Viscous Drag

A Level Further A

The acceleration of a particle is given by $-bv$ where v is the velocity at time t and b is a constant. The particle has an initial velocity v_0.

Part A Find an expression for v

Given that acceleration is the rate of change of velocity with time write down the differential equation describing the system and solve this to find an expression for v in terms of v_0, b and t.

Find an expression for v in terms of v_0, b and t.

The following symbols may be useful: `b, e, t, v, v_0`

Part B Find an expression for x

Now use the fact that velocity is equal to the rate of change of displacement x to write down the differential equation relating x and t. Solve this equation to find how x varies with t given that $x = 0$ when $t = 0$.

Find the equation for the displacement x as a function of time t.

The following symbols may be useful: `b, e, t, v_0, x`

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Acceleration and Displacement

Further A

P P P

A particle P is moving such that its acceleration is $(3 - \mathrm{e}^{-x})\ \mathrm{m\ s^{-2}}$, where $x\ \mathrm{m}$ is its displacement from the origin O. When P passes through O, its velocity is $2\ \mathrm{m\ s^{-1}}$ in the direction of x increasing.

Part A Velocity

Find the velocity, $v\ \mathrm{m\ s^{-1}}$, of P in terms of its displacement.

The following symbols may be useful: `e, v, x`

Part B Velocity when $x = 10$

Find the velocity of P when its displacement is $10\ \mathrm{m}$. Give your answer to 2 significant figures.

Part C Displacement with minimum velocity

Find the exact value of the displacement of P when its velocity takes its minimum value.

The following symbols may be useful: `ln(), log(), x`

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STEM SMART Double Maths 34 - Calculus in Kinematics, Gravitation & Oscillations

Acceleration and Velocity

Further A

A particle P is moving in a straight line with velocity $v\,\mathrm{m\,s^{-1}}$ and an acceleration of $(4-v^2)\,\mathrm{m\,s^{-2}}$. Its displacement from the origin O is $x\,\mathrm{m}$. When P passes through O it has a velocity of $\sqrt{3}\,\mathrm{m\,s^{-1}}$ in the direction of x increasing.

Part A Displacement in terms of velocity

Find the displacement of P from O in terms of its velocity.

The following symbols may be useful: `ln(), log(), v, x`

Part B Displacement when P is at rest

Find the exact value of the displacement when P is instantaneously at rest.

Part C Time to reach $\sqrt{3}\,\mathrm{m\,s^{-1}}$

Given that P is initially at rest, find the time that P takes to accelerate to $\sqrt{3}\,\mathrm{m\,s^{-1}}$. Give your answer to 3 significant figures.

Part D Displacement in terms of time

Find the displacement of P in terms of time, t.

The following symbols may be useful: `cosech()`, `cosh()`, `coth()`, `ln()`, `log()`, `sech()`, `sinh()`, `tanh()`

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7.4.5 Sedimentation II

Further AUniversity

Now solve the 7.4.1 Sedimentation, which is the simpler task of solving the equations that you have formulated. The problems need posing correctly. A problem reducible to a first order differential equation (one order of derivative) needs one “boundary” or “initial” condition, second order differential equations require two conditions, and so on.

A particle of mass m sediments through a fluid, receiving a retarding force proportional to its velocity. It starts from rest at the origin, that is its speed has $v(t=0)=0$ and $x(t=0)=0$.

Part A Speed of particle

Give the variation of speed v with time t during the motion, given that $v(t=0)=0$.

The following symbols may be useful: `e, g, k, m, t, v`

Part B Position of particle

Give the variation of the position x of the particle with time t, given that $x(t=0)=0$. Note that another condition has crept in. Why?

The following symbols may be useful: `e, g, k, m, t, x`

Part C Asymptote

Give the equation of the asymptote to the graph of x against t with x as the subject of the equation.

The following symbols may be useful: `e, g, k, m, t, x`

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Weight on Comet 67P

Objects sitting on Comet 67P weigh much less than they would on Earth because Comet 67P is both much less dense and much smaller than Earth. It has been said that the Philae lander, with a mass of $100\,\mathrm{kg}$, weighs about as much on 67P as a paperclip does on Earth.

