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Simplify Trig Expressions

A Level Further A
P P P P P P

Simplify the following trigonometric expressions.

Part A $\frac{1}{\cos^2 t - 1}$

Simplify the following expression to give a single trigonometric function:

$$\frac{1}{\cos^2 t - 1}$$

The following symbols may be useful: $\cos()$, $\operatorname{cosec}()$, $\cot()$, $\sec()$, $\sin()$, t , $\tan()$

Part B $\frac{1 - \sin^2 x}{\cos x}$

Simplify the following expression to give a single trigonometric function:

$$\frac{1 - \sin^2 x}{\cos x}$$

The following symbols may be useful: $\cos()$, $\operatorname{cosec}()$, $\cot()$, $\sec()$, $\sin()$, $\tan()$, x

Part C $\sin \alpha \tan \alpha - \frac{1}{\cos \alpha}$

Simplify the following expression to give a single trigonometric function:

$$\sin \alpha \tan \alpha - \frac{1}{\cos \alpha}$$

The following symbols may be useful: α , $\cos()$, $\operatorname{cosec}()$, $\cot()$, $\sec()$, $\sin()$, $\tan()$

Part D $\tan w - \frac{\cos w}{1 - \sin w}$

Simplify the following expression to give a single trigonometric function:

$$\tan w - \frac{\cos w}{1 - \sin w}$$

The following symbols may be useful: $\cos()$, $\operatorname{cosec}()$, $\cot()$, $\sec()$, $\sin()$, $\tan()$, w

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Functions: Reciprocal Trig 1ii

A Level



Part A $\sec \frac{1}{2}\alpha = 4$

Solve, for $0^\circ < \alpha < 180^\circ$, the equation $\sec \frac{1}{2}\alpha = 4$. Give your answer in degrees, to three significant figures.

Part B $\tan \beta = 7 \cot \beta$

Solve, for $0^\circ < \beta < 180^\circ$, the equation $\tan \beta = 7 \cot \beta$, and give the largest solution in degrees to three significant figures.

Give the smallest solution in degrees to three significant figures.

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Advanced Trig Identities 5ii



Solve, for $0^\circ < \theta < 360^\circ$, the equation $\sec^2 \theta = 4 \tan \theta - 2$.

Part A Smallest

Give the smallest solution to 3 significant figures.

Part B Second Smallest

Give the second smallest solution to 3 significant figures.

Part C Second Largest

Give the second largest solution to 3 significant figures.

Part D Largest

Give the largest solution to 3 significant figures.

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Advanced Trig Identities 2ii

A Level



Part A $2 \tan^2 \theta - \frac{1}{\cos \theta}$

Express $2 \tan^2 \theta - \frac{1}{\cos \theta}$ in terms of $\sec \theta$.

The following symbols may be useful: $\sec()$, θ

Part B **Solve**

Hence solve, for $0^\circ < \theta < 360^\circ$, the equation

$$2 \tan^2 \theta - \frac{1}{\cos \theta} = 4$$

Give the smallest solution to three significant figures.

Give the second smallest solution to four significant figures.

Give the second largest solution to four significant figures.

Give the largest solution to three significant figures.

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Sketching Reciprocal Trigonometric Functions

