

# STAT205\_HW5\_supp

June 7, 2022

```
[1]: !apt-get install texlive texlive-xetex texlive-latex-extra pandoc
!pip install pypandoc
```

```
Reading package lists... Done
Building dependency tree
Reading state information... Done
pandoc is already the newest version (1.19.2.4~dfsg-1build4).
pandoc set to manually installed.
The following package was automatically installed and is no longer required:
  libnvidia-common-460
Use 'apt autoremove' to remove it.
The following additional packages will be installed:
  fonts-droid-fallback fonts-lato fonts-lmodern fonts-noto-mono fonts-texgyre
  javascript-common libcupsfilters1 libcupsimage2 libgs9 libgs9-common
  libijs-0.35 libjbig2dec0 libjs-jquery libkpathsea6 libpotrace0 libptexenc1
  libruby2.5 libsynchronet1 libtexlua52 libtexlua52 libzzip-0-13 lmodern
  poppler-data preview-latex-style rake ruby ruby-did-you-mean ruby-minitest
  ruby-net-telnet ruby-power-assert ruby-test-unit ruby2.5
  rubygems-integration tluils tex-common tex-gyre texlive-base
  texlive-binaries texlive-fonts-recommended texlive-latex-base
  texlive-latex-recommended texlive-pictures texlive-plain-generic tipa
Suggested packages:
  fonts-noto apache2 | lighttpd | httpd poppler-utils ghostscript
  fonts-japanese-mincho | fonts-ipafont-mincho fonts-japanese-gothic
  | fonts-ipafont-gothic fonts-arphic-ukai fonts-arphic-uming fonts-nanum ri
  ruby-dev bundler debhelper gv | postscript-viewer perl-tk xpdf-reader
  | pdf-viewer texlive-fonts-recommended-doc texlive-latex-base-doc
  python-pygments icc-profiles libfile-which-perl
  libspreadsheet-parseexcel-perl texlive-latex-extra-doc
  texlive-latex-recommended-doc texlive-pstricks dot2tex prerex ruby-tcltk
  | libtcltk-ruby texlive-pictures-doc vprerex
The following NEW packages will be installed:
  fonts-droid-fallback fonts-lato fonts-lmodern fonts-noto-mono fonts-texgyre
  javascript-common libcupsfilters1 libcupsimage2 libgs9 libgs9-common
  libijs-0.35 libjbig2dec0 libjs-jquery libkpathsea6 libpotrace0 libptexenc1
  libruby2.5 libsynchronet1 libtexlua52 libtexlua52 libzzip-0-13 lmodern
  poppler-data preview-latex-style rake ruby ruby-did-you-mean ruby-minitest
  ruby-net-telnet ruby-power-assert ruby-test-unit ruby2.5
```

```

rubygems-integration tlutils tex-common tex-gyre texlive texlive-base
texlive-binaries texlive-fonts-recommended texlive-latex-base
texlive-latex-extra texlive-latex-recommended texlive-pictures
texlive-plain-generic texlive-xetex tipa
0 upgraded, 47 newly installed, 0 to remove and 45 not upgraded.
Need to get 146 MB of archives.
After this operation, 460 MB of additional disk space will be used.
Get:1 http://archive.ubuntu.com/ubuntu bionic/main amd64 fonts-droid-fallback
all 1:6.0.1r16-1.1 [1,805 kB]
Get:2 http://archive.ubuntu.com/ubuntu bionic/main amd64 fonts-lato all 2.0-2
[2,698 kB]
Get:3 http://archive.ubuntu.com/ubuntu bionic/main amd64 poppler-data all
0.4.8-2 [1,479 kB]
Get:4 http://archive.ubuntu.com/ubuntu bionic/main amd64 tex-common all 6.09
[33.0 kB]
Get:5 http://archive.ubuntu.com/ubuntu bionic/main amd64 fonts-lmodern all
2.004.5-3 [4,551 kB]
Get:6 http://archive.ubuntu.com/ubuntu bionic/main amd64 fonts-noto-mono all
20171026-2 [75.5 kB]
Get:7 http://archive.ubuntu.com/ubuntu bionic/universe amd64 fonts-texgyre all
20160520-1 [8,761 kB]
Get:8 http://archive.ubuntu.com/ubuntu bionic/main amd64 javascript-common all
11 [6,066 B]
Get:9 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libcupsfilters1
amd64 1.20.2-0ubuntu3.1 [108 kB]
Get:10 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libcupsimage2
amd64 2.2.7-1ubuntu2.9 [18.6 kB]
Get:11 http://archive.ubuntu.com/ubuntu bionic/main amd64 libijs-0.35 amd64
0.35-13 [15.5 kB]
Get:12 http://archive.ubuntu.com/ubuntu bionic/main amd64 libjbig2dec0 amd64
0.13-6 [55.9 kB]
Get:13 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libgs9-common
all 9.26~dfsg+0-0ubuntu0.18.04.16 [5,093 kB]
Get:14 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libgs9 amd64
9.26~dfsg+0-0ubuntu0.18.04.16 [2,265 kB]
Get:15 http://archive.ubuntu.com/ubuntu bionic/main amd64 libjs-jquery all
3.2.1-1 [152 kB]
Get:16 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libkpathsea6
amd64 2017.20170613.44572-8ubuntu0.1 [54.9 kB]
Get:17 http://archive.ubuntu.com/ubuntu bionic/main amd64 libpotrace0 amd64
1.14-2 [17.4 kB]
Get:18 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libptexenc1
amd64 2017.20170613.44572-8ubuntu0.1 [34.5 kB]
Get:19 http://archive.ubuntu.com/ubuntu bionic/main amd64 rubygems-integration
all 1.11 [4,994 B]
Get:20 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 ruby2.5 amd64
2.5.1-1ubuntu1.11 [48.6 kB]
Get:21 http://archive.ubuntu.com/ubuntu bionic/main amd64 ruby amd64 1:2.5.1

```

[5,712 B]

Get:22 <http://archive.ubuntu.com/ubuntu> bionic-updates/main amd64 rake all 12.3.1-1ubuntu0.1 [44.9 kB]

Get:23 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 ruby-did-you-mean all 1.2.0-2 [9,700 B]

Get:24 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 ruby-minitest all 5.10.3-1 [38.6 kB]

Get:25 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 ruby-net-telnet all 0.1.1-2 [12.6 kB]

Get:26 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 ruby-power-assert all 0.3.0-1 [7,952 B]

