Isaac Physics Skills

Linking concepts in pre-university physics

Lisa Jardine-Wright, Keith Dalby, Robin Hughes, Nicki Humphry-Baker,
Anton Machacek, Ingrid Murray and Lee Phillips
Isaac Physics Project

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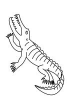
Use this collection of worksheets in parallel with the electronic version at http://isaacphysics.org/books/linking_concepts. Marking of answers and compilation of results is free on Isaac Physics. Register as a student or as a teacher to gain full functionality and support.

 used with kind permission of M. J. Rutter.

Linking Concepts – Notes for the Student and the Teacher

A basketball player trains for matches using fitness exercises and ball drills. A musician will play hours of scales, arpeggios and technical exercises in the course of achieving concert standard. In a similar way, a scientist, engineer or mathematician is able to solve problems in a creative way partly because they have practised with simpler questions.

Our book *Mastering Essential Pre-University Physics* contained many questions allowing students to practise applying a single concept of Physics to a variety of situations. However practice of this kind is not enough to solve the problems faced in professional life, advanced study, or even in an examination. For these, knowledge and understanding of different concepts need to be brought together to solve the problem. Furthermore, it is not always clear which prior knowledge is going to be helpful for a particular situation.

A member of our team noticed something which helped with revision for University exams; namely that questions often required particular concepts to be combined in similar ways. He made a list of the combinations, and the equations which could be obtained by putting those ideas together. He then practised these, and found it made facing the hard, novel questions in the exam more accessible, as there would always be something similar to one of the links he had practised. A similar approach works in pre-university study.

In this book you will find, on each double page spread, a particular link between Physics concepts (or between Physics and Mathematics). You will put the equations of the concepts together, and then apply this understanding to a variety of similar questions. By the end of the two pages, if you have done the questions, you should have no difficulty remembering the link and how to apply it.

We have three particular pieces of advice. Work through enough questions until the method of combining the concepts becomes second nature. Each time you start a new question, make the links afresh — do not copy out any algebraic derivation of a previous question. Instead, work from your fundamental equations (such as those which might be found on a data sheet) each time. This builds proficiency. Finally, in the run up to examinations, redo the first question from each double page spread to ensure that your knowledge of the links is sound. This is the equivalent of practising a bounce pass or an arpeggio.

Worked solutions to the first question in each section are given in the appendix of the book.

Isaac Physics Team
Cambridge, 2022

Contents

1 Gravitational potential and kinetic energy

Objects rising and falling exchange stores of gravitational potential and kinetic energy.

Example context: We can calculate the speed of objects after they have fallen. We can also work out the height to which a projected object rises. The analysis is particularly useful when balls bounce.

Quantities:	h_0 starting height (m)	v_0 starting speed $\left(\mathrm{m\,s^{-1}}\right)$
	h_1 final height (m)	v_1 final speed $\left(\mathrm{m\,s^{-1}}\right)$
	m mass (kg)	g gravitational field strength $\left(\mathrm{N\,kg^{-1}}\right)$
	E_K kinetic energy (J)	E_GP gravitational potential energy (J)
	η efficiency (no unit)	E_T total energy (J)

Equations: $E_\mathrm{K} = \frac{1}{2}mv^2$ $\quad E_\mathrm{GP} = mgh$ $\quad E_\mathrm{T} = E_\mathrm{K} + E_\mathrm{GP}$ $\quad E_\text{T, after} = \eta E_\text{T, before}$

1.1 In the absence of air resistance, use the equations to derive expressions for

a) the speed v_1 at the ground if an object was dropped from h_0,

b) the speed v_1 at a height h_1 if an object had speed v_0 at h_0,

c) the greatest height h_1 for an object projected up from the ground with speed v_0,

d) the greatest height h_1 for an object projected up from a height h_0 with speed v_0,

e) the greatest height h_1 above a hard surface reached by an object dropped from height h_0 if the efficiency of the bounce is η,

f) the speed v_1 just after a bounce from a hard surface if the speed just before was v_0,

Example 1 – *A 0.80 kg melon falls from 3.4 m. Calculate its speed just before striking the ground.*
At start: $E_\text{T,before} = E_\text{GP,before} = mgh_0 = 0.800 \times 9.81 \times 3.4 = 26.68$ J.
At end: $E_\text{T,after} = E_\text{K,after} = \frac{1}{2}mv_1^2 = \frac{1}{2} \times 0.800 \times v_1^2 = 0.400 v_1^2$.

$$E_\text{T,before} = E_\text{T,after} \rightarrow 26.68 = 0.400\, v_1^2 \rightarrow v_1 = \sqrt{\frac{26.68}{0.400}} = 8.2\ \mathrm{m\,s^{-1}}.$$

1.2 An 800 kg pumpkin falls from 3.4 m. Calculate its speed just before striking the ground.

1.3 A 60 g tennis ball is hit upwards at 27 m s^{-1}. How high will it rise?

1.4 A 60 g tennis ball is hit upwards at 27 m/s from a 25 m high rooftop. How fast will it be travelling when it passes the rooftop on the way down?

Example 2 – *Calculate the height reached by a* 0.15 *kg ball thrown up from a* 20 *m cliff with a speed of* 15 *m s*$^{-1}$.

$$\begin{aligned} E_{\text{T,before}} &= E_{\text{GP,before}} + E_{\text{K,before}} = mgh_0 + \tfrac{1}{2}mv_0^2 \\ &= 0.15 \times 9.81 \times 20 + \tfrac{1}{2} \times 0.15 \times 15^2 = 46.31 \text{ J} \\ E_{\text{T,after}} &= E_{\text{GP,after}} + E_{\text{K,after}} = mgh_1 + 0 \\ &= 0.15 \times 9.81 \times h_1 = 1.472\, h_1 \\ E_{\text{T,after}} &= E_{\text{T,before}} \to 1.472\, h_1 = 46.31 \to h_1 = \frac{46.31}{1.472} = 32 \text{ m} \end{aligned}$$

1.5 A 3.1 kg brick falls from scaffolding on a building site. A worker 3.5 m above the ground sees it fall past at 6.5 m/s. What is its kinetic energy just before striking the ground?

1.6 At what speed will a 4.2 kg lump of clay hit a potter's wheel if it is thrown downwards at 1.1 m s^{-1} from a height 40 cm above the wheel?

1.7 A worker at ground level throws a 2.2 kg drinks bottle upwards to a thirsty colleague 3.2 m above the ground. It just reaches him, but he fails to catch it, and it falls into an excavated trench 1.6 m below ground level.

a) At what speed did the worker need to throw the bottle if she threw it from the waist, 1.0 m above the ground?

b) How fast was it moving when it struck the base of the trench?

Example 3 – *A* 25 *g ball is thrown down to a hard surface at* 12.3 *m s*$^{-1}$. *How high will it rise after bouncing if* $\eta = 0.35$*?*

On hitting the surface $E_{\text{T,before}} = \frac{1}{2}mv_0^2 = \frac{1}{2} \times 0.025 \times 12.3^2 = 1.891$ J

$E_{\text{T,after}} = \eta E_{\text{T,before}} = 0.35 \times 1.891 = 0.6619$ J

$E_{\text{T,after}} = E_{\text{GP,final}} = mgh_1 = 0.025 \times 9.81 \times h_1 = 0.2453\, h_1$

So $0.6619 = 0.2453\, h_1 \to h_1 = \dfrac{0.6619}{0.2453} = 2.699$ m

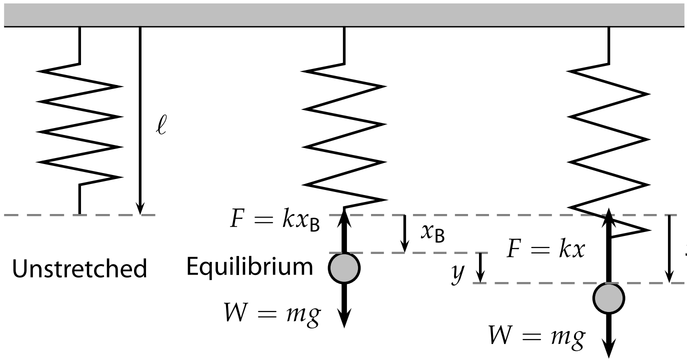
1.8 A 5.2 g ball is dropped from 90 cm onto a surface and bounces to a maximum height of 41 cm. Calculate the efficiency η.

1.9 How fast would the ball in question 1.8 rebound if it struck the surface at 2.5 m s^{-1}?

1.10 How high would a ball bounce if it struck an $\eta = 0.75$ surface at 13 m s^{-1}?

2 Gravitational, elastic and kinetic energy

Objects suspended from a spring exchange stores of kinetic, elastic potential and kinetic energy as they move up and down.



Quantities:

x spring extension (m)	ℓ spring natural length (m)
x_B equilibrium x (m)	y distance from equilibrium (m)
v speed $\left(\mathrm{m\,s^{-1}}\right)$	k spring constant $\left(\mathrm{N\,m^{-1}}\right)$
m mass (kg)	g gravitational field strength $\left(\mathrm{N\,kg^{-1}}\right)$
E_K kinetic energy (J)	E_{GP} gravitational potential energy (J)
E_T total energy (J)	E_{EP} elastic potential energy (J)
F spring tension (N)	W weight (N)

Equations:

$$E_K = \tfrac{1}{2}mv^2 \quad E_{GP} = -mgx \quad E_{EP} = \tfrac{1}{2}kx^2 \quad F = kx$$
$$E_T = E_K + E_{GP} + E_{EP} \quad W = mg \quad y = x - x_B$$

2.1 In the absence of air resistance, use the equations to derive expressions for

a) the total energy E_T in terms of x and v,

b) the value of x where the forces balance (we will call this x_B),

c) $E_{GP} + E_{EP}$ at the point where the forces balance (we will call this E_B),

d) the greatest value of x if you hold the mass at $x = 0$ and let it go,

e) (optional) the value of $E_{GP} + E_{EP}$ in terms of $y = x - x_B$.

2.2 Calculate E_{GP}, E_{EP}, E_K and E_T for a 2.5 kg mass when $x = 0.055$ m and $v = 0.25\ \mathrm{m\,s^{-1}}$ if $k = 600\ \mathrm{N\,m^{-1}}$.

2.3 Calculate x_B (the extension of the spring at the equilibrium point) for a 100 N weight hanging from a $k = 5000\ \mathrm{N\,m^{-1}}$ spring.

Example – *A* 60 *kg bungee jumper falls* 12 *m before their bungee is taut. How fast will they be moving after falling a further* 4.0 *m?* $k = 200$ *N m*$^{-1}$

At the start $x = -12$ m and $v = 0$, and $E_K = E_{EP} = 0$,

so $E_T = E_{GP} = -mgx = -60 \times 9.81 \times (-12) = 7063$ J.

When $x = 4.0$ m, $E_T = E_{EP} + E_{GP} + E_K = \frac{1}{2}kx^2 - mgx + \frac{1}{2}mv^2$

so $E_T = \frac{1}{2} \times 200 \times 4^2 - 60 \times 9.81 \times 4 + \frac{1}{2} \times 60 \times v^2$

and so $E_T = -754.4 + 30v^2$.

