Quadratic inequalities

GCSE C C C A Level P P P

Solve the following quadratic inequalities.

Part A $3x^2 - 2x - 8 \leq 0$

Solve the inequality $3x^2 - 2x - 8 \leq 0$. Firstly select the form of your answer from the choices given below, where a and b are constants and $a < b$, and then find a and/or b.

Select the form of your answer from the choices given below.

- $x < a$ only
- $x > b$ only
- a < x < b
- $x \leq a$ or $x \geq b$
- $a \leq x \leq b$
- $x \geq b$ only
- $x \leq a$ only
- $x < a$ or $x > b$

Given your deduction above, find a.

Given your deduction above, find b.

Part B $-2x^2 + 5 < 7x + 11$

Solve the inequality $-2x^2 + 5 < 7x + 11$. Firstly select the form of your answer from the choices given below, where c and d are constants, and then find c and/or d.

Select the form of your answer from the choices given below.

- $x > d$ only
- $x < c$ only
- $x < c$ or $x > d$
- $c \leq x \leq d$
- $x \geq d$ only
- $x \leq c$ only
- $x \leq c$ or $x \geq d$
- $c < x < d$

Given your deduction above, find c.

Given your deduction above, find d.

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Inequalities 2

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Solve the following inequalities.

Part A $7 - 4a \leq -5$

Solve the inequality $7 - 4a \leq -5$

The following symbols may be useful: `<, <=, >, >=, a, b`

Part B $3 - 2(b + 1) \geq 6 + 3(2b + 1)$

Solve the inequality $3 - 2(b + 1) \geq 6 + 3(2b + 1)$

The following symbols may be useful: `<, <=, >, >=, a, b`

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Inequalities 1

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Solve the following inequalities.

Part A $3m + 8 \geq 2$

Solve the inequality $3m + 8 \geq 2$.

The following symbols may be useful: `<, <=, >, >=, m`

Part B $2p + 5 < 4p - 7$

Solve the inequality $2p + 5 < 4p - 7$.

The following symbols may be useful: `<, <=, >, >=, p`

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Escape from a planet

GCSE A Level

A body of mass m and speed v can escape from a planet of mass M and radius R if the sum of its kinetic energy ($\frac{1}{2}mv^2$) and its gravitational potential energy ($-GMm/R$) is greater than or equal to zero i.e.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} \geq 0$$

(G is the universal constant of gravitation).

Part A Escape velocity

Find the range of speeds v over which it will escape. Give your answer as an inequality, with v on the left hand side.

The following symbols may be useful: `<, <=, >, >=, G, M, R, pi, rho, v, v_0`

Part B Escape radius

If the speed of the body has a fixed value, i.e. $v = v_0$, and the mass of the planet $M = \frac{4}{3}\pi R^3 \rho$, where ρ is its average density, find the range of radii R for which the body will escape.

The following symbols may be useful: `<, <=, >, >=, G, M, R, pi, rho, v, v_0`

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Condition for damping

GCSE C C C A Level C C C

A mass m is suspended on a spring with spring constant k in a medium which damps its motion. The condition that it will oscillate after it has been displaced from equilibrium is

$$\frac{k}{m} > \frac{b^2}{4m^2}$$

where b is called the damping constant.

Find the range of masses over which it will oscillate.

The following symbols may be useful: `<, <=, >, >=, b, k, m`

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Simplifying indices

GCSE P P P A Level P P P

Part A **Simplify** $(4a^2b^3)^{\frac{1}{2}} \times (9ab^2)^{-\frac{3}{2}}$

Simplify $(4a^2b^3)^{\frac{1}{2}} \times (9ab^2)^{-\frac{3}{2}}$

The following symbols may be useful: `a, b, p, q`

Part B **Simplify** $(8p^3q^2)^{\frac{2}{3}} \div \left(2p/q^{\frac{1}{3}}\right)^5$

Simplify $(8p^3q^2)^{\frac{2}{3}} \div \left(\frac{2p}{q^{\frac{1}{3}}}\right)^5$

The following symbols may be useful: `a, b, p, q`

Part C **Simplify** $(10^{-34})^{\frac{1}{2}}(10^{-10})^{\frac{1}{2}}(10^8)^{-\frac{5}{2}}$

Simplify $(10^{-34})^{\frac{1}{2}}(10^{-10})^{\frac{1}{2}}(10^8)^{-\frac{5}{2}}$

The following symbols may be useful: `a, b, p, q`

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Simplifying surds

GCSE P P P A Level P P P

Simplify the following expressions.

Part A $2\sqrt{20}+\sqrt{45}-5\sqrt{5}$

Simplify $2\sqrt{20}+\sqrt{45}-5\sqrt{5}$

Part B $4(\sqrt{3}+1)(\sqrt{3}-1)-2(2+\sqrt{2})(1+\sqrt{2})$

Simplify $4(\sqrt{3}+1)(\sqrt{3}-1)-2(2+\sqrt{2})(1+\sqrt{2})$

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Rationalisation

GCSE A Level

Rationalise the denominators of the following expressions.

Part A $(3\sqrt{6})/(2\sqrt{18})$

Rationalise the denominator of $\dfrac{3\sqrt{6}}{2\sqrt{18}}$.

Part B $(4-\sqrt{3})/(4+2\sqrt{3})$

Rationalise the denominator of $\dfrac{4-\sqrt{3}}{4+2\sqrt{3}}$.

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Introducing Dimensional Analysis

GCSE A Level

The dimensions of physical properties do not depend on specific units; here we use length L, time T and mass M as our fundamental dimensions. In any equation relating physical properties the dimensions must be the same on both sides.

For example force $=$ mass $\times$ acceleration.

Obviously mass has dimensions M. To deduce the dimensions of acceleration recall that acceleration $=$ change in velocity over time; velocity ($=$ change in displacement over time) has dimensions of LT^{-1} so acceleration has dimensions $(LT^{-1})(T^{-1}) = LT^{-2}$.

Thus force has dimensions MLT^{-2}.

Part A Dimensions of kinetic energy

The kinetic energy of a body of mass m moving with speed v is equal to $\frac{1}{2}mv^2$.

Find the dimensions of (kinetic) energy. Recall that the factor of $\frac{1}{2}$ in the expression is dimensionless.

The following symbols may be useful: `L, M, T`

Part B Planck unit

One type of "Planck unit" is defined as:

$$h^{\frac{1}{2}}G^{\frac{1}{2}}c^{-\frac{5}{2}}$$

where h is Planck's constant (dimensions ML^2T^{-1}),
G is the universal constant of gravitation (dimensions $M^{-1}L^3T^{-2}$) and
c is the speed of light (dimensions LT^{-1}).

Find the dimensions of this "Planck unit".

The following symbols may be useful: `L, M, T`

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