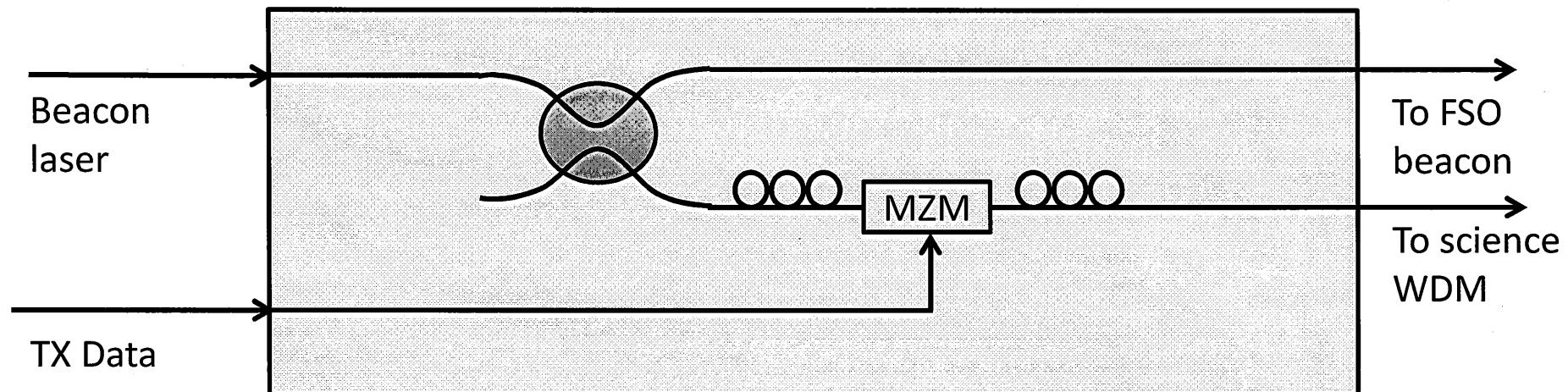
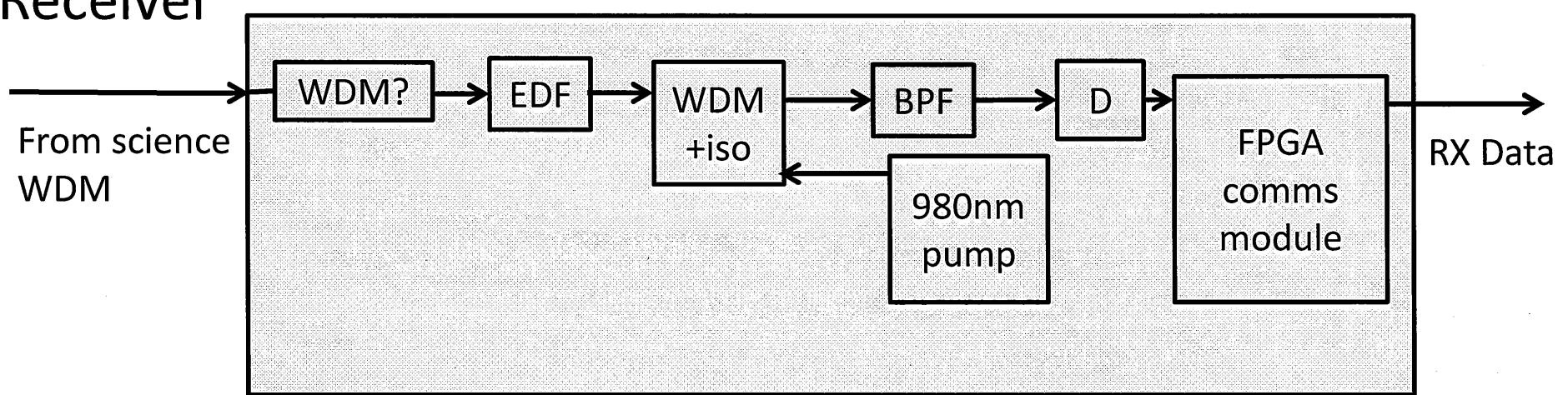


Incoherent comms, separate RX and TX

Transmitter

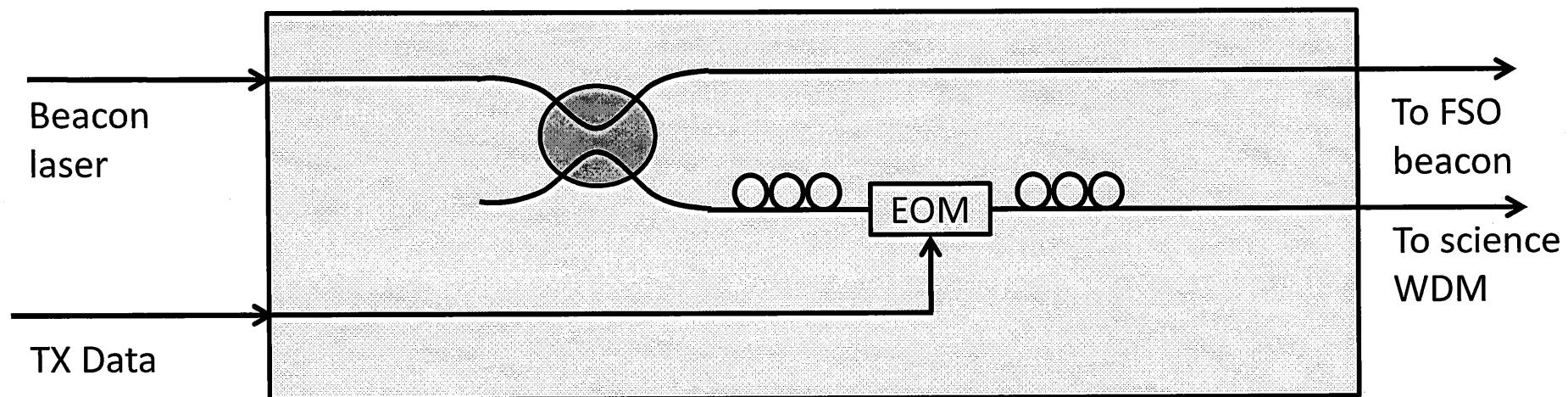


Receiver

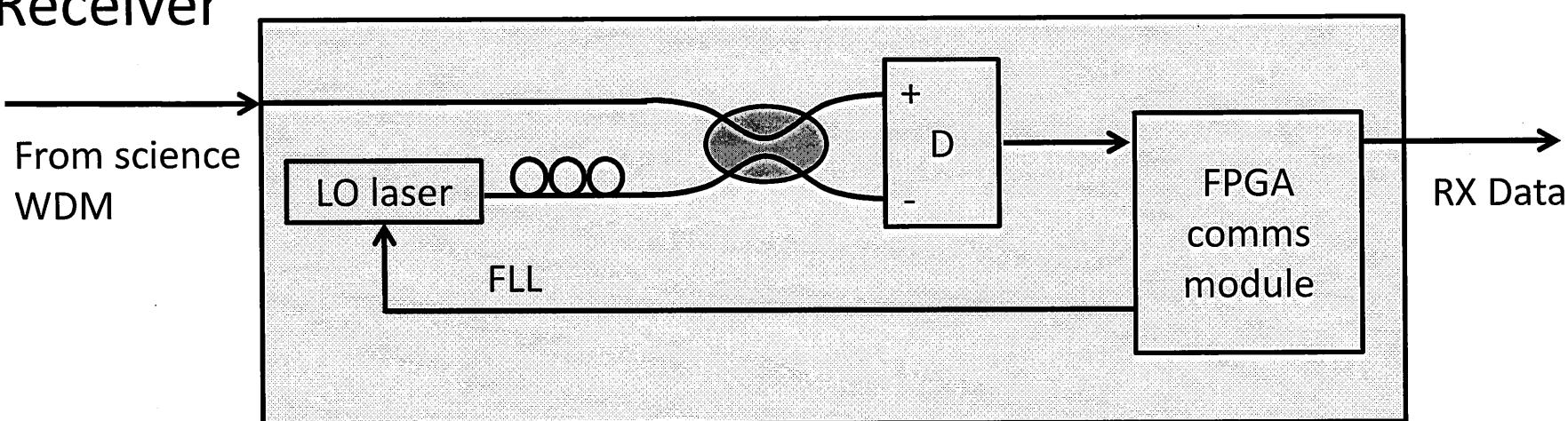


Coherent comms, separate RX and TX

Transmitter



Receiver



Noise on ϕ for 8 fractional bits:

$$\Delta = \frac{2\pi}{2^8}, \text{ rad}$$

$$V_\phi = \sqrt{\left(\frac{\Delta}{12}\right)^2} = 7 \text{ mrad}$$

$$\xi_\phi = \left(\frac{1 \text{ mrad}}{\sqrt{Hz}}\right)$$

$$\zeta_\phi = -120 \text{ dBc}$$

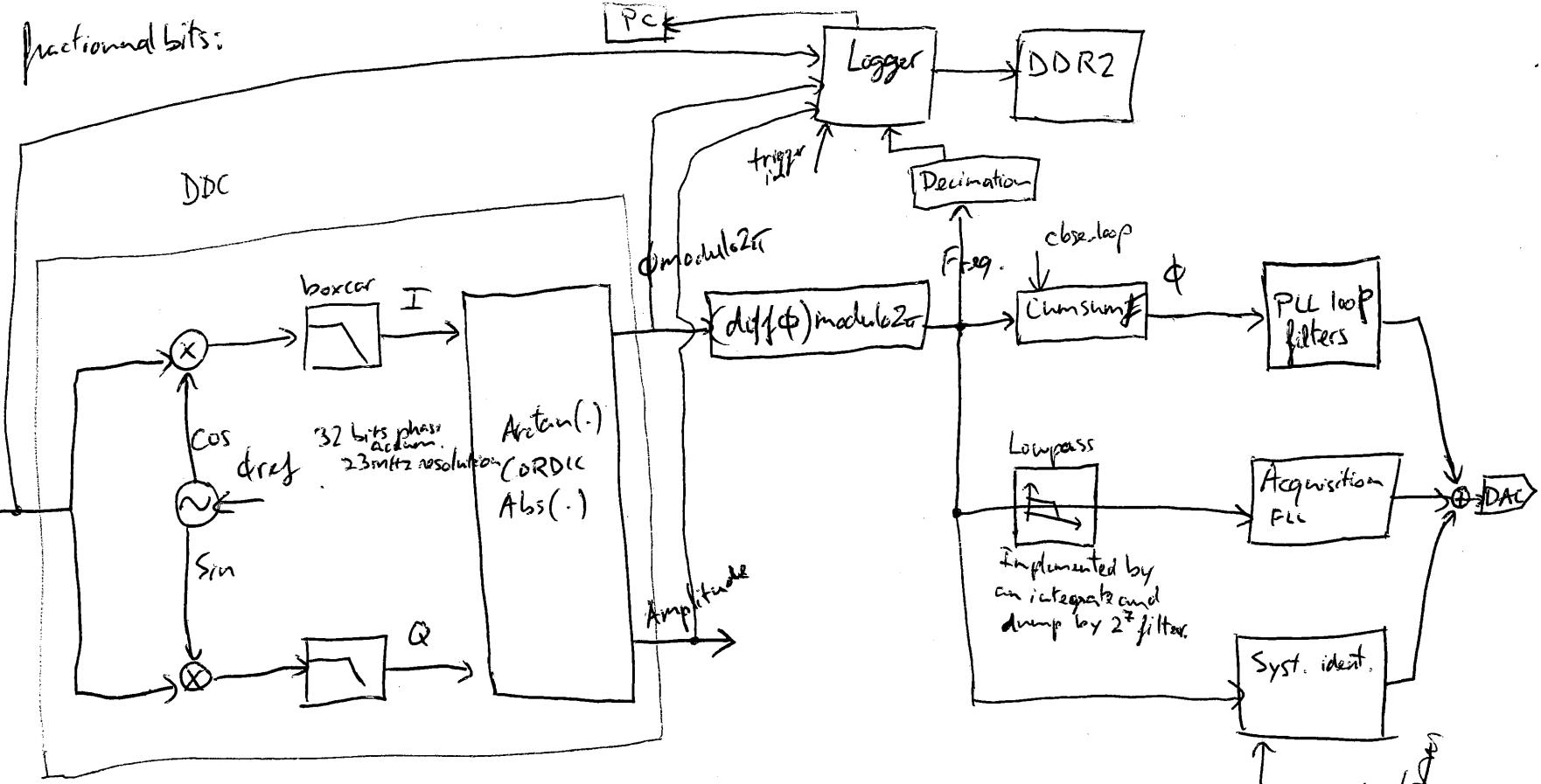


→ Phase noise of ADC at low offset frequencies?

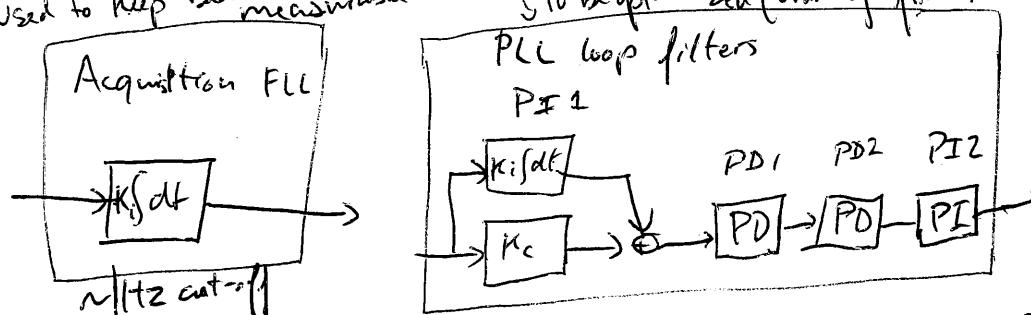
→ Online removal by measuring reference tone (f_{ref}) and feeding back to dref. (Later)

→ Can we enable a smooth change in PLL parameters

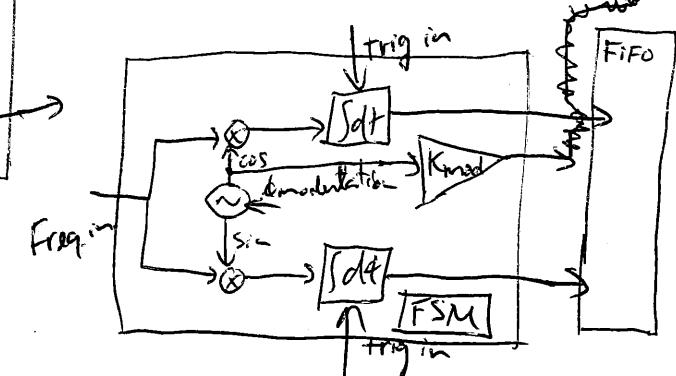
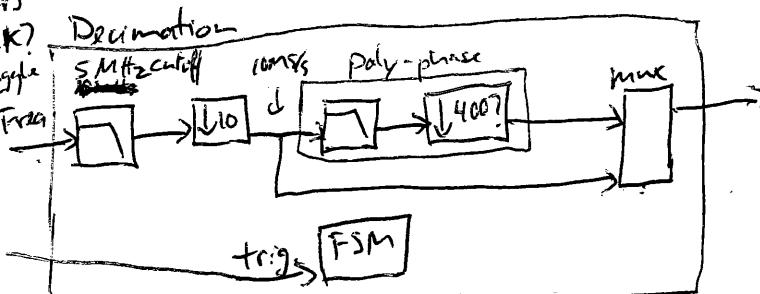
with breaking lock?
→ Twice the block toggle between the two after convergence?

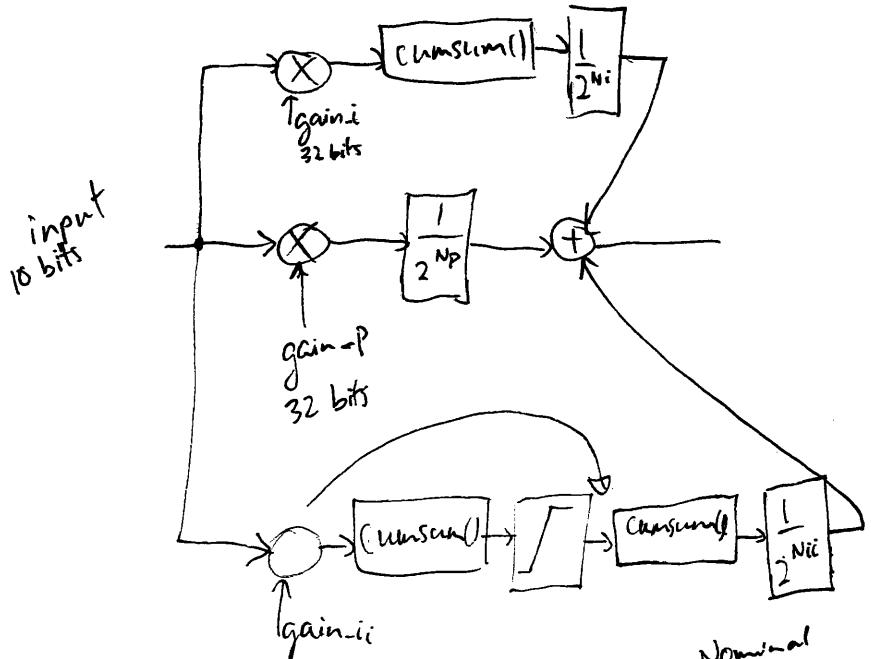


Used to keep beat note within measurable range
↳ to be optimized (order of filters, bit sizes)



System identification





$$H(f) = \frac{K_p}{2^{N_p}} + \frac{K_i}{2^{N_i}} \cdot H_{\text{cumsum}}(f)$$

$$H_{\text{cumsum}}(f) = \frac{j_s}{j^2 2\pi f}$$

P branch:

$$\text{min gain} = H_{p\min} = \frac{1}{2^{N_p}} = \frac{1}{2^{16}}$$

$$\text{max gain} = H_{p\max} = \frac{2^{31}-1}{2^{N_p}} \approx 2^{15}$$

I branch:

$$\text{min gain} = H_{i\min} = \frac{1}{2^{N_i}} \cdot \frac{j_s}{j^2 2\pi f}$$

$$N_i = 16 + 12 = 28$$

$$\text{max gain} = H_{i\max} = \frac{2^{31}}{2^{N_i}} \cdot \frac{j_s}{j^2 2\pi f} = \frac{2^3 \cdot j_s}{j^2 2\pi f}$$

I^2 branch:

$$H_{ii} = \frac{K_{ii}}{2^{N_{ii}}} \cdot (H_{\text{cumsum}}(f))^2$$

$$= \frac{K_{ii}}{2^{N_{ii}}} \cdot \frac{j_s^2}{j^2 (2\pi f)^2}$$

$$K_i = \frac{\text{gain-}i \cdot j_s}{2^{N_i}}$$

$$\text{Nominal } K_i = 95e6$$

$$2^{N_i} = \frac{\text{gain-}i \cdot 2^{24}}{95e6} \sim 2^{24}$$

$$K_{ii} = \frac{\text{gain-}ii \cdot j_s^2}{2^{N_{ii}}} \quad \text{Nominal } K_{ii} = 4e12$$

$$2^{N_{ii}} = 2^{24} \cdot 2500 = 2^{24+11} = 2^{35}$$

$$H_i = \frac{f_s \cdot K_i}{j^2 \pi f}$$

$$H_{ii} = \frac{f_s^2 \cdot K_{ii}}{j^2 (2\pi f)^2}$$

$$H_i = H_{ii} = K_i = \frac{f_s \cdot K_i}{j^2 \pi f}$$

$$K_{ii} = \frac{j^2 \pi f}{2^{N_{ii}}} \cdot K_i$$

vs

$$\frac{1}{2^{N_p}}$$

$$\frac{1}{2^{N_i}} \cdot \frac{j_s}{j^2 \pi f}$$

$$\frac{2^{31}}{2^{N_{ii}}} \cdot \frac{j_s}{j^2 \pi f}$$

Case 1: f_i relative to 0dB.



$$P_{\text{gain}} = k_p \cdot k_p \\ \text{integrator gain} = \frac{2\pi f_i}{2\pi f} \\ I^2 \text{ gain} = \frac{2\pi f_i}{2\pi f} \cdot \frac{2\pi f_{ii}}{2\pi f}$$

Critical points:

Actual transfer function is:

We want:

$$I: H_i = \text{gain}_i \cdot \frac{f_s}{2\pi f}$$

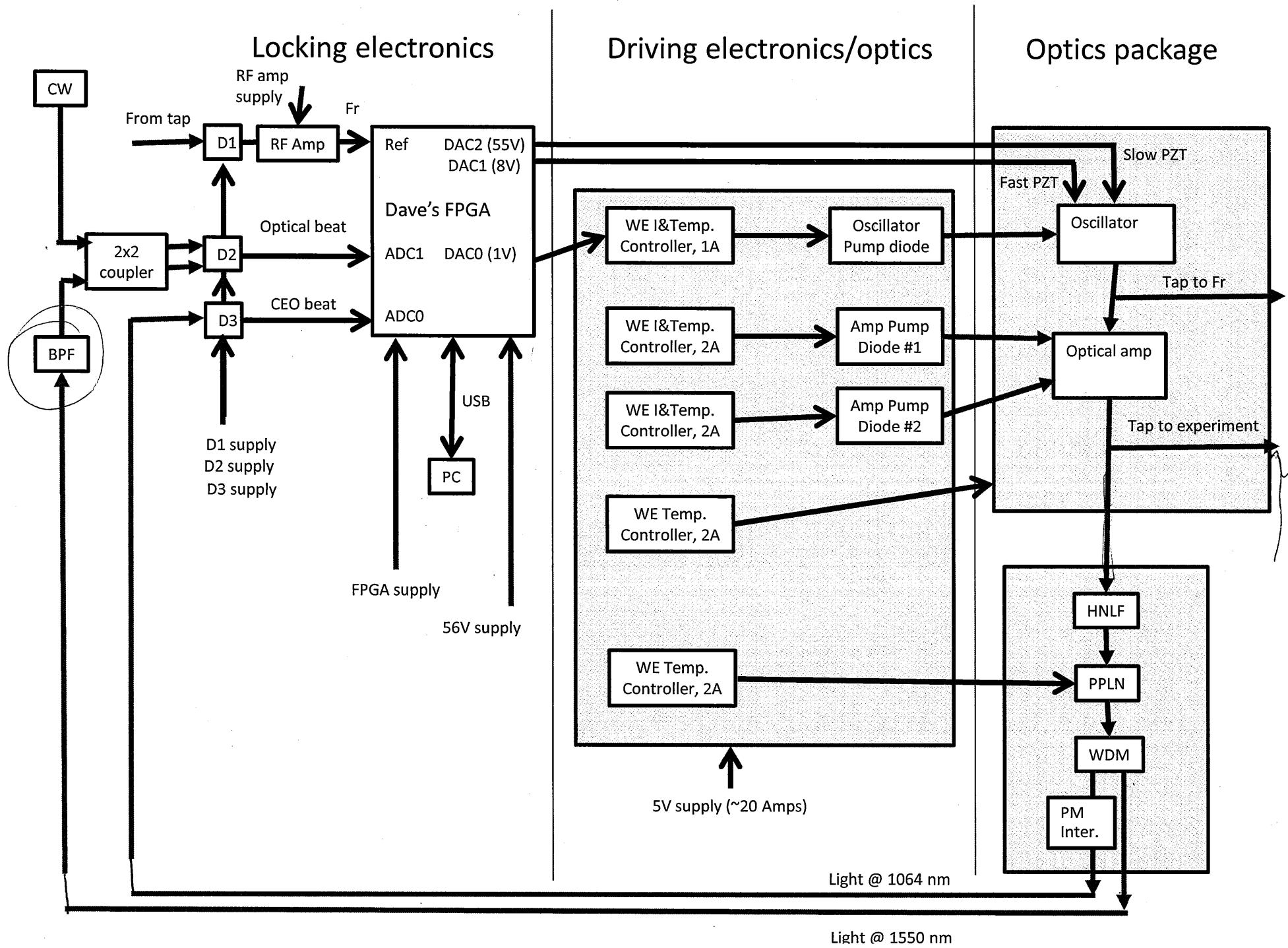
$$H_i = \frac{1}{k_c} \cdot \frac{2\pi f_i}{2\pi f} \Rightarrow \text{gain}_i = \frac{1}{k_c} \cdot \frac{2\pi f_i}{f_s}$$

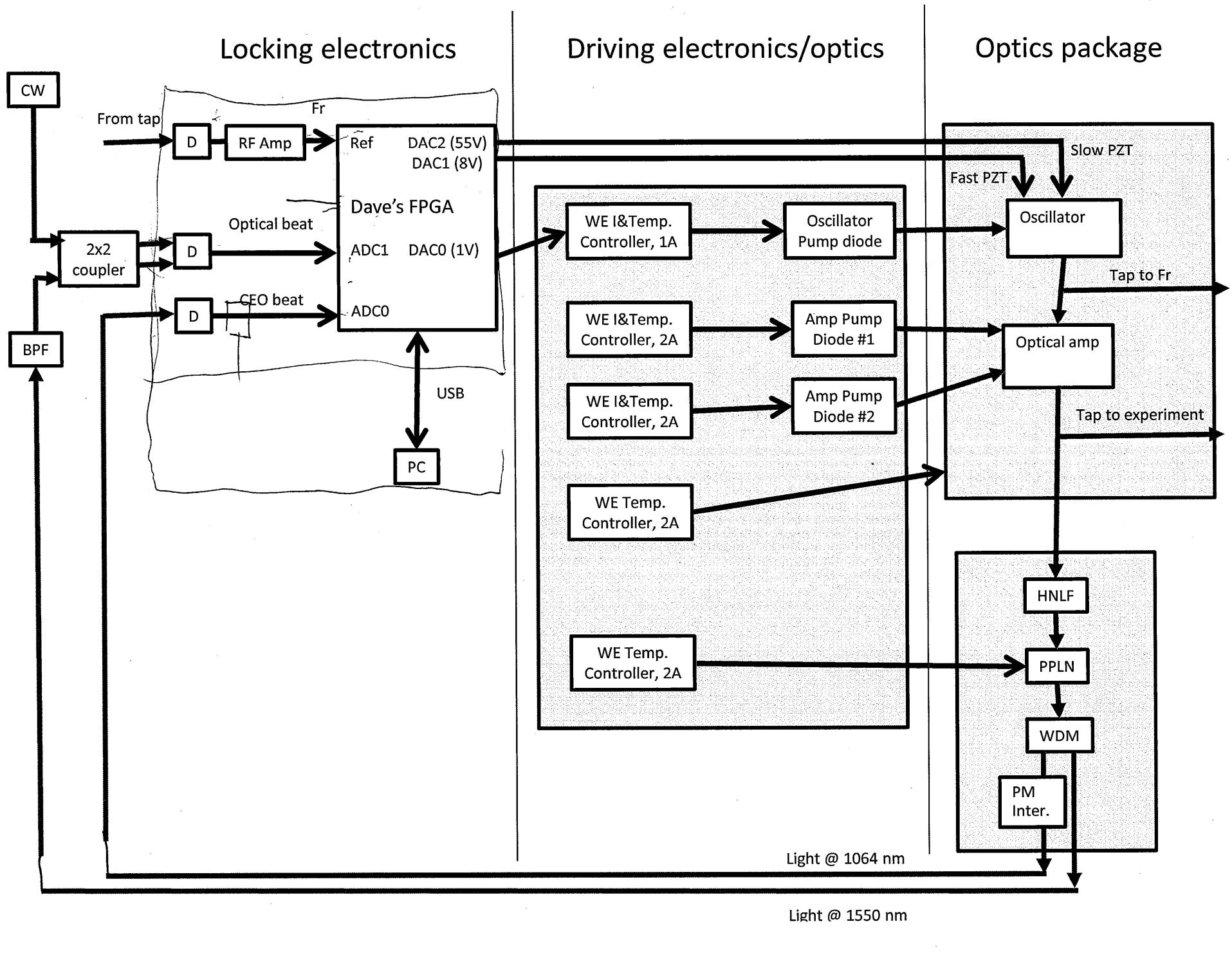
$$I^2: H_{ii} = \text{gain}_{ii} \cdot \left(\frac{1}{2\pi f} \right)^2$$

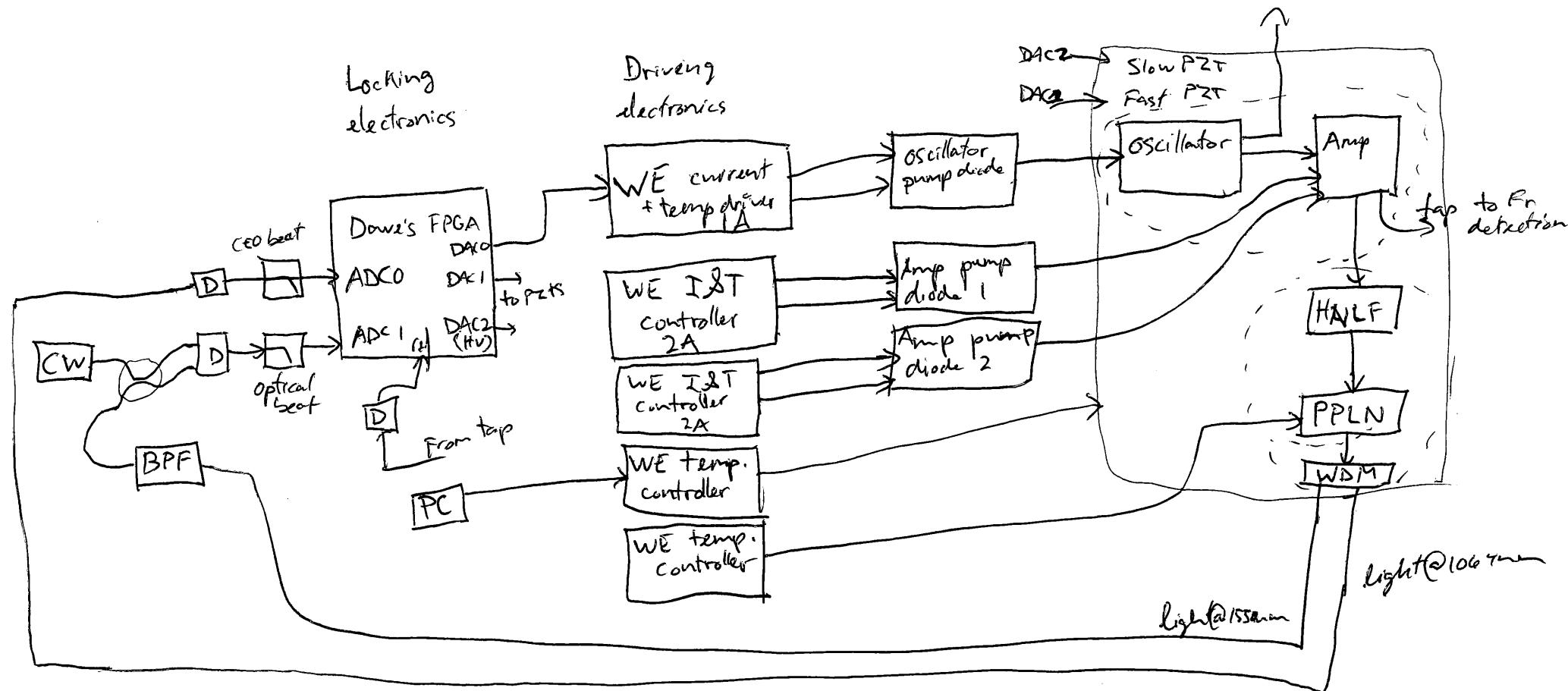
$$H_{ii} = \frac{1}{k_c} \cdot \frac{2\pi f_i}{2\pi f} \cdot \frac{2\pi f_{ii}}{2\pi f} \Rightarrow \text{gain}_{ii} = \frac{1}{k_c} \cdot f_i \cdot f_{ii} \cdot \left(\frac{2\pi}{f_s} \right)^2$$

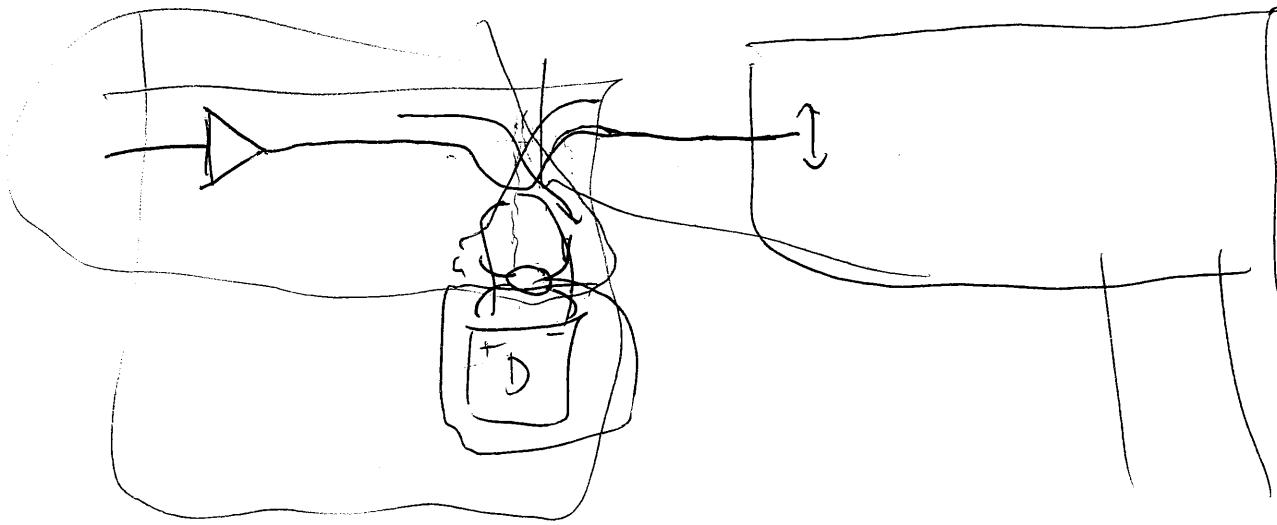
We want:

