



INTRODUCTION AND HASKELL I

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Outcomes



After today's lecture you will:

- Introduce the course and review the syllabus
- Learn the basics of Haskell
 - Running Haskell Scripts and using the GHCi
 - Haskell Types and Expressions
 - Basic Haskell Data Structures
 - Lists and List Comprehensions
 - Tuples
 - Functions



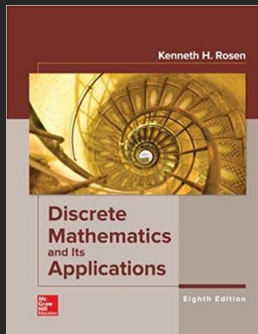
Syllabus Review

CS 1187

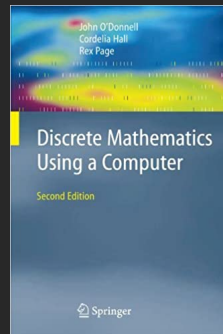
Course Introduction

CS 1187

What is Discrete Structures/Mathematics?



Discrete Mathematics
and Its Applications, 8th Edition
Rosen 2019



Discrete Mathematics Using
a Computer, 2nd Edition
O'Donnell, Hall and Page (2006)



- **Goals of this course:**

- To understand and be able to apply the following concepts of discrete mathematics to computing
 - Logic
 - Set Theory
 - Relations and Functions
 - Counting
 - Number Systems and Modular Arithmetic
 - Recursion and Induction
 - Graphs and Trees
- To be able to use symbolic mathematics, logical laws, and logical inference in mathematical reasoning
- Assess mathematical and logical arguments for validity, construct mathematical arguments and simple proofs, and apply definitions and theorems



- In considering this topic:
 - I wanted to better connect the goals of the previous slide to computing
 - I thought: “What better way than through programming itself?”
 - Additionally: “What if the language was a paradigm the students haven’t seen and one which is very close to the math itself?”
- Towards this, I opted to select the Haskell language, and then found the course book which already put all of this together.
- This adds four additional goals for this course, provide by Haskell:
 - Be able to understand and use the functional programming paradigm.
 - Be able to use the computer to help you to learn and understand mathematics.
 - Be able to use software tools to make it possible to use the mathematics more effectively.
 - Understand the widespread application of mathematics to computing.



- Why a functional language? Why Haskell?
 - Allows you to compute directly with fundamental objects of discrete mathematics
 - Haskell is powerful yet expressed simply
 - We reason about Haskell programs in the same way we reason about mathematics
 - Haskell is excellent for *rapid prototyping*
 - Haskell is stable, standard, and well-documented
 - Haskell implementations are free and available on most OSs
 - Haskell can be used interactively

- What is a **REPL**?
 - **Read** - reads in code from command prompt
 - **Evaluate** - evaluates the code read in
 - **Print** - prints the result of the evaluation to the terminal
 - **Loop** - loops on REP until the user exits
- Several REPLs exist for Haskell
 - **GHCI**
 - Hugs
 - nhc



- To start haskell interactively (with GHCi), do the following (assuming you have installed Haskell):
 1. Open a terminal (Linux/MacOS) or Command Prompt (Windows)
 2. At the prompt, execute the following command (Note the \$ is the prompt not a part of the command)

```
$ ghci
```

3. This should start the interactive haskell system. Additionally it should give you an intro message followed by a prompt, for example:

```
GHCi, version 8.8.4: https://www.haskell.org/ghc/  :? for help
Prelude>
```

where `Prelude>` is the prompt.

4. Personally, I don't like the `Prelude>` prompt, so I set it to `ghci>` using the following command:

```
:set prompt "ghci> "
```

- As noted previously, this prompt is the REPL
 - We can type in *expressions* and see their results (similar to a calculator)
- For example:

```
ghci> 1 + 2
3
ghci> 3 * 4
12
ghci>
```

- When working with haskell, we often need to use definitions stored in scripts
 - Haskell scripts end with the **.hs** extension
 - To load a file while in GHCi, we use the load file command:

```
:load <module> or :l <module>
```

Where <module> is the name of the file to load (**without the .hs extension**).