A Level

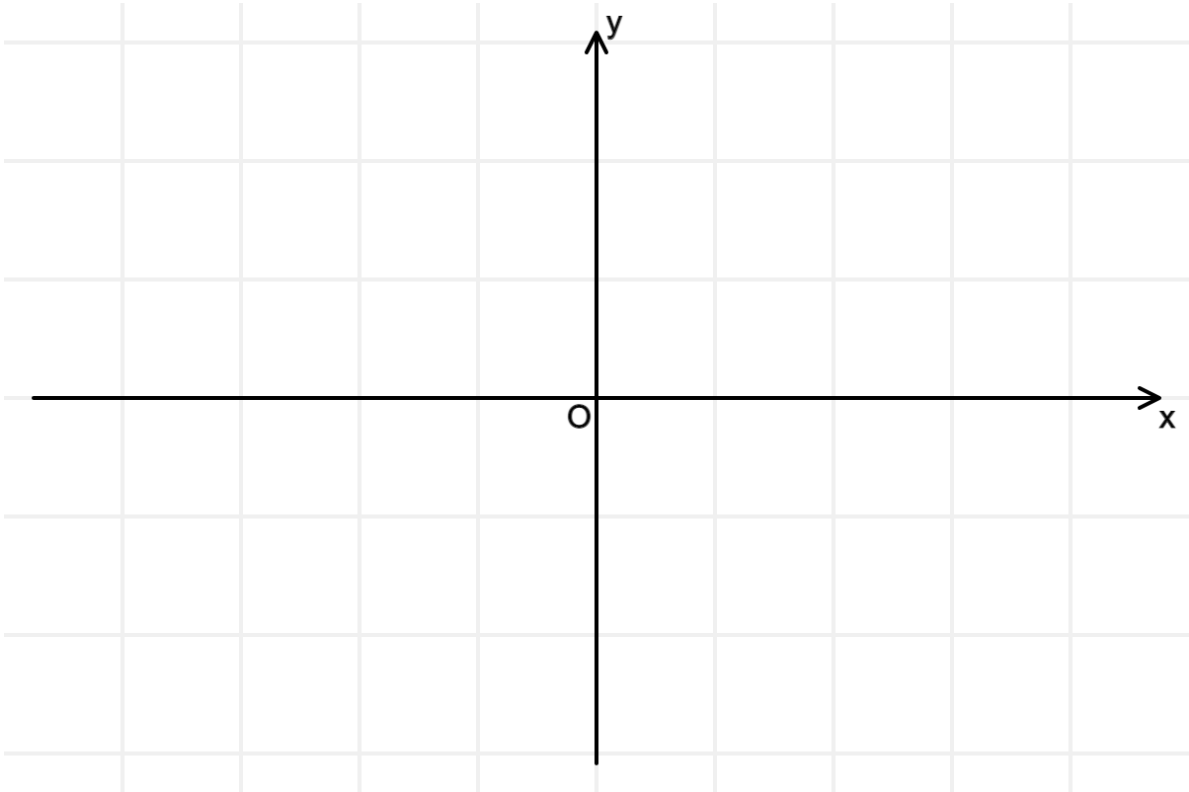
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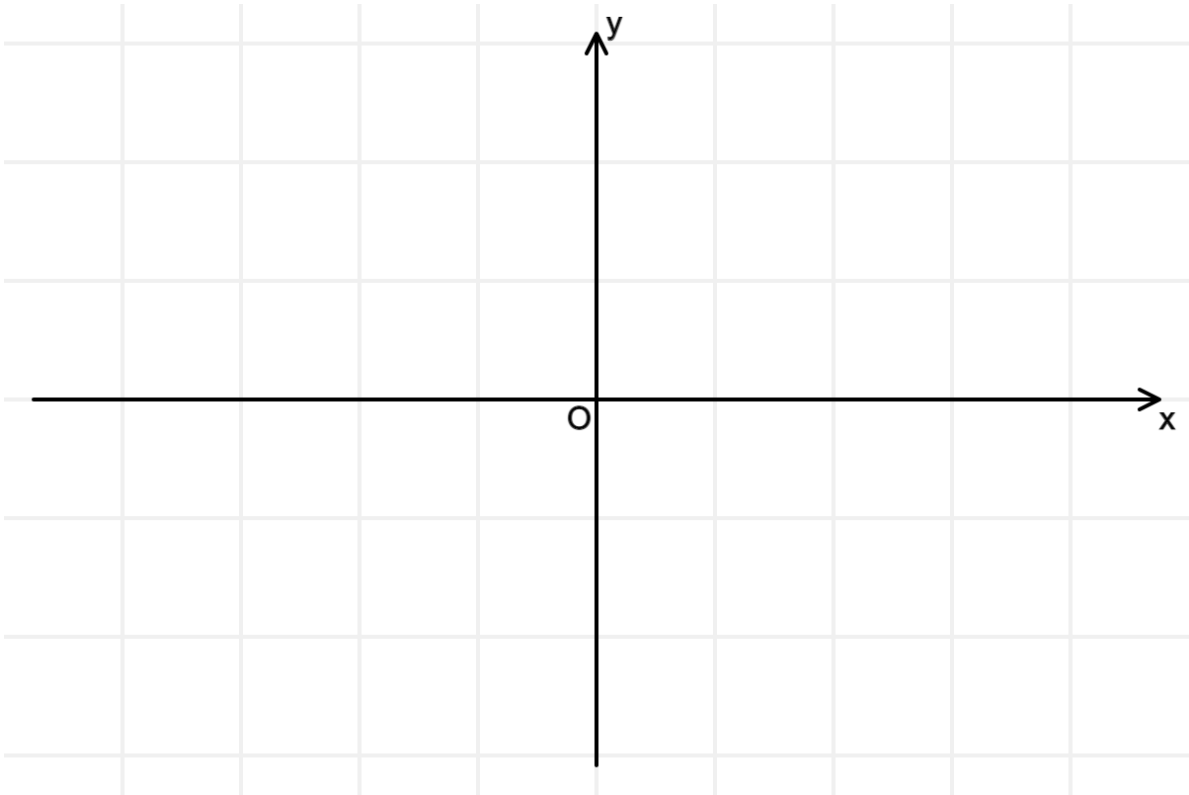
Part A Sketch $2 \sec x + 2$

