Functions and Algebra 5ii

A Level

The function f is defined by

$$f(x) = \frac{1}{\sqrt{x}} + 2, \quad x > 0.$$

The function g is defined for all real values of x by

$$g(x) = 10 - (x + 3)^2.$$

Part A   Range of f

State the range of $f(x)$ as an inequality.

The following symbols may be useful: `<, <=, >, >=, f(x), x, y`

Part B   Inverse of f

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, x`

Part C Range of g
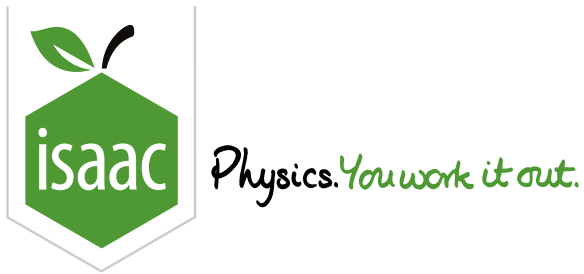
State the range of $g(x)$ as an inequality.

The following symbols may be useful: `<, <=, >, >=, g(x), x, y`

Part D Compound function of g

Find the value of $g(g(-1))$.

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Functions and Algebra 3i

A Level

The functions f and g are defined for all real values of x by

$$f(x) = \left|2x + a\right| + 3a \quad \text{and} \quad g(x) = 5x - 4a,$$

where a is a positive constant.

Part A Range

Find the range of $f(x)$.

What form does your answer take? Choose from the list below, where c and d are constants and $c < d$, and then find c and/or d.

○ $y < c$

○ $y \leq c$

○ $y > c$

  $y \geq c$

○ $c < y < d$

○ $c \leq y \leq d$

○ $y < c$ or $y > d$

○ $y \leq c$ or $y \geq d$

Write down the value of c (in terms of a).

The following symbols may be useful: `a, c`

Write down the value of d (or if your chosen form has no d, write "n").

The following symbols may be useful: `a, d, n`

Part A Range

Part B   Inverse function of $f(x)$

State why $f(x)$ has no inverse.

Easier question?

Part C   Inverse function of $g(x)$

Find an expression for $g^{-1}(x)$.

The following symbols may be useful: `a, g, x`

Part D   Solve for x

Solve for x the equation $g\big(f(x)\big) = 31a$.

Give the value of x furthest from 0.

The following symbols may be useful: `a, x`

Give the value of x that is closest to 0.

The following symbols may be useful: `a, x`

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Home   Gameboard   Maths   Functions: Graphs and Inverse Functions 2i

Functions: Graphs and Inverse Functions 2i

A Level

The functions f and g are defined for all real values of x by

$$f(x) = 3x - 2$$

$$g(x) = 3x + 7$$

Part A   Intersection of $f\left(g(x)\right)$ and x-axis

Find the exact x coordinate of the point at which the graph of $y = f\left(g(x)\right)$ meets the x-axis.

What is the x-coordinate?

The following symbols may be useful: x

Part B   Intersection of $y = g(x)$ and $y = g^{-1}(x)$

Find the exact coordinates of the point at which the graph of $y = g(x)$ meets the graph of $y = g^{-1}(x)$
.

What is the x-coordinate?

The following symbols may be useful: x

What is the y-coordinate?

The following symbols may be useful: y

Part C   Intersection of $y = \left| f(x) \right|$ and $y = \left| g(x) \right|$

Find the exact coordinates of the exact point where the graph of $y = \left| f(x) \right|$ meets the graph of $y = \left| g(x) \right|$.

What is the x-coordinate?

The following symbols may be useful: x

What is the y-coordinate?

The following symbols may be useful: y

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Part C   Intersection of $y = \left| f(x) \right|$ and $y = \left| g(x) \right|$

