Isaac Physics Skills

Linking concepts in pre-university physics

Lisa Jardine-Wright, Keith Dalby, Robin Hughes,
Anton Machacek, Jed Marshall, Ingrid Murray and Lee Phillips
Isaac Physics Project

Periphyseos Press
Cambridge, UK.

Linking Concepts – Notes for the Student and the Teacher

A basketball player trains for matches using fitness exercises and ball drills. A musician will play hours of scales, arpeggios and technical exercises in the course of achieving concert standard. In a similar way, a scientist, engineer or mathematician is able to solve problems in a creative way partly because they have practised with simpler questions.

Our book *Mastering Essential Pre-University Physics* contained many questions allowing students to practise applying a single concept of Physics to a variety of situations. However practice of this kind is not enough to solve the problems faced in professional life, advanced study, or even in an examination. For these, knowledge and understanding of different concepts need to be brought together to solve the problem. Furthermore, it is not always clear which prior knowledge is going to be helpful for a particular situation.

A member of our team noticed something which helped with revision for University exams; namely that questions often required particular concepts to be combined in similar ways. He made a list of the combinations, and the equations which could be obtained by putting those ideas together. He then practised these, and found it made facing the hard, novel questions in the exam more accessible, as there would always be something similar to one of the links he had practised. A similar approach works in pre-university study.

In this book you will find, on each double page spread, a particular link between Physics concepts (or between Physics and Mathematics). You will put the equations of the concepts together, and then apply this understanding to a variety of similar questions. By the end of the two pages, if you have done the questions, you should have no difficulty remembering the link and how to apply it.

We have three particular pieces of advice. Work through enough questions until the method of combining the concepts becomes second nature. Each time you start a new question, make the links afresh — do not copy out any algebraic derivation of a previous question. Instead, work from your fundamental equations (such as those which might be found on a data sheet) each time. This builds proficiency. Finally, in the run up to examinations, redo the first question from each double page spread to ensure that your knowledge of the links is sound. This is the equivalent of practising a bounce pass or an arpeggio.

Worked solutions to the first question in each section are given in the appendix of the book.

Isaac Physics Team
Cambridge, 2021

Contents

1 Momentum and Kinetic Energy

It is helpful to be able to calculate a momentum from a kinetic energy without first working out the speed.

Example context: In particle physics, the wavelength of a particle is related to its momentum. In a question you are more likely to be told its energy (eg. a 50 keV electron) than its speed.

Quantities:	p momentum $\left(\text{kg m s}^{-1}\right)$	E kinetic energy (J)
	m mass (kg)	λ wavelength (m)
	v speed $\left(\text{m s}^{-1}\right)$	q charge (C)
	V accelerating voltage (V)	

Equations: $p = mv \quad E = \frac{1}{2}mv^2 \quad E = qV \quad \lambda = \dfrac{h}{p}$

1.1 Use the equations to derive expressions without v for

a) the kinetic energy E in terms of p and m,

b) the momentum p in terms of E and m,

c) the momentum of an accelerated particle in terms of V, m and q,

d) the wavelength of an accelerated particle.

Example 1 – *Calculate the kinetic energy of a* 9 *kg pumpkin with a momentum of* 150 *kg m s*$^{-1}$.

$$E = \frac{m}{2}v^2 = \frac{m}{2}\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} = \frac{150^2}{2\times 9} = 1250\text{ J}$$

Example 2 – *calculate the wavelength of a* 1 *keV electron.*
Kinetic energy $E = qV$ where q is the charge on one electron and $V = 1000$ V. As $E = \frac{1}{2}mv^2$, the momentum will be

$$p = mv = m\sqrt{\frac{2E}{m}} = \sqrt{2mE} = \sqrt{2mqV}$$, so we calculate $\lambda = \dfrac{h}{p}$ as

$$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.63\times 10^{-34}}{\sqrt{2\times 9.1\times 10^{-31}\times 1.6\times 10^{-19}\times 1000}} = 1.22\times 10^{-12}\text{ m}$$

1.2 Calculate the kinetic energy of a $p = 23\,700$ kg m s^{-1}, 720 kg car.

1.3 Fill in the missing entries in the table below.

Mass / kg	Momentum / kg m s^{-1}	Kinetic energy / J
32	(a)	0.040
5.6	252	(b)
4.6 g	(c)	980
12 000	168 000	(d)

