- **Theorem 2.9**: use $|\langle x_n, y_n\rangle - \langle x, y\rangle| = |\langle x_n - x, y_n\rangle + \langle x, y_n\rangle - \langle x, y_n\rangle + \langle x, y_n - y\rangle|$ and Cauchy-Schwarz, reverse triangle inequality to show $\|y_n\| \to \|y\|$.
- **Theorem 2.14**: use linearity of inner product and orthogonal condition.
- **Theorem 2.15**:
  - Subspace:
    - For $y, z \in A^\perp$, $\lambda, \mu \in \mathbb{C}$, show $\forall x \in A$, $\lambda y + \mu z \in A^\perp$.
  - Closed:
    - Show if $(y_n) \subseteq A^\perp$, $y_n \to y$, then $y \in A^\perp$:
      - Let $x \in A$, then show $|\langle x, y\rangle| \to 0$ by squeezing, triangle inequality and Cauchy-Schwarz.
- **Theorem 2.16**:
  - Let $d = \inf\{\|x - z\| : z \in M\}$. Show that $\exists y \in M : \|x - y\| = d$:
    - There is sequence $(y_n) \subset M$ with $\|x - y_n\| \to d$. Show that (y_n) is Cauchy:
      - $\|y_m - y_n\|^2 + \|2x - y_m - y_n\|^2 = 2\|x - y_m\|^2 + 2\|x - y_n\|^2$ by parallelogram identity.
      - $\frac{y_m + y_n}{2} \in M$, so $\|2x - y_m - y_n\| \geq 2d$.
    - Deduce that $y_n \to y \in M$ and $\|x - y\| \to d$ by squeezing.
  - Uniqueness of y:
    - Let $\|x - y\| = d = \|x - y'\|$.
    - By parallelogram identity, $2\|x - y\|^2 + 2\|x - y'\|^2 = \|2x - y - y'\|^2 + \|y - y'\|^2$.
    - Use that $\frac{y + y'}{2} \in M$ to show $\|y - y'\| = 0$.
  - To show $z = x - y \perp M$:
    - For $w \in M$, write $\langle z, w\rangle = |\langle z, w\rangle|\ \lambda$ where $\lambda = e^{i\theta}$, set $u = \lambda w$.
    - Define $f(t) = \|z + tu\|^2$, show $t = 0$ is minimum of f and so $0 = f'(0)$, hence $z \in M^\perp$.
  - To show uniqueness of z:
    - Show for $y, y' \in M$ such that $x - y \perp M$ and $x - y' \perp M$, then $\langle y - y', w\rangle = 0$ for any $w \in M$. Set $w = y - y'$ to give $y = y'$.
- **Corollary 2.18**: by theorem 2.16.
- **Theorem 2.24**:
  - Prove for any finite $J \subseteq I$, then take supremum on LHS.
  - Show that

    $$\left\| x - \sum_{\alpha \in J} \langle x, u_\alpha\rangle u_\alpha \right\| = \|x\|^2 - \sum_{\alpha \in J} |\langle x, u_\alpha\rangle|^2$$

    using equation 2.2 and Pythagorean theorem.