Jury Duty

GCSE   A Level

`C` `C` `C`   `C` `C` `C`

You are serving on a jury in the Crown Court. The defendant has been accused of a serious crime, however the **only** evidence is that their DNA is a perfect match to the perpetrator's DNA found at the crime scene.

The expert in genetic analysis tells you that the chance of a false positive (i.e. an innocent person matching that DNA) is 1 in 3 000 000. The prosecution lawyer says in their summing up speech that this means that as the defendant matches the DNA, the chance that they are innocent is less than $0.000\,04\%$, This means that there is a $99.999\,96\%$ chance that the person is guilty, and as this is *beyond reasonable doubt* you and your jury colleagues should decide that the person is guilty.

Back in the jury room, the other jurors know that you have mathematical knowledge and ask you for your view on the matter.

If you think it relevant, you may also assume that the crime was definitely committed by a British person, and that the population of Britain is 67 000 000.

Based on the information given, what is your best estimate of the probability that the defendant is guilty? Give your answer to two significant figures.

Created for isaacphysics.org by Anton Machacek.

Essential GCSE Maths 56.7

The histogram below summarises the total annual payments (including expenses) made to employees in a company.
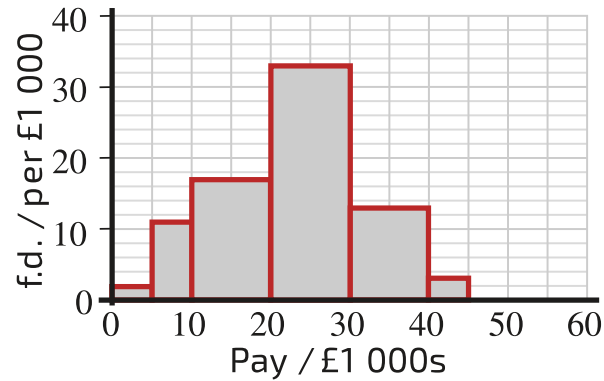

Figure 1: A histogram summarises the total annual payment made to employees.

Part A   Make a frequency table

Let the variable for pay be p. Make a frequency table for the data in the histogram.

A partially complete frequency table is given below. Find the missing values A, B and C.

Pay in £1 000, p	Frequency
$0 \le p < 5$	10
$5 \le p < 10$	A
$10 \le p < 20$	B
$20 \le p < 30$	330
$30 \le p < 40$	130
$40 \le p < 45$	C

What is the value of A?

What is the value of B?

What is the value of C?

Part B   What is the frequency density of the new class?

An extra class is added for $45 \le p < 60$. This class has a frequency of 15. What is the frequency density of this class?

Part C   Estimate the mean pay

Calculate an estimate of the mean amount payed out to an employee, including the extra class from part B. Give your answer to 3 s.f..

Data analysis 3.8

A Level

P · P · P

Consider the following data set:

$3.91, 4.29, 3.75, 4.28, 3.68, 4.13, 3.61, 4.19, 4.18, 4.01.$

Find the median, interquartile range, mean and standard deviation of the data set.

Part A   The median   ^

Find the median of the data set.

Part B   The interquartile range   ⌄

Find the interquartile range of the data set.

Part C   The mean   ⌄

Find the mean of the data set. Give your answer to 3 sf.

Part D   The standard deviation   ⌄

Find the standard deviation of the data set. Give your answer to 3 sf.

Probability 3.6

A Level
C C C

In some magnetic materials each atom or molecule is itself a permanent magnet. Consider a very simple one-dimensional model of such a material which consists of 20 such mini-magnets arranged in a line; each mini-magnet has two possible orientations which are equally likely and which we will call 'up' and 'down'. Find the probabilities of obtaining the following arrangements.

Part A   All pointing up   ⌃

Find the probability that all 20 mini-magnets are pointing up. Give your answer to $3\,\mathrm{sf}$.

Part B   Exactly 10 pointing up   ⌄

Find the probability that exactly 10 mini-magnets are pointing down. Give your answer to $3\,\mathrm{sf}$.

Part C   10 or more pointing up   ⌄

Find the probability that 10 or more are pointing up. Give your answer to $3\,\mathrm{sf}$.

Part D   Alternately up and down   ⌄

What is the probability that in the line the mini-magnets are alternately up (U) and then down (D) or down (D) and then up (U) i.e. UDUD... or DUDU...? Give your answer to $3\,\mathrm{sf}$.

