Physics. You work it out.

Home Gameboard Maths Curve Sketching and Combined Transformations 1ii

Curve Sketching and Combined Transformations 1ii

A Level

P P P

The curve $y = \ln x$ is transformed to the curve $y = \ln\left(\frac{1}{2}x - a\right)$ by means of a translation followed by a stretch. It is given that a is a positive constant.

Part A Translation

Give full details of the translation involved.

In which direction is the translation?

- Positive x-direction
- Negative x-direction
- Positive y-direction
- Negative y-direction

By how far is the translation?

The following symbols may be useful: `a`, `x`, `y`

Part B Stretch

Give full details of the stretch involved.

In which direction is the stretch?

- y-direction
- x-direction

By what factor is the stretch?

The following symbols may be useful: `a, x, y`

Part C Sketch (a)

Sketch the graph of $y = \ln\left(\frac{1}{2}x - a\right)$.

To see an example sketch, answer the following question: The graph is asymptotic to which line? Give the equation of the line in the form $x = p$ where p is an expression.

The following symbols may be useful: `a, x`

Part D Sketch (b)

Sketch the graph of $y = \left|\ln\left(\frac{1}{2}x - a\right)\right|$.

To see an example sketch, answer the following question: For what value of x does the graph touch the x-axis?

The following symbols may be useful: `a, x, y`

Part E Values for x

Find, in terms of a, the set of values of x for which $\left|\ln\left(\frac{1}{2}x-a\right)\right| = -\ln\left(\frac{1}{2}x-a\right)$, and give the upper bound in the form $x < c$ or $x \leq c$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `a`, `x`

Find, in terms of a, the set of values of x for which $\left|\ln\left(\frac{1}{2}x-a\right)\right| = -\ln\left(\frac{1}{2}x-a\right)$, and give the lower bound in the form $x > c$ or $x \geq c$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `a`, `x`

Physics. You work it out.

Home Gameboard Maths Algebraic Division 2ii

Algebraic Division 2ii

A Level
P P P

Part A Quotient and Remainder

Find the quotient and the remainder when $3x^3 - 2x^2 + x + 7$ is divided by $x^2 - 2x + 5$.

Give the quotient.

The following symbols may be useful: x

Give the remainder.

The following symbols may be useful: x

Part B Value of a and b

Hence, or otherwise, determine the values of the constants a and b such that, when $3x^3 - 2x^2 + ax + b$ is divided by $x^2 - 2x + 5$, there is no remainder.

Give the value of a.

The following symbols may be useful: a

Give the value of b.

The following symbols may be useful: b

Gameboard:
STEM SMART Double Maths 14 - A2 Pure Maths Revision (non-calculus)

Binomial: All Rational n 1i

A Level

Part A Partial Fractions

Given that $\frac{3x+4}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$, find A, B, and C.

Find A.

The following symbols may be useful: A

Find B.

The following symbols may be useful: B

Find C.

The following symbols may be useful: C

Part B Expand

Hence or otherwise expand $\frac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2.

The following symbols may be useful: x

Part C Values of x

State the set of values of x for which the expansion in the above part is valid.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $x < a$
- $x \leq a$
- $x > a$
- $x \geq a$
- $a < x < b$
- $a \leq x \leq b$
- $x < a$ or $x > b$
- $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

Gameboard:
STEM SMART Double Maths 14 - A2 Pure Maths Revision (non-calculus)

Physics. You work it out.

Functions: Graphs and Inverse Functions 3ii

A Level
C C C

The function $f(x)$ is defined by

$$f(x) = 1 + \sqrt{x} \text{ for } x \geqslant 0.$$

Part A Domain and Range

What is the domain of the inverse function $f^{-1}(x)$? Write your answer in the form of an inequality.

The following symbols may be useful: `<, <=, >, >=, f, x, y`

What is the range of the inverse function $f^{-1}(x)$? Write your answer in the form of an inequality.

The following symbols may be useful: `<, <=, >, >=, f, x, y`

Part B $f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, x, y`

Part C $f(x) = f^{-1}(x)$

Find the x-value that is the solution to the equation $f(x) = f^{-1}(x)$ to four significant figures.

Gameboard:
STEM SMART Double Maths 14 - A2 Pure Maths Revision (non-calculus)

Trigonometry: Combined Angles 4ii

A Level

C C C

Part A Combined Angles

Express $8\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.

Give the value of R.

The following symbols may be useful: R

Give the value of α to three significant figures.

Part B Solve

Hence solve, for $0^\circ < \theta < 360^\circ$, the equation $8\sin\theta - 6\cos\theta = 9$, giving your answers in degrees to three significant figures.

Give the smallest solution.

Give the largest solution.

Part C Maximum Value

Hence find the greatest possible value of

$$32\sin x - 24\cos x - (16\sin y - 12\cos y)$$

as the angles x and y vary.

Gameboard:

STEM SMART Double Maths 14 - A2 Pure Maths Revision (non-calculus)

Physics. You work it out.

Trigonometry: Double Angles 1ii

A Level

C C C

Part A The form $a\sin^2\theta + b\sin\theta + c = 0$

Express the equation $(\operatorname{cosec}\theta)(3\cos 2\theta + 7) + 11 = 0$ in the form $a\sin^2\theta + b\sin\theta + c = 0$, where $a,\ b,$ and c are constants.

Give the value of a.

The following symbols may be useful: a

Give the value of b.

The following symbols may be useful: b

Give the value of c.

The following symbols may be useful: c

Part B Solve

Hence solve, for $-180^\circ < \theta < 180^\circ$, the equation $(\operatorname{cosec}\theta)(3\cos 2\theta + 7) + 11 = 0$. Give your answers in degrees, to three significant figures.

Give the highest (most positive) solution.

Give the lowest (most negative) solution.

Gameboard:

STEM SMART Double Maths 14 - A2 Pure Maths Revision (non-calculus)

Series

A Level

C C C

A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 2 \text{ and } u_{n+1} = \frac{1}{1-u_n} \text{ for } n \geqslant 1.$$

Part A u_2

Write down the value of u_2.

The following symbols may be useful: u_2

Part B u_3

Write down the value of u_3.

The following symbols may be useful: u_3

Part C u_4

Write down the value of u_4.

The following symbols may be useful: u_4

Part D u_5

Write down the value of u_5.

The following symbols may be useful: `u_5`

Part E u_{200}

Deduce the value of u_{200}.

The following symbols may be useful: `u_200`

Part F $\sum_{n=1}^{200} u_n$

Find $\sum_{n=1}^{200} u_n$.

Part G Amount of Chemical

Sarah is carrying out a series of experiments which involve using increasing amounts of a chemical. In the first experiment she uses $6\,\mathrm{g}$ of a chemical and in the second experiment she uses $7.8\,\mathrm{g}$ of the chemical.

Given that the amounts of chemical used form an arithmetic progression, find the total amount of chemical used in the first 30 experiments.

Part H Number of experiments possible

Instead, it is given that the amounts of chemical used form a geometric progression. Sarah has a total of $1800\,\mathrm{g}$ of the chemical available. As N, the greatest number of experiments possible, satisfies the inequality

$$1.3^N \leqslant 91$$

Use logarithms to find the greatest value for N.

The following symbols may be useful: `N`

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