

Lecture 12: Fast RL Part III¹

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CS234 Reinforcement Learning

Winter 2022

¹With a few slides derived from David Silver

Refresh Your Knowledge Fast RL Part II

- The prior over arm 1 is Beta(1,2) (left) and arm 2 is a Beta(1,1) (right figure). Select all that are true.
 - 1 Sample 3 params: 0.1,0.5,0.3. These are more likely to come from the Beta(1,2) distribution than Beta(1,1).
 - 2 Sample 3 params: 0.2,0.5,0.8. These are more likely to come from the Beta(1,1) distribution than Beta(1,2).
 - 3 It is impossible that the true Bernoulli parameter is 0 if the prior is Beta(1,1).
 - 4 Not sure
- The prior over arm 1 is Beta(1,2) (left) and arm 2 is a Beta(1,1) (right). The true parameters are arm 1 $\theta_1 = 0.4$ & arm 2 $\theta_2 = 0.6$. Thompson sampling = TS
 - 1 TS could sample $\theta = 0.5$ (arm 1) and $\theta = 0.55$ (arm 2).
 - 2 For the sampled thetas (0.5,0.55) TS is optimistic with respect to the true arm parameters for all arms.
 - 3 For the sampled thetas (0.5,0.55) TS will choose the true optimal arm for this round.
 - 4 Not sure

Beta(1,2)

Beta(1,1)



Refresh Your Knowledge Fast RL Part II Solution

- The prior over arm 1 is Beta(1,2) (left) and arm 2 is a Beta(1,1) (right figure). Select all that are true.

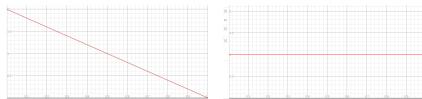
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- 3 It is impossible that the true Bernoulli parameter is 0 if the prior is Beta(1,1).
- 4 Not sure

1. True. 2. True. 3 False

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- 4 Not sure

1. True. 2. False. 3. True



Class Structure

- Last time: Fast Learning (Bayesian bandits to MDPs)
- **This time: Fast Learning III (MDPs)**
- Next time: Batch RL

Settings, Frameworks & Approaches

- Over these 3 lectures will consider 2 settings, multiple frameworks, and approaches
- Settings: Bandits (single decisions), MDPs
- Frameworks: evaluation criteria for formally assessing the quality of a RL algorithm. So far seen empirical evaluations, asymptotic convergence, regret, probably approximately correct
- Approaches: Classes of algorithms for achieving particular evaluation criteria in a certain set. So far for exploration seen: greedy, ϵ -greedy, optimism, Thompson sampling, for multi-armed bandits

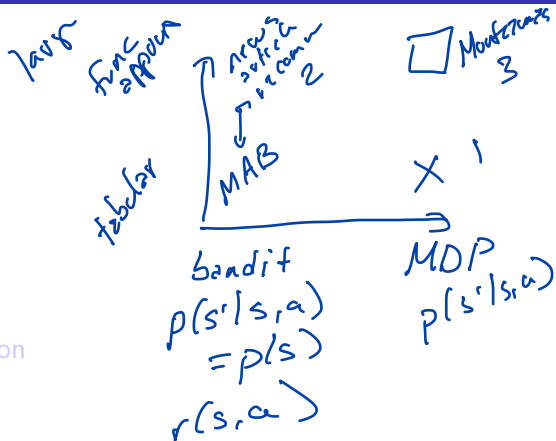
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3 Generalization and Exploration

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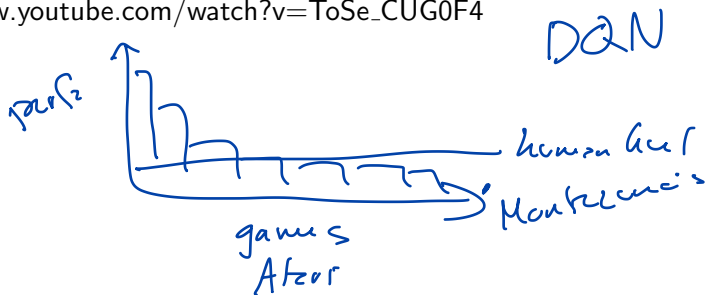
Fast RL in Markov Decision Processes

