Radians-problems involving area 5ii

A Level

Figure 1 shows a sector OAB of a circle, centre O and radius $8\,\text{cm}$. The angle AOB is $46\,^\circ$.

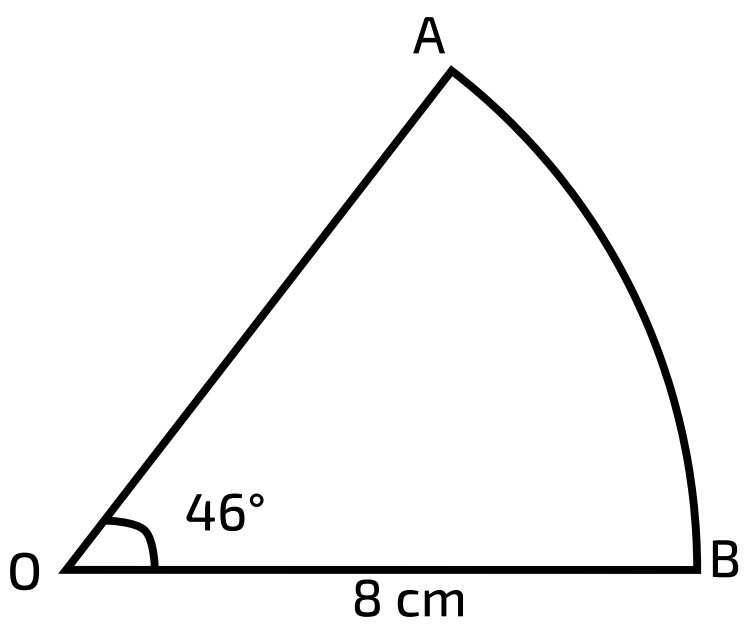

Figure 1: Sector AOB.

Part A Convert angle to radians

Express $46\,^\circ$ in radians, correct to 3 significant figures.

Part B Arc length

Find the length of the arc AB.

Part C Area of sector

Find the area of the sector OAB.

Gameboard:

Pure Maths Practice: Radians - Problems Involving Area

Radians-problems involving area 1ii

A Level

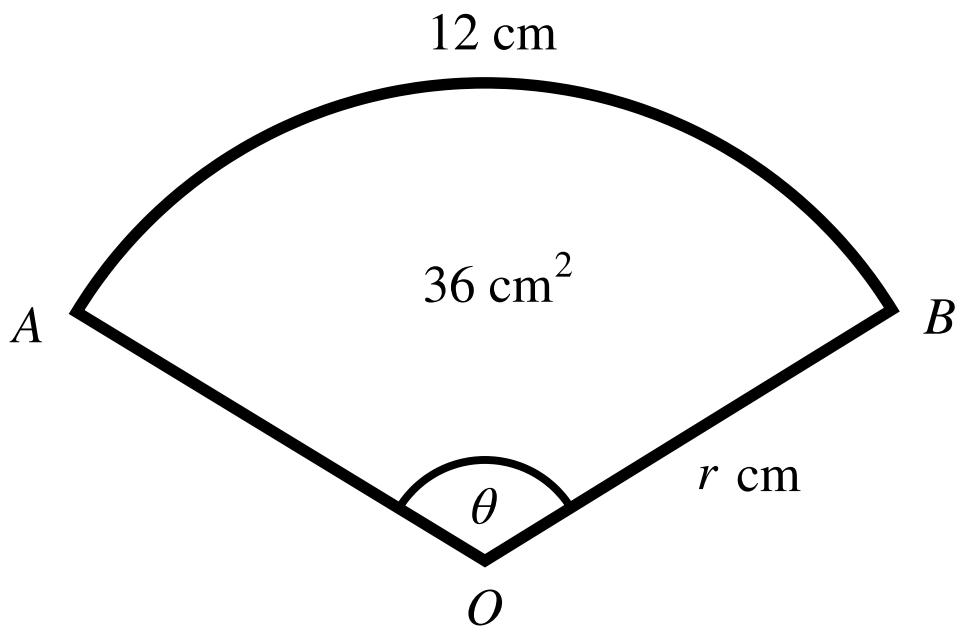

Figure 1: The sector OAB.