1.4 Calculate the momentum of a 200 g orange with 54 J of kinetic energy.

1.5 Calculate the momentum of a proton accelerated by 20 kV.

1.6 Calculate the kinetic energy of a neutron with a wavelength of 2.4 nm.

1.7 Calculate the wavelength of an 80 keV electron.

1.8 Calculate the accelerating voltage needed to produce protons with a wavelength of 3.5 pm.

1.9 Calculate the wavelength of a 50 MeV proton.

1.10 Calculate the wavelength of a 10 MeV alpha particle.

1.11 A 10 MeV particle in a particle detector travels on a curved path in a magnetic field. From the curvature, the momentum of the particle is calculated to be 7.31×10^{-20} kg m s^{-1}.

a) What is the mass of the particle?

b) What is the particle?

1.12 A 15 g bullet hits and stops within a 1.500 kg sandbag, which then swings up by a height of 5.1 cm. Work out the initial speed of the bullet. Hint: the height can be used to work out the gravitational potential energy, and hence the initial kinetic energy of the bag. The momentum of the bag just after the collision will be equal to the momentum of the bullet before it.

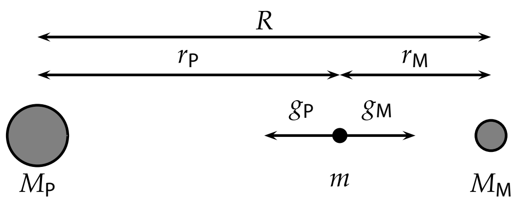
2 Vectors and Fields – between a planet and a moon

It is helpful to be able to calculate the gravitational field at points between a planet and its moon.

Example context: Satellites can be placed in orbits in complicated systems if we understand the overall gravitational field. The motion of stars in our galaxy can be used to measure the total mass of the galaxy, and thereby estimate the amount of dark matter in the galaxy.

Quantities:

- m mass of object (kg)
- M_P mass of planet (kg)
- M_M mass of moon (kg)
- g field at point $\left(\mathrm{N\,kg^{-1}}\right)$
- g_P field at point due to planet $\left(\mathrm{N\,kg^{-1}}\right)$
- g_M field at point due to moon $\left(\mathrm{N\,kg^{-1}}\right)$
- F force on object (N)
- r_P point – planet distance (m)
- r_M point – moon distance (m)
- R planet – moon distance (m)



Equations: $F = mg \quad g_\mathrm{P} = \dfrac{GM_\mathrm{P}}{r_\mathrm{P}^2} \quad g_\mathrm{M} = \dfrac{GM_\mathrm{M}}{r_\mathrm{M}^2}$ for magnitudes only

2.1 Taking the direction $\rightarrow$ as positive, write expressions

a) for the force on m due to the moon,

b) for the force on m due to the planet,

c) for the total force F on m,

d) for the total field g at the point where m is,

e) relating r_P and r_M for the location where $g = 0$.

2.2 Now repeat the first four parts of question 2.1 for the situation where $r_\mathrm{P} > R$, and the mass m is on the far side of the moon.

2.3 Calculate g_M on the surface of the Earth nearest the Moon. The radius of the Earth is 6.37×10^6 m, the mass of the Moon is $M_\mathrm{M} = 7.38 \times 10^{22}$ kg, and $R = 3.85 \times 10^8$ m

Example – *Calculate the distance from the centre of a planet $M_P = 5.0 \times 10^{24}$ kg at which there is no net force on an object. The planet's moon has a mass of 5.0×10^{22} kg and orbits at a radius of 4.4×10^{7} m.*

At this point total field $g = -\frac{GM_\text{P}}{r_\text{P}^2} + \frac{GM_\text{M}}{r_\text{M}^2} = 0$ so $\frac{M_\text{P}}{r_\text{P}^2} = \frac{M_\text{M}}{r_\text{M}^2}$

$$\frac{r_\text{M}}{r_\text{P}} = \sqrt{\frac{M_\text{M}}{M_\text{S}}} = \sqrt{\frac{5.0 \times 10^{22}}{5.0 \times 10^{24}}} = 0.1$$

$r_\text{P} + r_\text{M} = R$, therefore $\frac{r_\text{P}}{r_\text{P}} + \frac{r_\text{M}}{r_\text{P}} = \frac{R}{r_\text{P}}$. So $1 + 0.1 = \frac{R}{r_\text{P}}$

