Isaac Physics Skills

Linking concepts in pre-university physics

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TABLE OF PHYSICAL CONSTANTS

Quantity & Symbol		**Magnitude**	**Unit**
Permittivity of free space	ϵ_0	8.85×10^{-12}	$\mathrm{F\ m^{-1}}$
Electrostatic force constant	$1/4\pi\epsilon_0$	8.99×10^{9}	$\mathrm{N\ m^{2}\ C^{-2}}$
Speed of light in vacuum	c	3.00×10^{8}	$\mathrm{m\ s^{-1}}$
Specific heat capacity of water	c_{water}	4180	$\mathrm{J\ kg^{-1}\ K^{-1}}$
Charge of proton	e	1.60×10^{-19}	C
Gravitational field strength on Earth	g	9.81	$\mathrm{N\ kg^{-1}}$
Universal gravitational constant	G	6.67×10^{-11}	$\mathrm{N\ m^{2}\ kg^{-2}}$
Planck constant	h	6.63×10^{-34}	J s
Boltzmann constant	k_{B}	1.38×10^{-23}	$\mathrm{J\ K^{-1}}$
Mass of electron	m_{e}	9.11×10^{-31}	kg
Mass of neutron	m_{n}	1.67×10^{-27}	kg
Mass of proton	m_{p}	1.67×10^{-27}	kg
Mass of Earth	M_{Earth}	5.97×10^{24}	kg
Mass of Sun	M_{Sun}	2.00×10^{30}	kg
Avogadro constant	N_{A}	6.02×10^{23}	$\mathrm{mol^{-1}}$
Gas constant	R	8.31	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Radius of Earth	R_{Earth}	6.37×10^{6}	m

OTHER INFORMATION YOU MAY FIND USEFUL

Electron volt	1 eV	=	1.60×10^{-19} J
Unified mass unit	1 u	=	1.66×10^{-27} kg
Absolute zero	0 K	=	$-273\ ^\circ$C
Year	1 yr	=	3.16×10^{7} s
Light year	1 ly	=	9.46×10^{15} m
Parsec	1 pc	=	3.09×10^{16} m

PREFIXES

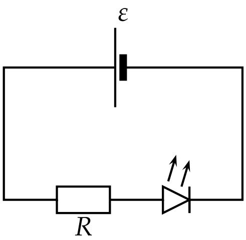
1 km = 1000 m	1 Mm = 10^{6} m	1 Gm = 10^{9} m	1 Tm = 10^{12} m
1 mm = 0.001 m	1 μm = 10^{-6} m	1 nm = 10^{-9} m	1 pm = 10^{-12} m

8 Potential dividers with LEDs

It is helpful to be able to calculate the resistances necessary to obtain a particular output voltage from a potential divider circuit containing an LED.

Example context: this section builds on **Section 7** about photon flux by considering the LED in a circuit in series with a fixed resistor. The fixed resistor is needed to make sure the LED receives the correct current.

Quantities: ε e.m.f. (V)
V p.d. across fixed resistor (V)
V_{LED} p.d. across LED (V)
I current through circuit (A)
R fixed resistor resistance (Ω)
E photon energy (J)
λ wavelength of emitted light (m)



Equations: $V = IR \quad \varepsilon = V_{\text{LED}} + V \quad V_{\text{LED}} = \dfrac{E}{e} \quad E = \dfrac{hc}{\lambda}$

8.1 Use the equations to derive expressions for

a) the resistance of the fixed resistor R in terms of the e.m.f. ε, the p.d. across the LED V_{LED} and the current I,

b) the resistance of the fixed resistor R in terms of the e.m.f. ε, the wavelength of the LED λ, the current I and the physical constants h, c and e.

