Parametric equations 3ii

A Level

Figure 1 shows the curve with parametric equations

$$x = a\sin\theta,\ y = a\theta\cos\theta,$$

where a is a positive constant and $-\pi \leq \theta \leq \pi$. The curve meets the positive y-axis at A and the positive x-axis at B.

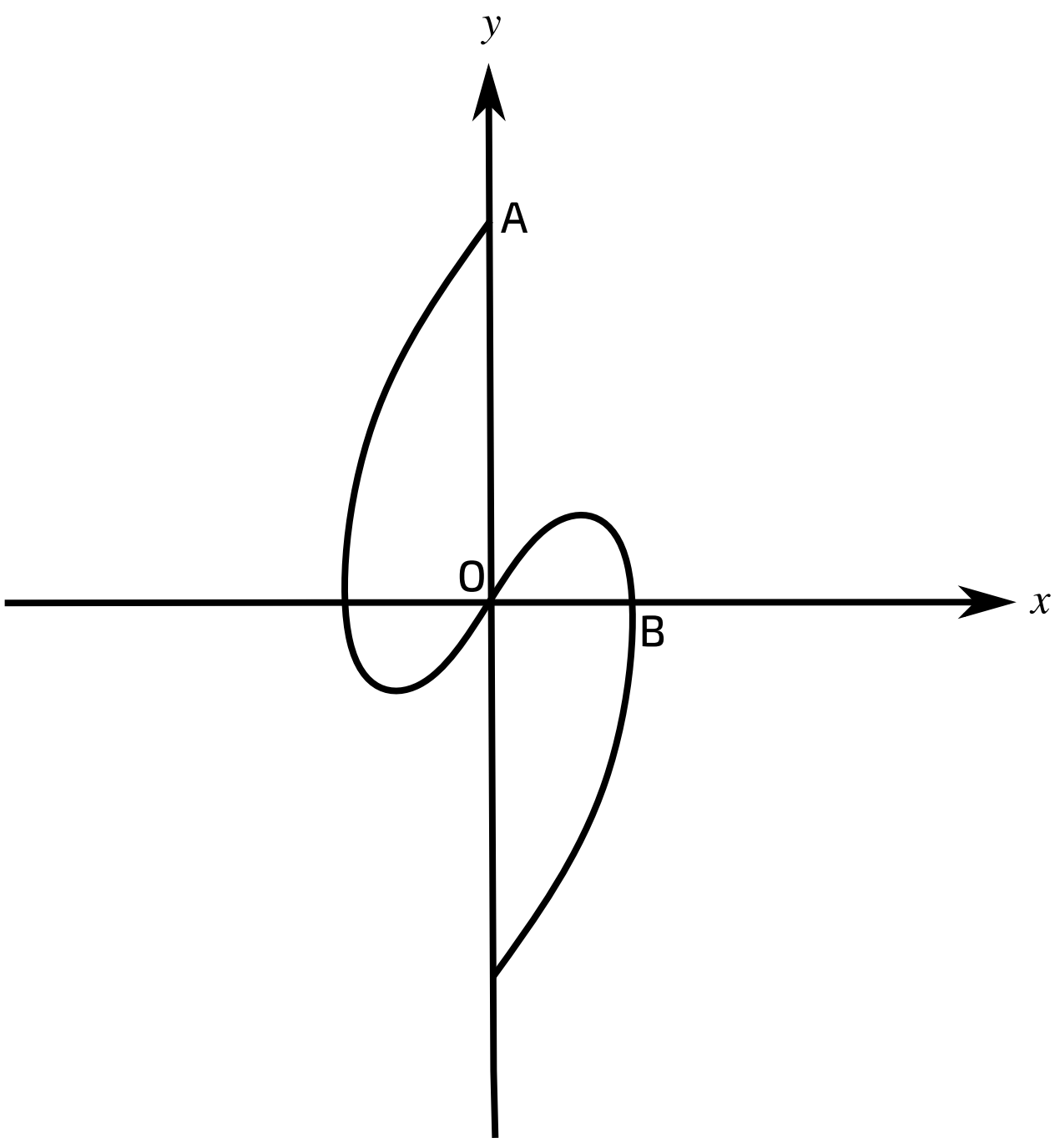


Figure 1: The graph defined by $x = a\sin\theta, y = a\theta\cos\theta$ for $-\pi \leq \theta \leq \pi$.

