

<u>Gameboard</u>

Maths

Complex Numbers

Complex Numbers



Part A De Moivre

The complex number $2+i\sqrt{5}$ can be expressed in the form $r\left(\cos\theta+i\sin\theta\right)$ where $-180^\circ<\theta<180^\circ.$

Find r.

The following symbols may be useful: r

Find an expression for $\tan \theta$.

The following symbols may be useful: tan(), theta

Part B Square roots

Use an algebraic method to find the square roots of the complex number $2+i\sqrt{5}$ in the form x+iy.

Give an expression for the solution with positive x and positive y.

The following symbols may be useful: i

Give an expression for the solution with negative x and negative y.

The following symbols may be useful: ${\tt i}$

Part C A complex quartic

Using your answer from the previous part, find the roots of the equation $z^4-4z^2+9=0$ in the form x+iy.

Give an expression for the root with both \boldsymbol{x} and \boldsymbol{y} positive.

The following symbols may be useful: i, z

Give an expression for the root with positive \boldsymbol{x} and negative \boldsymbol{y} .

The following symbols may be useful: i, z

Give an expression for the root with negative x and positive y.

The following symbols may be useful: i, z

Give an expression for the root with negative x and negative y.

The following symbols may be useful: i, z

Part D Argand diagram

Sketch the roots of the equation from the previous part on an Argand diagram.

Easier question?

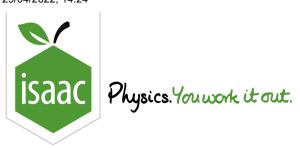
Part E Complex locus

Given that α is the root of the equation in part C) such that $0<\arg\alpha<\frac{1}{2}\pi$, sketch on the same Argand diagram the locus given by $|z-\alpha|=|z|$.

Easier question?

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Maths

Roots of Polynomials

Roots of Polynomials



This question is about manipulation of the roots of two polynomials.

$$x^2 + kx + 2k = 0$$

has the roots α and β , while

$$x^3 + 4x + 3 = 0$$

has the roots lpha', eta' and γ' . Take k
eq 0.

Part A Roots of the quadratic

Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

The following symbols may be useful: k, x

Part B Substitution

Starting from the cubic equation above, use the substitution $x = \sqrt{u}$ to obtain a cubic equation in u.

The following symbols may be useful: u

Part C Roots of the cubic

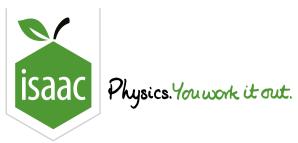
Find an expression for ${\alpha'}^4 + {\beta'}^4 + {\gamma'}^4 + {\alpha'}{\beta'}{\gamma'}$.

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Maths

Vectors

Vectors



The vector $\mathbf{u} = \frac{3}{13}\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is perpendicular to the vector $\mathbf{v} = 4\mathbf{i} + \mathbf{k}$ and to the vector $\mathbf{w} = 4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$.

Part A

Find c as a single rational fraction.

The following symbols may be useful: c

Part B b

Find b in exact form.

The following symbols may be useful: b

Part C |u|

Find $|\mathbf{u}|$.

Calculate, to the nearest degree, the angle between ${\bf v}$ and ${\bf w}$.

Part E n

Find a unit vector ${\bf n}$ in the direction of the common perpendicular to the vectors $(3{\bf i}-2{\bf j}+2{\bf k})$ and $(-{\bf i}+3{\bf j}-5{\bf k})$. Take ${\bf n}$ to have positive x,y and z.

Find the x component of \mathbf{n} as a single fraction.

The following symbols may be useful: x

Find the y component of \mathbf{n} as a single fraction.

The following symbols may be useful: y

Find the z component of ${\bf n}$ as a single fraction.

The following symbols may be useful: z

Part F Two lines

Determine whether the lines described by

$$\mathbf{r}_1 = (1+2\lambda)\mathbf{i} - \lambda\mathbf{j} + (3+5\lambda)\mathbf{k}$$

and

$$\mathbf{r}_2 = (\mu - 1)\mathbf{i} - (5 - \mu)\mathbf{j} + (2 - 5\mu)\mathbf{k}$$

are parallel, intersect or are skew.

() Intersect

Skew

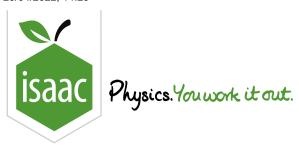
() Parallel

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Maths

Sequences

Sequences



The sequence u_1 , u_2 , u_3 . . . is defined by $u_1=3$ and $u_{n+1}=3u_n-2$ for $n\geq 1$.

Part A u_2 and u_3

Find u_2 .

The following symbols may be useful: u_2

Find u_3 .

The following symbols may be useful: u_3

Part B $rac{1}{2}(u_4-1)$

Find $\frac{1}{2}(u_4-1)$.

Part C u_n

Hence, find an expression for u_n and prove it with induction.

The following symbols may be useful: n, u_n

Part D Divisibility

Prove by induction that 5^n-2^n is divisible by 3 for all integers $n\geq 1$.



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