Mastering essential pre-university physics

Authors' hints and notes for teachers

A1 – Using and Re-arranging Equations

This is a vital skill. If the students can apply their GCSE mathematics knowledge to their Physics, and if they can recognize the equations, then there is much of A-level they can already tackle. It is also worth the student building up their physics alphabet, and you may wish to encourage them to compile a list of quantities, including the letters used to represent them and the units they are measured in. This can be added to during the course of their Sixth Form studies.

Do use the opportunity to insist that units are given, and that a sensible number of significant figures are used. Typically, the student should carry 4 significant figures throughout the working. Final answers should be given to the number of significant figures of the least accurate datum – nearly always 2 significant figures in sheet A1.

Do note that questions A1.9 onwards involve two stages. This may need to be pointed out to the student so that they don't become perplexed. The student can approach them in one of two ways. Taking A1.9 as an example, most students find it easier to use the method which would be found on an A-level mark scheme:

A1.9 $V = IR$ = 5.0 A × 2.0 Ω = 10 V, so P $= IV$ = 5.0 A × 10 V = 50 W.

However you can also keep the quantities algebraic until the end, as is more suitable for advanced problems and undergraduate study:

A1.9 $P = IV = I \times IR = I^2R$ = 5.0^2 × 2.0 = 50 W.

A2 – Derived and Base SI Units

Students will be new to algebra based on units, and will need practice to become familiar with this idea. However, it is very powerful, enables work to be checked quickly, and allows the student to spot new relationships which could be useful to them in independent research or in practical investigations.

As examples, do note how much easier it is to visualise momentum when you realise that its unit is the **N s**, and accordingly it gives the force needed to stop an object in one second; whereas kinetic energy is in **N m** and gives the force needed to stop an object in one metre (A2.33b).

Hopefully, students will notice that at the end of the table, the N C^{-1} and the V m^{-1} are the same, which will be useful to them when they cover electric fields.

You may also wish to use A2.33d as a springboard for discussion of the link between stress (which has the same units as pressure) and energy stored per unit volume in a stretched material. Equally, Bernoulli's equation $p + ½\rho v^2 =$ constant can be seen to make sense when considering a cubic metre of fluid. The total energy is constant, and this comprises energy involved in the compression (numerically equal to p for one cubic metre) and the motion.

A3 – Standard Form and Prefixes

This should all be standard stuff. While it is vital that the student does not rely on it as a crutch, do point out the role of the ENG button on their calculator, which can make A3.5 and A3.8 easier.

A4 – Converting Units

Please remind students that 1.34 mm^2 = 1.34 (mm × mm) = 1.34 (10^{-3} m × 10^{-3} m) = 1.34 × 10^{-6} m^2. It is not equal to 1.34 × 10^{-3} m^2.

Unit conversion ought to be a doddle, but is often the bane of students' and teachers' lives. The remedy is to give them a reliable method to solve the problem. Here is a suggestion for A4.9:

A4.9 9600 μm^2 = 9600 × (10^{-6} m)^2 = 9600 × 10^{-12} m^2 = 9.6 × 10^{-9} m^2.

1 cm^2 = (10^{-2} m)^2 = 10^{-4} m^2, therefore the answer is 9.6 × 10^{-9} / 10^{-4} cm^2 = 9.6 × 10^{-5} cm^2.

One final point. The base unit of mass is the kilogram. Accordingly, before substituting into an equation such as $F = ma$, it is vital to turn 3 mg into 3 × 10^{-6} kg, not 3 × 10^{-3} g.

A5 – Gradients and Intercepts of Graphs

Please be really strict with units! Graph A's gradient is 6.0 m s^{-1}, not just 6.

Alert your students to watch for tricks: Graph C's gradient is not –2 kg m s^{-2}, but rather –2000 kg m s^{-2} because the x-axis is calibrated in milliseconds.

Graph D's intercept is not 10.0 V, because the y-axis of this graph does not coincide with a zero current (i.e. the origin).

gradient: gradient = Δy / Δx = –2 V / 50 A = –0.04 Ω

y-intercept: V = I × gradient + intercept;

So, intercept = V – I × gradient = 10 V – 50 A × (–0.04 Ω) = 12 V.

Note the way in which the x-axis of graph E has been labelled. A student would be equally able to write it as Time^2 / s^2, but I think that the form given is clearer. Time^2 / s is of course wrong.

A6 – Equations of Graphs

Hopefully, after a reminder that straight lines have the functional form $y = mx + c$ where m is the gradient and c the y-intercept, the students should find this sheet straightforward.

It may help the student to replace the quantities plotted with x and y. Thus:

A6.3,4 $\quad s = ½\, gt^2$, so $y = ½gx = ½gx + 0$, so the gradient is $½g$ and the y-intercept is zero.

Then do extend the logic. For example, suppose you had $s = ut + ½at^2$, and had measured s and t (with a constant, but u unknown), challenge the students as to what they should plot. Hopefully they will realise that if you set $y = s/t$ and $x = t$, the equation becomes $y = u + ½ax$. The intercept is then the initial velocity, and the gradient is half the acceleration.

A7 – Area Under the Line on a Graph

Ensure the students are careful with units. Graph A has an area measured in m s^{-2} × s = m s^{-1}.

Take care with graph D – it doesn't have a 'true' x-axis – the base of the graph corresponds to a current of 6 A. There is thus 6 A × 25 s = 150 C of charge 'hidden' under the graph.

Graph E will require estimation. You can work out the worth of each rectangle as 500 N × 0.2 m = 100 J, and then count the rectangles which are more than half under the line.

A8 – Area Under the Line on a Graph II

All credit to Miss Crowter for recognising that students needed extra practice with this concept. Notice too that in this sheet, unit prefixes change:

Graph B displacement = mean velocity × time = 4 m s^{-1} × 0.4 μs = 1.6 μm = 1.6 × 10^{-6} m.

Graph E medium size rectangles worth 1 kN × 0.05 μs = 0.05 mN s = 5 × 10^{-5} kg m s^{-1}.

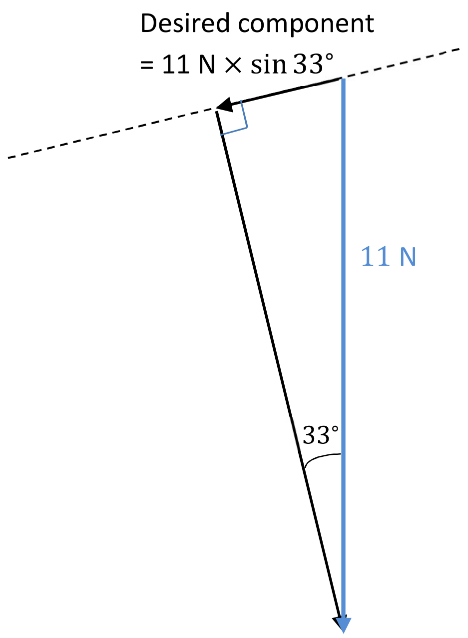
While technically frowned upon, it may be worth doing a bit of prefix algebra of the kind k × k = M or even k × μ = m, and so on. Note that $V = IR$ becomes even more practically useful when you measure I in mA and R in kΩ.

B1 – Components of a Vector

This is an essential skill that needs drumming into the head of any A-level physics student. Given that they passed GCSE Maths, they already have the knowledge – they just need to apply it reliably at speed in different contexts.

For example, in B1.1 - 1.4, you are finding the components (the hypotenuse is already known). It is simply a matter of knowing whether to multiply by sine or cosine. For this, note that COS is ClOSe. In B1.3, you are trying to find the side closest to the angle, so use cosine. The others use sine.

B1.6 is likely to cause trouble. How, after all, do you assign components to a vector (11 N) which is already vertical? However here we are looking for components perpendicular and parallel to the slope. A large diagram is essential to the solution.



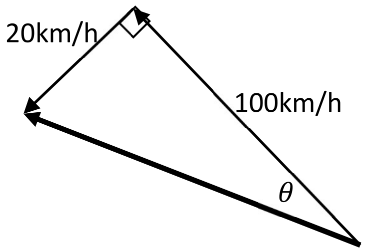
Similar vigilance is required in B1.9.

B1.10 has proved difficult to interpret. All the question is asking, is 'How much further south does the fly get each second?'

B2 – Adding Vectors

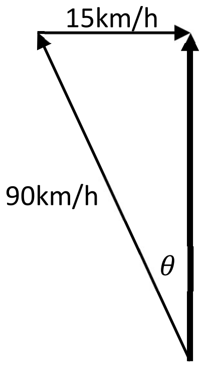
Please drum into students the need to draw a clear vector diagram for questions like this. When the question specifies that the answer is a direction, the student must be specific. "A bearing of 340°", or "3.4° N of W" or "32° above the horizontal" are all acceptable forms of answer. "9.2°" by itself is not, that is, unless it accompanies a labelled angle on a clear diagram.

Note the difference in situation between B2.2d and B2.3b. In the first case we are calculating the hypotenuse of the triangle, whereas in the second, the hypotenuse is 90 km/h.



B2.2d

In this case, if we wished to work out θ we would calculate $\mathrm{atan}(\frac{20}{100})$.



B2.3b

Now the hypotenuse is known, and we are calculating the North pointing vector using $\sqrt{90^2 - 15^2}$ km/h. The angle θ is now equal to $\mathrm{asin}(15/90)$. Contrast this with the inverse tangent used in B2.2d.

B2.3b is also instructive as a case where the vector sum is not the hypotenuse, and so it would seem that the ‘resultant’ is not the ‘resultant’. Confusion arises because in the context of Newton’s Second Law, we use ‘resultant’ to mean ‘vector sum’, but in questions like those in B1, we use ‘resultant’ in a very special case where it is the vector sum of two perpendicular components – in this context the resultant is always the hypotenuse. In B2.3b, the vector sum is the North pointing vector, but it looks very like a component rather than a resultant when the triangle is drawn. Personally, I try to avoid the use of the word ‘resultant’ altogether: except when I’m writing questions such as B2.4, that is. Feel free to castigate me in front of your class saying, “He should have said ‘vector sum’ shouldn’t he?”

B3 – Uniform Accelerated Motion in One Dimension

Little needs saying about this kind of question, as teachers are very well versed in preparing students for these.

Questions do arise as to whether $g = +9.8$ m s^{-2}, or $g = -9.8$ m s^{-2}. It is best to use common sense. In B3.1, we know it is going downwards, so I would usually regard s and v as positive for a falling object, and accordingly I can take g = +9.8 m s^{-2}. However, in B3.2, the rugby ball is initially going upwards, but with a downwards acceleration. Accordingly, I would usually take u to be positive, and g negative; however, you could also solve the problem with g = +9.8 m s^{-2}, but then in that case, you must take u = –16 m s^{-1}.

If you fancy a challenge, give your class these three facts and get them to solve all the questions just using these facts (no s, u, v, a, t):

- displacement = average velocity × time
- change in velocity = acceleration × time
- average velocity = (initial velocity + final velocity)/2

B3.1 Change in velocity = 9.8 m s^{-2} × 0.25 s = 2.45 m s^{-1}.
Average velocity = (0 + 2.45)/2 = 1.225 m s^{-1}.
Displacement = 1.225 m s^{-1} × 0.25 s = 0.31 m (2 sf)

B3.4 is really nasty if you go about it the wrong way. You can use $s = ut + ½at^2$, but then you end up solving a quadratic, which you might find distasteful. Alternatively, you can find the final velocity first using $v^2 = u^2 + 2as$, and then substitute this into $t = (v - u)/a$.

B3.7 repays further discussion. Runways actually have to be a lot longer than this to cover the braking distance should a pilot decide to cancel a take-off. However real runways are not quite long enough to allow a pilot to cancel at any time up to take-off. For any aircraft and local conditions, there is a 'decision speed' called V1. Once this speed is reached, the pilot has to commit to take off even though they haven't got enough speed to get off the ground yet. If an engine fails just after V1, the manoeuvre requires great skill on the part of the pilot. If you only have one engine, then it is a case of 'hello hedge'.

B4 – Trajectories

The key point here is that students must not confuse horizontal and vertical motion. Within each equation used, every term must refer the same kind of motion. Thus $s_v = u_v t + ½a_v t^2$, where 'v' means vertical. Alternatively, $s_h = u_h t + ½a_h t^2$, which usually simplifies to $s_h = u_h t$, since no horizontal acceleration is usually present. When working out B4.2, it is essential to take $u_v = 0$ and not $u_v = 4.0\ \mathrm{m\ s^{-1}}$.

The first stage in any problem is to work out the time. You will have to work out B4.4 before you can evaluate B4.3. Similarly B4.8 and B4.10 must be worked out before you can attempt B4.7 and B4.9.

B4.13 – B4.15 concern motion which is not initially horizontal. Some A-level syllabi will not test this scenario. Here the first stage is to work out the u_v and u_h by resolving the initial velocity into horizontal and vertical components.

B5 – Moments

Clear diagrams are essential for all problems. Without them, students all too easily assume that all distances are measured from 'the pivot', and they get really confused when there is more than one support (such as in B5.6).

Remind your class that the weight of an object acts as if it is all at the centre of gravity. For symmetrical objects, the centre of gravity will be at the centre or half way along. This may seem obvious, but in the absence of it being stated in the question, students sometimes forget to include the weight at all.

B5.5 makes students think because they want to measure to the pivot, and they don't know where it is.

> One approach is to say that the pivot is at position x. The weight is accordingly $(x - 10)$ cm from the pivot, while the centre of gravity is $(50 - x)$ cm from the pivot. To balance, we then have to solve
>
> $(x - 10) \times 2.0 = (50 - x) \times 0.92.$
>
> Simpler algebra is involved if we remember that the force upwards on the ruler at the pivot must be 2.92 N as it must support the weight and the ruler. If the pivot is distance y from the centre of the ruler, then we can neglect the 2 N force (it acts through the centre of our co-ordinates). The 0.92 N force is 40 cm away. Thus
>
> $0.92\ \mathrm{N} \times\ 40\ \mathrm{cm} = 2.92\ \mathrm{N} \times y.$
>
> Do remember that the answer to the question is not y. You need to specify the location on the ruler (e.g. at the $50\ \mathrm{cm} - y$ mark).

B5.8 Please do it the quick way by subtracting your answer for B5.7 from the total weight. Don't use moments again!

B5.10 The angle complicates matters. There are two methods of proceeding.

- i) The closest approach of the rod to the hinge is 80 cm × sin (30°) = 40 cm. Thus the 'perpendicular distance from the hinge to the line of action of the force' is 40 cm. Once you know this, you can write T × 40 cm = 30 kg × 9.8 N kg^{-1} × 40 cm.
- ii) The rod must exert an upwards force on the corner of the sign equal to half the weight of the sign (as the point is twice as far away from the hinge as the centre of gravity). This force is equal to T sin 30° (the vertical component of the tension in the rod). Thus half the weight of the sign is T sin 30°.

C1 – Combinations of Resistors

Students typically find C1.1 – C1.6 straightforward once they realize that you work up from the smallest combination. So in C1.2, you firstly combine the 1.0 Ω and 2.0 Ω resistors in series to make a 3.0 Ω resistor which is in parallel with the 4.0 Ω resistor. Application of the parallel resistance formula then gives the final result. If students wish to work these questions out quickly, mental arithmetic with parallel combinations is easier if they use the $R_T = R_1 R_2 / (R_1 + R_2)$ formula, although this formula does not extend easily to combinations of more than two resistors.

Questions C1.7 – C1.11 are straightforward applications of $R = \rho L/A$.

With C1.9 and C1.10, the radius should be converted into metres before calculating the area. Students must be careful with C1.10: remember that it is the diameter which is 1.0 cm (not the radius). It is 5.0×10^{-3} m which needs to be 'plugged in' to the formula $A = \pi r^2$.

C1.12 is challenging. In effect, the 15 copper wires and the 10 aluminium wires are all in parallel with each other. My own approach is to first work out the resistance of one copper wire (23.87 Ω) and one aluminium wire (39.79 Ω) using the formula $R = \rho L/A$. Fifteen copper wires in parallel have an overall resistance of 23.87/15 = 1.591 Ω, while ten aluminium wires in parallel have a resistance of 39.79/10 = 3.979 Ω. Finally the overall cable's resistance can be calculated as a parallel combination of 1.591 Ω and 3.979 Ω.

C2 – Charge Carriers

C2.1 – C2.4 do not require the use of the flow equation $I = NAvq$, but simply an understanding of current as a rate of flow of charge. Students get easily confused by the calculations, and I still fail to understand why a student can instinctively work out the number of 57-seat coaches needed to carry 800 pupils to a football match as 800/57, but struggles to calculate the number of 1.6×10^{-19} C electrons needed to carry 6.0 C as $6.0/1.6 \times 10^{-19}$.

With question C2.1, care is needed with the calculator. Ideally the '$\times 10^x$' button [next to 'Ans' on most calculators] should be used to enter the charge of an electron into the calculation. If the student uses any other buttons (e.g. the power or 10^x buttons), then it is imperative that brackets are used: $6.0/(1.6 \times 10^{-19})$, otherwise the answer is a very small fraction of an electron and is clearly wrong.

C2.5 – C2.9 are straightforward applications of the flow equation $I = NAvq$, however the usual caution with areas must be given. $2.5\ \text{mm}^2 = 2.5 \times (0.001\ \text{m})^2 = 2.5 \times 10^{-6}\ \text{m}^2$.

C2.10 is not as confusing as it first appears. Given the equal number of ions, the simplest method is to assume that half the current (i.e. 1.75 A) is being carried by the Cu^{2+}, and then to ignore the SO_4^{2-} from then on. It is worth pointing out that the two kinds of ions travel in opposite directions, but both contribute to an electric current in the same direction.

C3 – Charge Carriers II

Miss Crowter very correctly noted that ten practice questions were simply not enough for many students, hence the addition of a second similar sheet. Some of these questions have extra elements so students must be careful.

C3.12 is a useful question in scaling. $v = I/(NAq)$. The thick wire has three times the radius, so nine times the cross-sectional area. As they are in series, they have the same current; and as they are both made of the same material, they have the same N. Accordingly, A is the only 'variable' and the thick wire will have electrons with one ninth of the speed (on average).

