Calculus: Improper Integration

Further A

Evaluate $\int_0^{\infty} 2x\mathrm{e}^{-x}\mathrm{d}x$.

Note that $x\mathrm{e}^{-x} \to 0$ as $x \to \infty$.

Adapted with permission from UCLES, A Level, Sample Paper 2017, Paper Y541, Question 5.

Calculus: Improper Integration

Further A

A large cuboid block is covered with a tarpaulin.

The tarpaulin is placed symmetrically over the block.

It is in contact with the bock where the block is horizontal, and the shape of the tarpaulin where it hangs over the ends of the block can be modelled by the function:

$$f(x) = \frac{1}{x^2}$$

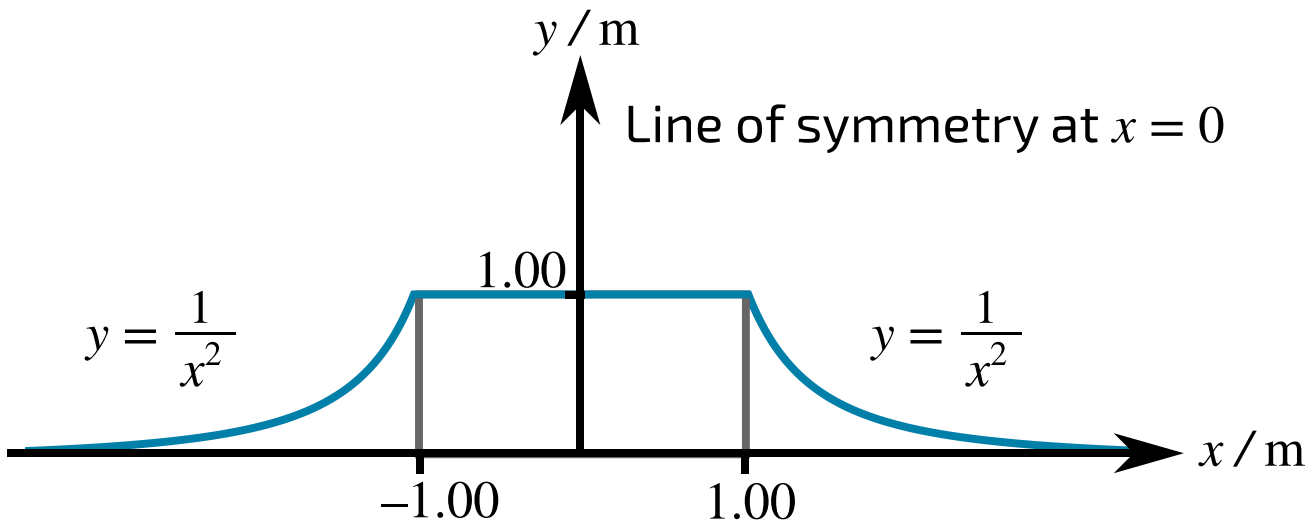

Figure 1: Diagram of the tarpaulin.

If the block is $2.00\,\mathrm{m}$ in length and $1.00\,\mathrm{m}$ in height, find the area of the cross-section of this system, which is shown in **Figure 1**.

Give your answer to 3 significant figures.

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Mean values 1

A Level Further A

C C C P P P

Find the mean values of the following.

Part A $\frac{x}{\sqrt{4-x^2}}$ between 0 and 2

Find the mean value of

$$\frac{x}{\sqrt{4-x^2}}$$

between $x = 0$ and $x = 2$.

Part B $\frac{\sin(2\theta)}{(1-\cos^2\theta)^3}$ between $\frac{\pi}{6}$ and $\frac{\pi}{2}$

Find the mean value of

$$\frac{\sin(2\theta)}{(1-\cos^2\theta)^3}$$

between $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{2}$, giving your answer in exact form.

The following symbols may be useful: `pi`

Part C $\frac{1}{1+e^{-2t}}$ between 0 and 3

Find the mean value of

$$\frac{1}{1+e^{-2t}}$$

between $t = 0$ and $t = 3$, giving your answer in exact form.

