Predicate Logic

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Predicate Logic

- Often we need to reason about statements of the form:
  - *everything has the property p*
  - *something has the property p*

- However, propositional logic is not expressive enough to support such reasoning

- Predicate logic, an extension to propositional logic, adds *quantifiers* to allow this type of reasoning.
  - If also includes all the definitions, inference rules, theorems, algebraic laws, etc.

Outline

- Language of Predicate Logic
- Translating English to Logic
- Computing with Quantifiers
- Inference with Predicates
- Algebraic Laws of Predicate Logic
- Proof Concepts

Language of Predicate Logic

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Predicates

- **Predicate:** A statement that an object *x* has a certain property
  - such statements may be either `True` or `False`
  - **Example:** `x > 5`
- Predicates may extend over multiple variables
  - **Example:** `x > y`
- Conditional expressions in programs are a form of predicate
- Predicates are typically written in the concise form $F(x)$
  - F is the predicate, x is the variable
  - $G(x, y)$ is a multivariate predicate
  - Can be thought of as a function that returns a Boolean

Predicates

- Any term in the form $F(x)$, where F is the predicate name and x is a variable name, is a WFF
  - $F(x_1, x_2, \ldots, x_k)$ is a WFF that is a predicate with k variables

- **Universe of Discourse (U)** - also called *universe*, is the set of possible values that the variables can have
  - Typically specified once, at the start of a piece of reasoning

- In predicate logic, the following standards apply
  - Universe is called U
  - Constants are lowercase letters (typically c and p)
  - Variables are lowercase letters (typically x, y, z)
  - Predicates are uppercase letters, i.e., $F(x)$
  - Generic predicates start with a lowercase letter, i.e., $f(x)$

Quantifiers

- **Universal Quantification ($\forall$):** If $F(x)$ is a WFF containing the variable x, then $\forall x.\ F(x)$ is a WFF
  - This is a statement that *everything* in the universe has a certain property
  - Says: "For all x in the universe, the predicate $F(x)$ holds"
  - Used to state required properties
- **Existential Quantification ($\exists$):** If $F(x)$ is a WFF containing the variable x, then $\exists x.\ F(x)$ is a WFF
  - This is a statement that *something* in the universe has a certain property
  - Says: "Some x has the property F"
  - Used to state a property must occur at least once
- **Note:** we can also nest quantifiers: $\forall x.\ \exists y.\ x < y$

Quantifiers

- Let U be the set of even numbers. Let $E(x)$ mean x is even. Then, $\forall x.\ E(x)$ is a WFF that is true

- Let U be the set of natural numbers. Let $E(x)$ mean x is even, then $\forall x.\ E(x)$ is a WFF that is false

- Let U be the set of natural numbers and $F(x, y)$ be $2x = y$, then
  - $\exists x.\ F(x, 6)$ is a WFF and is True
  - $\exists x.\ F(x, 7)$ is a WFF and is False

Expanding Quantified Expressions

- If we have a finite universe
  - Quantified expressions can be interpreted as ordinary terms in propositional logic
  - The quantifiers simply act as syntactic abbreviations

  Suppose: $U = \{c_1, c_2, \ldots, c_n\}$ (of size n), then

$$\forall x.\ F(x) \quad = \quad F(c_1) \wedge F(c_2) \wedge \ldots \wedge F(c_n) \quad = \quad \bigwedge_{i=1}^{n} F(c_i)$$

$$\exists x.\ F(x) \quad = \quad F(c_1) \vee F(c_2) \vee \ldots \vee F(c_n) \quad = \quad \bigvee_{i=1}^{n} F(c_i)$$

  - Quantifiers make reasoning practical

- If we have an infinite universe
  - It is impossible to expand quantifiers

- All WFFs must have a finite size (even if the universe itself is infinite)

Expanding Quantified Expressions

- Let $U = \{1, 2, 3\}$ with the following predicates

$$\begin{aligned} \textit{even } x &= (x \ \text{ mod } 2 = 0) \\ \textit{odd } x &= (x \ \text{ mod } 2 = 1) \end{aligned}$$