Total energy is constant so $7063 = -754.4 + 30v^2$,

therefore $7817.4 = 30v^2$ and $v = \sqrt{\dfrac{7817.4}{30}} = 16$ m s^{-1}

2.4 Calculate the speed of the bungee jumper in the example when

a) the bungee has stretched 5.0 m,

b) the bungee becomes slack on the way up.

2.5 Fill in the missing entries in the table below. This describes the motion of a 100 N weight ($m = 10.2$ kg), hanging from a $k = 5000$ N m^{-1} spring, which is released from rest at $x = 0$. You calculated x_B in question 2.3.

x	v	E_K	E_{GP}	E_{EP}	$E_{EP} + E_{GP}$	E_T	$y = x - x_B$
/ cm	/ m s^{-1}	/J					/ cm
1.0	(a)	(b)	(c)	(d)	(e)	0.0	(f)
2.0	(g)	(h)	(i)	(j)	(k) $= E_B$	0.0	0.0
3.0	(l)	(m)	(n)	(o)	(p)	0.0	(q)
4.0	(r)	(s)	(t)	(u)	(v)	0.0	(w)

2.6 For the system in question 2.5, state or calculate

a) the value of x where the total potential energy is a minimum,

b) the minimum total potential energy,

c) the total potential energy *relative to the minimum* when $y = 2.0$ cm,

d) the energy required to stretch a $k = 5000$ N m^{-1} spring by 2.0 cm.

2.7 Calculate how far the bungee jumper in the example falls before they first come to rest. You may assume that the *total* potential energy of the jumper relative to the equilibrium position is given by $\frac{1}{2}ky^2$.

3 Momentum and kinetic energy

It is helpful to be able to calculate a momentum from a kinetic energy without first working out the speed.

Example context: In particle physics, the wavelength of a particle is related to its momentum. In a question you are more likely to be told its energy (eg. a 50 keV electron) than its speed.

Quantities: p momentum $\left(\mathrm{kg\,m\,s^{-1}}\right)$, m mass (kg), v speed $\left(\mathrm{m\,s^{-1}}\right)$, V accelerating voltage (V), E kinetic energy (J), λ wavelength (m), q charge (C), h Planck constant (J s)

Equations: $p = mv \quad E = \frac{1}{2}mv^2 \quad E = qV \quad \lambda = \dfrac{h}{p}$

3.1 Use the equations to derive expressions without v for

a) the kinetic energy E in terms of p and m,

b) the momentum p in terms of E and m,

c) the momentum of an accelerated particle in terms of V, m and q,

d) the wavelength of an accelerated particle in terms of V and q.

Example 1 – *Calculate the kinetic energy of a 9 kg pumpkin with a momentum of 150 kg m s*$^{-1}$.

$$E = \frac{m}{2}v^2 = \frac{m}{2}\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} = \frac{150^2}{2 \times 9} = 1250\ \mathrm{J}$$

Example 2 – *calculate the wavelength of a 1 keV electron.*
Kinetic energy $E = qV$ where q is the charge on one electron and $V = 1000$ V. As $E = \frac{1}{2}mv^2$, the momentum will be

$$p = mv = m\sqrt{\frac{2E}{m}} = \sqrt{2mE} = \sqrt{2mqV}$$, so we calculate $\lambda = \dfrac{h}{p}$ as

$$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 10^3}} = 3.89 \times 10^{-11}\ \mathrm{m}$$

3.2 Calculate the kinetic energy of a $p = 23\,700$ kg m s^{-1}, 720 kg car.

3.3 Fill in the missing entries in the table below.

Mass / kg	Momentum / kg m s^{-1}	Kinetic energy / J
32	(a)	0.040
5.6	252	(b)
4.6 g	(c)	980
12 000	168 000	(d)

3.4 Calculate the momentum of a 200 g orange with 54 J of kinetic energy.

3.5 Calculate the momentum of a proton accelerated by 20 kV.

3.6 Calculate the kinetic energy of a neutron with a wavelength of 2.4 nm.

3.7 Calculate the wavelength of an 80 keV electron.

3.8 Calculate the accelerating voltage needed to produce protons with a wavelength of 3.5 pm.

3.9 Calculate the wavelength of a 50 MeV proton.

3.10 Calculate the wavelength of a 10 MeV alpha particle.

3.11 A 10 MeV particle in a particle detector travels on a curved path in a magnetic field. Its charge is 1.60×10^{-19} C. From the curvature, the momentum of the particle is calculated to be 7.31×10^{-20} kg m s^{-1}.

a) What is the mass of the particle?

b) What is the particle?

3.12 A 15 g bullet hits and stops within a 1.500 kg sandbag, which then swings up by a height of 5.1 cm. Work out the initial speed of the bullet. Hint: the height can be used to work out the gravitational potential energy, and hence the initial kinetic energy of the bag. The momentum of the bag just after the collision will be equal to the momentum of the bullet before it.

4 Elastic collisions

An elastic collision is one where the total kinetic energy is the same before and after the collision. Momentum is also conserved (as in all collisions). Solving these questions needs energy and momentum formulae.

Example context: many collisions of subatomic particles are elastic, especially if the speeds aren't high enough to trigger reactions. Collisions between snooker balls are also almost elastic.

Before collision | After collision

Quantities: p, P momentum $\left(\text{kg m s}^{-1}\right)$ k, K kinetic energy (J)
v, V velocity $\left(\text{m s}^{-1}\right)$ m, M mass (kg)

Equations: $p = mv \quad k = \frac{1}{2}mv^2 \quad P = MV \quad K = \frac{1}{2}MV^2$
$p_0 + P_0 = p_1 + P_1 \qquad k_0 + K_0 = k_1 + K_1$

4.1 Use the equations to derive expressions for

a) the final velocity V_1 of M if M was stationary at the beginning and the initial and final velocities of m (v_0 and v_1) are known,

b) V_1 if the masses are equal $(M = m)$, M begins at rest $(V_0 = 0)$, m is stopped by the collision $(v_1 = 0)$ and v_0 is known,

c) (optional) $k + K$ in terms of $p + P$, M, m and the relative velocity $r = v - V$. Hint: use $2\,(m + M)$ as a denominator for $k + K$, and then look for terms adding to give $(p + P)^2$ on the top.

Example 1 – *A 1 kg trolley moving at 1.2 m s^{-1} strikes a stationary 2 kg trolley, which then moves at 0.8 m s^{-1}. Calculate the final velocity of the 1 kg trolley.*

$$mv_0 + MV_0 = mv_1 + MV_1 \text{ (conservation of momentum)}$$
$$1 \times 1.2 + 2 \times 0 = 1 \times v_1 + 2 \times 0.8$$
$$1.2 - 1.6 = v_1 \text{ and hence } v_1 = -0.4\ \text{m s}^{-1}$$

4.2 Calculate the kinetic energy lost by the 1 kg trolley in Example 1.

4.3 Calculate the final speed of the 2 kg trolley in Example 1 assuming that it gains all of the kinetic energy lost by the 1 kg trolley.

4.4 Fill in the missing entries in the table below. For these collisions $v_0 \neq v_1$.

m	M	v_0	V_0	v_1	V_1	$K+k$	$K_1 - K_0$
/kg		/m s^{-1}				/J	
1.0	3.0	3.0	0.0	−1.5	(a)	(b)	(c)
0.050	0.050	1.5	0.0	0.0	(d)	(e)	(f)
2.0	3.0	3.0	(g)	(h)	(i)	15	0.0
0.010	0.99	50	0.0	(j)	1.0	(k)	(l)
0.010	9.99	50	0.0	(m)	0.10	(n)	(o)

4.5 In space, an elastic 'sling shot' collision is arranged between a stationary 6.4×10^{24} kg planet and a 6000 kg spacecraft moving at 4.5 km s^{-1}. By looking at the pattern in your answers to question 4.4 (j,m,l,o) estimate

a) the kinetic energy gained by the planet,

b) the final speed of the spacecraft.

In elastic collisions, the approach speed $|v_0 - V_0|$ and the separation speed $|V_1 - v_1|$ are equal. This is a consequence of question 4.1 part (c).

4.6 Repeat question 4.5b where the planet is also moving towards the spacecraft at 9.0 km s^{-1}.

Example 2 – *A neutron m with $v_0 = 1200$ m s^{-1} collides elastically with a stationary hydrogen molecule $M = 2m$. Calculate the velocity of the molecule after the collision.*
The two particles must separate at v_0, so if the molecule's final velocity is V_1, $v_1 = V_1 - v_0$. Conservation of momentum gives $mv_0 + 0 = mv_1 + 2mV_1$, so $v_0 = (V_1 - v_0) + 2V_1$, so $2v_0 = 3V_1$, and $V_1 = \frac{2}{3}v_0 = 800$ m s^{-1}.

4.7 A neutron (of mass m) travelling at 2.4×10^5 m s^{-1} collides elastically with a stationary carbon nucleus (mass $M = 12m$).

a) Calculate the final speed of the carbon nucleus.

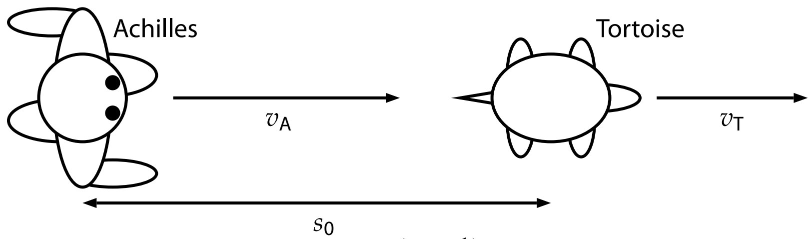
b) Calculate the percentage of the neutron's kinetic energy which is given to the nucleus.

4.8 Repeat question 4.7 for a neutron of the same speed colliding with an iron nucleus ($M = 65m$).

5 Vectors and motion – relative motion

It is helpful to be able to calculate the time it would take for two bodies to collide when they are travelling in the same direction, with the body in front is moving slower than the body behind.

Example context: Achilles chases after a tortoise. Achilles is faster than the tortoise; however, the by the time Achilles reaches where the tortoise was, it has moved forwards. When will Achilles catch up with the tortoise?



Quantities:

v_A velocity of Achilles $(\mathrm{m\,s^{-1}})$	s_0 initial displacement (m)
v_T velocity of tortoise $(\mathrm{m\,s^{-1}})$	s displacement (m)
T time for Achilles to catch up (s)	t time since start (s)

Equations: $v = \dfrac{s}{t}$

5.1 Use the equations to derive expressions for

a) the velocity of Achilles relative to the Tortoise v_{REL},

b) the time for Achilles to catch up with the tortoise T, in terms of v_A and v_T,

c) the displacement of the tortoise relative to Achilles as a function of time s.