The following symbols may be useful: `e, ln(), log()`

Part D $A\tan\phi$ between 0 and b

The mean value of the function $4b\tan\phi$ between $\phi = 0$ and $\phi = b$ is equal to $2\ln 2$, where $b < \frac{\pi}{2}$. Deduce the value of b.

The following symbols may be useful: `pi`

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Mean values 2

A Level Further A

C C C P P P

Find the mean values of the following.

Part A $\sin\left(\frac{\pi x}{a}\right)$ between 0 and a

Find the mean value of $\sin\left(\frac{\pi x}{a}\right)$ between $x = 0$ and $x = a$.

The following symbols may be useful: `a, pi`

Part B $\sin\left(\frac{\pi x}{a}\right)$ between $-a$ and a

Find the mean value of $\sin\left(\frac{\pi x}{a}\right)$ between $x = -a$ and $x = a$.

The following symbols may be useful: `a, pi`

Part C $\sin^2\left(\frac{\pi x}{a}\right)$ between 0 and a

Find the mean value of $\sin^2\left(\frac{\pi x}{a}\right)$ between $x = 0$ and $x = a$.

The following symbols may be useful: `a, pi`

Part D $\sin^2\left(\frac{\pi x}{a}\right)$ between $-a$ and a

Find the mean value of $\sin^2\left(\frac{\pi x}{a}\right)$ between $x = -a$ and $x = a$.

The following symbols may be useful: `a, pi`

Part E $x \sin^2\left(\frac{\pi x}{2a}\right)$ between 0 and $2a$

Find the mean value of $x \sin^2\left(\frac{\pi x}{2a}\right)$ between $x = 0$ and $x = 2a$.

The following symbols may be useful: `a, pi`

Part F $x \sin^2\left(\frac{\pi x}{2a}\right)$ between $-a$ and a

Find the mean value of $x \sin^2\left(\frac{\pi x}{2a}\right)$ between $x = -a$ and $x = a$.

The following symbols may be useful: `a, pi`

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Calculus: Inverse Trigonometry

Part A Derivative of $\arcsin x$

Find the derivative of $\arcsin x$

The following symbols may be useful: x

Part B Implicit differentiation

Given that

$$\arcsin 2x + \arcsin y = \frac{1}{2}\pi$$

find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \frac{1}{4}$.

Adapted with permission from UCLES, A Level, January 2009 , Paper 4726, Question .

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Integration using inverse trig 1

A Level C C C Further A P P P

Find the following integrals.

Part A $\frac{1}{\sqrt{1-x^2}}$

Find the indefinite integral

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}}$$

using a suitable trigonometric substitution.

The following symbols may be useful: `C, arccos(), arcsin(), arctan(), c, k, x`

Part B $\frac{5}{\sqrt{9-x^2}}$

Find the integral

$$\int_{\frac{3}{\sqrt{2}}}^{\frac{3\sqrt{3}}{2}} \frac{5\mathrm{d}x}{\sqrt{9-x^2}}$$

giving your answer in exact form.

The following symbols may be useful: `pi`

Part C $\frac{2}{\sqrt{1-2x^2}}$

Find the integral

$$\int_0^{\frac{1}{2}} \frac{2\mathrm{d}x}{\sqrt{1-2x^2}}$$

giving your answer in exact form.

The following symbols may be useful: `pi`

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Integration using inverse trig 2

A Level Further A

C C C P P P

Find the following integrals.

Part A $\frac{1}{(1+x^2)}$

Find the indefinite integral

$$\int \frac{\mathrm{d}x}{1+x^2}$$

using a suitable trigonometric substitution.

The following symbols may be useful: `C, arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), c, k, x`

Part B $\frac{4}{(4x^2+9)}$

Find the integral

$$\int_0^{\infty} \frac{4\mathrm{d}x}{4x^2+9}$$

giving your answer in exact form.