C4 – Kirchhoff's Laws

A-level textbooks are usually very good at defining the laws, but perhaps not so good at putting them into normal language. While the formal definitions are needed for exams and more complex scenarios, I find it helpful to give a simplified version to get people started – and it often takes the form of a table:

	Series	Parallel
Current	=	+
Voltage	+	=

where = means that all components have the same value at any particular time (which is also the value for the circuit as a whole), while + means that you add the values from the individual components to get the value for the circuit as a whole.

Do check that students realize that a 2 A lamp in series with a 2 A resistor draws 2 A from the battery not 4 A! Similarly, a resistor connected in parallel with a lamp to a 9 V battery will have 9 V across each component (not 4.5 V).

While not good as a rigorous statement, students more readily get to grips with the concepts when they realize that current is shared out between components in parallel, whereas it is the voltage which is shared out between components in series.

C4.11 is the hardest question here. Students regularly say it should be 5.0 V (as each of L and M have 2.0 V across them). However, we must regard L/M as a parallel combination which has 2.0 V across it as a whole, and accordingly resistor K has 9.0 V – 2.0 V = 7.0 V across it. The most helpful way of teaching Kirchhoff's Second Law as far as more complex circuits are concerned is to use the idea of a current 'loop'. In other words, you draw a complete path from one side of the battery to the other, choosing one route at any junctions. The sum of the voltages across the resistors/bulbs *on this path* will equal the battery voltage. Components not on the path are ignored.

C5 – Potential Dividers

By all means use the first four circuits to teach this concept instinctively. To give an example: in circuit B, the 2.0 kΩ resistor has one third of the total resistance, so will have one third of the voltage – namely 4.0 V, leaving 8.0 V for the 4.0 kΩ resistor. Alternatively, you can work out the current first (e.g. for circuit C the current is 24 V/23 Ω = 1.043 A), and then use *V=IR* to get the voltages (the voltage across the lower resistor = 8.0 Ω × 1.043 A = 8.35 V).

C5.6 should be tackled by first combining the parallel combination to make one 2.0 Ω resistor. This has one third of the total resistance, so will have one third of the 12 V. This leaves 8.0 V for the 4.0 Ω resistor. Please make it clear to the class that the voltage across the 6.0 Ω and 3.0 Ω resistor are *both* 4.0 V (not 2.0 V each). The question is also different in asking for the 'potential at G'. This means the voltage with respect to the '0 V' point, and is accordingly 8.0 V.

C5.7 and C5.8 cause problems. A break in the circuit has lots of resistance, and effectively the whole voltage of the battery will end up across this break. In C5.7, it is the top resistor which is removed, so there will be 12 V across the top [removed] resistor and 0 V across the lower [4.0 kΩ] one. Alternatively, you can say that there will be no current, so no voltage drop across any resistors, so the mid-point (which is connected to the 0 V but not to the 12 V) must have a potential of 0 V.

C5.10 Please put a 10 kΩ resistor in parallel with the 4 kΩ resistor. This gives an overall resistance of 2.86 kΩ. The voltage across this combination (equal to the voltage across the voltmeter) will therefore be 12 V × (2.86 / [2.86 + 2]).

C5.11 is best done by working it twice – once with the variable resistor set at 0 Ω and once set at 30 Ω.

C6 – Internal Resistance

C6.1 – C6.5 should be straightforward applications of the formulae $V = IR$ and $V = \varepsilon - Ir$ where V is the terminal p.d., ε is the emf, R is the load resistance and r is the internal resistance.

Some students may find it helpful to add a 'lost volts' column (equal to ε - V and also equal to Ir) for putting the intermediate stage answers into.

C6.6 – here R = 0, so we have a simple application of $r = \varepsilon / I$. There is only one component in the circuit so its resistance must equal the voltage across it (5 kV) divided by the current through it (5 mA).

C6.8 contains superfluous information. The emf is the no-current terminal p.d., and as such is 12.4 V.

C6.10 has an emf of 13.5 V (the terminal p.d. when no current is drawn).

D1 – Amplitude and Intensity

D1.1 to D1.4 are straightforward applications of Intensity = Power/Area

D1.5 to D1.7 – asks the student to apply the equation Intensity ∝ (Amplitude)^2. In D1.5 the second laser has an amplitude 1.5 × the amplitude of the first. Its intensity, all other things being equal, will therefore be 1.5^2 × (or 2.25×) the intensity of the first.

D1.8 asks the student to imagine the surface of a sphere. Intensity = Power/Area = Power / ($4\pi r^2$).

D2 – Polarization

D2.1 Transmitted amplitude = 200 V/m × cos (55°) = 115 V/m

Transmitted intensity = 53 W/m^2 × {cos(55°)}^{2} = 17.4 W/m^2

D2.3 and D2.4 The first polarizer lets it all through, as the light was vertically polarized, and the polarizer is oriented vertically. The second will reduce (by a multiplying factor) the amplitude by cos(20°), and the intensity by $\cos^2$(20°). The light will now be polarized at 20° to the vertical, so the final polarizer (set at 20° to the polarization of the light) will multiply the amplitude by another cos(20°) and the intensity by another $\cos^2$(20°). The final light will be polarized parallel to the final polarizer – vertically.

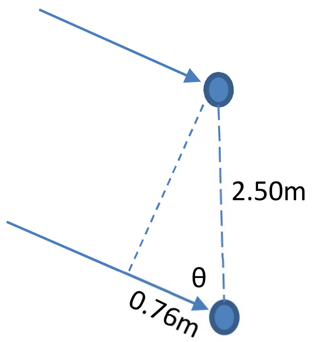
D3 – Path Difference

A good understanding of this section will help with further work on interference. Phase differences (path difference × 360°/λ) should be given in the range 0° - 360°, so D3.13 is not 3780° but 180° making it much clearer that there will be full destructive interference. The easiest way of doing this is to calculate path difference/λ first, and only keep the decimal fraction (so 10.5 becomes 0.5) after dividing by 360°.

D3.16 causes consternation however; although the actual beam zigs and zags, this need not be taken into account. You know the wavelength and the phase difference, so the path difference can be calculated from path difference = (phase difference/360°) × λ.

D3.17 The wavelength should be calculated using $\lambda = c/f$ = 2.40 m. The path difference is therefore 2.40 m × (114/360) = 0.76 m

D3.18



requires a careful diagram to be drawn using the information from the previous question, as below. Once this has been done, the 'bearing' of the signal is given clearly as θ = acos(0.76/2.5) either east of north or west of north (from the information, we can't be sure whether the waves are arriving from the East or the West).

D3.19 – with the observer 0.25 m from the mid-point, the sound from one speaker will have to travel an extra 0.25 m while the signal from the other will travel 0.25 m less. Accordingly, the path difference is 0.50 m. The wavelength can be calculated from $\lambda = c/f$ = 330 / 256, and the phase difference then worked out as (path difference × 360°)/λ.

D3.20 – here we need a path difference of half a wavelength, so the person must walk a quarter of a wavelength from the middle. (Again, path difference = 2 × distance of observer from mid-point).

D4 – Interference

D4.1 to D4.8 should be straightforward applications of the two formulae $y = \lambda D/s$ and $s \sin \theta = n \lambda$. In D4.6 and D4.7, the slit separation s must be calculated from the grating ruling, e.g. $s = 10^{-3}$ m / 600 = 1.67×10^{-6} m in D4.6.

D4.9 should be done in two parts – one angle calculated for each wavelength, and then the angles can be subtracted to give the angular separation.

D4.11 is probably best done by trial and error. Start by assuming n = 1 will work. It doesn't, as the angles for the two colours come out as 15.4° and 18.4°, so the difference is 3.0°. For small angles, the second order interferences will be at roughly twice the angles of the first, leading to a difference of about 6.0° which would be suitable. So we now guess n = 2 and calculate the actual difference to check.

D5 – Standing Waves

D5.1 to D5.6 Remember that while the amplitude will decrease as you go from antinode to node (and then increase again between the node and the next antinode), the phase remains constant until the node, where it suddenly changes by 180°.

D5.8 First, work out the wavelength. Antinode-node distances are always a quarter of λ.

D5.11 Closed end has a node, open end has an antinode, so the lowest sound has a wavelength four times longer than the tube.

D5.12 For a tube open at one and closed at the other, the tube must either be $\lambda/4$ or $3\lambda/4$ or $5\lambda/4$… Here it is $3\lambda/4$.

D5.14 The first stage is to calculate the wavelength, which is twice the antinode-antinode distance (8.0 cm). The frequency can then be obtained using $f = c/\lambda$.

D5.15 $\lambda = c/f = 330 / 125 = 2.64$ m. For this system, the oscillating air length is $\lambda/4 = 0.66$ m. Given that the tube is 1.0 m long, the water must occupy the bottom 0.34 m.

D5.16 First harmonic will have a wavelength one third of the fundamental for this system (see comment regarding D5.12), namely 0.88 m. Half a wavelength (corresponding to same amplitude but out of phase) will be 0.44 m, so the point in question is 0.44 m above the 0.08 m reference point, and as such is 0.52 m above the water.

D6 – The Photoelectric Effect

D6.1 to D6.4 should be straightforward applications of $E = hf = hc/\lambda$ for the photon, and then $KE = hf - \varphi$ for the KE, with the stopping potential $V = (hf - \varphi)/q_e$. Never give the max KE or stopping potential as negative. If it comes out below zero, then no photoelectrons are going to be emitted.

