



# Angles and Projection

A Level Further A



## Part A Finding $\cos \phi$ and $\tan \phi$

Find, without using a calculator,  $\cos \phi$  and  $\tan \phi$  given that  $\sin \phi = \frac{5}{13}$  and that  $\phi$  is an obtuse angle.

- ☐  $\cos \phi = -\frac{5}{12}, \tan \phi = -\frac{12}{13}$
- ☐  $\cos \phi = -\frac{12}{13}, \tan \phi = \frac{5}{12}$
- ☐  $\cos \phi = \frac{5}{12}, \tan \phi = \frac{12}{13}$
- ☐  $\cos \phi = \frac{12}{13}, \tan \phi = \frac{5}{12}$
- ☐  $\cos \phi = -\frac{12}{13}, \tan \phi = -\frac{5}{12}$

## Part B Value of $\theta$

A particle is projected into the air with a speed of  $50 \text{ m s}^{-1}$  at an angle of  $\theta$  to the horizontal. It lands a horizontal distance of  $250 \text{ m}$  away after  $6.4 \text{ s}$ . Assuming that it travels at a constant velocity in the horizontal direction find the value of  $\theta$ .



# Exact Values of Angles 1

---



For the range  $0 \leq \theta \leq 360^\circ$ , write down all the values of  $\theta$  which have the following: (a)  $\sin \theta = \sqrt{3}/2$ , (b)  $\sin \theta = -1/2$ .

- ☐ (a)  $60^\circ$ ,  $120^\circ$ , (b)  $210^\circ$ ,  $330^\circ$
- ☐ (a)  $30^\circ$ ,  $150^\circ$ , (b)  $240^\circ$ ,  $300^\circ$
- ☐ (a)  $30^\circ$ ,  $150^\circ$ , (b)  $210^\circ$ ,  $330^\circ$
- ☐ (a)  $60^\circ$ ,  $300^\circ$ , (b)  $150^\circ$ ,  $330^\circ$
- ☐ (a)  $60^\circ$ ,  $120^\circ$ , (b)  $240^\circ$ ,  $300^\circ$



## Addition of Forces 2



Forces  $\underline{F_1} = \begin{pmatrix} F_{1x} \\ F_{1y} \end{pmatrix}$  and  $\underline{F_2} = \begin{pmatrix} F_{2x} \\ F_{2y} \end{pmatrix}$  act in the  $x - y$  plane, having magnitudes  $F_1$  and  $F_2$  and making angles of  $\theta$  and  $\phi$  with the positive  $x$  axis respectively. Find expressions for (a)  $F_{1x}$ ,  $F_{1y}$ ,  $F_{2x}$  and  $F_{2y}$ , (b) the magnitude of the vector sum of the two forces and (c) the angle the vector sum of the two forces makes with the  $x$  axis.

### Part A The components of the forces

Write down an expression in terms of  $F_1$  and  $\theta$  for  $F_{1x}$ .

The following symbols may be useful:  $F_1$ ,  $\arccos()$ ,  $\arcsin()$ ,  $\arctan()$ ,  $\cos()$ ,  $\sin()$ ,  $\tan()$ ,  $\theta$

Write down an expression in terms of  $F_1$  and  $\theta$  for  $F_{1y}$ .

The following symbols may be useful:  $F_1$ ,  $\theta$

Write down an expression in terms of  $F_2$  and  $\phi$  for  $F_{2x}$ .

The following symbols may be useful:  $F_2$ ,  $\phi$

Write down an expression in terms of  $F_2$  and  $\phi$  for  $F_{2y}$ .

The following symbols may be useful:  $F_2$ ,  $\phi$

## Part B The magnitude

Consider the sums of the components of the two forces in the  $x$ -direction and the  $y$ -direction. Hence find an expression for the magnitude of the vector sum of the two forces.

The following symbols may be useful:  $F_{1x}$ ,  $F_{1y}$ ,  $F_{2x}$ ,  $F_{2y}$

---

## Part C The angle

Consider the sums of the components of the two forces in the  $x$ -direction and in the  $y$ -direction. Hence find an expression for the angle the vector sum of the two forces makes to the  $x$ -axis,.

The following symbols may be useful:  $F_{1x}$ ,  $F_{1y}$ ,  $F_{2x}$ ,  $F_{2y}$ ,  $\arccos()$ ,  $\arcsin()$ ,  $\arctan()$ ,  $\cos()$ ,  $\sin()$ ,  $\tan()$

---

Created for isaacphysics.org by Julia Riley.

All materials on this site are licensed under the [Creative Commons license](https://creativecommons.org/licenses/by/4.0/), unless stated otherwise.



# Values of Angles 1

---



For the range  $-180^\circ \leq \alpha \leq 180^\circ$ , consider all the values of  $\alpha$  which satisfy  $\sin \alpha = 0.2$ .

## Part A Values of $\alpha$

---

How many values of  $\alpha$ , satisfying the equation, are in this range?

---

## Part B Largest value of $\alpha$

---

What is the largest positive value of  $\alpha$  satisfying the equation in this range? Give your answer to 3.s.f.

---



# Simplify Trig Expressions



Simplify the following trigonometric expressions.

## Part A $1/(\cos^2 t - 1)$

Simplify the following expression to give a single trigonometric function:

$$\frac{1}{\cos^2 t - 1}$$

The following symbols may be useful:  $\cos()$ ,  $\operatorname{cosec}()$ ,  $\cot()$ ,  $\sec()$ ,  $\sin()$ ,  $t$ ,  $\tan()$

## Part B $(1 - \sin^2 x)/(\cos^2 t - 1)$

Simplify the following expression to give a single trigonometric function:

$$\frac{1 - \sin^2 x}{\cos x}$$

The following symbols may be useful:  $\cos()$ ,  $\operatorname{cosec}()$ ,  $\cot()$ ,  $\sec()$ ,  $\sin()$ ,  $\tan()$ ,  $x$

## Part C $\sin \alpha \cos \alpha - 1/(\cos^2 t - 1)$

Simplify the following expression to give a single trigonometric function:

$$\sin \alpha \tan \alpha - \frac{1}{\cos \alpha}$$

The following symbols may be useful:  $\alpha$ ,  $\cos()$ ,  $\operatorname{cosec}()$ ,  $\cot()$ ,  $\sec()$ ,  $\sin()$ ,  $\tan()$

**Part D**      $\tan w - \frac{\cos w}{1 - \sin w}$

Simplify the following expression to give a single trigonometric function:

$$\tan w - \frac{\cos w}{1 - \sin w}$$

The following symbols may be useful:  $\cos()$ ,  $\operatorname{cosec}()$ ,  $\cot()$ ,  $\sec()$ ,  $\sin()$ ,  $\tan()$ ,  $w$

---

Created for isaacphysics.org by Julia Riley.

All materials on this site are licensed under the [Creative Commons license](https://creativecommons.org/licenses/by/4.0/), unless stated otherwise.