Isaac Physics Skills

Linking concepts in pre-university physics

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TABLE OF PHYSICAL CONSTANTS

Quantity & Symbol		**Magnitude**	**Unit**
Permittivity of free space	ϵ_0	8.85×10^{-12}	$\mathrm{F\,m^{-1}}$
Electrostatic force constant	$1/4\pi\epsilon_0$	8.99×10^{9}	$\mathrm{N\ m^2\ C^{-2}}$
Speed of light in vacuum	c	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
Specific heat capacity of water	c_{water}	4180	$\mathrm{J\,kg^{-1}\,K^{-1}}$
Charge of proton	e	1.60×10^{-19}	C
Gravitational field strength on Earth	g	9.81	$\mathrm{N\,kg^{-1}}$
Universal gravitational constant	G	6.67×10^{-11}	$\mathrm{N\ m^2\ kg^{-2}}$
Planck constant	h	6.63×10^{-34}	J s
Boltzmann constant	k_{B}	1.38×10^{-23}	$\mathrm{J\,K^{-1}}$
Mass of electron	m_{e}	9.11×10^{-31}	kg
Mass of neutron	m_{n}	1.67×10^{-27}	kg
Mass of proton	m_{p}	1.67×10^{-27}	kg
Mass of Earth	M_{Earth}	5.97×10^{24}	kg
Mass of Sun	M_{Sun}	2.00×10^{30}	kg
Avogadro constant	N_{A}	6.02×10^{23}	$\mathrm{mol^{-1}}$
Gas constant	R	8.31	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Radius of Earth	R_{Earth}	6.37×10^{6}	m

OTHER INFORMATION YOU MAY FIND USEFUL

Electron volt	1 eV	=	1.60×10^{-19} J
Unified mass unit	1 u	=	1.66×10^{-27} kg
Absolute zero	0 K	=	-273 °C
Year	1 yr	=	3.16×10^{7} s
Light year	1 ly	=	9.46×10^{15} m
Parsec	1 pc	=	3.09×10^{16} m

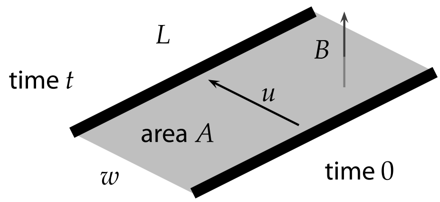
PREFIXES

1 km = 1000 m	1 Mm = 10^{6} m	1 Gm = 10^{9} m	1 Tm = 10^{12} m
1 mm = 0.001 m	1 μm = 10^{-6} m	1 nm = 10^{-9} m	1 pm = 10^{-12} m

21 Electromagnetic induction – moving wire

When a wire moves through a perpendicular magnetic field, cutting through the magnetic flux lines, a voltage appears across it.

Example context: We can calculate the voltage induced in any moving conductor, even if it is not a complete loop.



Quantities: B magnetic flux density (T) w distance moved by wire (m) L wire length (m) A area swept through (m^2) F_B magnetic force (N) E electric field ($N\,C^{-1}$) u speed of wire ($m\,s^{-1}$) V induced voltage (V) t time taken (s) q charge of carriers (C) F_E electric force (N)

Equations: $A = Lw \quad w = ut \quad V = \frac{d(BA)}{dt} = \frac{BA}{t}$

$F_B = quB \quad F_E = qE \quad E = V/L$

21.1 Use the equations to write down expressions for

a) the area A swept through by the wire using u, Δt and L,

b) the magnetic flux BA cut by the wire using u, Δt and L,

c) the rate of cutting flux $d(BA)/dt$,

d) the voltage V induced in the wire by Faraday's law,

e) the magnetic force on a charge q inside the wire,

f) the strength of an electric field E along the wire that could produce the same force on the charge,

g) the voltage V that would exist between the ends of the wire, if that electric field was uniform.

21.2 Find V if $B = 0.50$ T, $L = 0.050$ m and $u = 2.0\ m\,s^{-1}$.

Example – *At a certain point in the cycle of a generator, one 12 cm length of wire in the coil moves at 25 m s^{-1} perpendicular to its length and to a 0.70 T magnetic field. What voltage would be induced across this wire at that point?*

$$V = \frac{BA}{t} = BLu = 0.70 \times 0.12 \times 25 = 2.1\,\text{V}$$

21.3 In a magnetic brake system on a roller-coaster, a metal bar of width 35 mm on the carriage moves through an electromagnet attached to the rails, where $B = 0.4$ T. If the carriage is moving at 70 m s^{-1} when it enters the brake, what is the initial voltage induced across the width of the bar?

21.4 In an experiment, a student induces a voltage of 15 mV by moving a 0.30 m-long section of wire through a region of uniform perpendicular magnetic field 0.35 T. At what speed were they moving the wire?

