Parametrics & Implicit Differentiation

A Level

Part A Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

The parametric equations of a curve are

$$x = \frac{1}{\sqrt{2+t}} \text{ and } y = t^3 - 3t \text{ for } -2 < t \leqslant 0$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

The following symbols may be useful: `Derivative(y, x), t`

Part B Stationary Point

The parametric equations of a curve are

$$x = \frac{1}{\sqrt{2+t}} \text{ and } y = t^3 - 3t \text{ for } -2 < t \leqslant 0$$

Give the x and y coordinates of the stationary point. Write your answer in the form (x, y) with no spaces.

What is the nature of the stationary point?

- Point of inflection
- Minimum
- Maximum

Part C Domain and Range

The parametric equations of a curve are

$$x = \frac{1}{\sqrt{2+t}} \text{ and } y = t^3 - 3t \text{ for } -2 < t \leqslant 0$$

State the domain of the resultant function. Write your answer in the form $x \geq a$, $x > a$, $x \leq a$, or $x < a$.

The following symbols may be useful: `<, <=, >, >=, x`

State the upper bound of the range. Write your answer in the form $y \geq a$, $y > a$, $y \leq a$, or $y < a$.

The following symbols may be useful: `<, <=, >, >=, y`

Give the lower bound of the range. Write your answer in the form $y \geq b$, $y > b$, $y \leq b$, or $y < b$.

The following symbols may be useful: `<, <=, >, >=, y`

Part D Sketch

The parametric equations of a curve are

$$x = \frac{1}{\sqrt{2+t}} \text{ and } y = t^3 - 3t \text{ for } -2 < t \leqslant 0$$

Sketch the graph of this function.

Easier question?

Part E $\frac{\mathrm{d}y}{\mathrm{d}x}$

Figure 2 shows the curve with equation $x^2 + y^3 - 8x - 12y = 4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q.

Figure 2: A diagram of the curve $x^2 + y^3 - 8x - 12y = 4$.

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), x, y`

Part F Implicit Differentiation

The diagram shows the curve with equation $x^2 + y^3 - 8x - 12y = 4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q.

Figure 3: A diagram of the curve $x^2 + y^3 - 8x - 12y = 4$.

Give the x and y coordinates for point P. Write your answer in the form (x, y) without spaces.

Give the x and y coordinates for point Q. Write your answer in the form (x, y) without spaces.

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Integration & Exponential Equations

A Level

Part A Integration by Substitution

The substitution $u = \sqrt{x}$ transforms $\int \frac{1}{x(1+\sqrt{x})}\mathrm{d}x$ to $\int f(u)\mathrm{d}u$. Write out the expression that is equivalent to $f(u)$.

The following symbols may be useful: `f, u`

Part B Integration

Hence find the exact value of $\int_1^9 \frac{1}{x(1+\sqrt{x})}\mathrm{d}x$.

The following symbols may be useful: `ln()`

Part C Rate of change

Figure 1: A graph showing a circular pond of radius a being covered by weeds.

The surface of a circular pond of radius a is being covered by weeds. The weeds are growing in a circular region whose centre is at the centre of the pond. At time t the region covered by the weeds has a radius r and area A (see **Figure 1**). An ecologist models the growth of the weeds by assuming that the rate of increase of the area covered is proportional to the area of the pond not yet covered.

Show that $\frac{\mathrm{d}A}{\mathrm{d}t} = 2\pi r\frac{\mathrm{d}r}{\mathrm{d}t}$.

Easier question?

Part D Differential Equation

Hence show that the ecologist's model leads to the differential equation

$$2r\frac{dr}{\mathrm{d}t} = k(a^2 - r^2),$$

where k is a constant.

Easier question?

Part E Solution to equation

By solving the differential equation in part D, express r in terms of t, a and k, given that $r = 0$ when $t = 0$.

The following symbols may be useful: `a, e, k, r, t`

Part F Weeds vs Pond

Will the weeds ever cover the whole pond?

- No
- Yes

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Circles and Integration

A Level

Figure 1 shows two circles of radius $7\,\text{cm}$ with centres A and B. The distance AB is $12\,\text{cm}$ and the point C lies on both circles. The region common to both circles is shaded.

Figure 1: A diagram of two circles of radius $7\,\text{cm}$ with centres A and B.

Part A Angle CAB

Find the value of the angle CAB in radians. Give your answer to 3 significant figures.

Part B Perimeter

Find the perimeter of the shaded region. Give your answer to 3 significant figures.

Part C Area

Find the area of the shaded region. Give your answer to 3 significant figures.

Part D x Value at P

Figure 2: A diagram of the curve $y = \sqrt{4x - 3}$ and the normal to the curve at the point $(7, 5)$.

Figure 2 shows the curve $y = \sqrt{4x - 3}$ and the normal to the curve at the point $(7, 5)$. The shaded region is bounded by the curve, the normal and the x-axis. The point P marks the intersection of the normal with the x-axis.

What is the x-value at P?

The following symbols may be useful: x

Part E Shaded Area

Find the exact area of the shaded region in **Figure 2**.

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Definite integrals 2

A Level Further A

Find the following integrals.

Part A $\int_1^\infty (3/2x\sqrt{x})\mathrm{d}x$

Find $\displaystyle\int_1^\infty \frac{3}{2}\frac{1}{x\sqrt{x}}\mathrm{d}x$.

Part B $\int_{-8}^0 1/\sqrt[3]{x}\mathrm{d}x$

Find $\displaystyle\int_{-8}^0 \frac{1}{\sqrt[3]{x}}\mathrm{d}x$.

Part C $\int_{-1}^1 (1+x+x^2/2+x^3/6)\mathrm{d}x$

Find $\displaystyle\int_{-1}^1 1+x+\frac{x^2}{2}+\frac{x^3}{6}\ \mathrm{d}x$. Give your answer as an improper fraction.

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