Acceleration f(t) 3ii

A Level

A cyclist travels along a straight road. Her velocity $v\,\mathrm{m\,s^{-1}}$, at time t seconds after starting from a point O, is given by

$$v = 2 \text{ for } 0 \leq t \leq 10$$

$$v = 0.03t^2 - 0.3t + 2 \text{ for } t \geq 10$$

Part A Displacement at $t = 10$

Find the displacement of the cyclist from O when $t = 10$.

Part B Expression for displacement

Find an expression for the displacement of the cyclist from O as a function of time for $t \geq 10\,\mathrm{s}$. Give your answer using fractions, not decimals.

The following symbols may be useful: `t`

Part C Time

Find the time when the acceleration of the cyclist is $0.6\,\mathrm{m\,s^{-2}}$.

Part D Displacement

Find the displacement of the cyclist from O when her acceleration is $0.6\,\mathrm{m\,s^{-2}}$.

Used with permission from UCLES, A Level, June 2006, OCR M1, Question 4

Acceleration f(t) 4i

A Level

A car is travelling along a straight horizontal road with velocity $32.5\,\mathrm{m\,s^{-1}}$. The driver applies the brakes and the car decelerates at $(8 - 0.6t)\,\mathrm{m\,s^{-2}}$, where $t\,\mathrm{s}$ is the time which has elapsed since the brakes were first applied.

Part A Velocity

Find an expression for the velocity of the car when it is decelerating.

The following symbols may be useful: `t`

Part B Time taken

Find the time taken to bring the car to rest.

Part C Distance travelled

Find the total distance travelled by the car whilst it is decelerating.

Gameboard:

STEM SMART Double Maths 23 - Vector Equations of Motion

Calculus and Vectors 1ii

A Level

A particle P of mass $0.2\,\mathrm{kg}$ moves on a smooth horizontal plane. Initially it is projected with velocity $0.8\,\mathrm{m\,s^{-1}}$ from a fixed point O towards another fixed point A. At time $t\,\mathrm{s}$ after projection, P is $x\,\mathrm{m}$ from O and is moving with velocity $v\,\mathrm{m\,s^{-1}}$, with the direction OA being positive. A force of $(1.5t - 1)\,\mathrm{N}$ acts on P in the direction parallel to OA.

Part A Expression for v

Find an expression for v in terms of t.

The following symbols may be useful: `t, v`

Part B Time when $v = 0.8\,\mathrm{m, s^{-1}}$

Find the time (in seconds) when the velocity of P is next $0.8\,\mathrm{m\,s^{-1}}$.

Part C Times through O

Find the first time when P subsequently passes through O.

Find the second time when P subsequently passes through O.

Part D Distance in third second

Find the distance P travels in the third second of its motion.

Used with permission from UCLES, A Level, June 2013, OCR M3, Question 3

Gameboard:

STEM SMART Double Maths 23 - Vector Equations of Motion

Vectors & Calculus 2i

A Level

A projectile has velocity $\begin{pmatrix} A \\ 5 - gt \end{pmatrix} \mathrm{m\,s^{-1}}$.

Part A Displacement

Given that the particle is at $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$ when $t = 0$.

Find an expression for the x-component of the particle's displacement, in metres, as a function of t.

The following symbols may be useful: `A, g, t`

Find an expression for the y-component of the particle's displacement, in metres, as a function of t.

The following symbols may be useful: `A, g, t`

Part B Force

Find an expression for the force on the particle, given that it has mass $m\,\mathrm{kg}$. Give your answer in the form $a\underline{\boldsymbol{i}} + b\underline{\boldsymbol{j}}$ where $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$ are unit vectors in the x and y directions respectively.

The following symbols may be useful: `A, g, i, j, m`

Part C Value of A

The projectile hits a target at the coordinates $\begin{pmatrix} 20 \\ 0 \end{pmatrix}$.

What is the value of A? Give your answer to 2 significant figures. In your calculation, use the approximation $g \simeq 10\,\mathrm{m\,s^{-2}}$ and assume that the target is hit at $t > 0$.

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Gameboard:

STEM SMART Double Maths 23 - Vector Equations of Motion

Vectors & Calculus 1i

A Level

A planet moves through space. The force on the planet is given by

$$\underline{\boldsymbol{F}} = \begin{pmatrix} -mAB^2\cos Bt \\ -mAB^2\sin Bt \end{pmatrix}$$

where A and B are numerical constants and m is the mass of the planet.

Part A Velocity

Given that the velocity of the planet when $t = 0$ is $\begin{pmatrix} 0 \\ AB \end{pmatrix}$.