8.2 Fill in the missing entries in the table below.

e.m.f. / V	current / mA	fixed resistor resistance / Ω	LED p.d. / V
9.00	12.1	(a)	4.14
6.00	(b)	300	1.78
(c)	8.05	73.6	3.11
5.00	10.1	250	(d)
7.40	51.5	(e)	2.25
12.0	28.8	330	(f)

Example 1 – *Calculate the resistance R needed when a 652 nm LED is connected to a 6.00 V battery if the current is to be 50.0 mA.*

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{652 \times 10^{-9}} = 3.051 \times 10^{-19}\ \text{J},$$

so $V_{\text{LED}} = \dfrac{E}{e} = 1.904$ V. $V = \varepsilon - V_{\text{LED}} = 6.00 - 1.90 = 4.10$ V

$$R = \frac{V}{I} = \frac{4.10}{0.050} = 81.9\ \Omega.$$

8.3 A blue LED produces light of wavelength 480 nm. It is powered using a 9.00 V battery using the circuit design shown above. Assume that there is no internal resistance in the power supply and calculate

a) the p.d. across the LED,

b) the minimum value of R to ensure the current through the LED does not exceed 50.0 mA,

c) the resistance of the LED.

Example 2 – *Calculate the current through a 510 nm LED (with a p.d. of 2.44 V across it) connected to an e.m.f. of 5.00 V, in series with a $300\ \Omega$ resistor.*
P.d. is shared, so p.d. across the resistor must be $5.00 - 2.44 = 2.56$ V
Fixed resistor is ohmic, so use Ohm's law $I = \dfrac{V}{R} = \dfrac{2.56}{300} = 8.53$ mA
As resistor and LED are in series, currents are the same.

8.4 A red LED produces light of wavelength 680 nm. It is powered using a 7.4 V battery with no internal resistance. Calculate

a) the p.d. across the LED,

b) the current through the LED when its power is 102 mW *(use $P = IV$)*,

c) the resistance of the LED when its power is 102 mW,

d) the resistance of the fixed resistor R.

8.5 Two LEDs *(labelled A and B)* are connected in parallel to a 3.7 V cell. Each LED is protected by its own resistor in series. *LED A* is protected by a $330\ \Omega$ resistor, whereas *LED B* is protected by a $165\ \Omega$ resistor. Both LEDs produce light of wavelength 650 nm. Presenting your answer as a decimal, calculate

a) the ratio of the p.d. across *LED A* to the p.d. across *LED B*,

b) the ratio of the current through *LED A* to the current through *LED B*,

9 Current division

It is helpful to be able to calculate the fraction of an electric current which takes each branch of a parallel circuit.

Example context: voltmeters are not perfect insulators. When the voltage across a component is measured, a fraction of the current flows through the voltmeter, and this affects the circuit. A knowledge of the fraction of current no longer flowing through the component enables a correction to be made.

Quantities: I current (A) — V voltage (V)
R resistance (Ω) — G conductance (Ω^{-1} or S)
Subscripts $_{1,2}$ label components. Subscript $_C$ refers to the circuit.

Equations: $R = \dfrac{V}{I}$ $\quad G = \dfrac{I}{V} = \dfrac{1}{R}$ $\quad R_{\text{parallel}} = \left(\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots\right)^{-1}$

For components in parallel: $I_C = I_1 + I_2 + \ldots \quad V_1 = V_2 = \ldots$

9.1 Two resistors R_1 and R_2 are in parallel, and carry a total current I_C. Use the equations to write or derive expressions (in terms of I_C, R_1 and R_2) for

a) the voltage V across each resistor,

b) the current I_1 through resistor R_1,

c) the fraction of the total current which flows through R_1: $\dfrac{I_1}{I_C}$,

d) the conductance G_1 of resistor R_1,

e) the total conductance $G_C = G_1 + G_2$ of the two resistors

f) the fraction $\dfrac{G_1}{G_C}$.

> **Example** – *A 3.0 Ω resistor is wired in parallel with a 6.0 Ω resistor, and between them, they carry 24 mA. Calculate the current carried by the 6.0 Ω resistor.*
> Overall resistance $R_C = \left(3.0^{-1} + 6.0^{-1}\right)^{-1} = 2.0\,\Omega$
> Voltage across combination $V = I_C R_C = 0.024 \times 2.0 = 0.048$ V
> Current through the $6.0\,\Omega$ resistor $I_6 = \dfrac{V}{R} = \dfrac{0.048}{6} = 8.0$ mA

9.2 A $9.0\,\Omega$ resistor is connected in parallel with a $81\,\Omega$ resistor. What fraction of the total current flows through the $81\,\Omega$ resistor?

