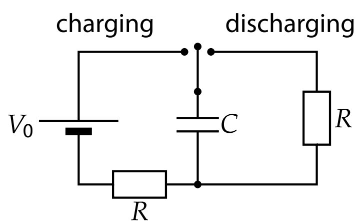
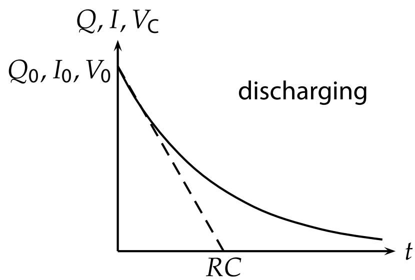
33 Capacitors and resistors

A capacitor can be charged or discharged gradually by connecting it in series with a resistor (and if charging, a voltage source). The voltages and currents in the circuit are decaying exponential functions of time.

Example context: Circuits containing capacitors and resistors in series are important in electronics applications, including signal processing and timing. You can calculate the capacitor charge, voltages and current in the circuit at any time.





Quantities:	R resistance (Ω)	V_C voltage across capacitor (V)
	C capacitance (F)	V_R voltage across resistor (V)
	t time (s)	V_0 initial or max voltage (V)
	Q charge on capacitor (C)	Q_0 initial or max charge (C)
	I current in circuit (A)	I_0 initial current (A)

Equations:	$Q = CV_C$	$V_R = IR$	$I = I_0 e^{-t/RC}$
	When discharging:	$V_C = V_R$	$V_C = V_0 e^{-t/RC}$
	When charging:	$V_C + V_R = V_0$	$V_C = V_0 (1 - e^{-t/RC})$

33.1 Use the equations to write down expressions for

a) the charge Q versus time, when discharging;

b) the charge Q versus time, when charging;

c) the initial charge Q_0 in terms of V_0 and C;

d) the voltage V_R across the resistor versus time, when discharging;

e) the voltage V_R across the resistor versus time, when charging;

f) Q in terms of I when discharging;

g) Q in terms of $dQ/dt = -I$ when discharging;

h) I_0 in terms of V_0 and R when discharging;

i) I_0 in terms of Q_0, R and C when discharging;

j) the time to completely discharge if the current were constant at I_0;

k) the fraction of Q_0 still on the capacitor after a time RC.

33.2 Find Q_0 and I_0 if $R = 200\ \Omega$, $C = 0.0010$ F and $V_0 = 5.0$ V when discharging.

33.3 A $47\ \mu$F capacitor discharges through a resistor. The initial charge on the capacitor is $9.4\ \mu$C and the initial current is 8.0 mA. Find the value of the resistor.

> **Example** – *A $600\ \mu$F capacitor is charged to 9.0 V and then discharged through a 70 kΩ resistor. How much charge is on the capacitor after 100 s?*
>
> $$Q = CV_C = CV_0e^{-t/RC} = (6 \times 10^{-4}) \times 9 \times e^{-100/(7\times10^4\times6\times10^{-4})} = 500\ \mu\text{C}$$

33.4 Fill in the missing entries in the table below for a capacitor discharging from an initial voltage of 10 V.

R / Ω	C / F	t / s	Q / C	V / V	I / A
550	8.0×10^{-3}	5.0		(a)	
550	8.0×10^{-3}	15		(b)	
2.2 MΩ	3.0×10^{-5}	10	(c)		(d)
2.2 MΩ	3.0×10^{-5}	20	(e)		(f)
2.2 MΩ	3.0×10^{-5}	50	(g)		(h)

33.5 A capacitor with $C = 10$ nF and initial charge 1.5×10^{-7} C is discharged through a resistor with $R = 10$ MΩ. What is the current after 0.25 s?

33.6 An initially uncharged 0.0020 F capacitor is connected to a 6.0 V battery via a $9.0\ \Omega$ resistor. How much charge has entered the capacitor after the first 0.02 s?

33.7 In a timing circuit, an initially uncharged 0.10 mF capacitor is connected to a 4.5 V source through a $80\ \Omega$ resistor.

a) What is the voltage across the capacitor after 5 ms?

b) After 5 ms, the capacitor is then disconnected from the source and connected across another $80\ \Omega$ resistor to discharge. What is the voltage across the resistor after another 5 ms?

(P) $\overline{u^2} = \dfrac{PV}{Nm} = \overline{v^2} = \overline{w^2}$ so $\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2} = \dfrac{3PV}{Nm}$ and so $PV = \dfrac{Nm\overline{c^2}}{3}$

32 Gas laws, density and kinetic energy

(a) $PV = nRT$ and $n = \dfrac{M}{M_\mathrm{M}}$ so $PV = \dfrac{MRT}{M_\mathrm{M}}$ and $P = \dfrac{MRT}{M_\mathrm{M}V}$

(b) From (a) $V = \dfrac{MRT}{M_\mathrm{M}P}$ so $\rho = \dfrac{M}{V} = \dfrac{M}{MRT/M_\mathrm{M}P} = \dfrac{M_\mathrm{M}P}{RT}$

(c) $\rho = \dfrac{M}{V} = \dfrac{Nm}{V} = \dfrac{Nm}{Nk_\mathrm{B}T/P} = \dfrac{mP}{k_\mathrm{B}T}$

(d) $PV = \dfrac{Nm\overline{c^2}}{3}$ so $P = \dfrac{Nm}{V} \cdot \dfrac{\overline{c^2}}{3} = \dfrac{\rho\overline{c^2}}{3}$ and $\rho = \dfrac{3P}{\overline{c^2}}$

(e) $PV = Nk_\mathrm{B}T = \frac{1}{3}Nm\overline{c^2}$ so $m\overline{c^2} = 3k_\mathrm{B}T$ and $\overline{K} = \dfrac{m\overline{c^2}}{2} = \dfrac{3k_\mathrm{B}T}{2}$

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(a) $Q = CV_0 e^{-t/RC}$ (or $Q_0 e^{-t/RC}$)

(b) $Q = CV_0(1 - e^{-t/RC})$ or $Q_0(1 - e^{-t/RC})$

(c) $Q_0/V_0 = C$ so $Q_0 = CV_0$

(d) $V_\mathrm{R} = V_\mathrm{C} = V_0 e^{-t/RC}$

(e) $V_\mathrm{R} = V_0 - V_\mathrm{C} = V_0 e^{-t/RC}$ i.e. the same as when discharging

(f) $Q = CV_\mathrm{C} = CV_\mathrm{R} = C(IR) = I \times RC$

(g) $Q = -RC\dfrac{\mathrm{d}Q}{\mathrm{d}t}$ so the differential equation is $\dfrac{\mathrm{d}Q}{\mathrm{d}t} = \dfrac{-Q}{RC}$

(h) $I_0 = \dfrac{V_0}{R}$

(i) $I_0 = \dfrac{(Q_0/C)}{R} = \dfrac{Q_0}{RC}$

(j) Time to discharge at constant current $= \dfrac{Q_0}{I_0} = RC$

(k) $t = RC$, so $\dfrac{Q}{Q_0} = e^{-1} = 0.37$