Straight Lines: Coordinates and Lengths 2i

A Level P P P

The points A, B, and C have coordinates $(5, 1)$, $(p, 7)$, and $(8, 2)$ respectively.

Part A Possible values of p

Given that the distance between the points A and B is twice the distance between points A and C, calculate the possible values of p. Enter the smallest possible value of p.

The following symbols may be useful: p

Part B Midpoint of AB

Given also that the line passing through A and B has equation $y = 3x - 14$, find the coordinates of the midpoint of AB. Enter the x and y coordinates below.

Enter the x coordinate:

The following symbols may be useful: x

Enter the y coordinate:

The following symbols may be useful: y

Used with permission from UCLES, A Level, January 2006, Paper 4721, Question 9.

Gameboard:

Pure Maths Practice: Straight Lines - Coordinates and Lengths

Physics. You work it out.

Home Gameboard Maths Straight Lines: Coordinates and Lengths 1ii

Straight Lines: Coordinates and Lengths 1ii

A Level
P P P

Part A Find coordinate

The line segment joining the points $(-2, 7)$ and $(-4, p)$ has gradient 4. Find the value of p.

The following symbols may be useful: p

Part B Find coordinates and midpoint

The line segment joining the points $(-2, 7)$ and $(6, q)$ has midpoint $(m, 5)$. Find m and q. Enter the values of m and q below.

Enter the value of m:

The following symbols may be useful: m

Enter the value of q:

The following symbols may be useful: q

Part C Find coordinate from length

The line segment joining the points $(-2, 7)$ and $(d, 3)$ has length $2\sqrt{13}$. Find the two possible values of of d. Enter the greatest possible value of d.

The following symbols may be useful: `d`

Used with permission from UCLES, A Level, January 2013, Paper 4721, Question 6.

Straight lines: gradients and normals 2i

A Level

P P P

A is the point $(2, 7)$ and B is the point $(-1, -2)$.

Part A Equation of line

Find the equation of the line through A parallel to the line $y = 4x - 5$, giving your answer in the form $y = mx + c$.

The following symbols may be useful: x, y

Part B Length of AB

Calculate the length of AB, giving your answer in simplified surd form.

Part C Find equation of line

Find the equation of the line which passes through the midpoint of AB, and which is perpendicular to AB. Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

The following symbols may be useful: x, y

Gameboard:

Pure Maths Practice: Straight Lines - Gradients and Normals

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Home Gameboard Maths Straight lines: gradients and normals 3ii

Straight lines: gradients and normals 3ii

A Level

P P P

The points $A(1,3)$, $B(7,1)$, and $C(-3,-9)$ are joined to form a triangle.

Part A Show right angle

Show that this triangle is right angled, and determine whether the right angle is located at A, B, or C.

- C
- B
- A

Part B Triangle in circle

The points A, B and C lie on the circumference of a circle.

Find the x coordinate of the centre of the circle.

The following symbols may be useful: x

Find the y coordinate of the centre of the circle.

The following symbols may be useful: y

Gameboard:

Pure Maths Practice: Straight Lines - Gradients and Normals

Logarithmic Plots 1

A Level

The logarithms to base 10 of two variables, x and y, are plotted against each other below.