5.2 Fill in the missing entries in the table below, using the diagram and quantities above to help.

s_0 / m	v_A / $\mathrm{m\,s^{-1}}$	v_T / $\mathrm{cm\,s^{-1}}$	T / s
(a)	5.81	6.71	15.0
1000	(b)	7.50	136
500	1.34	(c)	400
250	5.50	3.42	(d)

Example 1 – *The tortoise hops on a motor cycle and can travel at* 18.0 *m s*$^{-1}$, *whereas Achilles can* only *run at* 12.4 *m s*$^{-1}$. *They are initially* 50.0 *m apart. Calculate the time taken for them to be* 1.00 *km apart.*

$$s = s_0 - (v_A - v_T)\, t \text{ therefore } t = -\frac{s - s_0}{v_A - v_T} = -\frac{1000 - 50.0}{12.4 - 18.0} = 170\,\text{s}$$

5.3 Following on from **Example 1** above, when the tortoise travelling at 18.0 m s^{-1} is 1.00km away from Achilles, Achilles gets into a motor vehicle that can travel at 96.5 km h^{-1}. Calculate how far ahead of the tortoise Achilles is after 2 minutes.

5.4 The tortoise and Achilles decide to participate in a jousting competition, whereupon the two charge at each other as fast as they can. They are initially stood 50.0 m apart from each other. The tortoise charges towards Achilles at 5.00 m s^{-1}, and Achilles charges towards the tortoise at 15.0 m s^{-1}. Calculate

a) the time taken before they collide,

b) how far Achilles has travelled when they collide.

Example 2 – *Achilles and the tortoise start at the same location. Achilles travels due South at* 15.0 *m s*$^{-1}$, *and the tortoise travels due East at* 8.00 *m s*$^{-1}$. *Calculate how far apart they will be after* 10 *s.*
Tortoise moves 8.00 m s^{-1} $\times$ 10 s $=$ 80 m East.
Achilles moves 15.0 m s^{-1} $\times$ 10 s $=$ 150 m South.
Distance apart (using Pythagoras) $= \sqrt{150^2 + 80^2} = 170$ m

5.5 Achilles starts 50.0 m due North of the tortoise. The tortoise runs due East at 3.00 m s^{-1}. Achilles walks briskly at 4.24 m s^{-1} South-East. Calculate

a) how long until Achilles intercepts the tortoise,

b) How far Achilles has travelled in this time,

c) How far the tortoise has travelled in this time.

5.6 Achilles starts 100.0 m due North of the tortoise. The tortoise runs due East at 2.50 m s^{-1}. Achilles runs at 7.31 m s^{-1} on a bearing of $160°$. A squirrel starts 50.0 m due South of the tortoise and scurries due North at a speed of 8.90 m s^{-1}. Calculate

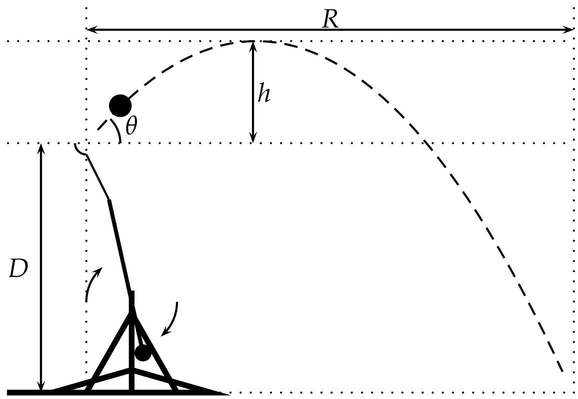
a) how long until Achilles intercepts the tortoise,

b) the distance between Achilles and the squirrel when Achilles intercepts the tortoise.

6 Vectors and motion – projectiles

It is useful to be able to calculate the maximum height and range of a body projected diagonally.

Example context: a trebuchet launches a missile towards the walls of a castle. The missile is massive enough that air resistance can be neglected.



Quantities: u initial velocity $\left(\mathrm{m\,s^{-1}}\right)$
a acceleration $\left(\mathrm{m\,s^{-2}}\right)$
s displacement (m)
t time (s)
h height increase (m)
v final velocity $\left(\mathrm{m\,s^{-1}}\right)$
R range of projectile (m)
D initial vertical displacement (m)
T time of flight (s)
θ projection angle $(^\circ)$
Subscripts $_\mathrm{x}$ and $_\mathrm{y}$ label horizontal and vertical components.

Equations: $v = u + at \quad s = \dfrac{v+u}{2}t \quad s = ut + \dfrac{1}{2}at^2 \quad v^2 = u^2 + 2as$

6.1 Use the equations to derive expressions for

a) the height increase h,

b) the final vertical component of velocity $v_{\mathrm{y,final}}$ of the missile in terms of h,

c) the time of flight of the projectile T,

d) the range of the projectile R.

Example 1 – *The projectile is launched at an angle of* 55.0° *and with an initial velocity of* $10.0\ \text{m s}^{-1}$ *from a height of* 4.00 *m. What is the maximum height of the projectile above the ground?*
Use $h = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{10.0^2 \sin^2 55.0}{2 \times 9.81} = 3.42$ m,
so maximum height $= D + h = 4.00 + 3.42 = 7.42$ m

6.2 Calculate the maximum height of a projectile launched from a trebuchet with the following initial values:

a) $u = 22.0\ \text{m s}^{-1}$ @ $\theta = 45.0^\circ$ from $s_y = 4.00$ m.

b) $u = 15.5\ \text{m s}^{-1}$ @ $\theta = 40.0^\circ$ from $s_y = 4.50$ m.

c) $u = 10.5\ \text{m s}^{-1}$ @ $\theta = 55.0^\circ$ from $s_y = 3.75$ m.

Example 2 – *Consider the same projectile as in* ***Example 1****. What is the vertical component of final velocity?*
Use $v_y = \sqrt{2 \times g \times (D + h)}$ because at the highest point $u_y = 0$.
$v_y = \sqrt{2 \times 9.81 \times (4.00 + 3.42)} = 12.1\ \text{m s}^{-1}$ $(12.07\ \text{m s}^{-1}$ to 4sf)

6.3 Calculate the vertical component of final velocity of the projectiles launched from a trebuchet with the same initial values as question 6.2:

Example 3 – *Consider the same projectile as in* ***Example 1****. What is the time of flight?*
Use $t = \dfrac{v_y - u_y}{a_y} = \dfrac{12.07 + 10.0 \sin 55.0^\circ}{9.81} = 2.07$ s (2.065 s to 4sf)
The positive sign is because the vertical components of the initial and final velocities are in opposite directions.

6.4 Calculate the time of flight of the projectiles launched from a trebuchet with the same initial values as question 6.2:

Example 4 – *Consider the same projectile as in* ***Example 1****. What is the horizontal range of the projectile?*
Use $s_x = u_x t = u \cos\theta \times t = 10.0 \cos 55.0 \times 2.065 = 11.8$ m
Note that the incorrect answer of 11.9 m is calculated if the rounded value $t = 2.07$ s is used.

6.5 Calculate the horizontal range of the projectiles launched from a trebuchet with the same initial values as question 6.2:

7 Photon flux for an LED

Photon flux (the number of photons per second) is closely related to intensity of light. Understanding how light is quantised and how current and photon flux are related in devices like LEDs and solar cells can be useful.

Example context: The energy levels in the material cause Light Emitting Diodes (LEDs) to emit light of particular wavelengths. The energy band levels correspond to the emitted photon energies, and therefore the wavelength of the emitted light. The potential difference across a component is how much energy per unit charge has been transferred by the component as the charge flows through it. Here we will assume that the drop in potential difference across an LED is entirely due to an electron changing energy state in the LED, releasing a photon in the process.

Quantities:

Φ_q photon flux $\left(\mathrm{s}^{-1}\right)$	V potential difference (V)
E photon energy (J)	e electron charge (magnitude) (C)
λ wavelength of light (m)	P LED power (W)
I electric current (A)	t duration (s)
e charge of electron (C)	n number of electrons or photons

Equations: $E = eV \quad E = \dfrac{hc}{\lambda} \quad \Phi_\mathrm{q} = \dfrac{n}{t} \quad ne = It \quad P = IV$

7.1 Use the equations to derive expressions for

a) the current I in terms of Φ_q and e,

b) the potential difference across a conducting LED V in terms of h, c, e, and λ,

c) The power of the LED P in terms of h, c, λ, and Φ_q.

Example – *Calculate the current through an LED of power rating* 88.8 *mW that produces light of wavelength* 700 *nm.*

$$P = IV = I\,\frac{hc}{e\lambda} \text{ so } \frac{e\lambda}{hc}\,P = I$$

$$I = \frac{Pe\lambda}{hc} = \frac{8.88 \times 10^{-2} \cdot 1.60 \times 10^{-19} \cdot 7.00 \times 10^{-7}}{6.63 \times 10^{-34} \cdot 3.00 \times 10^{8}} = 50.0 \text{ mA}$$

7.2 A 1.50 W Infra-Red LED produces electromagnetic radiation of wavelength 850 nm. Calculate

a) the potential difference across the LED,

b) the current that passes through the LED,

c) the photon flux emitted by the LED.

7.3 A UV-C (ultra-violet) LED emits 1.00×10^{19} photons per second when there is a potential difference of 6.22 V across it. Calculate

a) the current passing through the LED,

b) the LED's power,

c) the wavelength of electromagnetic radiation emitted.

7.4 Fill in the missing entries in the table below for different LEDs.

power P/mW	current I/mA	potential difference V/V	photon flux $\Phi_q/(10^{17}\,\mathrm{s}^{-1})$	wavelength λ/nm
	52.2		(a)	
75.0	(b)	2.26		(c)
	18.1	(d)	(e)	450
250	(f)	(g)	3.02	(h)

7.5 An LED has a power rating of 500 mW and produces blue light of wavelength 400 nm. Calculate

a) the potential difference across the LED,

b) the amount of charge that flows through the LED in one minute,

c) the number of photons emitted in one minute.

7.6 An LED has a potential difference across it of 2.07 V and emits 2.72×10^{17} photons each second. Calculate

a) the power of the LED,

b) the amount of charge that flows through the LED each second,

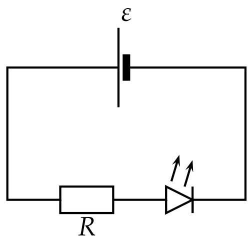
c) the amount of energy transferred by the LED in one hour.

8 Potential dividers with LEDs

It is helpful to be able to calculate the resistances necessary to obtain a particular output voltage from a potential divider circuit containing an LED.

Example context: this section builds on **Section 7** about photon flux by considering the LED in a circuit in series with a fixed resistor. The fixed resistor is needed to make sure the LED receives the correct current.

Quantities: ε e.m.f. (V)
V p.d. across fixed resistor (V)
V_{LED} p.d. across LED (V)
I current through circuit (A)
R fixed resistor resistance (Ω)
E photon energy (J)
λ wavelength of emitted light (m)



Equations: $V = IR \quad \varepsilon = V_{\text{LED}} + V \quad V_{\text{LED}} = \dfrac{E}{e} \quad E = \dfrac{hc}{\lambda}$

8.1 Use the equations to derive expressions for

a) the resistance of the fixed resistor R in terms of the e.m.f. ε, the p.d. across the LED V_{LED} and the current I,

b) the resistance of the fixed resistor R in terms of the e.m.f. ε, the wavelength of the LED λ, the current I and the physical constants h, c and e.