- Very similar set of frameworks and approaches are relevant for fast learning in reinforcement learning
- Frameworks
 - Regret
 - Bayesian regret
 - Probably approximately correct (PAC)
- Approaches
 - Optimism under uncertainty
 - Probability matching / Thompson sampling
- Framework: Probably approximately correct



Fast RL in Markov Decision Processes

- Montezuma's revenge
- https://www.youtube.com/watch?v=ToSe_CUG0F4



Model-Based Interval Estimation with Exploration Bonus (MBIE-EB)

(Strehl and Littman, J of Computer & Sciences 2008)

tabular MDPs

- 1: Given ϵ, δ, m
- 2: $\beta = \frac{1}{1-\gamma} \sqrt{0.5 \ln(2|S||A|m/\delta)}$
- 3: $n_{sas}(s, a, s') = 0, \forall s \in S, a \in A, s' \in S$
- 4: $rc(s, a) = 0, n_{sa}(s, a) = 0, \tilde{Q}(s, a) = 1/(1-\gamma), \forall s \in S, a \in A$
- 5: $t = 0, s_t = s_{init}$
- 6: **loop**
- 7: $a_t = \arg \max_{a \in A} \tilde{Q}(s_t, a)$
- 8: Observe reward r_t and state s_{t+1}
- 9: $n_{sa}(s_t, a_t) = n_{sa}(s_t, a_t) + 1, n_{sas}(s_t, a_t, s_{t+1}) = n_{sas}(s_t, a_t, s_{t+1}) + 1$
- 10: $rc(s_t, a_t) = \frac{rc(s_t, a_t)(n_{sa}(s_t, a_t) - 1) + r_t}{n_{sa}(s_t, a_t)}$
- 11: $\hat{R}(s_t, a_t) = rc(s_t, a_t)$ and $\hat{T}(s'|s_t, a_t) = \frac{n_{sas}(s_t, a_t, s')}{n_{sa}(s_t, a_t)}, \forall s' \in S$
- 12: **while** not converged **do**
- 13: $\tilde{Q}(s, a) = \hat{R}(s, a) + \gamma \sum_{s'} \hat{T}(s'|s, a) \max_{a'} \tilde{Q}(s', a) + \boxed{\frac{\beta}{\sqrt{n_{sa}(s, a)}}}, \forall s \in S, a \in A$
- 14: **end while**
- 15: **end loop**

$r \in [0, 1]$

Leslie Kaelbling

~1994

Rich Sutton
~1980s?

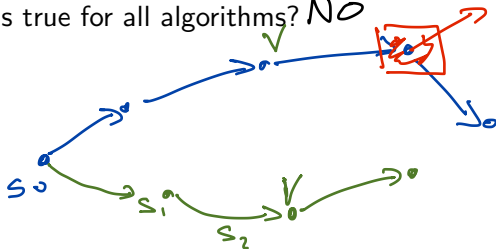
optimiser
under
uncertainty

optimiser
bonus

Framework: PAC for MDPs

- For a given ϵ and δ , A RL algorithm \mathcal{A} is PAC if on all but N steps, the action selected by algorithm \mathcal{A} on time step t , a_t , is ϵ -close to the optimal action, where N is a polynomial function of $(|S|, |A|, \gamma, \epsilon, \delta)$
- Is this true for all algorithms? **NO**

