Dr Lisa Jardine-Wright, Cavendish Laboratory

Single Slit Diffraction

Aims: To investigate the phenomena of diffraction through single slits of different widths using a laser as a light source.

Objective: To find the wavelength of the laser beam

Safety notes

- The laser beam can easily be reflected not only from the mirrors provided but also off any other metal surface in the experiment. These reflections can be dangerous please avoid causing them.
- Take care when using the tape measure as it is easy to cut yourself when it retracts.

1. *Theory:*

When waves pass through apertures or around obstacles, they spread out into regions which would be in shadow if they travelled in straight lines. This property is called diffraction and can be described in terms of Huygens Principle.

Huygens proposed that every point on a wavefront may be regarded as a source of secondary spherical wavelets. Where these waves cross they constructively and destructively add (figure 1). Diffraction is regarded as being due to the addition (superposition) of Huygens' secondary wavelets.

Imagine that a slit consists of strips of equal width, parallel to the length of the slit. The total effect in a particular direction is then found by adding the wavelets emitted in that direction by all the strips (figure 2).

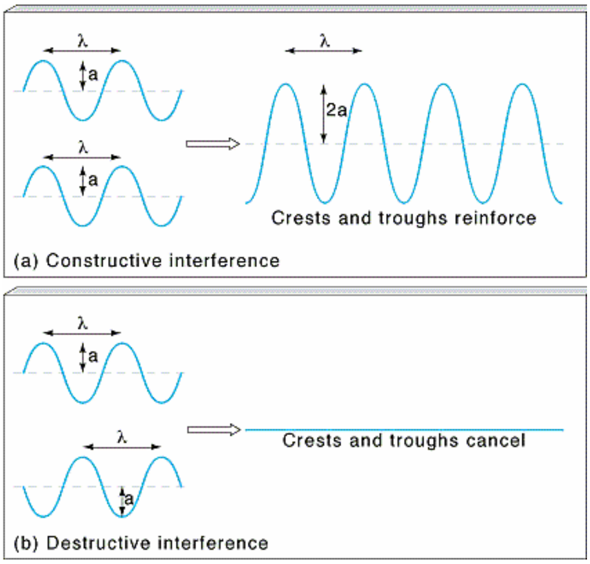
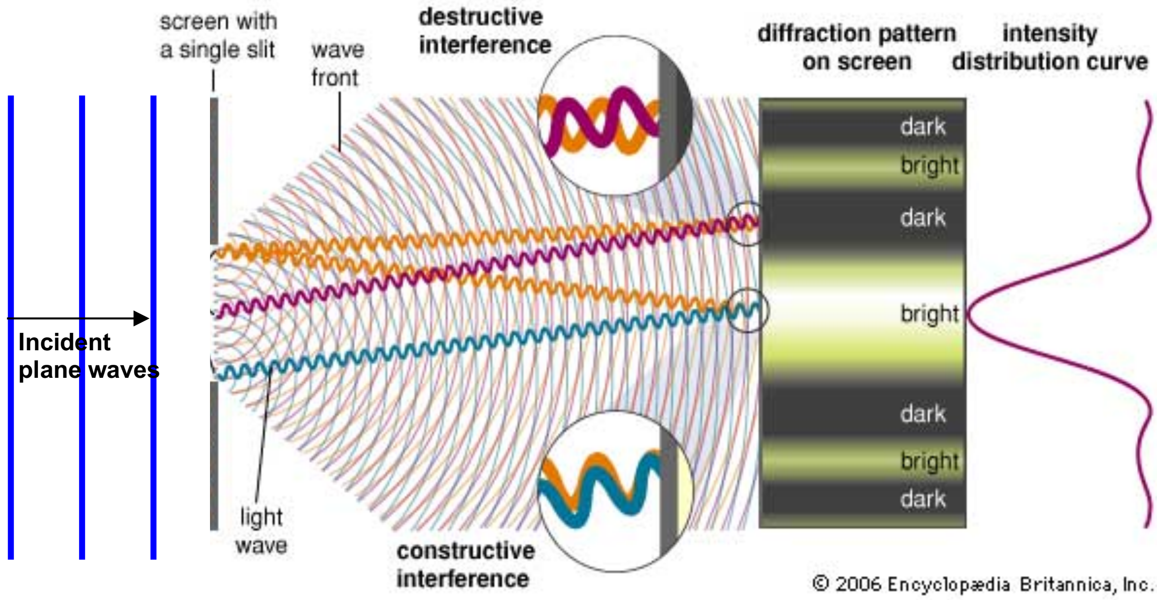


