

Lab class 9: Queues and Little's Law

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1 Cafeteria M/M/2

You're in a cafeteria. There's two checkout stations, and a shared line for both of them. People arrive to the line according to a Poisson process with rate 1 person per minute, and take $Exp(3)$ minutes to check out (an average of $E[Exp(3)] = 1/3$ minute = 20 seconds).

This is an M/M/2 queue, and we'll model it as a CTMC, where the state is the number of people present, in the queue or at a checkout station.

1. Draw the transition diagram for this CTMC.
2. Using the Birth-Death rule $\alpha_i Q_{i,i+1} = \alpha_{i+1} Q_{i+1,i}$ for the stationary distribution α , find the relationship between α_i and α_{i+1} .
3. Find the relationship between α_i and α_0 . Start by relating α_1 and α_0 , then α_2 and α_0 , then generalize.

4. Using the fact that $\sum_{i=0}^{\infty} \alpha_i = 1$, find α_0 .
5. What is the stationary distribution α ?
6. Suppose you arrive and see a given number of people n in the system?.What is your mean time in queue $E[T_Q | N = n]$? What is your mean response time $E[T | N = n]$?
7. Applying PASTA (Poisson Arrivals See Time Averages), what is $E[T]$ in terms of $E[N]$?
8. Applying Little's Law, solve for $E[T]$ and $E[N]$.
9. If you have extra time, replace 1 person/minute with λ people/minute and $Exp(3)$ with $Exp(\mu)$, and solve for the general result.