8.2 Fill in the missing entries in the table below.

e.m.f. / V	current / mA	fixed resistor resistance / Ω	LED p.d. / V
9.00	12.1	(a)	4.14
6.00	(b)	300	1.78
(c)	8.05	73.6	3.11
5.00	10.1	250	(d)
7.40	51.5	(e)	2.25
12.0	28.8	330	(f)

Example 1 – *Calculate the resistance R needed when a 652 nm LED is connected to a 6.00 V battery if the current is to be 50.0 mA.*

$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{652 \times 10^{-9}} = 3.051 \times 10^{-19}$ J,

so $V_{\text{LED}} = \frac{E}{e} = 1.904$ V. $V = \varepsilon - V_{\text{LED}} = 6.00 - 1.90 = 4.10$ V

$R = \frac{V}{I} = \frac{4.10}{0.050} = 81.9\,\Omega$.

8.3 A blue LED produces light of wavelength 480 nm. It is powered using a 9.00 V battery using the circuit design shown above. Assume that there is no internal resistance in the power supply and calculate

a) the p.d. across the LED,

b) the minimum value of R to ensure the current through the LED does not exceed 50.0 mA,

c) the resistance of the LED.

Example 2 – *Calculate the current through a 510 nm LED (with a p.d. of 2.44 V across it) connected to an e.m.f. of 5.00 V, in series with a $300\,\Omega$ resistor.*

P.d. is shared, so p.d. across the resistor must be $5.00 - 2.44 = 2.56$ V

Fixed resistor is ohmic, so use Ohm's law $I = \frac{V}{R} = \frac{2.56}{300} = 8.53$ mA

As resistor and LED are in series, currents are the same.

8.4 A red LED produces light of wavelength 680 nm. It is powered using a 7.4 V battery with no internal resistance. Calculate

a) the p.d. across the LED,

b) the current through the LED when its power is 102 mW *(use $P = IV$)*,

c) the resistance of the LED when its power is 102 mW,

d) the resistance of the fixed resistor R.

8.5 Two LEDs *(labelled A and B)* are connected in parallel to a 3.7 V cell. Each LED is protected by its own resistor in series. *LED A* is protected by a $330\,\Omega$ resistor, whereas *LED B* is protected by a $165\,\Omega$ resistor. Both LEDs produce light of wavelength 650 nm. Presenting your answer as a decimal, calculate

a) the ratio of the p.d. across *LED A* to the p.d. across *LED B*,

b) the ratio of the current through *LED A* to the current through *LED B*,

9 Current division

It is helpful to be able to calculate the fraction of an electric current which takes each branch of a parallel circuit.

Example context: voltmeters are not perfect insulators. When the voltage across a component is measured, a fraction of the current flows through the voltmeter, and this affects the circuit. A knowledge of the fraction of current no longer flowing through the component enables a correction to be made.

Quantities: I current (A) $\quad$ V voltage (V)
R resistance (Ω) $\quad$ G conductance (Ω^{-1} or S)
Subscripts $_{1,2}$ label components. Subscript $_{\text{C}}$ refers to the circuit.

Equations: $R = \dfrac{V}{I}$ $\quad G = \dfrac{I}{V} = \dfrac{1}{R}$ $\quad R_{\text{parallel}} = \left(\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots\right)^{-1}$

For components in parallel: $I_{\text{C}} = I_1 + I_2 + \ldots \quad V_1 = V_2 = \ldots$

9.1 Two resistors R_1 and R_2 are in parallel, and carry a total current I_{C}. Use the equations to write or derive expressions (in terms of I_{C}, R_1 and R_2) for

a) the voltage V across each resistor,

b) the current I_1 through resistor R_1,

c) the fraction of the total current which flows through R_1: $\dfrac{I_1}{I_{\text{C}}}$,

d) the conductance G_1 of resistor R_1,

e) the total conductance $G_{\text{C}} = G_1 + G_2$ of the two resistors

f) the fraction $\dfrac{G_1}{G_{\text{C}}}$.

Example – *A $3.0\,\Omega$ resistor is wired in parallel with a $6.0\,\Omega$ resistor, and between them, they carry 24 mA. Calculate the current carried by the $6.0\,\Omega$ resistor.*
Overall resistance $R_{\text{C}} = \left(3.0^{-1} + 6.0^{-1}\right)^{-1} = 2.0\,\Omega$
Voltage across combination $V = I_{\text{C}}R_{\text{C}} = 0.024 \times 2.0 = 0.048$ V
Current through the $6.0\,\Omega$ resistor $I_6 = \dfrac{V}{R} = \dfrac{0.048}{6} = 8.0$ mA

9.2 A $9.0\,\Omega$ resistor is connected in parallel with a $81\,\Omega$ resistor. What fraction of the total current flows through the $81\,\Omega$ resistor?

9.3 How much current flows through a $330\,\Omega$ resistor which is connected in parallel with a $68\,\Omega$ resistor which is carrying 40 mA by itself?

9.4 I am going to connect two resistors in parallel to share a 13 A current so that 5.0 A flows through one resistor. The resistor with the larger resistance is a $2.2\ \Omega$ resistor. Calculate the resistance of the other resistor.

9.5 Fill in the missing entries in the table below. In this circuit, three resistors (R_1, R_2, R_3) are connected in parallel.

R_1	R_2	R_3	I_1	I_2	I_3	I_C	V
/ Ω			/ A				/ V
1.0	2.0	3.0	(a)	(b)	(c)	2.4	(d)
5.0	15	20	(e)	(f)	(g)	(h)	12
48	(i)	7.5	5.0	20	(j)	(k)	(l)

9.6 A wire in an oven typically carries 20 A. I wish to put an LED in the circuit which will light up when the current is flowing. The LED requires a voltage of 1.8 V to light, and takes a current of 25 mA when it is lit. I will connect the LED in parallel with a resistor, and place the combination in series with the oven's heater element.

a) Calculate the resistance of the LED when it is lit.

b) Calculate the current through the resistor when the LED is lit.

c) Calculate the resistance of the resistor needed.

9.7 An ammeter designed for electricians has a resistance of 0.10 mΩ and it can measure a maximum of 200 A. I wish to adapt it so it can measure currents up to 1000 A by connecting a resistor in parallel with it.

a) What is the voltage across the ammeter when it carries 200 A?

b) Once the resistor is connected, what fraction of the total current should flow through the ammeter?

c) When the resistor is connected and the combination is carrying 1000 A, what is the current through the resistor?

d) Calculate the resistance of the resistor.

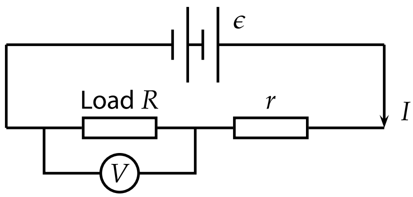
e) Using $P = IV$ calculate the power dissipated in the resistor when the combination is carrying 1000 A.

10 Power in a potential divider

It is helpful to be able to calculate the power (or fraction of the total power) dissipated in one part of a potential divider circuit.

Example context: Electrical generators have internal resistance. A power supply company wishes to maximise the efficiency of the system by ensuring that as much of the electricity generated as possible is passed on to customers.

Quantities: I current (A) | P load power (W)
R load resistance (Ω) | V voltage or p.d. across load (V)
r internal resistance (Ω) | η efficiency (no unit)
ϵ electromotive force (emf) of supply (V)



Equations: $P = IV = I^2R = \dfrac{V^2}{R}$ $\quad V = IR \quad \epsilon = V + Ir \quad \eta = \dfrac{P}{I\epsilon}$

10.1 Use the equations to derive expressions for

a) the current I in terms of ϵ, R and r,

b) the voltage V in terms of ϵ, R and r,

c) the power P in terms of ϵ, R and r,

d) the efficiency η in terms of ϵ, R and r.

Example 1 – *Calculate the efficiency if a 20 Ω resistor is supplied from a 12 V battery with an internal resistance of 4 Ω.*

Total resistance is $20 + 4 = 24\ \Omega$, so current $I = \dfrac{12\ \text{V}}{24\ \Omega} = 0.50$ A.

Power in load $P = IV = I \times IR = I^2R = 0.50^2 \times 20 = 5.0$ W.

Power supplied $I\epsilon = 0.50 \times 12 = 6.0$ W. Efficiency $= \dfrac{5.0\ \text{W}}{6.0\ \text{W}} = 0.83$

10.2 Calculate the load power P for an $\epsilon = 240$ V generator with internal resistance 2.5 Ω when it is supplying 4.2 A. Hint: use $\epsilon = V + Ir$

10.3 Calculate the efficiency η of the generator in question 10.2.

10.4 An $\epsilon = 12$ V battery has an internal resistance $r = 4.0\,\Omega$. Fill in the missing entries in the table below.

R / Ω	V / V	I / A	P / W	Efficiency η
0.10	(a)	(b)	(c)	(d)
2.0	(e)	(f)	(g)	(h)
4.0	(i)	(j)	(k)	(l)
6.0	(m)	(n)	(o)	(p)
50	(q)	(r)	(s)	(t)

10.5 Use your answers to question 10.4 to state the value of r/R which gives the greatest load power P for given, fixed values of ϵ and r.

10.6 Use your answers to question 10.4 (or other reasoning) to state the value of r/R which gives the greatest efficiency for given values of ϵ and r.

10.7 Calculate r if $P = 500$ MW, $V = 23$ kV and $\eta = 0.99$.

Example 2 – *The load resistor R in the circuit shown is replaced by $30\,\Omega$ and $60\,\Omega$ heaters wired in parallel. Calculate the power dissipated in the $30\,\Omega$ heater if $\epsilon = 230$ V and $r = 3.0\,\Omega$.*

Resistance of the two heaters in parallel $R = \left(30^{-1} + 60^{-1}\right)^{-1} = 20\,\Omega$.

Total circuit resistance $= 20 + 3 = 23\,\Omega$, so current $= \dfrac{230\text{ V}}{23\,\Omega} = 10$ A.

Voltage across the heaters is $V = IR = 10\text{ A} \times 20\,\Omega = 200$ V.

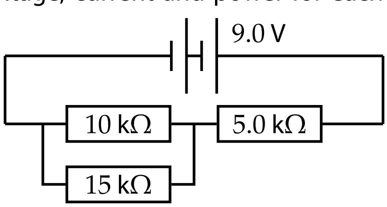
Power in $30\,\Omega$ heater is given by $\dfrac{\text{Voltage}^2}{\text{Resistance}} = \dfrac{200^2}{30} = 1300$ W to 2sf.

10.8 An $\epsilon = 5.4$ V power supply (with $r = 8.0\,\Omega$) powers a $50\,\Omega$ phone. A voltmeter (with resistance $200\,\Omega$) is connected to measure V.

a) How much voltage V is measured across the phone?

b) Calculate the power delivered to the phone.