Therefore $r_\text{P} = \frac{R}{1.1} = \frac{4.4 \times 10^7}{1.1} = 4.0 \times 10^7$ m

2.4 For the system in the example, calculate the total field g at the locations below. Take the direction from the planet to the moon as positive.

a) $r_\text{P} = 2.2 \times 10^7$ m
b) $r_\text{P} = 3.9 \times 10^7$ m
c) $r_\text{P} = 4.1 \times 10^7$ m
d) $r_\text{P} = 1.6 \times 10^6$ m

2.5 For the Earth - Moon system, $M_\text{P} = 81 M_\text{M}$ and $R = 3.85 \times 10^8$ m.

a) Calculate r_M/r_P for this system.
b) Evaluate r_M as a fraction of R.
c) Evaluate r_P.

2.6 Mars has a mass of 6.39×10^{23} kg, and its 1.06×10^{17} kg moon Phobos has an orbital radius of 9.38×10^6 m. Calculate the gravitational field strength 6.0 km from the centre of Phobos on its surface nearest to Mars. Does the field point towards or away from Mars?

2.7 The mass of the Sun is 2.00×10^{30} kg, and the Earth-Sun distance is 1.50×10^{11} m. You may also use data from questions 2.3 and 2.5. Work out the component of the field g due to the *Sun and the Moon* pointing towards the centre of the Earth at the locations below:

a) at the surface of the Earth, nearest the Moon, at a full Moon,
b) at the surface of the Earth, nearest the Sun, at a full Moon.
c) Given your answers, what is principally responsible for the Earth's tides? The Sun or the Moon?

3 Energy and Fields – Closest Approach

It is useful to work out how close a charged particle can get to an atomic nucleus from its speed, its kinetic energy or the temperature of the material.

Example context: In Michelson-Morley's gold foil experiment, alpha particles were fired at gold nuclei. Depending on how close they came to the nucleus, they were deflected by different amounts. In nuclear fusion, small nuclei will only fuse if they come close enough for the strong nuclear force to act.

Quantities:
- V electric potential (V)
- U energy (J)
- v initial speed $\left(\mathrm{m\,s^{-1}}\right)$
- m mass (kg)
- Q charge of nucleus (C)
- q charge of projectile (C)
- r minimum separation (m)
- T absolute temperature (K)

Equations: $V = \dfrac{Q}{4\pi\epsilon_0 r}$ $\quad U = qV \quad U = \dfrac{mv^2}{2} \quad U = \dfrac{3k_\mathrm{B}T}{2}$

3.1 Assume that the large nucleus is held stationary in the material by interatomic forces. Use the equations to derive expressions for

a) the distance of closest approach r in terms of energy U,

b) the distance of closest approach r given the initial speed v,

c) the distance of closest approach r given the temperature T.

Example 1 – *Calculate the distance of closest approach of a* 10 *MeV alpha particle to a gold nucleus (with 79 protons) in a direct collision where the nucleus remains stationary.*

We work out the energy of the alpha particle, remembering that it has two protons $U = qV = 2 \times 1.6 \times 10^{-19} \times 1.0 \times 10^{7} = 3.2 \times 10^{-12}$ J. Now we calculate the distance of closest approach

$$r = \frac{Qq}{4\pi\epsilon_0 U} = \frac{\left(2 \times 1.6 \times 10^{-19}\right)\left(79 \times 1.6 \times 10^{-19}\right)}{4\pi\epsilon_0 \times 3.2 \times 10^{-12}} = 1.14 \times 10^{-14}\ \mathrm{m}$$

3.2 Calculate the distance of closest approach of a proton travelling at 1.0×10^{7} $\mathrm{m\,s^{-1}}$ to a copper nucleus with 29 protons. Assume that the copper nucleus remains stationary.

3.3 Repeat question 3.2 for a proton with half the speed.

3.4 How much kinetic energy would a proton need to have to approach a gold nucleus (79 protons) and to come within 2 fm of it?

3.5 Fill in the missing entries in the table below for collisions with a stationary gold nucleus.

Projectile	Energy	Closest approach distance
Proton	3.0×10^{-12} J	(a)
Proton	(b)	7.2×10^{-15} m
Alpha particle	(c)	18 fm
Positron	50 keV	(d)

3.6 For collisions of an alpha particle with a gold nucleus held stationary, calculate

a) the distance of closest approach for $v = 1.5 \times 10^6$ m s^{-1},

b) the distance of closest approach for $v = 3.0 \times 10^6$ m s^{-1},

c) the initial speed needed for $r = 7.0$ fm,

d) the initial speed needed for $r = 3.5$ fm.

