

2/2

Stack and Queue

Data Structures
C++ for C Coders

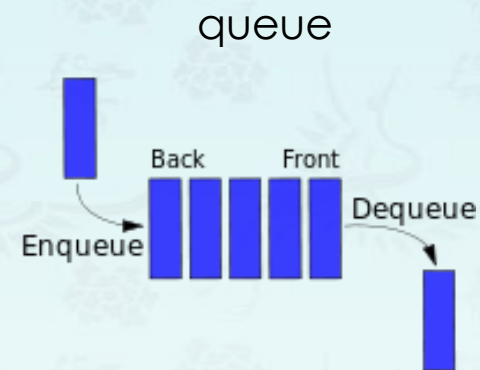
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*applications - **infix to postfix***



Queues

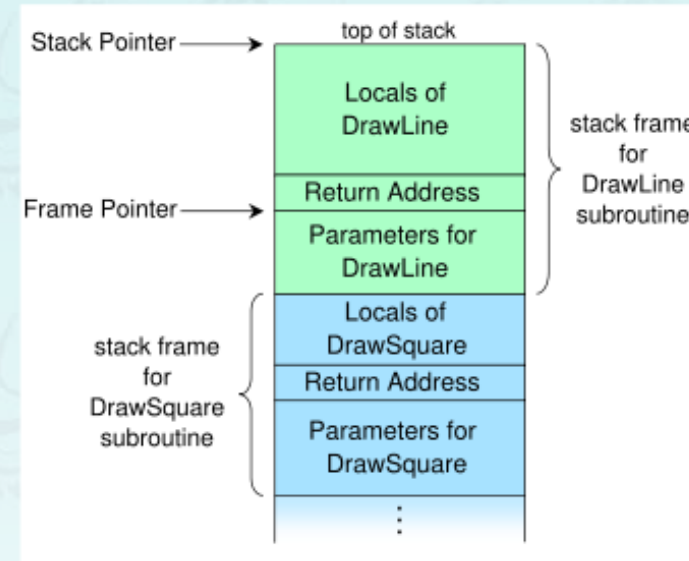
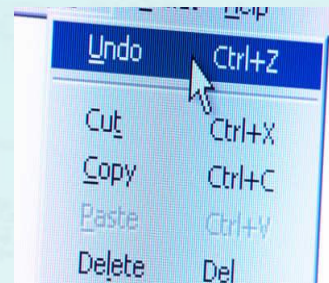
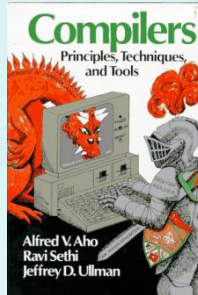
Queue: An ordered list in which **enqueues** (insertion or add) at the **rear** and **dequeues** (deletion or remove) take place at different end or **front**. It is also known as a First-in-first-out(**FIFO**) list.



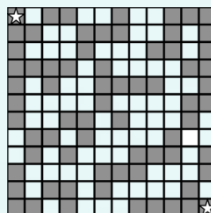
- ❖ Items can only be added at the **rear** of the queue and the only item that can be removed is the one at the **front** of the queue.



Stack and Queue Applications



- Parsing in a compiler. (p.127)
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Backtracking as in a maze (p.121)
- Implementing function calls in a compiler. (p.108)
- ...





Stack and Queue Applications

In a computer OS: Requests for services come in unpredictable order and timing, sometimes faster than they can be serviced .

- print a file
- need a file from the disk system
- send an email
- job scheduling

Arithmetic expression evaluation

Goal: Convert an **infix** expression to a **postfix** expression using a **stack**.

(1 + 2) * 3

operand operator

Stack: (
Output:

Stack: (
Output: 1

Stack: (+
Output: 1

Stack: (+
Output: 1 2

Stack:
Output: 1 2 +

Stack: *
Output: 1 2 +

Stack: *
Output: 1 2 + 3

Stack:
Output: 1 2 + 3 *

- Operands are output immediately
- Stack operators until right parens
- Unstack until left parens
Delete left parens
- In general, higher precedence operator must be output before lower one.)

fully parenth sized

infix

(1 + ((2 + 3) * (4 * 5)))

postfix

맨처음에 제일 안에 있는 괄호에 있는 식부터
(1+(2 3 +) * (4 5 *)))
그리고 하나씩 풀기
(1+(2 3 + 4 5 * *))
=>1 2 3 + 4 5 * * +

