Isaac Physics Skills

Linking concepts in pre-university physics

Lisa Jardine-Wright, Keith Dalby, Robin Hughes, Nicki Humphry-Baker,
Anton Machacek, Ingrid Murray and Lee Phillips
Isaac Physics Project

Periphyseos Press
Cambridge, UK.

TABLE OF PHYSICAL CONSTANTS

Quantity & Symbol		**Magnitude**	**Unit**
Permittivity of free space	ϵ_0	8.85×10^{-12}	$\mathrm{F\ m^{-1}}$
Electrostatic force constant	$1/4\pi\epsilon_0$	8.99×10^{9}	$\mathrm{N\ m^2\ C^{-2}}$
Speed of light in vacuum	c	3.00×10^{8}	$\mathrm{m\ s^{-1}}$
Specific heat capacity of water	c_{water}	4180	$\mathrm{J\ kg^{-1}\ K^{-1}}$
Charge of proton	e	1.60×10^{-19}	C
Gravitational field strength on Earth	g	9.81	$\mathrm{N\ kg^{-1}}$
Universal gravitational constant	G	6.67×10^{-11}	$\mathrm{N\ m^2\ kg^{-2}}$
Planck constant	h	6.63×10^{-34}	J s
Boltzmann constant	k_{B}	1.38×10^{-23}	$\mathrm{J\ K^{-1}}$
Mass of electron	m_{e}	9.11×10^{-31}	kg
Mass of neutron	m_{n}	1.67×10^{-27}	kg
Mass of proton	m_{p}	1.67×10^{-27}	kg
Mass of Earth	M_{Earth}	5.97×10^{24}	kg
Mass of Sun	M_{Sun}	2.00×10^{30}	kg
Avogadro constant	N_{A}	6.02×10^{23}	$\mathrm{mol^{-1}}$
Gas constant	R	8.31	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Radius of Earth	R_{Earth}	6.37×10^{6}	m

OTHER INFORMATION YOU MAY FIND USEFUL

Electron volt	1 eV	=	1.60×10^{-19} J
Unified mass unit	1 u	=	1.66×10^{-27} kg
Absolute zero	0 K	=	-273 °C
Year	1 yr	=	3.16×10^{7} s
Light year	1 ly	=	9.46×10^{15} m
Parsec	1 pc	=	3.09×10^{16} m

PREFIXES

1 km = 1000 m	1 Mm = 10^{6} m	1 Gm = 10^{9} m	1 Tm = 10^{12} m
1 mm = 0.001 m	1 μm = 10^{-6} m	1 nm = 10^{-9} m	1 pm = 10^{-12} m

4 Elastic collisions

An elastic collision is one where the total kinetic energy is the same before and after the collision. Momentum is also conserved (as in all collisions). Solving these questions needs energy and momentum formulae.

Example context: many collisions of subatomic particles are elastic, especially if the speeds aren't high enough to trigger reactions. Collisions between snooker balls are also almost elastic.

Before collision After collision

Quantities: p, P momentum $\left(\text{kg m s}^{-1}\right)$ k, K kinetic energy (J)
v, V velocity $\left(\text{m s}^{-1}\right)$ m, M mass (kg)

Equations: $p = mv \quad k = \frac{1}{2}mv^2 \quad P = MV \quad K = \frac{1}{2}MV^2$
$p_0 + P_0 = p_1 + P_1 \qquad k_0 + K_0 = k_1 + K_1$

4.1 Use the equations to derive expressions for

a) the final velocity V_1 of M if M was stationary at the beginning and the initial and final velocities of m (v_0 and v_1) are known,

b) V_1 if the masses are equal $(M = m)$, M begins at rest $(V_0 = 0)$, m is stopped by the collision $(v_1 = 0)$ and v_0 is known,

c) (optional) $k + K$ in terms of $p + P$, M, m and the relative velocity $r = v - V$. Hint: use $2\,(m + M)$ as a denominator for $k + K$, and then look for terms adding to give $(p + P)^2$ on the top.