Figure 1: Diagrams of constructive and destructive interference for a wave of wavelength "λ" and amplitude "a". *http://www.pas.rochester.edu/~afrank/A105/LectureVI/LectureVI.html*

Figure 2: Schematic showing the diffraction pattern of a single slit. Constructive and destructive interference across the slit leads to bright and dark bands in the pattern



Consider plane waves falling on a narrow slit of width *a* as in the diagram below.

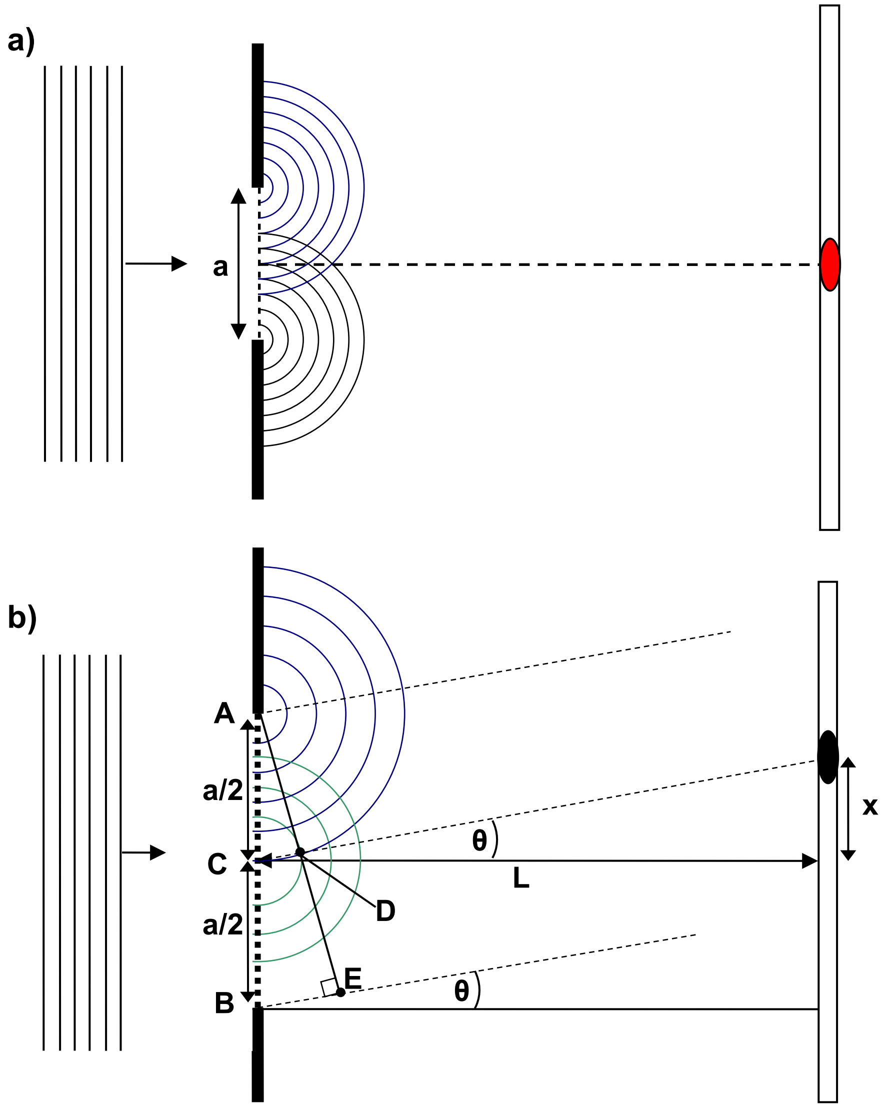


Figure 3: Huygens secondary wavelets as a way of describing the diffraction pattern observed for a single slit

a) Shows on axis constructive interference leading to the bright central maximum

b) Shows the destructive interference of pairs of wavelets which leads to the dark bands at an angle θ to the straight through direction.

