Manipulating Vectors in 2D

GCSE A Level

Two vectors $\underline{\boldsymbol{p}}$ and $\underline{\boldsymbol{q}}$ are given by $\underline{\boldsymbol{p}} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\underline{\boldsymbol{q}} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Find the following:

Part A Magnitude of p

The magnitude of $\underline{\boldsymbol{p}}$.

Part B Magnitude of q

The magnitude of $\underline{\boldsymbol{q}}$. Give your answer to 3.s.f

Part C Angle of p

The angle that $\underline{\boldsymbol{p}}$ makes with the x-axis. Give your answer to 3.s.f

Part D p + q

Find the vector $\underline{\boldsymbol{p}} + \underline{\boldsymbol{q}}$ in column vector form and give its magnitude. Give your answer to 3.s.f

Part E p - q

Find the column vector form of $\underline{\boldsymbol{p}} - \underline{\boldsymbol{q}}$ and give the angle this vector makes with the x-axis. Give your answer to 3.s.f

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Manipulating Vectors in 3D

A Level Further A

A vector $\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$ has a length of 4 units. For the following three cases, find u_x, u_y and u_z and answer the questions.

Part A Case 1

The vector lies in the (x, y)-plane, makes an angle of 30° with the x-direction and u_y is positive.

What is u_x? Give your answer to 3.s.f

Part B Case 2

The vector has $u_x = u_y = 2$ and u_z is negative.

What is u_z? Give your answer to 3.s.f

Part C Case 3

The vector is such that $u_z = 1$, $u_y = 2u_x$, and u_y is positive.

What is u_y? Give your answer to 3.s.f

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Vectors: Position, Distance and Problems 1i

A Level

An aircraft flies due north from A for a distance of $360\,\mathrm{km}$ $\left(3.6 \times 10^5\,\mathrm{m}\right)$ to point B. Its average speed between A and B is $170\,\mathrm{m\,s^{-1}}$. At B the aircraft is forced to change course and flies due east for a distance of $100\,\mathrm{km}$ to arrive at C.

Part A Journey time

Calculate the time of the journey from A to B.

Part B A vector triangle

Draw a labelled displacement triangle to represent the aircraft's journey.

Easier question?

Part C Distance A to C

Use the displacement triangle to determine the magnitude of the displacement in km of the aircraft at C from A.

Part D Bearing

Find the bearing of C from A.

Modified with permission from UCLES, OCR Physics A, June 2009, Question 1 part B.

Vectors: Positions and Problems 1ii

A Level

It is given that $\underline{\boldsymbol{a}} = 3\underline{\boldsymbol{i}} + 4\underline{\boldsymbol{j}}$, $\underline{\boldsymbol{b}} = -5\underline{\boldsymbol{i}} + 2\underline{\boldsymbol{j}}$, $\underline{\boldsymbol{c}} = 7\underline{\boldsymbol{i}} - 3\underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{d}} = 3\underline{\boldsymbol{a}} - 2\underline{\boldsymbol{b}} + \underline{\boldsymbol{c}}$.

Part A Vector $\underline{\boldsymbol{d}}$

Write down vector $\underline{\boldsymbol{d}}$ in terms of $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$. When you enter your answer, use ordinary i and j to represent the unit vectors.

The following symbols may be useful: `d, i, j`

Part B Magnitude of $\underline{\boldsymbol{d}}$

Find the magnitude of $\underline{\boldsymbol{d}}$. Give your answer to 3 significant figures.

Part C Direction of $\underline{\boldsymbol{d}}$

Find the direction of vector $\underline{\boldsymbol{d}}$, relative to $\underline{\boldsymbol{i}}$. If the direction is anticlockwise from $\underline{\boldsymbol{i}}$ give a positive angle, if it is clockwise from $\underline{\boldsymbol{i}}$ give a negative angle. Give your answer in degrees to 3 significant figures.

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Block on a Slope

A Level Further A

A block of weight W is on a slope which makes an angle α to the horizontal as shown in **Figure 1**

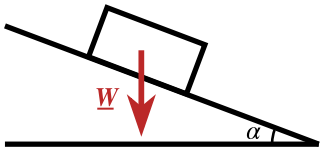

Figure 1: Block at rest on the slope.

