Isaac Physics Skills

Linking concepts in pre-university physics

Lisa Jardine-Wright, Keith Dalby, Robin Hughes, Nicki Humphry-Baker,
Anton Machacek, Ingrid Murray and Lee Phillips
Isaac Physics Project

Periphyseos Press
Cambridge, UK.

TABLE OF PHYSICAL CONSTANTS

Quantity & Symbol		Magnitude	Unit
Permittivity of free space	ϵ_0	8.85×10^{-12}	F m^{-1}
Electrostatic force constant	$1/4\pi\epsilon_0$	8.99×10^{9}	N m^2 C^{-2}
Speed of light in vacuum	c	3.00×10^{8}	m s^{-1}
Specific heat capacity of water	c_{water}	4180	J kg^{-1} K^{-1}
Charge of proton	e	1.60×10^{-19}	C
Gravitational field strength on Earth	g	9.81	N kg^{-1}
Universal gravitational constant	G	6.67×10^{-11}	N m^2 kg^{-2}
Planck constant	h	6.63×10^{-34}	J s
Boltzmann constant	k_{B}	1.38×10^{-23}	J K^{-1}
Mass of electron	m_{e}	9.11×10^{-31}	kg
Mass of neutron	m_{n}	1.67×10^{-27}	kg
Mass of proton	m_{p}	1.67×10^{-27}	kg
Mass of Earth	M_{Earth}	5.97×10^{24}	kg
Mass of Sun	M_{Sun}	2.00×10^{30}	kg
Avogadro constant	N_{A}	6.02×10^{23}	mol^{-1}
Gas constant	R	8.31	J mol^{-1} K^{-1}
Radius of Earth	R_{Earth}	6.37×10^{6}	m

OTHER INFORMATION YOU MAY FIND USEFUL

Electron volt	1 eV	=	1.60×10^{-19} J
Unified mass unit	1 u	=	1.66×10^{-27} kg
Absolute zero	0 K	=	-273 °C
Year	1 yr	=	3.16×10^{7} s
Light year	1 ly	=	9.46×10^{15} m
Parsec	1 pc	=	3.09×10^{16} m

PREFIXES

1 km = 1000 m	1 Mm = 10^{6} m	1 Gm = 10^{9} m	1 Tm = 10^{12} m
1 mm = 0.001 m	1 μm = 10^{-6} m	1 nm = 10^{-9} m	1 pm = 10^{-12} m

26 Orbits

An orbit is the path that an object follows in a gravitational or electromagnetic field. This includes the paths of the planets around the Sun.

Example context: The planets of the solar system orbit around the Sun due to the gravitational force of attraction between the planets and the Sun. To accelerate a particle in orbit in a particle accelerator the magnetic field strength must be increased so that the radius of the particles orbit remains the same and the particles do not collide with the walls of the accelerator.

Quantities:

- G Newton's gravitational constant $\left(\mathrm{N\,m^2\,kg^{-2}}\right)$
- g gravitational field strength $\left(\mathrm{N\,kg^{-1}}\right)$
- E electric field strength $\left(\mathrm{N\,C^{-1}}\right)$
- B magnetic flux density (T)
- a centripetal acceleration $\left(\mathrm{m\,s^{-2}}\right)$
- ϵ_0 permittivity of free space $\left(\mathrm{F\,m^{-1}}\right)$
- q, Q charge (C)
- r radius of orbit (m)
- F centripetal force (N)
- T orbital period (s)
- m, M mass (kg)
- v velocity $\left(\mathrm{m\,s^{-1}}\right)$

Equations:

$$g = \frac{GM}{r^2} \qquad E = \frac{Q}{4\pi\epsilon_0 r^2} \qquad a = \frac{v^2}{r} \qquad F = ma \qquad v = \frac{2\pi r}{T}$$

$$F = mg \qquad F = qE \qquad F = qvB \qquad r^3 \propto T^2$$

26.1 A moon of mass m moves at speed v in a circular orbit around a planet of mass M

a) Use the equations above to obtain v in terms of G, M and r.

b) Use the equations above to derive Kepler's Third Law: $r^3 \propto T^2$,

c) What is the constant of proportionality r^3/T^2 in terms of G and M?

