Exponential Rates

A Level

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An experiment involves two substances, Substance 1 and Substance 2, whose masses are changing. The mass, M_1 grams, of Substance 1 at time t hours is given by

$$M_1 = 400\mathrm{e}^{-0.014t}$$

.

The mass, M_2 grams, of Substance 2 is increasing exponentially and the mass at certain times is shown in the following table.

t (hours)	0	10	20
M_2 (grams)	75	120	192

A critical stage in the experiment is reached at time T hours when the masses of the two substances are equal.

Part A Rate of change of Substance 1

Find the rate at which the mass of Substance 1 is changing when $t = 10\,\mathrm{hours}$, giving your answer in grams per hour $\left(\mathrm{g\,hour^{-1}}\right)$ correct to 2 significant figures.

Part B Solving for T

Show that T is the root of an equation of the form $\mathrm{e}^{kt} = c$. State the values of the constants k and c.

What is the value of k?

What is the value of c? Please give your answer to 3 significant figures.

Part C Value of T

Find the value of T to 3 significant figures.

Used with permission from UCLES, June 2011, OCR C3 Paper 4723, question 8.

Area of isosceles triangle

A Level C C C Further A P P P

The isosceles triangle shown in **Figure 1** has a base of length $2b$ and perpendicular height h. The length p of the perimeter of the triangle is fixed. Find an expression in terms of p for the value of b which will maximise the area A of the triangle. Find an expression for this maximum area.

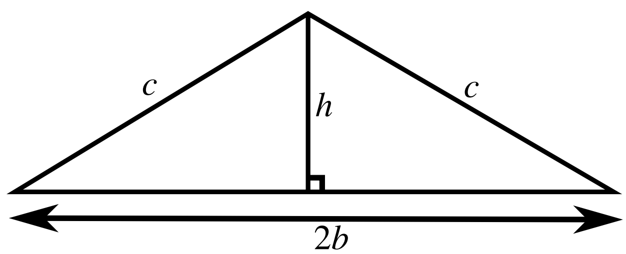

Figure 1: An isosceles triangle with a base of length $2b$, perpendicular height h and sides of length c.

Part A Area A and perimeter p

Write down the equation for the area A of the triangle in terms of b and h.

The following symbols may be useful: `A, b, h`

Find the equation for the perimeter p of the triangle in terms of b and h.

- ○ $p = 2\left(b + \sqrt{b^2 + h^2}\right)$
- ○ $p = b + 2\sqrt{b^2 + h^2}$
- ○ $p = 2b + \sqrt{b^2 + h^2}$
- ○ $p = b + \sqrt{b^2 + h^2}$
- ○ $p = 2b + 2\sqrt{4b^2 + h^2}$
- ○ $p = 2b + \sqrt{4b^2 + h^2}$

Using the above, obtain an equation for A in terms of p and b.

The following symbols may be useful: `A, b, p`

Part B Expressions for b and h

Using the equation for A you found in Part A, find an **expression** in terms of p for the value of b which will maximise the area A of the triangle. (Since p is fixed you may treat it as a constant.)

Hint: you may not know how to differentiate the expression for A, but note that since A is positive it will be a maximum when A^2 is a maximum.

The following symbols may be useful: `p`

Find, in terms of p, the expression for h corresponding to this value of b.

The following symbols may be useful: `p`

Part C The maximum area

Using your result from Part B, find an expression for the maximum area in terms of p.

The following symbols may be useful: p

Part D Check that the area is a maximum

Find, at the value of b deduced above, an expression in terms of p for the second derivative of A^2 with respect to b; convince yourself that the value of the second derivative indicates that the value of A^2, and hence of A, is a maximum.

The following symbols may be useful: p

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