For the remaining questions, remember 1 eV = 1.6×10^{-19} J.

D6.7 Work out the maximum KE, then use $KE = mv^2/2$ to get the maximum speed.

D6.8 The minimum speed is zero (always is – there will always be a deeper electron which required all of hf to enable it to escape).

D6.10 Hopefully the students did this without detailed calculation 5.0 – 3.4 = 1.6 V.

D7 – Quantum Calculations

D7.1 to D7.6 are all about light, and the formulae to use are $E = hf$, $c = f\lambda$.

D7.8 – we are working with a photon with 511 keV of energy. Use $E = hf$ to get the frequency.

D7.9 to D7.14 are for particles. Momentum is given by $p = h/\lambda$. The kinetic energy should be calculated from this momentum. The slow way is to use $v = p/m$ and then $KE = mv^2/2$. The faster way is to remember that $KE = p^2/2m$.

D7.15 Either use $KE = mv^2/2$ to work out the speed, and then use $p = mv$; or alternatively start with $KE = p^2/2m$ and re-arrange to give $p=\sqrt{(2m \times KE)}$. Before you do anything though, you will need to put the kinetic energy into joules by multiplying by 1.6×10^{-19} (J/eV)

D7.19 – you should get the 'wrong' answer of 1.0×10^6 m/s (i.e. half the true speed you started with in D7.18). If you want to research what has happened here, find out more about phase and group velocities. The electron may well be represented as a wave-packet, but in a dispersive medium (different frequencies of waves travel at different speeds) the speed of the 'packet' (the electron) is not necessarily the same as the speed of the individual peaks and troughs (the phase velocity).

D8 – Refraction and Total Internal Reflection

These should be reasonably straightforward questions using Snell's Law $n_1 \sin \theta_1 = n_2 \sin \theta_2$, and the formula for critical angle $n_1 \sin c = n_2$.

If students have already covered D7, then point out that $n = c/v = c/(f\lambda)$. Thus n, like p, is proportional to $1/\lambda$. Snell's law is, in effect, showing that the component of momentum of a photon parallel to a boundary is conserved as the photon crosses from one side to the other.

E1 – Absolute Uncertainties

The questions in this section can largely be answered by common sense, and many students will find that their instinct is correct. There are a few points worth mentioning.

- If a measurement is given to 'the nearest mm' then the uncertainty is ±0.5 mm (if you are measuring to the nearest millimetre, the actual value can't be more than 0.5 mm from the nearest mark on the ruler).
- Absolute uncertainties should usually be given to one significant figure (1sf). If the leading figure is a '1' then 2sf is justified, but rarely otherwise. In short, 3.5 ± 0.3 A means that the 'true' value is likely to lie within 0.3 A of 3.5 A. To say the measurement is 3.52 ± 0.28 A is disingenuous. If the error is more than a quarter of an amp, it makes no sense to give the current to the nearest hundredth of an amp. Furthermore, a statement that 'the answer is usually within 0.28 A' gives no extra information than 'the answer is usually within 0.3 A'.
- Where timing apparatus is operated manually, and the process being observed allows for preparation (e.g. when timing a moving object which you can see approaching the 'gate') it is usually acceptable to give the value to the nearest tenth of a second, with an absolute uncertainty of ±0.05 s. For objects with an element of surprise, the uncertainty would be much greater.
- Where measurements are added (or subtracted), the absolute uncertainty in the sum (or difference) is equal to the sum of the individual absolute uncertainties.
- Where you measure the time for 50 oscillations, say, to an uncertainty of ±0.05 s, then the uncertainty in the timing of one oscillation will be one fiftieth of this.
- Where you have a set of results, we usually assume that the absolute uncertainty is 'half the range'. In other words, if a voltage were measured four times with the values 3.2 V, 3.3 V, 3.4 V, 0.3 V we would say that 0.3 V was an anomaly and discount it. Of the remaining values, the highest is 3.4 V while the lowest is 3.2 V. The difference between these (0.2 V) is called the range. The mean is 3.3 V, and you can see that the measurements lie within half the range (0.1 V) of this. We would give this measurement as 3.3 ± 0.1 V.

E2 – Relative Uncertainties

Most of the questions on this sheet require the use of [Relative uncertainty] = [Absolute uncertainty] × 100% / [Measurement]. To give an example, the answer to E2.1a is calculated as 0.5 mm × 100% / 50.4 cm = 0.5 mm × 100% / 504 mm. Remember to ensure that both length measurements are put into the same unit (here mm) although it doesn't matter which unit is used.

E2.1b – here, 'nearest milliamp' means an absolute uncertainty of ±0.5 mA.

E2.5 requires the calculation of a 'percentage inaccuracy' which is the difference between the measurement and the true value expressed as a percentage of the true value. Here this is calculated as $0.24 \times 10^8 \times 100\% / 3 \times 10^8$.

E2.6 gives a measurement for g which is clearly too low. The relative uncertainty is ±1.5%, so we add 1.5% to the measured value (i.e. we calculate 1.015 × 9.62 m s^{-2}) and see if this is still smaller than g. If it is, then the true value of g does not lie within the range of this measurement, so it is inaccurate. If, however 1.015 × 9.62 m s^{-2} is larger than 9.81 m s^{-2}, then the measurement is accurate as the true value of g lies within the range of uncertainty.

E2.7 – for the car to fail the test, the braking distance will be too large. If, however it is within 3% of 15 m, it may still pass. To be sure it has failed, it must take more than 15 m + 3% = 15 m × 1.03.

E3 – Propagating Uncertainties

These questions involve propagating uncertainties – for example, working out the relative uncertainty in a speed if a distance is known to 2% and the time is known to 3%. In this case the answer would be 2% + 3% = 5%.

E3.1 – some students will naturally think of $V = IR$, and accordingly be puzzled that the error in V is smaller than the error in I ("shouldn't the error in V be the error in I plus the error in R?"). However, any calculated value will always have a larger error than any of the individual measurements it depends upon (unless there are roots involved). So here we have to re-arrange $R = V/I$, and the error in R is the percentage error in V and I added together (10%).

E3.2 – the usual rule for powers is that if the error in x is 2%, then x^n has an error of n × 2%. Here the power is –1, so a 2% error in T leads to a –2% error in f. In other words, the error is of the same magnitude, but underestimates are turned into overestimates and so on. However, given that relative uncertainties are prone to being too high or too low, we simply say that the error in frequency is 2%.

E3.3 – the student must remember that the volume calculation involves three distance measurements and as such has an error of 3 × 2% = 6%. The total error is then 6% + 0.1%. Technically this is 6.1%, but this level of 'accuracy' is not meaningful. We round to 1sf (the only time when we sometimes give uncertainties to more than 1sf is when the leading digit is a '1') and say the total uncertainty is 6%.

E3.4 – the student needs to remember to re-arrange the formula $t = \sqrt{(2s/g)}$. There is no error in 2 or g, and the distance is to the power 0.5, so the uncertainty in t is 0.5 × 4% = 2%.

E3.5 – the student must remember that the uncertainty in the area will be twice the uncertainty in the diameter (4%). The student must *not* halve this to take into account that $A = \pi r^2$ rather than πd^2, nor multiply anything by 3.14. There is no error in the factor of 2 which separates radius and diameter, nor in the value of π. Accordingly, they contribute 0% to the propagated uncertainty.

E3.6 – while there are various ways of doing this, the suggestion here is to evaluate the relative (%) uncertainty in the distance and time and then add them to get the uncertainty in the speed.

E3.7 – same procedure as in E3.6, but at the end multiply the calculated frequency (f = 320 m/s ÷ 0.322 m) by your relative uncertainty to get the absolute uncertainty in frequency.

E3.8 is a sneaky question as the resistance formula does not involve multiplication or division, so the 'add the relative uncertainty' rule does not apply – it is the absolute uncertainty which needs to be added. Each resistor has ±0.12 Ω, so the overall combination has ±0.24 Ω, which corresponds to a relative uncertainty of ±0.24 / 12 = 2%. It is just as if you buy lots of things in a sale, each with a 5% reduction. Overall, you save 5% on your bill.

E3.9 – each weighing is 'to the nearest 5 g' so there is absolute uncertainty of ±2.5 g. Accordingly, three weighings leads to an absolute uncertainty of ±7.5 g.

E3.10 needs to be done in stages. Firstly, we need to work out the relative uncertainties in the initial and final speeds (by adding the relative uncertainties in the distances and times). Next, the absolute uncertainty in each speed can be calculated. From here the relative uncertainty of the velocity change can be worked out (as the sum of the absolute uncertainties in the two speeds), and this can be added to the relative uncertainty in the 1.7 s of acceleration to give the relative uncertainty in the acceleration.

You may find it quicker to work out the expected acceleration (neglecting any of the uncertainties), and then the largest possible acceleration. The largest possible acceleration will have the trolley initially moving 99 mm in 1.79 s (to get the lowest starting speed), and then moving 101 mm in 0.73 s (to get the highest final speed) in 1.6 s (to give the more rapid acceleration). The percentage difference between this 'highest acceleration' and the expected acceleration should give the relative uncertainty in the acceleration. This method will not agree exactly with that suggested in the first paragraph because the 'add the relative uncertainties' is only an approximation. However, to 1sf there should be agreement.