A sector OAB of a circle of radius $r\,\text{cm}$ has angle $\theta\,\text{radians}$. The length of the arc of the sector is $12\,\text{cm}$ and the area of the sector is $36\,\text{cm}^2$ (see **Figure 1**).

Part A First equation

By considering the length of the arc of the sector, write down an equation involving r and θ, where one side of the equation is a numerical constant.

The following symbols may be useful: `r, theta`

Part B Second equation

By considering the area of the sector, write down another equation involving r and θ, where one side of the equation is a numerical constant.

The following symbols may be useful: `r, theta`

Part C Values of r and θ

Hence show that $r = 6\,\text{cm}$ and find the value of θ.

Part D Area of segment

Find the area of the segment bounded by the arc AB and the chord AB. Answer to 3 sf.

Radians and Trig Functions 1i

A Level

A curve has equation $y = \sin(ax)$, where a is a positive constant and x is in radians.

Part A Time Period

State the period of $y = \sin(ax)$, giving your answer in an exact form in terms of a.

The following symbols may be useful: `a, pi, t`

Part B $\sin(ax) = k$

Given that $x = \frac{1}{5}\pi$ and $x = \frac{2}{5}\pi$ are the two smallest positive solutions of $\sin(ax) = k$, where k is a positive constant, find the values of a and k.

Find the value of a.

The following symbols may be useful: `a`

Find the value of k.

The following symbols may be useful: `k`

Part C $\sin(ax) = \sqrt{3}\cos(ax)$

Given instead that $\sin(ax) = \sqrt{3}\cos(ax)$, find the two smallest positive solutions for x, giving your answers in an exact form in terms of a.

Give the smallest positive solution.

The following symbols may be useful: `a, pi, x`

Give the second smallest positive solution.

The following symbols may be useful: `a, pi, x`

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Gameboard:

Pure Maths Practice: Radians and Trig Functions

Small Angle Approximations 1ii

A Level

$$f(x) = \frac{\sin x + \tan(2x)}{\tan x + 2}$$

Part A Small angle approximation

Use the small angle approximation to write an approximate expression to second order for $f(x)$, valid when x is small.

The following symbols may be useful: `f, x`

Part B Estimation

Use your expression to estimate the value of $f(0.1)$ to 4 significant figures.

Part C Percentage error

What is the percentage error in this estimate? Give your answer to 3 significant figures.

Small Angle Approximations 1i

A Level

The small angle approximation is used when measuring distances in astronomy.

The two stars Alpha Centauri A and Alpha Centauri B are in a binary pair (they orbit one another). The distance between them is an average of 11 Astronomical Units, and they are an average of 4.4 light years from Earth.

$1\,\text{AU} = 1\,\text{Astronomical Unit} = 149\,597\,870\,700\,\text{m}$

$1\,\text{ly} = 1\,\text{Light Year} = 9.4607 \times 10^{15}\,\text{m}$

Assume that a telescope is pointing straight at Alpha Centauri A with the geometry shown in **Figure 1**.