Part A Points O, A and B

Write down the value of θ corresponding to the origin.

The following symbols may be useful: `arccos(), arcsin(), arctan(), cos(), cosec(), cot(), pi, sec(), sin(), tan(), theta`

State the y coordinate of A.

The following symbols may be useful: `a, arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), pi, sec(), sin(), tan(), y`

State the x coordinate of B.

The following symbols may be useful: `a, arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), pi, sec(), sin(), tan(), x`

Part B Gradient

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `, Derivative(y, x), arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), dx, dy, sec(), sin(), tan(), theta, x, y`

Part C Tangent equation

Find the equation for the tangent to the curve at the origin.

The following symbols may be useful: `x, y`

Parametric equations 1ii

A Level

A curve is defined by the parametric equations

$$x = \sin^2\theta, y = 4\sin\theta - \sin^3\theta$$

where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

Part A Differential $\frac{\mathrm{d}y}{\mathrm{d}x}$

Find an expression for $\frac{dy}{dx}$.

The following symbols may be useful: `Derivative(y, x)`, `arccos()`, `arccosec()`, `arccot()`, `arcsec()`, `arcsin()`, `arctan()`, `cos()`, `cosec()`, `cot()`, `dx`, `dy`, `sec()`, `sin()`, `tan()`, `theta`, `x`, `y`

Part B Point on the curve

Find the coordinates of the point on the curve at which the gradient is 2.

Find the x coordinate.

Find the y coordinate.

Part C Stationary points

Show that the curve has no stationary points.

More practice questions?

Part D Cartesian equation

Find a cartesian equation of the curve, giving your answer in the form $y^2 = f(x)$.

The following symbols may be useful: `x, y`

Gameboard:

STEM SMART Single Maths 34 - Parametric Equations & Projectiles

Physics. You work it out.

Sketching a Parametric Curve

A Level

A curve has parametric equations $x = 1 - \cos t$, $y = \sin t \sin 2t$, for $0 \leq t \leq \pi$.

Part A Coordinates

At how many different points does the curve meet the x-axis?

Enter the highest of the x-coordinates of the points where the curve meets the x-axis.

The following symbols may be useful: `x`

Part B Derivative

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

The following symbols may be useful: `Derivative(y, x), arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), sec(), sin(), t, tan(), x, y`

Part C Stationary points 1

Hence find, in an exact form, the coordinates of the stationary points.

Enter the exact x-coordinate of the stationary point with the lower x-coordinate.

The following symbols may be useful: x

Enter the exact y-coordinate of the stationary point with the lower x-coordinate.

The following symbols may be useful: y

Part D Stationary points 2

Hence find, in an exact form, the coordinates of the stationary points.

Enter the exact x-coordinate of the stationary point with the higher x-coordinate.

The following symbols may be useful: x

Enter the exact y-coordinate of the stationary point with the higher x-coordinate.