Get:27 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 ruby-test-unit all 3.2.5-1 [61.1 kB]

Get:28 <http://archive.ubuntu.com/ubuntu> bionic-updates/main amd64 libruby2.5 amd64 2.5.1-1ubuntu1.11 [3,072 kB]

Get:29 <http://archive.ubuntu.com/ubuntu> bionic-updates/main amd64 libsyntax1 amd64 2017.20170613.44572-8ubuntu0.1 [41.4 kB]

Get:30 <http://archive.ubuntu.com/ubuntu> bionic-updates/main amd64 libtexlua52 amd64 2017.20170613.44572-8ubuntu0.1 [91.2 kB]

Get:31 <http://archive.ubuntu.com/ubuntu> bionic-updates/main amd64 libtexluajit2 amd64 2017.20170613.44572-8ubuntu0.1 [230 kB]

Get:32 <http://archive.ubuntu.com/ubuntu> bionic-updates/main amd64 libzip-0-13 amd64 0.13.62-3.1ubuntu0.18.04.1 [26.0 kB]

Get:33 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 lmodern all 2.004.5-3 [9,631 kB]

Get:34 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 preview-latex-style all 11.91-1ubuntu1 [185 kB]

Get:35 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 t1utils amd64 1.41-2 [56.0 kB]

Get:36 <http://archive.ubuntu.com/ubuntu> bionic/universe amd64 tex-gyre all 20160520-1 [4,998 kB]

Get:37 <http://archive.ubuntu.com/ubuntu> bionic-updates/main amd64 texlive-binaries amd64 2017.20170613.44572-8ubuntu0.1 [8,179 kB]

Get:38 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 texlive-base all 2017.20180305-1 [18.7 MB]

Get:39 <http://archive.ubuntu.com/ubuntu> bionic/universe amd64 texlive-fonts-recommended all 2017.20180305-1 [5,262 kB]

Get:40 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 texlive-latex-base all 2017.20180305-1 [951 kB]

Get:41 <http://archive.ubuntu.com/ubuntu> bionic/main amd64 texlive-latex-recommended all 2017.20180305-1 [14.9 MB]

Get:42 <http://archive.ubuntu.com/ubuntu> bionic/universe amd64 texlive all 2017.20180305-1 [14.4 kB]

Get:43 <http://archive.ubuntu.com/ubuntu> bionic/universe amd64 texlive-pictures all 2017.20180305-1 [4,026 kB]

Get:44 <http://archive.ubuntu.com/ubuntu> bionic/universe amd64 texlive-latex-extra all 2017.20180305-2 [10.6 MB]

Get:45 <http://archive.ubuntu.com/ubuntu> bionic/universe amd64 texlive-plain-

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generic all 2017.20180305-2 [23.6 MB]
Get:46 http://archive.ubuntu.com/ubuntu bionic/universe amd64 tipa all 2:1.3-20
[2,978 kB]
Get:47 http://archive.ubuntu.com/ubuntu bionic/universe amd64 texlive-xetex all
2017.20180305-1 [10.7 MB]
Fetched 146 MB in 4s (33.9 MB/s)
Extracting templates from packages: 100%
Preconfiguring packages ...
Selecting previously unselected package fonts-droid-fallback.
(Reading database ... 155632 files and directories currently installed.)
Preparing to unpack .../00-fonts-droid-fallback_1%3a6.0.1r16-1.1_all.deb ...
Unpacking fonts-droid-fallback (1:6.0.1r16-1.1) ...
Selecting previously unselected package fonts-lato.
Preparing to unpack .../01-fonts-lato_2.0-2_all.deb ...
Unpacking fonts-lato (2.0-2) ...
Selecting previously unselected package poppler-data.
Preparing to unpack .../02-poppler-data_0.4.8-2_all.deb ...
Unpacking poppler-data (0.4.8-2) ...
Selecting previously unselected package tex-common.
Preparing to unpack .../03-tex-common_6.09_all.deb ...
Unpacking tex-common (6.09) ...
Selecting previously unselected package fonts-lmodern.
Preparing to unpack .../04-fonts-lmodern_2.004.5-3_all.deb ...
Unpacking fonts-lmodern (2.004.5-3) ...
Selecting previously unselected package fonts-noto-mono.
Preparing to unpack .../05-fonts-noto-mono_20171026-2_all.deb ...
Unpacking fonts-noto-mono (20171026-2) ...
Selecting previously unselected package fonts-texgyre.
Preparing to unpack .../06-fonts-texgyre_20160520-1_all.deb ...
Unpacking fonts-texgyre (20160520-1) ...
Selecting previously unselected package javascript-common.
Preparing to unpack .../07-javascript-common_11_all.deb ...
Unpacking javascript-common (11) ...
Selecting previously unselected package libcupsfilters1:amd64.
Preparing to unpack .../08-libcupsfilters1_1.20.2-0ubuntu3.1_amd64.deb ...
Unpacking libcupsfilters1:amd64 (1.20.2-0ubuntu3.1) ...
Selecting previously unselected package libcupsimage2:amd64.
Preparing to unpack .../09-libcupsimage2_2.2.7-1ubuntu2.9_amd64.deb ...
Unpacking libcupsimage2:amd64 (2.2.7-1ubuntu2.9) ...
Selecting previously unselected package libijs-0.35:amd64.
Preparing to unpack .../10-libijs-0.35_0.35-13_amd64.deb ...
Unpacking libijs-0.35:amd64 (0.35-13) ...
Selecting previously unselected package libjbig2dec0:amd64.
Preparing to unpack .../11-libjbig2dec0_0.13-6_amd64.deb ...
Unpacking libjbig2dec0:amd64 (0.13-6) ...
Selecting previously unselected package libgs9-common.
Preparing to unpack .../12-libgs9-common_9.26~dfsg+0-0ubuntu0.18.04.16_all.deb
...