Arithmetic expression evaluation

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Output: 1 2

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infix

(1 + ((2 + 3) * (4 * 5)))

postfix

1 2 3 + 4 5 * * +

Arithmetic expression evaluation

Goal: Evaluate infix expressions.

put parenthesis wherever possible

(1 + ((2 + 3) * (4 * 5)))

operator

operand

Two-stack algorithm. [E. W. Dijkstra]

Arithmetic expression evaluation

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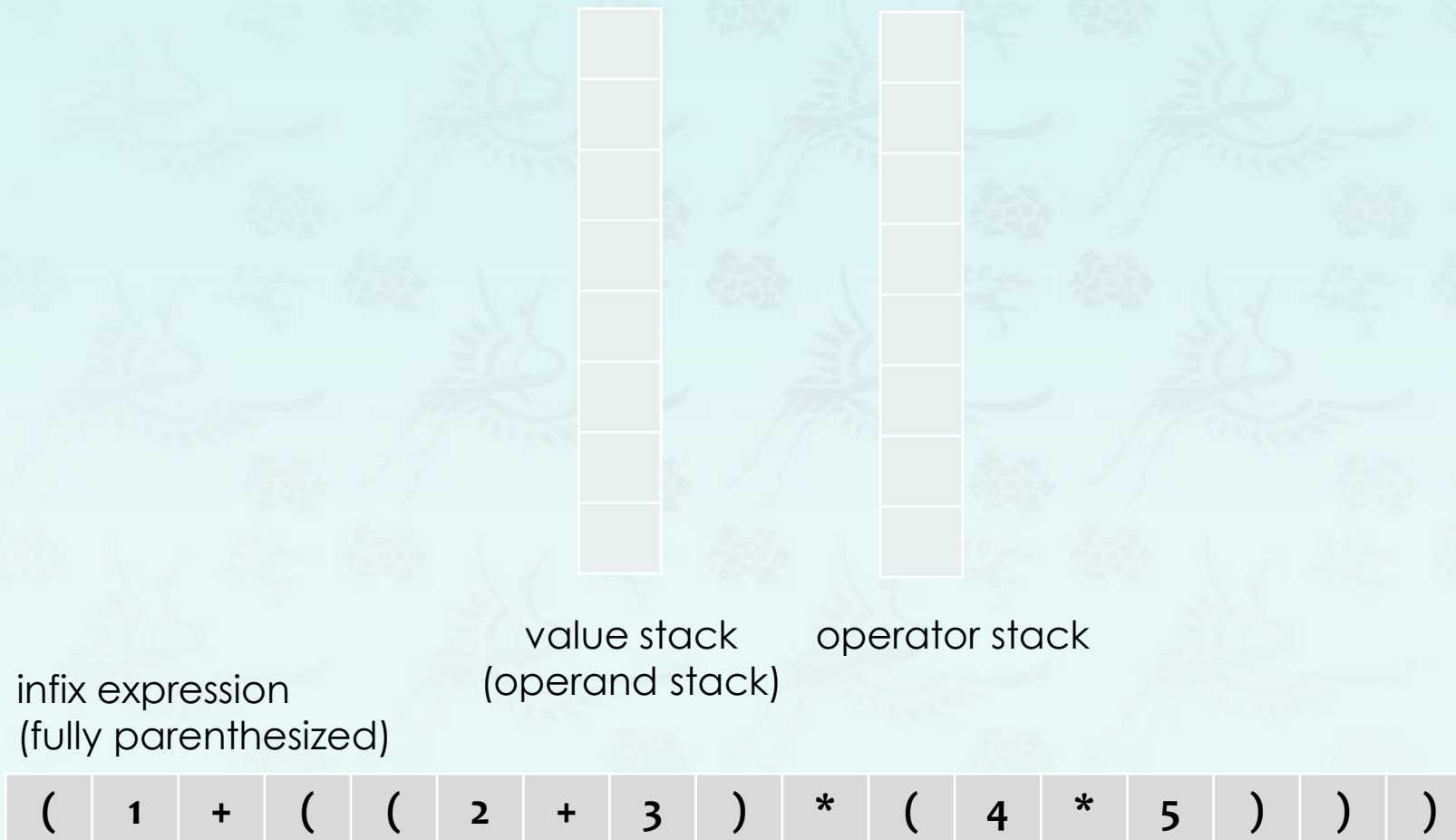
operator

operand

Two-stack algorithm. [E. W. Dijkstra]

