Essential GCSE Maths 50.9

GCSE C C C A Level P P P

A landowner has a triangular piece of land. They are planning to build a path along the boundary of the land, and plant trees in the centre. Each tree will need $50\,\mathrm{m}^2$ of land when it is mature. The landowner knows that some trees will not survive to maturity. They plant 30% more trees than the maximum suggested by an area calculation.

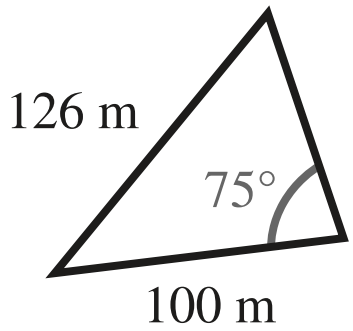

Figure 1: A plan of the piece of land that trees are going to be planted on.

Part A How long is the path?

How long is the path? Give your answers to 3 s.f..

Part B How many trees will be planted

Assuming that the landowner plants as many trees as possible, how many trees will be planted? Round your answer to the nearest whole tree.

Sine and Cosine Rules and Area 1i

A Level

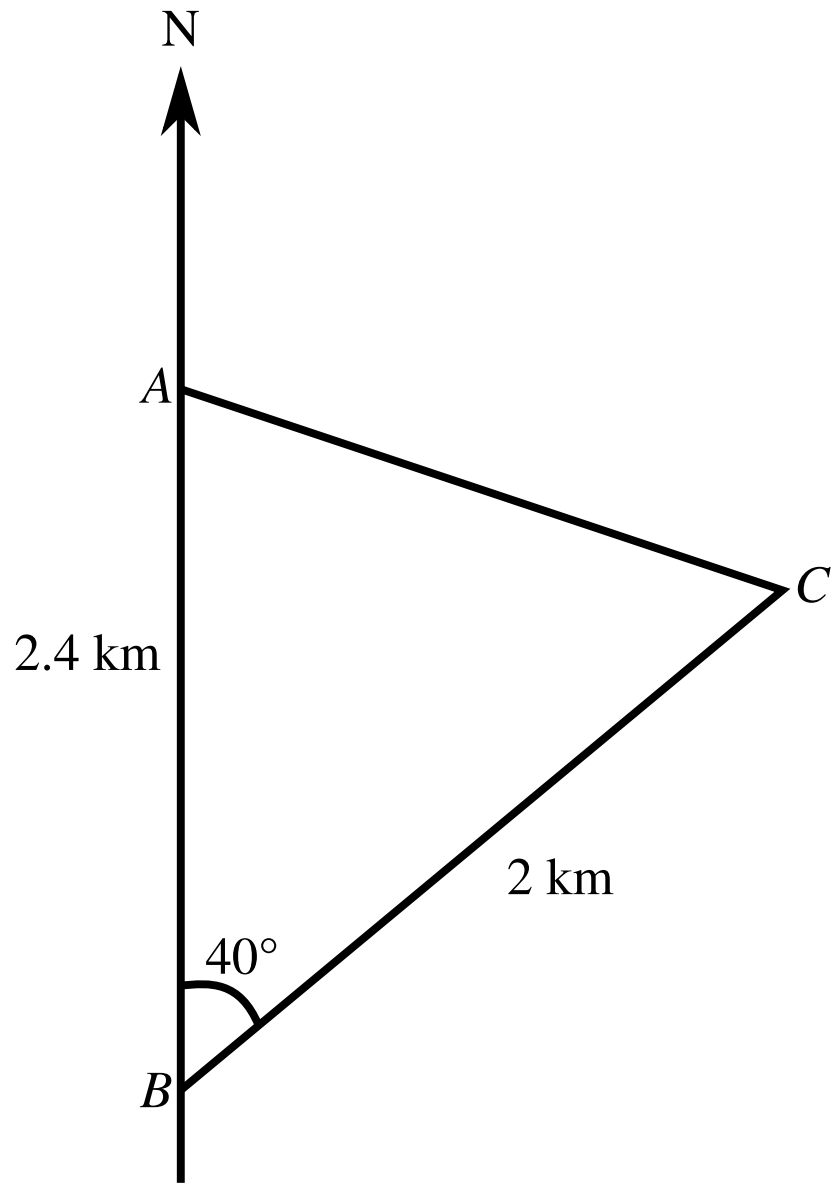

Figure 1: Positions of two points on a coastline, and a nearby ship.