Find the mass of an object that would have the same weight on Earth's surface as the Philae lander has on the surface of Comet 67P, given that Comet 67P has a mass of $1.0 \times 10^{13}\,\mathrm{kg}$ and the lander is sat on its surface a distance $2.0\,\mathrm{km}$ from the centre of mass of the comet. [Model the comet as spherical and give your answer to 2 significant figures].

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Physics. You work it out.

Essential Pre-Uni Physics F6.1

A Level

Physical constants which may be necessary to answer the problems on this page can be found within the hint tabs.

The Earth takes a year to go round the Sun in an orbit with radius $1.50 \times 10^{11}\ \mathrm{m}$. Calculate the Sun's mass.

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Freefall to the Moon

A Level

A stationary piece of cargo is released from a point P a distance $3R$ from the centre of the Moon which has radius R and mass M.

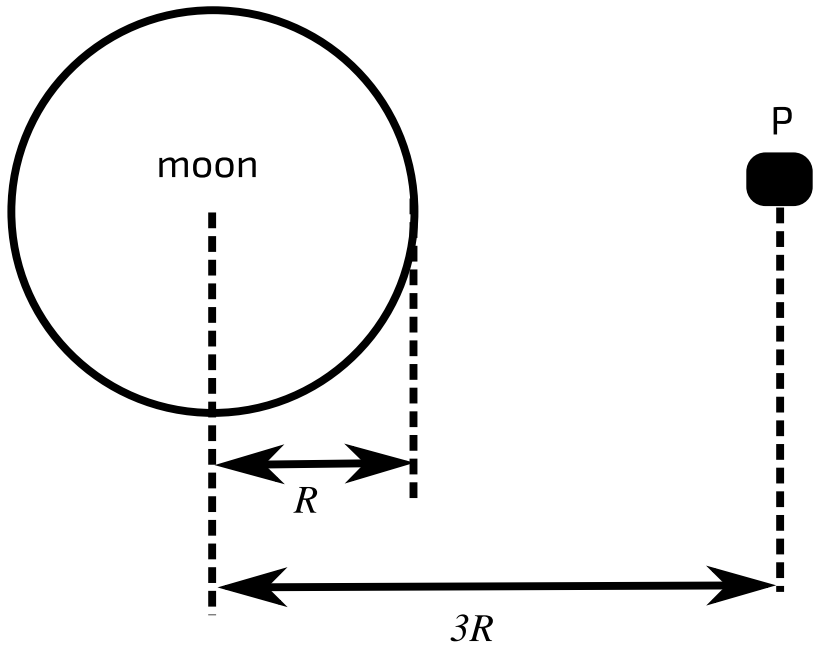

Figure 1: Diagram of the cargo above the moon.

Find an expression for the speed of the cargo upon hitting the moon.

The following symbols may be useful: `G, M, R, v`

Used with permission from UCLES, A Level Physics, June 1988, Paper 1, Question 18.

Gameboard:

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Physics. You work it out.