Universal Expansion

$$\begin{aligned} &\forall x.\ (\textit{even } x \to \neg(\textit{odd } x)) \\ &= (\textit{even } 1 \to \neg(\textit{odd } 1)) \land (\textit{even } 2 \to \neg(\textit{odd } 2)) \land (\textit{even } 3 \to \neg(\textit{odd } 3)) \\ &= (\text{False} \to \neg\text{True}) \land (\text{True} \to \neg\text{False}) \land (\text{False} \to \neg\text{True}) \\ &= \text{True} \land \text{True} \land \text{True} \\ &= \text{True} \end{aligned}$$

Existential Expansion

$$\begin{aligned} &\exists x.\ (\textit{even } x \land \textit{odd } x) \\ &= (\textit{even } 1 \land \textit{odd } 1) \lor (\textit{even } 2 \land \textit{odd } 2) \lor (\textit{even } 3 \land \textit{odd } 3) \\ &= (\text{False} \land \text{True}) \lor (\text{True} \land \text{False}) \lor (\text{False} \land \text{True}) \\ &= \text{False} \lor \text{False} \lor \text{False} \\ &= \text{False} \end{aligned}$$

Exercises

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- Let $U = \{1, 2, 3\}$, expand the following expression into propositional term
  - $\forall x.\ F(x)$
  - $\exists x.\ \forall y.\ G(x, y)$
- Expand the following expression
  $\forall x \in \{1, 2, 3, 4\}.\ \exists y \in \{5, 6\}.\ F(x, y)$

Exercises

- Let $U = \{1, 2, 3\}$, expand the following expression into propositional term
  - $\forall x.\ F(x)$

    $$\begin{aligned} &\forall x.\ F(x) \\ &= F(1) \land F(2) \land F(3) \end{aligned}$$
  - $\exists x.\ \forall y.\ G(x, y)$

    $$\begin{aligned} &\exists x.\ G(x, y) \\ &= G(1,1) \lor G(1,2) \lor G(1,3) \lor G(2,1) \lor G(2,2) \lor G(2,3) \lor G(3,1) \lor G(3,2) \lor G(3,3) \end{aligned}$$

- Expand the following expression
  $\forall x \in \{1, 2, 3, 4\}.\ \exists y \in \{5, 6\}.\ F(x, y)$

  $$\begin{aligned} &\forall x.\ \exists y.\ F(x, y) \\ &= (F(1,5) \lor F(1,6)) \land (F(2,5) \lor F(2,6)) \land (F(3,5) \lor F(3,6)) \land (F(4,5) \lor F(4,6)) \end{aligned}$$

Variable Bindings

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- **Binding:** Quantifiers *bind* variables by assigning them values from a universe
  - **Dangling expressions:** expressions without explicit quantification have no explicit binding: $x + 2$
- **Scope:** Extent of a binding
  - Convention: Quantifiers extend over the smallest sub-expression possible. So use parentheses to make meaning clear.
- **Example**: $\forall x, y.\ (p(x) \rightarrow q(x, y)) \land r(z)$
  - x, y are bound
  - z is free
  - All are in the scope of $\forall x, y$ except $r(z)$
  - $r(z)$ is a dangling expression

Translating English to Logic

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English $\rightarrow$ Logic

- When an English statement has no internal structure relevant to reasoning, we can use simple propositional variables:

$$\begin{aligned} A &\equiv \text{Elephants are big.} \\ B &\equiv \text{Cats are furry.} \\ C &\equiv \text{Cats are good pets.} \end{aligned}$$

- We can combine statements using the operators corresponding to connective words.

$$\begin{aligned} \neg A &\equiv \text{Elephants are small.} \\ A \vee B &\equiv \text{Elephants are big and cats are furry.} \\ B \rightarrow C &\equiv \text{If cats are furry then they make good pets.} \end{aligned}$$

English $\rightarrow$ Logic

- However, when general statements are made about classes of objects, then predicates and quantifiers are needed

- For example, these statements are difficult to model with propositional logic:

$$\begin{aligned} A &\equiv \text{Small animals are good pets.} \\ C &\equiv \text{Cats are animals.} \\ S &\equiv \text{Cats are small.} \end{aligned}$$

