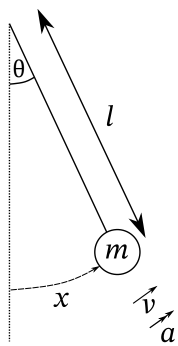
20 Simple pendulum

A simple pendulum has a mass (called a bob) on the end of a light string which, when displaced from the vertical, swings back and forth with a time period which varies only with the length of the string and the acceleration due to gravity.

Example context: Simple pendulums can be found all around us, for example a swing on a playground or the timing mechanism inside a clock. If we can neglect air resistance then a simple pendulum will continue back and forth with the same amplitude and with a consistent time period. It is for these reasons that pendulums have been used to keep accurate time in clocks since 1656.

Quantities: θ angular displacement (rad)
ω angular frequency $\left(\text{rad s}^{-1}\right)$
f frequency $\left(\text{s}^{-1}\right)$
x displacement (m)
v velocity $\left(\text{m s}^{-1}\right)$
T period (s)
a linear acceleration $\left(\text{m s}^{-2}\right)$
m mass (kg)
l length (m)
g acceleration due to gravity $\left(\text{m s}^{-2}\right)$



Equations: $T = \dfrac{2\pi}{\omega} \quad \omega = \sqrt{\dfrac{g}{l}} \quad a = -\omega^2 x \quad f = \dfrac{1}{T} \quad \omega = 2\pi f$

$\sin\theta \approx \theta$ for small θ if θ is in radians

20.1 Use the pendulum diagram provided to

a) Write down an expression for the arc length (distance) x of the mass m from the vertical in terms of l and θ in radians.

b) Calculate the distance the bob travels if it moves through an angle of 60° and the pendulum string has a length of 30 cm.

c) Write down the **magnitude** of the resultant force that acts perpendicular to the string on mass m.

d) Use your result from part (c) with Newton's Second Law derive an expression for the linear acceleration, a of the bob in terms of g and θ, taking care with the direction of the resultant force perpendicular to the string and the direction of positive acceleration shown on the diagram.

e) Use the small angle approximation for $\sin\theta$ to simplify your expression for a found in part (d).

f) By combining your result from part (e) with your answer for question (a) rewrite the linear acceleration a in terms of g, l and x.

g) Finally compare your answer from part (f) with the Simple Harmonic Motion equation for acceleration in terms of displacement, $a = -\omega^2 x$ to show that $\omega^2 = g/l$.

Example – *A clock maker wishes to make a clock such that it ticks once every* 2.0 *s rather than every second. How long will the length of their pendulum need to be?*

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{therefore} \quad l = g\left(\frac{T}{2\pi}\right)^2 = 9.81\left(\frac{2}{2\pi}\right)^2 \approx 1.0\text{ m}$$

20.2 An astronaut takes a pendulum on a mission to Mars to estimate their weight on the planet. Their pendulum bob has a mass of 50 g, the length of the string is 0.5 m and the astronaut has a mass of 70 kg. The astronaut measures the period of the pendulum to be 2.3 s on Mars. How heavy is the astronaut on Mars to the nearest newton?

20.3 A simple pendulum is made of a light string of length $l = 25$ cm with a bob of mass $m = 30$ g and is stationed on the Moon ($g_m = 1.63$ m s^{-2})

a) What is the time period t_p for this pendulum?

b) How many whole oscillations does the pendulum make in 1 min

c) Calculate the angular frequency of this pendulum using l and g and show that it is numerically equal to $2\pi f$.

d) What would the value of t_p and ω be if we doubled the mass of the bob to $2m$?

20.4 In a lecture demonstration three pendulums are set in motion. The first has a length l, the second has a length $4l$ and the third has a length $9l$. If they all begin at the same amplitude and at the same time, how many whole swings will the first pendulum have completed after the initial drop when all three pendulums are instantaneously back in sync?

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(a) $x = l\theta$ From the definition of the radian.

(b) $60^\circ = 60 \times \frac{2\pi}{360} = 1.047$ rad So, $x = l\theta = 30$ cm $\times\ 1.047 = 31.4$ cm

(c) Resultant force perpendicular to string has magnitude $=$ component of weight perpendicular to string $= mg\sin\theta$

(d) $ma = -mg\sin\theta$ so $a = -g\sin\theta$

(e) $a = -g\sin\theta \approx -g\theta$

(f) $\theta = \frac{x}{l}$ so $a \approx -g\theta = -\frac{gx}{l}$

(g) $a = -\frac{g}{l}x$ so if $a = -\omega^2 x$ then $\omega^2 = \frac{g}{l}$

21 Electromagnetic induction – moving wire

(a) $A = Lw = Lut$

(b) $BA = BLut$

(c) $\frac{\mathrm{d}(BA)}{\mathrm{d}t} = \frac{BA}{t} = BLu$

(d) $V = \frac{\mathrm{d}(BA)}{\mathrm{d}t} = BLu$

(e) Force $F_\mathrm{B} = quB$

(f) Electric field $E = \frac{\text{Force}}{q} = uB$

(g) $V = EL = (uB)L = BLu$ – i.e. the same as part (d)