Gas

59 Boyle's Law

Definition:

$$\text{pressure} = \frac{\text{force (N)}}{\text{area (m}^2\text{)}} \qquad P = \frac{F}{A}$$

The unit of pressure is the pascal (Pa). 1 Pa $= 1$ N/m^2.
Atmospheric pressure is approximately 1.01×10^5 Pa $= 101$ kPa.

The pressure a gas exerts on a wall depends on

- how often molecules hit the wall. This depends on the
  - size of the container (if the container is longer, molecules will take longer to cross it, and each molecule will collide with the wall less often).
  - speed of the molecules (this depends on the temperature).
  - number of molecules in the container.
- the momentum change when each molecule hits the walls. This depends on the
  - mass of the molecules.
  - speed of the molecules (which depends on the temperature).

If the temperature is constant (which is usually the case if the gas is compressed or expands slowly), the speed of the molecules doesn't change. Halving the volume of the container doubles the gas pressure because each molecule only takes half the time to cross it – so hits the walls twice as often.

Equation for Boyle's Law (constant temperature)

$$\text{pressure} \times \text{volume} = \text{constant} \qquad p_1 V_1 = p_2 V_2$$

where $_1$ means 'before the change' and $_2$ means 'after the change'

Example – 40 cm^3 of gas at atmospheric pressure is squeezed into a volume of 10 cm^3. What is the new pressure?
$p_1V_1 = p_2V_2$, so 101 kPa $\times$ 40 cm^3 $= p_2 \times 10$ cm^3, so $4\,040 = 10p_2$
$p_2 = 4\,040/10 = 404$ kPa.

The average kinetic energy of molecules in a gas depends on temperature. The average kinetic energy of molecules is proportional to the temperature, if the temperature is measured in kelvins (K), where

$$\text{temperature in kelvins (K)} = \text{temperature in degrees Celsius } (^\circ\text{C}) + 273.$$

The temperature of 0 K = -273 °C is called absolute zero. If you were able to cool a gas right down to this level, the molecules would be stationary. You couldn't cool it any further - this is the coldest temperature possible.

59.1 (a) What is 1.00 cm^2 in square metres?

(b) How much force does the atmosphere exert on a 1.00 cm^2 area?

59.2 A barometer measures the pressure of the atmosphere in millibar (mbar), where 1.0 mbar = 100 Pa. The surface area of the chamber in a barometer is 0.010 m^2, and the air pressure changes from 997 mbar to $1\,013$ mbar. What is the change in the force exerted by the air on the barometer's chamber?

59.3 Why does a gas exert a pressure on the walls of its container? Number the statements below to put them in the correct order to answer the question.

(a) bounce off with the same speed in a different direction.

(b) In a gas, molecules move around rapidly.

(c) By Newton's Third Law there must also have been a

(d) They frequently collide with the wall. When this happens they

(e) force exerted on the wall.

(f) their momentum must have changed, so

(g) there must have been a force on the molecule to cause this.

(h) This means that their velocity has changed and so

59.4 $20\ \text{cm}^3$ of gas is at 100 kPa.

(a) What will the pressure be on squeezing slowly down to $10\ \text{cm}^3$?

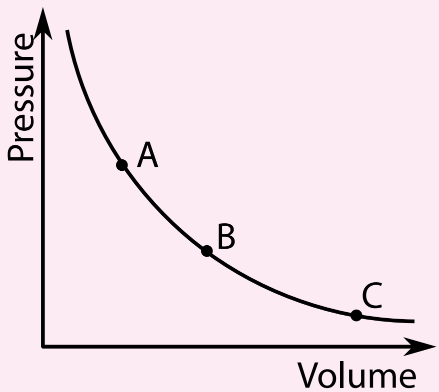
(b) What will it be if the gas is allowed to expand slowly to $40\ \text{cm}^3$?

(c) What will the volume be if the pressure is increased slowly to $1\,000$ kPa?

(d) Suppose the change in (a) were done really quickly. What would the effect be on the speed of the molecules?

(e) Would your final answer in (a) be lower or higher, if the change were instead done quickly?

59.5 The co-ordinates of a point A on the line of a pressure-volume graph constructed for a fixed mass of gas at constant temperature are $(40, 30)$. Point B $(60, y)$ and point C $(x, 10)$ also lie on the line. Calculate the values of x and y.



59.6 A certain car's suspension works by having a fixed mass of gas sealed inside a flexible capsule. Its pressure is usually 2.4×10^5 Pa and its volume is 2.0 litres. On a bumpy road, at one point, the gas inside the capsule is compressed to 1.5 litres. What is its pressure at this point? (Assume the gas temperature remains constant).

59.7 A child lets its helium-filled balloon float up, higher and higher into the air, becoming larger and larger. When it was at ground level, its volume was $4\,000\ \text{cm}^3$ and the helium was at a pressure of 1.5×10^5 Pa.

What would the helium's pressure become if the volume increased to $6\,000$ cm^3 with no change of temperature?

59.8 In Boyle's Law, pressure and volume are inversely proportional ($p = k/V$, where k is a constant). What graph could you plot against p which would give a straight line? *[Hint: suppose we plotted p on the y-axis, we could plot some function of V on the x-axis.]*

59.9 Suppose the speed of each molecule doubled. What would happen to the pressure if the volume were fixed?

59.10 Convert the following temperatures into the other unit:

(a) 23 °C (c) -50 °C (e) 4 K (g) 600 K

(b) 0 °C (d) 90 K (f) 0 K (h) $6\,000$ °C

Additional Boyle's Law Questions

59.11 The average kinetic energy of molecules in air at 300 K is 6.21×10^{-21} J.

(a) What is the average kinetic energy of molecules in air at 600 K? *Hint: average kinetic energy is proportional to kelvin temperature. See page 7 if you need help with calculations involving proportionality.*

(b) What is the average kinetic energy of molecules in air at 373 K?

(c) What is the average kinetic energy of molecules in air at 0 °C?

(d) At atmospheric pressure, nitrogen liquefies at the temperature where the average kinetic energy of gas molecules would be 1.86×10^{-21} J. What is the boiling temperature of nitrogen? Give your answer in kelvin.

59.12 The mass of a nitrogen molecule is 4.65×10^{-26} kg. Use Q59.11(c) to calculate (a) the average value for (speed)^2 of a nitrogen molecule in air at 0 °C, and (b) a typical speed for such molecules.

59.13 The mass of a helium atom is 6.64×10^{-27} kg. Using the method of Q59.12, calculate a typical speed for helium atoms in air at 0 °C.