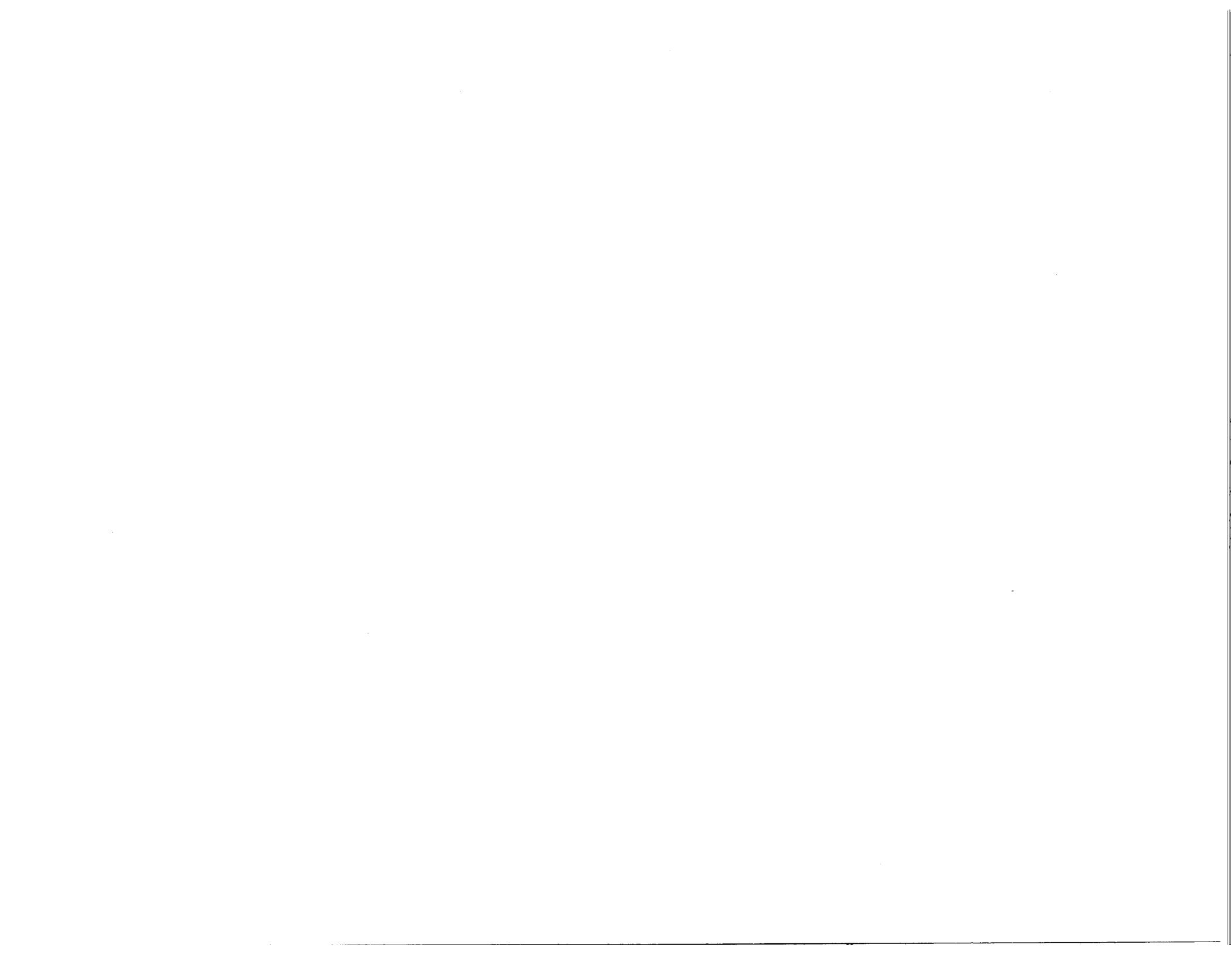
main
 $s1 = \text{SuperWidget}()$ (creates the object. first ref)
 $xdm = XEM_name_window(s1)$ keeps a reference to s1
 child widget: Loop, filters UI keeps a reference to s1

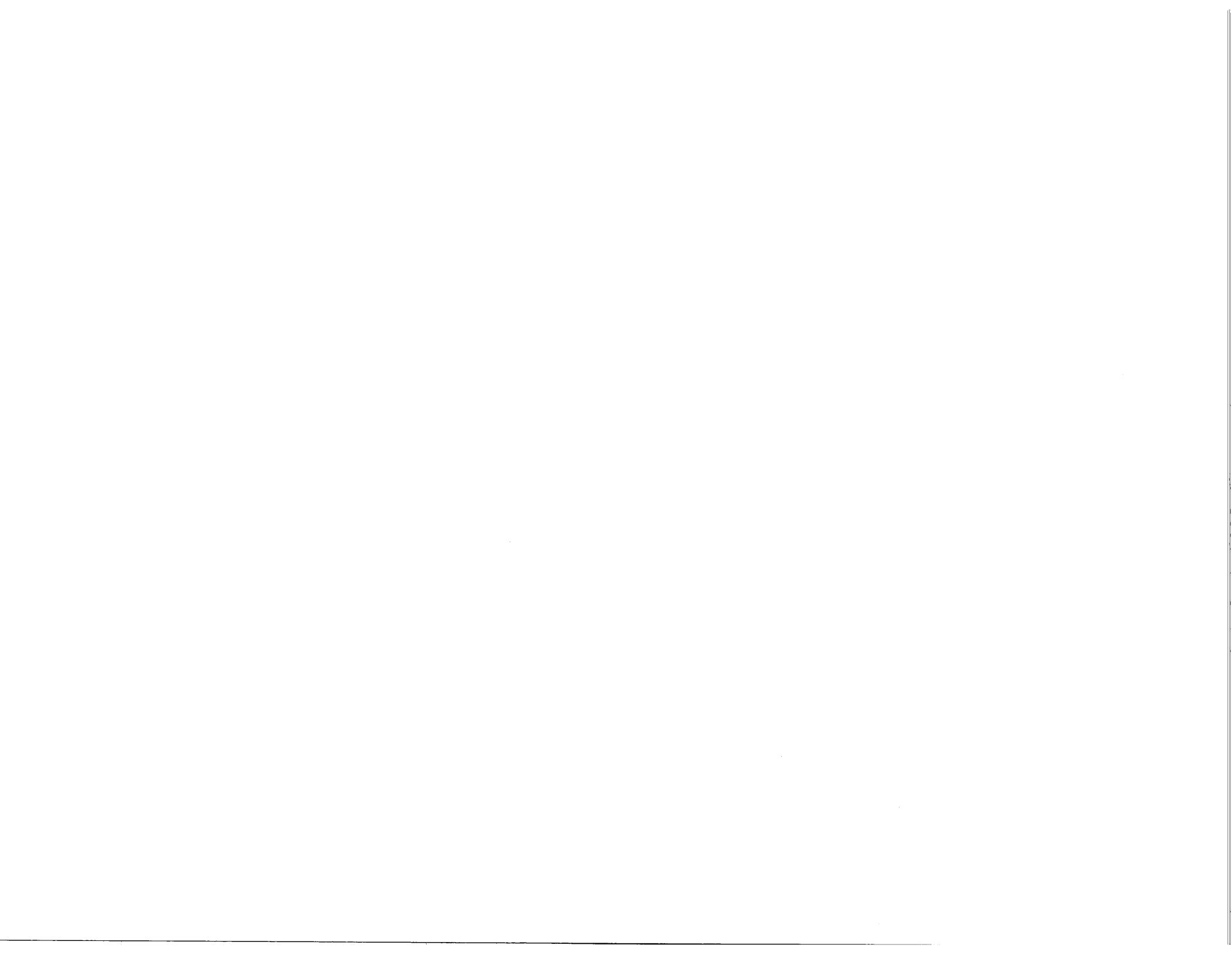




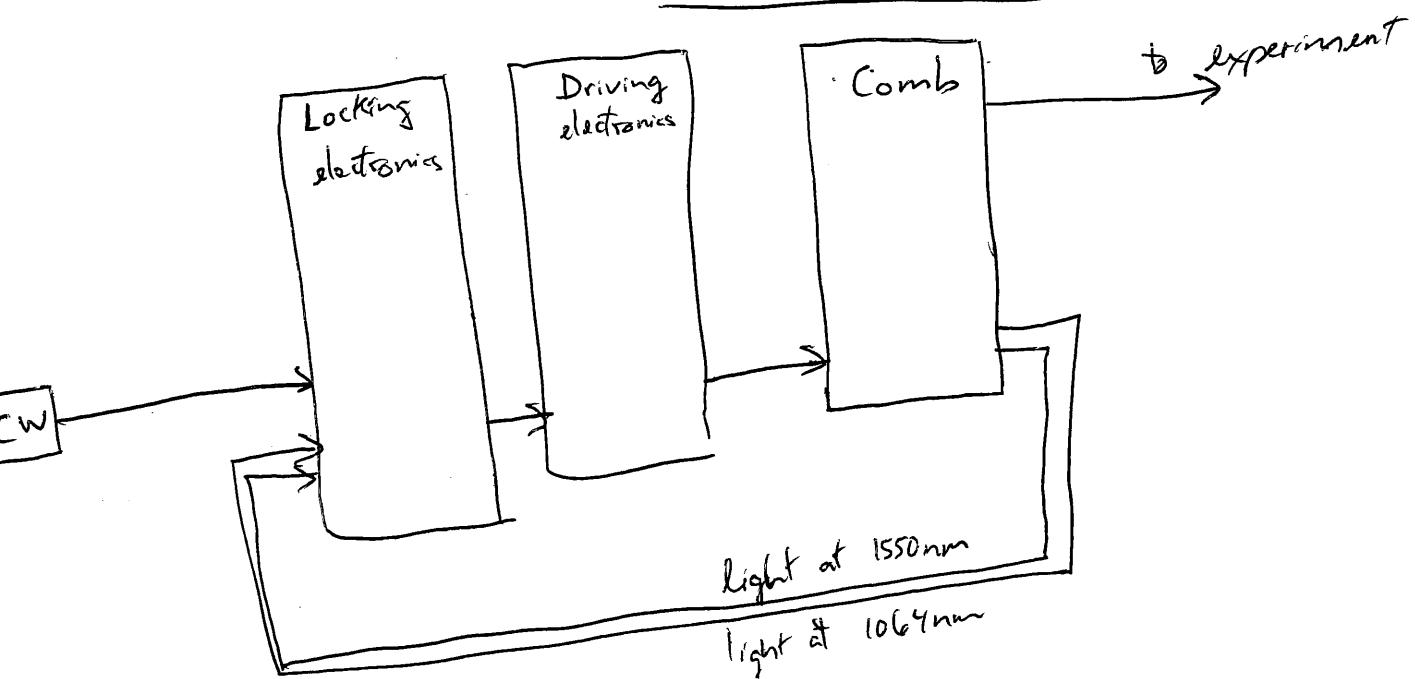


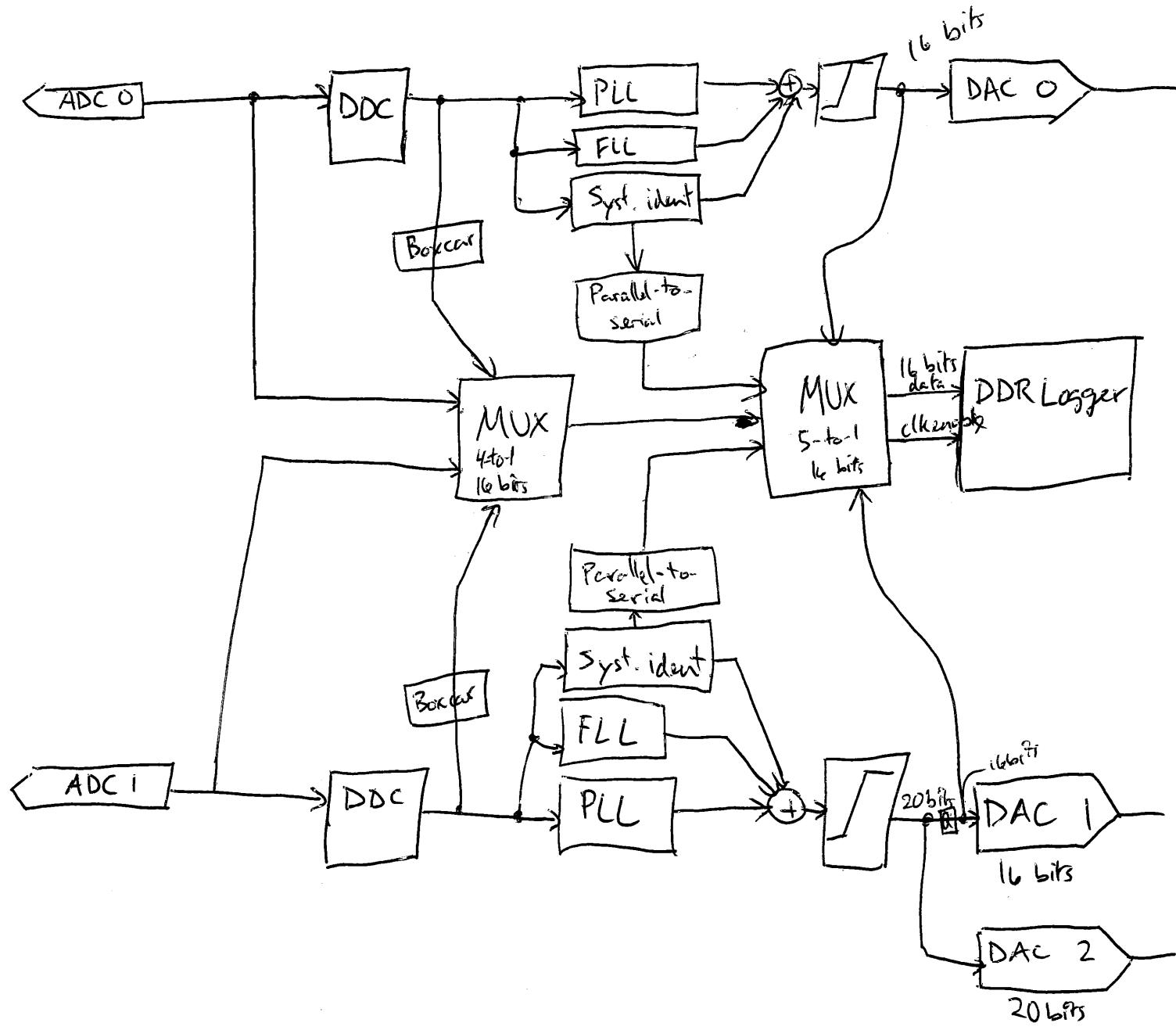


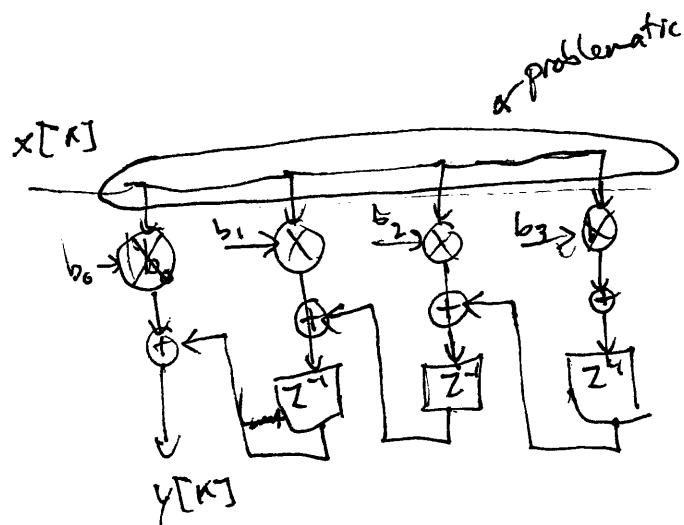




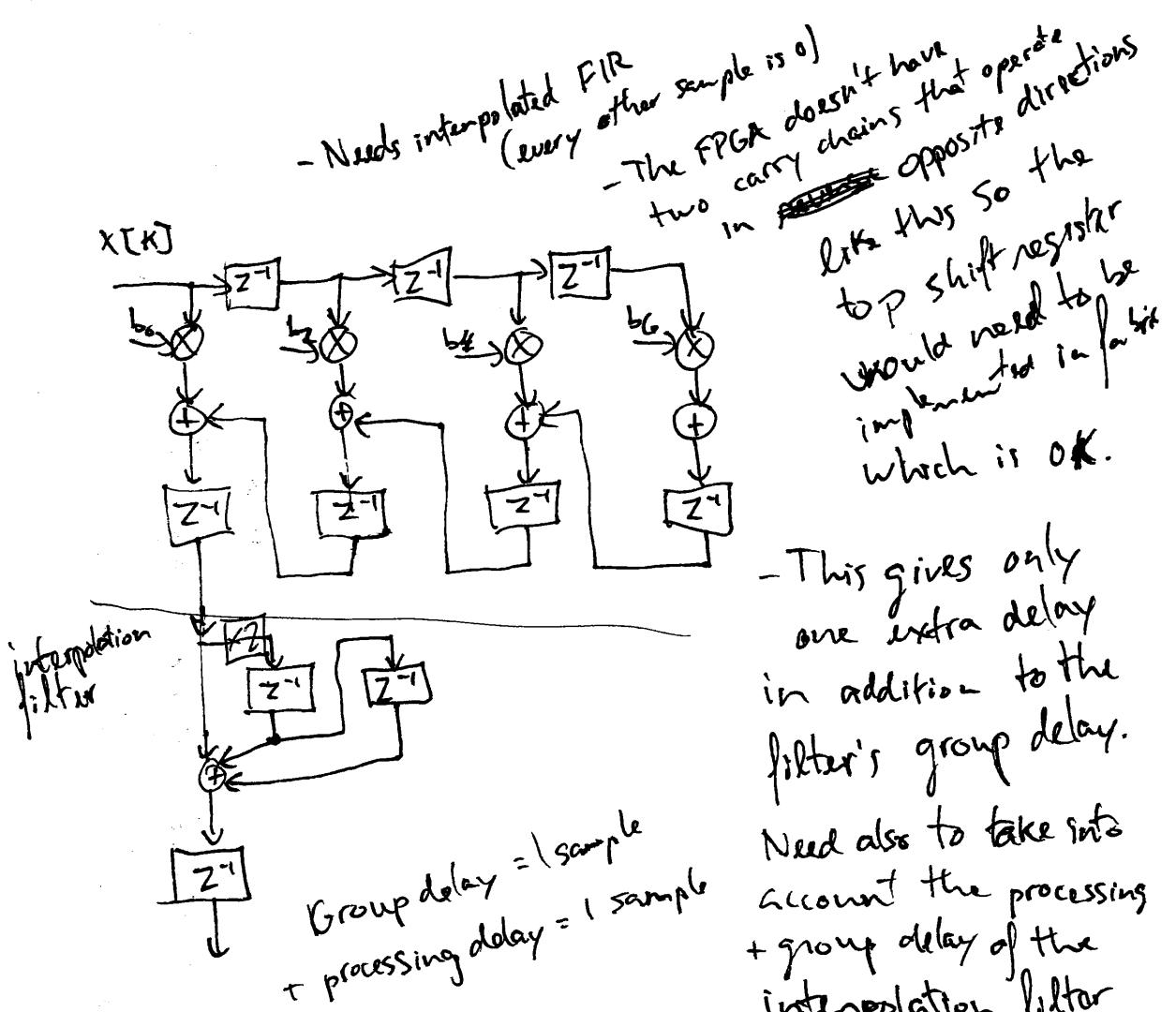
High-level diagram







- AM-to-PM conversion in the presence of out-of-band phase noise conversion factor is given by $\text{rms relative phase jitter over the full bandwidth}$ and slope of tone error.
 $\propto \Delta \theta_{\text{RMS}}$ of (maybe with 2π)
Probably ^{almost} same as static phase error.



- Needs interpolated FIR
(every other sample is 0)
 - The FPGA doesn't have two carry chains that operate in ~~opposite~~ opposite directions like this so the top shift register would need to be implemented in ~~a~~ a bit which is OK.
 - This gives only one extra delay in addition to the filter's group delay.

First-order filter:

$$H(f) = \frac{1}{1 + jf/f_c}$$

Phase below cut-off:

$$\phi(f) \approx -\pi f/f_c - \pi f/f_c$$

$$\text{Equivalent delay: } T_{eq} \approx \frac{1}{2\pi f_c}$$

or $f_c = 200 \text{ kHz}$,

$$T_{eq} \approx 800 \text{ ns}$$

*Gain increase
as double integrator
for freq below
(2nd pole)*

$$\begin{aligned} H_c(s) &= \left(\frac{T_1 s + 1}{T_1 s}\right) K \left(1 + \frac{1}{T_{ii} s}\right) \\ &= K \left(1 + (T_1 s)^{-1}\right) \left(1 + (T_{ii} s)^{-1}\right) \\ &= K \left(1 + \cancel{\frac{1}{T_1}} \left(T_1 + T_{ii}\right) s^{-1} + T_1 T_{ii} s^{-2}\right) \end{aligned}$$

$$K_p = K$$

$$K_i = \frac{K}{(T_1 + T_{ii})}$$

$$K_{ii} = \frac{K}{T_1 T_{ii}}$$

Pure delay:

$$H(f) = \exp(j2\pi f T)$$

Phase:

$$\phi(f) = -2\pi f T$$

Our current processing delay $T = 570 \text{ ns}$
(includes filters group delay)

$$H_p(f) = \frac{1}{1 + jf/f_1} \cdot \frac{1}{1 + jf/f_2}$$

$$H_c(f) = \frac{1 + jf/f_2}{1}$$

$$H_p(s) = \cancel{K_p T_1 s} \frac{1}{T_1 s + 1} \cdot \frac{1}{T_2 s + 1}$$

$$H_c(s) = \frac{T_2 s + 1}{1} \cdot \frac{T_1 s + 1}{\cancel{T_1 s}} \cdot K$$

*Compensation
for 2nd pole*

$$G(s) = H_p(s) H_c(s) = \frac{K}{T_2 s}$$

$$H_{cl}(s) = \frac{G(s)}{1 + G(s)} = \frac{(T_2 s)^{-1} K}{1 + (T_1 s)^{-1} K} = \frac{1}{\left(\frac{T_1}{K}\right) s + 1}$$

$$H_c(s) = K_p + \frac{K_i}{s} + K_d s$$

~~$$H_c(s) = \left(\frac{1}{T_1} + T_2 s^2 + T_1 s\right) \cdot \frac{1}{T_1 s}$$~~

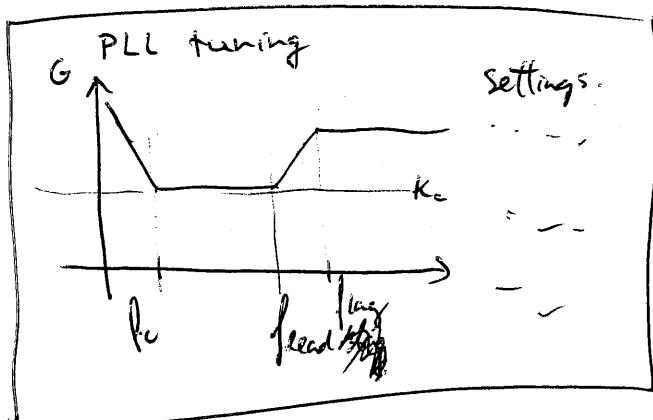
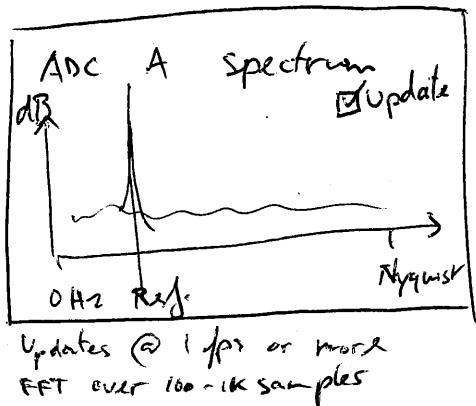
$$\begin{aligned} H_c(s) &= (T_2 s + 1)(1 + (T_1 s)^{-1}) K \\ &= \left(T_2 s + 1 + \frac{T_1}{T_2} + \frac{1}{T_1 s}\right) K \\ &= \left(1 + \frac{T_1}{T_2}\right) K + \frac{K}{T_1 s} + K \cdot T_2 s \end{aligned}$$

$$K_d = K T_2$$

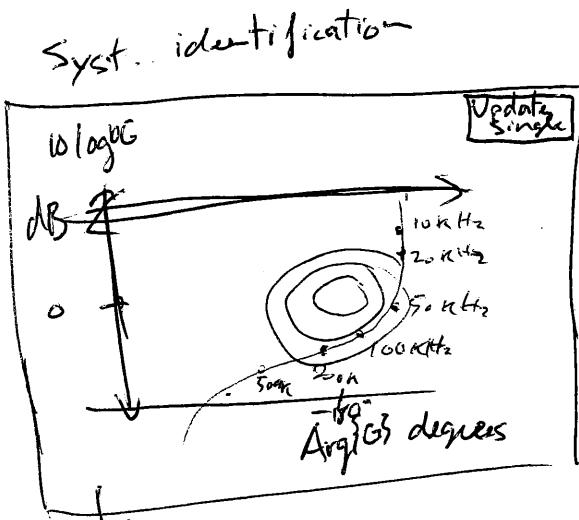
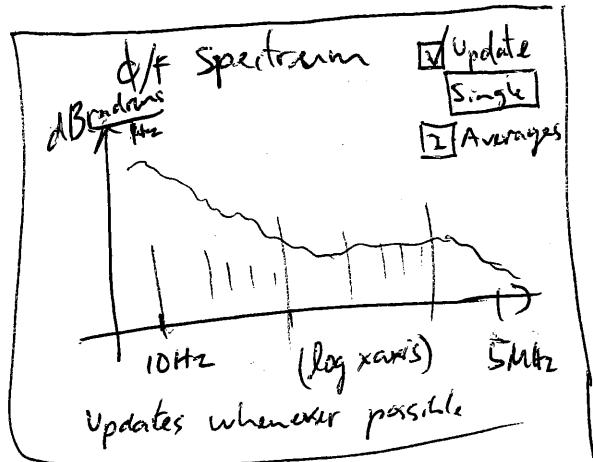
$$K_i = \frac{K}{T_1}$$

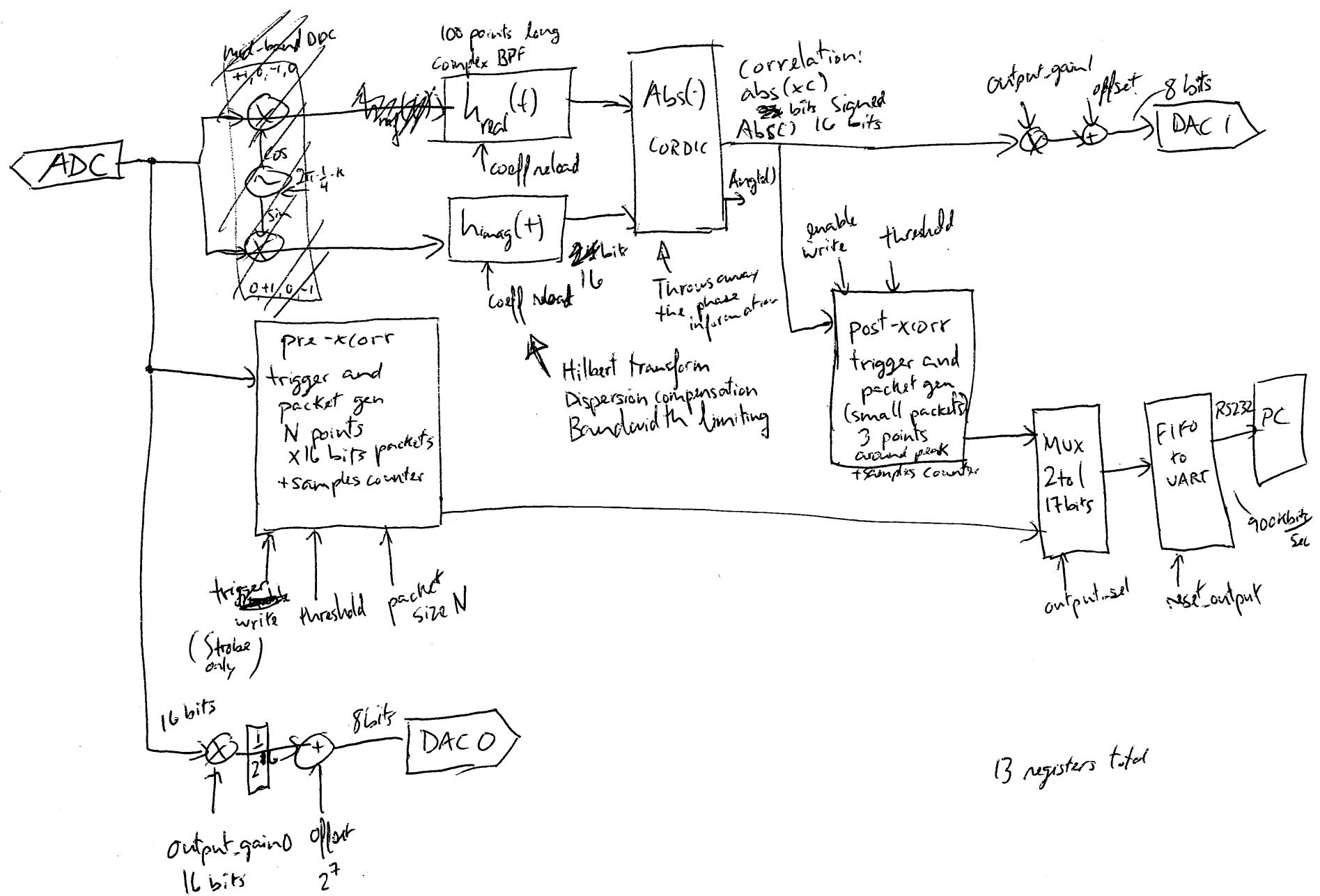
$$K_p = \left(1 + \frac{T_1}{T_2}\right) K_c$$

