Algebra

12 Writing and Using Algebra

Algebra has its own terminology:

$4\pi x^2$	A term is made up of constants (such as 4 and π) and variables (such as x). Together the leading constants are known as the coefficient (here 4π). Like terms contain the same variables, to the same powers. They differ only in the values of their coefficients.
$4\pi x^2 - 3x$	An expression is made up of terms linked by operators ($+, -, \times, \div$).
$x - 5 = 4\pi x^2 - 3x$	An equation links two expressions with an equals sign.

If an equation is true for all possible values of the variable it is called an identity, and the $\equiv$ sign may be used. One side of an identity is effectively just a re-arrangement of the other. For example, $(2x+3) - 4 \equiv 2x - 1$.

$7x^2 - 3x + 2$ and $5x^3 + 2x$ are polynomials. Polynomials have terms which contain only constants and positive, whole number powers of variables, linked together by addition or subtraction.

- If the highest power of the variable is 1, the polynomial is linear.
  For example, $2x - 5$ and $y + 3$.
- If the highest power of the variable is 2, the polynomial is quadratic.
  For example, $7x^2 + 4x - 2$ and $y^2 - 9$.
- If the highest power of the variable is 3, the polynomial is cubic.
  For example, $9x^3 + 2x^2 - 7x$ and $y^3 - 4$.

Writing algebra involves replacing words with variables, constants and operators. Brackets are often needed to ensure that applying the rules of BIDMAS when doing calculations will give correct answers.

Example 1 - Write the following as an equation:

"To find Q, add 6 to p, then divide by 5."

First add 6 to p: $p + 6$ Next divide everything by 5: $\dfrac{p+6}{5}$

This is equal to Q: $Q = \dfrac{p+6}{5}$

12.1 (a) Write the following as an equation: "To find y, multiply x by four then subtract three."

(b) When $x = 5$ what is y?

12.2 (a) Write the following statement as an algebraic equation: "y is found by adding eight to six x."

(b) Find y if $x = 10$.

(c) Find y if $x = -5$.

12.3 A child says "Two p and three q make z."

(a) Write this statement as an equation.

(b) Find z if $p = 9$ and $q = -7$.

12.4 The costs of pieces of fruit are: apple 30 p, pear 35 p, banana 28 p and orange 25 p.

(a) Write an equation to find the total cost, C p, of d apples, e pears, f bananas and g oranges.

(b) What is the change from £10.00 if $d = 4$, $e = 4$, $f = 7$ and $g = 6$?

12.5 A gardener walks up and down his garden sowing seeds. The garden has length L, and he makes twelve round trips. In total he walks 336 m.

(a) Write an equation for this information.

(b) What is the length of the garden, L?

Superscripts and subscripts perform different roles. A superscript, such as the 2 in x^2, is used to indicate that a number or variable is raised to a power. Subscripts are used purely as labels. For example, the initial speed of a vehicle might be written as v_0, v_S or even v_{Start}. Numbers in subscripts are part of the label, and do not indicate that a mathematical operation is taking place.

Example 2 - the velocity of a car at time t is given by

$$v_t = v_0 + at$$

where v_0 is the initial velocity of the car and a is the acceleration. Find the value of v_t when $v_0 = 5$ m/s, $a = 2$ m/s^2 and $t = 8$ s.

$$v_t = 5 + 2 \times 8 = 5 + 16 = 21 \text{ m/s}$$

12.6 Using the equation $v_t = v_0 + at$, find v_t if

(a) $v_0 = 0$ m/s, $a = 3$ m/s^2 and $t = 10$ s.

(b) $v_0 = 50$ mm/s, $a = 2$ mm/s^2 and $t = 4$ s.

(c) $v_0 = 0.7$ km/s, $a = -0.04$ km/s^2 and $t = 10$ s.

12.7 (a) If R is the number of rabbits now, and R_0 is the number of rabbits originally, write an equation for the statement "The number of rabbits now is twice the starting number of rabbits, minus 10 which have been sold."