Example 1 – *A 1 kg trolley moving at 1.2 m s^{-1} strikes a stationary 2 kg trolley, which then moves at 0.8 m s^{-1}. Calculate the final velocity of the 1 kg trolley.*

$$mv_0 + MV_0 = mv_1 + MV_1 \text{ (conservation of momentum)}$$
$$1 \times 1.2 + 2 \times 0 = 1 \times v_1 + 2 \times 0.8$$
$$1.2 - 1.6 = v_1 \text{ and hence } v_1 = -0.4\ \text{m s}^{-1}$$

4.2 Calculate the kinetic energy lost by the 1 kg trolley in Example 1.

4.3 Calculate the final speed of the 2 kg trolley in Example 1 assuming that it gains all of the kinetic energy lost by the 1 kg trolley.

4.4 Fill in the missing entries in the table below. For these collisions $v_0 \neq v_1$.

m	M	v_0	V_0	v_1	V_1	$K+k$	$K_1 - K_0$
/kg		/m s^{-1}				/J	
1.0	3.0	3.0	0.0	−1.5	(a)	(b)	(c)
0.050	0.050	1.5	0.0	0.0	(d)	(e)	(f)
2.0	3.0	3.0	(g)	(h)	(i)	15	0.0
0.010	0.99	50	0.0	(j)	1.0	(k)	(l)
0.010	9.99	50	0.0	(m)	0.10	(n)	(o)

4.5 In space, an elastic 'sling shot' collision is arranged between a stationary 6.4×10^{24} kg planet and a 6000 kg spacecraft moving at 4.5 km s^{-1}. By looking at the pattern in your answers to question 4.4 (j,m,l,o) estimate

a) the kinetic energy gained by the planet,

b) the final speed of the spacecraft.

In elastic collisions, the approach speed $|v_0 - V_0|$ and the separation speed $|V_1 - v_1|$ are equal. This is a consequence of question 4.1 part (c).

4.6 Repeat question 4.5b where the planet is also moving towards the spacecraft at 9.0 km s^{-1}.

Example 2 – *A neutron m with $v_0 = 1200$ m s^{-1} collides elastically with a stationary hydrogen molecule $M = 2m$. Calculate the velocity of the molecule after the collision.*
The two particles must separate at v_0, so if the molecule's final velocity is V_1, $v_1 = V_1 - v_0$. Conservation of momentum gives $mv_0 + 0 = mv_1 + 2mV_1$, so $v_0 = (V_1 - v_0) + 2V_1$, so $2v_0 = 3V_1$, and $V_1 = \frac{2}{3}v_0 = 800$ m s^{-1}.

4.7 A neutron (of mass m) travelling at 2.4×10^5 m s^{-1} collides elastically with a stationary carbon nucleus (mass $M = 12m$).

a) Calculate the final speed of the carbon nucleus.

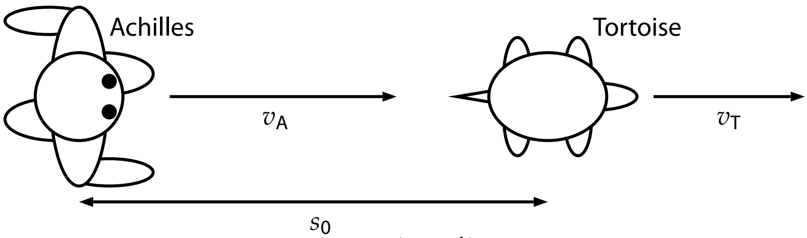
b) Calculate the percentage of the neutron's kinetic energy which is given to the nucleus.

4.8 Repeat question 4.7 for a neutron of the same speed colliding with an iron nucleus ($M = 65m$).

5 Vectors and motion – relative motion

It is helpful to be able to calculate the time it would take for two bodies to collide when they are travelling in the same direction, with the body in front is moving slower than the body behind.

Example context: Achilles chases after a tortoise. Achilles is faster than the tortoise; however, the by the time Achilles reaches where the tortoise was, it has moved forwards. When will Achilles catch up with the tortoise?



Quantities:
- v_A velocity of Achilles $\left(\mathrm{m\,s^{-1}}\right)$
- v_T velocity of tortoise $\left(\mathrm{m\,s^{-1}}\right)$
- T time for Achilles to catch up (s)
- s_0 initial displacement (m)
- s displacement (m)
- t time since start (s)

Equations: $v = \dfrac{s}{t}$

5.1 Use the equations to derive expressions for

a) the velocity of Achilles relative to the Tortoise v_{REL},

b) the time for Achilles to catch up with the tortoise T, in terms of v_A and v_T,

c) the displacement of the tortoise relative to Achilles as a function of time s.