Inverse Quadratic Function

A Level

Figure 1 shows the graph of $y = f(x)$, where

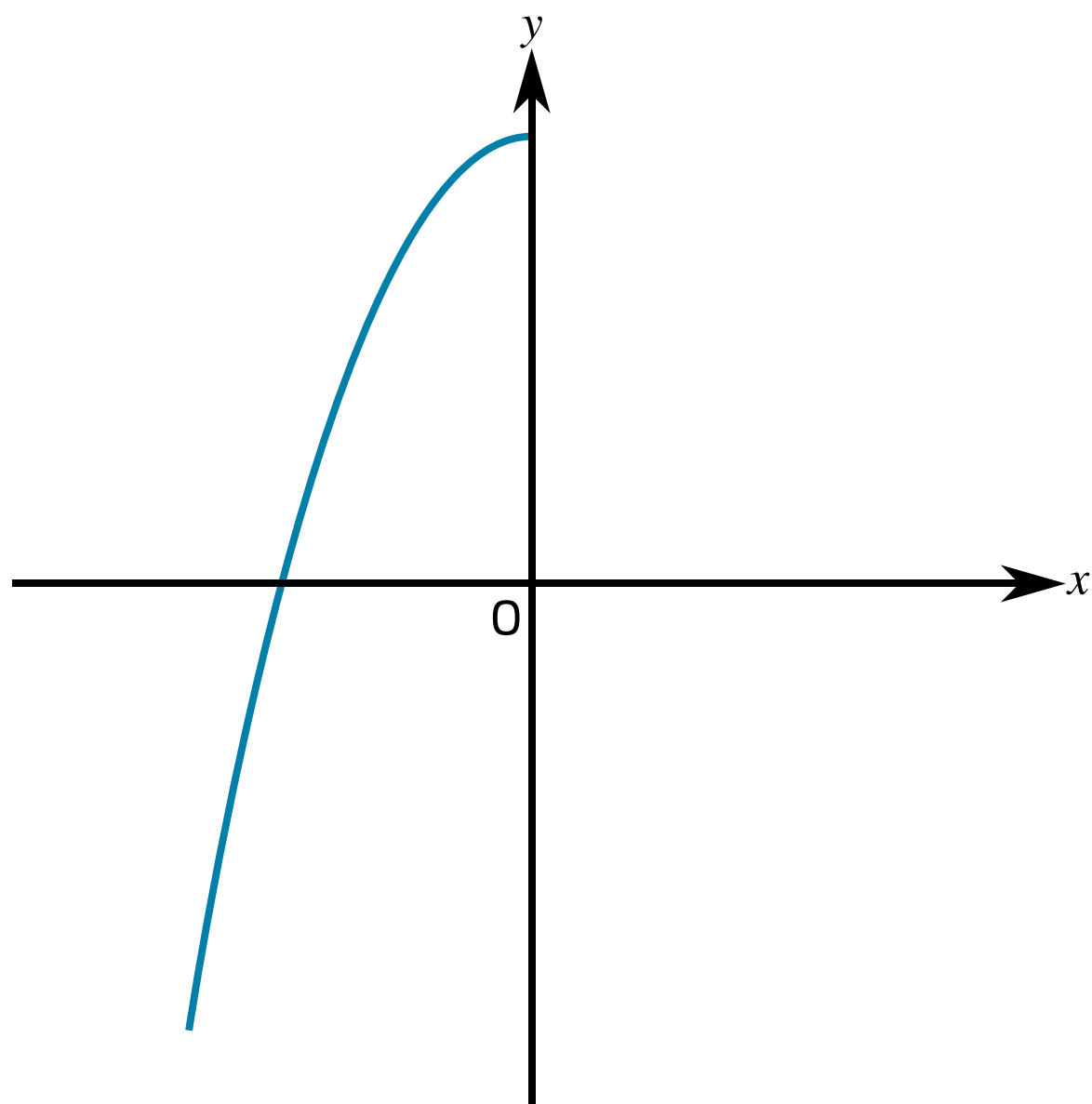$$f(x) = 2 - x^2, \quad x \leq 0$$

Figure 1: The graph of $y = f(x)$, for $x \leq 0$.

Part A $\quad f^2(-3)$
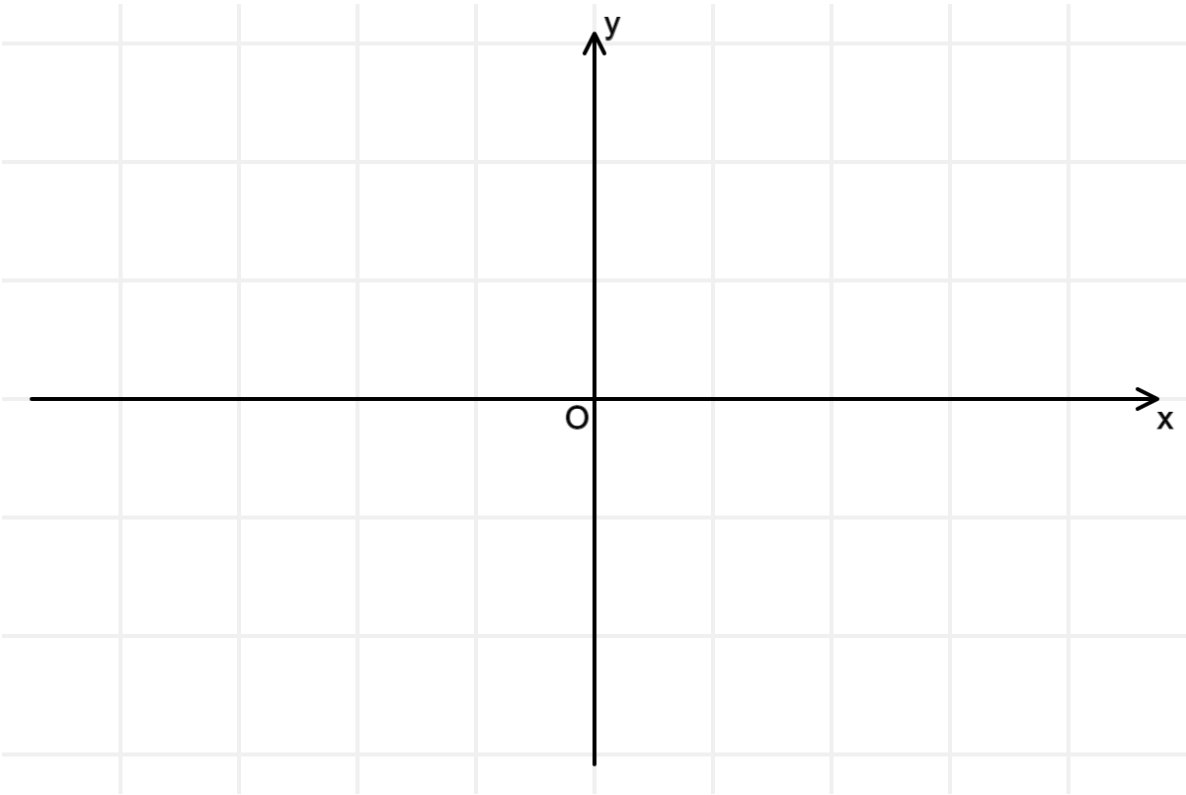
Evaluate $f^2(-3)$.

Part B　　$f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, x, y`

Part C　　Graph of $f^{-1}(x)$

Sketch the graph of $y = f^{-1}(x)$.