Tan θ = x / L

≈ sin θ (if θ is small)

Therefore $x / L = n\lambda / a$

So $x = n\lambda L / a$

where L is the distance of the screen from the slit and x is the distance on the screen from the central bright spot to the minima when n=1, 2, 3, 4…

In figure 3a) we consider Huygens wavelets emanating from the edges of the slit. The blue and black semi-circular lines represent the **crest** of the wavelets which along the dashed line constructively interfere (add) to make a bright band on the screen (see figure 1a).

Figure 3b) shows the Huygens wavelets from one edge of the slit and half way across the slit. This time we consider the crests from one point (blue) and the troughs from the middle point (green). At an angle of *θ* the first dark band will be formed – where the blue crest destructively interferes with the green trough (see figure 1b). For destructive interference one wave has to be half a wavelength ahead of the other wave. This happens for all corresponding pairs of points in **AC** and **AB** because the same path difference of half a wavelength exists.

Therefore when:

$$\mathrm{CD} = \frac{\lambda}{2} \quad \text{but} \quad \frac{\mathrm{CD}}{\mathrm{AC}} = \sin\theta \quad \text{so} \quad \frac{\mathbf{CD}}{\mathbf{a}/2} = \sin\theta \quad \text{therefore} \quad \frac{\lambda}{2} = \frac{\mathbf{a}}{2}\sin\theta$$

$$\boxed{\sin\theta = \frac{\lambda}{\mathbf{a}}}$$

We can show by similar "pairing" processes that other minima occur when $\boxed{\sin\theta = \frac{\mathbf{n}\lambda}{\mathbf{a}}}$

where n = ±1, ±2, ±3, ±4 … where ± means one on each side of the central maximum (bright band).

