Physics. You work it out.

Expand and simplify binomials

A Level Further A

Part A $(x+1)^4$

Expand and simplify $(x+1)^4$. (Give your answer in descending powers of x.)

The following symbols may be useful: x

Part B $(z+2a)^3$

Expand and simplify $(z+2a)^3$. (Give your answer in descending powers of z.)

The following symbols may be useful: a, z

Part C $(a-b)^5$

Expand and simplify $(a-b)^5$. (Give your answer in descending powers of a.)

The following symbols may be useful: a, b

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Find coefficients 2

A Level Further A

Without expanding the binomials, find:

Part A Coefficient of x^4y^6

The coefficient of x^4y^6 in the expansion of $(x^2+3y^2)^5$.

Part B Coefficient of x^{20}

The coefficient of x^{20} in the expansion of $(x^2+3x)^{12}$.

Part C The coefficient of ab^7

The coefficient of ab^7 in the expansion of $(a+\frac{1}{4}b)^8$.

Part D Constant term

The constant term in the expansion of $(\frac{x^2}{2}-\frac{8}{x})^9$.

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Group and expand

A Level Further A

C C C P P P

Expand $(1-2x+3x^2)^7$ in ascending powers of x as far as x^3.

The following symbols may be useful: x

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Binomial: All Rational n 4i

A Level

Part A Expansion 1

Find the first three terms of in the expansion of $(9-16x)^{\frac{3}{2}}$ in ascending powers of x.

The following symbols may be useful: x

Part B Expansion 1: Validity

Find the set of values for which the expansion in Part A is valid.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $x < a$
- $x \leq a$
- $x > a$
- $x \geq a$
- $a < x < b$
- $a \leq x \leq b$
- $x < a$ or $x > b$
- $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

Part C Expansion 2

Expand $(1 + 3x)^{-\frac{5}{3}}$ in ascending powers of x, up to and including the term x^3.

The following symbols may be useful: x

Gameboard:

Pure Maths Practice: Binomial - All Rational n

Binomial: All Rational n 2i

A Level

Part A Expansion

Expand $(1-4x)^{\frac{1}{4}}$ in ascending powers of x, up to and including the term in x^3.

The following symbols may be useful: x

Part B Values of a and b

The term of lowest degree in the expansion of

$$(1+ax)(1+bx^2)^7-(1-4x)^{\frac{1}{4}}$$

in ascending powers of x is the term x^3. Find the values of the constants a and b.

What is the value of a?

The following symbols may be useful: a

What is the value of b?

The following symbols may be useful: b

Gameboard:

Pure Maths Practice: Binomial - All Rational n

Physics. You work it out.

Maclaurin Series - Binomial

A Level Further A

Part A Expand $(1+r)^{1/3}$ and find $1.1^{1/3}$ and $9^{1/3}$

Expand $(1+r)^{1/3}$ up to the term in r^3.

The following symbols may be useful: `r`

Hence find, without using a calculator, $(1.1)^{1/3}$ to 3 decimal places.

Now find $9^{1/3}$ without using a calculator to 2 decimal places.

Part B Electric field on the axis of a charged sheet

The electric field E on the axis of a uniformly charged circular sheet at a distance z from the centre of the sheet is given by

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+a^2}}\right]$$

where σ is the charge per unit area on the sheet and a is the radius of the sheet. Show that in the limit when $z \gg a$ the field on the axis is such that $E \approx A/z^2$ and find A.

The following symbols may be useful: `A, a, epsilon_0, sigma, z`

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Maclaurin Series - Cos & Sin 1

Further A

C C C

Part A Find the cosine of the angle 0.2 rad

Find, using a Maclaurin expansion, the cosine of the angle $0.2\,\mathrm{rad}$, correct to 3 decimal places.

Part B Find the sine of the angle 0.08 rad

Find, using a Maclaurin expansion, the sine of the angle 0.08 rad, correct to 2 significant figures.

Part C Potential energy of mass on pendulum

A pendulum consists of a point mass m suspended on a light string of length l. When the string makes an angle of ϕ to the vertical its potential energy relative to the point where $\phi = 0$ is given by $mgl(1 - \cos\phi)$. Show that for $\phi \ll 1$ the potential energy is given approximately by $A_0\phi^2$ and find an expression for A_0.

The following symbols may be useful: `g, l, m`

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Gameboard:

STEM SMART Double Maths 21 - Binomial & Maclaurin Expansions

Maclaurin Series - ln

Further A

Part A Expand $\ln(1+z)$ and hence $\ln(2+4y)$

(i) Write down the Maclaurin expansion of $\ln(1+z)$ up to the term in z^3.

The following symbols may be useful: `z`

(ii) By re-writing $\ln(2+4y)$ in the form $A+\ln(1+z)$, where A is a constant, find the Maclaurin expansion of $\ln(2+4y)$ up to the term in y^3.

The following symbols may be useful: `y, z`

Part B Expand $\ln([1+q]/[1-q])$

Find the first 4 non-zero terms in the Maclaurin expansion of $\ln\left(\frac{1+q}{1-q}\right)$.

The following symbols may be useful: `q`

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Gameboard:

STEM SMART Double Maths 21 - Binomial & Maclaurin Expansions

Maclaurin Series - Exponentials 2

Further A

Part A Expand $Ae^{-\alpha t}$

Expand $Ae^{-\alpha t}$ up to the term in t^2.

The following symbols may be useful: `A, alpha, p, t`

Part B Expand $e^{p} - e^{-p}$

Find the first two non-zero terms in the Maclaurin expansion of $e^{p} - e^{-p}$.

The following symbols may be useful: `A, alpha, p, t`

Part C Energy decay in oscillations

A lightly damped oscillatory system has a period T. The total energy of the system at time t is given by $E(t)$. One period later its energy $E(t+T) = E(t)e^{-\gamma T}$.

(i) Find an expression for the fractional change in energy in one cycle.

The following symbols may be useful: `T, e, gamma`

(ii) On the assumption that $\gamma T \ll 1$ find an approximate expression for the fractional change in energy in one cycle.

The following symbols may be useful: `T, e, gamma`

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Gameboard:

STEM SMART Double Maths 21 - Binomial & Maclaurin Expansions

Maclaurin Series - Cos, Sin & Tan

Further A

C C C

Part A Expand $\tan\phi$

(i) Write down the first two non-zero terms in the Maclaurin expansion of $\cos\phi$.

The following symbols may be useful: `alpha, phi`

(ii) Using your result from (i) and the Binomial expansion find the first two non-zero terms in the series for $1/\cos\phi = (\cos\phi)^{-1}$.

The following symbols may be useful: `alpha, phi`

(iii) Hence, using $\tan\phi = \sin\phi/\cos\phi = \sin\phi(\cos\phi)^{-1}$, multiply the result from (ii) and the Maclaurin expansion of $\sin\phi$ to get the first two non-zero terms in the Maclaurin expansion of $\tan\phi$.

The following symbols may be useful: `alpha, phi`

Part B Expand $\sin(2\alpha)$

(i) Using the fact that $\sin(2\alpha) = 2\sin\alpha\cos\alpha$, multiply the Maclaurin expansions of $\cos\alpha$ and $\sin\alpha$ together to find the first three non-zero terms in the Maclaurin expansion of $\sin(2\alpha)$.

The following symbols may be useful: `alpha, phi`

(ii) Now find the Maclaurin expansion for $\sin(2\alpha)$ directly and verify that the first three non-zero terms in the series are the same as in (i). Find the coefficient of the α^7 term.

The following symbols may be useful: `alpha, phi`

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