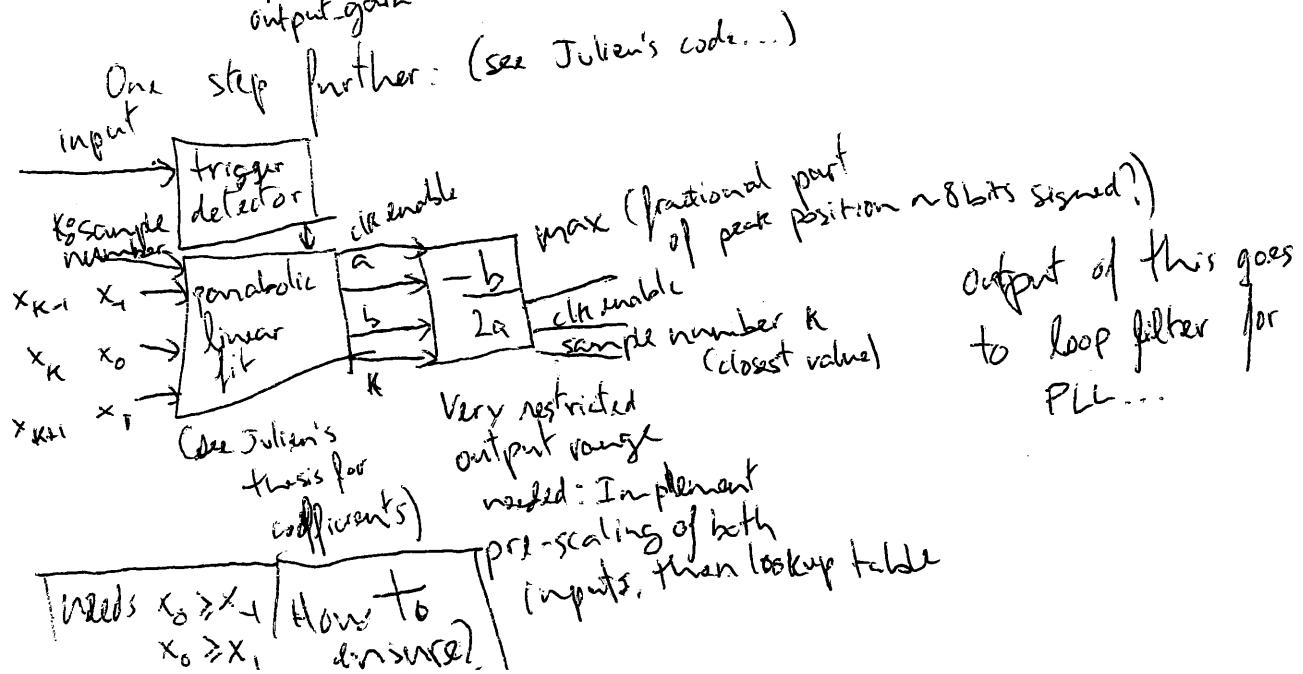
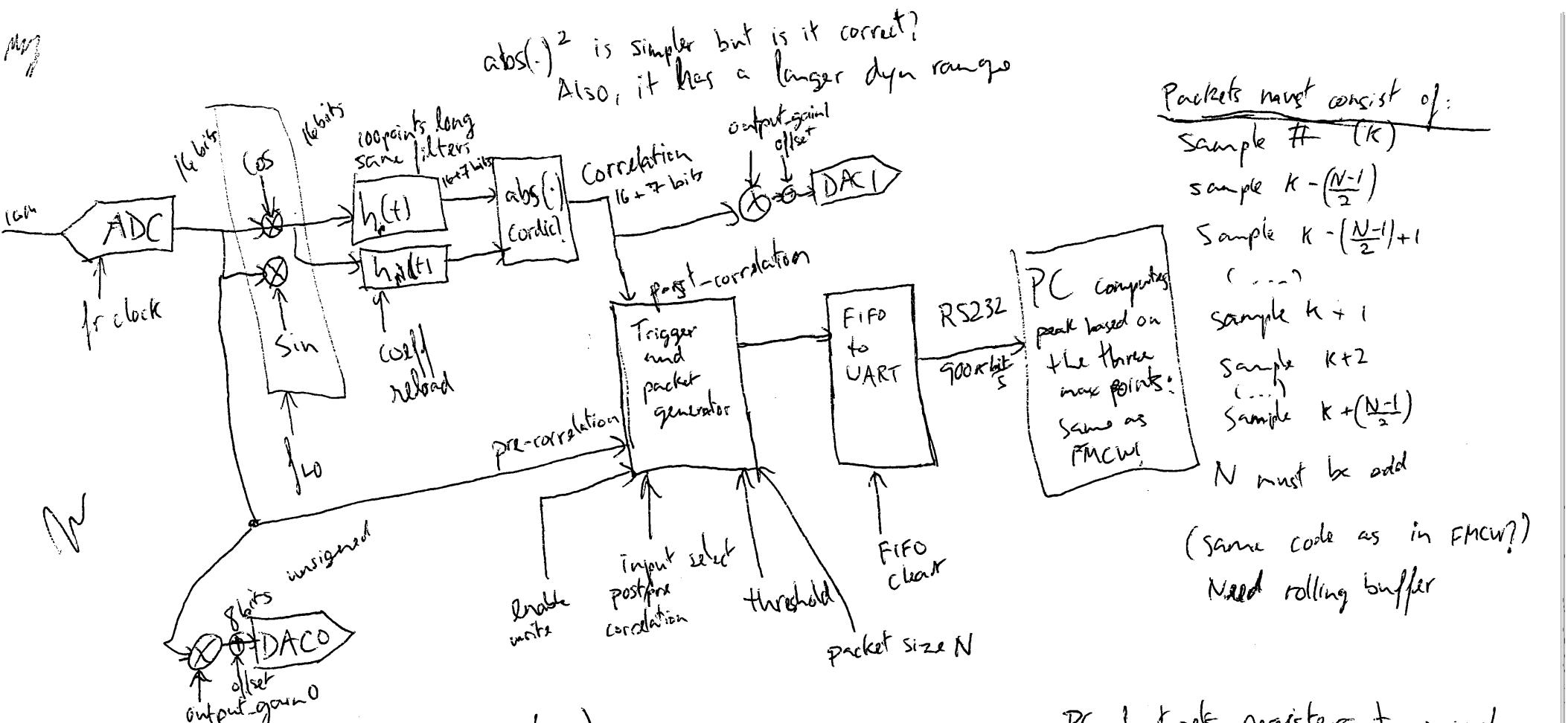
User interface (This works for a single lock)



Status	
Green Red	<input type="checkbox"/>
<input checked="" type="checkbox"/> Beat amplitude	<input type="checkbox"/>
<input checked="" type="checkbox"/> Beat SNR	<input type="checkbox"/>
<input checked="" type="checkbox"/> Frequency lock	<input checked="" type="checkbox"/> Enable
<input checked="" type="checkbox"/> Phase lock	<input type="checkbox"/> Enable







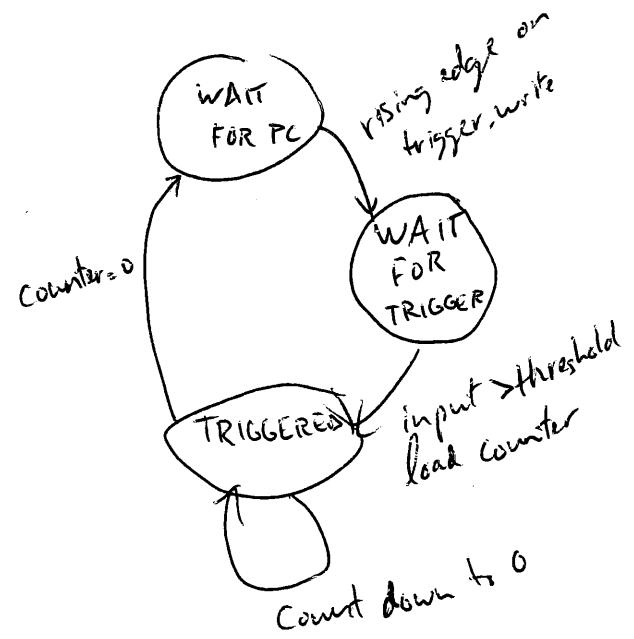
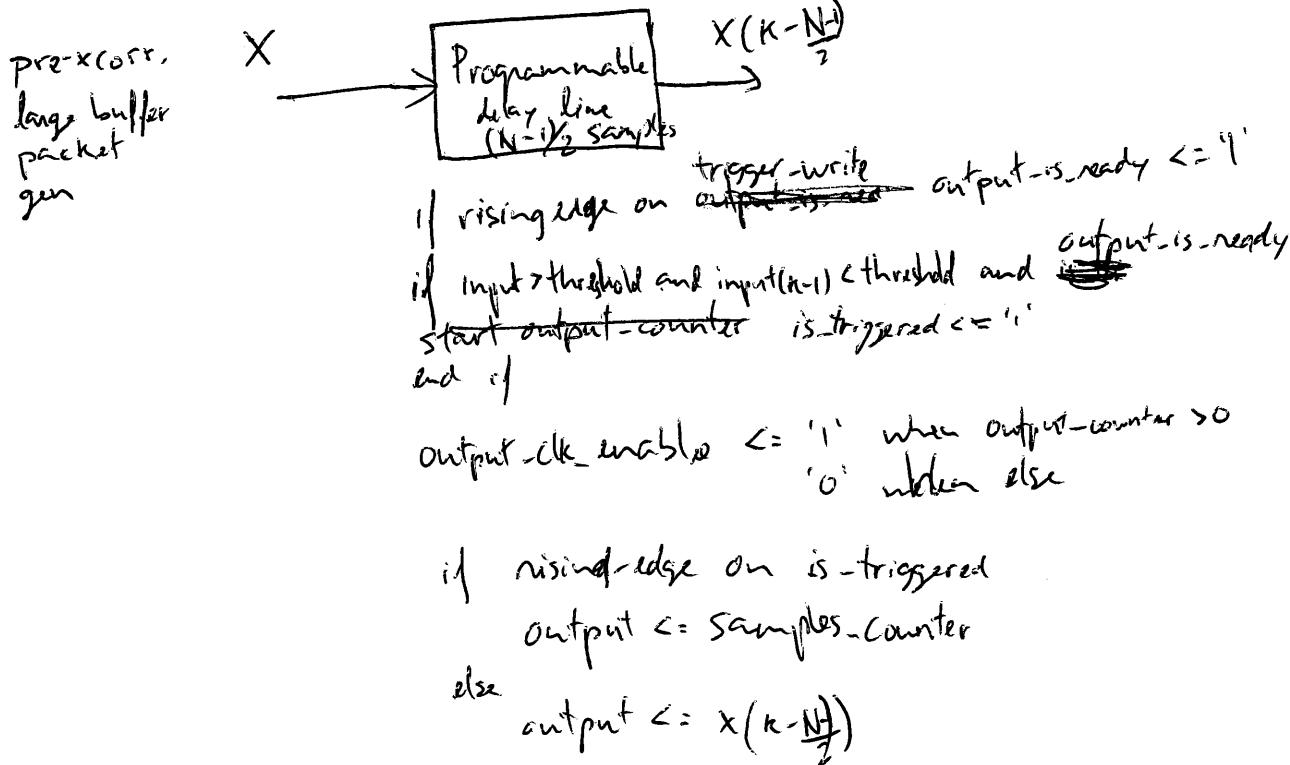
Packets must consist of:

- Sample k (K)
- sample $k - \frac{N-1}{2}$
- sample $k - \frac{N-1}{2} + 1$
- ...
- sample $k + 1$
- sample $k + 2$
- ...
- sample $k + \frac{N-1}{2}$

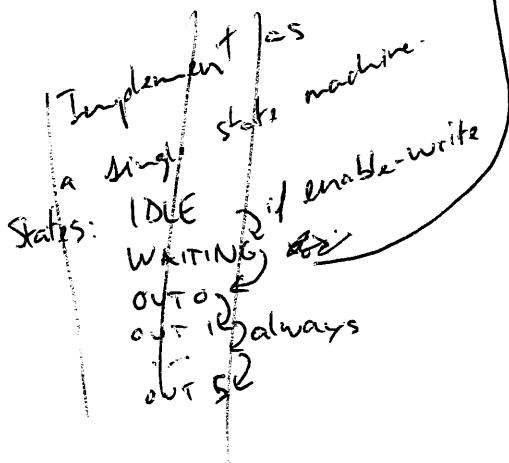
N must be odd

(same code as in FMCW?)
Need rolling buffer

PC first sets registers to record one 1GM, combines the filter coefficients to de-chirp the 1GM, then sends the coefficients back. Sets the trigger selection to post-correlation, a small number of points for realtime operation, then starts reading continuously.



Post-xcorr
trigger and packet gen



trigger if:

$x_0 \geq x_{-1}$ and $x_0 \geq x_{+1}$ and $x_0 > \text{threshold}$

~~if not triggered:~~

new shift register: $x_{-1} \leftarrow x_0$
 $x_0 \leftarrow x_{+1}$
 $x_{+1} \leftarrow \text{input}$

else if triggered:

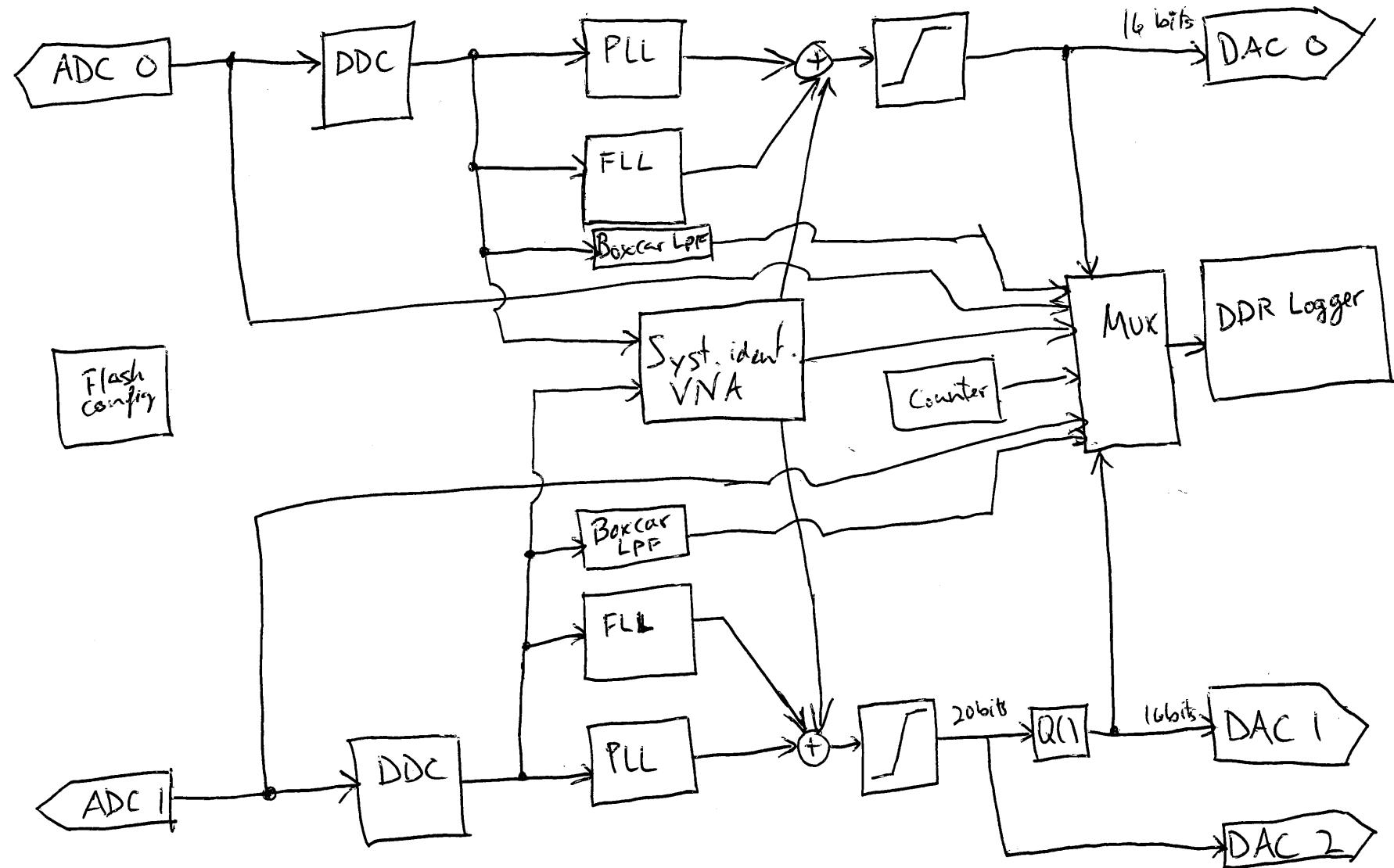
copy current shift register to internal registers.
 start serializing output to output bus
 4x 24 bits to 6x 16 bits
 samples-counter

Implement as two modules

-One does the triggering and outputs three points that satisfy: $x_0 \geq x_{-1}$, $x_0 \geq x_{+1}$, $x_0 > \text{threshold}$

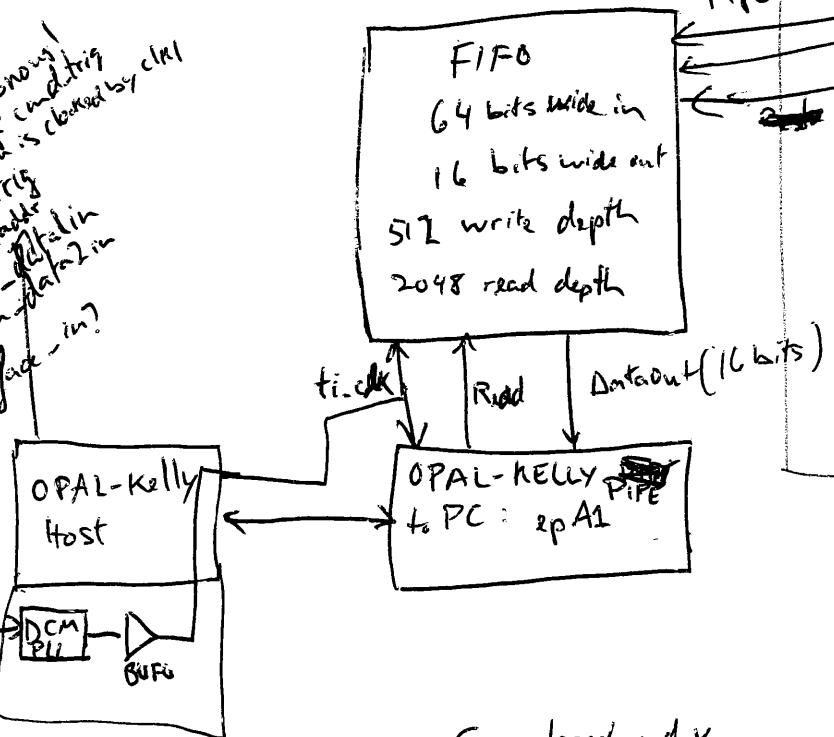
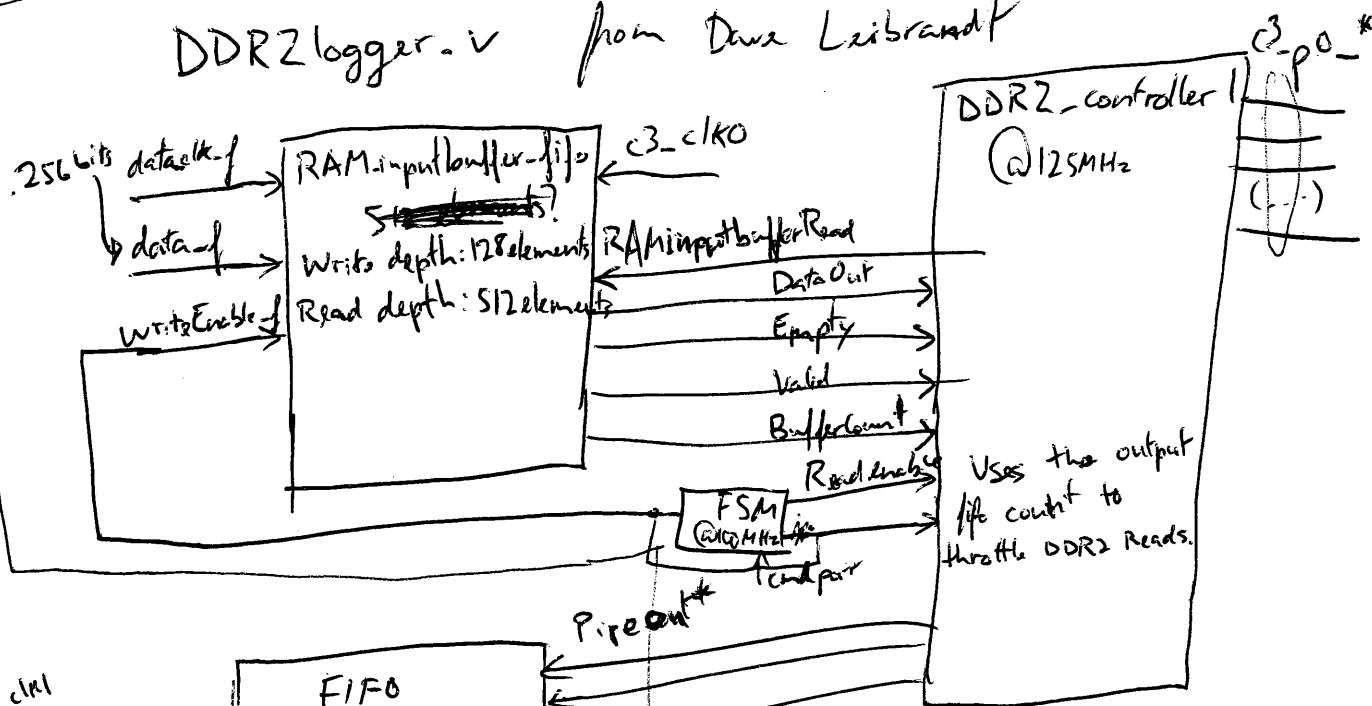
-One serializes the samples to 16 bits and ~~the sample counter~~.

-This way we can add the parabolic peak-finding in between.



DDR2 logger.v from Dave Leibrandt

Asynchronous!
All except command
(and trig)
and data-in
and data2-in
(and data2-in?)
Port-interface-in?



SuperlaserLand.v

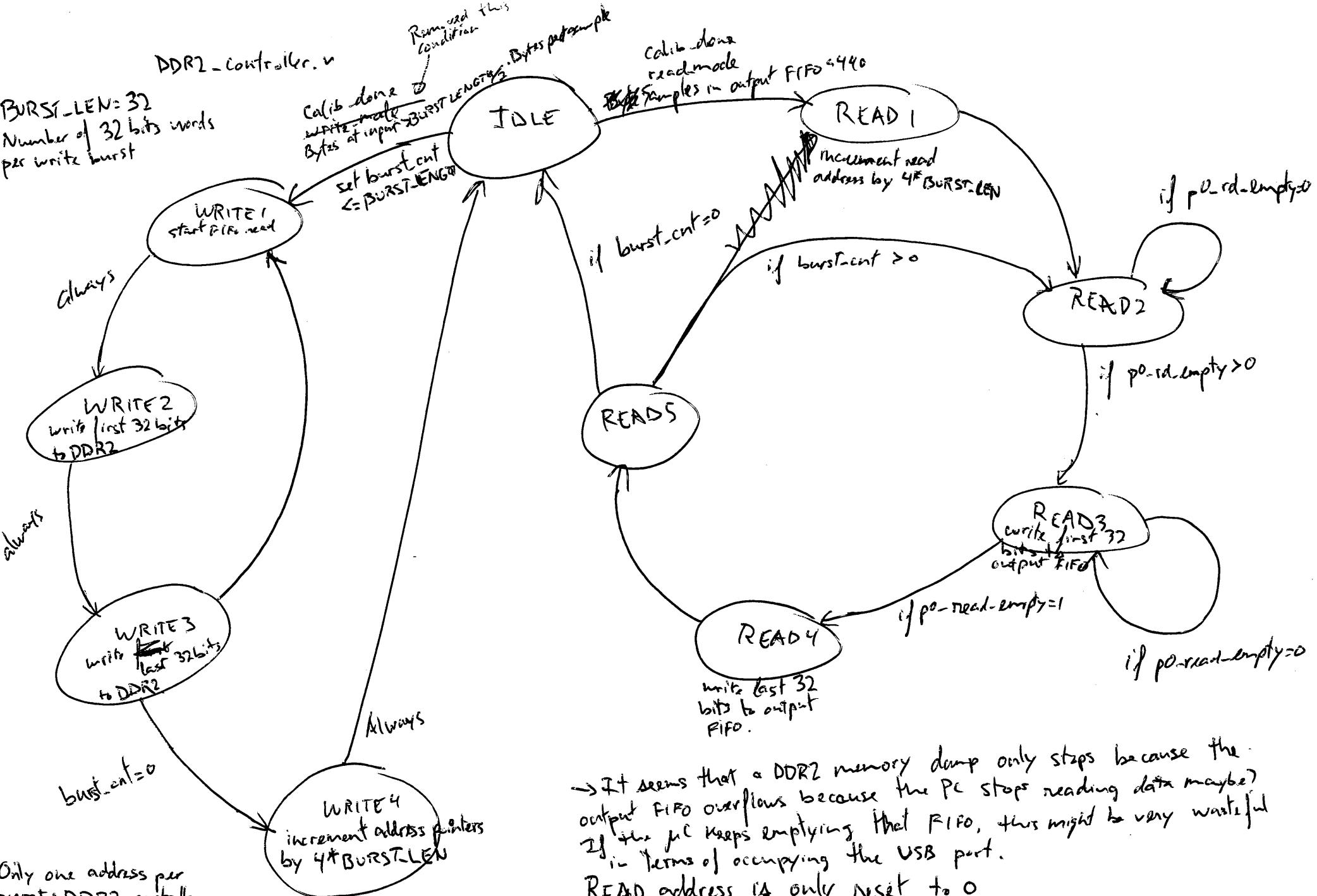
- Check if the DDR2 dump ever stops and maybe fix that.

ReadEnable-f initiates the dumping of the DDR2 contents to the FIFO which goes to a USB pipe (throttled by the FIFO count)

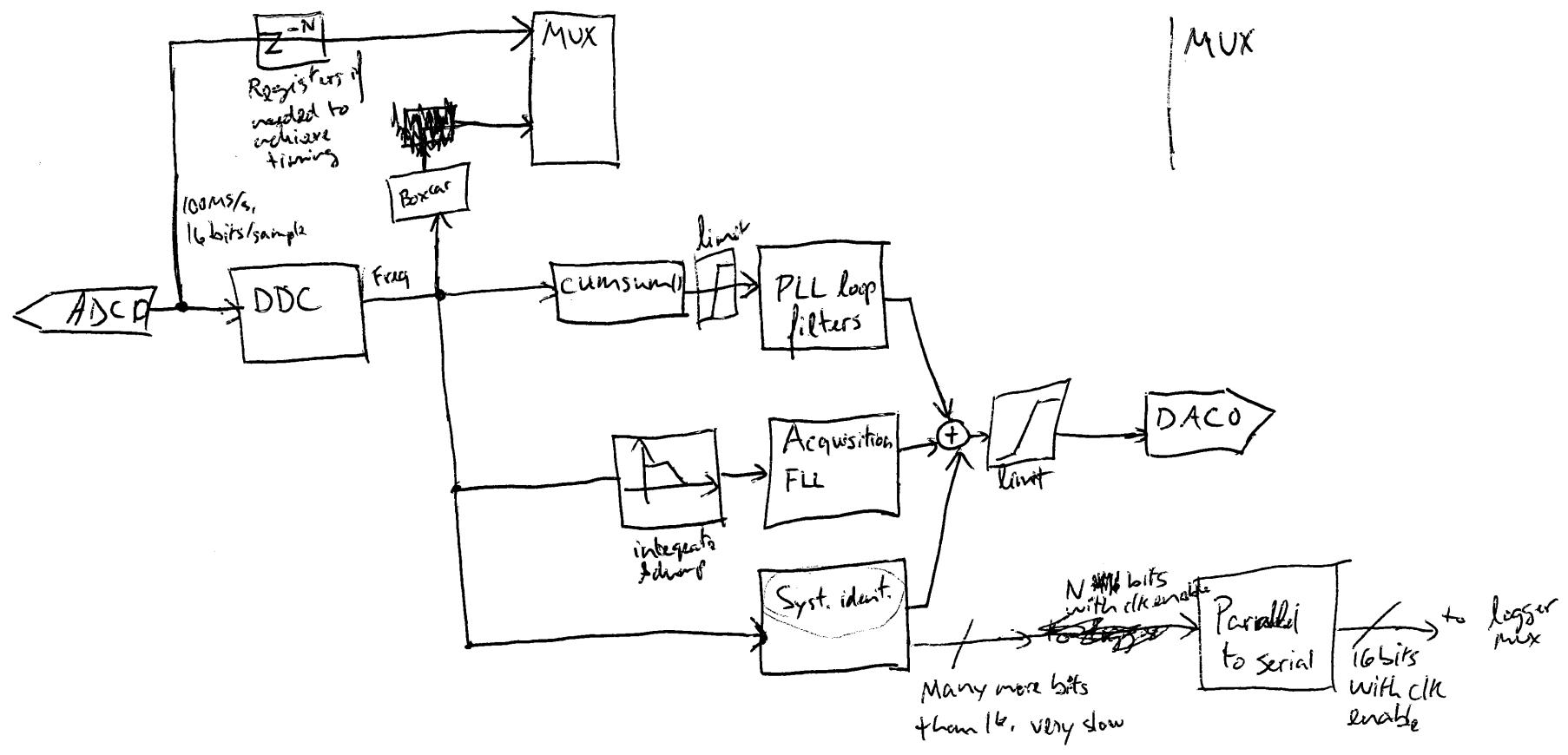
So far the only changes that we need to do are:

- Change the width of the input FIFO
- Change the FSM to clean up clocking issues (register output used as a click instead of a clock enable)
- Change the FSM to enable decimation = 1
- Add an input multiplexer instead of the very wide write port.

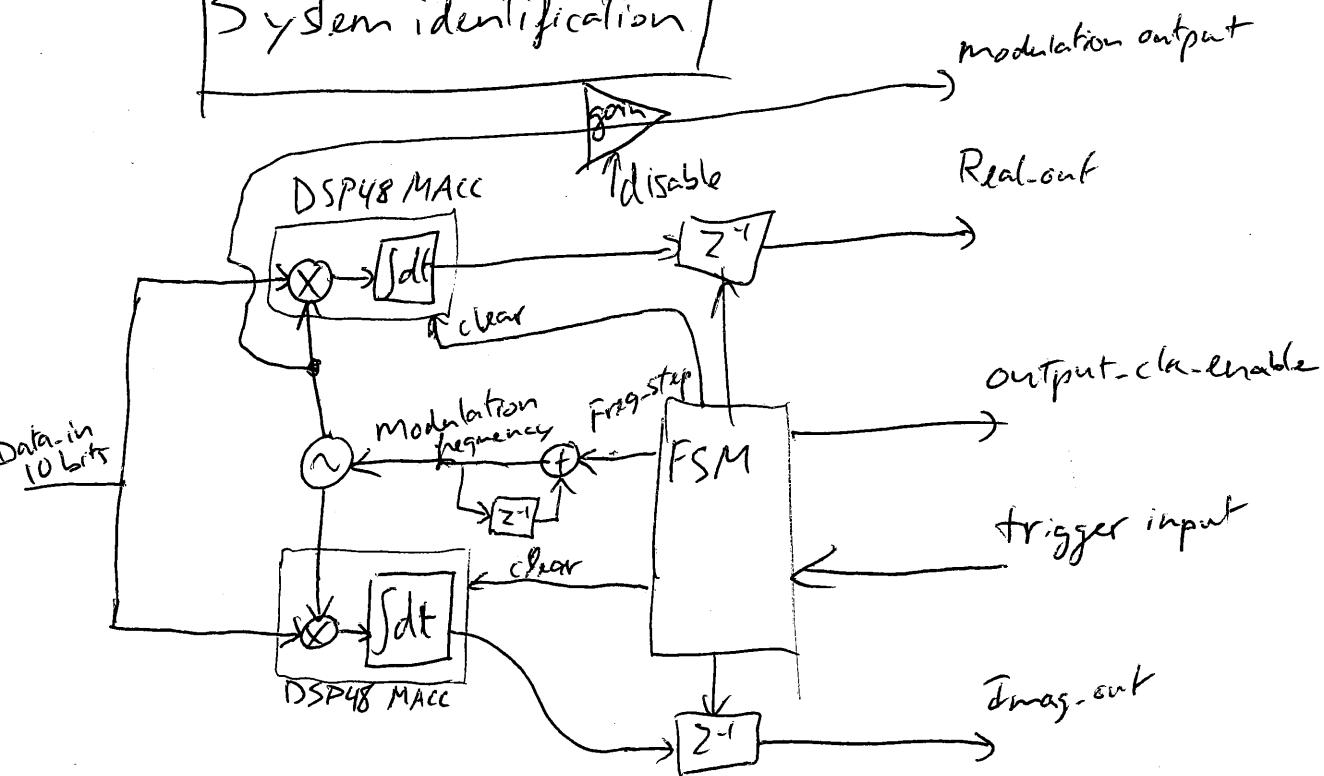
- Measure actual throughput of DDR in and USB out.
- Initiate dumping of RAM as soon as write is finished instead?



Only one address per burst: DDR2 controller memory3) probably takes care of addresses within a burst.
 → It seems that a DDR2 memory dump only stops because the output FIFO overflows because the PC stops reading data maybe? If the PC keeps emptying that FIFO, this might be very wasteful in terms of occupying the USB port.
 READ address is only reset to 0 when reset is asserted, not when read is initiated. This reset is indeed initiated by DDR2logger.v at the start of reception of a read or write command.



