

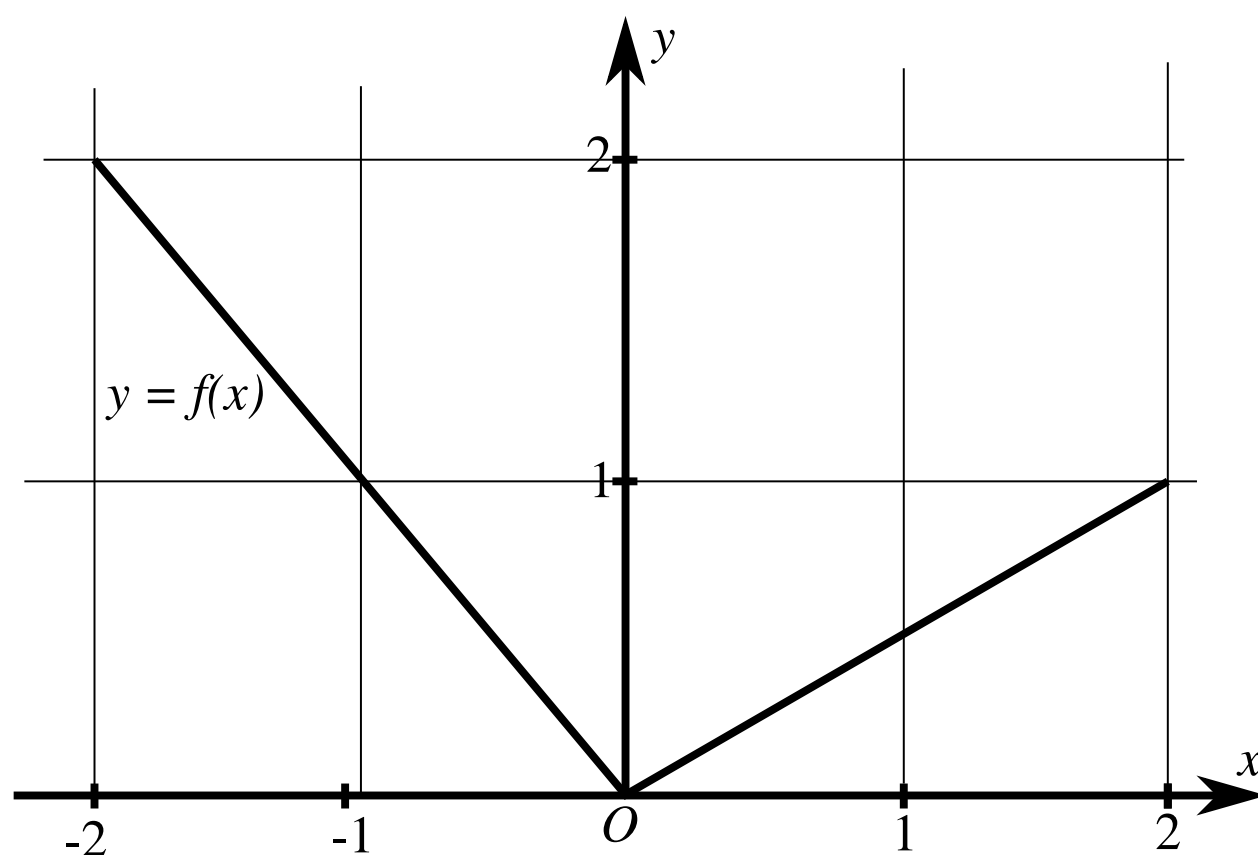


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# Transformations of Graphs 3ii

A Level



**Figure 1:** The graph of  $y = f(x)$  for  $-2 \leq x \leq 2$  is shown to the left.

## Part A Sketch $y = f(-x)$

Sketch the curve  $y = f(-x)$  for  $-2 \leq x \leq 2$ .

What is the  $y$ -value of the curve  $y = f(-x)$  when  $x = 1$ ?

The following symbols may be useful:  $y$

**Part B** Sketch  $y = f(-x) + 2$ 

Sketch the curve  $y = f(-x) + 2$  for  $-2 \leq x \leq 2$ .

