A PHYSICS ALPHABET
ABBREVIATIONS USED IN THIS BOOK, UNITS AND FORMULAE[1]

Quantity (with symbol)		Unit
Area (e.g. surface area)	A	m^2
Acceleration	a	m/s^2
Specific heat capacity	c	J/(kg °C)
Critical angle	i_c	°
Speed of light	c	m/s
Energy	E	J = Nm
Force	F	N = kg m/s^2
Frequency	f	Hz = 1/s
Focal length (of lens)	f	m
Gravitational field strength	g	m/s^2 or N/kg
Height	h	m
Current	I	A
Angle of incidence	i	°
Intensity	I	W/m^2
Spring constant	k	N/m
Specific latent heat	L	J/kg
Mass	m	kg
Number of turns on coil	N	
Refractive index	n	
Momentum	p	kg m/s or Ns

Quantity (with symbol)		Unit
Pressure	p	Pa = N/m^2
Power	P	W = J/s
Power (of lens)	P	D = 1/m
Charge	Q	C = As
Radius	r	m
Resistance	R	Ω = V/A
Angle of refraction	r	°
Distance or displacement	s	m
Time	t	s
Temperature	T	°C or K
Time period	T	s
Object distance (lens)	u	m
Speed before change	u	m/s
Speed or velocity	v	m/s
Image distance (lens)	v	m
Voltage	V	V = J/C
Volume	V	m^3
Weight	W	N = kg m/s^2
Extension	x	m
Wavelength	λ	m
Density	ρ	kg/m^3

1 km = 1000 m	1 Mm = 10^6 m	1 Gm = 10^9 m	
1 cm = 0.01 m	1 mm = 0.001 m	1 µm = 10^{-6} m	1 nm = 10^{-9} m

Units with powers. Note for example:
1 cm^2 means 1 cm × 1 cm = 0.01 m × 0.01 m = 10^{-4} m^2

[1] A list of formulae and data is given on the inside back cover.

FORMULAE AND DATA[2]

The meaning of all symbols in the formulae, and the units used, are given on the inside of the cover. If you need to revise a formula, turn to the page listed alongside it in this table.

Velocity and Acceleration	
$s = vt$	P 20
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Force and Acceleration	
$F = ma$	P 37
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$p = F/A$	P 49
$p = \rho g h$	P 51
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Energy or Work Done	
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$E = \frac{1}{2}mv^2$	P 106
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$E = mgh$	P 103
Energy and Temperature change	
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Energy and Change of State	
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Efficiency	
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Electricity	
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$V = IR$	P 75
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$P = V^2/R = I^2R$	P 84
$E = Pt = IVt$	P 82
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Intensity	
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Refraction (Snell's Law)	
$n_1 \sin(i) = n_2 \sin(r)$	P 142
Critical Angle	
$\sin(i_c) = 1/n$	P 144
Lenses	
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$1/v = 1/f - 1/u$	P 148
Gases	
$\frac{p_{\text{after}} V_{\text{after}}}{T_{\text{after}}} = \frac{p_{\text{before}} V_{\text{before}}}{T_{\text{before}}}$	P 182
Equivalence of Energy and Mass	
$E = mc^2$	P 167

In the questions on these worksheets, unless otherwise given, take

- **Gravitational field strength (g) as 10 N/kg**
- **Acceleration of a dropped object on Earth without air resistance (g) as 10 m/s^2**
- **Speed of light (c) as 3.00×10^8 m/s (in vacuum)**

Other data will be given on each worksheet when you need it.

[2] A list of quantities, symbols and data is given on the inside front cover.

Isaac Physics Skills

Mastering GCSE Physics

A.C. Machacek & K.O. Dalby
Westcliff High School for Boys

with extra questions written by R. Meikle

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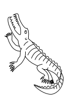
Use this collection of worksheets in parallel with the electronic version at `isaacphysics.org`. Marking of answers and compilation of results is free on Isaac Physics. Register as a student or as a teacher to gain full functionality and support.

 used with kind permission of M. J. Rutter.

Notes for the Student and the Teacher

Some students (and teachers) may seem daunted by the idea of performing calculations on momentum conservation, lens images, circular motion, nuclear energy and the like, below the age of 16. However, it is our conviction that all secondary school students can gain mastery of these concepts. By mastery, we mean that you feel confident that you understand them and can apply them to reasonably straightforward problems with accuracy. To begin, all you need is the ability to multiply and divide numbers (using a calculator), the willingness to give Physics a go, and the determination not to listen to any thoughts which say that it's going to be too difficult.

Each worksheet contains notes, explanations, worked examples and then questions. It is important that you can see where formulae come from, and accordingly explanations in places will go deeper than required for GCSE.

Once you have read the notes, you are ready to try the questions, they get harder as you go on. You can check your answers using the isaacphysics.org website.

We suggest that you revisit each sheet and its questions until you can answer at least three quarters of the material correctly; see the pass mark indicated in the square on each sheet. Until 75% is achieved, study further, then repeat a selection of questions. This is the mastery method here ensuring a good foundation is laid for a GCSE physics education. $\boxed{{}^{x}/_{y}}$

You may well also find this book helpful when you come to revise. When you revise, resist the temptation to move beyond a page until you have attempted a good selection of the questions and have got at least three-quarters of them correct. Be aware that this book is not written with a particular specification in mind - not all sections will be relevant to your exam, especially those marked ♡. A specification map is on isaacphysics.org. ♡

Also remember, this is a Physics book not an exam revision guide. Its aim is to help you understand the principles. This may well take longer than memorizing a few soundbites, but, once achieved, enables you to solve a wide range of problems with very little further 'learning'. Quite simply, as thousands of our students have found, once you 'get it' it is then 'obvious' and no further notes or books are needed. This is mastery - and this is what you should aim for.

ACM & KOD
Westcliff-on-Sea, 2016

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We are very grateful to the Isaac Physics team at the University of Cambridge for initiating this project, and for their continued advice, flexibility and encouragement. Particular thanks go to Aleksandr Bowkis, Ben Hanson, Rupert Fynn and Umberto Lupo for their great work in typesetting our sheets into a book. Michael Conterio and Bianca Andrei have closely checked the questions while making them available through the isaacphysics.org website, a remarkable resource for students and teachers alike. Laura Moat must be thanked for all of her expertise and help with the aesthetics of these worksheets, both on the covers and the content itself.

We must also thank Rob Meikle for his generosity in allowing us to use many of his excellent Physics questions from his rich resource 'Physics Examples' within many of the worksheets.

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Finally, we thank you, the student. We wish you well in your studies, and trust that with a bit of determination, you will use these to help you become a master of physics at GCSE level, and that this might encourage you to try more physics in the sixth form.

Soli Deo Gloria,

ACM & KOD
Westcliff-on-Sea, 2016

Suggestions for use in lessons

Traditional approach

- Introduce the concept you wish to teach – perhaps by giving an example of a situation where this is going to be useful in the solution. A problem could be set, or a short video of a relevant situation shown (I like showing a YouTube clip of a North American 'Fall' festival in which a 500 kg pumpkin is dropped on an old school bus, when about to discuss gravitational and kinetic energy, or the Mythbusters video in which a compressed gas cylinder bursts through a breeze block wall when its regulator is sheared off, as an introduction to gas pressure).
- If desired, you can project the relevant page from the teacher's section of IsaacPhysics. This includes notes with essential details, but with some key words and definitions missing, and spaces for certain explanations. Before students have opened their Isaac books, teach the main concepts, give the definitions and use class questioning and discussion to agree the answers to the 'cloze text' parts.
- Students then turn to the relevant page, and read the 'notes' section. Students may make their own notes in their exercise books if you wish.
- While many students will be ready to begin the questions straight away (and will be able to do so without further help from you), others will need your specific help in going over important points in the notes.
- By the time that the students who needed your help with the notes are ready to start the more straightforward questions, the others will have reached the more tricky questions, and will wish assistance. Questions can be answered in students' exercise books, showing working, with the final answer to an appropriate number of significant figures and with a unit. You may choose to make certain questions optional.
- Follow-up questions can be selected from the Isaac Physics website for homework.

Use with 'Flipped Lessons'

- Here, you would set a homework to study the notes of a particular page, and complete some of the more straightforward questions – you can check their progress using isaacphysics.org.

- Students then work on questions in class, as above. The teachers' version of the text (with spaces for the explanations and results of class discussion) could be projected onto the screen and discussed as the starter for the lesson to see how much students remember and understand from their own reading.

Using Isaac Physics with this book

Isaac Physics offers online versions of each sheet at:

`isaacphysics.org/gcsebook`

There, a student can enter answers as well as learn the concepts detailed in these worksheets by reading the online versions. This online tool will mark answers, giving immediate feedback to a student who, if registered on isaacphysics.org, can have their progress stored and even retrieved for their CV! Teachers can set a sheet for class homework as the appropriate theme is being taught, and again for pre-exam revision. Isaac Physics can return the fully assembled and analysed marks to the teacher, if registered for this free service. Isaac Physics aims to follow the significant figures (sf) rules on page (v), and warns if your answer has a sf problem. Isaac is stricter at A-level, in accordance with A-level examination practice.

Uncertainty and Significant Figures

In physics, numbers represent measurements that have uncertainty and this is indicated by the number of significant figures in an answer.

Significant figures
When there is a decimal point (dp), all digits are significant, except leading (leftmost) zeros: 2.00 (3 sf); 0.020 (2 sf); 200.1 (4 sf); 200.010 (6 sf)
Numbers without a dp can have an *absolute accuracy*: 4 people; 3 electrons.
Some numbers can be ambiguous: 200 could be 1, 2 or 3 sf (see below). Assume such numbers have the same number of sf as other numbers in the question.
Combining quantities
Multiplying or dividing numbers gives a result with a number of sf equal to that of the number with the smallest number of sf:
$x = 2.31$, $y = 4.921$ gives $xy = 11.4$ (3 sf, the same as x).
An absolutely accurate number multiplied in does not influence the above.
Standard form
Online, and sometimes in texts, one uses a letter 'x' in place of a times sign and ^ denotes "to the power of":
$1\,800\,000$ could be 1.80x10^6 (3 sf) and $0.000\,015\,5$ is 1.55x10^-5
(standardly, 1.80×10^6 and 1.55×10^{-5})
The letter 'e' can denote "times 10 to the power of": 1.80e6 and 1.55e-5.
Significant figures in standard form
Standard form eliminates ambiguity: In $n.nnn \times 10^n$, the numbers before and after the decimal point are significant:
$191 = 1.91 \times 10^2$ (3 sf); 191 is $190 = 1.9 \times 10^2$ (2 sf); 191 is $200 = 2 \times 10^2$ (1 sf).
Answers to questions
In these worksheets and online, give the appropriate number of sf:
For example, when the least accurate data in a question is given to 3 significant figures, then the answer should be given to three significant figures; see above.
Too many sf are meaningless; giving too few discards information. Exam boards require consistency in sf, so it is best to get accustomed to proper practices.

Contents

Skills

1 Units

In Physics, measurable quantities usually have a ________ and a ____. The ____ gives an indication of the size of that quantity and also information about what the quantity physically represents. This is best understood with examples.

A quantity such as 15 metres is clearly a ______; one cannot measure a mass or a time in metres. 15 metres is a _______ length than 15 miles, but a ______ length than 15 inches. Without the inclusion of a unit, a length of 15 is meaningless.

To facilitate global collaboration in science, seven units have been selected as the standard that all scientists should use. These are called ____________ (which comes from the French name: Système International d'unités). At GCSE Physics level, you are expected to know and be able to use the first six of these units.

Quantity	**Unit name**	**Unit symbol**
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

_______________ are units given in terms of the SI base units. A speed, for example, is always a ________________________. In SI derived units, a speed should be given in metres per second (m/s). A volume always includes the product of three lengths so, in SI derived units, a volume should be given in ____________ (m^3).

You can work out what the appropriate unit for any quantity is by considering the quantities that are combined in any equation for that quantity.

Units may also include a prefix. These are included between the number and the unit and tell you by how much the number should be multiplied.

Prefix	Multiply By
mega (M)	1 000 000
kilo (k)	1 000
centi (c)	0.01
milli (m)	0.001
micro (μ)	0.000 001
nano (n)	0.000 000 001

Additional Units Questions

2 Standard form

The radius of the Earth is $6\,400\,000$ m.
The speed of light is $300\,000\,000$ m/s.
The charge of one electron is $-0.000\,000\,000\,000\,000\,000\,16$ C.

Big and small numbers are inconvenient to write down – scientists and engineers use ____________ to make things clearer.

The above numbers in standard form look like this:

6.4×10^6 (or 6.4 e 6 on a computer).

3.0×10^8 (or 3.0 e 8 on a computer).

-1.6×10^{-19} (or -1.6 e -19 on a computer).

number in standard form $=$ mantissa $\times$ power of ten

The ________ is a number bigger than or equal to 1, but less than 10.
____________ are numbers you can make by starting with 1 and either multiplying or dividing as many times as you like by 10.
So 100, 0.01, $100\,000$, 10, 1 and $0.000\,1$ are all powers of ten,
but 30, 0.98 and $40\,000$ are not powers of ten.
Powers of ten can be written using _________ (e.g. 10^2 rather than 100).

$10\,000 = 10 \times 10 \times 10 \times 10 = 10^4$	exponent $= 4$
$1\,000 = 10 \times 10 \times 10 = 10^3$	exponent $= 3$
$100 = 10 \times 10 = 10^2$	exponent $= 2$
$10 = 10 = 10^1$	exponent $= 1$
$1 = 10^0$	exponent $= 0$
$0.1 = 1/10 = 1/10^1 = 10^{-1}$	exponent $= -1$
$0.01 = 1/100 = 1/10^2 = 10^{-2}$	exponent $= -2$
$0.001 = 1/1000 = 1/10^3 = 10^{-3}$	exponent $= -3$

Note that the ________ counts the number of times the decimal point must be moved to get from its starting point before the number is turned into a ________:

e.g. $0.\underset{\rightarrow}{0}\underset{\rightarrow}{0}\underset{\rightarrow}{0}\underset{\rightarrow}{3}2 = 3.2 \times 10^{-4}$, as the decimal point must be moved 4 times

to the right before it makes a 3.2.
Also $893 = 8\underset{\leftarrow}{9}\underset{\leftarrow}{3}.0 = 8.93 \times 10^2$ as the decimal point must be moved 2 times to the left before it makes an 8.93.

You key 3.4×10^{-9} into a calculator by pressing

$$\boxed{3}\ \boxed{.}\ \boxed{4}\ \boxed{\times 10^n}\ \boxed{-}\ \boxed{9}$$

3 Re-Arranging Equations

Whatever is done to one side of an equals sign must be done to the other also. Take, for example, the equation:

$$a = b + c$$

a is the subject. To make b the subject, one must look at what is done to b and do the ________ to both sides. In the above equation, c is added to b, so b is made the subject by ____________ c from both sides of the equals sign:

- Subtracting c: $a - c = b + c - c$
- Simplifying the right hand side: $a - c = b$
- Writing b as the subject: $b = a - c$

Addition and ____________ are inverse operations.

______________ and division are inverse operations.

Powers and ______ are inverse operations.

Example 1 – Make y the subject of $x = 2 \times y + z$

The last operation on y is the addition of z, so subtract z from both sides:

$$x - z = 2 \times y$$

y is multiplied by 2, so divide both sides of the equation by 2:

$$(x - z)/2 = y$$

Example 2 – Make g the subject of $5\sqrt{g} = h + j$

Divide by 5:

$$\sqrt{g} = (h + j)/5$$

Square both sides:

$$g = (h + j)^2/25$$

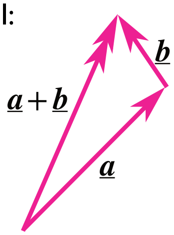
4 Vectors and Scalars

______ quantities have a magnitude (size) only, whereas ______ quantities have a magnitude and a direction.

Vectors can be represented graphically as ______. The ________________ indicates the magnitude of the vector. The ____________________ indicates the direction of the vector.