2. Experimental Set up:

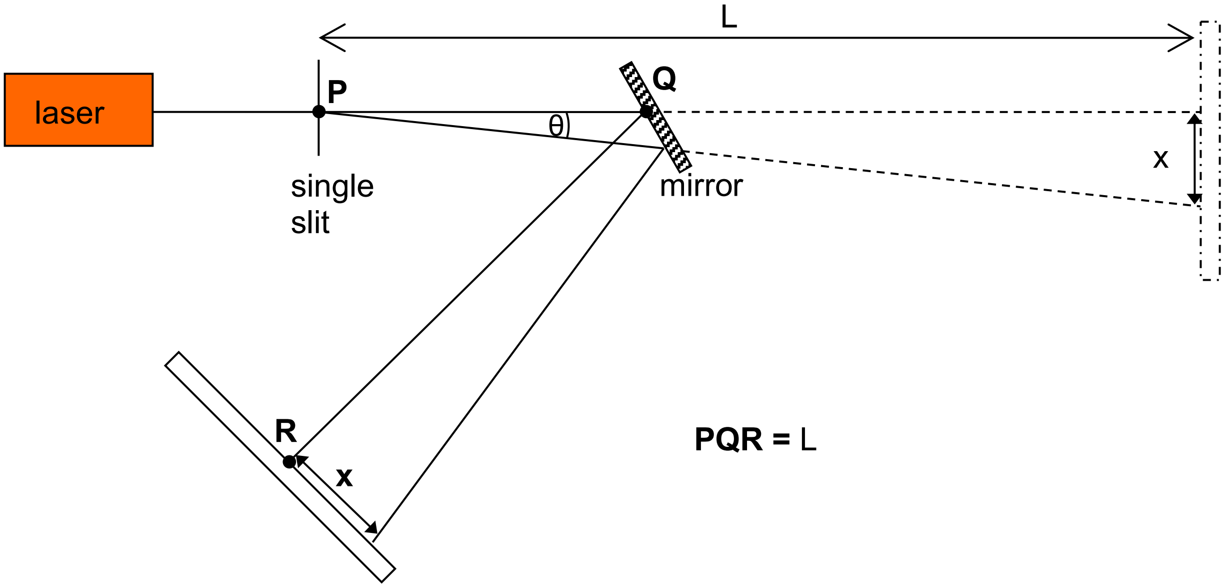


Figure 4: A Helium – Neon laser with a main beam 1-2mm in diameter provides a coherent source of light. The beam falls onto a small, transparent aperture in a metal-coated plate, the resulting diffraction pattern is observed on a translucent screen. The light path is "folded" using a mirror to increase its length.

Take care of the apparatus

- Never touch the middle of your aperture plate – finger prints are hard to remove.
- Do not touch the front (reflective surface) of the mirrors – unlike everyday mirrors the silvering is on the front surface and comes off easily!

Using the theory we looked at in section 1, we can work out where each of the dark bands should appear on the screen.

$$\tan\theta = \frac{x}{L}$$

For small angles $\tan\theta = \sin\theta$, so if x is the position of a minimum

$$\sin\theta = \frac{x}{L} = \frac{n\lambda}{a}$$

$$x = \frac{n\lambda L}{a}$$

As you widen the slit what do you expect to happen to the separation of the minima?

2.1. Qualitative description of diffraction by single slits.

The single slit apertures that you are going to use are shown here in figure 5. Examine in turn the diffraction patterns for the 5 single slits. Sketch what you see in the cases of **A1 and A5** showing the relative dimensions of the patterns in the two cases. Comment, qualitatively, on the relationship between the widths of the slits and the separation of the diffraction minima.

Figure 5: The single slit apertures used, with their dimensions. Black regions in this figure are transparent, the rest is nearly but not quite opaque

Sketch for slit A1 (inc. relative dimensions)**:**

Sketch for slit A3 (inc. relative dimensions)**:**

Qualitative description of diffraction by single slits:

A1: 0.48mm

A2: 0.24mm

A3: 0.12mm

Measuring the wavelength of the laser light

We have shown that the distance, ***x***, of the ***n**[th]* minimum from the centre of the diffraction pattern on the screen is given by

$$x = \frac{n\lambda L}{a}$$

where ***a*** is the width of the slit.

- Measure the distance from the aperture to the screen, *L*, using the tape measure
- The width of the slit *A2* = 0.24 ± 0.02mm
- Record the methods you used and results, giving the errors and units of measurement.

Method used to measure the distance from the aperture to the screen:

Distance between aperture and screen, *L* = **±**

Width of slit *A2, a* = 0.24 **± 0.01 mm**

- Measure the separation on the screen of **10** pairs of symmetrically placed minima (see figure 6)