What is the  $y$ -value of the curve  $y = f(-x) + 2$  when  $x = -2$ ?

The following symbols may be useful:  $y$

---

**Part C** Sketch  $y = -\frac{1}{x^2}$ 

Sketch the curve  $y = -\frac{1}{x^2}$ .

For large negative values of  $x$ , the curve  $y = -\frac{1}{x^2}$  becomes asymptotic to the horizontal line with which  $y$ -value?

The following symbols may be useful:  $y$

---

**Part D** Sketch  $y = 3 - \frac{1}{x^2}$ 

Sketch the curve  $y = 3 - \frac{1}{x^2}$ .

For large negative values of  $x$ , the curve  $y = 3 - \frac{1}{x^2}$  becomes asymptotic to the horizontal line with which  $y$ -value?

The following symbols may be useful:  $y$

---

**Part E** State the equation

The curve  $y = -\frac{1}{x^2}$  is stretched parallel to the  $y$ -axis by scale factor 2. State the equation of the transformed curve.

The following symbols may be useful:  $x$ ,  $y$

---

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# Transformations of Graphs 1i

A Level



## Part A Sketch $y$

Find the roots of the curve  $y = x^2(3 - x)$  and sketch it. You can check your sketch after entering your answer.

Give the value of the root at which  $y$  has a minimum.

The following symbols may be useful:  $x$

---

## Part B Translate $y$

The curve  $y = x^2(3 - x)$  is translated by two units in the positive direction parallel to the  $x$  axis.

State the equation of the curve after this transformation.

The following symbols may be useful:  $x$ ,  $y$

---

**Part C** Find transformation of  $y$ 

Which of these describes the transformation of the curve  $y = x^2(3 - x)$  to  $y = \frac{1}{2}x^2(3 - x)$ ?

- ☐ A stretch of scale factor  $\frac{1}{2}$  parallel to the  $y$ -axis.
- ☐ A stretch of scale factor  $\frac{1}{2}$  parallel to the  $x$ -axis.
- ☐ A stretch of scale factor 2 parallel to the  $y$ -axis.
- ☐ A stretch of scale factor 2 parallel to the  $x$ -axis.
- 

**Part D** Vertical translation of  $f(x)$ 

The curve  $y = f(x)$  passes through the point  $P$  with coordinates  $(2, 5)$ .

State the coordinates of the point corresponding to  $P$  on the curve  $y = f(x) + 2$ . Enter the  $x$  and  $y$  coordinates below.

Enter the  $x$  coordinate:

The following symbols may be useful:  $x$

---

Enter the  $y$  coordinate:

The following symbols may be useful:  $y$

---

**Part E** Lateral stretching of  $f(x)$ 

The curve  $y = f(x)$  passes through the point  $P$  with coordinates  $(2, 5)$ .

State the coordinates of the point corresponding to  $P$  on the curve  $y = f(2x)$ . Enter the  $x$  and  $y$  coordinates below.

Enter the  $x$  coordinate:

The following symbols may be useful:  $x$

---

Enter the  $y$  coordinate:

The following symbols may be useful:  $y$

---

**Part F** Find transformation of  $f(x)$ 

Which of the following describes the single transformation that maps the curve  $y = f(x)$  onto  $y = f(x + 4)$ ?

- ☐ A translation of 4 units parallel to the  $y$ -axis.
- ☐ A translation of 4 units parallel to the  $x$ -axis.
- ☐ A translation of  $-4$  units parallel to the  $x$ -axis.
- ☐ A translation of  $-4$  units parallel to the  $y$ -axis.
- 

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# Lateral and vertical translations

GCSE

A Level

Investigate the transformations of the following functions.

## Part A   Lateral translation

Consider the function  $f(x) = x^2 + 2x + 1$  . The function  $g(x) = f(x - a)$ , where  $a$  is a constant. If  $g(1) = 9$  find the value of  $a$ , given that it is positive.

The following symbols may be useful: a

## Part B   Vertical translation

Consider the function  $r(u) = \frac{2}{u - 2}$ . The function  $s(u) = r(u) + b$ , where  $b$  is a constant. If  $s(0) = 1$  , find the value of  $b$ .

