Isaac Physics Skills

Developing mastery of

Essential pre-university physics

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Periphyseos Press
Cambridge, UK.

Cambridge
University Press

Co-published in Cambridge, United Kingdom, by
Periphyseos Press and Cambridge University Press.

`www.periphyseos.org.uk` and `www.cambridge.org`

First published & reprinted 2015
Second edition & First reprint 2015
Second – Sixth reprints 2016, Seventh – Ninth reprints 2017, Tenth & Eleventh reprints 2018
Third edition 2018. First reprint 2019
Co-published adaptation, 2020

Printed and bound in the UK by Short Run Press Limited, Exeter.

Typeset in LaTeX

A catalogue record for this publication is available from the British Library

ISBN 978-1-8382160-0-9 Paperback

Use this book in parallel with the electronic version at `isaacphysics.org`.
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Register as a student or as a teacher to gain full functionality and support.

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Attaining Mastery – Notes for the Student and the Teacher

These sheets are of two kinds - skill sheets and fact sheets and are equally valuable in both the early stages of learning and in revision.

The skill sheets provide practice for the student in applying a single principle of physics to a range of reasonably straightforward situations - often starting out with the substitution of values into an equation. After the first few questions have enabled the student to gain confidence in using the equation, subsequent questions may require the use of more than one equation or principle, or insight to solve a novel problem.

The fact sheets test knowledge on the parts of sixth form physics courses which require reading. While eminently suitable for revision, they may also be used to verify that the student has performed prior reading on the operation of an MRI scanner, for example, before it is discussed in class.

While the manner of use is up to the teacher and the student, we recommend that until a pass mark (we suggest 75%, as indicated in the square by each skill sheet) is obtained, the student studies further, then repeats a selection of questions. This process is repeated until the student passes. This is our understanding of applying a mastery method to ensure a good foundation is laid for a pre-university physics education. The teacher's mark book records how many attempts the student has taken rather than the mark obtained on a sole attempt. Students, likewise, have a list of the skills, and tick them off when the required level of proficiency has been obtained. A grid is provided at the front of this book for this very purpose. In this way, all students achieve mastery of the skills, and do not move on until this has been achieved. We have found that all students who have the capacity to pass an A-level course have the capacity to attain mastery of all of these skills. However, we find that if such mastery has been obtained, the student goes on to gain a greater understanding of physics and a higher grade at A-level than would be expected otherwise.

$\boxed{{}^{n}/_{m}}$

We also recommend that an answer is not considered correct unless it is numerically accurate, is given to an appropriate number of significant figures, and incorporates a suitable unit: a student cannot be satisfied with their comprehension while they are still getting the final result incorrect. After all, this would be intolerable in any practical situation in which physics principles were applied. The Isaac Physics on-line version of this book, and its associated marking facility, follow these significant figure and unit requirements.

A mapping of each set of exercises in the book onto the 6 major school exam specifications in the UK is given on-line at `isaacphysics.org/pages/syllabus_map` where, with a click, the appropriate part of the book can be accessed and the problems solved or set as homework.

ACM & JJC
Westcliff-on-Sea & High Wycombe, 2015

Acknowledgements

These sheets, and the approach presented here for using them, were first devised for use in the Physics Department at the Royal Grammar School in High Wycombe, where we served as colleagues from 2010 to 2014. We are grateful to the students who have put them to such productive use, and have given valuable feedback. We are also grateful to colleagues in other schools who have also applied these methods and given us encouragement – particularly Keith Dalby at Westcliff High School for Boys.

However, our greatest thanks are to our colleagues in High Wycombe who tirelessly used these methods with their students to bring about better learning - to Peter Glendining, to Matthew Hale (who also devised an excellent spreadsheet for the monitoring of student progress on these sheets), and to Paula Dove who suggested the need for fact sheets.

We are also grateful to Prof. Mark Warner and his team at the Cavendish Laboratory for their assistance in turning a set of A4 problem sheets into a professional publication. Much credit is due to James Sharkey and Aleksandr Bowkis for the typesetting and editing. We are also grateful to Robin Hughes and Ally Davies for their thoughtful comments on the draft of this text.

Anton has three personal 'thank yous' to record: firstly to the Civil Aviation Authority. Its insistence that all student pilots gain mastery in all ground school topics inspired the development of a similar approach to A-level Physics teaching. Secondly, to Jennifer for her support and feedback in the implementation of the project, not to mention the enthusiasm with which she has championed its adoption, and her extensive written contributions. Most importantly, he wishes to acknowledge Helen Machacek. For not only has she remained the most supportive, encouraging and loving wife imaginable, but she bore much more than her fair share of the responsibility of looking after their children during the holiday periods when her husband was writing many of the sheets.

Jennifer wishes to thank Anton, for being such an inspiration, both as a physicist and as a friend. He has opened her eyes to so many ideas and possibilities in the teaching of Physics and in interactions with students, and this skills sheet approach is just one example of that. He has been unendingly supportive and his wisdom, advice and example have contributed hugely to the teacher and to the person she is today.

And finally, we thank you for being willing to try them out too. We wish you well with your studies, and hope that these sheets help you and your students attain understanding and success.

Soli Deo Gloria,

ACM & JJC
Westcliff-on-Sea & High Wycombe, 2015

Using Isaac Physics with this book

Isaac Physics offers on-line versions of each sheet at `isaacphysics.org/books/physics_skills_19` where a student can enter answers. This on-line tool will mark answers, giving immediate feedback to a student who, if registered on isaacphysics.org, can have their progress stored and even retrieved for their CV! Teachers can set a sheet for class homework as the appropriate theme is being taught, and again for pre-exam revision. Isaac Physics can return the fully assembled and analysed marks to the teacher, if registered for this free service. Isaac Physics zealously follows the significant figures (sf) rules below and warns if your answer has a sf problem.