- This leads to weak conclusions such as:
  - $A \wedge C$
  - $S \vee A$

English $\rightarrow$ Logic

- However, we can model the internal structure of these statements using predicates:

$$\begin{aligned} A(x) &\equiv x \text{ is an animal.} \\ C(x) &\equiv x \text{ is a cat.} \\ S(x) &\equiv x \text{ is small.} \\ GP(x) &\equiv x \text{ is a good pet.} \end{aligned}$$

- Using these we can produce the following richer translations of the english statements:

$$\begin{aligned} \forall x.\ C(x) \rightarrow A(x) &\equiv \text{Cats are animals.} \\ \forall x.\ C(x) \rightarrow S(x) &\equiv \text{Cats are small.} \\ \forall x.\ C(x) \rightarrow S(x) \wedge A(x) &\equiv \text{Cats are small animals.} \\ \forall x.\ S(x) \wedge A(x) \rightarrow GP(x) &\equiv \text{Small animals are good pets.} \end{aligned}$$

- Often the difficulty in translating English to Logic is found in determining what the original speaker meant to say.

Computing with Quantifiers

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Computing w/ Quantifiers

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- If the universe is finite, we can utilize computers to evaluate quantified expressions

- In the context of Haskell,
  - A *predicate* is a function returning a Boolean value
  - The `forall` function
    - takes in an `Int` list, representing the universe, and a predicate
    - it applies the predicate to each item in the universe and returns the results

```
forall :: [Int] -> (Int -> Bool) -> Bool
```

    - uses the `and` function
  - Similarly, the `exists` function
    - applies the `or` function

```
exists :: [Int] -> (Int -> Bool) -> Bool
```

- Both `forall` and `exists` are defined in the `Stdm` module

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Exercises

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- Write the predicate logic expressions corresponding to the following Haskell expressions. Then decide whether the value is `True` or `False`, and finally evaluate each using a computer.

```
forall [1, 2, 3] (==2)
```

```
exists [1, 2, 3] (<= 5)
```


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Exercises

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- Write the predicate logic expressions corresponding to the following Haskell expressions. Then decide whether the value is `True` or `False`, and finally evaluate each using a computer.

```
forall [1, 2, 3] (==2)
```

Solution:

$\forall x \in [1, 2, 3].\ x = 2$

`False`

```
exists [1, 2, 3] (<= 5)
```

Solution:

$\exists x \in [1, 2, 3].\ x \leq 5$

`True`

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Inference with Predicates

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Inference w/ Predicates

- Inference rules can be extended to predicate logic
- An additional four rules are required for dealing with quantifiers
- As noted before, if the universe if finite quantification acts as propositions
- However, if the universe is infinite, the inference rules of predicate logic allow deductions that are otherwise impossible with propositional logic.

Universal Introduction $\{\forall I\}$

Rule:

$$\frac{F(x) \qquad \{\text{x arbitrary}\}}{\forall x.\ F(x)}\{\forall I\}$$

Meaning:

- If the expression a (which may contain a variable x) can be proved for an *arbitrary x*, then we may infer the proposition $\forall x.\ a$

Example: $\vdash \forall x.\ E(x) \to (E(x) \lor \neg E(x))$

$$\cfrac{\cfrac{\cfrac{\cancel{E(p)}}{E(p) \lor \neg E(p)}\{\lor I_L\}}{E(p) \to E(p) \lor \neg E(p)}\{\to I\}}{\forall x.\ E(x) \to E(x) \lor \neg E(x)}\{\forall I\}$$

Universal Elimination $\{\forall E\}$

Rule:

$$\frac{\forall x.\ F(x)}{F(p)}\ \{\forall E\}$$

Meaning:

- If we have established $\forall x.\ F(x)$ and p is an element of the universe, then you can infer $F(p)$

Example: $\forall x.\ F(x) \to G(x) \vdash G(p)$

$$\frac{F(p) \qquad \dfrac{\forall x.\ F(x) \to G(x)}{F(p) \to G(p)}\ \{\forall E\}}{G(p)}\ \{\to E\}$$

Exercises

- Prove: $\forall x.\ F(x),\ \forall x.\ F(x) \rightarrow G(x) \vdash \forall x.\ G(x)$

Exercises

- Prove: $\forall x.\ F(x),\ \forall x.\ F(x) \rightarrow G(x) \vdash \forall x.\ G(x)$