9.3 How much current flows through a $330\,\Omega$ resistor which is connected in parallel with a $68\,\Omega$ resistor which is carrying 40 mA by itself?

9.4 I am going to connect two resistors in parallel to share a 13 A current so that 5.0 A flows through one resistor. The resistor with the larger resistance is a $2.2\ \Omega$ resistor. Calculate the resistance of the other resistor.

9.5 Fill in the missing entries in the table below. In this circuit, three resistors (R_1, R_2, R_3) are connected in parallel.

R_1	R_2	R_3	I_1	I_2	I_3	I_C	V
/ Ω			/ A				/ V
1.0	2.0	3.0	(a)	(b)	(c)	2.4	(d)
5.0	15	20	(e)	(f)	(g)	(h)	12
48	(i)	7.5	5.0	20	(j)	(k)	(l)

9.6 A wire in an oven typically carries 20 A. I wish to put an LED in the circuit which will light up when the current is flowing. The LED requires a voltage of 1.8 V to light, and takes a current of 25 mA when it is lit. I will connect the LED in parallel with a resistor, and place the combination in series with the oven's heater element.

a) Calculate the resistance of the LED when it is lit.

b) Calculate the current through the resistor when the LED is lit.

c) Calculate the resistance of the resistor needed.

9.7 An ammeter designed for electricians has a resistance of 0.10 mΩ and it can measure a maximum of 200 A. I wish to adapt it so it can measure currents up to 1000 A by connecting a resistor in parallel with it.

a) What is the voltage across the ammeter when it carries 200 A?

b) Once the resistor is connected, what fraction of the total current should flow through the ammeter?

c) When the resistor is connected and the combination is carrying 1000 A, what is the current through the resistor?

d) Calculate the resistance of the resistor.

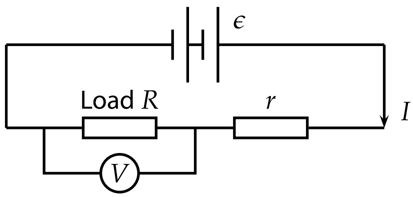
e) Using $P = IV$ calculate the power dissipated in the resistor when the combination is carrying 1000 A.

10 Power in a potential divider

It is helpful to be able to calculate the power (or fraction of the total power) dissipated in one part of a potential divider circuit.

Example context: Electrical generators have internal resistance. A power supply company wishes to maximise the efficiency of the system by ensuring that as much of the electricity generated as possible is passed on to customers.

Quantities: I current (A) — P load power (W)
R load resistance (Ω) — V voltage or p.d. across load (V)
r internal resistance (Ω) — η efficiency (no unit)
ϵ electromotive force (emf) of supply (V)



Equations: $P = IV = I^2R = \dfrac{V^2}{R}$ $\quad V = IR \quad \epsilon = V + Ir \quad \eta = \dfrac{P}{I\epsilon}$

10.1 Use the equations to derive expressions for

a) the current I in terms of ϵ, R and r,

b) the voltage V in terms of ϵ, R and r,

c) the power P in terms of ϵ, R and r,

d) the efficiency η in terms of ϵ, R and r.

Example 1 – *Calculate the efficiency if a* $20\,\Omega$ *resistor is supplied from a* 12 *V battery with an internal resistance of* $4\,\Omega$.

Total resistance is $20 + 4 = 24\,\Omega$, so current $I = \dfrac{12\text{ V}}{24\,\Omega} = 0.50$ A.

Power in load $P = IV = I \times IR = I^2R = 0.50^2 \times 20 = 5.0$ W.

Power supplied $I\epsilon = 0.50 \times 12 = 6.0$ W. Efficiency $= \dfrac{5.0\text{ W}}{6.0\text{ W}} = 0.83$

10.2 Calculate the load power P for an $\epsilon = 240$ V generator with internal resistance $2.5\,\Omega$ when it is supplying 4.2 A. Hint: use $\epsilon = V + Ir$

10.3 Calculate the efficiency η of the generator in question 10.2.

10.4 An $\epsilon = 12$ V battery has an internal resistance $r = 4.0\,\Omega$. Fill in the missing entries in the table below.

