Isaac Physics Skills

Linking concepts in pre-university physics

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TABLE OF PHYSICAL CONSTANTS

Quantity & Symbol		**Magnitude**	**Unit**
Permittivity of free space	ϵ_0	8.85×10^{-12}	$\mathrm{F\,m^{-1}}$
Electrostatic force constant	$1/4\pi\epsilon_0$	8.99×10^{9}	$\mathrm{N\ m^{2}\ C^{-2}}$
Speed of light in vacuum	c	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
Specific heat capacity of water	c_{water}	4180	$\mathrm{J\,kg^{-1}\,K^{-1}}$
Charge of proton	e	1.60×10^{-19}	C
Gravitational field strength on Earth	g	9.81	$\mathrm{N\,kg^{-1}}$
Universal gravitational constant	G	6.67×10^{-11}	$\mathrm{N\ m^{2}\ kg^{-2}}$
Planck constant	h	6.63×10^{-34}	J s
Boltzmann constant	k_{B}	1.38×10^{-23}	$\mathrm{J\,K^{-1}}$
Mass of electron	m_{e}	9.11×10^{-31}	kg
Mass of neutron	m_{n}	1.67×10^{-27}	kg
Mass of proton	m_{p}	1.67×10^{-27}	kg
Mass of Earth	M_{Earth}	5.97×10^{24}	kg
Mass of Sun	M_{Sun}	2.00×10^{30}	kg
Avogadro constant	N_{A}	6.02×10^{23}	$\mathrm{mol^{-1}}$
Gas constant	R	8.31	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Radius of Earth	R_{Earth}	6.37×10^{6}	m

OTHER INFORMATION YOU MAY FIND USEFUL

Electron volt	1 eV	$=$	1.60×10^{-19} J
Unified mass unit	1 u	$=$	1.66×10^{-27} kg
Absolute zero	0 K	$=$	-273 °C
Year	1 yr	$=$	3.16×10^{7} s
Light year	1 ly	$=$	9.46×10^{15} m
Parsec	1 pc	$=$	3.09×10^{16} m

PREFIXES

1 km = 1000 m	1 Mm = 10^{6} m	1 Gm = 10^{9} m	1 Tm = 10^{12} m
1 mm = 0.001 m	1 μm = 10^{-6} m	1 nm = 10^{-9} m	1 pm = 10^{-12} m

3 Momentum and kinetic energy

It is helpful to be able to calculate a momentum from a kinetic energy without first working out the speed.

Example context: In particle physics, the wavelength of a particle is related to its momentum. In a question you are more likely to be told its energy (eg. a 50 keV electron) than its speed.

Quantities:		
	p momentum $\left(\mathrm{kg\,m\,s^{-1}}\right)$	E kinetic energy (J)
	m mass (kg)	λ wavelength (m)
	v speed $\left(\mathrm{m\,s^{-1}}\right)$	q charge (C)
	V accelerating voltage (V)	h Planck constant $(\mathrm{J\,s})$

Equations: $p = mv \quad E = \frac{1}{2}mv^2 \quad E = qV \quad \lambda = \dfrac{h}{p}$

3.1 Use the equations to derive expressions without v for

a) the kinetic energy E in terms of p and m,

b) the momentum p in terms of E and m,

c) the momentum of an accelerated particle in terms of V, m and q,

d) the wavelength of an accelerated particle in terms of V and q.

Example 1 – *Calculate the kinetic energy of a 9 kg pumpkin with a momentum of 150 kg m s^{-1}.*

$$E = \frac{m}{2}v^2 = \frac{m}{2}\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} = \frac{150^2}{2 \times 9} = 1250\ \mathrm{J}$$

Example 2 – *calculate the wavelength of a 1 keV electron.*
Kinetic energy $E = qV$ where q is the charge on one electron and $V = 1000\ \mathrm{V}$. As $E = \frac{1}{2}mv^2$, the momentum will be

$$p = mv = m\sqrt{\frac{2E}{m}} = \sqrt{2mE} = \sqrt{2mqV}$$, so we calculate $\lambda = \dfrac{h}{p}$ as

$$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 10^3}} = 3.89 \times 10^{-11}\ \mathrm{m}$$

