Integration - Trig Manipulations 3ii

A Level

P P P

Find $\int_0^{\frac{\pi}{4}} \frac{1 - 2\sin^2 x}{1 + 2\sin x \cos x} \mathrm{d}x$, giving your answer in the form $a \ln b$.

Integration - Trig Manipulations 3i

A Level

P P P

Part A Simplify

Simplify as far as possible $\frac{1}{1-\tan x}-\frac{1}{1+\tan x}$.

The following symbols may be useful: `x`

Part B Integrate

Hence evaluate $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}}(\frac{1}{1-\tan x}-\frac{1}{1+\tan x})\mathrm{d}x$, giving your answer in the form $a\ln(b)$.

The following symbols may be useful: `pi`

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Gameboard:

STEM SMART Double Maths 29 - Differential Equations & Volumes of Revolution

Integration of Differential Equations 1i

A Level

Part A Partial Fractions

Express $\dfrac{1}{(3-x)(6-x)}$ in partial fractions.

The following symbols may be useful: x

Part B Value of n

In a chemical reaction, the amount x grams of a substance at time t seconds is related to the rate at which x is changing by the equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(3-x)(6-x),$$

where k is a constant. When $t=0$, $x=0$ and when $t=1$, $x=1$.

If $k = \frac{1}{3}\ln n$, find the exact value of n.

The following symbols may be useful: n

Part C Value of x

Find the value of x when $t=2$, to 3 s.f.

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Gameboard:

STEM SMART Double Maths 29 - Differential Equations & Volumes of Revolution

Integration of Differential Equations 4i

A Level

Part A Derivative

If $y = \operatorname{cosec} x$ then find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), arccos(), arccosec(), arccosech(), arccosh(), arccot(), arccoth(), arcsec(), arcsech(), arcsin(), arcsinh(), arctan(), arctanh(), cos(), cosec(), cosech(), cosh(), cot(), coth(), ln(), log(), sec(), sech(), sin(), sinh(), tan(), tanh(), x, y`

Part B Solve

Solve the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\sin x \tan x \cot t$$

given that $x = \frac{\pi}{6}$ when $t = \frac{\pi}{2}$.

The following symbols may be useful: `arccos(), arccosec(), arccosech(), arccosh(), arccot(), arccoth(), arcsec(), arcsech(), arcsin(), arcsinh(), arctan(), arctanh(), cos(), cosec(), cosech(), cosh(), cot(), coth(), ln(), log(), sec(), sech(), sin(), sinh(), t, tan(), tanh(), x`

Gameboard:

STEM SMART Double Maths 29 - Differential Equations & Volumes of Revolution

Modelling - Advanced 3ii

A Level

The height, h metres, of a shrub t years after planting is given by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{6-h}{20}$$

A shrub is planted when its height is $1\,\mathrm{m}$.

Part A Solution

Integrate the differential equation to find an expression for t in terms of h.

The following symbols may be useful: `h`, `ln()`, `log()`, `t`

Part B Time to reach a known height

How long after planting will the shrub reach a height of $2\,\mathrm{m}$? Give your answer to 3 significant figures.

Part C Height after a known time

Find the height of the shrub $10\,\mathrm{years}$ after planting. Give your answer to 3 significant figures.

Part D Maximum height

State the maximum possible height of the shrub.

Gameboard:

STEM SMART Double Maths 29 - Differential Equations & Volumes of Revolution

Modelling - Advanced 1i

A Level P P P

In the year 2000 the population density, P, of a village was 100 people per km^2, and was increasing at the rate of 1 person per km^2 per year. The rate of increase of the population density is thought to be inversely proportional to the size of the population density. The time in years after the year 2000 is denoted by t.

Part A Differential equation

Write down a differential equation to model this situation.

The following symbols may be useful: `Derivative(P, t), P, k, t`

Part B Solution $P(t)$

Solve the differential equation to express P in terms of t.

The following symbols may be useful: `P, t`

Part C Evaluate the model

In 2008 the population density of the village was 108 people per km^2 and in 2013 it was 128 people per km^2. Determine how well the model fits these figures.

