

oard Maths

Calculus

**Differential Equations** 

Integrating Factors 1

# **Integrating Factors 1**



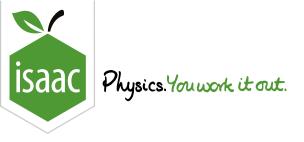
Find the general solution of the differential equation

$$xrac{\mathrm{d}y}{\mathrm{d}x}+(a+x)y=\mathrm{e}^{-x}.$$

Find the general solution for y as a function of x.

The following symbols may be useful: , a, c, e, k, x, y

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Calculus Maths

Differential Equations

RC Circuit (Integrating Factors)

# **RC Circuit (Integrating Factors)**



A circuit consists of a capacitor C, a resistor R and a switch in series with a battery of emf  $V_0$ . The switch is initially open and the capacitor is uncharged. At t=0 the switch is closed. The equation for the charge q on the capacitor as a function of time t after the switch is closed is

$$Rrac{\mathrm{d}q}{\mathrm{d}t}+rac{q}{C}=V_0.$$

Find how the charge on the capacitor varies with time t given that q=0 at t=0.

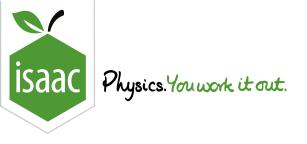
Find the equation for the charge q on the capacitor as a function of time t.

The following symbols may be useful: C, R, V\_0, e, q, t

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Undamped Pendulum (2nd Order)

#### **Undamped Pendulum (2nd Order)**



The equation describing the small-angle oscillations of a simple pendulum is

$$rac{\mathrm{d}^2 heta}{\mathrm{d}t^2} = -rac{g}{l} heta$$

where  $\theta$  is its angular displacement from the vertical at time t, l is the length of the pendulum and g is the acceleration due to gravity. Find an expression for heta as a function of t given that heta=lpha and  $rac{{
m d} heta}{{
m d}t}=eta$  at t=0.

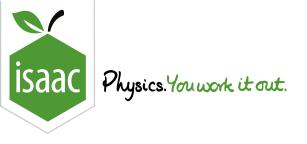
Find the equation for  $\theta$  as a function of t given that  $\theta=\alpha$  and  $\frac{\mathrm{d}\theta}{\mathrm{d}t}=\beta$  at t=0.

The following symbols may be useful: alpha, beta, g, 1, t, theta

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Mass on Spring (2nd Order)

#### Mass on Spring (2nd Order)



A mass m on a spring is subjected to a damping force. The equation describing its displacement x from its equilibrium position as a function of time t is

$$mrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx - brac{\mathrm{d}x}{\mathrm{d}t},$$

where -kx is the force from the spring and  $-b\frac{\mathrm{d}x}{\mathrm{d}t}$  is the force due to damping. The damping coefficient b is related to the spring constant k by  $k=\frac{4b^2}{25m}$ . Find an expression for the subsequent motion of the mass given that x=0 and  $\frac{\mathrm{d}x}{\mathrm{d}t}=V$  at t = 0.

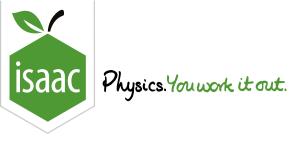
Find the equation describing the subsequent motion of the mass given that x=0 and  $\frac{\mathrm{d}x}{dt}=V$  at t=0.

The following symbols may be useful: V, b, e, m, t, x

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Damped Pendulum (2nd Order)

# Damped Pendulum (2nd Order)



The equation describing the displacement x of the bob of a damped pendulum from its equilibrium position is given by

$$rac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega_0^2 x - 2\gamma rac{\mathrm{d}x}{\mathrm{d}t}$$

where  $\omega_0$  is the angular frequency of undamped oscillations of the pendulum and  $\gamma$  is related to the damping. Assuming  $\omega_0>\gamma$  find an equation for x at time t given that x=X and  $\dfrac{\mathrm{d}x}{\mathrm{d}t}=0$  at t=0. (You will find it helpful to define a new constant  $\omega_1$  such that  $\omega_1^2=\omega_0^2-\gamma^2$ .)