Uncertainty and Significant Figures

In physics, numbers represent values that have uncertainty and this is indicated by the number of significant figures in an answer.

Significant figures

When there is a decimal point (dp), all digits are significant, except leading (leftmost) zeros: 2.00 (3 sf); 0.020 (2 sf); 200.1 (4 sf); 200.010 (6sf)

Numbers without a dp can have an *absolute accuracy*: 4 people; 3 electrons.

Some numbers can be ambiguous: 200 could be 1, 2 or 3 sf (see below). Assume such numbers have the same number of s.f. as other numbers in the question.

Combining quantities

Multiplying or dividing numbers gives a result with a number of sf equal to that of the number with the smallest number of sf:

$x = 2.31$, $y = 4.921$ gives $xy = 11.4$ (3 sf, the same as x).

An absolutely accurate number multiplied in does not influence the above.

Standard form

On-line, and sometimes in texts, one uses a letter 'x' in place of a times sign and ˆ denotes "to the power of":

1800000 could be 1.80x10ˆ6 (3 sf) and 0.0000155 is 1.55x10ˆ-5

(standardly, 1.80×10^6 and 1.55×10^{-5})

The letter 'e' can denote "times 10 to the power of": 1.80e6 and 1.55e-5.

Significant figures in standard form

Standard form eliminates ambiguity: In $n.nnn \times 10^n$, the numbers before and after the decimal point are significant:

$191 = 1.91 \times 10^2$ (3 sf); 191 is $190 = 1.9 \times 10^2$ (2 sf); 191 is $200 = 2 \times 10^2$ (1 sf).

Answers to questions

In this book and on-line, give the appropriate number of sf: For example, when the least accurate data in a question is given to 3 significant figures, then the answer should be given to three significant figures; see above. Too many sf are meaningless; giving too few discards information. Exam boards also require consistency in sf.

Physical Quantities

Quantity	**Magnitude**	**Unit**
Permittivity of free space (ϵ_0)	8.85×10^{-12}	$\mathrm{F\,m^{-1}}$
Electrostatic force constant ($1/4\pi\epsilon_0$)	8.99×10^{9}	$\mathrm{N\ m^2\ C^{-2}}$
Speed of light (*in vacuo*)	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
Universal gravitational constant (G)	6.67×10^{-11}	$\mathrm{N\ m^2\ kg^{-2}}$
Avogadro constant (N_A)	6.02×10^{23}	$\mathrm{mol^{-1}}$
Boltzmann constant (k_B)	1.38×10^{-23}	$\mathrm{J\,K^{-1}}$
Gas constant (R)	8.31	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Planck constant (h)	6.63×10^{-34}	$\mathrm{J\,s}$
Charge of proton (e)	1.60×10^{-19}	C
Electron volt (eV)	1.60×10^{-19}	J
Unified atomic mass unit (u)	1.66054×10^{-27}	kg
Mass of neutron (m_n)	1.67493×10^{-27}	kg
Mass of neutron (m_n)	1.00867	u
Mass of proton (m_p)	1.67262×10^{-27}	kg
Mass of proton (m_p)	1.00728	u
Mass of electron (m_e)	9.10938×10^{-31}	kg
Mass of electron (m_e)	5.48580×10^{-4}	u
0 °C	273.15	K
1 parsec (pc)	3.086×10^{16}	m
Acceleration due to gravity (g)	9.81	$\mathrm{m\,s^{-2}}$
Seconds per year	3.156×10^{7}	-
Light year	9.46×10^{15}	m
Specific heat capacity of water	4180	$\mathrm{J\,kg^{-1}\,K^{-1}}$

Contents

Checklists

Unit	Skill	
A1	Choose an appropriate equation from a list, re-arrange it, substitute numbers for variables, and calculate the unknown quantity.	
A2	Express units in terms of SI base units.	
A3	Express a measurement in standard form (2.43×10^{-8}) or using a prefix (24.3 ns) to a given number of significant figures.	
A4	Convert measurements from one unit to another.	
A5	Calculate the gradient or y-intercept of a straight line on a graph and give its unit.	
A7	Estimate the area under the line on a graph and give its unit.	
A8	Estimate areas under lines using units with prefixes.	
A9	Calculate changes in response to changes by factors & percentages.	
A10	Calculate changes when a proportionality is known.	
B1	Determine the horizontal and vertical components of a vector (displacement, velocity, force or the electric field in Malus' Law).	
B2	Determine the sum of two vectors by scale drawing or trigonometry where the triangle of vectors always has a 90° angle.	
B3	Solve problems of uniform accelerated motion in 1-dimension (SUVAT problems).	
B4	Solve trajectory problems using the independence of horizontal and vertical motion.	
B5	Find the missing force in a question using the principle of moments.	
B6	Stress, strain and Young's modulus.	
B7	Force & energy calculations for springs separately & combined.	
B8	Mechanical calculations of energy and power.	
B9	Energy and force in springs and stretched materials.	
C1	Resistors & resistivity.	
C2	Understands the relationship between charge carrier motion and electric current.	
C4	Solve circuit problems using Kirchhoff's Laws.	
C5	Find the voltage across components in a potential divider circuit.	
C6	Work out the terminal p.d. of a battery given the current supplied, e.m.f. and internal resistance.	
D1	Calculate amplitudes and intensities from power.	
D2	Intensities of light after passing a polarizer.	
D3	Determine whether a wave, traveling by two different routes to a detector will interfere constructively or destructively.	