System identification



Expected value in the integrator is:

$$N_{\text{Integration}} \cdot \frac{A_{\text{output}}}{2} \quad \text{For a loop back system}$$

$$A_{\text{output}} = \text{output_gain} \quad (\text{from } 1 \text{ to } 2^{15})$$

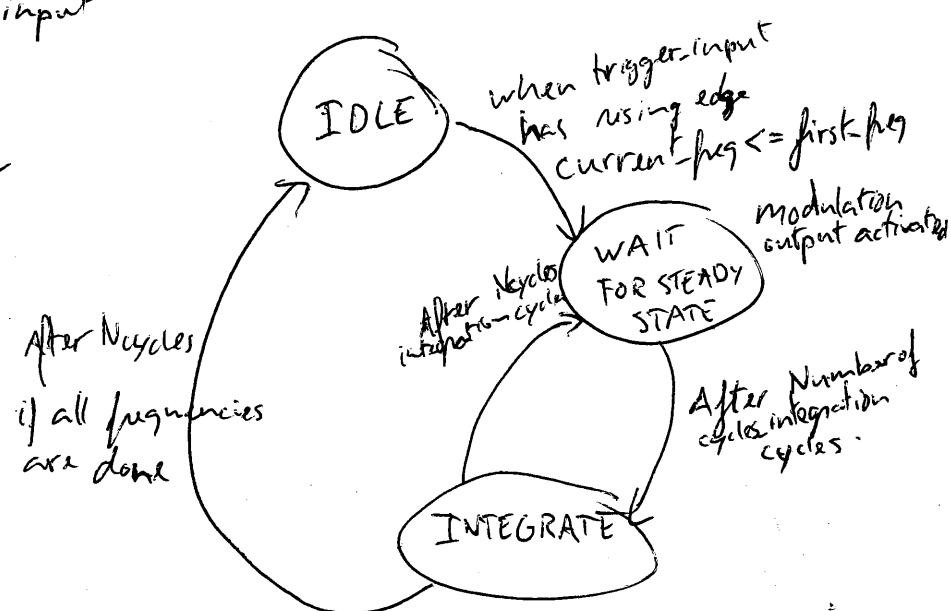
~~1111111111111111~~

4 DSP 48 Slices

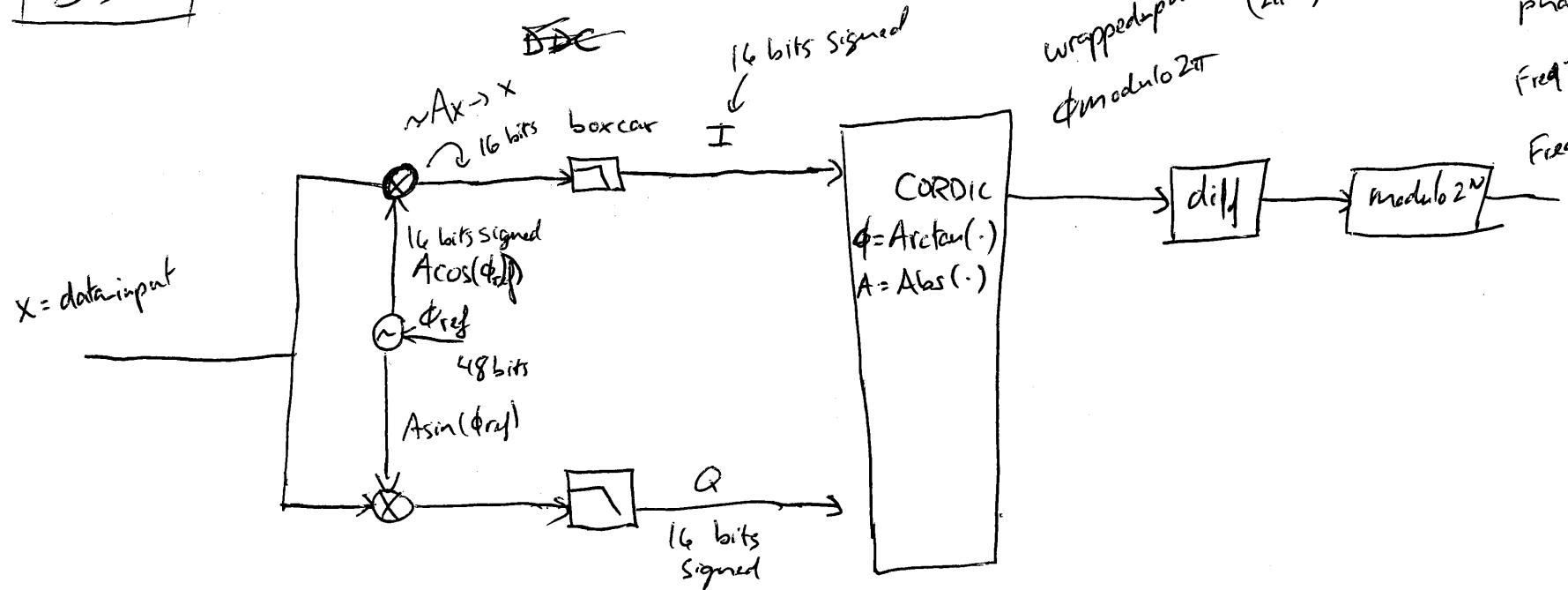
2 MACCS

1 for DDS (phase accum probably)
1 for output gain

FSM

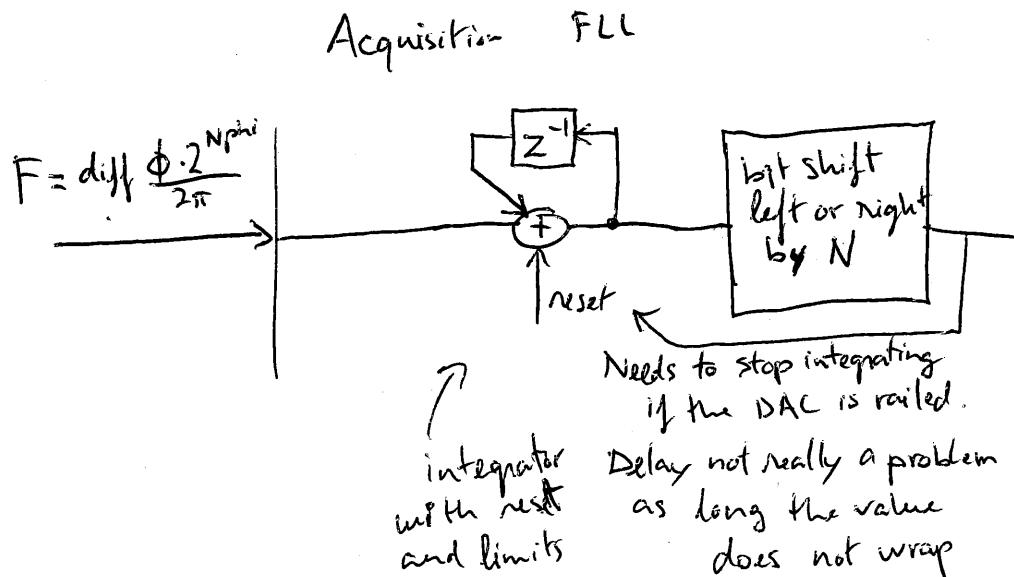


DDC

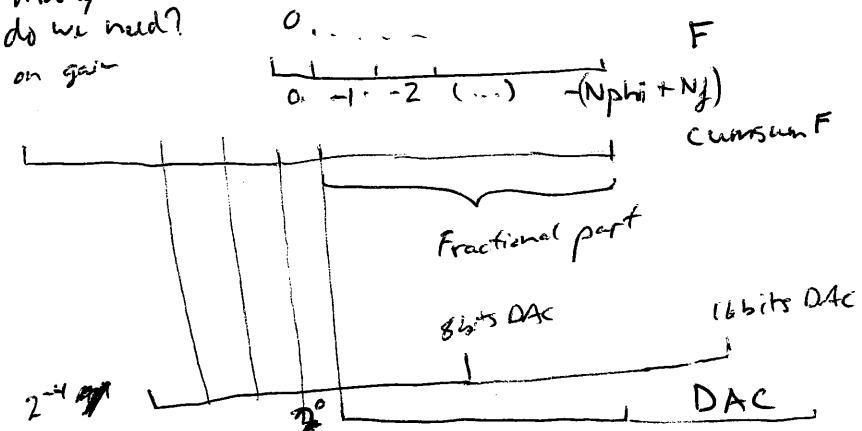


$$\text{wrappedphase} = \left(\frac{\phi \cdot 2^N}{2\pi} \right) \bmod 2^N$$
$$\phi \bmod 2\pi$$

$$N = 10 \text{ bits on phase}$$
$$\text{freq} = \text{diff} \left(\frac{2^N \phi}{2\pi} \right) \bmod 2^N$$
$$\text{Freq} = \frac{N}{2} \cdot \text{frequency}$$



How many extra bits do we need?
Depends on gain



Worst case is if DAC/vco gain is too high and thus we need to divide by a lot with K_i .

$$K_i = \frac{f_c}{2^{N\phi\text{hi}-NDAC}} \cdot \frac{f_s}{f_{\text{span}}} \quad f_s > 100 \text{ MHz}$$

* Lowest K_i : low f_c , high f_{span} , high $N\phi\text{hi}$, low $NDAC$
 1 Hz 1 GHz 10 bits 8 bits

$$K_i = \frac{1}{4} = 2^{-2}$$

Highest K_i : High f_c , low f_{span} , low $N\phi\text{hi}$, High $NDAC$
 1 kHz 100 MHz 8 bits 16 bits

$$K_i = 256e3 = 2^{18}$$

Anything 2^8 and higher makes little sense as this means that every LSB flip on the phase will rail the DAC...

$$\text{We have } \text{Freq} = \frac{\Delta\phi}{\Delta t} = f_s \Delta\phi/\phi \quad S_{\text{freq}} = \left(\frac{f_s}{2\pi}\right)^2 4 \sin^2\left(\frac{2\pi f}{2f_s}\right) S_\phi = \frac{4f_s^2}{\pi^2} \sin^2\left(\frac{2\pi f}{2f_s}\right) S_\phi$$

$$\text{Freq}(f) = \frac{f_s}{\pi} \sin\left(\frac{2\pi f}{2f_s}\right) \phi(f)$$

So noise floor goes from 0 to $\frac{4f_s^2}{\pi^2} S_\phi$, with $S_\phi = \left(\frac{1 \text{ mrad}}{\sqrt{Hz}}\right)^2$ for 8 fractional bits

$$\text{Thus } S_{\text{max}} = \left(\frac{3 \text{ Hz}}{\sqrt{Hz}}\right)^2$$

$$\text{CORDIC } \sigma_\phi = \frac{\pi}{2\sqrt{2^{N\text{iterations}}}}, \text{ for } N\text{iterations} = 8, \sigma_\phi = 4.3 \text{ mrad} \quad S_\phi = \frac{\sigma_\phi^2}{(f_s/2)} = \frac{\pi^2}{4f_s 2^{N\text{iterations}}}$$

$$\text{How much frequency noise in 1 kHz BW? } S_{\text{freq}}|_{1 \text{ kHz}} = \sqrt{\int_0^{1 \text{ kHz}} S_\phi(f) df} = \left(\int_0^{1 \text{ kHz}} \frac{f_s^2}{\pi^2} \sin^2\left(\frac{2\pi f}{2f_s}\right) \frac{\pi^2}{4f_s 2^{N\text{iter}}} df \right)^{1/2}$$

Gain required for the acquisition FLC?

$$\text{Bridge } F_{\text{digital}} = \frac{2\pi f_{\text{inst}} \cdot 2^{N\text{Phi}}}{f_s} \left[\frac{\text{Counts}}{\text{Hz}} \right] = K_{\text{discriminator}}$$

$$\text{DAC: Gain} = \frac{F_{\text{span}}}{2^{N\text{DAC}}} \left[\frac{\text{Hz}}{\text{counts}} \right] = K_{\text{DAC}} \quad K_{\text{DAC}} K_{\text{Discrim}} = \frac{2^{N\text{Phi}-N\text{DAC}}}{2^{N\text{Phi}}} \cdot \frac{F_{\text{span}}}{f_s}$$

$$G(f) = K_{\text{DAC}} \cdot K_{\text{discrim}} \cdot \frac{K_i}{2\pi f}$$

$$f_c = \frac{K_{\text{DAC}} K_{\text{discrim}} \cdot K_i}{2\pi} \Rightarrow K_i = \frac{2\pi f_c}{K_{\text{DAC}} K_{\text{discrim}}} \cdot \frac{2\pi \cdot 1 \text{ Hz}}{f_s} = \frac{f_c}{2^{N\text{Phi}-N\text{DAC}}} \cdot \frac{1}{f_s}$$

We have added a low-pass filter with gain: $K_{\text{filter}} = 2^N$ with $N=7$

$$G(f) = K_{\text{DAC}} \cdot K_{\text{discrim}} \cdot \frac{K_i}{2\pi f}, \quad K_{\text{discrim}} = \frac{2\pi f_{\text{inst}} \cdot 2^{N\text{Phi}+N\text{F}}}{f_s} \left[\frac{\text{Counts}}{\text{Hz}} \right]$$

$$\begin{aligned} &= \left(\frac{f_s}{4 \cdot 2^{N\text{iter}}} \int_0^{1 \text{ kHz}} \sin^2\left(\frac{2\pi f}{2f_s}\right) df \right)^{1/2} \\ &= \frac{f_s}{2^{(N\text{iter}+1)}} \left(\int_0^{1 \text{ kHz}} \sin^2\left(\frac{2\pi f}{2f_s}\right) df \right)^{1/2} \\ &= 1/2 \text{ Hz.} \end{aligned}$$

Nichols Chart

X axis: phase in degrees of $G: \text{Arg}(G(j))$

X axis: gain in dB of $G: 10 \log_{10}(|G(j)|^2)$

We want to add a grid which indicates locus of constant ~~closed~~ loop magnitude.

$$H(j) = \frac{G(j)}{1+G(j)}, \text{ solve: } Ae^{j\theta} = \frac{Be^{j\phi}}{1+Be^{j\phi}} \quad \cancel{Ae^{j\theta}} \quad Ae^{j\theta} = \frac{Be^{j\phi}}{1+Be^{j\phi}} \quad \text{for all } A, \theta$$

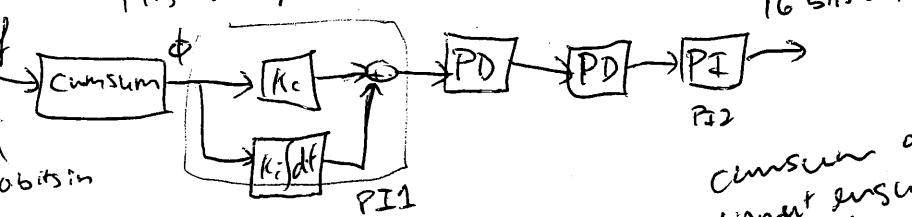
$$\cancel{H} \quad H(1+G) = G$$

$$\cancel{H} \quad H = G - HG = G(1-H)$$

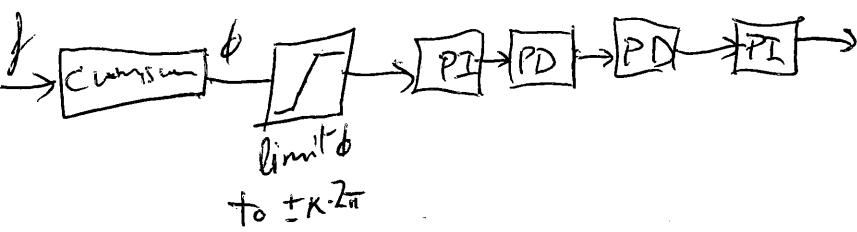
$$\frac{H}{1-H} = G \quad \text{for all } H = Ae^{j\phi}, \text{ lines of constant } A.$$

PLL Loop filters

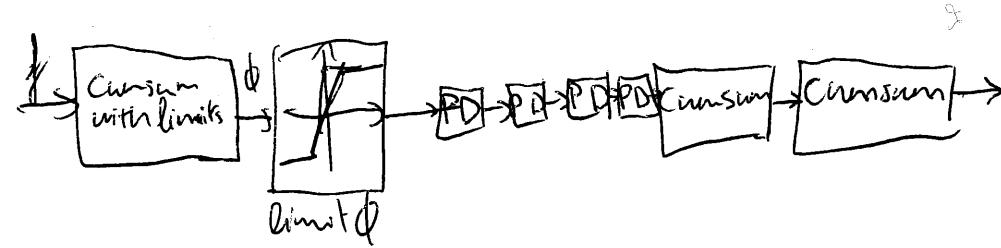
First design iteration



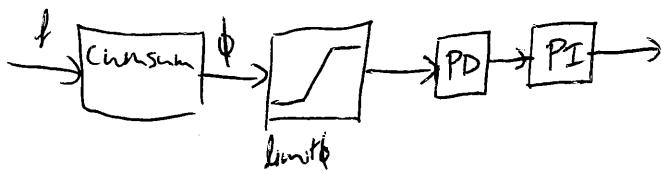
2nd design



3rd design



4th design



In normal operation (closed loop), the error signal first should be pretty small.

Same thing for the integral $\phi = \text{cumsum}(f)$, it should stay within $\pm 2\pi$ or better.

\rightarrow The 2nd design iteration has two operation modes: linear and saturated (non-linear).

In linear mode, which is the desired mode, everything works as expected.

In saturated mode, the error signal has the correct sign but is clamped at some value. The ~~output~~ will ramp as a parabola in the correct direction until the error de-saturates.

It basically does the same thing as a PFD (Phase-frequency detector) except that it keeps track of how many cycles were lost.

\rightarrow Having the output stage as an integrator makes it easy to add anti-windup and hold capabilities.

\rightarrow Can we do the whole chain in exact arithmetic? No quantization noise anywhere except at the end? Probably not unless we stick to "nice" values for all the filters coefficients.

What's the transfer function that we want anyway?

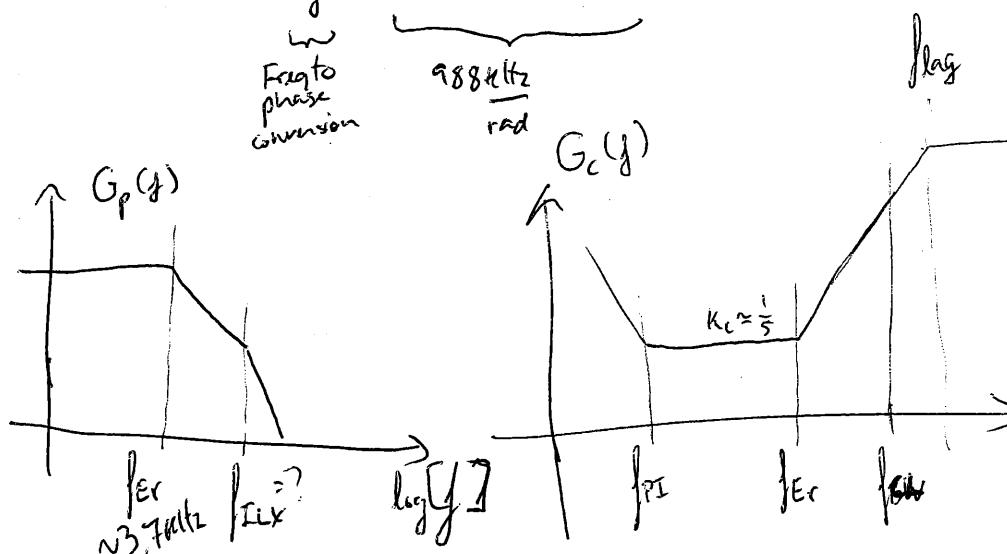
$$K_\phi = \frac{2^{N_{\text{phi}}}}{2\pi} \left[\frac{\text{counts}}{\text{rad}} \right] \quad N_{\text{phi}} = 10 \text{ bits} \Rightarrow K_\phi = 162 \left[\frac{\text{counts}}{\text{rad}} \right]$$

$$K_{\text{DAC}} = \frac{V_{\max}}{2^{N_{\text{DAC}}}} \left[\frac{V}{\text{count}} \right] \quad N_{\text{DAC}} = 16 \text{ bits}$$

$$K_{\text{VCO}} = \frac{F_{\max}}{V_{\max}} \left[\frac{\text{Hz}}{V} \right]$$

$$K_{\text{DAC}} K_{\text{VCO}} = \frac{F_{\max}}{2^{N_{\text{DAC}}}} \left[\frac{\text{Hz}}{\text{count}} \right] \quad K_{\text{DAC}} K_{\text{VCO}} = 6.1 \frac{\text{kHz}}{\text{count}}$$

$$\text{Loop gain } G(f) = \frac{1}{f} \cdot K_\phi \cdot K_{\text{DAC}} K_{\text{VCO}} \cdot \underbrace{G_c(f)}_{\text{loop}} \cdot G_p(f)$$



$G_c(f)$: Controller transfer function

$G_p(f)$: Actuator transfer function (normalized to **DC** gain = 1)

$G_c(f) G_p(f)$ needs to be proportional to $f \omega K_c$ around the cut-off frequency.

Below the loop BW, we can have an integrator
Let's assume that $G_c(f)$ is PI + PD

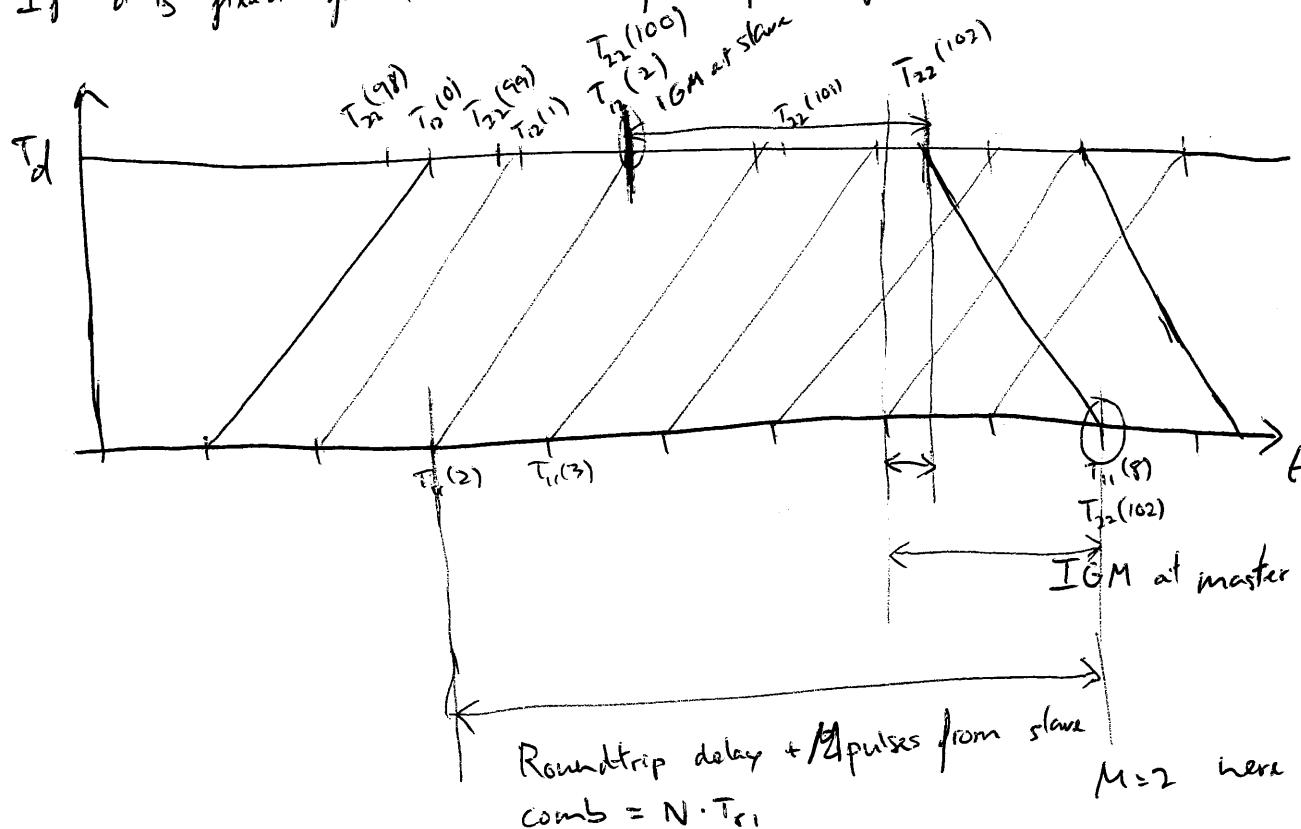
$$G_c(f) = \left(K_c + \frac{K_i K_c}{2\pi f} \right) G_{\text{lead lag}}(f), \text{ where } G_{\text{lead lag}}(f) \text{ cancels the } \text{BW roll-off}$$

For a closed loop BW of 200kHz we want $K_c \approx \frac{1}{5}$

$$\omega_c \approx 200 \text{ rad/s}$$

f_{flag} must be well past the cut-off

I) T_d is fixed for the duration of a pair of IGM:



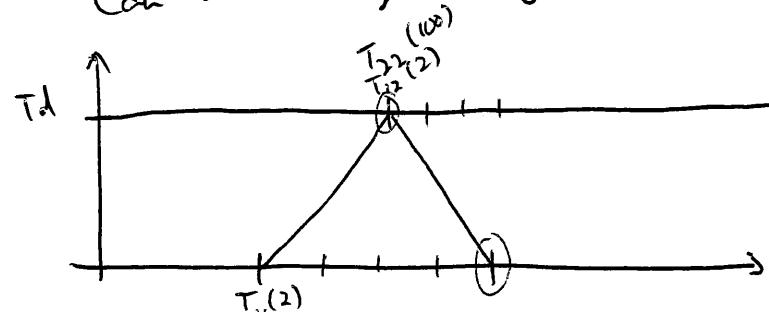
$$2T_d + \rho_{\text{AT}_{r2}} = N \cdot T_{ri}$$

$T_d + \Delta T_{r2} = N \cdot T_{ri}$
 T_{r2} can be known (on average) through delay between two loms at master site.

Tr. is assumed to be known perfectly

How well can T_{r2} be known? Depends on how much it changes between two IGMS.

Can we arrange things so that $M=0$?



This implies $MTr_1 = 2Td$

Can we expressly add a delay to T_d to make this equation match? Removes T_{rs} from the measurement of T_d . Something as looping back the master's pulses.
Actuator only needs $\frac{2T_d}{\pi}$ range.

\AVoC\Public\Landesk

Assume that T_d is slowly varying, thus constant over 2 adjacent GMS:

$$(1) T_{11}(N) = T_d + T_{22}(M)$$

$$(2) T_{22}(P) = T_d + T_{11}(Q)$$

$T_{11}(\cdot)$ is known

$T_{22}(\cdot)$ is unknown, but $T_{11}(Q) = T_{11}(N) + (Q-N)T_{r1}$

T_d is unknown ~~but~~

$$T_{22}(P) = T_{22}(M) + (P-M)T_{r2}$$

} How well does this require knowledge of T_{r1}, T_{r2} ?

$$(1) T_{11}(N) = T_d + T_{22}(M)$$

$$(2) T_{22}(M) + (P-M)T_{r2} = T_d + T_{11}(N) + (Q-N)T_{r1}$$

Solve for T_d : (1) - (2):

$$\cancel{T_{11}(N)} + \cancel{T_{22}(M)} + (P-M)T_{r2} = 2T_d + \cancel{T_{11}(N)} + \cancel{T_{22}(M)} + (Q-N)T_{r1}$$

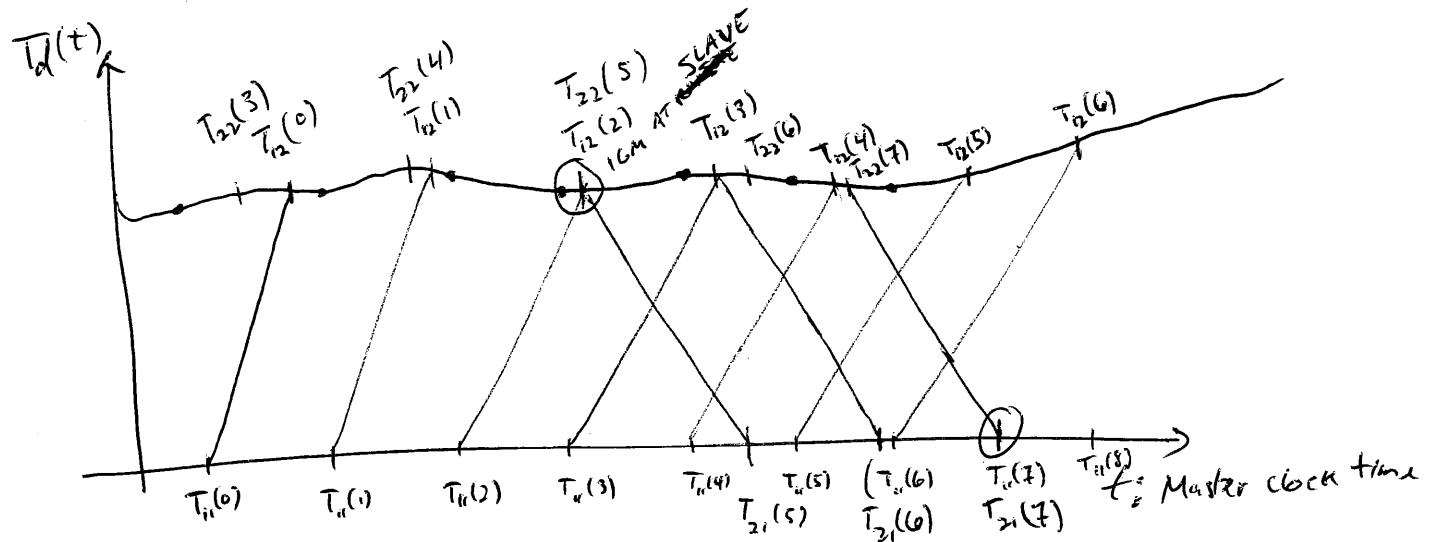
} Once again, requires T_{r1} and T_{r2}

$$(P-M)T_{r2} - (Q-N)T_{r1} = 2T_d$$

Solve for T_{22} : (1) - (2):

$$T_{11}(N) - T_{22}(M) - (P-M)T_{r2} = \cancel{T_d} + T_{22}(M) - \cancel{T_d} - T_{11}(N) + (Q-N)T_{r1}$$

$$2T_{11}(N) - (P-M)T_{r2} - (Q-N)T_{r1} = 2T_{22}(M) \quad \} \quad T_{r1} \text{ and } T_{r2}$$



Two equations to solve for two unknowns:

$$(1) \quad T_{11}(N) = T_d(T_{11}(N)) + T_{22}(M)$$

$$(2) \quad T_{22}(P) = T_d(T_{22}(P)) + T_{11}(Q)$$

IGM AT MASTER SITE
The pulse numbers are a coincidence.

$T_d(t)$ is slowly varying so we can replace ~~$T_{22}(P)$~~ by coarse estimate of $T_{22}(P)$: $T_{22}(P) \approx T_{22}^*(P) = T_{21}(P) - T_d(T_{21}(P))$

$$(2) \quad T_{22}(P) = T_d(T_{21}(P) - T_d^*) + T_{11}(Q)$$

$$T_{11}(Q) = T_{11}(N) + (Q-N)T_{ri}$$

$$(1) \quad T_{22}(P) = T_d^* + T_{11}(N) + (Q-N)T_{ri}$$

$$(2) \text{ and } (1): T_{22}(P) = T_d^* + T_d^* + T_{22}(M) + (Q-N)T_{ri}$$

$$\text{Interpolation: } T_{22}(M) = T_{22}(P) + (M-P)T_{ri}$$

Assume that only one sideband affects the phase per each frequency.

$$\theta = \text{Arg} \left\{ H_{\text{igen}}(f) + \frac{2\pi f_c A_{\text{Timing}} \exp(j(\phi_0 + \pi)) H_{\text{igen}}(f - f_{\text{Timing}})}{2} \right\}$$

$$= \text{Arg} \left\{ H_{\text{igen}}(f) + \frac{2\pi f_c A_{\text{Timing}} H_{\text{igen}}(f - f_{\text{Timing}}) (\cos(\phi_0 + \pi) + j \sin(\phi_0 + \pi))}{2} \right\} \quad \begin{matrix} \text{small deviations from } 0 \\ \text{phase.} \end{matrix}$$

$$= \frac{\text{Arg} \left\{ \frac{\text{Im} \{ \}}{\text{Re} \{ \}} \right\}}{\text{Arg} \left\{ \frac{\text{Re} \{ \}}{\text{Re} \{ \}} \right\}} = \frac{\text{Im} \{ \}}{\text{Re} \{ \}} = \frac{H_{\text{igen}}(f - f_{\text{Timing}}) \cdot \underbrace{A \cdot 2\pi f_c A_{\text{Timing}}}_{2} \cdot \sin(\phi_0 + \pi)}{H_{\text{igen}}(f)}$$

choose worst case ϕ_0 : $\sin(\phi_0 + \pi) = 1$

~~$$= \frac{2\pi f_c A_{\text{Timing}} \cdot H_{\text{igen}}(f - f_{\text{Timing}})}{H_{\text{igen}}(f)}$$~~

Thus the shape of the phase error
is given by the shape of the spectrum

$$F\left\{ A(t) \exp\left[j(2\pi f_c(t + A_{\text{timing}} \cos(2\pi f_{\text{timing}} t + \phi_0)))\right]\right\}$$

$$\exp(jx)$$

$$= \exp(j\pi) \exp(jx)$$

$$= \exp j(x + \pi)$$

$$= F\left\{ A(t) \exp(j2\pi f_c t) \exp(j2\pi f_c A_{\text{timing}} \cos(\dots))\right\}$$

$$= F\left\{ A(t)\right\} * F\left\{ \exp(j2\pi f_c t)\right\} * F\left\{ \exp(j2\pi f_c A_{\text{timing}} \cos(\dots))\right\}$$

$$= H_{\text{sign}}(f) * F\left\{ \exp(j2\pi f_c A_{\text{timing}} \cos(\dots))\right\}$$

$$= H_{\text{sign}}(f) * F\left\{ 1 + j2\pi f_c A_{\text{timing}} \cos(2\pi f_{\text{timing}} t + \phi_0)\right\}$$

$$= H_{\text{sign}}(f) * \left[\frac{\delta(f) + j2\pi f_c A_{\text{timing}} \left\{ \exp(j2\pi f_{\text{timing}} t + \phi_0) + \exp(-j2\pi f_{\text{timing}} t - j\phi_0) \right\}}{2} \right]$$

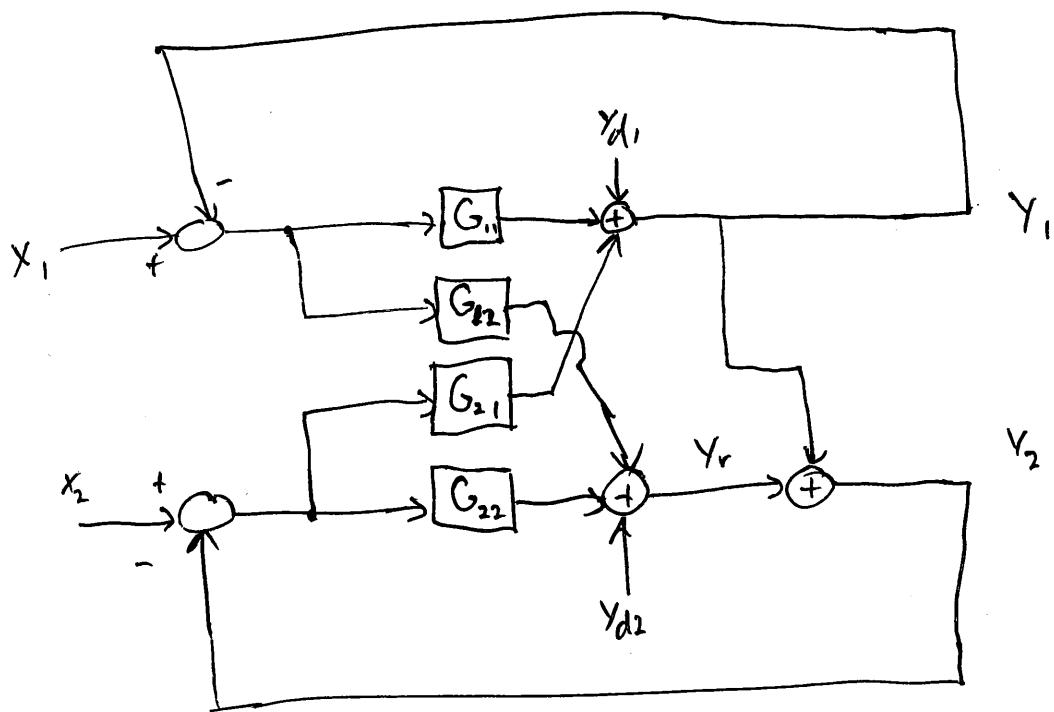
$$= H_{\text{sign}}(f) * \left[\delta(f) + j\pi f_c A_{\text{timing}} \left(\exp(j\phi_0) \delta(f - f_{\text{timing}}) + \exp(-j\phi_0) \delta(f + f_{\text{timing}}) \right) \right]$$

$$= H_{\text{sign}}(f) + j\pi f_c A_{\text{timing}} \exp(j\phi_0) H_{\text{sign}}(f - f_{\text{timing}}) + j\pi f_c A_{\text{timing}} \exp(-j\phi_0) H_{\text{sign}}(f + f_{\text{timing}})$$

~~$$= H_{\text{sign}}(f) + \frac{2\pi f_c A_{\text{timing}} \exp(j(\phi_0 + \pi))}{2} H_{\text{sign}}(f - f_{\text{timing}}) + \frac{2\pi f_c A_{\text{timing}} \exp(j(\pi - \phi_0))}{2} H_{\text{sign}}(f + f_{\text{timing}})$$~~

Assume that $\text{Arg } H_{\text{sign}}(f) = 0$ for all f .