The following symbols may be useful: y

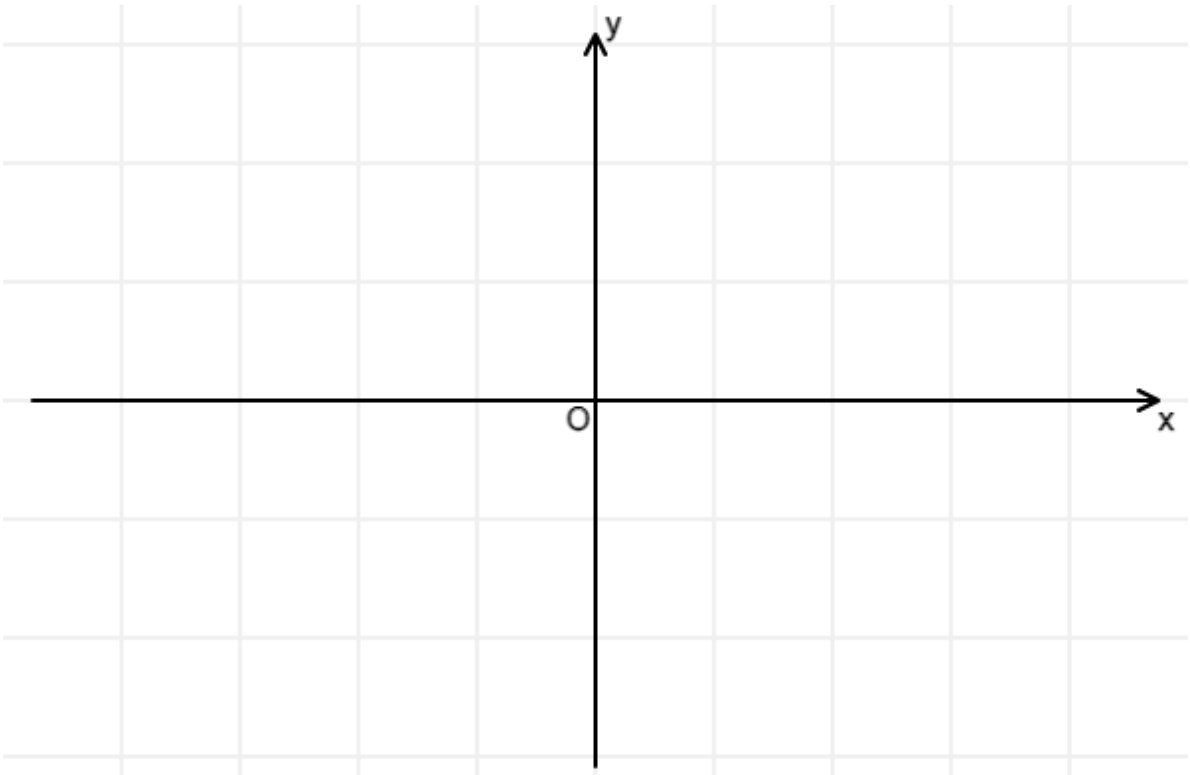
Part E Cartesian Equation

Find the cartesian equation of the curve. Give your answer in the form $y = f(x)$, where $f(x)$ is a polynomial.

The following symbols may be useful: x, y

Part F Sketch

Sketch the curve.



Gameboard:

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Parametric Equations 3i

A Level

The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, y = 4\sin\theta$$

and part of its graph is shown in **Figure 1**.

