Double Angles 2

A Level

C C C

Prove that

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Now prove that $\tan 4\theta = k/(1-6\tan^2\theta+\tan^4\theta)$ and give an expression for k in its simplest form in terms of $\tan\theta$.

The following symbols may be useful: `k, theta`

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Trigonometry: Double Angles 1ii

A Level

C C C

Part A The form $a\sin^2\theta + b\sin\theta + c = 0$

Express the equation $(\text{cosec}\,\theta)(3\cos 2\theta + 7) + 11 = 0$ in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b, and c are constants and $a > 0$.

Give the value of a.

The following symbols may be useful: `a`

Give the value of b.

The following symbols may be useful: `b`

Give the value of c.

The following symbols may be useful: `c`

Part B Solve

Hence solve, for $-180^\circ < \theta < 180^\circ$, the equation $(\text{cosec}\,\theta)(3\cos 2\theta + 7) + 11 = 0$. Give your answers in degrees, to three significant figures.

Give the highest (most positive) solution.

Give the lowest (most negative) solution.

Gameboard:

STEM SMART Single Maths 28 - Compound and Double-Angle Formulae

Trigonometry: Double Angles 2ii

A Level

C C C

Part A $\sin$ Double Angle

Write down the identity expressing $\sin 2\theta$ in terms of $\sin\theta$ and $\cos\theta$.

The following symbols may be useful: `theta`

Part B $\sin 2\alpha$

Given that $\sin\alpha = \frac{1}{4}$ and α is acute, find the exact value of $\sin 2\alpha$.

The following symbols may be useful: `alpha`

Part C Solve

Solve in degrees, for $0^\circ < \beta < 90^\circ$, the equation $5\sin 2\beta \sec\beta = 3$, giving your answer in degrees to three significant figures.

Gameboard:

STEM SMART Single Maths 28 - Compound and Double-Angle Formulae

Trigonometry: Combined Angles 5i

A Level

The value of $\tan 10^\circ$ is denoted by p. Find, in terms of p, the value of:

Part A $\tan 55^\circ$

$\tan 55^\circ$

The following symbols may be useful: `p`

Part B $\tan 5^\circ$

$\tan 5^\circ$

The following symbols may be useful: `p`

Part C $\tan \theta$

$\tan \theta$, where θ satisfies the equation $3 \sin (\theta + 10^\circ) = 7 \cos (\theta - 10^\circ)$.

The following symbols may be useful: `p, theta`

Gameboard:

STEM SMART Single Maths 28 - Compound and Double-Angle Formulae

Functions: Reciprocal Trig 2i

A Level

It is given that A and B are angles such that

$$\sec^2 A - \tan A = 13 \quad \text{and} \quad \sin B \sec^2 B = 27 \cos B \operatorname{cosec}^2 B.$$

Part A Largest value of $\tan(A - B)$

Find the largest possible exact value of $\tan(A - B)$.

The following symbols may be useful: `A, B`

Part B Smallest value of $\tan(A - B)$

Give the smallest possible value of $\tan(A - B)$.

The following symbols may be useful: `A, B`

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Gameboard:

STEM SMART Single Maths 28 - Compound and Double-Angle Formulae

Trigonometry and R-Form 1

A Level

C C C

Part A $5\cos x + 12\sin x$

Express $5\cos x + 12\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

State the value of R.

The following symbols may be useful: R

Give the value of α in degrees, to three significant figures.

Part B Transformations

Hence give details of a pair of transformations which transform the curve $y = \cos x$ to the curve $y = 5\cos x + 12\sin x$.

Available items

- Translation in the negative y direction by α.
- Stretch parallel to the y-axis by a factor of $\frac{1}{R}$.
- Translation in the positive y direction by α.
- Stretch parallel to the y-axis by a factor of R.
- Stretch parallel to the x-axis by a factor of R.
- Translation in the negative x direction by α.
- Translation in the positive x direction by α.
- Stretch parallel to the x-axis by a factor of $\frac{1}{R}$.