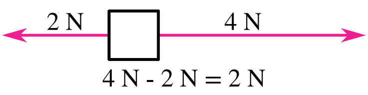
Quantity	Vector or Scalar?
Distance	Scalar
Time	Scalar
Displacement	Vector
Velocity	Vector
Acceleration	Vector
Speed	Scalar
Force	Vector
Gravitational potential energy	Scalar
Kinetic energy	Scalar
Momentum	Vector

When two vector quantities are added, the two arrows that represent the quantities are joined tip-to-tail:



Subtracting a vector is the same as adding a vector pointing in the opposite direction.

If two vectors are in opposite directions, they add to give a vector with magnitude equal to the _________ of the original vectors' magnitudes.



If two vectors are at right angles, the sum of their magnitudes can be calculated using ____________________.

5 Variables and Constants

Measurable quantities are either variables or constants. A ________ is a quantity whose value can change. A ________ is an unchanging quantity.

Commonly used constants include:

charge of the electron	-1.60×10^{-19} C
speed of light in a vacuum	3.00×10^{8} m/s

Some quantities *can* have different values (so they are ________), but within a particular experiment we do not expect their value to change. With these quantities, every effort should be taken to make sure their value remains as constant as possible. These are called ________________. Sometimes, deducing a value of a control variable and comparing this to an expected value is a useful way of testing the validity of the experiment. Common control variables include:

gravitational field strength at the surface of the Earth	9.81 N/kg taken as 10 N/kg at GCSE level
specific heat capacity of water	$4\,200$ J/(kg °C)
speed of sound in air	330 m/s
refractive index of glass	1.50

In any experiment, the value of one quantity must be systematically changed in order to measure its effect on another quantity. The quantity that the experimenter chooses to change is called the ____________________.

The quantity whose value changes in response to the change of independent variable value is called the ____________________.

Often, the independent variable and dependent variable values will be ______________ so that the relationship between the two can be deduced and predictions can be made and tested.

6 Straight Line Graphs

To be able to correctly predict the effect of changing one variable on the value of another, physicists write ________. Part of the process of writing an equation requires the physicist to draw a ______, which reveals how one variable relates to another. When drawing graphs, it is common practice to

plot the independent variable on the ______ (the ________ axis), and the dependent variable on the ______ (the ______ axis). Occasionally, it is more sensible to plot the variables on the axes the other way around. The equation for a straight line graph is:

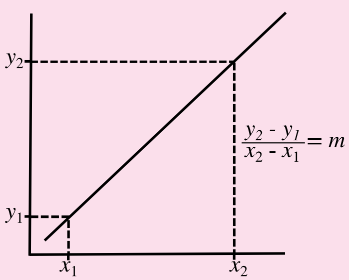
$$y = mx + c$$

where ____

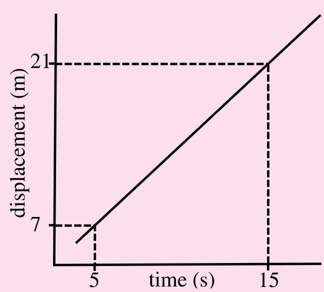
At GCSE level, the relationship between two chosen variables is often ______, which means a graph of one variable versus another produces a straight line graph and the above equation works. Most equations at GCSE level can be written in the form $y = mx + c$.

Example – If a student records every second how far something has travelled at constant speed, they can plot a graph distance on the y-axis and time on the x-axis. The gradient will be the speed.

The gradient of a straight line can be determined by considering two points on it:



e.g.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(21 - 7)\ \text{m}}{(15 - 5)\ \text{s}} = \frac{14\ \text{m}}{10\ \text{s}} = 1.4\ \text{m/s}$$

7 Proportionality

Physicists measure things, and then look for patterns in the numbers.

The most important pattern is called ____________ (also called direct proportion). If distance is proportional to time, it means that if the time doubles, the distance will _________. If the distance gets 10 times bigger, the time will get __________________. Mathematically, this is written as $s \propto t$.

Example 1 – A particular resistor passes a 25 mA current when the voltage across it is 5.5 V. If voltage is proportional to current, what will the voltage be when the current is 60 mA?

The new current is (60 mA / 25 mA) $= 2.4$ times larger than the old one. The new voltage will be 2.4 times larger than the old one: 5.5 V $\times$ 2.4 $=$ 13.2 V.

Using a formula

If s is proportional to t then s/t will always have the same value. If we call this fixed value k, it follows that $k = s/t$, and that $s = kt$. We can use this information to answer questions. The formula method is much clearer if there are more than two quantities involved.

Example 2 – A spring obeying Hooke's Law (its extension is proportional to the force) stretches by 14 mm when a 7.0 N load is applied. How far will it stretch with a 3.0 N load?

We write force $= k \times$ extension, so extension $=$ force$/k$.
$k =$ force$/$extension $= 7.0$ N$/14$ mm $= 0.50$ N/mm
For a 3.0 N load, extension $=$ force$/k = 3.0/0.50 = 6.0$ mm.

Example 3 – The energy transferred by an electric circuit in a fixed time is proportional to the voltage and also to the current $(E \propto V \times I)$. If the current is 3.2 A, and the voltage is 15 V and the energy transferred is 340 J. What current will be needed if we need to deliver 640 J using 12 V in the same time?
The equation is $E = kIV$, so $k = E/(IV) = 340$ J$/(3.2$ A $\times$ 15 V$)$ $=$

7.08 J/(AV)
Re-arranging gives $I = E/(kV) = 640/(7.08 \times 12) = 7.53 = 7.5$ A (2sf)

Inverse Proportionality

The time taken on a journey is inversely proportional to the speed. If you double the speed, the time ______. If you only go at a tenth of the speed, it takes ____________. We write this as $t \propto 1/v$, where v is the speed. In this case, $v \times t$ always has the same value.

Example 4 – The number of books printed each day is proportional to the number of printers owned, and inversely proportional to the number of pages in each book.
If $3\,000$ 300-page books can be printed in one day on 8 printers, how many 125-page books can they print on 6 printers in a day?

As books $\propto$ printers, and books $\propto 1/$pages,
then books $= k \times$ printers/pages.
$k =$ books $\times$ pages/printers $= 3\,000 \times 300/8 = 112\,500.$
books $= k \times$ printers/pages $= 112\,500 \times 6/125 = 5\,400.$

Additional Proportionality Questions

Mechanics

8 Speed, Distance and Time

When we study motion, ________ is a scalar quantity that is equal to how far an object has moved. It is measured in _______ in SI units. Other units include centimetres, inches, yards, miles and lightyears.

Time is central to the study of motion. It is measured in ________ in SI units. Other units include minutes, hours and days.

______ is a scalar quantity that is equal to how far an object has moved divided by the time taken. It is measured in __________________ in SI units. Other units include miles per hour, parsecs per jubilee and feet per Julian year: Any speed unit using a distance unit divided by a time unit is valid.

The equation for average speed is:

$$\text{average speed} = \text{total distance} / \text{total time} \quad [v = s/t]$$

In the equation, physicists use v for speed and s for distance. These symbols are useful for more advanced mechanics. Always define your symbols.

Typical speeds are: Walking: 1.5 m/s Running: 3 m/s Cycling: 6 m/s

Additional Speed, Distance and Time Questions

9 Displacement and Distance

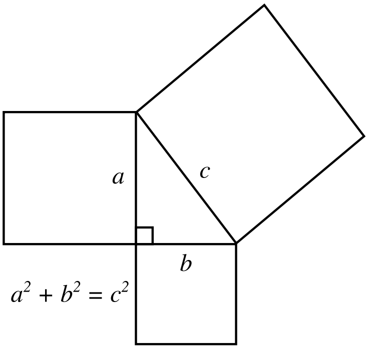
The straight line distance between an object's starting point and its end point - together with the ________ - is called its ____________. The length of the path along which the object moves is the ________.

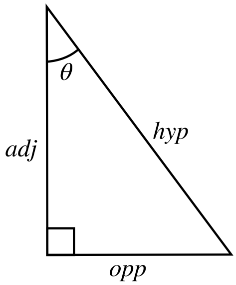
Displacement is a ______ since it has a direction associated with it. Distance is a _____; see Section 8.

Distance and displacement are both measured in metres (m) in SI units.

If an object moves in a circle, after one complete rotation, the displacement will equal ____ and the distance travelled will equal the ________________________.

If an object is displaced in two perpendicular steps, the magnitude of the displacement can be calculated using ___________________ and the direction can be calculated using trigonometry.





$$\sin\theta = \frac{\text{distance opposite the angle}}{\text{distance along the hypotenuse}}$$

$$\cos\theta = \frac{\text{distance adjacent to the angle}}{\text{distance along the hypotenuse}}$$

$$\tan\theta = \frac{\text{distance opposite the angle}}{\text{distance adjacent to the angle}}$$

10 Motion Graphs; Displacement–Time (s–t)

A displacement-time graph has displacement on the __-axis (the ________ axis) and time on the __-axis (the ___________ axis). The gradient of the line at any point is the ________________________.
To review gradient calculations, see Straight Line Graphs - P14.

Take particular care of the unit for the gradient. It will be equal to the unit on the __-axis divided by the unit on the __-axis. For example, if displacement is measured in ____ on the __-axis and time in _________ on the __-axis, the gradient would have units of ________________.

When displacement is on the y-axis, the direction of the displacement is ______ to the direction of the velocity, unless the gradient has a negative value, in which case the direction of the velocity is __________ to the direction of the displacement.

When distance is on the y-axis instead of displacement, the gradient equals _______ instead of velocity.

11 Acceleration

Acceleration means that there is a change of velocity – a change of speed or a change of direction of motion.

This could mean

- ____________
  - when the acceleration is in the same direction as the motion
- ______________________________________
  - here the acceleration is in the opposite direction to the motion
- ____
  - here the acceleration is at right angles to the motion

We measure acceleration in _________________________________ An acceleration of 3 m/s^2 means that each second the velocity changes by _____.

$$\text{acceleration (m/s}^2) = \text{change in velocity (m/s)} / \text{time taken (s)}$$

$$a = (v - u)/t$$

When the velocity changes we use u for the velocity at the start, and v for the velocity at the end.

> Example 1 – A car is travelling at 3.0 m/s. It accelerates at 2.5 m/s^2. How fast is it going 5.5 s later?
>
> Change in velocity $= a \times t = 2.5$ m/s^2 $\times$ 5.5 s $= 13.75$ m/s
> New velocity $= 3.0 + 13.75 = 17$ m/s (2sf)

> Example 2 – A car at 31 m/s stops in 6.8 s. Calculate the deceleration.
>
> Acceleration = $(v - u)/t =$ $(0\text{ m/s} - 31\text{ m/s})/(6.8\text{ s}) =$ $(-31\text{ m/s})/\,(6.8\text{ s}) = -4.56$ m/s^2 so deceleration = 4.6 m/s^2 (2sf)
> Here the velocity change is negative as the final velocity (0 m/s) is lower than the starting velocity (31 m/s), thus is a deceleration.

Example 3 – A car starts from rest. It accelerates backwards until it is reversing at 4.0 m/s. This takes 5.0 s. Calculate the acceleration.

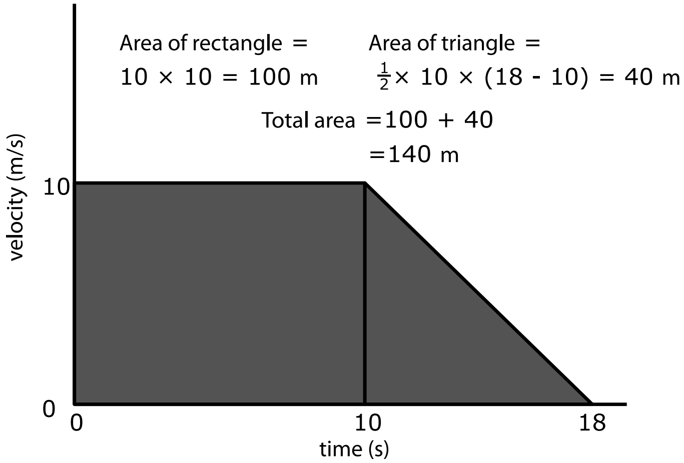
Acceleration = $(v - u)/t = (-4.0\,\text{m/s})/(5.0\,\text{s}) = -0.80\,\text{m/s}^2$.
The change in velocity is negative as the final velocity (-4.0 m/s) is lower than the starting velocity (0 m/s). However, although the acceleration is negative, this is not a deceleration as the car is speeding up (backwards).

12 Motion Graphs; Velocity–Time (v–t)

The displacement of an object moving with a constant velocity is equal to the product of the ________ and the amount of ____________________________.

To find the displacement when the velocity is changing, a velocity-time graph is needed. Normally, velocity is plotted on the _-axis (the _______ axis) and time is plotted on the _-axis (the __________ axis).

The area under the line on a velocity-time graph is equal to the _____________ of the object.



If the shape of the graph can be broken into simple geometric shapes, the total area under the line can be calculated by adding _________________________.

The area under a speed-time graph is the distance. Speed cannot be negative, and neither can the distance.

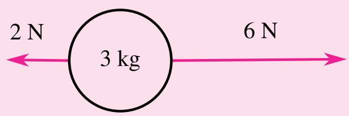
The area under a velocity-time graph is the displacement. Velocity can be negative if an object is moving backwards. The displacement can also be negative. An area beneath the x-axis has a negative value. An area above the x-axis has a positive value. Be careful when calculating the total displacement, when summing the displacements remember to _______ the $+$ and $-$ signs of the displacements.

13 Resultant Force and Acceleration

The resultant force on an object is:

- the force left over after equal and opposite forces have ___________;
- the one force which would have the same effect as ___________;
- the ___________ of the forces on the object.

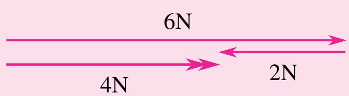
Example 1 – Calculate the resultant force on this object.



2 N force to left cancels out 2 N of the 6 N of the right force, leaving $6\text{ N} - 2\text{ N} = 4\text{ N}$ to the right left over.

Or you can answer: The two forces are $+6$ N and -2 N. Adding gives 4 N.

Or you can add the vector arrows 'nose to tail' to get a resultant 4 N answer:



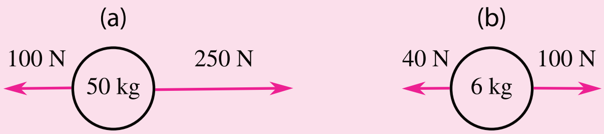
[A double arrow symbol here denotes a resultant vector.]

If you need more practice, turn back to Vectors and Scalars - P9 and try to balance the forces in Q4.9.

The acceleration of an object depends on the:

- ___________
- ___________

Example 2 – Which of these objects will have the greater acceleration?



(a) has resultant 150 N to the right, acting on 50 kg of mass. This means 150 N / 50 kg = 3 N/kg, i.e. 3 N acting on each kilogram.
(b) has resultant 60 N to the right, acting on 6 kg of mass. This means 60 N / 6 kg = 10 N/kg, i.e. 10 N acting on each kilogram.
Therefore, object (b) will have the greater acceleration.

Formula:

$$\text{acceleration } (\text{m/s}^2) = \text{resultant force } (\text{N}) \ / \ \text{mass } (\text{kg}) \qquad a = F/m$$

Usually written:

$$\text{resultant force } (\text{N}) = \text{mass } (\text{kg}) \times \text{acceleration } (\text{m/s}^2) \qquad F = ma$$

A resultant force in the direction of motion ____________________.
A resultant force opposite to the direction of motion ______________.
Zero resultant force means that the object ______________________.

Additional Resultant Force and Acceleration – on-line

14 Terminal Velocity

A falling object in the air, which is not influenced by wind or other sideways forces, has a maximum of ____ forces acting on it: _______ and ____________ (also known as drag). The weight ________________. The air resistance is ____ when the object is stationary but ________________________________.