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Function Types and Inverses

A Level

Figure 1 shows five different graphs, A, B, C, D and E, each for values of x such that $-a \le x \le a$ where a is a constant.
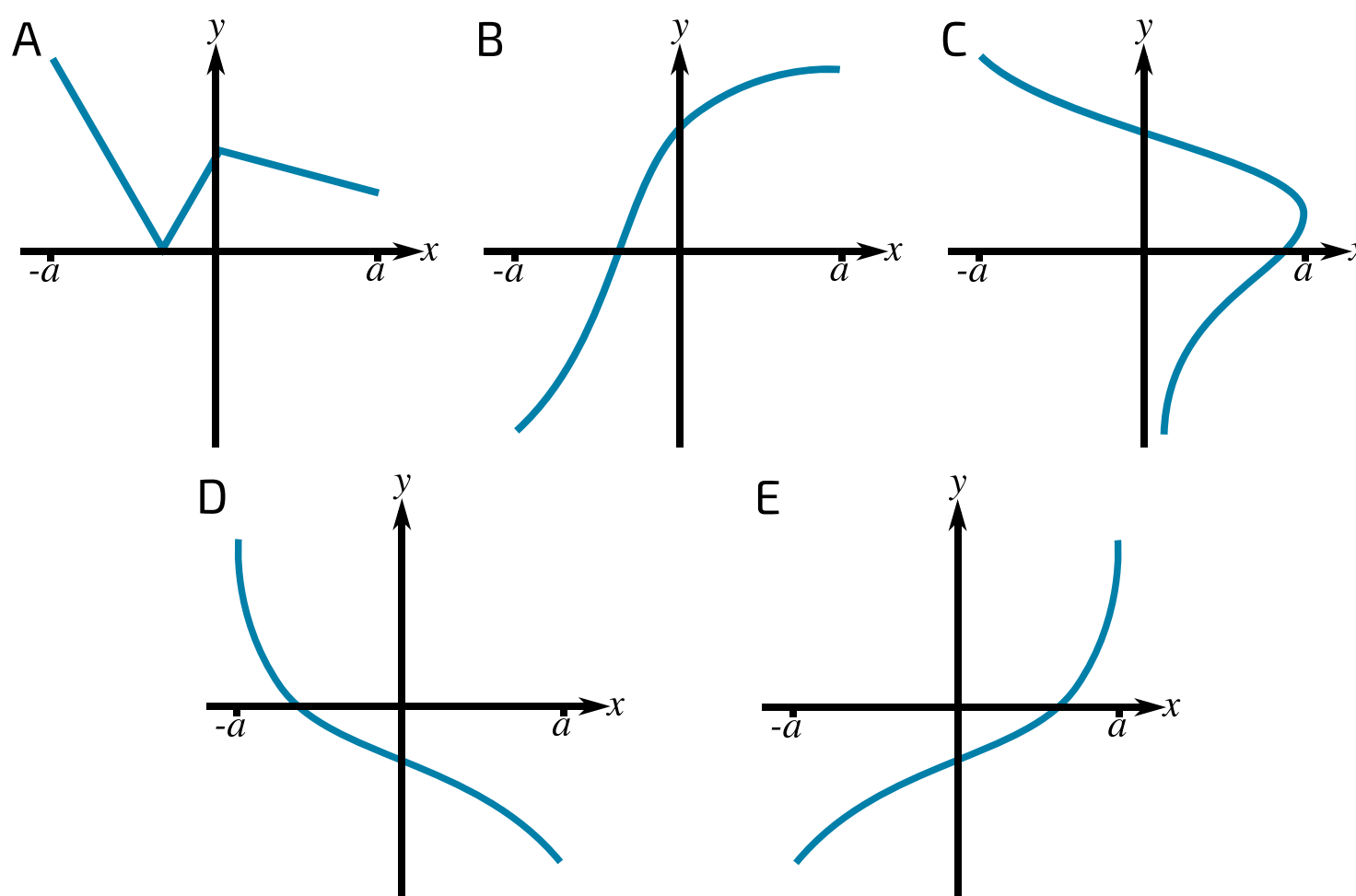

Figure 1: The set of five graphs, labelled A, B, C, D and E

Part A   Function

Which diagram does not show the graph of a function?

- ◯ A
- ◯ B
- ◯ C
- ◯ D
- ◯ E

Part B   One-to-one Function

Which diagram shows the graph of a function that is not one-to-one?

- ○ A

- ○ B

- ○ C

- ○ D

- ○ E

Part C   Inverses

It is given that two of the diagrams illustrate functions that are inverses of each other. Identify one of these two diagrams.

- ○ A

- ○ B

- ○ C

- ○ D

- ○ E

Part D   Sketch

The graph in E has equation $y = f(x)$. Sketch the graph of $y = \left| f(x) \right|$.

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.

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Part D   Sketch
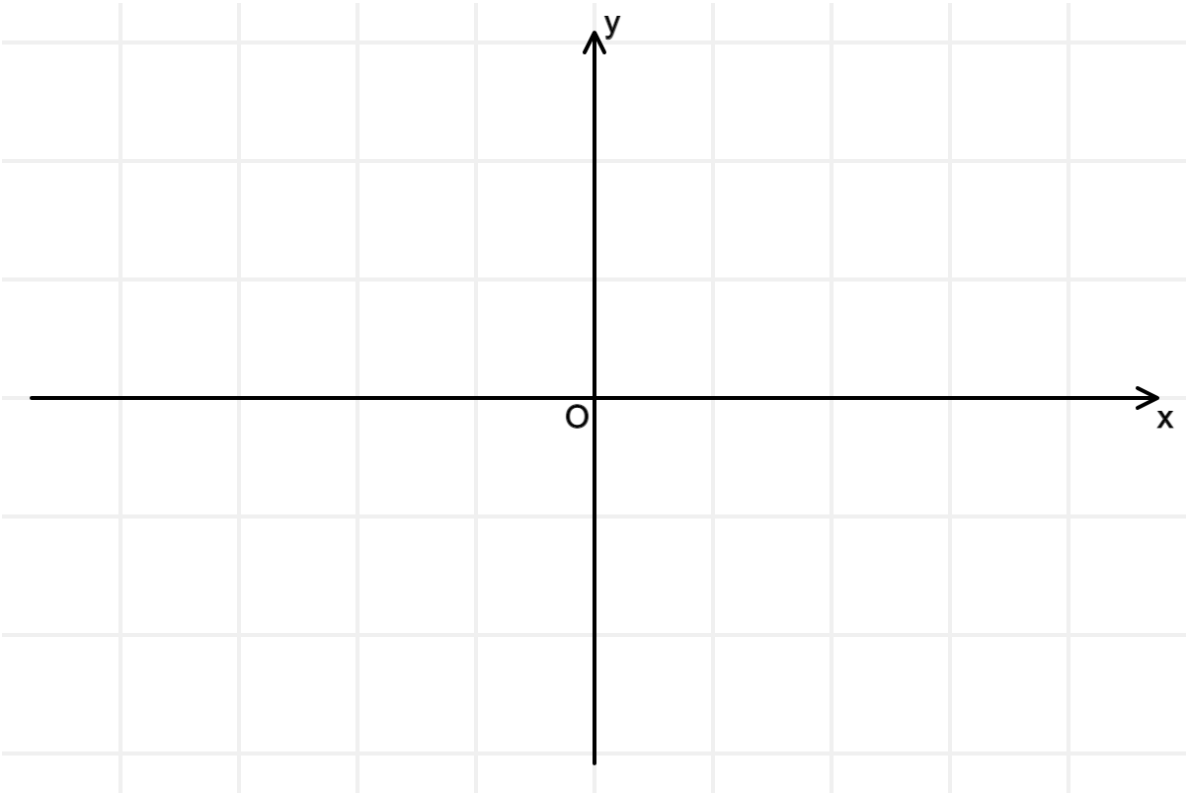
Combined Transformations