10.9 Calculate the voltage, current and power for each of the resistors in the circuit below.

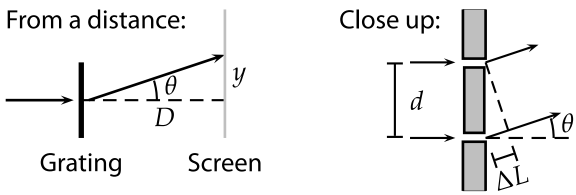


11 Path and phase difference

When waves of the same frequency arrive at a position from more than one source or route, it is helpful to calculate how they will interfere. The path and phase difference tell us whether they will interfere constructively or destructively.

Example context: a microphone placed between two speakers can receive either a strong or weak signal depending on where it is placed.

Quantities:		
	λ wavelength (m)	f frequency (Hz)
	v wave speed $\left(\mathrm{m\,s^{-1}}\right)$	ΔL path difference (m)
	D distance to screen (m)	$\Delta\phi$ phase difference (°)
	d slit separation (m)	N slits per mm $\left(\mathrm{mm^{-1}}\right)$
	θ angle from axis (°)	y distance from axis (m)
	n order of interference (no unit) $n = 0, 1, 2, 3...$ if constructive	



Equations: $v = f\lambda \quad \Delta\phi = \dfrac{\Delta L}{\lambda} \times 360^\circ \quad y = D\tan\theta \quad d = \dfrac{1\,\mathrm{mm}}{N}$

For slits: $\Delta L = d\sin\theta$ Small angles: $\tan\theta \approx \sin\theta$

11.1 Use the equations to derive expressions for

a) the phase difference $\Delta\phi$ in terms of d, θ and λ,

b) $\sin\theta$ for constructive interference in terms of λ, n and d,

c) $\sin\theta$ for constructive interference in terms of λ, n and N,

d) $\sin\theta$ for constructive interference in terms of n, N, f and v,

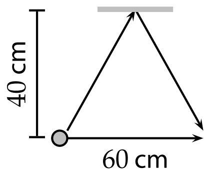
e) y for $n = 1$ in terms of λ, D and d if θ is small,

f) y for $n = 5$ in terms of f, v, D and d if θ is small,

g) ΔL for a microphone placed between two speakers connected to the same signal. The speakers are a distance D apart, and the microphone is a distance y from the mid point.

11.2 Calculate $\Delta\phi$ (as an angle less than 360°) for $\Delta L = 40.0$ cm if $\lambda = 3.6$ cm.

Example 1 – *Calculate $\Delta\phi$ between the two routes below. $\lambda = 12$ cm*



Longer route: $L = 2 \times \sqrt{30^2 + 40^2} = 100$ cm.
Direct route: $L = 60$ cm. $\Delta L = 100 - 60 = 40$ cm.
$\Delta\phi = \frac{\Delta L}{\lambda} \times 360^\circ = \frac{40}{12} \times 360^\circ = \left(3\frac{1}{3}\right) \times 360^\circ$.
Ignoring the 3 whole rotations, which do not affect the interference, $\Delta\phi = \frac{1}{3} \times 360^\circ = 120^\circ$.

11.3 A 440 Hz sound wave reaches a microphone by two routes. The sound travels 2.50 m directly and travels 4.00 m if it reflects off a wall on the way. Calculate the phase difference on arrival. Assume that $v = 330$ m s^{-1}.

Example 2 – *Light from a sodium lamp passes a grating with 650 lines mm^{-1} and then strikes a wall which is $D = 50.0$ cm from the grating. The grating and wall are both at right angles to the original ray. The first order ($n = 1$) interference hits the wall $y = 20.7$ cm from the centre. Calculate the wavelength.*

$d = \frac{1.0 \times 10^{-3}\text{ m}}{650} = 1.538 \times 10^{-6}$ m; $\theta = \tan^{-1}\left(\frac{0.207}{0.500}\right) = 22.49^\circ$

$n = 1$ so $\lambda = d \sin\theta = 1.538 \times 10^{-6}\text{ m} \times \sin(22.49^\circ) = 5.88 \times 10^{-7}$ m.

11.4 A grating with 450 lines mm^{-1} is 75.0 cm from a wall. Light shines perpendicular to both grating and wall onto the centre of the grating.

a) Calculate the angle θ for the $n = 1$ diffraction of 450 nm blue light.

b) Calculate the $n = 1$ distance y for 633 nm light.

c) Calculate λ for $n = 1$ light with $y = 27.8$ cm.

d) Visible light has wavelengths in the range 400 nm $< \lambda <$ 700 nm. How wide is the colourful $n = 1$ pattern on the wall?

11.5 20 GHz microwaves pass through a pair of narrow slits 10 cm apart. Calculate the fringe spacing (y when $n = 1$) on a screen 2.00 m behind the slits.

11.6 Calculate the smallest angle θ at which you would get destructive ($\Delta\phi = 180^\circ$) interference when 550 nm light passes two slits 50 μm apart.

11.7 Using $v = 330$ m s^{-1}, calculate $\Delta\phi$ for a microphone placed between two speakers which are 1.5 m apart if

a) $f = 440$ Hz and the microphone is 37.5 cm from one speaker,

b) $f = 660$ Hz and the microphone is 65 cm from one speaker.

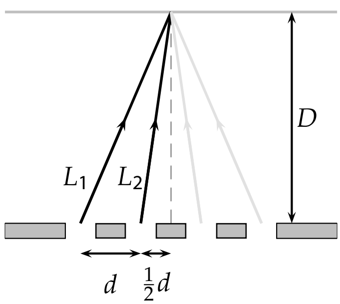
11.8 Two synchronised 10 GHz microwave transmitters face each other. How far from the mid point is the first place with $\Delta\phi = 60^\circ$?

12 Diffraction, interference and multiple slits

It is helpful to be able to determine the the path difference and phase difference of rays that originate from non-adjacent multiple slits.

Example context: multiple slits in a diffraction grating enable the interference fringes to be brighter and narrower. This is useful in spectroscopy.

Quantities:	L path length (m)	λ wavelength (m)
	ΔL path difference (m)	$\Delta\phi$ phase difference $(^\circ)$
	d slit separation (m)	D distance to screen (m)
	Subscripts $_{1,2,3,4}$ label different paths.	



Equations: $\Delta\phi = \dfrac{\Delta L}{\lambda} \times 360^\circ$ $\quad c^2 = a^2 + b^2$ (Pythagoras)

12.1 Use the equations and the diagram to derive expressions for

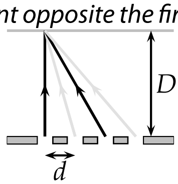
a) the path lengths L_1 and L_2 in terms of d and D,

b) the largest λ for destructive interference in terms of L_1 and L_2,

c) the largest λ for destructive interference in terms of d and D.

12.2 In the diagram above, $d = 1.00\,\mathrm{mm}$ and $D = 4.00\,\mathrm{m}$. Calculate the largest wavelength that produces total destructive interference in the centre of the screen.

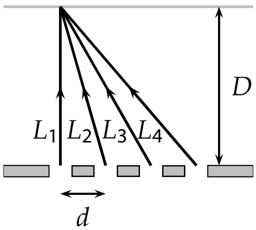
Example – *Calculate the phase difference between the first and third slit at the point opposite the first slit, when $d = 1.00$ mm, $D = 2.00$ m and $\lambda = 700$ nm.*



$$\frac{L_3 - L_1}{\lambda} = \frac{\sqrt{2.00^2 + (2.00 \times 10^{-3})^2} - 2.00}{700 \times 10^{-9}}$$

$= 1.429$, so $\Delta\phi = 360^\circ \times 1.429 = 514^\circ$.
Subtracting the whole cycle gives 154°.

12.3 In the diagram here, $D = 5.00$ m, $d = 1.00$ mm and $\lambda = 600$ nm. For the spot directly opposite slit 1, calculate



a) the ΔL between L_2 and L_3,

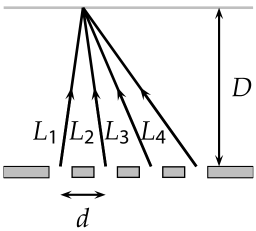
b) the $\Delta\phi$ between L_2 and L_3,

c) the ΔL between L_1 and L_4,

d) the $\Delta\phi$ between L_1 and L_4.

12.4 Describe the appearance of the spot opposite slit 1 in question 12.3.

12.5 In the diagram here, $D = 2.50$ m, $d = 1.00$ mm and $\lambda = 400$ nm. For the spot directly opposite the point half-way between slit 1 and 2, calculate



a) the ΔL between L_1 and L_3,

b) the $\Delta\phi$ between L_1 and L_3,

c) the ΔL between L_2 and L_4,

d) the $\Delta\phi$ between L_2 and L_4.

12.6 Describe the appearance of the spot directly opposite the point half-way between slit 1 and 2 in question 12.5.

12.7 The screen in question 12.6 is moved to a distance of 5.00 m from the slits, by the rest of the experimental setup remains the same. Calculate

a) the ΔL between L_1 and L_3,

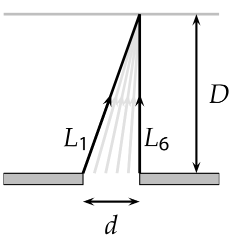
b) the $\Delta\phi$ between L_1 and L_3,

c) the ΔL between L_2 and L_4,

d) the $\Delta\phi$ between L_2 and L_4.

12.8 Describe the appearance of the spot directly opposite the point half-way between slit 1 and 2 in question 12.7.

12.9 A single slit can be thought of as six individual slits. In the diagram here, $D = 20.0$ cm, the width of the slit $d = 1.00$ mm and $\lambda = 600$ nm. For the spot directly opposite the individual slit 6, calculate



a) the $\Delta\phi$ between L_1 and L_4,

b) the $\Delta\phi$ between L_2 and L_5,

c) the $\Delta\phi$ between L_3 and L_6,

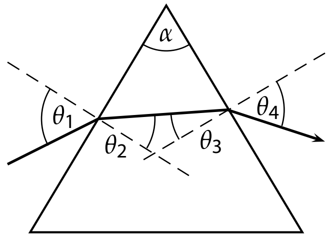
d) the appearance of the spot.

13 Reflection and refraction – angle of acceptance and prisms

It is helpful to be able to calculate the angle of acceptance for light entering an optical fibre; that is, the maximum angle of incidence that causes total internal reflection within the fibre. As a starting point, consider a prism.

Example context: A beam of light from a laser is directed towards a glass triangular prism. At large enough angles of incidence, the light passes through the prism. At small enough angles of incidence, the light undergoes total internal reflection within the prism.