26.2 A positron of charge $+q$ and mass m enters a magnetic field B travelling at a speed v perpendicular to the direction of the magnetic field.

a) Derive an expression for r in terms of q, B, m and v.

b) If we now change the particle from a positron to a proton, keeping the magnetic field and the velocity of the particle the same, what would happen?

26.3 Calculate the radius of the Moon's orbit around the Earth given that Moon takes approximately 27 days to orbit the Earth and the mass of the Earth is 6.0×10^{24} kg.

26.4 Astronauts on the International Space Station appear weightless because both they and the space station have the same centripetal acceleration and therefore there is no contact force between the astronauts and the floor of the space station. They are in free-fall. What is the centripetal acceleration of the international space station in orbit at a height $h = 400$ km above the surface of the Earth?

Example – *Venus takes* 225 *Earth days to orbit the Sun at an average distance of* 1.08×10^8 *km. What is the mass of the Sun according to this data?*

$$r^3 = \frac{GM}{4\pi^2} T^2 \text{ therefore } M = \frac{4\pi^2 r^3}{GT^2}$$

$$M = \frac{4\pi^2 (1.08 \times 10^{11})^3}{6.67 \times 10^{-11}\ (225 \times 24 \times 3600)^2} \approx 1.97 \times 10^{30} \text{ kg}$$

26.5 Calculate the orbital period of Jupiter in units of Earth years given that the mass of the Sun, $M = 2.0 \times 10^{30}$ kg, the mass of Jupiter, $m = 1.9 \times 10^{27}$ kg and the average radius of Jupiter's orbit around the sun is $R = 7.8 \times 10^8$ km.

26.6 Calculate the ratio of the radii of the orbits of Phobos and Deimos, which are the moons of Mars. The mass of Mars is $M = 6.4 \times 10^{23}$ kg, the mass of Phobos $m_1 = 11 \times 10^{15}$ kg and the mass of Deimos $m_2 = 1.5 \times 10^{15}$ kg. The period of Phobos's orbit is $T_1 = 7.7$ hours and of Deimos's orbit is $T_2 = 30.4$ hours.

26.7 61 Cygni is a wide binary star system. It contains two stars of nearly equal mass which orbit once around their mid point every 659 years. They are 1.26×10^{13} m apart. Assuming that the two stars have equal mass, calculate

a) the speed of the stars,

b) the total mass of the system.

26.8 Find an expression for the the ratio of the gravitational field to the electric field, g/E, for an electron that is in orbit at a radius r around the central proton of a hydrogen atom.

26.9 In a particle accelerator protons are accelerated in the $+x$-direction until they have a velocity of $v = 6.5 \times 10^6$ m s^{-1}. They then pass into a magnetic field of strength 0.1 T that is oriented in the $+y$-direction.

a) In which direction do the protons accelerate when they first enter the magnetic field?

b) What is the radius of the orbital path that the protons take?

24 Energy and fields – relativistic accelerator

(a) $\gamma = \frac{E}{mc^2} = \frac{K + mc^2}{mc^2} = \frac{K}{mc^2} + 1 = \frac{qV}{mc^2} + 1$

(b) $\gamma^{-2} = 1 - \frac{v^2}{c^2}$ so $\frac{v}{c} = \sqrt{1 - \gamma^{-2}}$ and $v = c\sqrt{1 - \gamma^{-2}}$

(c) $v = c\sqrt{1 - \gamma^{-2}} = c\sqrt{1 - \left(1 + \frac{qV}{mc^2}\right)^{-2}}$

(d) $$p^2 = \gamma^2 m^2 v^2 = \frac{m^2c^2\left(v^2/c^2\right)}{1 - v^2/c^2} = \frac{m^2c^2\left(v^2/c^2 - 1 + 1\right)}{1 - v^2/c^2}$$
$$= -m^2c^2 + \frac{m^2c^2}{1 - v^2/c^2} = -m^2c^2 + \gamma^2 m^2 c^2$$
therefore $p^2c^2 = -m^2c^4 + \gamma^2 m^2 c^4 = E^2 - m^2c^4$

(e) $$p^2 = \frac{E^2}{c^2} - m^2c^2 = \frac{\left(K + mc^2\right)^2 - m^2c^4}{c^2} = \frac{K^2 + 2Kmc^2}{c^2}$$
$$= \frac{K^2}{c^2} + 2Km = \frac{q^2V^2}{c^2} + 2qVm$$