Sketch the graph of $y = 2 \sec x + 2$ in the interval $0^\circ \leq x \leq 360^\circ$.



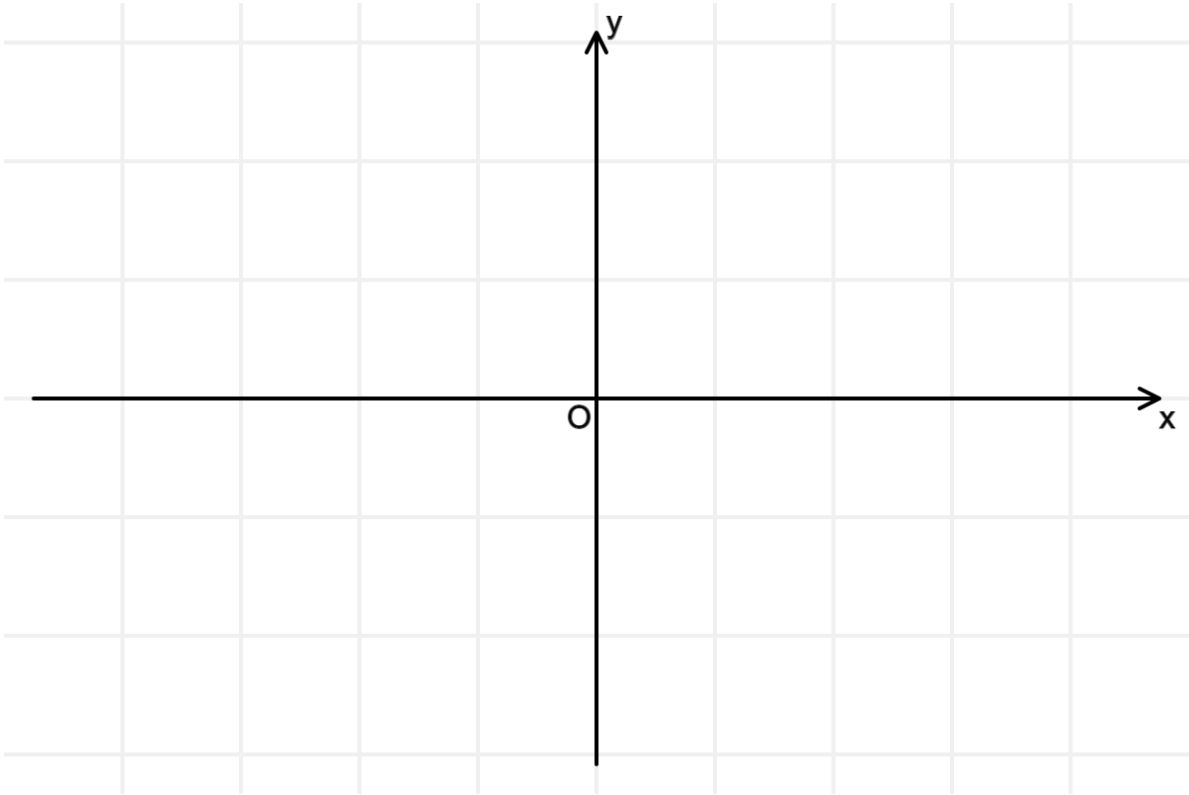
Part B Sketch $\cot \left(x + \frac{\pi}{4}\right) - 1$

Sketch the graph of $y = \cot \left(x + \frac{\pi}{4}\right) - 1$ in the interval $0 \leq x \leq 2\pi$.



Part C Sketch $-\operatorname{cosec} \frac{x}{2} - 3$

Sketch the graph of $y = -\operatorname{cosec} \frac{x}{2} - 3$ in the interval $-2\pi \leq x \leq 2\pi$.