Figure 1 shows two points A and B on a straight coastline, with A being $2.4\,\mathrm{km}$ due north of B. A stationary ship is at a point C, on a bearing of 040° and at a distance of $2\,\mathrm{km}$ from B.

Part A Find AC

Find the distance AC in kilometres, giving your answer correct to three significant figures.

Part B Find θ

The bearing of C from A is $\theta\,^\circ$. Find the value of θ correct to three significant figures.

Part C Shortest distance

Find the shortest distance from the ship to the coastline, giving your answer in kilometres correct to three significant figures.

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STEM SMART Double Maths 3 - Trigonometry

Trigonometry: Basic Functions 2i

A Level

Part A Trigonometric functions 1

Given that α is the acute angle such that $\tan\alpha = \frac{2}{5}$, find the exact value of $\cos\alpha$.

- $\frac{5\sqrt{29}}{29}$
- $\frac{5\sqrt{2}}{8}$
- $\frac{5}{29}$
- $\frac{\sqrt{2}}{2}$

Part B Trigonometric functions 2

Given that β is the obtuse angle such that $\sin\beta = \frac{3}{7}$, find the exact value of $\cos\beta$.

- $-\frac{2\sqrt{10}}{3}$
- $-3\sqrt{10}$
- $\frac{2\sqrt{10}}{5}$
- $-\frac{\sqrt{40}}{7}$

Part C A triangle

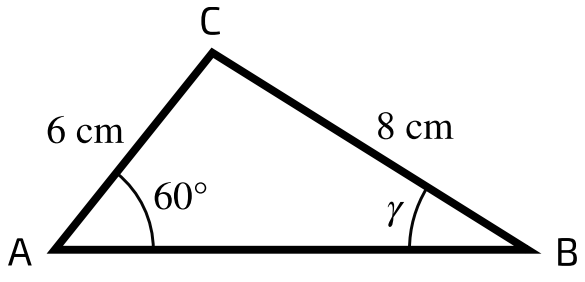

Figure 1: Triangle ABC.

Figure 1 shows a triangle ABC with AC$= 6\,\mathrm{cm}$, BC$= 8\,\mathrm{cm}$, angle BAC$= 60^\circ$ and angle ABC$= \gamma$.

Find the exact value of $\sin\gamma$, simplifying your answer.

- $\frac{3}{\sqrt{5}}$
- $\frac{2\sqrt{5}}{3}$
- $\frac{3\sqrt{3}}{8}$
- $2\sqrt{3}$

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STEM SMART Double Maths 3 - Trigonometry

Trigonometry: Basic Functions 1i

A Level

P P P

Part A Sketch a trig function

Sketch the graph of $y = 2\cos x$ for values of x such that $0° \leq x \leq 360°$, indicating the coordinates of any points where the curve meets the axes. You can check your sketch after giving the correct answer.

Give the value of the smallest root in degrees.

Part B A trig equation

Solve the equation $2\cos x = 0.8$, giving the highest value of x between $0°$ and $360°$ to 3 significant figures.

Part C Equating trig functions

Solve the equation $2\cos x = \sin x$, giving the value of x between $-180°$ and $180°$ that has the largest negative value.

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STEM SMART Double Maths 3 - Trigonometry

Trigonometry: Basic Functions 1ii

A Level

Part A $\sin\left(\frac{1}{2}x\right) = 0.8$

Solve $\sin\left(\frac{1}{2}x\right) = 0.8$, for $0° \leq x \leq 360°$.

