Polynomials 2

Part A   Partial Fractions

Express

$$\frac{(x-7)(x-2)}{(x+2)(x-1)^2}$$

in partial fractions.

The following symbols may be useful: x

Part B   Polynomial Division

Find the quotient and remainder when $6x^4 + 12x^3 - 3x^2 - 11x - 2$ is divided by $2x^2 + 4x + 1$.

What is the quotient?

The following symbols may be useful: x

What is the remainder?

The following symbols may be useful: x

Part C   Integration

Hence, find

$$\int_0^3 \frac{6x^4 + 12x^3 - 3x^2 - 11x - 2}{2x^2 + 4x + 1}\,\mathrm{d}x.$$

Write your answer in the form $A + B \ln C$ where A, B and C are constants to be found.

Part D   **Approximation of** $\frac{x}{(1-x)^3}$

Show that $\frac{x}{(1-x)^3} \approx x + 3x^2 + 6x^3$ for small values of x.

<div style="border:1px solid black; padding:10px; text-align:center;">Easier question?</div>

Part E   **Decimal estimate of** $\frac{100}{729}$

Use the result from the above part, and $x = 0.1$, to obtain a decimal estimate of the value of $\frac{100}{729}$ to three significant figures.

Part F   **First three terms of the binomial expansion**

Show that

$$\frac{x}{(1-x)^3} = -\frac{1}{x^2}\left(1 - \frac{1}{x}\right)^{-3}.$$

Hence find the first three terms of the binomial expansion of $\frac{x}{(1-x)^3}$ in powers of $\frac{1}{x}$.
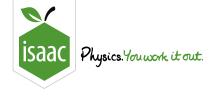
The following symbols may be useful: x

Part G   **Suitability of** x **in expansion**

Comment on the suitability of substituting the value of x from Part E into the expansion in Part F, to estimate the value of $\frac{100}{729}$.

<div style="border:1px solid black; padding:10px; text-align:center;">Easier question?</div>

Series

A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 2 \ \text{ and } \ u_{n+1} = \frac{1}{1 - u_n} \text{ for } n \geqslant 1.$$

Part A u_2

Write down the value of u_2.

The following symbols may be useful: u_2

Part B u_3

Write down the value of u_3.

The following symbols may be useful: u_3

Part C u_4

Write down the value of u_4.

The following symbols may be useful: u_4

Part D u_5

Write down the value of u_5.

The following symbols may be useful: u_5

Part E u_{200}

Deduce the value of u_{200}.

The following symbols may be useful: u_200

Part F $\sum_{n=1}^{200} u_n$

Find $\sum_{n=1}^{200} u_n$.

Part G   **Amount of Chemical**

Sarah is carrying out a series of experiments which involve using increasing amounts of a chemical. In the first experiment she uses $6\,\text{g}$ of a chemical and in the second experiment she uses $7.8\,\text{g}$ of the chemical.

Given that the amounts of chemical used form an arithmetic progression, find the total amount of chemical used in the first 30 experiments.

Part H   **Number of experiments possible**

Instead, it is given that the amounts of chemical used form a geometric progression. Sarah has a total of $1800\,\text{g}$ of the chemical available. As N, the greatest number of experiments possible, satisfies the inequality

$$1.3^N \leqslant 91$$

Use logarithms to find the greatest value for N.

The following symbols may be useful: N