Integration – Trig Manipulations 1ii

Use integration to find the exact value of $\displaystyle\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} \left(9 - 6\cos^2 4x\right) \mathrm{d}x$.

The following symbols may be useful: `pi`

Integration – Trig Manipulations 3ii

Find $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1 - 2\sin^2 x}{1 + 2\sin x \cos x}\,\mathrm{d}x$, giving your answer in the form $a \ln b$.

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Gameboard:

STEM SMART Single Maths 33 - Integration by Parts & Differential Equations

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Integration – Trig Manipulations 3i

A Level

P P P

Part A   Simplify

Simplify as far as possible $\dfrac{1}{1-\tan x} - \dfrac{1}{1+\tan x}$.

The following symbols may be useful: x

Part B   Integrate

Hence evaluate $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \left(\dfrac{1}{1-\tan x} - \dfrac{1}{1+\tan x}\right)\mathrm{d}x$, giving your answer in the form $a\ln(b)$.

The following symbols may be useful: pi

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Gameboard:

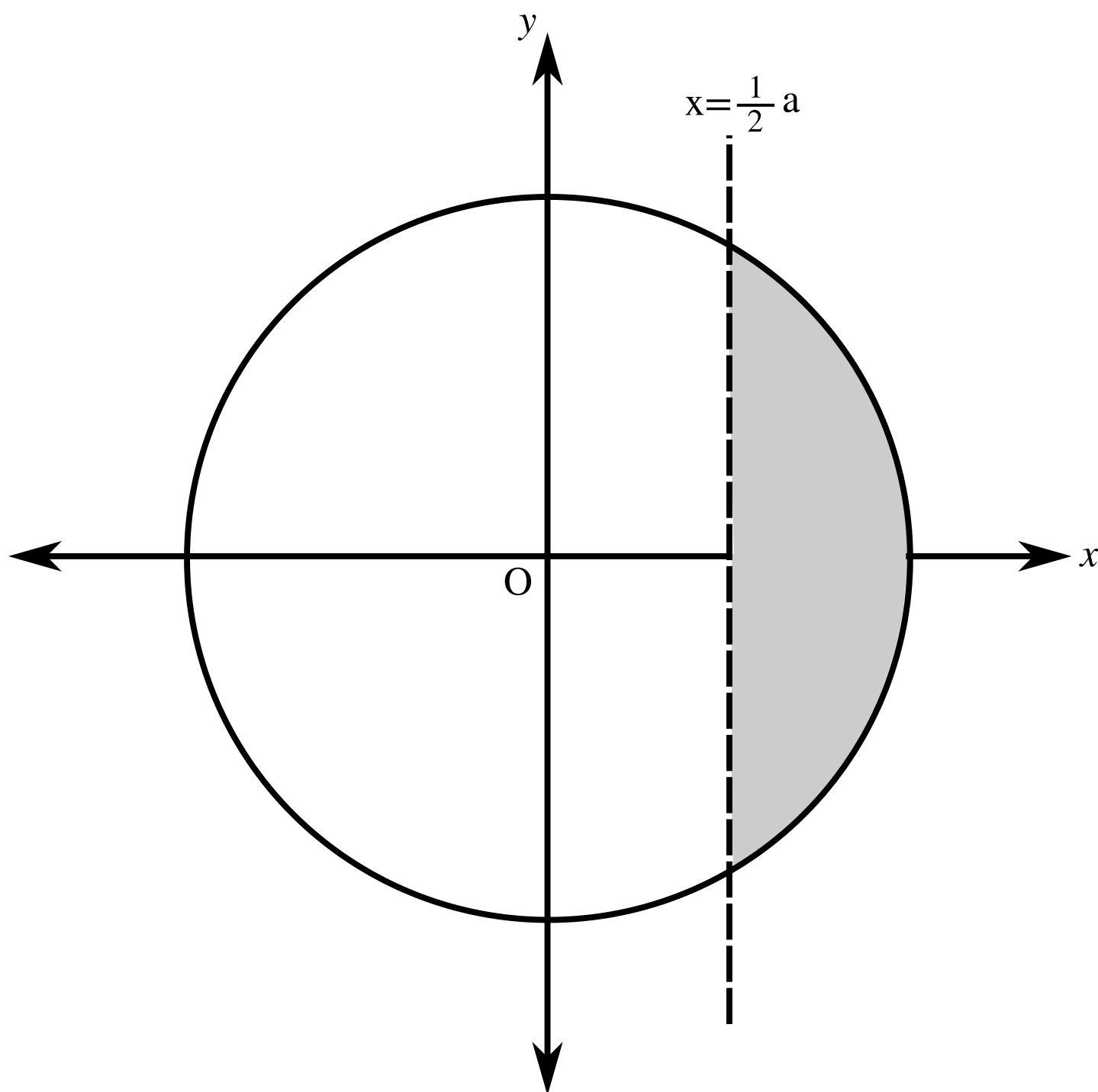**STEM SMART Single Maths 33 - Integration by Parts & Differential Equations**