The resultant (net) force the falling object experiences is equal to the force of gravity minus the air resistance. [$F = mg - Drag$]

Newton's Second Law states that the acceleration of the object is _______________ the resultant force on the object when its mass is ________.

- The longer the object falls for, ____,
- but then the greater the ____________, which increases with speed.
- Eventually, however, the air resistance upwards will equal the ______________ downwards.
- At this point, the resultant force is ____,
- hence, the velocity remains constant. This is called the ________________ of the object.

An object with a constant force acting on it in the direction it is travelling and a frictional force (related to the object's velocity) acting in the opposite direction, will reach terminal velocity given enough time. This is true for cars driving along a road or an anchor falling through the water towards the sea floor.

15 Stopping With and Without Brakes

Formulae:

$$\begin{aligned} \text{distance travelled} &= \text{average speed} \times \text{time} & s &= vt \\ \text{resultant force} &= \text{mass} \times \text{acceleration} & F &= ma \\ \text{change in velocity} &= \text{acceleration} \times \text{time} & v - u &= at \\ \text{kinetic energy} &= \tfrac{1}{2} \times \text{mass} \times \text{speed}^2 & E &= \tfrac{1}{2}mv^2 \\ \text{energy transfer} &= \text{force} \times \text{distance} & E &= Fs \end{aligned}$$

Data:

To convert miles / hr to m / s, multiply by $1\,609/3\,600 = 0.447$.

With Brakes

The shortest distance taken to stop a car from the moment when the driver first notices a problem is called the ________________. This is made of two parts – the distance the car travels while the driver reacts and first applies the brakes ____________________, and the distance the brakes take to stop the car ____________________.

The Highway Code estimates that a typical reaction time of a driver is two thirds of a second; and that once applied, brakes will give a car a 6.67 m/s^2 deceleration. This reaction time may seem very long - but it takes into account the fact that during a long drive a driver may not be fully alert, and that the action of moving your foot from the accelerator to the brake pedal and stamping takes longer than pressing a button with your finger.

Example 1 – Calculate the thinking distance at 30 mph.

Conversion: 30 mph $= 30 \times 0.447 = 13.4$ m/s.
Thinking distance $=$ reaction time $\times$ speed $= 0.667$ s $\times$ 13.4 m/s $=$ 8.9 m.

Example 2 – Calculate the braking distance from 30 mph.

Conversion: 30 mph $= 30 \times 0.447 = 13.4$ m/s.
Velocity drop $=$ deceleration $\times$ braking time, so
braking time $=$ velocity reduction / deceleration $= 13.4/6.67 = 2.0$ s.
Average speed on decelerating from 13.4 m/s to 0 m/s is $(13.4 + 0)$ / 2 $= 6.7$ m/s.
Braking distance $=$ braking time $\times$ average speed $= 2.0$ s $\times$ 6.7 m/s $= 13.4$ m.

In your answer to (c), going at twice the speed, you cover ________________ during your reaction time, as the reaction time ______________.
In your answer to (d), going at twice the speed, it takes you ____________________.
However, you are going twice as fast, leading to an overall multiplication by _.

$^{16}/_{20}$

In wet conditions, drivers should allow *at least* twice these distances.

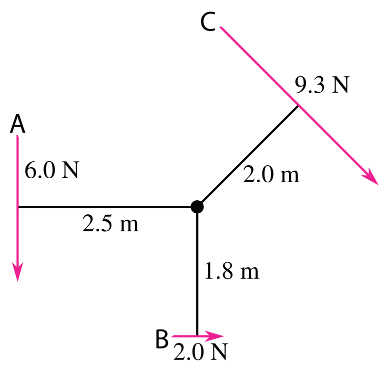
Without Brakes

16 Moments, Turning and Balancing

The turning or twisting effect of a force is called its ________. The moment of a force depends on:

- the ____ of the force;
- ________ the force is from the pivot or axle (the distance is measured from the pivot to the line of action of the force, at right angles to the force);
- the _________ of the force. Moments can be anticlockwise (AC) or clockwise (C).

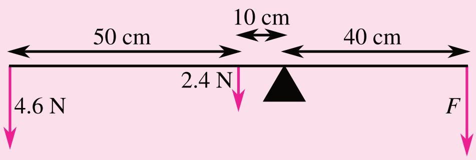
moment (Nm) $=$ force (N) $\times$ perpendicular distance to axle (m)



Example 1
Moments of the forces in the diagram are:
A: 6 N $\times$ 2.5 m $=$ __ Nm AC
B: 2 N $\times$ 1.8 m $=$ ___ ____ AC
C: 9.3 N $\times$ 2 m $=$ __________
In this case, the two AC moments added together equal the C moment. Because they are equal and in opposite directions, the system will not turn.

Principle of Moments:
An object will ________ and not start turning if the total of the anticlockwise (AC) moments _______ the total of the clockwise (C) moments.

Example 2 – What force, F, is needed to make the rod balance?



Moment of a 4.6 N force is 4.6 N $\times$ 60 cm $=$ 276 Ncm AC
Moment of ______________________________
Total AC moment is ___ Ncm. To balance, the moment of F must be ________ clockwise, so
$F =$ ____________________
In this question, the 2.4 N force is the weight of the rod. Weights are always drawn downwards from the centre of gravity - which is always in the centre of symmetric, uniform objects like rods.

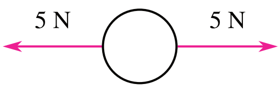
17 Pressure, Hydraulic Systems, Density and Depth

$$\text{pressure} = \text{force (N)} / \text{area (m}^2\text{)} \qquad p = F/A$$

The unit of pressure is the pascal (Pa). 1 Pa = 1 N/m^2

Example 1 – What is the pressure on a wall when a drawing pin, with a point of cross sectional area of 2.0 mm^2, is pushed in with a force of 8.0 N?
Pressure $=$ force / area $= 8.0$ N / 2.0 mm^2 $= 4.0$ N/mm^2.
Notice that 1 mm$^2 = 1$ mm $\times\ 1$ mm $= 10^{-3}$ m $\times\ 10^{-3}$ m $= 10^{-6}$ m^2.
Pressure $=$ force (N) / area (m^2) $= 8.0$ N / 2×10^{-6} m^2 $= 4 \times 10^6$ Pa.

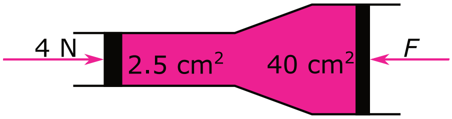
Pressure in fluids



A solid object will stay still if the _____ pulling from each side is equal.

A section of a fluid (liquid or gas) will stay still if the ________ on both sides is equal.

In a hydraulic system, two pistons of different area push on the same fluid, exerting pressures on it. The fluid is in equilibrium if ______________________.

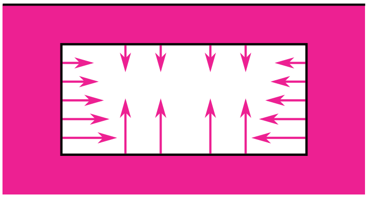


The pressure on the left $=$ 4 N / 2.5 cm^2 $=$ __________.
In equilibrium, the pressure on the right will also be __________, so the force on the right must be 1.6 N/cm^2 $\times\ 40$ cm^2 $= 64$ N.

Density ____

Density gives the mass of a material per cubic metre (or cubic centimetre). 1.00 kg of water has a volume of 0.00100 m^3. density = mass / volume = 1.00 kg / 0.00100 m^3 = 1000 kg/m^3.

Pressure at Depth



As you go deeper in a fluid, the pressure rises because of the increased weight of fluid above you. However, any surface in the fluid has a force on it regardless of its angle. A box held under water has forces on it from all sides, all pushing inwards at right angles to each surface.

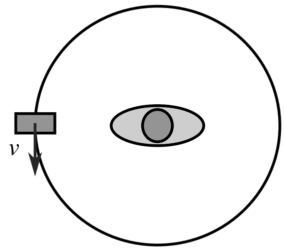
The formula for the extra pressure at a depth is:

To calculate the total pressure at that depth, the pressure at the surface (e.g. atmospheric pressure) must be added.

Example 2 – Calculate the total pressure at a depth of 8.0 m in oil of density 850 kg/m^3 if atmospheric pressure is 101 kPa.
Extra pressure $= \rho g h = 850$ kg/m^3 $\times\ 10$ N/kg $\times\ 0.8$ m $= 68\,000$ Pa $=$ 68 kPa
Total pressure $=$ pressure at surface $+\ 68$ kPa $=\ 101$ kPa $+\ 68$ kPa $=$ 169 kPa

18 Moving in a Circle

I swing a bung around in a horizontal circle above my head using a string. From above, it looks like this, and we can ignore the effect of gravity.



There is a force from the string acting on the bung (not shown in the diagram) acting ________________. Redraw the diagram above in your book to include this force.

The bung is neither speeding up nor slowing down, yet there is an unbalanced force acting on it. This force must therefore be ________________

- The bung's ________ is changing so that it can go round in a circle.
- This means that its ________ is changing, so it must be ________.
- This ________ requires a ________.

Any force which causes something to go round in a circle can be labelled as a ________ force. Three factors which affect the centripetal force are:

- ________________
- ________
- ________________

Formulae:

$$\text{centripetal acceleration} = \frac{\text{speed}^2}{\text{radius}} \qquad a = \frac{v^2}{r}$$

$$\text{centripetal force} = \text{mass} \times \text{acceleration} \qquad F = \frac{mv^2}{r}$$

The orbit of the Earth around the Sun is approximately circular. The force holding the Earth in this motion is the ________________, which acts as a ________________ in this case.

Draw another arrow on the diagram in your book to show where the bung would go next if the string were cut whilst the bung is at the position shown.

19 Introducing Momentum and Impulse

Momentum measures how much 'motion' an object has, taking into account its mass and velocity.

$$\text{momentum} = \text{mass (kg)} \times \text{velocity (m/s)} \qquad p = mv$$

The unit of momentum is ______________________________

The ____ of the momentum (plus or minus) tells you the direction.
In these one dimensional problems, positive momentum means 'travelling East' and negative momentum means 'travelling West'.

Momentum is a ______ – it has a direction.

> Example 1 – A 3.0 kg motion trolley is moving at 2.0 m/s East. A force of 4.2 N acts on it for 6.0 s.
> Acceleration = force / mass = ______________ ___ m/s_
> Velocity change = acceleration × time = ________________________
> New velocity = ______________________________
> Original momentum = mass × velocity = ______________________________
> New momentum = ______________________________
> Change in momentum = ____________________________________
> Notice that force × time = 4.2 N × 6.0 s = +25.2 Ns

The last line of the example suggests:

$$\text{change in momentum (kg m/s)} = \text{force (N)} \times \text{time (s)}$$

$$p_{\text{after}} - p_{\text{before}} = Ft$$

> Example 2 - First row of table above
> Momentum before $= mu = 1.0$ kg $\times\ 0.0 = 0.0$ kg m/s
> Momentum change $= Ft = 3.0$ N $\times\ 60$ s $= 180$ kg m/s
> Momentum afterwards $= 0.0 + 180 = 180$ kg m/s
> Velocity afterwards = momentum / mass $= 180 / 1.0 = 180$ m/s

Impulse

We define impulse $(\text{Ns}) = \text{force } (\text{N}) \times \text{time } (\text{s})$

so impulse $(\text{Ns}) = $ change in momentum (kg m/s)

So, a moving object with 400 kg m/s of momentum would need a 400 N force to stop it in one _______.

Newton's 2nd Law: resultant force $=$ rate of change of momentum.

Additional Introducing Momentum and Impulse Questions

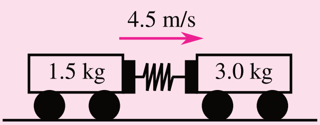
20 Momentum Conservation

$$\text{momentum (kg m/s)} = \text{mass (kg)} \times \text{velocity (m/s)} \qquad p = mv$$

$$\text{change in momentum (kg m/s)} = \text{force (N)} \times \text{time (s)} \qquad p_{\text{after}} - p_{\text{before}} = Ft$$

The sign of the momentum (plus or minus) tells you the direction.
In these one dimensional problems, positive momentum means 'travelling East' and negative momentum means 'travelling West'.

Example 1



Two motion trolleys are moving East. The spring expands, and pushes the trolleys apart. It pushes the 3.0 kg trolley forwards with a 2.5 N force for 1.2 s. What is the change in momentum change of each trolley?

Momentum change of 3.0 kg trolley = Ft = ______________________

By ______________, force on 1.5 kg trolley must be 2.5 N West (−2.5 N).

Momentum change of 1.5 kg trolley = Ft = _________________________.

The momentum gained by the 3.0 kg trolley is _____ to the momentum lost by the 1.5 kg trolley. So the total momentum ____________

So, when forces act between objects, their total momentum is conserved.

Example 2 – Calculate the new velocity of the 1.5 kg trolley.
Momentum = old momentum + change = $1.5 \times 4.5 - 3 = 3.75$ kg m/s
New velocity = momentum / mass = $3.75/1.5 = 2.5$ m/s (2.5 m/s East)

Example 3 – A 2.5 kg mass travelling at 2.5 m/s collides with and sticks to a 7.5 kg mass which is stationary. Calculate the velocity afterwards.
Total initial momentum: $2.5\text{ kg} \times 2.5\text{ m/s} + 7.5\text{ kg} \times 0\text{ m/s} = 6.25$ kg m/s
Total final momentum must be the same = 6.25 kg m/s
Final velocity = momentum / mass = $6.25 / 10.0 = 0.63$ m/s (2 sf)

21 Motion with Constant Acceleration

The equations we will develop and practise here can be used in any situation where the acceleration does not change. This includes:

- anything falling, providing ____________________;
- anything speeding up because an engine is providing a ________ on it;
- anything slowing down because brakes are providing a ________.

We start with three principles:

1. Displacement $=$ average velocity $\times$ time
2. Velocity change $=$ acceleration $\times$ time
3. If the acceleration is constant, then the velocity will rise steadily. This means that the average velocity will be half way between the starting and final velocities (it will be the mean of the starting and final velocities).

In this book, we use five letters to represent the quantities.

Letter	**Quantity**	**Unit**
s	Displacement	__
u	Starting velocity	____
v	Final velocity	____
a	Acceleration	_____
t	Time taken	_

We can write our three principles as equations using these letters. Firstly, the third principle means that average velocity $= 1/2\,(u+v)$.

1. $s = \left(\dfrac{u+v}{2}\right) t$
2. $v - u = at$

Now rearrange equation (2) to make v the subject; and then substitute this into equation (1). This gives

$$v = u + at \quad \text{so} \quad s = \left(\frac{u + u + at}{2}\right) t = \underline{\hspace{4cm}}$$

Next, rearrange equation (2) to make t the subject; and then substitute this into equation (1). Finally, rearrange it to make v^2 the subject. This gives

$$t = \frac{v - u}{a} \quad \text{so} \quad s = \left(\frac{u + v}{2}\right) \times \left(\frac{v - u}{a}\right) = \underline{\hspace{5cm}}$$

$$\text{so} \quad v^2 = \underline{\hspace{1.5cm}}$$

Let's look at our four equations, often given in examination formula sheets.

$v = u + at$	has no s	$v^2 = u^2 + 2as$	has no t
$s = \left(\frac{u+v}{2}\right) t$	has no a	$s = ut + \frac{1}{2}at^2$	has no v

Example 1 – An aeroplane requires a speed of 26 m/s to take off. If its acceleration is 2.3 m/s^2, how much runway does it 'use up' before it lifts off? Assume it starts at rest.

