Cambridge
Sutton Trust Physics Summer School

INVESTIGATING MOMENTS OF INERTIA

1. Aims

- To investigate the effect of moment of inertia on the time taken for rotating objects to roll down the slope.
- Practise keeping clear experimental notes and reporting findings.
- Be able to analyse findings
- Be able to use graphical methods to determine the power law linking moment of inertia and rolling time.

We would like you to submit a report describing your experimental setup, a results, a discussions sections and a conclusion. Feedback will be given on your report.

Please submit it with the following file name format: surname_first name.

You can submit it as a word document, a pdf or a picture of your report ***if you do not have access to a laptop or tablet.***

SAFETY WARNING: Please make sure you **have the consent of the owner** of the various objects, table/shelf, before conducting the experiment.

- Avoid using very heavy objects so as not to break your slope or the floor, or hurt yourself or anyone else helping you.
- Do not use fragile objects such as things made of glass.
- If someone is catching the objects for you, make sure their feet are not directly under the edge of the slope so as not to have any hard objects land if they miss.
- Aim to stand to the side to avoid any falling objects.
- Create a soft landing for objects, where appropriate to avoid breaking any objects or apparatus (table or floor).

2. Introduction

If an object **slides** down a **smooth** slope, its speed when it reaches the bottom is independent of its mass. However, the speed that the object reaches at the bottom does depend on the steepness of the slope. You can calculate this relationship using conservation of energy. If the object of mass m, starts at a height h, and its final velocity is v, then conservation of energy implies that the gravitational potential energy is converted to linear kinetic energy

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

What happens when an object rolls down a rough slope without slipping?

When an object rolls down a rough slope, the gravitational potential energy is converted to linear kinetic energy **and** rotational kinetic energy.

The rotational kinetic energy, RKE, is given by

$$RKE = \frac{1}{2}I\omega^2$$

where $\boldsymbol{I}$ is the ***moment of inertia*** of the object and ω is the angular velocity of the object.

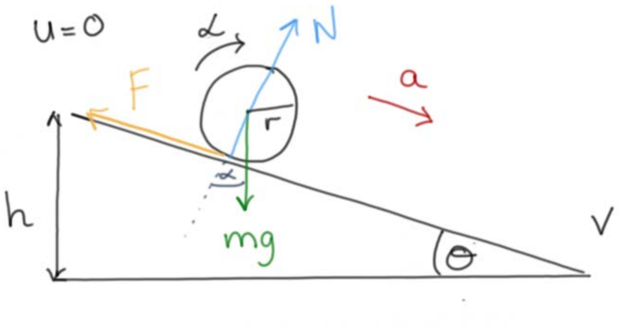
The moment of inertia depends on how the object's mass is distributed about the axis of rotation. The moment of inertia of a body for rotational mechanics is analogous to the mass of an object for linear motion and can be written as

$$I = kmr^2$$

where r is the radius of the object, m is the mass of the object and k is a numerical coefficient that depends on the axis of rotation and the shape of the object.

In this experiment we will determine the moment of inertia of an everyday object by rolling that object down a slope and measuring the time it takes to get from the top to the bottom for different angles of slope.

3. Theory



We start an object rolling from rest at a height, h. The object rolls without sliding and accelerates linearly and rotationally to reach a linear velocity h at the bottom of the slope. The distance that the object moves along the slope is L. A frictional force F, acts up the slope and causes the object to roll.

The linear acceleration is a and the angular acceleration is α and is related to a as

$$a = \alpha r$$

Using Newton's Second Law linearly parallel to the slope:

$$mg \sin\theta - F = ma \qquad (1)$$

Using Newton's Second Law for rotational motion

$$Fr = I\alpha$$

$$Fr = kmr^2\alpha$$

$$F = kmr\alpha$$

$$F = kma \qquad (2)$$

Therefore substituting (2) in (1)

$$g \sin\theta = (k+1)a$$

Acceleration,

$$a = {}^{g\sin\theta}\!/_{(k+1)}$$

Using the SUVAT equation with the initial velocity equal to zero, $L = \frac{1}{2}at^2$ we can find a relationship for how time varies with the angle of the slope for a rotating body.

$$L = \frac{g\sin\theta}{2(k+1)}t^2$$

$$g \sin\theta = \frac{2L(k+1)}{t^2} \qquad (3)$$

The quantity that you can vary is the angle of the slope θ and the quantity that you can measure is the time t. Given that we wish to compare our results with the equation of a straight line $y = mx + c$

1. **Design a table for collecting your data -**
2. **Determine what your y, x, and m are in equation 3 to use on your graph. You may want to rearrange it slightly**.

4. Experimental Method

For this experiment you will need:

- **A slope:** a shelf that can be easily removed and propped up on the floor, a long board, an ironing board which isn't too soft, a table,… a book is a little too short.
- An object to roll down the slope: tin can, ping-pong balls, tennis balls, reel of tape, empty drinks bottle…
- A ruler or tape measure
- A stopwatch: can use an app on your phone
- Pencil
- Paper
- A computer or device to record your measurements and plot a graph

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NOTE: If you are unable to perform the experiment yourself, you can watch the video of the experiment being performed here. You can then use the angles and the timings on that video to find k.

Setting up your experiment

a) Take notes of what you used, how you performed the experiment

b) Sketch and/or take a photograph of your experimental setup.

c) Create a slope by propping one end of the table/shelf/board up. Make sure that your surface is not too smooth. The objects need to roll down the slope without sliding. Create a landing area for the objects to fall onto.

d) Start with the angle of the slope quite shallow. Does the object roll reliably? How steep can you make the slope and reliably time the object rolling?

5. Results

a) Draw a table (or use excel or Google sheets) using your design from Section 3.

b) Consider how you are going to minimise the error in each timing.

c) Do you need to modify your table to help with (b)?

d) Decide on how many readings you can reliably take.

e) It is **VITAL** that you plot your results as you take them. If you see a point that is anomalous compared with the rest you can re-measure straight away which will save time and effort.

f) Take a rough timing for the shallowest slope and a rough timing for your steepest slope – use these timings to work out the scale for the axes on your graph.

g) Find the line of best fit for your graph

h) Determine the value of k for your object.

6. Conclusions

Look up the moments of inertia for different 3-d shapes (hollow cylinder, solid cylinder, solid sphere, hollow sphere…..).

How do the values of k for these objects compare with the object you used?

Can you explain why your answer may be similar or different to these values?

7. Extension

Roll the same type of shape but of various sizes and mass down the slope at the same time, for example different hollow cylinders, different hollow spheres, different solid cylinders. Is this what you expected? Could you have predicted this from the equation (3)?

If you have time repeat the experiment with a different shape (if you used a full can – try an empty one).

Can you derive the moment of inertia for a hollow cylinder and a solid cylinder?