3.2 Calculate the kinetic energy of a $p = 23\,700\ \mathrm{kg\,m\,s^{-1}}$, 720 kg car.

3.3 Fill in the missing entries in the table below.

Mass / kg	Momentum / kg m s^{-1}	Kinetic energy / J
32	(a)	0.040
5.6	252	(b)
4.6 g	(c)	980
12 000	168 000	(d)

3.4 Calculate the momentum of a 200 g orange with 54 J of kinetic energy.

3.5 Calculate the momentum of a proton accelerated by 20 kV.

3.6 Calculate the kinetic energy of a neutron with a wavelength of 2.4 nm.

3.7 Calculate the wavelength of an 80 keV electron.

3.8 Calculate the accelerating voltage needed to produce protons with a wavelength of 3.5 pm.

3.9 Calculate the wavelength of a 50 MeV proton.

3.10 Calculate the wavelength of a 10 MeV alpha particle.

3.11 A 10 MeV particle in a particle detector travels on a curved path in a magnetic field. Its charge is 1.60×10^{-19} C. From the curvature, the momentum of the particle is calculated to be 7.31×10^{-20} kg m s^{-1}.

a) What is the mass of the particle?

b) What is the particle?

3.12 A 15 g bullet hits and stops within a 1.500 kg sandbag, which then swings up by a height of 5.1 cm. Work out the initial speed of the bullet. Hint: the height can be used to work out the gravitational potential energy, and hence the initial kinetic energy of the bag. The momentum of the bag just after the collision will be equal to the momentum of the bullet before it.

(e) $$\begin{aligned} E_{\mathrm{GP}} + E_{\mathrm{EP}} &= -mgx + \tfrac{1}{2}kx^2 = -mg\left(x_{\mathrm{B}} + y\right) + \tfrac{1}{2}k\left(x_{\mathrm{B}} + y\right)^2 \\ &= -mg\left(\frac{mg}{k} + y\right) + \frac{k}{2}\left(\frac{mg}{k} + y\right)^2 \\ &= -\frac{m^2g^2}{k} - mgy + \frac{m^2g^2}{2k} + mgy + \frac{ky^2}{2} \\ &= \frac{ky^2}{2} - \frac{m^2g^2}{2k} = \frac{ky^2}{2} + E_{\mathrm{B}} \end{aligned}$$

3 Momentum and kinetic energy

(a) $p = mv$ so $v = \dfrac{p}{m}$. Therefore $E = \dfrac{m}{2}v^2 = \dfrac{m}{2}\left(\dfrac{p}{m}\right)^2 = \dfrac{p^2}{2m}$

(b) $E = \dfrac{mv^2}{2}$ so $v = \sqrt{\dfrac{2E}{m}}$. Now $p = mv = m\sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2Em^2}{m}} = \sqrt{2mE}$

(c) $p = \sqrt{2mE} = \sqrt{2mqV}$ as $E = qV$

(d) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mqV}}$

4 Elastic collisions

(a) $p_0 + P_0 = p_1 + P_1$ so $mv_0 + 0 = mv_1 + MV_1$ and $V_1 = \dfrac{m\left(v_0 - v_1\right)}{M}$

(b) $p_0 + P_0 = p_1 + P_1$ so $mv_0 + 0 = 0 + mV_1$ and $V_1 = v_0$

Part (b) could also be completed using energy conservation.

For the third and optional part (c), the algebra is much more complicated, but we show it so that you can see why approach and separation speeds are the same in elastic collisions. Remember that r is defined as the approach speed ($v - V = r$), so $v = V + r$.

(c) $$\begin{aligned} P + p &= MV + mv = MV + m\left(V + r\right) = \left(M + m\right)V + mr \\ \left(P + p\right)^2 &= \left(M + m\right)^2 V^2 + 2\left(M + m\right)mrV + m^2r^2 \\ K + k &= \frac{MV^2}{2} + \frac{mv^2}{2} = \frac{M^2V^2 + MmV^2 + m^2v^2 + Mmv^2}{2\left(M + m\right)} \end{aligned}$$