The following symbols may be useful: b

## Part C Lateral and vertical translation

Consider the function  $p(r) = \frac{1}{r}$ . The function  $q(r) = p(r - c) + d$ , where  $c$  and  $d$  are constants.

If  $q(0) = 1$  and  $q(2) = 3$ , find the values of  $c$  and  $d$ .

Find the value of  $c$ .

The following symbols may be useful:  $c$

---

Find the value of  $d$ .

The following symbols may be useful:  $d$

---

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# Reflection and symmetry

GCSE

A Level



The following questions ask you to deduce the symmetry properties of a number of functions. There are three choices:

- even - a function for which  $f(x) = f(-x)$  which is also described as being symmetric about the vertical axis,
- odd - a function for which  $f(x) = -f(-x)$  which is also described as being antisymmetric about the vertical axis (or symmetric about zero),
- neither even nor odd.

For more details see the section on Symmetry in

[Graph interpreting - Level 2: More powers of  \$x\$  and general polynomials](#)

Where relevant you may assume that  $a$  and  $b$  are non-zero constants.

## Part A Even functions

In one of the following lists of functions all the functions are even. Pick the correct option from the choices below.

- ☐  $ax^2, ax^2 + b, x^2(a + bx), \frac{a}{x^2} + b, (x - a)(x + a), a \cos x$
- ☐  $ax^2, ax^2 + b, ax^2 + bx^4, \frac{a}{x^2} + b, (x - a)(x + b) (a \neq b), a \sin x$
- $ax^2, ax^2 + b, ax^2 + bx^4, \frac{a}{x^2} + bx^2, (x - a)(x + a), a \cos x$
- ☐  $ax^2, a(x + b)^2, ax^2 + bx^4, \frac{a}{x^2} + b, (x - a)(x + b) (a \neq b), a \sin x$
- ☐  $ax^2, a(x + b)^2, x^2(a + bx), \frac{a}{x^2} + bx^2, (x - a)(x + a), a \sin x$
- ☐  $ax^2, a(x + b)^2, x^2(a + bx), \frac{a}{x^2} + bx^2, (x - a)(x + b) (a \neq b), a \cos x$

Part B    Odd functions

In one of the following lists of functions all the functions are odd. Pick the correct option from the choices below.

- ☐  $ax, \frac{a}{x} + b, \frac{a}{x} + \frac{b}{x^3}, x(a + bx^2), (x + a)^{1/3}, a \sin x$
- ☐  $ax, \frac{a}{x}, \frac{a}{x} + \frac{b}{x^3}, x^2(a + bx), x^{1/3}, a \sin x$
- ☐  $ax, \frac{a}{x}, \frac{a}{x} + bx^3, x^2(a + bx), (x + a)^{1/3}, a \tan x$   
 $ax, \frac{a}{x}, \frac{a}{x} + bx^3, x(a + bx^2), x^{1/3}, a \sin x$
- ☐  $ax, \frac{a}{x} + b, \frac{a}{x} + \frac{b}{x^3}, x(a + bx^2), x^{1/3}, a \tan x$
- ☐  $ax, \frac{a}{x} + b, \frac{a}{x} + bx^3, x^2(a + bx), (x + a)^{1/3}, a \tan x$

Part C    Neither odd nor even functions

In one of the following lists of functions all the functions are neither odd nor even. Pick the correct option from the choices below.

- ☐  $ax - b, x^2(ax + b), (x - a)(x + a)^2, a \frac{1}{x^2} - \frac{1}{b^2}, a(b - x)^{1/2}, \cos x + \sin x$
- ☐  $ax - b, x(ax^2 + b), (x - a)(x + a), \frac{a}{(x - b)^2}, a(b - x)^{1/2}, a \tan(x + 45^\circ)$
- ☐  $ax - b, x(ax^2 + b), (x - a)(x + a)^2, a \frac{1}{x^2} - \frac{1}{b^2}, a(b - x)^{1/2}, \cos x + \sin x$
- ☐  $ax - b, x^2(ax + b), (x - a)(x + a), a \frac{1}{x^2} - \frac{1}{b^2}, ax^{1/2}, a \tan(x + 45^\circ)$   
 $ax - b, x^2(ax + b), (x - a)(x + a)^2, \frac{a}{(x - b)^2}, ax^{1/2}, a \tan(x + 45^\circ)$
- ☐  $ax - b, x(ax^2 + b), (x - a)(x + a), \frac{a}{(x - b)^2}, ax^{1/2}, \cos x + \sin x$