The following symbols may be useful: `pi`

Part C $\frac{3}{x\sqrt{4x^2-1}}$

Find the integral

$$\int_1^{\infty} \frac{3\mathrm{d}x}{x\sqrt{4x^2-1}}$$

giving your answer in exact form.

The following symbols may be useful: `pi`

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Polar Coordinates: General

Further A

A curve C has the cartesian equation $x^3 + y^3 = axy$, where $x \geqslant 0, y \geqslant 0$ and $a > 0$.

Part A Polar equation of C

Express the polar equation of C in the form $r = \mathrm{f}(\theta)$.

The following symbols may be useful: `a, cos(), r, sin(), tan(), theta`

Part B Range of θ

$0 \leqslant \theta \leqslant \beta$.

Find β.

The following symbols may be useful: `pi`

Part C Line of symmetry

The line $\theta = \alpha$ is a line of symmetry of C.

Find and simplify an expression for $r = f\left(\frac{1}{2}\pi - \theta\right)$.

The following symbols may be useful: `a, cos(), r, sin(), tan(), theta`

Hence find the value of α.

The following symbols may be useful: `pi`

Part D $\theta = \frac{1}{4}\pi$

Find the value of r when $\theta = \frac{1}{4}\pi$.

The following symbols may be useful: a

Part E **Sketch of C**

Sketch the curve C.

Which curve in **Figure 1** is closest to your sketch?

