Differentiation: Products 4ii

A Level

P P P

Differentiate with respect to x, simplifying your answers where possible.

Part A $\sin x \tan x$

Differentiate $\sin x \tan x$.

The following symbols may be useful: x

Part B $x^2(x+1)^6$

Differentiate $x^2(x+1)^6$.

The following symbols may be useful: x

Used with permission from UCLES A-level Maths papers, 2003-2017.

Differentiation: Products 2ii

A Level

P P P

Given that $y = 4x^2 \ln x$, answer the following.

Part A First Derivative

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part B Second Derivative

Find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, when $x = e^2$.

The following symbols may be useful: `Derivative(y , x , x), ln(), log(), x, y`

Used with permission from UCLES A-level Maths papers, 2003-2017.

Gameboard:

STEM SMART Single Maths 30 - The Product Rule & Implicit Differentiation

Differentiation: Quotients 2ii

A Level

Differentiate with respect to x, simplifying your answers where possible.

Part A Differentiation (a)

$$y = \frac{\ln x}{x}$$

The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part B Differentiation (b)

$$y = \frac{x^2}{\ln x}$$

The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part C Differentiation (c)

Determine the exact x-coordinate of the stationary point of the curve $y = \frac{x^2}{\ln(x)}$.

The following symbols may be useful: `e, ln(), x`

Used with permission from UCLES A-level Maths papers, 2003-2017.

Gameboard:

STEM SMART Single Maths 30 - The Product Rule & Implicit Differentiation

Differentiation: Quotients 3i

A Level

A curve has equation $y = \frac{x^2+4}{x+2}$.

Part A Derivative

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

The following symbols may be useful: `Derivative(y, x), x, y`

Part B Normal

Find the equation of the normal to the curve at the point $(1, \frac{5}{3})$, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

The following symbols may be useful: `x, y`

Gameboard:
STEM SMART Single Maths 30 - The Product Rule & Implicit Differentiation

Differentiation: Products 1i

A Level

Figure 1 shows the curve with equation

$$x = (y+4)\ln(2y+3).$$

The curve crosses the x-axis at A and the y-axis at B.

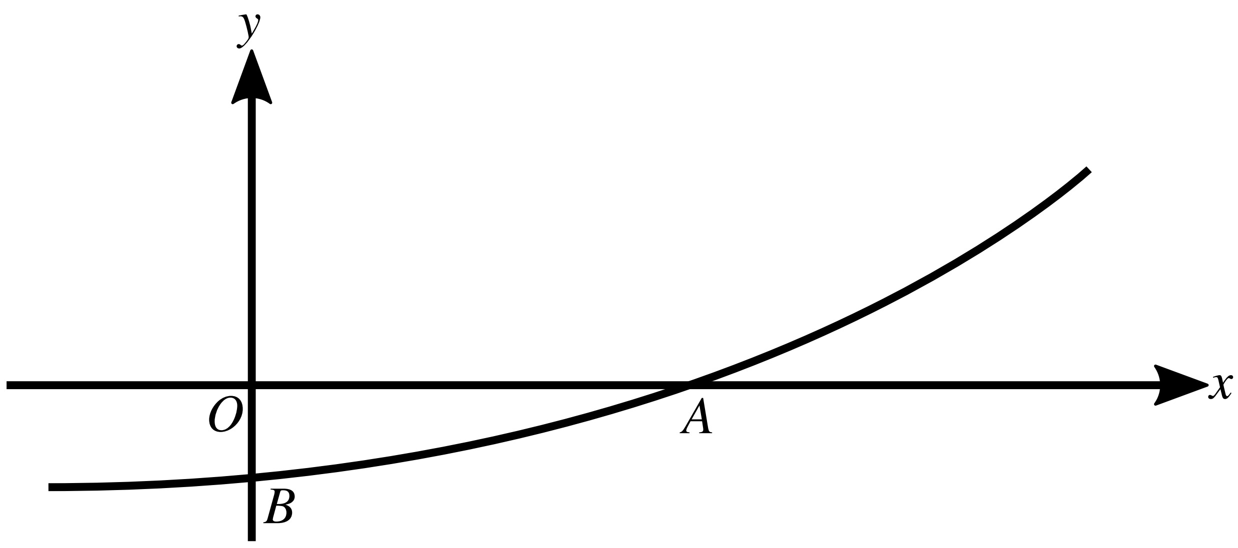

Figure 1: The curve $x = (y+4)\ln(2y+3)$.

