01) Dados os pontos, encontre o polinomio de terceiro grau que passa pelos pontos.

i	x_i	y_i
0	1	2
1	2	5
2	3	10
3	4	23

$$L(x) = 2\frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)} + 5\frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)} + 10\frac{(x-1)(x-2)(x-4)}{(3-1)}$$

Por newton: $P_n(x) = 2 + \left(\frac{2}{1-2} + \frac{5}{2-1}\right)(x-1) + \left(\frac{2}{(1-2)(1-3)} + \frac{5}{(2-1)(2-3)}\right.$

$$\left. + \frac{10}{(3-1)(3-2)}\right)((x-1)(x-2)) + \frac{2}{} + \frac{5}{} + \frac{10}{}$$

$$L(x) = 2\frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)} + 5\frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)} + 10\frac{(x-1)(x-2)(x-4)}{(3-1)}$$

Por newton: $P_n(x) = 2 + \left(\frac{2}{1-2} + \frac{5}{2-1}\right)(x-1) + \left(\frac{2}{(1-2)(1-3)} + \frac{5}{(2-1)(2-3)} + \right.$

$$\left. + \frac{10}{(3-1)(3-2)}\right)((x-1)(x-2)) + \frac{2}{(1-2)(1-3)(1-4)} + \frac{5}{(2-1)(2-3)(2-4)} + \frac{10}{(3-1)(3-2)(3-4)} +$$

$$\left. + \frac{23}{(4-1)(4-2)(4-3)}\right)((x-1)(x-2)(x-3))$$

$$= \frac{2}{-1} + \frac{5}{1}(x-1) + \left(\frac{2}{-2+3} + \frac{5}{-2+1} + \frac{10}{3-1}\right)(x^2-2x-x+2) + \left(\frac{2}{-1\cdot 6} + \frac{5}{6-2} + \frac{10}{3-4}\right.$$

$$\left.\frac{23}{5-4}\right)(x^3-6)$$, simplificando temos $\boxed{x^3 - 5x^2 + 11x - 5}$

(2) Polinomio de quarto grau.

a. Lagrange
b. Newton
c. Gregory-Newton

x_i	
0	7
1	2
2	4
3	6
4	?

a. $L_0(x) = \dfrac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}$

$= \dfrac{(x-29)(x+37)(x+65)(x-719)}{(71-29)(71+37)(71+65)(71-719)} =$

Simplificando, temos: **29,0625 //**

(48)

b. Newton: $71 + \left(\frac{71}{0-2} + \frac{29}{2-0}\right)(x-0) + \left(\frac{71}{(0-2)(0-4)} + \frac{29}{(2-0)(2-4)} + \frac{-37}{(4-0)(4-2)}\right)$

$((x-0)(x-2))\left(+\frac{71}{(0-2)(0-4)(0-6)} + \frac{29}{(2-0)(2-4)(2-6)} + \frac{-37}{(4-0)(4-2)(4-6)} + \frac{65}{(6-0)(6-2)(6-4)}\right)(x-0)(x-2)(x-4)$. Simplificando temos

$$\frac{89}{48}x^4 - \frac{503}{24}x^3 + \frac{425}{6}x^2 - \frac{281}{3}x + 71 //$$

④

x_i	y_i
0	8
1	-3
2	-16
3	-25
5	-7
6	32
7	99

Ⓘ $\int_0^3 f(x)dx \approx \frac{3}{8}(8+3\cdot(-3)+3\cdot(-16)+(-25)) =$

$= -27{,}75$

Ⓘ$\!\!$Ⓘ $\int_5^7 f(x)dx \approx \frac{1}{3}(-7+4(32)+99) =$

$= 73{,}3333333333$

Ⓘ$\!\!$Ⓘ$\!\!$Ⓘ $\int_3^5 f(x)dx \approx \frac{2}{2}(-25-7) = -32$

$\Rightarrow \int_0^7 f(x)dx \approx -27{,}75 + 7{,}3333 - 32 \approx$

$\approx 13{,}583333333$