$\frac{1}{\epsilon}, \frac{1}{\delta}$



MBIE-EB is a PAC RL Algorithm

Theorem 2. Suppose that ϵ and δ are two real numbers between 0 and 1 and $M = \langle S, A, T, \mathcal{R}, \gamma \rangle$ is any MDP. There exists an input $m = m(\frac{1}{\epsilon}, \frac{1}{\delta})$, satisfying $m(\frac{1}{\epsilon}, \frac{1}{\delta}) = O(\frac{|S|}{\epsilon^2(1-\gamma)^4} + \frac{1}{\epsilon^2(1-\gamma)^4} \ln \frac{|S||A|}{\epsilon(1-\gamma)^\delta})$, and $\beta = (1/(1-\gamma))\sqrt{\ln(2|S||A|m/\delta)/2}$ such that if MBIE-EB is executed on MDP M , then the following holds. Let \mathcal{A}_t denote MBIE-EB's policy at time t and s_t denote the state at time t . With probability at least $1 - \delta$, $V_M^{\mathcal{A}_t}(s_t) \geq V_M^*(s_t) - \epsilon$ is true for all but $O(\underbrace{\frac{|S||A|}{\epsilon^3(1-\gamma)^6}(|S| + \ln \frac{|S||A|}{\epsilon(1-\gamma)^\delta}) \ln \frac{1}{\delta} \ln \frac{1}{\epsilon(1-\gamma)}}_{\text{timesteps}}) t$.

A Sufficient Set of Conditions to Make a RL Algorithm PAC

- Strehl, A. L., Li, L., & Littman, M. L. (2006). Incremental model-based learners with formal learning-time guarantees. In Proceedings of the Twenty-Second Conference on Uncertainty in Artificial Intelligence (pp. 485-493)

A Sufficient Set of Conditions to Make a RL Algorithm PAC

- 1 Optimism
 - 2 Accuracy
 - 3 Bounded learning complexity: number of updates of the state-action Q values, and number of times visit a (s,a) pair for which don't have an accurate estimate of its reward and/or dynamics model.
- Note: the above assumed a tabular domain (finite state and action space). But these ideas relate back to the ideas we saw in UCB, and also are relevant later for function approximation.

UCB
pigeonhole
principle

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Refresher: Bayesian Bandits

- **Bayesian bandits** exploit prior knowledge of rewards, $p[\mathcal{R}]$
- They compute posterior distribution of rewards $p[\mathcal{R} \mid h_t]$, where $h_t = (a_1, r_1, \dots, a_{t-1}, r_{t-1})$
- Use posterior to guide exploration
 - Upper confidence bounds (Bayesian UCB)
 - Probability matching (Thompson Sampling)
- Better performance if prior knowledge is accurate

Refresher: Bernoulli Bandits

- Consider a bandit problem where the reward of an arm is a binary outcome $\{0, 1\}$ sampled from a Bernoulli with parameter θ
 - E.g. Advertisement click through rate, patient treatment succeeds/fails, ...
- The Beta distribution $Beta(\alpha, \beta)$ is conjugate for the Bernoulli distribution

$$\underline{p(\theta|\alpha, \beta)} = \theta^{\alpha-1}(1-\theta)^{\beta-1} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

where $\Gamma(x)$ is the Gamma function.

- Assume the prior over θ is a $Beta(\alpha, \beta)$ as above
- Then after observed a reward $r \in \{0, 1\}$ then updated posterior over θ is $Beta(r + \alpha, 1 - r + \beta)$

posterior

Thompson Sampling for Bandits

-
- 1: Initialize prior over each arm a , $p(\mathcal{R}_a)$
 - 2: **loop**
 - 3: For each arm a **sample** a reward distribution \mathcal{R}_a from posterior
 - 4: Compute action-value function $Q(a) = \mathbb{E}[\mathcal{R}_a]$
 - 5: $a_t = \arg \max_{a \in \mathcal{A}} Q(a)$
 - 6: Observe reward r
 - 7: Update posterior $p(\mathcal{R}_a|r)$ using Bayes law
 - 8: **end loop**
-

Bayesian Model-Based RL

- Maintain posterior distribution over **MDP** models
- Estimate both transition and rewards, $p[\mathcal{P}, \mathcal{R} \mid h_t]$, where $h_t = (s_1, a_1, r_1, \dots, s_t)$ is the history
- Use posterior to guide exploration
 - Upper confidence bounds (Bayesian UCB)
 - Probability matching (Thompson sampling)

Thompson Sampling: Model-Based RL

- Thompson sampling implements probability matching

$$\begin{aligned}\pi(s, a \mid h_t) &= \mathbb{P}[Q(s, a) \geq Q(s, a'), \forall a' \neq a \mid h_t] \\ &= \mathbb{E}_{\mathcal{P}, \mathcal{R} \mid h_t} \left[\mathbb{1}(a = \arg \max_{a \in \mathcal{A}} Q(s, a)) \right]\end{aligned}$$