Solution:

$$\dfrac{\dfrac{\dfrac{\forall x.\ F(x)}{F(p)}\{\forall E\} \qquad \dfrac{\forall x.\ F(x) \rightarrow G(x)}{F(p) \rightarrow G(p)}\{\forall E\}}{G(p)}\{\rightarrow E\} \qquad \{\text{arbitrary p}\}}{\forall x.\ G(x)}\{\forall I\}$$

Existential Introduction $\{\exists I\}$

Rule:

$$\frac{f(p)}{\exists x.\ f(x)}\ \{\exists I\}$$

Meaning:

- If $f(p)$ has been established for a particular p, then you can infer $\exists x.\ f(x)$

Example: $\forall x.\ F(x) \vdash \exists x.\ F(x)$

$$\cfrac{\cfrac{\forall x.\ F(x)}{F(p)}\ \{\forall E\}}{\exists x.\ F(x)}\ \{\exists I\}$$

Existential Elimination $\{\exists E\}$

Rule:

$$\cfrac{\exists x.\ F(x) \qquad F(x) \vdash A \qquad \{x \text{ arbitrary}\}}{A}\ \{\exists E\}$$

Meaning:

- If we know $\exists x.\ F(x)$ holds for some x, and furthermore that A must hold if $F(x)$ holds for *arbitrary x*, then A can be inferred.

Example: $\exists x.\ P(x),\ \forall x.\ P(x) \to Q(x) \vdash \exists x.\ Q(x)$

$$\cfrac{\cfrac{\exists x.\ P(x) \qquad \cfrac{\cancel{P(c)} \qquad \cfrac{\forall x.\ P(x) \to Q(x)}{P(c) \to Q(c)}\ \{\forall E\}}{Q(c)}\ \{\to E\}}{Q(c)}\ \{\exists E\}}{\exists x.\ Q(x)}\ \{\exists I\}$$

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Exercises

- Prove: $\exists x.\ \exists y.\ F(x, y) \vdash \exists y.\ \exists x.\ F(x, y)$

Exercises

- Prove: $\exists x.\ \exists y.\ F(x,y) \vdash \exists y.\ \exists x.\ F(x,y)$

  Solution:

$$\dfrac{\exists x.\ \exists y.\ F(x,y) \qquad \dfrac{\dfrac{\dfrac{\cancel{\exists y.\ F(p,y)}}{F(p,q)}\{\exists E\}}{\exists x.\ F(x,q)}\{\exists I\}}{\exists y.\ \exists x.\ F(x,y)}\{\exists I\}}{\exists y.\ \exists x.\ F(x,y)}\{\exists E\}$$

Algebraic Laws of Predicate Logic

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Algebraic Laws

- Just as with propositional logic, we have an alternate style of reasoning based on a set of algebraic laws
  - All previous propositional laws apply, as well as new ones we will discuss
- Not a minimal, nor a complete set of laws
  - Some correspond to inference rules
  - Some are provable as theorems
- Here, we focus on practical use, rather than theoretical foundations

Algebraic Laws

- These laws are related to rules of inference

Laws:

$$
\begin{aligned}
\forall x.\ f(x) &\rightarrow f(c) && (7.3)\\
f(c) &\rightarrow \exists x.\ f(x) && (7.4)
\end{aligned}
$$

Where:

- x is bound by the quantifier
- c is a fixed element in the universe

Example:

- Prove: $\forall x.\ f(x) \rightarrow \exists x.\ f(x)$

$$
\begin{aligned}
\forall x.\quad & f(x) & \\
& \rightarrow f(c) & \{7.3\}\\
& \rightarrow \exists x.\ f(x) & \{7.4\}
\end{aligned}
$$

Algebraic Laws

- These laws focus on the effects of negation on quantifiers

Laws:

$$
\begin{aligned}
\forall x.\ \neg f(x) &= \neg\exists x.\ f(x) \quad (7.5) \\
\exists x.\ \neg f(x) &= \neg\forall x.\ f(x) \quad (7.6)
\end{aligned}
$$

Algebraic Laws

- These laws are concerned with how a predicate $f(x)$ combines with a proposition q that does not contain x