- Throughout this course, we will be using the book's `stdm.hs` file, so you should download it
 - Additionally, I would suggest creating a folder for your Haskell work
 - In this folder should be the `stdm.hs` file
 - This folder is the working directory where you should be prior to starting your Haskell interactive sessions

- Along with loading prior definitions created by others, we will also want to create our own code.
- Using a text editor, create a new file `mydefs.hs` and add the following to it:

```
y = x + 1  
x = 2 * 3
```

- Save the file to your Haskell working folder. Then load it as follows:

```
ghci> :l mydefs  
[1 of 1] Compiling Main                ( mydefs.hs, interpreted )  
Ok, one module loaded.  
ghci>
```

- After loading `mydefs.hs` both `x` and `y` in the file are definitions, which are now available to us
 - So we could do something like:

```
ghci> x
6
ghci> y
7
ghci> x * y
42
ghci>
```

- If we make any changes to the file
 - then we would need to reload it for those changes to take affect in GHCi
 - using the `:reload` or `:r` command, or
 - load again with the `:load` or `:l` command



- All GHCi commands start with ":" which tells the system that we are executing a command for the environment not entering an expression
- The commands we will be using the most are:
 - `:load <module>` or `:l <module>` which loads the specified module (or file in our case)
 - `:reload` or `:r` which reloads the current module set
 - `:type <expr>` or `:t <expr>` which identifies the type of the given expression
 - `:cd <dir>` change the current working directory to the one specified
 - `:set prompt <prompt>` which sets the prompt to the given string
 - `:quit` or `Ctrl+D` (on some systems) which exits GHCi

- Haskell has an apparent lack of structure
 - No extra punctuation (colons, semicolons, braces, begin, end, etc.)
 - Instead, structure is controlled by line endings and indentation
 - Haskell will figure the rest out for us

- Comments

- **line comments** start with `--` and everything after it is ignored

```
x = 2 + 2 -- the result should be 4
```

- **multiline comments**: `{- text -}` where the `text` is the comment, and anything between `{-` and `-}` will be ignored

```
{-  
This is a multiline comment  
-}  
x = 2 + 2
```


§ Haskell Expressions

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- We can do a lot with just expressions in Haskell
- The following discusses useful kinds of expressions organized according to value **type**
 - Integer and Int expressions
 - Rational and Floating Number Expressions
 - Boolean
 - Character and String Expressions

- Integer constants are simply a sequence of digits:
2, 0, 12345, -72
- **Operators:**
 - Addition (+) : $4 + 3$
 - Subtraction (-) : $4 - 3$
 - Multiplication (*) : $2 * 3$
 - Exponentiation (^) : 2^3
 - Division ``div`` : $4 \text{ `div` } 2$ (note the backticks)

- Haskell has two Integer types
 - `Int` - a whole number whose maximum size fits within a *word* in memory (i.e., 64 bits on a 64bit machine)
 - `Integer` - a type representing mathematical integers
 - Using `Integer` type allows arithmetic operations to satisfy algebraic laws
- The *has type* operator `(::)`
 - Can be used to force the type specification rather than allow Haskell to infer the type
 - Example: `2::Int` or `2::Integer`

- Types of Floating Point numbers:
 - Float - single precision
 - Double - double precision
- Operators:
 - Addition (+)
 - Subtraction (-)
 - Multiplication (*)
 - Division (/)
 - Exponentiation (**) : `4.0 ** 2`

Floats are Approximations



- Floating points are approximations
 - Cannot guarantee satisfaction of algebraic laws
 - Cannot directly compare two floating point numbers for equality
- **Procedure to compare Floating Point numbers:**
 - for two floating point numbers x and y
 - compare the absolute value of the difference: $|x - y|$ to some small error tolerance
 - Mathematically: $|x - y| < 0.001$
 - In Haskell:

```
isEqual :: Float -> Float -> Bool  
isEqual x y = (abs (x - y)) < 0.001
```



- To get around the limitations of Floating Point arithmetic, Haskell also supports exact arithmetic on rational numbers
- To use the exact form we must work with fractions in which both the numerator and denominator are integers
- The type of these numbers is `Fractional` and is written as `num/denom`, for example:
 - $2/3$
 - $2/3 + 1/6 \rightarrow 5/6$
- **Note:** Haskell will automatically reduce fractions

- Booleans are represented by the `Bool` type which can be one of two values (note the capitalization)
 - `True`
 - `False`

Comparing Two Numbers

- `==` -- equality
- `/=` -- not equal
- `<` -- less than
- `<=` -- less than or equal
- `>` -- greater than
- `>=` -- greater than or equal