Using basic principles:
Time = velocity gained / acceleration = 26 m/s ÷ 2.3 m/s^2 = 11.3 s
Average velocity = $\frac{1}{2}$ (0.0 m/s + 26 m/s) = 13 m/s
Displacement = av. velocity × time = 13 m/s × 11.3 s = 150 m (2 sf)

Using the equations:

$u = 0$ m/s $\qquad v = 26$ m/s $\qquad a = 2.3$ m/s^2 $\qquad$ we want to know s

We use the equation with no t as we don't know t.
$v^2 = u^2 + 2as$, so $26^2 = 0^2 + 2 \times 2.3 \times s$
so $676 = 4.6 \times s$, so $s = 676/4.6 = 150$ m (2 sf)

Example 2 – How much time does it take a ball to fall 30 cm if it is accelerating downwards at 10 m/s^2 after being dropped?

NB: 'dropped' means it isn't moving to start with, so $u = 0$.

Using the equations:

$s = 0.30$ m $u = 0$ m/s $a = 10$ m/s^2 we want to know t

We use the equation with no v as we don't know v:
$s = ut + {}^1/_2\, at^2$, so $0.30 = 0t + {}^1/_2\, 10t^2$, so $0.3 = 5t^2$
$t^2 = 0.3/5 = 0.06$ so $t = \sqrt{0.06} = 0.24$ s

Using basic principles (where we use t to represent the time):
Velocity change $=$ acceleration $\times$ time $= 10t$
Final velocity $=$ initial velocity $+$ velocity change $= 0 + 10t = 10t$
Average velocity $= {}^1/_2\,(0 + 10t) = 5t$
Displacement $=$ average velocity $\times$ time $= 5t \times t = 5t^2 = 0.30$
so $t^2 = 0.30/5 = 0.06$ so $t = \sqrt{0.06} = 0.24$ s.

Electricity

22 Charge and Current

Electric charge is a property of matter. There are two types of electric charge, which are conventionally labelled ________ and ________. Two objects that have the same charge exert ________ forces on each other. Two objects that have opposite charges exert ________ forces on each other.

Electric charge is measured in ________ (C) and has the symbol Q.

Electric charge is ________, which means any object can only have an integer multiple of a certain value of charge. The smallest value of the magnitude of charge an object can have is equal to the magnitude of the charge of an ________, which is approximately ____________. Electrons are negatively charged. If a neutral object loses electrons, it becomes more ________ charged. If a neutral object gains electrons, it becomes more ________ charged.

Current is the ________________________. Current can be caused by the flow of electrons, ions or other charged particles. Electrons are negatively charged, so the direction electrons flow is the ________ direction to current.

The equation relating electric charge, current and time is:

$$\text{electric charge} = \text{electric current} \times \text{time} \quad [Q = It]$$

In an electric circuit, electric current flows from the ________ terminal of a power supply to the ________ terminal or ________, or from the ________ to a ________ terminal.

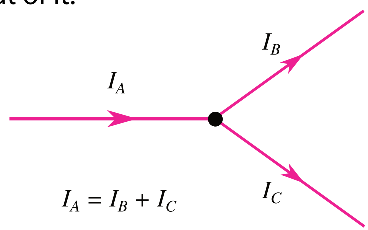
Electric current is measured in amperes (A) and has the symbol I.

Additional Charge and Current Questions

23 Current and Voltage - Circuit Rules

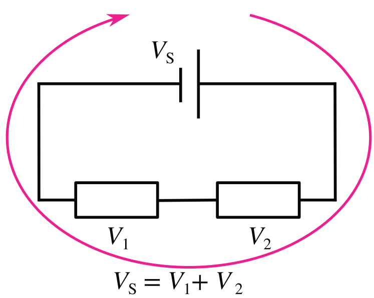
Current is the ______________________________. Current is not used up in a circuit; at all points in a series circuit, current has ________________.

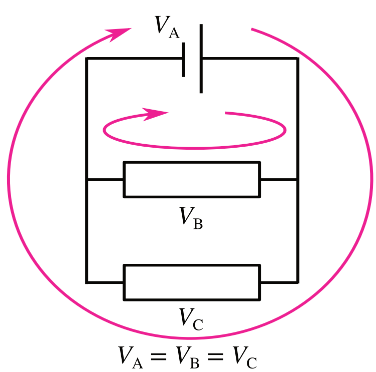
If a circuit has a branch, the current flowing into the junction must ______ the current flowing out of it.



In the diagram above, the value of Current A is equal to ________ of the values of Current B and Current C.

Voltage is also known as potential difference. The voltage across a component is the ____________________________ in driving the charge through the component. In a circuit loop, the sum of the voltages across the power supplies is always equal to ________ of the voltages across the rest of the components; see the left figure below.





In the right diagram above, the value of the voltage across the cell is ______ to the value of the voltage across the top resistor (the top loop of the circuit) and also to the value of the voltage across the bottom resistor (the bottom loop of the circuit).

This means components in _______ have equal voltage, and components in _____ divide the available voltage between them.

In these questions, assume that voltmeters and ammeters are perfect. A perfect voltmeter carries no current, and a perfect ammeter has zero potential difference across it.

24 Resistance

Resistance measures how difficult it is for electric current to pass through a component (or through an object) for an applied voltage. A resistor is a circuit component that dissipates energy thermally when work is done in driving a current through it.

So an iron nail has ___ resistance than a plastic pen.

Resistance is measured in ohms (Ω – the upper case Greek letter "omega").

Formula:

resistance (Ω) $=$ voltage across component (V)/current through it (A)

$$R = V/I, \text{ so}$$

$$V = IR$$

Notice from your answer from Q24.3 that the equation $V = IR$ also works if you measure I in mA and R in kΩ. This is useful as the mA and the kΩ are more convenient units in electronics than the amp and ohm.

25 Characteristics

Component characteristic graphs can be used to predict the amount of current drawn by an electrical component when a certain ________________ is across it. With these two values, the resistance of the component can be calculated using the equation,

$$\text{resistance} = \text{voltage}/\text{current} \qquad [R = V/I]$$

A voltage-current graph that is a straight line through the origin shows that the __________ of the component is independent of the potential difference across it, or the current flowing through it.

A graph with a curved line shows that the __________ depends on the potential difference applied across it; the __________ does not have a ______________.

Characteristic graphs are typically drawn with the current (I) on the ______ (the _______ axis) and the potential difference (V) on the ______ (the __________ axis), although they can also be drawn the other way around.

A negative value for V means that the supply is connected to the component the other way round. You then get a negative value for I meaning that the current is now flowing the opposite way through it.

26 Power Calculations

Power is the ______________________________, or the ______________________.
It is calculated using the equation:

$$\text{power} = \text{work done}/\text{time} \qquad \left[P = \frac{W}{t}\right]$$

$$\text{or} \quad \text{power} = \text{energy transferred}/\text{time} \qquad \left[P = \frac{E}{t}\right]$$

The unit of power is the watt (W).

$$1 \text{ watt} = 1 \text{ joule per second}$$

$$1 \text{ W} = 1 \text{ J/s}$$

Electrical power

Potential difference, or voltage, across a component is the amount of __________ per ____, i.e.

$$\text{potential difference} = \text{work done}/\text{charge} \qquad \left[V = \frac{W}{Q}\right]$$

Electric current is the amount of __________________________ per _______, i.e.

$$\text{current} = \text{charge}/\text{time} \qquad \left[I = \frac{Q}{t}\right]$$

Multiplying these quantities together gives:

$$I \times V = \left(\frac{Q}{t}\right) \times \left(\frac{W}{Q}\right)$$

The Qs cancel, giving:

$$I \times V = \frac{W}{t} = P$$

which is equal to power (first equation on the page). So, the equation for electrical power is:

$$\text{power} = \text{current} \times \text{potential difference} \qquad [P = I \times V]$$

27 Resistance and Power

Equations:

$$\text{voltage} = \text{current} \times \text{resistance} \qquad V = IR$$
$$\text{power} = \text{current} \times \text{voltage} \qquad P = IV$$

Example 1 – Calculate the power dissipated in a 6.0 Ω resistor carrying 3.5 A.
Voltage $= IR = 3.5$ A $\times$ 6.0 $\Omega = 21$ V
Power $= IV = 3.5$ A $\times$ 21 V $= 73.5$ W $= 74$ W (2 sf)

Eliminating V, we have:

$$\begin{aligned} P &= I \times V = I \times (IR) = I^2R : \quad \text{rearranging gives} \\ I^2 &= P/R \quad \text{and} \quad R = P/I^2. \end{aligned}$$

Example 2 – Calculate the resistance of a heater if it needs to carry 13 A when dissipating 3 100 W.
$R = P/I^2$, so $R = 3100/169 = 18\ \Omega$ (2 sf)

Eliminating I, we have:

$$\begin{aligned} P &= I \times V = (V/R) \times V = V^2/R : \quad \text{rearranging gives} \\ V^2 &= PR \quad \text{and} \quad R = V^2/P. \end{aligned}$$

Example 3 – Calculate the power dissipated when a 200 Ω resistor is connected to a 240 V supply.
$P = V^2/R = 240^2/200 = 290$ W (2 sf)

Example 4 – Calculate the resistance of a 50 W light bulb connected to a 12 V supply.
$R = 12^2/50 = 2.9\ \Omega$ (2 sf)

28 E-M Induction and Generators

When a wire is moved in a magnetic field, a voltage is ________, providing that the wire is moved so that it cuts the ____________________.

You can reverse the direction of the voltage by

- moving the wire in the ___________________, or by
- reversing the direction of the _______________.

You can increase the voltage by

- moving the wire _____________, or by
- using a stronger _______________.

When a magnet is moved into a coil of wire, a voltage is ________.
You can make it larger by

- moving the magnet _____________,
- using a _________________, or by
- using a coil with more _____________ on it.

You can reverse the direction of the voltage by

- moving the magnet in the ___________________, or by
- using a magnet ___________________________.

In both cases, the voltage is _____________ to the magnetic field strength, the speed of movement and the length of wire in the field. The energy to make the electricity comes from the _______ and not the magnetic field itself. Generators turn ______________ into _________________ – they do not reduce the energy stored in the magnetic field of the magnet. This means that if there is no relative motion (the magnet is stationary in the coil, or the wire is stationary in the magnetic field) no voltage is induced - no matter how strong the field is (providing the field strength is not changing).

29 Transformers

While a very strong magnet held stationary inside a coil will not induce a voltage (there is no ______ energy), if the magnetic field ____________ a voltage will be induced. This is because the increase in magnetic field at the coil could have been caused by an ordinary magnet ____________. Permanent magnets cannot change strength readily, but you can change the strength of an _____________ if you change the ______ flowing in it.

This is the principle of the transformer. Transformers only work on ____________
The current in the primary coil causes it to become an electromagnet. The continually changing current produces a ___________________ magnetic field in an iron core. This in turn induces a continually changing _______ in the nearby secondary coil wound round the iron core. A transformer won't work on _________________ because a stationary magnet will only produce a steady magnetic field - and steady, stationary magnetic fields do not ______________.
A transformer does not change the frequency of the alternating current.

Transformers have two coils

- the _______ coil, connected to an a.c. supply of known voltage, and
- the _________ coil, which supplies electrical energy to other components using energy from the primary.

The voltage across the secondary coil, V_s, is not usually the same as the primary coil's supply voltage, V_p. It could be greater (a _______ transformer) if the number of turns on the secondary is greater, $N_s > N_p$, or less if the number of turns on the secondary is fewer, $N_s < N_p$.

$$\text{secondary (a.c.) voltage} = \text{primary (a.c.) voltage} \times \frac{\text{no. of turns on secondary}}{\text{no. of turns on primary}}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \qquad \text{or} \qquad \frac{N_p}{V_p} = \frac{N_s}{V_s}$$

This means that the number of 'turns per volt' is the same on both coils.

Example 1 – A transformer has an input voltage of 240 V a.c. and output of 48 V. If there are $3\,000$ turns on the primary coil, how many are there on the secondary?

$V_{\mathrm{s}}/V_{\mathrm{p}} = N_{\mathrm{s}}/N_{\mathrm{p}}$, so $48/240 = N_{\mathrm{s}}/3\,000$.
Thus $0.2 = N_{\mathrm{s}}/3\,000$, so $N_{\mathrm{s}} = 0.2 \times 3\,000 = 600$ turns.
Or, you could solve it like this: the primary coil has $3\,000/240 = 12.5$ turns/volt
So the secondary must have $48 \times 12.5 = 600$ turns.

Energy

Energy analysis determines whether some processes are possible. It involves calculating the amounts of energy stored in different places and in different ways. An energy analysis is one of many ways of examining physical processes. If we want to explain how a microphone works, there is lots to discuss but little benefit from mentioning energy. However, if we want to know how much fuel is needed to lift a satellite into space, then we must perform calculations based on energy.

You will calculate energy as it is stored: thermally, gravitationally, elastically, as kinetic energy, and as nuclear energy. Whilst the energy stored in these different ways may differ before and after a physical change, the **total** energy is the same.

30 Thermal Energy

Hot objects (or substances) store energy ________.

The energy is associated with the _______, thermal _______ of a substance's __________________. The physical processes of __________________________________ can result in increases or decreases of a thermal energy store.

If two objects at different ____________ are in contact with each other, then energy is __________________ their particles. After some time the thermal energy store of the hotter object will be _________ (and its temperature will have _________) and the thermal energy of the cooler object will be _________ (and its temperature will have _________). This thermal process is called __________. When the objects reach the same ___________ we say they are in __________________ and the thermal energy of each remains constant.

Hot objects emit _______________________. After a period of time the thermal energy store associated with a hot object will have _________ (and the temperature of the hot object will be _____ than it was).

Thermal energy is measure in ________.

Heating involves the _______ of thermal energy from a _______ object to a ______ object.

The amount of thermal energy required to increase the ___________ of an object (of a certain substance) by $1\ ^\circ$C is called the ____________. The heat capacity per kilogram is called the ___________________ (of that substance).

change in thermal energy $=$ mass $\times$ specific heat capacity $\times$ change in temp.

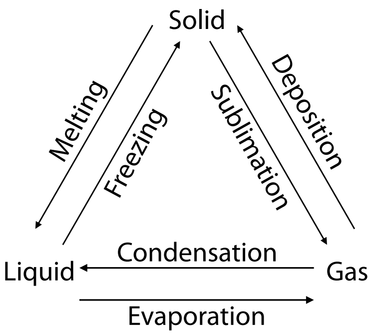
$$\Delta Q = mc\Delta T$$

Specific heat capacity is measured in J/(kg $^\circ$C) or the equivalent unit J/(kg K).

The specific heat capacity of pure water is _____________.

Additional Thermal Energy Questions

31 Latent Heat



The three most commonly encountered states of matter are ____________.

When a substance changes state, it does not change ________ but thermal energy is still transferred.

The energy needed to change the state of a substance is called ________.

Specific latent heat of fusion, L, is the energy transferred from 1 kg of a substance changing from __________ at a ________ pressure. [unit: J/kg]

Specific latent heat of vaporisation is the energy transferred to 1 kg of a substance changing from __________ at a ________ pressure.

Equation:

thermal energy transferred for a change of state $=$ mass $\times$ specific latent heat

$$Q = mL$$

	Latent heat of fusion	**Latent heat of vaporisation**
Melting	____________	
Freezing	____________	
Evaporating		____________
Condensing		____________

Example – The specific latent heat of fusion of ice is 3.36×10^5 J/kg.

How much thermal energy is transferred to melt 2.00 kg of ice?
$Q = mL = 2.00 \times 336\,000 = 672\,000$ J $= 672$ kJ

32 Payback Times

Domestic photovoltaic solar panels or small scale wind turbines are popular additions to many people's homes. Given the ever rising cost of fossil fuels and their environmental impact, individuals are investing significant sums of money in the hope of saving money in the long run.

____________: the time to save as much money as the initial investment.

Example 1 – A wind turbine costs £1 000 including installation. Since its installation, it has saved the owner £50 per year on their electricity bill. How long will it take for the owner to be in profit?
£1 000/£50 per year $=$ 20 years.