A Level

P P P

The function f is defined by $f(x) = \sqrt{mx + 7} - 4$, where $x \geqslant -\frac{7}{m}$ and m is a positive constant. **Figure 1** shows the curve $y = f(x)$.
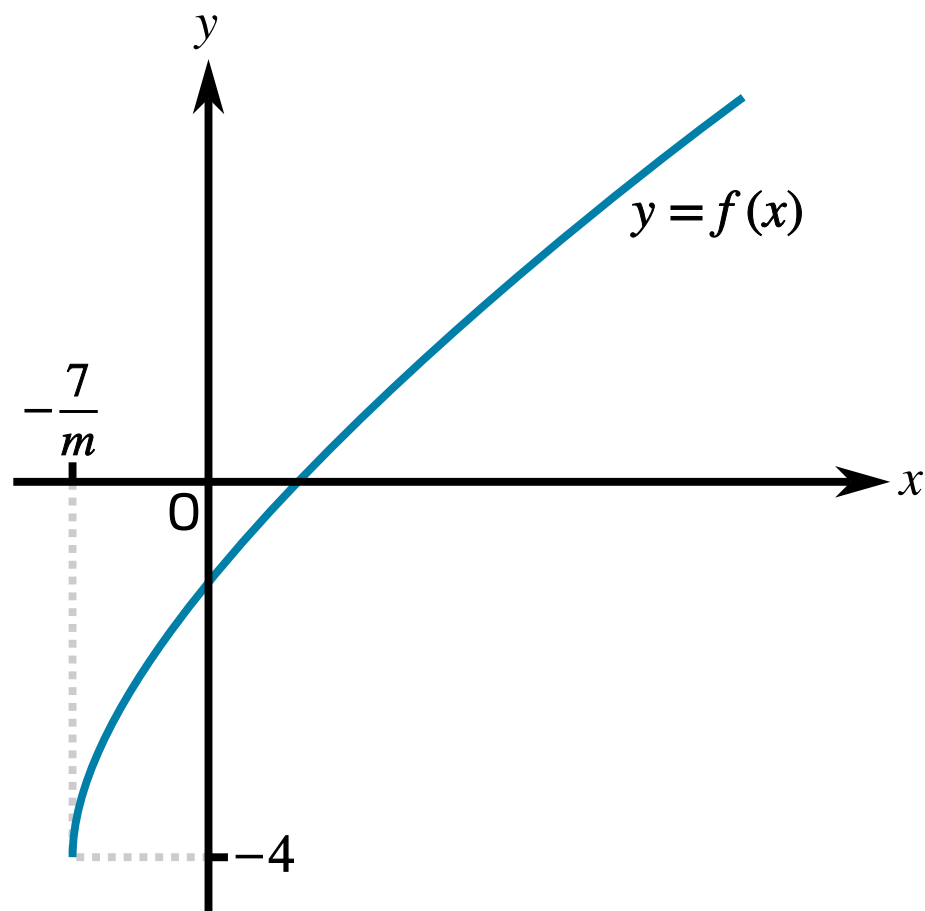

Figure 1: The curve $y = f(x)$

Part A   Translation of the curve $y = \sqrt{x}$

A sequence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations.

Available items

Translate the curve 4 units in the negative y direction.

Translate the curve 7 units in the positive x direction.

Stretch the curve in the y direction by a factor of $\frac{1}{m}$.

Translate the curve 4 units in the negative x direction.

Translate the curve 4 units in the positive y direction.

Stretch the curve in the x direction by a factor of m.

Stretch the curve in the x direction by a factor of $\frac{1}{m}$.

Translate the curve 7 units in the negative x direction.

Translate the curve 7 units in the negative y direction.

Part B $f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, m, x`

Part C   Values of m

It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Thus it can be deduced that neither curve meets the line $y = x$. Hence determine the set of possible values of m, and give the upper bound in the form $m < a$ or $m \leq a$.

The following symbols may be useful: `<, <=, >, >=, m`

Give the lower bound in the form $m > a$ or $m \geq a$.

The following symbols may be useful: `<, <=, >, >=, m`

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Part C   Values of m

Curve Sketching and Combined Transformations 3i

A Level

The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x)$$

where k is a positive constant. **Figure 1** shows the curve with equation $y = f(x)$.
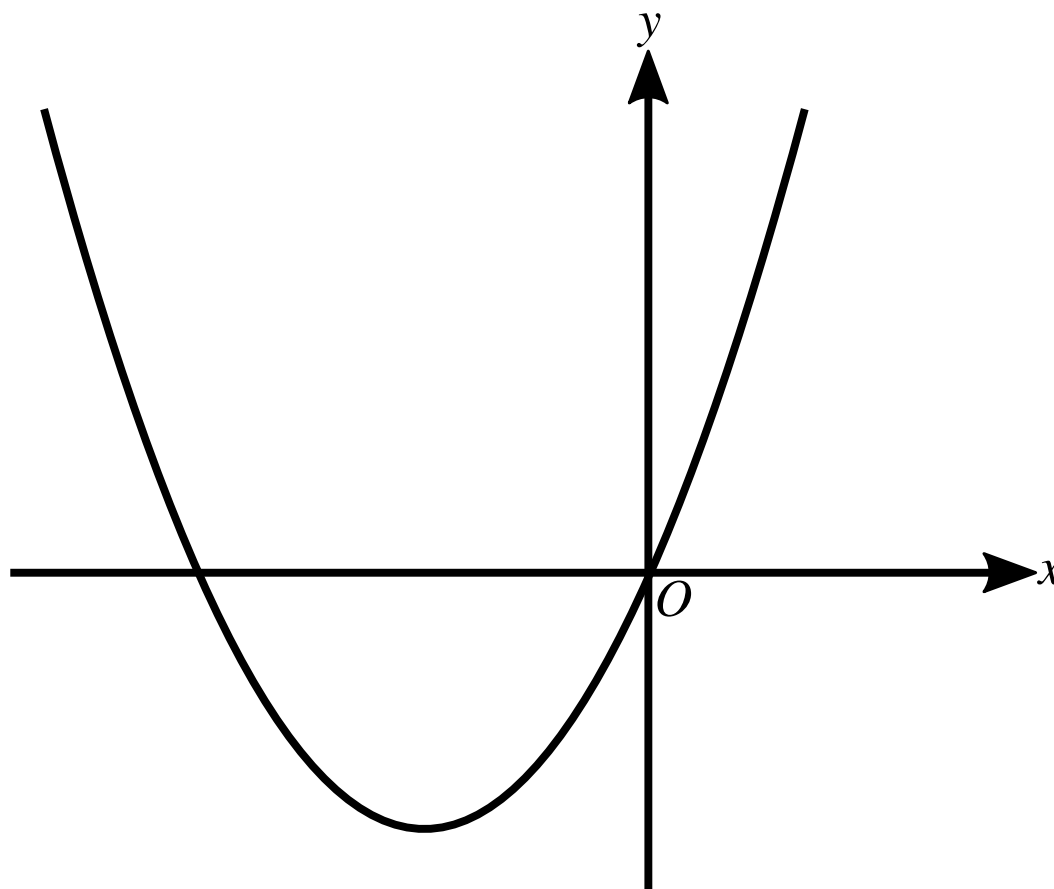

