Crossing Paths

A Level Further A

A person is walking northwards with velocity $\underline{\boldsymbol{v}} = 1.0\hat{\underline{\boldsymbol{j}}}\,\mathrm{m\,s^{-1}}$ from a point $\underline{\boldsymbol{r}} = 500\hat{\underline{\boldsymbol{i}}}\,\mathrm{m}$. At the same time a second person starts walking from a point $\underline{\boldsymbol{s}} = -500\hat{\underline{\boldsymbol{i}}} + 500\hat{\underline{\boldsymbol{j}}}\,\mathrm{m}$ with velocity $\underline{\boldsymbol{u}} = 1.0\hat{\underline{\boldsymbol{i}}} + u_y\hat{\underline{\boldsymbol{j}}}\,\mathrm{m\,s^{-1}}$.

Part A Time Taken

Find the time T in seconds that passes between the walkers setting off, and their paths crossing.

Part B Required Speed

Find the speed u_y required for the two people to meet.

Ferry and Current

A Level Further A

A ferry is to cross the Sound of Islay from Port Askaig on Islay to Feolin on Jura which is due east of Port Askaig. The tidal current in the Sound of Islay is strong and the water is flowing at in a northerly direction. The ferry travels at a speed of relative to the water.

Part A Which direction?

In what direction should the ferry set out? Give your answer as a bearing.

Part B How long to cross

How long, in seconds, will the ferry take to make the crossing?

Forces in Equilibrium

A Level Further A

A body is acted on by three forces $\underline{\boldsymbol{T}}_1$, $\underline{\boldsymbol{T}}_2$ and $\underline{\boldsymbol{T}}_3$ in the (x, y)-plane as shown in the diagram. $\underline{\boldsymbol{T}}_3 = 20.0\underline{\hat{\boldsymbol{j}}}\,\mathrm{N}$, $\phi = 20.0^\circ$ and the body is in equilibrium.

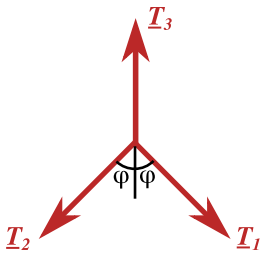

Figure 1: A vector diagram of the forces $\underline{\boldsymbol{T}}_1$, $\underline{\boldsymbol{T}}_2$ and $\underline{\boldsymbol{T}}_3$ acting on a body.

Part A Magnitude of T1

Find the magnitude of $\underline{\boldsymbol{T}_1}$.

Part B Horizontal Component of T2

Find the horizontal component of T_2, taking left-to-right to be the positive direction.

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Shooting a Balloon

A Level

The points A and B are a distance a apart and AB is horizontal. A shot is fired from A with velocity $\sqrt{\lambda ag}$ at an angle θ to the horizontal. At the same instant a balloon is released from rest at B and rises vertically with constant acceleration $(\mu - 1)g$, where $1 < \mu < \lambda$. The shot hits the balloon.

For $\lambda = 10$ and $\mu = 5.0$ find the possible values of $\tan\theta$.

Part A Higher value

Give the higher of the two values of $\tan\theta$ (to 2 significant figures).

Part B Lower value

Give the lower of the two values of $\tan\theta$ (to 2 significant figures).

Triangular Flying Course

A Level

Two identical aeroplanes, P_1 and P_2, can both complete a course in the form of an equilateral triangle ABC in time $3T$ at a speed V when there is no wind. They then fly over the same course in a wind of velocity kV parallel to the side AB and blowing from B to A, where $k < 1$.

With both planes starting at A, the pilot of P_1 thinks that he can complete the course faster by initially flying along AC (completing the course ACBA). However, the pilot of P_2 thinks that it will be faster to first travel along AB (completing the course ABCA). This is shown in **Figure 1**.

