



## Manipulating Vectors in 3D

A Level Further A



A vector  $\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$  has a length of 4 units. For the following three cases, find  $u_x$ ,  $u_y$  and  $u_z$  and answer the questions.

### Part A Case 1

The vector lies in the  $(x, y)$ -plane, makes an angle of  $30^\circ$  with the  $x$ -direction and  $u_y$  is positive.

What is  $u_x$ ? Give your answer to 3.s.f

### Part B Case 2

The vector has  $u_x = u_y = 2$  and  $u_z$  is negative.

What is  $u_z$ ? Give your answer to 3.s.f

### Part C Case 3

The vector is such that  $u_z = 1$ ,  $u_y = 2u_x$ , and  $u_y$  is positive.

What is  $u_y$ ? Give your answer to 3.s.f



## Vectors: Position, Distance and Problems 1i



An aircraft flies due north from A for a distance of 360 km ( $3.6 \times 10^5$  m) to point B. Its average speed between A and B is  $170 \text{ m s}^{-1}$ . At B the aircraft is forced to change course and flies due east for a distance of 100 km to arrive at C.

### Part A Journey time

Calculate the time of the journey from A to B.

### Part B A vector triangle

Draw a labelled displacement triangle to represent the aircraft's journey.

Easier question?

### Part C Distance A to C

Use the displacement triangle to determine the magnitude of the displacement in km of the aircraft at C from A.

### Part D Bearing

Find the bearing of C from A.





## Vectors: Positions and Problems 1ii



It is given that  $\underline{a} = 3\underline{i} + 4\underline{j}$ ,  $\underline{b} = -5\underline{i} + 2\underline{j}$ ,  $\underline{c} = 7\underline{i} - 3\underline{j}$  and  $\underline{d} = 3\underline{a} - 2\underline{b} + \underline{c}$ .

### Part A   Vector $\underline{d}$

Write down vector  $\underline{d}$  in terms of  $\underline{i}$  and  $\underline{j}$ . When you enter your answer, use ordinary  $i$  and  $j$  to represent the unit vectors.

The following symbols may be useful:  $d$ ,  $i$ ,  $j$

### Part B   Magnitude of $\underline{d}$

Find the magnitude of  $\underline{d}$ . Give your answer to 3 significant figures.

### Part C   Direction of $\underline{d}$

Find the direction of vector  $\underline{d}$ , relative to  $\underline{i}$ . If the direction is anticlockwise from  $\underline{i}$  give a positive angle, if it is clockwise from  $\underline{i}$  give a negative angle. Give your answer in degrees to 3 significant figures.



## 3D Vectors 1i



The points  $A$ ,  $B$  and  $C$  are three of the vertices of a parallelogram.

The point  $P$  lies at the location  $8\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$ , relative to the fixed origin  $O$ . Relative to  $P$ , the locations of  $A$ ,  $B$  and  $C$  are

$$\vec{PA} = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}, \quad \vec{PB} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad \vec{PC} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

### Part A   Three locations

Find the one of the three possible locations for  $D$  (ie, the fourth point of the parallelogram). Give your answer in the form  $(x, y, z)$  with the commas and without spaces.

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### Part B   One location

You are given the extra information that point  $D$  lies on a sphere of radius  $\sqrt{29}$  centred on  $P$ .

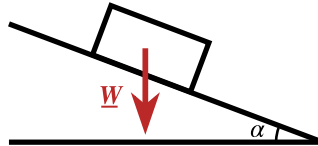
What is the location of  $D$ ? Give your answer in the form  $(x, y, z)$  with the commas and without spaces.

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## Block on a Slope

A block of weight  $\underline{W}$  is on a slope which makes an angle  $\alpha$  to the horizontal as shown in **Figure 1**



**Figure 1:** Block at rest on the slope.

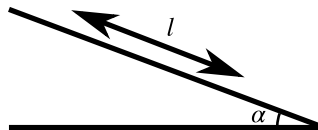
### Part A Perpendicular Component of Weight

What is the component of the block's weight  $\underline{W}$  that acts perpendicular to the slope?

