Development of Linear Battery Model for Path Planning with Mixed Integer Linear Programming: Simulated and Experimental Validation

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Abstract: Mixed Integer Linear Programs (MILPs) are often used in the path planning of both ground and aerial vehicles. Such a formulation of the path planning problem requires a linear objective function and constraints, limiting the fidelity of the the tracking of vehicle states. One such state often used is the the charge level of the on board battery. High-fidelity battery state estimation requires nonlinear differential equations to be solved. This state estimation is vital in path planning to ensure flyable paths, however when using a linear path planning problem cannot implement these nonlinear models. Poor accuracy in battery estimation during the path planning runs the risk of the planned path being feasible by the estimation model but in reality will deplete the battery to a critical level, resulting in a dangerous planned path. To the end of higher accuracy battery estimation within a linear framework, we test a simple linear battery model which predicts the change in state-of-charge (SOC) of a battery given a power draw, time duration, and current SOC in the context of an a-priori path plannign problem. This context differentiates itself from real-time estimation. In ahead-of-time path planning, the changes to battery draw are often assumed as a series of constant power draws as opposed to rapidly changing power draw which may occur in real-time battery tracking and estimation. The linear battery model is presented and then tested against alternate models in both numerical and in experimental tests. Further, the effect of the proposed linear model on the time-to-solve a resource constrained shortest path problem is also evaluated, where two different algorithms are used to solve the path planning problem. It is seen that the linear model performs well in battery state estimation while remaining implementable in a Linear Program or MILP, with minimal effect on the time-to-solve. This provides what we consider to be a worthwhile trade-off in improved accuracy relative to increased time-to-solve.

1. INTRODUCTION

Path and trajectory planning are problems which show up in many domains, including vehicle routing, Unmanned Aerial Vehicle (UAV) planning, and general robotics. In applications where a battery is used to power the vehicle, the tracking of battery state-of-charge (SOC) along the path is important to ensure planned paths are feasible on the real system. If the battery capacity is limited relative to the expected returned path distance, there lies the risk of planning a path which exceeds the vehicle's battery limits but is calculated to be feasible by the battery model used in the planning. This danger is especially present in the case of small UAVs (sUAV) and electric or hybrid-fuel ground vehicles. Tracking battery SOC along a path in a Mixed Integer Linear Program (MILP), an often used class to model the path planning problem, requires a linear function for the battery. However battery SOC in reality changes in a very nonlinear manner. To track with a higher fidelity nonlinear battery model, the problem ceases to be linear and thus must be solved with more costly *nonlinear* optimization techniques. However, the cost of increased problem complexity may not be outweighed by the improved battery tracking, depending on the exact problem at hand. For example, multi-agent cooperative planning, in a MILP form, is generally very computationally expensive and quickly becomes infeasible to solve as the problem sizes increase. If transformed to a nonlinear programming problem, to the end of improved battery tracking accuracy, the time-to-solve scaling is even harsher. We present here a simple linear battery model which accounts for voltage changes due to power draw and drop in battery SOC changes. This aims

[1] Approved for public release, distribution unlimited: AFRL-2023-1752

primarily to replace a constant-voltage assumption, where the battery voltage changes due to SOC and power load are ignored. While those two effects are nonlinear, a simple fitted linear model can account for both of those phenomenon while retaining a linear path-planning problem formulation. Such models have been studied prior, however we seek to analyze these models in the context of ahead-of-time path planning. There is utility in comparing these well-studied battery models in the context of a-priori path planning, where there is an absence of rapidly changing battery currents in which higher order battery dynamics come into play. In such cases, a real-time model may be useful. However, with these rapid fluctuations in battery current removed, the higher order models lose some of their normal utility. Thus, the improved accuracy they provide over simpler models is expected to be reduced in an a-priori planning context. The linear model is compared in terms of accuracy of SOC tracking to several alternate battery models in both numerical tests and experimental tests. Further, its effect on time-to-solve a resource constrained shortest path problem (RCSPP) is evaluated in comparison with a constant-voltage model using two different path planning algorithms.

