Differentiating Powers 1

A Level Further A

Part A Differentiate $y = x^4$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x^4$.

The following symbols may be useful: x

Part B Differentiate $x = t^2$

Find the gradient of the curve $x = t^2$ at the points $t = 0$, $t = 3$ and $t = -3$.

Find the gradient at $t = 0$.

Find the gradient at $t = 3$.

Find the gradient at $t = -3$.

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Differentiating Powers 2

A Level Further A

Part A Gradient of $v = \frac{1}{u}$

Find $\frac{\mathrm{d}v}{\mathrm{d}u}$ if $v = \frac{1}{u}$.

The following symbols may be useful: `u`

Part B Tangent to $v = \frac{1}{u}$

Find the equation of the tangent to this curve (i.e. $v = \frac{1}{u}$) at the point $u = 2$.

The following symbols may be useful: `u, v`

Part C Derivative of $F = Ar^3$

Find $\frac{\mathrm{d}F}{\mathrm{d}r}$ if $F = Ar^3$ where A is a constant.

The following symbols may be useful: `A, r`

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Differentiating Powers 3

A Level C C C Further A P P P

Part A Derivative of $v = Bu^{-3}$

Find $\frac{\mathrm{d}v}{\mathrm{d}u}$ if $v = Bu^{-3}$.

The following symbols may be useful: `B, u`

Part B Force if potential $V = \frac{q^2}{(4\pi\epsilon_0 r)}$

The electrostatic potential energy V of two equal charges q a distance r apart is given by $V = \frac{q^2}{(4\pi\epsilon_0 r)}$.

The force between the two charges is given by $-\frac{\mathrm{d}V}{\mathrm{d}r}$; find an expression for this force.

The following symbols may be useful: `epsilon_0, pi, q, r`

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Differentiating Powers 4

A Level Further A

Part A Gradient of curve $t = 4s^{-3/4}$

Find the gradient of the curve $t = 4s^{-3/4}$ at the point $s = 16$.

Part B First derivative of $x = bt^{3/2}$

Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ if $x = bt^{3/2}$.

The following symbols may be useful: `b, t`

Part C Second derivative of $x = bt^{3/2}$

Find $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ if $x = bt^{3/2}$.

The following symbols may be useful: `b, t`

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Differentiating Sums and Differences 3

A Level C C C Further A P P P

Part A Velocity if $s = ut + bt^2$

A particle is moving in one dimension. Its displacement s at time t is given by $s = ut + bt^2$. The velocity v of the particle at time t is given by the rate of change of displacement with time, i.e. $v = \dfrac{\mathrm{d}s}{\mathrm{d}t}$.

Find an expression for the velocity.

The following symbols may be useful: `b, t, u, v`

Part B Acceleration if $s = ut + bt^2$

A particle is moving in one dimension. Its displacement s at time t is given by $s = ut + bt^2$. The acceleration a of the particle at time t is given by the rate of change of velocity with time.

Find an expression for the acceleration.

The following symbols may be useful: `a, b, t, u`

Part C Velocity if $x = \alpha t + \beta t^3$

The displacement of a body at time t is given by $x = \alpha t + \beta t^3$ where $\alpha = 4\,\mathrm{m\,s^{-1}}$ and $\beta = 5\,\mathrm{m\,s^{-3}}$. Use the fact that the velocity is the rate of change of displacement to find the velocity of the body at $t = 2\,\mathrm{s}$.

Find the velocity of the body at $t = 2\,\mathrm{s}$.

Part D Acceleration if $x = \alpha t + \beta t^3$

The displacement of a body at time t is given by $x = \alpha t + \beta t^3$ where $\alpha = 4\,\mathrm{m\,s^{-1}}$ and $\beta = 5\,\mathrm{m\,s^{-3}}$. Use the fact that the acceleration is the rate of change of velocity to find the acceleration of the body at $t = 2\,\mathrm{s}$.

