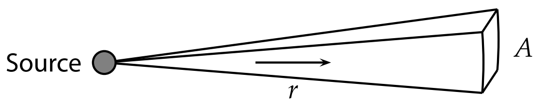
16 Inverse square intensity

It is helpful to be able to calculate the intensity of a wave at different distances as it spreads out from a source.

Example context: the distance to a star can be calculated from a knowledge of its luminosity (power) and its brightness when seen from Earth. We can also work out the exposure to ionising radiation at different distances from a source.

Quantities: P Power (W) A Surface area $\left(\text{m}^2\right)$
I Intensity $\left(\text{W m}^{-2}\right)$ r Distance from source (m)
C Count rate $\left(\text{Bq} = \text{s}^{-1}\right)$
Subscripts label different locations, so I_1 is measured at r_1.



Equations: $A_{\text{sphere}} = 4\pi r^2 \quad I = \dfrac{P}{A}$

16.1 For a source which radiates in all directions, use the equations to derive expressions for

a) the intensity I at a distance r from a source of power P,

b) The distance d at which a source P has intensity I.

c) the intensity I_2 at a distance r_2 from a source if the intensity at distance r_1 is I_1.

> **Example 1** – *The desk in a room is 1.7 m directly below a perfectly efficient light bulb. To read a 0.0625 m^2 (A4) sheet of paper comfortably, at least 27 mW of light must hit it. Calculate the minimum power of light bulb required.*
> Intensity required $I = \dfrac{27 \times 10^{-3}\text{ W}}{0.0625\text{ m}^2} = 0.432\text{ W m}^{-2}$.
> If light spreads out 1.7 m in all directions, it will illuminate a spherical shape of area $A_{\text{sphere}} = 4\pi \times (1.7\text{ m})^2 = 36.3\text{ m}^2$.
> The power needed is $P = IA_{\text{sphere}} = 0.432 \times 36.3 = 16$ W.

16.2 Calculate the intensity if 130 W of light from a spotlight hits a 4.0 m $\times$ 3.0 m region of a stage.

16.3 The rear light on a bicycle gives off 0.15 W of light in all backwards directions. Calculate the intensity of light 23 m behind the lamp.

16.4 Someone can see a lamp providing the intensity is larger than 2.5×10^{-7} W m^{-2}. How far away could this person see a 500 W warning lamp which gives off light in all directions?

16.5 When sunlight shines perpendicularly on a 0.6 m $\times$ 1.2 m solar panel, 195 W of electricity is generated. This is 20% of the total radiation incident on it.

a) Calculate the intensity of the sunlight at the panel.

b) The Earth is 1.50×10^{11} m from the Sun. Calculate the Sun's power.

c) How close to your eye would you need to hold a 150 W light bulb for it to appear as bright as the Sun? Assume that the Sun and the light bulb make visible light with the same efficiency.

Example 2 – *A gamma source is* 12.3 *cm from a detector, which records* 942 *background-corrected counts in* 30.0 *s. How many counts would you expect from the same detector in* 40.0 *s at a distance of* 16.8 *cm?*
The count rate C (in Bq) will be proportional to I at each distance.
Count rate at 12.3 cm $= \dfrac{942}{30} = 31.4$ Bq. Cr^2 is the same at all distances (it is proportional to P), so 31.4 Bq $\times$ $(12.3\text{ cm})^2 = C \times (16.8\text{ cm})^2$
$C = \dfrac{31.4 \times 12.3^2}{16.8^2} = 16.8$ Bq. Counts expected in 40 s $= 16.8 \times 40 = 673$.

16.6 When dentists take X-rays, they stand by the door, or outside the room. Calculate the intensity at 3.5 m from the source as a fraction of the intensity 0.32 m from it.

16.7 The background count in a laboratory is 36 counts in 40 s. When a gamma source is placed 1.5 m from the detector, there are 236 counts each minute.

a) Calculate the background-corrected count rate in Bq.

b) Calculate the expected background-corrected count rate 15 cm from the source.

16.8 On a very dark night, an astronomer can see a 5.3×10^{28} W star with their unaided eye providing the intensity is larger than 1.8×10^{-10} W m^{-2}.

a) How far away would the star be if it is only just visible?

b) What is the minimum visible intensity with a telescope of diameter 7.5 cm rather than an eye with a pupil diameter of 0.75 cm.

c) Another star of the same power is just visible using the telescope. How far away is it?

15 Standing waves on a string

(a) $$f_1 = \frac{c}{\lambda} = \frac{\sqrt{\frac{T}{\mu}}}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$$

(b) $$f_1 = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

(c) $$f_n = \frac{c}{\lambda} = \frac{\sqrt{\frac{T}{\mu}}}{\frac{2\ell}{n}} = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$$

16 Inverse square intensity

(a) Area illuminated is $A_{\text{sphere}} = 4\pi r^2$, so $I = \dfrac{P}{A_{\text{sphere}}} = \dfrac{P}{4\pi r^2}$

(b) Using (a) $I = \dfrac{P}{4\pi r^2}$ so $r^2 = \dfrac{P}{4\pi I}$ and $r = \sqrt{\dfrac{P}{4\pi I}}$

(c) $P = I_1 A_1 = I_2 A_2$, so $I_1 \times 4\pi r_1^2 = I_2 \times 4\pi r_2^2$, so $I_2 = \dfrac{I_1\, r_1^2}{r_2^2}$

In (c) we assumed that the radiation spread equally in all directions (so $A = 4\pi r^2$). The reasoning is also true for radiation which spreads in all **relevant** directions. In this case, P will not be the power of the source, but the power of a source which could shine this brightly in all directions.

17 Banked tracks for turning

(a) Resolving vertically, $N\cos\theta = mg$, so $N = \dfrac{mg}{\cos\theta}$

(b) $\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{mv^2/r}{mg} = \dfrac{v^2}{rg}$. So, $v = \sqrt{rg\tan\theta}$

(c) $$t_{\text{p}} = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{rg\tan\theta}} = 2\pi\sqrt{\frac{r}{g\tan\theta}}$$