We are motivated by a prior work in Scott et al. (2022) where noise-aware path planning is done for a hybrid-fuel UAV which combines a generator and battery pack as power sources. In that work, a simple equation is used to update the battery SOC where a constant (nominal) voltage is assumed, such that the change in SOC along an edge is constant regardless of power draw, the current SOC, hysteresis effects, or any environmental factors. This is a simplification made to keep the path planning problem easier, where in reality the change in battery SOC for a given flight leg depends on many factors, two major ones being electrical load and the current battery SOC.

The primary contributions of this paper are: i) Numerical and experimental testing of a linear battery model, on both single-cell and multi-cell batteries in the context of a linear a-priori path planning problem ii) analysis of the effect the linear model has on the time-to-solve a RCSPP variant as compared to a constant voltage assumption. The paper is organized as follows. A survey of related literature is presented in Section 2. The battery tracking problem of concern is presented in Section 3. Section 4 presents the linear model, along with the alternate models. Section 5 presents the results of a numerical test comparing the linear model with the alternate models on an 18650 battery cell. Section 6 presents the results of experimental tests on a Lithium Polymer (LiPo) battery. An evaluation of time-to-solve a RCSPP with and without the linear battery model is presented in Section 7. This RCSPP is solved with two algorithms: i) Branch-and-bound ii) Labeling Algorithm.

2. LITERATURE REVIEW

To the best of the authors' knowledge, a linear battery model specifically for use in path planning with MILPs has not been presented and tested in the existing literature. However, there exist numerous models for battery systems. These are nonlinear and seek to model the battery behavior as accurately as possible. There is also much work done on real-time estimation of SOC and battery health.

Battery models are divided into 3 main types: i) Electro-chemical models; ii) Equivalent-circuit models; iii) Data-driven models. Electro-chemical models seek to model the behavior of the battery at a chemical level, whereas the equivalent-circuit models represent the battery as a circuit system, often a resistor-capacitor (RC) circuit, with behavior that mimics that of the real battery. Data-driven models develop a numerical model of the battery system based on gathered data. Battery state estimation techniques either seek to determine the state-of-energy, being the remaining energy that can be pulled from the battery, state-of-charge estimation, power capability, or state-of-health (SOH). A comprehensive survey of the above estimation and model types is given in Wang et al. (2020).

Tracking battery state in path planning problems has been studied before, primarily in the case of Electric Vehicles (EVs) and sUAVs where the only power source is a battery. In Di Franco and Buttazzo (2016), a model is presented to check if a given path is feasible with regard to battery SOC. Here, a path is first produced and then checked for battery feasibility after the fact in an online fashion. In Schacht-Rodríguez et al. (2018), the path planning of a UAV is done while accounting for battery SOH. UAV path planning is considered in Hovenburg et al. (2020) where an algorithm is proposed which includes a sUAV performance model and battery model within the path planning, which was solved using a particle swarm optimization algorithm.

There is also existing work on Vehicle Routing Problem (VRP) variants applied to electric vehicles (EV). The Electric VRP (EVRP) is that of finding minimum global cost routes for a fleet of electric vehicles, often with additional constraints such as recharging events. An extensive survey of the problems and solution approaches is given in Erdelić and Carić (2019). There are several studies on hybrid vehicle routing Verma (2018); Doppstadt et al. (2016); Vincent et al. (2017); Hiermann et al. (2019) where a constant charge/discharge is used for the battery, ignoring the effect battery SOC has on the change in battery SOC for the same edge. Another study Scott et al. (2022) involves a battery/generator hybrid UAV, where the battery is discharged and charged throughout the flight, where the battery is also tracked using a constant charge/discharge for a given edge. These problems, all formulated as MILPS, can be improved to have greater accuracy in battery modeling using the model presented in this paper without the significant increase in computational cost when using nonlinear constraints.