Part C Solve

Solve, for $0° < x < 360°$, the equation $5\cos x + 12\sin x = 2$, giving your answers correct to the nearest $0.1°$

.

Give the smallest solution, in degrees, to four significant figures.

Give the largest solution, in degrees, to four significant figures.

Gameboard:

STEM SMART Single Maths 28 - Compound and Double-Angle Formulae

Trigonometry: Combined Angles 5ii

A Level

C C C

Part A Proof

Simplify as far as possible $\frac{\sin(\theta-\alpha)+3\sin\theta+\sin(\theta+\alpha)}{\cos(\theta-\alpha)+3\cos\theta+\cos(\theta+\alpha)}$. Your final answer should be in terms of θ only.

The following symbols may be useful: `cos(), sin(), tan(), theta`

Part B Exact Value

Find the exact value of $\frac{4\sin 149^\circ+12\sin 150^\circ+4\sin 151^\circ}{3\cos 149^\circ+9\cos 150^\circ+3\cos 151^\circ}$

Part C Solve

It is given that k is a positive constant. Solve, in terms of k, for $0^\circ<\theta<60^\circ$, the equation

$$\frac{\sin(6\theta-15^\circ)+3\sin 6\theta+\sin(6\theta+15^\circ)}{\cos(6\theta-15^\circ)+3\cos 6\theta+\cos(6\theta+15^\circ)}=k.$$

Give the smallest solution.

The following symbols may be useful: `arccos(), arcsin(), arctan(), k, pi`

Give the largest solution.

The following symbols may be useful: `arccos(), arcsin(), arctan(), k, pi`

Gameboard:

STEM SMART Single Maths 28 - Compound and Double-Angle Formulae

Addition of Angles 5

A Level Further A
C C C P P P

Show that you can express $A\cos(\omega t + \phi)$ in the form $B\cos\omega t + C\sin\omega t$, where B and C are expressions to be found.

Part A Expression for B

Give an expression for B in terms of A and ϕ.

The following symbols may be useful: `A, B, phi`

Part B Expression for C

Also give an expression for C in terms of A and ϕ.

The following symbols may be useful: `A, C, phi`

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Gameboard:
STEM SMART Single Maths 28 - Compound and Double-Angle Formulae

Trigonometry: Combined Angles 3i

A Level

In **Figure 1**, $ABCD$ represents a rectangular table with sides $3.5\,\mathrm{m}$ and $1.5\,\mathrm{m}$. It has been turned so it wedges in a passage of width $2.5\,\mathrm{m}$.

