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Acceleration f(t) 1i

A Level
P P P

A particle P moves in a straight line. At time $t\,\mathrm{s}$ after passing through a point O of the line the displacement of P from O is $x\,\mathrm{m}$ where $x = 0.06t^3 - 0.45t^2 - 0.24t$.

Part A Velocity of P

Find the velocity of P when $t = 0$.

Part B Acceleration of P

Find the acceleration of P when $t = 0$.

Part C Minimum velocity of P

Find the speed of P when it is at its minimum velocity. Give your answer to 3 significant figures.

Part D Positive value of t

Find the positive value of t when the direction of motion of P changes. Give your answer to 3 significant figures.

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Acceleration f(t) 4ii

A Level

A particle moves in a straight line. Its velocity $t\,\mathrm{s}$ after leaving a fixed point on the line is $v\,\mathrm{m\,s^{-1}}$, where $v = t + 0.1t^2$.

Part A Acceleration

Find an expression for the acceleration of the particle at time t.

The following symbols may be useful: `t`

Part B Distance travelled

Find the distance travelled by the particle from time $t = 0$ until the instant when its acceleration is $2.8\,\mathrm{m\,s^{-2}}$. Give your answer to 3 significant figures.

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STEM SMART Physics 32 - Writing Differential Equations

Acceleration f(t) 3ii

A Level
P P P

A cyclist travels along a straight road. Her velocity $v\,\mathrm{m\,s^{-1}}$, at time t seconds after starting from a point O, is given by

$$v = 2 \text{ for } 0 \leq t \leq 10$$

$$v = 0.03t^2 - 0.3t + 2 \text{ for } t \geq 10$$

Part A Displacement at $t = 10$

Find the displacement of the cyclist from O when $t = 10$.

Part B Expression for displacement

Find an expression for the displacement of the cyclist from O as a function of time for $t \geq 10\,\mathrm{s}$. Give your answer using fractions, not decimals.

The following symbols may be useful: `t`

Part C Time

Find the time when the acceleration of the cyclist is $0.6\,\mathrm{m\,s^{-2}}$.

Part D Displacement

Find the displacement of the cyclist from O when her acceleration is $0.6\,\mathrm{m\,s^{-2}}$.

Used with permission from UCLES, A Level, June 2006, OCR M1, Question 4

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7.4.3 Falling Through Air I

Further A University

A ball of mass m is dropped and moves quickly through the air, receiving a drag force proportional to the square of its speed. The constant of proportionality is q.

Part A A diagram

Draw a diagram showing all the forces acting on the ball.

Part B Differential equation of motion

Write a differential equation in terms of v that describes the motion of the ball. Write you answer in a form with zero on the right hand side. Take down to be the positive direction.

The following symbols may be useful: `Derivative(_, t), g, m, q, t, v`

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7.4.1 Sedimentation I

Further A University

A spherical particle of effective mass m sediments through a fluid, receiving a retarding force proportional to its velocity. Denote its speed by v and the relevant constant of proportionality by k. Ignore buoyancy forces.

A "particle of **effective mass** m" suggests Newton's Second Law and inertia, and also suggests gravity, weight force etc.; effective suggests there may be other effects acting, but that you can ignore them by incorporating them into m – here it is Archimedian upthrust

"**sediments**" suggests falling under gravity, but slowly (and thus a drag proportional to speed – but you will be given that here)

"receiving a **retarding**" means decelerating, oppositely directed to the velocity

"**force**" suggests Newton's Second Law, equations of motion

"**proportional to its velocity**" means a force directed along the motion, and clearly "retarding" means the proportionality is negative overall, that is, the force is directed against the motion

Part A Diagram

Draw a diagram showing all the forces that are acting on the particle at time t.

When drawing your diagram it is important to consider the directions that each force is acting on the particle. You should also consider where each force is acting from. For example, the weight of the particle must pass through the centre of mass.

Part B Equation of motion

Write down a differential equation for the velocity of the particle, taking the downward direction to be positive.

The following symbols may be useful: `Derivative(_, t), g, k, m, t, v`

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7.4.2 Mass on a Spring I

Further AUniversity

P P P P P P

A mass m lies on a smooth horizontal surface and is attached to one end of a massless spring of constant k, the other end of which is anchored. The displacement of m, away from where the spring takes its natural length, is x which can take positive and negative values.

Part A Diagram

Draw a diagram showing the forces acting on the mass.

Part B Differential equation

Write down the equation of motion for $x(t)$ in the form $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + cx = 0$. Where c is a constant made up of k and m.

The following symbols may be useful: `A, B, Derivative(_,t,t), cos(), k, m, omega_0, sin(), x`

Part C Retarding Force

The mass now suffers a retarding frictional force proportional to its velocity (with constant of proportionality q). Write down the differential equation of motion for the mass in the form $a\frac{\mathrm{d}^2x}{\mathrm{d}t^2} \pm b\frac{dx}{dt} \pm cx = 0$ where a, b and c are constants, and $\pm$ means choose a suitable sign.

The following symbols may be useful: `Derivative(_,t), Derivative(_,t,t), cos(), d, k, m, omega_0, q, sin(), t, x`

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7.4.4 A Rocket I

Further AUniversity

A rocket, currently of mass m, burns fuel at a steady rate α (mass per unit time). The fuel leaves the rocket with a speed v_e relative to the rocket. The rocket initially has mass m_0.

