17   Pressure, Hydraulic Systems, Density and Depth

$$\text{pressure} = \text{force (N)}/\text{area (m}^2) \qquad p = F/A$$

The unit of pressure is the pascal (Pa). $1 \text{ Pa} = 1 \text{ N/m}^2$

Example 1 – What is the pressure on a wall when a drawing pin, with a point of cross sectional area of 2.0 mm^2, is pushed in with a force of 8.0 N?
Pressure $= \text{force}/\text{area} = 8.0 \text{ N}/2.0 \text{ mm}^2 = 4.0 \text{ N/mm}^2$.
Notice that $1 \text{ mm}^2 = 1 \text{ mm} \times 1 \text{ mm} = 10^{-3} \text{ m} \times 10^{-3} \text{ m} = 10^{-6} \text{ m}^2$.
Pressure $= \text{force (N)}/\text{area (m}^2) = 8.0 \text{ N}/2 \times 10^{-6} \text{ m}^2 = 4 \times 10^6 \text{ Pa}$.

17.1   Calculate the value of the missing quantities in the table.

Pressure (Pa)	Force (N)	Area (m²)
(a)	10	2.0
(b)	1.0	4.0×10^{-5}
(c)	10^6	2.5×10^{-5}
10^5	400	(d)
5.0×10^6	2 000	(e)
4.0×10^{-3}	8.0×10^{-3}	(f)
2.5×10^5	(g)	0.020
10^7	(h)	10^{-3}

17.2   Assume the flat end of the drawing pin in Example 1 has an area of 1.2 cm^2. Calculate the pressure on the person's finger who is pushing in the nail with a force of 8.0 N in

(a) N/cm²;   (b) Pa.

17.3   My weight is 670 N, and each of my shoes has a sole area of 200 cm^2.

(a) What will be the pressure when I stand on the ground?

(b) A plank 15 cm wide and 1.5 m long is laid across a muddy path. What will be the pressure on the mud when I stand on the plank?

(c) Compare your answers for (a) and (b). How and why do you think the plank affects whether I sink into the ground?
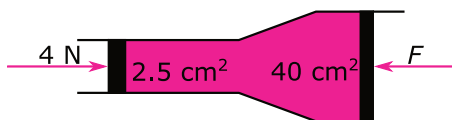
Pressure in fluids
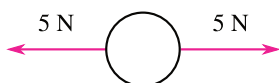
A solid object will not accelerate if the force pulling from each side is equal.

A section of a fluid (liquid or gas) will not accelerate if the pressure on both sides is equal.

In a hydraulic system, two pistons of different area push on the same fluid, exerting pressures on it. The fluid is in equilibrium if the pressures are equal.

The pressure on the left $= 4\,\text{N}/2.5\,\text{cm}^2 = 1.6\,\text{N/cm}^2$. In equilibrium, the pressure on the right will also be $1.6\,\text{N/cm}^2$, so the force on the right must be $1.6\,\text{N/cm}^2 \times 40\,\text{cm}^2 = 64\,\text{N}$.

17.4 Complete the table of hydraulic systems in equilibrium.

Left Piston		Pressure	Right Piston	
Force	Area		Force	Area
3.0 N	0.60 cm^2	(a)	(b)	3.6 cm^2
65 N	15 cm^2	(c)	(d)	0.50 cm^2
45 N	4.5 cm^2	(e)	(f)	25 cm^2
10 N	1.0 cm^2	(g)	(h)	1.0 m^2
35 N	25 cm^2	(i)	7 000 N	(j)

17.5 At a garage, a car (8.0 kN weight) is going to be lifted on four hydraulic jacks, each with a cross sectional area of 25 cm^2. Fluid is forced into the jacks by a compressor. If you want to support the car on the jacks, what is the pressure in the fluid?

**Density**   density $=$ mass/volume $\rho = m/V$

Density gives the mass of a material per cubic metre (or cubic centimetre). 1.00 kg of water has a volume of 0.00100 m^3. density = mass / volume = 1.00 kg / 0.00100 m^3 = 1000 kg/m^3.

17.6 Complete the table using the formula for density. Give answers in the units requested, shown in brackets in the column headings.

Substance	Water (g/cm^3)	Gold (g/cm^3)	Iron (g)	Ice (kg)	Nitrogen gas (m^3)
Density	(a)	(b)	7.87 g/cm^3	920 kg/m^3	1.13 kg/m^3
Mass	250 g	5.4 g	(c)	(d)	2000 g
Volume	250 cm^3	0.28 cm^3	300 cm^3	434 cm^3	(e)

Remember that 1 cm^3 = (0.01 m)^3 = (10^{-2} m)^3 = 10^{-6} m^3.

Pressure at Depth
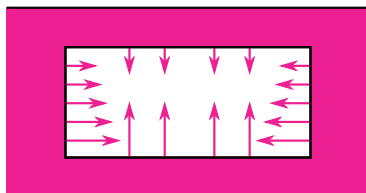
As you go deeper in a fluid, the pressure rises because of the increased weight of fluid above you. However, any surface in the fluid has a force on it regardless of its angle. A box held under water has forces on it from all sides, all pushing inwards at right angles to each surface.

The formula for the extra pressure at a depth is:

pressure $=$ density $\times$ gravitational field strength $\times$ depth $p = \rho g h$

To calculate the total pressure at that depth, the pressure at the surface (e.g. atmospheric pressure) must be added.

Example 2 – Calculate the total pressure at a depth of 8.0 m in oil of density 850 kg/m^3 if atmospheric pressure is 101 kPa.
Extra pressure $= \rho g h = 850$ kg/m^3 $\times$ 10 N/kg $\times$ 0.8 m $= 68\,000$ Pa $=$ 68 kPa
Total pressure $=$ pressure at surface $+ 68$ kPa $= 101$ kPa $+ 68$ kPa $=$ 169 kPa

For these questions assume water has a density of $1\,000$ kg/m^3, the gravitational field strength is 10 N/kg and atmospheric pressure is 101 kPa.

17.7 A beaker has a cross sectional area of 0.080 m^2 and is filled to a depth of 0.12 m.

   (a) Calculate the volume of water in m^3.

   (b) Calculate the mass of water in kg.

   (c) Calculate the weight of water in N.

   (d) Calculate the pressure of the water on the base in Pa.

   (e) What would your answers to the previous parts be if the beaker had a cross sectional area of 0.80 m^2, but the same depth of water?

17.8 A watch states that it is 'water resistant to 30 m'.

   (a) What extra pressure can it withstand before leaking?

   (b) What is the extra pressure on the watch at a depth of 10 m?

17.9 The deepest part of the Pacific Ocean, the Mariana Trench, has a depth of $10\,994$ m. The density of sea water is $1\,030$ kg/m^3.

   (a) What is the total pressure at that depth?

   (b) What would be the inwards force on a 10 cm by 10 cm window in a submarine at the pressure calculated in (a)?

17.10 Mercury has a density of $13\,600$ kg/m^3.

   (a) What depth of mercury gives an extra pressure equal to atmospheric pressure?

   (b) At what depth in mercury would the pressure be the same as at a depth of 68 cm in water?

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Additional Pressure, Density and Depth Questions

17.11 An ocean temperature probe is lowered from a survey ship into the water. The maximum pressure that the probe is designed to withstand is 100 MPa. What is the greatest depth to which the probe could be safely lowered? The density of sea water is $1\,030$ kg/m^3.