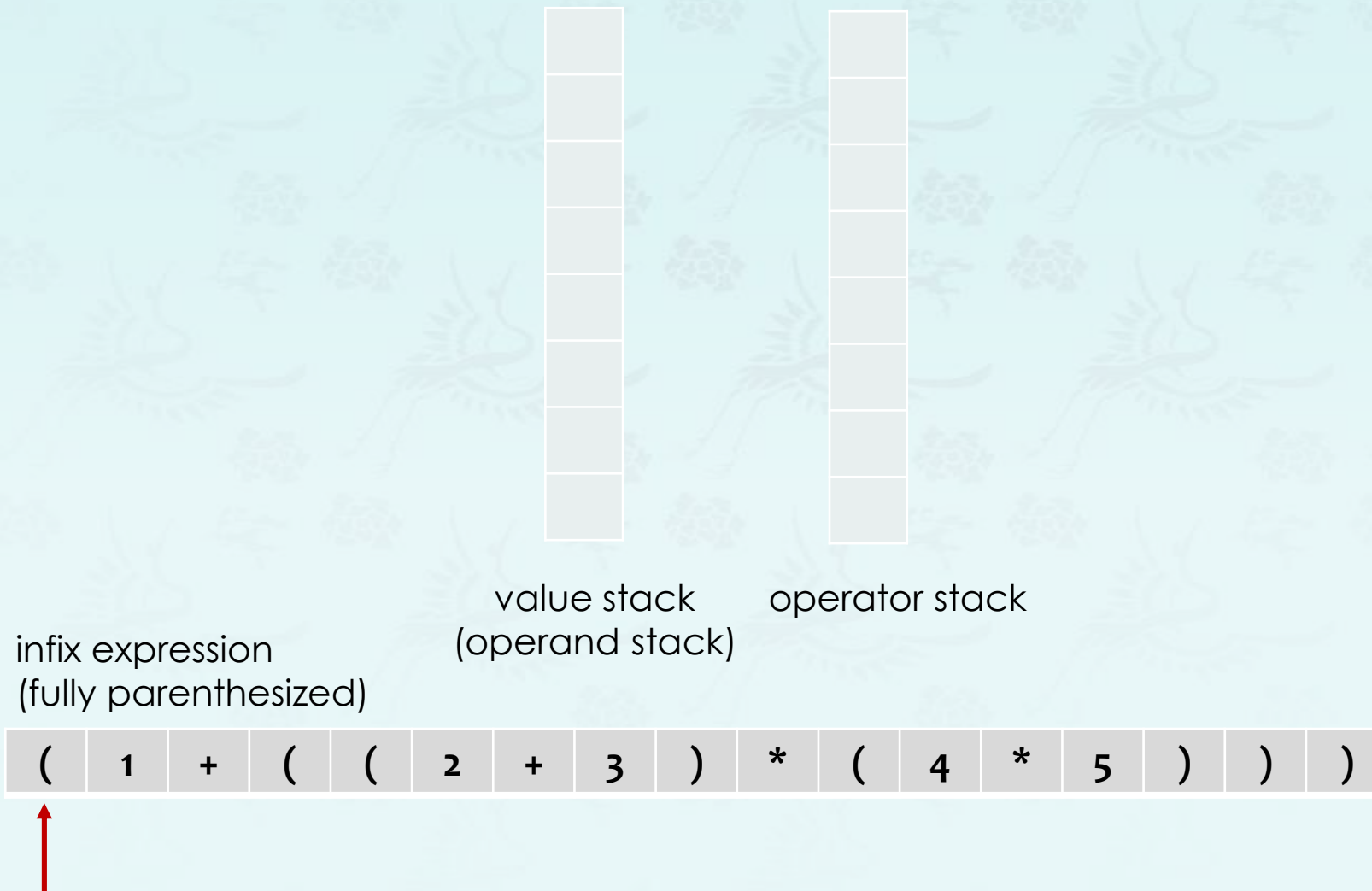
- **Value:** push onto the **value stack**.
- **Operator:** push onto the **operator stack**.
- **Left parenthesis:** ignore.
- **Right parenthesis:**
 - pop operator and two values;
 - push the result of applying that operator to those values onto the value stack.