The following symbols may be useful:  $W$ ,  $\alpha$

### Part B Horizontal Component of Displacement

The block slides a distance  $l$  along the slope as shown in **Figure 2**.



**Figure 2:** The distance the block moves down the slope.

How far has the block moved horizontally?

The following symbols may be useful:  $\alpha$ ,  $l$

The block slides a distance  $l$  along the slope as in Part B. How much gravitational potential energy has the block lost?

The following symbols may be useful:  $W$ ,  $\alpha$ ,  $l$

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# Position of Boat

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A Level Further A



A boat sails 4.00 km at a bearing of  $210^\circ$ .

## Part A Displacement South

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How far south of its starting point is its final position?

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## Part B Displacement West

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How far west of its starting point is its final position?

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## Vectors: Position, Distance and Problems 2i



A man drives his friend to a station, A, to catch a train. When the man and his friend arrive at station A, the train is ready to leave and the friend cannot catch the train.

The man knows the train goes to another station D on a journey which can be modelled as a straight line. The train travels at an average speed of 30 mph and arrives at station D 22 minutes after leaving station A.

The man knows a road route from station A to station D that can be modelled as three straight roads, going via road junctions, B and C, such that

$$\vec{AB} = 4\mathbf{i} + 3\mathbf{j} \quad \vec{BC} = 2\mathbf{j}, \quad \vec{CD} = -4\mathbf{i} + 6\mathbf{j}$$

where the numbers are distances in miles.

The man can drive with an average road speed of 45 mph. Can the man drive his friend from station A to station D in time for his friend to catch the train from station D? Show clearly how you arrive at your answer.

How many minutes spare does the friend have or by how many minutes has he missed it by? If he missed the train, give a negative number of minutes.

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Written for Isaac Physics by Sally Waugh



## 3D Vectors 1ii

A Level



$ABCD$  is a parallelogram. The position vectors of  $A$ ,  $B$  and  $C$  are given respectively by

$$\underline{a} = 2\underline{i} + \underline{j} + 3\underline{k} \qquad \underline{b} = 3\underline{i} - 2\underline{j} \qquad \underline{c} = \underline{i} - \underline{j} - 2\underline{k}$$

### Part A   Position of $D$

Find the position vector of  $D$ . Give your answer using  $ijk$  notation.

The following symbols may be useful:  $\underline{i}$ ,  $\underline{j}$ ,  $\underline{k}$

### Part B   Unit vector

Find the unit vector in the direction  $\vec{OD}$  where  $O$  is the fixed origin. Give your answer using  $ijk$  notation.

The following symbols may be useful:  $\underline{i}$ ,  $\underline{j}$ ,  $\underline{k}$

Adapted with permission from UCLES, A Level, June 2005, OCR C4, Question 5

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## 3D Vectors 3i

A Level



The position vectors of the points  $A$  and  $B$ , relative to a fixed origin  $O$ , are  $6\mathbf{i} + 4\mathbf{j} - \mathbf{k}$  and  $8\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$  respectively.

### Part A Vector $\underline{AB}$

Find the vector  $\vec{AB}$ . Give your answer in the form  $(x, y, z)$  with the commas and without spaces.

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Find the length  $AB$ .

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### Part B Line $\underline{l}$

A point  $P$  lies on the line

$$\underline{l} = (8 + 2\lambda)\mathbf{i} + (5 + \lambda)\mathbf{j} - (3 + 2\lambda)\mathbf{k}$$

Find the direction vector of  $\underline{l}$ . Give your answer in the form  $(x, y, z)$  with the commas and without spaces.

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How does the line  $\underline{l}$  relate to the points  $A$  and  $B$ ?

- ☐ It passes through  $A$
- ☐ It passes through  $B$
- ☐ It is in the direction of  $AB$
- ☐ All of the above

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## Vectors: Positions and Problems 2ii

Figure 1 shows a ship **S** being pulled by two tug-boats.