- Use Bayes law to compute posterior distribution $p[\mathcal{P}, \mathcal{R} \mid h_t]$
- Sample** an MDP \mathcal{P}, \mathcal{R} from posterior
- Solve MDP using favorite planning algorithm to get $Q^*(s, a)$
- Select optimal action for sample MDP, $a_t = \arg \max_{a \in \mathcal{A}} Q^*(s_t, a)$

Thompson Sampling for MDPs

-
- 1: Initialize prior over the dynamics and reward models for each (s, a) , $p(\mathcal{R}_{as}), p(\mathcal{T}(s'|s, a))$
 - 2: Initialize state s_0
 - 3: **loop**
 - 4: Sample a MDP \mathcal{M} : for each (s, a) pair, sample a dynamics model $\mathcal{T}(s'|s, a)$ and reward model $\mathcal{R}(s, a)$
 - 5: Compute $Q_{\mathcal{M}}^*$, optimal value for MDP \mathcal{M}
 - 6: $a_t = \arg \max_{a \in \mathcal{A}} Q_{\mathcal{M}}^*(s_t, a)$
 - 7: Observe reward r_t and next state s_{t+1}
 - 8: Update posterior $p(\mathcal{R}_{a_t s_t} | r_t), p(\mathcal{T}(s' | s_t, a_t) | s_{t+1})$ using Bayes rule
 - 9: $t = t + 1$
 - 10: **end loop**
-

Check Your Understanding: Fast RL III

tabular cases

- Strategic exploration in MDPs (select all):

① Doesn't really matter because the distribution of data is independent of the policy followed F

T ② Can involve using optimism with respect to both the possible dynamics and reward models in order to compute an optimistic Q function

F ③ Is known as PAC if the number of time steps on which a less than near optimal decision is made is guaranteed to be less than an ~~exponential~~ ^{polynomial} function of the problem domain parameters (state space cardinality, etc).

④ Not sure

$$N < \frac{1}{\epsilon^2(1-\gamma)^4} |S|^2 |A|$$

- In Thompson sampling for tabular MDPs in the shown algorithm:

F ① TS samples the reward model parameters and could use the empirical average for the dynamics model parameters and obtain the same performance

② Must perform MDP planning everytime the posterior is updated

③ Has the same computational cost each step as Q-learning F

④ Not sure

certainty
↓
exploration

T in
diag
sum

Check Your Understanding: Fast RL III Solutions

- Strategic exploration in MDPs (select all):

- 1 Doesn't really matter because the distribution of data is independent of the policy followed
- 2 Can involve using optimism with respect to both the possible dynamics and reward models in order to compute an optimistic Q function
- 3 Is known as PAC if the number of time steps on which a less than near optimal decision is made is guaranteed to be less than an exponential function of the problem domain parameters (state space cardinality, etc).
- 4 Not sure

1. False. 2. True. 3. False (needs to be a polynomial function)

- In Thompson sampling for tabular MDPs in the shown algorithm:

- 1 TS samples the reward model parameters and could use the empirical average for the dynamics model parameters and obtain the same performance
- 2 Must perform MDP planning everytime the posterior is updated
- 3 Has the same computational cost each step as Q-learning
- 4 Not sure

1. False. 2. True in algorithm shown, but could imagine alternatives. 3. False: doing planning with sampled model, again there could be alternatives

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Generalization and Strategic Exploration

- Active area of ongoing research: combine generalization & strategic exploration
- Many approaches are grounded by principles outlined here
 - Optimism under uncertainty
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Generalization and Strategic Exploration