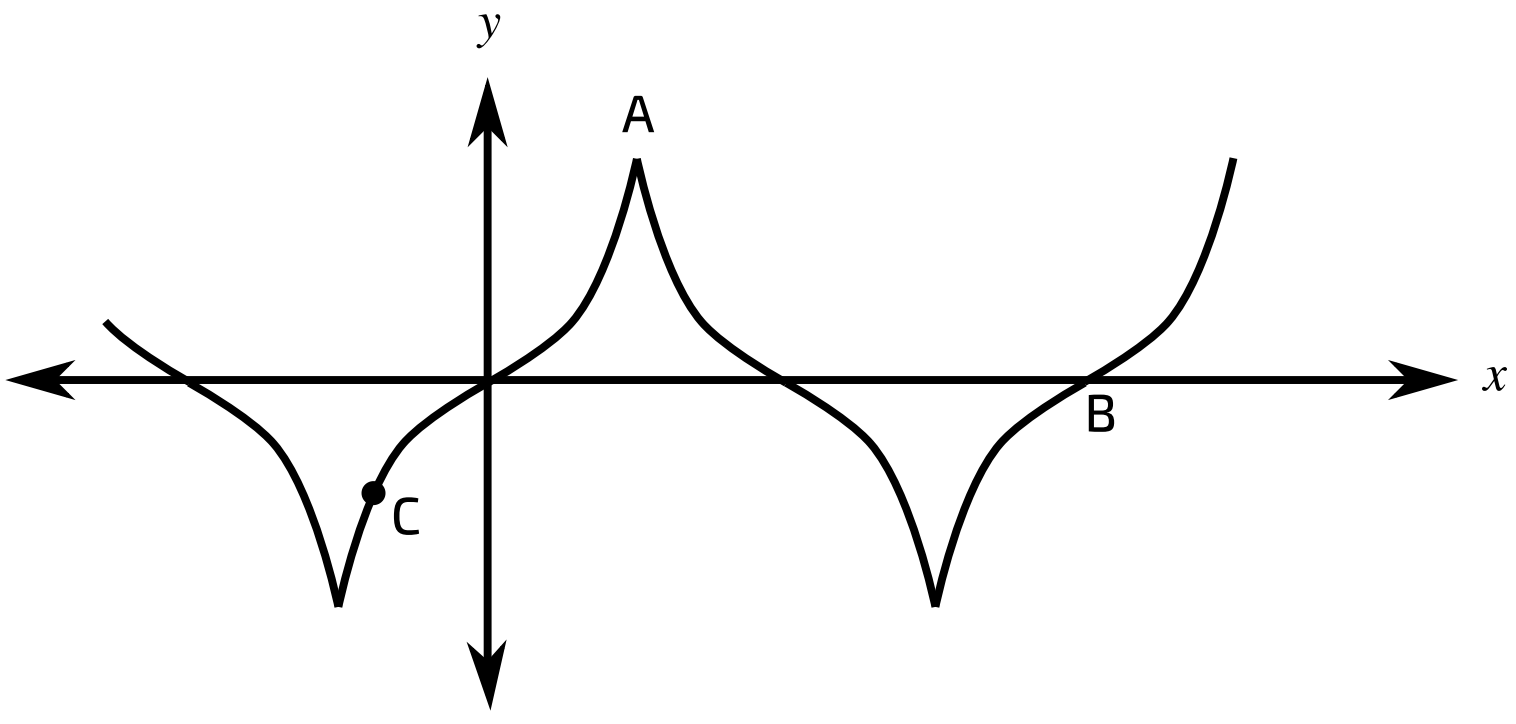

Figure 1: A sketch of the curve.

Part A Value of θ

Find the value of θ at A.

The following symbols may be useful: `pi, theta`

Find the value of θ at B.

The following symbols may be useful: `pi, theta`

Part B Derivative

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ.

The following symbols may be useful: `Derivative(y, x), arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), sec(), sin(), tan(), theta, x, y`

Part C Coordinates

At the point C on the curve the gradient is 2. Find the coordinates of C, giving your answer in an exact form.

Find the x-coordinate.

The following symbols may be useful: `pi, x`

Find the y coordinate.

The following symbols may be useful: `pi, y`

Part D Nature of Origin

Point O is at the origin. State the nature of point O, justifying your answer by reference to suitable values of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

Easier question?

Gameboard:
STEM SMART Single Maths 34 - Parametric Equations & Projectiles

Parametric Equations 2i

A Level

A curve has parametric equations

$$x = \frac{1}{t+1}, y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

Part A Value of t

Show that the value of t at one of these points is -2 and find the value of t at the other point.

The following symbols may be useful: `t`

Part B Normal

Find the equation of the normal to the curve at the point for which $t = -2$, giving your answer in the form $y = f(x)$.

The following symbols may be useful: `x, y`

Part C Value of t

Find the value of t at the point where this normal meets the curve again.

The following symbols may be useful: `t`

Part D Cartesian Equation

Find a cartesian equation of the curve, giving your answer in the form $y = f(x)$.

The following symbols may be useful: `x, y`

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Parametric Integration 1

A Level

The curve C has parametric equations

$$x = 2t^2 - 3 \qquad y = t(4 - t^2)$$

The curve crosses the x-axis at the points A and B and the region R is enclosed by the loop of the curve, as shown in **Figure 1**.

