6 Percentages

Fractions, percentages and decimals are closely related. Percentage means "out of one hundred", so 20% is the fraction $\frac{20}{100}$, which can be cancelled down to $\frac{1}{5}$. $\frac{1}{5}$ can be written as the decimal 0.2.

Example 1 – Express 38% as (i) a fraction (ii) a decimal.

(i) $38\% = \frac{38}{100} = \frac{19}{50}$ (ii) $38\% = \frac{38}{100} = 38 \div 100 = 0.38$

Example 2 – Express the following as percentages: (i) $\frac{17}{20}$ (ii) 0.348.

(i) $\frac{20}{20}$ would be 100%, so $\frac{17}{20} = \frac{17}{20} \times 100\% = 85\%$

(ii) The number of digits 0.348 has after the decimal point is 3.

$$\therefore 0.348 = \frac{348}{1000} \qquad \frac{348}{1000} \times 100\% = \left(\frac{348}{10}\right)\% = 34.8\%$$

To calculate percentage changes, you must decide if you are starting with 100% of a quantity, or if you are given a different percentage and must calculate 100%.

Example 3 – There is a 25%-off sale. What is the sale price of a coat which was originally priced at £120?

In this example you are starting with 100%. If 25% is taken off, you will pay 75% of the starting cost, or $\frac{75}{100}$ of £120. $\frac{75}{100}$ is a scale factor, so multiply.

$$\frac{75}{100} \times £120 = £90$$

Example 4 – In the same 25%-off sale, there is a pair of shoes priced at £48. What was the original price?

This time you know what 75% of the original price is. Scale down to find 1%, then scale up to find 100%.

So the multiplier is $\frac{100}{75}$, or $\frac{4}{3}$. The original price was $\frac{4}{3} \times £48 = £64$.

6.1 Write these percentages as fractions.

(a) 30% (b) 5% (c) 0.5%

6.2 Write these percentages as decimals.

(a) 25% (b) 0.7% (c) 0.003%

6.3 Write these decimals as percentages.

(a) 0.10 (b) 0.01 (c) 0.005 (d) 2.00

6.4 Write these fractions as percentages.

(a) $\frac{3}{4}$ (b) $\frac{5}{8}$ (c) $\frac{7}{350}$

6.5 Which is larger in each case?

(a) 21.5% or $\frac{9}{20}$ (b) $\frac{7}{6}$ or 1.16 (c) 112% or $\frac{20}{18}$

6.6 Rank the following in order of size, starting with the smallest.

$$62\% \qquad \frac{5}{8} \qquad 0.629$$

6.7 Evaluate the following:

(a) 20% of £16 (b) 65% of 400 g (c) 160% of \$240

6.8 Calculate

(a) 27% of £24 000 (b) 15% of 75 (c) 7.5% of 6 kg

6.9 Write the first quantity as a percentage of the second.

(a) 3 minutes out of 2 hours

(b) £1.40 out of £40.00

(c) 366 g out of 3 kg

6.10 A family needs to buy a new washing machine and have it delivered.

Company A sells a machine at 15% off an original purchase price of £275, and charges £25 for delivery.

Company B sells the same machine. Their usual price is £300, but the machine is on sale at 20% off. Delivery is free.

From which company would it be cheaper to buy the machine?

6.11 Rohit saves 5% when purchasing a coat originally costing $£100$, and 8% on a sofa which originally costs $£200$. What is his overall saving as a percentage?

6.12 Rank the following in order of size, starting with the largest.

$$\frac{17}{20} \qquad 87\% \qquad \frac{7}{8} \qquad 0.889 \qquad \frac{349}{400}$$

6.13 A new process reduces the time to manufacture a product by 25%. If it takes 2 hours 21 minutes to make the product using the new process, how long did it take to make the item with the old process?

In a simple interest scheme, the interest added to an account each year is calculated as a percentage of the original amount deposited. The same amount of interest is added each year.

Example 5 – $£250$ is invested in a simple interest scheme at an interest rate of 4% per year. Calculate the total amount of money at the end of 5 years.

The amount of interest paid after each year is 4% of the starting $£250$. This is

$$4\% \times £250 = \frac{4}{100} \times £250 = \frac{4 \times £250}{100} = \frac{£1000}{100} = £10$$

After 5 years the amount of interest paid is $5 \times £10 = £50$.
The total amount of money in the account is therefore

$$£250 + £50 = £300$$

In a compound interest scheme, the interest added to an account each year is calculated as a percentage of the amount in the account during that year. Assuming that no money is taken out, the amount of interest added increases from year to year.

If the interest rate is $I\%$ per year, then after one year the amount in an account is $(100 + I)\%$ of the original amount. This is equivalent to multiplying the starting amount by the fraction $\frac{100+I}{100}$.

Example 6 – £250 is invested in a compound interest scheme at an interest rate of 4% per year. Calculate the total amount of money at the end of 5 years to the nearest penny.

For an interest rate of 4%, the multiplier is $\frac{100+I}{100} = \frac{104}{100} = 1.04$.

To find the amount after 5 years, apply the multiplier 5 times. To the nearest penny the amount in the account is £250 $\times$ 1.04^5 = £304.16.

6.14 Calculate, to the nearest penny, the amount that will be in an account if

(a) £1 000 is invested for one year in a simple interest scheme with a 5% interest rate.

(b) £1 500 is invested for two years in a simple interest scheme with a 3% interest rate.

(c) £4 000 is invested for one year in a compound interest scheme with a 4% interest rate.

(d) £4 500 is invested for three years in compound interest scheme with a 2.5% interest rate.

6.15 For each of the percentages below, calculate the amount in an account after 10 years if £3 000 is invested in a scheme with (i) simple interest (ii) compound interest. Give your answers to the nearest penny.

(a) 0.1% (b) 1% (c) 3% (d) 6%

6.16 Dan uses a balance to find the mass of some objects. The machine has an offset error, so it registers a mass of 9.0 g even when there is no mass on it.
Find the percentage error in his measurements when he weighs objects which have a true mass of

(a) 90 g (b) 720 g (c) 36.0 g

6.17 A carpenter is cutting a long, thin plank of wood into shorter lengths using a band saw. The saw blade is 2 mm wide, so each cut wastes 2 mm of wood as sawdust.

(a) How many 10 cm lengths can they cut from a plank with a total length of 2 m?

(b) What percentage of the original plank will not be used to make 10 cm lengths?

(c) What percentage of the plank will be turned into sawdust?

6.18 The average attendance at a sporting fixture went up by 40% every year. In year 2 the attendance was $35\,000$. Find the attendance
(a) in year 3 (b) in year 1

6.19 Uranium has many isotopes. 99.274% of natural uranium is the isotope U_{238}. 720 out of $100\,000$ atoms of natural uranium are the isotope U_{235}. What percentage of natural uranium is accounted for by the other isotopes?

6.20 For the formula $P = IV$, what is the percentage change in P if the current increases by 3.0% and the voltage falls by 10.0% at the same time?