Physics. You work it out.

Powers using Chain Rule 2

A Level Further A

P P P P P P

Part A First and second derivatives of $u = 2(3-2v)^{3/2}$

Find $\frac{\mathrm{d}u}{\mathrm{d}v}$ and $\frac{\mathrm{d}^2u}{\mathrm{d}v^2}$ when $u = 2(3-2v)^{3/2}$.

Find $\frac{\mathrm{d}u}{\mathrm{d}v}$ when $u = 2(3-2v)^{3/2}$.

The following symbols may be useful: v

Find $\frac{\mathrm{d}^2u}{\mathrm{d}v^2}$ when $u = 2(3-2v)^{3/2}$.

The following symbols may be useful: v

Part B Differentiate $p = \frac{1}{\sqrt{3a+8}}$

Find $\frac{\mathrm{d}p}{\mathrm{d}a}$ if $p = \frac{1}{\sqrt{3a+8}}$.

Find $\frac{dp}{\mathrm{d}a}$ if $p = \frac{1}{\sqrt{3a+8}}$.

The following symbols may be useful: a

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Physics. You work it out.

Powers using Chain Rule 3

A Level Further A

P P P P P P

Part A Stationary point of $y = (2-3x)^4 + 4$

Find the coordinates and nature of the stationary point of the function $y = (2-3x)^4 + 4$.

Find the x coordinate of the stationary point of the function $y = (2-3x)^4 + 4$.

The following symbols may be useful: x

Find the y coordinate of the stationary point of the function $y = (2-3x)^4 + 4$.

The following symbols may be useful: y

By considering the behaviour of the function when x is very large and positive and also when it is very large and negative deduce the nature of the stationary point of the function $y = (2-3x)^4 + 4$.

- Minimum
- Maximum

Part B Stationary points of $q = 4(2p-1)^3 - 3(2p-1)^4$

Consider the function $q = 4(2p-1)^3 - 3(2p-1)^4$.

Find the stationary points of the function. How many are there? The stationary point with the lowest value of p is at (p_1, q_1) and the stationary point with the second lowest value of p is at (p_2, q_2). Find the values of p and q at (p_1, q_1) and (p_2, q_2).

How many stationary points are there?

- 1
- 0
- 3
- 2
- 4

Find p_1, the p coordinate of the stationary point with the lowest value of p.

The following symbols may be useful: `p_1, q_1`

Find p_2, the p coordinate of the stationary point with the second lowest value of p.

The following symbols may be useful: `p_2, q_2`

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Home Maths Calculus Differentiation Powers using Chain Rule 1

Powers using Chain Rule 1

A Level Further A

P P P P P P

Part A Differentiate $w = (4s+3)^3$

Find $\dfrac{\mathrm{d}w}{\mathrm{d}s}$ if $w = (4s+3)^3$.

Find $\dfrac{\mathrm{d}w}{\mathrm{d}s}$ if $w = (4s+3)^3$.

The following symbols may be useful: `s`

Part B First derivative of $z = (b - aw)^4$

Find $\dfrac{\mathrm{d}z}{\mathrm{d}w}$ and $\dfrac{\mathrm{d}^2 z}{\mathrm{d}w^2}$ when $z = (b-aw)^4$ and a and b are constants.

Find $\dfrac{\mathrm{d}z}{\mathrm{d}w}$ when $z = (b-aw)^4$.

The following symbols may be useful: `a`, `b`, `w`

Part C Second derivative of $z = (b - aw)^4$

Find $\dfrac{\mathrm{d}^2 z}{\mathrm{d}w^2}$ when $z = (b-aw)^4$.

The following symbols may be useful: `a`, `b`, `w`

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Differentiating Sums and Differences 1

A Level P P P Further A P P P

Part A Differentiate $ax^3 + (b/x) + c$

Differentiate $ax^3 + (b/x) + c$ with respect to x (a, b and c are constants).

The following symbols may be useful: `a, b, c, x`

Part B Differentiate $(2m + 3)(m - 1)$

Differentiate $(2m + 3)(m - 1)$ with respect to m.

Differentiate $(2m + 3)(m - 1)$ with respect to m.

The following symbols may be useful: `m`

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Differentiating Sums and Differences 2

A Level Further A

Part A Gradient of curve $w = z^{1/2} + z^{-1/2}$ at $z = 1/4$

Find the gradient of the curve $w = z^{1/2} + z^{-1/2}$ at $z = 1/4$.

Part B Gradient of curve $w = z^{1/2} + z^{-1/2}$ at $z = 1$

Find the gradient of the curve $w = z^{1/2} + z^{-1/2}$ at $z = 1$.

Part C Gradient of curve $w = z^{1/2} + z^{-1/2}$ at $z = 4$

Find the gradient of the curve $w = z^{1/2} + z^{-1/2}$ at $z = 4$.

Part D First root of curve $f(u) = u^3 - 4u$

Find the lowest u value at which the curve $f(u) = u^3 - 4u$ crosses the u axis.

The following symbols may be useful: u

Part E Second root of curve $f(u) = u^3 - 4u$

Find the second lowest u value at which the curve $f(u) = u^3 - 4u$ crosses the u axis.

