Physics. You work it out.

3D Vectors 2ii

A Level

The points A and B have position vectors $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ relative to an origin O, where $\underline{\boldsymbol{a}} = 4\underline{\boldsymbol{i}} + 3\underline{\boldsymbol{j}} - 2\underline{\boldsymbol{k}}$ and $\underline{\boldsymbol{b}} = -7\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}} + 4\underline{\boldsymbol{k}}$.

Part A Length AB

Find the length of $\vec{AB}$. Give you answer as an exact surd.

Part B Unit vector

Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. Give your answer using ijk notation.

The following symbols may be useful: `i, j, k`

Adapted with permission from UCLES, A Level, January 2007 & June 2011, OCR C4

Gameboard:

Mechanics Practice: 3D Vectors

3D Vectors 1ii

A Level

$ABCD$ is a parallelogram. The position vectors of A, B and C are given respectively by

$$\underline{\boldsymbol{a}} = 2\underline{\boldsymbol{i}} + \underline{\boldsymbol{j}} + 3\underline{\boldsymbol{k}} \qquad \underline{\boldsymbol{b}} = 3\underline{\boldsymbol{i}} - 2\underline{\boldsymbol{j}} \qquad \underline{\boldsymbol{c}} = \underline{\boldsymbol{i}} - \underline{\boldsymbol{j}} - 2\underline{\boldsymbol{k}}$$

Part A Position of D

Find the position vector of D. Give your answer using ijk notation.

The following symbols may be useful: `i, j, k`

Part B Unit vector

Find the unit vector in the direction $\vec{OD}$ where O is the fixed origin. Give your answer using ijk notation.

The following symbols may be useful: `i, j, k`

Adapted with permission from UCLES, A Level, June 2005, OCR C4, Question 5

Vectors in 3D

A Level

Given that $\underline{\boldsymbol{a}} = 6\underline{\boldsymbol{i}} + (p-10)\underline{\boldsymbol{j}} + (3p-5)\underline{\boldsymbol{k}}$, and that $|\underline{\boldsymbol{a}}| = 11$, find the possible values of p.

Part A Smaller value of p

Enter the smaller value of p:

Part B Larger value of p

Enter the larger value of p:

Adapted for Isaac Physics from NST IA Biology preparation work

Physics. You work it out.

Angles Between a 3D Vector and the Axes

A Level

Find the angles between the vector $\underline{\boldsymbol{i}} + 2\underline{\boldsymbol{j}} + 3\underline{\boldsymbol{k}}$ and the x, y and z coordinate axes.

Part A Angle with x axis

What is the angle in degrees between the vector and the x axis? Give your answer to 3s.f.

Part B Angle with y axis

What is the angle in degrees between the vector and the y axis? Give your answer to 3s.f.

Part C Angle with z axis

What is the angle in degrees between the vector and the z axis? Give your answer to 3s.f.

Adapted for Isaac Physics from NST IA Biology preparation work

Vectors: Diagrams and Proof 2ii

A Level

Part A Resultant vector

Find the resultant of the vectors $4\mathbf{i} - \mathbf{j}$ and $-2\mathbf{i} + 5\mathbf{j}$.

The following symbols may be useful: `i, j`

Part B $\vec{MN}$

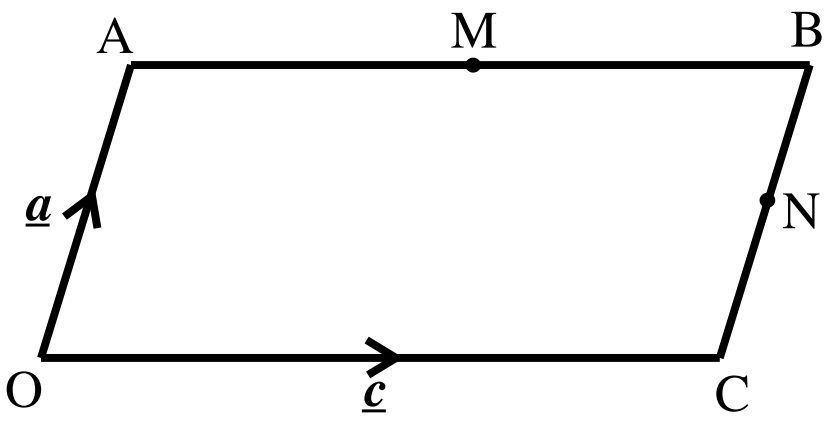

Figure 1: A parallelogram $OABC$.

In **Figure 1**, $OABC$ is a parallelogram. M is the midpoint of $\vec{AB}$. N is the midpoint of $\vec{BC}$. $\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$.