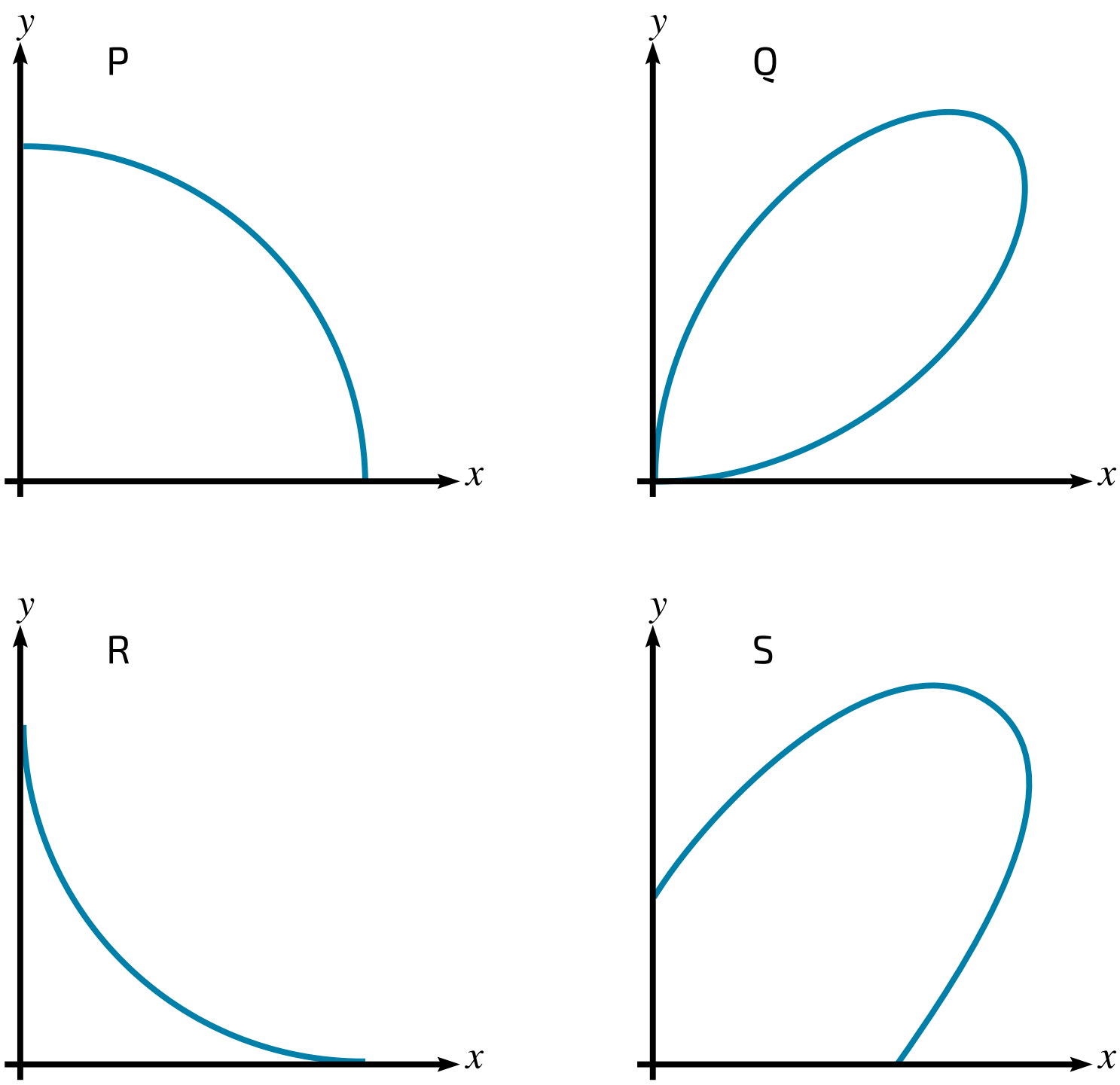

Figure 1: Four curves.

- Curve P
- Curve Q
- Curve R
- Curve S

Adapted with permission from UCLES, A Level, June 11, Paper 4726, Question 4.

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STEM SMART Double Maths 30 - Inverse Trigonometric Calculus & Polar Coordinates

Polar Coordinates: General

Further A

A curve has polar equation $r = \cos\theta \sin 2\theta$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

Part A Maximum value of r

Find the maximum value of r.

Part B Cartesian equation of the curve

Find a cartesian equation of the curve.

Give your answer in the form $(x^2 + y^2)^2 = f(x, y)$

The following symbols may be useful: `x, y`

Adapted with permission from UCLES, A Level, June 2012, Paper 4726, Question 2.

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Polar Coordinates: Area

Further A

The equation of a curve, in polar coordinates, is

$$r = 2\cos 2\theta \quad (-\pi < \theta \leqslant \pi).$$

Part A Tangents at the poles

Find the values of θ which give the directions of the tangents at the pole.

Give your answers in order of lowest to highest (most negative to most positive).

Find the lowest value, θ_1.

The following symbols may be useful: `pi`

Find the second-lowest value, θ_2.

The following symbols may be useful: `pi`

Find the third-lowest value, θ_3.

The following symbols may be useful: `pi`

Find the highest value, θ_4.

The following symbols may be useful: `pi`

Part B Area enclosed by one loop

A loop of the curve is shown in the diagram.

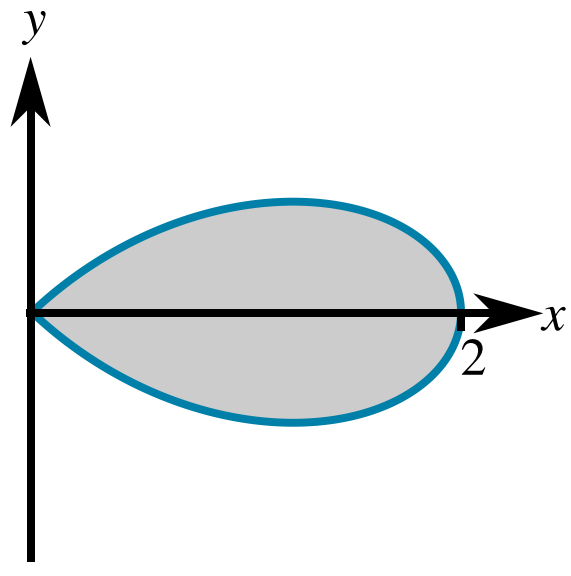

Figure 1: One loop of $r = 2\cos 2\theta$.

Find the exact value of the area of the region enclosed by the loop.

The following symbols may be useful: `pi`