Boolean Logic

- `&&` -- Boolean and (True only if both are True)
- `||` -- Boolean or (False only if both are False)
- `not` -- Boolean not (Returns opposite truth value)

- Characters have type `Char`
- Constants are written using single quotes: `'a'`
- **Useful Operations**
 - Comparison operators can be used
 - `toUpper` - converts lowercase to uppercase (must import `Data.Char`)
 - `toLower` - converts uppercase to lowercase (must import `Data.Char`)
 - Examples:

```
ghci> 'c' < 'Z'
False
ghci> import Data.Char
ghci> toUpper 'w'
'W'
ghci> toLower 'Q'
'q'
```

- Special Character: *newline* which causes a line break when printed
 - Written as `'\n'`

- A `String` is a sequence of zero or more characters.
- Constants are written inside double quotes:
 - `"tree"`
- **Useful Operators and Operations**
 - Concatenation (`++`) - joins two strings together
 - Example: `"abc" ++ "defg" => "abcdefg"`
 - Example: `"Here is a line" ++ "\n"`
 - **Note:** will not join a `String` to any other type
 - `length` - operation which counts the length of the string
 - Example: `length "abc" → 3`
 - Example: `length "" → 0`

§ Basic Data Structures

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- Tuples provide the capability to store multiple values in a single variable
 - (1, 2)
 - ('a', 2, "cab")
- A tuple type is defined by two characteristics
 - The number of items to be stored, which is fixed once defined
 - The order of types to be stored, types do not need to be homogenous
- Examples:

```
("dog", "cat") :: (String, String)
(True, 5) :: (Bool, Int)
('a', "b") :: (Char, String)
("bat", (3.14, False)) :: (String, (Double, Bool))
```

- The general name is an *n-tuple* where *n* the number of components (i.e., 4-tuple)
 - a 2-tuple is also called a *pair*
 - a 3-tuple is also called a *triple*
 - no such thing as a 1-tuple
 - However, there is a special 0-tuple in Haskell
 - Written as `()`
 - Often used as a dummy value
- Pairs are a commonly used data structure, common operations include:
 - `fst (a, b)` - where the argument to the function is the tuple `(a, b)`, will return the value `a`
 - `snd (a, b)` - where the argument to the function is the tuple `(a, b)`, will return the value `b`

- Lists are the most common data structure used in functional programming
 - Size: unlimited
 - Type: all elements must be the same type
 - Examples:

```
[1, 2, 3]
['c', 'a', 'b']
[] -- empty list
```

- Type is written as: `[A]` where `A` is the type of the contained elements

```
[13,9,-2,100] :: [Int]
["cat", "dog"] :: [String]
[[1,2], [3,7,1], [], [900]] :: [[Int]]
```

- If we recall, a String is a sequence of characters, well this is just a list:

```
"string" == ['s','t','r','i','n','g']
```

- Additionally, we define a list using a range:

```
[1..10] == [1,2,3,4,5,6,7,8,9,10]  
[0,2..10] == [0,2,4,6,8,10]  
[10,9..0] == [10,9,8,7,6,5,4,3,2,1,0]
```

- Ranges also work for characters

```
['a'..'z'] == "abcdefghijklmnopqrstuvwxyz"  
['0'..'9'] == "0123456789"
```



List Notation and (:)

The **Cons Operator** - (:)

- We can construct new lists with the `:` operator
- This is an infix binary operator
 - Left argument is an element to add to the list
 - Right argument is a list
 - Type: `(:) :: a -> [a] -> [a]`
- Examples:

```
1 : [2, 3] => [1, 2, 3]
1 : [] -> [1]
```




List Notation and (:)

- Thus, we can write a list a series of cons operations:

```
[1, 2, 3, 4] == 1 : (2 : (3 : (4 : [])))  
"abc" == 'a' : ('b' : ('c' : []))
```

- However, since (:) is right-associative, we can drop the parentheses

```
[1, 2, 3, 4] == 1 : 2 : 3 : 4 : []  
"abc" == 'a' : 'b' : 'c' : []
```

- **Note:** that the end of the cons sequence is always the empty list

- **List Comprehension** - a simple but powerful syntax to directly define a list
 - based on set comprehensions from mathematics
 - Example set comprehension: $\{x^2 | x \in \mathcal{S}\}$
 - does not require a program to build a list

