56 Energy from the Nucleus – Radioactivity & Fission

You can calculate the energy released by a nuclear process if you know the total mass of the nuclei present before and the total mass of those present after the process.

$$\text{energy} = \text{change of mass} \times (\text{speed of light})^2 \qquad E = mc^2$$

In this equation, the mass is measured in kilograms, the energy is measured in joules and the speed of light is 3.00×10^8 m/s.
In a nuclear reaction, the products (once they have slowed to normal speeds) have less mass in total than the reactants had: Although mass-energy is conserved, the energy released by the reaction has a mass equivalent.

$$\text{energy released} = \text{mass 'lost'} \times (\text{speed of light})^2$$

Example – Calculate the energy released during the reaction:

$$^{241}_{95}\text{Am} \longrightarrow {}^{237}_{93}\text{Np} + {}^{4}_{2}\alpha$$

The masses of the nuclei are given in the table below.

^{241}Am	$4.001\,98 \times 10^{-25}$ kg
^{237}Np	$3.935\,43 \times 10^{-25}$ kg
α	6.645×10^{-27} kg

$$\begin{aligned}
\text{Mass of reactants} &= \text{mass of } {}^{241}\text{Am} = 4.001\,98 \times 10^{-25}\text{ kg}\\
\text{Mass of products} &= \text{mass of } {}^{237}\text{Np} + \text{mass of } \alpha\\
&= 3.935\,43 \times 10^{-25}\text{ kg} + 6.645 \times 10^{-27}\text{ kg}\\
&= 4.001\,88 \times 10^{-25}\text{ kg}\\
\text{Difference in masses} &= 4.001\,98 \times 10^{-25}\text{ kg} - 4.001\,88 \times 10^{-25}\text{ kg}\\
&= 0.000\,10 \times 10^{-25}\text{ kg} \equiv 1.0 \times 10^{-29}\text{ kg}\\
\text{Energy released} &= mc^2 = 1.0 \times 10^{-29}\text{ kg} \times (3.00 \times 10^8)^2\\
&= 9.0 \times 10^{-13}\text{ J}
\end{aligned}$$

This may seem a very small amount per reaction, but it is over 5 000 000 times larger than the energy given out in chemical reactions.

Some masses of nuclei for use in the questions:

${}^{1}_{0}$n	$0.016\,749 \times 10^{-25}$ kg	α	6.645×10^{-27} kg
${}^{87}_{35}$Br	$1.443\,031 \times 10^{-25}$ kg	${}^{103}_{40}$Zr	$1.708\,773 \times 10^{-25}$ kg
${}^{134}_{54}$Xe	$2.223\,061 \times 10^{-25}$ kg	${}^{147}_{57}$La	$2.439\,291 \times 10^{-25}$ kg
${}^{189}_{81}$Tl	$3.137\,255 \times 10^{-25}$ kg	${}^{193}_{83}$Bi	$3.203\,808 \times 10^{-25}$ kg
${}^{206}_{82}$Pb	$3.419\,541 \times 10^{-25}$ kg	${}^{206}_{84}$Po	$3.419\,623 \times 10^{-25}$ kg
${}^{210}_{84}$Po	$3.486\,084 \times 10^{-25}$ kg	${}^{210}_{86}$Rn	$3.486\,179 \times 10^{-25}$ kg
${}^{212}_{83}$Bi	$3.519\,444 \times 10^{-25}$ kg	${}^{216}_{85}$At	$3.586\,032 \times 10^{-25}$ kg
${}^{234}_{90}$Th	$3.885\,568 \times 10^{-25}$ kg	${}^{235}_{92}$U	$3.902\,162 \times 10^{-25}$ kg
${}^{238}_{92}$U	$3.952\,090 \times 10^{-25}$ kg	${}^{239}_{94}$Pu	$3.968\,700 \times 10^{-25}$ kg

56.1 Using the previous information calculate the energy released during the alpha decays of:

(a) ${}^{193}_{83}$Bi
(b) ${}^{210}_{84}$Po
(c) ${}^{216}_{85}$At
(d) ${}^{210}_{86}$Rn
(e) ${}^{238}_{92}$U

[Hint: use the method in the Section 56, Example 1]

56.2 When a uranium nucleus fissions, there are various products which can be made. One typical reaction is

$${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{147}_{57}\mathrm{La} + {}^{87}_{35}\mathrm{Br} + 2{}^{1}_{0}\mathrm{n}$$

(a) Calculate the total mass of the reactants.

(b) Calculate the total mass of the products.

(c) The mass 'lost' is the energy lost to the nuclei. This energy is released in the form of kinetic energy. Calculate the mass lost.

(d) Use the equation $E = mc^2$ to work out how much energy has been lost from the nuclei (and gained in kinetic energy).

(e) The energy you calculated in (d) was released when one nucleus of uranium was fissioned. Use the mass of this nucleus to work out how much energy you could get out of 1.0 kg of uranium if you fissioned all of the nuclei.

(f) A nuclear power station has a thermal power output of 3.0×10^9 W. Calculate how much energy is generated in one year of continuous operation.

(g) Use your answers to 56.1(e) and (f) calculate the minimum amount of uranium you would need to fuel the power station for a year.

(h) Why must the reaction make at least two neutrons?

(i) The combustion of one carbon atom (mass = 2.0×10^{-26} kg) releases 6.6×10^{-20} J of energy. Calculate the mass of carbon (e.g. coal) which would need to be burnt each day to have the same thermal power output as the 3.0×10^9 W nuclear station.

(j) The mass of high-level nuclear waste produced per year by the nuclear power station will be similar to your answer to (g). If the material had a density of 6000 kg/m^3, work out the volume of nuclear waste produced in one year's operation.

(k) A cube-shaped underground chamber is to hold the waste produced by the power station over its 20 year operating lifetime. Use your answer to (j) to work out the side length of the cube.

(l) Since fission products absorb too many neutrons, nuclear fuel has to be removed from a reactor when only 5% of the fissile uranium has been used. Repeat part (k) on the assumption that a country does not 'reprocess' its nuclear fuel to remove the fission products, and that fuel is thrown away when only 5% of it has been used up.

(m) Work out the mass of the waste products of burning the coal in (i). When you burn 12 g of carbon, you need 32 g of oxygen, and so for every 12 g of coal burnt, 44 g of waste is made.

(n) Work out the volume of oxygen required to burn the coal in (i). Assume that every 12 g of carbon needs 32 g of oxygen to burn, and that 32 g of oxygen gas has a volume of 0.024 m^3.

56.3 A Plutonium-239 fission is by the following reaction:

$$^{239}_{94}\mathrm{Pu} + ^{1}_{0}\mathrm{n} \longrightarrow ^{134}_{54}\mathrm{Xe} + ^{103}_{40}\mathrm{Zr} + x\,^{1}_{0}\mathrm{n}$$

Give the value of x, on the right hand side of this equation, and calculate the kinetic energy of the products.