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Functions: Inverse Trig 3ii

A Level

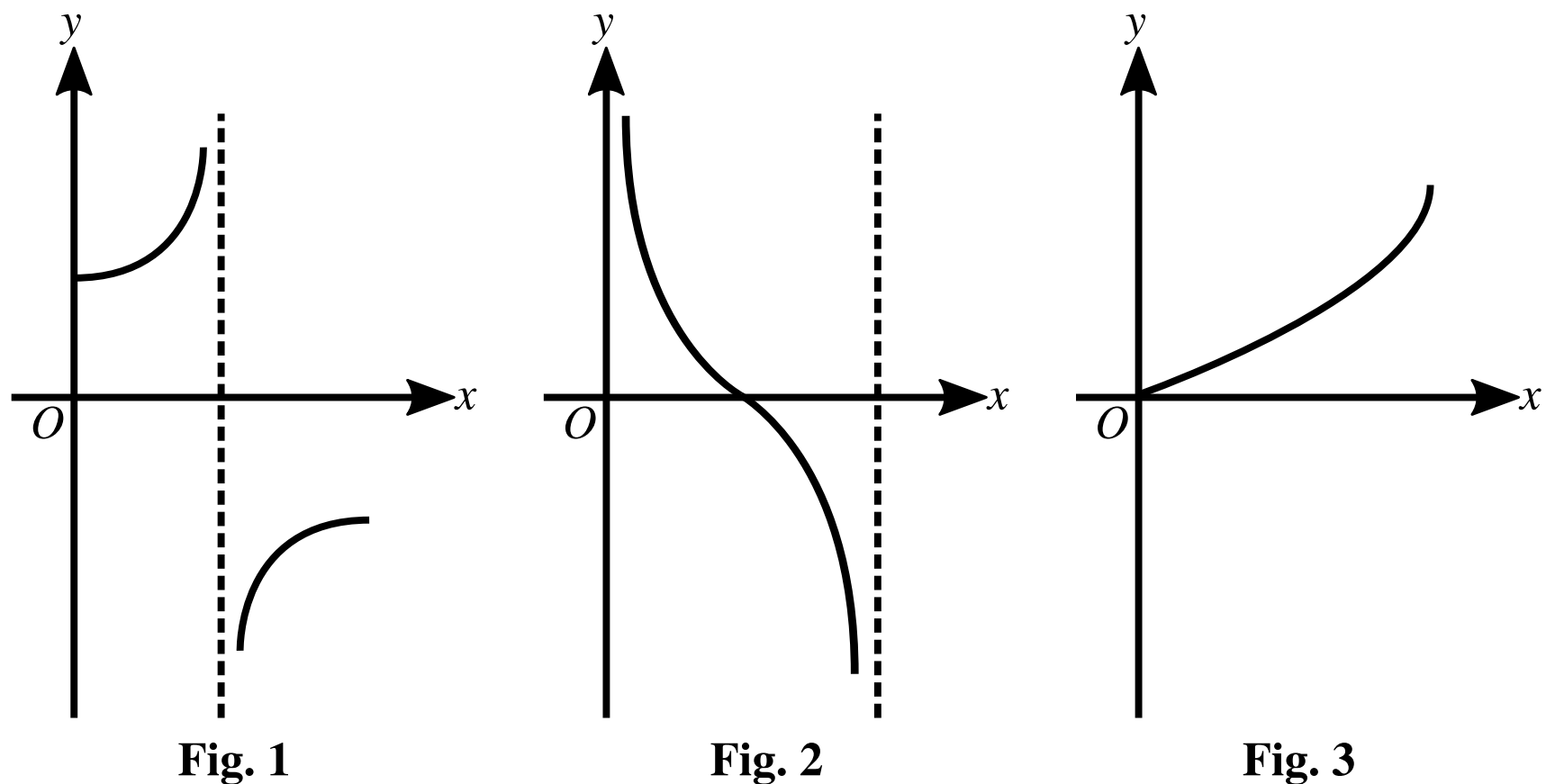


Figure 1: A diagram showing sections of curves

Each diagram in **Figure 1** shows part of a curve, the equation of which is one of the following:

$$y = \sin^{-1} x, \quad y = \cos^{-1} x, \quad y = \tan^{-1} x, \quad y = \sec x, \quad y = \operatorname{cosec} x, \quad y = \cot x$$

Part A **Fig. 1**

Which equation corresponds to **Fig. 1**?

- ☐ $y = \tan^{-1} x$
- ☐ $y = \sec x$
- ☐ $y = \cot x$
- ☐ $y = \operatorname{cosec} x$
- ☐ $y = \sin^{-1} x$
- ☐ $y = \cos^{-1} x$

Part B Fig. 2

Which equation corresponds to **Fig. 2**?