R / Ω	V / V	I / A	P / W	Efficiency η
0.10	(a)	(b)	(c)	(d)
2.0	(e)	(f)	(g)	(h)
4.0	(i)	(j)	(k)	(l)
6.0	(m)	(n)	(o)	(p)
50	(q)	(r)	(s)	(t)

10.5 Use your answers to question 10.4 to state the value of r/R which gives the greatest load power P for given, fixed values of ϵ and r.

10.6 Use your answers to question 10.4 (or other reasoning) to state the value of r/R which gives the greatest efficiency for given values of ϵ and r.

10.7 Calculate r if $P = 500$ MW, $V = 23$ kV and $\eta = 0.99$.

Example 2 – *The load resistor R in the circuit shown is replaced by $30\,\Omega$ and $60\,\Omega$ heaters wired in parallel. Calculate the power dissipated in the $30\,\Omega$ heater if $\epsilon = 230$ V and $r = 3.0\,\Omega$.*

Resistance of the two heaters in parallel $R = \left(30^{-1} + 60^{-1}\right)^{-1} = 20\,\Omega$.

Total circuit resistance $= 20 + 3 = 23\,\Omega$, so current $= \dfrac{230\text{ V}}{23\,\Omega} = 10$ A.

Voltage across the heaters is $V = IR = 10\text{ A} \times 20\,\Omega = 200$ V.

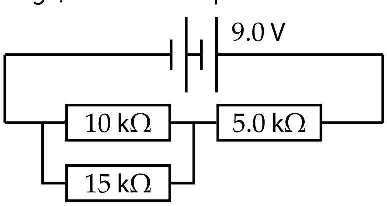
Power in $30\,\Omega$ heater is given by $\dfrac{\text{Voltage}^2}{\text{Resistance}} = \dfrac{200^2}{30} = 1300$ W to 2sf.

10.8 An $\epsilon = 5.4$ V power supply (with $r = 8.0\,\Omega$) powers a $50\,\Omega$ phone. A voltmeter (with resistance $200\,\Omega$) is connected to measure V.

a) How much voltage V is measured across the phone?

b) Calculate the power delivered to the phone.

10.9 Calculate the voltage, current and power for each of the resistors in the circuit below.



(b) Analysing motion from high point to end $s_{\mathrm{y}} = h + D,\ u_{\mathrm{y}} = 0,\ a_{\mathrm{y}} = g$

$v_{\mathrm{y,final}}^2 = u_{\mathrm{y}}^2 + 2a_{\mathrm{y}}s_{\mathrm{y}} = 0 + 2g\,(h + D)$, so $v_{\mathrm{y,final}} = \sqrt{2g\,(h+D)}$

(c) Using $v_{\mathrm{y}} = u_{\mathrm{y}} + a_{\mathrm{y}}t$ over the whole motion, $v_{\mathrm{y,final}} = -u\sin\theta + gT$

$$T = \frac{v_{\mathrm{y,final}} + u\sin\theta}{g} = \frac{\sqrt{2g\,(h+D)} + u\sin\theta}{g}$$
$$= \frac{\sqrt{u^2\sin^2\theta + 2gD} + u\sin\theta}{g}$$

(d) $u_{\mathrm{x}} = v_{\mathrm{x}}$ because $a_{\mathrm{x}} = 0$

$$R = u_{\mathrm{x}}T = u\cos\theta \times \frac{\sqrt{u^2\sin^2\theta + 2gD} + u\sin\theta}{g}$$

7 Photon flux for an LED

(a) $I = \dfrac{\text{charge}}{t} = \dfrac{ne}{t} = \dfrac{n}{t}\cdot e = \Phi_{\mathrm{q}}e$

(b) $V = \dfrac{E}{e} = \dfrac{hc}{\lambda}\cdot\dfrac{1}{e} = \dfrac{hc}{e\lambda}$