List Comprehension Syntax



General Form: `[expression | generator, ..., filter, ...]`

- Read the `|` as *such that*
- `generator` - defines a sequences of values that a variable will take on and is written in the form `var <- list`
 - **Note:** there may be more than one generator, one for each variable in the expression (acts like loop nesting)
- `expression` - evaluated for each value that the generator variable(s)
- `filter` - are `Bool` expressions that apply to one or more generator variables in order to determine if the value will be included or not
 - If the expression evaluates to `False` then the value is thrown out
 - Filters are optional and there can be more than one filter, each separated by commas

List Comprehension Examples



- List of the product of each pair from another list

```
[a * b | (a, b) <- [(1, 2), (10, 20), (6, 6)]]  
=> [2, 200, 36]
```

- expression* - `a * b`
 - generator* - `(a, b) <- [(1, 2), (10, 20), (6, 6)]`
- List of numbers that are divisible by 5 from the range 1 to 1000

```
[a `mod` 5 | a <- [1..1000]]  
=> [5, 10, 15, 20, ...]
```

- expression* - `a `mod` 5`
- generator* - `a <- [1..1000]`

List Comprehension Examples



```
[(x, y) | x <- [1,2,3], y <- ['a', 'b']]  
=> [(1, 'a'), (1, 'b'), (2, 'a'), (2, 'b'), (3, 'a'), (3, 'b')]
```

```
[x | x <- [0..100], x `mod` 2 == 0 && x `mod` 7 == 0]  
=> [0,14,28,42,56,70,84,98]
```

```
[x | x <- [1..12], y <- [1..12], x*y == 12]  
=> [1,2,3,4,6,12]
```

Exercises



Work out the values of the following list comprehensions; then check your results by evaluating them with the computer

```
[x | x <- [1,2,3], False]
```

```
[not (x && y) | x <- [False, True],  
              y <- [False, True]]
```

```
[x || y | x <- [False, True],  
          y <- [False, True],  
          x /= y]
```

Exercises



Work out the values of the following list comprehensions; then check your results by evaluating them with the computer

```
[x | x <- [1,2,3], False]
=> []
```

```
[not (x && y) | x <- [False, True],
               y <- [False, True]]
=> [False, False, False, True]
```

```
[x || y | x <- [False, True],
          y <- [False, True],
          x /= y]
=> [True, True]
```

§ Functions

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- **Function Application** - when an expression uses a function
 - A function is **applied** to its arguments
 - Syntax: `func arg1 arg2`
 - `func` is the function name
 - `arg1` and `arg2` are the arguments (may be 0 or more of these all separated by spaces)
- Example:

```
sqrt 9.0
3.4 + sqrt 25 * 100
2 * sqrt (pi * 5 * 5) + 10
```

- Functions like data have types, and these types are quite important
- **Function type** - for a function, f , with an argument of type a and a return type of b has a *function type* of $a \rightarrow b$ (read a arrow b).
 - Thus we would write: $f :: a \rightarrow b$
- **Example Function Types**

```
sqrt :: Double -> Double
max  :: Integer -> Integer -> Integer -- first two are the args
not  :: Bool -> Bool
toUpper :: Char -> Char
```

- **Operator** - a function which takes exactly two arguments
 - **Infix notation** - when the operator is written between the arguments
 - `(+)` operator - `2 + 4`
 - `min` function - `2 `min` 4` the backticks allow it to act as an operator
 - **Prefix notation** - when the operator
 - `(+)` operator - `(+) 2 4` when used like this or by itself it must be enclosed in parentheses
 - `min` operator - `min 2 4`
- All operators are functions, and thus have a function type
 - `(+) :: Integer -> Integer -> Integer`
 - `(&&) :: Bool -> Bool -> Bool`

- A function definition has two parts:

1. **Type declaration** which has the following form:

function_name :: *argType*₁ → *argType*₂ → ... → *argType*_n → *resultType*

2. **Defining Equation** which has the following form:

function_name *arg*₁ *arg*₂ ... *arg*_n = *expression using the arguments*

- Function definitions should be written in a Haskell script file

- To use the functions, the file should be loaded into GHCi

- Example Function Definition

```
square :: Integer -> Integer
square x = x * x
```

For Next Time



- Review DMUC Chapter 1.1 - 1.5
- Review this Lecture
- Come To Lecture
- Read DMUC Chapter 1.6 - 1.10





Are there any questions?