

$$2^4$$

$$c=2$$

$$n=4$$

CS 312

HW 1

$$g(n) = 1 + c + c^2 + \dots + c^n$$

1) 0.1)

- a) $f(n) = \Theta(g(n))$
- b) $f(n) = O(g(n))$
- c) $f(n) = \Theta(g(n))$
- d) $f(n) = \Theta(g(n))$
- e) $f(n) = \Theta(g(n))$
- f) $f(n) = \Theta(g(n))$

m) $f(n) = O(g(n))$

2) 0.2)

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|------------------------------------|----------------------------------|
| 1) $\Theta(1)$ if $c < 1$ | proof assumed |
| 2) $\Theta(n)$ if $c = 1$ | assumed |
| 3) $\Theta(c^n)$ if $c > 1$ | assumed |
| 4) $c^n = 1$ if $c = 1$ | |
| 5) $g(n) = n$ | #4 & $g(n)$ |
| 6) $\Theta(g(n)) = c^n$ if $c > 1$ | n^a dominates n^b if $a > b$ |
| 7) if $c < 1$, c must be > 0 | c must be positive & real |
| 8) $\Theta(g(n)) = \Theta(1)$ | #7 |

3) a)

function fab1(n)

if $n = 0, 1, 2$
 return 1
 return fab1(n-1) + fab1(n-2) * fab1(n-3)

This gives an upper bound Big-O of $O(3^n)$. The time is exponential w/ a base of 3. This is b/c we have 3 recursive calls each time we call fab1(n), except when $n \leq 2$. Since each call calls 3 functions, those 3 functions multiply for each call, thus 3^n .

b)

function fab2(n)

if $n = 0, 1, 2$
 return 1
 create array myArray[0...n]
 $f[0, 1, 2] = 1$
 for ($i = 3 \dots n$)
 $myArray[i] = myArray[i-1] + myArray[i-2] * myArray[i-3]$
 return myArray[n]

As a function of n , the exact number of adds & multiplies is

$f(n) = 2n - 3$ → for each n , you have to perform addition & 1 multiplication, except for $n = 0, 1, 2$