(c) $P = IV = \Phi_{\mathrm{q}}e\cdot\dfrac{hc}{e\lambda} = \Phi_{\mathrm{q}}\dfrac{hc}{\lambda}$

8 Potential dividers with LEDs

(a) $V = IR$ so $R = \dfrac{V}{I} = \dfrac{\varepsilon - V_{\mathrm{LED}}}{I}$

(b) $R = \dfrac{\varepsilon - V_{\mathrm{LED}}}{I} = \dfrac{\varepsilon}{I} - \dfrac{hc}{Ie\lambda}$

9 Current division

(a) $V = I_{\mathrm{C}}R_{\mathrm{parallel}} = I_{\mathrm{C}}\left(R_1^{-1} + R_2^{-1}\right)^{-1} = \dfrac{I_{\mathrm{C}}}{R_1^{-1} + R_2^{-1}}$

(b) $I_1 = \dfrac{V}{R_1} = VR_1^{-1} = \dfrac{I_{\mathrm{C}}R_1^{-1}}{R_1^{-1} + R_2^{-1}}$

(c) $\dfrac{I_1}{I_C} = I_1 \times \dfrac{1}{I_C} = \dfrac{I_C R_1^{-1}}{R_1^{-1} + R_2^{-1}} \times \dfrac{1}{I_C} = \dfrac{R_1^{-1}}{R_1^{-1} + R_2^{-1}}$

(d) $G_1 = \dfrac{I_1}{V} = \dfrac{1}{R_1} = R_1^{-1}$

(e) $G_C = G_1 + G_2 = R_1^{-1} + R_2^{-1}$

(f) $\dfrac{G_1}{G_C} = \dfrac{R_1^{-1}}{R_1^{-1} + R_2^{-1}}$

We hope you noticed that $\dfrac{G_1}{G_C} = \dfrac{I_1}{I_C}$. If one resistor has two thirds of the conductance, it will carry two thirds of the current.

10 Power in a potential divider

(a) $I = \dfrac{\epsilon}{\text{Circuit resistance}} = \dfrac{\epsilon}{R+r}$

(b) $V = IR = \dfrac{\epsilon}{R+r} \times R = \dfrac{\epsilon R}{R+r}$

(c) $P = IV = \dfrac{\epsilon}{R+r} \times \dfrac{\epsilon R}{R+r} = \dfrac{\epsilon^2 R}{(R+r)^2}$

(d) $\eta = \dfrac{P}{I\epsilon} = P \times \dfrac{1}{\epsilon I} = \dfrac{\epsilon^2 R}{(R+r)^2} \times \dfrac{1}{\epsilon \times \epsilon / (R+r)} = \dfrac{R}{R+r}$

11 Path and phase difference

(a) $\Delta\phi = \dfrac{\Delta L}{\lambda} \times 360^\circ = \dfrac{d \sin\theta}{\lambda} \times 360^\circ$

(b) $\sin\theta = \dfrac{\Delta L}{d} = \dfrac{n\lambda}{d}$ for constructive interference

(c) $\sin\theta = \dfrac{n\lambda}{d} = \dfrac{n\lambda}{1\,\text{mm}/N} = \dfrac{nN\lambda}{1 \times 10^{-3}\,\text{m}}$

(d) $\sin\theta = \dfrac{nN\lambda}{1 \times 10^{-3}\,\text{m}} = \dfrac{nN\,(v/f)}{1 \times 10^{-3}\,\text{m}} = \dfrac{nNv}{1 \times 10^{-3}\,\text{m} \times f}$

(e) $y = D\tan\theta \approx D\sin\theta = D \times \dfrac{n\lambda}{d} = \dfrac{1 \times \lambda D}{d} = \dfrac{\lambda D}{d}$

(f) $y = D\tan\theta \approx D\sin\theta = D \times \dfrac{n\lambda}{d} = \dfrac{5 \times (v/f)\,D}{d} = \dfrac{5vD}{df}$

(g) $\Delta L = \left(\tfrac{1}{2}D + y\right) - \left(\tfrac{1}{2}D - y\right) = 2y$