Figure 1: The graph of $y = f(x)$

Part A　Transformations

The curve $y = x^2$ can be transformed to the curve $y = f(x)$ by the following sequence of transformations

$$\text{a translation parallel to the } x\text{-axis,}$$

$$\text{a translation parallel to the } y\text{-axis,}$$

$$\text{a stretch.}$$

Give details, in terms of k where appropriate, of these transformations.

Give the number of units that the curve is translated in the x direction, assuming right to be positive.

The following symbols may be useful: k

Give the number of units that the curve is translated in the y direction, assuming up to be positive.

The following symbols may be useful: k

Give the stretch factor of the transformation.

The following symbols may be useful: k

Part B　Range

Find the range of $f(x)$ as a single inequality in terms of k.

The following symbols may be useful: <, <=, >, >=, f, k, x, y

Part C $\left|f(x)\right| = 20$

It is given that there are three distinct values of x which satisfy the equation $\left|f(x)\right| = 20$. Find the value of k and determine exactly the three values of x which satisfy the equation in this case.

State the value of k.

The following symbols may be useful: k

Give the rational value of x which satisfies this equation.

The following symbols may be useful: x

Give one of the irrational solutions for x in its simplest exact form.

The following symbols may be useful: x

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Part C $\left|f(x)\right| = 20$

Modulus 3ii

A Level

P P P

Solve the inequality $\left| 2x - 5 \right| > \left| x + 1 \right|$.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- ○ $x < a$

- ○ $x \leq a$

- ○ $x > a$

- ○ $x \geq a$

- ○ $a < x < b$

- ○ $a \leq x \leq b$

-    $x < a$ or $x > b$

- ○ $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

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Home   Gameboard   Maths   Functions   General Functions   Modulus Functions 2

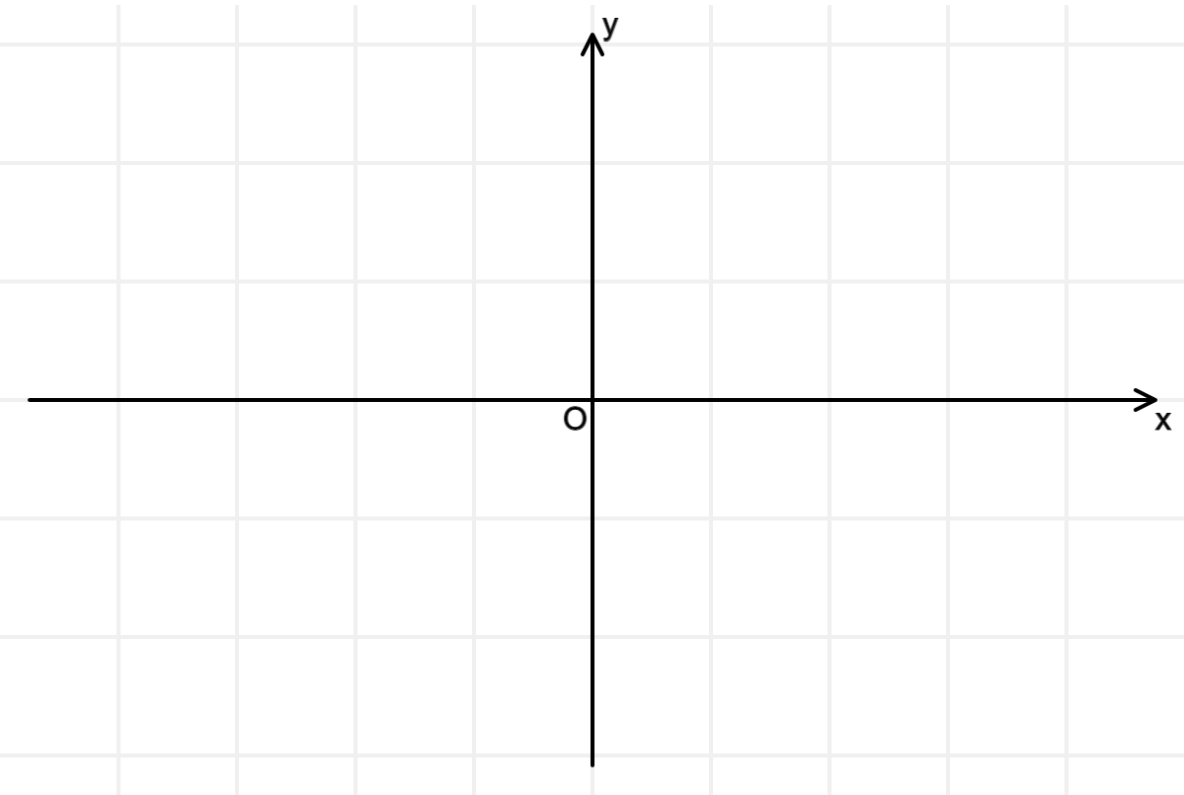# Modulus Functions 2

A Level Further A

Part A   **Divergence of** $y = \left| \frac{1}{x} \right|$

Does the function $y = \left| \dfrac{1}{x} \right|$ diverge anywhere? Where?