- ☐ $y = \cos^{-1} x$
- ☐ $y = \tan^{-1} x$
- ☐ $y = \operatorname{cosec} x$
- ☐ $y = \sin^{-1} x$
- ☐ $y = \cot x$
- ☐ $y = \sec x$

Part C Fig. 3

Which equation corresponds to **Fig. 3**?

- ☐ $y = \cos^{-1} x$
- ☐ $y = \operatorname{cosec} x$
- ☐ $y = \tan^{-1} x$
- ☐ $y = \cot x$
- ☐ $y = \sin^{-1} x$
- ☐ $y = \sec x$

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Gameboard:

Pure Maths Practice: Functions - Inverse Trig

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Functions: Inverse Trig 1ii

A Level



Figure 1 shows the graph of $y = -\sin^{-1}(x - 1)$.

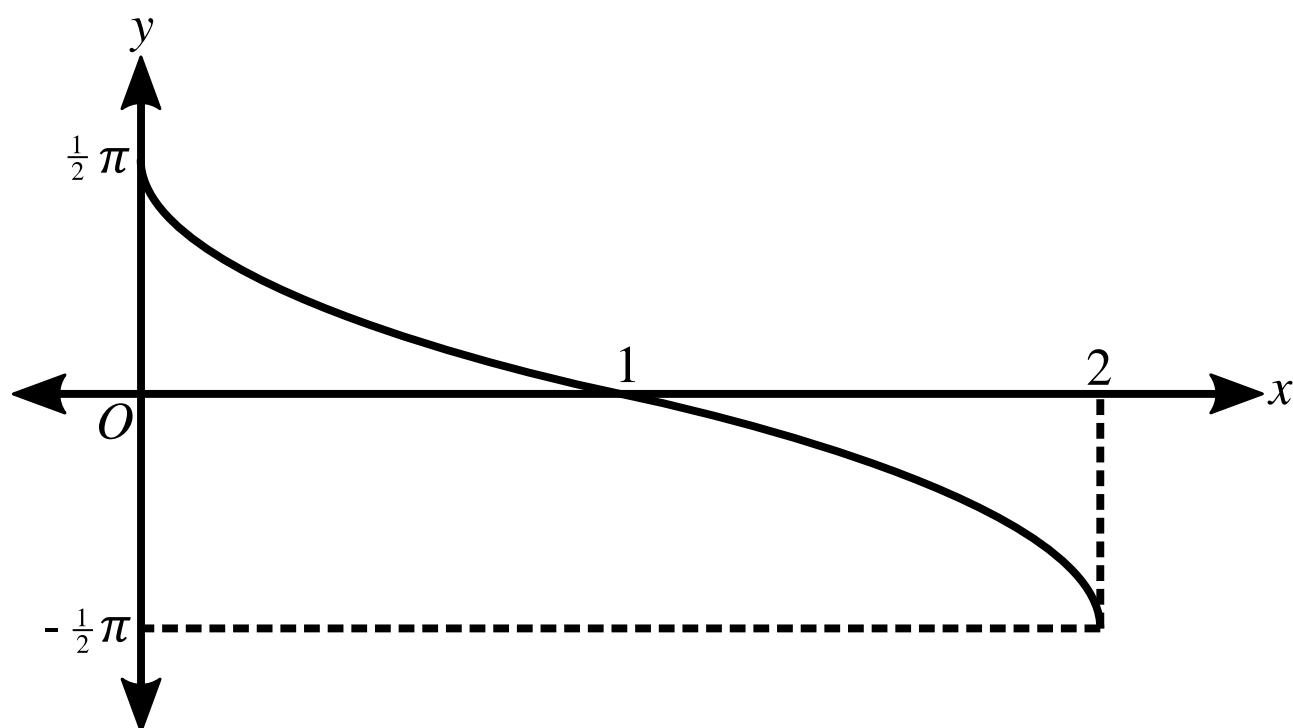


Figure 1: A graph of the function $y = -\sin^{-1}(x - 1)$

Part A Transformations

Give details of the pair of geometrical transformations which transform the graph of $y = -\sin^{-1}(x - 1)$ to the graph of $y = \sin^{-1} x$.

