Isaac Physics Skills

Linking concepts in pre-university physics

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TABLE OF PHYSICAL CONSTANTS

Quantity & Symbol		**Magnitude**	**Unit**
Permittivity of free space	ϵ_0	8.85×10^{-12}	$\mathrm{F\ m^{-1}}$
Electrostatic force constant	$1/4\pi\epsilon_0$	8.99×10^{9}	$\mathrm{N\ m^{2}\ C^{-2}}$
Speed of light in vacuum	c	3.00×10^{8}	$\mathrm{m\ s^{-1}}$
Specific heat capacity of water	c_{water}	4180	$\mathrm{J\ kg^{-1}\ K^{-1}}$
Charge of proton	e	1.60×10^{-19}	C
Gravitational field strength on Earth	g	9.81	$\mathrm{N\ kg^{-1}}$
Universal gravitational constant	G	6.67×10^{-11}	$\mathrm{N\ m^{2}\ kg^{-2}}$
Planck constant	h	6.63×10^{-34}	J s
Boltzmann constant	k_{B}	1.38×10^{-23}	$\mathrm{J\ K^{-1}}$
Mass of electron	m_{e}	9.11×10^{-31}	kg
Mass of neutron	m_{n}	1.67×10^{-27}	kg
Mass of proton	m_{p}	1.67×10^{-27}	kg
Mass of Earth	M_{Earth}	5.97×10^{24}	kg
Mass of Sun	M_{Sun}	2.00×10^{30}	kg
Avogadro constant	N_{A}	6.02×10^{23}	$\mathrm{mol^{-1}}$
Gas constant	R	8.31	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Radius of Earth	R_{Earth}	6.37×10^{6}	m

OTHER INFORMATION YOU MAY FIND USEFUL

Electron volt	1 eV	=	1.60×10^{-19} J
Unified mass unit	1 u	=	1.66×10^{-27} kg
Absolute zero	0 K	=	-273 °C
Year	1 yr	=	3.16×10^{7} s
Light year	1 ly	=	9.46×10^{15} m
Parsec	1 pc	=	3.09×10^{16} m

PREFIXES

1 km = 1000 m	1 Mm = 10^{6} m	1 Gm = 10^{9} m	1 Tm = 10^{12} m
1 mm = 0.001 m	1 μm = 10^{-6} m	1 nm = 10^{-9} m	1 pm = 10^{-12} m

23 Energy and fields – accelerator

It is helpful to be able to calculate the kinetic energy, momentum or speed of a charged particle which has been accelerated by a known voltage.

Example context: many particle accelerators, and the electron guns in older TVs and oscilloscopes, produce beams of charged particles using an electric field. A knowledge of the accelerating voltage enables the speed to be calculated.

Quantities:
m mass (kg)
p momentum $\left(\mathrm{kg\,m\,s^{-1}}\right)$
u initial speed $\left(\mathrm{m\,s^{-1}}\right)$
v final speed $\left(\mathrm{m\,s^{-1}}\right)$
F force (N)
λ wavelength (m)
q charge (C)
K kinetic energy (J)
V accelerating voltage (V)
E electric field $\left(\mathrm{N\,C^{-1}}\right)$
L length of accelerating region (m)

Equations:
$$p = mv \qquad \Delta K = K_{\text{final}} - K_{\text{initial}} = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2 \qquad \Delta K = qV$$
$$\lambda = \frac{h}{p} \qquad F = qE \qquad \Delta K = FL \qquad 1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$$

23.1 Use the equations to derive expressions for

a) the momentum p in terms of V, m and q if $u = 0$,

b) the speed v in terms of V, m and q if $u = 0$,

c) the speed v if $u \neq 0$,

d) the additional kinetic energy ΔK in terms of E, L and q,

e) the electric field E in terms of V and L,

f) the momentum p in terms of E, L, m and q if $u = 0$,

g) the wavelength λ in terms of V, m and q when $u = 0$.