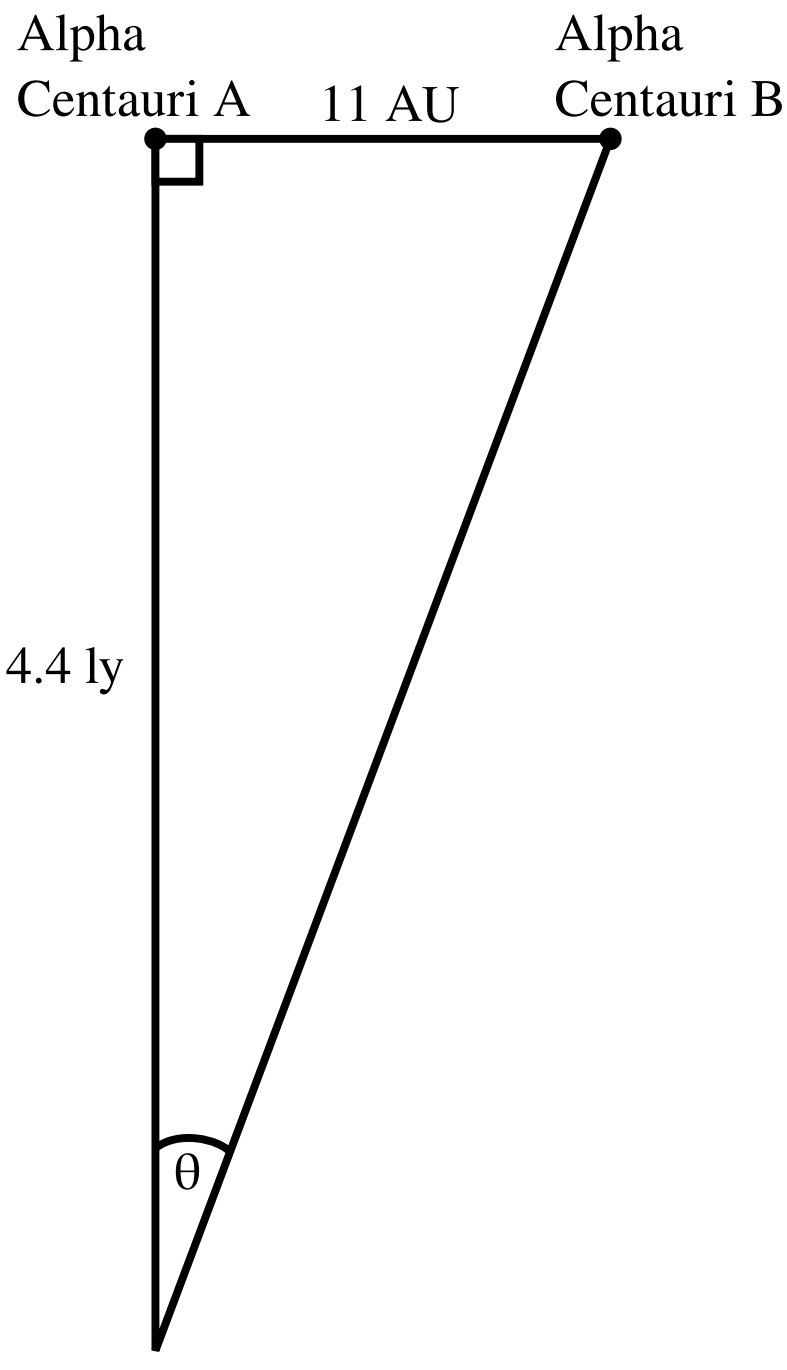

Figure 1: A telescope pointing straight at Alpha Centauri A

Use the small angle approximation to estimate θ, the angular separation between the stars as seen by the telescope. Give your answer to 2 significant figures.

Part A Radians

Give the answer in radians.

Part B Degrees

Give the answer in degrees.

Part C Arc Seconds

Give the answer in Arc Seconds. (Where 1 arc second is one $\frac{1}{3600}^{\text{th}}$ of a degree.)

Gameboard:

Pure Maths Practice: Small Angle Approximations

Advanced Trig Identities 2ii

A Level

Part A $2\tan^2\theta - \frac{1}{\cos\theta}$

Express $2\tan^2\theta - \frac{1}{\cos\theta}$ in terms of $\sec\theta$.

