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# Complex Numbers: Manipulations 3i

Further A



The complex number  $2 + i$  is denoted by  $z$ , and the complex conjugate of  $z$  is denoted by  $z^*$ .

## Part A $z^2$

Express  $z^2$  in the form  $x + iy$ , where  $x$  and  $y$  are exact real numbers.

The following symbols may be useful:  $i$

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## Part B $4z - z^2$

Express  $4z - z^2$  in the form  $x + iy$ , where  $x$  and  $y$  are exact real numbers.

The following symbols may be useful:  $i$

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## Part C $zz^*$

Express  $zz^*$  in the form  $x + iy$ , where  $x$  and  $y$  are exact real numbers.

The following symbols may be useful:  $i$

---

## Part D $\frac{z+1}{z-1}$

Express  $\frac{z+1}{z-1}$  in the form  $x + iy$ , where  $x$  and  $y$  are exact real numbers.

The following symbols may be useful:  $i$

---

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# Complex Numbers: Manipulations 1i

Further A

P

P

P

The complex number  $z$  has modulus  $2\sqrt{3}$  and argument  $-\frac{\pi}{3}$ .

Part A    $z$

Find  $z$  in the form  $z = x + iy$ , where  $x$  and  $y$  are exact real numbers.

The following symbols may be useful:  $i$ ,  $z$

Part B    $\frac{1}{(z^* - 5i)^2}$

Find  $\frac{1}{(z^* - 5i)^2}$  in the form  $x + iy$ , where  $x$  and  $y$  are exact real numbers.

The following symbols may be useful:  $i$

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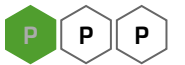


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# Complex Numbers: $x+iy$ and Euler $3i$

Further A



The complex number  $z$  satisfies the equation

$$z + 2iz^* = 12 + 9i$$

## Part A $z$

Find  $z$  in the form  $z = x + iy$ .

The following symbols may be useful:  $i$ ,  $z$

## Part B Modulus-Argument

$z$  can also be expressed in the form

$$z = r(\cos \theta + i \sin \theta)$$

Find  $r$ .

The following symbols may be useful:  $r$

Find  $\theta$  to 3 significant figures in radians.

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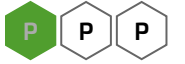


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# Complex Numbers: Equations to Quartics 1ii

Further A



## Part A Square roots

The square roots of the complex number  $5 + 12i$  can be expressed in the form  $x + iy$ .

Give the square root with positive  $x$  and positive  $y$ .

The following symbols may be useful:  $i$

---

Give the square root with negative  $x$  and negative  $y$ .

The following symbols may be useful:  $i$

---

## Part B $(3 - 2i)^2$

Find  $(3 - 2i)^2$  in the form  $x + iy$  where  $x$  and  $y$  are exact.

The following symbols may be useful:  $i$

---

## Part C    Roots of quartic

The answers to the previous part can be used to solve the quartic

$$z^4 - 10z^2 + 169 = 0$$

The roots to the quartic can be expressed in the form  $x + iy$ .

Give the root with positive  $x$  and positive  $y$ .

The following symbols may be useful:  $i$

---

Give the root with positive  $x$  and negative  $y$ .

The following symbols may be useful:  $i$

---

Give the root with negative  $x$  and positive  $y$ .

The following symbols may be useful:  $i$

---

Give the root with negative  $x$  and negative  $y$ .

The following symbols may be useful:  $i$

---

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# Applying Complex Numbers 2ii

Further A



One root of the cubic equation  $x^3 + bx^2 + cx - 15 = 0$ , where  $b$  and  $c$  are real, is the complex number  $2 + i$ .

## Part A Complex root

Find the other complex root in the form  $x + iy$ .

The following symbols may be useful:  $i$

---

## Part B Real root

Find the real root.

---

## Part C $b$

Find  $b$ .

The following symbols may be useful:  $b$

---

## Part D $c$

Find  $c$ .