Essential GCSE Maths 55.11

A technician is given a list of measurements in cm, correct to the nearest $0.1\,cm$. He is told that the mean of the values is $3.3\,cm$, but when he checks the calculation he finds a different value. Here is the list:

$$3.6, 3.4, 3.2, 2.9, 3.8, 3.4, 3.6, 3.2, 3.3, 3.6$$

Part A   What is the mean of the values?   ⌃

What is the mean of these values?

Part B   What is the value of the missing numbers?   ⌄

To find the source of the discrepancy, the technician checks the list he was given against the original data for the experiment, and finds two identical numbers are missing. What is the value of these numbers?

Probability 3.8

A Level

P P P

A bag contains four red and five yellow objects. Three of the objects are removed and not replaced. Deduce the probability distribution and the cumulative probability distribution for X, the number of red objects removed, giving your answers in their simplest exact form.

Part A **The probability distribution** ⌄

The probability distribution $f(x)$ can be defined as

$$f(x) = \mathrm{P}(X = x)$$

where X is the number of red objects removed. Find $f(x)$ for $x = 0,\ 1,\ 2,\ 3$.

Find $f(0)$.

Find $f(1)$.

Find $f(2)$.

Find $f(3)$.

Part B   The cumulative probability distribution

The cumulative probability distribution $F(x)$ can be defined as

$$F(x) = P(X \leq x)$$

where X is the number of red objects removed. Find $F(x)$ for $x = 0,\ 1,\ 2,\ 3$.

Find $F(0)$.

Find $F(1)$.

Find $F(2)$.

Find $F(3)$.

Essential GCSE Maths 56.8

A company conducts plant growth trials of two varieties of chilli pepper, A and B. The graph shows cumulative frequency plots for the heights of both types of pepper after 13 weeks.
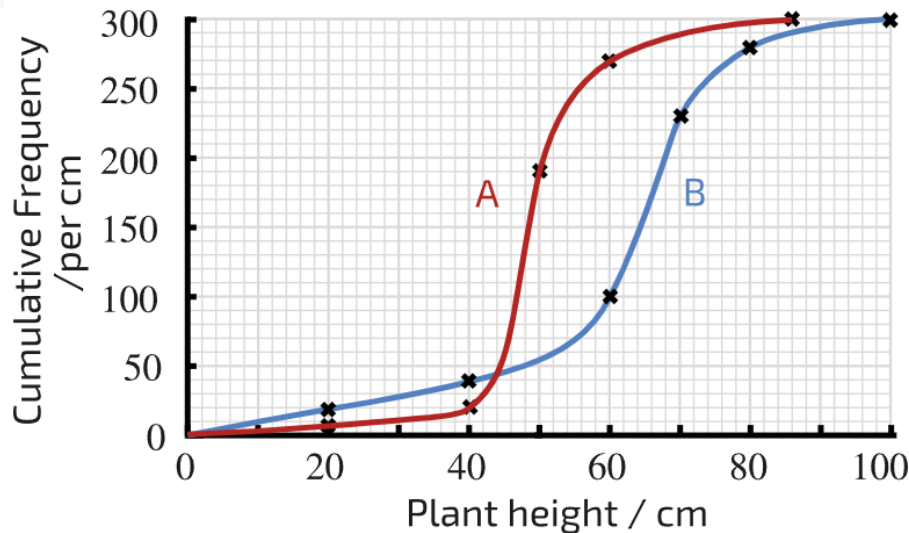

Figure 1: A cumulative frequency plot for the height of two types of chilli pepper.