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Unpacking libgs9-common (9.26~dfsg+0-0ubuntu0.18.04.16) ...
Selecting previously unselected package libgs9:amd64.
Preparing to unpack .../13-libgs9_9.26~dfsg+0-0ubuntu0.18.04.16_amd64.deb ...
Unpacking libgs9:amd64 (9.26~dfsg+0-0ubuntu0.18.04.16) ...
Selecting previously unselected package libjs-jquery.
Preparing to unpack .../14-libjs-jquery_3.2.1-1_all.deb ...
Unpacking libjs-jquery (3.2.1-1) ...
Selecting previously unselected package libkpathsea6:amd64.
Preparing to unpack .../15-libkpathsea6_2017.20170613.44572-8ubuntu0.1_amd64.deb
...
Unpacking libkpathsea6:amd64 (2017.20170613.44572-8ubuntu0.1) ...
Selecting previously unselected package libpotrace0.
Preparing to unpack .../16-libpotrace0_1.14-2_amd64.deb ...
Unpacking libpotrace0 (1.14-2) ...
Selecting previously unselected package libptexenc1:amd64.
Preparing to unpack .../17-libptexenc1_2017.20170613.44572-8ubuntu0.1_amd64.deb
...
Unpacking libptexenc1:amd64 (2017.20170613.44572-8ubuntu0.1) ...
Selecting previously unselected package rubygems-integration.
Preparing to unpack .../18-rubygems-integration_1.11_all.deb ...
Unpacking rubygems-integration (1.11) ...
Selecting previously unselected package ruby2.5.
Preparing to unpack .../19-ruby2.5_2.5.1-1ubuntu1.11_amd64.deb ...
Unpacking ruby2.5 (2.5.1-1ubuntu1.11) ...
Selecting previously unselected package ruby.
Preparing to unpack .../20-ruby_1%3a2.5.1_amd64.deb ...
Unpacking ruby (1:2.5.1) ...
Selecting previously unselected package rake.
Preparing to unpack .../21-rake_12.3.1-1ubuntu0.1_all.deb ...
Unpacking rake (12.3.1-1ubuntu0.1) ...
Selecting previously unselected package ruby-did-you-mean.
Preparing to unpack .../22-ruby-did-you-mean_1.2.0-2_all.deb ...
Unpacking ruby-did-you-mean (1.2.0-2) ...
Selecting previously unselected package ruby-minitest.
Preparing to unpack .../23-ruby-minitest_5.10.3-1_all.deb ...
Unpacking ruby-minitest (5.10.3-1) ...
Selecting previously unselected package ruby-net-telnet.
Preparing to unpack .../24-ruby-net-telnet_0.1.1-2_all.deb ...
Unpacking ruby-net-telnet (0.1.1-2) ...
Selecting previously unselected package ruby-power-assert.
Preparing to unpack .../25-ruby-power-assert_0.3.0-1_all.deb ...
Unpacking ruby-power-assert (0.3.0-1) ...
Selecting previously unselected package ruby-test-unit.
Preparing to unpack .../26-ruby-test-unit_3.2.5-1_all.deb ...
Unpacking ruby-test-unit (3.2.5-1) ...
Selecting previously unselected package libruby2.5:amd64.
Preparing to unpack .../27-libruby2.5_2.5.1-1ubuntu1.11_amd64.deb ...
Unpacking libruby2.5:amd64 (2.5.1-1ubuntu1.11) ...

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Selecting previously unselected package libsyntax1:amd64.
Preparing to unpack .../28-libsyntax1_2017.20170613.44572-8ubuntu0.1_amd64.deb
...
Unpacking libsyntax1:amd64 (2017.20170613.44572-8ubuntu0.1) ...
Selecting previously unselected package libtexlua52:amd64.
Preparing to unpack .../29-libtexlua52_2017.20170613.44572-8ubuntu0.1_amd64.deb
...
Unpacking libtexlua52:amd64 (2017.20170613.44572-8ubuntu0.1) ...
Selecting previously unselected package libtexluaajit2:amd64.
Preparing to unpack
.../30-libtexluaajit2_2017.20170613.44572-8ubuntu0.1_amd64.deb ...
Unpacking libtexluaajit2:amd64 (2017.20170613.44572-8ubuntu0.1) ...
Selecting previously unselected package libzip-0-13:amd64.
Preparing to unpack .../31-libzip-0-13_0.13.62-3.1ubuntu0.18.04.1_amd64.deb ...
Unpacking libzip-0-13:amd64 (0.13.62-3.1ubuntu0.18.04.1) ...
Selecting previously unselected package lmodern.
Preparing to unpack .../32-lmodern_2.004.5-3_all.deb ...
Unpacking lmodern (2.004.5-3) ...
Selecting previously unselected package preview-latex-style.
Preparing to unpack .../33-preview-latex-style_11.91-1ubuntu1_all.deb ...
Unpacking preview-latex-style (11.91-1ubuntu1) ...
Selecting previously unselected package t1utils.
Preparing to unpack .../34-t1utils_1.41-2_amd64.deb ...
Unpacking t1utils (1.41-2) ...
Selecting previously unselected package tex-gyre.
Preparing to unpack .../35-tex-gyre_20160520-1_all.deb ...
Unpacking tex-gyre (20160520-1) ...
Selecting previously unselected package texlive-binaries.
Preparing to unpack .../36-texlive-
binaries_2017.20170613.44572-8ubuntu0.1_amd64.deb ...
Unpacking texlive-binaries (2017.20170613.44572-8ubuntu0.1) ...
Selecting previously unselected package texlive-base.
Preparing to unpack .../37-texlive-base_2017.20180305-1_all.deb ...
Unpacking texlive-base (2017.20180305-1) ...
Selecting previously unselected package texlive-fonts-recommended.
Preparing to unpack .../38-texlive-fonts-recommended_2017.20180305-1_all.deb ...
Unpacking texlive-fonts-recommended (2017.20180305-1) ...
Selecting previously unselected package texlive-latex-base.
Preparing to unpack .../39-texlive-latex-base_2017.20180305-1_all.deb ...
Unpacking texlive-latex-base (2017.20180305-1) ...
Selecting previously unselected package texlive-latex-recommended.
Preparing to unpack .../40-texlive-latex-recommended_2017.20180305-1_all.deb ...
Unpacking texlive-latex-recommended (2017.20180305-1) ...
Selecting previously unselected package texlive.
Preparing to unpack .../41-texlive_2017.20180305-1_all.deb ...
Unpacking texlive (2017.20180305-1) ...
Selecting previously unselected package texlive-pictures.
Preparing to unpack .../42-texlive-pictures_2017.20180305-1_all.deb ...