Find an expression for the x-component of the velocity of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Find an expression for the y-component of the velocity of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part B Displacement

Given that the displacement of the planet when $t = 0$ is $\begin{pmatrix} A \\ 0 \end{pmatrix}$.

Find an expression for the x-component of the displacement of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Find an expression for the y-component of the displacement of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part C Modulus

Find an expression for the modulus of the displacement. Simplify your answer as far as possible.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part D Shape of path

What is the shape of the path that the planet follows?

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STEM SMART Double Maths 23 - Vector Equations of Motion

Projectiles: Trajectories 3i

A Level

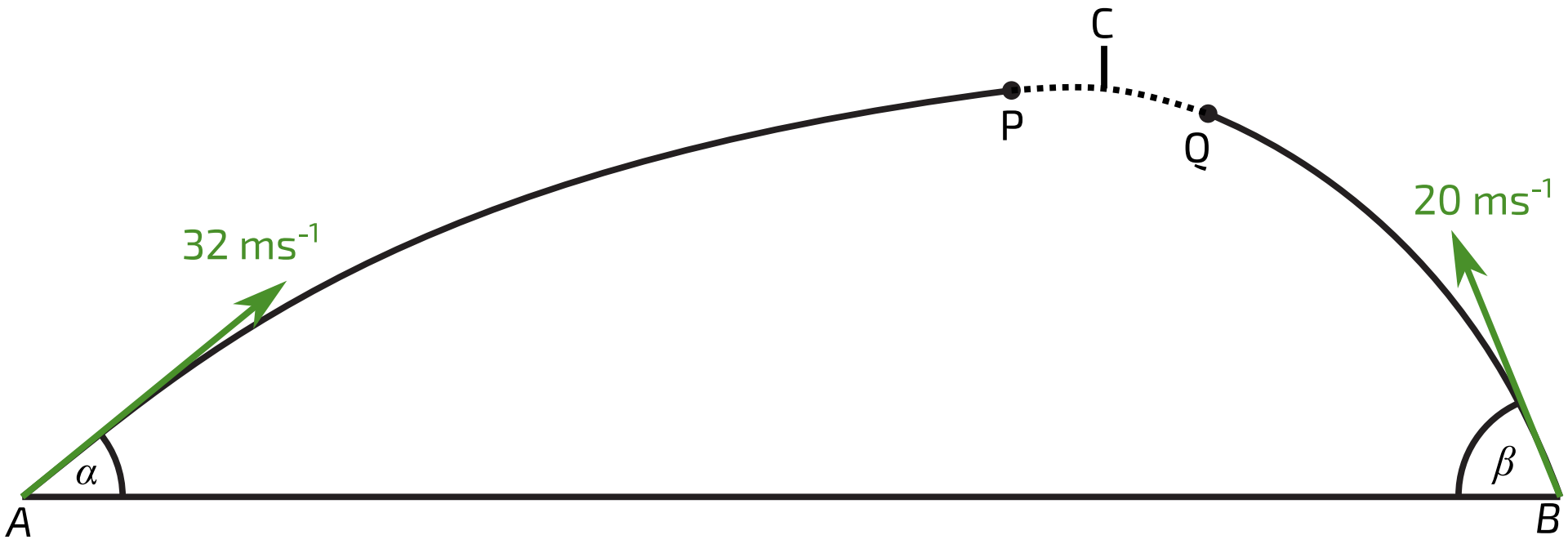

Figure 1: The trajectory of a particle P.

A particle P is projected with speed $32\,\mathrm{m\,s^{-1}}$ at an angle of elevation α, where $\sin\alpha = \frac{3}{5}$, from a point A on horizontal ground. At the same instant a particle Q is projected with speed $20\,\mathrm{m\,s^{-1}}$ at an angle of elevation β, where $\sin\beta = \frac{24}{25}$, from a point B on the same horizontal ground. The particles move freely under gravity in the same vertical plane and collide with each other at the point C at the instant when they are travelling horizontally.

Part A Velocities of P and Q

Express the velocity of P in vector form using the unit vectors $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$, where $\underline{\boldsymbol{i}}$ is a unit vector in the direction of $\overrightarrow{AB}$ and $\underline{\boldsymbol{j}}$ is a unit vector vertically upwards.

The following symbols may be useful: `i, j, k`

Express the velocity of Q in vector form using the unit vectors $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$, where $\underline{\boldsymbol{i}}$ is a unit vector in the direction of $\overrightarrow{AB}$ and $\underline{\boldsymbol{j}}$ is a unit vector vertically upwards.

