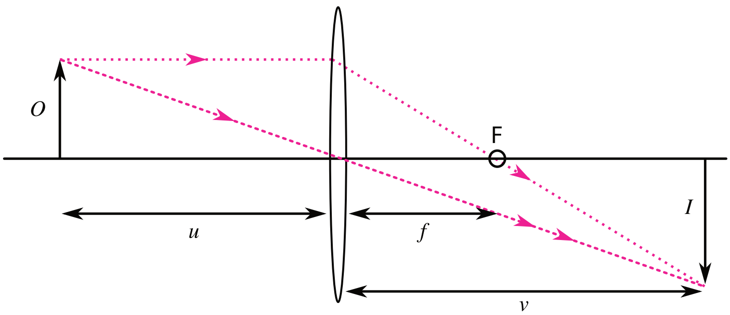
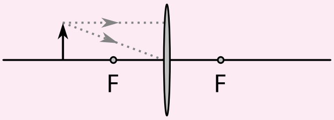
48 Convex Lenses ♡

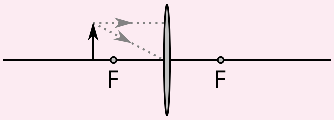
In the diagram below, the object has size O, the image size I, and the convex lens has a focal length f. We can work out the location of the image by drawing two rays through the system.

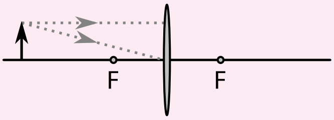
1. A ray passing through the centre of the lens is not bent.
2. A ray travelling parallel to the axis will bend at the lens, so that it crosses the axis at the focal point F (distance f behind the lens).
3. The image I is where the rays meet. If the rays are diverging (spreading apart) after the lens, extend both back to the left to find a place where the lines meet – this will be a virtual image.
4. The object distance is labelled u. The image distance is labelled v.

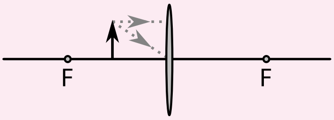


48.1 Copy and complete the ray diagrams by drawing the path of each ray of light after passing through the lens. Hence find the position, size and orientation of the image formed. Is it real or virtual?

(a)


(b)


(c)


(d)


Power: The "strength" with which a lens focuses a parallel beam to a point (a focus) is measured as its power. Lens power is measured in dioptres (D).

$$\text{power in dioptres (D)} = (\text{focal length in metres})^{-1}$$
$$P = \frac{1}{f} = f^{-1}$$

Example 1 – Calculate the power of a lens with a 10 cm focal length.

$$10 \text{ cm} = 0.1 \text{ m} \quad P = 1/f = 1/0.1 = 10 \text{ D}$$

48.2 Use the equation $P = f^{-1}$ to work out the power in dioptres (D) of the following lenses.

(a) A convex lens with a 1.6 m focal length.

(b) A convex lens with a 50 cm focal length.

(c) A convex lens with a 5.0 cm focal length.

48.3 Calculate the focal lengths in metres (m) for the following lenses.

(a) A +2.5 D lens.

(b) A +1.5 D lens.

(c) A +20 D lens.

48.4 Calculate the focal lengths, in centimetres, of these converging lenses.

(a) +20 D

(b) +2.5 D

(c) +10 D

(d) +14 D

(e) +2.0 D

(f) +5.0 D

48.5 Calculate the power, in dioptres, of the following lenses.

(a) A converging lens with a focal length of 40 cm.

(b) A lens which brings parallel rays of light to a focus at a distance of 20 cm from the centre of the lens on the axis.

48.6 Which lens has the greater optical power: a lens of focal length 10 cm or one with a focal length of 50 cm?

Working out the lens equation: We use similar triangles on the diagram of page 144 to form two equations for O/I.

Using the ray through the middle of the lens we know that:

$$\frac{O}{I} = \frac{u}{v}$$

Using the other diagonal ray, and the two triangles it forms, we can also write:

$$\frac{O}{I} = \frac{f}{v-f}$$

Equating the two expressions:

$$\frac{u}{v} = \frac{f}{v-f} \Rightarrow \frac{v}{u} = \frac{v-f}{f} \Rightarrow \frac{v}{u} = \frac{v}{f} - 1$$

$$\Rightarrow \frac{1}{u} = \frac{1}{f} - \frac{1}{v} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

This can be worked out more easily on a calculator like this:

$$v^{-1} = f^{-1} - u^{-1}, \text{ so } v = (f^{-1} - u^{-1})^{-1}.$$

Example 2 – Calculate the image distance (v) of a 5.0 D lens, if the object distance (u) is 30 cm.

$$P = 1/f, \quad \text{so } f = 1/P = 1/5.0 = 0.2 \text{ m}$$

$$1/v = 1/f - 1/u = 1/0.2 - 1/0.3 = 1.667,$$

$$\text{so } v = 1/1.667 = 0.6 \text{ m} = 60 \text{ cm}$$

48.7 Remember that $\frac{1}{4}$ and $1 \div 4$ and 4^{-1} all mean the same thing. Use the x^{-1} button on your calculator to calculate these values.

(a) 2^{-1}
(b) 4^{-1}
(c) 0.5^{-1}
(d) 20^{-1}
(e) $\frac{1}{50}$
(f) $1 \div 7$

48.8 Calculate the following using the x^{-1} button on your calculator.

(a) $3^{-1} + 4^{-1}$
(b) $2^{-1} - 4^{-1}$
(c) $(2^{-1} - 4^{-1})^{-1}$
(d) $(4^{-1} + 12^{-1})^{-1}$
(e) $(\frac{1}{3} - \frac{1}{12})^{-1}$
(f) $\frac{1}{(1/2-1/5)}$

48.9 Use the formula to work out the image distance for each situation. Assume that the focal length of the lens is 5.0 cm.

(a) Object distance = 20 cm
(b) Object distance = 10 cm
(c) Object distance = 15 cm
(d) Object distance = 7.0 cm
(e) Object distance = 1.0 cm
(f) Object distance = 4.0 cm

In the final two cases, v is negative (the image is to the left of the lens). This is a virtual image, meaning that no rays actually cross there. A screen at that position would not show a bright spot.

$$\text{magnification} = \text{image height} \ / \ \text{object height} = \frac{I}{O} = \frac{v}{u}$$

A magnification of 2.0 means that the image is twice as tall as the object. Magnification of 1.0 gives an image the same height as the object. The magnification number does not say whether the image is 'upside down'.

48.10 Work out the image distance and magnification for the following convex lenses, and state whether the image is real or virtual.

(a) $f = 10$ cm, object distance $= 20$ cm
(b) $P = 4.0$ D, object distance $= 10$ cm
(c) $f = 50$ cm, object distance $= 75$ cm
(d) $P = 5.0$ D, object distance $= 4.0$ cm

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A convex lens makes a virtual image if the object distance < focal length ($u < f$).
A convex lens makes a real, magnified image if f < object distance < $2f$.
A convex lens makes a real, diminished image if $u > 2f$.