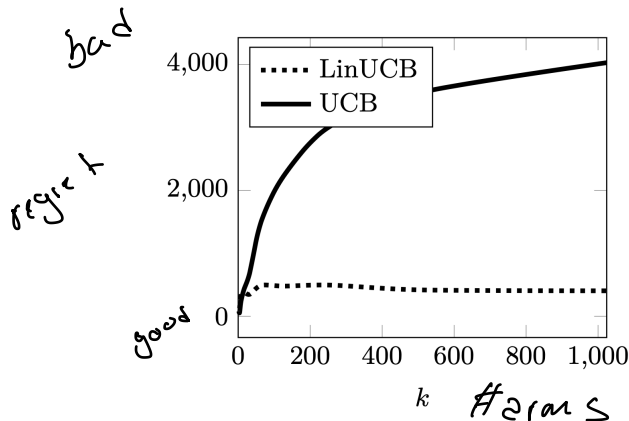
- Active area of ongoing research: combine generalization & strategic exploration
- Many approaches are grounded by principles outlined here
 - Optimism under uncertainty
 - Thompson sampling
- These issues are important for large state spaces and large action spaces, in bandits and Markov decision processes
- In homework 4 you will explore contextual multi-armed bandits
- Rest of today: brief discussion of **contextual bandits**, then MDPs

Contextual Multiarmed Bandits

- Multi-armed bandit is a tuple of $(\mathcal{A}, \mathcal{R})$, where \mathcal{A} : known set of m actions (arms)
 - $\mathcal{R}^a(r) = \mathbb{P}[r \mid a]$ is an unknown probability distribution over rewards
 - At each step t the agent selects an action $a_t \in \mathcal{A}$
 - The environment generates a reward $r_t \sim \mathcal{R}^{a_t}$
 - Goal: Maximize cumulative reward $\sum_{\tau=1}^t r_\tau$ / minimize total regret
- Contextual bandits: context/state space \mathcal{S} and action space \mathcal{A}
 - $\mathcal{R}^{a,s}(r) = \mathbb{P}[r \mid a, s]$ is an unknown probability distribution over rewards, for a particular state and action
 - If the state and/or action space is very large, it is common to use a function to represent the relationship between the input state and action and the output rewards

$$p(s' | s, a) = p(s)$$

Benefits of Generalization: Bandits vs Contextual Multiarmed Bandits:



- k is the number of arms, y-axis is the regret. [Figure is Figure 19.1, Lattimore and Szepesvari, Bandit Algorithms]

Contextual Multiarmed Bandits

- Contextual bandits: context/state space \mathcal{S} and action space \mathcal{A}
- $\mathcal{R}^{a,s}(r) = \mathbb{P}[r \mid a, s]$ is an unknown probability distribution over rewards, for a particular state and action
- If the state and/or action space is very large, it is common to use a function to represent the relationship between the input state and action and the output rewards
- Common to model reward as a linear function¹ of input features $\phi(s, a)$ vector
- $\underbrace{r}_{\text{unknown}} = \underbrace{\theta}_{\text{unknown}} \phi(s, a) + \epsilon$ where $\epsilon \sim \mathcal{N}(0, \sigma^2)$
 \uparrow might be known / unknown

¹Notation alert! \vec{x} state state action

Disjoint Linear Contextual Multi-armed Bandits

- Assumes that each arm a has its own θ_a parameter
- $r(s, a) = \theta_a \phi(s) + \epsilon$ where $\epsilon \sim \mathcal{N}(0, \sigma^2)$
- Check your understanding: can $r = \theta \phi(s, a) + \epsilon$ represent a disjoint linear model? *yes*

$$\begin{bmatrix} \phi(s) & \phi(s) & \dots & - \end{bmatrix}$$
$$\theta = \begin{bmatrix} \theta_{a_1} \\ \theta_{a_2} \\ \vdots \\ \theta_{a_{|A|}} \end{bmatrix}$$

Learning in Linear Contextual Multiarmed Bandits

- $r = \theta\phi(s, a) + \epsilon$
- Previously we used Hoeffding's inequality to represent uncertainty over a scalar reward
- We would like to now represent uncertainty over r through uncertainty over θ (check your understanding: why is this sufficient to capture uncertainty over r ?)
- Requires us to compute an uncertainty set over a vector θ
- This can be done in a computationally tractable way, see e.g. [A Contextual-Bandit Approach to Personalized News Article Recommendation, WWW 2010](#) or Chapter 19 in Lattimore and Szepesvari)

Generalization and Strategic Exploration

- Active area of ongoing research: combine generalization & strategic exploration
- Many approaches are grounded by principles outlined here
 - Optimism under uncertainty
 - Thompson sampling
- These issues are important for large state spaces and large action spaces, in bandits and Markov decision processes
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- Rest of today: brief discussion of contextual bandits, then **MDPs**

Generalization and Optimism

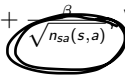
- Recall MBIE-EB algorithm for finite state and action domains
- What needs to be modified for continuous / extremely large state and/or action spaces?

