful: `A, a, b, c`

From your equation for V deduce an expression for b in terms of V, a and c. Hence, by substitution, obtain an equation for A in terms of V, a and c.

The following symbols may be useful: `A, V, a, c`

Part B Expressions for a and b

Differentiate with respect to a the expression for A you found in Part A (since V and c are fixed you may treat them as constants). Hence find in terms of V and c an expression for the value of a for which the area A is minimised.

The following symbols may be useful: `V`, `c`

Find, in terms of V and c, the expression for b corresponding to this value of a.

The following symbols may be useful: `V`, `c`

Part C The minimum area

Find an expression for the minimum area in terms of V and c.

The following symbols may be useful: `V`, `c`

Part D Check that the area is a minimum

Find, at the value of a deduced in Part B, an expression in terms of V and c for the second derivative of A with respect to a; convince yourself that the value of the second derivative indicates that the value of A is a minimum at this point.

The following symbols may be useful: `V`, `c`

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Stationary Points 4ii

A Level

P P P

Part A Find coordinates

Find the coordinates of the stationary point on the curve $y = x^4 + 32x$. Enter the x and y coordinates below.

Enter x coordinate:

The following symbols may be useful: x

Enter y coordinate:

The following symbols may be useful: y

Part B Maxima or Minima

Determine whether this stationary point is a maximum or a minimum.

- Maximum
- Minimum

Part C Range of x

For what range of values of x does $x^4 + 32x$ increase as x increases? Give your answer in the form of an inequality.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `x`

Home Maths Calculus Differentiation Differentiating Natural Logs

Differentiating Natural Logs

A Level Further A

Part A Differentiate $u = \ln(2v + 3)$

Find $\dfrac{\mathrm{d}u}{\mathrm{d}v}$ if $u = \ln(2v + 3)$.

The following symbols may be useful: v

Part B Stationary point of $p = 2\ln(2q) - 3q$

Find the coordinates and nature of the stationary point of the function $p = 2\ln(2q) - 3q$.

Find the q coordinate of the stationary point.

The following symbols may be useful: q

Find the p coordinate of the stationary point.

The following symbols may be useful: p

Determine the nature of the stationary point.

- Minimum
- Maximum

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Stationary Points 1ii

A Level

P P P

The curve $y = x^3 - kx^2 + x - 3$ has two stationary points.

Part A Differentiate

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: k, x

Part B Find k

Given that there is a stationary point when $x = 1$, find the value of k.

The following symbols may be useful: k

Part C Differentiate twice

Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

The following symbols may be useful: x

Hence determine whether the stationary point is a minimum or a maximum.

- Maximum
- Minimum

Part D Find coordinate

Find the x-coordinate of the other stationary point.

The following symbols may be useful: x

Used with permission from UCLES, A Level, June 2009, Paper 4721, Question 10.

Stationary Points 1i

A Level

P P P

Part A Find stationary points

Find the coordinates of the stationary points on the curve $y = 2x^3 - 3x^2 - 12x - 7$. Enter the x and y coordinates of the stationary point with the largest x coordinate.

Enter the x coordinate:

The following symbols may be useful: x

Enter the y coordinate:

The following symbols may be useful: y

Part B Nature of stationary points

Determine whether each stationary point is a minimum or maximum point. Identify the nature of the stationary point whose coordinates you have entered in Part A.

- Maximum
- Minimum

Part C Expand and simplify

Expand and simplify $(x+1)^2(2x-7)$.

The following symbols may be useful: x

Part D Sketch

Hence sketch the curve $y = 2x^3 - 3x^2 - 12x - 7$, indicating the coordinates of all stationary points and intercepts with the axes. In order to check your answer, give the value of the intercept with the y-axis.

The following symbols may be useful: y

Used with permission from UCLES, A Level, Paper 4721 specimen, Question 8.

Differentiating Exponentials 3

A Level Further A

P P P P P P

Part A Tangent to $y = e^{2x} - e^{-2x}$

Find the equation of the tangent to the curve $y = e^{2x} - e^{-2x}$ at the point $x = \frac{1}{2}$.

The following symbols may be useful: e, x, y

Part B Stationary point of $u = 2e^{3v} - 3v$

Find the coordinates and nature of the stationary point of the function $u = 2e^{3v} - 3v$.

Find the v coordinate of the stationary point.

The following symbols may be useful: v

Find the u coordinate of the stationary point.

The following symbols may be useful: u

Determine the nature of the stationary point.

- Minimum
- Maximum

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