Integration by substitution 2i

Part A   **The substitution** $x = a \sin \theta$

By using the substitution $x = a \sin \theta$, find the exact value of

$$\int_{\frac{1}{2}a}^{a} \sqrt{(a^2 - x^2)}\, dx$$

The following symbols may be useful: `a, pi`

Part B Area of a segment

The diagram shows the circle $x^2 + y^2 = a^2$ and the line $x = \frac{1}{2}a$. Find the area of the shaded region, giving your answer in an exact form.

The following symbols may be useful: `a, pi`

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Integration by Parts 3ii

Evaluuate $\displaystyle\int_0^{\frac{\pi}{2}} x \cos x \, \mathrm{d}x$, giving your answer in an exact form.

The following symbols may be useful: `pi`

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Integration by Parts 6

Find, by integrating by parts twice, $\displaystyle\int_0^{\pi/3} \mathrm{e}^{-x} \sin x \mathrm{d}x$.

The following symbols may be useful: `e, pi`

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Integration by Parts 2ii

A Level

Find the exact value of $\displaystyle\int_1^8 \frac{1}{\sqrt[3]{x}}\ln(x)\,\mathrm{d}x$, giving your answer in the form $A\ln(2) + B$, where A and B are constants to be found.

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Integration of Differential Equations 1ii

The gradient of a curve at the point (x, y), where $x > -2$, is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3y^2(x+2)}$$

The points $(1, 2)$ and $(q, 1.5)$ lie on the curve. Find the value of q, giving your answer correct to 3 significant figures.

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Integration of Differential Equations 4i

Part A   Derivative

If $y = \operatorname{cosec} x$ then find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), arccos(), arccosec(), arccosech(), arccosh(), arccot(), arccoth(), arcsec(), arcsech(), arcsin(), arcsinh(), arctan(), arctanh(), cos(), cosec(), cosech(), cosh(), cot(), coth(), ln(), log(), sec(), sech(), sin(), sinh(), tan(), tanh(), x, y`

Part B   Solve

Solve the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\sin x \tan x \cot t$$

given that $x = \frac{\pi}{6}$ when $t = \frac{\pi}{2}$.

The following symbols may be useful: `arccos(), arccosec(), arccosech(), arccosh(), arccot(), arccoth(), arcsec(), arcsech(), arcsin(), arcsinh(), arctan(), arctanh(), cos(), cosec(), cosech(), cosh(), cot(), coth(), ln(), log(), sec(), sech(), sin(), sinh(), t, tan(), tanh(), x`

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Constructing Differential Equations 3i

A container in the shape of an inverted cone of radius $3\,\mathrm{m}$ and vertical height $4.5\,\mathrm{m}$ is initially filled with liquid fertiliser. This fertiliser is released through a hole in the bottom of the container at a rate of $0.01\,\mathrm{m^3s^{-1}}$. At time t seconds the fertiliser remaining in the container forms an inverted cone of height h metres.

The volume of a cone is $V = \frac{1}{3}\pi r^2 h$

Part A   Differential equation

Find an expression for $\frac{\mathrm{d}h}{\mathrm{d}t}$.

The following symbols may be useful: `Derivative(h, t), h, pi, t`

Part B   Expression for h

Express h in terms of t.

The following symbols may be useful: `h, pi, t`

Part C   Time

Find the time it takes to empty the container, giving your answer in minutes to the nearest minute.

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