Abstract

This paper proves that push-pull block puzzles in 3D are PSpace-Complete to solve and push-pull block puzzles in 2D with thin walls are NP-Hard to solve. Push-pull block puzzles are a game, and motion planning problem, similar to Sokoban, that involves moving a 'robot' on a grid with obstacles. The obstacles cannot be traversed by the robot, but some can be pushed or pulled by the robot into adjacent squares. Thin wall prevent movement between two adjacent squares. This work follows in a long line of algorithms and complexity work on similar problems [10] [2] [8] [5] [4] [6] [3] [1] [7] [9]. The 2D push-pull block puzzle shows up in a number of video games, including The Legend of Zelda, thus implying a number of other results. This variant of block pushing puzzles is of particular interest to the authors because it is fully reversible, meaning the inverse of any action that has been taken is valid.

References

[1] J. C. Culberson. Sokoban is PSPACE-complete. In *Proceedings International Conference on Fun with Algorithms (FUN98)*, pages 65–76, Waterloo, Ontario, Canada, June 1998. Carleton Scientific.

[2] Erik D. Demaine, Martin L. Demaine, and Joseph O'Rourke. PushPush and Push-1 are NP-hard in 2D. In *Proceedings of the 12th Annual Canadian Conference on Computational Geometry (CCCG 2000)*, pages 211–219, Fredericton, New Brunswick, Canada, August 16–18 2000.

[3] Erik D. Demaine, Robert A. Hearn, and Michael Hoffmann. Push-2-f is pspace-complete. In *Proceedings of the 14th Canadian Conference on Computational Geometry (CCCG 2002)*, pages 31–35, Lethbridge, Alberta, Canada, August 12–14 2002.

[4] Erik D. Demaine and Michael Hoffmann. Pushing blocks is NP-complete for noncrossing solution paths. In *Proceedings of the 13th Canadian Conference on Computational Geometry (CCCG 2001)*, pages 65–68, Waterloo, Ontario, Canada, August 13–15 2001.

[5] Erik D. Demaine, Michael Hoffmann, and Markus Holzer. Pushpush-k is pspace-complete. In *Proceedings of the 3rd International Conference on Fun with Algorithms (FUN 2004)*, pages 159–170, Isola d'Elba, Italy, May 26–28 2004.

[6] A. Dhagat and J. ORourke. Motion planning amidst movable square blocks. In *Proceedings of the 4th Canadian Conference on Computational Geometry (CCCG 1992)*, 1992.

[7] D. Dor and U. Zwick. Sokoban and other motion planning problems. *Computational Geometry*, 13(4), 1996.

[8] M. Hoffman. Push-* is np-hard. In *Proceedings of the 12th Canadian Conference on Computational Geometry (CCCG 2000)*, Lethbridge, Alberta, Canada, 2000.

[9] Marcus Ritt. Motion planning with pull moves. *CoRR*, abs/1008.2952, 2010.

[10] Gordon Wilfong. Motion planning in the presence of movable obstacles. *Annals of Mathematics and Artificial Intelligence*, 3(1):131–150, 1991.