Indefinite integrals 1

A Level Further A

Find the following indefinite integrals.

Part A Integrate $(3x-1)(x+1)$

Find $\int (3x-1)(x+1)\mathrm{d}x$.

The following symbols may be useful: `k, x`

Part B Integrate $(\sqrt{p}-(1/p))^2$

Find $\displaystyle\int \left(\sqrt{p}-\frac{1}{p}\right)^2 \mathrm{d}p$.

The following symbols may be useful: `k, p`

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Equation of curve

A Level C C C Further A P P P

A function $v(u)$ is such that $\frac{\mathrm{d}v}{\mathrm{d}u} = \frac{1}{3}u^{\frac{1}{3}}\ 1 - \frac{1}{u}$ and $v(8) = -1$.

Find the equation of the function $v(u)$.

The following symbols may be useful: `u, v`

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Indefinite integrals 2

A Level Further A

Find the following indefinite integrals.

Part A Integrate $(q^2+3)/q^{\frac{5}{2}}$

Find $\int \frac{q^2+3}{q^{5/2}}\mathrm{d}q$.

The following symbols may be useful: `k, q`

Part B Integrate $2z(z^2-1)(z^2+1)$

Find $\int 2z(z^2-1)(z^2+1)\mathrm{d}z$.

The following symbols may be useful: `k, z`

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Area under a curve 3

A Level Further A

A graph of the functions $\frac{}{\sqrt{}}$ and $\sqrt{}$ for is shown in **Figure 1**. Find the area of the shaded region OPQR.

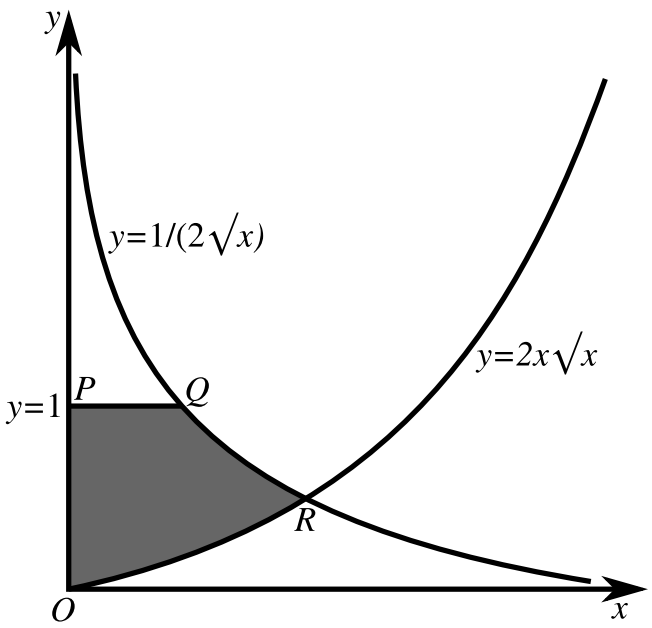

Figure 1: A graph of the functions $\frac{}{\sqrt{}}$ and $\sqrt{}$ for . The shaded region OPQR is bounded by the line , the line , the curve $\frac{}{\sqrt{}}$ and the curve $\sqrt{}$.

Part A The coordinate of Q

Deduce the coordinate of the point Q.

Part B The coordinate of R

Find the coordinate of the point R.

Part C The area of OPQR

Find the area of the shaded region OPQR, giving your answer in an exact form.

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Definite integrals 3

A Level Further A

For the integrals given below, find the values of a and m respectively.

Part A $\int_{-1}^{a} x^{-4/5}\mathrm{d}x = 10$

Given that the definite integral $\int_{-1}^{a} x^{-4/5}\mathrm{d}x = 10$, find the value of a.

The following symbols may be useful: a

Part B $\int_{0}^{4} [(m+1)/2] x^{m}\mathrm{d}x = 4$

Given that the integral $\int_{0}^{4} \frac{(m+1)}{2} x^{m}\mathrm{d}x = 4$, find the value of m.

The following symbols may be useful: m

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Area under a curve 1

A Level Further A

A graph of the functions $y = (x-2)(x+1)$ and $y = x+1$ is shown in **Figure 1**. Find the areas of the shaded regions labelled A and B. A is the region between P and Q enclosed by the curve $y = (x-2)(x+1)$ and the x-axis; B is the region between Q and R below the curve $y = (x-2)(x+1)$ and above the x-axis.

