Essential GCSE Maths 12.14

GCSE C C C A Level P P P

A postman delivers mail to four houses. House 1 receives $3s$ letters and t parcels. House 2 receives $7s$ letters. House 3 receives $5s$ letters and $2t$ parcels. House 4 receives t parcels.

Part A Write the information as an equation

Write an equation for the total number of items the four houses receive, N. Simplify your answer as far as possible.

The following symbols may be useful: `N, s, t`

Part B Write an equation for C

Assuming that the cost to send a letter is $80\,\text{pence}$ and the cost to send a parcel is $£5.50$, write an equation for C, the total cost in pounds to send all the items that were delivered.

The following symbols may be useful: `C, s, t`

Essential GCSE Maths 15.8

GCSE P P P A Level P P P

Simplify the following, factorising if possible.

Part A $3x^2 \times 2a \times ax^3$

$$3x^2 \times 2a \times ax^3$$

The following symbols may be useful: `a, x`

Part B $7p \times \frac{1}{2}x^2 \div \frac{p}{4} - 7x$

$$7p \times \frac{1}{2}x^2 \div \frac{p}{4} - 7x$$

The following symbols may be useful: `p, x`

Part C $3c \times \left(\frac{1}{2}x\right)^2 \times 8c^2x + 4c^3$

$$3c \times (\frac{1}{2}x)^2 \times 8c^2x + 4c^3$$

The following symbols may be useful: `c, x`

Gameboard:

STEM SMART Double Maths 38 - Algebra, Indices & Matrices

Physics. You work it out.

Simplifying Indices

GCSE P P P A Level P P P

Part A Simplify $(4a^2b^3)^{\frac{1}{2}} \times (9ab^2)^{-\frac{3}{2}}$

Simplify $(4a^2b^3)^{\frac{1}{2}} \times (9ab^2)^{-\frac{3}{2}}$

The following symbols may be useful: `a, b, p, q`

Part B Simplify $(8p^3q^2)^{\frac{2}{3}} \div \left(\frac{2p}{q^{\frac{1}{3}}}\right)^5$

Simplify $(8p^3q^2)^{\frac{2}{3}} \div \left(\frac{2p}{q^{\frac{1}{3}}}\right)^5$

The following symbols may be useful: `a, b, p, q`

Part C Simplify $(10^{-34})^{\frac{1}{2}}(10^{-10})^{\frac{1}{2}}(10^8)^{-\frac{5}{2}}$

Simplify $(10^{-34})^{\frac{1}{2}}(10^{-10})^{\frac{1}{2}}(10^8)^{-\frac{5}{2}}$

The following symbols may be useful: `a, b, p, q`

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Physics. You work it out.

Simplifying Surds

GCSE A Level

Simplify the following expressions.

Part A $2\sqrt{20}+\sqrt{45}-5\sqrt{5}$

Simplify $2\sqrt{20}+\sqrt{45}-5\sqrt{5}$

Part B $4(\sqrt{3}+1)(\sqrt{3}-1)-2(2+\sqrt{2})(1+\sqrt{2})$

Simplify $4(\sqrt{3}+1)(\sqrt{3}-1)-2(2+\sqrt{2})(1+\sqrt{2})$

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Rationalisation

GCSE P P P A Level P P P

Rationalise the denominators of the following expressions.

Part A $\frac{3\sqrt{6}}{2\sqrt{18}}$

Rationalise the denominator of $\frac{3\sqrt{6}}{2\sqrt{18}}$.

Part B $\frac{4-\sqrt{3}}{4+2\sqrt{3}}$

Rationalise the denominator of $\frac{4-\sqrt{3}}{4+2\sqrt{3}}$.

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Algebraic Manipulation 4.1

GCSE C C C A Level P P P

Rearrange each of the following equations to make the indicated symbol the subject.

Part A Find b if $\frac{1}{4a} - \frac{c}{3b} = 1$

Make b the subject of the equation $\dfrac{1}{4a} - \dfrac{c}{3b} = 1$.