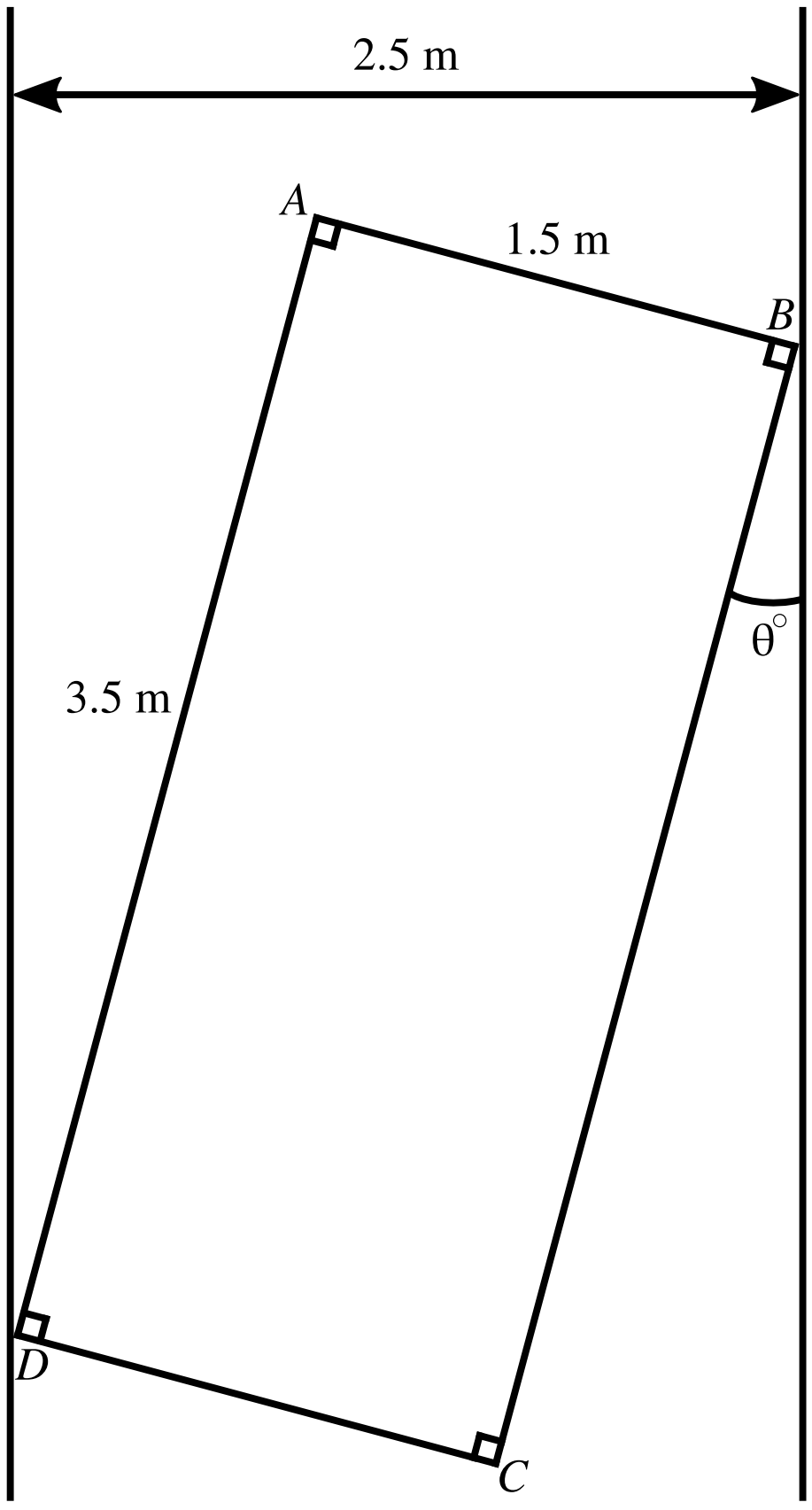

Figure 1: The rectangular table $ABCD$.

Part A Value of $7\sin\theta^\circ + 3\cos\theta^\circ$

Given that θ is the acute angle between the longer side and the passage, as shown in the diagram, find the exact value of $7\sin\theta + 3\cos\theta$.

The following symbols may be useful: `cos(), sin(), tan(), theta`

Part B The form $R\sin\left(\theta^\circ + \alpha^\circ\right)$

Express $7\sin\theta + 3\cos\theta$ in the form $R\sin\left(\theta + \alpha\right)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.

Give the exact value of R.

The following symbols may be useful: `R`

Give the value of α to 3 significant figures.

Part C Find θ

Find θ, to 3 significant figures.

Gameboard:

STEM SMART Single Maths 28 - Compound and Double-Angle Formulae

Addition of Angles 7

A Level Further A

Two waves

$$\psi_1 = A\cos\left(2\pi f t - \left(\frac{2\pi}{\lambda}\right)x + \phi\right)$$

and

$$\psi_2 = A\cos\left(2\pi f t - \left(\frac{2\pi}{\lambda}\right)x - \phi\right)$$

interfere, such that the resultant wave is given by $\psi = \psi_1 + \psi_2$. Express ψ as the product of two terms.

Express ψ as the product of two terms.

The following symbols may be useful: `A, f, lambda, phi, pi, t, x`

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