Maxima and Minima: Problems 1ii

A Level

Figure 1: The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is x metres, and the area of the enclosure is $A\,\mathrm{m}^2$.

Part A Express as equation

Show that A can be expressed in the form $px - qx^2$, and find this expression.

The following symbols may be useful: x

Part B Use differentiation

Use differentiation to find the maximum value of A.

The following symbols may be useful: A

Used with permission from UCLES, A Level, June 2007, Paper 4721, Question 5.

Maxima and Minima: Problems 2ii

A Level

A curve has equation $y = 3x^3 - 7x + \frac{2}{x}$

Part A Verify stationary point

Verify the curve has a stationary point when $x = 1$.

Part B Nature of stationary point

Determine the nature of this stationary point.

- Maximum
- Minimum
- Neither/inconclusive

Part C Tangent to curve

The tangent to the curve at this stationary point meets the y-axis at the point Q. Find the y-coordinate of Q.

Used with permission from UCLES, A Level, June 2014, Paper 4721, Question 8.

Gameboard:

STEM SMART Single Maths 12 – Curves

Maxima and Minima: Problems 1i

A Level

A cuboid has an volume of exactly $8\,\mathrm{m}^3$. The base of the cuboid is a square with side length x metres. The surface area of the cuboid is $A\,\mathrm{m}^2$.

Part A Find expression for A

Show that A can be expressed in the form $ax^2 + \frac{b}{x}$, where a and b are constants, and find this expression.

The following symbols may be useful: x

Part B Find $\frac{\mathrm{d}A}{\mathrm{d}x}$

Find $\frac{\mathrm{d}A}{\mathrm{d}x}$.

The following symbols may be useful: x

Part C Find minimum

Find the value of x which gives the smallest surface area of the cuboid.

The following symbols may be useful: x

Used with permission from UCLES, A level, June 2006, Paper 4721, Question 8.

Gameboard:

STEM SMART Single Maths 12 – Curves

Minimising the area

A Level Further A

C C C P P P

A rectangular cuboid has a base with sides of length a and b and a height c. Its volume V and height c are fixed. By following the steps below find expressions in terms of V and c for the values of a and b which will minimise the surface area A of the cuboid, find an expression for this minimum surface area and check that this is indeed a minimum.

Part A Volume V and surface area A

Write down the equation for the volume V of the rectangular cuboid in terms of a, b and c.

The following symbols may be useful: `V, a, b, c`

Write down the equation for the surface area A of the rectangular cuboid in terms of a, b and c.

The following symbols may be useful: `A, a, b, c`

From your equation for V deduce an expression for b in terms of V, a and c. Hence, by substitution, obtain an equation for A in terms of V, a and c.

The following symbols may be useful: `A, V, a, c`

Part B Expressions for a and b

Differentiate with respect to a the expression for A you found in Part A (since V and c are fixed you may treat them as constants). Hence find in terms of V and c an expression for the value of a for which the surface area A is minimised.

The following symbols may be useful: `V, c`

Find, in terms of V and c, the expression for b corresponding to this value of a.

The following symbols may be useful: `V, c`

Part C The minimum area

Find an expression for the minimum surface area in terms of V and c.

The following symbols may be useful: `V, c`

Part D Check that the area is a minimum

Find, at the value of a deduced in Part B, an expression in terms of V and c for the second derivative of A with respect to a; convince yourself that the value of the second derivative indicates that the value of A is a minimum at this point.

The following symbols may be useful: `V, c`

Created for isaacphysics.org by Julia Riley

Gameboard:

STEM SMART Single Maths 12 – Curves

Stationary Points 1ii

A Level

The curve $y = x^3 - kx^2 + x - 3$ has two stationary points.

Part A Differentiate

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `k, x`

Part B Find k

Given that there is a stationary point when $x = 1$, find the value of k.

The following symbols may be useful: `k`

Part C Differentiate twice

Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

The following symbols may be useful: x

Hence determine whether the stationary point is a minimum or a maximum.

- Minimum
- Maximum

Part D Find coordinate

Find the x-coordinate of the other stationary point.

The following symbols may be useful: x

Used with permission from UCLES, A Level, June 2009, Paper 4721, Question 10.

Gameboard:

STEM SMART Single Maths 12 – Curves

Stationary Points 1i

A Level

Part A Find stationary points

Find the coordinates of the stationary points on the curve $y = 2x^3 - 3x^2 - 12x - 7$. Enter the x and y coordinates of the stationary point with the largest x coordinate.

Enter the x coordinate:

The following symbols may be useful: x

Enter the y coordinate:

The following symbols may be useful: y

Part B Nature of stationary points

Determine whether each stationary point is a minimum or maximum point. Identify the nature of the stationary point whose coordinates you have entered in Part A.

- Maximum
- Minimum

Part C Expand and simplify

Expand and simplify $(x+1)^2(2x-7)$.

The following symbols may be useful: x

Part D Sketch

Hence sketch the curve $y = 2x^3 - 3x^2 - 12x - 7$, indicating the coordinates of all stationary points and intercepts with the axes. In order to check your answer, give the value of the intercept with the y-axis.

The following symbols may be useful: y

Used with permission from UCLES, A Level, Paper 4721 specimen, Question 8.

Gameboard:

STEM SMART Single Maths 12 – Curves

Stationary Points 2i

A Level

Part A Find minimum

Find the coordinates of the minimum point of the curve $y = (x + 2)(x^2 - 3x + 5)$. Enter the x and y coordinates below.

Enter the x-coordinate:

The following symbols may be useful: x

Enter the y-coordinate:

The following symbols may be useful: y

Part B Finding nature of stationary point

How did you know that the stationary point in part A was a minimum point?

- At this point, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is positive.
- At this point, $\frac{\mathrm{d}y}{\mathrm{d}x}$ is zero.
- At this point, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is negative.

Part C Calculate discriminant

Calculate the discriminant of $x^2 - 3x + 5$. Enter the exact value.

Part D Explain

Explain why $(x + 2)(x^2 - 3x + 5)$ is always positive whenever $x > -2$.

Easier question?

Used with permission from UCLES, A Level, January 2012, Paper 4721, Question 7

Gameboard:

STEM SMART Single Maths 12 – Curves

Physics. You work it out.

Stationary Points 2ii

A Level

Part A Find coordinate

Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 + 4$. Enter the x and y coordinates of the stationary point with the greatest x coordinate.

Enter the x-coordinate:

The following symbols may be useful: x

Enter the y-coordinate:

The following symbols may be useful: y

Part B Stationary point

Determine whether the stationary point whose coordinates you entered is a maximum point or a minimum point.

- Minimum
- Maximum
- Inconclusive

Part C Range of x

For which range of values of x does $x^3 - 3x^2 + 4$ decrease as x increases?

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $x < a$
- $x \leq a$
- $x > a$
- $x \geq a$
- $a < x < b$
- $a \leq x \leq b$
- $x < a$ or $x > b$
- $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: `n`

Gameboard:

STEM SMART Single Maths 12 – Curves

Physics. You work it out.

Stationary Points 4ii

A Level

Part A Find coordinates

Find the coordinates of the stationary point on the curve $y = x^4 + 32x$. Enter the x and y coordinates below.

Enter x coordinate:

The following symbols may be useful: x

Enter y coordinate:

The following symbols may be useful: y

Part B Maxima or Minima

Determine whether this stationary point is a maximum or a minimum.

Minimum

Maximum

Part C Range of x

For what range of values of x does $x^4 + 32x$ increase as x increases? Give your answer in the form of an inequality.

The following symbols may be useful: `<, <=, >, >=, x`