5.2 Fill in the missing entries in the table below, using the diagram and quantities above to help.

s_0 / m	v_A / $\mathrm{m\,s^{-1}}$	v_T / $\mathrm{cm\,s^{-1}}$	T / s
(a)	5.81	6.71	15.0
1000	(b)	7.50	136
500	1.34	(c)	400
250	5.50	3.42	(d)

Example 1 – *The tortoise hops on a motor cycle and can travel at* 18.0 *m* s^{-1}*, whereas Achilles can* only *run at* 12.4 *m* s^{-1}*. They are initially* 50.0 *m apart. Calculate the time taken for them to be* 1.00 *km apart.*

$$s = s_0 - (v_\mathrm{A} - v_\mathrm{T})\,t \text{ therefore } t = -\frac{s - s_0}{v_\mathrm{A} - v_\mathrm{T}} = -\frac{1000 - 50.0}{12.4 - 18.0} = 170\,\mathrm{s}$$

5.3 Following on from **Example 1** above, when the tortoise travelling at $18.0\,\mathrm{m\,s^{-1}}$ is 1.00km away from Achilles, Achilles gets into a motor vehicle that can travel at $96.5\,\mathrm{km\,h^{-1}}$. Calculate how far ahead of the tortoise Achilles is after 2 minutes.

5.4 The tortoise and Achilles decide to participate in a jousting competition, whereupon the two charge at each other as fast as they can. They are initially stood 50.0 m apart from each other. The tortoise charges towards Achilles at $5.00\,\mathrm{m\,s^{-1}}$, and Achilles charges towards the tortoise at $15.0\,\mathrm{m\,s^{-1}}$. Calculate

a) the time taken before they collide,

b) how far Achilles has travelled when they collide.

Example 2 – *Achilles and the tortoise start at the same location. Achilles travels due South at* 15.0 *m* s^{-1}*, and the tortoise travels due East at* 8.00 *m* s^{-1}*. Calculate how far apart they will be after* 10 *s.*

Tortoise moves $8.00\,\mathrm{m\,s^{-1}} \times 10\,\mathrm{s} = 80\,\mathrm{m}$ East.

Achilles moves $15.0\,\mathrm{m\,s^{-1}} \times 10\,\mathrm{s} = 150\,\mathrm{m}$ South.

Distance apart (using Pythagoras) $= \sqrt{150^2 + 80^2} = 170\,\mathrm{m}$

5.5 Achilles starts 50.0 m due North of the tortoise. The tortoise runs due East at $3.00\,\mathrm{m\,s^{-1}}$. Achilles walks briskly at $4.24\,\mathrm{m\,s^{-1}}$ South-East. Calculate

a) how long until Achilles intercepts the tortoise,

b) How far Achilles has travelled in this time,

c) How far the tortoise has travelled in this time.

5.6 Achilles starts 100.0 m due North of the tortoise. The tortoise runs due East at $2.50\,\mathrm{m\,s^{-1}}$. Achilles runs at $7.31\,\mathrm{m\,s^{-1}}$ on a bearing of $160°$. A squirrel starts 50.0 m due South of the tortoise and scurries due North at a speed of $8.90\,\mathrm{m\,s^{-1}}$. Calculate

a) how long until Achilles intercepts the tortoise,

b) the distance between Achilles and the squirrel when Achilles intercepts the tortoise.

(e) $$\begin{aligned} E_{\mathrm{GP}} + E_{\mathrm{EP}} &= -mgx + \tfrac{1}{2}kx^2 = -mg\left(x_{\mathrm{B}} + y\right) + \tfrac{1}{2}k\left(x_{\mathrm{B}} + y\right)^2 \\ &= -mg\left(\frac{mg}{k} + y\right) + \frac{k}{2}\left(\frac{mg}{k} + y\right)^2 \\ &= -\frac{m^2g^2}{k} - mgy + \frac{m^2g^2}{2k} + mgy + \frac{ky^2}{2} \\ &= \frac{ky^2}{2} - \frac{m^2g^2}{2k} = \frac{ky^2}{2} + E_{\mathrm{B}} \end{aligned}$$