E4 – Accuracy, Percentage Difference and Reliability

The questions here give considerable guidance. The over-riding point is that if the 'true' value is not within the acceptable range (measured value ± uncertainty) then we say that the measurement is not accurate. Terminology between exam boards varies, and the teacher is encouraged to make sure that they use these questions in a way similar to that required by the examiner.

F1 – Force and Momentum

These questions should be straightforward practice of $Ft = m(v - u)$. As the sheet develops, the student may have to do a two-stage calculation.

F1.4 requires the student to remember to take into account the change of direction – the velocity change is 3.5 m/s not 0.5 m/s.

F2 – Conservation of Momentum

This section requires use and knowledge of the equation $p = mv$ and awareness that the total momentum before collision = total momentum afterwards.

Students are recommended to draw a before-and-after diagram with the momenta written on them.

F2.4 – with an unknown mass, the question needs to be solved algebraically: call Beth's mass m. Then the momentum before the collision is $120 \times 3.0 - 31m$, while the momentum afterwards is $(120 + m) \times 2$. Equating these expressions allows a solution for m to be found.

F3 – Units of Rotary Motion

This section requires knowledge of $f = 1/T$, $\omega = 2\pi f = 2\pi/T$, and distance = θr and $v = \omega r$.

Please remind students that final answers need to be given as decimals (not as $\pi/2$, for example). Students may also not have come across the rpm before (revolutions per minute, not radians per minute).

F3.18 – note that in radians this calculation is very easy: angle = 10 000 m / 0.3 m.

F4 – Centripetal Acceleration

This section requires use and knowledge of $a = v^2/r = r\,\omega^2$.

F4.11 may be seen by some as trivial, and by others as a trick. Gravity holds the Earth on its orbit – so the answer is gravitational force or weight. 'Centripetal force' describes the job that gravity is doing in this particular situation. Personally, I prefer not to talk of centripetal (or centrifugal) forces at all. Objects going on a circular path are accelerating, the centripetal acceleration can be calculated by v^2/r, and good old $F = ma$ can be used to work out the force needed.

F5 – Newtonian Gravity

This section requires use and knowledge of $F = GMm/r^2$ and $g = F/m = GM/r^2$.

F5.2 can be done using the formula $g = GM/r^2$, but students should try to work it out from a knowledge of the inverse square law. Compared to the first row, the mass hasn't changed and the radius has doubled. As $g \propto M/r^2$, this means that g will have been quartered.

F5.6 should also be done using ratios $g \propto M/r^2$. The distance has multiplied by 5, so the field strength will have been divided by 25.

F5.7 requires care to be taken. The height is 100 km, so the radius is 2.4×10^6 m.

F6 – Gravity and Orbits

This section requires students to combine the formulae $a = v^2/r = r\omega^2$ for circular motion and $g = F/m = GM/r^2$ for gravitational attraction.

F6.2 can be calculated using the full formula, however a ratio argument is quicker. Combining $r\omega^2 = a = F/m = GM/r^2$, we have that $T^2 \propto 1/\omega^2 \propto r^3$. Thus $T \propto r^{3/2}$. It follows that that the new orbital period is $2^{3/2}$ times bigger than one year.

F6.3 requires care to be taken – the question asks for the height of the orbit above the Earth's surface. Once the radius of the orbit has been worked out, the Earth radius (6400 km as stated in section F5) needs to be subtracted to get the final answer.

F6.7 are appended as the ability to derive these equations at speed is vital for many A-level courses, and the results are needed for the earlier questions. Practising derivations is not only helpful preparation for the examinations, it becomes a vital revision technique for undergraduate physics or engineering courses.

F7 – Oscillators

This section tests competence with the standard formulae of simple harmonic motion, such as $\omega = 2\pi/T = \sqrt{(k/m)}$, $v_{max} = \omega A$, $a = -\omega^2 x$ and so on.

F7.2 requires some care, as students must put $m = 0.3$ kg, not $m = 300$ g.

F7.6 looks much worse than it is. Let us measure the height of the water at time t measured in hours after high-tide (7am). The height is a maximum at $t = 0$, so we use a cosine function to describe the height. The time period is 12 hours, so $\omega = 2\pi/12 = \pi/6$ rad/hr. The formula for water height in metres then becomes height = $3.5 + 1.6 \cos(\pi t/6)$. Putting the numbers in gives the correct answer, providing that the student has remembered to put their calculator in radian mode.

F7.7 – quick method is to work out ω first as $2\pi/T$, then to use $v_{max} = \omega A$.

F7.10 – this question touches upon the idea of resonance. For the machine to lurch about, we need to design it with a natural frequency $f = \omega/2\pi = \sqrt{(k/m)}/2\pi$ of 20 Hz (thus 1200 rpm). Students find it amusing to watch the YouTube video of a brick being put into a spinning washing machine (https://www.youtube.com/watch?v=8_jLpd-_gdY), however, it is worth pointing out that the conductor of this particular prank took the heavy blocks out of his machine first to make it more spectacular.

G1 – Kelvin Scale of Temperature

This should be the work of a few minutes, but make sure that students take care to give the correct unit symbol (and don't miss it off). Anyone putting °K needs talking to.

G2 – Gas Laws

This section requires knowledge of $pV = nRT = NkT$, as well as the use of ratios $p_1V_1/T_1 = p_2V_2/T_2$, and accordingly, students do need to be aware of the concept of a mole.

Many questions here catch students out because the temperatures are frequently given in °C. The first stage in these questions must always be to convert to kelvin first.

You may prefer to ask students to do G2.4 – G2.7 first before trying the 'written' questions.

G2.2 is found tricky as it involves density. Students should simply use $pV = nRT$ to work out the volume of one mole ($n = 1$), and then use $\rho = m/V$ to get the density using the mass provided.

G2.9 – the number of molecules of gas $\propto pV/T$. Accordingly, if pV/T 'after' is divided by pV/T 'before' you get the fraction of molecules which remain. On subtraction from one, the fraction which have leaked is obtained.

G3 – Heat Capacity

This section requires an understanding of $E = mc\,\Delta T$. For certain questions, $P = E/t$ is also needed, and G3.8 onwards involve mixtures (not needed for all syllabi). The mixtures questions are harder, and I would suggest that unless this is being used as a revision exercise, that it is not set in one go – but that G3.1 – G3.7 is reviewed before progressing onto the mixtures.

G3.4 asks the student to work out the energy from $E = mc\,\Delta T$, then use $t = E/P$ to get the time.

G3.5 is easiest if the student thinks about a time period of one second. The energy is thus 4200 J, and they just need to find $m = E / (c\,\Delta T)$ as the mass heated each second.

G3.7 reminds the student that 'heat capacity' and 'specific heat capacity' are not the same. This distinction may need reinforcing.

For the mixtures questions (G3.8 onwards), teachers have different preferred methods. Most seem to prefer the approach 'Energy gained by the substance warming up = energy lost by substance cooling down'. In the case of G3.8 this becomes

Energy gained by Paraffin = Energy lost by Water

$4.3\text{kg} \times 2130\ \text{J/(kg K)} \times (t - 18\ ^\circ\text{C}) = 3.2\ \text{kg} \times 4180\ \text{J/(kg K)} \times (83^\circ\text{C} - t)$

where t is the unknown final temperature, and then re-arranging to find the value for t.

G4 – Latent Heat and Heat Capacity

This section builds on the content of G3 (Heat Capacity) and also requires the use of $E = mL$.

G4.3 requires the student to first work out the mass of ice from the 100 J of latent heat. Once the mass is known, the initial temperature can be calculated from the knowledge that it took 10 J to heat it up to 0 °C.

G4.5 will hopefully give the student an appreciation of the high value for the specific latent heat of vaporization of water.

G4.10 involves a mixture. Initially we have to guess the final state of the water – let's say liquid. We then use our normal method for mixtures; remember that the heat gained by the ice will be 0.35 kg × 2030 J/(kg K) × 15 °C + 0.35 kg × 335 kJ/kg + 0.35 kg × 4180 J/(kg K) × t where t is the final temperature in °C. This will be equal to 0.61 kg × 4180 J/(kg K) × (59 –t).

G4.11 corresponds to the situation where the final mixture is all ice at 0 °C, so energy required to heat 0.35 kg of ice from –15 °C to 0 °C is equal to the energy given out by our mystery mass of water in cooling from 59 °C to 0 °C and then turning to solid.

G4.12 In this situation, the final mixture is all water at 0 °C. Here the energy given out by the mystery mass of water in cooling from 59 °C to 0 °C is equal to the energy needed to heat the 0.35 kg of ice to 0 °C and then melt all of it.

H1 – Uniform Electric Fields

Questions H1.1 to H1.3 only require the use of $F = Eq$. From H1.4 onwards use of $E = V/d$ is also required. Please remind students that V is the voltage *difference* between the plates. It is worth drumming $E = V/d$ into students' minds, as frequently it is forgotten in the heat of 'live' examination questions.

H1.4 Students should note that the relevant voltage is 2500 V (the voltage difference).

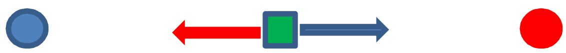
H1.6 is the first 'two stage' question. First work out the electric field strength from $E = V/d$, then use $F = Eq$ to work out the force on the drop.