- ☐ Reflect in $y = -x$, translate by 1 in $+x$ -direction
- ☐ Reflect in x -axis, translate by 1 in $-x$ -direction
- ☐ Reflect in y -axis, translate by 1 in $-x$ -direction
- ☐ Reflect in x -axis, translate by 1 in $+x$ -direction
- ☐ Reflect in y -axis, translate by 1 in $-y$ -direction
- ☐ Reflect in y -axis, translate by 1 in $+y$ -direction
- ☐ Reflect in line $y = x$, translate by 1 in $+y$ -direction

Part B Sketch

Sketch the graph of $y = \left| -\sin^{-1}(x - 1) \right|$.

To see an example sketch, answer the following question: For what value of y does the curve meet the y -axis?

The following symbols may be useful: π

Part C Solutions

Find the exact solutions of the equation $\left| -\sin^{-1}(x - 1) \right| = \frac{\pi}{3}$.

Give the largest exact solution.

The following symbols may be useful: x

Give the smallest exact solution.

The following symbols may be useful: x

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Functions: Inverse Trig 3i

A Level



Figure 1 shows the graphs of $y = \cos^{-1} x$ and $y = \tan^{-1} x$, for $-1 \leq x \leq 1$ in each case. The graphs intersect at the point with coordinates (p, q) .

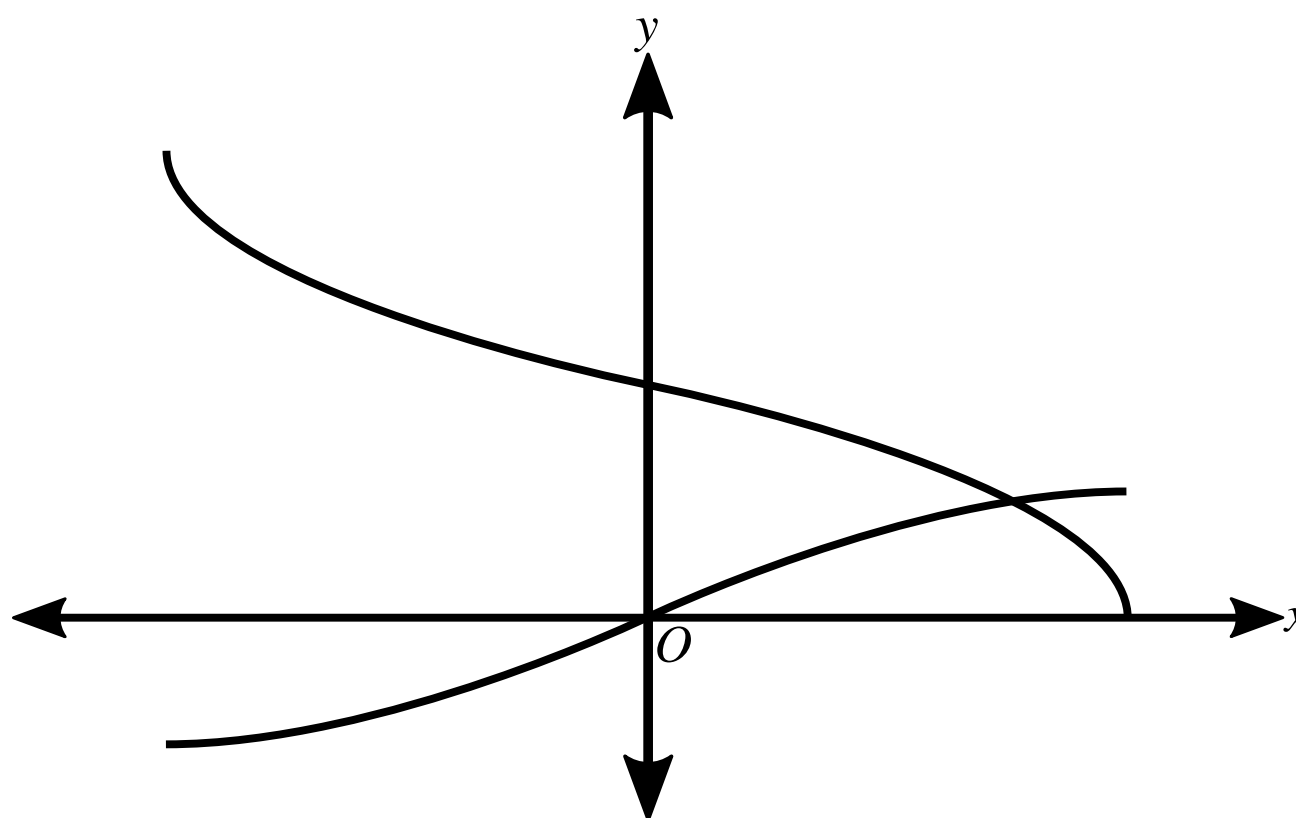


Figure 1: The graphs of $y = \cos^{-1} x$ and $y = \tan^{-1} x$, for $-1 \leq x \leq 1$.