Easier question?

Gameboard:
STEM SMART Double Maths 29 - Differential Equations & Volumes of Revolution

Calculus: Volume of Revolution

Further A

P P P

Part A Re-arranging for x

Given that $y = \frac{1}{4}(2 + \sqrt[5]{x})$, show that x may be expressed in the form $(ay + b)^5$, where the values of the constants a and b are to be found.

Write your answer in the form $x = (ay + b)^5$.

The following symbols may be useful: `x`, `y`

Part B Volume of revolution

The diagram shows a sketch of the curve $y = \frac{1}{4}(2 + \sqrt[5]{x})$. The shaded region is bounded by part of the curve and the lines $x = 0$ and $y = 1$. The shaded region is rotated through four right angles about the y-axis.

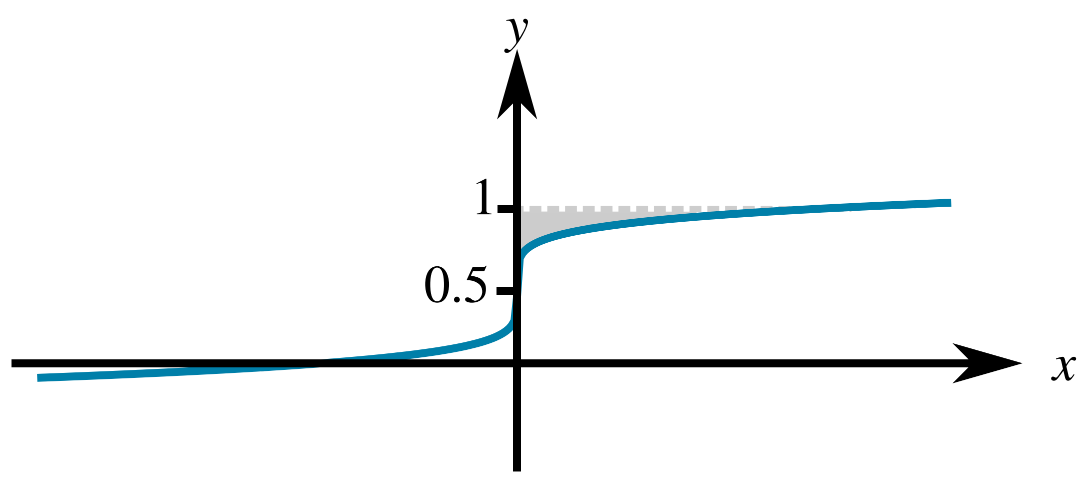

Figure 1: Diagram of $y = \frac{1}{4}(2 + \sqrt[5]{x})$ with the shaded region shown.

Find the exact volume of the solid produced.

The following symbols may be useful: `pi`

Adapted with permission from UCLES, A Level, January 2005, Paper 2632, Question 7.

Gameboard:

STEM SMART Double Maths 29 - Differential Equations & Volumes of Revolution

Calculus: Volume of Revolution

Further A

P P P

Figure 1 shows the curves $y = \mathrm{e}^{3x}$ and $y = (2x-1)^4$. The shaded region is bounded by the two curves and the line $x = \frac{1}{2}$. The shaded region is rotated completely about the x-axis.