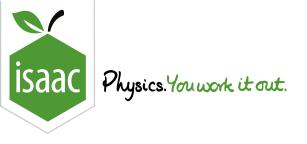
Find an equation for x at time t given that x=X and  $\dfrac{\mathrm{d}x}{\mathrm{d}t}=0$  at t=0.

The following symbols may be useful: X, e, gamma,  $omega_1$ , t, x

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Inhomogeneous Equation (2nd Order)

# Inhomogeneous Equation (2nd Order)



Find the solution of the equation

$$rac{\mathrm{d}^2 p}{\mathrm{d}q^2} - 4rac{\mathrm{d}p}{\mathrm{d}q} + 3p = 3q-1$$

given that p=2 and  $rac{\mathrm{d}p}{\mathrm{d}q}=-1$  when q=0.

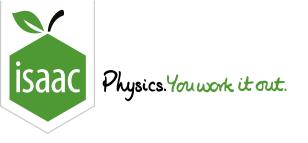
Find the solution of the equation.

The following symbols may be useful: e, p, q

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Forced Oscillator (2nd Order)

#### Forced Oscillator (2nd Order)



The equation of motion of a forced oscillator is given by

$$rac{\mathrm{d}^2z}{\mathrm{d}t^2}+\omega_0^2z=Z_0\sin(\omega_1t)$$

Given that  $\omega_0 
eq \omega_1$  find the solution for z given that z=0 and  $\frac{\mathrm{d}z}{\mathrm{d}t}=0$  at t=0.

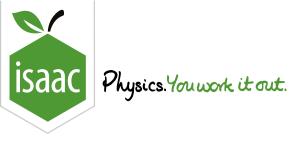
Fnd the solution for z given that z=0 and  $\frac{\mathrm{d}z}{\mathrm{d}t}=0$  at t=0.

The following symbols may be useful: Z\_0, omega\_0, omega\_1, t, z

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Differential Equations: General Applications 2i

### Differential Equations: General Applications 2i

During an industrial process substance X is converted into substance Z. Some of the substance X goes through an intermediate phase, and is converted into substance Y, before being converted into substance Z. The situation is modelled by

$$rac{\mathrm{d}y}{\mathrm{d}t} = 0.3x - 0.2y \quad ext{ and } \quad rac{\mathrm{d}z}{\mathrm{d}t} = 0.2y + 0.1x$$

where x, y and z are the amounts in kg of X, Y and Z at time t hours after the process starts.

Initially there is  $10 \,\mathrm{kg}$  of substance X and nothing of substances Y and Z. The amount of substance X decreases exponentially. The initial rate of decrease is  $4 \,\mathrm{kgh^{-1}}$ .

#### Part A Expression for x

Find an expression for x.

The following symbols may be useful: e, t

Part B	$\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\mathrm{d}z}{\mathrm{d}t}$
S	how that $rac{\mathrm{d}x}{\mathrm{d}t}+rac{\mathrm{d}y}{\mathrm{d}t}+rac{\mathrm{d}z}{\mathrm{d}t}=k$ where $k$ is a constant.
S	tate the value of $k.$
С	comment on this result in the context of the industrial process.
	The total amount of all three substances increases throughout the process.
	The total amount of all three substances decreases throughout the process.
	The total amount of all three substances is zero throughout the process.
	The total amount of all three substances is constant throughout the process.
Part C	Expression for $y$
Fi	ind an expression for $y$ in terms of $t.$
Th	ne following symbols may be useful: e,t
Part D	Maximum amount of $Y$
D	etermine the maximum amount of substance $Y$ present during the process.
Part E	Time to produce $9\mathrm{kg}$ of substance $Z$

How long does it take to produce  $9\,\mathrm{kg}$  of substance Z? Give your answer to 3 significant figures.