

- **Lemma 7.8:**

- Consider I for which statement does not hold, with $N(I)$ minimal, then there are $b, b' \notin I$ but $bb' \in I$.

- **Theorem 7.9:**

- Define $A = \{x \in K : xP \subseteq \mathcal{O}_K\}$, show A is fractional ideal and $R \subseteq A$
- Show $A \neq \mathcal{O}_K$:
 - Choose $0 \neq \alpha \in P$, choose prime ideals such that $P_1 \cdots P_t \subseteq (\alpha)$ and t is minimal.
 - Choose $\beta \in P_2 \cdots P_t$ and $\beta \notin (\alpha)$, show that $\frac{\beta}{\alpha} \in A - R$.
- Show that $P \neq AP$, using Theorem 4.6.
- Use fact that P is maximal to conclude $AP = R$.

- **Lemma 8.4:**

- Clear when I or J is \mathcal{O}_K so assume both are proper.
- Sufficient to show for when J is prime (why?)
- Use that $N(IP) = |R/(IP)| = |R/I| \cdot |I/(IP)|$.
- Show that $|I/(IP)| = |R/P|$:
 - Show $I/(IP)$ is one-dimensional vector space over R/P :
 - Show $I \neq IP$ and choose $x \in I - (IP)$.
 - Show $(x, IP) = I$ using unique factorisation.