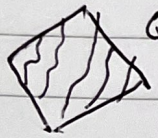
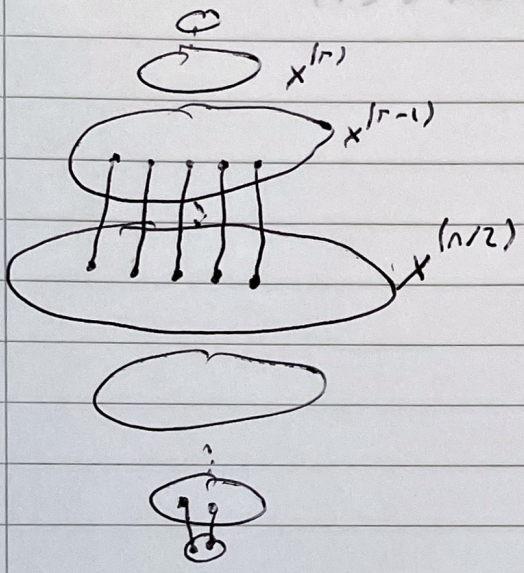
COMBINATORICS

Sperner lemma proof idea:

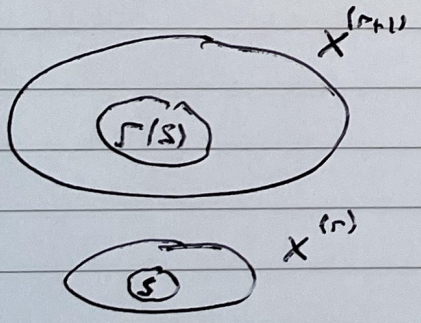


decomposition into chains

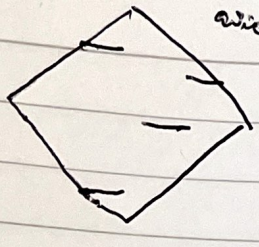


we match layers towards the middle layer

$(X^{(r)} \to X^{(r+1)}$ if $r < \frac{n}{2}$, $X^{(r)} \to X^{(r-1)}$ if $r \geq \frac{n}{2}$)



$\Gamma(S)$ = all neighbours of S.



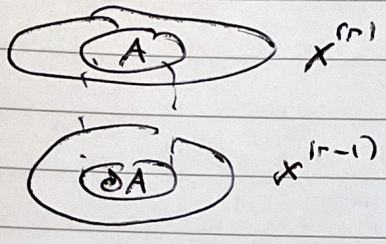
We want to show that if A is antichain, then

$$\sum_{r \geq 0} \frac{|A \cap X^{(r)}|}{\#(X^{(r)})} \leq 1$$

[the proportion of each layer occupied added to ≤ 1]

This trivially implies Sperner.

Proof of Local LYM



Proof of LYM inequality

Method 1