Find $\vec{MN}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

The following symbols may be useful: `a, c`

Adapted with permission from UCLES, A Level, 2012

Gameboard:

Pure Maths Practice: Vectors - Diagrams and Proof

Vectors: Diagrams and Proof 1ii

A Level

Part A $2\underline{s} - \underline{t}$

Given that $\underline{s} = 3\underline{i} + 4\underline{j}$ and $\underline{t} = 6\underline{i} - \underline{j}$, find $2\underline{s} - \underline{t}$.

The following symbols may be useful: `i, j`

Part B In terms of $\underline{p}$

Figure 1: Three points P, X and Q.

Figure 1 shows three points P, X and Q such that $\vec{XQ} = 3\vec{PX}$.

Given that $\vec{PX} = \underline{p}$, find $\vec{XQ}$ in terms of $\underline{p}$

The following symbols may be useful: `p`

Given that $\vec{PX} = \underline{p}$, find $\vec{QP}$ in terms of $\underline{p}$

The following symbols may be useful: `p`

Part C Proving $AMCN$ is a parallelogram

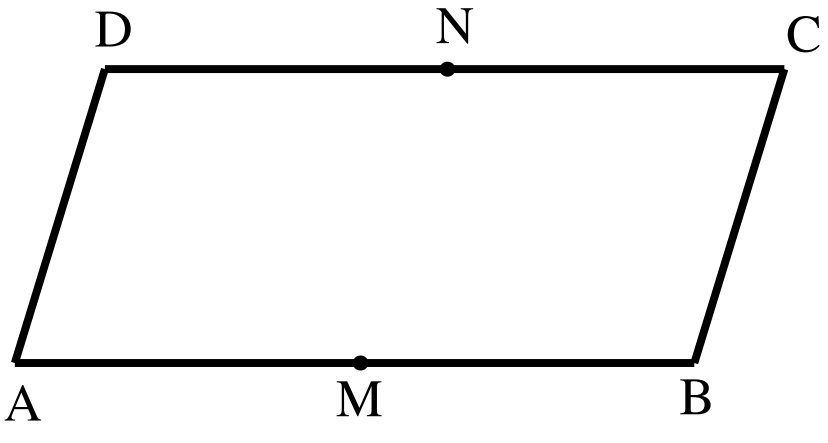

Figure 2: $ABCD$ is a parallelogram

In **Figure 2** $ABCD$ is a parallelogram. M and N are the mid-points of AB and DC. $\vec{AB} = \underline{\boldsymbol{a}}$ and $\vec{AD} = \underline{\boldsymbol{b}}$. Use a vector method to prove that $AMCN$ is also a parallelogram.

Choose four items from the left and put them into order on the right to create a proof.

Available items

1. A parallelogram has two pairs of sides which are parallel and of equal length.

3. $\vec{AN} = \vec{MC} = \frac{1}{2}\underline{\boldsymbol{a}} + \underline{\boldsymbol{b}}$. Therefore $\vec{AN}$ is parallel to $\vec{MC}$ and has the same length.

2. $\vec{MB} = \vec{NC} = \frac{1}{2}\underline{\boldsymbol{a}}$. Therefore $\vec{MB}$ is parallel to $\vec{NC}$ and has the same length.

1. A parallelogram has two pairs of sides which are parallel. All sides of a parallelogram are the same length.

3. $\vec{AD} = \vec{BC} = \underline{\boldsymbol{b}}$. Therefore $\vec{AD}$ is parallel to $\vec{BC}$ and has the same length.

4. $AMCN$ has two pairs of sides which are parallel and of equal length. Hence, $AMCN$ is a parallelogram.

4. $AMCN$ has four sides which are parallel and of equal length. Hence, $AMCN$ is a parallelogram.

2. $\vec{AM} = \vec{NC} = \frac{1}{2}\underline{\boldsymbol{a}}$. Therefore $\vec{AM}$ is parallel to $\vec{NC}$ and has the same length.

Adapted with permission from UCLES, A Level, 2000

Vectors: Diagrams and Proof 2i

A Level

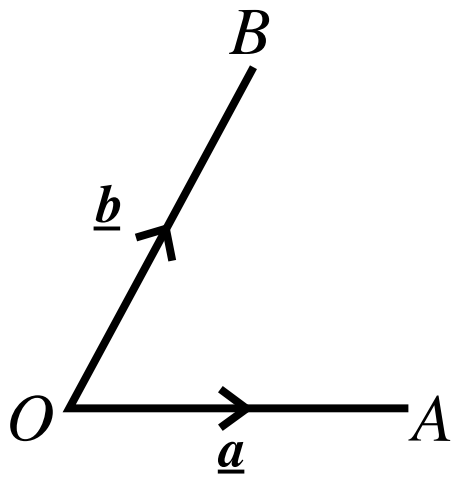

Figure 1: Points A and B and their position vectors with respect to the origin O.