H2 – Electric Field near Point Charges

These questions also require the use of $F = Eq$, but also the use of $E = kQ/r^2$ or $F = kQq/r^2$ (some syllabi prefer $(4\pi\epsilon_0)^{-1}$ in place of k).

H2.1 is best done with $F = kQq/r^2$, as is H2.3, however H2.2 requires $E = kQ/r^2$.

H2.5 should be done after drawing a diagram showing the two charges, the mid-point where we need the field and the two fields produced by the two charges at that point.



You could work out the field due to each of the charges (the distance to each charge will be 0.5 mm, so the field will be $E = kQ/r^2 = 8.99 \times 10^9 \times 10^{-9} / (5 \times 10^{-4})^2$ from each charge). However, the fields will be equal but in opposite directions, so will cancel out. I hope that your students notice this before they spend lots of time calculating!

H2.6 The aim of this question, in addition to giving calculation practice, is to demonstrate that one coulomb is a very, very large charge in electrostatic terms. The actual calculation is not too difficult – students should rearrange $F = kQq/r^2$ to make r the subject, where $Q =$ – 0.31 C (if doing it from Mr M's perspective, Mrs M sets up the field which causes the irresistible force on Mr M, so it is her charge which is Q and his which is q), q = +0.31 C, F = 200 kN (the original version did not mention buses, but simply stated that this was a suitable magnitude of a force irresistible to a human being) and $k = 8.99 \times 10^9$ N m^2/C^2 as normal. There is the subsidiary point that impulses caused by human emotion do not always obey Newton's Third Law (i.e. that attraction must be mutual), but in Mr & Mrs M's case they do, for which we are both very grateful.

H2.7 should be done without calculating Qq but rather using ratios. Moving them four times further apart reduces the force by $1/4^2$.

H2.10 requires a diagram showing the charges, the point where the field needs calculating and the fields due to the two charges at that point.

The field due to the +1 pC charge will be $E = kQ/r^2 = 8.99 \times 10^9 \times 10^{-12} / 0.2^2$, while the field due to the –1 pC charge will be in the opposite direction but four times stronger (it is twice as close). A vector sum gives the final answer. Note that the question merely asked for the 'strength of the electric field' and accordingly does not need a direction. This question has a simplifying aspect in that the two fields come out of the kQ/r^2 calculation with opposite signs and are indeed in opposite directions. However, a diagram is always recommended as where the point is between the charges (as in H2.5) it is when the fields from kQ/r^2 have the *same* sign that they point in opposite directions! You can not rely on the sign of kQ/r^2 to give you the direction of a field (unless you say + means away from the generating charge and – means towards it).

H3 – Speed of Electron in an Electric Field

The introduction should be read carefully – as some of the questions require the student to recognize that they can not answer them without relativistic equations (e.g. H3.3c). Anyone getting an answer of 6×10^{10} m/s should think before underlining it and moving on to the next question. Most A-level syllabi simply require students to recognize the limits of classical mechanics rather than attempt the relativistic analysis.

The questions require a knowledge of 1 eV = 1.6×10^{-19} J, and use of the equation $qV = mv^2/2$. Encourage students to ensure that their v and V are written differently so that they don't get confused. If there is a problem, you may wish to encourage them to use u for speed so that confusion does not arise.

Students who wish to do the relativistic calculations should use the equation $qV = (\gamma - 1)c^2$ where $\gamma = \{1 - (v/c)^2\}^{-1/2}$.

H3.5 requires some care. Given the double charge of the alpha particle, the kinetic energy will be 3 MeV.

H3.6 is simpler than some students may fear. As deuterium is singly charged, the k.e. in eV is equal to the accelerating voltage. All that is needed is to convert the energy required into eV. No speeds are needed.

H3.7 should be done without any calculation. It is 5 MV of course.

H4 – Force on a Conductor in a Magnetic Field

Most of these questions require simple application of the formula $F = BIL \sin \theta$ where θ is the angle between wire and field. It tends to be the directionality which causes trouble in understanding, so a thorough treatment of Fleming's Left Hand rule or an alternative method is required (if bored with Fleming, and your syllabus does not require it, you may prefer what I believe to be the Canadian method – hold up your right hand as if stopping traffic. The thumb (pointing left) represents the current, the fingers (pointing up) represent field, and the direction of shoving (forwards) represents the force.

H4.8 the wire is running North South, and accordingly is going to be unaffected by the horizontal component of the Earth's magnetic field (which is parallel), but will be affected by the full value of the vertical component.

H4.9 requires the student to be aware that the vertical component of the Earth's magnetic field is downwards in Britain.

H5 – Force on a Particle in a Magnetic Field

These questions should be reasonably straightforward applications of $F = Bqv \sin \theta$ where θ is the angle between field and velocity.

H5.6 is relevant to many A-level syllabi. The important fact here is that if the forces cancel out, then $F = Eq$ must be equal in magnitude to $F = Bqv$, and accordingly $E = Bv$. This idea is used in the velocity selectors of some mass spectrometers to measure the speed of accelerated particles.

H6 – Circular Paths of Particles in Magnetic Fields

H6.1 to H6.4 ought to be straightforward applications of $F = Bqv = mv^2/r$, so $r = mv/Bq$.

H6.5 requires the student to recognize that $r = 0.06$ m and also to calculate the speed of the electron from its kinetic energy using the methods of sheet H3. Once you have the speed, the magnetic flux density can be evaluated.

H6.6 again has a 12 cm diameter, so $r = 0.06$ m. Use of $r = mv/Bq$ gives as v the component of the velocity involved in the circular motion (perpendicular to the field), and accordingly v = speed × sin 70°, so speed = v / sin 70°.

H6.8 should help the student realise that because $r = mv/Bq$, the primary attribute of the motion is the momentum not the mass or speed. Because the electron has the same momentum as the muon in H6.7, the answer is the same.

H7 – Magnetic Flux and Faraday's Law

This is a topic which students typically find difficult. From my experience, they find it more straightforward if they initially have a strong understanding of flux linkage, and are able to calculate flux linkages for different geometries, competently. The induced voltage is then calculated as the change in flux linkage (the students would have already calculated these separately) divided by the time for the change.

To make things worse, angle conventions here differ from previous magnetic work. In $F = BIL \sin \theta$ or $F = Bqv \sin \theta$, the θ represents the angle between magnetic field and current or velocity. Accordingly, the maximum force, when field is perpendicular to current or motion, is at θ = 90°.

With magnetic flux linkage, the formula in most A-level courses is $\Phi = NBA \cos \theta$, which mirrors undergraduate texts. Here θ is the angle between the field and the coil's *normal* or *perpendicular*. The maximum flux linkage is when the coil presents the most area to the field, which is when *all* of the wires of the coil are at right angles to the magnetic field, and accordingly the coil's normal is parallel to the field. This is the θ = 0° situation.

It is essential that students are made aware of this transition, or they are likely to get very confused. They have a point – "you mean the situation where everything is nice and perpendicular is called 0°, and when things are parallel you label it 90°?" "Yes, indeed."

In this sheet, from H7.1 onwards, the angle is explicitly given as the 'angle between the plane of the coil and the magnetic field lines' NOT the angle between the coil's perpendicular and the field. This is more intuitive for the student familiar with the work of H4 to H6, but it does mean that this angle mentioned in the text is not the θ of $\Phi = NBA \cos \theta$. So in H7.1, where the angle is given as 90°, this is the nice perpendicular situation, so $\Phi = NBA$.

In H7.2, the angle between plane of coil and field lines is 60°, so the angle between coil perpendicular and field is 30°, so we use $\Phi = NBA \cos \theta$ with θ = 30°.

H7.4 involves $\Phi = NBA \cos \theta$ with θ = 60°.

In H7.5 everything is nice and perpendicular, so $\Phi = NBA$ can be used. The complicating factor here is that Φ needs to be calculated before and after the change (after the change $\Phi = 0$), and then the rate of change calculated. The answer should really be given in Wb.turns/s (Wb/s is acceptable but not advised as the unit is the only place where a distinction can be made between flux and flux linkage).

H7.6 is the first question which introduces Faraday's Law. Its answer is equal to that of H7.5 but now with the unit switched from Wb.turns/s to V. The two are of course equivalent, and students may enjoy exploring why:

1 Wb = 1 Tm^2, from the definition of $\Phi = NBA \cos \theta$

1 T = 1 N/(A m), from $F = BIL \sin \theta$ which defines B in tesla.

Thus 1 Wb = 1 N m^2/(A m) = 1 N m/A = 1 J/A.

The rate of change of magnetic flux density therefore has units of Wb/s = J/(A s) = J/C = V.

H7.7 and H7.8 are intended to show the student that Faraday's Law is easy – just subtract the initial flux linkage from the final one, and then divide by the time.

H7.9 has an impossibly large magnetic field, and should lead to the better students not bothering with any calculation. There is no motion, there is no change, there is no voltage. It may help to mention that in a generator, the source of energy to 'make' the electricity comes not from the store of energy in the magnetic field (if it did, generators with permanent magnets in them would have to have their magnets replaced very frequently), but from the kinetic energy of the rotation. This is why the generator has to be continually forced round by a turbine. Otherwise its store of kinetic energy would be depleted and it would slow down.