3. BATTERY TRACKING PROBLEM

As discussed above, the primary challenge in tracking the battery SOC within a MILP path planning problem arises due to the nonlinear behavior of batteries. Depending on the chemical principles used for modeling, this nonlinear behavior can be more pronounced. The path planning, when done on a graph, returns a path, optimal by some metric, as a series of edges. An example graph is given in Fig. 1, in a case where edges are defined by their power requirement and time to travel along. Graphs may not be parameterized in this exact manner, but by metrics which track energy or power usage in some alternate manner. This path planning is often formulated as a MILP, where the battery SOC is tracked by a linear equation that is a function of the graph parameters and decision variables. An example of such is given in Equation (1), a simplified equation from Scott et al. (2022).

$$b_j \leq b_i + \Delta(b_i, P_{ij}, t_{ij}) + M(1 - x_{ij}) \quad \forall (i,j) \in E \quad (1)$$

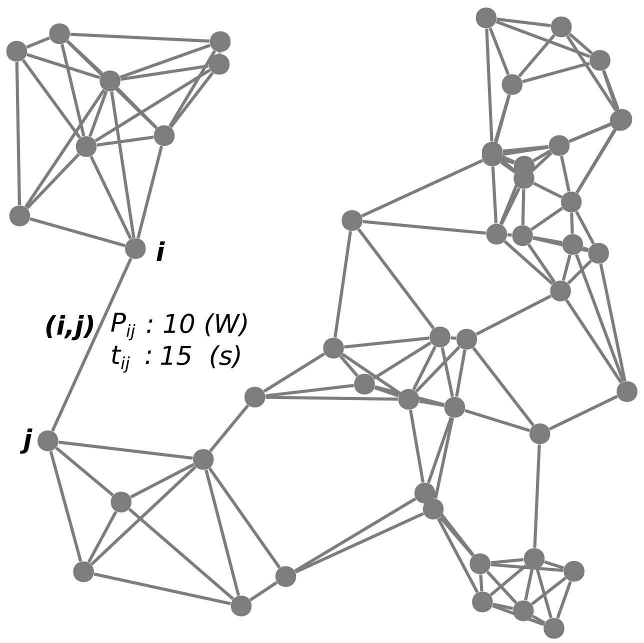

Fig. 1. Graph with Power Consumption Parameters

where b_i is the SOC of the battery at node i, x_{ij} is a binary decision variable for usage of the edge from node i to node j, and E is the set of all edges in the problem. The value M is a large constant such that if edge (i, j) is not used in the solution then the constraint is trivially satisfied. $\Delta(b_i, P_{ij}, t_{ij})$ is the function which determines the change in battery SOC from node i to node j given the power draw P and time duration t for the given edge. The change in SOC Δ will be nonlinear. The aim of the linear model is to estimate the Δ function while still being implementable in a MILP as a linear constraint. The above equation is for a shortest path variant, whereas a similar constraint for a VRP or Traveling Salesman Problem variant would require adaptation to the tour format rather than path format.

4. BATTERY UPDATE EQUATIONS

The change in SOC with respect to time is found with Equation (2)

$$\frac{dS}{dt} = -\frac{I}{C_m} = -\frac{I(S,P)}{C_m} \tag{2}$$

where S is battery SOC normalized by the maximum charge in Coulombs C_m such that at full SOC $S = 1$, t is time, and I is the current through the battery which is positive if the battery is being drained. Assuming there is a known power draw P, the current can be calculated using Equation (3).

$$I(S,P) = \frac{P}{V(S,P)} \tag{3}$$

where $V(S,P)$ is a function of battery SOC and power draw.

Substituting yields the change in SOC with respect to time, given in Equation (4).

$$\frac{dS}{dt} = -\frac{P}{V(S,P)C_m} \tag{4}$$

Integrating Equation (4) gives a function for $S(t)$, given in Equation (5).

$$S(t) = S_0 - \int_{t_0}^{t} \frac{P}{V(S(\tau),P)C_m}\, d\tau \tag{5}$$

where a positive power value P is that which drains the battery. This can be solved numerically with methods such as Runge-Kutta techniques or similar. The solution obtained from the numerical methods can be compared to the linear model estimation, which uses a single time-step in calculation of $S(t)$. For comparison against the linear model, three alternate battery models are used. The first is a simple Ohmic drop model and the second is a 1st order RC model, both from Hu et al. (2012). These are models (2) and (7) respectively as referred to in the cited study. The Ohmic drop model was chosen due to its simplicity, comparable to that of the linear model presented here. The 1st order RC model was chosen as its accuracy was comparable to more complex models presented in Hu et al. (2012), where the higher complexity models saw diminishing returns in accuracy improvements. The third alternate model used is one which assumes constant voltage. Here, this is taken to be the nominal voltage of the battery types testing in Section 5 and 6.