Next: Include non-orthogonal actuators:



$$\begin{aligned}
 \frac{1}{sT+1} &= G \\
 H &= \frac{G}{1+G} \\
 &= \frac{1}{\frac{1}{G} + 1} \\
 &\approx \frac{1}{sT_h + 1} \\
 &\approx \frac{0,5}{sT_h + 1}
 \end{aligned}$$

$$Y_r = \frac{1}{1+G_2} Y_{d2} + \frac{G_2}{1+G_2} \left[(\alpha-1) Y_1 + \alpha Y_{m1} + X_1 - Y_{m2} \right]$$

$$= \frac{1}{1+G_2} Y_{d2} + \frac{G_2}{1+G_2} \left[(\alpha-1) \left(\frac{1}{1+G_1} Y_{d1} + \frac{G_1}{1+G_1} X_1 - Y_{m1} \frac{G_1}{1+G_1} \right) + \alpha Y_{m1} + X_1 - Y_{m2} \right]$$

We can drop X_1 and X_2 since they usually are 0.

$$Y_r = \frac{1}{1+G_2} Y_{d2} + \frac{G_2}{1+G_2} \left[(\alpha-1) \underbrace{\frac{1}{1+G_1} Y_{d1}}_{\text{High-passed}} + \underbrace{\left(\alpha - (\alpha-1) \frac{G_1}{1+G_1} \right) Y_{m1}}_{\text{low-passed}} - Y_{m2} \right]$$

~~First loop (EO loop)~~ First loop (EO loop) residuals

~~So if~~

$$Y_r = \frac{1}{1+G_2} Y_{d2} + \frac{G_2}{1+G_2} \left[(\alpha-1) \cdot \underbrace{\frac{1}{1+G_1} Y_{d1}}_{\text{High-passed}} + \alpha Y_{m1} - (\alpha-1) \frac{G_1}{1+G_1} Y_{m1} - Y_{m2} \right]$$

So that means that if $\alpha = 1$ (we decouple the two loops), instead of putting all the residuals of the first loop onto the re-rate, we leave them untouched ~~on~~ on the optical phase.

→ What happens if the first loop has a "servo bump"? That means that $\frac{1}{1+G_1}$ and $\frac{G_1}{1+G_1}$ are resonating, i.e. G_1 is close to -1, yielding something like $\frac{1}{1+\text{close to } 1} \approx \frac{1}{\text{small}} = \text{big}$. Same thing for $\frac{G_1}{1+G_1} \approx \frac{-\text{close to } 1}{1-\text{close to } 1} \approx \frac{\text{close to } 1}{\text{small}} \approx \text{big}$.

In this case, the residuals of the EO loop are amplified by both servo bumps, unless $\alpha = 1$.

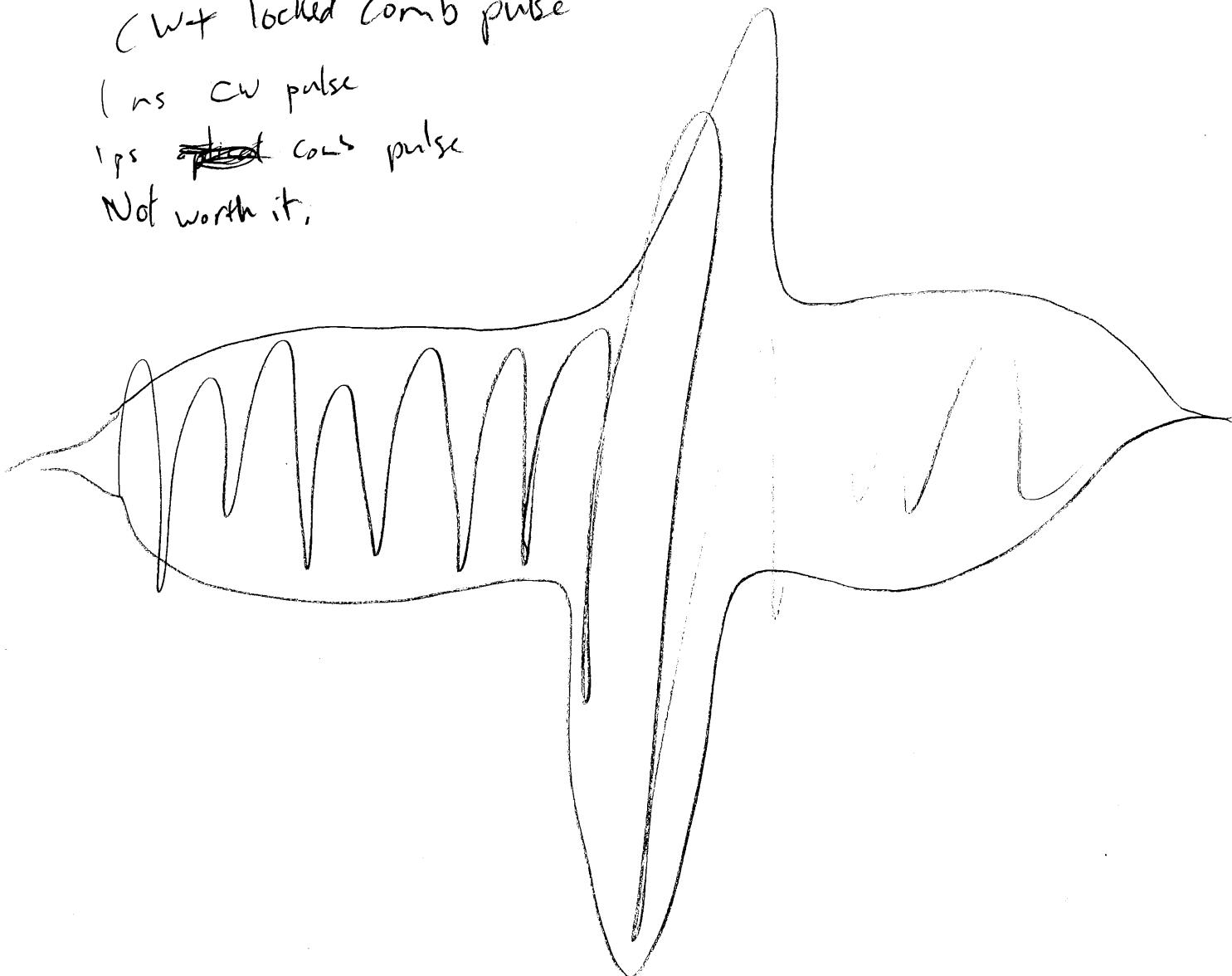
The nice thing about putting $\alpha = 1$, is that the phase noise should now be more uniform across all the comb modes, instead of hitting a minimum at the optical lock point and increasing from there. That is of course in addition to minimizing the timing jitter...

CW+locked comb pulse

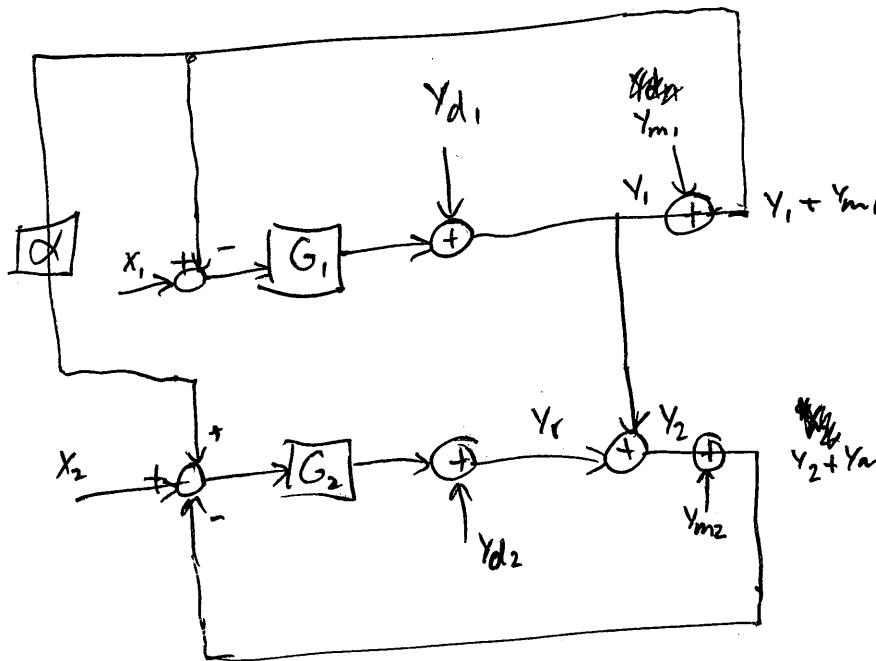
1 ns CW pulse

1 ps ~~locked~~ comb pulse

Not worth it.



How different is it if we explicitly remove the coupling:



Y_{d1} would be Y_1 measurement noise

Y_{d1} and Y_{d2} are the actual disturbances on the phases.

Y_{m1} and Y_{m2} are the measurement noises.

So Y_r is completely decoupled from whatever happens in the second loop.

$$Y_1 = Y_{d1} + G_1(X_1 - (Y_1 + Y_{m1}))$$

$$Y_1(1+G_1) = Y_{d1} + G_1X_1 - G_1Y_{m1}$$

$$Y_1 = \frac{1}{1+G_1}Y_{d1} + \frac{G_1}{1+G_1}X_1 - \frac{G_1}{1+G_1}Y_{m1}$$

$$Y_1 = \underbrace{\frac{1}{1+G_1}Y_{d1}}_{\substack{\text{high-pass on} \\ \text{disturbances} \\ (\text{because } G_1 \neq 1)}} + \underbrace{\frac{G_1}{1+G_1}X_1}_{\substack{\text{low-pass} \\ \text{w/ unit gain} \\ \text{on phaserf.}}} - \underbrace{\frac{G_1}{1+G_1}Y_{m1}}_{\substack{\text{measurement} \\ \text{noise is added} \\ \text{with low-pass TF.} \\ \cancel{\text{further}} \\ \cancel{\text{to output}}}}$$

$$Y_2 = Y_1 + Y_{d2} + G_2(X_2 + \alpha Y_1 + \alpha Y_{m1} - Y_2 - Y_{m2})$$

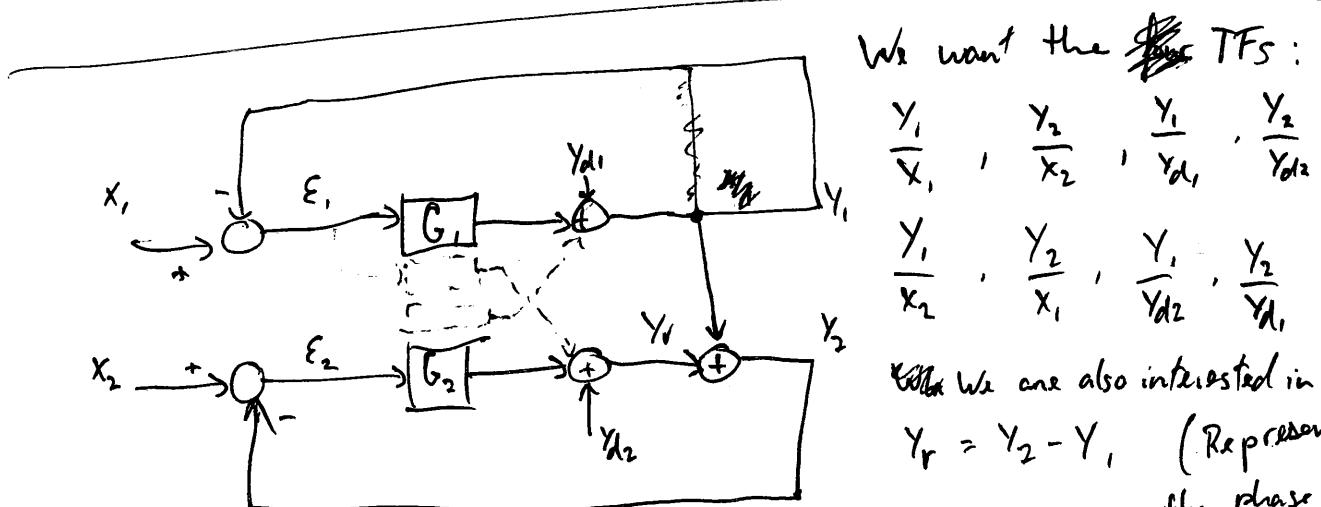
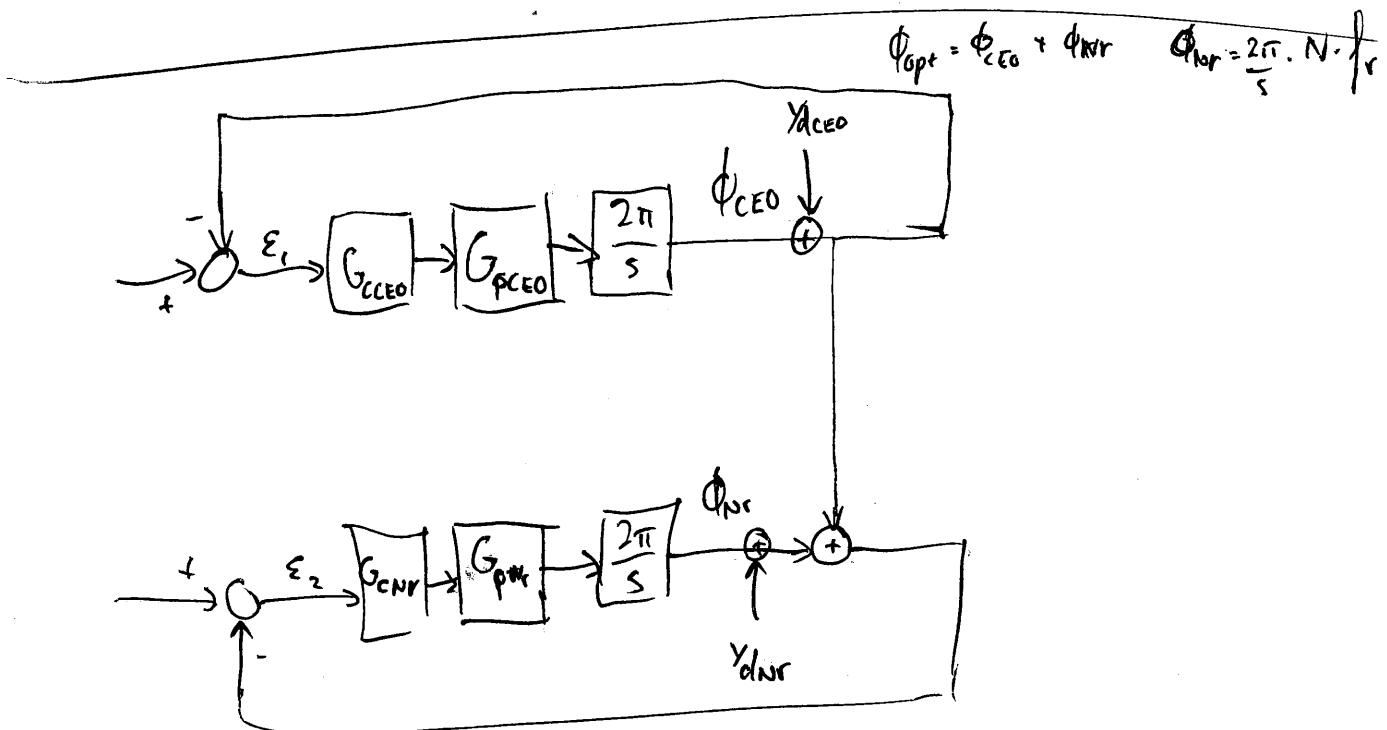
$$Y_2(1+G_2) = Y_1(1+\alpha G_2) + Y_{d2} + G_2X_2 + G_2\alpha Y_{m1} - G_2Y_{m2}$$

$$Y_2 = \frac{1+\alpha G_2}{1+G_2}Y_1 + \frac{1}{1+G_2}Y_{d2} + \frac{G_2}{G_2+1}X_2 + \frac{\alpha G_2}{1+G_2}Y_{m1} - \frac{G_2}{1+G_2}Y_{m2}$$

We care more about $Y_r = Y_2 - Y_1 = \frac{1+\alpha G_2}{1+G_2}Y_1 + \frac{1}{1+G_2}Y_{d2} + \frac{G_2}{1+G_2}X_2 + \frac{\alpha G_2}{1+G_2}Y_{m1} - \frac{G_2}{1+G_2}Y_{m2}$

$$Y_r = (\alpha-1)\frac{G_2}{1+G_2}Y_1 + \frac{1}{1+G_2}Y_{d2} + \frac{G_2}{1+G_2}X_2 + \frac{\alpha G_2}{1+G_2}Y_{m1} - \frac{G_2}{1+G_2}Y_{m2}$$

- Temp control, set point + settable from PC
 Needs ~2A current output, thermistor input.



$$y_1 = y_{d1} + G_1 \epsilon_1 = y_{d1} + G_1(x_1 - y_1)$$

$$y_2 = y_{d2} + y_1 + G_2 \epsilon_2 = y_{d2} + y_1 + G_2(x_2 - y_2)$$

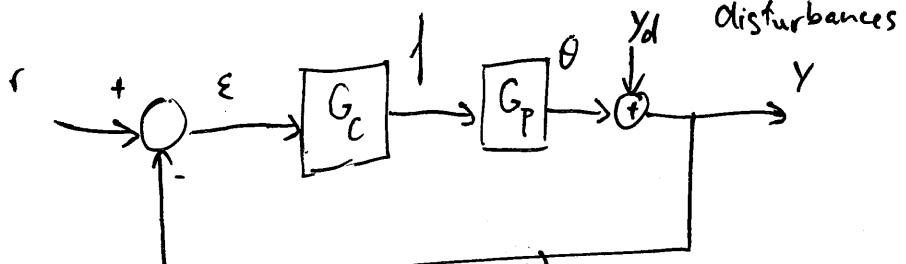
$$\Rightarrow y_1(1+G_1) = y_{d1} + G_1 x_1$$

$$y_1 = \frac{y_{d1}}{1+G_1} + \frac{G_1}{1+G_1} x_1$$

$$y_2(1+G_2) = y_{d2} + y_1 + G_2 x_2$$

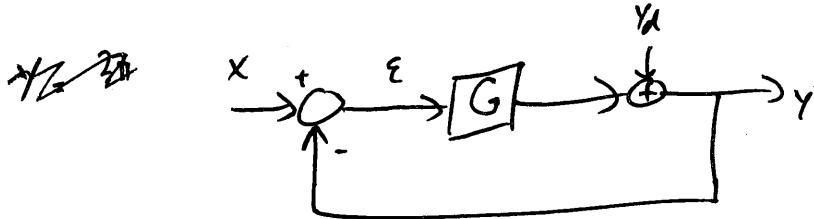
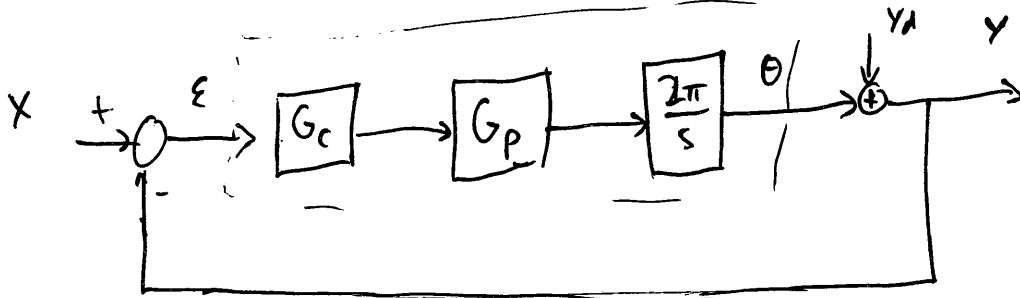
$$y_2 = \frac{y_{d2}}{1+G_2} + \frac{y_1}{1+G_2} + \frac{G_2}{1+G_2} x_2$$

So y_1 shows up as perturbations on the second loop and no TF is modified in any way (because there are no feedback paths). What about $y_r = y_2 - y_1$?
 $y_r = \frac{y_{d2}}{1+G_2} + \frac{G_2}{1+G_2} x_2 + \left(\frac{1}{1+G_2} - 1\right) y_1$



$$\theta = 2\pi \int_{-\infty}^t f(\tau) d\tau * h(t) \Rightarrow \theta(s) = \frac{2\pi F(s)}{s}$$

$$G_p(s) = \underline{G}$$



$$y = GE = G(x - y)$$

$$y(1+G) = GX$$

$$\frac{y}{x} = \frac{G}{1+G} = H$$

$$y = y_d + -GY$$

$$y(1+G) = y_d$$

$$\frac{y}{y_d} = \frac{1}{1+G}$$

In our case, G_p is hopefully a first-order lowpass $\frac{G_0}{1+s\tau}$, $\tau \approx 50\mu s$ or so (Er Riffstone)
 // We always have an integrator due to phasor being the integral of inst. freq.

$$H(1+G) = G$$

$$H + HG = G$$

$$H = G(1-H)$$

$$\frac{H}{1-H} = G$$

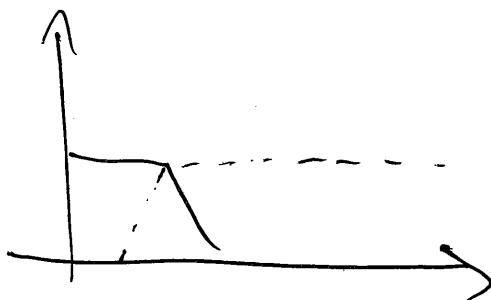
What if G_c contains ~~an integrator~~ a delayed P term plus another P term with less delay (can approximate to 0 delay for now).

$$G_c = K_1 e^{-j\omega T_1} + K_2 \quad K_2 \text{ higher than } K_1.$$

The first term uses a ~~high~~ high dynamic range phase detector for capture, while the second term uses a faster phase detector with only $\pm\pi$ of approximate range.

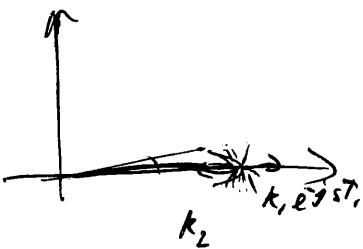
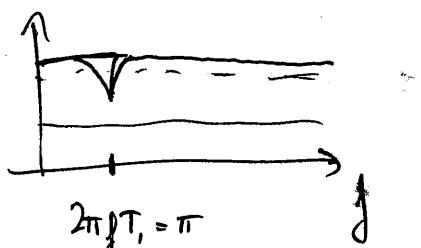
The important points are where the ~~the~~ open-loop transfer function reaches 1 of gain, and also where the relative phase between the two P terms reaches π , where they will subtract instead of summing. If their amplitude is different, the larger one will still dominate. Can we make the faster be high-pass only? Some sort of cross-over design like for a loudspeaker?

Make the crossover before the delay accumulates too much phase.

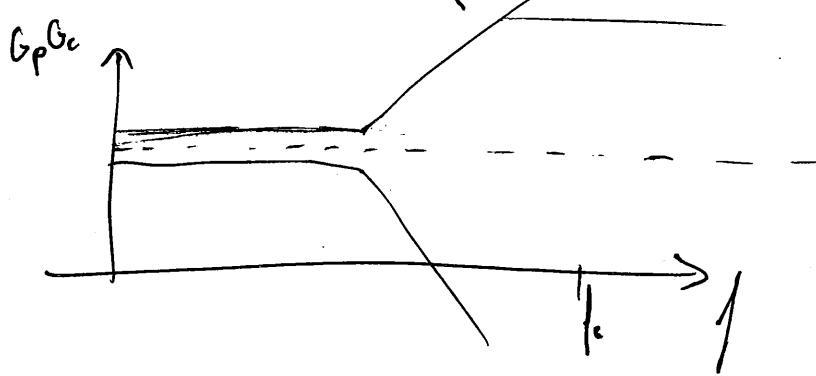
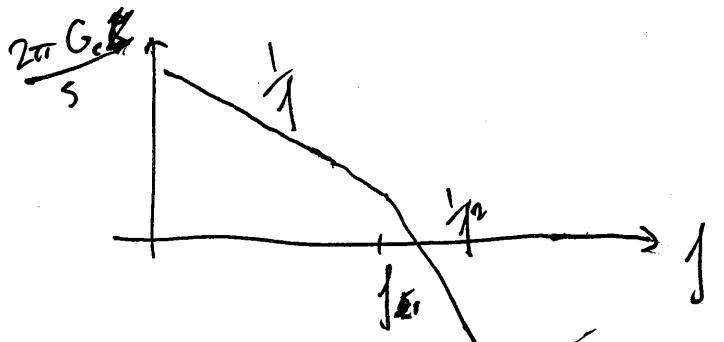
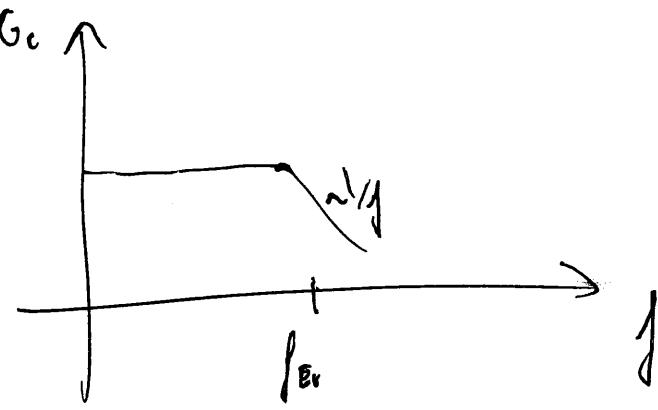


$$G_c = \frac{K_1 e^{-j\omega T_c}}{1 + j\omega T_c} + \frac{j\omega T_c}{1 + j\omega T_c} K_2$$

Ultimately we want the closed-loop BW to be higher than what we could get with just the slow term, which implies that we want $K_2 > K_1$. Is the cross-over design still workable? Do we even have to care to put filters in?



This will make the total phase oscillate with $\approx \frac{K_2}{K_1}$ radians of amplitude.
Minimum at $2\pi f T_c = \pi \Rightarrow f T_c = \frac{1}{2} \Rightarrow f = \frac{1}{2\pi T_c}$

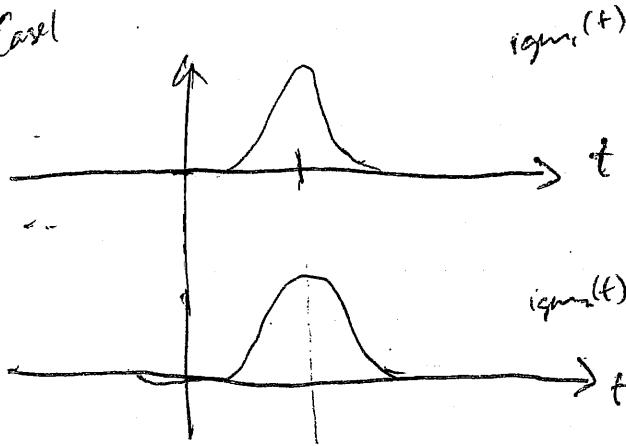


We want the total open-loop transfer function to be an integrator (around the critical frequency)

f_c is where total loop gain is equal to 1. We want as much phase margin as possible to have low servo bump.

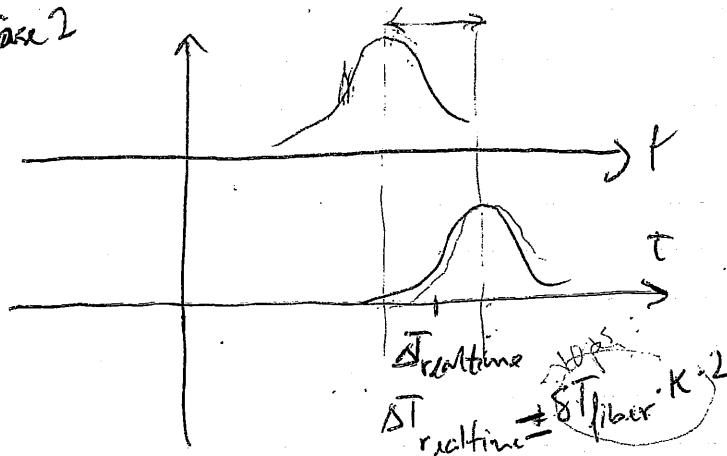
$$\left(1 + j\frac{f}{f_p}\right) \frac{1}{\left(1 + j\frac{f}{f_p}\right)^2}$$

Case 1



Link shorted, common
timelbase, paths adjusted
so that 10ms line up.

Case 2



Add link delay, common
timelbase.

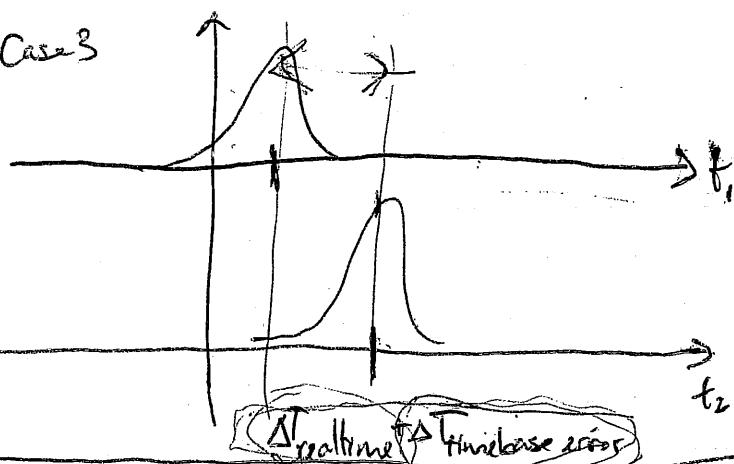
Assume link delay

$$\Delta T_{link} = \Delta T_{realtime} \cdot fr$$

$$\Delta T_{link} = \frac{\Delta T_{realtime}}{\text{Compression factor}}$$

$$K = \frac{\Delta T_{realtime}}{\Delta T_{link}} \quad \text{Compression factor} = \frac{fr}{fr}$$

Case 3



Has link delay, two separate
timelbases.