Unit	Skill	
D4	Select the right equation to use for two-source interference problems, re-arrange it, and solve it to obtain the correct answer.	
D5	Understand standing waves.	
D6	Select an equation to use in solving photoelectric effect problems, re-arrange it, and solve it to obtain the correct answer.	
D7	Perform calculations relevant to quantum physics.	
D8	Calculate refractive indices, angles of refraction, and critical angles.	
D9	Calculate electromagnetic spectra from atomic energy schemes.	
E1	Estimate absolute uncertainties.	
E2	Calculate relative uncertainties.	
E3	Estimate the relative uncertainty in a calculated result from the uncertainties of the original measurements.	
E4	Assess whether measurements are accurate or reliable.	
F1	Calculate the force needed to change an object's momentum in a given time.	
F2	Solve a 1-d problem in conservation of momentum.	
F3	Convert angles from degrees to radians, can convert ordinary speeds into angular speeds, can convert between frequency, time period & angular frequency.	
F4	Work out the size and direction of the force needed to keep an object in uniform circular motion.	
F5	Work out the gravitational force on an object using Newton's law of gravity, either directly (knowing M) or by comparison with another object where the force is known.	
F6	Work out the time period of a circular orbit from its radius (or vice versa) without looking up Kepler's 3rd Law.	
F7	Perform calculations related to oscillators.	
G1	Convert Celsius into kelvin, and know when K must be used.	
G2	Use the right form of the gas law (pV=nRT or $pV = N$kT or pV/T=const) to solve a problem involving gases.	
G3	Calculate energies required to cause temperature changes, and to calculate the final temperature of mixtures.	
G4	Calculate energies required to cause changes of state.	
H1	Work out the force (direction & magnitude) on an electron or alpha particle between two charged plates.	
H2	Calculate the electric field E near one or two point charges.	
H3	Find the speed of electrons accelerated from rest in an electric field.	

Unit	Skill	
H4	Work out the force (direction & magnitude) on a wire carrying a current in a magnetic field.	
H5	Work out the force (direction & magnitude) on a moving charged particle in a magnetic field.	
H6	Work out the radius of the circular path followed by a charged particle in a magnetic field.	
H7	Work out the e.m.f. (magnitude & direction) induced in a coil of wire when it is moved in a magnetic field.	
H8	Calculate the voltage on the secondary of a transformer.	
H9	Calculate energies and potentials of charges in electric fields.	
I1	Work out the charge & energy stored on a capacitor from the charging voltage.	
I2	Work out the capacitance of a network of capacitors.	
I3	Sketch the current / voltage / charge on a capacitor as a function of time as it discharges through a resistor, labelling key points.	
J1	Complete a nuclear equation (including beta decay and neutrinos.)	
J2	Calculate the half life of a radioactive sample from a knowledge of number of nuclei & activity.	
J3	Sketch the no. of nuclei remaining in / activity of a radioactive source as a function of time as it decays, labelling key points.	
J4	Calculate the energy released in a nuclear reaction from the masses of reactants & products OR a graph of binding energy per nucleon.	
K1	Work out galaxy velocity from spectral shift and thus its distance using Hubble's law.	
K2	Perform a variety of exponential calculations.	

Explanation Checklist

Unit	Skill	
L1	How a mass spectrometer works.	
L2	The main categories of fundamental particles.	
L3	The construction of a nuclear (fission) reactor.	
L4	What happens when X-rays hit tissue, and how X-ray images can be improved.	
L5	How an ultrasound image is taken, the significance of acoustic impedance, and the difference between A and B scans.	
L6	How MRI works, and the advantages & disadvantages of MRI in comparison with other techniques (X-ray, PET).	
L7	The life cycle of a star.	
L8	The history of the Universe according to the Big Bang model.	

Chapter A

General Questions

The boxed fraction shows how many questions need to be answered correctly to achieve mastery.

A1 Using and Rearranging Equations

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Use the following equations:

$$s = ut \qquad a = \frac{(v - u)}{t} \qquad F = ma \qquad v = f\lambda$$

$$V = IR \qquad P = IV \qquad E = Pt \qquad Q = It$$

where the letters have the following meanings:

s = distance	u, v = velocity	t = time	m = mass
V = voltage	I = current	F = force	a = acceleration
Q = charge	E = energy	P = power	f = frequency
λ = wavelength	R = resistance		

A1.1 a) $F = 3.0$ N, $m = 2.0$ kg, what is a?

b) $I = 0.20$ A, $t = 200$ s, what is Q?

A1.2 Calculate the resistance needed if you want 0.030 A to flow through a component when a 9.0 V battery is connected to it.

A1.3 Calculate the distance travelled by a car going at 30 m s^{-1} in 2.0 minutes.

A1.4 Calculate the wavelength of a wave that travels at 3.0×10^8 m s^{-1} if its frequency is 2.0 GHz (2.0×10^9 Hz).

A1.5 a) Calculate the power of a 0.25 A, 240 V light bulb.

b) Calculate the power if 5.0 A flows through a 2.0 Ω resistor.

A1.6 A Corsa accelerates from 15 m s^{-1} to 25 m s^{-1} in 8.0 s. Calculate the acceleration.

A1.7 If a jet has a maximum acceleration of 20 m s^{-2}, what is the time it would take to get from 0 m s^{-1} to 100 m s^{-1}?

A1.8 My kettle needs to be able to give 672 000 J of heat energy to water in 240 s. Assuming that it is connected to the 240 V mains, what current is needed?