21.5 When 20 cm of straight wire is moved between the poles of a superconducting magnet at 25 mm s^{-1}, a 35 mV voltage is induced. Find the magnetic flux density between the poles.

21.6 Fill in the missing entries in the table below for a wire moving through a perpendicular magnetic field.

w / m	t / s	u / m s^{-1}	L / m	B / T	V / V
		6.0	0.50	1.2	(a)
		100	(b)	0.08	0.40
0.05	0.075	(c)	0.045	(d)	3.0×10^{-3}
1.35	(e)	15	1.5	5.0	(f)

21.7 In a motor, each long side of a turn in the coil acts like a 2.0 cm wire moving at 7.3 m s^{-1}. The strongest field it experiences during a cycle is 300 mT. If each turn has two long sides in series, what is the minimum number of turns in series needed to get a peak voltage greater than 3 V?

21.8 A metal aircraft with a 15.0 m wingspan is flying North at 450 km h^{-1}. What voltage could be induced between the wingtips, if the Earth's magnetic field has strength 60.0 μT and is pointing:

a) vertically up from the Earth's surface?

b) inclined at 20.0° from the vertical towards the horizontal South-North direction? [Hint: the flux lines will be spaced farther apart.]

22 Electromagnetic induction – rotating coil

It is helpful to be able to calculate the voltage, or electromotive force (EMF) induced by a rotating coil in a magnetic field.

Example context: most generators contain a coil of wire rotating uniformly in a uniform magnetic field. Whenever a there is a conductor in a changing magnetic field, an EMF is induced.

Quantities:
- ε EMF (V)
- N number of turns
- ϕ magnetic flux (Wb)
- B flux density (T)
- A_0 coil area (m^2)
- t time (s)
- A component of coil area linking flux (m^2)
- ω angular frequency $(\mathrm{rad\,s^{-1}})$
- Subscript $_{\mathrm{rms}}$ represents root mean square values
- $\frac{\mathrm{d}}{\mathrm{d}t}$ means *rate of change of a quantity*

Equations:

$$\varepsilon = -N\frac{\mathrm{d}\phi}{\mathrm{d}t} \qquad \phi = BA \qquad A = A_0 \cos \omega t$$

$$\varepsilon_{\mathrm{rms}} = \sqrt{(\varepsilon^2)_{\mathrm{mean}}} \qquad \frac{\mathrm{d}\cos\omega t}{\mathrm{d}t} = -\omega \sin \omega t$$

22.1 Use the equations to derive expressions for

a) the magnetic flux ϕ in terms of B, A_0 and t,

b) the EMF ε in terms of B, A_0, N, ω and t,

c) the maximum EMF $\varepsilon_{\mathrm{max}}$,

d) the root mean squared EMF $\varepsilon_{\mathrm{rms}}$ in terms of $\varepsilon_{\mathrm{max}}$.

22.2 Fill in the missing entries in the table below.

$\varepsilon_{\mathrm{max}}$ / V	N	B / mT	A_0 / cm^2	ω / rad s^{-1}
(a)	100	50.0	5.00	31.4
2.50	(b)	80	10.0	157
1.70	50.0	(c)	12.5	62.8
325	25.0	100	(d)	314
325	1000	103	100	(e)

Example 1 – *A single circular loop of wire has a diameter of* 30.0 *cm and is rotated at* 1500 *rpm in a uniform magnetic field of flux density* 150 *mT. Calculate the maximum EMF induced.*

$$\phi = BA = BA_0 \cos \omega t \text{ so } \varepsilon = -NBA_0 \tfrac{\mathrm{d}}{\mathrm{d}t} \cos \omega t = NBA_0 \omega \sin \omega t$$

$$\varepsilon_{\text{max}} = NBA_0\omega = 1 \times 0.150 \times \frac{\pi \times 0.300^2}{4} \times 1500 \frac{2\pi}{60} = 1.67 \text{ V}$$

22.3 A 5.00 cm long square coil with 10 turns is slowly rotated in a magnetic field of 80.0 mT at a rate of 20.0 rpm (revolutions per minute). Calculate

a) the angular frequency in rad s^{-1},

b) the magnitude of the EMF induced 1.00 s after the EMF was zero,

c) the magnitude of the maximum EMF induced.

22.4 A circular coil of diameter 10.0 cm with 50 turns is rotated in a magnetic field at 100 Hz. Calculate the flux density B that would induce a peak EMF of

a) 6.00 V, b) 3.00 V, c) 1.50 V.

22.5 A circular coil of radius 12.0 cm with 100 turns is rotated in a magnetic field of 500 mT. Calculate the angular frequency ω for a peak EMF of

a) 2.00 V, b) 4.00 V, c) 8.00 V.