Part A **Exact values (a)**

For the curve $y = \cos^{-1} x$, state the exact values of y for $x = -1$, 0 and 1.

State the exact value of y when $x = -1$.

The following symbols may be useful: π , y

State the exact value of y when $x = 0$.

The following symbols may be useful: π , y

State the exact value of y when $x = 1$.

The following symbols may be useful: π , y

Part B **Exact values of y**

For the curve $y = \tan^{-1} x$, state the exact values of y for $x = -1$, 0 and 1.

State the exact value of y when $x = -1$.

The following symbols may be useful: π , y

State the exact value of y when $x = 0$.

The following symbols may be useful: π , y

State the exact value of y when $x = 1$.

The following symbols may be useful: π , y

Part C The graphs $y = \cos x$ and $y = \tan x$

Write down, in terms of p and q , the coordinates of the corresponding point of intersection of the graphs of $y = \cos x$ and $y = \tan x$, and hence show that $\cos^2 q = \sin q$.

Easier question?

Part D $p^4 + p^2 - 1$

What is the value of $p^4 + p^2 - 1$?

Part E Solution of the equation

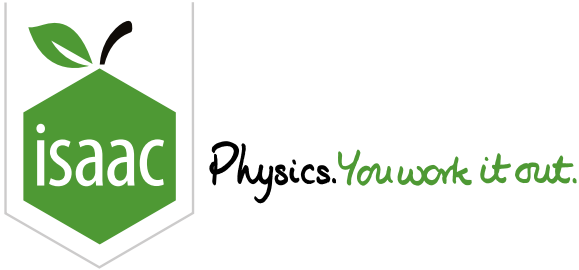
Hence find, in exact form, the solution of the equation

$$\cos^{-1} x = \tan^{-1} x.$$

The following symbols may be useful: p , x

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Sketching Inverse Trigonometric Functions