A1.9 Calculate the force needed if my 750 kg car needs to accelerate from rest to 13 m s^{-1} in 5.0 s.

A1.10 Calculate the electrical energy used by a 240 V light bulb with a resistance of 60 Ω in 600 s.

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A2 Derived and Base SI Units

Express the following derived units in terms of the SI base units. The first one has been done for you:

	Derived Unit	in Base Units	Power of each base unit			
			m	s	kg	A
	m s^{-2}	m s^{-2}	1	-2	0	0
A2.1	J		(a)	(b)	(c)	(d)
A2.2	N		(a)	(b)	(c)	(d)
A2.3	C		(a)	(b)	(c)	(d)
A2.4	V		(a)	(b)	(c)	(d)
A2.5	Ω		(a)	(b)	(c)	(d)
A2.6	Pa		(a)	(b)	(c)	(d)
A2.7	N C^{-1}		(a)	(b)	(c)	(d)
A2.8	V m^{-1}		(a)	(b)	(c)	(d)

Express the following derived units in terms of the unit specified and base units. The first one has been done for you.

A2.9 a) Express the ohm in terms of the volt and base units: $\Omega = \mathrm{V\,A^{-1}}$

b) Express the joule in terms of the newton and base unit(s).

c) Express the pascal in terms of the joule and base unit(s).

d) The answer to (c) means that pressure in effect measures an amount of energy per unit ________________

e) Express the $\mathrm{V\,m^{-1}}$ in terms of the joule and base unit(s).

f) Express the unit of density in newtons and base unit(s).

A3 Standard Form and Prefixes

$^{9}/_{12}$

You will be penalized if you give the wrong number of significant figures where the question specifies the required number of significant figures. [NOTE: standard form means that there is always one non-zero digit before the decimal point.]

A3.1 Write the following as 'normal' numbers:

a) 3×10^4 b) 4.89×10^6

A3.2 Write the following as 'normal' numbers:

a) 3.21×10^{-3} b) 2×10^0

A3.3 Write the following in standard form to three significant figures:

a) 2 000 000 b) 34 580

A3.4 Write the following in standard form to three significant figures:

a) 23.914 b) 0.000 005 638

A3.5 Write the following as 'normal' numbers with the unit (but without the prefix):

a) 3 kJ b) 20 mA

A3.6 Write the following using the most appropriate prefixes:

a) 5×10^7 m b) 6×10^{-10} s

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A4 Converting Units

Convert between units as specified. Express your answer in standard form if the power of ten is ≥ 3, or ≤ -3. Your answer must include units, as indeed it must in *all* questions with units in this book.

Convert:

A4.1 a) 34.5 mm to nm b) 34.5 mm to pm

A4.2 2.4 ps to ms

A4.3 a) 465 μA to mA b) 465 μA to kA

A4.4 43×10^{-7} GW to μW

A4.5 34 m^2 to cm^2

A4.6 58 N m to N cm

A4.7 9600 μm^2 to cm^2

A4.8 0.035 N cm^{-2} to Pa

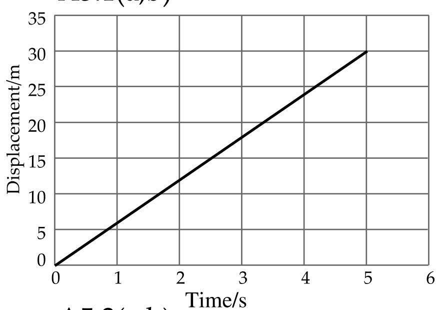
A4.9 450 kg m^{-3} to kg mm^{-3}

A5 Gradients and Intercepts of Graphs

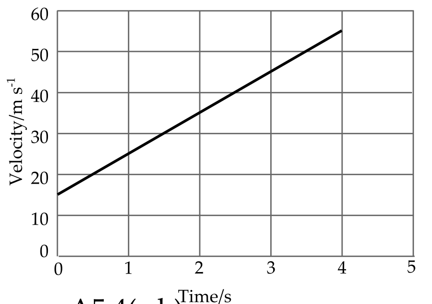
$^{8}/_{10}$

Work out the physical quantity corresponding to the gradient and y-intercept.

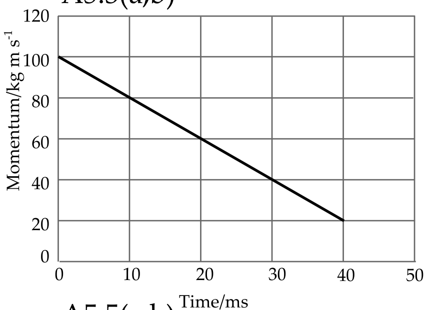
A5.1(a,b)



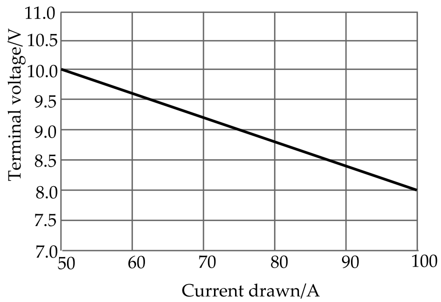
A5.2(a,b)



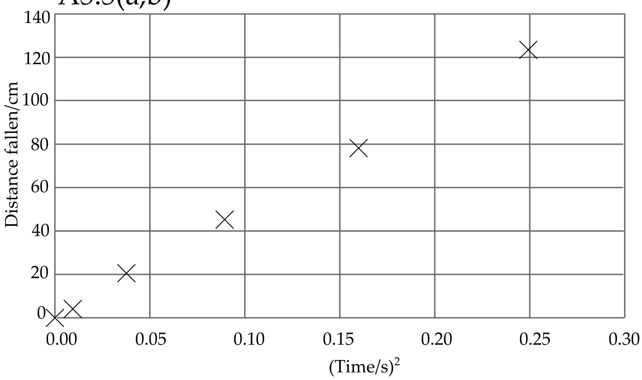
A5.3(a,b)



A5.4(a,b)



A5.5(a,b)



Chapter F

Mechanics

$^{8}/_{10}$ F1 Force and Momentum

In these questions ignore the effects of friction & drag.