Part A   Create box plots

Create box plots for both varieties of pepper. Choose which of the following figures is correct.
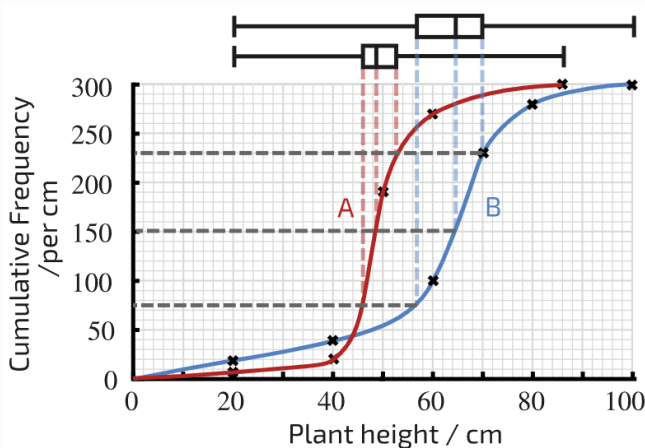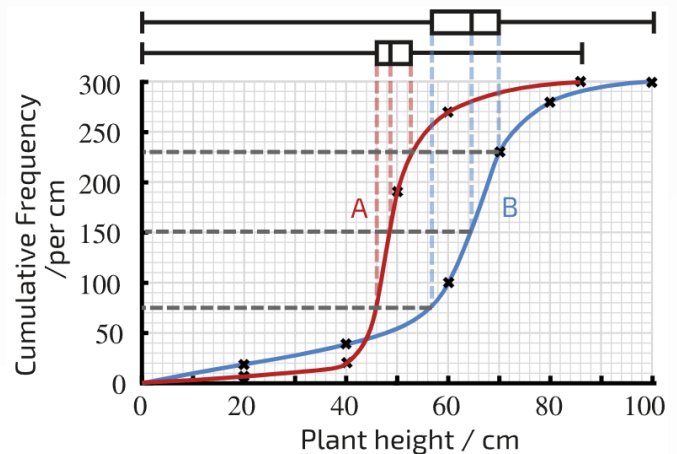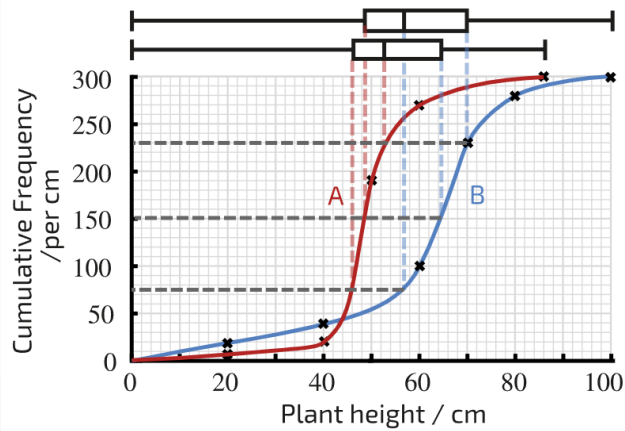

Figure 2: Option A.

Figure 3: Option B.

Figure 4: Option C.

○ Option A

○ Option B

○ Option C

Part B   Which variety produced fewer failures?

The company defines failures as plants which do not reach $40\,\mathrm{cm}$ in height. Which variety produced fewer failures?

○ Type A

○ Type B

Part C   For which plant was the spread of plant heights greater?

For which plant was the spread of plant heights greater? Explain your answer.

○ Type A

○ Type B

Essential GCSE Maths 57.5

The nuclei of atoms contain protons and neutrons. In this question p is the number of protons and n is the number of neutrons.

Plot a graph with p on the x-axis and n on the y-axis for the following selected light nuclei. Then choose which of the options below is the best.

Element	He	Be	C	N	F	Mg	Cl	Ca
p	2	4	6	7	9	12	17	20
n	2	5	6	7	10	12	18	20
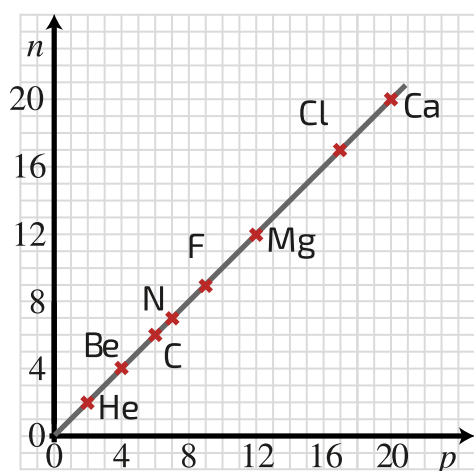
Figure 1: Option A.
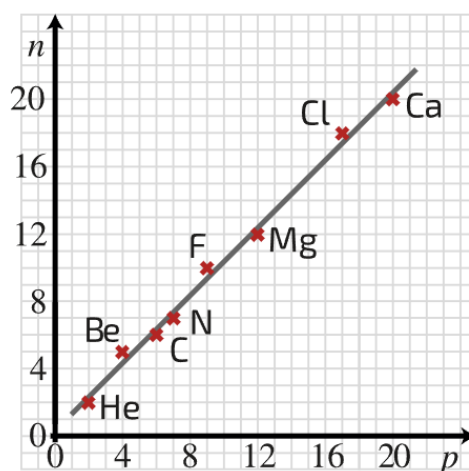
Figure 2: Option B.
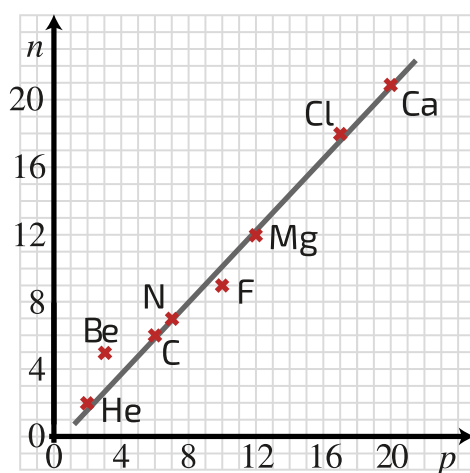