In **Figure 1**, the points A and B have position vectors $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ with respect to the origin O.

Part A Sketch

Make a sketch of the diagram, and mark on the points C, D and E such that $\vec{OC} = 2\underline{\boldsymbol{a}}$, $\vec{OD} = 2\underline{\boldsymbol{a}} + \underline{\boldsymbol{b}}$ and $\vec{OE} = \frac{1}{3}\vec{OD}$.

Which of the sketches below correctly shows this information?

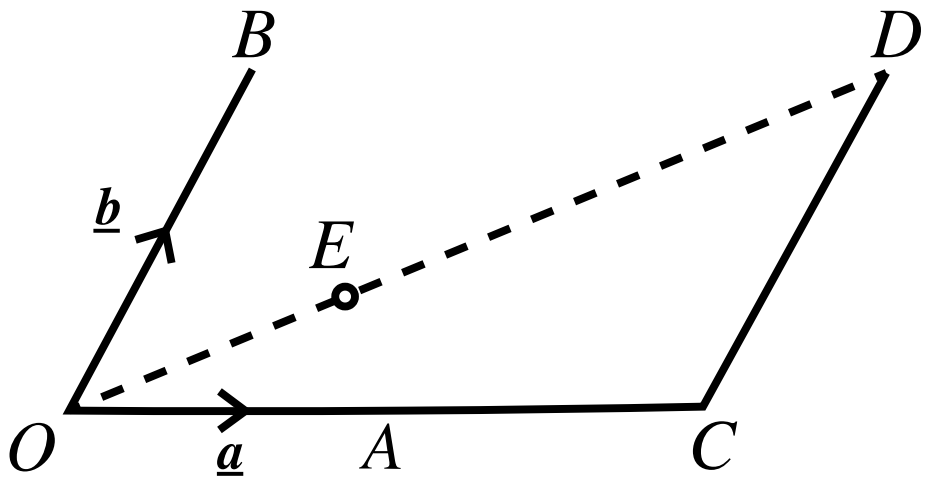

Figure 2: Option A

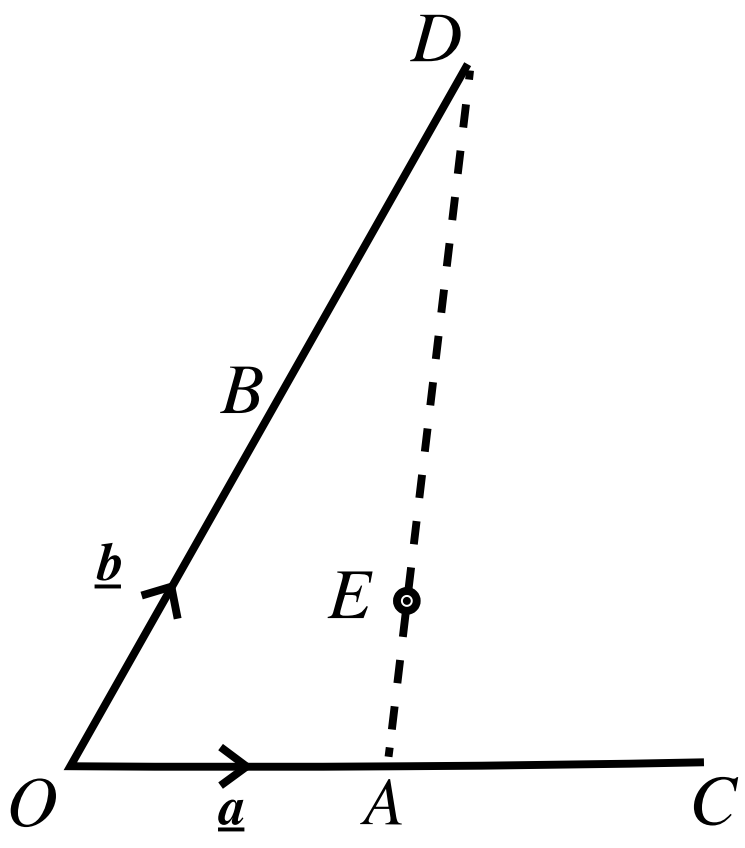

Figure 3: Option B

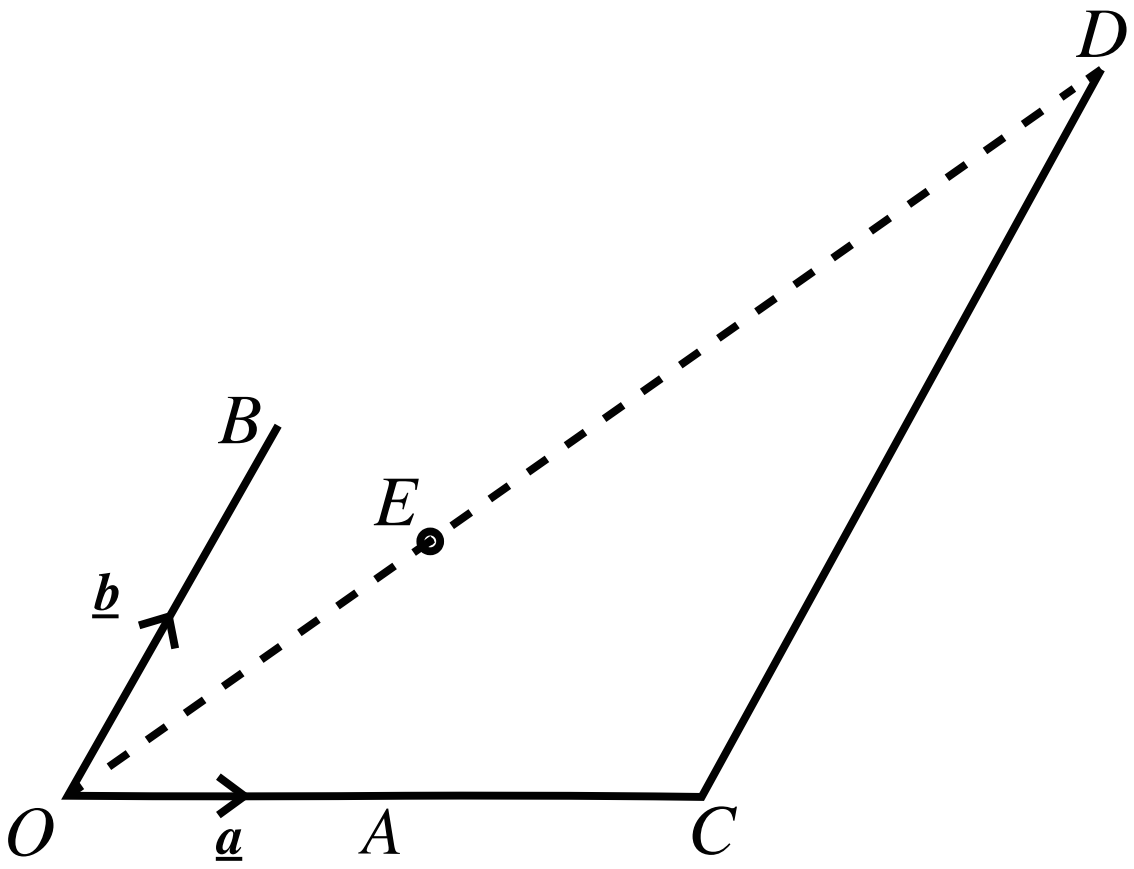

Figure 4: Option C

- Option A
- Option B
- Option C

Part B Position vector of E

Give the position vector of E with respect to A.

The following symbols may be useful: `a, b`

Part C Proof

Hence prove that E lies on the line joining A and B.

Choose three items from the choices on the left and put them into order on the right to create a proof.

Available items

3. $\vec{AE}$ is a scalar multiple of $\vec{AB}$. Hence, E lies on the line joining A and B.

2. $\vec{AE} = \frac{1}{3}(\underline{\boldsymbol{b}} - \underline{\boldsymbol{a}})$ and $\vec{AB} = \underline{\boldsymbol{b}} - \underline{\boldsymbol{a}}$. Therefore, $\vec{AE} = \frac{1}{3}\vec{AB}$.

3. $\vec{OE}$ is a scalar multiple of $\vec{OD}$. Hence, E lies on the line joining O and D.

1. Point E lies on the line joining A and B if the vector $\vec{OE}$ is a scalar multiple of the vector $\vec{OB}$, i.e. $\vec{OE} = k\vec{OB}$.

2. $\vec{AE} = \underline{\boldsymbol{b}} - \underline{\boldsymbol{a}}$ and $\vec{AB} = \frac{1}{3}(\underline{\boldsymbol{b}} - \underline{\boldsymbol{a}})$. Therefore, $\vec{AE} = 3\vec{AB}$.