F1.1 What is the momentum of a 750 kg car going at 31 m s^{-1}?

F1.2 What is the momentum of an electron (mass = 9.1×10^{-31} kg) travelling at 3.0×10^{7} m s^{-1}?

F1.3 If a 20 000 kg bus accelerates from 10 m s^{-1} to 25 m s^{-1}, what is the change in momentum?

F1.4 A 50 g ball is travelling at 2.0 m s^{-1} when it hits a wall and rebounds at 1.5 m s^{-1}. Calculate the change in momentum.

F1.5 A 750 kg car takes 15.3 s to accelerate from 5.0 m s^{-1} to 31 m s^{-1}. Calculate the force needed to do this.

F1.6 A 70 kg person jumps in the air and is travelling downwards at 2.0 m s^{-1} when their feet touch the ground. If it takes the person 0.30 s to stop, calculate the resultant force on them.

F1.7 I am trying to push start a car which has stopped. If the biggest force with which I can push the car is 420 N, and the car has a mass of 1025 kg, how fast will it be going after 8.0 s of pushing?

F1.8 Calculate the force needed to accelerate a 50 000 kg spacecraft from rest to 7000 m s^{-1} in four minutes.

F1.9 An alpha particle (mass = 6.7×10^{-27} kg) is fired at the nucleus in a gold atom with a speed of 3.5×10^{6} m s^{-1}. It bounces off at the same speed in the opposite direction. If the collision takes 10^{-19} s, what is the average force?

F1.10 How long would it take a 637 N force to accelerate a 65 kg physics teacher from rest up to a speed of 100 m s^{-1}? (NB this is over 200 mph)

F2 Conservation of Momentum

$^{8}/_{10}$

F2.1 Two masses, called Alfie and Beth, collide and stick together under four different circumstances, as shown in the four rows of the table below. Calculate the missing measurements:

Before collision				After collision
Alfie's mass /kg	Alfie's velocity /m s^{-1}	Beth's mass /kg	Beth's velocity /m s^{-1}	Velocity /m s^{-1}
30	+2.0	40	+1.5	(a)
60	-1.4	30	+2.8	(b)
120	+1.5	80	(c)	0.0
120	+3.0	(d)	-31	+2.0

F2.2 Charlie is driving her 20 000 kg bus. She stops at a roundabout. Percy is driving his 750 kg Corsa at 15 m s^{-1} behind her. He fails to stop and rams into the back of the bus, sticking to it. The impact releases the brakes on the bus. How fast will the smashed up wreck be travelling immediately after the collision?

F2.3 A neutron (mass = 1 u) is moving at 300 m s^{-1} when it smacks into a stationary ^{235}U nucleus (mass = 235 u), and sticks to it. What will the velocity of the combined particle be?

F2.4 A 7.90 g bullet is travelling at 200 m s^{-1}. It hits a 3.00 kg sack of sand which is hanging by a rope from the ceiling. The bullet goes into the sack, and is stopped inside it by friction with the sand. How fast is the sack going immediately after the bullet has "stopped" inside it[1]? NB you must give your answer to 3 significant figures to be awarded the mark.

F2.5 A rocket (containing a space probe) is travelling at 7000 m s^{-1} in outer space. The 2000 kg probe is ejected from the front of the rocket (forwards) using a big spring. If the speed of the probe afterwards is 7200 m s^{-1}, and the rest of the rocket has a mass of 6000 kg, what is the speed of the rest of the rocket?

[1] "stopped" means stopped relative to the sand, not stopped relative to a stationary observer.

F2.6 In a strange form of billiards, the cue ball is one third the mass of the other balls, which are stripey. There is no spin, and I hit a stripey ball centrally with the cue ball (travelling at 1.4 m s^{-1}) such that the cue ball rebounds in the opposite direction with half of its initial speed. What is the speed of the stripey ball?

F2.7 I am stranded, stationary, in space, but near to my spacecraft. I detach my 30 kg oxygen cylinder, and fling it away from the spacecraft with a speed of 3.0 m s^{-1}. If my mass (without the cylinder) is 80 kg, how fast will I travel in the other direction towards my spacecraft?

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F3 Units of Rotary Motion

F3.1 How big is 3 rad, when expressed in degrees?

F3.2 How many radians are there in 90°?

Complete the questions in the table by converting the units:

	Time period /s	Frequency /Hz	Angular Velocity /rad s^{-1}	Revolutions per Minute (rpm)
F3.3	0.50	(a)	(b)	(c)
F3.4	(a)	(b)	3.0	(c)
F3.5	(a)	(b)	(c)	3800
F3.6	(a)	50	(b)	(c)
F3.7	2700	(a)	(b)	(c)

F3.8 A car travels 10 km. One of its wheels has a radius of 30 cm. Calculate the angle the wheel turns as the car travels this distance (answer in radians).

F3.9 An astronaut's training centrifuge has a radius of 4.0 m. If it goes round once every 2.5 s, calculate the velocity of the end of the centrifuge arm (4.0 m from the pivot).