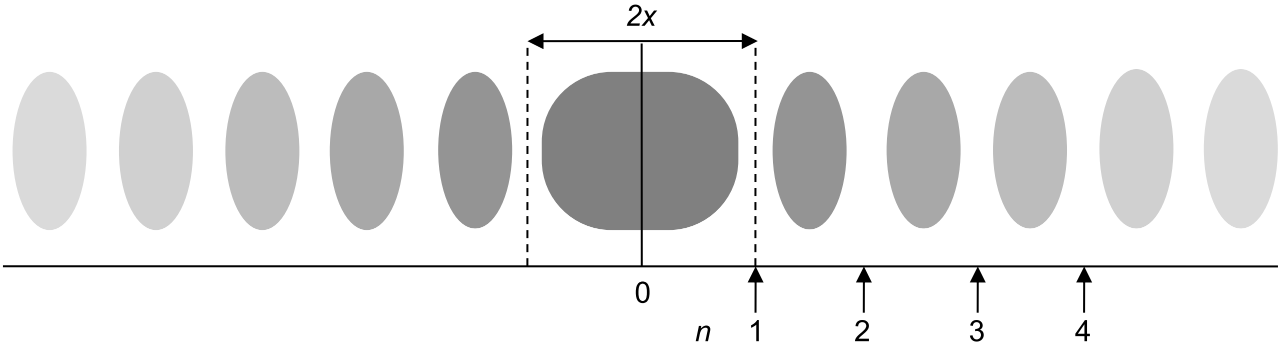


Figure 6: A sketch of the diffraction pattern of a single slit. The dark regions represent the bright regions in the diffraction pattern.

- Record the value of n, and the separation $2x$ in the table below.

Separation of the diffraction minima, $2x$, as a function of n.

n	***2x / cm***	***x / cm***

- Plot a graph of ***x* against *n*** to test the relationship

$$x = \frac{n\lambda L}{a}$$

If the theory is correct, this graph will be a straight line with slope _________________.

Rules for plotting graphs

- Give an informative title, with axes labelled using the Royal Society convention (e.g. x / *cm*) since the co-ordinates of each point are **pure numbers**.
- Draw each data point as a small horizontal/vertical cross (**+**) with a sharp pencil; try to get the position right to 0.5 mm or less
- Often the vertical size of the cross is used to indicate the "error bar" associated with that point.
- Annotate important features of your graph (e.g. gradients).

- Hence deduce the value of λ (turn to page 7 for a step by step method to calculate this).

Deriving a value for the wavelength, λ:

The slope of the graph is given algebraically by **:**

My measured value for the slope of the graph, **slope =**

Therefore, λ **= slope x** **=**

Rough estimate of the error in λ (assuming the error in the slope of the graph is negligible):

The wavelength of the laser is 632.8 nm. Comment on whether your result agrees with this.

2.3. Measuring the diameter of a hair

- Keeping the same arrangements as before remove the aperture plate.
- Tape a hair into the cardboard frame and place it in a holder in the position previously occupied by the aperture plate.
- Measure the separation of an appropriate pair of symmetrical minima and the corresponding value of *n*.
- Use this to estimate the diameter of the hair – turn to page 8 for the steps you need to estimate this value.

Describe and sketch the diffraction pattern of a hair; how does it compare with the diffraction pattern of a slit?

Finding the diameter of a hair, using the fact that $x=n\lambda L/a$:

Value of n =

Separation of the diffraction minima measured, $2x$ =.

The diameter of the hair =

Specific skills learnt in this experiment

- Presenting data graphically.
- Labelling graphs correctly.
- Using a projection microscope.
- Qualitative observation and quantitative measurement of some diffraction phenomena

Appendix A:

The following is a mathematical description of single slit diffraction that goes way beyond anything that you will study at A-level. However, it may be interesting to those with a strong mathematical inclination and is an extremely powerful method which enables us to predict diffraction patterns of complex structures. The method is called **Fourier Transforms** and uses an understanding of complex numbers (i.e. the idea that √(-1) can be written as *i*) and the exponential function *e* and how one might integrate it

$$\textbf{Amplitude} = \int_{-\mathbf{a}/2}^{\mathbf{a}/2} \mathbf{e}^{-\mathbf{iwz}}\mathbf{dz} = \left[-\frac{1}{\mathbf{iw}}\mathbf{e}^{-\mathbf{iwz}} \right]_{-\mathbf{a}/2}^{\mathbf{a}/2} = \frac{1}{\mathbf{iw}}\left[-\mathbf{e}^{-\mathbf{iwa}/2} + \mathbf{e}^{\mathbf{iwa}/2} \right]$$

$$= \frac{1}{\mathbf{iw}}\left[\mathbf{e}^{\mathbf{iwa}/2} - \mathbf{e}^{-\mathbf{iwa}/2} \right]$$