Safe Passage

A Level

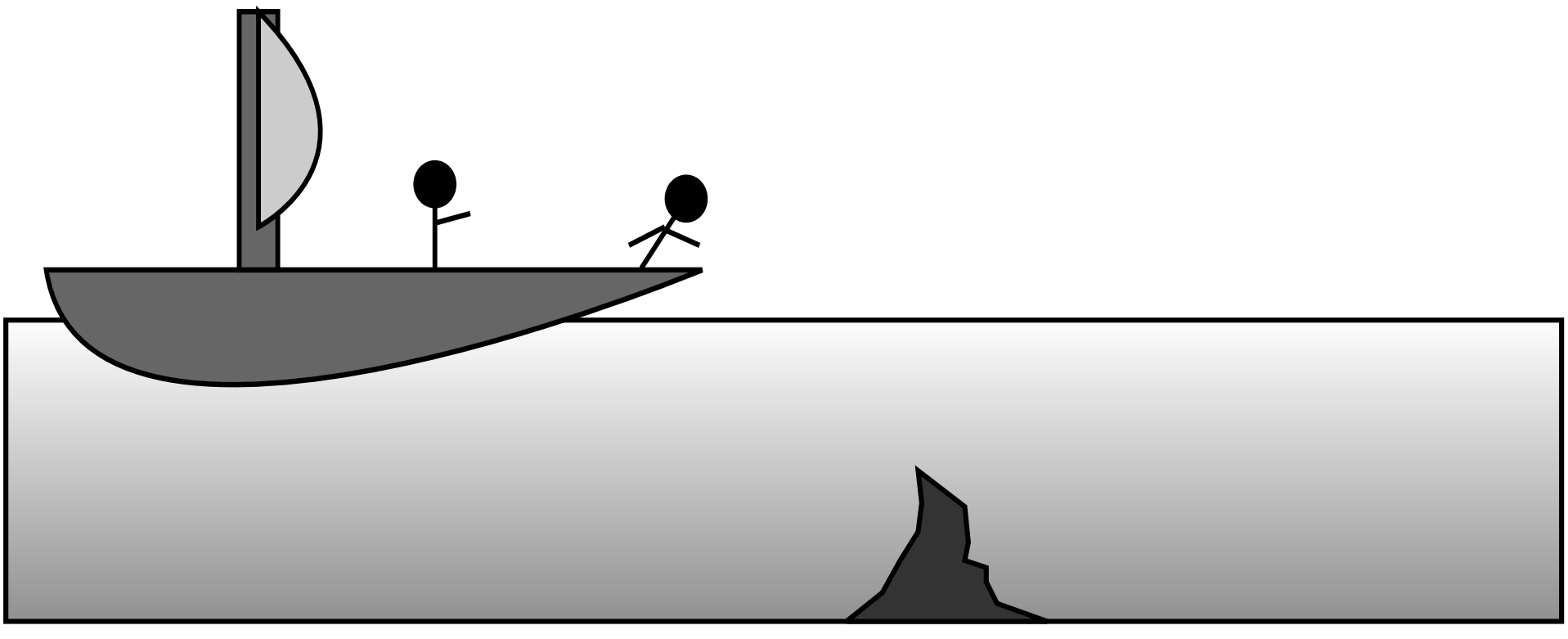

Figure 1: The river is at its shallowest for a very short distance.

For safe passage a ship needs $9.0\,\mathrm{m}$ of water. At the shallowest point of a tidal river in which the ship is travelling, the depth of the water at low tide, which occurs at 12 noon, is $6.0\,\mathrm{m}$ and the depth of the water at high tide, which occurs at 6:20pm, is $11.0\,\mathrm{m}$.

What is the earliest time in the afternoon at which the ship can pass this point safely, assuming that the water surface moves up and down in simple harmonic motion? Give your answer 24 hour time format, e.g. 0000 for midnight or 1600 for 4 pm. Give your answer correct to the nearest minute.

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Mass on a Horizontal Spring

A Level

An object of mass $m = 1.0\,\mathrm{kg}$ is attached to an ideal horizontal spring. The spring is initially stretched by $x_0 = 0.10\,\mathrm{m}$, and the object is released from rest there. It proceeds to move without friction. The next time the speed of the object is zero is $0.50\,\mathrm{s}$ later.

What is the maximum speed of the object?

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Physics. You work it out.

SHM of a Simple Scale

A Level

C C C

The upper end of an ideal vertical spring is fixed and its lower end is hanging freely. A scale pan of mass $20.0\,\mathrm{g}$ is then attached to the free end without extending the spring.

If the scale pan is then released from rest then the resulting motion is simple harmonic.

Part A Amplitude

Calculate the amplitude of the oscillation if the spring constant is $98.1\,\mathrm{N\,m^{-1}}$.

Part B Period

Calculate the period of the oscillation.

Part C Modified amplitude

The oscillating pan is held at the equilibrium and then released so that it is at rest. A mass of $20.0\,\mathrm{g}$ is introduced into the scale pan so that it once again moves with simple harmonic motion.

Find the new amplitude.

Part D Modified period

Find the new period after the mass of $20.0\,\mathrm{g}$ is introduced into the scale pan when it was previously at equilibrium.

Used with permission from UCLES, A Level Physics, June 1951, Paper 1, Question 3.