Part A Derivative

Find an expression for $\frac{dx}{dy}$ in terms of y.

The following symbols may be useful: `Derivative(x, y), ln(), log(), x, y`

Part B Gradients

Find the gradient of the curve at each of the points A and B, giving each answer correct to two decimal places.

Give the gradient at A.

Give the gradient at B.

Gameboard:

STEM SMART Single Maths 30 - The Product Rule & Implicit Differentiation

Implicit Differentiation 1

A Level Further A

C C C P P P

Part A Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $x^2 + y^2 = r^2$.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $x^2 + y^2 = r^2$, giving your answer as a simple function of x and y.

The following symbols may be useful: x, y

Part B Find gradient of tangent to $x^2 - xy + y^2 = 7$

Consider the curve $x^2 - xy + y^2 = 7$.

(i) Find as a function of x and y the gradient of the tangent to the curve $x^2 - xy + y^2 = 7$.

The following symbols may be useful: x, y

(ii) Using the equation for the gradient of the tangent to the curve $x^2 - xy + y^2 = 7$ from part (a) evaluate the slope at the point (-1, 2).

Created for isaacphysics.org by Julia Riley

Gameboard:

STEM SMART Single Maths 30 - The Product Rule & Implicit Differentiation

Physics. You work it out.

Differentiation: Products 4i

A Level

The equation of a curve has the form $y = \mathrm{e}^{x^2}\left(ax^2 + b\right)$, where a and b are non-zero constants.

Part A First Derivative

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), a, b, e, ln(), log(), x, y`

Part B Second Derivative

Find an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

The following symbols may be useful: `Derivative(y, x, x), a, b, e, ln(), log(), x, y`

Part C a in terms of b

It is given that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ can be expressed in the form $\mathrm{e}^{x^2}(cx^4 + d)$, where c and d are non-zero constants. Find an expression for a in terms of b.

The following symbols may be useful: `a, b`

Used with permission from UCLES A-level Maths papers, 2003-2017.

Gameboard:

STEM SMART Single Maths 30 - The Product Rule & Implicit Differentiation

Differentiation: Quotients 2i

A Level

P P P

Part A Derivative

Given that $y = \frac{4\ln(x)-3}{4\ln(x)+3}$, find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part B Gradient

Give the exact value of the gradient of the curve $y = \frac{4\ln(x)-3}{4\ln(x)+3}$ at the point where it crosses the x-axis.

The following symbols may be useful: `Derivative(y , x), e`

Part C Area

Figure 1 shows part of the curve with equation

$$y = \frac{2}{x^{\frac{1}{2}}(4\ln(x)+3)}.$$

The region shaded in the diagram is bounded by the curve and the lines $x=1$, $x=\mathrm{e}$, and $y=0$. Find the exact value of the integral I where