H7.10 still involves the impossibly large magnetic field, but there is a voltage this time as the coil area has changed, and therefore the flux linkage has changed.

H7.11 has $\theta = 90°$, so there is no possibility of a magnetic flux linkage, and therefore no possibility of a change of magnetic flux linkage, so no voltage.

H7.12 is best thought of as a flux linkage problem with the change of 'area' being the area 'swept out' by the spoke as it goes around the circle once. The flux linkage change in one such rotation is therefore $BA = \Phi = 1.95 \times 10^{-5}$ Wb. Given that this change occurs six times a second, the time for one revolution is 1/6 = 0.167, so V = change in Φ / time = 1.95×10^{-5} / 0.167.

Students who prefer to use the equation $V = BLu$ for the situation in H7.12 are going to have to be careful as each part of the spoke is going round at a different speed. The simple way is to note that the voltage is proportional to the speed, so the mean speed (the speed of a point half way out from axle to rim) is the speed which should be used in $V = BLu$. Alternatively, an integral can be performed, but it is a rare A-level course which would expect this.

There is no change if there were 20 spokes – each will trace the same area as if they were the only one. You could even think of it as a situation with 20 identical 'cells' in parallel. The 'total' voltage is equal to the voltage of the individual cell.

H8 – Transformers

This sheet should in the main (up to H8.13) be a revision of the GCSE equation $V_s/V_p = N_s/N_p$ (if you prefer, the 'turns per volt' is the same on both coils).

H8.12 requires care. There is a battery, so DC, so the transformer won't work.

H8.13 – indeed, the voltage has not changed. This begs the question as to why a transformer is put in the circuit at all – and it is to ensure safety. A person in the bathroom who touches any one part of the circuit will not get a shock as there is no complete path for the charge as there is no electrical connection from primary to secondary. However, an unfortunate person without a transformer who touches the 'live' 230 V side will receive a shock as their feet provide a 'route back' to the 0 V earthing terminal at the property or at the substation. This is the one situation where earthing (ensuring one conductor of a supply is permanently fixed to 0 V) makes things more dangerous.

H8.14 deals with current. In this situation, with the primary having 50× as many coils, the primary will have 50× the voltage. If the transformer were perfectly efficient, the power ($P = IV$) on both sides would be the same, so the secondary would have 50× the current. As it is only 90% efficient, the secondary power must be reduced by 10%, and so the current is 45× the primary current. Inefficiencies do not affect the voltage (although in practice a transformer would have an internal resistance, but we do not consider this at A-level).

H8.15 introduces the role of transformers in audio systems. We have a 10:1 transformer with a secondary voltage of 10 V, so the primary must have 100 V. Assuming that the transformer is perfectly efficient, the input current = Power / Voltage, and the Power must equal the secondary power of 10 V × 1.25 A = 12.5 W. Thus primary (or input) current is 12.5 W / 100 V = 0.125 A. This enables the apparent resistance on the primary side to be $R = V/I$ = 100 V / 0.125 A = 800 Ω. Or put another way, the resistance ratio is the square of the voltage (or turn) ratio. This high resistance is a help to the amplifier and power transmission system, as it will be much larger than the resistance of the cables therefore ensuring that the energy lost as heat in the cables is not a large fraction of the total.

I1 – Charge and Energy stored on a Capacitor

This sheet should require only the ability to use the equations $Q = CV$ and $E = CV^2/2$.

I1.11 should be done making simplifying assumptions. Calculate the charge using $Q = It$, then the voltage from $V = Q/C$.

I1.12 should ideally be done using ratios rather than working out the capacitance first. $E \propto V^2$. So halving the voltage will quarter the energy.

I2 – Capacitor Networks

The methodology of I2.1 is very similar to that of C1.1 to C1.6, except that the formulae are the other way around (the formula for capacitors in series is identical to that for resistors in parallel and vice-versa). Students are encouraged to draw a diagram before calculating.

I2.2 – the capacitors are in series so will have the same charge. No calculation is needed.

I2.3 – the capacitors are in series so will have the same charge. The charge on the 200 μF capacitor will be 200 μF × 12 V = 2.4 mC, so the voltage on the other capacitor will be Q/C = 2.4 mC / 2200 μF. Alternatively, you can recognize that $V \propto 1/C$. The larger capacitor has 11× the capacitance, so will have one eleventh of the voltage.

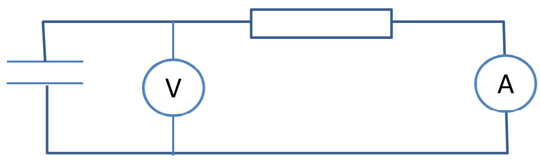
I2.4 has various methods of solution, the easiest one being hinted at in the question. The total charge will not change (it can't go anywhere – anything 'lost' by one capacitor will be picked up by the other) and will be 4.7 mC. The new capacitance is 690 μF, and with a charge and a capacitance, you can calculate the voltage.

I2.5 has capacitors in parallel, which will accordingly have identical voltages. No calculation should be necessary here.

I3 – Discharge of a Capacitor

I3.1 to I3.8 should be straightforward applications of Time constant = RC, and the halving time = $RC \times \ln 2$.

I3.9



I3.10 – the current is the initial voltage divided by the resistance.

I3.11 requires the student to work out the charge held by the capacitance at the start and then to divide this by the answer to I3.10. The answer should give the time constant.

I3.12 – hopefully students will realise that 22 s is the time constant, and accordingly the voltage will be 12 V × e^{-1}.

I3.13 – given the resistance is constant, if the voltage has halved, the current will also have halved.

I3.14 requires an application of $V = V_0\, e^{-t/RC}$, only here we need t as the subject. It is vital that students can make the necessary algebraic manipulation confidently:

$V/V_0 = e^{-t/RC}$, so $\ln(V/V_0) = -t/RC$, so $t = -RC \times \ln(V/V_0)$.

In this particular case, the voltage is one quarter of the initial voltage, so the time is twice the halving time.

I3.15 requires a calculation of halving time (RC ln 2).

I3.16 requires the formula $V = V_0\, e^{-t/RC}$. Here $t = 2RC$, so the answer is $V_0\, e^{-2}$.

I3.17 – the voltage will have reduced to $e^{-1} \approx 37\%$ of its original value, and the charge will have done the same as charge is proportional to voltage ($Q = CV$).

I3.18 requires $I = I_0\, e^{-t/RC}$, where the initial current I_0 can be calculated from the initial voltage using $I = V/R = 20\ \mathrm{V}/10\ \mathrm{k\Omega}$.

I3.19 requires the use of $t = -RC \times \ln(V/V_0)$ as in I3.14.

I3.20 is the only question on the sheet to involve a capacitor being charged. The final voltage will be 12 V, and the initial voltage is 0 V (it is uncharged). The formula for voltage in cases like this is [voltage difference from final voltage] = [constant] × $e^{-t/RC}$. The voltage difference from the final value is $12 - V$. Given that we need $V = 0$ at $t = 0$, and $e^{-t/RC} = 1$ when $t = 0$, it follows that the constant must be 12. This gives us a formula which is $12 - V = 12e^{-t/RC}$. This can be re-arranged to make V the subject, and the necessary substitutions made to evaluate the answer.

J1 – Nuclear Equations

This should enable revision of GCSE concepts, the only differences are likely to be the need to put the right kind of neutrino on the beta decays and the addition of beta plus.

J1.9 – the student will not get this equation to balance unless they put the right number of neutrons in (there may be more than one on the right hand side).

J2 – Activity and Decay

J2.1 to J2.4 test the use of the equation Half life = ln 2 / λ. Half lives in years must be converted into seconds before being inserted into the formula in order to give decay constants in s^{-1} as required.

J2.5 will be zero as this is not a radioactive isotope of carbon.

J2.5 to J2.12 enable the student to practise the equations $A = \lambda N$, and also $N = N_A M / M_m$ where N_A is the Avogadro number, M is the mass of the sample and M_m is the mass of one mole.

J2.11 can not be worked out until J2.12 has been calculated.

J2.13 – we do not have a molar mass, so we assume it is 14 g (or 0.014 kg). The number of moles is therefore 5 mg/14 g = 0.005/14 or 5×10^{-6} kg/0.014 kg. If the student chooses to use formal SI units, then they must remember that 5 mg counts as 5×10^{-6} not 5×10^{-3}, as they need to express it in kilograms.

J2.17 requires the student to complete a number of stages. Firstly, they need to work out the number of moles of uranium (3 mg / 234 g = 0.003/234), then multiply this by 6.02×10^{23} to get the number of nuclei (N), then use $A = \lambda N$ to find the decay constant λ, and finally use the formula Half life = ln 2 / λ to get the half life. It will come out of the equation in seconds, but will make more sense if then converted into years. Leaving the answer in seconds is perfectly acceptable, however.

J2.18 also requires a number of stages. Each second, 200 J of electrical energy is required. The number of decays required each second (the activity) is therefore 200 J / 2.5×10^{-12} J. The equation $A = \lambda N$ can then be used to get the number of nuclei needed. If this is divided by 6.02×10^{23}, the number of moles is found, and this can be multiplied by 0.236 kg to get the mass of the substance needed on board the spacecraft.