3 Momentum and kinetic energy

(a) $p = mv$ so $v = \dfrac{p}{m}$. Therefore $E = \dfrac{m}{2}v^2 = \dfrac{m}{2}\left(\dfrac{p}{m}\right)^2 = \dfrac{p^2}{2m}$

(b) $E = \dfrac{mv^2}{2}$ so $v = \sqrt{\dfrac{2E}{m}}$. Now $p = mv = m\sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2Em^2}{m}} = \sqrt{2mE}$

(c) $p = \sqrt{2mE} = \sqrt{2mqV}$ as $E = qV$

(d) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mqV}}$

4 Elastic collisions

(a) $p_0 + P_0 = p_1 + P_1$ so $mv_0 + 0 = mv_1 + MV_1$ and $V_1 = \dfrac{m\left(v_0 - v_1\right)}{M}$

(b) $p_0 + P_0 = p_1 + P_1$ so $mv_0 + 0 = 0 + mV_1$ and $V_1 = v_0$

Part (b) could also be completed using energy conservation.

For the third and optional part (c), the algebra is much more complicated, but we show it so that you can see why approach and separation speeds are the same in elastic collisions. Remember that r is defined as the approach speed ($v - V = r$), so $v = V + r$.

(c) $$\begin{aligned} P + p &= MV + mv = MV + m\left(V + r\right) = \left(M + m\right)V + mr \\ \left(P + p\right)^2 &= \left(M + m\right)^2 V^2 + 2\left(M + m\right)mrV + m^2r^2 \\ K + k &= \frac{MV^2}{2} + \frac{mv^2}{2} = \frac{M^2V^2 + MmV^2 + m^2v^2 + Mmv^2}{2\left(M + m\right)} \end{aligned}$$

$$\begin{aligned}
K+k &= \frac{M^2V^2 + MmV^2 + m^2\left(V+r\right)^2 + Mm\left(V+r\right)^2}{2\left(M+m\right)} \\
&= \frac{M^2V^2 + 2MmV^2 + m^2V^2 + 2m^2Vr + m^2r^2 + 2MmVr + Mmr^2}{2\left(M+m\right)} \\
&= \frac{\left(M+m\right)^2 V^2 + 2\left(M+m\right)mVr + m^2r^2 + Mmr^2}{2\left(M+m\right)} \\
&= \frac{\left(P+p\right)^2 + Mmr^2}{2\left(M+m\right)} \\
&= \frac{\left(P+p\right)^2}{2\left(M+m\right)} + \frac{Mm}{2\left(M+m\right)}r^2
\end{aligned}$$

In an elastic collision $k+K$ will be the same before and after the collision. As the total momentum $p+P$ will also be conserved, it follows that r^2 will not change either. Therefore $|r_1| = |r_0|$, so for a one-dimensional collision, $r_1 = \pm r_0$. In the $r_1 = r_0$ case, nothing has changed (there has been no collision), so in collisions $r_1 = -r_0$. In other words, when an elastic collision is viewed from the perspective of one object, the other object bounces off it at the same speed as it arrived.

5 Vectors and motion – relative motion

(a) $v_{\mathsf{REL}} = v_{\mathsf{A}} - v_{\mathsf{T}}$

(b) $v_{\mathsf{REL}} = \dfrac{s_0}{T} \longrightarrow T = \dfrac{s_0}{v_{\mathsf{REL}}} = \dfrac{s_0}{v_{\mathsf{A}} - v_{\mathsf{T}}}$

(c) $s = s_0 - \left(v_{\mathsf{A}} - v_{\mathsf{T}}\right)t$

6 Vectors and motion – projectiles

(a) $v_{\mathsf{y}}^2 = u_{\mathsf{y}}^2 + 2a_{\mathsf{y}}s_{\mathsf{y}}$ using the vertical components of the vectors.

$s_{\mathsf{y}} = -h$ when $v_{\mathsf{y}} = 0$ and $a_{\mathsf{y}} = g$ (downwards is positive)

$u_{\mathsf{y}} = -u\sin\theta$ (as upwards is negative)

$$-h = \frac{v_{\mathsf{y}}^2 - u_{\mathsf{y}}^2}{2a_{\mathsf{y}}} = \frac{0 - u^2\sin^2\theta}{2g}$$

$$h = \frac{u^2\sin^2\theta}{2g}$$