

Integration by Substitution 1

A Level Further A



Part A Integrate $\sin(c\theta)$

Find $\int \sin(c\theta) d\theta$, where c is a constant.

The following symbols may be useful: c , k , θ

Part B Integrate $e^{\alpha v}$

Find $\int e^{\alpha v} dv$, where α is a constant.

The following symbols may be useful: α , e , k , v

Integration by Substitution 2

A Level **Further A**

Part A Integrate $(bv + c)^2$

Find $\int (bv + c)^2 dv$, where b and c are constants.

The following symbols may be useful: b , c , k , v

Part B Integrate $a(y - b)^3$

Find $\int_0^b a(y - b)^3 dy$, where a and b are constants.

The following symbols may be useful: a , b

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Integration: General 4ii

Part A Integrate $(4 - 3x)^7$

Find $\int (4 - 3x)^7 dx$.

The following symbols may be useful: c , x

Part B Integrate $(4 - 3x)^{-1}$

Find $\int (4 - 3x)^{-1} dx$.

The following symbols may be useful: c , x

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Integration by Substitution 5ii

Use the substitution $u = 2x - 5$ to find the exact value of $\int_{\frac{5}{2}}^3 (4x - 8)(2x - 5)^7 dx$.

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Integration by Substitution 5i

Use the substitution $u = 2x + 1$ to evaluate $\int_0^{\frac{1}{2}} \frac{4x - 1}{(2x + 1)^5} dx$.

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Integration by Substitution 4ii

Use the substitution $u = \sqrt{x+2}$ to find the exact value of $\int_{-1}^7 \frac{x^2}{\sqrt{x+2}} dx$.

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Integration by Substitution 4i

Use the substitution $u = 1 + \sqrt{x}$ to find the exact value of $\int_4^9 \frac{1}{1 + \sqrt{x}} dx$ in the form $a + b \ln(c)$, where a , b , and c are positive constants to be found.

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Integration by Substitution 3

Part A Integrate $\frac{3}{(z+1)^2}$

Find $\int_0^2 \frac{3}{(z+1)^2} dz$.

Part B Integrate $\frac{e^{-\alpha x}}{(1+e^{-\alpha x})^4}$

Find $\int \frac{e^{-\alpha x}}{(1+e^{-\alpha x})^4} dx$, where α is a constant.

The following symbols may be useful: alpha, e, k, x

Integration by Substitution 2i

In this question, I denotes the definite integral $\int_2^5 \frac{5-x}{2+\sqrt{x-1}} dx$. The value of I can be found using two different methods.

Part A Substitution

Show that the substitution $u = \sqrt{x-1}$ transforms I to $\int_1^2 (4u - 2u^2) du$ and hence find the exact value of I .

The following symbols may be useful: \int

Part B Rationalisation

Simplify as far as possible $(2 + \sqrt{x-1})(2 - \sqrt{x-1})$.

The following symbols may be useful: \times

By first multiplying the numerator and denominator of $\frac{5-x}{2+\sqrt{x-1}}$ by $2 - \sqrt{x-1}$, find the exact value of I .

The following symbols may be useful: \int

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Integration: General 5i

Part A Integrate $\frac{18}{\sqrt{6x+1}}$

Find $\int_0^4 \frac{18}{\sqrt{6x+1}} dx$ correct to two significant figures.

Part B Integrate $(e^x + 2)^2$

Find $\int_0^1 (e^x + 2)^2 dx$, giving your answer in terms of e.

The following symbols may be useful: e

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