Example 2 – A wind turbine can generate an average of 100 W throughout the year [1 W = 1 J/s; see section 33]. Electricity suppliers charge for each kilowatt-hour (kW h):
1 kW h = 1 kW x 1 hour = 1000 W x 3600 s = 3.6 $\times 10^6$ J
The owner usually pays 20p per kW h. If she wants to be "in profit" within 5.0 years, what is the maximum cost of the turbine?

100 W/1 000 $\times$ 365 days $\times$ 24 hours per day $=$ ________________

____

____

Assume one year $=$ 365 days in the following questions.

33 Doing Work, Potential Energy and Power

Doing work always requires a force. However, applying a force does not necessarily mean that work is done.

A force does need an energy supply if the force's point of application is ____________

Example of a force which does require an energy supply:

____

Example of a force which does not require an energy supply:

____

If the force is in the same direction as the motion, ____________ and the object will speed up (unless other forces act).

If the force on the object is in the opposite direction to its motion, ____________ and it will slow down (unless other forces act).

When work is done, one energy store will decrease, and another will increase. Work is measured in joules (J) - the same unit as energy.

If the force is perpendicular to the motion, the object ____________ No _____ is done, and there is no ____________. However the object will change ________ and accelerate.

$$\text{work done} = \text{force} \times \text{distance moved parallel to force}$$

$$E = Fs$$

So, lifting a 1 N weight 1 m upwards requires work of 1 J.
Lifting a 1 N weight 2 m upwards requires work of 2 J.
Lifting a 2 N weight 2 m upwards requires work of 4 J.
Lifting a 10 N weight 4.0 m upwards onto a shelf, and then sliding it sideways by 2.0 m against a friction force of 2.5 N requires work of $40 + 5 = 45$ J.

The energy change each second is called the power, measured in watts (W).

$$\text{power} = \text{energy transfer} / \text{time}$$

$$P = \frac{E}{t} \quad \text{or} \quad P = \frac{W}{t}$$

Example 1 – Calculate the power needed to push a car 12.5 m along a road with a force of 2 340 N in 15.0 s.

Work done $= Fs = 2\,340$ N $\times$ 12.5 m = 29 250 J
Power $= E/t = 29\,250$ J/15.0 s $= 1\,950$ W

Example 2 –Calculate the energy transfer when a 20 kg sack of flour is winched 13.5 m upwards in a mill.

Force = weight = mass $\times g = 20.0$ kg $\times$ 10 N/kg = 200 N
Energy transfer $= Fs = 200$ N $\times$ 13.5 m $= 2\,700$ J

Notice that the work done during lifting equals the increase in gravitational potential energy.

$$\text{gravitational potential energy (GPE)} = \text{mass} \times g \times \text{height}$$

$$E = mgh$$

Q33.8(d) should show you that there is another useful equation:

$$\text{power} = \text{force parallel to motion} \times \text{speed}$$

$$P = Fv$$

34 Kinetic Energy

The kinetic energy associated with a ________ object depends upon ____________________.

Kinetic energy is a ______ quantity, which means that it ____________________________

Kinetic energy is measured in _________.

Numerically, if an object has 400 J of kinetic energy it will require a 400 N force to stop it in ____ as work done = force $\times$ distance ($W = Fd$).

Formula:

$$\text{kinetic energy} = \tfrac{1}{2} \times \text{mass} \times \text{speed}^2$$
$$E = \tfrac{1}{2}\,mv^2$$

(you can see where this formula comes from if you do Q34.9)

Suppose an object has 400 J of kinetic energy.

- The energy of an object with twice the mass, but the same speed, would be _______________ because kinetic energy is proportional to _____.
- The energy of an object with twice the speed, but the same mass, would be ____________________ because kinetic energy is proportional to _________________________.

Example 1 – A 2.00 kg carton of milk is falling at 2.50 m/s. Calculate its kinetic energy.

E = $\frac{1}{2}\,mv^2 = \frac{1}{2} \times 2.00$ kg $\times$ (2.50 m/s)^2 = 6.25 J

Example 2 – A 4.50 kg rolling skateboard has a kinetic energy of 32.0 J. How fast is it going?

E = $\frac{1}{2}\,mv^2$, so 32.0 J = $\frac{1}{2} \times 4.50$ kg $\times\, v^2$
Therefore $32.0 = 2.25v^2$

$32/2.25 = v^2 = 14.2$, so $v = 3.77$ m/s

Example 3 – How much force will it take if you wish to stop a 930 kg car going at 14.5 m/s in a distance of 23.0 m?

E = $\frac{1}{2}\,mv^2 = \frac{1}{2} \times 930$ kg $\times$ $(14.5$ m/s$)^2 = 97\,800$ J
Energy transferred $=$ force $\times$ distance, so $97\,800$ J $= F \times 23.0$ m,
$F = 97\,800$ J$/23.0$ m $= 4\,250$ N

35 Efficiency

To carry out an energy analysis of a physical event or process, we need to identify a clear start point and an end point. We then consider and calculate the changes in calculate the changes in the energy stores at the start point and at the end point.

It is always true that there is no overall change in the total of all the energy stores - energy is conserved.

However, it is often the case that a process results in an overall increase in less-useful thermal stores (and a corresponding decrease in the total of the more-useful stores). What is meant by the word 'useful' depends on the situation. Sometimes it will be fairly obvious; sometimes you may be told; in some situations, you may have to think carefully.

For any given process (or system) we can calculate its efficiency. Efficiency has no units. It is usually written as a decimal (generally between 0.00 and 1.00), as a fraction or as a percentage.

$$\text{efficiency} = \text{useful energy transferred} / \text{total energy transferred}$$

To express efficiency as a percentage, multiply the decimal answer by 100.

Sometimes the total energy transferred is the total electrical or mechanical work done.

Example 1 - An electric current drives an electric motor to raise a 25 N weight by a vertical distance of 1.2 m. The electrical work done by the power supply is 47 J. Calculate the efficiency of this process.

$$\text{efficiency} = \text{useful energy transferred} / \text{total energy transferred}$$

$$= \text{GPE gained (or work done against gravity)} / \text{electrical work done}$$

$$= 25 \times 1.2/47 = 0.64 \ (2\text{sf}) \quad \text{or} \quad 64\%$$

Example 2 - A battery powered motor is used to lift a load. As the load is lifted, the increase in gravitational potential energy is 230 J. The decrease in the energy stored chemically in the battery is 290 J. Calculate the efficiency of this process.

$$\text{efficiency} = \text{useful energy transferred} / \text{total energy transferred}$$

$$= \text{increase in gravitational store} / \text{decrease in chemical store}$$

$$= 230/290 = 0.79\ (2\text{sf}) \quad \text{or} \quad 79\%$$

Efficiency can also be calculated by considering power for a process. Then

$$\text{efficiency} = \text{useful power output} / \text{total power input}$$

To express the efficiency as a percentage, again multiply the decimal answer by 100.

Example 3 - An electric water heater heats water with an output power of 2 050 W whilst its electrical power input is 2 200 W.

$$\text{efficiency} = \text{useful power output} / \text{total power output}$$

$$= 2\,050/2\,200$$

$$= 0.93\ (2\text{sf}) \quad \text{or} \quad 93\%$$

Additional Efficiency Questions

36 Power and the Human Body

Formulae:

$$\text{work done (J)} = \text{force (N)} \times \text{distance parallel to direction of force (m)}$$

$$E = Fs$$

$$\text{power (W)} = \text{energy (J)}/\text{time (s)}$$

$$P = {}^{E}/_{t}$$

37 Springs and Elastic Deformation

extension (e) = ____

For a spring or any material below its limit of proportionality, the force stretching (or compressing) the material is __________ to its extension (or compression). Twice the force causes twice the extension. This is Hooke's Law.

When the force is removed, it goes back to its ____________. This is called ________________.

The spring constant k measures the force needed to stretch it by 1 cm or 1 m. The unit of the spring constant is N/cm or N/m.

Formula:

force (N) $=$ spring constant (N/m) $\times$ extension (m) $\qquad F = kx$

Example 1 – A spring is 5.0 cm long when unstretched. With a 6.0 N force stretching it, it becomes 13 cm long. What is the spring constant?

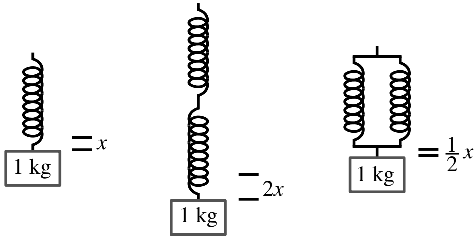
extension $= 13.0$ cm $- 5.0$ cm $= 8.0$ cm.

spring constant $=$ force / extension $= 6.0$ N / 8.0 cm $= 0.75$ N/cm
or 6.0 N / 0.080 m $= 75$ N/m

Example 2 – What will the length of this spring be when it is stretched with a 9.0 N force?

extension $=$ force / spring constant $= 9.0$ N / 0.75 N/cm $= 12$ cm
length $= 5.0$ cm $+ 12$ cm $= 17$ cm.

When two springs support a weight in series (one hanging off the other), they each carry the full weight of the load. When two identical springs support a weight in parallel, they share the weight of the load, but have the same extension as each other.



Potential energy stored in a stretched spring

$$\text{work done when stretching a spring} = \text{force} \times \text{distance}$$

However, the force changes as you stretch the spring. To start with, very little force is needed to stretch it. At the end, the force is $F = kx$ where x is the final extension. The average force is $\frac{1}{2}kx$.

$$\begin{aligned}\text{work done in stretching a spring} &= \text{average force} \times \text{distance}\\ &= \tfrac{1}{2}kx \times x = \tfrac{1}{2}kx^2\end{aligned}$$

$$\text{elastic potential energy (J)} = \frac{1}{2} \times \text{spring constant} \times \text{extension}^2$$

$$E = \frac{1}{2}kx^2$$

Example 3 – Calculate the elastic potential energy stored when a $1\,000$ N/m spring is stretched by 3.0 cm from its natural length.

With energy calculations, you should always use distances in metres.
Energy $= \frac{1}{2}kx^2 = \frac{1}{2} \times 1\,000\text{ N/m} \times (0.030\text{ m})^2 = 0.45$ J

Waves and Optics

38 Wave Properties and Basic Equations

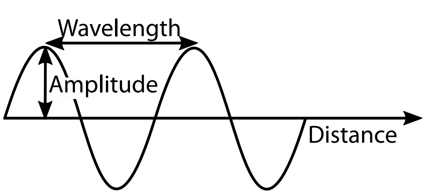
Waves can transfer ________ (or ______________) without transferring __________.

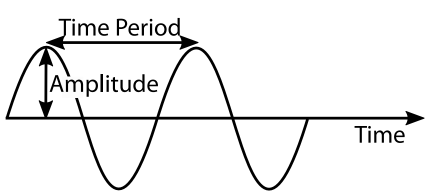
All waves involve _____________ (repeating motions back and forth).

Longitudinal Wave	Examples
Oscillations ____________ the direction of energy transfer	________ _____________ __________________

Transverse Wave	Examples
Oscillations ____________________ the direction of energy transfer	______ ____ __________________

Wavelength	The distance from one peak to the next
Time period	The time for one whole wave to go past you
Amplitude	The height of the wave's peaks
Frequency	The number of waves going past each second
Peak	The highest point on the wave
Trough	The lowest point on the wave
Speed	How fast the wave goes





Formulae:

$$\text{wave speed} = \text{frequency} \times \text{wavelength} \qquad v = f\lambda$$

Reason: Length of wave made each second $=$ *number of waves made each second* $\times$ *length of each wave.*

$$\text{frequency} = 1/\text{time Period} \qquad f = 1/T \qquad \text{so} \qquad T = 1/f$$

Reason: the frequency tells you how many time periods there are in one second, so multiplying the time period by the frequency will always give the answer 1.

When a wave moves from one material to another, the frequency does not change. If the speed changes, the wavelength will change too.

Additional Wave properties and basic equations

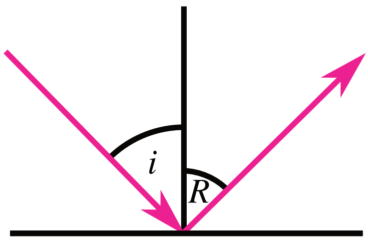
39 Reflection – Plane Mirrors

Reflections can be of two types: ________ and ________. ________ reflections are from rough surfaces, where the light rays are ________ in all directions. ________ reflections are from smooth surfaces, where the ____________ can be easily verified.

The law of reflection states that the ____

The ______________ is the angle between the incident ray and the normal.

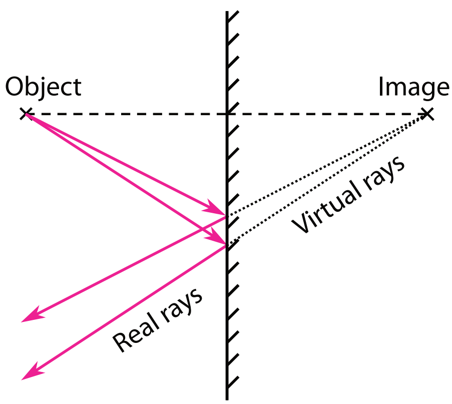
The ______________ is the angle between the reflected ray and the normal at the point where the reflection occurs.



$i = R$

A normal is an imaginary line that is ___ to the surface.

An image is a point in space from where light rays can be considered to ____________. Plane mirrors produce a ______ image - that is, an image that ____; the light rays appear to meet but they actually do not.

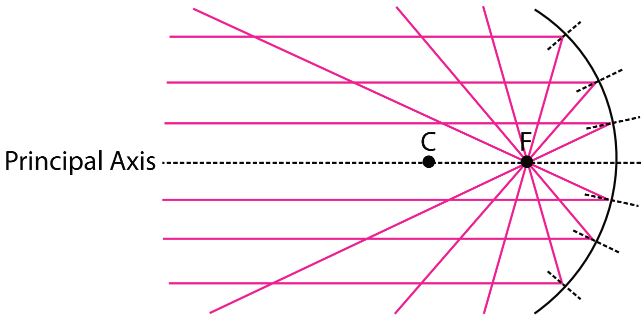


Virtual rays are extrapolated real rays. Light does not actually emerge from a virtual image, but an observer does not know that just by looking.

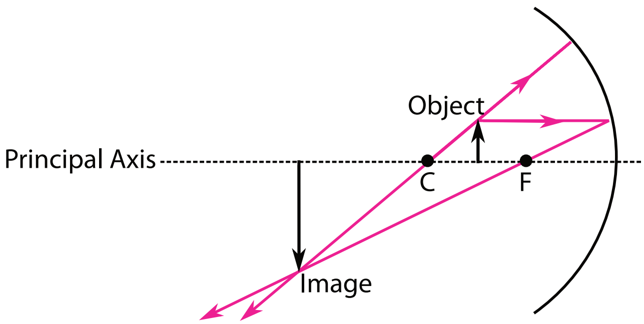
40 Reflection – Concave Mirrors ♡

The most commonly encountered curved mirrors are spherical. Spherical mirrors are either ________ or _______.

Concave mirrors produce a ____ image at the focal point (labelled F in the diagram below) when parallel rays are incident parallel to the principal axis, which passes through the centre of curvature of the mirror (labelled C). The distance from the mirror to C is always _______ the distance from the mirror to F for spherical mirrors. In the diagram, arrows have not been included because the rays would follow the same geometric path in the reverse direction. An object at F will produce an image at _______.



An object placed at C produces a __________ at C because the rays are always incident on the mirror with zero angle of incidence. Combining these two ideas, the image of an object placed anywhere between C and F can be found graphically thus:



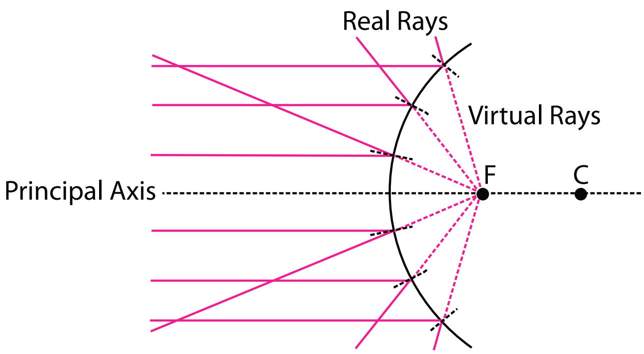
The three rules:

1. Rays passing through C reflect back through C.
2. Rays parallel to the principal axis reflect through the focal point.
3. Rays passing through the focal point reflect parallel to the principal axis.