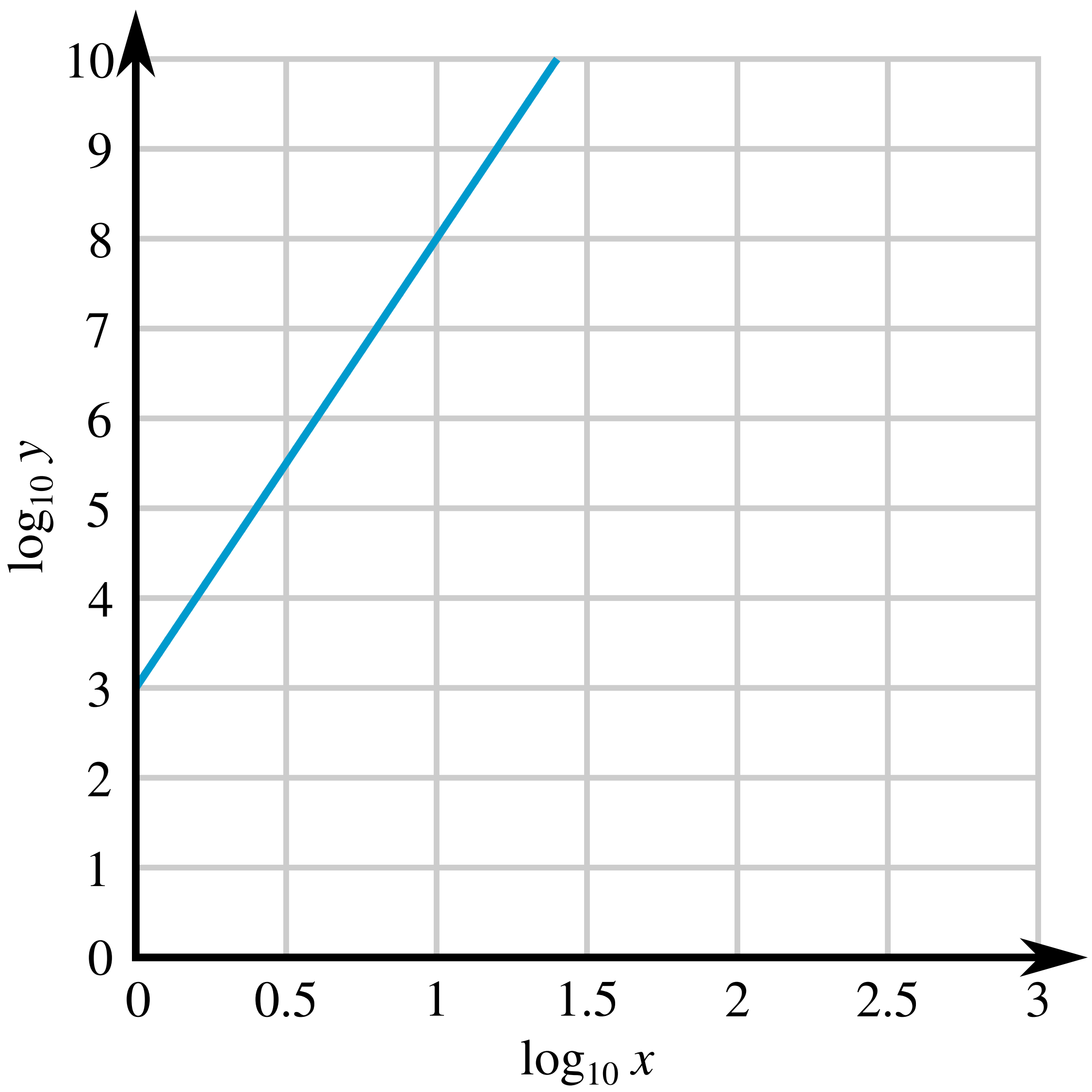

Figure 1: A plot of $\log_{10} y$ against $\log_{10} x$.

Use this plot to determine the relationship between x and y. Give your answer in the form $y = ax^b$, where a and b are constants.

The following symbols may be useful: x, y

Adapted for Isaac Physics from NST IA Biology preparation work

Physics. You work it out.

Home Maths Functions General Functions Logarithmic Plots 3

Logarithmic Plots 3

A Level
P P P

By plotting a graph of $\ln F$ against $\ln r$, a student finds that the relationship between the gravitational force, F, on a pair of objects with fixed masses is given by

$$F = \frac{10^8}{r^2}$$

where r is the separation between them.

Part A Find the gradient

What was the gradient of the graph?

Part B Find the intercept

What was the intercept of the graph? Give your answer to 2 significant figures.