J3 – Nuclear Decay with Time

J3.1 to J3.16 test knowledge of $A = \lambda N$, $N = N_0 e^{-\lambda t}$, $A = A_0 e^{-\lambda t}$ and half life = ln 2 / λ.

For questions about the decay of nuclei or activity after a particular time when the half life is known, it is sometimes quicker to use the formula $N = N_0 / 2^{t/\text{half life}}$.

J3.18 can catch people out. 75% of the nuclei remain, and therefore $N/N_0 = e^{-\lambda t} = 0.75$.

J3.19 requires the student to calculate the decay constant from ln2 / half life before the activity can be calculated. The time for all decays to happen if this rate were maintained would be 10^{20} / initial activity. You should find that this is equal to $1/\lambda$.

J3.20 after a time equal to $1/\lambda$, the number remaining $N = N_0\,e^{-1}$, so the fraction remaining is $e^{-1} \approx 37\%$.

J3.21 states that 1.5% of the nuclei have decayed, so 98.5% are still in their original form. $N = N_0\,e^{-\lambda t}$, so $N/N_0 = e^{-\lambda t}$ must be 0.985. Given that λ can be calculated from the half life, t can be worked out.

J4 – Energy in Nuclear Reactions

As stated in the header, it is vital that students keep all the significant figures in their calculations until the subtraction is made.

In J4.1 and J4.2 it is recommended that the mass of the atomic components are taken in unified mass units (u) from the table on page iv, and once the total has been found, the mass of the atom or nucleus in u subtracted from it. The mass can then be converted into kilograms using the conversion factor on page iv (1 u = 1.66054×10^{-27} kg)

J4.2 requires the student to subtract the mass of the atom from the total mass of 6 protons, 6 neutrons *and 6 electrons*. Do not forget the electrons.

J4.3 requires the answer to J4.1 to be turned into joules by multiplication by c^2, then into MeV by division by 1.602×10^{-13}.

J4.4 requires the answer to J4.2 not only to be converted into MeV as in J4.3, but then needs to be divided by 12 to get the binding energy per nucleon.

J4.5 requires the total mass of products to be subtracted from the total mass of reactants (the mass of the neutron in u is on page iv). Once this has been done, the mass difference in u can be converted into kg (by multiplication by 1.66054×10^{-27} kg/u) then into J (by multiplication by c^2), then into MeV (by division by 1.602×10^{-13} J/MeV). Alternatively, some syllabi encourage students to know that 1 u is equivalent to 931 MeV which permits a more direct route.

J4.6 is very similar to J4.5 however the masses are already in kilograms. The mass of the neutron in kg will need to be looked up. The fact that these are atomic masses rather than nuclear masses does not make the calculation any more difficult – it just means that the electrons have been included on both sides of the equation.

J4.7 and J4.9 require use of the same techniques used in J4.1 to J4.4.

J4.8 and J4.10 employ a different method of working out the energy release. This has a benefit over the methods of J4.5 and J4.6 in that it doesn't involve a subtraction of very similar numbers with the subsequent risk of loss of accuracy. Here you work out the binding energy of each nucleus (e.g. for ^{147}La it will be 147 × 8.2227 MeV), and then the energy release = total binding energy of products – total binding energy of reactants. The loose neutrons do not need to be counted as they have no binding energy.

K1 – Red Shift and Hubble's Law

K1.1 to K1.15 should be straightforward manipulations of the equations [Red Shift] = [Received wavelength] – [Emitted wavelength] and $v = c$ × [Red Shift]/[Emitted wavelength].

Students should take care to read the comment regarding K1.15 printed on p78.

K1.17 is a question of proportionality. A Hubble constant of 70 km s^{-1} Mpc^{-1} means that a galaxy moving at 70 km/s will be 1 Mpc away from us. The galaxy of K1.16 has a speed of 100 km/s, so will be (100 / 70) Mpc away. Students may need a reminder of the size of a megaparsec (Mpc), i.e. that this is about 3 million light years. The galaxy in this case is clearly part of the local group, and as such would be unlikely to fit with Hubble's Law.

K1.19 is a good question to check that students are able to convert units well. A suggested approach is given below:

$$\frac{70\text{km/s}}{10^{6}\,\text{pc}} = \frac{7\times10^{4}\,\text{m/s}}{10^{6}\times3.09\times10^{16}\,\text{m}} = 2.27\times10^{-18}\,\text{s}^{-1}$$

K1.20 can be done as suggested, with the size of the parsec used to calculate the distance to the galaxy as 2 × 10^{8} × 3.09 × 10^{16} m. This can then be multiplied by the Hubble constant in s^{-1} to get the speed in metres per second. Arguably it may be quicker to use the constant in K1.17 to get the speed in km/s as 70 × 200. From this it is easy to work out the speed in m/s.

K2 – Exponential Extrapolation

K2.1 should be a matter of rearranging $N = N_0e^{-\lambda t}$ to obtain $\lambda = -\ln(N/N_0)/t = -\ln(0.55)/50$. Note that if 45% have decayed, 55% remain. It is the fraction remaining which needs to be inserted into the formula.

K2.2 uses $I = I_0e^{-\mu x}$ where μ is the attenuation co-efficient. I suggest that you work in mm. Re-arranging gives $\mu = -\ln(I/I_0)/x = -\ln(0.7)/5$ in mm^{-1}.

K2.3 involves a re-arrangement of $Q = Q_0e^{-t/RC}$ to give $RC = -t/\ln(Q/Q_0) = -180/\ln(0.2)$.

K2.4 involves very similar mathematics to K2.2.

K2.5 – while the formula $A = A_0e^{-\lambda t}$ can be used, this is not the most straightforward way. A fifteen minute period involves the activity reducing by a factor of 1230/3300 = 0.373. A further fifteen minutes will reduce it by the same factor again to give an answer of 1230 × 0.373.

K2.6 – one hour leads to a reduction by the factor of 7.2/11.5 = 0.626. Three hours will lead to this reduction three times over, so the answer will be $11.5\text{ V} \times 0.626^3$.

K2.7 requires you to think like this: one filter reduces the intensity by a factor of 0.02. Therefore n filters will reduce the intensity to a fraction 0.02^n of the original. We therefore need to solve the equation $10^{-5} = 0.02^n$. This has solution $n = \log(10^{-5})/\log(0.02)$ where you can use logs of any base (10 or e) as long as you use the same kind of logs in both calculations.

K2.8 reminds the student that 1 Bq is essentially the same thing as 1 s^{-1}. We have to find the value of x in $5.0 = 5.0 \times 10^5\, e^{-2.4x}$. Hence $x = -\ln(10^{-5})/2.4$, and the answer will be in mm.

K2.9 is best done using an attenuation co-efficient. We start with $0.5 = e^{-\mu x}$ where $x = 3 \times 10^{19}$ m. It follows that $\mu = -\ln(0.5)/(3 \times 10^{19}) = 2.31 \times 10^{-20}\text{ m}^{-1}$. Given the minute distance in the water (comparatively) it is more accurate to calculate the reduction fraction in 100 m of water by using $\mu x = 2.31 \times 10^{-20}\text{m}^{-1} \times 100 = 2.31 \times 10^{-18}$, rather than attempting to use $I/I_0 = e^{-\mu x}$ where μx is going to be so tiny.

K2.10 is best viewed that the debt at the end of the month is 1.03× larger than it was at the beginning. Accordingly after 3 years (36 months) the debt will be $£150 \times 1.03^{36}$. This is quite expensive, and may well prompt a moralizing comment from the teacher on the perils of debt – particularly at these levels of interest. Credit cards and payday lenders frequently give their interest as a monthly percentage which can make it seem deceptively low. A credit card always paid on time, of course, can be a wonderful thing to spread the cost of an exceptional purchase over a few weeks and to gain insurance for it – but only in the hands of someone who is ruthlessly disciplined.

If your students are up for the challenge, you could tell them the current [monthly] interest on a typical mortgage and then see if they can work out the monthly payment if they borrow £100 000 over 300 months (the usual 25-year payback window). If the annual interest is 4% (annual increase in debt by 1.04), then the monthly debt increase is $(1.04)^{1/12}$ = 1.0033, so the monthly interest rate is 0.33%.

Given the high level of relevance of this calculation (and its usefulness over the years to Mr and Mrs M in avoiding bad loan deals) – we append one method of working it.

We shall imagine that debt (amount A = £100 000) is simply allowed to multiply for n = 300 months, while meanwhile we pay amount P into a bank account with the same interest each month. At the end of 300 months, we wish the debt to equal the amount in the savings account. Of course this is not what actually happens – unless you have an endowment mortgage, however there is a problem there in that the interest amounts in the two accounts will not be the same and can go relatively pear shaped very easily.

The debt will now be $A \times i^n$ where i is the monthly interest fraction (e.g. 1.0033 in the example above).

On the other hand, the savings account will have the most recent payment P without interest, the previous one with one month's interest (Pi), the previous one with two month's interest (Pi^2), and so on until the first payment which has n – 1 months interest on it (Pi^{n-1}).

The total amount in this fictitious account is accordingly

$P + Pi + Pi^2 + Pi^3 + \ldots + Pi^{n-1} = P(i^n - 1)/(I - 1)$ using the usual formulae for geometric series.

It follows that $Ai^n = P(i^n - 1)/(I - 1)$, and from this P can be calculated.