Quantities: θ_1 incidence angle (°) | θ_2 refraction angle (°)
θ_3 incidence angle (°) | θ_4 refraction angle (°)
n_A refractive index of air | n_G refractive index of glass
α prism angle (°)



The dotted lines are normals

Equations: $n_A \sin\theta_1 = n_G \sin\theta_2$ $\quad n_G \sin\theta_3 = n_A \sin\theta_4$ $\quad \alpha = \theta_2 + \theta_3$

13.1 Use the diagram above and the equations to derive expressions for

a) the refracted angle θ_2 in terms of θ_1 and the refractive indices,

b) the incidence angle θ_3 in terms of α, θ_1 and the refractive indices,

c) the refraction angle θ_4 in terms of α, θ_1 and the refractive indices.

13.2 Fill in the missing entries in the table below.

n_G	n_A	$\alpha/°$	$\theta_1/°$	$\theta_2/°$	$\theta_3/°$	$\theta_4/°$
1.52	1.00	40.0	40.0	(a)	(b)	(c)
1.38	1.00	65.0	35.0	(d)	(e)	(f)
1.50	1.00	60.0	28.0	(g)	(h)	(i)

Example – *Calculate the outgoing refraction angle θ_4 for a prism ($\alpha = 60.0°$) made of glass ($n_G = 1.50$) in air ($n_A = 1.00$) when the incoming incidence angle $\theta_1 = 40°$.*

$$\theta_2 = \sin^{-1}\left(\frac{n_\mathrm{A}}{n_\mathrm{G}}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.00}{1.50} \times \sin 40°\right) = 25.37°$$

$$\theta_3 = \alpha - \theta_2 = 60.0° - 25.37° = 34.63°$$

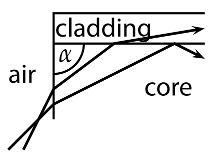
$$\theta_4 = \sin^{-1}\left(\frac{n_\mathrm{G}}{n_\mathrm{A}}\sin\theta_3\right) = \sin^{-1}\left(\frac{1.50}{1.00} \times \sin 34.63°\right) = 58.5°$$

13.3 Total internal reflection occurs when the incidence angle is greater than the critical angle. The critical angle is the incidence angle that would produce a refraction angle of $90.0°$. Calculate the critical angle for the glass-air boundary when $n_\mathrm{G} = 1.50$.

13.4 For a $60.0°$ prism made of the same glass as question 13.3, calculate the minimum incidence angle θ_1 that does not produce total internal reflection within the prism. *(Hint, let $\theta_4 = 90.0°$)*

Optical fibres are similar to prisms, except instead of air, the glass core is clad with a slightly different glass. The same equations as were used so far can be used for optical fibres, where n_A becomes the cladding refractive index and n_G becomes the core refractive index.

13.5 Calculate the critical angle for core-cladding optical fibre boundary when $n_\mathrm{G} = 1.51$ and $n_\mathrm{A} = 1.49$.



13.6 An optical fibre can be thought of like a $90°$ prism, except θ_1 is in air and θ_4 is in the cladding. If the core and cladding have the same refractive indices as question 13.5, calculate the maximum incidence angle θ_1 that can produce total internal reflection within the fibre. (This is called the angle of acceptance)

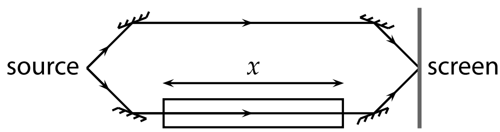
13.7 An optical fibre has a core with refractive index 1.55 and cladding with refractive index 1.49. Calculate the angle of acceptance.

14 Optical path

It is helpful to be able to calculate the optical path length for a ray of light passing through a medium. This enables us to study interference in different media.

Example context: a beam of light passes through a beam splitter that causes 50% of the intensity to travel along a path in the air, and the other 50% of the intensity to pass through the same distance path through water. When the two beams are brought back together, they interfere.

Quantities: n refractive index; ℓ optical path length (m); x geometric path length (m); $\Delta\phi$ phase difference (°); $\Delta\ell$ optical path difference (m); λ wavelength (m)
Subscripts $_{1,2}$ label different paths



Equations: $\Delta\phi = \dfrac{\Delta\ell}{\lambda} \times 360^\circ \quad \ell = n\,x \quad \Delta\ell = \ell - x$

14.1 Use the equations and the diagram above to derive expressions for

a) the phase difference $\Delta\phi$ in terms of ℓ, x and λ,

b) the phase difference $\Delta\phi$ in terms of n, x and λ,

c) the shortest geometric path length x through a medium of refractive index n that causes destructive interference.

Example 1 – *Using the diagram above, calculate the shortest geometric path length through water ($n = 1.35$) that causes red light ($\lambda = 700$ nm) to destructively interfere.*

$\Delta\ell = \frac{1}{2}\lambda$ so $nx - x = \frac{1}{2}\lambda$ and $x = \dfrac{\lambda}{2\,(n-1)} = \dfrac{700 \times 10^{-9}}{2\,(1.35-1)} = 1.00\,\mu$m

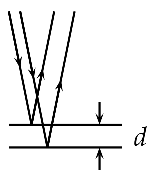
14.2 Using the diagram above, calculate the shortest geometric path length through glass ($n = 1.50$) that causes purple light ($\lambda = 400$ nm) to destructively interfere.

14.3 Using the diagram on page 27, complete the table below.

$\Delta\phi$ / °	λ / nm	n	x / μm
(a)	700	1.35	1.00
360	(b)	1.40	1.50
270	550	(c)	0.500
90.0	630	2.20	(d)

Example 2 – *Normal rays reflect off the two sides of a sheet of glass ($n = 1.50$), thickness $d = 1.20\ \mu$m. The rays have a wavelength $\lambda = 654$ nm. Ray 1 (see diagram) has a phase change of $180°$ on reflection, Ray 2 does not. Calculate the phase difference between the reflected rays 1 and 2 outside the glass.*

1 2Not drawn normal for clarity



$$\phi_1 = 180° \text{ while } \phi_2 = \frac{\Delta\ell}{\lambda} \times 360° = \frac{2nd}{\lambda} \times 360°.$$

$$\Delta\phi = \phi_2 - \phi_1 = \left(\frac{2 \times 1.50 \times 1.20 \times 10^{-6}}{654 \times 10^{-9}} - \frac{1}{2}\right) \times 360°$$

$= 5.0 \times 360°$. Equivalent to $\Delta\phi = 0°$ (constructive).

14.4 A thin film of soapy water ($n = 1.25$) is illuminated with coherent monochromatic yellow light ($\lambda = 580$ nm) normal to the surface. Calculate the minimum thickness of soapy water film for destructive interference.

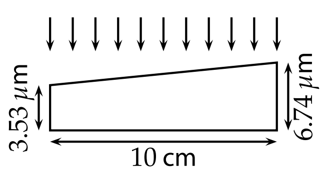
14.5 A thin film of soapy water ($n = 1.25$) is illuminated with coherent blue light ($\lambda_1 = 450$ nm) and coherent orange light ($\lambda_2 = 600$ nm) normal to the surface. Calculate the minimum thickness of soapy water film that causes destructive interference of

a) blue light,

b) orange light,

c) both blue and orange light.

14.6 A sheet of glass ($n = 1.50$) is slightly thicker at one side (see diagram). It is illuminated from above with coherent monochromatic red light ($\lambda = 700$ nm) normal to the surface. Calculate the number of dark fringes that would be observed from above the wedge.



15 Standing waves on a string

Standing waves (also known as stationary waves) appear in many places in physics, so it is useful to be able to work out the frequency of a stationary wave for different harmonics.

Example context: On a string bound at both ends and held under tension, standing waves can exist at certain discrete frequencies. The lowest frequency of standing wave is called the fundamental mode or the first harmonic. The second harmonic has double the frequency of the first harmonic.

Quantities:

- f frequency (Hz)
- n harmonic (no unit)
- λ wavelength (m)
- T tension in string (N)
- ℓ length of vibrating string (m)
- μ linear mass density $\left(\text{kg m}^{-1}\right)$
- c speed of progressive wave $\left(\text{m s}^{-1}\right)$
- M mass of vibrating string (kg)

Equations: $c^2 = \dfrac{T}{\mu}$ $\quad \mu = \dfrac{M}{\ell}$ $\quad \lambda = \dfrac{2\ell}{n}$ $\quad c = f\lambda$

15.1 Use the equations to derive expressions for

a) the fundamental frequency f_1 in terms of λ, μ and T *(Hint: $n = 1$)*,

b) the fundamental frequency f_1 in terms of ℓ, μ and T,

c) the frequency of the n^{th} harmonic f_n in terms of ℓ, n, μ and T.

15.2 Fill in the missing entries in the table below for different standing waves.

Tension T	Linear mass density μ	frequency f	wavelength λ
(a)	5.00 g m^{-1}	50.0 Hz	28.3 cm
5.00 N	(b)	50.0 Hz	28.3 cm
5.00 N	5.00 g m^{-1}	(c)	28.3 cm
5.00 N	5.00 g m^{-1}	50.0 Hz	(d)

Example – *Calculate the frequency of the 5th harmonic on a vibrating string of length* 1.00 *m under* 5.00 *N tension with a linear mass density of* 1.00 *g m*$^{-1}$

$$f_n = \frac{c}{\lambda} = \frac{\sqrt{T/\mu}}{2\ell/n} = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{5}{2\times 1.00}\sqrt{\frac{5.00}{0.001}} = 177\text{ Hz}$$

15.3 Fill in the missing entries in the table below for different standing waves.

Frequency f	Harmonic n	Length ℓ	Tension T	Linear mass density μ
82.4 Hz	1	64.8 cm	(a)	5.78 g m^{-1}
313 Hz	3	64.8 cm	56.4 N	(b)
523 Hz	(c)	33.0 cm	98.0 N	3.29 g m^{-1}
824 Hz	4	(d)	650 N	36.4 g m^{-1}

15.4 A standing wave has 4 nodes including the two at each end. The length of the vibrating string is 85.0 cm, the tension in the string is 75.0 N, and it vibrates at a frequency of 50 Hz. Calculate the linear mass density μ of the string.

15.5 A 2.00 m long string has a mass of 10.9 g. It is used in an experiment where two bridges are placed horizontally 90 cm apart. The string is kept under tension by suspending an unknown mass on the end of the string, which passes over a low-friction pulley wheel. The other end of the string is clamped in place. A large speaker nearby produces vibrations of 50.0 Hz, which causes the string to resonate with 3 nodes between the bridges. Calculate the mass suspended on the string.

15.6 Two strings (*string A* and *string B*) are set up on a benchtop alongside each other. Each string has two bridges along its length the same distance apart. The two strings have equal tension. A nearby loudspeaker produces sound at 440 Hz. *String A* shows three nodes between the bridges, *string B* shows two nodes between the bridges. Calculate μ_A / μ_B where μ_A and μ_B are the linear mass densities of *string A* and *string B*.

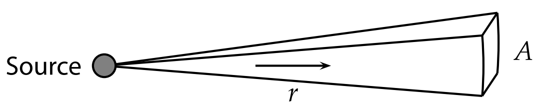
15.7 A string is held under 5.00 N of tension, with a distance between two bridges of 50.0 cm. A signal generator can produce vibrations in the string, but is broken and does not work for frequencies below 100 Hz. Resonance is observed at 125 Hz, 187.5 Hz, and 250 Hz. Calculate the speed of the progressive wave along the string.