The following symbols may be useful: x

Part B   **Graph of** $y = \left| \frac{1}{x} \right|$

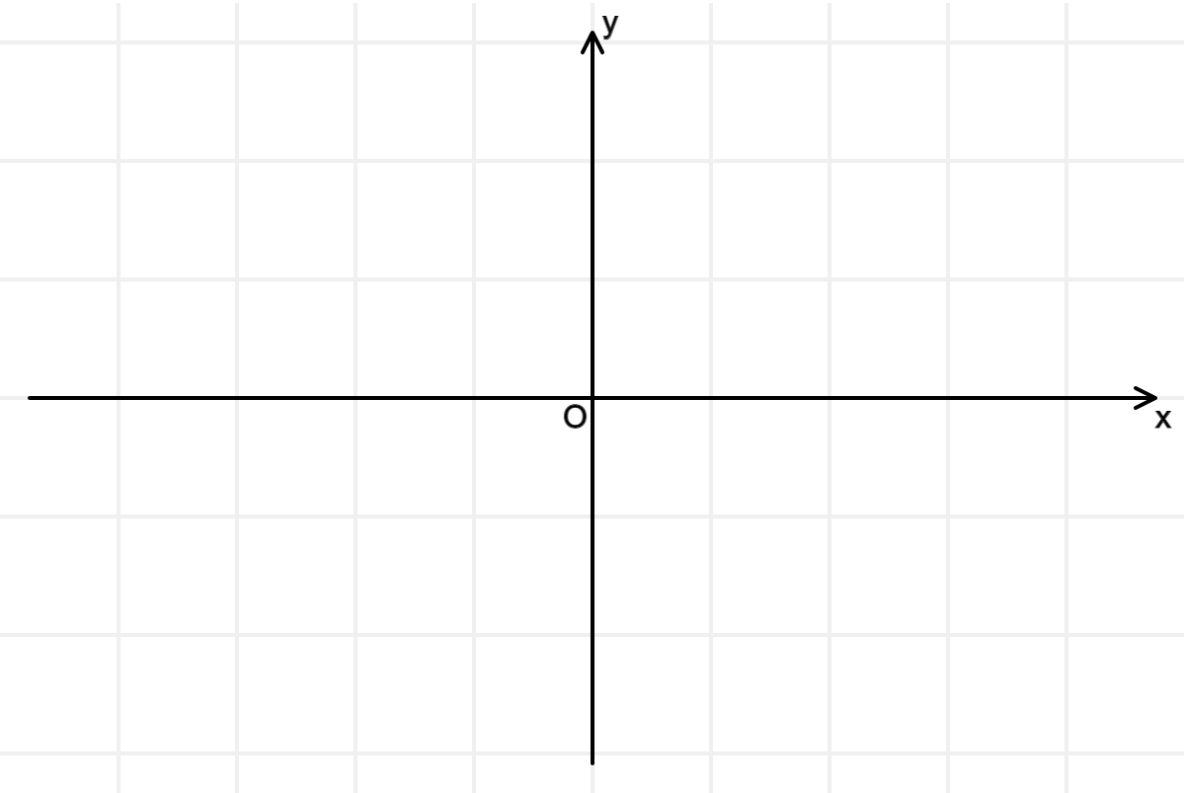Sketch the graph of $y = \left| \dfrac{1}{x} \right|$.

Part C   Divergence of $y = \left| \frac{1}{x^2-4} \right|$

Does the function $y = \left| \dfrac{1}{x^2 - 4} \right|$ diverge anywhere? Where?

The following symbols may be useful: x, ±

Part D   Graph of $y = \left| \frac{1}{x^2-4} \right|$

Sketch the graph of $y = \left| \dfrac{1}{x^2 - 4} \right|$.

Part E   Solve equation graphically

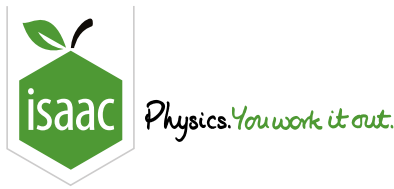Solve the equation $|x| = \left| \dfrac{1}{x} \right|$ graphically and give the solution as a single expression.

The following symbols may be useful: x, ±

Physics. *You work it out.*

Home   Gameboard   Maths   Functions   Graph Sketching   Sketching a Cubic Modulus Function

Sketching a Cubic Modulus Function

A Level

C

The function f is defined by $f(x) = 2x^3 - x^2 - 4x - 4$.

Part A   Quotient

Find the quotient when $f(x)$ is divided by $x - 2$.

The following symbols may be useful: x

Part B   First stationary point of $f(x)$

Find the co-ordinates and nature of the stationary point of $f(x)$ with the larger x-coordinate.

The stationary point ([_____] , [_____]) is a [_____] .

Items:

| -3 | -7 | -2 | -4 | 92 | minimum point | -55 | 29 | -16 | 3 | -132 | -1 | 0 | 2 | 1 | maximum point | point of inflection | 4 |

Part C   Second stationary point of $f(x)$

Find the co-ordinates and nature of the stationary point of $f(x)$ with the smaller x-coordinate.

The stationary point ([_____] , [_____]) is a [_____] .

Items:

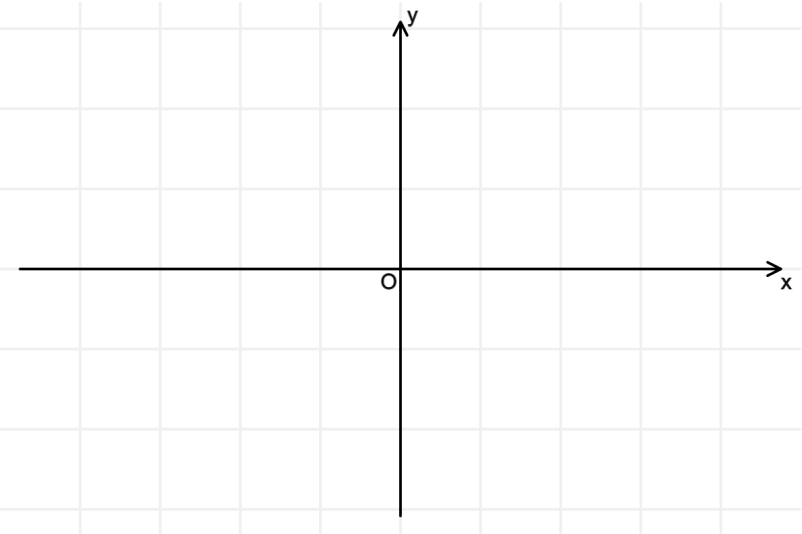| $-\frac{77}{27}$ | maximum point | $-\frac{3}{2}$ | $\frac{64}{27}$ | $\frac{1}{3}$ | -1 | -7 | $-\frac{1}{2}$ | $\frac{3}{2}$ | $-\frac{176}{27}$ | $\frac{145}{27}$ | 1 | minimum point | -7 | $-\frac{2}{3}$ | $-\frac{1}{3}$ | $\frac{1}{2}$ | -6 | $\frac{2}{3}$ | $-\frac{5}{2}$ | point of |

Part D   Sketch of $y = f(x)$

Sketch the graph of $y = f(x)$.

