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Maths

Sequences and Series 1i

Sequences and Series 1i



A sequence of terms $u_1,\,u_2,\,u_3,\,...$ is defined by

$$u_1 = 2 \ {
m and} \ u_{n+1} = 1 - u_n$$

for
$$n \geqslant 1$$

Part A Values

Give the values of u_2 , u_3 and u_4 .

Give the value of u_2 .

The following symbols may be useful: u_2

Give the value of u_3 .

The following symbols may be useful: u_3

Give the value of u_4 .

The following symbols may be useful: u_4

Part B Behaviour

Describe the behaviour of the sequence.

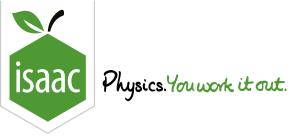
| / \ _ | | | | | | | |
|------------------|--------------|--------|------------------|-----------------|----------------|-------------|----------|
| l he sequence | is periodic. | with a | period of three. | . It cycles thi | rough values o | f 2 . $-$ | 1 and 1. |

- The sequence is periodic, with a period of two. It alternates between values of 2 and -1.
- The sequence is periodic, with a period of four. The first two values that repeat are 2 and -1.
- It is a geometric sequence, with first term 2 and constant ratio $-\frac{1}{2}$.

Part C Sum

Find
$$\sum_{n=1}^{100} u_n$$
 .

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Maths Arithmetic Series 1ii

Arithmetic Series 1ii



| Da | rt | Δ | V | / 2 | h | 0 | of | $\boldsymbol{\gamma}$ |
|----|-----|------------------|---|------------|----|---|----|-----------------------|
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The first three terms of an arithmetic progression are 2x, x + 4, and 2x - 7 respectively. Find the value of x.

The following symbols may be useful: \times

An Arithmetic Progression Part B

The $20^{\rm th}$ term of an arithmetic progression is 10 and the $50^{\rm th}$ term is 70.

What is the first term?

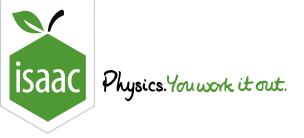
What is the common difference?

Calculate the sum of the first 29 terms.

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Maths

Arithmetic Series 1i

Arithmetic Series 1i



In an arithmetic progression the first term is 5 and the common difference is 3. The $n^{\rm th}$ term of the progression is denoted by u_n .

Part A Value of u_{20}

Find the value of u_{20} .

The following symbols may be useful: u_20

Part B Sum

Find the value of
$$\sum_{n=10}^{20} u_n$$
 .

${\bf Part \, C} \qquad {\bf Value \, of \, } N$

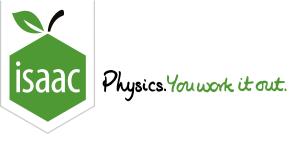
Find the value of N such that $\displaystyle \sum_{n=N}^{2N} u_n = 2750.$

The following symbols may be useful: N

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Maths

Geometric Series 1ii

Geometric Series 1ii



Records are kept of the number of copies of a certain book that are sold each week. In the first week after publication, 3000 copies were sold, and in the second week 2400 copies were sold. The publisher forecasts future sales by assuming that the number of copies sold each week will form a geometric progression with first two terms 3000 and 2400. Calculate (to the nearest number of whole books) the publisher's forecasts for:

| Part A | $20^{ m th}$ | Mook |
|--------|--------------|-------|
| Part A | 700 | vveek |

The number of copies that will be sold in the $20^{
m th}$ week after publication.

Part B Total copies sold in 20 weeks

The total number of copies sold during the first 20 weeks after publication.

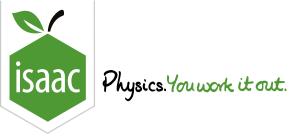
Part C Total sold copies

The total number of copies that will ever be sold.

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Part A

Geometric Series 2ii

Geometric Series 2ii

Maths

Geometric Progression 1



| In a geometric progression, the sum to infinity is four times the | <u> </u> |
|---|-----------|
| | mot term. |
| Find the common ratio. | |

Given that the third term is 9, find the first term.

Find the sum of the first twenty terms. (To three significant figures.)

