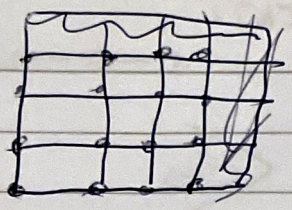
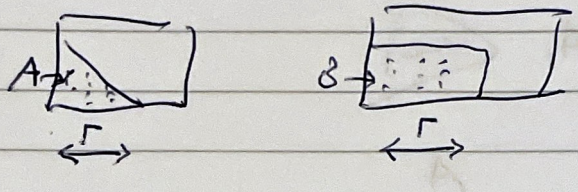
COMB1

Grid $[4]^2$:



Which $A \subseteq [k]^n$ minimise $|N(A)|$

e.g. in $[k]^2$, each will choose

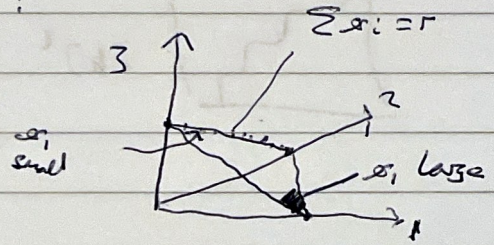


For A: $|\partial(A)| \approx r \approx \sqrt{4|A|}$

For B: $|\partial(B)| = 2r = 2\sqrt{|B|}$

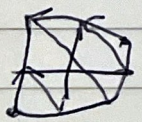
Which pairs do we take if $|\{|\alpha| \le r\}| < |A| < |\{|\alpha| \le r+1\}|$

e.g. in $[k]^3$:

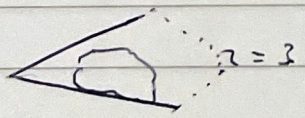


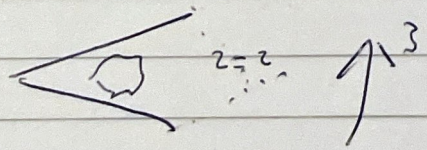
$\Rightarrow$ "keep x_i large"

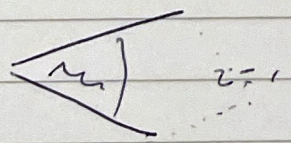
Simplicial on $[3]^2$:



in terms of A:

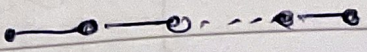
A_3  $z = 3$

A_2  $z = 2$

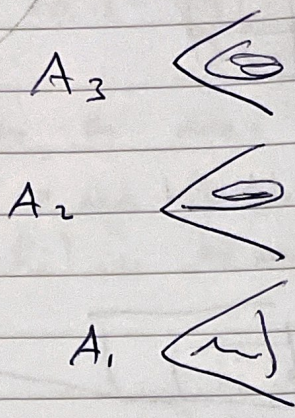
A_1  $z = 1$

$A \subseteq [k]^3$

Proof of theorem $n=1$: $[k]^1$:



Proof of claim :



Case $n=2$: