Isaac Physics Skills

Linking concepts in pre-university physics

Lisa Jardine-Wright, Keith Dalby, Robin Hughes, Nicki Humphry-Baker,
Anton Machacek, Ingrid Murray and Lee Phillips
Isaac Physics Project

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1 Gravitational potential and kinetic energy

Objects rising and falling exchange stores of gravitational potential and kinetic energy.

Example context: We can calculate the speed of objects after they have fallen. We can also work out the height to which a projected object rises. The analysis is particularly useful when balls bounce.

Quantities:	h_0 starting height (m)	v_0 starting speed $\left(\mathrm{m\,s^{-1}}\right)$
	h_1 final height (m)	v_1 final speed $\left(\mathrm{m\,s^{-1}}\right)$
	m mass (kg)	g gravitational field strength $\left(\mathrm{N\,kg^{-1}}\right)$
	E_K kinetic energy (J)	E_{GP} gravitational potential energy (J)
	η efficiency (no unit)	E_T total energy (J)

Equations: $E_K = \frac{1}{2}mv^2 \quad E_{GP} = mgh \quad E_T = E_K + E_{GP} \quad E_{T,\,after} = \eta E_{T,\,before}$

1.1 In the absence of air resistance, use the equations to derive expressions for

a) the speed v_1 at the ground if an object was dropped from h_0,

b) the speed v_1 at a height h_1 if an object had speed v_0 at h_0,

c) the greatest height h_1 for an object projected up from the ground with speed v_0,

d) the greatest height h_1 for an object projected up from a height h_0 with speed v_0,

e) the greatest height h_1 above a hard surface reached by an object dropped from height h_0 if the efficiency of the bounce is η,

f) the speed v_1 just after a bounce from a hard surface if the speed just before was v_0,

> **Example 1** – *A 0.80 kg melon falls from 3.4 m. Calculate its speed just before striking the ground.*
> At start: $E_{T,before} = E_{GP,before} = mgh_0 = 0.800 \times 9.81 \times 3.4 = 26.68$ J.
> At end: $E_{T,after} = E_{K,after} = \frac{1}{2}mv_1^2 = \frac{1}{2} \times 0.800 \times v_1^2 = 0.400v_1^2$.
> $E_{T,before} = E_{T,after} \rightarrow 26.68 = 0.400\,v_1^2 \rightarrow v_1 = \sqrt{\frac{26.68}{0.400}} = 8.2\ \mathrm{m\,s^{-1}}$.

1.2 An 800 kg pumpkin falls from 3.4 m. Calculate its speed just before striking the ground.

1.3 A 60 g tennis ball is hit upwards at 27 m s^{-1}. How high will it rise?

1.4 A 60 g tennis ball is hit upwards at 27 m/s from a 25 m rooftop. How fast will it be travelling when it passes the rooftop on the way down?

Example 2 – *Calculate the height reached by a* 0.15 *kg ball thrown up from a* 20 *m cliff with a speed of* 15 *m s*$^{-1}$.

$$
\begin{aligned}
E_{\text{T,before}} &= E_{\text{GP,before}} + E_{\text{K,before}} = mgh_0 + \tfrac{1}{2}mv_0^2 \\
&= 0.15 \times 9.81 \times 20 + \tfrac{1}{2} \times 0.15 \times 15^2 = 46.31\ \text{J} \\
E_{\text{T,after}} &= E_{\text{GP,after}} + E_{\text{K,after}} = mgh_1 + 0 \\
&= 0.15 \times 9.81 \times h_1 = 1.472\, h_1 \\
E_{\text{T,after}} &= E_{\text{T,before}} \rightarrow 1.472\, h_1 = 46.31 \rightarrow h_1 = \frac{46.31}{1.472} = 32\ \text{m}
\end{aligned}
$$

1.5 A 3.1 kg brick falls from scaffolding on a building site. A worker 3.5 m above the ground sees it fall past at 6.5 m/s. What is its kinetic energy just before striking the ground?

1.6 At what speed will a 4.2 kg lump of clay hit a potter's wheel if it is thrown downwards at 1.1 m s^{-1} from a height 40 cm above the wheel?

1.7 A worker at ground level throws a 2.2 kg drinks bottle upwards to a thirsty colleague 3.2 m above the ground. It just reaches him, but he fails to catch it, and it falls into an excavated trench 1.6 m below ground level.

a) At what speed did the worker need to throw the bottle if she threw it from the waist, 1.0 m above the ground?

b) How fast was it moving when it struck the base of the trench?

Example 3 – *A* 25 *g ball is thrown down to a hard surface at* 12.3 *m s*$^{-1}$. *How high will it rise after bouncing if* $\eta = 0.35$*?*
On hitting the surface $E_{\text{T,before}} = \frac{1}{2}mv_0^2 = \frac{1}{2} \times 0.025 \times 12.3^2 = 1.891$ J
$E_{\text{T,after}} = \eta E_{\text{T,before}} = 0.35 \times 1.891 = 0.6619$ J
$E_{\text{T,after}} = E_{\text{GP,final}} = mgh_1 = 0.025 \times 9.81 \times h_1 = 0.2453\, h_1$
So $0.6619 = 0.2453\, h_1 \rightarrow h_1 = \dfrac{0.6619}{0.2453} = 2.699$ m

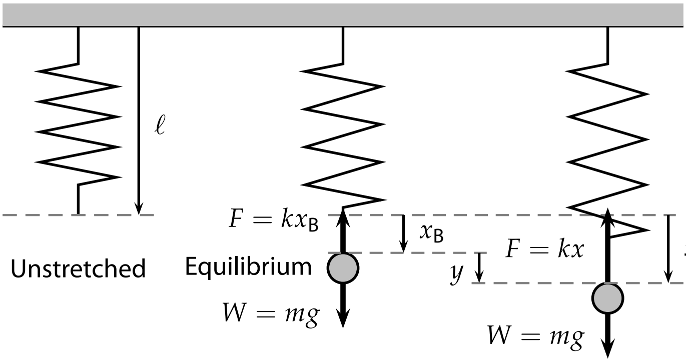
1.8 A 5.2 g ball is dropped from 90 cm onto a surface and bounces to a maximum height of 41 cm. Calculate the efficiency η.

1.9 How fast would the ball in question 1.8 rebound if it struck the surface at 2.5 m s^{-1}?

1.10 How high would a ball bounce if it struck an $\eta = 0.75$ surface at 13 m s^{-1}?

2 Gravitational, elastic and kinetic energy

Objects suspended from a spring exchange stores of kinetic, elastic potential and kinetic energy as they move up and down.



Quantities:	x spring extension (m)	ℓ spring natural length (m)
	x_B equilibrium x (m)	y distance from equilibrium (m)
	v speed $\left(\mathrm{m\,s^{-1}}\right)$	k spring constant $\left(\mathrm{N\,m^{-1}}\right)$
	m mass (kg)	g gravitational field strength $\left(\mathrm{N\,kg^{-1}}\right)$
	E_K kinetic energy (J)	E_{GP} gravitational potential energy (J)
	E_T total energy (J)	E_{EP} elastic potential energy (J)
	F spring tension (N)	W weight (N)

Equations: $E_K = \frac{1}{2}mv^2 \quad E_{GP} = -mgx \quad E_{EP} = \frac{1}{2}kx^2 \quad F = kx$

$E_T = E_K + E_{GP} + E_{EP} \quad W = mg \quad y = x - x_B$

2.1 In the absence of air resistance, use the equations to derive expressions for

a) the total energy E_T in terms of x and v,

b) the value of x where the forces balance (we will call this x_B),

c) $E_{GP} + E_{EP}$ at the point where the forces balance (we will call this E_B),

d) the greatest value of x if you hold the mass at $x = 0$ and let it go,

e) (optional) the value of $E_{GP} + E_{EP}$ in terms of $y = x - x_B$.

2.2 Calculate E_{GP}, E_{EP}, E_K and E_T for a 2.5 kg mass when $x = 0.055$ m and $v = 0.25\ \mathrm{m\,s^{-1}}$ if $k = 600\ \mathrm{N\,m^{-1}}$.

2.3 Calculate x_B (the extension of the spring at the equilibrium point) for a 100 N weight hanging from a $k = 5000\ \mathrm{N\,m^{-1}}$ spring.

Example – *A* 60 *kg bungee jumper falls* 12 *m before their bungee is taut. How fast will they be moving after falling a further* 4.0 *m?* $k = 200$ *N m*$^{-1}$
At the start $x = -12$ m and $v = 0$, and $E_\mathrm{K} = E_\mathrm{EP} = 0$,
so $E_\mathrm{T} = E_\mathrm{GP} = -mgx = -60 \times 9.81 \times (-12) = 7063$ J.
When $x = 4.0$ m, $E_\mathrm{T} = E_\mathrm{EP} + E_\mathrm{GP} + E_\mathrm{K} = \frac{1}{2}kx^2 - mgx + \frac{1}{2}mv^2$
so $E_\mathrm{T} = \frac{1}{2} \times 200 \times 4^2 - 60 \times 9.81 \times 4 + \frac{1}{2} \times 60 \times v^2$
and so $E_\mathrm{T} = -754.4 + 30v^2$.
Total energy is constant so $7063 = -754.4 + 30v^2$,
therefore $7817.4 = 30v^2$ and $v = \sqrt{\dfrac{7817.4}{30}} = 16$ m s^{-1}

2.4 Calculate the speed of the bungee jumper in the example when

a) the bungee has stretched 5.0 m,

b) the bungee becomes slack on the way up.

2.5 Fill in the missing entries in the table below. This describes the motion of a 100 N weight ($m = 10.2$ kg), hanging from a $k = 5000$ N m^{-1} spring, which is released from rest at $x = 0$. You calculated x_B in question 2.3.

x	v	E_K	E_GP	E_EP	$E_\mathrm{EP} + E_\mathrm{GP}$	E_T	$y = x - x_\mathrm{B}$
/ cm	/ m s^{-1}	/J					/ cm
1.0	(a)	(b)	(c)	(d)	(e)	0.0	(f)
2.0	(g)	(h)	(i)	(j)	(k) $= E_\mathrm{B}$	0.0	0.0
3.0	(l)	(m)	(n)	(o)	(p)	0.0	(q)
4.0	(r)	(s)	(t)	(u)	(v)	0.0	(w)

2.6 For the system in question 2.5, state or calculate

a) the value of x where the total potential energy is a minimum,

b) the minimum total potential energy,

c) the total potential energy *relative to the minimum* when $y = 2.0$ cm,

d) the energy required to stretch a $k = 5000$ N m^{-1} spring by 2.0 cm.

2.7 Calculate how far the bungee jumper in the example falls before they first come to rest. You may assume that the *total* potential energy of the jumper relative to the equilibrium position is given by $\frac{1}{2}ky^2$.

36 Solutions to first questions

1 Gravitational potential and kinetic energy

(a) $E_{\text{T,before}} = E_{\text{T,after}}$, so $E_{\text{P,before}} = E_{\text{K,after}}$, and $mgh_0 = \frac{1}{2}mv_1^2$.
Therefore $v_1 = \sqrt{2gh_0}$

(b) $E_{\text{T,before}} = E_{\text{T,after}}$, so $E_{\text{P,before}} + E_{\text{K,before}} = E_{\text{P,after}} + E_{\text{K,after}}$
So $mgh_0 + \frac{1}{2}mv_0^2 = mgh_1 + \frac{1}{2}mv_1^2$, and $v_1 = \sqrt{2g\left(h_0 - h_1\right) + v_0^2}$

(c) $E_{\text{T,before}} = E_{\text{T,after}}$, so $E_{\text{K,before}} = E_{\text{P,after}}$, and $\frac{1}{2}mv_0^2 = mgh_1$.
Therefore $h_1 = \dfrac{v_0^2}{2g}$

(d) $E_{\text{T,before}} = E_{\text{T,after}}$, so $E_{\text{P,before}} + E_{\text{K,before}} = E_{\text{P,after}}$
So $mgh_0 + \frac{1}{2}mv_0^2 = mgh_1$, and $h_1 = h_0 + \dfrac{v_0^2}{2g}$

(e) $E_{\text{T,after}} = \eta E_{\text{T,before}}$, so $E_{\text{P,after}} = \eta E_{\text{P,before}}$, and $mgh_1 = \eta mgh_0$.
Therefore $h_1 = \eta h_0$

(f) $E_{\text{T,after}} = \eta E_{\text{T,before}}$, so $E_{\text{K,after}} = \eta E_{\text{K,before}}$, and $\frac{1}{2}mv_1^2 = \frac{1}{2}\eta mv_0^2$.
Therefore $v_1 = \sqrt{\eta}\, v_0$

2 Gravitational, elastic and kinetic energy

(a) $E_{\text{T}} = E_{\text{K}} + E_{\text{GP}} + E_{\text{EP}} = \frac{1}{2}mv^2 - mgx + \frac{1}{2}kx^2$

(b) $kx_{\text{B}} = mg$ so $x_{\text{B}} = \dfrac{mg}{k}$

(c) $E_{\text{B}} = E_{\text{GP}} + E_{\text{EP}} = -mgx_{\text{B}} + \frac{1}{2}kx_{\text{B}}^2 = -\dfrac{m^2g^2}{k} + \dfrac{k}{2}\dfrac{m^2g^2}{k^2} = -\dfrac{m^2g^2}{2k}$

(d) $E_{\text{T}} = 0$ and $E_{\text{K}} = 0$, so $0 = E_{\text{GP}} + E_{\text{EP}} = -mgx + \frac{1}{2}kx^2$
$= x\left(\frac{1}{2}kx - mg\right)$ so $x = \dfrac{2mg}{k}$

(e) $$\begin{aligned} E_{\text{GP}} + E_{\text{EP}} &= -mgx + \tfrac{1}{2}kx^2 = -mg\left(x_{\text{B}} + y\right) + \tfrac{1}{2}k\left(x_{\text{B}} + y\right)^2 \\ &= -mg\left(\frac{mg}{k} + y\right) + \frac{k}{2}\left(\frac{mg}{k} + y\right)^2 \\ &= -\frac{m^2g^2}{k} - mgy + \frac{m^2g^2}{2k} + mgy + \frac{ky^2}{2} \\ &= \frac{ky^2}{2} - \frac{m^2g^2}{2k} = \frac{ky^2}{2} + E_{\text{B}} \end{aligned}$$

3 Momentum and kinetic energy

(a) $p = mv$ so $v = \dfrac{p}{m}$. Therefore $E = \dfrac{m}{2}v^2 = \dfrac{m}{2}\left(\dfrac{p}{m}\right)^2 = \dfrac{p^2}{2m}$

(b) $E = \dfrac{mv^2}{2}$ so $v = \sqrt{\dfrac{2E}{m}}$. Now $p = mv = m\sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2Em^2}{m}} = \sqrt{2mE}$

(c) $p = \sqrt{2mE} = \sqrt{2mqV}$ as $E = qV$

(d) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mqV}}$

4 Elastic collisions

(a) $p_0 + P_0 = p_1 + P_1$ so $mv_0 + 0 = mv_1 + MV_1$ and $V_1 = \dfrac{m\left(v_0 - v_1\right)}{M}$

(b) $p_0 + P_0 = p_1 + P_1$ so $mv_0 + 0 = 0 + mV_1$ and $V_1 = v_0$

Part (b) could also be completed using energy conservation.

For the third and optional part (c), the algebra is much more complicated, but we show it so that you can see why approach and separation speeds are the same in elastic collisions. Remember that r is defined as the approach speed ($v - V = r$), so $v = V + r$.

(c) $$\begin{aligned} P + p &= MV + mv = MV + m\left(V + r\right) = \left(M + m\right)V + mr \\ \left(P + p\right)^2 &= \left(M + m\right)^2 V^2 + 2\left(M + m\right)mrV + m^2r^2 \\ K + k &= \frac{MV^2}{2} + \frac{mv^2}{2} = \frac{M^2V^2 + MmV^2 + m^2v^2 + Mmv^2}{2\left(M + m\right)} \end{aligned}$$

TABLE OF PHYSICAL CONSTANTS

Quantity & Symbol		**Magnitude**	**Unit**
Permittivity of free space	ϵ_0	8.85×10^{-12}	F m^{-1}
Electrostatic force constant	$1/4\pi\epsilon_0$	8.99×10^{9}	$\text{N m}^2 \text{ C}^{-2}$
Speed of light in vacuum	c	3.00×10^{8}	m s^{-1}
Specific heat capacity of water	c_{water}	4180	$\text{J kg}^{-1} \text{ K}^{-1}$
Charge of proton	e	1.60×10^{-19}	C
Gravitational field strength on Earth	g	9.81	N kg^{-1}
Universal gravitational constant	G	6.67×10^{-11}	$\text{N m}^2 \text{ kg}^{-2}$
Planck constant	h	6.63×10^{-34}	J s
Boltzmann constant	k_{B}	1.38×10^{-23}	J K^{-1}
Mass of electron	m_{e}	9.11×10^{-31}	kg
Mass of neutron	m_{n}	1.67×10^{-27}	kg
Mass of proton	m_{p}	1.67×10^{-27}	kg
Mass of Earth	M_{Earth}	5.97×10^{24}	kg
Mass of Sun	M_{Sun}	2.00×10^{30}	kg
Avogadro constant	N_{A}	6.02×10^{23}	mol^{-1}
Gas constant	R	8.31	$\text{J mol}^{-1} \text{ K}^{-1}$
Radius of Earth	R_{Earth}	6.37×10^{6}	m

OTHER INFORMATION YOU MAY FIND USEFUL

Electron volt	1 eV	=	1.60×10^{-19} J
Unified mass unit	1 u	=	1.66×10^{-27} kg
Absolute zero	0 K	=	-273 °C
Year	1 yr	=	3.16×10^{7} s
Light year	1 ly	=	9.46×10^{15} m
Parsec	1 pc	=	3.09×10^{16} m

PREFIXES

1 km = 1000 m	1 Mm = 10^{6} m	1 Gm = 10^{9} m	1 Tm = 10^{12} m
1 mm = 0.001 m	1 μm = 10^{-6} m	1 nm = 10^{-9} m	1 pm = 10^{-12} m