Geometric Progression 2 Part B

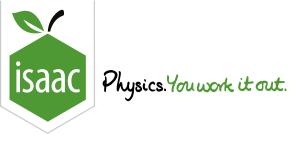
The first term of a geometric progression is 6 and the sum to infinity is 10.

Find the common ratio.

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Maths

Geometric Series 4ii

Geometric Series 4ii



In a geometric progression, the first term is 5 and the second term is 4.8.

Sum to Infinity Part A

Find the sum to infinity.

Value of nPart B

The sum of the first n terms is greater than 124. By showing that

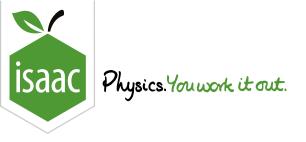
 $0.96^n < 0.008$

and using logarithms, calculate the smallest possible value of n.

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Maths

Series: Summation - Standard Results 2ii

Series: Summation - Standard Results 2ii



Find

$$\sum_{r=1}^n \left(4r^3+6r^2+2r
ight),$$

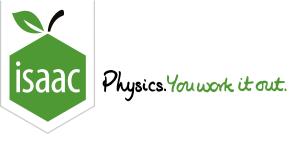
expressing your answer in a fully factorised form.

The following symbols may be useful: n

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Series: Summation - Standard Results 1i

Series: Summation - Standard Results 1i



Part A
$$\sum_{r=n}^{2n} r^3$$

Express $\sum_{r=n}^{2n} r^3$ in terms of n, giving your answer in fully factorised form.

The following symbols may be useful: n

Part B
$$\sum_{r=n}^{2n} r \left(r^2-2
ight)$$

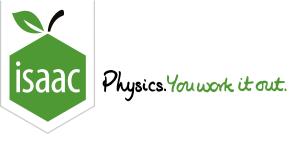
Hence find $\sum_{r=n}^{2n} r\left(r^2-2
ight)$, giving your answer in a fully factorised form.

The following symbols may be useful: n

Adapted with permission from UCLES, A Level, June 2014, Paper 4725, Question 8.

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Maths

Series: Method of Differences 2i

Series: Method of Differences 2i



Part A
$$(r+2)! - (r+1)!$$

Show that (r+2)!-(r+1)!=f(r) imes r! where f(r) is a function to be found.

What is f(r)?

The following symbols may be useful: r

Part B Expression for a series

Hence find an expression, in terms of n, for

$$2^2 imes 1! + 3^2 imes 2! + 4^2 imes 3! + \ldots + (n+1)^2 imes n!$$

Your answer can be written as g(n)! - 2.

What is g(n)?

The following symbols may be useful: n

Part C Convergence

State, giving a brief reason, whether the series

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \dots$$

converges. Fill in the gaps in the argument (you can use an item more than once).

We can express this series as a summation as n o n. This is the limit of the partial sum n o n.

From Part A we can write the partial sum as _____, and from Part B we know that the partial sum evaluates to _____.

Items:

$$\left[\sum_{r=1}^{\infty} (r+1)^2 \, r! \right] \left[\sum_{r=1}^{\infty} r^2 (r+1)! \right] \left[\sum_{r=1}^{n} (r+1)^2 r! \right] \left[\sum_{r=1}^{n} r^2 (r+1)! \right] \left[0 \right] \left[1 \right] \left[\sum_{r=1}^{n} \left[(r+2)! - (r+1)! \right] \right]$$

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Maths

Series: Method of Differences 1i

Series: Method of Differences 1i



Rewriting a fraction Part A

Express $\frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2}$ as a single fraction.

The following symbols may be useful: r

Part B Sum of a series

Hence find an expression, in terms of n, for

$$\sum_{r=1}^n rac{3r+4}{r(r+1)(r+2)}.$$

The following symbols may be useful: n

Limit as $n o \infty$ Part C

Hence write down the value of

$$\sum_{r=1}^{\infty}rac{3r+4}{r(r+1)(r+2)}.$$

${\bf Part \ D} \qquad {\bf Solve \ for \ } N$

Given that

$$\sum_{r=N+1}^{\infty} rac{3r+4}{r(r+1)(r+2)} = rac{7}{10}$$

find the value of N.

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