The following symbols may be useful: `a, b, c`

Part B Find q if $p = \frac{2}{q^2} + \frac{3}{r}$

Consider the equation $p = \dfrac{2}{q^2} + \dfrac{3}{r}$. Show that if you make q the subject of this equation it can be written in the form $q = \pm S$ and find an expression for S.

The following symbols may be useful: `S, p, r`

Part C Find x if $\frac{1}{x^2} - \frac{a}{z^2} = b$

Consider the equation $\dfrac{1}{x^2} - \dfrac{a}{z^2} = b$. Show that if you make x the subject of this equation it can be written in the form $x = \pm Y$ and find an expression for Y.

The following symbols may be useful: `Y, a, b, z`

Part D Find m if $\frac{m}{a}+\frac{n}{b}=c$

Make m the subject of the equation $\dfrac{m}{a}+\dfrac{n}{b}=c$.

The following symbols may be useful: `a, b, c, m, n`

Part E Find s if $\frac{2}{r}-\frac{5}{s}=6$

Make s the subject of the equation $\dfrac{2}{r}-\dfrac{5}{s}=6$.

The following symbols may be useful: `r, s`

Part F Find r if $\frac{1}{p}=\frac{1}{q}+\frac{1}{r}$

Make r the subject of $\dfrac{1}{p}=\dfrac{1}{q}+\dfrac{1}{r}$.

The following symbols may be useful: `p, q, r`

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Matrices: nxm Rules 2i

Further A

P P P

The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$

Part A $\mathbf{AB}$

The matrix $\mathbf{AB}$ can be written as the 1×1 matrix a.

Find a.

The following symbols may be useful: a

Part B $\mathbf{BA} - 4\mathbf{C}$

Give the first row of the matrix given by $\mathbf{BA} - 4\mathbf{C}$ in the form $x\ y$ with a single space between x and y.

Give the second row of the matrix given by $\mathbf{BA} - 4\mathbf{C}$ in the form $x\ y$ with no spaces at the beginning or end.

Adapted with permission from UCLES, A Level, June 2010, Paper 4725, Question 2.

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2x2 Determinants and Inverses 1ii

Further A

The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -4 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}$. $\mathbf{I}$ denotes the 2×2 identity matrix.

Part A $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$

Give the first row of the matrix given by $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$ in the form $x\ y$ with a single space between x and y.

Give the second row of the matrix given by $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$ in the form $x\ y$ with a single space between x and y.

Part B $\mathbf{A}^{-1}$

$\mathbf{A}^{-1}$ can be written in the form $\mathbf{A}^{-1} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$.

Find $\alpha + \beta + \gamma + \delta$ in exact form.

Part C $\left(\mathbf{AB}^{-1}\right)^{-1}$

$\left(\mathbf{AB}^{-1}\right)^{-1}$ can be written in the form $\left(\mathbf{AB}^{-1}\right)^{-1} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$.

Find $\alpha + \beta + \gamma + \delta$ in exact form.

Adapted with permission from UCLES, A Level, Jan 2014, Paper 4725, Question 3.

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Matrices: 3x3 Determinants and Inverses 1i

Further A

P P P

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$. The matrix $\mathbf{B}$ is such that $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$.

Part A $\det \mathbf{AB}$

Find $\det \mathbf{AB}$.

The following symbols may be useful: a

Part B $(\mathbf{AB})^{-1}$

Give the first row of $(\mathbf{AB})^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Give the second row of $(\mathbf{AB})^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Give the third row of $(\mathbf{AB})^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Part C $\mathbf{B}^{-1}$

Give the first row of $\mathbf{B}^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Give the second row of $\mathbf{B}^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Give the third row of $\mathbf{B}^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 10.

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Algebraic Manipulation 5.4

GCSE C C C A Level C C C

Consider the following equations

$$w_x = \frac{c\cos\theta - v}{1 - \frac{v\cos\theta}{c}}$$

$$w_y = \frac{c\sin\theta}{\gamma\left(1 - \frac{v\cos\theta}{c}\right)}$$

where $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$.

Find an expression for $w = \sqrt{w_x^2 + w_y^2}$ in terms of one of either v, c or θ.

The following symbols may be useful: `c, cos(), sin(), tan(), theta, v, w`

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