Essential GCSE Maths 41.9

The diagram shows a ship near the coast. The ship is at point A, $200\,\mathrm{m}$ from a buoy at B. On the cliff top there is a lighthouse. The tip of the lighthouse (point C) is $y\,\mathrm{m}$ above the level of the sea at D.

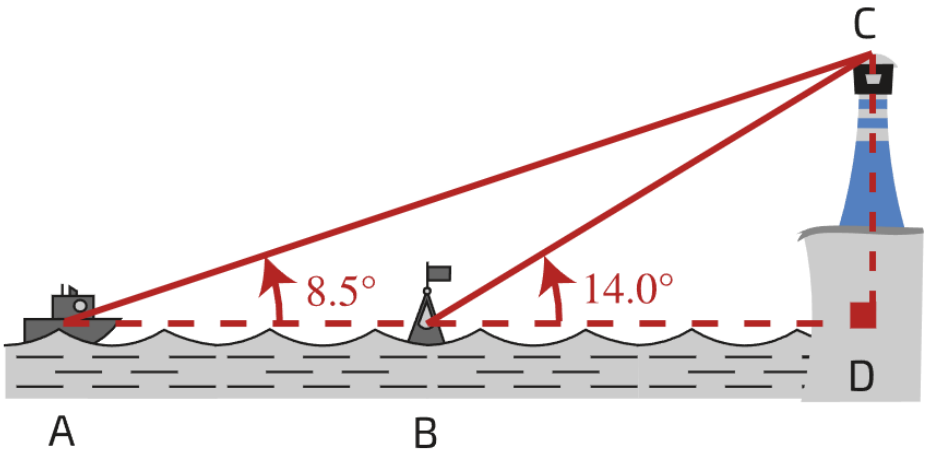

Figure 1: A diagram of the lighthouse, the ship and the buoy.

The angle of elevation of the top of the lighthouse is 8.5° at A and 14.0° at B.

What is the value of y?

Sine and Cosine Rules and Area 1i

A Level

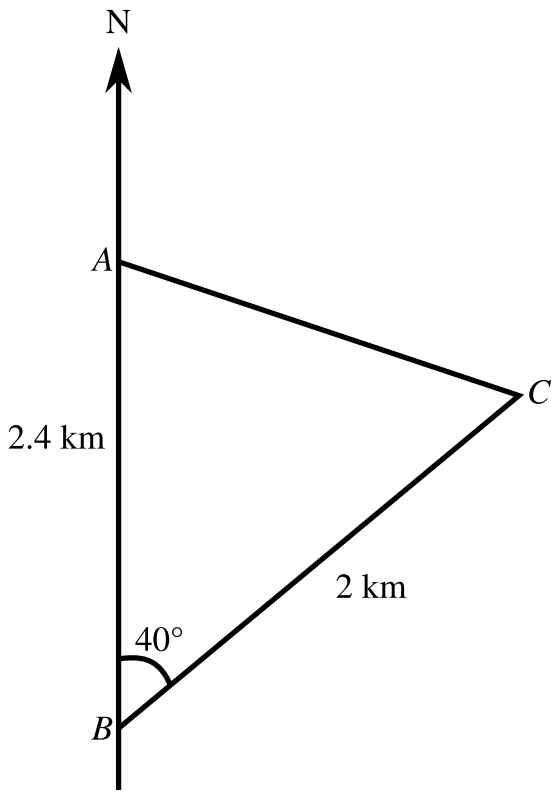

Figure 1: Positions of two points on a coastline, and a nearby ship.

Figure 1 shows two points A and B on a straight coastline, with A being $2.4\,\mathrm{km}$ due north of B. A stationary ship is at a point C, on a bearing of $040\,^\circ$ and at a distance of $2\,\mathrm{km}$ from B.

Part A **Find AC**

Find the distance AC in kilometres, giving your answer correct to three significant figures.

Part B **Find θ**

The bearing of C from A is $\theta\,^\circ$. Find the value of θ correct to three significant figures.

Part C Shortest distance

Find the shortest distance from the ship to the coastline, giving your answer in kilometres correct to three significant figures.

Trigonometry: Basic Functions 1ii

A Level

Part A $\sin(\frac{1}{2}x) = 0.8$

Solve $\sin(\frac{1}{2}x) = 0.8$, for $0^\circ \leqslant x \leqslant 360^\circ$.

What is the lowest (smallest) solution? Give your answer in degrees, to 3 significant figures.

What is the highest (largest) solution? Give your answer in degrees, to 3 significant figures.

Part B $\sin(x) = 3\cos(x)$

Solve $\sin(x) = 3\cos(x)$, for $0^\circ \leqslant x \leqslant 360^\circ$.

What is the lowest (smallest) solution? Give your answer in degrees, to 3 significant figures.

What is the highest (largest) solution? Give your answer in degrees, to 3 significant figures.

Addition of Angles 6

A Level C C C Further A C C C

Show that $X\cos\alpha t + Y\sin\alpha t$ can be expressed as $E\sin(\alpha t + \theta)$, where E and θ are expressions to be found.

Part A Expression for E

Give an expression for E in terms of X and Y.

The following symbols may be useful: `E, X, Y, alpha t`

Part B Expression for θ

Also give an expression for θ in terms of X and Y.

