Differentiating Powers 1

A Level P P P Further A P P P

Part A Differentiate $y = x^4$

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x^4$.

The following symbols may be useful: x

Part B Differentiate $x = t^2$

Find the gradient of the curve $x = t^2$ at the points $t = 0$, $t = 3$ and $t = -3$.

Find the gradient at $t = 0$.

Find the gradient at $t = 3$.

Find the gradient at $t = -3$.

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Differentiating Powers 3

A Level Further A

C C C P P P

Part A Derivative of $v = Bu^{-3}$

Find $\dfrac{\mathrm{d}v}{\mathrm{d}u}$ if $v = Bu^{-3}$.

The following symbols may be useful: `B, u`

Part B Force if potential $V = \frac{q^2}{(4\pi\epsilon_0 r)}$

The electrostatic potential energy V of two equal charges q a distance r apart is given by $V = \dfrac{q^2}{(4\pi\epsilon_0 r)}$.

The force between the two charges is given by $-\dfrac{\mathrm{d}V}{\mathrm{d}r}$; find an expression for this force.

The following symbols may be useful: `epsilon_0, pi, q, r`

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Differentiating logarithms

Further A University

Part A Differentiate $t = (1/3)\log_4(s^2)$

Find $\dfrac{\mathrm{d}t}{\mathrm{d}s}$ if $t = \dfrac{1}{3}\log_4(s^2)$, giving your answer in its simplest possible form.

The following symbols may be useful: s

Part B Differentiate $\ln(uv) = u\mathrm{e}^v$

Differentiate $\ln(uv) = u\mathrm{e}^v$ to show that $\dfrac{\mathrm{d}v}{\mathrm{d}u}$ can be written as

$$\frac{\mathrm{d}v}{\mathrm{d}u} = \frac{f(u,v)\,v}{u\,(1 - uve^v)}$$

where $f(u,v)$ is a function of u and v.

Find an expression for $f(u,v)$.

The following symbols may be useful: e, u, v

Part C Differentiate $q = (p+1)^{2(p+1)}$

Differentiate $q = (p+1)^{2(p+1)}$ to show that $\dfrac{\mathrm{d}q}{\mathrm{d}p}$ can be written as

$$\frac{\mathrm{d}q}{\mathrm{d}p} = g(p)(p+1)^{2(p+1)}$$

where $g(p)$ is a function of p.

Find an expression for $g(p)$.

The following symbols may be useful: p

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Differentiating Trig Functions 2

A Level Further A

Part A Differentiate $s = r\sin(\alpha\theta)$

Find $\frac{\mathrm{d}s}{\mathrm{d}\theta}$ if $s = r\sin(\alpha\theta)$ and r and α are constants.

Find $\frac{\mathrm{d}s}{\mathrm{d}\theta}$ if $s = r\sin(\alpha\theta)$ and r and α are constants.

The following symbols may be useful: `alpha`, `r`, `theta`

Part B Differentiate $q = l\cos(\alpha - 2\beta\theta)$

Find $\frac{\mathrm{d}q}{\mathrm{d}\theta}$ if $q = l\cos(\alpha - 2\beta\theta)$ and l, α and β are constants.

Find $\frac{\mathrm{d}q}{\mathrm{d}\theta}$ if $q = l\cos(\alpha - 2\beta\theta)$ and l, α and β are constants.

The following symbols may be useful: `alpha`, `beta`, `l`, `theta`

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Differentiating Trig Functions 3

A Level Further A

C C C P P P

Part A Velocity and acceleration if $x = A\cos(\omega t + \phi)$

The displacement x of an oscillating particle at time t is given by $x = A\cos(\omega t + \phi)$ where A, ω and ϕ are constants; find expressions for the velocity (the rate of change of displacement) and acceleration (the rate of change of velocity) of the particle.

Find an expression for the velocity (the rate of change of displacement) of the particle.