What is the lowest (smallest) solution? Give your answer in degrees, to 3 significant figures.

What is the highest (largest) solution? Give your answer in degrees, to 3 significant figures.

Part B $\sin x = 3\cos x$

Solve $\sin x = 3\cos x$, for $0° \leq x \leq 360°$.

What is the lowest (smallest) solution? Give your answer in degrees, to 3 significant figures.

What is the highest (largest) solution? Give your answer in degrees, to 3 significant figures.

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STEM SMART Double Maths 3 - Trigonometry

Trigonometry: Identities and Equations 3i

A Level

Part A Quadratic equation

Write $15\cos^2\theta = 13 + \sin\theta$ as a quadratic equation in $\sin\theta$.

The following symbols may be useful: `cos(), sin(), tan(), theta`

Part B Solve equation

Solve the equation $15\cos^2\theta = 13 + \sin\theta$ giving the second largest value in the range $0^\circ \leqslant \theta \leqslant 360^\circ$, in degrees to 4 significant figures.

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STEM SMART Double Maths 3 - Trigonometry

Addition of Angles 1

A Level Further A

Without using a calculator, find exact expressions for:

Part A $\sin 15^\circ$

$\sin 15^\circ$

Part B $\cos 165^\circ$

$\cos 165^\circ$

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Trigonometry: Double Angles 1i

A Level

It is given that $f(\theta) = \sin\left(\theta + 30^\circ\right) + \cos\left(\theta + 60^\circ\right)$.

Part A Double Angles

Show that $f(\theta) = \cos\theta$. Hence find an expression for $f(4\theta) + 4f(2\theta)$, in terms of $\cos\theta$.

The following symbols may be useful: `theta`

Part B $\frac{1}{f(4\theta)+4f(2\theta)+7}$

Hence determine the greatest and least values of $\dfrac{1}{f(4\theta) + 4f(2\theta) + 7}$ as θ varies.

Give the greatest value.

Give the smallest value.

Part C Solve

Solve the equation

$$\sin\left(12\alpha + 30^\circ\right) + \cos\left(12\alpha + 60^\circ\right) + 4\sin\left(6\alpha + 30^\circ\right) + 4\cos\left(6\alpha + 60^\circ\right) = 1$$

for $0^\circ < \alpha < 60^\circ$, in degrees, to three significant figures.

Give the smallest solution.

Give the largest solution.

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Trigonometry: Combined Angles 3i

A Level

In **Figure 1**, $ABCD$ represents a rectangular table with sides $3.5\,\mathrm{m}$ and $1.5\,\mathrm{m}$. It has been turned so it wedges in a passage of width $2.5\,\mathrm{m}$.

