0.1. Prerequisites

- $I \subset R$ is an ideal if $\forall (a, b) \in \mathbb{R}^2, ab \in I \Longrightarrow a \in I \vee b \in I$.
- I is maximal if $I \neq R$ and there is no ideal $J \subset R$ such that $I \subset J$.
- $p \in \mathbb{Z}$ is prime iff $\langle p \rangle = \langle p \rangle_{\mathbb{Z}}$ is a prime ideal.
- For commutative ring R:
  - $I \subset R$ is prime ideal iff R/I is an integral domain.
  - I is maximal iff R/I is a field.
- Let R be PID and $a \in R$ irreducible. Then $\langle a \rangle = \langle a \rangle_R$ is maximal.
- **Theorem**: let F be field, $f(x) \in F[x]$ irreducible. Then $F[x]/\langle f(x) \rangle$ is a field and a vector space over F with basis $B = \left\{1, \overline{x}, ..., \overline{x}^{n-1}\right\}$ where $n = \deg(f)$. That is, every element in $F[x]/\langle f(x) \rangle$ can be uniquely written as a linear combination

$$a_0 + a_1 \overline{x} + \cdots + a_{n-1} \overline{x}^{n-1}$$