Physics. You work it out.

Home Gameboard Maths Curves and Integration

Curves and Integration

A Level

C C C

Part A Working back from $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

A curve has an equation which satisifies $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 3x^{-\frac{1}{2}}$. The point $P(4, 1)$ lies on the curve, and the gradient of the curve at P is 5.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(x , y), x, y`

Part B The curve

Now find the equation of the curve from your answer in Part A.

The following symbols may be useful: `x, y`

Part C Linear factor of $f(x)$

Figure 1 shows the curve $y = f(x)$, where $f(x) = -4x^3 + 9x^2 + 10x - 3$.

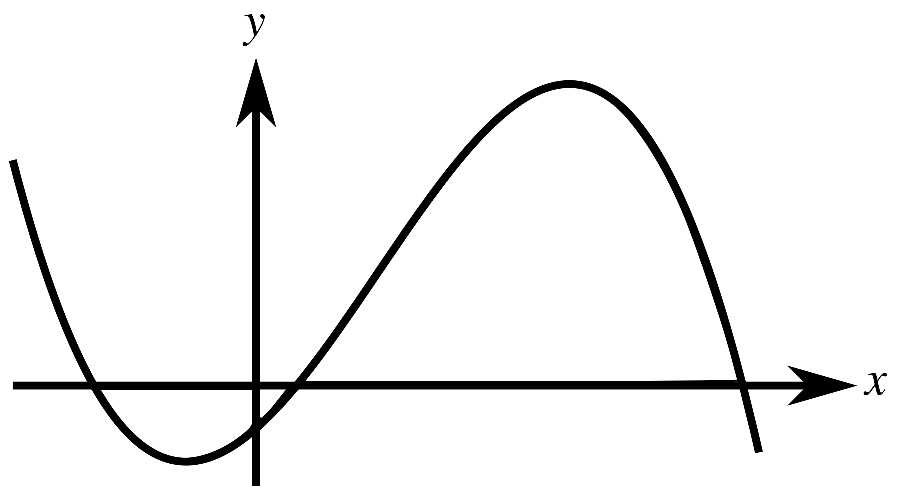

Figure 1: Diagram of the curve $y = f(x)$, where $f(x) = -4x^3 + 9x^2 + 10x - 3$.

Verify that the curve crosses the x-axis at $(3, 0)$ and hence state a factor of $f(x)$.

The following symbols may be useful: `x`

Part D Quadratic factor of $f(x)$ and roots

Using your result from Part C, express $f(x)$ as the product of a linear factor and a quadratic factor.

The following symbols may be useful: `f`, `x`

Part E The most negative root

Hence find the other two points of intersection of the curve with the x-axis and enter the most negative value as your answer.

The following symbols may be useful: `x`

Part F Area under the curve

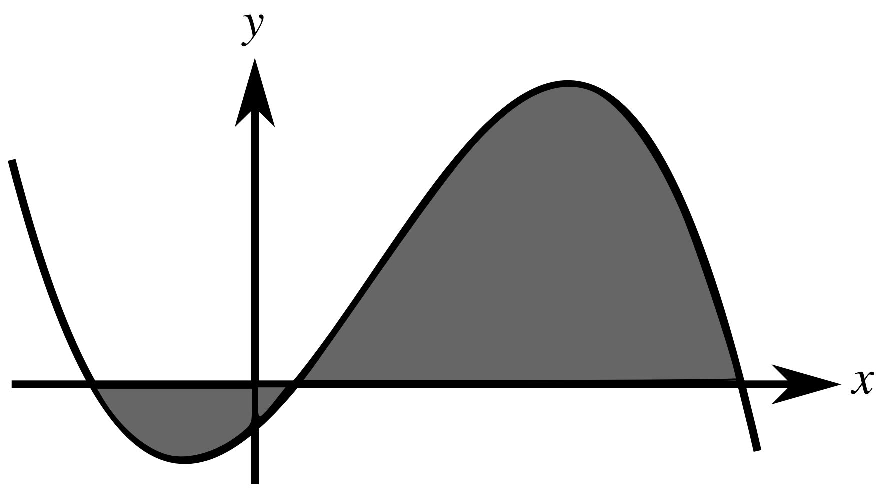

Figure 2: Diagram of the curve $y = f(x)$, where $f(x) = -4x^3 + 9x^2 + 10x - 3$.

The region enclosed by the curve and the x-axis is shaded in **Figure 2**.

Use integration to find the total area of this region. Enter your answer as a number to 3 significant figures.

Adapted with permission from UCLES, A Level, June 2015 Paper 4722 Question 5, and January 2011 Paper 4722 Question 9.

Calculus

A Level

C C C

Part A Integrating a factorised expression

Find $\int (x^2 + 9)(x - 4)\mathrm{d}x$.

The following symbols may be useful: `c, x`

Part B Differentiation

A curve has the equation $y = \frac{1}{3}x^3 - 9x$.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), x, y`

Part C Stationary points

Find the coordinates of the stationary points of the curve $y = \frac{1}{3}x^3 - 9x$. Enter the x and y coordinates of the stationary point with the largest x coordinate.

