Lab class 3: Conditioning and DTMCs

Izzy Grosof

October 11, 2024

1 Random distributions with random rates

Suppose you have a coin. Let H be the indicator random variable for whether it lands on heads. 1 if heads, 0 if tails. The coin is not necessarily a fair coin – it flips heads with probability X. You're not sure which way it's biased – X is either $\{0, 1/4, 1/2, 3/4, 1\}$, with equal probability.

a. What's the conditional probability distribution $P(H = 1 \mid X = x)$, for each possible value of X?

b. What's the joint pmf $P(H = h, X = x)$, for all pairs (h, x), $h \in \{0, 1\}, x \in \{0, 1/4, 1/2, 3/4, 1\}$?

c. What's the chance of an unlikely outcome – $H = 1$ and $X < 1/2$, or $H = 0$ and $X > 1/2$?

2   Modeling as a Discrete-Time Markov Chain

You're running a manufacturing facility with two locations, A and B. You have 2 trucks that you use to carry parts and supplies between these two locations. Each hour, a call will come in. It might request parts to be taken from A to B, or from B to A. There's 1/2 probability of each, and it's independent of past calls.

If you have a truck at the starting point, you send the truck, with the parts, to the other location. The truck parks at its destination and waits for the next hour.

If you don't have a truck at the starting point, one of the trucks from the other location comes over and then goes back. The truck has to do a round trip. It's annoying, you don't like it when that happens.

All trucks are either at location A or location B, whenever the hour rolls over and a new call comes in.

We're going to model this scenario as a Discrete-Time Markov chain.

a. What's the state of the system? In other words, where can the trucks be when a call comes in each hour? How many states are there?

b. What are the transition probabilities? Draw the Markov chain two ways: Transition diagram (circles and arrows) and Transition matrix (grid of probabilities)

c. Suppose at time 0, just before the first call comes in, we're in the state where there's one truck in each location. What's the probability of being in each state at time 1?

d. What's the chance that on time step 1, a truck has to do a round trip?