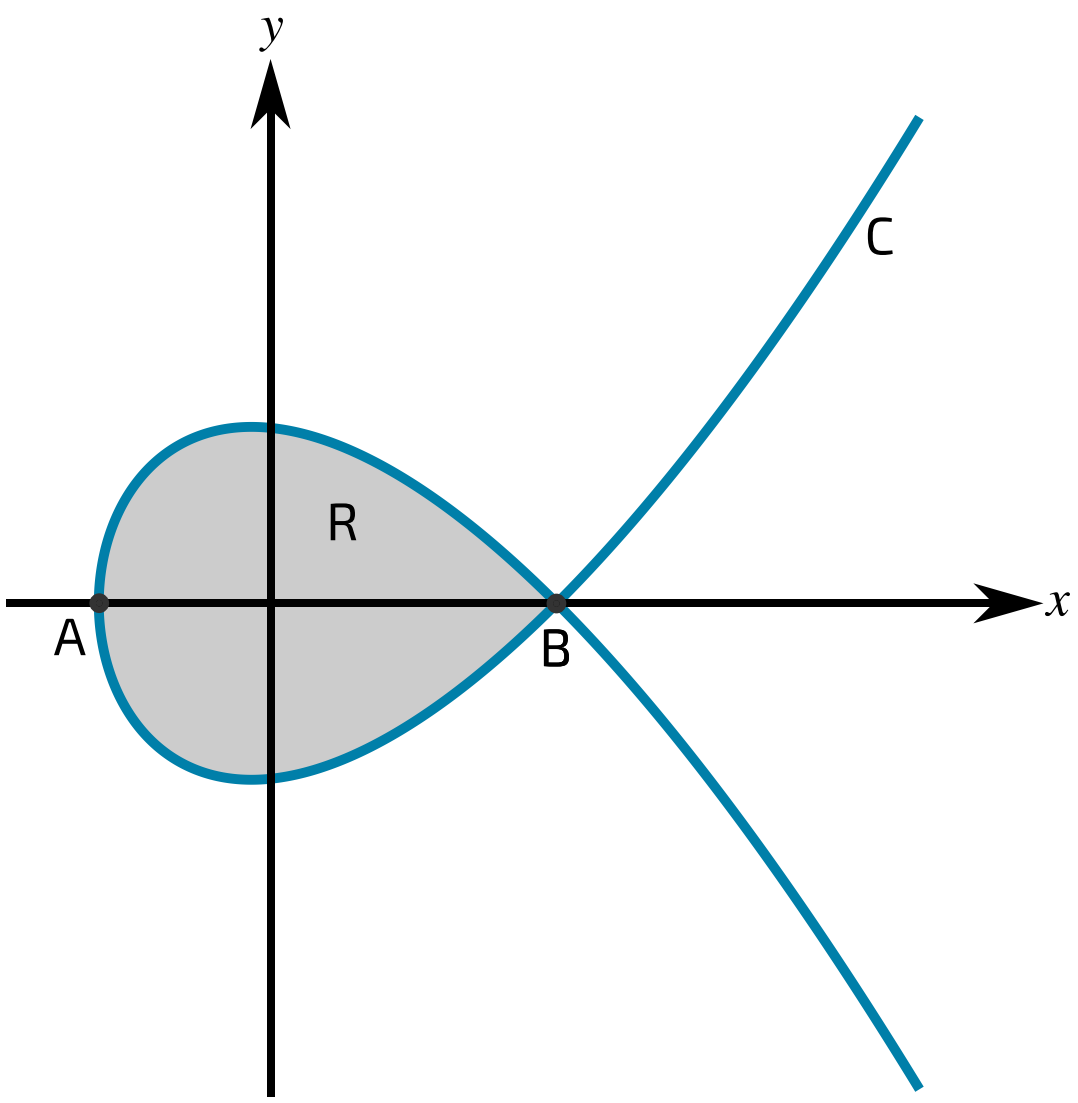

Figure 1: A graph of the curve C.

Part A Point A

Find the x-coordinate of the point A.