Part E   **Sketch of** $y = |f(x)|$

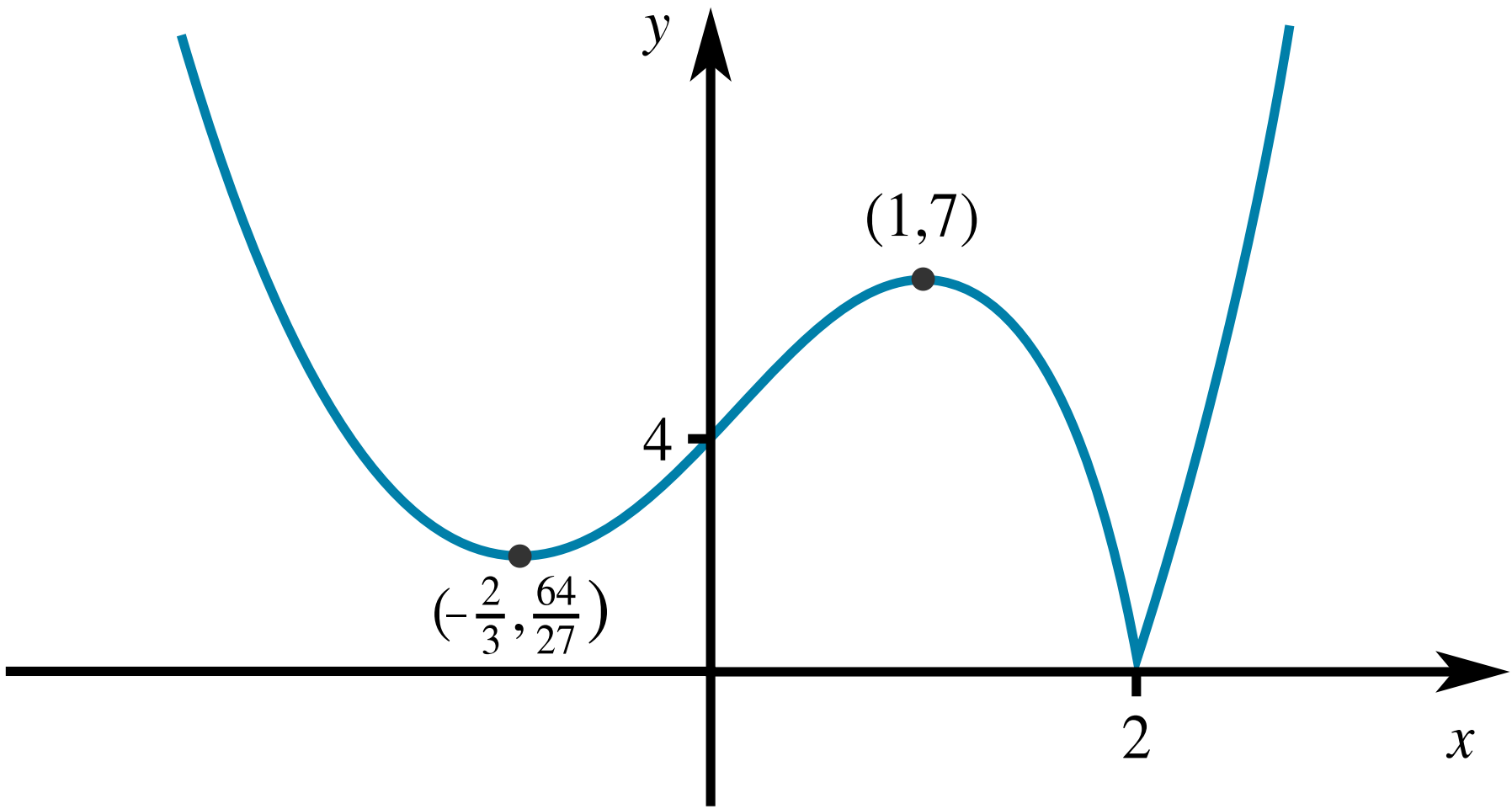Sketch the graph of $y = |f(x)|$, then pick the graph that corresponds to $y = |f(x)|$ from the options below.
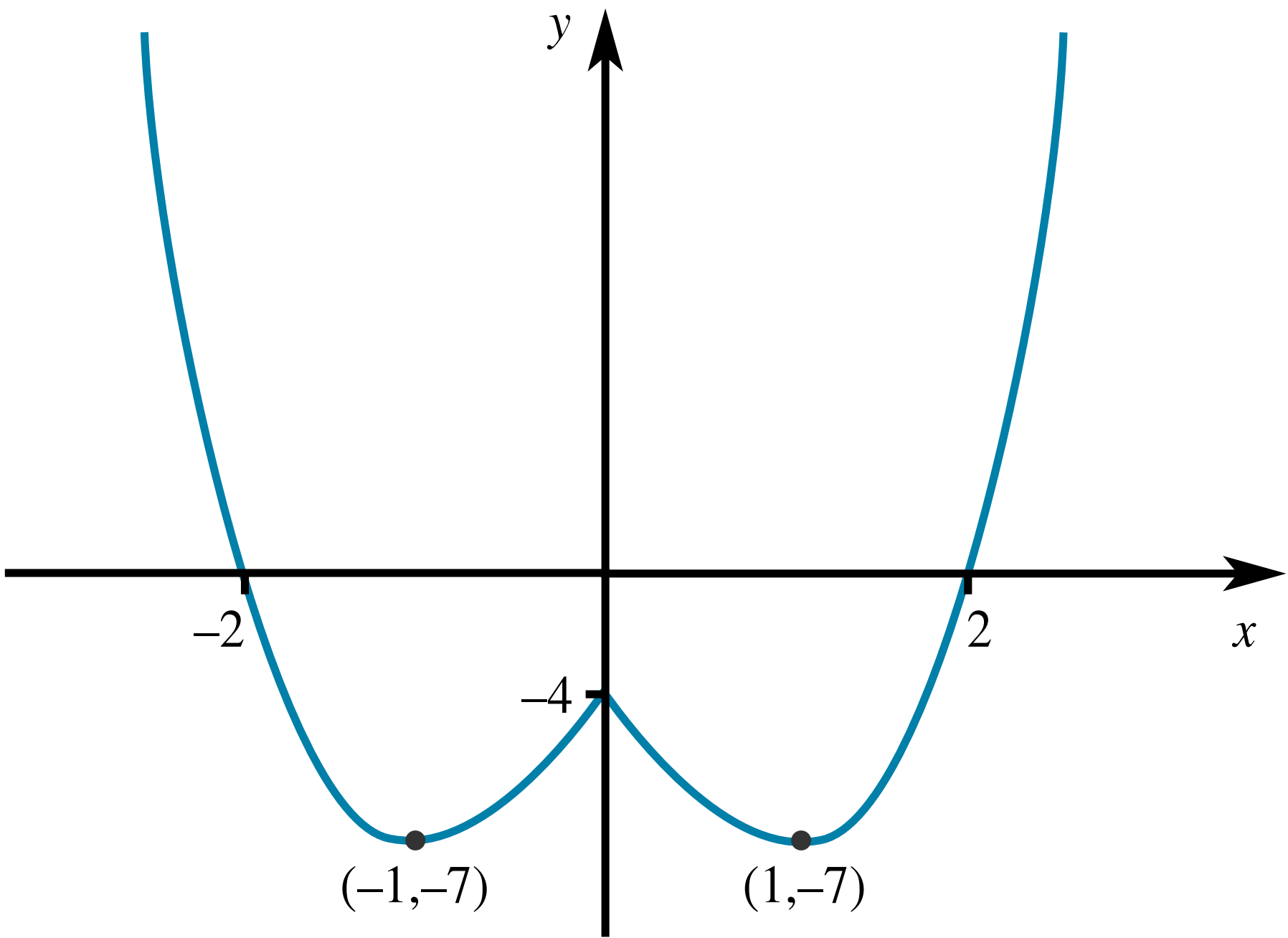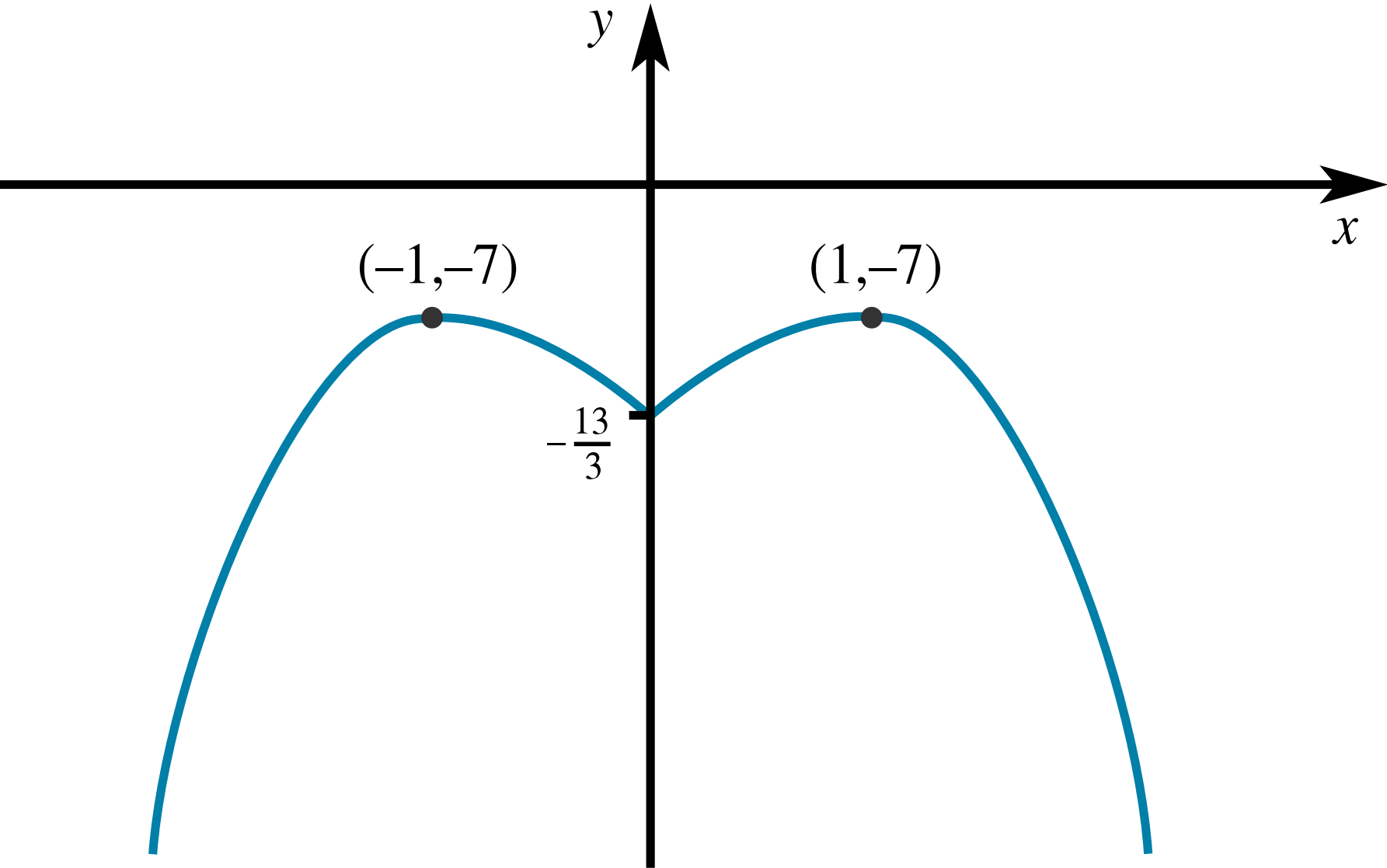

Figure 1: Option (i)

Figure 2: Option (ii)

Figure 3: Option (iii)

○ Option (i)

○ Option (ii)

○ Option (iii)

Part F   **Sketch of** $y = f(|x|)$

Sketch the graph of $y = f(|x|)$.

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.

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