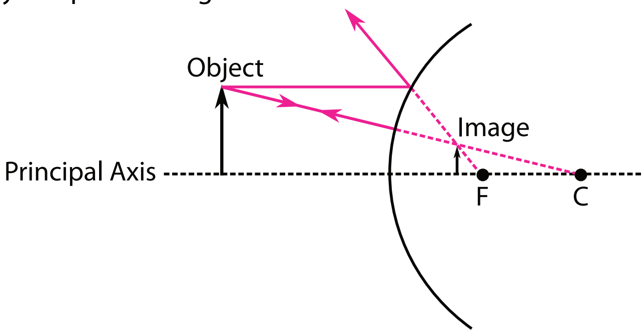
41 Reflection – Convex Mirrors ♡

The image formed from a convex mirror is always ________, ___________ and ______, regardless of where the object is placed.

Rays that are parallel to the principal axis reflect in a direction directly away from the focal point, which is _________________ the mirror and the centre of curvature for the mirror. When drawing ray diagrams, virtual rays can be drawn to correctly determine the path of the reflected rays.



As with concave mirrors, a ray incident on the mirror with an angle of incidence of zero (through the centre of curvature) will reflect in the opposite direction with zero angle of reflection. The virtual ray extrapolated from the incident ray will pass through the centre of curvature.



The two rules:
1. Rays which are incident in the direction of C reflect away from C.
2. Rays parallel to the principal axis reflect away from the virtual focal point.

42 Refraction

Light bends as it enters a glass block because the light travels ______ in glass. This causes the wavelength of the light to get ______, and causes the direction of the light to change.

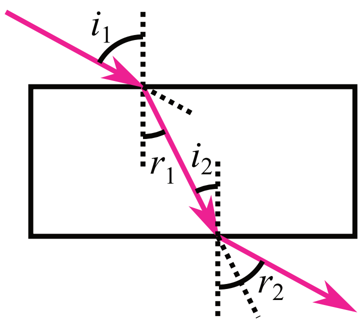
We say light bends 'towards the normal' when it __________,
and bends 'away from the normal' when it _________.

Remember 'Light goes **FAST**.'

When it goes **F**aster
it bends **A**way from the normal

When it goes **S**lower
it bends **T**owards the normal.

Formulae for refraction are explained in Refractive Index & Snell's Law - P140.



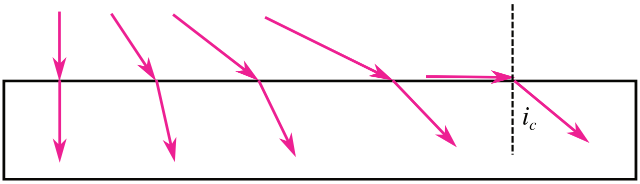
The direction can be correctly predicted by viewing the incoming light as a car whose wheels travel more slowly once they've crossed the boundary. If the front right wheel hits the boundary and slows down first, the car will turn right until the front left wheel also reaches the boundary.

$$\text{refractive index} = \text{speed of light in air} / \text{speed of light in material}$$

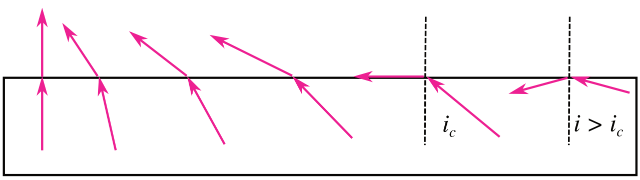
The refractive index is always greater than or equal to 1.

43 Total Internal Reflection

The diagrams below show rays of light ________ a glass block at different angles. The last one shows light hitting the boundary at a very glancing angle.



The next diagram shows the situation where light is _______ a glass block. Notice that these are identical to the rays shown above but with the direction reversed.



Where the angle of incidence is greater than i_c (the critical angle), the light cannot refract, and so it all _______ back inside the material.

Total internal reflection occurs when light attempts to _____ a glass or Perspex block with an _________________ bigger than the ____________. None of the light refracts. None of it leaves.

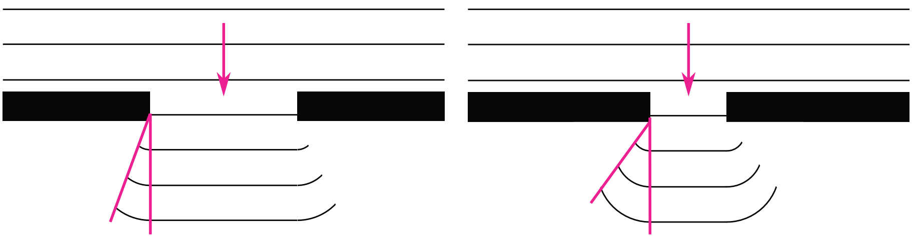
The critical angle for light leaving a glass block into air is $42°$.
The critical angle for light leaving water into air is $49°$.
The critical angle for light leaving diamond into air is $24°$.
The critical angle for light leaving cubic zirconia into air is $28°$.

The slower light travels in a material, the ______ its refractive index, and the _______ its critical angle at an air boundary.

44 Diffraction

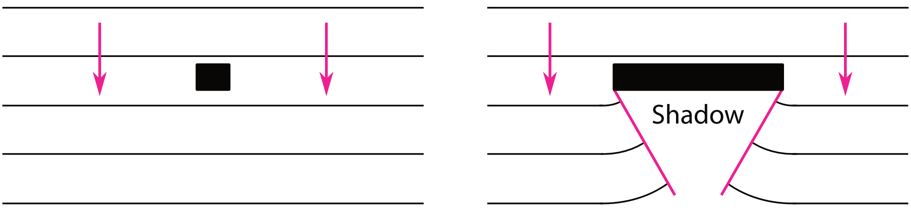
When waves encounter an obstacle or aperture (a gap), ______________.

For gaps, the amount the waves spread out depends on the __________ divided by the width of the aperture. In the two images below, waves are travelling from the top of the image to the bottom. The __________ is the same in both images (the distance between the waves fronts is the same), but the width of the gap is different. Notice that the diffraction angle, marked with the coloured lines, is greater for the ________ gap.



When waves are incident on an obstacle that is smaller than the wavelength of the wave, the waves ______ around the obstacle so very little shadow can be seen.

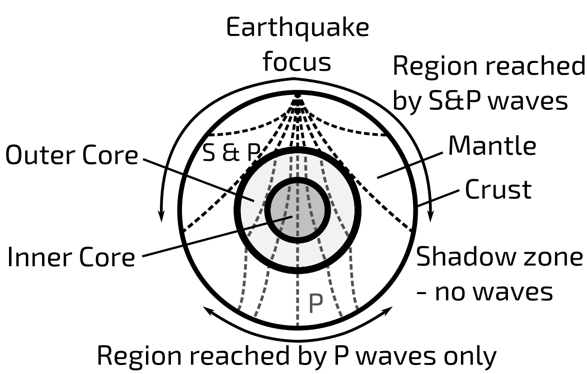
When waves are incident on an obstacle that is larger than the wavelength of the wave, the waves ______ around the edges of the obstacle but some of the wave energy ______ back from the obstacle and there is a shadow behind the obstacle.



45 Seismic Waves and Earthquakes

When a geological plate moves suddenly during an earthquake, it sets off waves which travel through the rocks. Some waves travel along the Earth's surface (e.g. Love waves). Others can travel through the Earth. These are the _ and _ waves.

<table>
<tr><th>Primary (P) Waves</th><th>Secondary (S) Waves</th></tr>
<tr><td>________________ - oscillations parallel to direction of energy transfer.</td><td>______________ - oscillations perpendicular to direction of energy transfer.</td></tr>
<tr><td>Faster (typical speed near surface of 8 km/s). These are the first waves to reach seismometers - this gives them their name ________.</td><td>Slower (typical speed near surface of 5 km/s). These are the second waves to reach seismometers - this gives them their name __________.</td></tr>
<tr><td>Can travel through liquids (such as the Earth's outer core).</td><td>Can not travel through liquids (such as the Earth's outer core) - this is because they are _________.</td></tr>
<tr><td colspan="2">Can be ________ at any boundary between two different regions.</td></tr>
<tr><td colspan="2">Can be ________ (or bent) by any change in rock compressibility or density. Generally as waves get deeper (and the pressure rises), their speed rises and they bend _________ the 'normal' (the vertical). When passing a boundary into a deeper (more dense) phase, they slow down, and bend _______ the vertical.</td></tr>
</table>



Typical speeds of seismic waves and rock densities are shown in the table.

Region	Depth (km)	Density (kg/m^3)	Speed (km/s)	
			P	S
Crust	$0 \sim 10$	3.0×10^3	8.0	5.0
Mantle	$\sim 10 - 2\,900$	$(3.0 - 5.0) \times 10^3$	$8.0 - 13$	$5.0 - 8.0$
Outer Core	$2\,900 - 5\,200$	10^4	$8.0 - 10$	-
Inner Core	$5\,200 - 6\,400$	1.2×10^4	11	$3.0 - 4.0$

(eqseis.geosc.psu.edu/~cammon/HTML/Classes/IntroQuakes/Notes/waves_and_interior.html)

Example 1 – What will be the delay between receiving P and S waves at a seismometer 200 km from the earthquake's focus?
Time for P-wave = Distance / Speed = 200 km/8.0 km/s = 25 s.
Time for S-wave = Distance / Speed = 200 km/5.0 km/s = 40 s.
Delay = 40 s − 25 s = 15 s.

Example 2 – If the delay between receiving P and S waves is 5 s, how far away is the earthquake's focus?
We call the distance d, taken in km where time will be in seconds.
For the P wave, the time take to arrive is $t_p = d/8$.
For the S wave, the time taken to arrive is $t_s = d/5$.
We are told the delay is 5 s. Thus $t_s - t_p = 5$.
Therefore $d/5 - d/8 = 5$. So, $0.2d - 0.125d = 5$, and accordingly $0.075d = 5$.
We finally get $d = 5/0.075 = 67$ km (70 km to 1sf).

46 Refractive Index & Snell's Law

Data:

$$\text{refractive index of glass} = 1.50$$
$$\text{refractive index of water} = 1.34$$
$$\text{refractive index of diamond} = 2.42$$
$$\text{refractive index of cubic zirconia} = 2.16$$
$$\text{refractive index of air} = 1.00$$
$$\text{speed of light in a vacuum} = 3.00 \times 10^8 \text{ m/s}$$

Refraction describes the change of ________ of light on entering or leaving a material when it crosses the ________.

Refraction is caused by the difference in the ____________ in the materials. To compare the speed of light in different materials, we compare their refractive indices.

$$\text{refractive index} = \frac{\text{speed of light in a vacuum}}{\text{speed of light in a material}} \qquad n = \frac{c}{v}$$

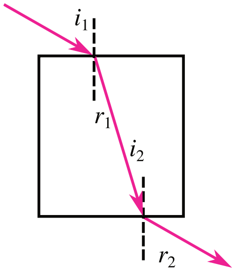
Air has a refractive index of 1.00, so the speed of light in air is very similar to the speed of light in a vacuum.

The larger the refractive index, the ______ light travels.

Example 1 – Calculate the speed of light in diamond.

$$n = \frac{c}{v} \quad \text{so} \quad v = \frac{c}{n} = \frac{3 \times 10^8}{2.42} = 1.24 \times 10^8 \text{ m/s}$$

Snell's Law enables us to calculate the angles when light refracts.



For light entering a material from air

$$\sin(r_1) = \frac{\sin(i_1)}{n} \quad \text{so} \quad r_1 = \sin^{-1}\left(\frac{\sin(i_1)}{n}\right)$$

For light leaving a material to pass into air

$$\sin(i_2) = n \times \sin(r_2) \quad \text{so} \quad r_2 = \sin^{-1}(n \times \sin(i_2))$$

These formulae are explained on P143.

Example 2 – Light enters glass with an incident angle of 25°. Calculate the angle of refraction.

$$r = \sin^{-1}\left(\frac{\sin(i)}{n}\right) = \sin^{-1}\left(\frac{\sin(25^\circ)}{1.50}\right) = \sin^{-1}(0.28) = 16^\circ$$

When light passes from one material (with refractive index n_1) to a second material (refractive index n_2), then the general formula is

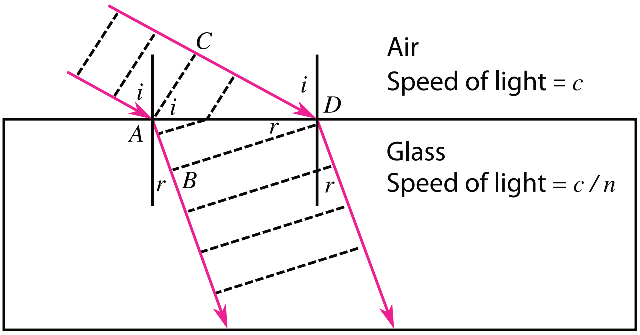
$$n_2 \sin(r) = n_1 \sin(i)$$

Notice that if the first material is air, $n_1 = 1$, then $n_2 \sin(r) = \sin(i)$.

If the second material is air, $n_2 = 1$, then $\sin(r) = n_1 \sin(i)$.

These agree with our earlier formulae, as well as the answer for Q46.10.

Reasoning behind Snell's Law



The wavefronts meet the rays at right angles.
In the time (t) that lights travels from C to D, light also travels from A to B.

$$t = \frac{AB}{(c/n)} = \frac{CD}{c}$$

So $CD = n \times AB$ and $CD/AB = n$.

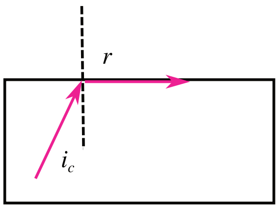
$$\begin{array}{llll} \angle CAD = i & \text{and} \quad CD = AD \sin(\angle CAD) & \text{so} & CD = AD \sin(i) \\ \angle ADB = r & \text{and} \quad AB = AD \sin(\angle ADB) & \text{so} & AB = AD \sin(r) \end{array}$$

Dividing these equations gives $\dfrac{CD}{AB} = \dfrac{\sin(i)}{\sin(r)}$ but $\dfrac{CD}{AB} = n$.

Therefore $n = \dfrac{\sin(i)}{\sin(r)}$, so $\sin(r) = \dfrac{\sin(i)}{n}$.

This is Snell's Law.

47 Calculating Critical Angles



The conditions for total internal reflection are that the light

- must be attempting to _____ a material into air, or more generally
  - crossing from a ______ to _____ refractive index material
  - this means that the light crosses a boundary where it __________
- and the angle of incidence must be ______ the critical angle (i_c).

If the angle of incidence were exactly critical, then the angle of refraction would be a __________.

So $i = i_c$ and $r = 90°$. Remember, $\sin(90°) = 1$
Snell's Law for light leaving a material into air is

$$\sin(r) = n\sin(i)$$

In this case $\sin(90°) = n\sin(i_c)$. So $\sin(i_c) = \frac{1}{n}$ and $i_c = \sin^{-1}\left(\frac{1}{n}\right)$.

The refractive index $n = \frac{1}{\sin(i_c)}$.