4.1 *Simple Ohmic Drop Model*

The open-circuit voltage (OCV) of a battery can be obtained experimentally and utilized by a look-up table to find OCV for a given SOC. However, when a load is applied to the battery, the actual voltage supplied by the battery drops. This effect, called Ohmic drop, increases with increasing load on the battery. For higher loads, the battery is supplying a lower voltage and thus, for the same power draw, a larger current. Therefore, the change in SOC depends on both the current SOC and the power. The simple Ohmic drop model is defined as:

$$V(S,I) = OCV(S) - IR_0 \tag{6}$$
$$P = V(S,I)I \tag{7}$$

where R_0 is the internal resistance of the battery. The current I is assumed to be a function of power and voltage, as in our case there is a constant power draw through which the current I is defined, whereas in most cases the current is assumed to be known directly. Substituting (7) into (6) gives the following where voltage is now a function of SOC and power draw:

$$V(S,P) = \frac{OCV(S) + \sqrt{OCV(S)^2 - 4PR_0}}{2} \tag{8}$$

Equation (8) is the simple Ohmic drop model when constant power is used. Here, $V(S,P)$ will always have a real, positive solution so long as the current is not so large to cause a negative voltage. Thus, the cases of complex solutions are ignored.

4.2 *1st Order RC Model*

The first-order model is defined in Equations (9) and (10).

$$V_k = OCV(SOC_k) - I_k R_0 - U_k \tag{9}$$

$$U_{k+1} = \exp(-\tfrac{\Delta t}{\tau})U_k + R_1[1 - \exp(-\tfrac{\Delta t}{\tau})]\frac{P_k}{V_k} \tag{10}$$

where τ is the time constant of the RC circuit which models the battery behavior, R_0 is the internal resistance of the battery, k is the numerical step, R_1 and τ are the resistance and time constant of the simulated circuit, and U_k is a secondary variable tracking the hysteresis effects over time.

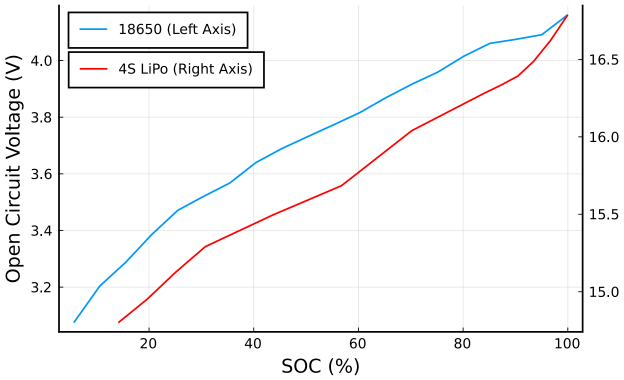

Fig. 2. OCV Curve - 4S LiPo and 18650 Cell

4.3 Nominal Voltage Only

The third estimation model used is one which uses the nominal battery voltage exclusively, and determines change in SOC based off this nominal voltage, power draw, and time duration of the load. This type of model is used often in path planning, as discussed in Section 2. This estimation model is solved using a single time step as a fair comparison to the linear model, as both can be implemented in a MILP in this manner. The update equation is given in Equation (11), where V_{nom} is the nominal battery voltage.

$$S(t) = S_0 - \frac{P(t - t_0)}{C_m V_{nom}} \tag{11}$$

4.4 Linear Model

The linear model to implement Equation (5) into a MILP constraint is now presented. A linear fit is made utilizing the simple Ohmic drop in Equation (6). However, since the change in SOC depends on $\frac{1}{V}$, a linear fit will be made of the inverse of the voltage given in Equation (6). The linear model which accounts for the power draw and current SOC is defined in Equation (12).

$$S(t) = S_0 - P(AS_0 + BP + C)(t - t_0)/C_m \tag{12}$$

where (A,B,C) are the fit parameters of the linear model. This is a linear fit of $\frac{1}{V(S,P)}$ where $V(S,P)$ is calculated with Equation (8). Note that over the time period (t_0, t), the voltage will change as the battery is discharged. This is addressed in the RC model and simple Ohmic drop model via the Runge-Kutta techniques to solve Equation (5). As the linear model is to be implemented in a MILP, such techniques cannot be used and instead a single time-step is used, where the voltage, which changes over the time period, is taken to be constant as the voltage at time t_0.

