

Partial Fractions 3ii

The equation of a curve is $y = f(x)$, where $f(x) = \frac{3x + 1}{(x + 2)(x - 3)}$.

Part A Partial Fractions

Hence express $f(x)$ in partial fractions.

The following symbols may be useful: x

Part B Derivative

Hence find $f'(x)$.

The following symbols may be useful: `Derivative(y, x)`, `ln()`, `log()`, x , y

Part C Deduction

Hence deduce that the gradient of the curve is negative for all points on the curve.

More practice questions?

Partial Fractions 2ii

Part A Partial Fractions

Express $\frac{x - 1}{x(x + 1)}$ in partial fractions.

The following symbols may be useful: x

Part B Integral

Hence find the exact value of $\int_1^2 \frac{x - 1}{x(x + 1)} dx$.

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Partial Fractions 1i

Part A Partial Fractions

Express $\frac{2+x^2}{(1+2x)(1-x)^2}$ in the form $\frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$.

The following symbols may be useful: x

Part B Integration

Hence find $\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2} dx$ in exact form.

The following symbols may be useful: $\ln()$, $\log()$

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Partial Fractions 4ii

Part A Partial Fractions

Express $\frac{7-2x}{(x-2)^2}$ in the form $\frac{A}{x-2} + \frac{B}{(x-2)^2}$, where A , and B are constants.

The following symbols may be useful: x

Part B Integral

Hence find the exact value of $\int_4^5 \frac{7-2x}{(x-2)^2} dx$.

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Integration by Substitution 4

A Level Further A



Part A Integrate $\frac{1}{b(x+a)}$

Find $\int_0^a \frac{1}{b(x+a)} dx$, where a and b are constants.

The following symbols may be useful: a , b , k , x

Part B Integrate $\frac{x}{1+x^2}$

Find $\int_0^1 \frac{x}{1+x^2} dx$.

The following symbols may be useful: k , x

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Integration by Substitution 5

Part A Integrate $x^3/(a^5 + ax^4)$

Find $\int_a^{2a} \frac{x^3}{a^5 + ax^4} dx$.

The following symbols may be useful: a

Part B Integrate $\tan \beta$

By writing $\tan \beta = \frac{\sin \beta}{\cos \beta}$, find $\int_0^{\pi/4} \tan \beta d\beta$.

Integration by Substitution 1i

Let $I = \int \frac{1}{x(1 + \sqrt{x})^2} dx$.

Part A Substitution

Using the substitution $u = \sqrt{x}$, transform I into the integral $\int f(u) du$. Give the function $f(u)$ in terms of u .

The following symbols may be useful: \int , u

Part B Partial Fractions

Express $\frac{2}{u(1+u)^2}$ in the form $\frac{A}{u} + \frac{B}{1+u} + \frac{C}{(1+u)^2}$.

The following symbols may be useful: u

Part C Integrate

Hence find I .

The following symbols may be useful: \int , c , x

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Integration by Substitution 1ii

Part A Partial Fractions

Given that $\frac{2t}{(t+1)^2}$ can be expressed in the form $\frac{A}{t+1} + \frac{B}{(t+1)^2}$, find the values of the constants A and B .

State the value of A .

The following symbols may be useful: A

State the value of B .

The following symbols may be useful: B

Part B Show

Show that the substitution $t = \sqrt{2x - 1}$ transforms $\int \frac{1}{x + \sqrt{2x - 1}} dx$ to $\int \frac{2t}{(t + 1)^2} dt$.

More practice questions?

Part C Exact Value

Hence find the exact value of $\int_1^5 \frac{1}{x + \sqrt{2x - 1}} dx$.

Algebraic Division 1ii

Part A Quotient and Remainder

Find the quotient when $3x^3 - x^2 + 10x - 3$ is divided by $x^2 + 3$.

The following symbols may be useful: x

Give the remainder.

The following symbols may be useful: x

Part B Integral

Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3} dx.$$

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Integration With Partial Fractions 2

A Level **Further A**







Write the function $\frac{2z^2 - z - 3}{(z + 2)(z^2 - 2z - 1)}$ in the form $\frac{A}{z + 2} + \frac{B + Cz}{z^2 - 2z - 1}$. Hence find $\int_1^2 \frac{2z^2 - z - 3}{(z + 2)(z^2 - 2z - 1)} \, dz$.

Part A Find A

Find the constant A

Part B Find B

Find the constant B .

Part C Find C

Find the constant C .

Part D Integrate

Hence find $\int_1^2 \frac{2z^2 - z - 3}{(z + 2)(z^2 - 2z - 1)} \, dz$.

The following symbols may be useful: $\cos()$, $\operatorname{cosec}()$, $\operatorname{cosech}()$, $\cosh()$, $\cot()$, $\coth()$, $\ln()$, $\log()$, $\sec()$, $\operatorname{sech}()$, $\sin()$, $\sinh()$, $\tan()$, $\tanh()$, z