The following symbols may be useful:  $c$

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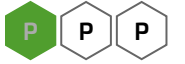


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# Complex Numbers: Equations to Quartics 1i

Further A



One root of the quadratic equation  $z^2 + ax + b = 0$ , where  $a$  and  $b$  are real, is  $16 - 30i$ .

## Part A Other root

Give the other root in the form  $x + iy$ .

The following symbols may be useful:  $i$

---

## Part B $a$ and $b$

Find the value of  $a$

The following symbols may be useful:  $a$

---

Find the value of  $b$ .

The following symbols may be useful:  $b$

---

## Part C    Quartic

The quartic equation  $z^4 + az^2 + b = 0$  has roots in the form  $x + iy$ .

Give the root with positive  $x$  and positive  $y$ .

The following symbols may be useful:  $i$

---

Give the root with positive  $x$  and negative  $y$ .

The following symbols may be useful:  $i$

---

Give the root with negative  $x$  and positive  $y$ .

The following symbols may be useful:  $i$

---

Give the root with negative  $x$  and negative  $y$ .

The following symbols may be useful:  $i$

---

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# Argand Diagrams and Simple Loci 2i

Further A

P

P

P

The complex number  $a$  is denoted by  $1 + i\sqrt{3}$ .

Part A    $a$

Find the value of  $|a|$ .

Find  $\arg a$  in exact form.

The following symbols may be useful:  $\pi$

Part B   Loci

Sketch the loci given by  $|z - a| = |a|$  and  $\arg(z - a) = \frac{1}{2}\pi$  on a single Argand diagram.

Easier question?

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# Argand Diagrams: Using Loci 2i

Further A

P

P

P

The loci  $C_1$  and  $C_2$  are given by

$$|z| = |z - 4i|$$

and

$$\arg z = \frac{\pi}{6}$$

respectively.

**Part A**   Loci of  $C_1$  and  $C_2$

Sketch the loci of  $C_1$  and  $C_2$  on a single Argand diagram.

Easier question?

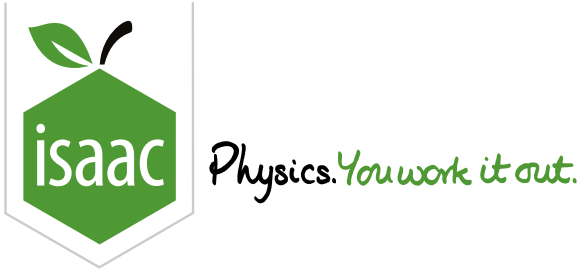
**Part B**   Intersection

Hence find, in the form  $x + iy$ , the complex number represented by the point of intersection of  $C_1$  and  $C_2$ . Give your answer as in exact form.

The following symbols may be useful:  $i$

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Argand Diagrams: Solving Inequalities 1ii

Further A



The loci  $L_1$  and  $L_2$  are given by

$$|z| = 2$$

and

$$\arg(z - 3 - i) = \pi$$

respectively.

Part A    Equation of  $L_1$

Give the equation of  $L_1$  in the form  $(x - a)^2 + (y - b)^2 = c^2$ .

The following symbols may be useful:  $x$ ,  $y$

Part B    Loci

Sketch  $L_1$  and  $L_2$  on a single Argand diagram.

More practice questions?

Part C    Inequalities

Indicate, by shading, the region of the Argand diagram for which

$$|z| \leq 2 \text{ and } 0 \leq \arg(z - 3 - i) \leq \pi$$

More practice questions?

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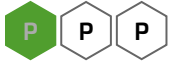


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# Argand Diagrams: Solving Inequalities 4ii

Further A



The loci  $L_1$  and  $L_2$  are given by

$$|z - 3 + 4i| = 5$$

and

$$|z| = |z - 6|$$

respectively.

## Part A Equation of $L_1$

Give the equation of  $L_1$  in the form  $(x - a)^2 + (y - b)^2 = c^2$ .

The following symbols may be useful:  $x$ ,  $y$

## Part B Inequalities

Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 + 4i| \leq 5 \text{ and } |z| \geq |z - 6|$$

**More practice questions?**

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