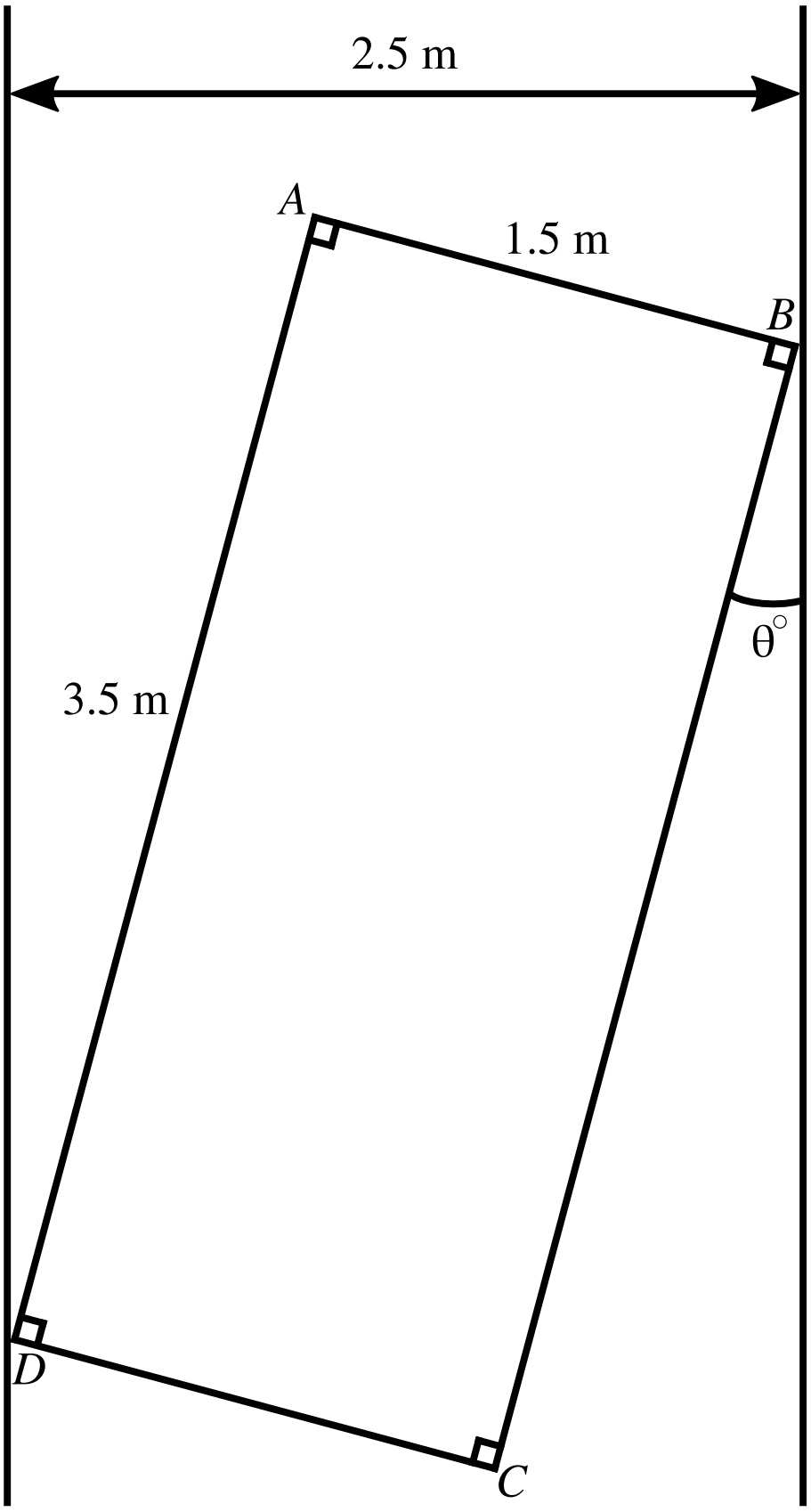

Figure 1: The rectangular table $ABCD$.

Part A Value of $7\sin\theta^\circ + 3\cos\theta^\circ$

Given that θ is the acute angle between the longer side and the passage, as shown in the diagram, find the exact value of $7\sin\theta + 3\cos\theta$.

The following symbols may be useful: `cos(), sin(), tan(), theta`

Part B The form $R\sin\left(\theta^\circ + \alpha^\circ\right)$

Express $7\sin\theta + 3\cos\theta$ in the form $R\sin\left(\theta + \alpha\right)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.

Give the exact value of R.

The following symbols may be useful: `R`

Give the value of α to 3 significant figures.

Part C Find θ

Find θ, to 3 significant figures.

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STEM SMART Double Maths 3 - Trigonometry

Addition of Angles 7

A Level Further A

Two waves

$$\psi_1 = A\cos\left(2\pi ft - \left(\frac{2\pi}{\lambda}\right)x + \phi\right)$$

and

$$\psi_2 = A\cos\left(2\pi ft - \left(\frac{2\pi}{\lambda}\right)x - \phi\right)$$

interfere, such that the resultant wave is given by $\psi = \psi_1 + \psi_2$. Express ψ as the product of two terms.

Express ψ as the product of two terms.

The following symbols may be useful: `A, f, lambda, phi, pi, t, x`

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