Cálculo numérico

01 - Dados os pontos, encontre o polinômio de terceiro grau que passa pelos pontos.

i	x_i	y_i
0	1	2
1	2	5
2	3	10
3	4	23

$$L(x) = 2\,\frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4} + 5\,\frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)} + 10\,\frac{(x-1)(x-2)(x-4)}{(3-1)}$$

Por newton: $P_n(x) : 2 + \left(\frac{2}{1-2} + \frac{5}{2-1}\right)(x-1) + \left(\frac{2}{(1-2)(1-3)} + \frac{5}{(2-1)(2-3)} + \frac{10}{(3-1)(3-2)}\right)((x-1)(x-2)) + \left(\frac{2}{(1-2)(1-3)(1-4)} + \frac{5}{(2-1)(2-3)(2-4)} + \frac{10}{((3-1)(3-2)(3-4)} + \frac{23}{(4-1)(4-2)(4-3)}\right)((x-1)(x-2)(x-3))$

$$= \frac{2}{-1} + \frac{5}{1}(x-1) + \left(\frac{2}{-2+3} + \frac{5}{-2+1} + \frac{10}{3-1}\right)(x^2-2x-x+2) + \left(\frac{2}{1\cdot 6} + \frac{5}{6\cdot 2} + \frac{10}{3-4} \frac{23}{5\cdot 4}\right)(x^3-0$$

simplificando temos $\boxed{x^3 - 5x^2 + 11x - 5}$

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② Polinomio de quarto grau.

a. Lagrange

b. Newton

c. Gregory-Newton

	x_i	y_i
0	0	71
1	2	29
2	4	-37
3	6	-65
4	8	719

$$a \cdot L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}$$

$$= \frac{(x-29)(x+37)(x+65)(x-719)}{(71-29)(71+37)(71+65)(71-719)} =$$

Simplificando, temos. $\boxed{29,0625}$ //

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b. newton: $71 + \left(\frac{71}{0-2} + \frac{29}{2-0}\right)(x-0) + \left(\frac{71}{(0-2)(0-4)} + \frac{29}{(2-0)(2-4)} + \ldots\right.$

$((x-0)(x-2))\left(+\frac{71}{(0-2)(0-4)(0-6)} + \frac{29}{(2-0)(2-4)(2-6)} + \frac{-37}{(4-0)(4-2)(4-6)} +\right.$

$\left.\frac{65}{(6-0)(6-2)(6-4)}\right)(x-0)(x-2)(x-4)$. Simplificando temos

$\frac{89}{48}x^4 - \frac{503}{24}x^3 + \frac{425}{6}x^2 - \frac{281}{3}x + 71$ //

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03 ro

$\int_0^6 f(x)dx$

x_i	y_i
0	98
1	80,5
2	46
3	0,5
4	-38
5	-39,5
6	38

③

a. $I = I_2 + (I_2 - I_1)\dfrac{n_1^2}{n_2^2 - n_1^2}$ | $n1 = 3$, $n2 = 6$

$I_1 = \dfrac{h}{2}(y_0 + y_1) = \dfrac{6}{2} \times (98 + 38)$

$I_1 = 3 \cdot 136 = 408$ //

$I_2 = h_1 = 1 \Rightarrow I_2 = \left(\dfrac{98}{2} + 80,5 + 46 + 0,5 - 38 - 39,5 + \dfrac{38}{2}\right) = 49 + 49,5 + 19 = 117,5$

$I = 117,5 + (117,5 - 408)\dfrac{9}{36 - 9}$

$I = 117,5 - 290,5 \cdot 0,3333$

$I = 117,5 - 57,006609$ //

2^+

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$5b - 9$

$I = 117{,}5 - 290{,}5 \cdot 0{,}3333$

$-57{,}1606609$ //

3) b) $= I = I_2 + (I_2 - I_1)\dfrac{h_1^4}{h_2^4 - h_1^4}$

$I_1 = \dfrac{3}{3} h \, (y_0 + 4y_1 + y_2)$

$= \; (98 + 4 \cdot 0{,}5 + 38)$

$= \cancel{296 + 6\ldots}$

$= \cancel{63}$ // $= 138$

$I_2 = \dfrac{1}{3} h \, (98 + 4 \cdot 80{,}5 + 2 \cdot 46 + 4 \cdot 0{,}5 + 2 \cdot (-38) + 4 \cdot (-39{,}5) + 38)$

$I_2 = \; \left(\dfrac{98}{3} + \dfrac{322}{3} + \dfrac{92}{3} + \dfrac{2}{3} + \dfrac{76}{3} + \dfrac{158}{3} + \dfrac{38}{3}\right) = 262$ //

$I = 262 + (262 - 138)\dfrac{2^4}{6^4 - 2^4}$

$= 386 \cdot \dfrac{16}{1296 - 6}$

$\boxed{= 4{,}78759689}$ //

4

x_i	y_i
0	8
1	-3
2	-16
3	-25
5	-7
6	32
7	99

I

I $\int_0^3 f(x)\,dx \cong \frac{3}{8}\left(8 + 3\cdot(-3) + 3\cdot(-16) + (-25)\right) =$

$= -27{,}75$

II $\int_5^7 f(x)\,dx \cong \frac{1}{3}\left(-7 + 4(32) + 99\right) =$

$= 73{,}3333333333$

III $\int_3^5 f(x)\,dx \cong \frac{2}{2}\left(-25 - 7\right) = -32$

$\Rightarrow \int_0^7 f(x)\,dx \cong -27{,}75 + 73{,}3333 - 32 \cong$

$\boxed{\cong 13{,}5833333333}$

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