Look at photo, what are possible valuations of w_1, w_2, w_3 if $\mathrm{val}(x) = \mathrm{val}(y) = 0$ assuming they satisfy the equations (assuming that they lie in the intersections of the vanishing sets).

Thoughts:

- Just wanted to clarify why we can conclude if $\mathrm{val}(y) = 2$, $\mathrm{val}(x) = 1$, then $\mathrm{val}(w_1) = 1$? Is it because $w_1 - x - y = 0 \implies w_1 = x + y$?
- Let $w_1, w_2, w_3, x, y \in \mathrm{trop}(V(w_1 - x - y) \cap V(w_2 - x - y^{-1}) \cap V(w_3 - x - y^2))$.
- $$\mathrm{trop}(V(w_1 - x - y) \cap V(w_2 - x - y^{-1}) \cap V(w_3 - x - y^2)) \subseteq$$ $$\mathrm{trop}(V(w_1 - x - y)) \cap \mathrm{trop}(V(w_2 - x - y^{-1})) \cap \mathrm{trop}(V(w_3 - x - y^2))$$
- Let $K = \mathbb{C}\{\{t\}\}$, $x, y \in K$. Let $x = \sum_{m=0}^{\infty} a_m t^{m/n_0}$, $y = \sum_{m=0}^{\infty} b_m t^{m/n_1}$
- $\mathrm{val}(1) = \mathrm{val}(yy^{-1}) = \mathrm{val}(y) + \mathrm{val}(y^{-1}) = 0 \implies \mathrm{val}(y^{-1}) = 0$.
- $\mathrm{val}(y^2) = \mathrm{val}(y) + \mathrm{val}(y) = 0 + 0 = 0$
- So $\mathrm{val}(w_1), \mathrm{val}(w_2), \mathrm{val}(w_3) \geq 0$
- If $n_0 = n_1$, $\mathrm{val}(w_1) = \min\{m \in \mathbb{N} : a_m \neq -b_m\}$.
- If $n_0 \neq n_1$, $$\mathrm{val}(w_1) = \min\left(\left\{\frac{m}{n_0} \in \mathbb{Q} : \forall k \in \mathbb{N}, \frac{m}{n_0} \neq \frac{k}{n_1} \text{ or } a_m \neq b_k\right\}\right.$$ $$\left.\cup \left\{\frac{k}{n_1} \in \mathbb{Q} : \forall m \in \mathbb{N} : \frac{m}{n_0} \neq \frac{k}{n_1} \text{ or } a_m \neq b_k\right\}\right)$$
- $$y^2 = \left(\sum_{m=0}^{\infty} b_m t^{m/n_1}\right)^2 = \sum_{m=0}^{\infty} \sum_{i+j=m} b_i b_j t^{m/n_1}$$