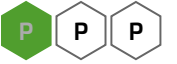


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# Circles 1ii

A Level



The circle with equation  $x^2 + y^2 - 6x - k = 0$  has radius 4.

The points  $A(3, a)$  and  $B(-1, 0)$  lie on the circumference of the circle, with  $a > 0$ .

## Part A Centre

By completing the square for  $x$  and  $y$  find the coordinates of the centre of the circle. Enter the  $x$  and  $y$  coordinates below.

Enter the  $x$ -coordinate:

The following symbols may be useful:  $x$

---

Enter the  $y$  coordinate:

The following symbols may be useful:  $y$

---

## Part B Value of $k$

Find the value of  $k$ .

The following symbols may be useful:  $k$

---

**Part C**   Length  $AB$ 

Calculate the length of  $AB$ , giving your answer in simplified surd form.

---

**Part D**   Equation

Find the equation of the line  $AB$ . Give your answer in the form  $y = mx + c$ .

The following symbols may be useful:  $x$ ,  $y$

---

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# Circles 3ii

A Level



A circle has centre  $(3, 1)$  and radius 5, and a line has equation  $y = 2x$ .

## Part A Circle equation

Write down the equation of the circle.

The following symbols may be useful:  $x$ ,  $y$

---

## Part B Intersection point

Find the coordinates of the point of intersection of the line and the circle with the largest  $x$  value.

Give the  $x$ -coordinate.

The following symbols may be useful:  $x$

---

Give the  $y$ -coordinate.

The following symbols may be useful:  $y$

---

**Part C**    **Point on the line**

Find the coordinates of the point on the line which is closest to the centre of the circle.

Give the  $x$ -coordinate.

The following symbols may be useful:  $x$

---

Give the  $y$ -coordinate.