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Unpacking texlive-pictures (2017.20180305-1) ...
Selecting previously unselected package texlive-latex-extra.
Preparing to unpack .../43-texlive-latex-extra_2017.20180305-2_all.deb ...
Unpacking texlive-latex-extra (2017.20180305-2) ...
Selecting previously unselected package texlive-plain-generic.
Preparing to unpack .../44-texlive-plain-generic_2017.20180305-2_all.deb ...
Unpacking texlive-plain-generic (2017.20180305-2) ...
Selecting previously unselected package tipa.
Preparing to unpack .../45-tipa_2%3a1.3-20_all.deb ...
Unpacking tipa (2:1.3-20) ...
Selecting previously unselected package texlive-xetex.
Preparing to unpack .../46-texlive-xetex_2017.20180305-1_all.deb ...
Unpacking texlive-xetex (2017.20180305-1) ...
Setting up libgs9-common (9.26~dfsg+0-0ubuntu0.18.04.16) ...
Setting up libkpathsea6:amd64 (2017.20170613.44572-8ubuntu0.1) ...
Setting up libjs-jquery (3.2.1-1) ...
Setting up libtexlua52:amd64 (2017.20170613.44572-8ubuntu0.1) ...
Setting up fonts-droid-fallback (1:6.0.1r16-1.1) ...
Setting up libsynchronet1:amd64 (2017.20170613.44572-8ubuntu0.1) ...
Setting up libptexenc1:amd64 (2017.20170613.44572-8ubuntu0.1) ...
Setting up tex-common (6.09) ...
update-language: texlive-base not installed and configured, doing nothing!
Setting up poppler-data (0.4.8-2) ...
Setting up tex-gyre (20160520-1) ...
Setting up preview-latex-style (11.91-1ubuntu1) ...
Setting up fonts-texgyre (20160520-1) ...
Setting up fonts-noto-mono (20171026-2) ...
Setting up fonts-lato (2.0-2) ...
Setting up libcupsfilters1:amd64 (1.20.2-0ubuntu3.1) ...
Setting up libcupsimage2:amd64 (2.2.7-1ubuntu2.9) ...
Setting up libjbig2dec0:amd64 (0.13-6) ...
Setting up ruby-did-you-mean (1.2.0-2) ...
Setting up t1utils (1.41-2) ...
Setting up ruby-net-telnet (0.1.1-2) ...
Setting up libijs-0.35:amd64 (0.35-13) ...
Setting up rubygems-integration (1.11) ...
Setting up libpotrace0 (1.14-2) ...
Setting up javascript-common (11) ...
Setting up ruby-minitest (5.10.3-1) ...
Setting up libbzip2-0.13:amd64 (0.13.62-3.1ubuntu0.18.04.1) ...
Setting up libgs9:amd64 (9.26~dfsg+0-0ubuntu0.18.04.16) ...
Setting up libtexluaajit2:amd64 (2017.20170613.44572-8ubuntu0.1) ...
Setting up fonts-lmodern (2.004.5-3) ...
Setting up ruby-power-assert (0.3.0-1) ...
Setting up texlive-binaries (2017.20170613.44572-8ubuntu0.1) ...
update-alternatives: using /usr/bin/xdvi-xaw to provide /usr/bin/xdvi.bin
(xdvi.bin) in auto mode
update-alternatives: using /usr/bin/bibtex.original to provide /usr/bin/bibtex

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(bibtex) in auto mode
Setting up texlive-base (2017.20180305-1) ...
mktexlsr: Updating /var/lib/texmf/ls-R-TEXLIVEDIST...
mktexlsr: Updating /var/lib/texmf/ls-R-TEXMFMAIN...
mktexlsr: Updating /var/lib/texmf/ls-R...
mktexlsr: Done.
tl-paper: setting paper size for dvips to a4:
/var/lib/texmf/dvips/config/config-paper.ps
tl-paper: setting paper size for dvipdfmx to a4:
/var/lib/texmf/dvipdfmx/dvipdfmx-paper.cfg
tl-paper: setting paper size for xdvi to a4: /var/lib/texmf/xdvi/XDvi-paper
tl-paper: setting paper size for pdftex to a4:
/var/lib/texmf/tex/generic/config/pdftexconfig.tex
Setting up texlive-fonts-recommended (2017.20180305-1) ...
Setting up texlive-plain-generic (2017.20180305-2) ...
Setting up texlive-latex-base (2017.20180305-1) ...
Setting up lmodern (2.004.5-3) ...
Setting up texlive-latex-recommended (2017.20180305-1) ...
Setting up texlive-pictures (2017.20180305-1) ...
Setting up tipa (2:1.3-20) ...
Regenerating '/var/lib/texmf/fmtutil.cnf-DEBIAN'... done.
Regenerating '/var/lib/texmf/fmtutil.cnf-TEXLIVEDIST'... done.
update-fmtutil has updated the following file(s):
    /var/lib/texmf/fmtutil.cnf-DEBIAN
    /var/lib/texmf/fmtutil.cnf-TEXLIVEDIST
If you want to activate the changes in the above file(s),
you should run fmtutil-sys or fmtutil.
Setting up texlive (2017.20180305-1) ...
Setting up texlive-latex-extra (2017.20180305-2) ...
Setting up texlive-xetex (2017.20180305-1) ...
Setting up ruby2.5 (2.5.1-1ubuntu1.11) ...
Setting up ruby (1:2.5.1) ...
Setting up ruby-test-unit (3.2.5-1) ...
Setting up rake (12.3.1-1ubuntu0.1) ...
Setting up libruby2.5:amd64 (2.5.1-1ubuntu1.11) ...
Processing triggers for mime-support (3.60ubuntu1) ...
Processing triggers for libc-bin (2.27-3ubuntu1.3) ...
/sbin/ldconfig.real: /usr/local/lib/python3.7/dist-
packages/ideep4py/lib/libmkldnn.so.0 is not a symbolic link