1. Point E lies on the line joining A and B if the vector $\vec{AE}$ is a scalar multiple of the vector $\vec{AB}$, i.e. $\vec{AE} = k\vec{AB}$.

Adapted with permission from UCLES, A Level, Specimen Paper

3D Vectors 2i

A Level

$ABCD$ is a quadrilateral. You are given four pieces of information:

- Relative to a fixed origin O, the position vector of A is $2\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}} + 8\underline{\boldsymbol{k}}$.
- Relative to a fixed origin O, the position vector of B is $5\underline{\boldsymbol{i}} + 9\underline{\boldsymbol{j}} + 8\underline{\boldsymbol{k}}$.
- The vector $\vec{BC} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$.
- The vector $\vec{BD} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$.

Part A Finding AB

Find the vector $\vec{AB}$. Give your answer in the form `(x,y,z)` with the commas and without the spaces.

Part B Finding CD

Find the vector $\vec{CD}$. Give your answer in the form `(x,y,z)` with the commas and without the spaces.

Part C Finding AD

Find the vector $\vec{AD}$. Give your answer in the form `(x,y,z)` with the commas and without the spaces.

Part D Type of quadrilateral

The shape $ABCD$ lies in a plane. What type of quadrilateral is $ABCD$?

- An irregular quadrilateral
- A square
- A rhombus
- A rectangle
- A kite
- A trapezium
- A parallelogram

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3D Vectors 3i

A Level

The position vectors of the points A and B, relative to a fixed origin O, are $6\underline{\boldsymbol{i}} + 4\underline{\boldsymbol{j}} - \underline{\boldsymbol{k}}$ and $8\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}} - 3\underline{\boldsymbol{k}}$ respectively.

Part A Vector $\underline{\boldsymbol{AB}}$

Find the vector $\vec{AB}$. Give your answer in the form `(x,y,z)` with the commas and without spaces.

Find the length AB.

Part B Line $\underline{\boldsymbol{l}}$

A point P lies on the line

$$\underline{\boldsymbol{l}} = (8 + 2\lambda)\underline{\boldsymbol{i}} + (5 + \lambda)\underline{\boldsymbol{j}} - (3 + 2\lambda)\underline{\boldsymbol{k}}$$

Find the direction vector of $\underline{\boldsymbol{l}}$. Give your answer in the form `(x,y,z)` with the commas and without spaces.

How does the line $\underline{\boldsymbol{l}}$ relate to the points A and B?

- It passes through A
- It passes through B
- It is in the direction of AB
- All of the above

Used with permission from UCLES, A Level, January 2002, OCR P3, Question 6

3D Vectors 3ii

A Level

Two points A and B have position vectors $3\underline{\boldsymbol{i}} - \underline{\boldsymbol{j}} + 2\underline{\boldsymbol{k}}$ and $2\underline{\boldsymbol{j}} + 3\underline{\boldsymbol{k}}$ respectively.

Part A Vector AB

Find the vector $\vec{AB}$. Give your answer using ijk notation.

The following symbols may be useful: `i, j, k`

Hence find the length of $\vec{AB}$. Give your answer as an exact surd.

Part B Intersection

Show that the line through A and B does not intersect the line through the origin parallel to the vector i.

One way to prove this is to use proof by contradiction. Fill in the blanks to complete the proof below.

Opening statement:

The line through the origin parallel to the vector i is the x-axis. On the x-axis, $y = z =$ [ ]. If the line through A and B intersects the x-axis, then there is a value of λ such that

$$\vec{OA} + \lambda\vec{AB} = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

where μ is the value of x where the line intersects the x-axis.

Calculations:

Putting in expressions for $\vec{OA}$ and $\vec{AB}$,

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \lambda \boxed{} = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

This gives three equations, one for each of the x, y and z components:

$$3 - 3\lambda = \mu, \quad -1 + 3\lambda = 0 \quad \text{and} \quad 2 + \lambda = 0$$

The second of these equations re-arranges to $\lambda =$ [ ], but the third equation rearranges to $\lambda =$ [ ]. Hence, these equations are inconsistent and we have reached a contradiction.

Conclusion:

There is no point on the line through A and B for which y and z are both zero, so this line does not intersect the x-axis, and hence this line does not intersect the line through the origin parallel to the vector i.

Items:

-3 -2 -1 0 1 2 3 $\frac{1}{3}$ $\frac{2}{3}$ $\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$

Used with permission from UCLES, A Level, January 2004, OCR P3, Question 5