(b) Find R if $R_0 = 210$.

Greek letters are commonly used in algebra in mathematics and the sciences. They can be manipulated in exactly the same way as Roman letters such as x and y. The table below shows those that are used most often and their names. On the left are lower case letters, and on the right are a smaller number of upper case letters.

	Name		Name		Name		Name
α	alpha	θ	theta	ρ	rho	Δ	delta
β	beta	λ	lambda	σ	sigma	Λ	lambda
γ	gamma	μ	mu	ϕ	phi	Σ	sigma
δ	delta	ν	nu	ω	omega	Φ	phi
ϵ, ε	epsilon	π	pi			Ω	omega

Example 3 - The resistance of a piece of wire, R, is equal to the resistivity of the wire ρ multiplied by the length of the wire l and divided by the wire's cross-sectional area A.

Multiply the wire's resistivity by its length. $\rho \times l$

Next divide by the cross-sectional area. $\frac{\rho \times l}{A}$

This is equal to the wire's resistance. $R = \frac{\rho \times l}{A}$

12.8 Λ is equal to ϕ minus ω.

(a) Write an equation for Λ.

(b) Find Λ for $\phi = 45°$ and $\omega = 15°$.

12.9 (a) Write this information as an equation: "To find γ start with 24 and subtract 4 times α, then divide the answer by 3."

(b) Find γ when $\alpha = 3$.

Simplifying "tidies up" algebra into a neater form. Simplifying includes collecting like terms together; using the rules of indices to combine different powers of a variable; and cancelling a common factor in the numerator and denominator of a fraction.

Example 4 - Simplify $\frac{1}{2} \times 4x^2 + 2p + 3\frac{p^2}{p}$.

The first term can be simplified by multiplying the $\frac{1}{2}$ and the 4 together. The third term can be simplified by cancelling a factor of p in the top and bottom of the fraction. Finally, combine like p terms.

$$\frac{1}{2} \times 4x^2 + 2p + 3\frac{p^2}{p} = 2x^2 + 2p + 3p = 2x^2 + 5p$$

In general it is good practice to simplify algebra whenever possible, even if not explicitly asked to do so.

12.10 Simplify:

(a) $3\alpha + 2\alpha$ (b) $5\lambda - \pi - 2\pi - \lambda$

(c) $M = M_0 + 3m + 5m - 6m + 4m$

12.11 Simplify:

(a) $3p - 6s + 2t - p + s$

(b) $\frac{3}{4}vw + \frac{1}{4}vw$

(c) $fg + gf + 2hj + jh$

12.12 Simplify:

(a) $2p \times 3q^2r + 4r \times 2pq^2$

(b) $\frac{1}{2} \times 2x^9 \div x^7 - 2x + x^2 + 20x$

12.13 A bar-tender is counting cans for stock-taking. He has x 4-packs, y 12-packs and z single cans.

(a) Write this information as an equation to find the total number of cans T.

(b) What is T if $x = 11$, $y = 10$ and $z = 7$?

12.14 A postman delivers mail to four houses. House 1 receives $3l$ letters and p parcels. House 2 receives $7l$ letters. House 3 receives $5l$ letters and $2p$ parcels. House 4 receives p parcels.

(a) Write an equation for the total number of items the four houses receive, T. Simplify your answer as far as possible.

(b) Assuming that the weight of a letter is 80 g and the weight of a parcel is 550 g, write an equation for W, the total weight in kilograms of the items delivered to the four houses.

12.15 A quantity called the discriminant is used in the calculation of solutions of quadratic equations.

(a) Using δ for the discriminant, write the following as an equation: "The discriminant is found by subtracting four times a times c from the square of b."

(b) Find δ if $b = 16$, $a = 1$ and $c = 4$.

(c) Find δ if $b = 100$, $a = 3$ and $c = 7$.

12.16 Write the following statements in algebra.

(a) α is twice β.

(b) α cubed is the same as γ squared.

$\beta = 2$ and γ is a positive integer.

(c) Find the value of γ.