$$I = \int_1^{\mathrm{e}} \pi y^2 \mathrm{d}x.$$

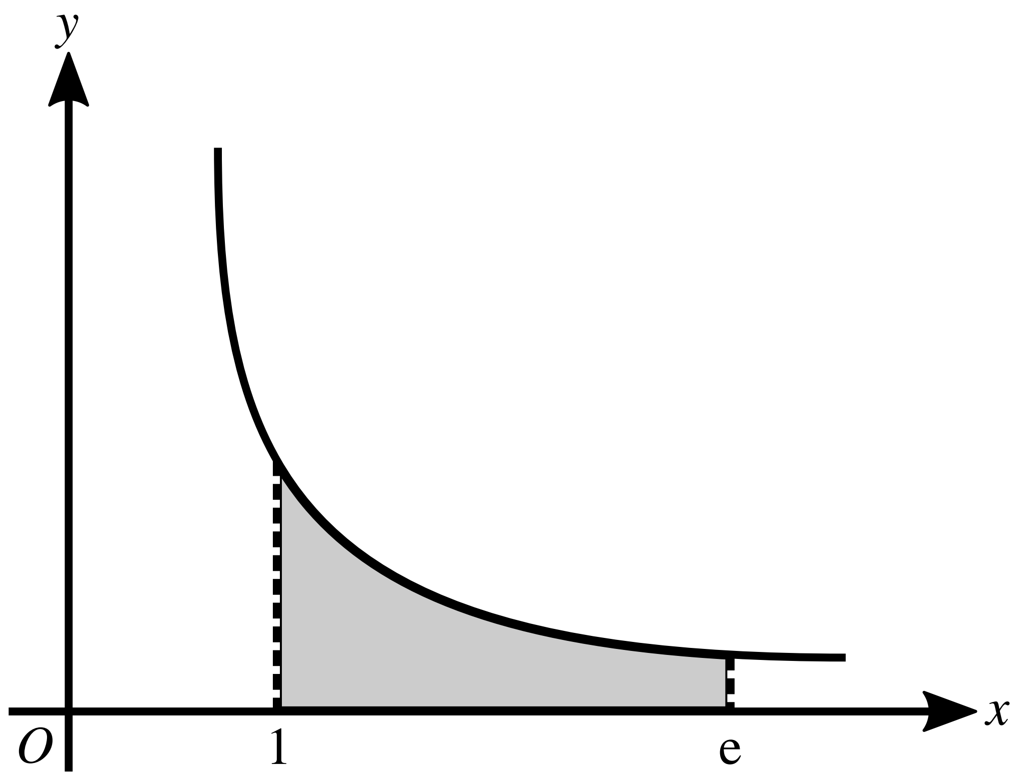

Figure 1: A diagram showing part of the curve with equation $y = \frac{2}{x^{\frac{1}{2}}(4\ln(x)+3)}$.

Give the exact value of I.

The following symbols may be useful: `I, pi`

Gameboard:
STEM SMART Single Maths 30 - The Product Rule & Implicit Differentiation

Calculus: Inverse Trigonometry

Part A Derivative of $\arcsin x$

Find the derivative of $\arcsin x$

The following symbols may be useful: x

Part B Implicit differentiation

Given that

$$\arcsin 2x + \arcsin y = \frac{1}{2}\pi$$

find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \frac{1}{4}$.

Adapted with permission from UCLES, A Level, January 2009 , Paper 4726, Question .

Gameboard:

STEM SMART Single Maths 30 - The Product Rule & Implicit Differentiation

Implicit Differentiation 2

A Level Further A

C C C C C C

Part A Find $\frac{\mathrm{d}p}{\mathrm{d}V}$ for a Van der Waal's gas

One modification to the perfect gas equation of state ($pV = RT$) which takes account of the finite sizes of the molecules and intermolecular attractions is Van der Waals' equation, given by $\left(p + \frac{a}{V^2}\right)(V - b) = RT$, where a and b are constants.

For a gas obeying Van der Waals' equation, find an expression for $\frac{\mathrm{d}p}{\mathrm{d}V}$ assuming T is a constant. Give your answer as a function of a, b, R, p and V only.

The following symbols may be useful: `R, V, a, b, p`

Part B Find $\frac{\mathrm{d}V}{\mathrm{d}T}$ for a gas obeying Dieterich's equation

One modification to the perfect gas equation of state ($pV = RT$) which takes account of the finite sizes of the molecules and intermolecular attractions is Dieterich's equation, given by $p(V - b) = RT\mathrm{e}^{-\frac{a}{RTV}}$, where a and b are constants.

For a gas obeying Dieterich's equation, $p(V - b) = RT\mathrm{e}^{-\frac{a}{RTV}}$. Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}T}$ assuming p is a constant.

The following symbols may be useful: `, R, T, V, a, b, e, p`

Created for isaacphysics.org by Julia Riley