F3.10 My washing machine has a spin speed of 1200 rpm, and a drum radius of 20 cm. Calculate how fast clothes go, when up against the side of the drum.

Isaac Physics

Developing problem solving skills.

L. Jardine-Wright
Director, Isaac Physics Project

About the author

Lisa Jardine-Wright is an astrophysicist, and a tutor and director of studies in physics at Churchill College, Cambridge. She lectures first year maths and physics within the Cambridge Natural Sciences degree, and has also directed and developed the educational outreach programmes of the Cavendish Laboratory.

In 2017, the University of Cambridge awarded her the Pilkington Prize for her outstanding and sustained contribution to excellent teaching in Physics and Mathematics both within the University and beyond and in 2019, she was jointly awarded the IOP Lawrence Bragg Medal and Prize, with Prof. Mark Warner, for the foundation and development of the Isaac Physics project. She was elected as the Vice-President for Education of the Institute of Physics in 2020.

Periphyseos Press
Cambridge, UK.

Acknowledgements

In this book we provide a sample of physics problems, selected from thousands that are available at `isaacphysics.org`. The Isaac Physics project and I would like to thank Cambridge Assessment for their permission to include selections of their previous examination questions on the Isaac platform, an Open Platform for Active Learning (OPAL).

I would like to record grateful thanks to all of the Isaac team past and present, and to the computer scientists that so ably produce the software and infrastructure that makes isaacphysics.org unique.

LJW, 2020

The Isaac Physics Project

The Isaac Physics project is funded by the Department for Education in England and hosted by the Cavendish Laboratory and the Department of Computer Science & Technology of the University of Cambridge.

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Contents

Developing Physics Problem Solving

Isaac Physics - You work it out `isaacphysics.org`

The key concepts and numerical manipulations that you practise in Chapters A – L are the vital step on the road to studying physics, engineering or any STEM course at university. To further advance your thinking as a physicist, you also need to enhance these skills and this analysis by combining multiple concepts and techniques, in order to solve longer and more involved problems. These multi-step, multi-stranded questions can be challenging for experienced physicists, but completion of such problems gives a real sense of achievement and success.

The main mission of Isaac Physics is to provide problem solving questions for students at GCSE (or equivalent) through to those who have finished their A-levels and are preparing for university. We have over 850 questions graded from Level 1 (post-GCSE) through to Level 6 (pre-university and beyond – NOT for the faint-hearted).

In the following chapters you will find:

- advice on how to problem-solve,
- sample solutions to two problems from our site (Level 3 Dynamics: Pop-up Toy, and Level 5 Statics: Prism)
- sample questions from mechanics and fields, from our six levels on isaacphysics.org
  Online, we also have the topics: waves, circuits and physical chemistry. They too span all six levels, with questions for you to try.
- next to each topic heading, a url link to a webpage containing this and the other questions from the same level and topic.

Take up the challenge and put your new found skills to the test – but before you do, make sure you pick up a large **pad of paper** and **a pencil** so that ***you*** can work it out.

A Guide to Solving Physics Problems

`isaacphysics.org/solving_problems`

Physicists develop highly desirable skills through their extensive experience of problem solving – logic, determination, resilience and mathematical ability to name just a few. You will become an ace problem solver by answering *lots* of questions – these 5 key steps will help you develop a logical, structured method and universal approach.

5 key steps to problem solving

- Step 1: Keywords
- Step 2: Diagram
- Step 3: Concepts
- Step 4: Symbols
- Step 5: Dimensions & Numbers

When faced with a new question, we employ a strategy to break the problem down into a series of 5 steps to digest and analyse it. Each step helps us to understand the information given in the question and establish what it is that we are being asked to calculate or discover. Using these steps for each new question we attempt can, with practice, make solving physics problems extremely satisfying and rewarding.

- **Step 1: Keywords**
  Are there words in the question that contain additional information about the problem? Frictionless; light; uniform… Highlight these words so that they stand out – they will help to simplify the solution and allow us to neglect concepts that we don't need to consider.

- **Step 2: Draw a diagram**
  The action of drawing a diagram helps us to digest and summarise the information in a question. Drawing a diagram will save time and effort later and is the key to finding a solution. When drawing the diagram, label the quantities that are given with symbols (e.g. u for a velocity, d for a displacement); staying in symbols rather than using numbers is vital – see Step 4.

- **Step 3: Key concepts & mathematics**
  Fluency with mathematical rearrangements is essential but we need to

be sure that we are logical with our approach and consider the physical concepts that we need before throwing algebra at the problem. Identify which concepts are relevant to the problem. Write down the relevant physical principles and equations that might be useful – particularly if they connect the quantities that are given in the question.

- **Step 4: Stay in symbols**
  Even if the question gives numerical values, represent each of them with a symbol. This may appear to overcomplicate the problem, but it really helps when checking the solution or trying to find a mistake.

  For example, imagine that as part of the calculation we want to find the magnitude of the displacement, $|\underline{\boldsymbol{d}}|$, of a cyclist who has travelled 13 km West and 5.0 km North.

  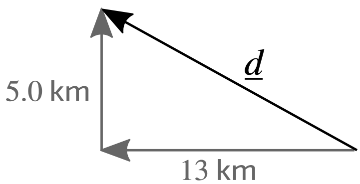

  Figure 1: The displacement of a cyclist who has travelled 13 km West and 5.0 km North.

