0.1. Integration and measure

- Dirichlet's function: $f : [0,1] \to \mathbb{R}$,

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{otherwise} \end{cases}$$

1. The real numbers

- $a \in \mathbb{R}$ is an **upper bound** of $E \subseteq \mathbb{R}$ if $\forall x \in E, x \leq a$.
- $c \in \mathbb{R}$ is a **least upper bound (supremum)** if $c \leq a$ for every upper bound a.
- $a \in \mathbb{R}$ is an **lower bound** of $E \subseteq \mathbb{R}$ if $\forall x \in E, x \geq a$.
- $c \in \mathbb{R}$ is a **greatest lower bound (supremum)** if $c \geq a$ for every upper bound a.
- **Completeness axiom of the real numbers**: every subset E with an upper bound has a least upper bound. Every subset E with a lower bound has a greatest lower bound.
- **Archimedes' principle**:

$$\forall x \in \mathbb{R}, \exists n \in \mathbb{N} : n > x$$

- Every non-empty subset of $\mathbb{N}$ has a minimum.
- **The rationals are dense in the reals**:

$$\forall x < y \in \mathbb{R}, \exists r \in \mathbb{Q} : r \in (x, y)$$

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