A Level

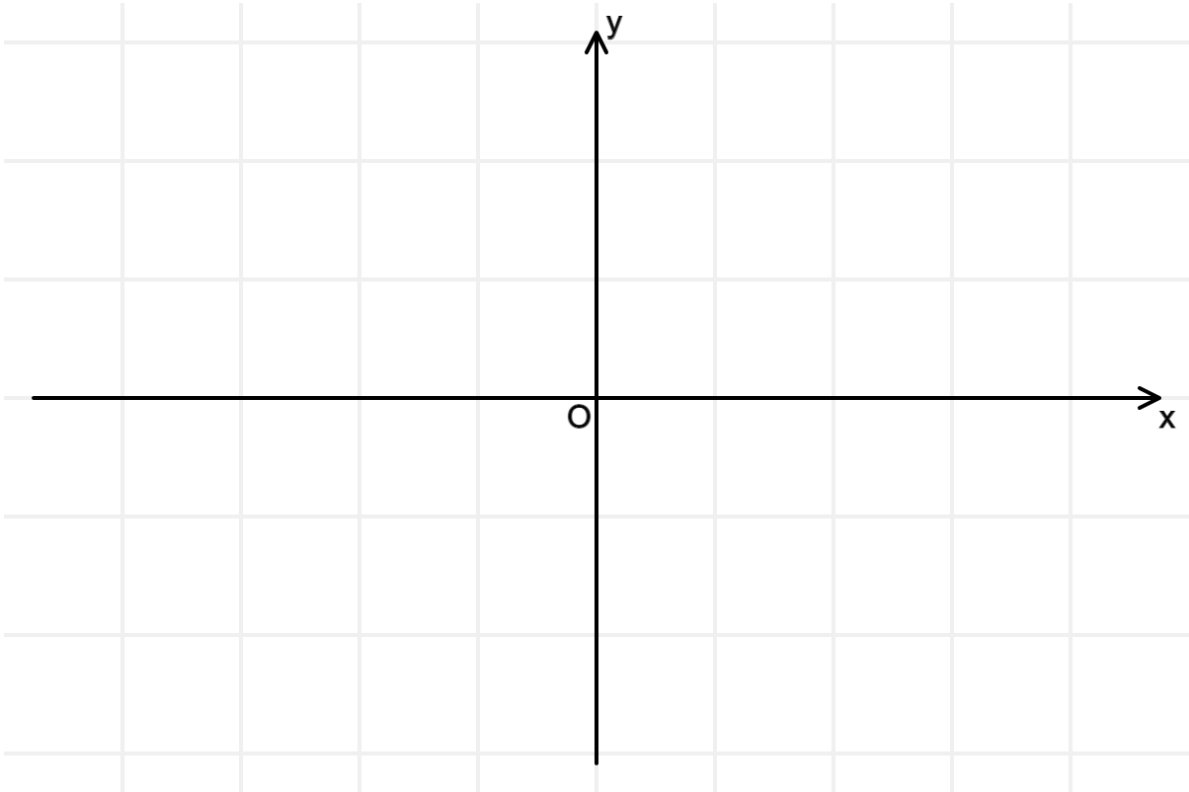
P

P

P

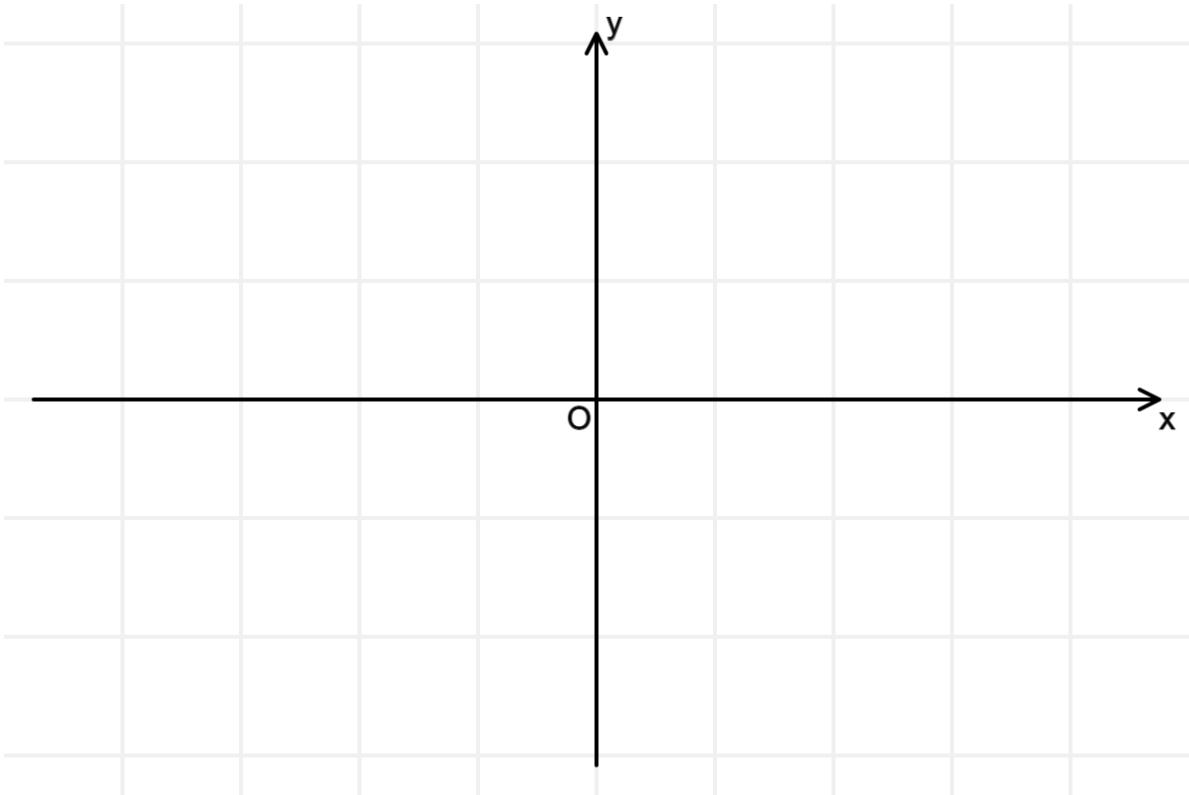
Part A Sketch $\arcsin \frac{x}{3} + \frac{\pi}{2}$

Sketch the graph of $y = \arcsin \frac{x}{3} + \frac{\pi}{2}$.



Part B Sketch $2 \arctan (x - 1)$

Sketch the graph of $y = 2 \arctan (x - 1)$.



Part C Sketch $\arccos \left(-\frac{x}{2} + \frac{1}{2}\right) - \frac{\pi}{3}$

Sketch the graph of $y = \arccos \left(-\frac{x}{2} + \frac{1}{2}\right) - \frac{\pi}{3}$.

