Quadratic inequalities

GCSE   A Level

C C C   P P P

Solve the following quadratic inequalities.

Part A $3x^2 - 2x - 8 \leq 0$

Solve the inequality $3x^2 - 2x - 8 \leq 0$. Firstly select the form of your answer from the choices given below, where a and b are constants and $a < b$, and then find a and/or b.

Select the form of your answer from the choices given below.

- $x < a$ only
- $x < a$ or $x > b$
- $x \leq a$ only
- $x > b$ only
- a < x < b
- $x \leq a$ or $x \geq b$
- $x \geq b$ only
- $a \leq x \leq b$

Given your deduction above, find a.

Given your deduction above, find b.

Solve the inequality $-2x^2 + 5 < 7x + 11$. Firstly select the form of your answer from the choices given below, where c and d are constants, and then find c and/or d.

Select the form of your answer from the choices given below.

- ◯ $x < c$ or $x > d$
- ◯ $x > d$ only
- ◯ $x \leq c$ or $x \geq d$
- ◯ $x \leq c$ only
- ◯ $c < x < d$
- ◯ $x < c$ only
- ◯ $c \leq x \leq d$
- ◯ $x \geq d$ only

Given your deduction above, find c.

Given your deduction above, find d.

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Condition for damping

A mass m is suspended on a spring with spring constant k in a medium which damps its motion. The condition that it will oscillate after it has been displaced from equilibrium is

$$\frac{k}{m} > \frac{b^2}{4m^2}$$

where b is called the damping constant.

Find the range of masses over which it will oscillate.

The following symbols may be useful: <, <=, >, >=, b, k, m

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Log Laws 2ii

A Level

Part A   **Express as a single log (i)**

Express $\log_a 2 + \log_a 3$ as a single logarithm.

The following symbols may be useful: `a, log()`

Part B   **Express as a single log (ii)**

Express $2\log_{10} x - 3\log_{10} y$ as a single logarithm.

The following symbols may be useful: `log(), x, y`

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Log Laws 1i

A Level

P P P

Part A   Log laws

Given that $log_a(x) = p$ and $log_a(y) = q$, express $log_a(xy)$ in terms of p and q.

The following symbols may be useful: `p`, `q`

Part B   Simplify 1

Given that $log_a(x) = p$ and $log_a(y) = q$, express $log_a(\frac{a^2 x^3}{y})$ in terms of p and q.

The following symbols may be useful: `p`, `q`

Part C   Simplify 2

Express $log_{10}(x^2 - 10) - log_{10}x$ as a single logarithm

The following symbols may be useful: `log()`, `x`

Part D   Solve equation

Solve the equation $log_{10}(x^2 - 10) - log_{10}x = 2log_{10}3$.

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Simplifying indices

GCSE   A Level

Part A   **Simplify** $(4a^2b^3)^{\frac{1}{2}} \times (9ab^2)^{-\frac{3}{2}}$

Simplify $(4a^2b^3)^{\frac{1}{2}} \times (9ab^2)^{-\frac{3}{2}}$

The following symbols may be useful: a, b, p, q

Part B   **Simplify** $(8p^3q^2)^{\frac{2}{3}} \div \left(2p/q^{\frac{1}{3}}\right)^5$

Simplify $(8p^3q^2)^{\frac{2}{3}} \div \left(\dfrac{2p}{q^{\frac{1}{3}}}\right)^5$

The following symbols may be useful: a, b, p, q

Part C   **Simplify** $(10^{-34})^{\frac{1}{2}}(10^{-10})^{\frac{1}{2}}(10^8)^{-\frac{5}{2}}$

Simplify $(10^{-34})^{\frac{1}{2}}(10^{-10})^{\frac{1}{2}}(10^8)^{-\frac{5}{2}}$

The following symbols may be useful: a, b, p, q

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Simplifying surds

Simplify the following expressions.

Part A $2\sqrt{20} + \sqrt{45} - 5\sqrt{5}$

Simplify $2\sqrt{20} + \sqrt{45} - 5\sqrt{5}$

Part B $4(\sqrt{3} + 1)(\sqrt{3} - 1) - 2(2 + \sqrt{2})(1 + \sqrt{2})$

Simplify $4(\sqrt{3} + 1)(\sqrt{3} - 1) - 2(2 + \sqrt{2})(1 + \sqrt{2})$

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Rationalisation

Rationalise the denominators of the following expressions.

Part A $(3\sqrt{6})/(2\sqrt{18})$

Rationalise the denominator of $\dfrac{3\sqrt{6}}{2\sqrt{18}}$.

Part B $(4 - \sqrt{3})/(4 + 2\sqrt{3})$

Rationalise the denominator of $\dfrac{4 - \sqrt{3}}{4 + 2\sqrt{3}}$.

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Log Laws 1ii

A Level

Part A   Express in terms of $\log_2(x)$ (i)

Express $\log_2(x^2)$ in terms of $\log_2(x)$.

The following symbols may be useful: `ln()`, `log()`, `x`

Part B   Express in terms of $\log_2(x)$ (ii)

Express $\log_2(8x^2)$ in terms of $\log_2(x)$.

The following symbols may be useful: `ln()`, `log()`, `x`

Part C   Find $\log_3 y$

Given that $y^2 = 27$, find the value of $\log_3 y$.

The following symbols may be useful: `ln()`, `log()`, `y`

Used with permission from UCLES, A Level Maths, Specimen paper, OCR C2, Question 3

Introducing Dimensional Analysis

The dimensions of physical properties do not depend on specific units; here we use length L, time T and mass M as our fundamental dimensions. In any equation relating physical properties the dimensions must be the same on both sides.

For example force = mass × acceleration.

Obviously mass has dimensions M. To deduce the dimensions of acceleration recall that acceleration = change in velocity over time; velocity (= change in displacement over time) has dimensions of LT^{-1} so acceleration has dimensions $(LT^{-1})(T^{-1}) = LT^{-2}$.

Thus force has dimensions MLT^{-2}.

Part A   Dimensions of kinetic energy

The kinetic energy of a body of mass m moving with speed v is equal to $\frac{1}{2}mv^2$.

Find the dimensions of (kinetic) energy. Recall that the factor of $\frac{1}{2}$ in the expression is dimensionless.

The following symbols may be useful: L, M, T

Part B   Planck unit

One type of "Planck unit" is defined as:

$$h^{\frac{1}{2}} G^{\frac{1}{2}} c^{-\frac{5}{2}}$$

where h is Planck's constant (dimensions ML^2T^{-1}),
G is the universal constant of gravitation (dimensions $M^{-1}L^3T^{-2}$) and
c is the speed of light (dimensions LT^{-1}).

Find the dimensions of this "Planck unit".

The following symbols may be useful: L, M, T

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Log Laws 1i

A Level

It is given that $\ln x = p + 2$ and $\ln y = 3p$

Part A   Log laws

Express $\ln(xy)$ in terms of p.

The following symbols may be useful: p

Part B   Simplify 1

Express $\ln(x^3)$ in terms of p.

The following symbols may be useful: p

Part C   Simplify 2

Express $\ln\left(\frac{y}{e}\right)$ in terms of p.

The following symbols may be useful: p

Part D   Solve equation

Express y in terms of x and e, simplifying your answer.

The following symbols may be useful: e, x, y

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