Processing triggers for man-db (2.8.3-2ubuntu0.1) ...
Processing triggers for fontconfig (2.12.6-0ubuntu2) ...
Processing triggers for tex-common (6.09) ...
Running updmap-sys. This may take some time... done.
Running mktexlsr /var/lib/texmf ... done.
Building format(s) --all.
    This may take some time... done.
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-

```

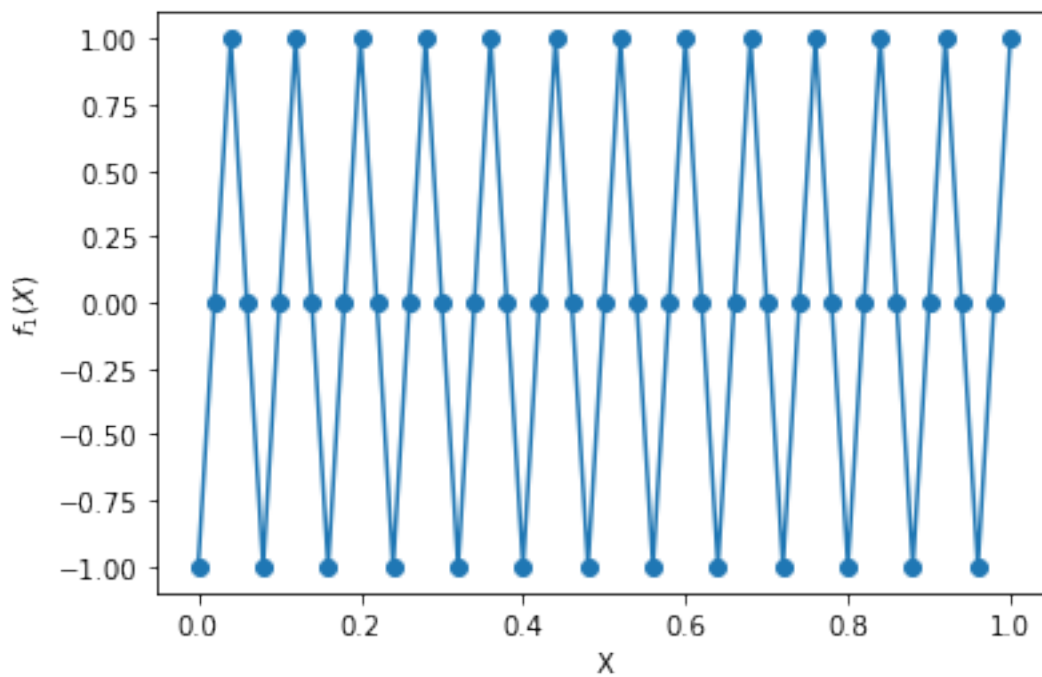


```
wheels/public/simple/  
Collecting py pandoc  
  Downloading py pandoc-1.8.1-py3-none-any.whl (20 kB)  
Installing collected packages: py pandoc  
Successfully installed py pandoc-1.8.1
```

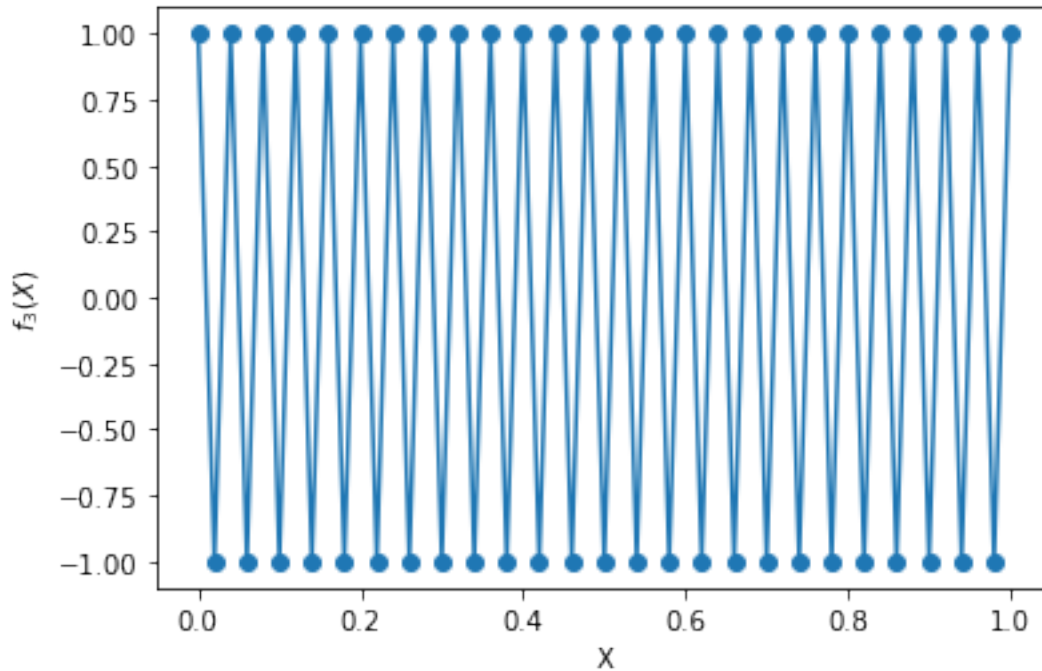
```
[2]: import numpy as np  
import matplotlib.pyplot as plt
```

### 0.0.1 2E

```
[3]: ### make toy dataset, with interpolating zeros halfway between  
N = 25  
X = np.arange(0, (2 * N) + 1) / (2 * N)  
Y = -np.zeros_like(X)  
Y[0::4] = -1.  
Y[2::4] = 1.  
  
plt.plot(X, Y)  
plt.scatter(X, Y)  
plt.xlabel("X")  
plt.ylabel("$f_1(X)$")  
plt.show()
```