The following symbols may be useful: `sec(), theta`

Part B Solve

Hence solve, for $0^\circ < \theta < 360^\circ$, the equation

$$2\tan^2\theta - \frac{1}{\cos\theta} = 4$$

Give the smallest solution to three significant figures.

Give the second smallest solution to four significant figures.

Give the second largest solution to four significant figures.

Give the largest solution to three significant figures.

Gameboard:

Pure Maths Practice: Advanced Trig Identities

Functions: Inverse Trig 1ii

A Level

Figure 1 shows the graph of $y = -\sin^{-1}(x-1)$.

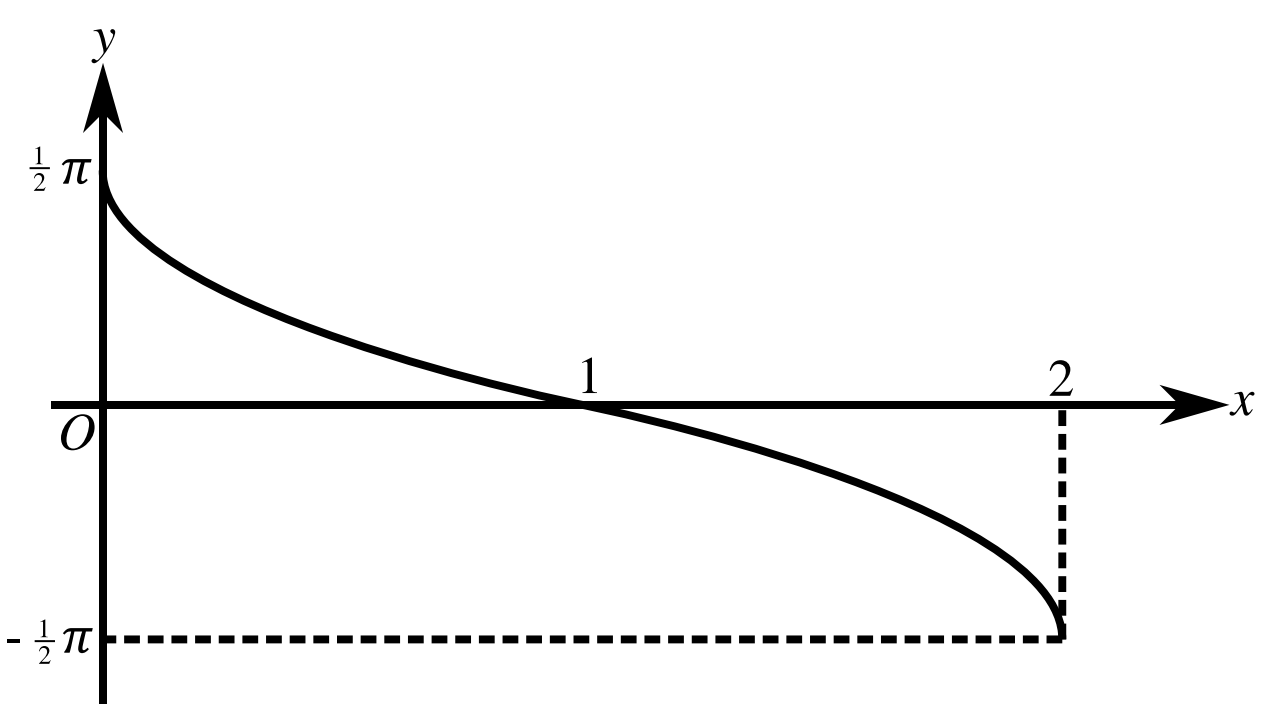

Figure 1: A graph of the function $y = -\sin^{-1}(x-1)$

Part A Transformations

Give details of the pair of geometrical transformations which transform the graph of $y = -\sin^{-1}(x-1)$ to the graph of $y = \sin^{-1} x$.