Laws:

$$
\begin{aligned}
(\forall x.\ f(x)) \wedge q &= \forall x.\ (f(x) \wedge q) && (7.7)\\
(\forall x.\ f(x)) \vee q &= \forall x.\ (f(x) \vee q) && (7.8)\\
(\exists x.\ f(x)) \wedge q &= \exists x.\ (f(x) \wedge q) && (7.9)\\
(\exists x.\ f(x)) \vee q &= \exists x.\ (f(x) \vee q) && (7.10)
\end{aligned}
$$

Algebraic Laws

- These laws concern the combination of quantifiers with $\wedge$ and $\vee$

Laws:

$$
\begin{aligned}
\forall x.\ f(x) \wedge \forall x.\ g(x) &= \forall x.\ (f(x) \wedge g(x)) && (7.11)\\
\forall x.\ f(x) \vee \forall x.\ g(x) &\rightarrow \forall x.\ (f(x) \vee g(x)) && (7.12)\\
\exists x.\ (f(x) \wedge g(x)) &\rightarrow \exists x.\ f(x) \wedge \exists x.\ g(x) && (7.13)\\
\exists x.\ f(x) \vee \exists x.\ g(x) &= \exists x.\ (f(x) \vee g(x)) && (7.14)
\end{aligned}
$$

Examples

- Prove: $\forall x.\ (f(x) \land \neg g(x)) = \forall x.\ f(x) \land \neg \exists x.\ g(x)$

$$
\begin{aligned}
& \forall x. && (f(x) \land \neg g(x)) & \\
& = && \forall x.\ f(x) \land \forall x.\ \neg g(x) & \{7.11\} \\
& = && \forall x.\ f(x) \land \neg \exists x.\ g(x) & \{7.5\}
\end{aligned}
$$

- Prove: $\exists x.\ (f(x) \rightarrow g(x)) \land (\forall x.\ f(x)) \rightarrow \exists x.\ g(x)$

$$
\begin{aligned}
& \exists x. && (f(x) \rightarrow g(x)) \land (\forall x.\ f(x)) & \\
& = && (\exists x.\ (f(x) \rightarrow g(x))) \land (\forall y.\ f(y)) & \text{change of var} \\
& = && \exists x.\ ((f(x) \rightarrow g(x)) \land (\forall y.\ f(y))) & \{7.9\} \\
& = && \exists x.\ ((f(x) \rightarrow g(x)) \land f(x)) & \{7.3\} \\
& \rightarrow && \exists x.\ g(x) & \{ModusPonens\}
\end{aligned}
$$

Mathematical Arguments

- Often include steps where both a rule of inference for propositions and a rule of inference for quantifiers are used.

- For example, universal instantiation and *modus ponens* are often used together

- When these rules of inference are combined, for example:
  - $\forall x.\ (P(X) \to Q(x))$
  - $P(x)$ // where c is a member of the universe of discourse
  - $Q(x)$

Proof Concepts

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Methods of Proving Theorems

- Direct Proof
- Indirect Proofs
  - Proof by contraposition
  - Vacuous and trivial proofs
  - Proof by contradiction

Direct Proofs

- **Example:** Prove that "if n is an odd integer, then n^2 is an odd integer"
  - **Proof:**
    n is odd $\rightarrow n = 2k + 1$
    $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2k(2k+2) + 1$ which is odd

- **Example:** Prove that the sum of two rational numbers is rational
  - rational number $= p/q \ (q \neq 0)$
  - **Proof:**

$$\begin{aligned} r &= \tfrac{p}{q} \ (q \neq 0) \\ s &= \tfrac{t}{u} \ (u \neq 0) \\ r + s &= \tfrac{p}{q} + \tfrac{t}{u} = \tfrac{pu+tq}{qu} \end{aligned}$$

    which is rational.

Indirect Proofs

- Proof by contraposition
- Vacuous and Trivial Proofs
- Proof by contradiction

Proof by Contraposition

- Based on the idea: $p \to q \equiv \neg q \to \neg p$
- Makes use of the fact that the conditional statement $p \to q$ is equivalent to $\neg q \to \neg p$
- The first step is to take $\neg q$ as a hypothesis and then using axioms, statements we assume to be true, definitions, and previously proven theorems together with rules of inference, we show that $\neg p$ must follow.