Figure 3: Option C.

○ Option A

○ Option B

○ Option C

Part B   What sort of correlation do you see?   ⌄

What sort of correlation do you see?

○ Positive linear correlation

○ No correlation

○ Correlation that is not linear

○ Negative linear correlation

Part C   What can you conclude from the graph?   ⌄

What can you conclude from the graph about the value of the ratio $n : p$ for the nuclei given?

○ There is no relation between the number of protons (p) and the number of neutrons (n).

○ The number of neutrons is roughly the same as the number of protons. For these light nuclei $n : p \approx 1 : 1$

○ The number of neutrons is generally larger than the number of protons. $n > p$.

○ The number of neutrons is generally smaller than the number of protons. $n < p$.

Plot a graph with p on the x-axis and n on the y-axis for the following selected heavier nuclei, and then choose which of the options below is the best.

Element	Pd	Cs	Pr	Tb	W	Pt	Au	Pb
p	46	55	59	65	74	78	79	82
$p+n$	106	133	141	159	184	195	197	207
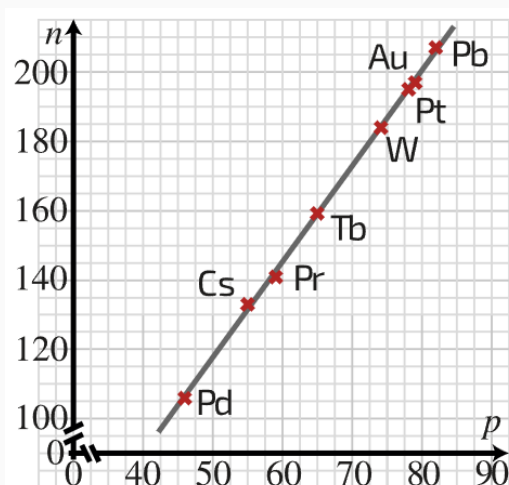
Figure 4: Option A.
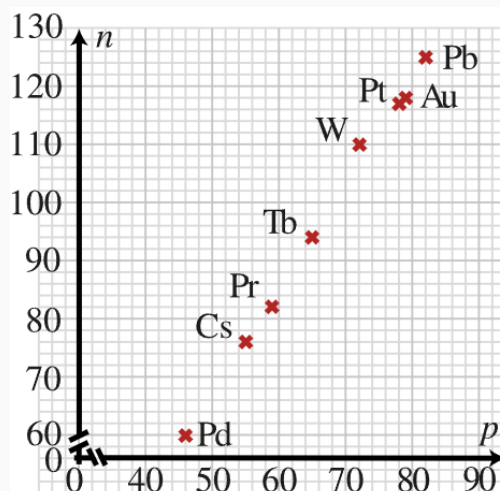
Figure 5: Option B.
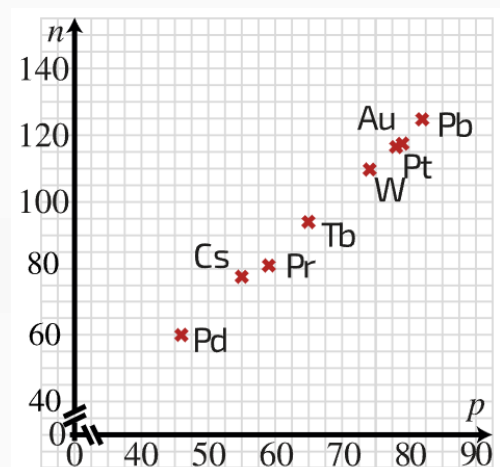
○ Option A

○ Option B

○ Option C

Figure 6: Option C.

