Differentiation: Implicit 3ii

A Level

P P P

Part A Derivative

For the curve $2x^2 + xy + y^2 = 14$, find $\frac{dy}{dx}$ in terms of x and y.

The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part B Stationary Points

How many points are there on the curve $2x^2 + xy + y^2 = 14$ at which the tangents are parallel to the x-axis?

Part C Coordinates 1

Find the coordinates of the points at which the tangents to the curve $2x^2 + xy + y^2 = 14$ are parallel to the x-axis.

Give the x-coordinate of the point with the highest (most positive) x-value.

The following symbols may be useful: `x`

Give the y-coordinate of the same point.

The following symbols may be useful: `y`

Part D Coordinates 2

(continued from Part C)

Find the coordinates of the points at which the tangents to the curve $2x^2 + xy + y^2 = 14$ are parallel to the x-axis.

Give the x-coordinate of the point with the lowest (most negative) x-value.

The following symbols may be useful: x

Give the y-coordinate of the same point.

The following symbols may be useful: y

Differentiation: Implicit 5i

A Level

P P P

Part A Derivative

Given that $y \sin 2x + \frac{1}{x} + y^2 = 5$, find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

The following symbols may be useful: `Derivative(y, x), cos(), cosec(), cot(), sec(), sin(), tan(), x, y`

Part B Gradient

Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point $(1, 2)$.

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Differentiation: Implicit 3i

A Level

P P P

The equation of a curve is $xy^2 = 2x + 3y$.

Part A Implicit Differentiation

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

The following symbols may be useful: `Derivative(y, x), x, y`

Part B Tangents

Give the number of tangents to this curve which are parallel to the y-axis.

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Sketching a Parametric Curve

A Level

P P P

A curve has parametric equations $x = 1 - \cos t$, $y = \sin t \sin 2t$, for $0 \leq t \leq \pi$.

Part A Coordinates

At how many different points does the curve meet the x-axis?

Enter the highest of the x-coordinates of the points where the curve meets the x-axis.

The following symbols may be useful: `x`

Part B Derivative

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

The following symbols may be useful: `Derivative(y, x)`, `arccos()`, `arccosec()`, `arccot()`, `arcsec()`, `arcsin()`, `arctan()`, `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `t`, `tan()`, `x`, `y`

Part C Stationary points 1

Hence find, in an exact form, the coordinates of the stationary points.

Enter the exact x-coordinate of the stationary point with the lower x-coordinate.

The following symbols may be useful: x

Enter the exact y-coordinate of the stationary point with the lower x-coordinate.

The following symbols may be useful: y

Part D Stationary points 2

Hence find, in an exact form, the coordinates of the stationary points.

Enter the exact x-coordinate of the stationary point with the higher x-coordinate.

The following symbols may be useful: x

Enter the exact y-coordinate of the stationary point with the higher x-coordinate.

