Sequences and Series 1i

A Level

A sequence of terms u_1, u_2, u_3, ... is defined by

$$u_1 = 2 \text{ and } u_{n+1} = 1 - u_n$$

$$\text{for } n \geqslant 1$$

Part A Values

Give the values of u_2, u_3 and u_4.

Give the value of u_2.

The following symbols may be useful: u_2

Give the value of u_3.

The following symbols may be useful: u_3

Give the value of u_4.

The following symbols may be useful: u_4

Part B Behaviour

Describe the behaviour of the sequence.

- The sequence is periodic, with a period of three. It cycles through values of 2, -1 and 1.
- The sequence is periodic, with a period of two. It alternates between values of 2 and -1.
- The sequence is periodic, with a period of four. The first two values that repeat are 2 and -1.
- It is a geometric sequence, with first term 2 and constant ratio $-\frac{1}{2}$.

Part C Sum

Find $\sum_{n=1}^{100} u_n$.

Adapted with permission from UCLES A-level Maths papers, 2003-2017.

Arithmetic Series 1ii

A Level

P P P

Part A Value of x

The first three terms of an arithmetic progression are $2x$, $x+4$, and $2x-7$ respectively. Find the value of x.

The following symbols may be useful: x

Part B An Arithmetic Progression

The 20^{th} term of an arithmetic progression is 10 and the 50^{th} term is 70.

What is the first term?

What is the common difference?

Calculate the sum of the first 29 terms.

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Gameboard:

STEM SMART Double Maths 35 - Sequences, Series & Summation

Arithmetic Series 1i

A Level

P P P

In an arithmetic progression the first term is 5 and the common difference is 3. The n^{th} term of the progression is denoted by u_n.

Part A Value of u_{20}

Find the value of u_{20}.

The following symbols may be useful: `u_20`

Part B Sum

Find the value of $\sum_{n=10}^{20} u_n$.

Part C Value of N

Find the value of N such that $\sum_{n=N}^{2N} u_n = 2750$.

The following symbols may be useful: `N`

Gameboard:

STEM SMART Double Maths 35 - Sequences, Series & Summation

Geometric Series 1ii

A Level

Records are kept of the number of copies of a certain book that are sold each week. In the first week after publication, 3000 copies were sold, and in the second week 2400 copies were sold. The publisher forecasts future sales by assuming that the number of copies sold each week will form a geometric progression with first two terms 3000 and 2400. Calculate (to the nearest number of whole books) the publisher's forecasts for:

Part A 20^{th} Week

The number of copies that will be sold in the 20^{th} week after publication.

Part B Total copies sold in 20 weeks

The total number of copies sold during the first 20 weeks after publication.

Part C Total sold copies

The total number of copies that will ever be sold.

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Geometric Series 2ii

A Level

P P P

Part A Geometric Progression 1

In a geometric progression, the sum to infinity is four times the first term.

Find the common ratio.

Given that the third term is 9, find the first term.

Find the sum of the first twenty terms. (To three significant figures.)

Part B Geometric Progression 2

The first term of a geometric progression is 6 and the sum to infinity is 10.

Find the common ratio.

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STEM SMART Double Maths 35 - Sequences, Series & Summation

Geometric Series 4ii

A Level

In a geometric progression, the first term is 5 and the second term is 4.8.

Part A Sum to Infinity

Find the sum to infinity.

Part B Value of n

The sum of the first n terms is greater than 124. By showing that

$$0.96^n < 0.008$$

and using logarithms, calculate the smallest possible value of n.

Gameboard:
STEM SMART Double Maths 35 - Sequences, Series & Summation

Series: Summation - Standard Results 2ii

Further A

P P P

Find

$$\sum_{r=1}^{n}\left(4r^3+6r^2+2r\right),$$

expressing your answer in a fully factorised form.

The following symbols may be useful: n

Adapted with permission from UCLES, A Level, January 2009, Paper 4725, Question 3.

Gameboard:

STEM SMART Double Maths 35 - Sequences, Series & Summation

Series: Summation - Standard Results 1i

Further A

P P P

Part A $\sum_{r=n}^{2n} r^3$

Express $\sum_{r=n}^{2n} r^3$ in terms of n, giving your answer in fully factorised form.

The following symbols may be useful: n

Part B $\sum_{r=n}^{2n} r\left(r^2-2\right)$

Hence find $\sum_{r=n}^{2n} r\left(r^2-2\right)$, giving your answer in a fully factorised form.

The following symbols may be useful: n

Adapted with permission from UCLES, A Level, June 2014 , Paper 4725, Question 8.

Gameboard:

STEM SMART Double Maths 35 - Sequences, Series & Summation

Series: Method of Differences 2i

Further A

P P P

Part A $(r+2)! - (r+1)!$

Show that $(r+2)! - (r+1)! = f(r) \times r!$ where $f(r)$ is a function to be found.

What is $f(r)$?

The following symbols may be useful: r

Part B Expression for a series

Hence find an expression, in terms of n, for

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \ldots + (n+1)^2 \times n!$$

Your answer can be written as $g(n)! - 2$.

What is $g(n)$?

The following symbols may be useful: n

Part C Convergence

State, giving a brief reason, whether the series

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \dots$$

converges. Fill in the gaps in the argument (you can use an item more than once).

We can express this series as a summation as ______. This is the limit of the partial sum ______ as $n \to$ ______.

From Part A we can write the partial sum as ______, and from Part B we know that the partial sum evaluates to ______.

As $n \to \infty$, the partial sum tends to ______, so the series ______ converge.

Items:

$\sum_{r=1}^{\infty}(r+1)^2\, r!$ | $\sum_{r=1}^{\infty} r^2\,(r+1)!$ | $\sum_{r=1}^{n}(r+1)^2 r!$ | $\sum_{r=1}^{n} r^2\,(r+1)!$ | 0 | 1 | ∞ | $\sum_{r=1}^{n}[(r+2)!-(r+1)!]$

$\sum_{r=1}^{n}[(r+1)!-r!]$ | $(n+2)!-2$ | $(n+1)!-1$ | **does** | **does not**

Adapted with permission from UCLES, A Level, January 2007 , Paper 4725, Question 8.

Gameboard:

STEM SMART Double Maths 35 - Sequences, Series & Summation

Series: Method of Differences 1i

Further A

P P P

Part A Rewriting a fraction

Express $\frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2}$ as a single fraction.

The following symbols may be useful: r

Part B Sum of a series

Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{3r+4}{r(r+1)(r+2)}.$$

The following symbols may be useful: n

Part C Limit as $n \to \infty$

Hence write down the value of

$$\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)(r+2)}.$$

Part D Solve for N

Given that

$$\sum_{r=N+1}^{\infty} \frac{3r+4}{r(r+1)(r+2)} = \frac{7}{10}$$

find the value of N.

Adapted with permission from UCLES, A Level, January 2008 , Paper 4725, Question 10