Write down an equation for the acceleration a of the rocket as seen by a stationary observer. Neglect gravity, drag, steering etc.

The following symbols may be useful: `a, alpha, m_0, t, v_e`

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7.3.1 Discharging a Metal Sphere

A Level Further A

A metal sphere/dome is charged with positive charge using a high voltage generator; see **Figure 1**. The dome is continuously charged such that a constant current flows through a resistor $R = 1.0 \times 10^{11}\Omega$. The potential shown on the voltmeter is $150,000\,\mathrm{V}$.

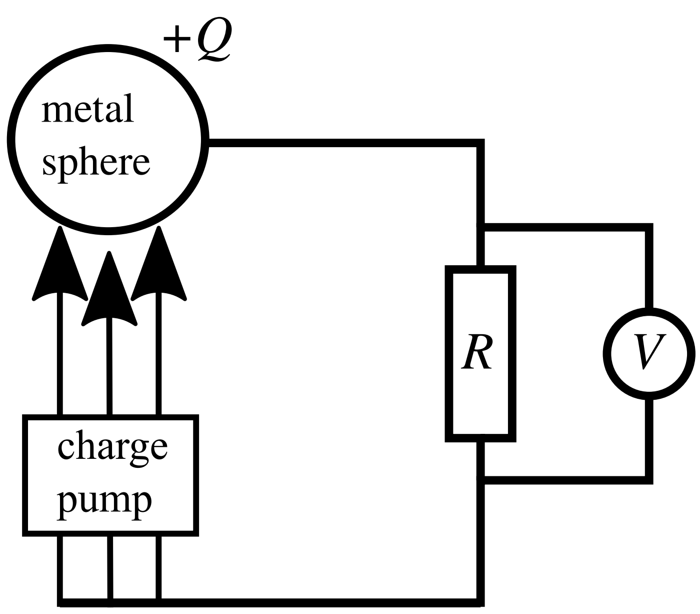

Figure 1: A positively charged metal sphere discharging through a resistor.

Part A Current flow

Calculate the current I flowing through the resistor.

Part B Leakage

The generator is now stopped at time $t = 0$, so that the charge on the dome starts to leak away. What is the initial rate at which the charge leaks away through the resistor?

Part C Initial charge

The potential V on the dome is proportional to the charge on the dome,with constant of proportionality C (the capacitance), that is $Q = VC$. Use the relation between V and I to turn this into an equation linking Q and I.

The following symbols may be useful: `C`, `I`, `Q`, `R`, `V`

Part D Rate of change of charge

Relate I to V and to rates of change of Q to derive an expression for $\mathrm{d}Q/\mathrm{d}t$.

The following symbols may be useful: `C`, `Derivative(Q, t)`, `Q`, `R`, `V`, `e`, `ln()`, `log()`, `t`

Part E Time constant

Recognise the form of the solution to this equation is $f(t/\tau)$, and give the time constant τ characterising changes in charge remaining on the dome.

The following symbols may be useful: `C`, `I`, `Q`, `R`, `V`, `e`, `ln()`, `log()`, `tau`

Part F Solution for $Q(t)$

What is the solution $Q(t)$, given $Q = Q_0$ at $t = 0$.

The following symbols may be useful: `C`, `I`, `Q(t)`, `Q_0`, `R`, `V`, `e`, `ln()`, `log()`, `t`, `tau`

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7.3.2 People

A Level Further A

C C C C C C

Without intervention, the rate of population growth is proportional to the population, P, at the time. Malthus first recognised that this dependence must lead to disaster(s), the nature of which he, and others since, have speculated on. This is known as the Malthusian catastrophe.

Part A Examples

What are examples of these disasters?

Part B Form of P

What form does $P(t)$ take if the rate of growth is $r\,\%$ per year? Assume that $P(t=0)=P_0$ and t is in years. Give your answer in the form $P = \ldots$.

The following symbols may be useful: `P`, `P_0`, `e`, `r`, `t`

Part C USA doubling

In the USA currently the growth rate $r = 1.5\,\%$ per year. What is the population doubling time? (Give your answer in years.)

Part D Avoid catastrophe

How can Malthusian catastrophes be avoided?

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Maximum Energy Transfer

A Level Further A
C C C P P P

A particle of mass M collides head on with a stationary particle of mass m. No energy is lost in the collision.

The ratio $R(m)$ of the kinetic energy of the mass m after the collision to the initial kinetic energy of mass M is given by

$$R(m) = \frac{4Mm}{(M+m)^2}.$$

Find, in terms of M, an expression for the value of m for which this ratio R is a maximum i.e. the fraction of the kinetic energy transferred from the mass M to the mass m is a maximum. Find the corresponding maximum value of R. Check that R is indeed a maximum for the value of m you found.

Hint: you may not know how to differentiate $\frac{4Mm}{(M+m)^2}$ but note that R will be a maximum when its reciprocal is a minimum.

Part A The value of m

Find, in terms of M, an expression for the value of m for which the fraction of the kinetic energy transferred to the mass m is a maximum.

The following symbols may be useful: `M, m`

Part B The maximum value of R

Using your result from Part A find the maximum value for the fraction of the kinetic energy transferred to mass m.

The following symbols may be useful: `R`

Part C Check R is a maximum

Find, at the value of m deduced in Part A, an expression in terms of M for the second derivative of the reciprocal of R; convince yourself that the value of the second derivative indicates that the value of R is a maximum at this point.

The following symbols may be useful: `M`

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