The following symbols may be useful: y

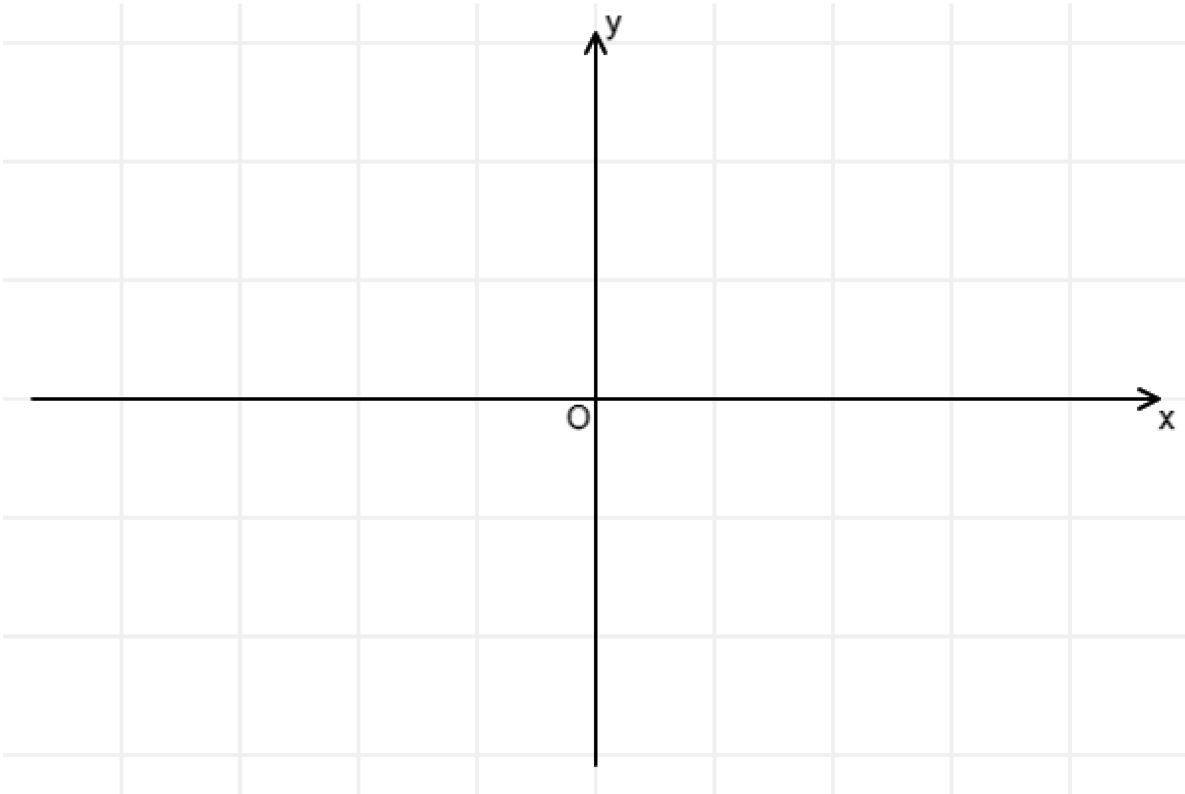
Part E Cartesian Equation

Find the cartesian equation of the curve. Give your answer in the form $y = f(x)$, where $f(x)$ is a polynomial.

The following symbols may be useful: x, y

Part F Sketch

Sketch the curve.



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Parametric Equations 2i

A Level

A curve has parametric equations

$$x = \frac{1}{t+1}, y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

Part A Value of t

Show that the value of t at one of these points is -2 and find the value of t at the other point.

The following symbols may be useful: `t`

Part B Normal

Find the equation of the normal to the curve at the point for which $t = -2$, giving your answer in the form $y = f(x)$.

The following symbols may be useful: `x, y`

Part C Value of t

Find the value of t at the point where this normal meets the curve again.

The following symbols may be useful: `t`

Part D Cartesian Equation

Find a cartesian equation of the curve, giving your answer in the form $y = f(x)$.

The following symbols may be useful: `x`, `y`

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Parametric Equations 3i

A Level

The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, y = 4\sin\theta$$

and part of its graph is shown in **Figure 1**.

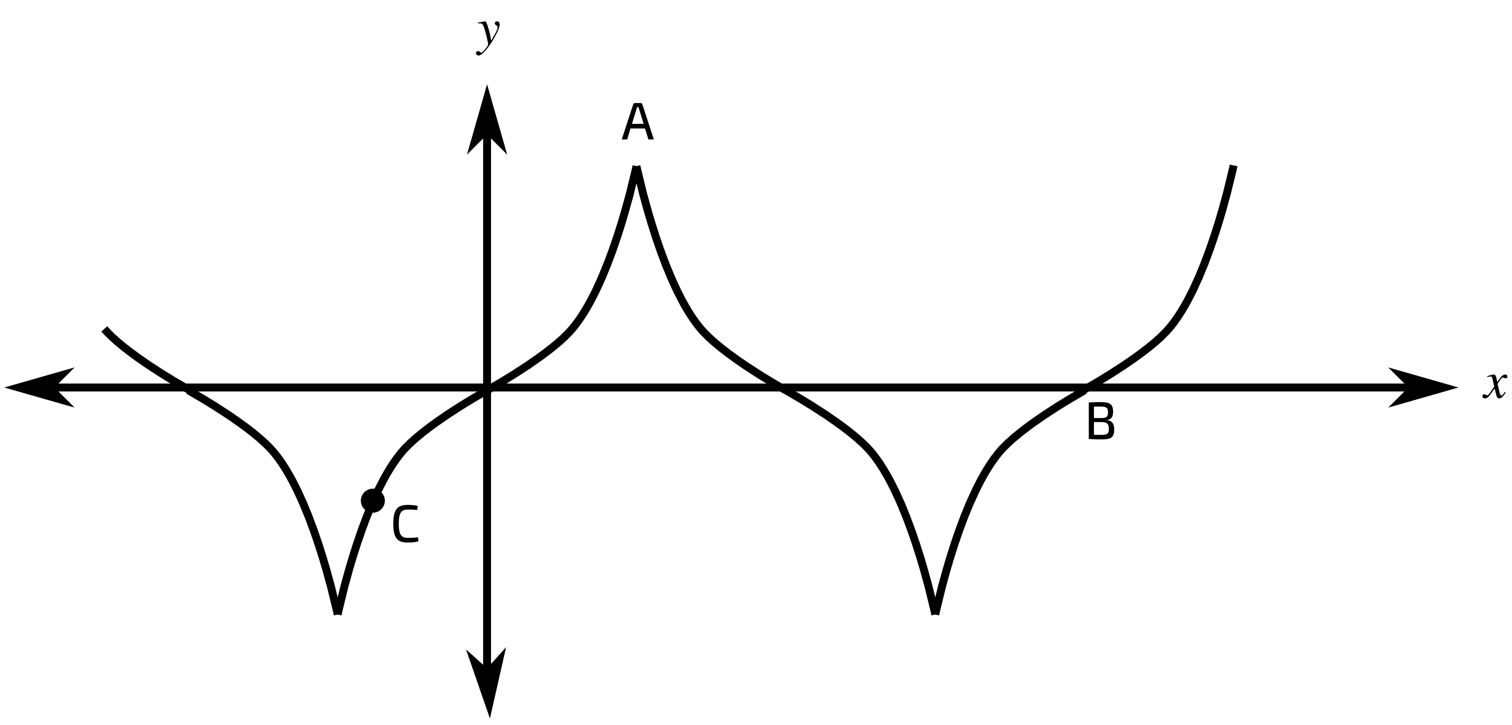

Figure 1: A sketch of the curve.

Part A Value of Theta

Find the value of θ at A.

The following symbols may be useful: `pi`, `theta`

Find the value of θ at B.

The following symbols may be useful: `pi`, `theta`

Part B Derivative

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ.

The following symbols may be useful: `Derivative(y, x)`, `arccos()`, `arccosec()`, `arccot()`, `arcsec()`, `arcsin()`, `arctan()`, `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `tan()`, `theta`, `x`, `y`

Part C Coordinates

At the point C on the curve the gradient is 2. Find the coordinates of C, giving your answer in an exact form.

Find the x-coordinate.

The following symbols may be useful: `pi`, `x`

Find the y coordinate.

The following symbols may be useful: `pi`, `y`

Part D Nature of Origin

Point O is at the origin. State the nature of point O, justifying your answer by reference to suitable values of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

Easier question?

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Parametric Integration 1

A Level

The curve C has parametric equations

$$x = 2t^2 - 3 \qquad y = t(4 - t^2)$$

The curve crosses the x-axis at the points A and B and the region R is enclosed by the loop of the curve, as shown in **Figure 1**.

