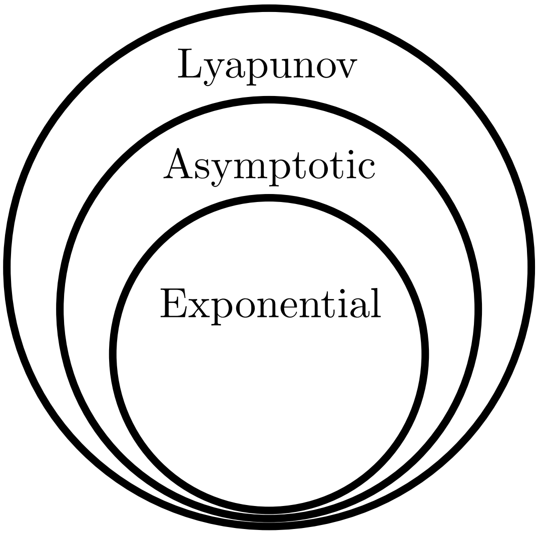
Autonomous



Local / Global

Invariant Set:
A state in an invariant set, you never leave the invariant set. Limit cycles are a classical example of invariant sets that are stable under Lyapunov.

Local Lyapunov Stability
$V(\mathbf{x}) > 0 \; \forall \mathbf{x} \in \Omega$
$\dot{V}(\mathbf{x}) \leq 0 \; \forall \mathbf{x} \in \Omega$

Local Asymptotic Stability
$V(\mathbf{x}) > 0 \; \forall \mathbf{x} \in \Omega$
$\dot{V}(\mathbf{x}) < 0 \; \forall \mathbf{x} \in \Omega$

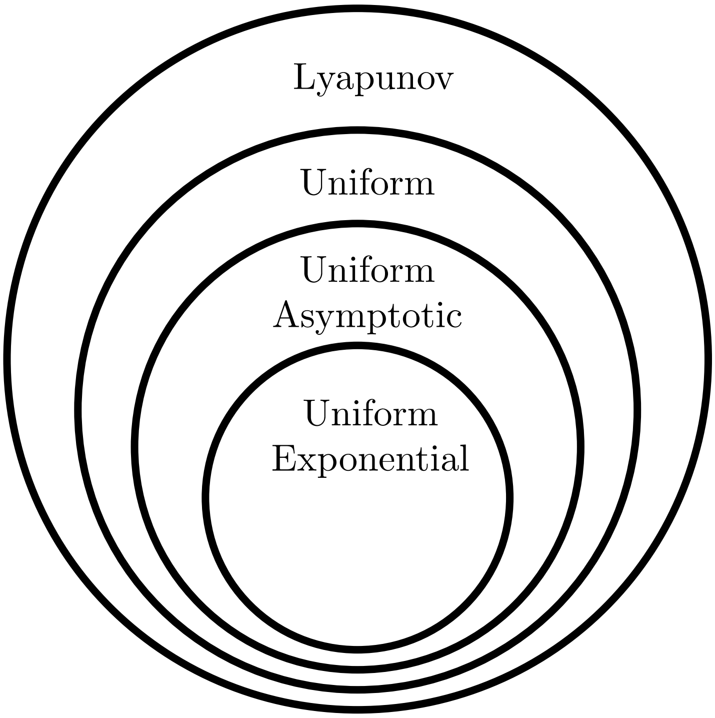
Local Exponential Stability
$V(\mathbf{x}) > 0 \; \forall \mathbf{x} \in \Omega$
$\dot{V}(\mathbf{x}) < 0 \; \forall \mathbf{x} \in \Omega$
$\forall \, t > 0 \; \exists \, \alpha > 0, \exists \, \lambda > 0$ such that:
$||\mathbf{x}(t)|| \leq \alpha ||\mathbf{x}(0)|| e^{-\lambda t}$

Gloabl Lyapunov Stability
$V(\mathbf{x}) > 0 \; \forall \mathbf{x} \neq 0$
$\dot{V}(\mathbf{x}) \leq 0 \; \forall \mathbf{x}$
As $||\mathbf{x}|| \to \infty \Rightarrow V(\mathbf{x}) \to \infty$