```
[4]: plt.plot(X, 2 * np.abs(Y) - 1)
plt.scatter(X, 2 * np.abs(Y) - 1)
plt.xlabel("X")
plt.ylabel("$f_3(X)$")
plt.show()
```



**Piecewise Linearity:** As we see,  $f_3$  remains piecewise linear -- we've simply folded the linear interpolation that was  $f_1$  (also piecewise linear) on itself.

**Knot Count:** The number of knots increases from  $N+1$  (for the original  $0, \dots, N$ ) to  $(N+1)+N$ , with the additional zeros (originally part of the interpolation, but become knots on folding) added via  $f_2$ .

**Additional Commentary:** Here,  $f_3$  has not only let the  $x_i$  retain their positions as knots in the linear interpolation; it's also introduced knot's at the original  $f_1(x) = 0$  points, as the L1 norm makes these zeros "turnaround" points for the otherwise function -- i.e. (prior to scaling down) the L1 norm guarantees that all other transformed points, descend to, before turning around and beginning their ascent. In other words, the  $2|x| - 1$  function makes all zeros knots, which effectively doubles the number of knots in the interpolation.

### Explanation

Sentence 1: "A Two-Hidden Layer ReLu Deepnet on  $x \in R$  with any coefficients whatsoever can still be represented as a piecewise linear spline." This follows from identical argumentation to that outlined in the  $f_3$  piecewise linear question above: if you just keep adding and composing piecewise linear functions, you'll get a piecewise linear function at every step. Then, over the whole dataset, you'll have a "spline."

Sentence 2: *"If some coefficients of  $f_1$  or  $f_2$  are nonpositive, the spline might involve more knots than would have been required in the positive-coefficient case"* This follows from the initial parts of problem 2 -- when you flip the sign, you may squash part of the output to zero, or you may unsquash part of the output from zero. This act of squashing/unsquashing will induce a kink, and hence another potential knot.

Sentence 3: *"Letting  $n_i$  denote the number of knots in  $f_i$ , it might involve roughly as many as  $(n_1 + 1) \cdot (n_2 + 1)$  knots"*. First, in  $f_1$ , we'll have approximately  $n_1$  knots from the first layer: we get this by summing  $n_1$  piecewise linear functions/relus. Then, for each  $j = 1, \dots, n_2$ , we examine

$$c_j^2 \cdot \text{relu}(f_1(x) - b_j^2).$$

This will take the  $n_1$  knots from  $f_1$ , and potentially adjust each not (depending on  $c_j^2, b_j^2$ ) to get  $n_1$  "updated" knots that are specific to the  $j$  index. Do this  $n_2$  times for each  $j$ , and now you have up to  $n_2$  sets of  $n_1$  "j-specific" knots. Hence, it's  $\mathcal{O}(n_1 \cdot n_2)$  total knots.

3.)

A.)

```
[5]: import itertools
import math

plt.rcParams["figure.figsize"] = (7, 7)

U_fn = lambda i: np.array([
    math.cos(2 * math.pi * (i - 1) / 3),
    math.sin(2 * math.pi * (i - 1) / 3)
])

U = np.vstack([U_fn(1), U_fn(2), U_fn(3)])

X = np.vstack(
    [
        (i, j)
        for (i, j)
        in itertools.product(
            np.linspace(-3, 3, 250),
            np.linspace(-3, 3, 250)
        )
    ]
)

def relu(x):
    x = x.copy()
    x[x < 0] = 0.
    return x

def f1(x):
```

```

x = x.copy()
x[:, 0] -= 1
x[:, 1] += 1
return relu(x)

def f2(x):
    return relu(x @ (U.T) - 1.).sum(axis=1)

```

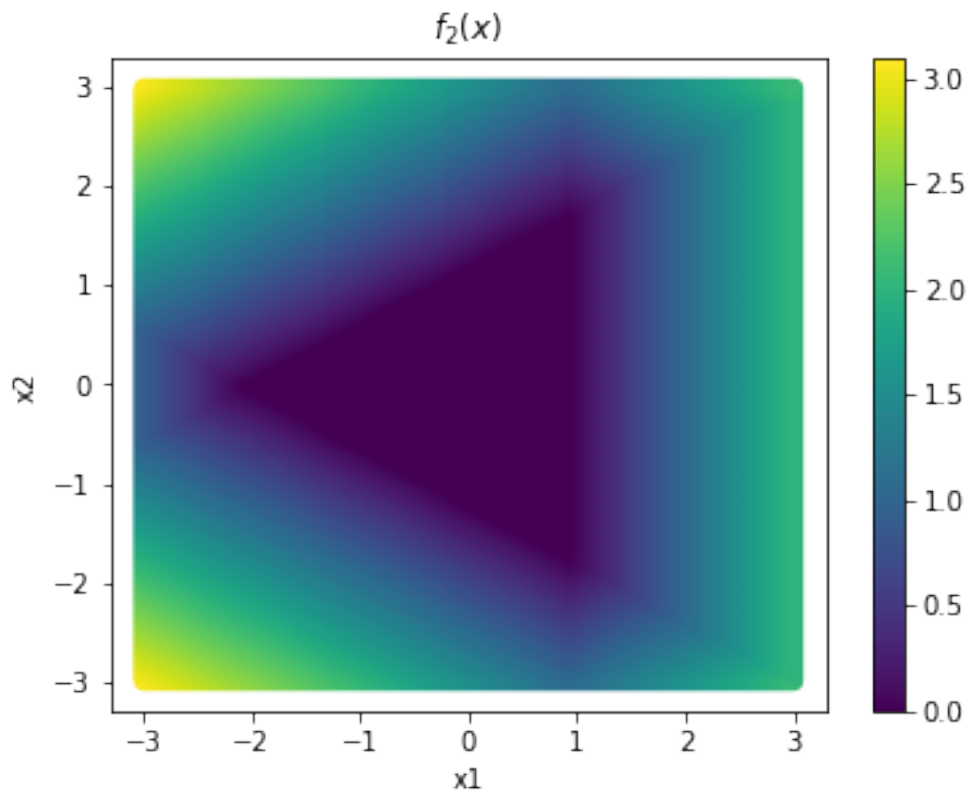
```
[6]: plt.rcParams["figure.figsize"] = (6, 4.5)
```

```

y2 = f2(X)