Model-Based Interval Estimation with Exploration Bonus (MBIE-EB)

(Strehl and Littman, J of Computer & Sciences 2008)

- 1: Given ϵ, δ, m
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- 10: $rc(s_t, a_t) = \frac{rc(s_t, a_t)(n_{sa}(s_t, a_t) - 1) + r_t}{n_{sa}(s_t, a_t)}$
- 11: $\hat{R}(s_t, a_t) = rc(s_t, a_t)$ and $\hat{T}(s'|s_t, a_t) = \frac{n_{sas}(s_t, a_t, s')}{n_{sa}(s_t, a_t)}, \forall s' \in S$
- 12: **while** not converged **do**
- 13: $\tilde{Q}(s, a) = \hat{R}(s, a) + \gamma \sum_{s'} \hat{T}(s'|s, a) \max_{a'} \tilde{Q}(s', a) + \frac{\beta}{\sqrt{n_{sa}(s, a)}}, \forall s \in S, a \in A$
- 14: **end while**
- 15: **end loop**

$n(s, a)$ maybe



Generalization and Optimism

- Recall MBIE-EB algorithm for finite state and action domains
- What needs to be modified for continuous / extremely large state and/or action spaces?
- Estimating uncertainty
 - Counts of (s,a) and (s,a,s') tuples are not useful if we expect only to encounter any state once

Recall: Value Function Approximation with Control

- For Q-learning use a TD target $r + \gamma \max_{a'} \hat{Q}(s', a'; \mathbf{w})$ which leverages the max of the current function approximation value

$$\Delta \mathbf{w} = \alpha(r(s) + \gamma \max_{a'} \hat{Q}(s', a'; \mathbf{w}) - \hat{Q}(s, a; \mathbf{w})) \nabla_{\mathbf{w}} \hat{Q}(s, a; \mathbf{w})$$

- Modify to:

$$\Delta \mathbf{w} = \alpha(r(s) + \underbrace{r_{\text{bonus}}(s, a)}_{\text{bonus}} + \gamma \max_{a'} \hat{Q}(s', a'; \mathbf{w}) - \hat{Q}(s, a; \mathbf{w})) \nabla_{\mathbf{w}} \hat{Q}(s, a; \mathbf{w})$$

Recall: Value Function Approximation with Control

- For Q-learning use a TD target $r + \gamma \max_{a'} \hat{Q}(s', a'; \mathbf{w})$ which leverages the max of the current function approximation value

$$\Delta \mathbf{w} = \alpha(r(s) + r_{bonus}(s, a) + \gamma \max_{a'} \hat{Q}(s', a'; \mathbf{w}) - \hat{Q}(s, a; \mathbf{w})) \nabla_{\mathbf{w}} \hat{Q}(s, a; \mathbf{w})$$

- $r_{bonus}(s, a)$ should reflect uncertainty about future reward from (s, a)
- Approaches for deep RL that make an estimate of visits / density of visits include: Bellemare et al. NIPS 2016; Ostrovski et al. ICML 2017; Tang et al. NIPS 2017
- Note: bonus terms are computed at time of visit. During episodic replay can become outdated.

Benefits of Strategic Exploration: Montezuma's revenge

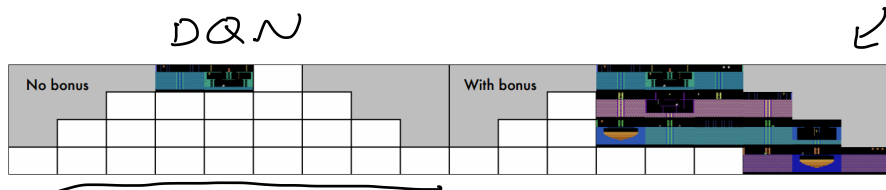


Figure 3: “Known world” of a DQN agent trained for 50 million frames with (**right**) and without (**left**) count-based exploration bonuses, in MONTEZUMA’S REVENGE.