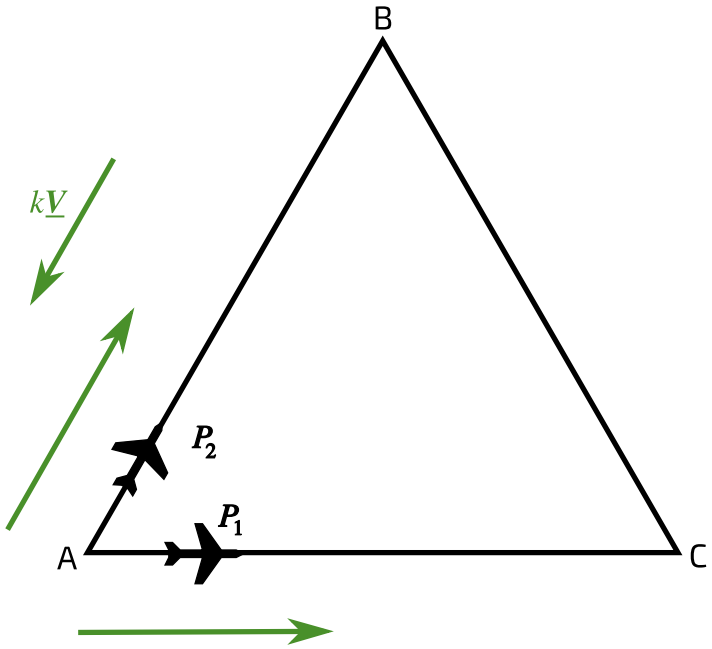

Figure 1: Pilot P_1 flies along AC first. Pilot P_2 flies along AB first.

Find an expression for the difference between the times taken by each pilot to complete the course.

The following symbols may be useful: `T`, `V`, `k`

Adapted with permission from UCLES, Higher School Certificate Applied Mathematics, June 1936, Question 6.

The Harbour Entrance

A Level

The entrance to a harbour is a channel of length a which runs between two sandbanks a distance b apart. The banks and the channel can be assumed to be rectangular. On this particular day, there is a current of constant speed v flowing from one sandbank to the other. A yacht travelling at a constant speed relative to the water of u wants to enter the harbour.

Find the least value of u needed for the yacht to safely enter the harbour.

The following symbols may be useful: `a, b, u, v`

Used with permission from UCLES, A Level Further Maths, Syllabus C, June 1989, Special Paper, Question 7.

Highway Pursuit

GCSE C C C A Level P P P

In a high speed highway pursuit, the police are chasing some gangsters. The police car is travelling at $45\,\mathrm{m\,s^{-1}}$ and the gangsters are travelling at $47\,\mathrm{m\,s^{-1}}$.

By considering the velocity of the gangsters relative to the police, what is the distance between the two cars after 90 seconds? Assume they start at the same point (the crime scene).

- $2\,\mathrm{m}$
- $4050\,\mathrm{m}$
- $180\,\mathrm{m}$
- $4230\,\mathrm{m}$
- $90\,\mathrm{m}$

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Overtaking on the road

GCSE A Level

A driver on a single carriageway road (one where there is only one lane in each direction) needs to ensure that the way ahead is clear before they overtake. In this question, you will work out how far ahead the road needs to be clear of traffic coming the other way in order for overtaking to be safe.

Part A Time to overtake

A car is travelling at the speed limit on a single carriageway road in England $\left(60\,\mathrm{mph} = 26.8\,\mathrm{m\,s^{-1}}\right)$. It comes up behind a lorry travelling at its speed limit $\left(50\,\mathrm{mph} = 22.3\,\mathrm{m\,s^{-1}}\right)$.

In order to honour the approved stopping distances, the car moves onto the other side of the road before getting closer than $73\,\mathrm{m}$ to the back of the lorry. The lorry is $15\,\mathrm{m}$ long, and the car does not pull back into the left hand lane until it is $53\,\mathrm{m}$ in front of the lorry. You may neglect the length of the car.

How much time will the car spend on the other side of the road?

Part B Clear distance

Assume that all traffic travelling in the opposite direction does so at $60\,\mathrm{mph} = 26.8\,\mathrm{m\,s^{-1}}$. Calculate the distance ahead of the car which must be clear in the right hand lane at the point the car moves over into that lane in order to overtake the lorry as described in the last question.

Love Bugs

A Level

A bug sits at each corner of a regular polygon with N sides of length l. Each bug always walks directly towards the next bug around, counterclockwise.

How long does it take for all of the bugs to meet in the middle if they walk with a speed $v = 1.0\,\mathrm{cm\,s^{-1}}$, $l = 10\,\mathrm{cm}$ and $N = 4$?