Example – *Calculate the voltage needed to accelerate an electron to* 1.2×10^7 *m s*$^{-1}$ *from rest.*
$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 9.11 \times 10^{-31} \times \left(1.2 \times 10^7\right)^2 = 6.552 \times 10^{-17}\text{ J}$$
$$V = \frac{K}{q} = \frac{6.552 \times 10^{-17}}{1.60 \times 10^{-19}} = 410\text{ V}$$

23.2 Calculate the voltage needed to accelerate a proton to 3.5×10^6 m s^{-1} from rest.

23.3 Calculate the voltage needed to accelerate an electron to 3.5×10^6 m s^{-1} from rest.

23.4 A 1.00 MeV proton has a kinetic energy of 1.0×10^6 eV.

a) Express this energy in joules.

b) Calculate the speed of the proton.

c) What is the accelerating voltage needed to produce it?

d) Calculate its momentum.

e) Calculate its wavelength.

23.5 The electron gun in an old TV accelerates electrons from rest with 3.0 kV.

a) Calculate the final speed of the electrons.

b) Calculate the momentum of the electrons.

c) Calculate the wavelength of the electrons.

23.6 Fill in the missing entries in the table below.

Particle	Energy / MeV	Momentum / kg m s^{-1}	Speed / m s^{-1}
Electron	$0.001\,50$	(a)	(b)
Proton	10.0	(c)	(d)
Electron	(e)	4.55×10^{-24}	(f)
Proton	(g)	8.35×10^{-21}	(h)
Alpha particle	5.0	(i)	(j)

23.7 Calculate the final speed of a proton which was travelling at 2.5×10^6 m s^{-1} before being accelerated through 1.4 MV.

23.8 Calculate the accelerating voltage required to accelerate particles from rest to achieve the desired wavelength.

a) Electrons of wavelength 2.0 nm.

b) Electrons of wavelength 20 nm.

c) Protons of wavelength 1.5×10^{-13} m.

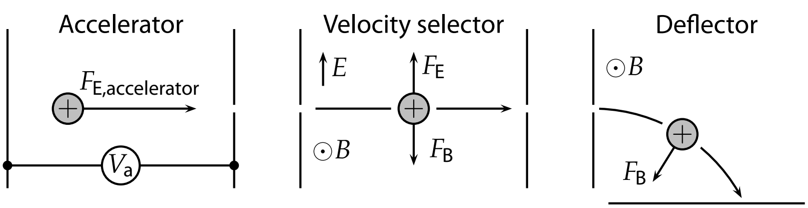
d) Alpha particles of wavelength 1.5×10^{-14} m.

30 Vectors and fields – mass spectrometer

A mass spectrometer is used to measure the mass/charge ratio of ions or particles. An understanding of electric and magnetic fields enables us to analyse the data.

Example context: the radius of the path in a magnetic field, coupled with a knowledge of the accelerating voltage, enables us to measure the mass of a carbon ion. Multiple measurements allow a measurement of the fraction of ${}^{14}_{6}\mathrm{C}$ in the sample.

Quantities:
- m particle mass (kg)
- q particle charge (C)
- v particle speed $\left(\mathrm{m\,s^{-1}}\right)$
- V_s velocity selector voltage (V)
- V_a accelerating voltage (V)
- d velocity selector plate gap (m)
- r path radius (m)
- B magnetic flux density (T)
- E electric field in velocity selector $\left(\mathrm{N\,C^{-1} = V\,m^{-1}}\right)$
- F_E electric force (N)
- F_B magnetic force (N)



Equations: $F = ma \quad F_\mathrm{E} = qE \quad F_\mathrm{B} = Bqv \quad qV_\mathrm{a} = \frac{1}{2}mv^2$ (see page 45)

$$E = \frac{V_\mathrm{s}}{d} \qquad a = \frac{v^2}{r}$$

30.1 Use the equations to derive expressions for

a) the radius r of the path in the magnetic field in terms of B, v, q and m,

b) the radius r in terms of B, V_a, q and m,

c) the specific charge q/m in terms of B, r and v,

d) the specific charge q/m in terms of B, r and V_a,

e) the voltage V_s across the plates in the velocity selector so that particles of speed v are not deflected.