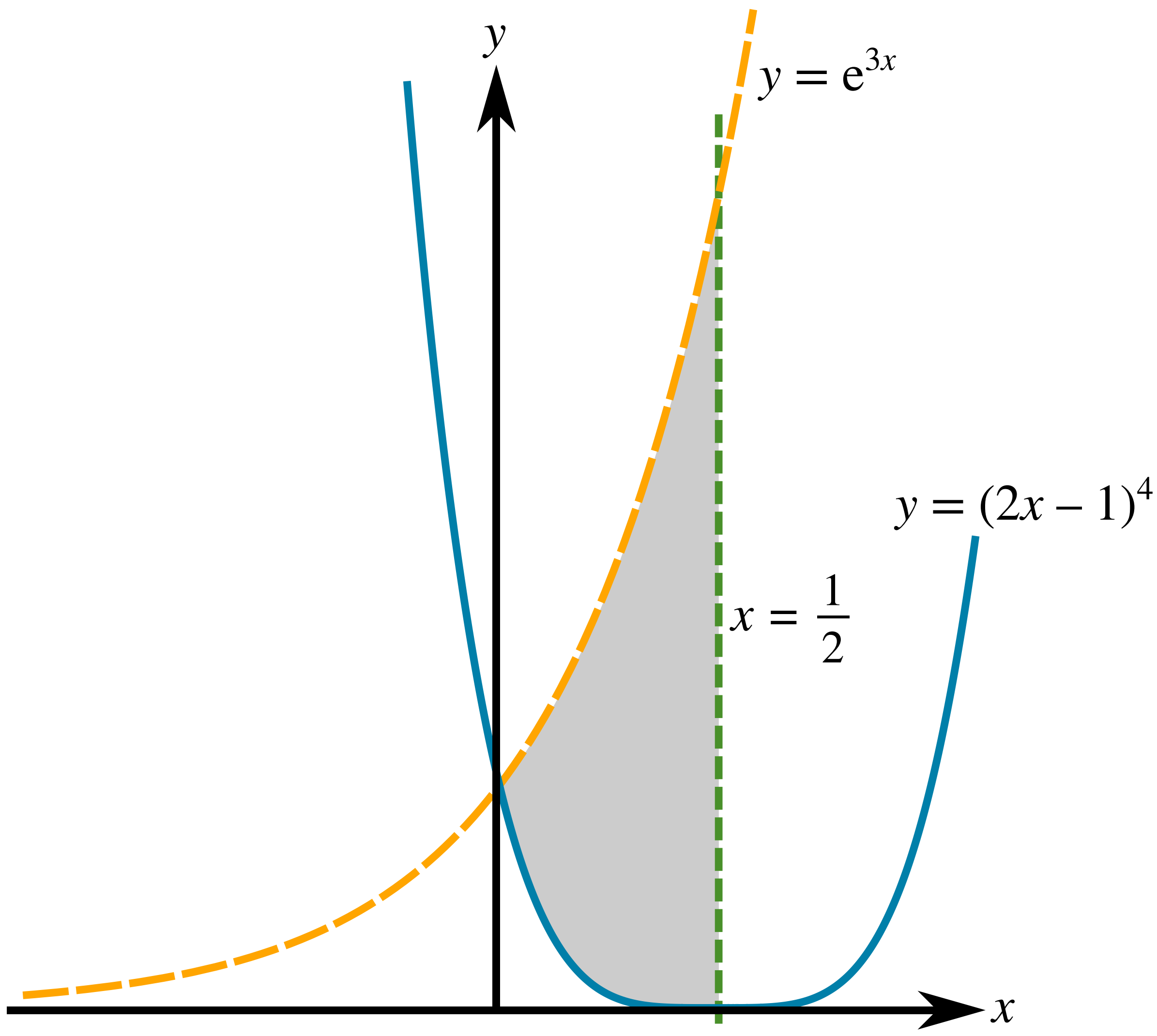

Figure 1: Curves $y = \mathrm{e}^{3x}$ and $y = (2x-1)^4$ and the line $x = \frac{1}{2}$.

Find the exact volume of the solid produced.

The following symbols may be useful: `e`, `pi`

Adapted with permission from UCLES, A Level, June 2008, Paper 4723, Question 6.

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STEM SMART Double Maths 29 - Differential Equations & Volumes of Revolution

Parametric volumes 1

A Level Further A

The parametric equation of a curve C between $x = 0$ and $x = 8$ is given by

$$x = 2t^2 \qquad y = 2 - t$$

The region R is bounded by C, the x-axis and the y-axis, as shown in **Figure 1**.