  You identify that you need to use Pythagoras' theorem

  $$|\underline{\boldsymbol{d}}| = \sqrt{144} = 12\,\text{km} \qquad \text{WRONG} \qquad (1)$$

  The numbers hide information about the calculation that letters would not. Let $w = 13$ km and $n = 5$ km

  $$|\underline{\boldsymbol{d}}| = \sqrt{w^2 - n^2} = 12\,\text{km} \qquad \text{WRONG} \qquad (2)$$

  You can see straight away that the square of the numbers have accidentally been subtracted rather than added.

- **Step 5: Check dimensions, then put in numbers, check if reasonable**
  Before putting numbers into an algebraic expression, experienced physicists check whether their answer has the correct *dimensions* (you may have seen this before as checking your units). For example, if we are trying to find a quantity of time, *each term* in our expression must have dimensions of *only* time – no length, or mass or charge!

  Imagine that we are trying to calculate how fast the Earth travels around the Sun and we know that the radius of the Earth's orbit is $r = 1.50 \times 10^{11}$ m and that it takes $t = 365.25$ days. We make a mistake and write down that the distance travelled by the Earth is $2\pi r^2$. If we check our dimensions on both sides of the equation, we can see that this is incorrect.

$$\text{speed} = \frac{2\pi r^2}{t} \tag{3}$$

$$\text{Dimensions:} \quad \frac{[\mathrm{L}]}{[\mathrm{T}]} \neq \frac{[\mathrm{L}]^2}{[\mathrm{T}]} \tag{4}$$

The length dimension, $[\mathrm{L}]$, that we have on the top of the left hand side of expression (4) does not match the length squared, $[\mathrm{L}]^2$ on the top of the right hand side of expression, so we determine that there is a mistake with our r^2 part of the expression.

Lastly, we put in the numbers and, vitally, check the magnitude of our answers – are they reasonable?

Now we have the steps, we apply them to two example problems from Levels 3 and 5 of isaacphysics.org.

Example Solution - Level 3 Dynamics: Pop-up Toy

`isaacphysics.org/s/i8OQ9P`

Q: A pop-up toy consists of a head and sucker of combined mass $m = 1.5$ kg stuck to the top of a light spring of natural length $l_0 = 0.30$ m and spring constant $k = 250$ N m^{-1}. The centre of mass of the system can be taken to be at the top of the spring. The spring is compressed to length $l_1 = 0.10$ m when the pop-up toy is stuck to the ground.

What height above the ground does the bottom of the unstretched spring jump to when it is smoothly released?

Step 1: Keywords

Highlight the key words in the text above.
What are the key words in the question that will help you to draw your diagram and understand the physical concepts that might be useful? Note also what the question is asking for.

Solution:
- combined mass at the top $-$ we can consider the sucker and mass as one object of mass m at the top of the spring.
- centre of mass $-$ the point at which we should consider the weight of the combined mass to act.

- light — we can consider the spring to have no mass or weight, it is negligible.
- spring — is extensible.
- smoothly released — the toy is not caused to jolt in anyway so there is no energy converted to heat or sound; we conserve kinetic and potential energies.
- **Q:** Height of the bottom of the spring.

Step 2: Draw a diagram

Draw and annotate a diagram with all of the information from the question, in particular draw each stage of the toy's behaviour.

Solution:

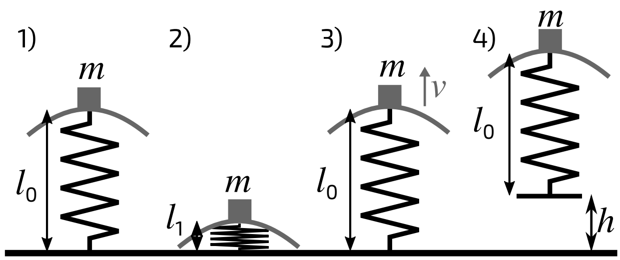

Figure 2: The 4 stages of the toy's behaviour. 1) Before compression. 2) Compressed and ready to jump. 3) Returned to its natural length, l_0, but now the mass and sucker has a velocity, v. 4) At maximum height, the velocity of the mass and sucker = 0.

Steps 3 & 4: Concepts, mathematics & symbols

Identify the concepts that you think might be useful in answering the question. Doing this gets you started. Some ideas might not get used.

Solution: Hooke's Law, spring constant, forces, work done, gravitational potential energy (GPE), elastic potential energy (EPE), kinetic energy (KE) and conservation of energy.

Method 1: Energy

How do you know whether to analyse a problem using forces (method 2) or energy? Using conservation of energy in this problem is simpler, as the total amount of energy remains the same throughout all the stages of motion and therefore we can consider just the beginning and end situations (stage 2 and stage 4).

Write down an expression for the conservation of energy. The total energy of the toy at stage 2 is equal to its total energy at stage 4.

- **Take care to define a position of zero gravitational potential energy.**

Solution: For the whole of this solution I will choose for the ground level to be the position of zero gravitational potential energy.

$$
\begin{aligned}
\text{Total energy at stage 2} &= \text{Total energy at stage 4} \\
\text{GPE}_2 + \text{EPE}_2 &= \text{GPE}_4 \\
mgl_1 + \frac{1}{2}k(l_0 - l_1)^2 &= mg(h + l_0) \\
h + l_0 &= l_1 + \frac{1}{2mg}k(l_0 - l_1)^2 \\
h &= l_1 - l_0 + \frac{1}{2mg}k(l_0 - l_1)^2 \qquad (5)
\end{aligned}
$$

Method 2: Force and Work Done

Using a force method for the pop-up toy is more challenging because the force due to the spring is not constant throughout the motion. However, analysing the problem through forces should give us the same answer.