Part B Point B

Find the x-coordinate of the point B.

Part C Area of R

The region R is enclosed by the loop of the curve, as shown in **Figure 1**. Find the exact value of the area of R.

Created for isaacphysics.org by Matthew Rihan

Gameboard:

STEM SMART Single Maths 34 - Parametric Equations & Projectiles

Projectiles: Trajectories 1i

A Level

A stone is projected horizontally with speed $7\,\mathrm{m\,s^{-1}}$ from a point O on the edge of a vertical cliff. The horizontal and upward vertical displacements of the stone from O at any subsequent time, t seconds, are $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively. Assume that there is no air resistance.

Part A x in terms of y

In this question, use the value $g = 9.8\,\mathrm{m\,s^{-2}}$ for the acceleration under gravity.

By first expressing x and y in terms of t, find an expression for y in terms of x.

The following symbols may be useful: `x, y`

Part B Distance between cliff and stone

The stone hits the sea at a point which is $20\,\mathrm{m}$ below the level of O.

Find the distance between the foot of the cliff and the point where the stone hits the sea. Give your answer to 3 significant figures.

Part C Speed and direction of motion

Find the speed of the stone immediately before it hits the sea. Give your answer to 2 significant figures.

Find the direction of motion of the stone immediately before it hits the sea. Give your answer as an angle below the horizontal to 3 significant figures.

Gameboard:

STEM SMART Single Maths 34 - Parametric Equations & Projectiles

Projectiles: Trajectories 4i

A Level

A particle is projected with speed $7\,\mathrm{m\,s^{-1}}$ at an angle of elevation of $30\,^\circ$ from a point O and moves freely under gravity. The horizontal and vertically upwards displacements of the particle from O at any subsequent time $t\,\mathrm{s}$ are $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively.

Part A x&y in terms of t

In this question, use the value $g = 9.8\,\mathrm{m\,s^{-2}}$ for the acceleration under gravity.

Express x in terms of t.

The following symbols may be useful: `cos(), sin(), t, tan(), x`

Express y in terms of t. When entering your answer, use fractions rather than decimals.

The following symbols may be useful: `cos(), sin(), t, tan(), y`

Part B y in terms of x

Hence find the equation, y in terms of x, for the trajectory of the particle.

The following symbols may be useful: `x, y`

Part C Values of x

Calculate the smaller of two values of x when $y = 0.6$. Give your answer as an exact surd.

Calculate the larger of two values of x when $y = 0.6$. Give your answer as an exact surd.

Part D Direction of motion

Find the direction of motion of the particle when $y = 0.6$ and the particle is rising. Give your answer as an angle from the horizontal and to 3 significant figures.

Adapted with permission from UCLES, A Level, OCR M2, June 2011, Question 5

Gameboard:

STEM SMART Single Maths 34 - Parametric Equations & Projectiles

Projectiles: Trajectories 2ii

A Level

A particle is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal from a point O. At time $t\,\mathrm{s}$ after projection, the horizontal and vertically upwards displacements of the particle from O are $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively.

Part A Equations of motion

In this question, use g to represent the (positive) acceleration under gravity.

Express x in terms of u, t and θ.

The following symbols may be useful: `cos(), sin(), t, tan(), theta, u, x`

Express y in terms of u, t and θ.

The following symbols may be useful: `cos(), g, sin(), t, tan(), theta, u, x`

Hence an equation for y in terms of u, x and θ.

The following symbols may be useful: `cos(), cosec(), cot(), g, sec(), sin(), tan(), theta, u, x, y`

Part B Value of θ

In this part, use $g = 9.8\,\mathrm{m\,s^{-2}}$.

In a shot put competition, a shot is thrown from a height of $2.1\,\mathrm{m}$ above horizontal ground. It has initial velocity of $14\,\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal. The shot travels a horizontal distance of $22\,\mathrm{m}$ before hitting the ground.

Find the value of θ correct to 3 significant figures.

Part C Time of flight

Find the time of flight of the shot correct to 3 significant figures.