The following symbols may be useful: `X, Y, alpha t, arccos(), arcsin(), arctan(), theta`

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Addition of Angles 7

A Level C C C Further A C C C

Two waves

$$\psi_1 = A\cos\left(2\pi ft - \left(\frac{2\pi}{\lambda}\right)x + \phi\right)$$

and

$$\psi_2 = A\cos\left(2\pi ft - \left(\frac{2\pi}{\lambda}\right)x - \phi\right)$$

interfere, such that the resultant wave is given by $\psi = \psi_1 + \psi_2$. Express ψ as the product of two terms.

Express ψ as the product of two terms.

The following symbols may be useful: `A, f, lambda, phi, pi, t, x`

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Sine and Cosine Rules and Area 3i

A Level

A landmark L is observed by a surveyor from three points A, B and C on a straight horizontal road, where $AB = BC = 200\,\mathrm{m}$. Angles LAB and LBA are $65\,^\circ$ and $80\,^\circ$ respectively (see **Figure 1**).

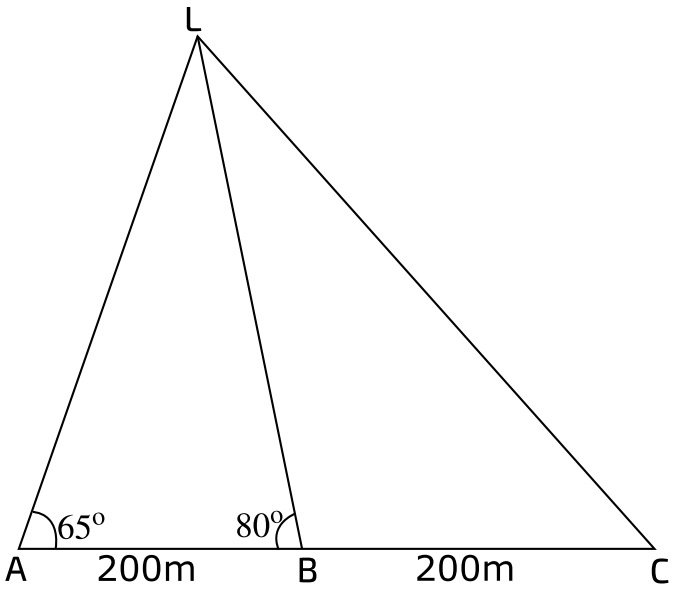

Figure 1: A triangle where $AB = BC$ and B connects to L

Part A Shortest distance

Calculate the shortest distance from L to the road. Give your answer in metres, to the nearest metre.

Part B Distance LC

Calculate the distance LC. Give your answer in metres, to the nearest metre.

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Essential GCSE Maths 50.9

A landowner has a triangular piece of land. They are planning to build a path along the boundary of the land, and plant trees in the centre. Each tree will need $50\,\mathrm{m}^2$ of land when it is mature. The landowner knows that some trees will not survive to maturity. They plant 30% more trees than the maximum suggested by an area calculation.

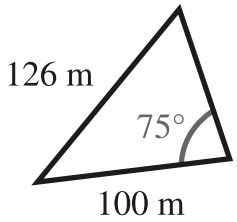

Figure 1: A plan of the piece of land that trees are going to be planted on.

Part A How long is the path?

How long is the path? Give your answers to 3 s.f..

Part B How many trees will be planted

Assuming that the landowner plants as many trees as possible, how many trees will be planted? Round up to the nearest whole tree.

Addition of Angles 1

A Level Further A

Without using a calculator, find exact expressions for:

Part A $\sin 15°$

$\sin 15\,°$

Part B $\cos 165°$

$\cos 165\,°$

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Trigonometry: Basic Functions 2i

A Level

Part A Trigonometric functions 1

Given that α is the acute angle such that $\tan\alpha = \frac{2}{5}$, find the exact value of $\cos\alpha$.

- $\frac{\sqrt{2}}{2}$
- $\frac{5\sqrt{2}}{8}$
- $\frac{5}{29}$
- $\frac{5\sqrt{29}}{29}$

Part B Trigonometric functions 2

Given that β is the obtuse angle such that $\sin\beta = \frac{3}{7}$, find the exact value of $\cos\beta$.

- $-\frac{\sqrt{40}}{7}$
- $\frac{2\sqrt{10}}{5}$
- $-\frac{2\sqrt{10}}{3}$
- $-3\sqrt{10}$

Part C A triangle

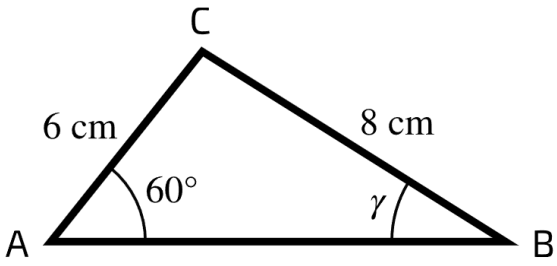

Figure 1: Triangle ABC.

Figure 1 shows a triangle ABC with AC$= 6\,\text{cm}$, BC$= 8\,\text{cm}$, angle BAC$= 60^\circ$ and angle ABC$= \gamma$.

Find the exact value of $\sin\gamma$, simplifying your answer.

- $2\sqrt{3}$
- $\frac{3}{\sqrt{5}}$
- $\frac{3\sqrt{3}}{8}$
- $\frac{2\sqrt{5}}{3}$