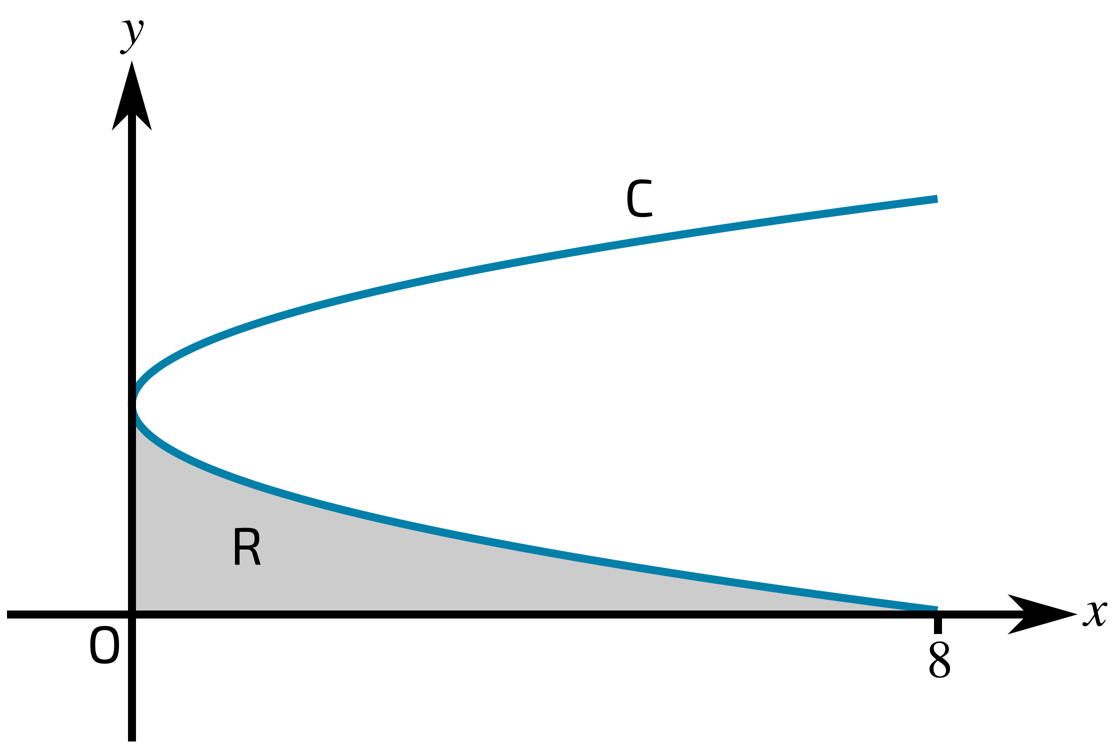

Figure 1: Graph of the curve C, showing the region R.

Part A Expression for R about x-axis

Find an expression for the volume of the solid of revolution formed when R is rotated completely about the x-axis. The required integral can be written in the form

$$\int f(t)\,\mathrm{d}t.$$

Give an expression for $f(t)$.

The following symbols may be useful: `f`, `pi`, `t`

Part B Volume for R about x-axis

Using your integral from part A, with appropriate limits, find the volume of the solid of revolution that is created when R is rotated completely about the x-axis, giving your answer in exact form.

The following symbols may be useful: `pi`

Part C Expression for R about y-axis

Find an expression for the volume of the solid of revolution formed when R is instead rotated completely about the y-axis. The required integral can be written in the form

$$\int g(t)\,\mathrm{d}t.$$

Give the expression for the function $g(t)$.

The following symbols may be useful: `g`, `pi`, `t`

Part D Volume for R about y-axis

Using your integral from part C, with appropriate limits, find the volume of the solid of revolution that is created when R is rotated completely about the y-axis, giving your answer in exact form.

The following symbols may be useful: `pi`

Part E   Volume for S about x-axis

The region S is bounded by C and the line $x = 8$, as shown in **Figure 2**.