16 Inverse square intensity

It is helpful to be able to calculate the intensity of a wave at different distances as it spreads out from a source.

Example context: the distance to a star can be calculated from a knowledge of its luminosity (power) and its brightness when seen from Earth. We can also work out the exposure to ionising radiation at different distances from a source.

Quantities: P Power (W) A Surface area $\left(\text{m}^2\right)$
I Intensity $\left(\text{W m}^{-2}\right)$ r Distance from source (m)
C Count rate $\left(\text{Bq} = \text{s}^{-1}\right)$
Subscripts label different locations, so I_1 is measured at r_1.



Equations: $A_{\text{sphere}} = 4\pi r^2 \quad I = \dfrac{P}{A}$

16.1 For a source which radiates in all directions, use the equations to derive expressions for

a) the intensity I at a distance r from a source of power P,

b) The distance d at which a source P has intensity I.

c) the intensity I_2 at a distance r_2 from a source if the intensity at distance r_1 is I_1.

Example 1 – *The desk in a room is 1.7 m directly below a perfectly efficient light bulb. To read a 0.0625 m^2 (A4) sheet of paper comfortably, at least 27 mW of light must hit it. Calculate the minimum power of light bulb required.*
Intensity required $I = \dfrac{27 \times 10^{-3}\ \text{W}}{0.0625\ \text{m}^2} = 0.432\ \text{W m}^{-2}$.
If light spreads out 1.7 m in all directions, it will illuminate a spherical shape of area $A_{\text{sphere}} = 4\pi \times (1.7\ \text{m})^2 = 36.3\ \text{m}^2$.
The power needed is $P = IA_{\text{sphere}} = 0.432 \times 36.3 = 16$ W.

16.2 Calculate the intensity if 130 W of light from a spotlight hits a $4.0\ \text{m} \times 3.0\ \text{m}$ region of a stage.

16.3 The rear light on a bicycle gives off 0.15 W of light in all backwards directions. Calculate the intensity of light 23 m behind the lamp.

16.4 Someone can see a lamp providing the intensity is larger than 2.5×10^{-7} W m^{-2}. How far away could this person see a 500 W warning lamp which gives off light in all directions?

16.5 When sunlight shines perpendicularly on a 0.6 m $\times$ 1.2 m solar panel, 195 W of electricity is generated. This is 20% of the total radiation incident on it.

a) Calculate the intensity of the sunlight at the panel.

b) The Earth is 1.50×10^{11} m from the Sun. Calculate the Sun's power.

c) How close to your eye would you need to hold a 150 W light bulb for it to appear as bright as the Sun? Assume that the Sun and the light bulb make visible light with the same efficiency.

Example 2 – *A gamma source is* 12.3 *cm from a detector, which records* 942 *background-corrected counts in* 30.0 *s. How many counts would you expect from the same detector in* 40.0 *s at a distance of* 16.8 *cm?*
The count rate C (in Bq) will be proportional to I at each distance.
Count rate at 12.3 cm $= \dfrac{942}{30} = 31.4$ Bq. Cr^2 is the same at all distances (it is proportional to P), so 31.4 Bq $\times$ $(12.3\text{ cm})^2 = C \times (16.8\text{ cm})^2$
$C = \dfrac{31.4 \times 12.3^2}{16.8^2} = 16.8$ Bq. Counts expected in 40 s $= 16.8 \times 40 = 673$.

16.6 When dentists take X-rays, they stand by the door, or outside the room. Calculate the intensity at 3.5 m from the source as a fraction of the intensity 0.32 m from it.

16.7 The background count in a laboratory is 36 counts in 40 s. When a gamma source is placed 1.5 m from the detector, there are 236 counts each minute.

a) Calculate the background-corrected count rate in Bq.

b) Calculate the expected background-corrected count rate 15 cm from the source.

16.8 On a very dark night, an astronomer can see a 5.3×10^{28} W star with their unaided eye providing the intensity is larger than 1.8×10^{-10} W m^{-2}.

a) How far away would the star be if it is only just visible?

b) What is the minimum visible intensity with a telescope of diameter 7.5 cm rather than an eye with a pupil diameter of 0.75 cm.

c) Another star of the same power is just visible using the telescope. How far away is it?

Solutions to first questions

1 Gravitational potential and kinetic energy

(a) $E_{\text{T,before}} = E_{\text{T,after}}$, so $E_{\text{P,before}} = E_{\text{K,after}}$, and $mgh_0 = \frac{1}{2}mv_1^2$.
Therefore $v_1 = \sqrt{2gh_0}$

(b) $E_{\text{T,before}} = E_{\text{T,after}}$, so $E_{\text{P,before}} + E_{\text{K,before}} = E_{\text{P,after}} + E_{\text{K,after}}$
So $mgh_0 + \frac{1}{2}mv_0^2 = mgh_1 + \frac{1}{2}mv_1^2$, and $v_1 = \sqrt{2g(h_0 - h_1) + v_0^2}$

(c) $E_{\text{T,before}} = E_{\text{T,after}}$, so $E_{\text{K,before}} = E_{\text{P,after}}$, and $\frac{1}{2}mv_0^2 = mgh_1$.
Therefore $h_1 = \dfrac{v_0^2}{2g}$

(d) $E_{\text{T,before}} = E_{\text{T,after}}$, so $E_{\text{P,before}} + E_{\text{K,before}} = E_{\text{P,after}}$
So $mgh_0 + \frac{1}{2}mv_0^2 = mgh_1$, and $h_1 = h_0 + \dfrac{v_0^2}{2g}$

(e) $E_{\text{T,after}} = \eta E_{\text{T,before}}$, so $E_{\text{P,after}} = \eta E_{\text{P,before}}$, and $mgh_1 = \eta mgh_0$.
Therefore $h_1 = \eta h_0$

(f) $E_{\text{T,after}} = \eta E_{\text{T,before}}$, so $E_{\text{K,after}} = \eta E_{\text{K,before}}$, and $\frac{1}{2}mv_1^2 = \frac{1}{2}\eta mv_0^2$.
Therefore $v_1 = \sqrt{\eta}\, v_0$

2 Gravitational, elastic and kinetic energy

(a) $E_{\text{T}} = E_{\text{K}} + E_{\text{GP}} + E_{\text{EP}} = \frac{1}{2}mv^2 - mgx + \frac{1}{2}kx^2$

(b) $kx_{\text{B}} = mg$ so $x_{\text{B}} = \dfrac{mg}{k}$

(c) $E_{\text{B}} = E_{\text{GP}} + E_{\text{EP}} = -mgx_{\text{B}} + \frac{1}{2}kx_{\text{B}}^2 = -\dfrac{m^2g^2}{k} + \dfrac{k}{2}\dfrac{m^2g^2}{k^2} = -\dfrac{m^2g^2}{2k}$

(d) $E_{\text{T}} = 0$ and $E_{\text{K}} = 0$, so $0 = E_{\text{GP}} + E_{\text{EP}} = -mgx + \frac{1}{2}kx^2$
$= x\left(\frac{1}{2}kx - mg\right)$ so $x = \dfrac{2mg}{k}$

(e)
$$
\begin{aligned}
E_{\mathrm{GP}} + E_{\mathrm{EP}} &= -mgx + \tfrac{1}{2}kx^2 = -mg\left(x_{\mathrm{B}} + y\right) + \tfrac{1}{2}k\left(x_{\mathrm{B}} + y\right)^2 \\
&= -mg\left(\frac{mg}{k} + y\right) + \frac{k}{2}\left(\frac{mg}{k} + y\right)^2 \\
&= -\frac{m^2g^2}{k} - mgy + \frac{m^2g^2}{2k} + mgy + \frac{ky^2}{2} \\
&= \frac{ky^2}{2} - \frac{m^2g^2}{2k} = \frac{ky^2}{2} + E_{\mathrm{B}}
\end{aligned}
$$

3 Momentum and kinetic energy

(a) $p = mv$ so $v = \dfrac{p}{m}$. Therefore $E = \dfrac{m}{2}v^2 = \dfrac{m}{2}\left(\dfrac{p}{m}\right)^2 = \dfrac{p^2}{2m}$

(b) $E = \dfrac{mv^2}{2}$ so $v = \sqrt{\dfrac{2E}{m}}$. Now $p = mv = m\sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2Em^2}{m}} = \sqrt{2mE}$

(c) $p = \sqrt{2mE} = \sqrt{2mqV}$ as $E = qV$

(d) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mqV}}$

4 Elastic collisions

(a) $p_0 + P_0 = p_1 + P_1$ so $mv_0 + 0 = mv_1 + MV_1$ and $V_1 = \dfrac{m\left(v_0 - v_1\right)}{M}$

(b) $p_0 + P_0 = p_1 + P_1$ so $mv_0 + 0 = 0 + mV_1$ and $V_1 = v_0$

Part (b) could also be completed using energy conservation.

For the third and optional part (c), the algebra is much more complicated, but we show it so that you can see why approach and separation speeds are the same in elastic collisions. Remember that r is defined as the approach speed $(v - V = r)$, so $v = V + r$.

(c)
$$
\begin{aligned}
P + p &= MV + mv = MV + m\left(V + r\right) = \left(M + m\right)V + mr \\
\left(P + p\right)^2 &= \left(M + m\right)^2 V^2 + 2\left(M + m\right)mrV + m^2r^2 \\
K + k &= \frac{MV^2}{2} + \frac{mv^2}{2} = \frac{M^2V^2 + MmV^2 + m^2v^2 + Mmv^2}{2\left(M + m\right)}
\end{aligned}
$$

$$
\begin{aligned}
K+k &= \frac{M^2V^2 + MmV^2 + m^2\left(V+r\right)^2 + Mm\left(V+r\right)^2}{2\left(M+m\right)} \\
&= \frac{M^2V^2 + 2MmV^2 + m^2V^2 + 2m^2Vr + m^2r^2 + 2MmVr + Mmr^2}{2\left(M+m\right)} \\
&= \frac{\left(M+m\right)^2V^2 + 2\left(M+m\right)mVr + m^2r^2 + Mmr^2}{2\left(M+m\right)} \\
&= \frac{\left(P+p\right)^2 + Mmr^2}{2\left(M+m\right)} \\
&= \frac{\left(P+p\right)^2}{2\left(M+m\right)} + \frac{Mm}{2\left(M+m\right)}r^2
\end{aligned}
$$

In an elastic collision $k + K$ will be the same before and after the collision. As the total momentum $p + P$ will also be conserved, it follows that r^2 will not change either. Therefore $|r_1| = |r_0|$, so for a one-dimensional collision, $r_1 = \pm r_0$. In the $r_1 = r_0$ case, nothing has changed (there has been no collision), so in collisions $r_1 = -r_0$. In other words, when an elastic collision is viewed from the perspective of one object, the other object bounces off it at the same speed as it arrived.

