Lab class 10: Final Review

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1 Variability of rain

Jory is a farmer, and it's important for them to estimate how much rain will fall in the next month, which is 30 days long.

The rainfall is concentrated into storms. These storms arrive according to a Poisson Process with a random rate. Some months are chaotic, and the arrival rate of storms is $\frac{1}{2}$ storms per day. Some months are more peaceful, and the arrival rate is $\frac{1}{4}$ per day. Jory estimates that there's a $\frac{1}{3}$ chance that the next month will be chaotic, and a $\frac{2}{3}$ chance that it will be peaceful. Call the number of storms in the next month N.

a. What is the mean number of storms in the next month, $E[N]$?

b. What is the mean squared number of storms in the next month, $E[N^2]$?
Note: If $X \sim Poisson(\lambda)$, $E[X^2] = \lambda^2 + \lambda$.

c. What is the variance of the number of storms in the next month, $Var(N)$?

2 Wandering

Sam is on a hike in the woods, and he's a bit lost. There's four possible locations he could be: In the woods on the north side, in the woods on the south side, exited to the north, and exited to the south. On each time step, if he's in the woods, he walks north with probability 1/3, stays where he is with probability 1/3, and walks south with probability 1/3, independent of previous steps. If he's out of the woods, he stays where he is.

Let's model this situation as a DTMC.

a. Draw the transition diagram for this DTMC.

b. What is the transition matrix for this DTMC?

c. In this DTMC, which states are transient, which states are absorbing, and which states are neither?

d. Suppose that Sam is initially in the woods on the north side. What are the distributions of Sam's position after 1, 2, and 3 time steps?

e. Does this DTMC has no stationary distribution, a unique stationary distribution, or many stationary distributions?

3 Running

Alex is out for a run. Their run takes them through several regions near where they live. They start in region A, then go to regions B, C, D, and E, before they go back to region A. Region A is their favorite, so they stay there the longest, $Exp(1/10)$ minutes. Region B is their next favorite, where they stay for $Exp(2/10)$ minutes. Then region C, $Exp(3/10)$ minutes, region D, $Exp(4/10)$ minutes, and region E, $Exp(5/10)$ minutes.

Model Alex's run as a CTMC.

1. Draw the transition diagram for this CTMC.

2. How long, on average, is each cycle of the Alex's run, from starting in region A to next returning to region A?

3. In the long run, what fraction of time does Alex spend in each region?