30.2 Calculate the speed electrons emerge from a 95 V accelerator. Assume that the electrons start from rest.

30.3 Calculate the radius of curvature of a 2.5×10^6 m s^{-1} electron in a 1.5 mT magnetic field.

30.4 Repeat question 30.3 for a proton of the same speed in the same field.

> **Example** – *Calculate the radius of curvature of a proton accelerated to 25 MV in a 0.75 T magnetic field.*
>
> We use $qV_\mathrm{a} = \frac{1}{2}mv^2$ to calculate the speed $v = \sqrt{\dfrac{2qV_\mathrm{a}}{m}}$
>
> $$v = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 2.5 \times 10^{7}}{1.67 \times 10^{-27}}} = 6.921 \times 10^{7}\ \mathrm{m\,s^{-1}}.$$
>
> In the magnetic field $F_\mathrm{B} = ma$ so $Bqv = \dfrac{mv^2}{r}$ and $r = \dfrac{mv}{Bq}$
>
> $$r = \frac{1.67 \times 10^{-27} \times 6.921 \times 10^{7}}{0.75 \times 1.60 \times 10^{-19}} = 0.96\ \text{m to 2sf.}$$

30.5 Fill in the missing entries in the table below for a proton with $B = 2.2$ T.

V_a / V	v / m s^{-1}	r / m
	2.5×10^5	(a)
1.2×10^6	(b)	(c)
	(d)	0.014
(e)		0.12

30.6 Calculate the specific charge q/m of a particle travelling at 2.0×10^6 m s^{-1} in a magnetic field if $r = 11.9$ mm and $B = 0.175$ T.

30.7 Calculate V_s needed in a velocity selector to pass 1.6×10^6 m s^{-1} electrons in a 2.2 T magnetic field if $d = 6.5$ cm.

30.8 Protons pass through a velocity selector with $B = 1.5$ T and $d = 8.0$ cm when $V_\mathrm{s} = 420$ kV. Calculate their speed.

30.9 Repeat question 30.8 for electrons with the same values for B, d and V_s.

30.10 Calculate the radius of the path of a $^{235}_{92}\mathrm{U}$ nucleus travelling at 4.2×10^6 m s^{-1} in a 1.25 T magnetic field. Assume that $m = 235$ u where 1 u $= 1.66 \times 10^{-27}$ kg.

30.11 A singly charged ion is accelerated by a 650 kV potential before passing into a region with a 1.25 T magnetic field. It curves with a radius of 0.322 m. Calculate its mass.

30.12 Express your mass from question 30.11 in terms of atomic mass units u where 1 u $= 1.66 \times 10^{-27}$ kg.

30.13 Calculate the radius of curve expected for a singly charged ion of $^{14}_{6}\mathrm{C}$ in the mass spectrometer of question 30.11. Assume that $m = 14$ u.

22 Electromagnetic induction – rotating coil

(a) $\phi = BA = BA_0 \cos \omega t$

(b) $$\varepsilon = -N\frac{\mathrm{d}\phi}{\mathrm{d}t} = -N\frac{\mathrm{d}}{\mathrm{d}t}\left(BA_0 \cos \omega t\right) = -NBA_0\frac{\mathrm{d}}{\mathrm{d}t}\cos \omega t$$
$$= NBA_0\omega \sin \omega t$$

(c) maximum value $\sin \omega t$ can take is 1, so $\varepsilon_{\text{max}} = NBA_0\omega$