5. NUMERICAL TESTS

Here we present the results of a numerical simulation for a single 18650 battery cell, where the constant-power flight legs of the path-planning problem are simulated. In the path-planning problem, the battery SOC is to be tracked along a path, where constant power is drawn for some known duration. The path, in this form, is a queue of these flight legs or edges, and thus can be represented as a series of constant power draws

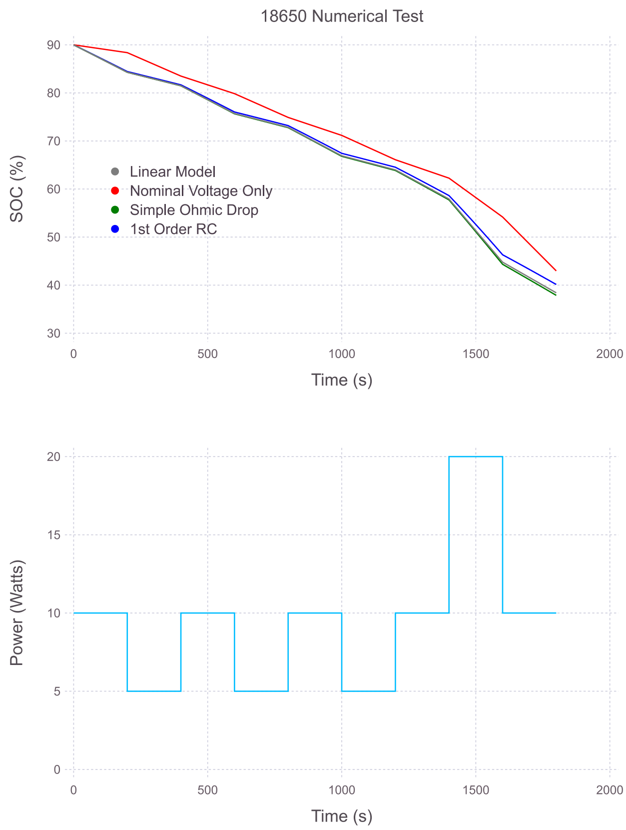

Fig. 3. Numerical Test - 18650 cell

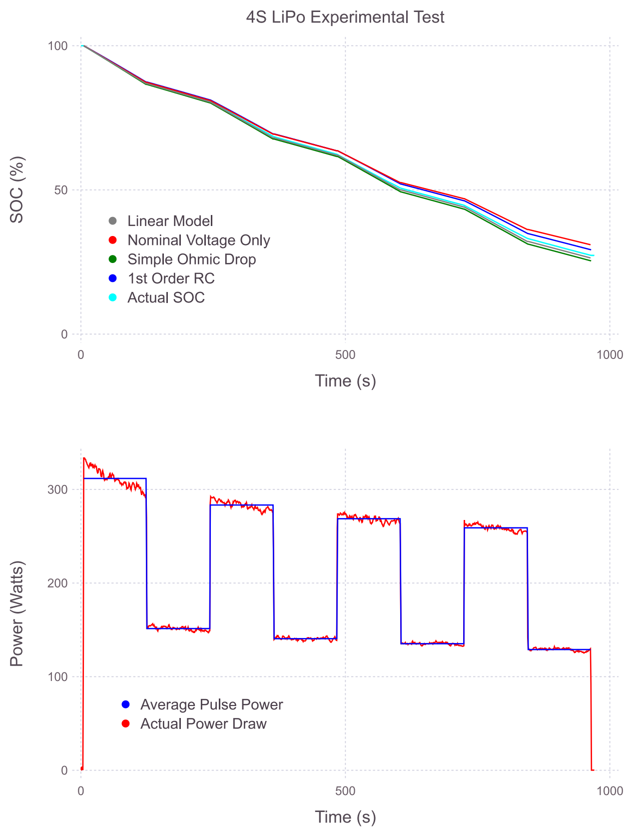

Fig. 4. Experimental Test - 4S LiPo

from the battery for some duration. An 18650 lithium-ion (Li-Ion) battery with 2500 mAh capacity is tested, with the OCV-SOC look up table taken from Elmahdi et al. (2021). Plots of the OCV-discharge curves are given in Fig. 2. In each set of tests the 4 estimation models from Section 4 are used: i) simple Ohmic drop model in Equation (6) ii) 1st Order RC model iii) Linear Model presented iv) Nominal Voltage only

The results of the simulation for the 18650 Li-Ion cell are given in Fig. 3. Here, the power pulses are applied such that the battery is nearly depleted by the end of the simulation, so that the models are tested over nearly the entire SOC of the battery. It can be seen that all models perform similarly, with a few percent-SOC difference in the final estimations. Because the power applied is constant, and thus the current changing very slowly only as a result of change in battery voltage as the battery is depleted, the transient effects of the battery are minimal in this application. Cases were the power changes more frequently may show improved performance of the 1st order RC model relative to the others as the transient effects will have a greater influence.

6. EXPERIMENTAL TESTS

The results of using the models to predict battery SOC as compared to real data from a 4S LiPo battery are presented in this section. A 4S LiPo was used, with 5000 mAh capacity, and a 50C maximum discharge rate. Measurements for power, voltage, and current were used with an 1585 RC Benchmark test stand. The battery load was applied via a motor and propeller, with the motor controlled by an Electronic Stability Control (ESC) Pulse-Width Modulation (PWM) setting. In these tests, a constant ESC frequency was applied to achieve approximately the desired power draw. As seen in Fig. 4, the power draw was not constant, including both the noise and slow reduction in power draw over time. The drop in power is due to the constant ESC