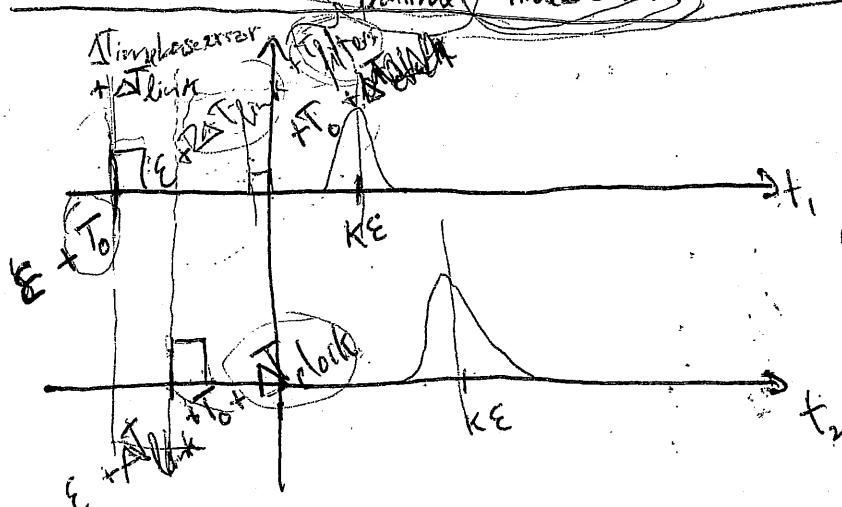
We make an error in link
delay: $\Delta T_{link} - \Delta T_{link, measured}$

$$= \frac{\Delta T_{timelbase error}}{\text{Compression factor}}$$

$$\text{Assume } AF = 20\text{Hz}, Fr = 200\text{Hz}$$

$$K = \frac{200}{20} = 10^2$$

$$\Delta T_{timelbase} = 1\text{ns}, \Delta T_{link error} = 10\text{ns}, \frac{10}{10^2} = 10^{-2}\text{s}.$$



$$t_2 = t_1 + \Delta T_{clock}$$

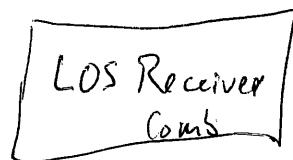
How do we sync the timebases?

Simple fact: When we get an IGM at one site, we know that the pulses overlap on that detector to $\leq 100\text{ fs}$. The time offset between the pulses at the remote site is Δt_{link} .

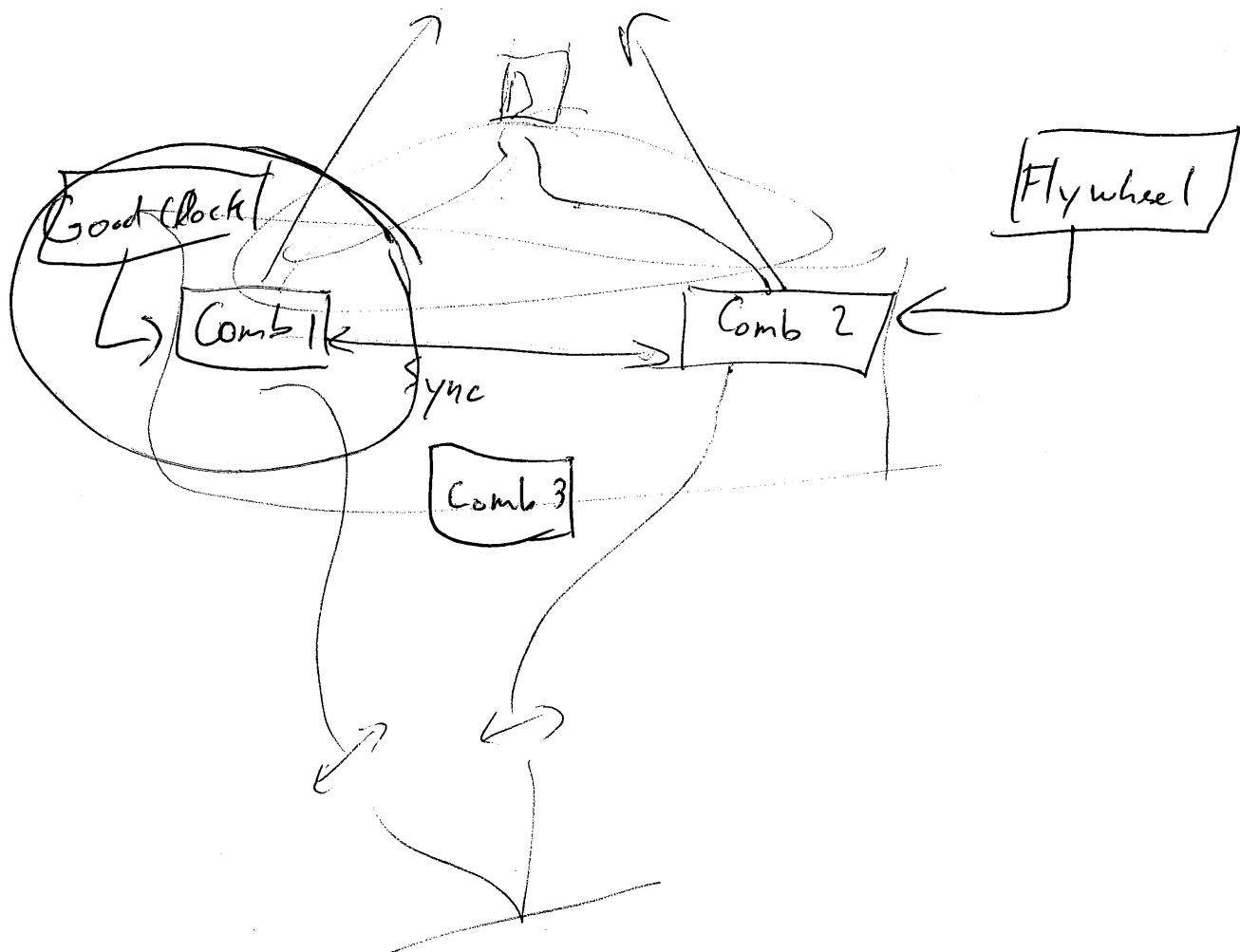
Link: 100km $300m \rightarrow 1\mu s$
 $100\text{ km} \rightarrow 1\mu s \cdot 360 = 360\mu s$

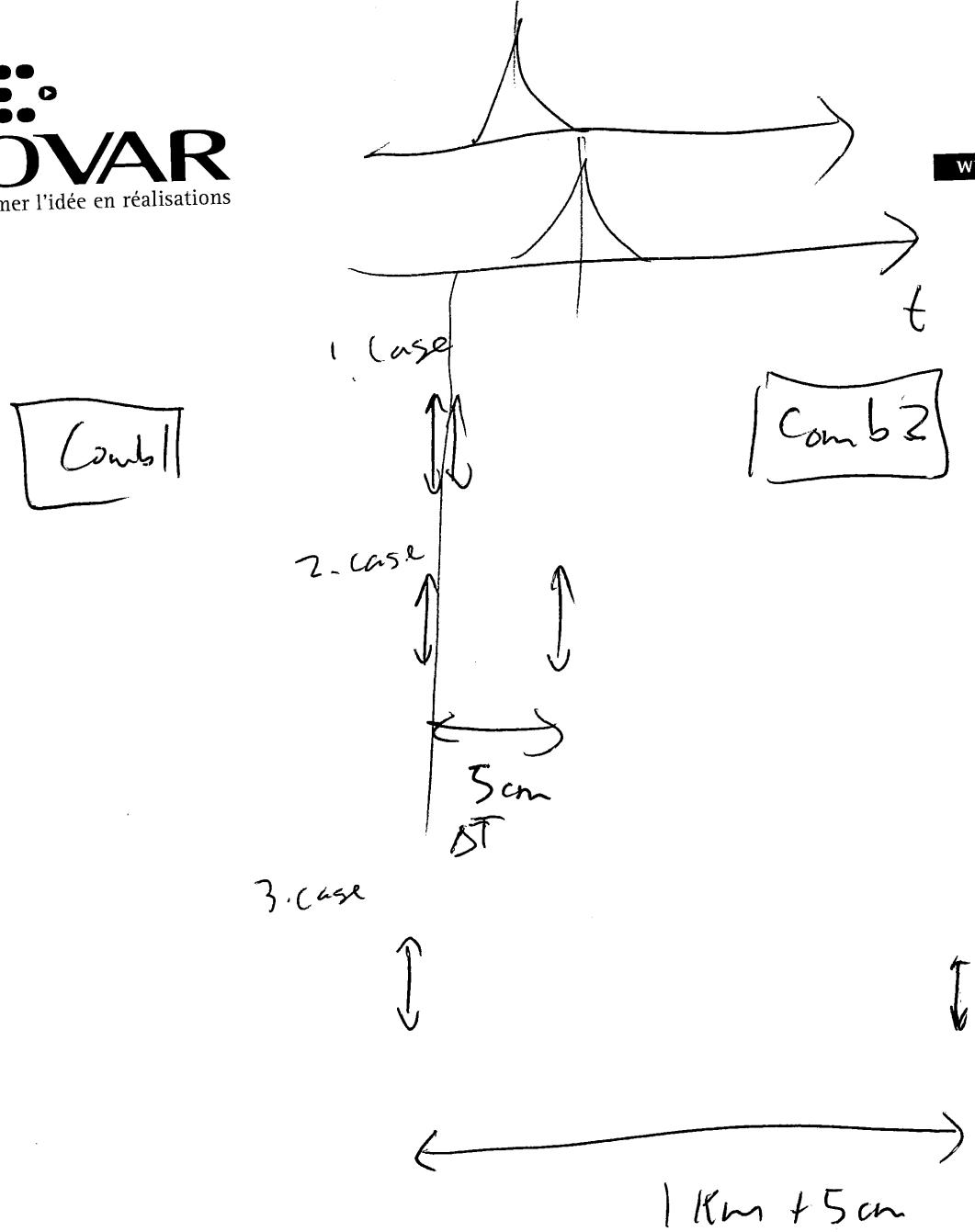
Plane at ~~1000m/s~~ 300 m/s (almost Mach 1)

$$\frac{300\text{ m}}{\text{s}} \cdot 300\mu s = 3e2 \cdot 3e2 \cdot 1e-6 = 10 \cdot 1e4 \cdot 1e-6 = 1e5 \cdot 1e-6 = 0,1\text{m}$$

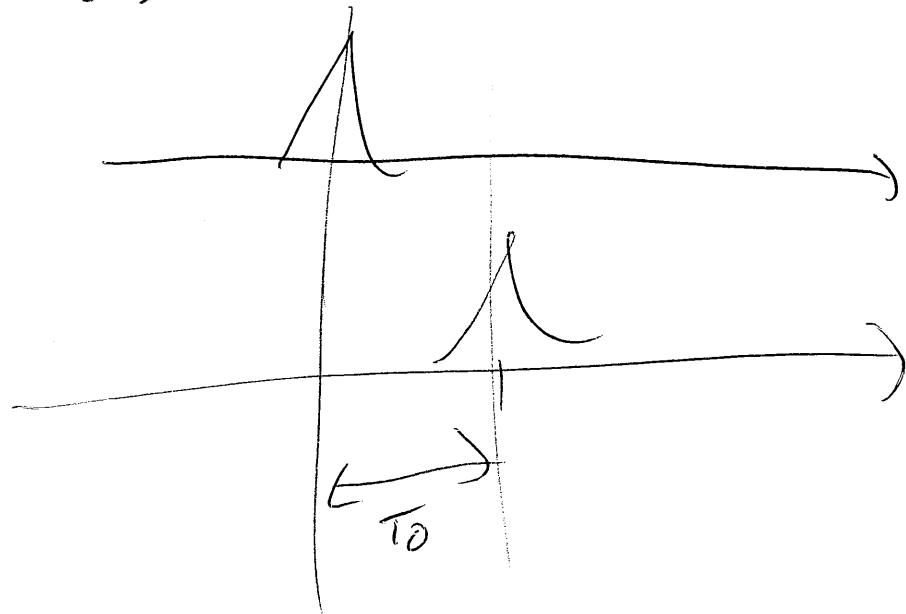


Outputs time difference
Can output position

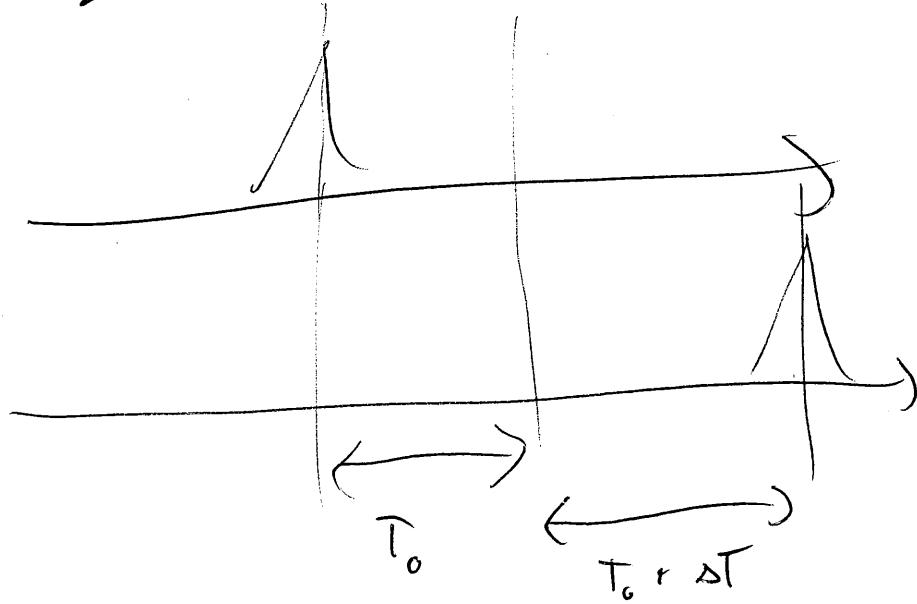




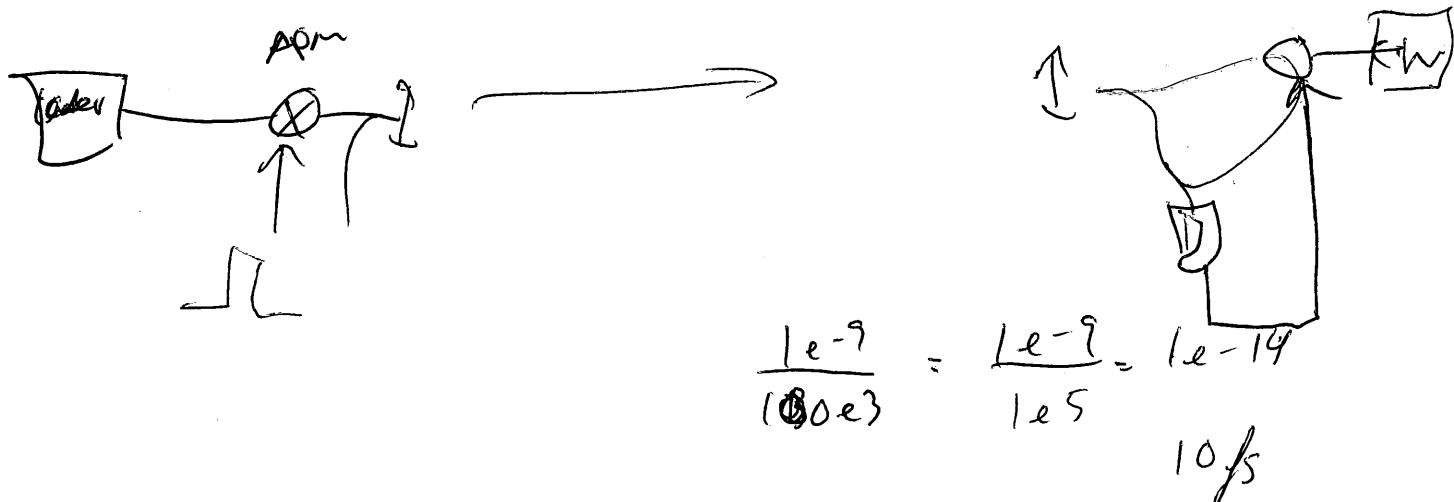
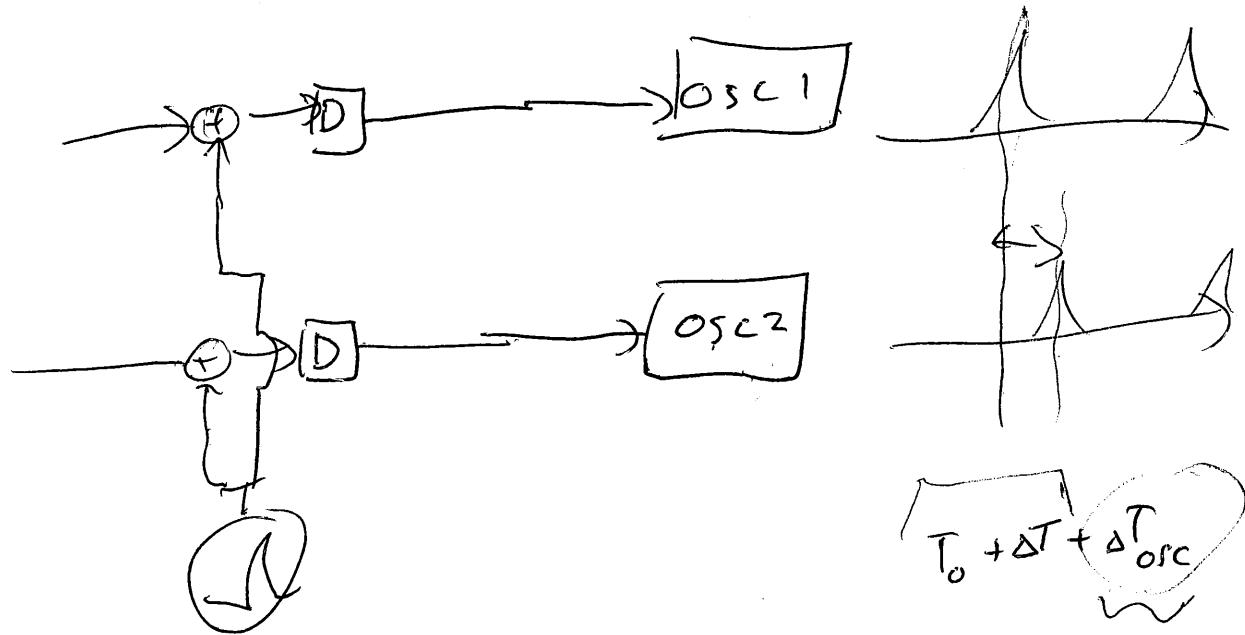
1. case



2. case



3. case



$$p(I_{obj}, \Theta_k) = \frac{\sqrt{I_{obj}}}{2\sqrt{\pi} \langle I_{obj} \rangle^{3/2} \sigma_z} \exp\left(-\frac{I_{obj}}{\langle I_{obj} \rangle} \left(1 + \frac{\Theta_k^2}{4\sigma_z^2}\right)\right), \quad I_{obj} > 0$$

$$= b \sqrt{x} \exp(ax)$$

$$P(\Theta_k) = \int_{-\infty}^{\infty} p(I_{obj}, \Theta_k) dI_{obj} = \int_0^{\infty} p(I_{obj}, \Theta_k) dI_{obj}$$

$$= b \int_0^{\infty} \sum_{I_{obj}} \exp(aI_{obj}) dI_{obj} = b \left[\frac{1}{a} \sqrt{I_{obj}} \exp(aI_{obj}) + \frac{j\sqrt{\pi}}{2a^{3/2}} \operatorname{erf}\left(\frac{\sqrt{a}I_{obj}}{2}\right) \right]_0^{\infty}$$

$$a = -\frac{1}{\langle I_{obj} \rangle} \left(1 + \frac{\Theta_k^2}{4\sigma_z^2}\right)$$

$$b = \frac{1}{2\sqrt{\pi} \langle I_{obj} \rangle^{3/2} \sigma_z}$$

$$P(\Theta_k) = b \left[\frac{1}{a} \sqrt{I_{obj}} \exp(a) - \frac{1}{a} \sqrt{0} \exp(0) + \frac{j\sqrt{\pi}}{2a^{3/2}} \left[\operatorname{erf}\left(\frac{\sqrt{a}I_{obj}}{2}\right) - \operatorname{erf}(0) \right] \right]$$

$$= b \left[\frac{j\sqrt{\pi}}{2a^{3/2}} \right] = b \cdot \frac{j\sqrt{\pi}}{2} \cdot \left(\frac{-1}{\langle I_{obj} \rangle} \left(1 + \frac{\Theta_k^2}{4\sigma_z^2}\right) \right)^{-3/2}$$

$$= \frac{b \cdot j\sqrt{\pi}}{2} \cdot (-1) \cdot \left(\frac{1}{\langle I_{obj} \rangle} \left(1 + \frac{\Theta_k^2}{4\sigma_z^2}\right) \right)^{-3/2}$$

$$= \frac{b\sqrt{\pi}}{2} \left(\frac{1}{\langle I_{obj} \rangle} \left(1 + \frac{\Theta_k^2}{4\sigma_z^2}\right) \right)^{-3/2}$$

$$= \frac{1}{2\cdot\sqrt{\pi} \langle I_{obj} \rangle^{3/2} \sigma_z} \cdot \frac{1}{\langle I_{obj} \rangle^{-3/2}} \cdot \left(\frac{1 + \Theta_k^2}{4\sigma_z^2} \right)^{-3/2}$$

$$= \frac{1}{4\sigma_z^2} \left(\frac{4\sigma_z^2 + \Theta_k^2}{4\sigma_z^2} \right)^{-3/2}$$

$$P(\Theta_k) = \frac{\sqrt{\pi}}{4\sigma_z} \cdot \left(\frac{4\sigma_z^2}{4\sigma_z^2 + \Theta_k^2} \right)^{3/2}$$

- Fix expression for $p(\Theta_k)$: Θ_k normalized

- Compute second moment $\langle \Theta_k^2 \rangle = \int_{-\infty}^{\infty} \Theta_k^2 p(\Theta_k) d\Theta_k$

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$p(\Theta_k)$ is almost Lorentzian distribution having no higher moments.

Fixed.

Has to be wrong because of
Scaling with $\langle I_{obj} \rangle$.

Is $p(I_{obj}, \Theta_k)$ even normalized?

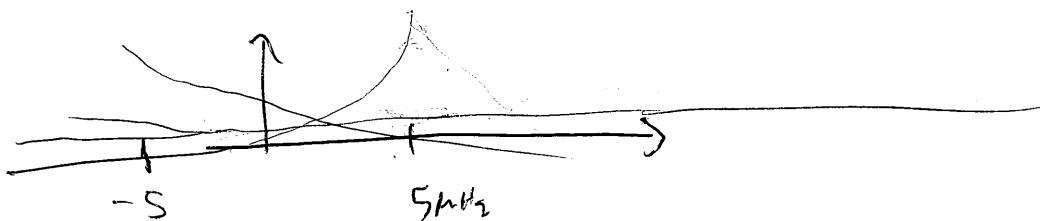
$$\begin{aligned}
 p(\theta_K) &= \frac{1}{4\sigma_z^2} \left(\frac{4\sigma_z^2}{4\sigma_z^2 + \theta_K^2} \right)^{3/2} \\
 &\approx \frac{1}{4\sigma_z^2} \left(1 + \frac{\theta_K^2}{4\sigma_z^2} \right)^{-3/2} \\
 &= \frac{1}{4\sigma_z^2} \left(\frac{1}{\frac{4\sigma_z^2}{\theta_K^2}} \cdot \frac{1}{1 + \left(\frac{\theta_K}{2\sigma_z} \right)^2} \right)^{3/2} \\
 &= \frac{1}{4\sigma_z^2} \left(\frac{1}{1 + \left(\frac{\theta_K}{2\sigma_z} \right)^2} \right)^{3/2}
 \end{aligned}$$

$$p(x) = \frac{1}{\pi(1+x^2)} \quad \text{is Lorentzian}$$

$$\langle \theta_K \rangle^2 = \int_{-\infty}^{\infty} p(\theta_K) \theta_K^2 d\theta_K = \int_{-\infty}^{\infty} \frac{1}{4\sigma_z^2} \left(\frac{1}{1 + \left(\frac{\theta_K}{2\sigma_z} \right)^2} \right)^{3/2} \theta_K^2 d\theta_K$$

Could do ~~do~~
~~2~~ $\int_{-\infty}^{\infty} d\theta_K$

$$u = \left(\frac{\theta_K}{2\sigma_z} \right)^2 \quad du = \frac{2\theta_K}{4\sigma_z^2} d\theta_K \quad \text{does not converge ...}$$



Signal is third order polynomial

We do a first order least-squares fit

→ Which is really just a linear combination of our points

How much does third order term affect the slope that we find?

But first: What is derivative of our third order polynomial

$R(z)$ = gaussian with std deviation = σ_R ~~width~~

$$h(t) = \sum_{k=1}^N a_k \delta(t - z_k)$$

z_k are Gaussian with std dev = σ_z , are one? we don't really care

$$\text{IM}(z) = R(z) \exp\left(j2\pi \frac{z}{\lambda}\right) \quad \text{IGM for a single scatterer at } z=0.$$

$$I_h(z) = \cancel{\text{IM}(z)} \star h(t) = \sum_{k=1}^N a_k R(z - z_k) \exp\left(j2\pi \frac{(z - z_k)}{\lambda}\right)$$

$$I_h(z) = R(z) * \sum_{k=1}^N a_k \exp\left(j2\pi \frac{(z - z_k)}{\lambda}\right) = R(z) * h_c(z)$$

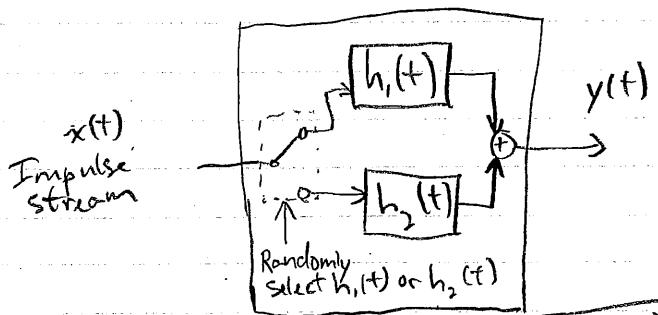
$R(z)$ is smooth compared to $h_c(z)$

~~$R(z) \approx R_1(z) + R_2(z)$~~

We look for the peak in the envelope of $I_h(z)$, or the envelope squared

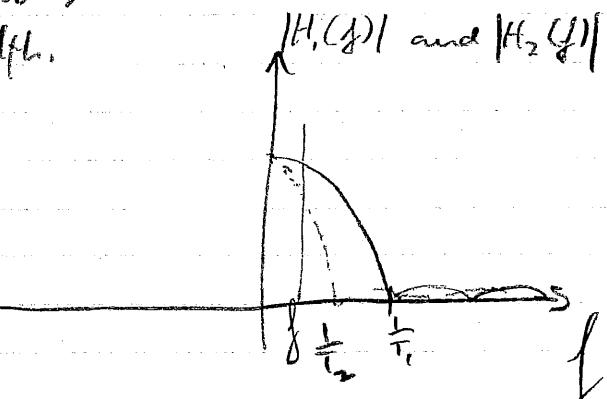
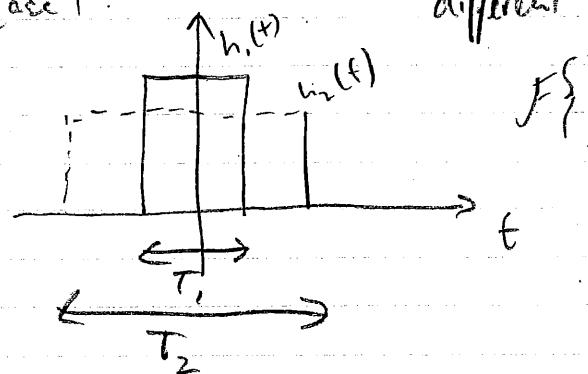
$$|I_h(z)|^2 = \left| \sum_{k=1}^N a_k R(z - z_k) \exp\left(j2\pi \frac{(z - z_k)}{\lambda}\right) \right|^2$$

$$\frac{\#_z}{V} \cdot \frac{\text{Counts}}{\Delta z} \cdot \frac{\lambda}{\text{Counts}} = \frac{\text{counts}}{\text{counts}}$$



Simple model: send Dirac impulses to a stochastic system, which chooses either $h_1(t)$ or $h_2(t)$ as the impulse response for each.

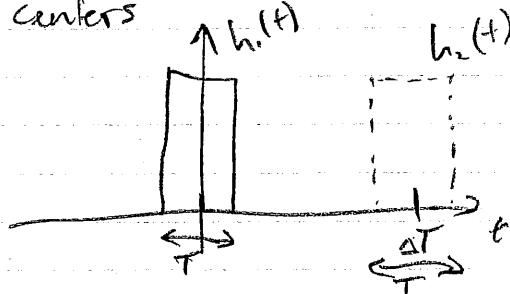
Case 1: $h_1(t)$ and $h_2(t)$ have same center but different widths.



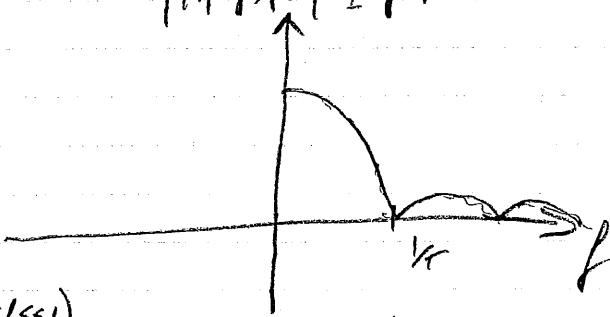
Results:

- Gives a lot of excess amplitude noise at f .
- Gives no extra jitter (center of mass of $h(t)$ doesn't change)
- Excess noise factor is high at f $F_{H(Y)} > 0$.

Case 2: $h_1(t)$ and $h_2(t)$ have same width but different centers



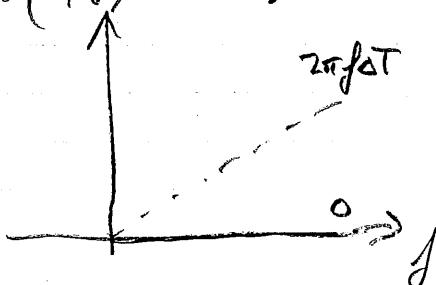
$$|H_1(f)| = |H_2(f)|$$



Results:

- No excess amplitude noise (for $\Delta T \ll 1$)
- Excess timing jitter equal to $\frac{\Delta T}{2}$.
- Excess noise factor is $F_{H(Y)} = 0$.

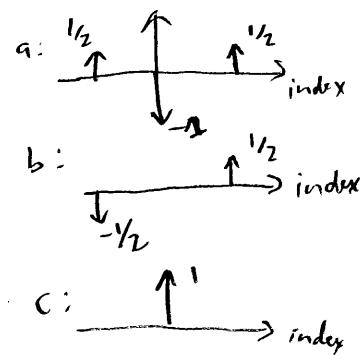
$$-\text{Angle}(H_1(f)) \text{ and } -\text{Angle}(H_2(f))$$



$$a = \frac{1}{2} [y(-1) - 2y(0) + y(1)] = \frac{1}{2} [y_{-1} - 2y_0 + y_1]$$

$$b = \frac{1}{2} [y(1) - y(-1)] = \frac{1}{2} [y_1 - y_{-1}]$$

$$c = y(0) = y_0$$



$$K_{\text{peak}} = \frac{-b}{2a} = \frac{1}{2} \frac{y_1 - y_{-1}}{y_{-1} - 2y_0 + y_1}$$

$$Y_{\text{peak}} = aK_{\text{peak}}^2 + bK_{\text{peak}} + c$$

$$= \frac{b^2}{2a} + \frac{-b^2}{2a} + c$$

noiseless measurement vector

Assume three noise samples: $y_{-1} = \tilde{y}_{-1} + n_{-1}$
 $y_0 = \tilde{y}_0 + n_0$
 $y_1 = \tilde{y}_1 + n_1$

Transformation from the measurement vector $[y_{-1}, y_0, y_1]$ to the peak position is: $K_{\text{peak}} = \frac{1}{2} \frac{y_1 - y_{-1}}{y_{-1} - 2y_0 + y_1}$

or it is non-linear, but we approximate it with the following:

$$K_{\text{peak}} = \frac{1}{2} \frac{\tilde{y}_1 - \tilde{y}_{-1} + n_{-1} - n_1}{\tilde{y}_{-1} + n_{-1} + -2\tilde{y}_0 - 2n_0 + \tilde{y}_1 + n_1} = \frac{1}{2} \frac{\tilde{y}_{-1} - \tilde{y}_1 + n_{-1} - n_1}{\alpha (1 + \frac{n_{-1} - 2n_0 + n_1}{\alpha})} = \frac{1}{2} \frac{\beta (1 + \frac{n_{-1} - n_1}{\beta})}{\alpha (1 + \frac{n_{-1} - 2n_0 + n_1}{\alpha})}$$

$$\approx \frac{1}{2} \frac{\beta}{\alpha} \left(1 + \frac{n_{-1} - n_1}{\beta} - \frac{n_{-1} - 2n_0 + n_1}{\alpha} \right) = \tilde{K}_{\text{peak}} \left(1 + \frac{n_{-1} - n_1}{\beta} - \frac{n_{-1} - 2n_0 + n_1}{\alpha} \right)$$

with $\beta = \tilde{y}_{-1} - \tilde{y}_1$ $\tilde{K}_{\text{peak}} = \frac{1}{2} \frac{\beta}{\alpha}$ (the true peak position)
 $\alpha = \tilde{y}_{-1} - 2\tilde{y}_0 + \tilde{y}_1$

~~$$K_{\text{peak}} = \frac{1}{2} \frac{\beta}{\alpha} \left(\frac{n_{-1} - n_1}{\beta} - \frac{n_{-1} - 2n_0 + n_1}{\alpha} \right)$$~~

$$\text{Error} = \frac{1}{2} \left(\frac{n_{-1} - n_1}{\alpha} - \frac{n_{-1} - 2n_0 + n_1}{\alpha^2} \beta \right)$$

$$= \frac{1}{2} \left[n_{-1} \left(\frac{1}{\alpha} - \frac{\beta}{\alpha^2} \right) + n_0 \left(\frac{-2\beta}{\alpha^2} \right) + n_1 \left(\frac{-1}{\alpha} - \frac{\beta}{\alpha^2} \right) \right]$$

The coefficients in parenthesis should be $\frac{\partial \text{error}}{\partial n_{-1}}$, $\frac{\partial \text{error}}{\partial n_0}$, $\frac{\partial \text{error}}{\partial n_1}$.

