21 Motion with Constant Acceleration

The equations we will develop and practise here can be used in any situation where the acceleration does not change. As long as drag forces are small enough to be ignored, this includes:

- anything falling freely;
- anything speeding up because an engine is providing a steady force on it;
- anything slowing down because brakes are providing a fixed force.

We start with three principles:

1. Displacement $=$ average velocity $\times$ time
2. Velocity change $=$ acceleration $\times$ time
3. If the acceleration is constant, then the velocity will rise steadily. This means that the average velocity will be half way between the starting and final velocities (it will be the mean of the starting and final velocities).

In this book, we use five letters to represent the quantities.

Letter	Quantity	Unit
s	Displacement	m
u	Starting velocity	m/s
v	Final velocity	m/s
a	Acceleration	m/s^2
t	Time taken	s

We can write our three principles as equations using these letters. Firstly, the third principle means that average velocity $= 1/2\,(u+v)$.

1. $s = \left(\dfrac{u+v}{2}\right) t$
2. $v - u = at$

Now rearrange equation (2) to make v the subject; and then substitute this into equation (1). This gives

$$v = u + at \quad \text{so} \quad s = \left(\frac{u + u + at}{2}\right) t = \frac{2ut + at^2}{2} = ut + \tfrac{1}{2}\,at^2$$

Next, rearrange equation (2) to make t the subject; and then substitute this into equation (1). Finally, rearrange it to make v^2 the subject. This gives

$$t = \frac{v - u}{a} \quad \text{so} \quad s = \left(\frac{u + v}{2}\right) \times \left(\frac{v - u}{a}\right) = \frac{(u + v)(v - u)}{2a} = \frac{v^2 - u^2}{2a}$$

$$\text{so} \quad v^2 = u^2 + 2as$$

Let's look at our four equations, often given in examination formula sheets.

$v = u + at$	has no s	$v^2 = u^2 + 2as$	has no t
$s = \left(\dfrac{u + v}{2}\right) t$	has no a	$s = ut + \frac{1}{2}\,at^2$	has no v

Example 1 – An aeroplane requires a speed of 26 m/s to take off. If its acceleration is 2.3 m/s^2, how much runway does it 'use up' before it lifts off? Assume it starts at rest.

Using basic principles:
Time = velocity gained / acceleration $= 26$ m/s $\div$ 2.3 m/s^2 $= 11.3$ s
Average velocity $= \frac{1}{2}\,(0.0$ m/s $+ 26$ m/s$) = 13$ m/s
Displacement = av. velocity $\times$ time $= 13$ m/s $\times$ 11.3 s $= 150$ m (2 sf)

Using the equations:

$u = 0$ m/s $\qquad v = 26$ m/s $\qquad a = 2.3$ m/s^2 $\qquad$ we want to know s

We use the equation with no t as we don't know t.
$v^2 = u^2 + 2as$, so $26^2 = 0^2 + 2 \times 2.3 \times s$
so $676 = 4.6 \times s$, so $s = 676/4.6 = 150$ m (2 sf)

Example 2 – How much time does it take a ball to fall 30 cm if it is accelerating downwards at 10 m/s^2 after being dropped?

NB: 'dropped' means it isn't moving to start with, so $u = 0$.

Using the equations:

$s = 0.30$ m $\quad u = 0$ m/s $\quad a = 10$ m/s^2 $\quad$ we want to know t

We use the equation with no v as we don't know v:
$s = ut + {}^1\!/_2\, at^2$, so $0.30 = 0t + {}^1\!/_2\, 10t^2$, so $0.3 = 5t^2$
$t^2 = 0.3/5 = 0.06$ so $t = \sqrt{0.06} = 0.24$ s

Using basic principles (where we use t to represent the time):
Velocity change $=$ acceleration $\times$ time $= 10t$
Final velocity $=$ initial velocity $+$ velocity change $= 0 + 10t = 10t$
Average velocity $= {}^1\!/_2\,(0 + 10t) = 5t$
Displacement $=$ average velocity $\times$ time $= 5t \times t = 5t^2 = 0.30$
so $t^2 = 0.30/5 = 0.06$ so $t = \sqrt{0.06} = 0.24$ s.

21.1 Complete the table, where each row is a separate question.

s (m)	u (m/s)	v (m/s)	a (m/s^2)	t (s)
	2.0	(a)	3.0	6.0
(b)	2.0		3.0	6.0
(c)	0.0		10	0.20
(d)	0.0		10	0.40
16	1.5	(e)		10
0.82	14	0.0		(f)
	31	0.0	-6.7	(g)

21.2 An old £5 note is 135 mm long. A friend has a crisp £5 note, and holds the bottom of the note in line with (and between) your thumb and index finger. She drops it, and if you grab it without moving your hand downwards, you are allowed to keep it. How quickly do you have to react to win your prize?

21.3 The Highway Code assumes that a car with its brakes on fully has an acceleration of -6.7 m/s^2. Calculate the

(a) time taken to stop a car from 30 mph (13.4 m/s);

(b) distance taken to stop a car at 30 mph;

(c) time taken to stop a car from 70 mph (31 m/s);

(d) distance taken to stop a car from 70 mph.

21.4 You throw a cricket ball up into the air at 10 m/s. [Hint: if you take $u = 10$ m/s then $a = -10$ m/s^2 as the acceleration is in the opposite direction to the initial velocity.]

(a) How much time elapses before it reaches the highest point of its motion? [Hint: at the top, v=0.]

(b) How high does it go?

21.5 If there were no air resistance, how much time would it take for a dropped parcel to fall $2\,000$ m?

21.6 What is the deceleration of a train which takes 2.3 km to stop from a speed of 67 m/s?

21.7 How much time does it take to stop an oil tanker if its speed is 8.0 m/s to start with, and the stopping distance is 5.0 miles? One mile is about $1\,600$ m.

21.8 The Eiffel Tower is 300 m high. A coin is dropped from the top; how fast is it going when it hits the pavement? Assume no air resistance.

21.9 How fast would you have to shoot a scientific instrument upwards if you wanted it to rise 200 km above the Earth's surface ignoring air resistance?

21.10 The acceleration of dropped objects on the Moon is 1.6 m/s^2. How long does it take a feather to fall 0.70 m? [There is no air resistance!]