The following symbols may be useful: `A, a, omega, phi, t, v`

Find an expression for the acceleration (the rate of change of velocity) of the particle.

The following symbols may be useful: `A, a, omega, phi, t, v`

Part B Stationary points of the function $y = \cos(\omega t) + \sin(\omega t)$

Consider the function $y = \cos(\omega t) + \sin(\omega t)$, where ω is a positive constant.

Find the stationary points of the function in the range $0 < t < \frac{2\pi}{\omega}$. How many are there? The stationary point with the lowest value of t is at (t_1, y_1) and the stationary point with the second lowest value of t is at (t_2, y_2). Find the values of t and y at (t_1, y_1) and (t_2, y_2).

How many stationary points are there?

- 4
- 3
- 1
- 0
- 2

Find t_1, the t coordinate of the stationary point with the lowest value of t in the range $0 < t < \frac{2\pi}{\omega}$.

The following symbols may be useful: `omega, pi, t_1, y_1`

Find t_2, the t coordinate of the stationary point with the second lowest value of t in the range $0 < t < \frac{2\pi}{\omega}$.

The following symbols may be useful: `omega, pi, t_2, y_2`

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Physics. You work it out.

Chain Rule 2

A Level Further A

P P P P P P

Part A Differentiate $E = B\sin^2(\omega t)$.

Find $\frac{\mathrm{d}E}{\mathrm{d}t}$ if $E = B\sin^2(\omega t)$.

The following symbols may be useful: `B, E, omega, t`

Part B Differentiate $y = \mathrm{e}^{-x^2/(2\sigma^2)}$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \mathrm{e}^{-x^2/(2\sigma^2)}$.

The following symbols may be useful: `e, sigma, x`

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Chain Rule 1

A Level Further A

P P P P P P

Part A Differentiate $p=(4t^2+3)^{-3}$

Find $\dfrac{\mathrm{d}p}{\mathrm{d}t}$ where $p=(4t^2+3)^{-3}$.

The following symbols may be useful: t

Part B Differentiate $p=\frac{1}{[(q+1)^2+(q-1)^2]}$

Find $\dfrac{\mathrm{d}p}{\mathrm{d}q}$ where $p=\dfrac{1}{[(q+1)^2+(q-1)^2]}$.

The following symbols may be useful: q

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Chain Rule & Product Rule

A Level Further A

P P P P P P

Using the chain and product rules etc., find the derivatives of the following where a is a positive constant.

Part A $y = \sin(x^2)$

$$y = \sin(x^2)$$

The following symbols may be useful: `cos()`, `cosec()`, `sec()`, `sin()`, `tan()`, `x`

Part B $y = a^x$

(hint: take logs)

The following symbols may be useful: `a`, `x`

Part C $y = \ln(x^a + x^{-a})$

The following symbols may be useful: `a`, `x`

Part D $y = x^x$

The following symbols may be useful: `x`

Part E $y = \sin^{-1} x$

The following symbols may be useful: `x`

Product Rule 2

A Level Further A

P P P P P P

Part A Differentiate $y = at^2\ \mathrm{e}^{\beta t}$

Find $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ where $y = at^2\ \mathrm{e}^{\beta t}$.

The following symbols may be useful: `a`, `beta`, `e`, `t`

Part B Differentiate $\tan\theta$

Find the derivative w.r.t. θ of $\tan\theta$ by writing it as $\dfrac{\sin\theta}{\cos\theta}$.

The following symbols may be useful: `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `tan()`, `theta`

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Product Rule 1

A Level Further A

P P P P P P

Part A Differentiate $(t+1)(3-t^2)$

Use the product rule to find the derivative w.r.t. t of $(t+1)(3-t^2)$.

The following symbols may be useful: `t`

Part B Differentiate $s = \frac{t}{(1+t^3)}$

Find $\frac{\mathrm{d}s}{\mathrm{d}t}$ if $s = \frac{t}{(1+t^3)}$.

The following symbols may be useful: `t`

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