The following symbols may be useful:  $y$

---

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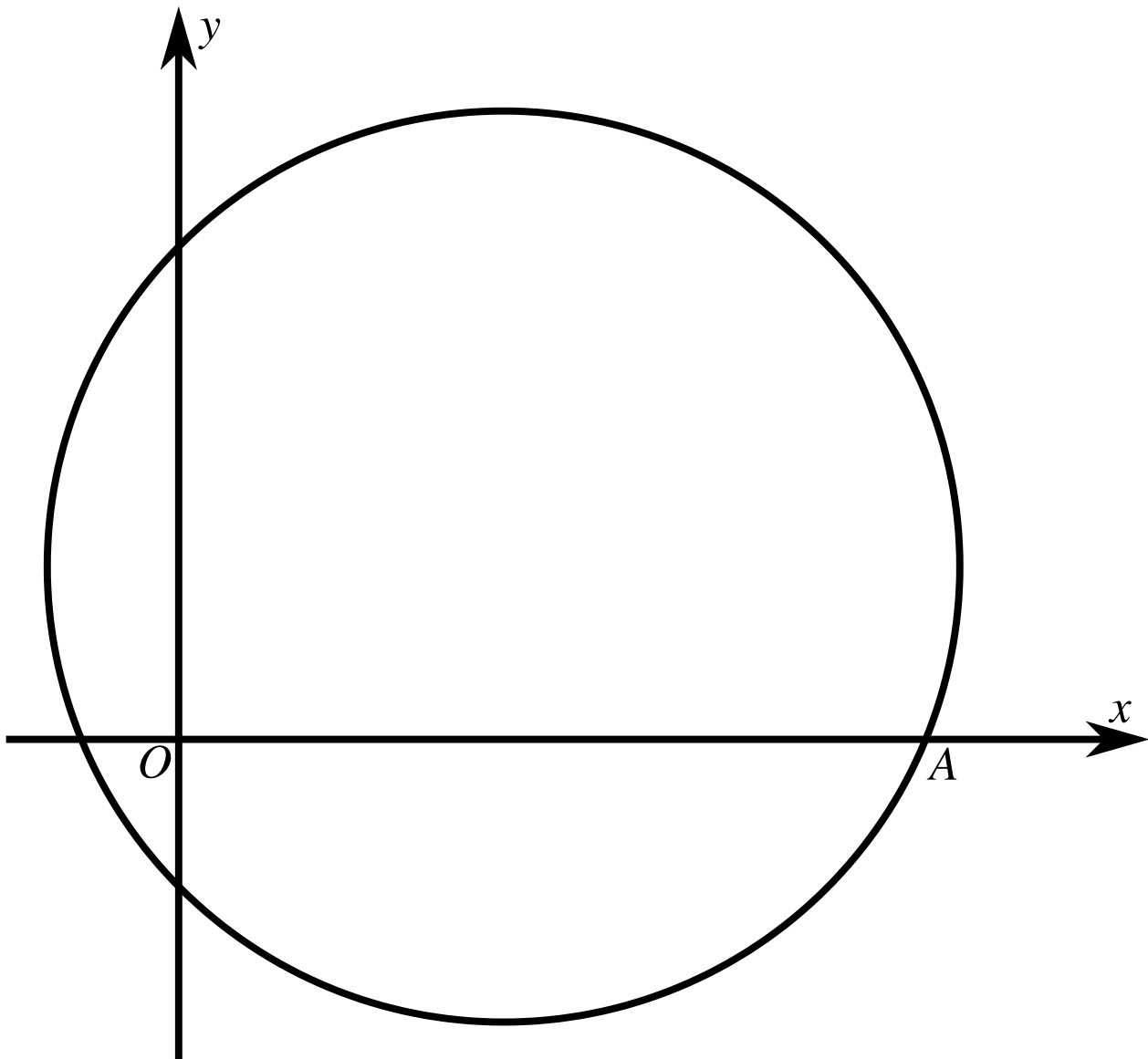
# Circles 2i

A Level

P

P

P



**Figure 1:** The circle with equation  $x^2 + y^2 - 8x - 6y - 20 = 0$ .

**Figure 1** shows the circle with equation  $x^2 + y^2 - 8x - 6y - 20 = 0$ . The circle crosses the positive  $x$  axis at point  $A$ .

Part A Find  $C$

By completing the square for  $x$  and  $y$  find the coordinates of the centre of the circle. Enter the  $x$  and  $y$  coordinates below.

Enter the  $x$  coordinate:

The following symbols may be useful:  $x$

---

Enter the  $y$  coordinate:

The following symbols may be useful:  $y$

---

Part B Find radius

Find the radius of the circle.

---

Part C Tangent to the circle 1

Find the equation of the tangent to the circle at  $A$ . Give your answer in the form  $y = mx + c$ .

The following symbols may be useful:  $x$ ,  $y$

---



## Part D Tangent to the circle 2

A second tangent to the circle is parallel to the tangent at  $A$ . Find the equation of this second tangent in the form  $y = mx + c$ .

The following symbols may be useful:  $x$ ,  $y$

---

## Part E Find a radius

Another circle has its centre at the origin  $O$  and radius  $r$ . This circle lies wholly inside the first circle. Find the set of possible values of  $r$ . Give your answer as an inequality.

The following symbols may be useful:  $<$ ,  $<=$ ,  $>$ ,  $>=$ ,  $r$

---

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# Transformations and Area 2i

Further A



## Part A A

The matrix **A** represents an enlargement, centre  $(0, 0)$ , with scale factor  $\sqrt{2}$ .

Give the first row of **A** in the form  $x \ y$  with a space between  $x$  and  $y$ .  $x$  and  $y$  are in exact form.

Write the square root of a number like this: `sqrt(5)`

---

Give the second row of **A** in the form  $x \ y$  with a space between  $x$  and  $y$ .  $x$  and  $y$  are in exact form.

