Exponential Rates

A Level

C C C

An experiment involves two substances, Substance 1 and Substance 2, whose masses are changing. The mass, M_1 grams, of Substance 1 at time t hours is given by

$$M_1 = 400\mathrm{e}^{-0.014t}$$

.

The mass, M_2 grams, of Substance 2 is increasing exponentially and the mass at certain times is shown in the following table.

t (hours)	0	10	20
M_2 (grams)	75	120	192

A critical stage in the experiment is reached at time T hours when the masses of the two substances are equal.

Part A Rate of change of Substance 1

Find the rate at which the mass of Substance 1 is changing when $t = 10\,\text{hours}$, giving your answer in grams per hour $\left(\text{g}\,\text{hour}^{-1}\right)$ correct to 2 significant figures.

Part B Solving for T

Show that T is the root of an equation of the form $\mathrm{e}^{kt} = c$. State the values of the constants k and c.

What is the value of k?

What is the value of c? Please give your answer to 3 significant figures.

Part C Value of T

Find the value of T to 3 significant figures.

Expansions and Algebra

A Level

Part A Indices and surds

Simplify $2x^{\frac{2}{3}} \times 3x^{-1}$.

The following symbols may be useful: x

Part B Indices

Express $2^{40} \times 4^{30}$ in the form 2^n.

The following symbols may be useful: n

Part C Simplifying expressions

Express $\frac{26}{4-\sqrt{3}}$ in the form $a + b\sqrt{3}$.

The following symbols may be useful: a, b

Part D Binomial expansions

Find the first four terms in the expansion, in ascending powers of x, of

$$(1 + 3x)^8 \ .$$

The following symbols may be useful: x

Part E Summing binomial expansions

Show that, if terms involving x^4 and higher powers of x may be ignored,

$$(1+3x)^8 + (1-3x)^8 = a + bx^2$$

Enter $a + bx^2$, substituting in the values for a and b.

The following symbols may be useful: x

Part F Estimating

Use the equation from the previous part $(1+3x)^8 + (1-3x)^8 = a + bx^2$ to solve this question.

Find the value of $1.000\,003^8 + 0.999\,997^8$ correct to 12 decimal places.

Adapted with permission from UCLES, A Level, January 2008, Paper 4722, Question 10, and June 2002, Paper 2, Question 5.