Example 2 – *What temperature would be needed in order for two deuterium* $\left({}^2_1\mathrm{H}\right)$ *nuclei of typical speeds to come within* 1.0 *fm of each other?*
We need to remember that half of the energy for the approach comes from each nucleus. The total kinetic energy will be $U = 2 \times \frac{3}{2}k_\mathrm{B}T = 3k_\mathrm{B}T$.

$$3k_\mathrm{B}T = U = \frac{Qq}{4\pi\epsilon_0 r}, \text{ so } T = \frac{qQ}{4\pi\epsilon_0 r}\frac{1}{3k_\mathrm{B}} = \frac{Qq}{12\pi\epsilon_0 k_\mathrm{B} r}$$

$$T = \frac{\left(1.6 \times 10^{-19}\right)^2}{12\pi\epsilon_0 k_\mathrm{B} \times 1.0 \times 10^{-15}} = 5.6 \times 10^9 \text{ K}$$

3.7 What is the distance of closest approach of two gold nuclei if the they have kinetic energies typical of material at 1.4×10^7 K?

3.8 What would be the wavelength of a proton which could come within 3.0 fm of a uranium nucleus with 92 protons which was held stationary? Hint: you may wish to revise the link on page 1.

4 Solutions to first questions

1 Momentum and Kinetic Energy

$$p = mv \text{ so } v = \frac{p}{m}\text{. Therefore } E = \frac{m}{2}v^2 = \frac{m}{2}\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$$

$$E = \frac{mv^2}{2} \text{ so } v = \sqrt{\frac{2E}{m}}\text{. Therefore } p = mv = m\sqrt{\frac{2E}{m}} = \sqrt{\frac{2Em^2}{m}} = \sqrt{2mE}$$

$$p = \sqrt{2mE} = \sqrt{2mqV} \text{ as } E = qV$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

2 Vectors and Fields – between a planet and a moon

$$F_{\mathsf{M}} = mg_{\mathsf{M}} = +\frac{GM_{\mathsf{M}}m}{r_{\mathsf{M}}^2}$$

$$F_{\mathsf{P}} = mg_{\mathsf{P}} = -\frac{GM_{\mathsf{P}}m}{r_{\mathsf{P}}^2}$$

$$F = F_{\mathsf{M}} + F_{\mathsf{P}} = +\frac{GM_{\mathsf{M}}m}{r_{\mathsf{M}}^2} - \frac{GM_{\mathsf{P}}m}{r_{\mathsf{P}}^2}$$

$$g = \frac{F}{m} = +\frac{GM_{\mathsf{M}}}{r_{\mathsf{M}}^2} - \frac{GM_{\mathsf{P}}}{r_{\mathsf{P}}^2}$$

$$g = 0 \text{ so } \frac{GM_{\mathsf{M}}}{r_{\mathsf{M}}^2} = \frac{GM_{\mathsf{P}}}{r_{\mathsf{P}}^2} \text{ therefore } \frac{M_{\mathsf{M}}}{r_{\mathsf{M}}^2} = \frac{M_{\mathsf{P}}}{r_{\mathsf{P}}^2} \text{ and } \frac{r_{\mathsf{P}}}{r_{\mathsf{M}}} = \sqrt{\frac{M_{\mathsf{P}}}{M_{\mathsf{S}}}}$$

3 Energy and Fields – Closest approach

$$U = qV = \frac{Qq}{4\pi\epsilon_0 r}\text{, so } r = \frac{Qq}{4\pi\epsilon_0 U}$$

$$r = \frac{Qq}{4\pi\epsilon_0 U} \text{ and } U = \frac{mv^2}{2}\text{, so } r = \frac{Qq}{4\pi\epsilon_0}\frac{2}{mv^2} = \frac{Qq}{2\pi\epsilon_0 mv^2}$$

$$r = \frac{Qq}{4\pi\epsilon_0 U} \text{ and } U = \frac{3k_{\mathsf{B}}T}{2}\text{, so } r = \frac{Qq}{4\pi\epsilon_0}\frac{2}{3k_{\mathsf{B}}T} = \frac{Qq}{6\pi\epsilon_0 k_{\mathsf{B}}T}$$