- **Lemma 4.19**:
  - Well-defined: show that $\lambda_1 \simeq \lambda_2 \implies f \circ \lambda_1 \simeq f \circ \lambda_2$ (use definition of $\lambda_1 \simeq \lambda_2$).
  - Homomorphism: use that $f \circ (\lambda * \mu) = (f \circ \lambda) * (f \circ \mu)$.
- **Lemma 4.20**: for loop λ in (X, x_0), find based homotopy between $f \circ \lambda$ and $g \circ \lambda$ in terms of based homotopy H between f and g.
- **Lemma 4.21**: straightforward, just use the definition of f_*.
- **Corollary 4.22**: use definition of homotopy equivalent based spaces and lemma 4.21, to show the induced homomorphisms of the homotopy equivalences are inverse to each other.
- **Theorem 4.23**:
  - There is path p from x_0 to x_1.
  - Let λ loop in X based at x_0.
  - Define $\overline{p}(s) = p(1-s)$, define loop λ_p in X based at x_1 by

$$\lambda_p(s) = \begin{cases} \overline{p(3s)} & \text{if } s \in [0, 1/3] \\ \lambda(3s-1) & \text{if } s \in [1/3, 2/3] \\ p(3s-2) & \text{if } s \in [2/3, 1] \end{cases}$$

  - Claim:

$$\Phi_p : \pi_1(X, x_0) \to \pi_1(X, x_1), \quad \Phi([\lambda]) = [\lambda_p]$$

  is isomorphism.
    - Well-defined: show if λ, μ loops based at x_0, $\lambda \simeq \mu \implies \lambda_p \simeq \mu_p$ by homotopy diagram (merge $\overline{p}, \lambda, p$ on bottom and $\overline{p}, \mu, p$ on top).