plt.scatter(X[:, 0], X[:, 1], c=y2)
plt.colorbar()
plt.xlabel("x1")
plt.ylabel("x2")
plt.title("$f_2(x)$")
plt.show()

```

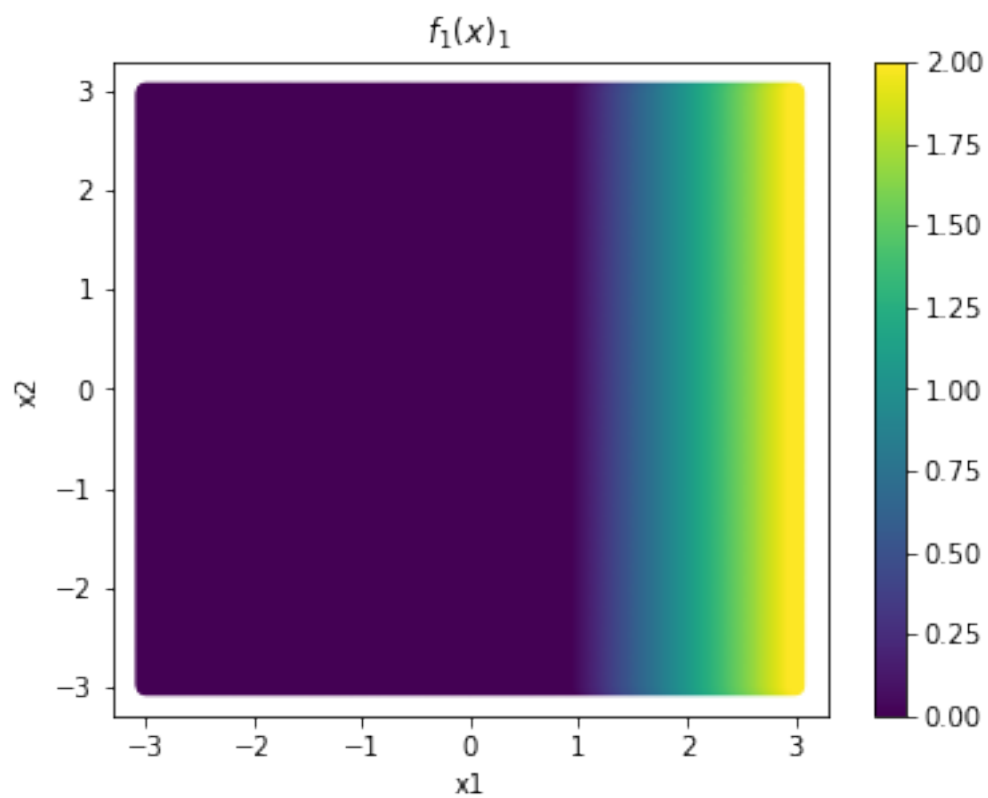


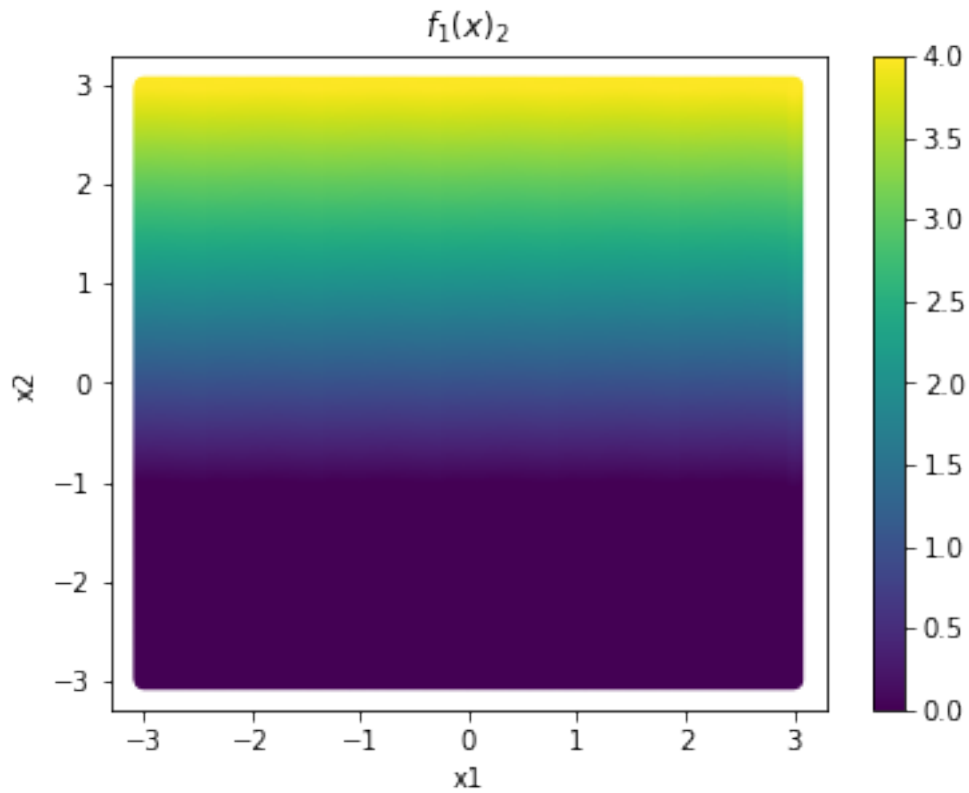
### 0.0.2 B.)

```
[7]: y1 = f1(X)

plt.scatter(X[:, 0], X[:, 1], c=y1[:, 0])
plt.colorbar()
plt.xlabel("x1")
plt.ylabel("x2")
plt.title("$f_1(x)_1$")
plt.show()