setting rather than power draw. As the battery discharged and heated up throughout the test, the voltage decreases, which then reduces power draw for the same ESC PWM setting. From the battery data, a constant power draw was calculated for each "pulse". This constant power draw, along with time duration of that specific pulse, was given to each model to predict the battery discharge along that pulse.

The results of the model predictions and the actual battery SOC are given in Fig. 4. It can be seen that the models compare similarly among themselves, as was the case in the simulation of the 18650 battery in Section 5. Between the linear model and the actual battery SOC there was a 0.84% SOC difference.

The linear model, due to the limitations of being implemented in a MILP, only uses a single time step to calculate change in state of charge. When compared to the prediction model which also uses a single time step but only uses the nominal voltage of the battery to predict change in state of charge, the linear model is more accurate while remaining implementable in a MILP for path planning or mission planning. The other models tested here, described in Section 4, used Runge-Kutta with 100 time-steps to solve Equation (5) whereas the linear model uses a single time step. However, despite the drastic difference in time-steps, the linear model is still able to predict battery SOC with high accuracy for the path planning battery prediction problem in both the tests.

7. MILP PATH PLANNING TEST

By design, the linear model is implementable in a MILP for path planning, however increased complexity in MILP constraints can have a significant effect on computation time. This effect can also vary between algorithms used to solve the problem. We implement the linear battery model in a RCSPP Joksch (1966) and compare the time to solve with the same problems using nominal battery voltage only. The MILP formulation is given in Equations (13)-(20).

$$\min_{x} \sum_{i} \sum_{j} (C_{ij} x_{ij}) \tag{13}$$

$$\sum_{j} x_{Sj} = 1 \tag{14}$$

$$\sum_{j} x_{jF} = 1 \tag{15}$$

$$\sum_{j} x_{ij} - \sum_{j} x_{ji} = 0 \qquad \forall i \in N \setminus \{S, F\} \tag{16}$$

$$0 \le b_j \le B_m \qquad \forall j \in N \setminus S \tag{17}$$

$$b_S = B_0 \tag{18}$$

where x_{ij} is a binary decision variable which is 1 if edge from node $i$to node j is used in the solution and 0 otherwise, b_i is the battery SOC at node i, and S and F are the start and goal nodes respectively. The constraints to track battery state are given as follows:

$$\begin{aligned} b_j \le b_i - P_{ij} t_{ij} (A b_i + B P_{ij} + C) \\ + M(1 - x_{ij}) \qquad \forall (i,j) \in E \end{aligned} \tag{19}$$

$$b_j \le b_i - \frac{P_{ij} t_{ij}}{V_{nom}} + M(1 - x_{ij}) \qquad \forall (i,j) \in E \tag{20}$$

where P_{ij} is the power draw, t_{ij} is the time to travel along the edge, (A, B, C) are the linear model parameters, and V_{nom} is the nominal voltage of the battery. The linear battery model is used as the resource constraint as implemented with Equation (19). The alternate resource constraint, using the nominal voltage model, is given in (20).

The effect on running time when using the linear model over the nominal voltage only model is evaluated when solving with 2 different algorithms: i) Branch-and-bound ii) Labeling algorithm. The MILP solver in IBM CPLEX v12.9 is used to solve with branch-and-bound and the labeling algorithm presented in Scott et al. (2022) is modified for this RCSPP problem. Problems are made from 5 nodes up to 100 nodes, with 30 instances generated for each problem size. Nodes are randomly placed in a 2D space and each node is connected to the 4 closest nodes, by Euclidean distance. Edge costs are the Euclidean distance between nodes. The battery parameters for the problem simulate the 4S LiPo battery described in Section 6. The averaged results for time-to-solve with the branch-and-bound is given in Fig. 5 and for the labeling algorithm in Fig. 6.

The time to solve is increased in both the branch-and-bound and the labeling algorithm when using the linear model. However, this increase in computation time is more pronounced in the branch-and-bound as compared to the labeling algorithm. This is expected, as the constraint complexity can have a significant effect on time-to-solve when using a branch-and-bound method. The labeling algorithm, which utilizes dynamic programming, has the computation time primarily affected by the number of feasible, undominated paths and sub-paths that exist

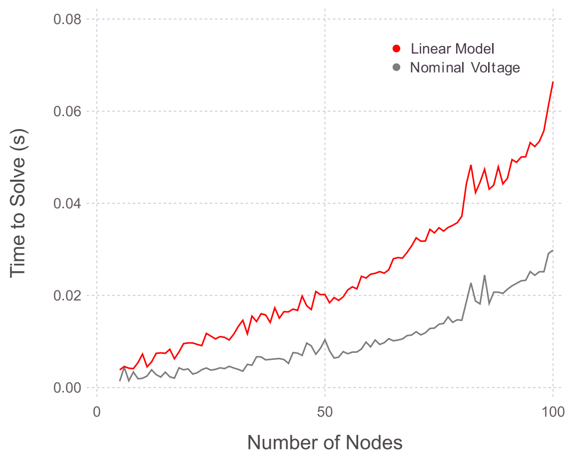

Fig. 5. Time-to-Solve with Branch-and-Bound - Battery Model Comparison

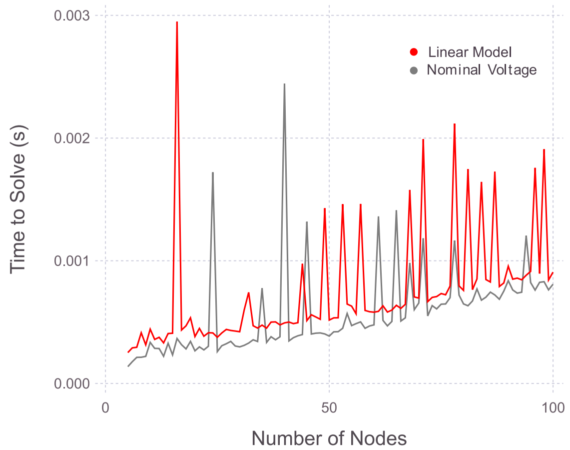

Fig. 6. Time-to-Solve with Labeling Algorithm- Battery Model Comparison

within the problem. While the linear model increases the battery constraint complexity, it has little effect on the number of feasible, undominated paths within the problem. Therefore, the battery constraint is expected to have little effect on the computational time when using the labeling algorithm. The spikes seen in the labeling algorithm performance are expected, where infeasible and loosely constrained problems require many more iterations to complete compared to the average problem for a given problem size. Labeling algorithms and their use in RCPP and similar is given in Desrosiers et al. (1995).