Adapted for Isaac Physics from NST IA Biology preparation work

Physics. You work it out.

Home Maths 3 Simultaneous Equations 3i

3 Simultaneous Equations 3i

Further A

P P P

The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

Part A a

Find the value of a in exact form, given that $\mathbf{B}$ is singular.

The following symbols may be useful: a

Part B $\mathbf{B}^{-1}$

$\mathbf{B}^{-1}$ can be written in the form $\mathbf{B}^{-1} = \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \zeta \\ \eta & \theta & \iota \end{pmatrix}$. You are given that $\mathbf{B}$ is non-singular.

Give an expression for $\alpha - \beta + \gamma - \delta + \epsilon - \zeta + \eta - \theta + \iota$ in terms of a.

The following symbols may be useful: a

Part C Simultaneous equations

x, y and z satisfy the following simultaneous equations

$$-x + y + 3z = 1$$

$$2x + y - z = 4$$

$$y + 2z = -1$$

Use matrix methods to solve this question only.

Find x in exact form.

The following symbols may be useful: x

Find y in exact form.

The following symbols may be useful: y

Find z in exact form.

The following symbols may be useful: z

Adapted with permission from UCLES, A Level, June 2005, Paper 4725, Question 7.

Matrices - intersecting lines

Further A University

Two lines are described by

$$3x - 4y - 1 = 0$$
$$2x + py - 10 = 0.$$

where p is a constant. Use matrix notation to find the coordinates of the point of intersection of these two lines.

Part A Write in matrix form

Write these equations in matrix form $\mathbf{Ax} = \mathbf{b}$.

If the matrix $\mathbf{A}$ is written in the form

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

give the values of these matrix elements.

(a) Give the value of a_{11}.

(b) Give the value of a_{12}.

(c) Give the value of a_{21}.

(d) Give the value of a_{22}.

The following symbols may be useful: p

Part B Condition for no intersection

Use the matrix to find the value of p for which the lines do not intersect. Give your answer as an improper fraction.

The following symbols may be useful: p

Part C The inverse matrix

Find $\mathbf{A}^{-1}$, the inverse of $\mathbf{A}$.

If the matrix $\mathbf{A}^{-1}$ is written in the form

$$\mathbf{A}^{-1} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$$

give the values of these matrix elements

(a) Give an expression for α_{11}.

The following symbols may be useful: p

(b) Give an expression for α_{12}.

The following symbols may be useful: p

(c) Give an expression for α_{21}.

The following symbols may be useful: p

(d) Give an expression for α_{22}.

The following symbols may be useful: p

Part D Components of point of intersection

Using $\mathbf{A}^{-1}$ obtain expressions for the x and y components for the point of intersection.

(a) Give an expression for the x-component of the point of intersection.

The following symbols may be useful: p

(b) Give an expression for the y-component of the point of intersection.

The following symbols may be useful: p

Part E A value for p

If the y-component of the point of intersection is equal to 2, find the value of p.

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Home Maths Algebra Matrices Matrices - linear equations 2

Matrices - linear equations 2

Further A University

Use matrix notation to solve the following set of three equations for x, y and z:

$$x + cy = c$$
$$x - y + 3z = -c$$
$$2x - 2y - z = 2.$$

Part A Determinant of the matrix

Write these equations in matrix form $\mathbf{R}\mathbf{x} = \mathbf{p}$. Hence deduce the determinant of $\mathbf{R}$ and find the value of c for which there is no unique solution.

(a) Find the determinant of $\mathbf{R}$.

The following symbols may be useful: c

(b) Deduce the value of c for which there is no unique solution.

Part B The inverse matrix

Find the inverse matrix $\mathbf{R}^{-1}$.

If the matrix $\mathbf{R}^{-1}$ is written in the form

$$\mathbf{R}^{-1} = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{pmatrix}$$

give expressions for the elements of $\mathbf{R}^{-1}$ on the leading diagonal i.e. ρ_{11}, ρ_{22} and ρ_{33}.