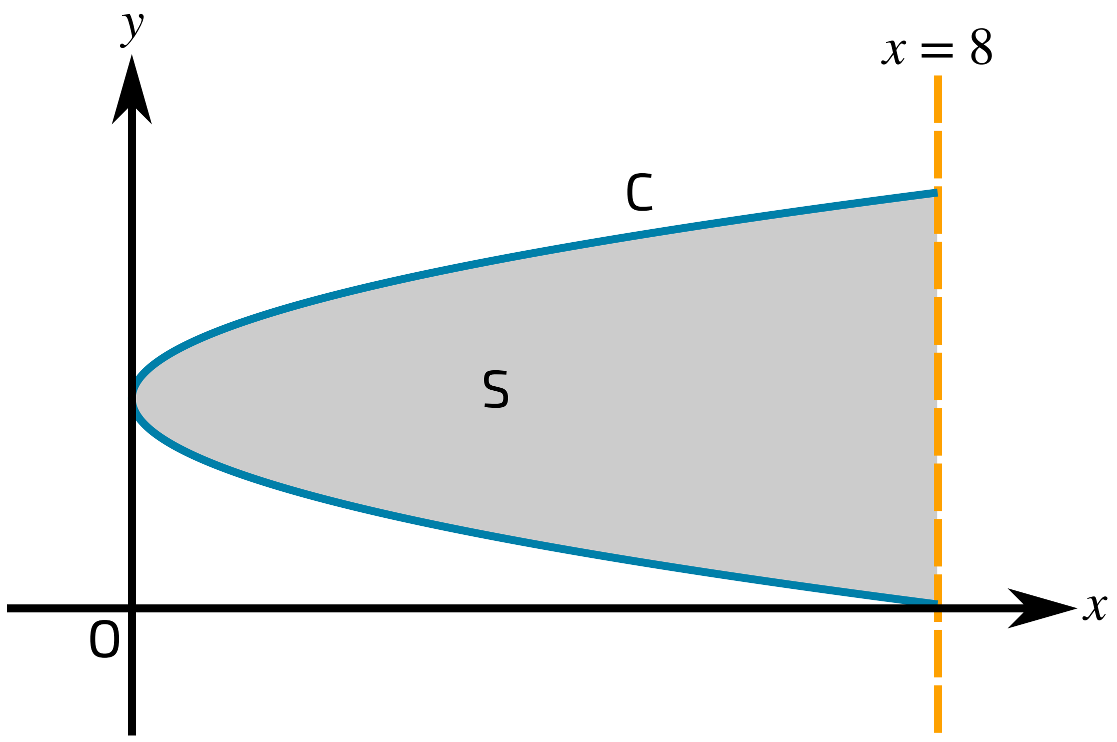

Figure 2: Graph of the curve C, showing the region S.

Find the volume of the solid of revolution formed when S is rotated completely about the x-axis, giving your answer in exact form.

The following symbols may be useful: `pi`

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STEM SMART Double Maths 29 - Differential Equations & Volumes of Revolution

Parametric volumes 3

A Level Further A

The parametric equation of an ellipse, C, with major axis along the x-axis of length $2a$ and minor axis along the y-axis of length $2b$, is given by

$$x = a\cos\theta \qquad y = b\sin\theta$$

where $0 \leq \theta < 2\pi$.

The region R is formed from the positive quadrant of the ellipse, i.e. the region $0 \leq \theta < \frac{\pi}{2}$, as shown in **Figure 1**. Find the volumes of revolution of this region about the x- and y-axes. Hence deduce the volumes of prolate spheroids (formed by rotation about the major of the ellipse) and oblate spheroids (formed by rotation about the minor axis of the ellipse).