How does the force exerted by the spring on the mass cause the toy leap off the ground?

Solution: Hooke's Law tells us that the magnitude of the force, F, needed to compress a spring by an amount x, is given by $F = kx$. To compress the spring by an additional small amount δx we need to do work **on** the spring. The amount of work done is $\delta W = F\delta x$. As δx becomes very small (tends to zero), δW tends to dW.

We can then find the total work done to compress the spring from stage 1 to stage 2 by integrating with respect to dx between the limits of the compression at stage 1 to the compression at stage 2.

$$
\begin{aligned}
&\text{Stage 1, compression} = 0 \\
&\text{Stage 2, compression} = (l_1 - l_0).
\end{aligned}
$$

The total work done on the spring between these two stages is then released between stages 2 and 4, and converted to gravitational potential energy.

$$
\begin{aligned}
\text{Work done from 1 to 2} &= \text{change in GPE from 2 to 4} \\
\int_0^{(l_1 - l_0)} (kx)\,\mathrm{d}x &= \text{GPE}_2 - \text{GPE}_4 \\
\left[\tfrac{1}{2}kx^2\right]_0^{(l_1 - l_0)} &= mgl_1 - mg(h + l_0)
\end{aligned}
$$

$$h + l_0 = l_1 + \frac{1}{2mg}k(l_1 - l_0)^2$$
$$h = l_1 - l_0 + \frac{1}{2mg}k(l_1 - l_0)^2$$

Our result is indeed consistent with that of method 1, equation (5).

This is not the only force method we could use, the problem can also be solved using Newton's Second Law to equate the resultant force on the toy, at a general time, to its acceleration. The acceleration must be written in terms of $\frac{\mathrm{d}v}{\mathrm{d}x}$ rather than $\frac{\mathrm{d}v}{\mathrm{d}t}$ so that we can then integrate to find an expression for v^2 which can be related to the height h.

Step 5: Dimensions & numbers

The question asks for a height, which has dimensions of length $[\mathrm{L}]$ $-$ each term on the right of our expression (5) should therefore also have dimensions of length.

Solution: Not all of the terms have dimensions that we can just write down; for example, what are the dimensions of k and g?

$$\begin{aligned}
g &= \text{acceleration} = [\mathrm{L}][\mathrm{T}]^{-2} \\
k &= \frac{\text{force}}{\text{length}} = \frac{[\mathrm{M}][\mathrm{L}][\mathrm{T}]^{-2}}{[\mathrm{L}]} = [\mathrm{M}][\mathrm{T}]^{-2} \\
h &= l_1 - l_0 + \frac{1}{2mg}k(l_0 - l_1)^2 \\
&= [\mathrm{L}] + [\mathrm{L}] + \frac{[1]}{[\mathrm{M}][\mathrm{L}][\mathrm{T}]^{-2}}[\mathrm{M}][\mathrm{T}]^{-2}([\mathrm{L}])^2 \\
&= [\mathrm{L}] + [\mathrm{L}] + [\mathrm{L}] = \text{correct}
\end{aligned}$$

Now we substitute the values given in the question and consider carefully the number of significant figures we should give in our answer.

$m = 1.5$ kg, $l_0 = 0.30$ m, $l_1 = 0.10$ m, $k = 250$ N m^{-1} and $g = 9.81$ m s^{-2}.

Solution:

$$\begin{aligned}
h &= l_1 - l_0 + \frac{1}{2mg}k(l_0 - l_1)^2 \\
h &= 0.10 - 0.30 + \frac{1}{2 \times 1.5 \times 9.81} \times 250 \times (0.30 - 0.10)^2 = 0.14\,\mathrm{m}
\end{aligned}$$

Is this answer reasonable? 14cm is indeed a realistic height for the bottom of the spring to reach.

Level 1: Mechanics

Each link by a topic heading will take you to nine more similar problems.

Statics

isaacphysics.org/s/Dbv5Dj

Q: Bed of Nails

A uniform rod of weight 500 N is supported by two pegs at either end of the rod.

Part A: Draw a free body diagram showing the forces acting on the rod. What is the magnitude of the forces exerted by each of the pegs on the rod?

Part B: If there are now eight pegs evenly spaced along the rod to support its weight, what force is applied by each peg on the rod?

Part C: Using the previous answers, explain how it is possible to lie on a bed of nails, but putting weight on one nail is extremely painful.

Dynamics

isaacphysics.org/s/CMjHTE

Q: A Hamburger

A hamburger has 2.2 MJ of chemical potential energy which can be released by eating it. (Take $g = 9.81$ m s^{-2})

Part A: How much energy is needed to lift a 50 kg student upwards through a height of 0.40 m?

Part B: How many step-ups of 0.40 m would the student need to make in order to burn off the energy of a hamburger?

Part C: How many hamburgers would the student need to consume in order to reach the top of Mount Everest, a height of 8.8 km?

Part D: If the energy of a hamburger could be used by itself to propel itself upwards, how high could it rise? (The mass of a hamburger is 0.22 kg; assume this remains constant. Assume that g also remains constant).

Kinematics

isaacphysics.org/s/IUV78T

Q: A Strange Planet

A lost astronaut lands her spaceship on an unknown planet. She decides to work out the value of the acceleration due to gravity on this planet so that she can check her onboard computer and find out her location. She knows that on Earth (where $g = 9.81$ m s^{-2} downwards) she takes 1.0 s to jump up and land again. On this planet, a jump takes 1.4 s.
What is the magnitude of the downward acceleration due to gravity on the strange planet? Assume she can jump up at the same speed on any planet.