Figure: Bellemare et al. “Unifying Count-Based Exploration and Intrinsic Motivation”

- Enormously better than standard DQN with ϵ -greedy approach

Generalization and Strategic Exploration: Thompson Sampling

- Leveraging Bayesian perspective has also inspired some approaches
- One approach: Thompson sampling over representation & parameters (Mandel, Liu, Brunskill, Popovic IJCAI 2016)

Generalization and Strategic Exploration: Thompson Sampling

- For scaling up to very large domains, again useful to consider model-free approaches
- Non-trivial: would like to be able to sample from a posterior over possible Q^*
- Bootstrapped DQN (Osband et al. NIPS 2016)
 - Train C DQN agents using bootstrapped samples
 - When acting, choose action with highest Q value over any of the C agents
 - Some performance gain, not as effective as reward bonus approaches

Generalization and Strategic Exploration: Thompson Sampling

- Leveraging Bayesian perspective has also inspired some approaches
- One approach: Thompson sampling over representation & parameters (Mandel, Liu, Brunskill, Popovic IJCAI 2016)
- For scaling up to very large domains, again useful to consider model-free approaches
- Non-trivial: would like to be able to sample from a posterior over possible Q^*
- Bootstrapped DQN (Osband et al. NIPS 2016)
- Efficient Exploration through Bayesian Deep Q-Networks (Azizzadenesheli, Anandkumar, NeurIPS workshop 2017)
 - Use deep neural network
 - On last layer use Bayesian linear regression
 - Be optimistic with respect to the resulting posterior
 - Very simple, empirically much better than just doing linear regression on last layer or bootstrapped DQN, not as good as reward bonuses in some cases

Theoretical Results

- We discussed regret bounds for bandits, & PAC bounds for tabular MDPs

Theoretical Results

- Discussed regret bounds for bandits, & PAC bounds for tabular MDPs
- Now exist tight (in dominant term) minimax results for regret and PAC for tabular MDPs
 - Azar, Mohammad Gheshlaghi, Ian Osband, and Rémi Munos. Minimax regret bounds for reinforcement learning. ICML 2017 (regret)
 - Dann, C., Li, L., Wei, W., and Brunskill, E. Policy certificates: Towards accountable reinforcement learning. ICML 2019 (PAC)
- Also exist instance-dependence bounds for tabular MDPs, e.g.:
 - Zanette and Brunskill. Tighter problem-dependent regret bounds in reinforcement learning without domain knowledge using value function bounds. ICML 2019
 - Simchowitz and Jamieson. Non-asymptotic gap-dependent regret bounds for tabular MDPs. NeurIPS 2019.

Theoretical Results: Function Approximation & RL

- Do there exist strong theoretical bounds for RL with function approximation?
- Active area of recent work
 - Jin, Yang, Wang, and Jordan. "Provably efficient reinforcement learning with linear function approximation." COLT 2020.
 - Many others, including our work (lead by Andrea Zanette), and Mengdi Wang's lab.

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Summary: What You Are Expected to Know

- Define the tension of exploration and exploitation in RL and why this does not arise in supervised or unsupervised learning
- Be able to define and compare different criteria for "good" performance (empirical, convergence, asymptotic, regret, PAC)
- Be able to map algorithms discussed in detail in class to the performance criteria they satisfy
- Understand the UCB proof sketch
- Be able to implement UCB and TS for linear contextual bandit. See references on website or in assignment 4 which include [A Contextual-Bandit Approach to Personalized News Article Recommendation, WWW 2010](#) or Chapter 19 in Lattimore and Szepesvari)

Class Structure

- Last time: Fast Learning (Bayesian bandits to MDPs)
- **This time: Fast Learning III (MDPs)**
- Next time: Batch RL