Complex Numbers

Further A

C C C

Part A De Moivre

The complex number $2 + i\sqrt{5}$ can be expressed in the form $r\left(\cos\theta + i\sin\theta\right)$ where $-180^\circ < \theta < 180^\circ$.

Find r.

The following symbols may be useful: `r`

Find an expression for $\tan\theta$.

The following symbols may be useful: `tan(), theta`

Part B Square roots

Use an algebraic method to find the square roots of the complex number $2 + i\sqrt{5}$ in the form $x + iy$.

Give an expression for the solution with positive x and positive y.

The following symbols may be useful: `i`

Give an expression for the solution with negative x and negative y.

The following symbols may be useful: `i`

Part C A complex quartic

Using your answer from the previous part, find the roots of the equation $z^4 - 4z^2 + 9 = 0$ in the form $x + iy$.

Give an expression for the root with both x and y positive.

The following symbols may be useful: `i, z`

Give an expression for the root with positive x and negative y.

The following symbols may be useful: `i, z`

Give an expression for the root with negative x and positive y.

The following symbols may be useful: `i, z`

Give an expression for the root with negative x and negative y.

The following symbols may be useful: `i, z`

Part D Argand diagram

Sketch the roots of the equation from the previous part on an Argand diagram.

Easier question?

Part E Complex locus

Given that α is the root of the equation in part C) such that $0 < \arg\alpha < \frac{1}{2}\pi$, sketch on the same Argand diagram the locus given by $|z - \alpha| = |z|$.

Easier question?

Adapted with permission from UCLES, A Level, June 2009, Paper 4725, Question 10.

Roots of Polynomials

A Level

This question is about manipulation of the roots of two polynomials.

$$x^2 + kx + 2k = 0$$

has the roots α and β, while

$$x^3 + 4x + 3 = 0$$

has the roots α', β' and γ'. Take $k \neq 0$.

Part A Roots of the quadratic

Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

The following symbols may be useful: `k, x`

Part B Substitution

Starting from the cubic equation above, use the substitution $x = \sqrt{u}$ to obtain a cubic equation in u.

The following symbols may be useful: `u`

Part C Roots of the cubic

Find an expression for $\alpha'^4 + \beta'^4 + \gamma'^4 + \alpha'\beta'\gamma'$.

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 8 and June 2015, Paper 4725, Question 10.

Gameboard:

STEM SMART Double Maths 14 - AS Pure Further Maths Revision (non-calculus)

Vectors

A Level

C C C

The vector $\mathbf{u} = \frac{3}{13}\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is perpendicular to the vector $\mathbf{v} = 4\mathbf{i} + \mathbf{k}$ and to the vector $\mathbf{w} = 4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$.

Part A c

Find c as a single rational fraction.

The following symbols may be useful: `c`

Part B b

Find b in exact form.

The following symbols may be useful: `b`

Part C $|\mathbf{u}|$

Find $|\mathbf{u}|$.

Part D Angle between $\mathbf{v}$ and $\mathbf{w}$

Calculate, to the nearest degree, the angle between $\mathbf{v}$ and $\mathbf{w}$.

Part E $\mathbf{n}$

Find a unit vector $\mathbf{n}$ in the direction of the common perpendicular to the vectors $(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and $(-\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})$. Take $\mathbf{n}$ to have positive x, y and z.

Find the x component of $\mathbf{n}$ as a single fraction.

The following symbols may be useful: x

Find the y component of $\mathbf{n}$ as a single fraction.

The following symbols may be useful: y

Find the z component of $\mathbf{n}$ as a single fraction.

The following symbols may be useful: z

Part F Two lines

Determine whether the lines described by

$$\mathbf{r}_1 = (1 + 2\lambda)\mathbf{i} - \lambda\mathbf{j} + (3 + 5\lambda)\mathbf{k}$$

and

$$\mathbf{r}_2 = (\mu - 1)\mathbf{i} - (5 - \mu)\mathbf{j} + (2 - 5\mu)\mathbf{k}$$

are parallel, intersect or are skew.

- Intersect
- Skew
- Parallel

Adapted with permission from UCLES, A Level, June 2009, Paper 4724, Question 7, UCLES, A Level, Jan 2012, Paper 4724, Question 3 and Sally Waugh.

Gameboard:
STEM SMART Double Maths 14 - AS Pure Further Maths Revision (non-calculus)

Sequences

A Level

The sequence $u_1, u_2, u_3 \ldots$ is defined by $u_1 = 3$ and $u_{n+1} = 3u_n - 2$ for $n \geq 1$.

Part A u_2 and u_3

Find u_2.

The following symbols may be useful: `u_2`

Find u_3.

The following symbols may be useful: `u_3`

Part B $\frac{1}{2}(u_4 - 1)$

Find $\frac{1}{2}(u_4 - 1)$.

Part C u_n

Hence, find an expression for u_n and prove it with induction.

The following symbols may be useful: `n, u_n`

Part D Divisibility

Prove by induction that $5^n - 2^n$ is divisible by 3 for all integers $n \geq 1$.

Easier question?

Adapted with permission from UCLES, A Level, June 2009, Paper 4725, Question 10.