plt.scatter(X[:, 0], X[:, 1], c=y1[:, 1])
plt.colorbar()
plt.xlabel("x1")
plt.ylabel("x2")
plt.title("$f_1(x)_2$")
plt.show()
```





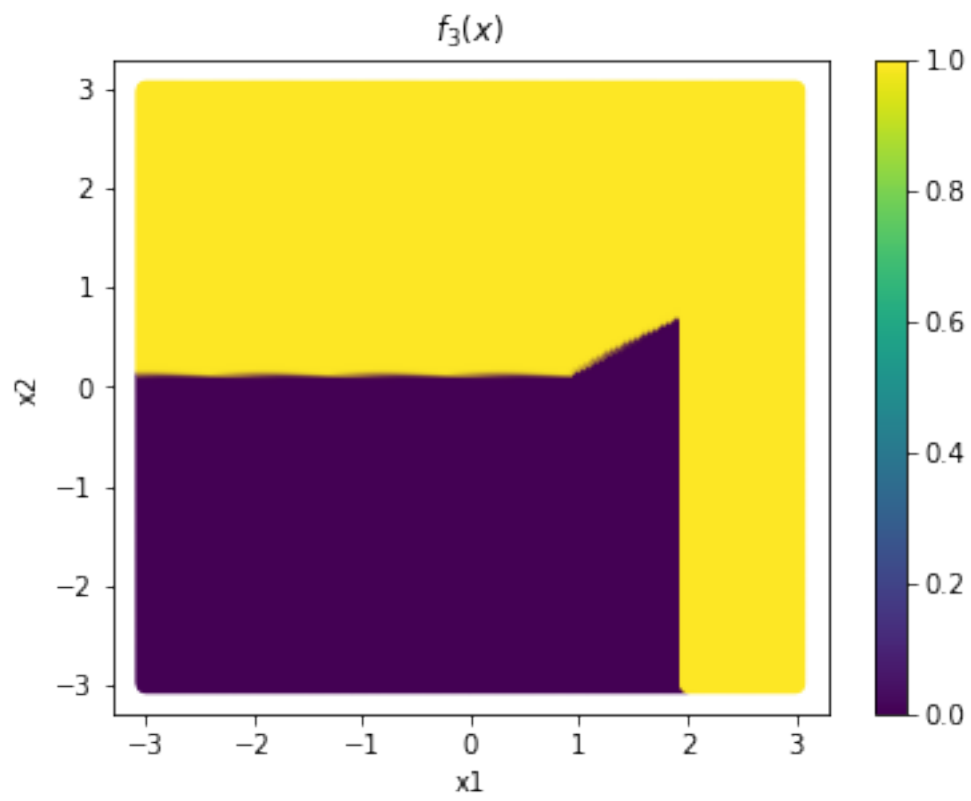
### 0.0.3 3C.

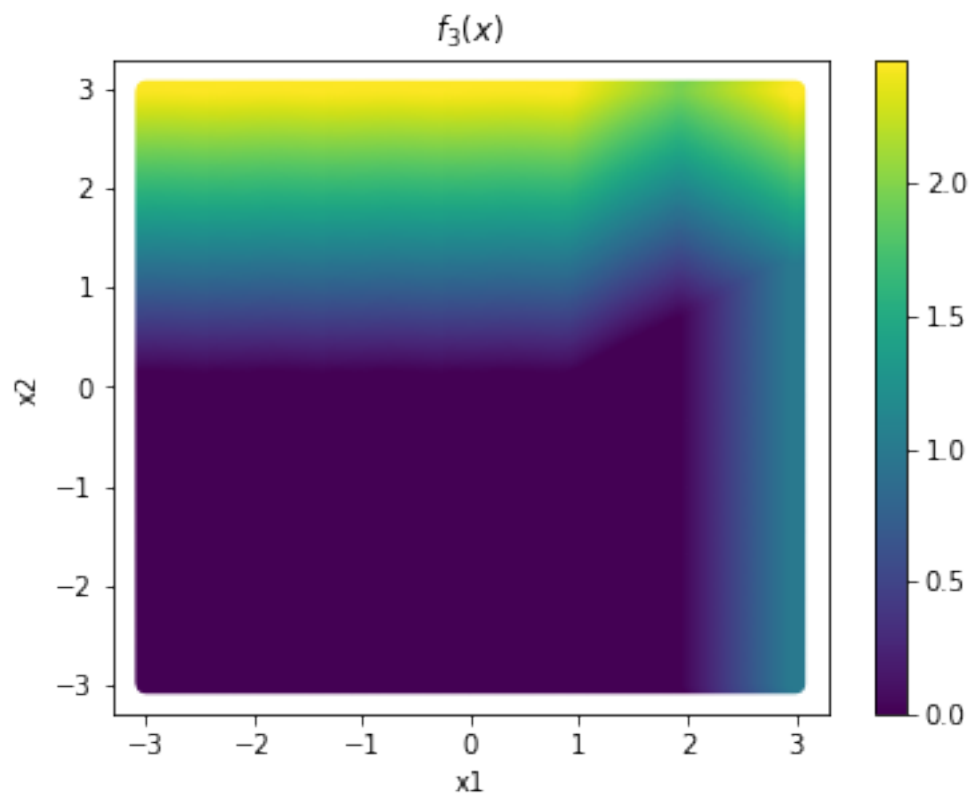
```
[8]: y3 = (f2(f1(X)) > 0 ).astype(int)

plt.scatter(X[:, 0], X[:, 1], c=y3)
plt.colorbar()
plt.xlabel("x1")
plt.ylabel("x2")
plt.title("$f_3(x)$")
plt.show()

y3 = f2(f1(X))

plt.scatter(X[:, 0], X[:, 1], c=y3)
plt.colorbar()
plt.xlabel("x1")
plt.ylabel("x2")
plt.title("$f_3(x)$")
plt.show()
```





#### 0.0.4 D.)

Given the simplicity of the ReLU, the cell decomposition is straightforward:

- The line  $x_2 = -1$  splits  $f_1(x)_2$  into two cells.
- The line  $x_1 = 1$  splits  $f_1(x)_1$  into two cells.

#### 0.0.5 E.)

Observe



$$f_2(x) = Relu(cos(0)x_1 + sin(0)x_2 - 1) \quad (1)$$

$$+ Relu(cos(2\pi/3)x_1 + sin(2\pi/3)x_2 - 1) \quad (2)$$

$$+ Relu(cos(4\pi/3)x_1 + sin(4\pi/3)x_2 - 1) \quad (3)$$

$$= Relu(x_1 - 1) \quad (4)$$

$$+ Relu(-\frac{1}{2}x_1 + \frac{\sqrt{3}}{2}x_2 - 1) \quad (5)$$

$$+ Relu(-\frac{1}{2}x_1 - \frac{\sqrt{3}}{2}x_2 - 1). \quad (6)$$

From this, we see that the cell boundaries (i.e. activation thresholds) must correspond to the lines

- $x_1 - 1 > 0 \implies x_1 = 1$
- $-\frac{1}{2}x_1 + \frac{\sqrt{3}}{2}x_2 - 1 > 0 \implies x_2 = \frac{x+2}{\sqrt{3}}$
- $-\frac{1}{2}x_1 - \frac{\sqrt{3}}{2}x_2 - 1 > 0 \implies x_2 = -\frac{x+2}{\sqrt{3}}.$

Hence the lines

$$\left\{ x_1 = -1, x_2 = \frac{x+2}{\sqrt{3}}, x_2 = -\frac{x+2}{\sqrt{3}} \right\}$$

define the cell decomposition.

### 0.0.6 F.)

As we are free to choose any subset/region  $A$  on  $\mathbb{R}^2$ , consider  $A = \{x : x_1 < 1, x_2 < -1\}$ . By the properties of  $f_1$ , we shall always have for  $x \in A$

$$f_1(x) = [Relu(x_1 - 1), Relu(x_2 + 1)] = [0, 0].$$

In turn, we are guaranteed

$$f_2([0, 0]) = Relu(0 - 1) + Relu(0 - 1) + Relu(0 - 1) = 0$$

, so for any  $x \in A = \{x : x_1 < 1, x_2 < -1\}$ , we will have

$$f_3(0) = 0 + \mathbf{0}^T x.$$

```
[ ]: from google.colab import drive
drive.mount('/content/drive')
```

```
[ ]: !cp "drive/MyDrive/Colab Notebooks/STAT205_HW5_supp.ipynb" ./
```

```
[ ]: !jupyter nbconvert --to PDF "STAT205_HW5_supp.ipynb"
```