7 Proportionality

Physicists measure things, and then look for patterns in the numbers.

The most important pattern is called proportionality (also called direct proportion). If distance is proportional to time, it means that if the time doubles, the distance will double too. If the distance gets 10 times bigger, the time will get 10 times bigger as well. Mathematically, this is written as $s \propto t$.

Example 1 – A particular resistor passes a 25 mA current when the voltage across it is 5.5 V. If voltage is proportional to current, what will the voltage be when the current is 60 mA?

The new current is (60 mA / 25 mA) $= 2.4$ times larger than the old one. The new voltage will be 2.4 times larger than the old one: 5.5 V $\times\ 2.4 =$ 13.2 V.

Using a formula

If s is proportional to t then s/t will always have the same value. If we call this fixed value k, it follows that $k = s/t$, and that $s = kt$. We can use this information to answer questions. The formula method is much clearer if there are more than two quantities involved.

Example 2 – A spring obeying Hooke's Law (its extension is proportional to the force) stretches by 14 mm when a 7.0 N load is applied. How far will it stretch with a 3.0 N load?

We write force $=$ k $\times$ extension, so extension $=$ force$/k$.
$k =$ force$/$extension $=$ 7.0 N$/14$ mm $=$ 0.50 N/mm
For a 3.0 N load, extension $=$ force$/k = 3.0/0.50 = 6.0$ mm.

Example 3 – The energy transferred by an electric circuit in a fixed time is proportional to the voltage and also to the current $(E \propto V \times I)$. If the current is 3.2 A, and the voltage is 15 V and the energy transferred is 340 J. What current will be needed if we need to deliver 640 J using 12 V in the same time?
The equation is $E = kIV$, so $k = E/(IV) =$ 340 J$/(3.2$ A $\times\ 15$ V$) =$

7.08 J/(AV)
Rearranging gives $I = E/(kV) = 640/(7.08 \times 12) = 7.53 = 7.5$ A (2sf)

7.1 A cyclist can travel 9.0 km in 30 minutes on level ground. Assume that their speed is constant.

(a) How far will they go in 120 minutes?

(b) How far do they go in 20 minutes?

(c) How much time will it take them to cover 27 km?

(d) How much time will it take them to cover 15 km?

7.2 One day, €1.00 is worth £0.83. On that day

(a) How many pounds would be needed to receive €200 when exchanging your money?

(b) How many euros could I get for £150 (to the nearest €)?

(c) A sandwich in a popular tourist city costs €6.50. How much is that in pounds (to the nearest penny)?

(d) A railway ticket costs £23.50. How much is that in euros (to the nearest cent)?

7.3 The UK minimum wage was £3.87 per hour for someone under the age of 18. Assume that your employer paid you this wage.

(a) How much did you earn for 20 hours of work?

(b) You worked 100 minutes a day after school. How much did you earn a day?

(c) How many hours would you have had to work to save £200?

7.4 The number of widgets made in a factory each week is proportional to the number of workers and the number of hours each worker works. When the factory employs 25 staff, each working 35 hours/week, $65\,400$ widgets were made each week.

(a) How many widgets would be made each week if 40 staff worked for 30 hours per week?

(b) If we need $130\,000$ widgets made each week, and the staff will work 42 hours/week, how many workers are needed?

7.5 The merchandiser at a warehouse sends stock to stores in proportion to their sales. She has 670 pairs of mauve trousers to dispatch. Her sales figures tell her that 124 pairs of trousers were sold in total last week, with the New Town branch selling 18 of them. How many pairs of trousers should she send to New Town?

7.6 A watch is set to the correct time at noon on 1^{st} January and put in a drawer. When it is checked at noon on 1^{st} February, it reads $11{:}51{:}20$. What did it read at $6{:}00$am on 24^{th} January?

Inverse Proportionality

The time taken on a journey is inversely proportional to the speed. If you double the speed, the time halves. If you only go at a tenth of the speed, it takes $10\times$ as long. We write this as $t \propto 1/v$, where v is the speed. In this case, $v \times t$ always has the same value.

Example 4 – The number of books printed each day is proportional to the number of printers owned, and inversely proportional to the number of pages in each book.
If $3\,000$ 300-page books can be printed in one day on 8 printers, how many 125-page books can they print on 6 printers in a day?

As books $\propto$ printers, and books $\propto 1/$pages,
then books $= k \times$ printers$/$pages.
$k =$ books $\times$ pages$/$printers $= 3\,000 \times 300/8 = 112\,500$.
books $= k \times$ printers$/$pages $= 112\,500 \times 6/125 = 5\,400$.

7.7 A cyclist's journey to work takes them 32 minutes at 19 km/h. [*Hint: time* $\times$ *speed* $= \frac{32}{60} \times 19 = 608$.]

(a) How long would it take at 15 km/h?

(b) How fast would they have to go to reduce the time to 25 minutes?

7.8 An interest free loan for a luxury sofa takes 15 months to pay back at £80/month. Monthly charge $\propto$ number of sofas/duration of loan. What would the monthly charge be if I bought 3 sofas and paid for them over one year?

7.9 The current through a resistor is inversely proportional to its resistance. With a $330\ \Omega$ resistor, the current is 25 mA. What value of resistance is needed if you wish a 55 mA current to pass?

7.10 The braking force required to stop a car is inversely proportional to the time taken to stop it. If a $5\,500$ N force can stop the car in 8.0 s, how much force would be needed to stop it in 3.5 s?

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Additional Proportionality Questions

7.11 Which two criteria must be met for a line graph to indicate direct proportionality between two quantities?

7.12 For each of the following equations state whether the two stated variables are directly proportional, inversely proportional or neither. [If there are other values in the question, they are kept constant.]

(a) $W = mg$ - W and m

(b) $pV = kT$ - p and V

(c) $p = mv$ - p and v

(d) $F = k\dfrac{Q_1Q_2}{r^2}$ - F and r

(e) $T(\text{K}) = T(^\circ\text{C}) + 273$ - T(K) and $T(^\circ$C)

(f) $a = 4\pi^2 r f^2$ - a and f^2