- Reflect in y-axis, translate by 1 in $-x$-direction
- Reflect in line $y = x$, translate by 1 in $+y$-direction
- Reflect in y-axis, translate by 1 in $-y$-direction
- Reflect in y-axis, translate by 1 in $+y$-direction
- Reflect in $y = -x$, translate by 1 in $+x$-direction
- Reflect in x-axis, translate by 1 in $-x$-direction
- Reflect in x-axis, translate by 1 in $+x$-direction

Part B Sketch

Sketch the graph of $y = \quad -\sin^{-1}(x-1)$.

To see an example sketch, answer the following question: For what value of y does the curve meet the y-axis?

The following symbols may be useful: `pi`

Part C Solutions

Find the exact solutions of the equation $\quad -\sin^{-1}(x-1) \quad = \frac{\pi}{3}$.

Give the largest exact solution.

The following symbols may be useful: `x`

Give the smallest exact solution.

The following symbols may be useful: `x`

Functions: Inverse Trig 3i

A Level

Figure 1 shows the graphs of $y = \cos^{-1} x$ and $y = \tan^{-1} x$, for $-1 \leqslant x \leqslant 1$ in each case. The graphs intersect at the point with coordinates (p, q).

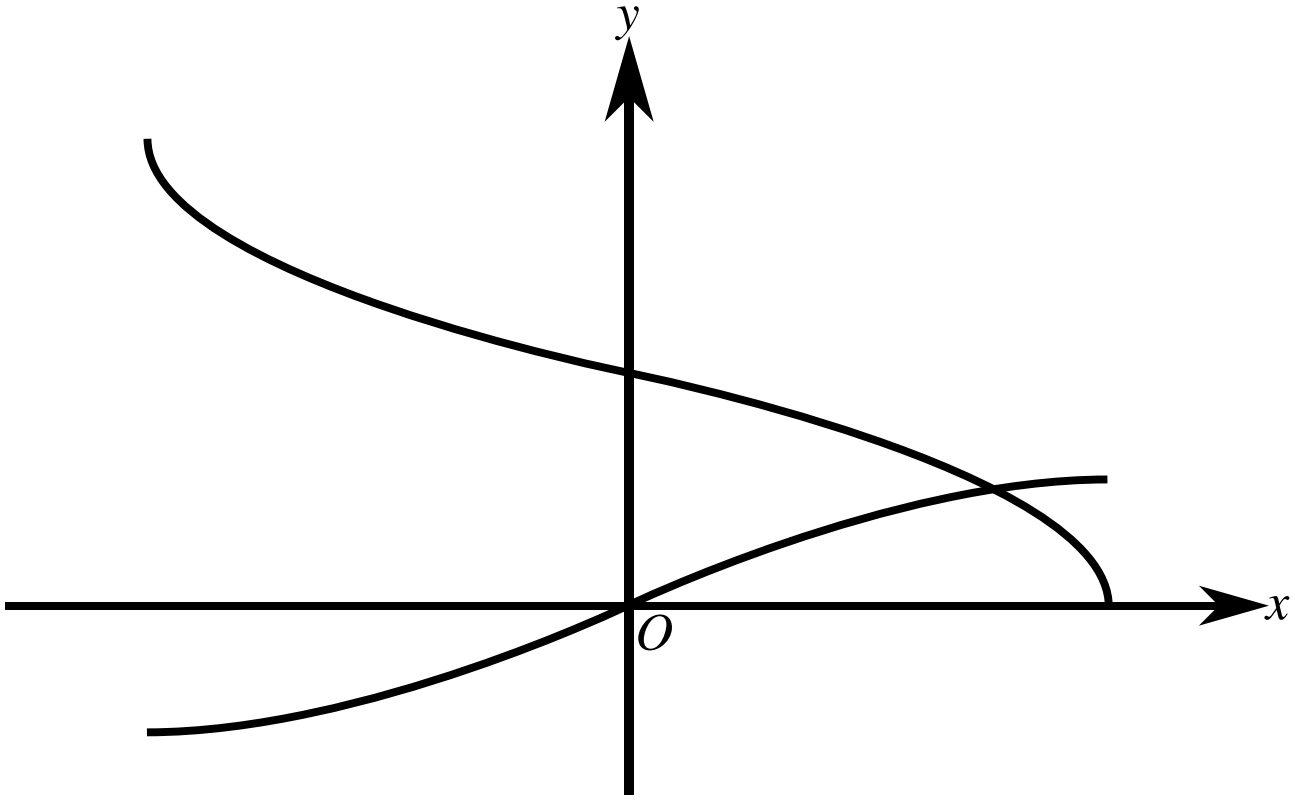

Figure 1: The graphs of $y = \cos^{-1} x$ and $y = \tan^{-1} x$, for $-1 \leqslant x \leqslant 1$.

Part A Exact values (a)

For the curve $y = \cos^{-1} x$, state the exact values of y for $x = -1,\ 0$ and 1.

State the exact value of y when $x = -1$.

The following symbols may be useful: `pi, y`

State the exact value of y when $x = 0$.

The following symbols may be useful: `pi, y`

State the exact value of y when $x = 1$.

The following symbols may be useful: `pi, y`

Part B Exact values of y

For the curve $y = \tan^{-1} x$, state the exact values of y for $x = -1,\ 0$ and 1.

State the exact value of y when $x = -1$.

The following symbols may be useful: `pi, y`

State the exact value of y when $x = 0$.

The following symbols may be useful: `pi, y`

State the exact value of y when $x = 1$.

The following symbols may be useful: `pi, y`

Part C The graphs $y = \cos x$ and $y = \tan x$