Find the acceleration of the body at $t = 2\,\mathrm{s}$.

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Stationary Points 1

A Level Further A

Part A Number of stationary points of $y = 2x^3 - 24x - 5$

Find the position and nature of the stationary points of the function $y = 2x^3 - 24x - 5$.

How many stationary points are there?

- 1
- 2
- 0
- 3
- 4

Part B First stationary point of $y = 2x^3 - 24x - 5$

Find the position and nature of the stationary points of the function $y = 2x^3 - 24x - 5$.

Find x_1, the x coordinate of the stationary point with the lowest value of x.

The following symbols may be useful: `x_1, y_1`

Find y_1, the y coordinate of the stationary point (x_1, y_1).

The following symbols may be useful: `x_1, y_1`

What is the nature of this stationary point?

- Minimum
- Maximum

Part C Second stationary point of $y = 2x^3 - 24x - 5$

Find the position and nature of the stationary points of the function $y = 2x^3 - 24x - 5$.

Find x_2, the x coordinate of the stationary point with the second lowest value of x.

The following symbols may be useful: `x_2, y_2`

Find y_2, the y coordinate of the stationary point (x_2, y_2).

The following symbols may be useful: `x_2, y_2`

What is the nature of this stationary point?

- Minimum
- Maximum

Part D Number of stationary points of $y = 2x^3 - 5x^2 + 4x + 6$

Find the position and nature of the stationary points of the function $y = 2x^3 - 5x^2 + 4x + 6$.

How many stationary points are there?

- 1
- 3
- 2
- 0
- 4

Part E First stationary point of $y = 2x^3 - 5x^2 + 4x + 6$

Find the position and nature of the stationary points of the function $y = 2x^3 - 5x^2 + 4x + 6$.

Find x_1, the x coordinate of the stationary point with the lowest value of x.

The following symbols may be useful: `x_1, y_1`

Find y_1, the y coordinate of the stationary point (x_1, y_1). (Give your answer in the form of an improper fraction.)

The following symbols may be useful: `x_1, y_1`

What is the nature of this stationary point?

- Minimum
- Maximum

Part F Second stationary point of $y = 2x^3 - 5x^2 + 4x + 6$

Find the position and nature of the stationary points of the function $y = 2x^3 - 5x^2 + 4x + 6$.

Find x_2, the x coordinate of the stationary point with the second lowest value of x.

The following symbols may be useful: `x_2, y_2`

Find y_2, the y coordinate of the stationary point (x_2, y_2).

The following symbols may be useful: `x_2, y_2`

What is the nature of this stationary point?

- ○ Maximum
- ○ Minimum

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Stationary Points 3

A Level Further A

C C C P P P

Part A Find the maximum height of a projectile

A particle is fired upwards into the air with a speed w and moves subsequently under the influence of gravity with an acceleration g downwards, such that its height h at time t is given by $h = wt - \frac{1}{2}gt^2$. Find an expression for its maximum height above its initial position.

A particle is fired upwards into the air with a speed w and moves subsequently under the influence of gravity with an acceleration g downwards, such that its height h at time t is given by $h = wt - \frac{1}{2}gt^2$. Find an expression for its maximum height above its initial position.

The following symbols may be useful: `g, h, w`

Part B Examine the potential energy of two molecules

The potential energy of two molecules separated by a distance r is given by

$$U = U_0 \left(\left(\frac{a}{r} \right)^{12} - 2 \left(\frac{a}{r} \right)^{6} \right)$$

where U_0 and a are positive constants. The equilibrium separation of the two molecules occurs when the potential energy is a minimum; find expressions for the equilibrium separation and the value of the potential energy at this separation.

(a) Find an expression for the equilibrium separation of the molecules.

The following symbols may be useful: `U, U_0, a, r`

(b) Find an expression for the potential energy when the molecules are at their equilibrium separation.

The following symbols may be useful: `U, U_0, a, r`

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