Part A Perpendicular Component of Weight

What is the component of the block's weight W that acts perpendicular to the slope?

The following symbols may be useful: `W, alpha`

Part B Horizontal Component of Displacement

The block slides a distance l along the slope as shown in **Figure 2**.

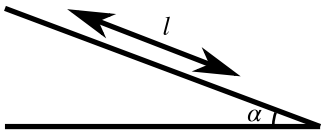

Figure 2: The distance the block moves down the slope.

How far has the block moved horizontally?

The following symbols may be useful: `alpha, l`

Part C GPE Lost

The block slides a distance l along the slope as in Part B. How much gravitational potential energy has the block lost?

The following symbols may be useful: `W, alpha, l`

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Position of Boat

A Level P P P Further A P P P

A boat sails $4.00\,\mathrm{km}$ at a bearing of 210°.

Part A Displacement South

How far south of its starting point is its final position?

Part B Displacement West

How far west of its starting point is its final position?

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Vectors: Position, Distance and Problems 3i

A Level

Four points A, B, C and D are such that $\vec{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, $\vec{BC} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and $\vec{CD} = \begin{pmatrix} m \\ m \end{pmatrix}$. $\vec{AD}$ is parallel to the x-axis.

Part A Vector $\vec{AD}$

Find the vector $\vec{AD}$.

Give the x component of $\vec{AD}$.

Give the y component of $\vec{AD}$.

Part B Magnitude of $\vec{BC}$

What is the magnitude of vector $\vec{BC}$?

Part C Direction of $\vec{BC}$

Find the direction of the vector $\vec{BC}$ relative to the x-axis. If vector is above the x-axis, give a positive angle, if it is below the x-axis, give a negative angle. This corresponds to anticlockwise being positive.

Modified with permission from UCLES, OCR GCSE Maths A, A502/02, November 2011, question 12.

Vectors: Position, Distance and Problems 2i

A Level

A man drives his friend to a station, A, to catch a train. When the man and his friend arrive at station A, the train is ready to leave and the friend cannot catch the train.

The man knows the train goes to another station D on a journey which can be modelled as a straight line. The train travels at an average speed of $30\,\text{mph}$ and arrives at station D $22\,\text{minutes}$ after leaving station A.

The man knows a road route from station A to station D that can be modelled as three straight roads, going via road junctions, B and C, such that

$$\vec{AB} = 4\underline{\boldsymbol{i}} + 3\underline{\boldsymbol{j}} \quad \vec{BC} = 2\underline{\boldsymbol{j}}, \quad \vec{CD} = -4\underline{\boldsymbol{i}} + 6\underline{\boldsymbol{j}}$$

where the numbers are distances in miles.

The man can drive with an average road speed of $45\,\text{mph}$. Can the man drive his friend from station A to station D in time for his friend to catch the train from station D? Show clearly how you arrive at your answer.

How many minutes spare does the friend have or by how many minutes has he missed it by? If he missed the train, give a negative number of minutes.

Written for Isaac Physics by Sally Waugh

Vectors: Positions and Problems 2ii

A Level

Figure 1 shows a ship **S** being pulled by two tug-boats.