Example 2 – *Calculate the magnetic flux density* B *necessary to generate* $V_{rms} =$ 230 *V at* 50.0 *Hz with a square coil of length* 1.00 *m and 50 turns.*

$$B = \frac{\sqrt{2}\varepsilon_{\text{rms}}}{NA_0\omega} = \frac{\sqrt{2} \times 230}{50 \times 1^2 \times (2\pi \times 50.0)} = 20.7 \text{ mT}$$

22.6 A circular coil with 1000 turns has a diameter of 5.00 cm and is rotating at 50.0 Hz in a uniform magnetic field of flux density 100 mT. Calculate

a) the magnitude of the EMF 2.50 ms after it was zero,

b) the magnitude of the EMF 5.00 ms after it was zero,

c) the time after the EMF was zero when the EMF reaches its maximum magnitude,

d) the root mean squared EMF.

22.7 Two identical coils rotate at identical rates. *Coil A* is in a uniform magnetic field strength that is double that of *coil B*. Calculate the ratio of the root mean square EMF of *coil A* compared to *coil B*.

20 Simple pendulum

(a) $x = l\theta$ From the definition of the radian.

(b) $60^\circ = 60 \times \dfrac{2\pi}{360} = 1.047$ rad So, $x = l\theta = 30$ cm $\times\, 1.047 = 31.4$ cm

(c) Resultant force perpendicular to string has magnitude $=$ component of weight perpendicular to string $= mg\sin\theta$

(d) $ma = -mg\sin\theta$ so $a = -g\sin\theta$

(e) $a = -g\sin\theta \approx -g\theta$

(f) $\theta = \dfrac{x}{l}$ so $a \approx -g\theta = -\dfrac{gx}{l}$

(g) $a = -\dfrac{g}{l}x$ so if $a = -\omega^2 x$ then $\omega^2 = \dfrac{g}{l}$

21 Electromagnetic induction – moving wire

(a) $A = Lw = Lut$

(b) $BA = BLut$

(c) $\dfrac{\mathrm{d}(BA)}{\mathrm{d}t} = \dfrac{BA}{t} = BLu$

(d) $V = \dfrac{\mathrm{d}(BA)}{\mathrm{d}t} = BLu$

(e) Force $F_\mathrm{B} = quB$

(f) Electric field $E = \dfrac{\text{Force}}{q} = uB$

(g) $V = EL = (uB)L = BLu$ – i.e. the same as part (d)

22 Electromagnetic induction – rotating coil

(a) $\phi = BA = BA_0 \cos \omega t$

(b) $$\varepsilon = -N\frac{\mathrm{d}\phi}{\mathrm{d}t} = -N\frac{\mathrm{d}}{\mathrm{d}t}(BA_0 \cos \omega t) = -NBA_0 \frac{\mathrm{d}}{\mathrm{d}t} \cos \omega t$$
$$= NBA_0 \omega \sin \omega t$$

(c) maximum value $\sin \omega t$ can take is 1, so $\varepsilon_{\text{max}} = NBA_0 \omega$

(d) $$\varepsilon^2 = N^2 B^2 A_0^2 \omega^2 \sin^2 \omega t$$
$$\left(\varepsilon^2\right)_{\text{mean}} = N^2 B^2 A_0^2 \omega^2 \left(\sin^2 \omega t\right)_{\text{mean}} = N^2 B^2 A_0^2 \omega^2 \times \tfrac{1}{2}$$
$$\sqrt{(\varepsilon^2)_{\text{mean}}} = \varepsilon_{\text{rms}} = NBA_0 \omega \times \sqrt{0.5} = \frac{1}{\sqrt{2}} NBA_0 \omega \quad \text{hence,}$$
$$\varepsilon_{\text{rms}} = \frac{1}{\sqrt{2}} \varepsilon_{\text{max}}$$

23 Energy and fields – accelerator

(a) $p = mv = m\sqrt{\dfrac{2K}{m}} = \sqrt{2mK} = \sqrt{2mqV}$

(b) $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2qV}{m}}$

(c) $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2}{m}\left(\dfrac{mu^2}{2} + qV\right)} = \sqrt{u^2 + \dfrac{2qV}{m}}$

(d) $\Delta K = FL = qEL$

(e) $E = \dfrac{F}{q} = \dfrac{\Delta K}{qL} = \dfrac{V}{L}$

(f) $p = mv = m\sqrt{\dfrac{2K}{m}} = \sqrt{2mK} = \sqrt{2mqV} = \sqrt{2mqEL}$

(g) $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{h}{m\sqrt{2K/m}} = \dfrac{h}{\sqrt{2Km}} = \dfrac{h}{\sqrt{2mqV}}$