- **Example:** prove that "if $3n + 2$ is odd, then n is odd"
  - **Proof:** Suppose n is even. Then $n = 2k$
    $3n + 2 = 3(2k) + 2 = 6k + 2 = 2(3k + 1)$ which is even
    $\therefore$ by contraposition if, $3n + 2$ is odd, then n is odd.

- **Example:** prove that if n^2 is odd, then n is odd
  - **Proof:** Suppose n is even. Then $n = 2k$
    $n^2 = (2k)^2 = 4k^2 = 2(2k)$ which is even

Vacuous and Trivial Proofs

- **Vacuous Proofs:** if we can show that q is false, then, $p \rightarrow q$ will always be true.
- **Trivial Proofs:** we can quickly prove $p \rightarrow q$ if we know q is true.

Proof by Contradiction

- Suppose we want to show that a statement p is true
- Suppose we can find a contradiction q such that $\neg p \to q$ is true
- Because q is false, but $\neg p \to q$ is true, we can conclude $\neg p$ is false and therefore p is true
- How to find the contradiction q to help us in this way:
  - Because the statement $r \wedge \neg r$ is a contradiction if r is a proposition, we can prove that p is true if we can show that $\neg p \to (r \wedge \neg r)$ is true for some proposition r

- **Example:** Prove that $\sqrt{2}$ is irrational
  - **Proof:** Suppose that $\sqrt{2}$ is rational
    Then: $\sqrt{2} = \frac{a}{b}$ where a and b have no common factor
    square both sides: $2 = \frac{a^2}{b^2}$
    $2b^2 = a^2 \to a$ is even
    $a = 2c$
    $\therefore 2b^2 = 4c^2$
    $b^2 = 2c^2 \to b$ is even
    $\therefore$ The assumption that a and b have no common factor is false so there is a contraction
    $\to \sqrt{2}$ is irrational.

Proof Methods and Strategy

- Exhaustive proofs
- Proof-by-cases
- Existence Proofs
- Uniqueness Proofs

Exhaustive Proof and Proof-by-cases

- There are times when we cannot prove a theorem using a single argument that holds for all cases.

- By considering different cases separately we can prove a theorem.

- This is based on the following rule of inference:
  $(p_1 \lor p_2 \lor p_3 \lor \ldots \lor p_n) \rightarrow q$

  The tautology: $[(p_1 \lor p_2 \lor \ldots \lor p_n) \rightarrow q] \leftrightarrow [(p_1 \rightarrow q) \land (p_2 \rightarrow q) \land \ldots \land (p_n \rightarrow q)]$

Exhaustive Proof

- Can be proved by examining a relatively small number of examples.

- Called exhaustive proof, since these proofs proceed by exhausting all possibilities

- It is a special case of proof-by-cases where each case involves checking a single example

Proof-by-Cases

- Must cover all possible cases that arise in a theorem
- Generally, we look for a proof-by-cases when there is no obvious way to begin a proof
- **Without Loss of Generality (WLOG)**
  - Used in a proof, we assert that by proving one case of a theorem, no additional argument is required to prove other specified cases

Existence Proofs

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- Many theorems are assertions that objects of a particular type exists
- Such a theorem is a proposition of the form $\exists x.\ P(x)$, where P is a predicate
- Proving this proposition is called an **existence proof**
- An existence proof of the form $\exists x.\ P(x)$ can be given by finding an element, a, such that $P(a)$ is true. This type of existence proof is called a **constructive proof**.
- It is also possible to give a **non-constructive proof**.
  - That is we do not find an element a such that $P(a)$ is true, but rather prove that $\exists x.\ P(x)$ is true in some other way.

Uniqueness Proofs

- Some theorems assert the existence of a unique element with a particular property (one element with this property)

- To prove this we need to show that an element with this property exists and that no other element has this property.

- The two parts of a **uniqueness proof** are:
  - **Existence:** we show that an element x with the desired property exists.
  - **Uniqueness:** we show that if $y \neq x$, then y does not have the desired property.

- Equivalently we can show that if x and y both have the desired property, then $x = y$

- This is the equivalent (proving the uniqueness proof) of proving the statement: $\exists x.\ (P(x) \land \forall y.\ (y \neq x \to \neg P(y)))$

For Next Time

- Review DMUC Chapter 7
- Review this Lecture
- Read DMUC Chapter 8
- Come To Lecture

Are there any questions?