5 Vectors and motion – relative motion

(a) $v_{\mathsf{REL}} = v_{\mathsf{A}} - v_{\mathsf{T}}$

(b) $v_{\mathsf{REL}} = \dfrac{s_0}{T} \longrightarrow T = \dfrac{s_0}{v_{\mathsf{REL}}} = \dfrac{s_0}{v_{\mathsf{A}} - v_{\mathsf{T}}}$

(c) $s = s_0 - \left(v_{\mathsf{A}} - v_{\mathsf{T}}\right)t$

6 Vectors and motion – projectiles

(a) $v_{\mathsf{y}}^2 = u_{\mathsf{y}}^2 + 2a_{\mathsf{y}}s_{\mathsf{y}}$ using the vertical components of the vectors.

$s_{\mathsf{y}} = -h$ when $v_{\mathsf{y}} = 0$ and $a_{\mathsf{y}} = g$ (downwards is positive)

$u_{\mathsf{y}} = -u\sin\theta$ (as upwards is negative)

$$
\begin{aligned}
-h &= \frac{v_{\mathsf{y}}^2 - u_{\mathsf{y}}^2}{2a_{\mathsf{y}}} = \frac{0 - u^2\sin^2\theta}{2g} \\
h &= \frac{u^2\sin^2\theta}{2g}
\end{aligned}
$$

(b) Analysing motion from high point to end $s_{\text{y}} = h + D,\ u_{\text{y}} = 0,\ a_{\text{y}} = g$

$v_{\text{y,final}}^2 = u_{\text{y}}^2 + 2a_{\text{y}}s_{\text{y}} = 0 + 2g\,(h+D)$, so $v_{\text{y,final}} = \sqrt{2g\,(h+D)}$

(c) Using $v_{\text{y}} = u_{\text{y}} + a_{\text{y}}t$ over the whole motion, $v_{\text{y,final}} = -u\sin\theta + gT$

$$T = \frac{v_{\text{y,final}} + u\sin\theta}{g} = \frac{\sqrt{2g\,(h+D)} + u\sin\theta}{g}$$
$$= \frac{\sqrt{u^2\sin^2\theta + 2gD} + u\sin\theta}{g}$$

(d) $u_{\text{x}} = v_{\text{x}}$ because $a_{\text{x}} = 0$

$$R = u_{\text{x}}T = u\cos\theta \times \frac{\sqrt{u^2\sin^2\theta + 2gD} + u\sin\theta}{g}$$

7 Photon flux for an LED

(a) $I = \dfrac{\text{charge}}{t} = \dfrac{ne}{t} = \dfrac{n}{t}\cdot e = \Phi_{\text{q}}e$

(b) $V = \dfrac{E}{e} = \dfrac{hc}{\lambda}\cdot\dfrac{1}{e} = \dfrac{hc}{e\lambda}$

(c) $P = IV = \Phi_{\text{q}}e\cdot\dfrac{hc}{e\lambda} = \Phi_{\text{q}}\dfrac{hc}{\lambda}$

8 Potential dividers with LEDs

(a) $V = IR$ so $R = \dfrac{V}{I} = \dfrac{\varepsilon - V_{\text{LED}}}{I}$

(b) $R = \dfrac{\varepsilon - V_{\text{LED}}}{I} = \dfrac{\varepsilon}{I} - \dfrac{hc}{Ie\lambda}$

9 Current division

(a) $V = I_{\text{C}}R_{\text{parallel}} = I_{\text{C}}\left(R_1^{-1} + R_2^{-1}\right)^{-1} = \dfrac{I_{\text{C}}}{R_1^{-1} + R_2^{-1}}$

(b) $I_1 = \dfrac{V}{R_1} = VR_1^{-1} = \dfrac{I_{\text{C}}R_1^{-1}}{R_1^{-1} + R_2^{-1}}$

(c) $\dfrac{I_1}{I_C} = I_1 \times \dfrac{1}{I_C} = \dfrac{I_C R_1^{-1}}{R_1^{-1} + R_2^{-1}} \times \dfrac{1}{I_C} = \dfrac{R_1^{-1}}{R_1^{-1} + R_2^{-1}}$

(d) $G_1 = \dfrac{I_1}{V} = \dfrac{1}{R_1} = R_1^{-1}$

(e) $G_C = G_1 + G_2 = R_1^{-1} + R_2^{-1}$

(f) $\dfrac{G_1}{G_C} = \dfrac{R_1^{-1}}{R_1^{-1} + R_2^{-1}}$

We hope you noticed that $\dfrac{G_1}{G_C} = \dfrac{I_1}{I_C}$. If one resistor has two thirds of the conductance, it will carry two thirds of the current.

10 Power in a potential divider

(a) $I = \dfrac{\epsilon}{\text{Circuit resistance}} = \dfrac{\epsilon}{R+r}$

(b) $V = IR = \dfrac{\epsilon}{R+r} \times R = \dfrac{\epsilon R}{R+r}$

(c) $P = IV = \dfrac{\epsilon}{R+r} \times \dfrac{\epsilon R}{R+r} = \dfrac{\epsilon^2 R}{(R+r)^2}$

(d) $\eta = \dfrac{P}{I\epsilon} = P \times \dfrac{1}{\epsilon I} = \dfrac{\epsilon^2 R}{(R+r)^2} \times \dfrac{1}{\epsilon \times \epsilon / (R+r)} = \dfrac{R}{R+r}$

11 Path and phase difference

(a) $\Delta\phi = \dfrac{\Delta L}{\lambda} \times 360° = \dfrac{d\,\sin\theta}{\lambda} \times 360°$

(b) $\sin\theta = \dfrac{\Delta L}{d} = \dfrac{n\lambda}{d}$ for constructive interference

(c) $\sin\theta = \dfrac{n\lambda}{d} = \dfrac{n\lambda}{1\,\text{mm}/N} = \dfrac{nN\lambda}{1 \times 10^{-3}\,\text{m}}$

(d) $\sin\theta = \dfrac{nN\lambda}{1 \times 10^{-3}\,\text{m}} = \dfrac{nN\,(v/f)}{1 \times 10^{-3}\,\text{m}} = \dfrac{nNv}{1 \times 10^{-3}\,\text{m} \times f}$

(e) $y = D\tan\theta \approx D\sin\theta = D \times \dfrac{n\lambda}{d} = \dfrac{1 \times \lambda D}{d} = \dfrac{\lambda D}{d}$

(f) $y = D\tan\theta \approx D\sin\theta = D \times \dfrac{n\lambda}{d} = \dfrac{5 \times (v/f)\,D}{d} = \dfrac{5vD}{df}$

(g) $\Delta L = \left(\tfrac{1}{2}D + y\right) - \left(\tfrac{1}{2}D - y\right) = 2y$

12 Diffraction, interference and multiple slits

(a) $$L_2^2 = D^2 + \left(\frac{1}{2}d\right)^2 \quad \text{therefore} \quad L_2 = \sqrt{D^2 + \frac{d^2}{4}}$$

$$L_1^2 = D^2 + \left(\frac{3}{2}d\right)^2 \quad \text{therefore} \quad L_1 = \sqrt{D^2 + \frac{9d^2}{4}}$$

(b) $$L_2 - L_1 = \frac{1}{2}\lambda$$

(c) $$\sqrt{D^2 + \frac{9d^2}{4}} - \sqrt{D^2 + \frac{d^2}{4}} = \frac{1}{2}\lambda$$

13 Reflection and refraction – angle of acceptance and prisms

(a) $n_\mathsf{A} \sin\theta_1 = n_\mathsf{G} \sin\theta_2$ therefore

$$\theta_2 = \sin^{-1}\left(\frac{n_\mathsf{A}}{n_\mathsf{G}}\sin\theta_1\right)$$

(b) $n_\mathsf{A} \sin\theta_1 = n_\mathsf{G} \sin\theta_2$ and $\theta_3 = \alpha - \theta_2$ therefore

$$\theta_3 = \alpha - \sin^{-1}\left(\frac{n_\mathsf{A}}{n_\mathsf{G}}\sin\theta_1\right)$$

(c) $$n_\mathsf{A} \sin\theta_4 = n_\mathsf{G} \sin\theta_3$$

$$= n_\mathsf{G} \sin\left[\alpha - \sin^{-1}\left(\frac{n_\mathsf{A}}{n_\mathsf{G}}\sin\theta_1\right)\right] \quad \text{therefore}$$

$$\theta_4 = \sin^{-1}\left\{\frac{n_\mathsf{G}}{n_\mathsf{A}}\sin\left[\alpha - \sin^{-1}\left(\frac{n_\mathsf{A}}{n_\mathsf{G}}\sin\theta_1\right)\right]\right\}$$

(Or, more sensibly, do it in three stages, as it is done in the example.)

14 Optical path

(a) $$\Delta\phi = \frac{\Delta\ell}{\lambda} \times 360^\circ = \frac{\ell - x}{\lambda} \times 360^\circ$$

(b) $$\Delta\phi = \frac{\Delta\ell}{\lambda} \times 360^\circ = \frac{\ell - x}{\lambda} \times 360^\circ = \frac{n \cdot x - x}{\lambda} \times 360^\circ = \frac{(n-1)\,x}{\lambda} \times 360^\circ$$

(c) $$180^\circ = \frac{(n-1)\,x}{\lambda} \times 360^\circ \quad \text{therefore} \quad \frac{1}{2} = \frac{(n-1)\,x}{\lambda} \quad \text{so} \quad x = \frac{\lambda}{2\,(n-1)}$$

15 Standing waves on a string

(a) $$f_1 = \frac{c}{\lambda} = \frac{\sqrt{\frac{T}{\mu}}}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$$

(b) $$f_1 = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

(c) $$f_n = \frac{c}{\lambda} = \frac{\sqrt{\frac{T}{\mu}}}{\frac{2\ell}{n}} = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$$

16 Inverse square intensity

(a) Area illuminated is $A_{\text{sphere}} = 4\pi r^2$, so $I = \dfrac{P}{A_{\text{sphere}}} = \dfrac{P}{4\pi r^2}$

(b) Using (a) $I = \dfrac{P}{4\pi r^2}$ so $r^2 = \dfrac{P}{4\pi I}$ and $r = \sqrt{\dfrac{P}{4\pi I}}$

(c) $P = I_1 A_1 = I_2 A_2$, so $I_1 \times 4\pi r_1^2 = I_2 \times 4\pi r_2^2$, so $I_2 = \dfrac{I_1 r_1^2}{r_2^2}$

In (c) we assumed that the radiation spread equally in all directions (so $A = 4\pi r^2$). The reasoning is also true for radiation which spreads in all **relevant** directions. In this case, P will not be the power of the source, but the power of a source which could shine this brightly in all directions.

17 Banked tracks for turning

(a) Resolving vertically, $N\cos\theta = mg$, so $N = \dfrac{mg}{\cos\theta}$

(b) $\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{mv^2/r}{mg} = \dfrac{v^2}{rg}$. So, $v = \sqrt{rg\tan\theta}$

(c) $$t_{\text{p}} = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{rg\tan\theta}} = 2\pi\sqrt{\frac{r}{g\tan\theta}}$$