Globally Asymptotic Stability
$V(\mathbf{x}) > 0 \; \forall \mathbf{x} \neq \mathbf{0}$
$\dot{V}(\mathbf{x}) < 0 \; \forall \mathbf{x} \neq \mathbf{0}$
As $||\mathbf{x}|| \to \infty \Rightarrow V(\mathbf{x}) \to \infty$

Global Exponential Stability
$V(\mathbf{x}) > 0 \; \forall \mathbf{x} \neq \mathbf{0}$
$\dot{V}(\mathbf{x}) < 0 \; \forall \mathbf{x} \neq \mathbf{0}$
$\forall \, t > 0 \; \exists \, \alpha > 0, \exists \, \lambda > 0$ such that:
$||\mathbf{x}(t)|| \leq \alpha ||\mathbf{x}(0)|| e^{-\lambda t}$
As $||\mathbf{x}|| \to \infty \Rightarrow V(\mathbf{x}) \to \infty$

Non-Autonomous



Local / Global

Decrescent
$V(\mathbf{0}, t) = 0$
$\forall \, t \geq 0, \exists V_1(\mathbf{x}) \ni V(\mathbf{x}, t) \leq V_1(\mathbf{x})$

Uniform
Function independent of t_0

Barbalat's Lemma:
$V(\mathbf{x}, t)$ is lower bounded
$\dot{V}(\mathbf{x}, t)$ is negative semi-definite
$\dot{V}(\mathbf{x}, t)$ is uniformly continuous in time
Then: $\dot{V}(\mathbf{x}, t) \to 0$ as $t \to \infty$

Local Lyapunov Stability
$V(\mathbf{x}, t) > 0 \; \forall \mathbf{x} \in \Omega$
$\dot{V}(\mathbf{x}, t) \leq 0 \; \forall \mathbf{x} \in \Omega$

Local Uniform Stability
$V(\mathbf{x}, t) > 0 \; \forall \mathbf{x} \in \Omega$
$\dot{V}(\mathbf{x}, t) \leq 0 \; \forall \mathbf{x} \in \Omega$
V is Decrescent

Local Asymptotic Uniform Stability
$V(\mathbf{x}) > 0 \; \forall \mathbf{x} \in \Omega$
$\dot{V}(\mathbf{x}) < 0 \; \forall \mathbf{x} \in \Omega$
V is Decrescent

Local Uniform Exponential Stability
$V(\mathbf{x}, t) > 0 \; \forall \mathbf{x} \neq \mathbf{0}$
$\dot{V}(\mathbf{x}, t) < 0 \; \forall \mathbf{x} \neq \mathbf{0}$
$\forall \, t > 0 \; \exists \, \alpha > 0, \exists \, \lambda > 0$ such that:
$||\mathbf{x}(t)|| \leq \alpha ||\mathbf{x}(0)|| e^{-\lambda t}$
V is Decrescent

Global Uniform Asymptotic Stability
$V(\mathbf{x}, t) > 0 \; \forall \mathbf{x} \neq \mathbf{0}$
$\dot{V}(\mathbf{x}, t) < 0 \; \forall \mathbf{x} \neq \mathbf{0}$
V is Decrescent
As $||\mathbf{x}|| \to \infty \Rightarrow V(\mathbf{x}, t) \to \infty$

Global Uniform Exponential Stability
$V(\mathbf{x}, t) > 0 \; \forall \mathbf{x} \neq \mathbf{0}$
$\dot{V}(\mathbf{x}, t) < 0 \; \forall \mathbf{x} \neq \mathbf{0}$
$\forall \, t > 0 \; \exists \, \alpha > 0, \exists \, \lambda > 0$ such that:
$||\mathbf{x}(t)|| \leq \alpha ||\mathbf{x}(0)|| e^{-\lambda t}$
V is Decrescent
As $||\mathbf{x}|| \to \infty \Rightarrow V(\mathbf{x}) \to \infty$