1. Motivation

1.1. Plane curves

- Curves mainly parametrised: $\alpha : I \to \mathbb{R}^2$, $I \subset \mathbb{R}$ interval, with a direction.
- **Four vertex theorem**: every closed plane curve has at least 4 vertices.

1.2. Surfaces

- Surfaces are 2-dimensional subsets of $\mathbb{R}^3$.

2. Regular curves in $\mathbb{R}^n$

2.1. Regular curves, length and tangent vectors

- Let I be open interval, then $\underline{\alpha} : I \to \mathbb{R}^n$ is **parametrised curve**.
  - $\underline{\alpha}$ is **smooth** if $\underline{\alpha}(u) = (\alpha_1(u), ..., \alpha_n(u))$ where all $\alpha_i : I \to \mathbb{R}$ are smooth maps.
  - Image $\underline{\alpha}(I) \subset \mathbb{R}^n$ is the **trace**.
  - **Tangent vector of $\boldsymbol{\alpha}$ at $\boldsymbol{u}$** is

$$\underline{\alpha}'(u) = (\alpha_1'(u), ..., \alpha_n'(u))$$

  - $\underline{\alpha}$ is **regular** if $\forall u \in I, \underline{\alpha}'(u) \neq 0$. $\underline{\alpha}$ is **singular at $\boldsymbol{u}$** if $\underline{\alpha}'(u) = 0$.
  - If $\underline{\alpha}$ is regular, **unit tangent vector of $\boldsymbol{\alpha}$ at $\boldsymbol{u}$** is

$$\underline{t}(u) = \underline{\alpha}' \frac{u}{\|\underline{\alpha}'(u)\|}$$

  - If $\forall u \in I, \|\underline{\alpha}'(u)\| = 1$ then $\underline{\alpha}$ is a **unit speed curve**. If $\forall u \in I, \|\underline{\alpha}'(u)\| = c$, $\underline{\alpha}$ is **constant speed curve**.