25 Energy and fields – closest approach

(a) $U = qV = \frac{Qq}{4\pi\epsilon_0 r}$, so $r = \frac{Qq}{4\pi\epsilon_0 U}$

(b) $r = \frac{Qq}{4\pi\epsilon_0 U}$ and $U = \frac{mv^2}{2}$, so $r = \frac{Qq}{4\pi\epsilon_0}\frac{2}{mv^2} = \frac{Qq}{2\pi\epsilon_0 mv^2}$

(c) $r = \frac{Qq}{4\pi\epsilon_0 U}$ and $U = \frac{3k_B T}{2}$, so $r = \frac{Qq}{4\pi\epsilon_0}\frac{2}{3k_B T} = \frac{Qq}{6\pi\epsilon_0 k_B T}$

26 Orbits

(1a) Newton's Second Law: $m\frac{v^2}{r} = \frac{GMm}{r^2}$ so $v^2 = \frac{GM}{r}$

(1b) From (a) $v^2 = \dfrac{GM}{r}$ We also know $v = \dfrac{2\pi r}{T}$ so $v^2 = \dfrac{4\pi^2 r^2}{T^2}$

Therefore $\dfrac{GM}{r} = \dfrac{4\pi^2 r^2}{T^2}$ and so $4\pi^2 r^3 = GMT^2$

(1c) From (b) $\dfrac{r^3}{T^2} = \dfrac{GM}{4\pi^2}$

(2a) Newton's Second Law: $m\dfrac{v^2}{r} = qvB$ so $r = \dfrac{mv}{Bq}$

27 Vectors and fields – between a planet and a moon

(a) $F_\mathsf{M} = mg_\mathsf{M} = +\dfrac{GM_\mathsf{M}m}{r_\mathsf{M}^2}$

(b) $F_\mathsf{P} = mg_\mathsf{P} = -\dfrac{GM_\mathsf{P}m}{r_\mathsf{P}^2}$

(c) $F = F_\mathsf{M} + F_\mathsf{P} = +\dfrac{GM_\mathsf{M}m}{r_\mathsf{M}^2} - \dfrac{GM_\mathsf{P}m}{r_\mathsf{P}^2}$

(d) $g = \dfrac{F}{m} = +\dfrac{GM_\mathsf{M}}{r_\mathsf{M}^2} - \dfrac{GM_\mathsf{P}}{r_\mathsf{P}^2}$

(e) $g = 0$ so $\dfrac{GM_\mathsf{M}}{r_\mathsf{M}^2} = \dfrac{GM_\mathsf{P}}{r_\mathsf{P}^2}$ therefore $\dfrac{M_\mathsf{M}}{r_\mathsf{M}^2} = \dfrac{M_\mathsf{P}}{r_\mathsf{P}^2}$ and $\dfrac{r_\mathsf{P}}{r_\mathsf{M}} = \sqrt{\dfrac{M_\mathsf{P}}{M_\mathsf{M}}}$

28 Vectors and fields – electric deflection

(a) $a_\mathsf{y} = \dfrac{F_\mathsf{E}}{m} = \dfrac{qE}{m} = \dfrac{qV}{dm}$

(b) $s_\mathsf{y} = \dfrac{1}{2}\left(\dfrac{qV}{dm}\right)t^2 = \dfrac{1}{2}\left(\dfrac{qV}{dm}\right)\left(\dfrac{s_\mathsf{x}}{v_\mathsf{x}}\right)^2 = \dfrac{qVs_\mathsf{x}^2}{2dmv_\mathsf{x}^2}$

(c) $\theta = \tan^{-1}\left(\dfrac{v_\mathsf{y}}{v_\mathsf{x}}\right) = \tan^{-1}\left(\dfrac{a_\mathsf{y}t}{v_\mathsf{x}}\right) = \tan^{-1}\left(\dfrac{qVs_\mathsf{x}}{dmv_\mathsf{x}^2}\right)$

(d) $s_\mathsf{y} = \dfrac{1}{2}\left(\dfrac{qE}{m}\right)t^2 = \dfrac{qEt^2}{2m}$

(e) $\theta = \tan^{-1}\left(\dfrac{a_\mathsf{y}t}{v_\mathsf{x}}\right) = \tan^{-1}\left(\dfrac{qEt}{mv_\mathsf{x}}\right)$