Potential complication: coefficients depend on the values of β and α , which depends on the subsample peak position.

Let's decompose the noise differently:

$$a = \frac{1}{2} [Y_1 - 2Y_0 + Y_2]$$

$$b = \frac{1}{2} [Y_1 - Y_2]$$

$$K_{\text{peak}} = \frac{-b}{2a}$$

$$\frac{\partial K_{\text{peak}}}{\partial b} = \frac{-1}{2a}$$

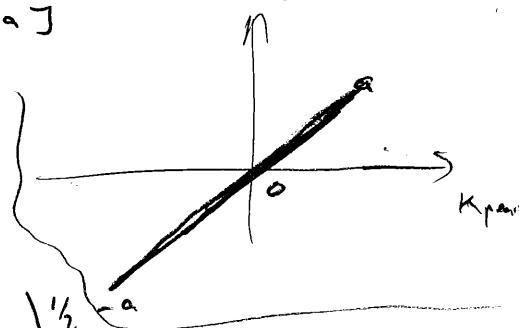
$$\frac{\partial K_{\text{peak}}}{\partial a} = \frac{+b}{2a^2}$$

a is pretty much constant with respect to K_{peak}

4. but b varies linearly with K_{peak} , so the b noise gets converted to K_{peak} noise differently depending on the subsample position, while the b noise has pretty much always the same conversion coefficient, $\frac{-1}{2a}$.

5. Assume that we know a for our signal, and K_{peak} varies within $[-0.5, 0.5]$:
 How does b scale? $-2aK_{\text{peak}} = b \in [-a, a]$
 What is the RMS conversion coefficients?

$$\sqrt{\left\langle \left(\frac{\partial K_{\text{peak}}}{\partial b} \right)^2 \right\rangle} = \sqrt{\frac{1}{4a^2}} = \frac{1}{2a}$$



RMS value of $\frac{\partial K_{\text{peak}}}{\partial a}$

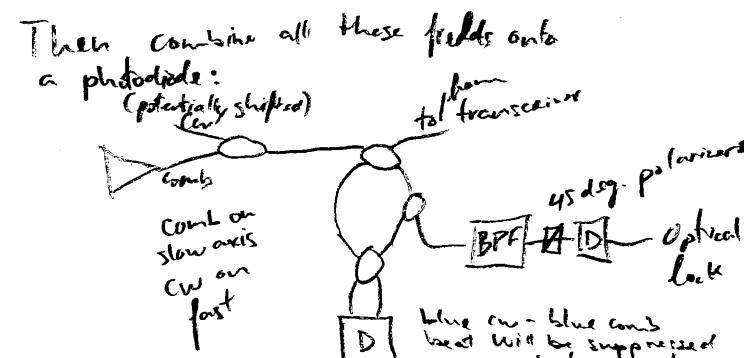
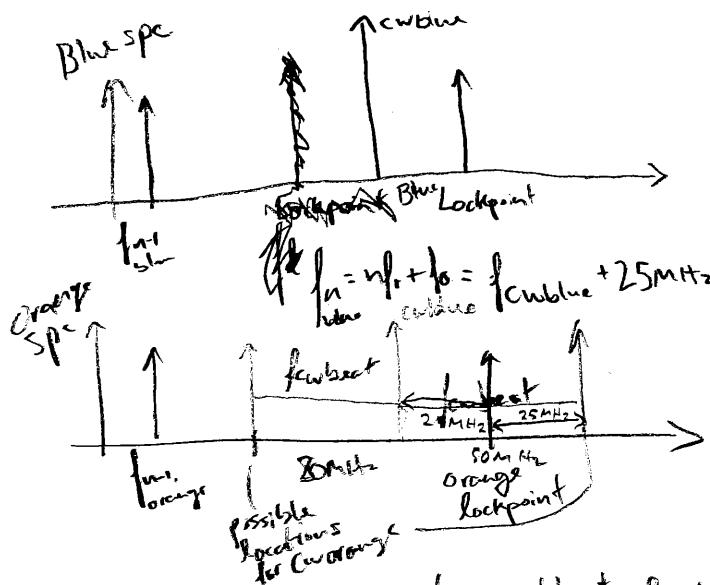
$$\sqrt{\left\langle \left(\frac{\partial K_{\text{peak}}}{\partial a} \right)^2 \right\rangle} = \sqrt{\left(\frac{1}{2a^2} \int_{-0.5}^{0.5} (-2aK_{\text{peak}})^2 dK_{\text{peak}} \right)^{1/2}} = \sqrt{\left(4a^2 \int_{-0.5}^{0.5} K_{\text{peak}}^2 dK_{\text{peak}} \right)^{1/2}} \cdot \frac{1}{2a^2}$$

$$= \frac{2a}{2a^2} \left(\frac{1}{3} K_{\text{peak}}^3 \Big|_{-0.5}^{0.5} \right)^{1/2} \cdot \frac{2a}{\sqrt{3} \cdot 2a^2} = \frac{2a}{\sqrt{3}} \frac{2}{2a^2} \frac{1}{2} = \frac{4ax}{4\sqrt{3}a^2} = \frac{1}{4\sqrt{3}|a|}$$

$$\text{std}(K_{\text{peak}}) = \sqrt{\left\langle \left(\frac{\partial K_{\text{peak}}}{\partial b} \right)^2 \right\rangle} \cdot \text{std}(b) +$$

$$3. \quad \text{Var}(K_{\text{peak}}) = \left\langle \left(\frac{\partial K_{\text{peak}}}{\partial b} \right)^2 \right\rangle \cdot \text{Var}(b) + \left\langle \left(\frac{\partial K_{\text{peak}}}{\partial a} \right)^2 \right\rangle \cdot \text{Var}(a)$$

$$2. \quad \text{Var}(K_{\text{peak}}) = \text{Var}\left(\frac{\partial K_{\text{peak}}}{\partial b} \cdot b_{\text{noise}}\right) + \text{Var}\left(\frac{\partial K_{\text{peak}}}{\partial a} \cdot a_{\text{noise}}\right)$$

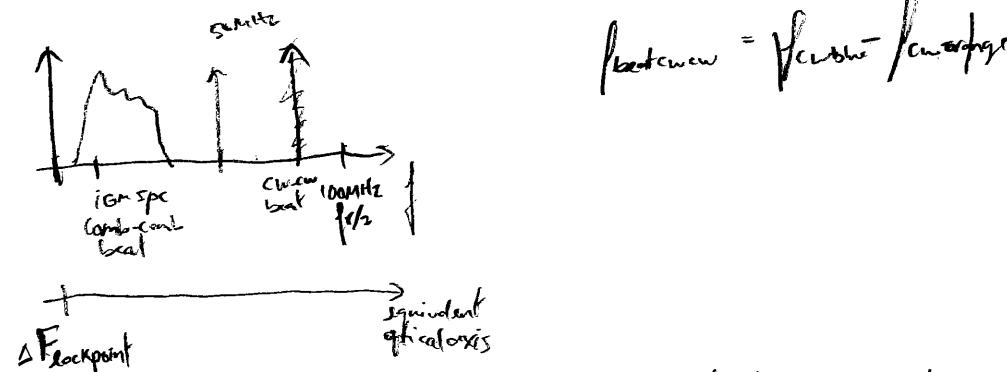


Blue cw - blue comb
beat will be suppressed somewhat by balancing.

+ combs fields are on one polarization while cw fields are on a different one.

↳ doesn't work because transceivers will let only 1 polarization through.

We want a beat spectrum that looks like:



On the IGM detector, we want:

100μW local CW

100μW local comb in 10THz BW

On the optical Beat detector, we want ideally:

100μW local CW

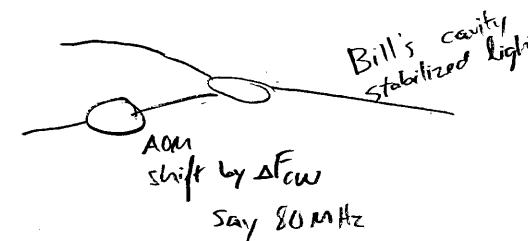
100μW local comb in 100 GHz BW

But we'll have only $\sim 100\mu\text{W} - \frac{100\text{GHz}}{10\text{THz}} \sim 1\mu\text{W}$ in 100 GHz

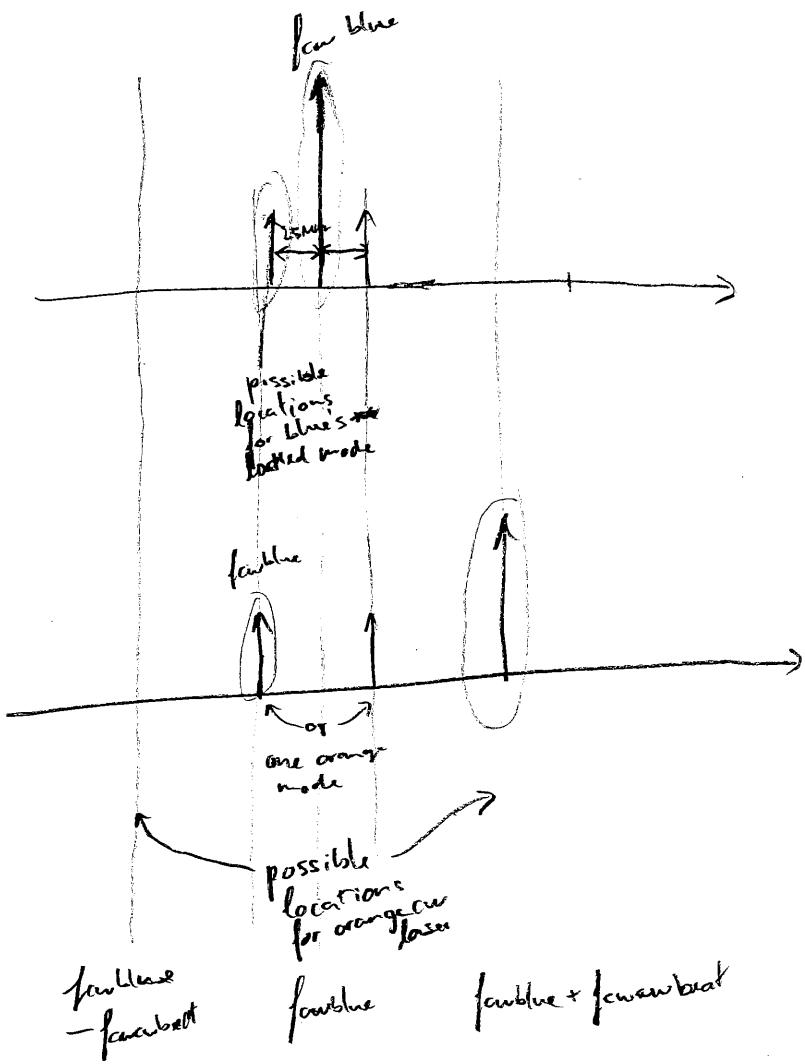
i.e. $100\text{GHz} = N \cdot 200\text{MHz} \Rightarrow N = 500\text{modes}$

$$\frac{1\mu\text{W}}{500\text{modes}} = 2\text{nW/mode}$$

Can use higher gain detector to have lower NEP or a GATOR. (GATOR paper used 20pW/mode)



which is nice because we need an AOM anyway for Doppler cancellation (needed on both so we just use that frequency offset).



We would like $f_{cw beat}$ to be well on one side of the beat spectrum, for example $f_{cw beat} = 75\text{MHz}$ or even 95MHz

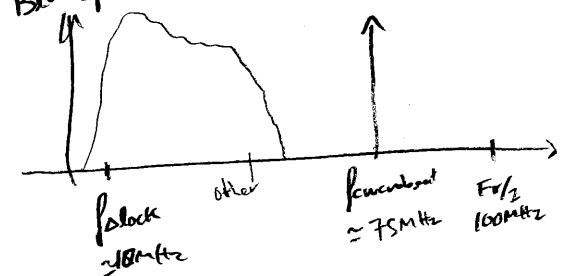
OK assume

$$f_{cw beat} = f_{orange} - f_{cw blue} \quad (\text{orange cw laser is higher})$$

$$\text{locked blue comb} = f_{cw blue} - 25\text{MHz} \quad f_{lock} = 25\text{MHz}$$

Assume we want one mode from the orange laser to be

$\sim 10\text{MHz}$ offset from blue's laser



$$f_{lock} = f_{lock orange} - f_{lock blue}$$

$$\begin{aligned} f_{locked orange} &= f_{locked blue} + f_{lock} \\ &= f_{locked blue} + 10\text{MHz} \\ &= f_{cw blue} - 25\text{MHz} + 10\text{MHz} \end{aligned}$$

$$f_{locked orange} = f_{cw blue} + f_{lock} + f_{lock}$$

$$f_{locked orange} = f_{orange} \pm f_{lock}$$

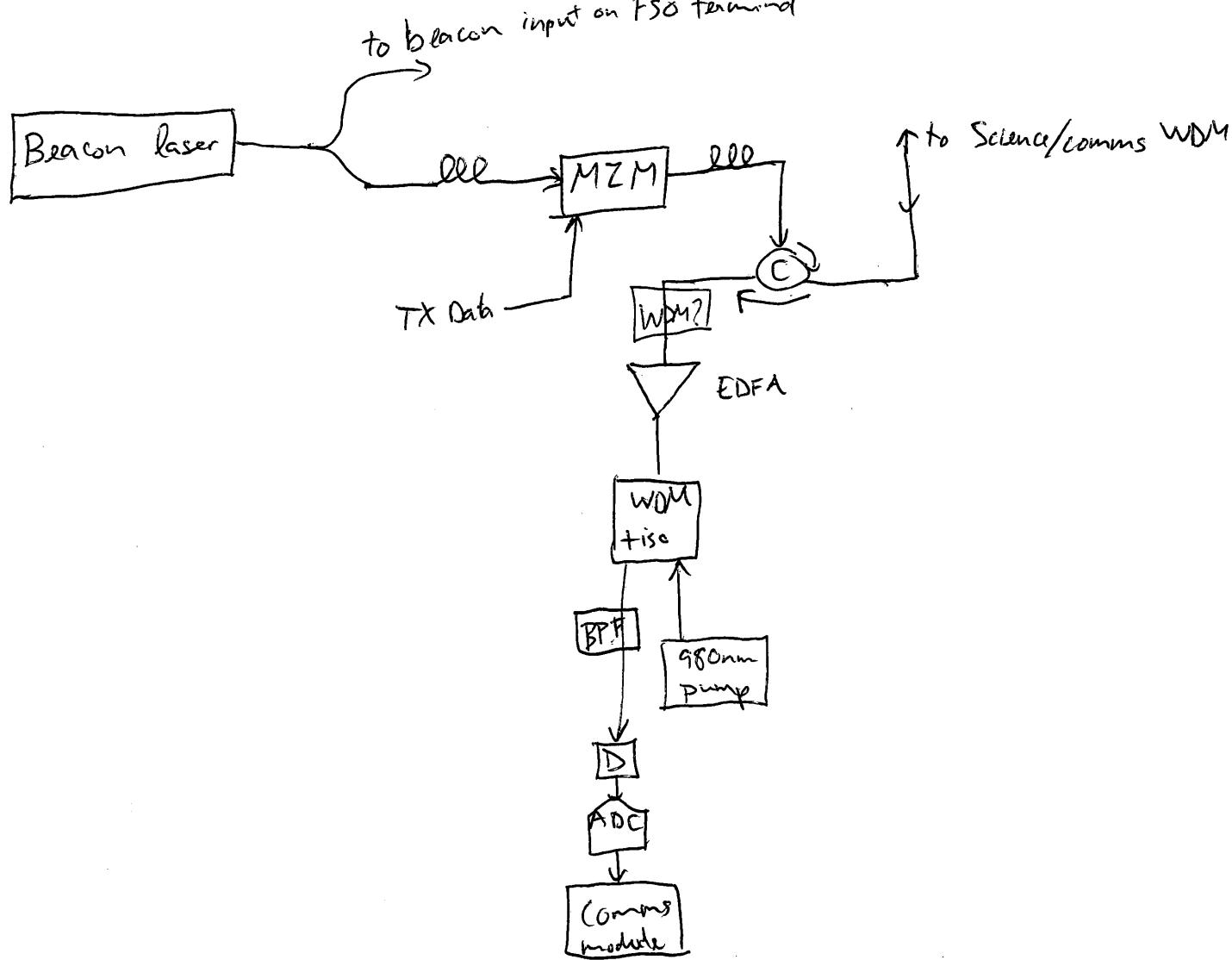
So we need $f_{cw blue} - f_{lock} + f_{lock} = f_{orange} \pm f_{lock}$
 We already have a constraint for $f_{cw blue} - f_{orange}$:

$$f_{cw beat} - f_{lock} = f_{lock} = -f_{lock}$$

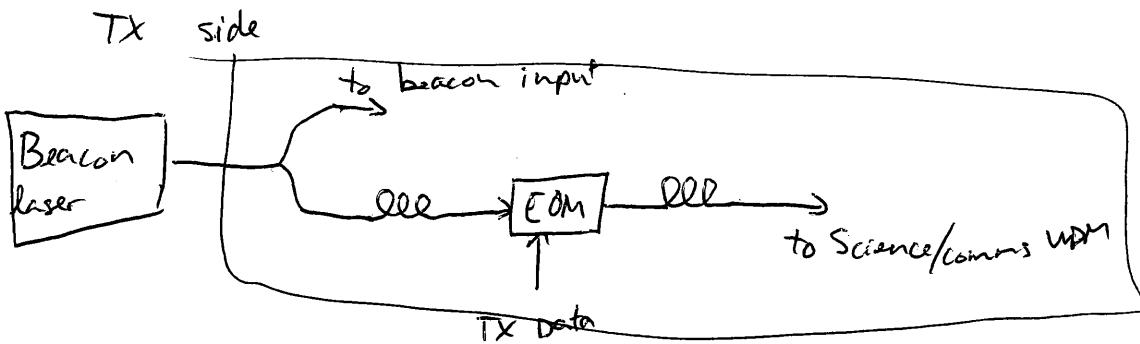
Let's put numbers

$$\begin{aligned} f_{cw beat} - 25\text{MHz} \pm 25\text{MHz} &= -10\text{MHz} \text{ or } 50\text{MHz} \\ 75\text{MHz} - (50\text{MHz} \text{ or } 0) &= -10\text{MHz} \end{aligned}$$

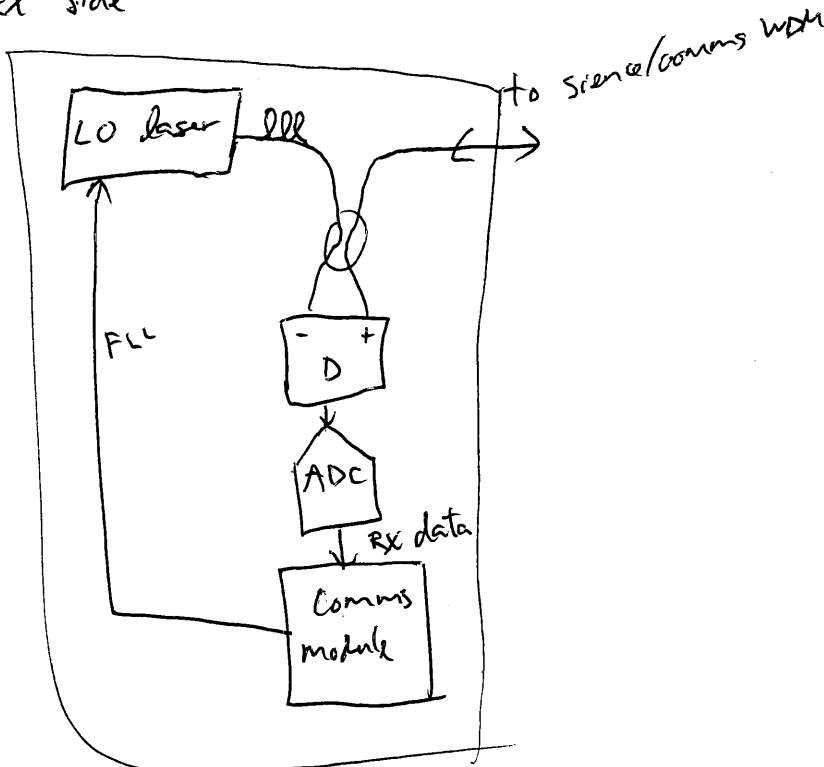
Optical comms package w/ optical pre-amp (incoherent detection), Rx+Tx

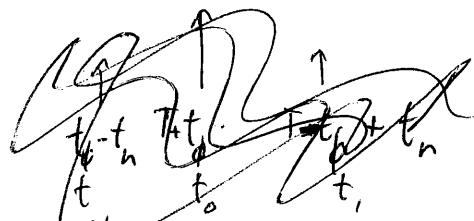


Coherent comms, ~~no~~ separate RX and TX



Rx side





We want a linear combination of t_{-1} , t_0 and t_1 and t_ϕ , and a gain of 1 through

which gives 0 sensitivity to t_n .

$$\begin{aligned} t_1 &= T - t_\phi - t_n \\ t_0 &= T + T_\phi \end{aligned}$$

$$t_1 = T - T_\phi + t_n$$

$$\frac{1}{2}(t_{-1} + t_1) = T - T_\phi$$

$$t_0 + \frac{1}{2}(t_{-1} + t_1) = 2T$$

$$\frac{1}{2}\left(t_0 + \frac{1}{2}(t_{-1} + t_1)\right) = \frac{1}{2}t_0 + \frac{1}{4}t_{-1} + \frac{1}{4}t_1 = T$$

Definitions used by Stage 32:

	<u>Symbol</u>	<u>Units</u>
Phase (really time)	$x[i]$	[seconds]
(Fractional) Frequency $y[i] = \frac{x[i]}{T_0}$	$y[i]$	[unitless]
Sampling period	T_{car}, T_0	
Real phase	$\phi(t)$	[radians]

To go from real phase to time: $\phi(t) = x(t) \cdot 2\pi\nu_0 = x(t) \cdot \frac{2\pi}{\text{Period}}$

$$x(t) = b(t) \cdot \frac{\text{Period}}{2\pi} = \frac{\phi(t)}{2\pi\nu_0}$$

To go from frequency to phase:

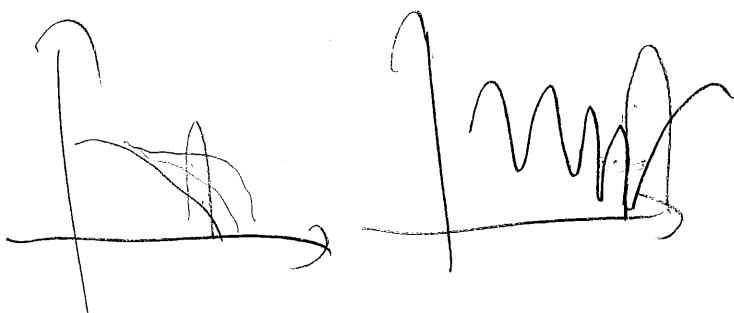
$$\phi(t) = 2\pi \int_{-\infty}^t \Delta f(u) du \quad \Delta f(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

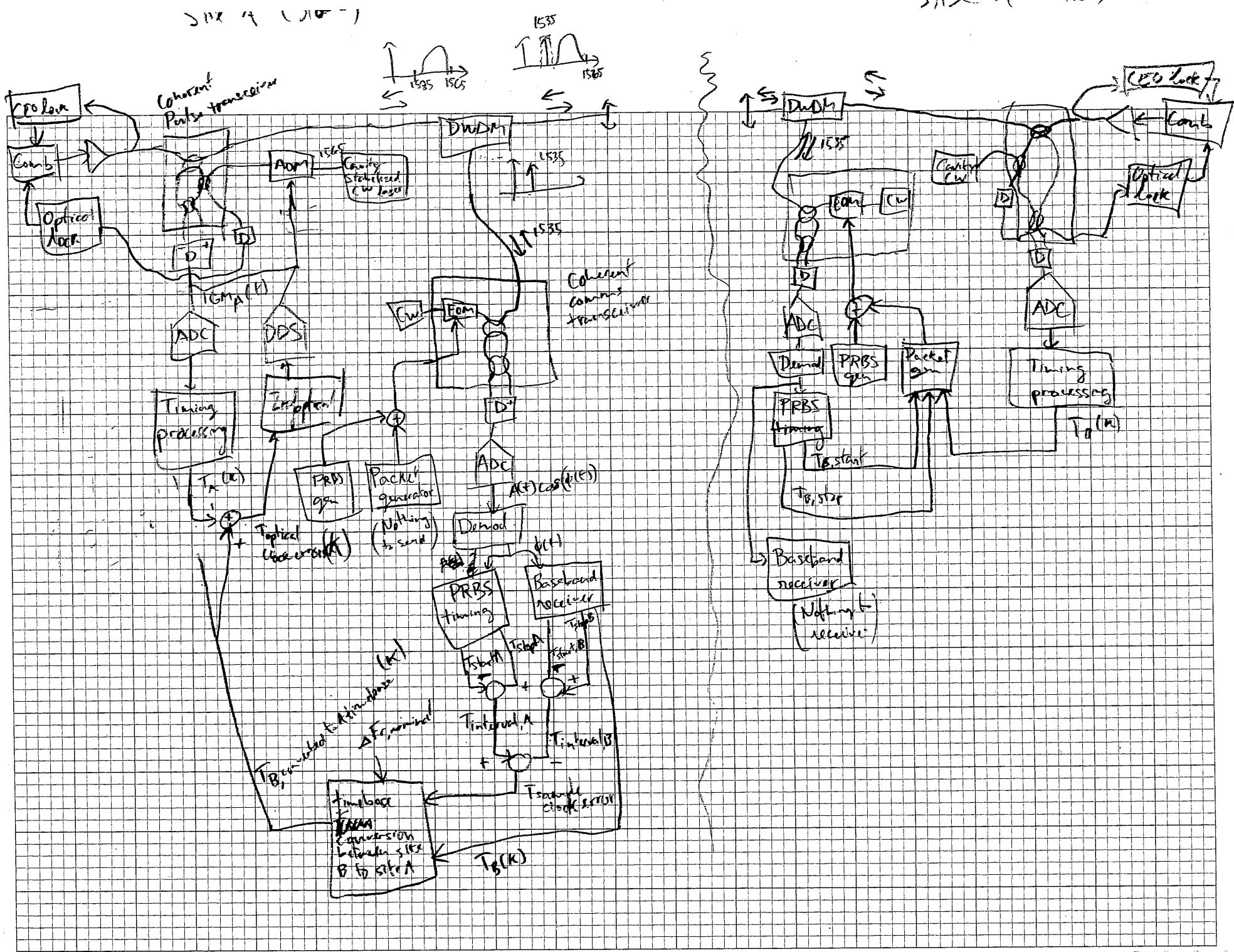
Between normalized frequency and phase and time:

$$y = \frac{\Delta f(t)}{\nu_0} = \frac{1}{2\pi\nu_0} \frac{d\phi(t)}{dt} = \frac{1}{2\pi\nu_0} \frac{d(x(t) \cdot 2\pi\nu_0)}{dt} = \frac{dx(t)}{dt}$$

This is why it doesn't need the carrier frequency if it has time.

$$y = \frac{d \text{delay}(t)}{dt} = \text{Delay slope in } \frac{\text{seconds}}{\text{seconds}}$$

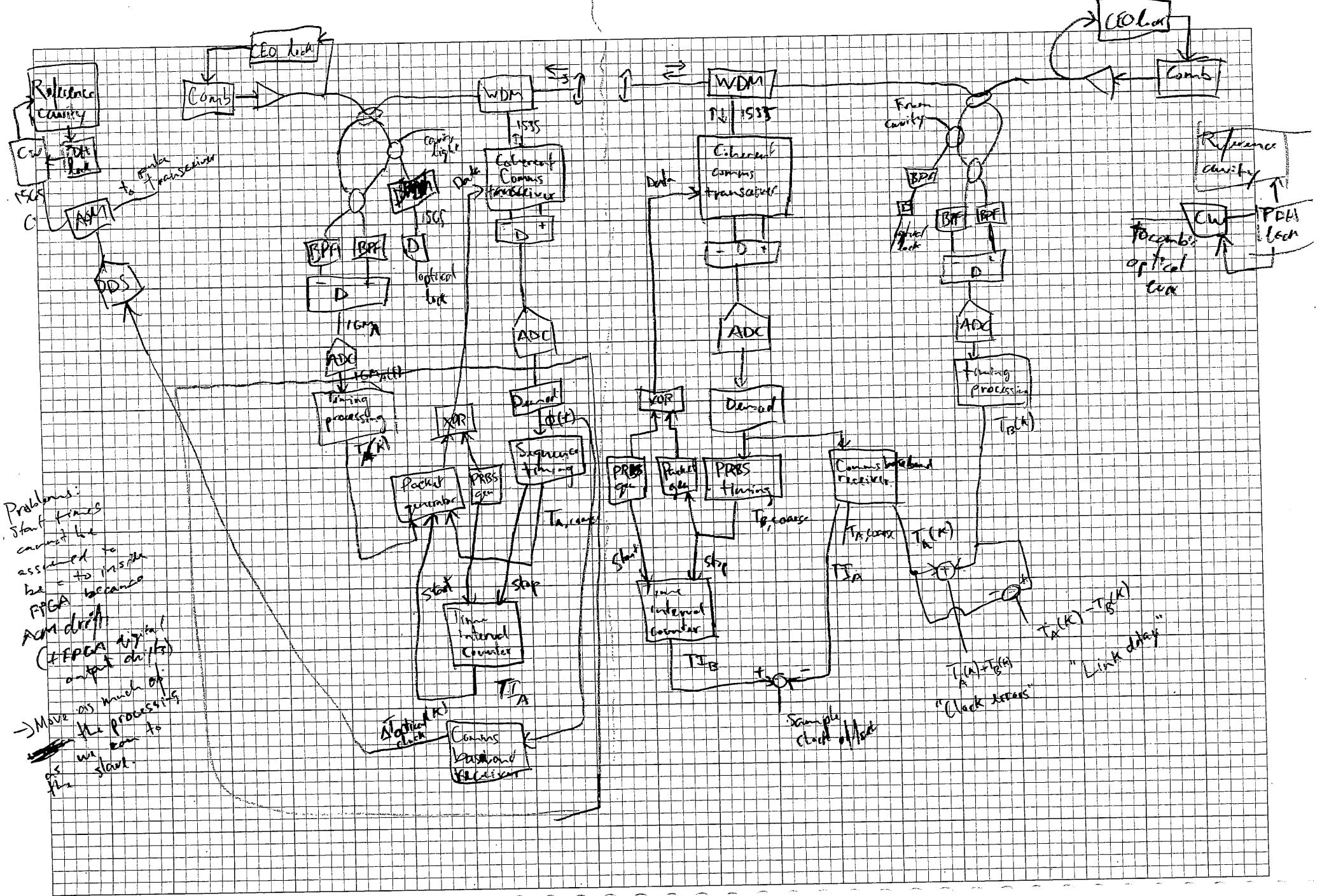




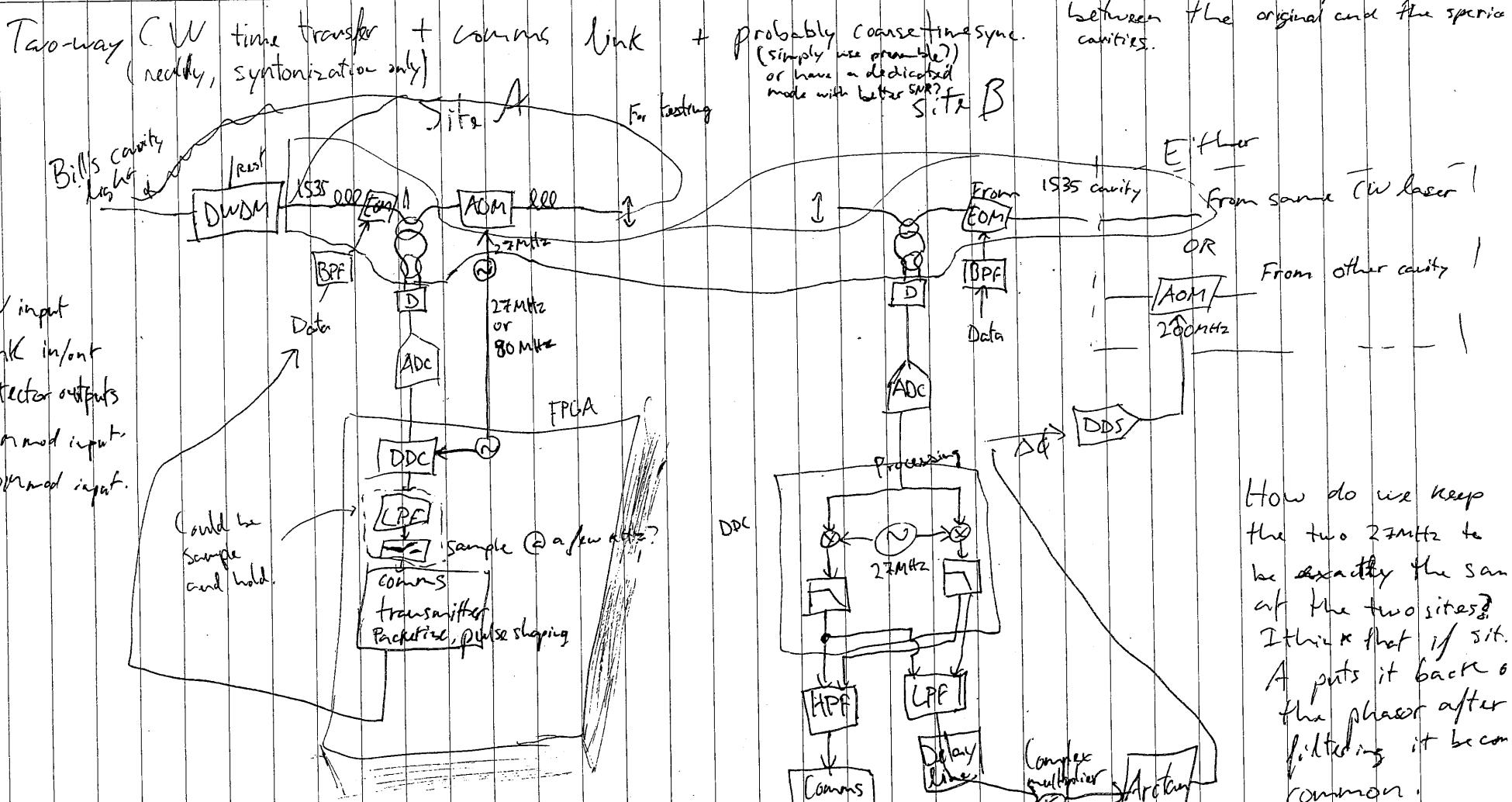
Site A (Slave)

Site B

Master



Bill's 1535 lasers are offset by 204m



- 1 CW input
- 2 Link in/out
- 4 Detector outputs
- 2 EOM mod inputs
- 1 AOM mod input.