8. CONCLUSIONS AND FUTURE WORK

The linear battery model was presented to the end of accurate battery SOC estimation for ahead-of-time path planning problems concerned with EVs and battery-powered UAVs. Tests were done to simulate the series of steady-state power draws along graph edges which occur in a-priori path planning problems, differentiating the application from standard real-time SOC tracking. Beyond the accuracy testing, changes to the time-to-solve a RCSPP were also studied. It was seen that the linear model provides greater accuracy in SOC estimation over a constant nominal voltage model with little effect on computational time, while remaining a implementable in a linear planning problem. Future work includes further testing on the linear model under different circumstances, and integration with a power or energy consumption model to accurately predict power usage along an edge, as power model accuracy runs the risk of planned paths which are in reality infeasible.

REFERENCES

Desrosiers, J., Dumas, Y., Solomon, M.M., and Soumis, F. (1995). Time constrained routing and scheduling. *Handbooks in operations research and management science*, 8, 35–139.

Di Franco, C. and Buttazzo, G. (2016). Coverage path planning for UAVs photogrammetry with energy and resolution constraints. *Journal of Intelligent & Robotic Systems*, 83(3), 445–462.

Doppstadt, C., Koberstein, A., and Vigo, D. (2016). The hybrid electric vehicle–traveling salesman problem. *European Journal of Operational Research*, 253(3), 825–842.

Elmahdi, F., Ismail, L., and Noureddine, M. (2021). Fitting the OCV-SOC relationship of a battery lithium-ion using genetic algorithm method. In *E3S Web of Conferences*, volume 234, 00097. EDP Sciences.

Erdelić, T. and Carić, T. (2019). A survey on the electric vehicle routing problem: variants and solution approaches. *Journal of Advanced Transportation*, 2019, 1–48.

Hiermann, G., Hartl, R.F., Puchinger, J., and Vidal, T. (2019). Routing a mix of conventional, plug-in hybrid, and electric vehicles. *European Journal of Operational Research*, 272(1), 235–248.

Hovenburg, A.R., de Alcantara Andrade, F.A., Hann, R., Rodin, C.D., Johansen, T.A., and Storvold, R. (2020). Long-range path planning using an aircraft performance model for battery-powered sUAS equipped with icing protection system. *IEEE Journal on Miniaturization for Air and Space Systems*, 1(2), 76–89.

Hu, X., Li, S., and Peng, H. (2012). A comparative study of equivalent circuit models for Li-ion batteries. *Journal of Power Sources*, 198, 359–367.

Joksch, H.C. (1966). The shortest route problem with constraints. *Journal of Mathematical analysis and applications*, 14(2), 191–197.

Schacht-Rodríguez, R., Ponsart, J.C., García-Beltrán, C.D., Astorga-Zaragoza, C.M., Theilliol, D., and Zhang, Y. (2018). Path planning generation algorithm for a class of UAV multirotor based on state of health of lithium polymer battery. *Journal of Intelligent & Robotic Systems*, 91(1), 115–131.

Scott, D., Manyam, S.G., Casbeer, D.W., Kumar, M., Rothenberger, M.J., and Weintraub, I.E. (2022). Power management for noise aware path planning of hybrid UAVs. In *American Control Conference (ACC)*, 4280–4285. Atlanta, GA, USA.

Verma, A. (2018). Electric vehicle routing problem with time windows, recharging stations and battery swapping stations. *EURO Journal on Transportation and Logistics*, 7(4), 415–451.

Vincent, F.Y., Redi, A.P., Hidayat, Y.A., and Wibowo, O.J. (2017). A simulated annealing heuristic for the hybrid vehicle routing problem. *Applied Soft Computing*, 53, 119–132.

Wang, Y., Tian, J., Sun, Z., Wang, L., Xu, R., Li, M., and Chen, Z. (2020). A comprehensive review of battery modeling and state estimation approaches for advanced battery management systems. *Renewable and Sustainable Energy Reviews*, 131, 110015.