The following symbols may be useful: `i, j, k`

Part B Height of C

Calculate the height of C above the ground. Give your answer to 3 significant figures.

Part C Time in air

Find the time, t, between projection and collision. Give your answer to 3 significant figures.

Part D Distance AB

Calculate the distance AB. Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, June 2016, OCR M2, Question 7

Gameboard:

STEM SMART Double Maths 23 - Vector Equations of Motion

Particles Moving on a Surface

A Level

C C C

A particle Q of mass $0.2\,\mathrm{kg}$ is projected horizontally with velocity $4\,\mathrm{m\,s^{-1}}$ from a fixed point A on a smooth horizontal surface. At time $t\,\mathrm{s}$ after projection Q is $x\,\mathrm{m}$ from A and is moving away from A with velocity $v\,\mathrm{m\,s^{-1}}$. There is a force of $3\cos 2t\,\mathrm{N}$ acting on Q in the positive x-direction.

Part A Expression for velocity

Find an expression for the velocity of Q at time t.

The following symbols may be useful: `cos(), sin(), t, tan(), v`

Part B Maximum and minimum

State the maximum value of the velocity of Q as t varies. Give your answer to 3 significant figures.

State the minimum value of the velocity of Q as t varies. Give your answer to 2 significant figures.

Part C Average velocity

Find the average velocity of Q between the times $t = \pi$ and $t = \frac{3}{2}\pi$. Give your answer to 3 significant figures.

Part D Particle's velocity

A particle P moves in a plane. Its displacement from the starting point, R, varies with time, t, as follows:

$$R = \begin{pmatrix} 2t^2 \sin \pi t - 1 \\ 1 + t^3 \end{pmatrix}$$

Where displacement is measured in metres and time is measured in seconds.

What is the x-component of the particle's velocity?

The following symbols may be useful: `cos(), pi, sin(), t, tan()`

What is the y-component of the particle's velocity?

The following symbols may be useful: `cos(), pi, sin(), t, tan()`

Part E Speed of particle

Find the speed of the particle when $t = 2$. Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, June 2016, OCR M3, Question 2

Gameboard:

STEM SMART Double Maths 23 - Vector Equations of Motion

Ships at Sea

A Level

C C C

Part A Unit vector

Find the unit vector in the same direction as $\underline{\boldsymbol{p}} = 6\underline{\boldsymbol{i}} — 3\underline{\boldsymbol{j}} + 2\underline{\boldsymbol{k}}$ in $\underline{\boldsymbol{i}}, \underline{\boldsymbol{j}}, \underline{\boldsymbol{k}}$ format.

The following symbols may be useful: `i, j, k, p`

Hence, find a vector of magnitude 4 parallel to $\underline{\boldsymbol{p}}$.

The following symbols may be useful: `i, j, k, p`

Part B S and T

In an experiment two ships, S and T, move in a calm sea.

S and T are considered to be represented by single points in space. Which single word describes an object for which such an assumption is made?

Part C Position of S

S starts from the origin and moves with constant velocity $\underline{\boldsymbol{v_1}} = 4\underline{\boldsymbol{i}}—3\underline{\boldsymbol{j}}$, where $\underline{\boldsymbol{v_1}}$ is measured in $\mathrm{km\,h^{-1}}$.

Write down the position vector of S in terms of t using ijk notation.

The following symbols may be useful: `i, j, k, t`

Part D Calm sea assumption

T starts from the point with position vector $(3\underline{\boldsymbol{i}}—5\underline{\boldsymbol{j}})$ and moves with constant velocity $\underline{\boldsymbol{v_2}} = (\underline{\boldsymbol{i}} + 4\underline{\boldsymbol{j}})$ where $\underline{\boldsymbol{v_2}}$ is measured in $\mathrm{km\,h^{-1}}$.

Using the information given, justify limiting the velocity vectors of S and T to two dimensions.

Easier question?

Part E Do they meet?

Do S and T ever meet? If so, find the time in seconds. If not, enter in 0.

Part F Minimum distance

Find the minimum distance between S and T. Give your answer using exact fractions and surds.

Part G Finding y

A second experiment is undertaken. The behavior of S is unchanged. T starts from the same point as before, but this time it is the intention that S and T meet. The required constant velocity to set for T is of the form $(\underline{\boldsymbol{i}} + y\underline{\boldsymbol{j}})$.

Find the value of y.

Part H Percentage accuracy

The ships are each approximately $40\,\mathrm{m}$ long. Estimate the maximum percentage error in the calculation of the time it takes for the ships to meet due to using the assumption described in Part B. Give your answer to 1 significant figure.

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