The following symbols may be useful: u

Part F Third root of curve $f(u) = u^3 - 4u$

Find the greatest u value at which the curve $f(u) = u^3 - 4u$ crosses the u axis.

The following symbols may be useful: u

Part G Gradient at first root of $f(u) = u^3 - 4u$

Find the gradient at the lowest u value at which the curve $f(u) = u^3 - 4u$ crosses the u axis.

Part H Gradient at second root of $f(u) = u^3 - 4u$

Find the gradient at the second lowest u value at which the curve $f(u) = u^3 - 4u$ crosses the u axis.

Part I Gradient at third root of $f(u) = u^3 - 4u$

Find the gradient at the greatest u value at which the curve $f(u) = u^3 - 4u$ crosses the u axis.

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Differentiating Sums and Differences 3

A Level Further A

C C C P P P

Part A Velocity if $s = ut + bt^2$

A particle is moving in one dimension. Its displacement s at time t is given by $s = ut + bt^2$. The velocity v of the particle at time t is given by the rate of change of displacement with time, i.e. $v = \frac{\mathrm{d}s}{\mathrm{d}t}$.

Find an expression for the velocity.

The following symbols may be useful: `b, t, u, v`

Part B Acceleration if $s = ut + bt^2$

A particle is moving in one dimension. Its displacement s at time t is given by $s = ut + bt^2$. The acceleration a of the particle at time t is given by the rate of change of velocity with time.

Find an expression for the acceleration.

The following symbols may be useful: `a, b, t, u`

Part C Velocity if $x = \alpha t + \beta t^3$

The displacement of a body at time t is given by $x = \alpha t + \beta t^3$ where $\alpha = 4\,\mathrm{m\,s^{-1}}$ and $\beta = 5\,\mathrm{m\,s^{-3}}$. Use the fact that the velocity is the rate of change of displacement to find the velocity of the body at $t = 2\,\mathrm{s}$.

Find the velocity of the body at $t = 2\,\mathrm{s}$.

Part D Acceleration if $x = \alpha t + \beta t^3$

The displacement of a body at time t is given by $x = \alpha t + \beta t^3$ where $\alpha = 4\,\mathrm{m\,s^{-1}}$ and $\beta = 5\,\mathrm{m\,s^{-3}}$. Use the fact that the acceleration is the rate of change of velocity to find the acceleration of the body at $t = 2\,\mathrm{s}$.

Find the acceleration of the body at $t = 2\,\mathrm{s}$.

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Stationary Points 3

A Level Further A

C C C P P P

Part A Find the maximum height of a projectile

A particle is fired upwards into the air with a speed w and moves subsequently under the influence of gravity with an acceleration g downwards, such that its height h at time t is given by $h = wt - \frac{1}{2}gt^2$. Find an expression for its maximum height above its initial position.

A particle is fired upwards into the air with a speed w and moves subsequently under the influence of gravity with an acceleration g downwards, such that its height h at time t is given by $h = wt - \frac{1}{2}gt^2$. Find an expression for its maximum height above its initial position.

The following symbols may be useful: `g, h, w`

Part B Examine the potential energy of two molecules

The potential energy of two molecules separated by a distance r is given by

$$U = U_0 \left(\left(\frac{a}{r} \right)^{12} - 2 \left(\frac{a}{r} \right)^{6} \right)$$

where U_0 and a are positive constants. The equilibrium separation of the two molecules occurs when the potential energy is a minimum; find expressions for the equilibrium separation and the value of the potential energy at this separation.

(a) Find an expression for the equilibrium separation of the molecules.

The following symbols may be useful: `U, U_0, a, r`

(b) Find an expression for the potential energy when the molecules are at their equilibrium separation.

The following symbols may be useful: `U, U_0, a, r`

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Differentiating Exponentials 2

A Level Further A

P P P P P P

Part A Differentiate $3\mathrm{e}^{4x+2}$

Differentiate $3\mathrm{e}^{4x+2}$ with respect to x.

Differentiate $3\mathrm{e}^{4x+2}$ with respect to x.

The following symbols may be useful: `e`, `x`

Part B Differentiate $x = X(\mathrm{e}^{\gamma t} + \mathrm{e}^{-\gamma t})$

Find the rate of change of x with respect to t where $x = X(\mathrm{e}^{\gamma t} + \mathrm{e}^{-\gamma t})$ and X and γ are constants.

Find the rate of change of x with respect to t where $x = X(\mathrm{e}^{\gamma t} + \mathrm{e}^{-\gamma t})$ and X is a constant.

The following symbols may be useful: `X`, `e`, `gamma`, `t`

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Differentiating Exponentials 1

A Level Further A

P P P P P P

Part A Differentiate $\beta \mathrm{e}^{-\alpha t}$

Differentiate $\beta \mathrm{e}^{-\alpha t}$ with respect to t where α and β are constants.

Differentiate $\beta \mathrm{e}^{-\alpha t}$ with respect to t.

The following symbols may be useful: `alpha, beta, e, t`

Part B Differentiate $C\mathrm{e}^{\beta m} + D$

Differentiate $C\mathrm{e}^{\beta m} + D$ with respect to m where β, C and D are constants.

Differentiate $C\mathrm{e}^{\beta m} + D$ with respect to m.

The following symbols may be useful: `C, D, beta, e, m`

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