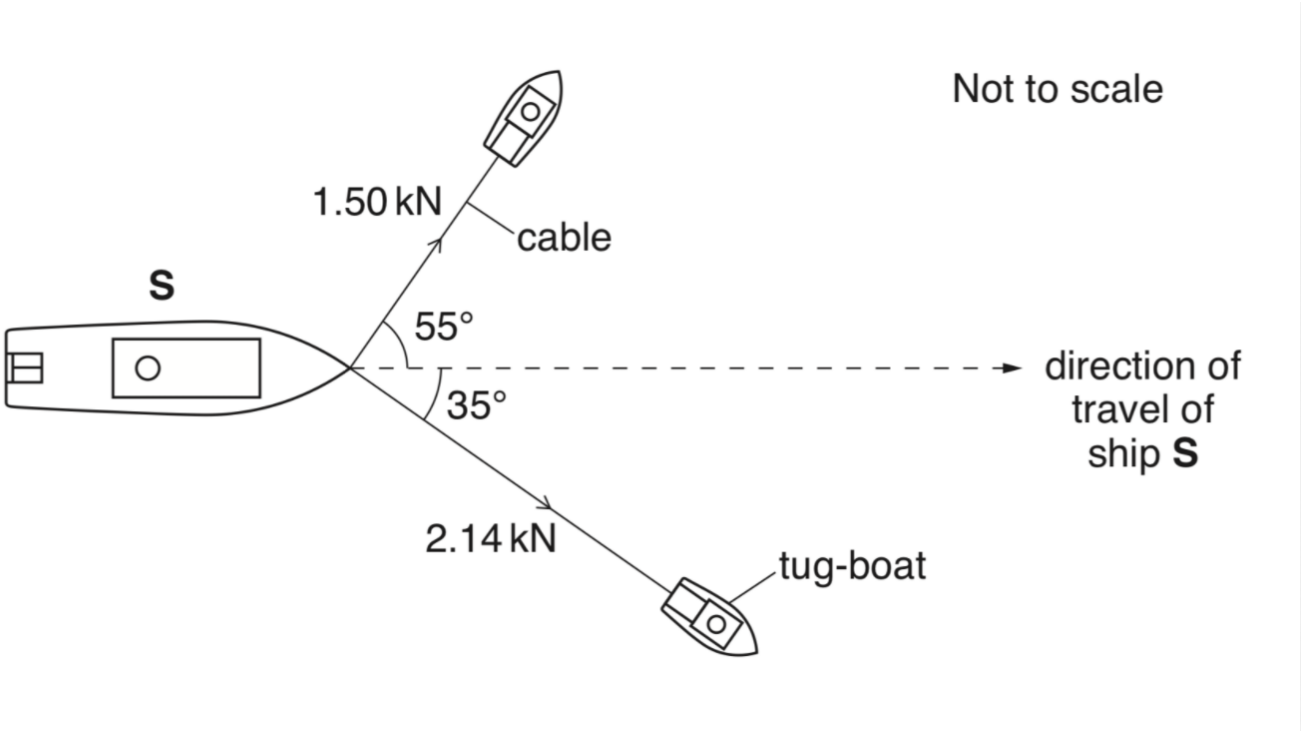

Figure 1: The ship S is being pulled by two tug-boats.

The ship is travelling at a constant velocity. The tensions in the cables and the angles made by these cables to the direction in which the ship travels are shown in **Figure 1**.

Part A Vector triangle

Draw a vector triangle of the forces on the boat due to the tug-boats. What angle does the resultant force make to the direction of travel of the ship?

Part B Magnitude of the resultant force

Determine the magnitude of the resultant force provided by the two cables. Give your answer to 3 significant figures.

Part C Drag force on the ship

State the value of the drag force acting on the ship, correct to 3 significant figures.

Part D Direction of drag force

What is the direction of the drag force and explain your answer?

- It is directly opposite to the resultant force due to the tug boats.
- It is in the same direction as the resultant force due to the tug boats.
- It is in the opposite direction as the top tug-boat.
- It is in the same direction as the bottom tug-boat.
- It is in the same direction as the top tug-boat.
- It is in the opposite direction to the bottom tug-boat.

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Vectors: Positions and Problems 3ii

A Level

These questions will help you practise vector additions.

Part A Finding the resultant

Find the resultant of $\begin{pmatrix} 8 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$.

What is the magnitude of the resultant? Give your answer in surd form.

Part B Direction of the resultant

What angle does the resultant make with the x-axis? If it is anticlockwise from the axis, give it as a positive angle, if it is clockwise, give it as a negative angle.

Part C Vector sum 1

You are given that $\underline{\boldsymbol{p}} + \underline{\boldsymbol{q}} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Write the following as column vectors and find the exact magnitude of the resultant vector.

$$\left[\begin{pmatrix} 3 \\ 7 \end{pmatrix} + \underline{\boldsymbol{p}}\right] + \underline{\boldsymbol{q}}$$

Part D Vector sum 2

As before, taking $\underline{\boldsymbol{p}} + \underline{\boldsymbol{q}} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$, find the resultant as a column vector to

$$-4\left(\underline{\boldsymbol{q}} + \underline{\boldsymbol{p}}\right).$$

What is its magnitude?

Adapted from UCLES, OCR GCSE Maths, A502/02, June 2015, question 12