Enter the x-coordinate of the stationary point with the largest (most positive) x:

The following symbols may be useful: x

Enter its corresponding y coordinate:

The following symbols may be useful: y

Part D Nature of stationary point

Determine the nature of the stationary point with the largest x-coordinate.

- Maximum
- Neither/Inconclusive
- Minimum

Part E Tangent to the curve

Given that $24x + 3y + 2 = 0$ is the equation of the tangent to the curve $y = \frac{1}{3}x^3 - 9x$ at the point (p, q), find the values of p and q.

(i) Enter value of p:

The following symbols may be useful: p

(ii) Enter value of q:

The following symbols may be useful: q

Part F Normal to the curve

Find the equation of the normal to the curve $y = \frac{1}{3}x^3 - 9x$ at the point (p, q) you found in Part E.

Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers

The following symbols may be useful: x, y

Modified by Sally Waugh with permission from UCLES, A Level, June 2005, Paper 4721, Question 10.

Gameboard:
STEM SMART Single Maths 15 - Pure Revision (calculus)

Exponential Rates

A Level

An experiment involves two substances, Substance 1 and Substance 2, whose masses are changing. The mass, M_1 grams, of Substance 1 at time t hours is given by

$$M_1 = 400\mathrm{e}^{-0.014t}$$

.

The mass, M_2 grams, of Substance 2 is increasing exponentially and the mass at certain times is shown in the following table.

t (hours)	0	10	20
M_2 (grams)	75	120	192

A critical stage in the experiment is reached at time T hours when the masses of the two substances are equal.

Part A Rate of change of Substance 1

Find the rate at which the mass of Substance 1 is changing when $t = 10\,\mathrm{hours}$, giving your answer in grams per hour $(\mathrm{g\,hour^{-1}})$ correct to 2 significant figures.

Part B Solving for T

Show that T is the root of an equation of the form $\mathrm{e}^{kt} = c$. State the values of the constants k and c.

What is the value of k?

What is the value of c? Please give your answer to 3 significant figures.

Part C Value of T

Find the value of T to 3 significant figures.

Used with permission from UCLES, June 2011, OCR C3 Paper 4723, question 8.

Gameboard:
STEM SMART Single Maths 15 - Pure Revision (calculus)

Area of isosceles triangle

A Level Further A

C C C P P P

The isosceles triangle shown in **Figure 1** has a base of length $2b$ and perpendicular height h. The length p of the perimeter of the triangle is fixed. Find an expression in terms of p for the value of b which will maximise the area A of the triangle. Find an expression for this maximum area.

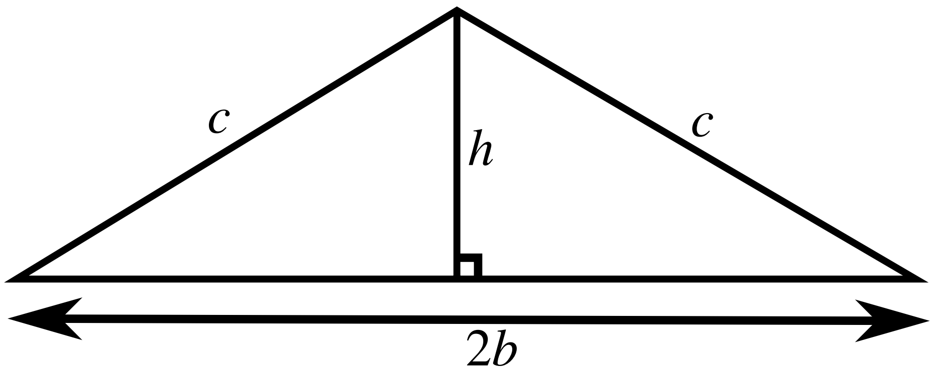

Figure 1: An isosceles triangle with a base of length $2b$, perpendicular height h and sides of length c.

Part A Area A and perimeter p

Write down the equation for the area A of the triangle in terms of b and h.

The following symbols may be useful: `A, b, h`

Find the equation for the perimeter p of the triangle in terms of b and h.

- $p = 2b + \sqrt{b^2 + h^2}$
- $p = b + \sqrt{b^2 + h^2}$
- $p = b + 2\sqrt{b^2 + h^2}$
- $p = 2b + \sqrt{4b^2 + h^2}$
- $p = 2\,(b + \sqrt{b^2 + h^2})$
- $p = 2b + 2\sqrt{4b^2 + h^2}$

Using the above, obtain an equation for A in terms of p and b.

The following symbols may be useful: `A, b, p`

Part B Expressions for b and h

Using the equation for A you found in Part A, find an **expression** in terms of p for the value of b which will maximise the area A of the triangle. (Since p is fixed you may treat it as a constant.)

Hint: you may not know how to differentiate the expression for A, but note that since A is positive it will be a maximum when A^2 is a maximum.

The following symbols may be useful: `p`

Find, in terms of p, the expression for h corresponding to this value of b.

The following symbols may be useful: `p`

Part C The maximum area

Using your result from Part B, find an expression for the maximum area in terms of p.

The following symbols may be useful: p

Part D Check that the area is a maximum

Find, at the value of b deduced above, an expression in terms of p for the second derivative of A^2 with respect to b; convince yourself that the value of the second derivative indicates that the value of A^2, and hence of A, is a maximum.

The following symbols may be useful: p

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