(a) Give an expression for ρ_{11}

The following symbols may be useful: c

(b) Give an expression for ρ_{22}

The following symbols may be useful: c

(c) Give an expression for ρ_{33}.

The following symbols may be useful: c

Part C Solution to the set of equations if $c = 1$

Using $\mathbf{R}^{-1}$, find the solutions for x, y and z if $c = 1$.

(a) Find the value of x.

(b) Find the value of y.

(c) Find the value of z.

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Home Maths Algebra Matrices Matrices - linear equations 3

Matrices - linear equations 3

Further A University

C C C P P P

A system consists of three masses m_1, m_2 and m_3 in a line; they each have the same mass m. The mass m_2 is in the centre and connected by springs of spring constant k to m_1 on the left and m_3 on the right. The masses are all performing simple harmonic motion at the same angular frequency ω such that their equations of motion are

$$\begin{aligned} -kx_1 + kx_2 &= -m\omega^2 x_1 \\ kx_1 - 2kx_2 + kx_3 &= -m\omega^2 x_2 \\ kx_2 - kx_3 &= -m\omega^2 x_3. \end{aligned}$$

where x_1, x_2 and x_3 are the displacements of m_1, m_2 and m_3 respectively.

These equations can be written in matrix form

$$\begin{aligned} \mathbf{A}\mathbf{x} &= -m\omega^2\mathbf{x} \\ &= -m\omega^2\mathbf{I}\mathbf{x} \\ \Rightarrow (\mathbf{A} + m\omega^2\mathbf{I})\mathbf{x} &= 0 \end{aligned}$$

A matrix equation of this sort only has solutions if $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$. Use this to find the possible values of ω^2. For each value of ω find the relationship between x_1, x_2 and x_3.

Part A The matrix $\mathbf{A}$

If the matrix $\mathbf{A}$ is written in the form

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

deduce the expressions for the following elements of $\mathbf{A}$.

(a) Give the expression for a_{11}.

The following symbols may be useful: `k, m`

(b) Give the expression for a_{21}.

The following symbols may be useful: `k, m`

(c) Give the expression for a_{22}.

The following symbols may be useful: `k, m`

(d) Give the expression for a_{31}.

The following symbols may be useful: `k, m`

Part B The possible values of ω^2

Write down the matrix $\mathbf{A} + m\omega^2\mathbf{I}$. Using the fact that solutions to the equation $\mathbf{A} + m\omega^2\mathbf{I} = 0$, require that $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$ deduce the three values of ω^2. The three values, ω_1^2, ω_2^2 and ω_3^2, are such that $\omega_1^2 < \omega_2^2 < \omega_3^2$.

(a) Give an expression for the 11 component (i.e. the component in row 1, column 1) of $\mathbf{A} + m\omega^2\mathbf{I}$.

The following symbols may be useful: `k, m, omega`

(b) Find an expression for ω_1^2.

The following symbols may be useful: `k, m`

(c) Find an expression for ω_2^2.

(d) Find an expression for ω_3^2.

The following symbols may be useful: `k, m`

Part C The relationship between x_1, x_2 and x_3

Since the determinant of the matrix is zero there are no unique solutions to the set of three equations; however, for each value of ω^2, x_1, x_2 and x_3 have a fixed relationship to each other. On the assumption that $x_1 = 1$, find x_2 and x_3 for each of the three frequencies deduced in Part B. Give your answers using the format 1,a,b with no spaces, where $x_1 = 1$, $x_2 = a$ and $x_3 = b$.

(a) Given that $x_1 = 1$, find x_2 and x_3 for ω_1^2.

(b) Given that $x_1 = 1$, find x_2 and x_3 for ω_2^2.

(c) Given that $x_1 = 1$, find x_2 and x_3 for ω_3^2.

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