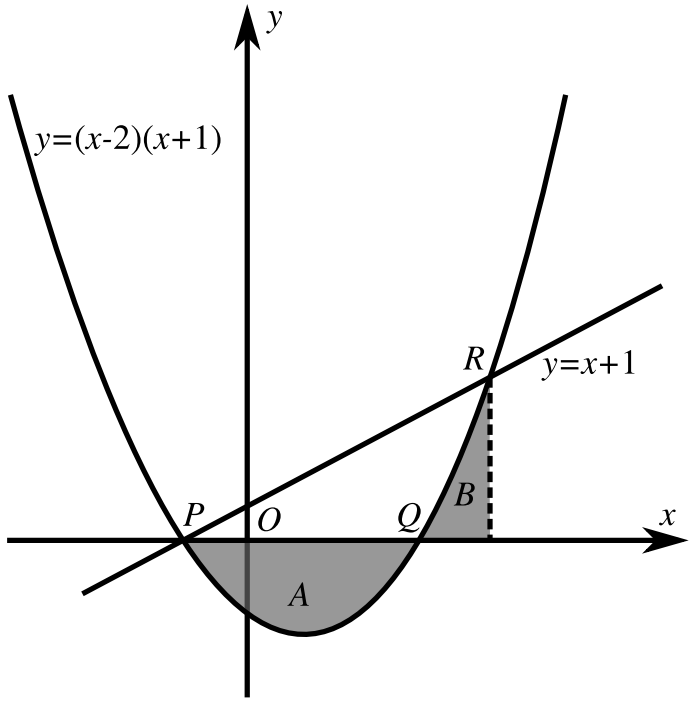

Figure 1: A graph of the functions $y = (x-2)(x+1)$ and $y = x+1$. A is the region between P and Q enclosed by the curve $y = (x-2)(x+1)$ and the x-axis; B is the region between Q and R below the curve $y = (x-2)(x+1)$ and above the x-axis.

Part A Region A

Find the area of the region A. Give your answer in the form of an improper fraction.

Part B Region B

Find the area of the region B. Give your answer in the form of an improper fraction.

Integrating Powers 4

A Level C C C Further A C C C

Part A Integrate $(A/r^7 - B/r^{13})$

Find $\int_a^\infty \left(\frac{A}{r^7} - \frac{B}{r^{13}}\right) \mathrm{d}r$.

(The force between, for example, two atoms of an inert gas, a distance r apart is given by $\left(\frac{A}{r^7} - \frac{B}{r^{13}}\right)$, where A and B are (negative) constants; the first term is the attractive force between them (the van der Waals interaction, due to their fluctuating induced dipoles) and the second is the repulsive force due to the overlap of their electron shells. The integral describes the potential energy of such a system i.e. the work done bringing one atom from infinity to within a distance a of the other atom.)

Find $\int_a^\infty \left(\frac{A}{r^7} - \frac{B}{r^{13}}\right) \mathrm{d}r$.

The following symbols may be useful: `A, B, a`

Part B Integrate $(C/x^2 + D)$

Find $\int_{x_1}^{x_2} \left(\frac{C}{x^2} + D\right) \mathrm{d}x$.

(The function $\left(\frac{C}{x^2} + D\right)$ could describe the component of an electric field in the x-direction due to a combination of the field due to a point charge at the origin and a uniform field in the x-direction. The integral is then the potential difference between two points x_1 and x_2 on the x-axis.)

Find $\int_{x_1}^{x_2} \left(\frac{C}{x^2} + D\right) \mathrm{d}x$.

The following symbols may be useful: `C, D, x_1, x_2`

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Integrating Powers 2

A Level Further A

Part A Integrate ax^{-8}

Find the indefinite integral of ax^{-8}, where a is a constant.

The following symbols may be useful: `a, k, x`

Part B Integrate $4/x^2$

Find $\int_1^2 \frac{4}{x^2}\mathrm{d}x$.

The following symbols may be useful: `k, x`

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Integrating Powers 1

A Level Further A

Part A Integrate $4x^3$

Find the indefinite integral of $4x^3$.

The following symbols may be useful: `k, x`

Part B Integrate αx.

Find $\int_0^{x_0} \alpha x \mathrm{d}x$, where α is a constant.

The following symbols may be useful: `alpha, x, x_0`

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Definite integrals 2

A Level Further A

Find the following integrals.

Part A $\int_1^\infty (3/2x\sqrt{x})\mathrm{d}x$

Find $\int_1^\infty \frac{3}{2}\frac{1}{x\sqrt{x}}\mathrm{d}x$.

Part B $\int_{-8}^0 1/\sqrt[3]{x}\mathrm{d}x$

Find $\int_{-8}^0 \frac{1}{\sqrt[3]{x}}\mathrm{d}x$.

Part C $\int_{-1}^1 (1+x+x^2/2+x^3/6)\mathrm{d}x$

Find $\int_{-1}^1 \left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)\mathrm{d}x$. Give your answer as an improper fraction.

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