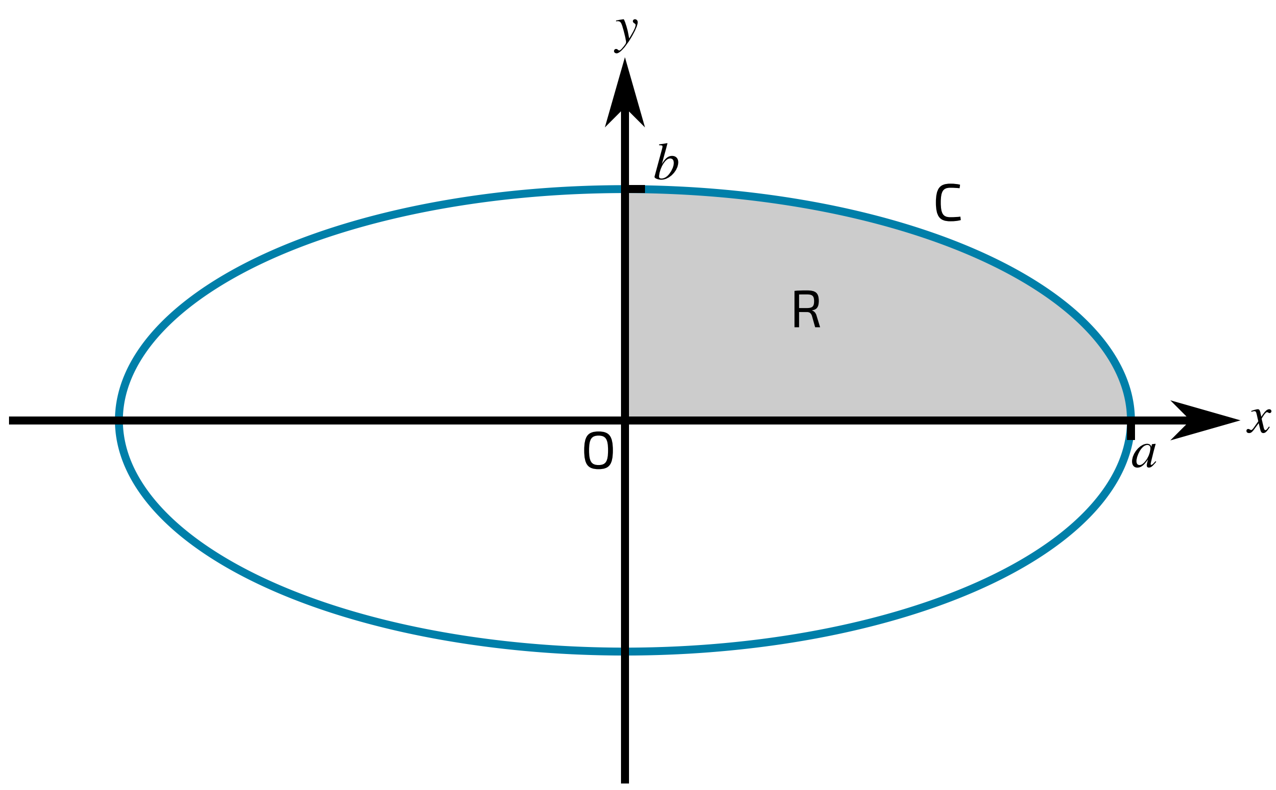

Figure 1: Graph of the curve C, showing the region R, bounded by C, the positive x-axis and the positive y-axis.

Part A Expression for R about x-axis

Find an expression for the volume of the solid of revolution formed when R is rotated completely about the x-axis. The required integral can be written in the form

$$\int f(\theta)\mathrm{d}\theta.$$

Give the expression for $f(\theta)$.

The following symbols may be useful: `a, b, cos(), f, pi, sin(), tan(), theta`

Part B Volume for R about x-axis

Using your integral from part A, with appropriate limits, find the volume of the solid of revolution that is created when R is rotated completely about the x-axis, giving your answer in exact form.

The following symbols may be useful: `a, b, pi`

Part C Volume of a prolate spheroid

Hence deduce the volume of a prolate spheroid formed from C.

The following symbols may be useful: `a, b, pi`

Part D Expression for R about y-axis

Find an expression for the volume of the solid of revolution formed when R is rotated completely about the y-axis. The required integral can be written in the form

$$\int g(\theta)\mathrm{d}\theta.$$

Give the expression for $g(\theta)$.

The following symbols may be useful: `a, b, cos(), g, pi, sin(), tan(), theta`

Part E Volume for R about y-axis

Using your integral from part D, with appropriate limits, find the volume of the solid of revolution that is created when R is rotated completely about the y-axis, giving your answer in exact form.

The following symbols may be useful: `a`, `b`, `pi`

Part F Volume of an oblate spheroid

Hence deduce the volume of an oblate spheroid formed from C.

The following symbols may be useful: `a`, `b`, `pi`

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