(d) $$\varepsilon^2 = N^2B^2A_0^2\omega^2 \sin^2 \omega t$$
$$\left(\varepsilon^2\right)_{\text{mean}} = N^2B^2A_0^2\omega^2 \left(\sin^2 \omega t\right)_{\text{mean}} = N^2B^2A_0^2\omega^2 \times \tfrac{1}{2}$$
$$\sqrt{(\varepsilon^2)_{\text{mean}}} = \varepsilon_{\text{rms}} = NBA_0\omega \times \sqrt{0.5} = \frac{1}{\sqrt{2}}NBA_0\omega$$ hence,
$$\varepsilon_{\text{rms}} = \frac{1}{\sqrt{2}}\varepsilon_{\text{max}}$$

23 Energy and fields – accelerator

(a) $p = mv = m\sqrt{\dfrac{2K}{m}} = \sqrt{2mK} = \sqrt{2mqV}$

(b) $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2qV}{m}}$

(c) $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2}{m}\left(\dfrac{mu^2}{2} + qV\right)} = \sqrt{u^2 + \dfrac{2qV}{m}}$

(d) $\Delta K = FL = qEL$

(e) $E = \dfrac{F}{q} = \dfrac{\Delta K}{qL} = \dfrac{V}{L}$

(f) $p = mv = m\sqrt{\dfrac{2K}{m}} = \sqrt{2mK} = \sqrt{2mqV} = \sqrt{2mqEL}$

(g) $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{h}{m\sqrt{2K/m}} = \dfrac{h}{\sqrt{2Km}} = \dfrac{h}{\sqrt{2mqV}}$

29 Vectors and fields – helix in magnetic field

(a) $F = ma$, so $qv_\perp B = \dfrac{mv_\perp^2}{r}$ and $qvB\sin\theta = \dfrac{mv^2\sin^2\theta}{r}$.

Rearranging gives $r = \dfrac{mv\sin\theta}{qB}$

(b) From a): $r = \dfrac{mv\sin\theta}{qB}$ and $v_\perp = \dfrac{2\pi r}{T} = v\sin\theta$.

So $r = \dfrac{mv\sin\theta}{qB} = \dfrac{T}{2\pi}v\sin\theta$. Therefore $T = \dfrac{2\pi m}{qB}$

(c) $v_\perp^2 + v_\parallel^2 = v^2\sin^2\theta + v^2\cos^2\theta = v^2\left(\sin^2\theta + \cos^2\theta\right) = v^2$.

Thus $v = \sqrt{v_\perp^2 + v_\parallel^2}$.

(d) From c): $T = \dfrac{2\pi m}{qB}$ and $s_\mathrm{p} = v_\parallel T = v\cos\theta\dfrac{2\pi m}{qB}$.

Re-arranging gives $q/m = \dfrac{2\pi}{Bs_\mathrm{p}}v\cos\theta$.

30 Vectors and fields – mass spectrometer

(a) $F_\mathrm{B} = ma$ so $Bqv = \dfrac{mv^2}{r}$. Rearranging gives $r = \dfrac{mv}{Bq}$

(b) $qV_\mathrm{a} = \frac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2qV_\mathrm{a}}{m}}$. Now using our result for r from (a),

$$r = \frac{mv}{Bq} = \frac{m}{Bq}\sqrt{\frac{2qV_\mathrm{a}}{m}} = \sqrt{\frac{2mV_\mathrm{a}}{B^2q}}$$

(c) From (a): $r = \dfrac{mv}{Bq}$ so $\dfrac{q}{m} = \dfrac{v}{Br}$

(d) From (b): $r^2 = \dfrac{2mV_\mathrm{a}}{B^2q}$ so $\dfrac{q}{m} = \dfrac{2V_\mathrm{a}}{B^2r^2}$

(e) $F_\mathrm{E} = F_\mathrm{B}$ so $qE = qvB$ and $E = vB$. So $V_\mathrm{s} = Ed = vBd$