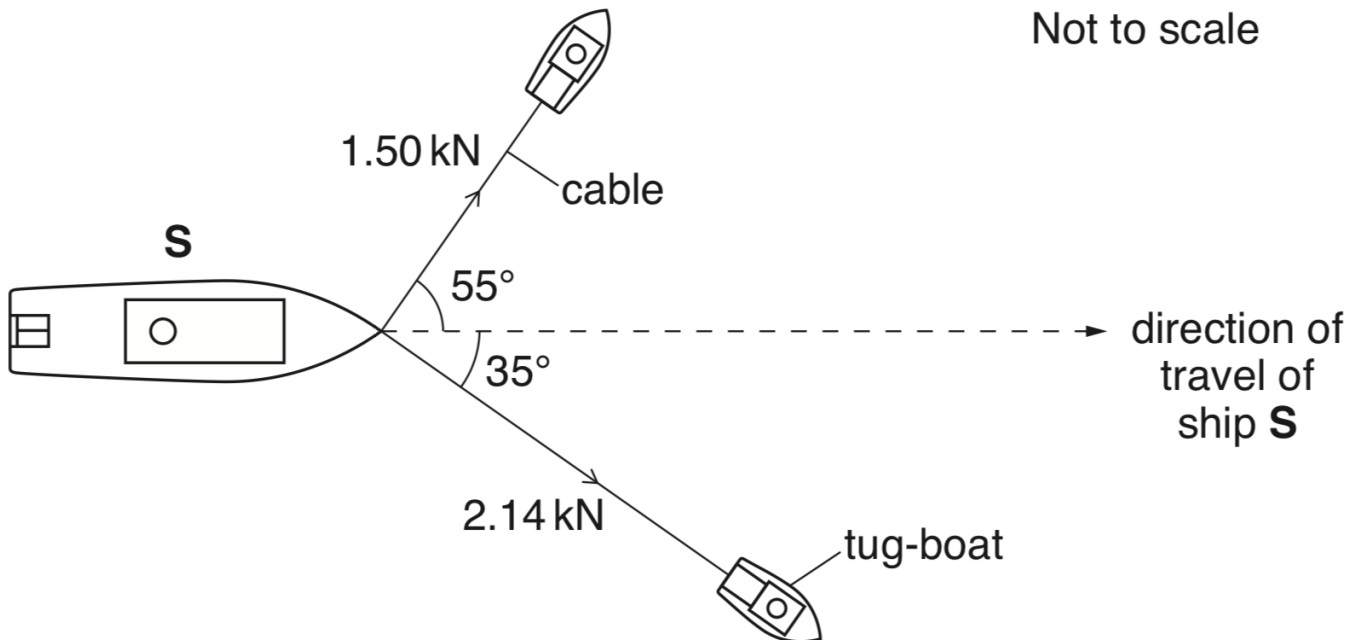


Figure 1: The ship **S** is being pulled by two tug-boats.

The ship is travelling at a constant velocity. The tensions in the cables and the angles made by these cables to the direction in which the ship travels are shown in **Figure 1**.

### Part A Vector triangle

Draw a vector triangle of the forces on the boat due to the tug-boats. What angle does the resultant force make to the direction of travel of the ship?

### Part B Magnitude of the resultant force

Determine the magnitude of the resultant force provided by the two cables. Give your answer to 3 significant figures.

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### Part C Drag force on the ship

State the value of the drag force acting on the ship, correct to 3 significant figures.

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### Part D Direction of drag force

What is the direction of the drag force and explain your answer?

- ☐ It is in the same direction as the resultant force due to the tug boats.
  - ☐ It is in the same direction as the top tug-boat.
  - ☐ It is directly opposite to the resultant force due to the tug boats.
  - ☐ It is in the opposite direction to the bottom tug-boat.
  - ☐ It is in the opposite direction as the top tug-boat.
  - ☐ It is in the same direction as the bottom tug-boat.
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