Write the square root of a number like this: `sqrt(5)`

---

## Part B B

The matrix **B** is given by  $\mathbf{B} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$ .

Which of the following transformations is represented by **B**?

- ☐ Reflection in the line  $y = \frac{x}{\sqrt{2}}$
- ☐ Enlargement, centre  $(0, 0)$ , scale factor  $\frac{1}{\sqrt{2}}$ .
- ☐ Stretch, scale factor  $\frac{\sqrt{2}}{2}$  parallel to the  $y$  axis.
- ☐ Rotation, about the origin,  $45^\circ$  clockwise.

---

Part C *C*

**C** is given by **C** = **AB**.

Give the first row of **C** in the form *x y* with a space between *x* and *y*. *x* and *y* are in exact form.

---

Give the second row of **C** in the form *x y* with a space between *x* and *y*. *x* and *y* are in exact form.

---

Part D Transformed area

Find the area of the image of the unit square under transformation *C*.

---

Part E Unit square

Draw a diagram showing the unit square and its image under the transformation represented by **C**.

Easier question?

---

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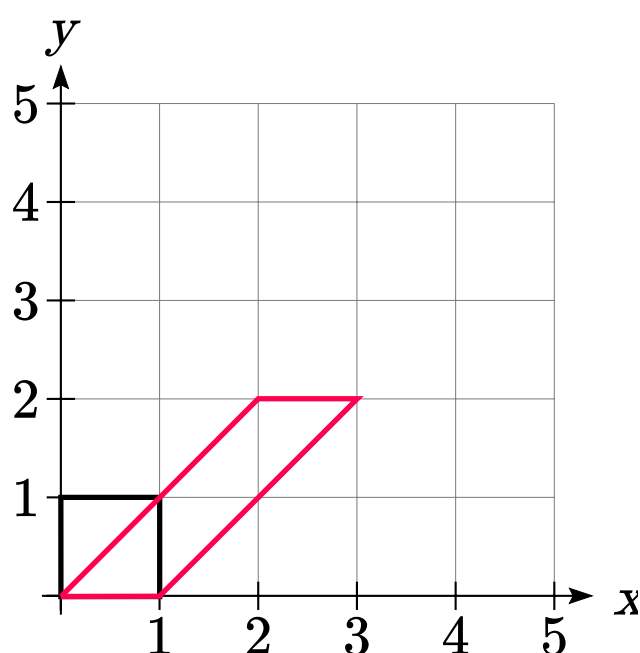


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# Transformations - Successive 3i

Further A



**Figure 1:** The unit square before and after transformation is shown in black and red respectively.

The matrix that represents this transformation is given by  $\mathbf{X}$ , which is also equivalent to a transformation  $\mathbf{P}$  followed by a transformation  $\mathbf{Q}$ .

## Part A   $\mathbf{X}$

The matrix that represents this transformation is given by  $\mathbf{X}$ .

Give the first row of  $\mathbf{X}$  in the form  $x \ y$  with a space between  $x$  and  $y$ .  $x$  and  $y$  are in exact form.

Give the second row of  $\mathbf{X}$  in the form  $x \ y$  with a space between  $x$  and  $y$ .  $x$  and  $y$  are in exact form.

Part B Transformation

Which of the following fully describe the transformation represented by **P** and **Q**?

- ☐ Stretch, scale factor 2 parallel to the  $y$  axis and shear,  $x$  axis invariant.
- ☐ Enlargement, centre  $(0, 0)$ , scale factor 2 and rotation, about the origin,  $45^\circ$  clockwise.
- ☐ Rotation, about the origin,  $46.3^\circ$  clockwise, and reflection in the  $y$  axis.
- ☐ Reflection in the line  $y = \frac{x}{\sqrt{3}}$  and enlargement, centre  $(0, 0)$ , scale factor  $-2$ .

Part C **P**

There are 2 possible matrices that represent **P**.

Give the first row of **P** in the form  $x \ y$  with a space between  $x$  and  $y$ .  $x$  and  $y$  are in exact form.