Write down, in terms of p and q, the coordinates of the corresponding point of intersection of the graphs of $y = \cos x$ and $y = \tan x$, and hence show that $\cos^2 q = \sin q$.

Easier question?

Part D $p^4 + p^2 - 1$

What is the value of $p^4 + p^2 - 1$?

Part E Solution of the equation

Hence find, in exact form, the solution of the equation

$$\cos^{-1} x = \tan^{-1} x.$$

The following symbols may be useful: `p, x`

Functions: Reciprocal Trig 1i

A Level

Part A Sketch

Sketch the graph of $y = \operatorname{cosec} x$ for $0 < x < 4\pi$.

Easier question?

Part B β in terms of α

It is given that $\operatorname{cosec} \alpha = \operatorname{cosec} \beta$, where $\frac{1}{2}\pi < \alpha < \pi$ and $2\pi < \beta < \frac{5}{2}\pi$. By using your sketch, or otherwise, express β in terms of α.

The following symbols may be useful: `alpha, beta, pi`

Part C Double angle $\tan$ properties

Write down the identity giving $\tan 2\theta$ in terms of $\tan \theta$.

The following symbols may be useful: `theta`

Part D $\tan \phi \cot 2\phi \tan 4\phi$

Given that $\cot \phi = 4$, find the exact value of $\tan \phi \cot 2\phi \tan 4\phi$.

The following symbols may be useful: `phi`

Gameboard:

Pure Maths Practice: Functions - Reciprocal Trig

Physics. You work it out.

t-Formulae Substitution

Further A

Part A Substitution

Using the substitution $t = \tan\frac{\theta}{2}$, write the equation $2\cos\theta - 5\sin\theta = 2 - 5\tan\theta$ in the form $f(t) = 0$ where $f(t)$ is a polynomial with integer coefficients and degree 4.

The following symbols may be useful: `cos(), cosec(), cot(), sec(), sin(), t, tan()`

Part B Solutions

Hence find all the solutions to the equation $2\cos\theta - 5\sin\theta = 2 - 5\tan\theta$ in the range $0 \leq \theta < 2\pi$.

Give the smallest solution.

Give the second smallest solution to 3 significant figures.

Give the largest solution to 3 significant figures.

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