Data:

refractive index of glass $= 1.50$
refractive index of water $= 1.34$

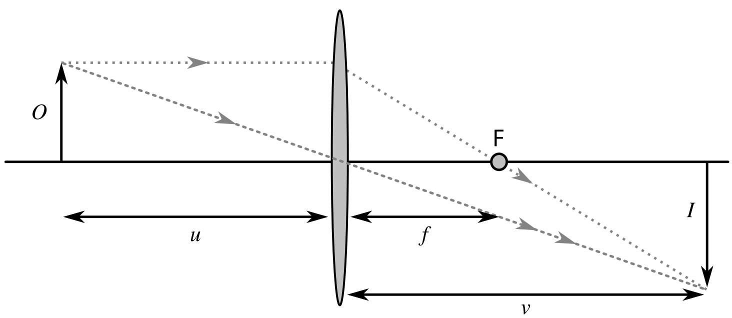
Where light goes from one material (refractive index n_1) to another (n_2), we use the more general form of Snell's Law.

$$
\begin{aligned}
n_1 \sin(i) &= n_2 \sin(r) \\
n_1 \sin(i_c) &= n_2 \sin(90^\circ) = n_2 \\
\Rightarrow \sin(i_c) &= \frac{n_2}{n_1}
\end{aligned}
$$

48 Convex Lenses

In the diagram below, the object has size O, the image size I, and the convex lens has a focal length f. We can work out the location of the image by drawing two rays through the system.

1. A ray passing through the centre of the lens __________.
2. A ray travelling parallel to the axis will bend at the lens, so that it crosses the axis at the ____________ (distance f behind the lens).
3. The ______ I is where the rays meet. If the rays are diverging (spreading apart) after the lens, extend both back to the left to find a place where the lines meet – this will be a virtual image.
4. The object distance is labelled u. The image distance is labelled v.



Power: The "strength" with which a lens focuses a parallel beam to a point (a focus) is measured as its power. Lens power is measured in dioptres (D).

$$\text{power in } \underline{\hspace{3cm}} = (\text{focal length in metres})^{-1}$$

$$P = \frac{1}{f} = f^{-1}$$

> Example 1 – Calculate the power of a lens with a 10 cm focal length.
>
> $$10\text{ cm} = 0.1\text{ m} \quad P = 1/f = 1/0.1 = 10\text{ D}$$

Working out the lens equation: We use similar triangles on the diagram of page 146 to form two equations for O/I.

Using the ray through the middle of the lens we know that:

$$\frac{O}{I} = \frac{u}{v}$$

Using the other diagonal ray, and the two triangles it forms, we can also write:

$$\frac{O}{I} = \frac{f}{v - f}$$

Equating the two expressions:

$$\frac{u}{v} = \frac{f}{v-f} \Rightarrow \frac{v}{u} = \frac{v-f}{f} \Rightarrow \frac{v}{u} = \frac{v}{f} - 1$$

$$\Rightarrow \frac{1}{u} = \frac{1}{f} - \frac{1}{v} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

This can be worked out more easily on a calculator like this:

$$v^{-1} = f^{-1} - u^{-1}, \text{ so } v = (f^{-1} - u^{-1})^{-1}.$$

Example 2 – Calculate the image distance (v) of a 5.0 D lens, if the object distance (u) is 30 cm.

$$P = 1/f, \quad \text{so} f = 1/P = 1/5.0 = 0.2 \text{ m}$$

$$1/v = 1/f - 1/u = 1/0.2 - 1/0.3 = 1.667,$$

$$\text{so } v = 1/1.667 = 0.6 \text{ m} = 60 \text{ cm}$$

In the final two cases, v is ________ (the image is to the left of the lens). This is a ______ image, meaning that _________________________. A screen at that position would not show a bright spot.

$$\text{magnification} = \text{image height} \;/\; \text{object height} = \frac{I}{O} = \frac{v}{u}$$

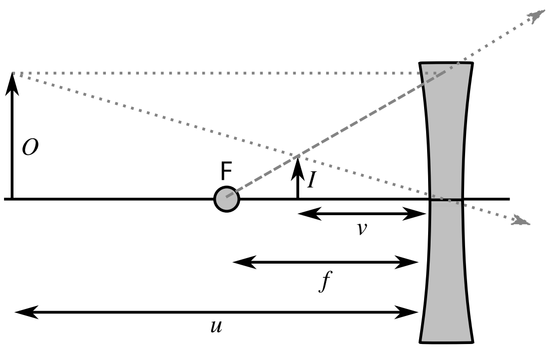
A magnification of 2.0 means that the image is ________________________. Magnification of 1.0 gives an image _____________________________. The magnification number does not say whether the image is 'upside down'.

A convex lens makes a virtual image if ____.
A convex lens makes a real, magnified image if _______________________.
A convex lens makes a real, diminished image if _______.

49 Concave Lenses

In the diagram below, the object has size O, the image size I, and the lens has a focal length f. The lens now causes the rays to ________. We can work out the location of the image by drawing two rays through the system.

1. A ray passing through the centre of the lens that does not bend.
2. A ray travelling parallel to the axis will bend at the lens so that it appears to come from the focal point F (distance f from the lens). On the diagram, we draw a dotted line from F to the lens, and a solid line from there on.
3. The ______ I is drawn where the lines cross.
4. The object distance is labelled u. The image distance is labelled v.



The power formula $P = 1/f$ is used for concave lenses, just as it is for convex lenses. However concave lens powers are negative. When giving the power of a lens, always give the sign to make it clear whether you mean a convex or concave lens.

Example 1 – Calculate the power of a concave lens with a focal length of 4.0 cm.

$$4.0\text{ cm} = 0.04\text{ m} \quad P = 1/f = 1/0.040 = 25\text{ D}$$

This is a concave lens, so we use a negative power $P = -25$ D.

Example 2 – Calculate the focal length of a -0.8 D lens. The power is negative, so this is a concave lens.

$$P = 1/f, \text{ so } f = 1/P = 1/(0.8\,\text{D}) = 1.25\,\text{m}$$

We use similar triangles in the diagram of page 150 to form two equations for O/I. The ray through the middle of the lens yields:

$$\frac{O}{I} = \frac{u}{v}$$

Using the line from F to the lens via the top of I, we can also write:

$$\frac{O}{I} = \frac{f}{f-v}$$

Equating the two expressions:

$$\frac{u}{v} = \frac{f}{f-v} \Rightarrow \frac{v}{u} = \frac{f-v}{f} \Rightarrow \frac{v}{u} = 1 - \frac{v}{f}$$
$$\Rightarrow \frac{1}{u} = \frac{1}{v} - \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

This is the same as the equation on page 148 for a convex lens, if you flip the signs of f and v. Making v negative makes sense given that our image is to the left of the lens. Making f negative also makes sense as we remember that $P = 1/f$ and concave lenses have negative powers.

- For all lenses $1/v = 1/f - 1/u$, where
- a negative v means that the image is to the left of the lens, and
- a negative value of f means that the lens is concave.

As with convex lenses, the magnification = $I/O = v/u$, where we ignore the sign of v. A concave lens makes a parallel beam if $u = f$. Otherwise it makes a ______, __________ image.

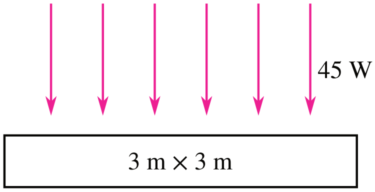
50 Intensity and Radiation

The intensity of light, sound or other radiation depends on the

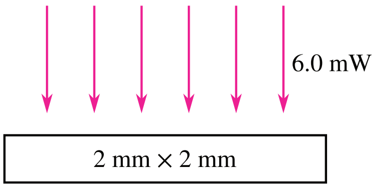
- ______ of the wave, and
- the size of the ____ in which the waves are focused.

Formula:

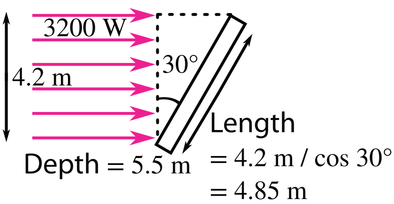
$$\text{intensity (W/m}^2\text{)} = \text{power (W)}/\text{area (m}^2\text{)} \qquad I = P/A$$



Example 1
Intensity $= P/A = 45\text{ W} \div 9\text{ m}^2$
$= 5\text{ W/m}^2$

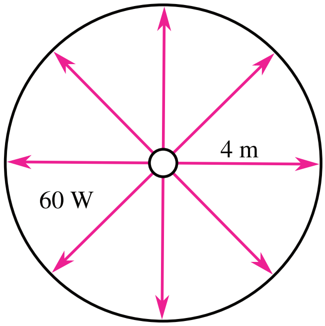


Example 2
Area $= 2\text{ mm} \times 2\text{ mm} = 0.002\text{ m} \times 0.002\text{ m} = 4 \times 10^{-6}\text{ m}^2$
Intensity $= P/A$
$= (6 \times 10^{-3}\text{ W}) \div (4 \times 10^{-6}\text{ m}^2)$
$= 1.5 \times 10^3\text{ W/m}^2$
$= 1\,500\text{ W/m}^2$



Example 3
Area lit $= 5.5\text{ m} \times 4.85\text{ m} = 26.7\text{ m}^2$
Intensity $= P/A$
$= 3\,100\text{ W} \div 26.7\text{ m}^2 = 120\text{ W/m}^2$

Point Sources
To work out the intensity at a distance from a point source, we imagine it shining light in all directions, making the shape of a ______.

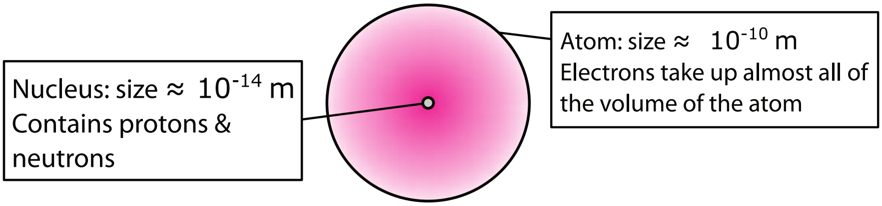


Intensity 4 m from the source
$=$ power / area illuminated
$=$ power / surface area of a 4 m sphere
$= P/(4\pi r^2)$
$= 60/(4\pi \times 4^2) = 60/201 = 0.30$ W/m^2.

Nuclear

51 Atomic Numbers and Nomenclature

All matter is made up of atoms.



Particles:

Name	Symbol	Relative charge	Relative mass
Proton			
Electron[1]			
Neutron			
Positron			

No internal structure inside an electron has been found; it is a ____________ particle.
Every particle has an anti-particle of opposite charge but identical mass. The anti-electron is called a ________. If a particle meets its antiparticle, the two __________ each other, and their energy is given out as gamma rays.

The _________________ is the number of _______ in a nucleus.

The _______________ is the number of ____________________ in a nucleus.

$^{14}_{6}C$ (also written as carbon-14) is an isotope of carbon with a mass number of $A = 14$. It has $Z = 6$ protons, 6 electrons and $14 - 6 = 8$ (= A - Z) neutrons.

All atoms with the same number of protons belong to the same ________. They will behave identically in any ________ process.

[1] Beta (β^-) radiation consists of free electrons moving very quickly. Beta particles are electrons emitted from nuclei- so not all electrons are beta particles.

Two atoms are said to be ________ of the same element if they have the same number of ________ but different numbers of ________. They will, consequently, have different ________.

Protons and neutrons are made of ________. Up quarks (u) have charge $+2/3$, while down quarks (d) have charge $-1/3$.

52 Radioactive Decay

Some nuclei are _____, and will remain as they are for ever. Others are _____. After an unpredictable period of time, unstable nuclei will change. This change is called _____. When a nucleus decays, it gives out highly energetic, _____ The main forms of ionizing radiation are _____, _____ and _____.

Type of decay	Particle given out	Penetrating ability	Ionising ability	Change to the original nucleus
Alpha	$^{4}_{2}\alpha$ - _____ nucleus (_ protons + _ neutrons)	_____ - stopped by _ cm of ___, by ____ or _____	_____	Mass number _____ by _ Atomic number _____ by _
Beta minus	$^{0}_{-1}\beta$ - _____ speed _____ produced when a _____ turns into a _____	_____ - can pass _ mm of _____, but stopped by _ cm	_____	Mass number _____ Atomic number _____ by _
Beta plus	$^{0}_{+1}\beta$ - _____ speed _____ produced when a _____ turns into a _____	Very ____ - _____ on contact with _____	N/A	Mass number _____ Atomic number _____ by _
Gamma	$^{0}_{0}\gamma$ - _____ frequency _____	_____ - can pass through _____ cm of _____	_____	Mass number _____ Atomic number _____ Excess _____ potential _____ is _____

Example 1 - Write the equation for the alpha decay of $^{241}_{95}Am$ into Np.

The symbol for the alpha particle is $^{4}_{2}\alpha$. We write the equation $^{241}_{95}Am \longrightarrow Np + ^{4}_{2}\alpha$ to show the decay.

Next, we need to put mass and atomic numbers on the Np. We do this using the rules in the table: $^{241}_{95}Am \longrightarrow ^{237}_{93}Np + ^{4}_{2}\alpha$.

Notice that once the equation is complete the numbers on the top balance ($214 = 237 + 4$), as do the numbers on the bottom ($95 = 93 + 2$).

Example 2 - Write the equation for the beta minus decay of $^{3}_{1}H$ into He.

Firstly, we write $^{3}_{1}H \longrightarrow He + ^{0}_{-1}\beta$, then put numbers on He to balance it: $^{3}_{1}H \longrightarrow ^{3}_{2}He + ^{0}_{-1}\beta$.

Again notice that the top row balances ($3 = 3 + 0$) and so does the bottom ($1 = 2 - 1$).

53 Half Life

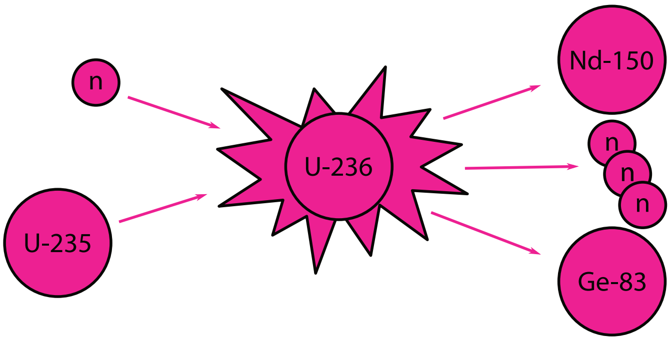
Nuclear decay is ________. You can not predict when an individual nucleus will ______. However, if you have many millions of nuclei, you can make a good prediction of how many will decay in a certain amount of time.
The _______ is the average time taken for the number of unstable nuclei to halve.
The half life is also the average time taken for the _______ (number of decays each _______) to halve.

Example - The half life of $^3_1$H is 12 years. A source starts with an activity of 150 Bq (150 decays per second). Estimate the activity 12 and 24 years after the start.

After 12 years, one half life has passed, so the activity will halve to 75 Bq. After 24 years, a second half life has passed, halving the activity again to $75 \times 0.5 = 37.5$ Bq.

54 Fission – The Process

Nuclear fission is the process by which one atomic nucleus _____ to form two atomic nuclei. If the nucleus that splits has an atomic number above __ (the atomic number of iron, Fe), the nuclear reaction ________ energy.



Heavy nuclei can often be made even less stable by absorbing an additional ________. Uranium−235, for example, has _____________ in the nucleus. If a uranium-235 nucleus absorbs a neutron, it quickly fissions (splits) into two _________ nuclei and a few free neutrons. The two daughter nuclei tend to have a mass ratio close to ___; however, this is random, and two or three free neutrons are also released. The free neutrons can hit other uranium nuclei, and could cause them to split. If these neutrons cause uranium nuclei to fission, releasing further neutrons which cause other uranium nuclei to fission, we call this a ______________.

The total number of neutrons before a fission is _____ to the total number of neutrons after a fission.

Similarly, the total number of protons before a fission is _____ to the total number of protons after a fission.
The fission products are usually highly radioactive.

Example - Balanced nuclear fission reactions:

$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{150}_{60}Nd + ^{83}_{32}Ge + 3\, ^{1}_{0}n$

Check mass (top) numbers balance: $235 + 1 = 150 + 83 + (3 \times 1)$

Check atomic (bottom) numbers balance: $92 + 0 = 60 + 32 + (3 \times 0)$

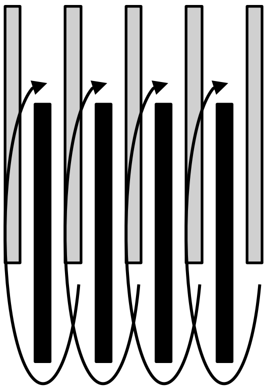
$^{239}_{94}Pu + ^{1}_{0}n \longrightarrow ^{149}_{58}Ce + ^{88}_{36}Kr + 3\, ^{1}_{0}n$

[1]

[1] For an interactive periodic table where you can check isotopes, masses, half lives etc, see www.ptable.com.

55 Fission – The Reactor

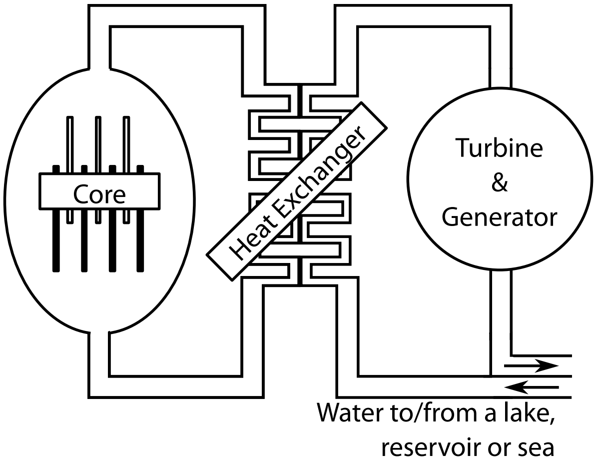
Nuclear fission reactors convert nuclear energy to ____________. The nuclear energy is locked away in the nuclei of atoms with large atomic masses. The most common nuclear fuel is ____________. When the nucleus of a uranium−235 atom ______, it becomes two smaller nuclei plus two or three free neutrons.



The neutrons that are released from a fission reaction are too fast to be absorbed by other uranium−235 nuclei. To slow them down, a __________, such as water or graphite, is used. The __________ converts the extra kinetic energy of the fast neutrons into ____________, so they require a coolant to carry the ____________ away. If water is used as a __________, the water itself can be the ______.

If one spare neutron from each fission reaction is slowed down enough and absorbed by another uranium−235 nucleus, the reaction is a self-sustaining chain reaction. If too many neutrons are absorbed, the reaction rate can ________________ - this is what happens when a nuclear fission bomb is detonated. To prevent the reaction rate increasing, __________ made from boron or cadmium are included in the reactor to absorb spare free neutrons.

The nuclear fuel rods, __________, _______ and ___________ are all in the nuclear reactor core, which is contained in a concrete domed building. Heat exchangers carry the energy out of the core.



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56 Energy from the Nucleus – Radioactivity & Fission

You can calculate the energy released by a nuclear process if you know the mass of each of the nuclei involved.

The most important equation is

$$\text{energy} = \text{mass} \times (\text{speed of light})^2 \qquad E = mc^2$$

In this equation, the mass is measured in __________, the energy is measured in ______ and the speed of light is 3.00×10^8 m/s.
In a nuclear reaction, the products (once they have slowed to normal speeds) have ____ mass in total than the reactants had: ____________________

$$\text{energy released} = \text{mass 'lost'} \times (\text{speed of light})^2$$

Example – Calculate the energy released during the reaction:

$$^{241}_{95}\text{Am} \longrightarrow {}^{237}_{93}\text{Np} + {}^{4}_{2}\alpha$$

The masses of the nuclei are given in the table below.

^{241}Am	$4.001\,98 \times 10^{-25}$ kg
^{237}Np	$3.935\,43 \times 10^{-25}$ kg
α	6.645×10^{-27} kg

$$\begin{aligned}
\text{Mass of reactants} &= \text{mass of } {}^{241}\text{Am} = 4.001\,98 \times 10^{-25}\text{ kg}\\
\text{Mass of products} &= \text{mass of } {}^{237}\text{Np} + \text{mass of } \alpha\\
&= 3.935\,43 \times 10^{-25}\text{ kg} + 6.645 \times 10^{-27}\text{ kg}\\
&= 4.001\,88 \times 10^{-25}\text{ kg}\\
\text{Difference in masses} &= 4.001\,98 \times 10^{-25}\text{ kg} - 4.001\,88 \times 10^{-25}\text{ kg}\\
&= 0.000\,10 \times 10^{-25}\text{ kg} \equiv 1.0 \times 10^{-29}\text{ kg}\\
\text{Energy released} &= mc^2 = 1.0 \times 10^{-29}\text{ kg} \times (3.00 \times 10^8)^2\\
&= 9.0 \times 10^{-13}\text{ J}
\end{aligned}$$

This may seem a very small amount of energy, but it is over 5 000 000 times larger than the energy given out in chemical reactions.

Some masses of nuclei for use in the questions:

$^{1}_{0}n$	$0.016\,749 \times 10^{-25}$ kg	α	6.645×10^{-27} kg
$^{87}_{35}Br$	$1.443\,031 \times 10^{-25}$ kg	$^{103}_{40}Zr$	$1.708\,773 \times 10^{-25}$ kg
$^{134}_{54}Xe$	$2.223\,061 \times 10^{-25}$ kg	$^{147}_{57}La$	$2.439\,291 \times 10^{-25}$ kg
$^{189}_{81}Tl$	$3.137\,255 \times 10^{-25}$ kg	$^{193}_{83}Bi$	$3.203\,808 \times 10^{-25}$ kg
$^{206}_{82}Pb$	$3.419\,541 \times 10^{-25}$ kg	$^{206}_{84}Po$	$3.419\,623 \times 10^{-25}$ kg
$^{210}_{84}Po$	$3.486\,084 \times 10^{-25}$ kg	$^{210}_{86}Rn$	$3.486\,179 \times 10^{-25}$ kg
$^{212}_{83}Bi$	$3.519\,444 \times 10^{-25}$ kg	$^{216}_{85}At$	$3.586\,032 \times 10^{-25}$ kg
$^{234}_{90}Th$	$3.885\,568 \times 10^{-25}$ kg	$^{235}_{92}U$	$3.902\,162 \times 10^{-25}$ kg
$^{238}_{92}U$	$3.952\,090 \times 10^{-25}$ kg	$^{239}_{94}Pu$	$3.968\,700 \times 10^{-25}$ kg

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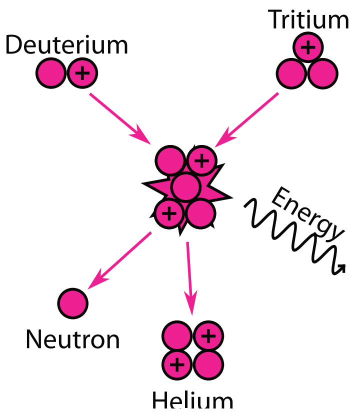
57 Fusion – The Process

Nuclear fusion is the process by which two atomic nuclei combine to form a single atomic nucleus. If the two nuclei that go into a fusion reaction have an atomic number below __ (the atomic number of iron), the nuclear reaction can release energy.



Atomic nuclei are positively charged. Like charges _____ each other. The strength of the ______________________________ increases as the distance between the nuclei ___________. This force prevents the two nuclei getting close enough to fuse unless the nuclei are moving very fast. If the nuclei are moving very fast, the electrostatic repulsive force cannot stop two nuclei moving towards each other until it is too late; they are close enough to fuse, under the action of the strong nuclear force, and become a single nucleus. This barrier to nuclear fusion is called the __________________.

Once the Coulomb barrier is breached, far more energy is _________ than the energy required to breach the Coulomb barrier in the first place.

In stars, atomic nuclei are given sufficient energy to breach the Coulomb barrier because the ______________ in the star is so high. On Earth, experimental fusion reactor designs focus on energy-efficient ways of increasing the ______ of the atomic nuclei and on novel methods for reducing the _________________. To date, no fusion reactor on Earth has released energy at a self-sustaining rate for more than a fraction of a second.

The best atomic nuclei to use as a fuel in a fusion reactor are __________, which has the fewest number of protons of any atomic nucleus. The ________________ can be more easily overcome by using isotopes of hydrogen that contain neutrons, such as __________ (one proton, one neutron) and _______ (one proton, two neutrons).

Hydrogen is a readily available fuel because it is present in water, which covers approximately 70% of the Earth's surface (and growing).

58 Energy from the Nucleus – Fusion ♡

The methods needed for working out the energy released are explained in full on Energy from the Nucleus – Radioactivity & Fission - P167.

The most promising fusion reaction, as far as power stations are concerned, is this:

$$^{2}_{1}\text{H} + ^{3}_{1}\text{H} \longrightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$$

The masses of some nuclei are given in the table below:

$^{1}_{0}\text{n}$	1.6749×10^{-27} kg	$^{2}_{1}\text{H}$	3.3436×10^{-27} kg
$^{3}_{1}\text{H}$	5.0074×10^{-27} kg	$^{4}_{2}\text{He}$	6.6447×10^{-27} kg

To answer the next two questions, you will need your answers to the Energy from the Nucleus – Radioactivity & Fission worksheet questions, P168.

Gas

59 Boyle's Law

Definition:

$$\text{pressure} = \frac{\text{force (N)}}{\text{area (m}^2\text{)}} \qquad P = \frac{F}{A}$$

The unit of pressure is the pascal (Pa). 1 Pa $= 1$ N/m^2.
Atmospheric pressure is approximately 1.01×10^5 Pa $= 101$ kPa.

The pressure a gas exerts on a wall depends on

- how often molecules hit the wall. This depends on the
  - ____________________ (if the container is longer, molecules will take longer to cross it, and each molecule will collide with the wall less often).
  - _______________________ (this depends on the temperature).
  - ______________________________________.
- the momentum change when each molecule hits the walls. This depends on the
  - _____ of the molecules.
  - ______ of the molecules (___________________________________).

If the temperature is constant (which is usually the case if the gas is compressed or expands slowly), the _______________________ doesn't change. Halving the volume of the container doubles the gas pressure because each molecule only takes _____________ to cross it – so hits the walls ______________.

Equation for Boyle's Law (constant temperature)

$$\text{pressure} \times \text{volume} = \text{constant} \qquad p_1V_1 = p_2V_2$$

where $_1$ means 'before the change' and $_2$ means 'after the change'

Example – 40 cm^3 of gas at atmospheric pressure is squeezed into a volume of 10 cm^3. What is the new pressure?
$p_1 V_1 = p_2 V_2$, so 101 kPa $\times$ 40 cm^3 $= p_2 \times 10$ cm^3, so $4\,040 = 10 p_2$
$p_2 = 4\,040/10 = 404$ kPa.

The average kinetic energy of molecules in a gas depends on ____________.
The average kinetic energy of molecules is proportional to the ________________________
where

Temperature in kelvins (K) $=$ Temperature in degrees Celsius ($^\circ$C) $+ 273$.

The temperature of 0 K = ________ is called _____________. If you were able to cool a gas right down to this level, the molecules would be __________. You couldn't cool it any further - this is the ____________________________.

Additional Boyle's Law Questions

60 The Pressure Law

In this situation, the ________ is fixed (we use a rigid container). The gas is heated, and the pressure increases.

As the temperature of the gas goes up, the ______________ and ______________ of the molecules increases.

This means that each second, ______________________________________, and also that on each collision there is a ______________________________________ for the molecule, leading to a greater _____ on the wall.

The equation is

$$\frac{p_{\text{after}}}{T_{\text{after}}} = \frac{p_{\text{before}}}{T_{\text{before}}}$$

where T must be in kelvins.

Example – Starting with some gas at $20.0\,^\circ$C at a pressure of 101 kPa and heating it to $100\,^\circ$C, what is the new pressure if the gas' volume is fixed?

1^{st} stage: convert the temperatures to kelvins.

$$20.0\,^\circ\text{C} + 273 = 293\ \text{K} \qquad 100\,^\circ\text{C} + 273 = 373\ \text{K}$$

2^{nd} stage: put the numbers into the equation.

$$\frac{p_{\text{after}}}{373\ \text{K}} = \frac{101\ \text{kPa}}{293\ \text{K}}$$

3^{rd} stage: rearrange the equation so that the thing you want to know is the subject, and calculate it.

$$p_{\text{after}} = 101\ \text{kPa} \times \frac{373}{293} = 129\ \text{kPa}$$

4^{th} stage: put the temperatures back in $^\circ$C if necessary (not needed here).

61 Charles' Law

In this situation, the ________ is fixed (we use a container with a free-running piston). The gas is heated, and the _______ increases.

As the temperature of the gas goes up, the ____
This means that each time a molecule strikes the wall, its ______________________,
so the _____ on the wall is ______.

However if the container expands, each molecule strikes the wall _________,
leading to the same ________ as before.

The equation is

$$\frac{V_{\text{after}}}{T_{\text{after}}} = \frac{V_{\text{before}}}{T_{\text{before}}}$$

where T must be in kelvins.

Example – If I start with $30.0\ \text{cm}^3$ gas at $20.0\ ^\circ\text{C}$ and heat it up to $100\ ^\circ\text{C}$, what will the new volume be if I don't let the pressure build up?

1st stage: convert the temperatures to kelvins.

$$20.0\ ^\circ\text{C} + 273 = 293\ \text{K} \qquad 100\ ^\circ\text{C} + 273 = 373\ \text{K}$$

2nd stage: put the numbers into the equation.

$$\frac{V_{\text{after}}}{373\ \text{K}} = \frac{30.0\ \text{cm}^3}{293\ \text{K}}$$

3rd stage: rearrange the equation so that the thing you want to know is the subject, and calculate it.

$$V_{\text{after}} = 30.0\ \text{cm}^3 \times \frac{373}{293} = 38.2\ \text{cm}^3$$

4th stage: put the temperatures back in °C if necessary (not needed here).

62 The General Gas Law

Experiments have taught us…

Law	For fixed	In words	Formula
Boyle	______	Halving volume ________________	$pV = k_1$
Pressure	___	Doubling temperature ________________	$p = k_2 T$
Charles	_____	Doubling temperature ______________	$V = k_3 T$

k_1, k_2 and k_3 are constants. The value of k_1, k_2, k_3 would depend on the control variable and the amount of gas in the experiment.

We must use the ______ scale so that zero (0 K) is the temperature of absolute zero. Temperature (K) = Temperature (°C) +273.

If you combine the three rules, you get

$$\text{pressure} \times \text{volume} = \text{constant} \times \text{temperature} \qquad pV = \text{const.} \times T$$

$$\Rightarrow \frac{\text{pressure} \times \text{volume}}{\text{temperature}} = \text{constant} \qquad \frac{pV}{T} = \text{const.}$$

Given that the constant must be the same before and after the process (as long as no gas leaks),

$$\frac{p_{\text{after}} V_{\text{after}}}{T_{\text{after}}} = \frac{p_{\text{before}} V_{\text{before}}}{T_{\text{before}}},$$

where T must be in kelvins.

Example – If I start with $10\ \text{cm}^3$ of gas at 20°C at a pressure of 101 kPa and heat it to 100 °C, what will the new pressure be if I let it expand to $12\ \text{cm}^3$?

1^{st} stage: convert the temperatures to kelvins.

$$20\,^\circ\text{C} + 273 = 293\ \text{K} \qquad 100\,^\circ\text{C} + 273 = 373\ \text{K}$$

2nd stage: put the numbers into the equation.

$$\frac{p_{\text{after}} \times 12\ \text{cm}^3}{373\ \text{K}} = \frac{101\ \text{kPa} \times 10\ \text{cm}^3}{293\ \text{K}}$$

3rd stage: rearrange the equation so that the thing you want to know is the subject, and calculate it.

$$p_{\text{after}} = 101\ \text{kPa} \times \frac{10 \times 373}{12 \times 293} = 107\ \text{kPa}$$

4th stage: put the temperatures back in °C if necessary (not needed here).