37 Springs and Elastic Deformation

extension = (length of a spring or material) − (its unstretched length)

For a spring or any material below its limit of proportionality, the force stretching (or compressing) the material is proportional to its extension (or compression). Twice the force causes twice the extension. This is Hooke's Law.

When the force is removed, it goes back to its original length. This is called elastic deformation.

The spring constant k measures the force needed to stretch it by 1 cm or 1 m. The unit of the spring constant is N/cm or N/m.

Formula:

$$\text{force (N)} = \text{spring constant (N/m)} \times \text{extension (m)} \qquad F = kx$$

For extension, two symbols are commonly used, e and x.

Example 1 – A spring is 5.0 cm long when unstretched. With a 6.0 N force stretching it, it becomes 13 cm long. What is the spring constant?

extension $= 13.0$ cm $- 5.0$ cm $= 8.0$ cm.

spring constant $=$ force/extension $= 6.0$ N$/8.0$ cm $= 0.75$ N/cm
or 6.0 N$/0.080$ m $= 75$ N/m

Example 2 – What will the length of this spring be when it is stretched with a 9.0 N force?

extension $=$ force/spring constant $= 9.0$ N$/(0.75$ N/cm$) = 12$ cm
length $= 5.0$ cm $+ 12$ cm $= 17$ cm.

37.1 A spring takes a force of 5.0 N to extend it by 2.0 cm.

(a) What is the spring constant in N/m?

(b) What force is needed to extend it by 10 cm?

(c) What force is needed to extend it by 12 cm?

(d) What force is needed to extend it by 5.0 mm?

(e) What is the extension when the force is 45 N?

(f) What is the extension when the force is 7.0 N?

37.2 A spring is 6.0 cm long when it is not stretched, and 10 cm long when a 7.0 N force is applied.

(a) What is the spring constant in N/m?

(b) How long will it be with a 21 N force?

(c) How long will it be with a 5.0 N force?

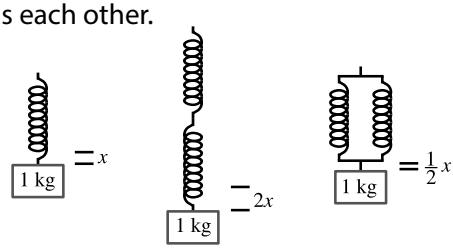
(d) What force is needed to make it 20 cm long?

37.3 A company is making newton meters using 150 N/m springs.

(a) How far from the 0.0 N mark will the 10 N mark need to be?

(b) How far apart will the 4.0 N and 9.0 N marks be?

When two springs support a weight in series (one hanging off the other), they each carry the full weight of the load. When two identical springs support a weight in parallel, they share the weight of the load, but have the same extension as each other.



37.4 Two identical springs each have a spring constant of 200 N/m. A mass of 9.7 kg is hung from one spring, which is supported by the other. The top spring is supported by a strong beam.

(a) What is the total extension of the springs?

(b) The two springs are now set up in parallel to share the load.

What is the extension of each spring when the mass is supported?

Potential energy stored in a stretched spring

$$\text{work done when stretching a spring} = \text{force} \times \text{distance}$$

However, the force changes as you stretch the spring. To start with, very little force is needed to stretch it. At the end, the force is $F = kx$ where x is the final extension. The average force is $\frac{1}{2}kx$.

$$\begin{aligned}\text{work done in stretching a spring} &= \text{average force} \times \text{distance} \\ &= \tfrac{1}{2}kx \times x = \tfrac{1}{2}kx^2\end{aligned}$$

$$\begin{aligned}\text{elastic potential energy (J)} &= \tfrac{1}{2} \times \text{spring constant} \times \text{extension}^2 \\ E &= \tfrac{1}{2}kx^2\end{aligned}$$

Example 3 – Calculate the elastic potential energy stored when a $1\,000$ N/m spring is stretched by 3.0 cm from its natural length.

With energy calculations, you should always use distances in metres.
Energy $= \frac{1}{2}kx^2 = \frac{1}{2} \times 1\,000$ N/m $\times (0.030\text{ m})^2 = 0.45$ J

37.5 Calculate the elastic potential energy stored when a $1\,000$ N/m spring is stretched by 6.0 cm from its natural length.

37.6 Calculate the elastic potential energy stored when a $1\,000$ N/m spring is stretched by a 20 N force. [Hint: first work out the extension.]

37.7 Calculate the extension of a 500 N/m spring when storing 3.0 J.

37.8 *[Harder]* Calculate the elastic potential energy when a 40 N force stretches a spring by 0.50 cm.

37.9 How much strain energy is

(a) stored in a 40 kN/m spring when it is stretched by 3.0 cm?

(b) released on now relaxing to a stretch of 1.5 cm?