Give the second row of **P** in the form  $x \ y$  with a space between  $x$  and  $y$ .  $x$  and  $y$  are in exact form.

Part D **Q**

There are 2 possible matrices that represent **Q**.

Give the first row of **Q** in the form  $x \ y$  with a space between  $x$  and  $y$ .  $x$  and  $y$  are in exact form.

Give the second row of **Q** in the form  $x \ y$  with a space between  $x$  and  $y$ .  $x$  and  $y$  are in exact form.

**Part E**   **PQ**

Give the first row of the matrix that represents transformation **Q** followed by transformation **P**. in the form  $x \ y$  with a space between  $x$  and  $y$ .  $x$  and  $y$  are in exact form.

---

Give the second row of the matrix that represents transformation **Q** followed by transformation **P**. in the form  $x \ y$  with a space between  $x$  and  $y$ .  $x$  and  $y$  are in exact form.

---

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# Matrices - transformations 1

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$\mathbf{P}$  and  $\mathbf{Q}$  are  $3 \times 3$  matrices which carry out a reflection in the plane  $y = 0$  and a rotation about the  $x$ -axis, respectively. The matrix  $\mathbf{R} = \mathbf{QP}$ .

## Part A Reflection in the $y = 0$ plane

---

The  $3 \times 3$  matrix  $\mathbf{P}$  carries out a reflection in the plane  $y = 0$ . Write down the matrix  $\mathbf{P}$ .

Give your answer by writing the elements in each row in brackets in the form  $(p_{m1}, p_{m2}, p_{m3})$  where  $m = 1, 2$  or  $3$ . Thus, if  $p_{21} = 1$ ,  $p_{22} = 2$  and  $p_{23} = 0$ , type: (1,2,0) with no spaces.

Give the elements in the top row ( $m = 1$ ) of the matrix, writing them in the form indicated above.

---

Give the elements in the second row ( $m = 2$ ) of the matrix, writing them in the form indicated above.

---

Give the elements in the bottom row ( $m = 3$ ) of the matrix, writing them in the form indicated above.

---

**Part B**    Rotation about the  $x$ -axis

The  $3 \times 3$  matrix **Q** carries out an anticlockwise rotation about the  $x$ -axis through an angle  $A$ . Write down the matrix **Q**.

Give your answer by writing the elements in each row in brackets in the form  $(q_{m1},q_{m2},q_{m3})$  where  $m = 1, 2$  or  $3$ . Thus, if  $q_{11} = 1$ ,  $q_{12} = 2$  and  $q_{13} = \sin A$ , type:  $(1,2,\sin A)$  with no spaces.

Give the elements in the top row ( $m = 1$ ) of the matrix, writing them in the form indicated above.

---

Give the elements in the second row ( $m = 2$ ) of the matrix, writing them in the form indicated above.

---

Give the elements in the bottom row ( $m = 3$ ) of the matrix, writing them in the form indicated above.

---

**Part C**    Reflection followed by rotation

Find the  $3 \times 3$  matrix **R** = **QP** which carries out a reflection in the plane  $y = 0$  followed by a rotation anticlockwise about the  $x$ -axis through an angle  $A$ .

Give your answer by writing the elements in each row in brackets in the form  $(r_{m1},r_{m2},r_{m3})$  where  $m = 1, 2$  or  $3$ . Thus, if  $r_{31} = 1$ ,  $r_{32} = 2$  and  $r_{33} = \cos A$ , type:  $(1,2,\cos A)$  with no spaces.

Give the elements in the top row ( $m = 1$ ) of the matrix, writing them in the form indicated above.

---

Give the elements in the second row ( $m = 2$ ) of the matrix, writing them in the form indicated above.

---

Give the elements in the bottom row ( $m = 3$ ) of the matrix, writing them in the form indicated above.

---



## Part D Reflection in the $z = 0$ plane

If the value of  $A$  is such that the  $3 \times 3$  matrix  $\mathbf{R}$  matrix represents a reflection in the plane  $z = 0$ , find the angle  $A$ . Assume  $0 \leq A < 2\pi$  and give your answer in radians.

The following symbols may be useful:  $A$ ,  $\pi$

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