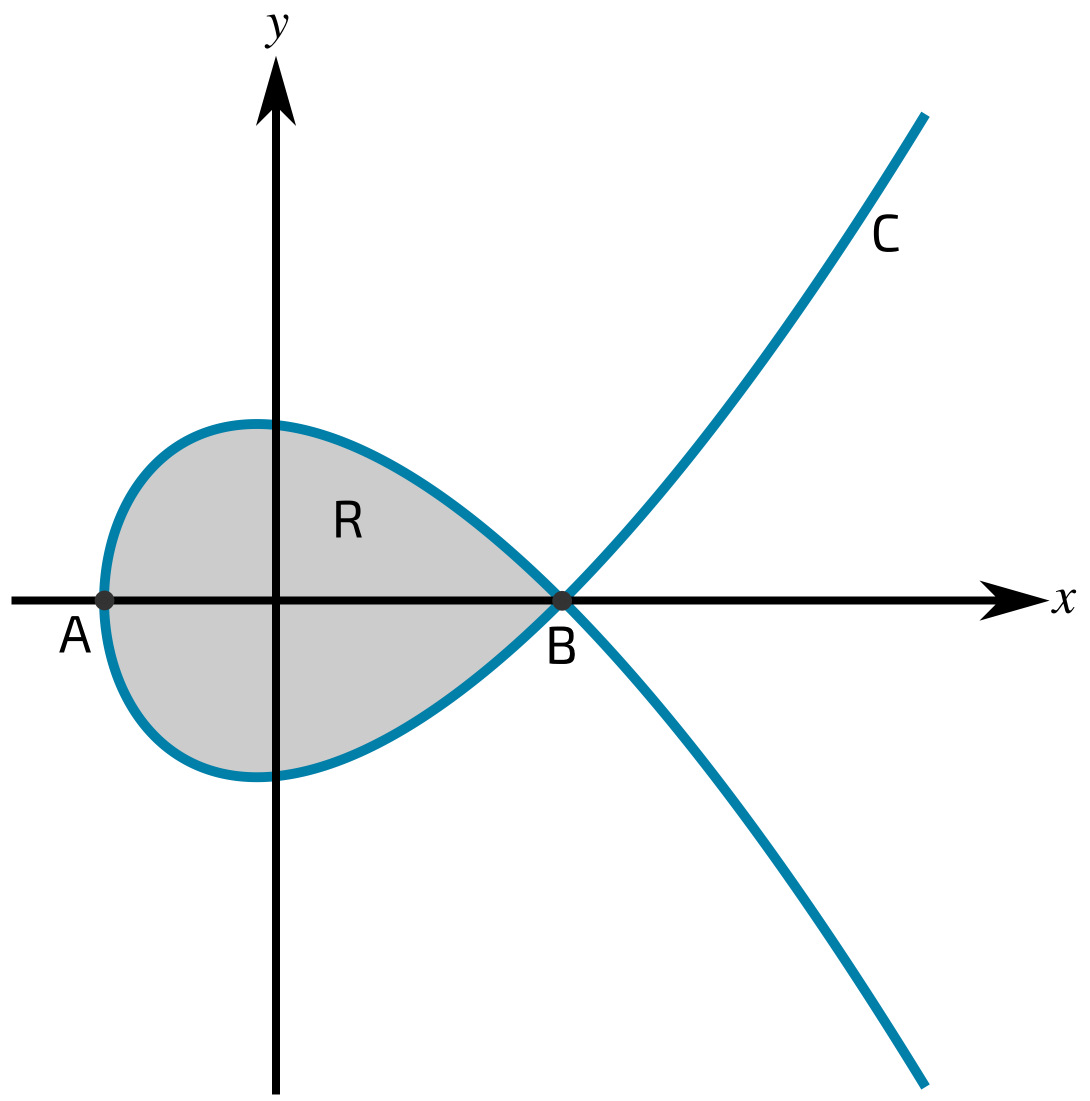

Figure 1: A graph of the curve C.

Part A Point A

Find the x-coordinate of the point A.

Part B Point B

Find the x-coordinate of the point B.

Part C Area of R

The region R is enclosed by the loop of the curve, as shown in **Figure 1**. Find the exact value of the area of R.

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Partial Fractions 3ii

A Level

The equation of a curve is $y = f(x)$, where $f(x) = \dfrac{3x+1}{(x+2)(x-3)}$.

Part A Partial Fractions

Hence express $f(x)$ in partial fractions.

The following symbols may be useful: `x`

Part B Derivative

Hence find $f'(x)$.

The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part C Deduction

Hence deduce that the gradient of the curve is negative for all points on the curve.

More practice questions?

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Partial Fractions 1i

A Level

P P P

Part A Partial Fractions

Express $\frac{2+x^2}{(1+2x)(1-x)^2}$ in the form $\frac{A}{1+2x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$.

The following symbols may be useful: x

Part B Integration

Hence find $\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2} \mathrm{d}x$ in exact form.

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Integration With Partial Fractions 2

A Level Further A

P P P P P P

Write the function $\dfrac{2z^2-z-3}{(z+2)(z^2-2z-1)}$ in the form $\dfrac{A}{z+2}+\dfrac{B+Cz}{z^2-2z-1}$. Hence find $\displaystyle\int_1^2 \frac{2z^2-z-3}{(z+2)(z^2-2z-1)}\,\mathrm{d}z$.

Part A Find A

Find the constant A

Part B Find B

Find the constant B.

Part C Find C

Find the constant C.

Part D Integrate

Hence find $\displaystyle\int_1^2 \frac{2z^2-z-3}{(z+2)(z^2-2z-1)}\,\mathrm{d}z$.

The following symbols may be useful: `cos()`, `cosec()`, `cosech()`, `cosh()`, `cot()`, `coth()`, `ln()`, `log()`, `sec()`, `sech()`, `sin()`, `sinh()`, `tan()`, `tanh()`, `z`

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