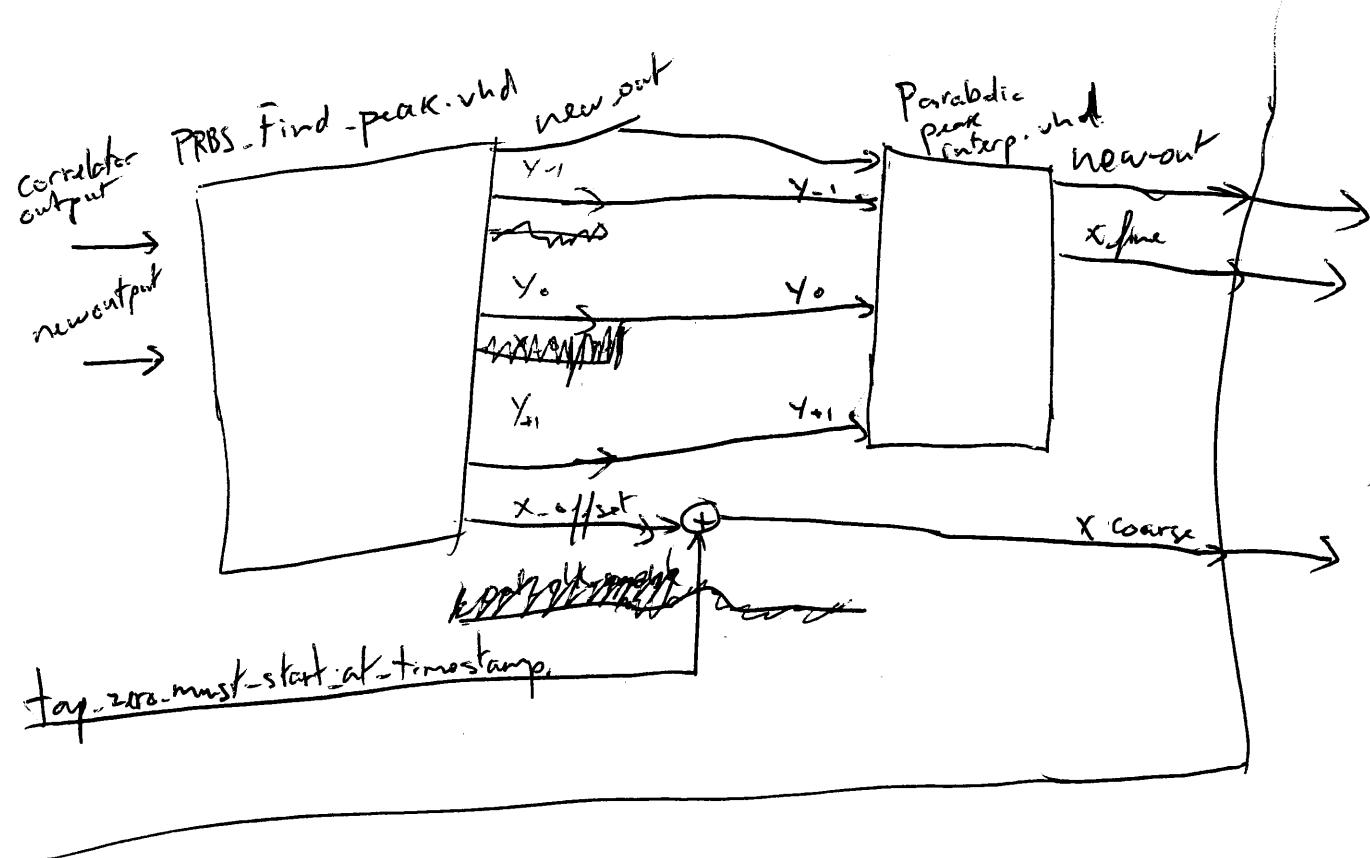
Could be sample and hold.

I think what this implicitly needs "coarse" time sync to phase lock the oscillators that provide the AOM offsets.

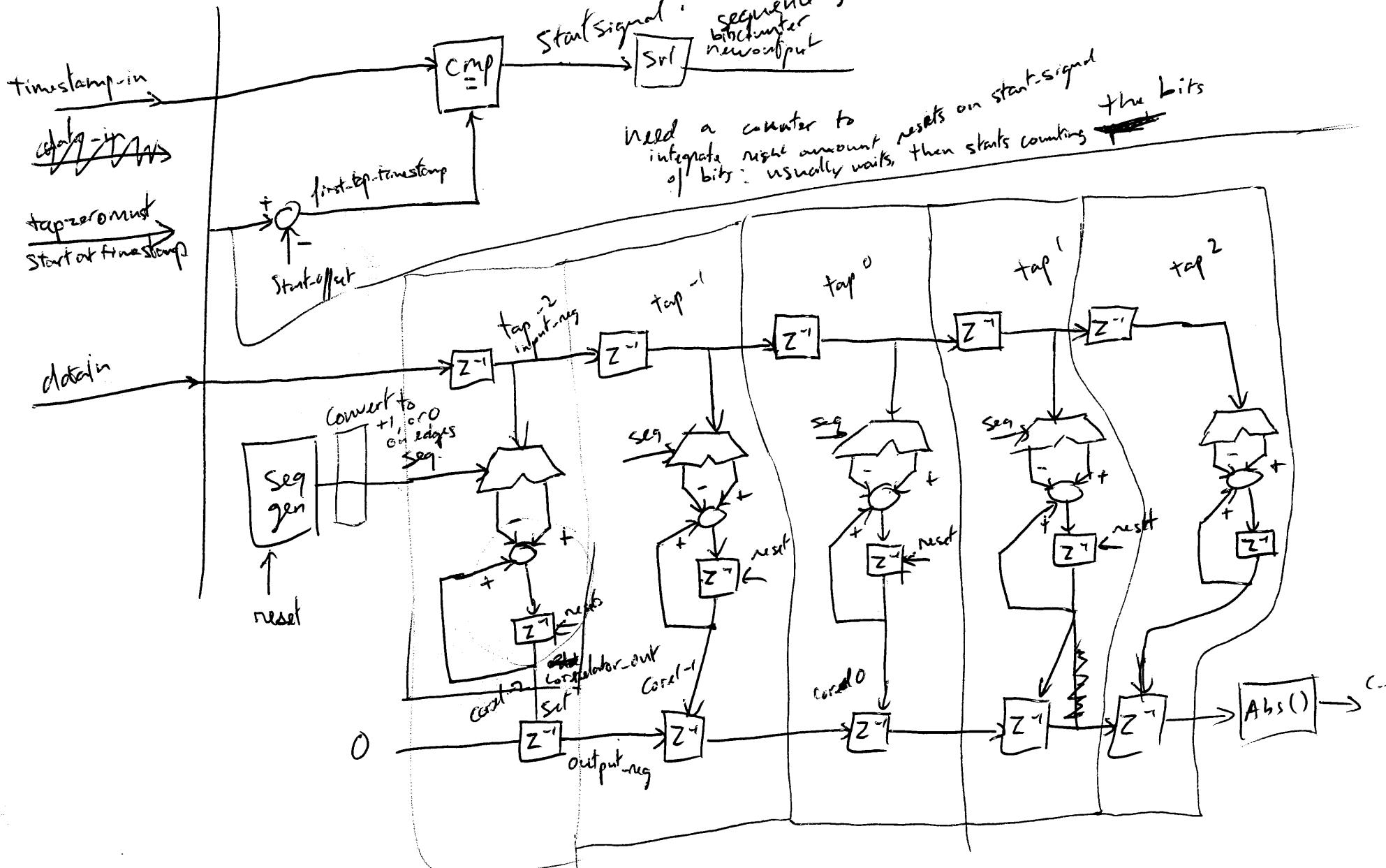
If we move the AOM to before the split, does this become in-loop?

How do we keep the two 27MHz to be exactly the same at the two sites? I think that if site A puts it back after filtering, it becomes common.

But then can we still transmit at a few kHz and be fine?



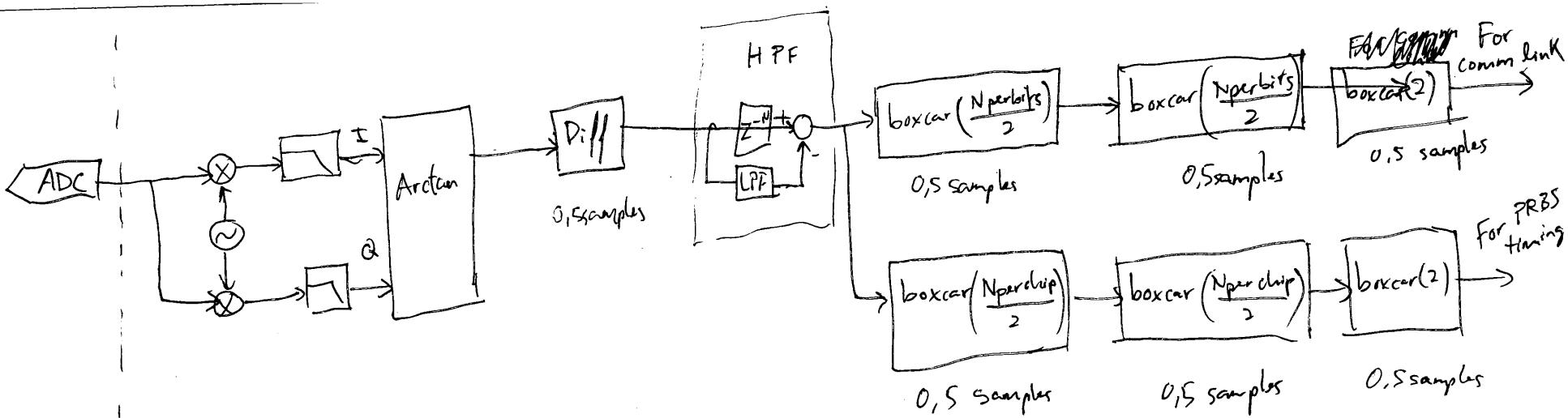
PRBS - Correlator. vhd



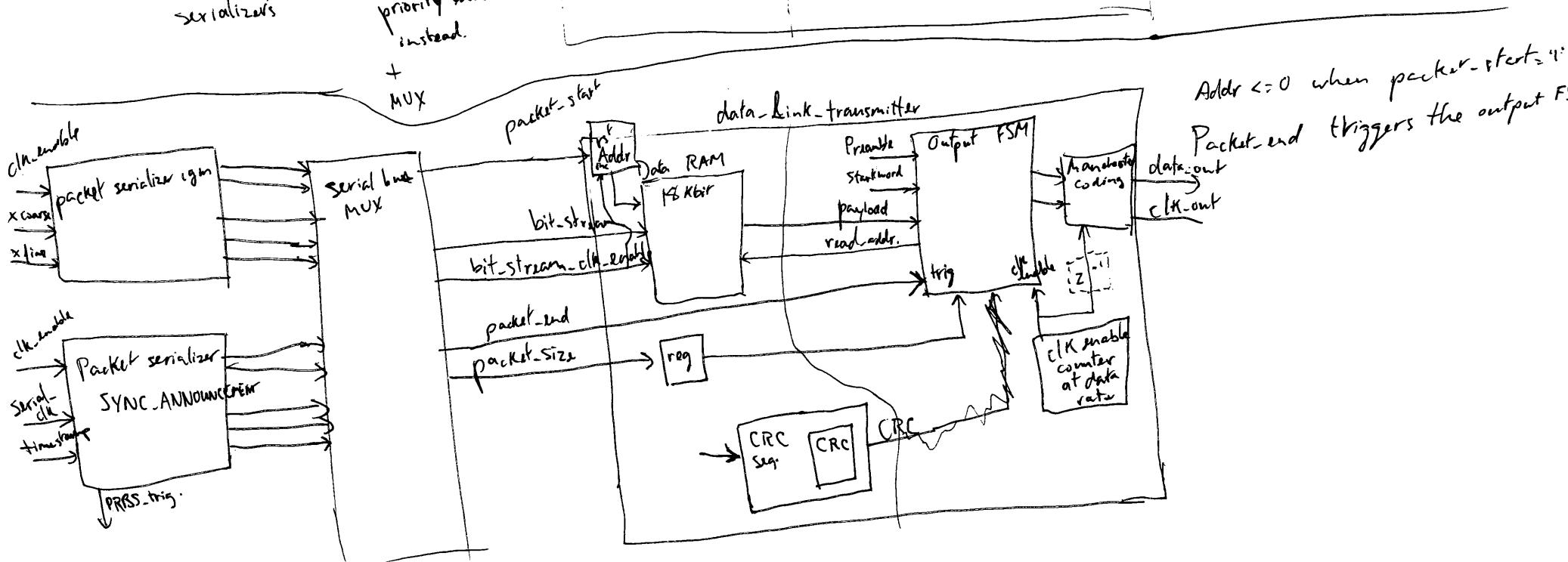
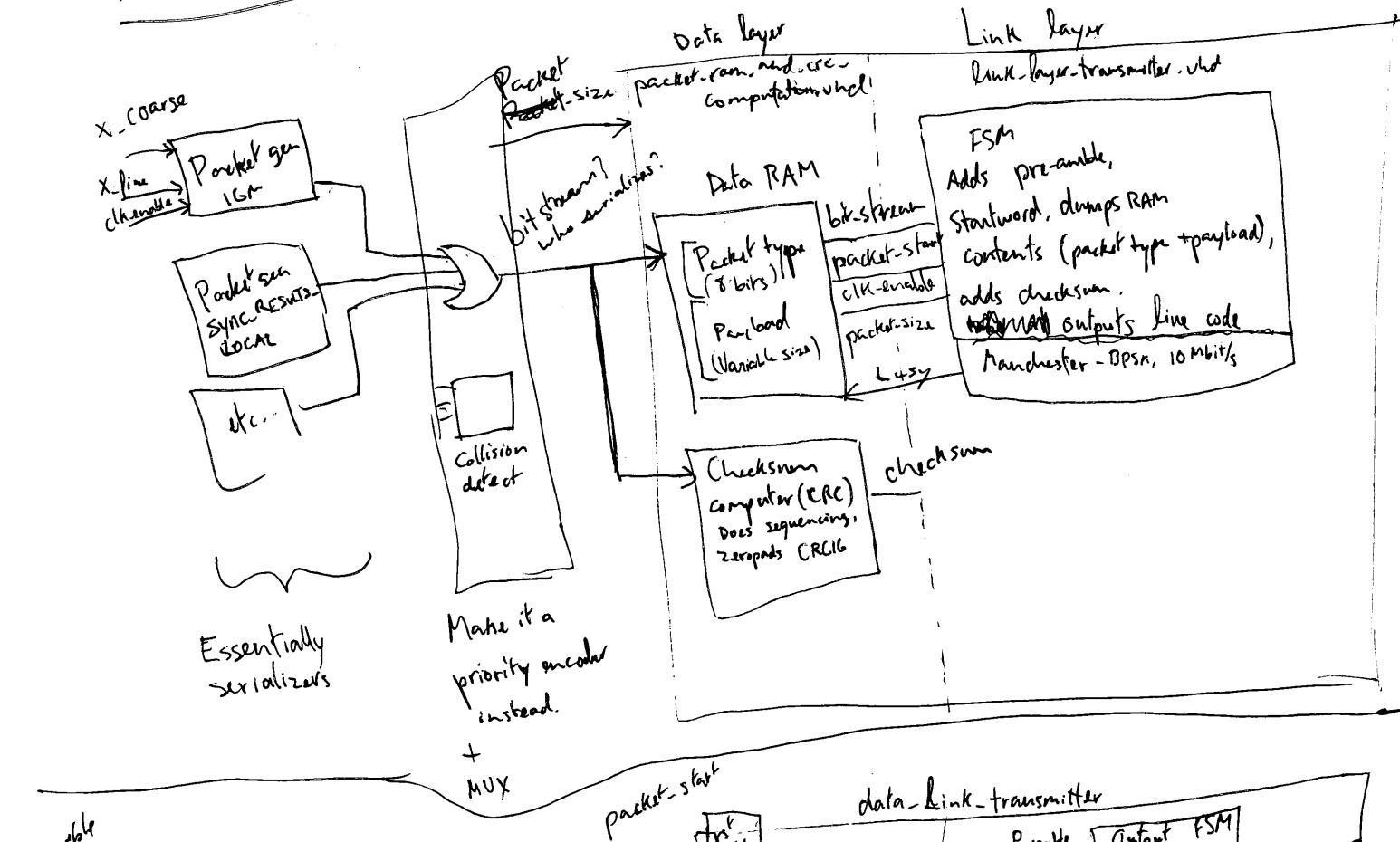
DTs - Correlator tap. whd
integrate functions

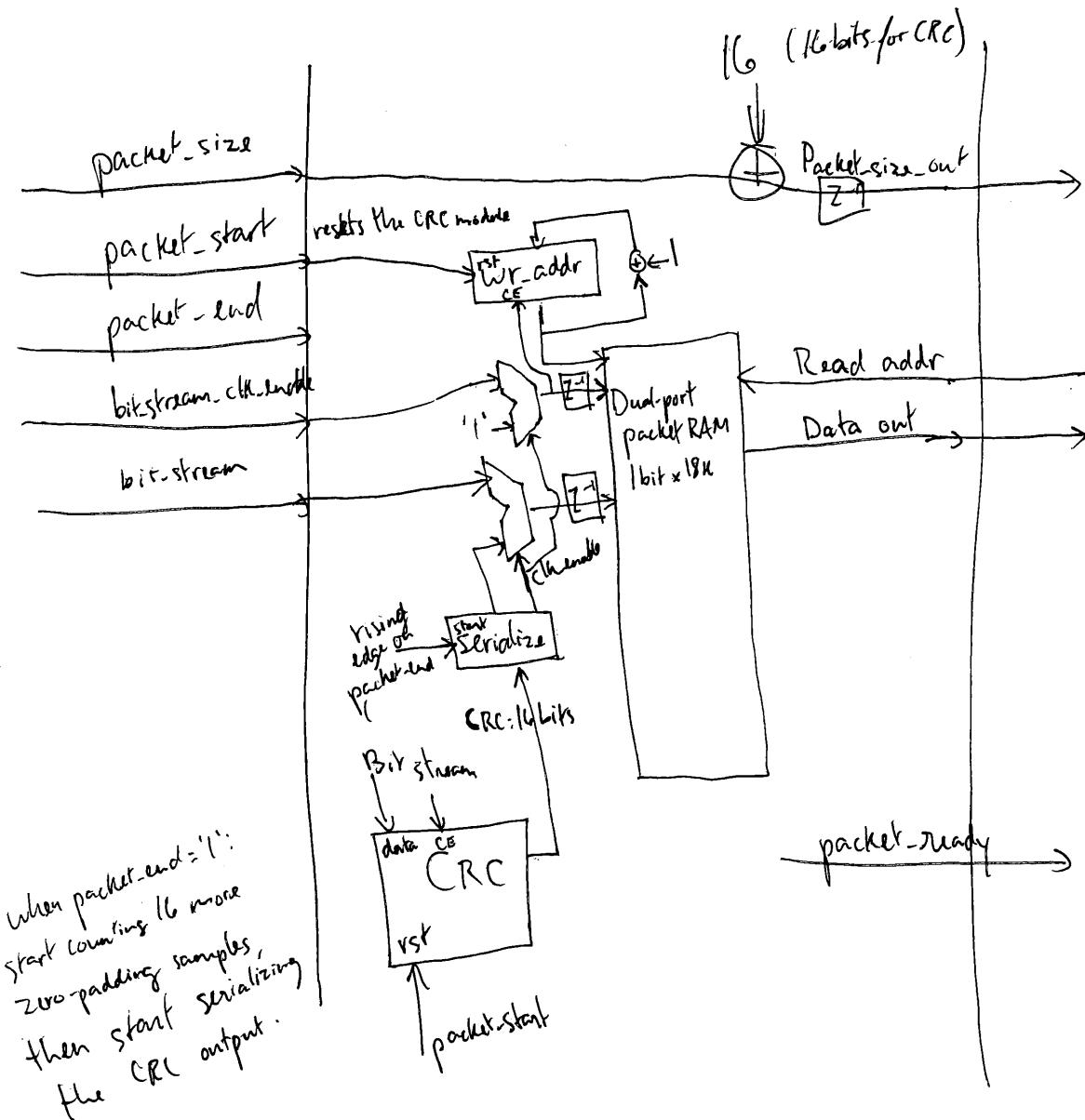
reset
hold
add
sub

Neutral carry chain:
lead app integrator
load carry



Transmitter design



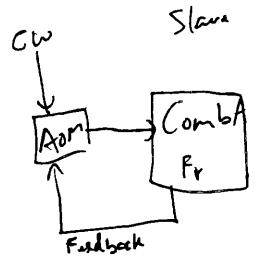


Test system:

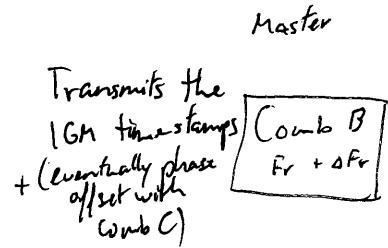
Looks at packet ready flag, then
reads out the RAM at 10 Mbit/s

from addr 0 to packet-size - 1

→ Test a bit error tool!



Needs to be able to translate timestamps from the master site to our own.

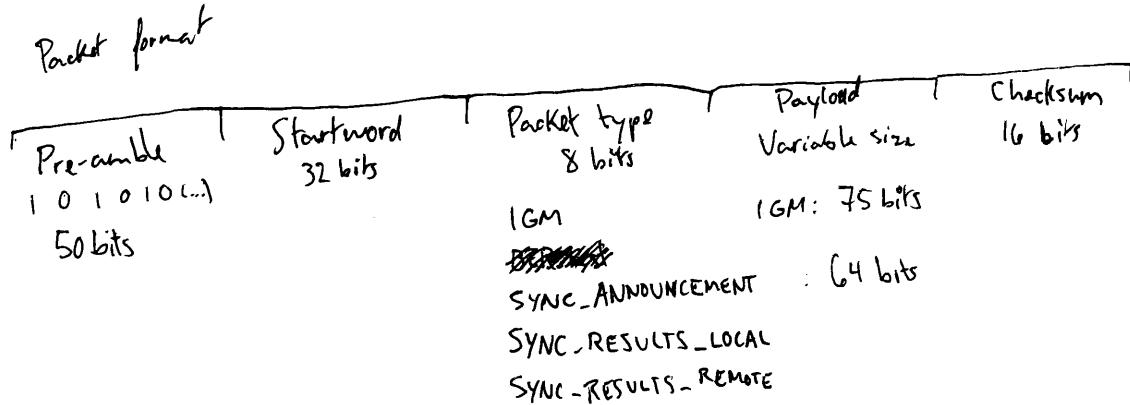
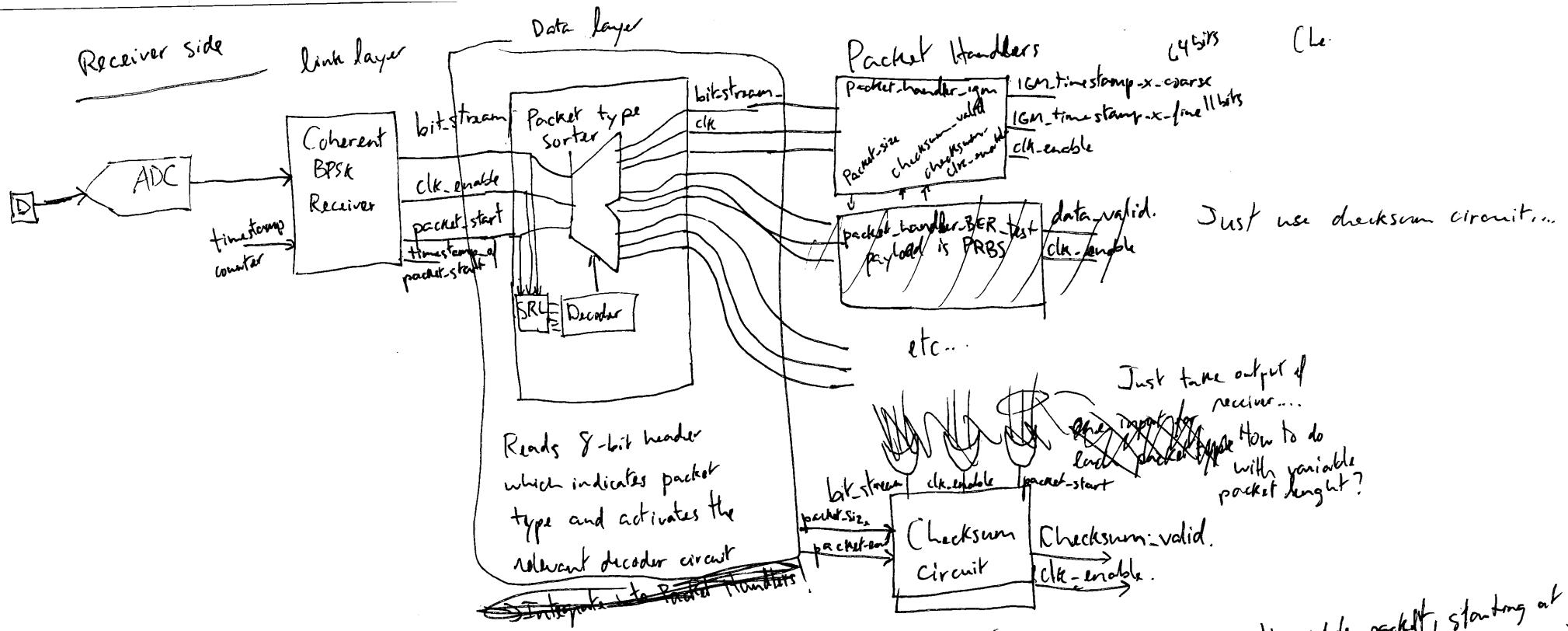


Protocol: Master is "active"; initiates all commands.

1. Master detects an IGM, sends results packet.
2. Master checks if we need to run sync protocol (how long he is listening)

Sync protocol:

1. Master sends SYNC-ANNOUNCEMENT packet, indicating that PRBS will follow after N cycles after the start of the announcement.
2. Master sends the PRBS at pre-determined time.
3. Master sends local arrival timestamp of the PRBS.
4. Once slave receives master's timestamp, he sends his own SYNC-ANNOUNCEMENT.
5. Slave sends PRBS as announced.
6. Master receives the slave's PRBS, sends result packet.
7. Slave finally has all the required information to translate master's timestamp to its own clock.



(Checksum computed over the whole packet starting at startword or packet type, use CRC, bitserial algorithm.)
total packet size for IGM: 181 bits (1.8 μs @ 10 Mbit/s)
41% packed

CCITT-CRC:

16 bits, polynomial:

$$x^{16} + x^{12} + x^5 + 1$$

In Hex: 0x1021 = ~~10000010000001~~

Implicit

$$\begin{array}{ccccccc} & 1 & 0 & 0 & 0 & 1 & \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$
$$+ x^{16} + x^{12} + x^5 + 1$$

USB 2.0 - CRC16:

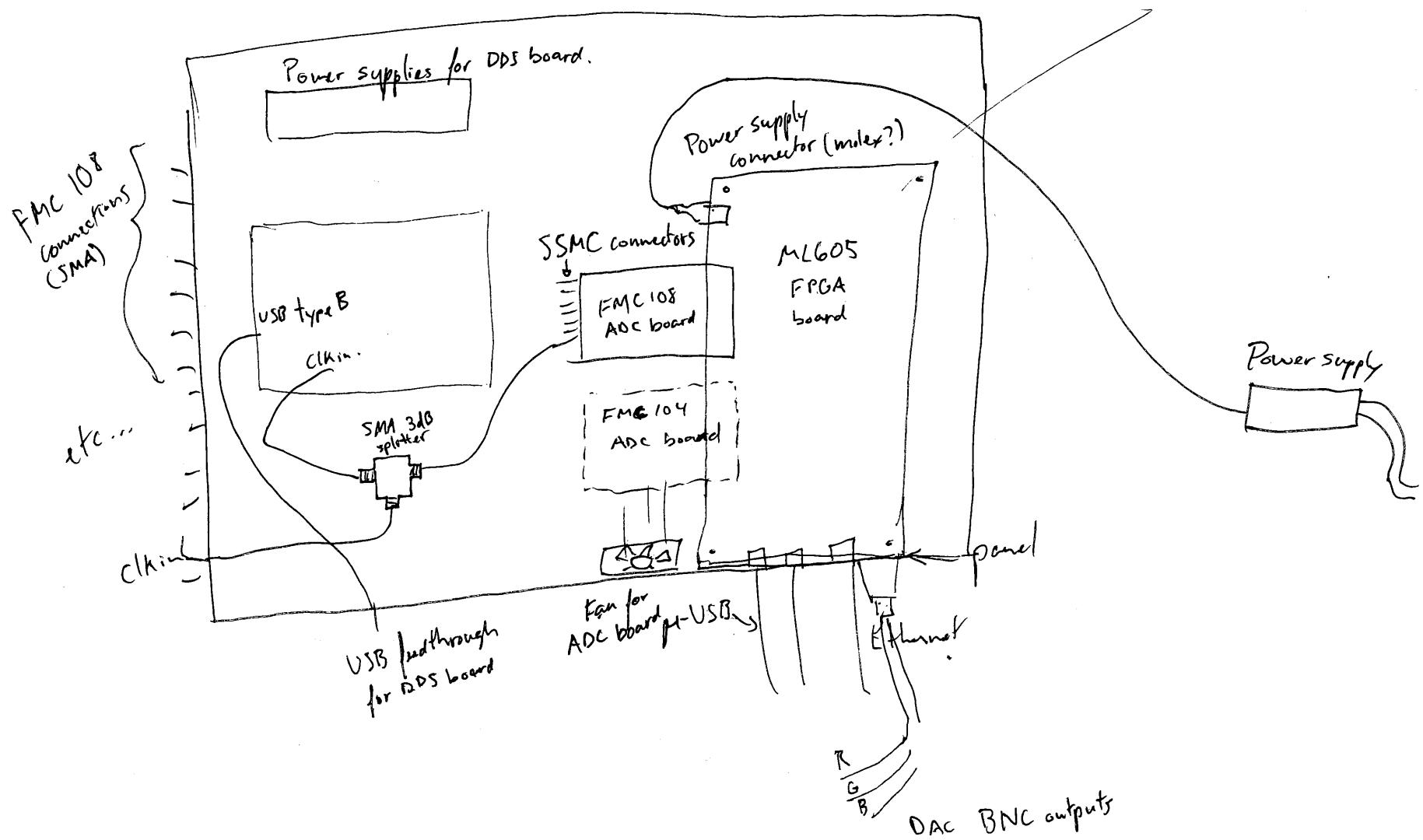
16 bits, polynomial:

$$x^{16} + x^{15} + x^2 + 1$$

Normal Hex: 0x8005 = 1 1000 0000 0000 0101

Implicit

$$\begin{array}{c} \downarrow \\ x^{16} + x^{15} \end{array} + x^2 + 1$$



- ML605 I/Os : (Virtex 6 FPGA eval board)

- FMC108 ADC board:
 - Trig in
 - Clock in
 - 8 data channels

} Total: 10 SMA's

- 4 SMA GPIOs (could be used by DDS board internally)

- 3 BNC outputs for VGA DAC (needs adapter DVI-VGA-BNC)

- Gigabit Ethernet port (mount flush to panel)

- LEDs on the panel could be nice (27 ohms series current-limiting resistor, no GND connection, or header)

- 2x USB 2.0 (mounted flush to panel)

(micro-USB)

- DDS board IOs (AD9912 eval board)

- 4 SMA outputs

- Clock input shared with FMC108 through an infrared splitter

- USB

- Need after for the FMC108/FMC104 board.