Part E   What sort of correlation do you see?

What sort of correlation do you see?

○ Negative linear correlation

○ Correlation that is not linear

○ Positive linear correlation

○ No correlation

Part F   Draw a line of best fit

Find the gradient.

Essential GCSE Maths 57.4

Look at the following sketches and work out which equation describes the line of best fit.

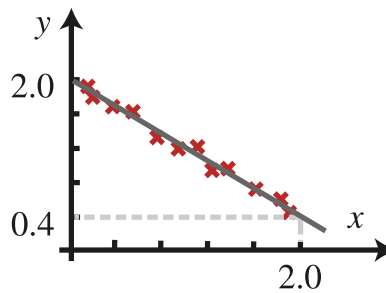**Part A**   **Which equation describes the line of best fit?**   ⌃

Figure 1: A graph of some data points and a line of best fit.

Which equation describes the line of best fit?

○ $y = -1.8x + 4.0$

○ $y = -0.8x + 2.0$

○ $y = 0.8x - 2.0$

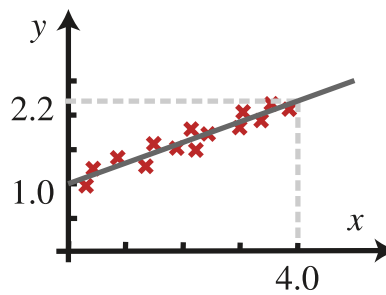**Part B**   Which equation describes the line of best fit?

Figure 2: A graph of some data points and a line of best fit.

Which equation describes the line of best fit?

○ $y = 0.4x - 1.5$

○ $y = 0.4x + 1.5$

○ $y = 0.3x + 1.0$

Part C   Which equation describes the line of best fit?

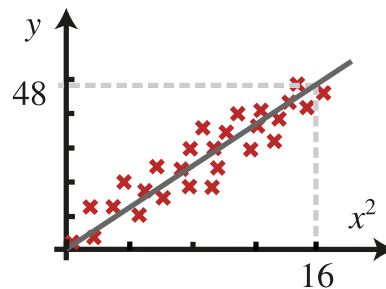**Figure 1:** A graph of some data points and a line of best fit.

Which equation describes the line of best fit?

○ $y = 3.0x^2$

○ $y = 3.0x$

○ $y = (3.0x)^2$

Essential GCSE Maths 52.8

An unbiased 6-sided die is rolled twice.
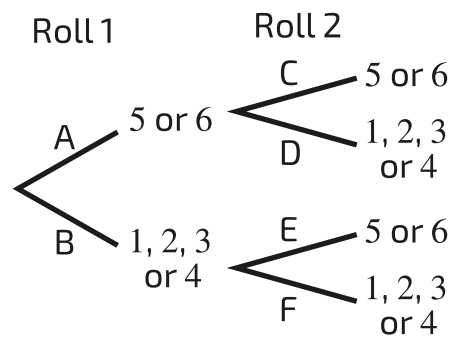
Put the missing probabilities on the diagram.

Roll 1   Roll 2

C — 5 or 6

5 or 6

A

D — 1, 2, 3 or 4

1, 2, 3 or 4

B

E — 5 or 6

F — 1, 2, 3 or 4

Figure 1: A tree diagram that needs the probabilities added.

Part A   **Put the missing probabilities on the diagram, find A**

Find probability A.

Part B   **Put the missing probabilities on the diagram, find B**

Find probability B.

Part C   **Put the missing probabilities on the diagram, find C**

Find probability C.

Part D   Put the missing probabilities on the diagram, find D ⌄

Find probability D.

Part E   Put the missing probabilities on the diagram, find E ⌄

Find probability E.

Part F   Put the missing probabilities on the diagram, find F ⌄

Find probability F.

Part G   What is the probability of rolling 5 or 6 twice in a row?   ⌄

An unbiased 6-sided die is rolled twice.

What is the probability of rolling 5 or 6 twice in a row?

Part H   What is the probability of rolling a 5 or 6 exactly once?   ⌄

An unbiased 6-sided die is rolled twice.

What is the probability of rolling a 5 or 6 exactly once?

Part I   What is the probability of rolling a 5 or 6 at least once?   ⌄

An unbiased 6-sided die is rolled twice.

What is the probability of rolling a 5 or 6 at least once?