Dijkstra's two-stack algorithm



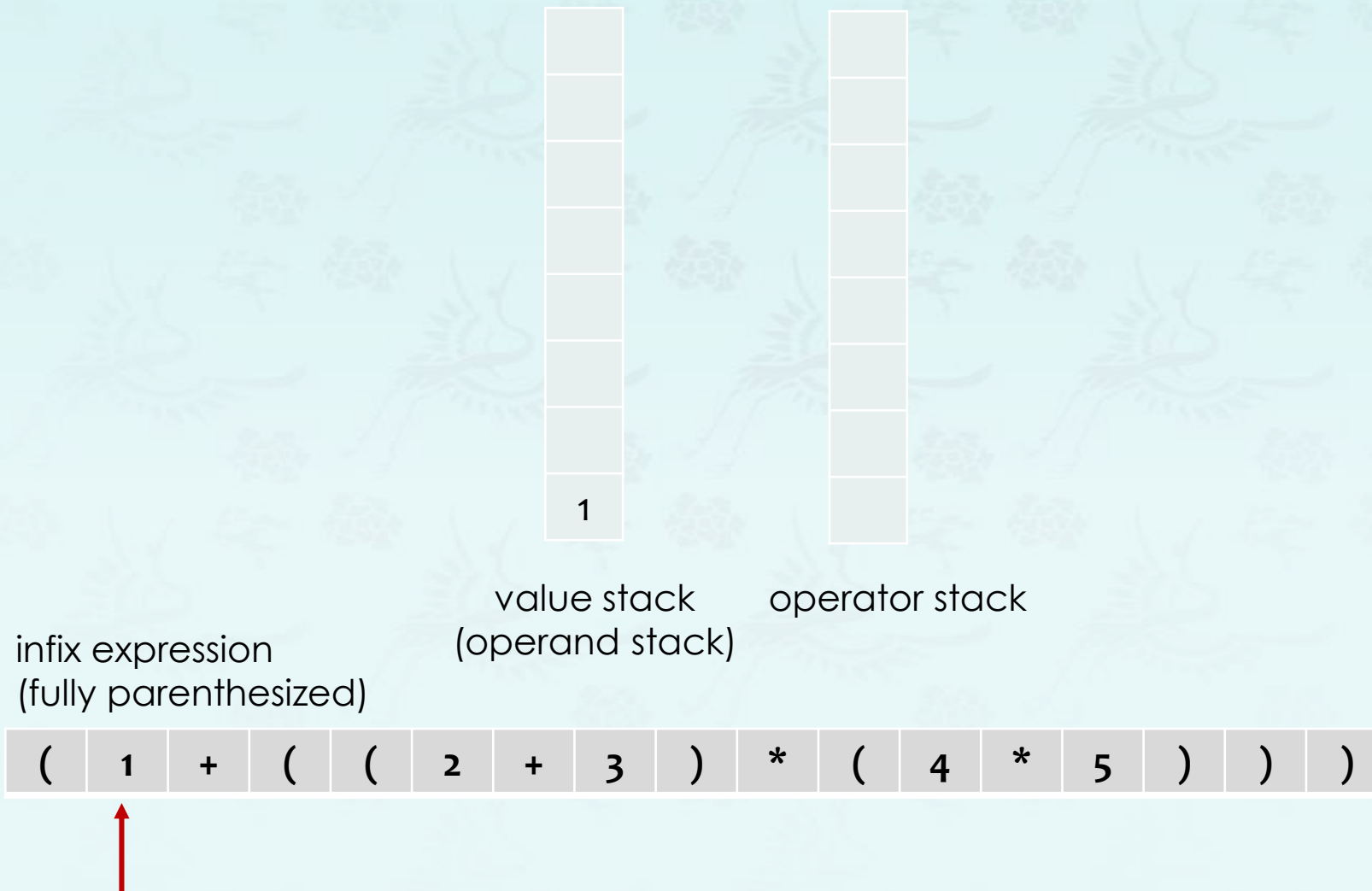
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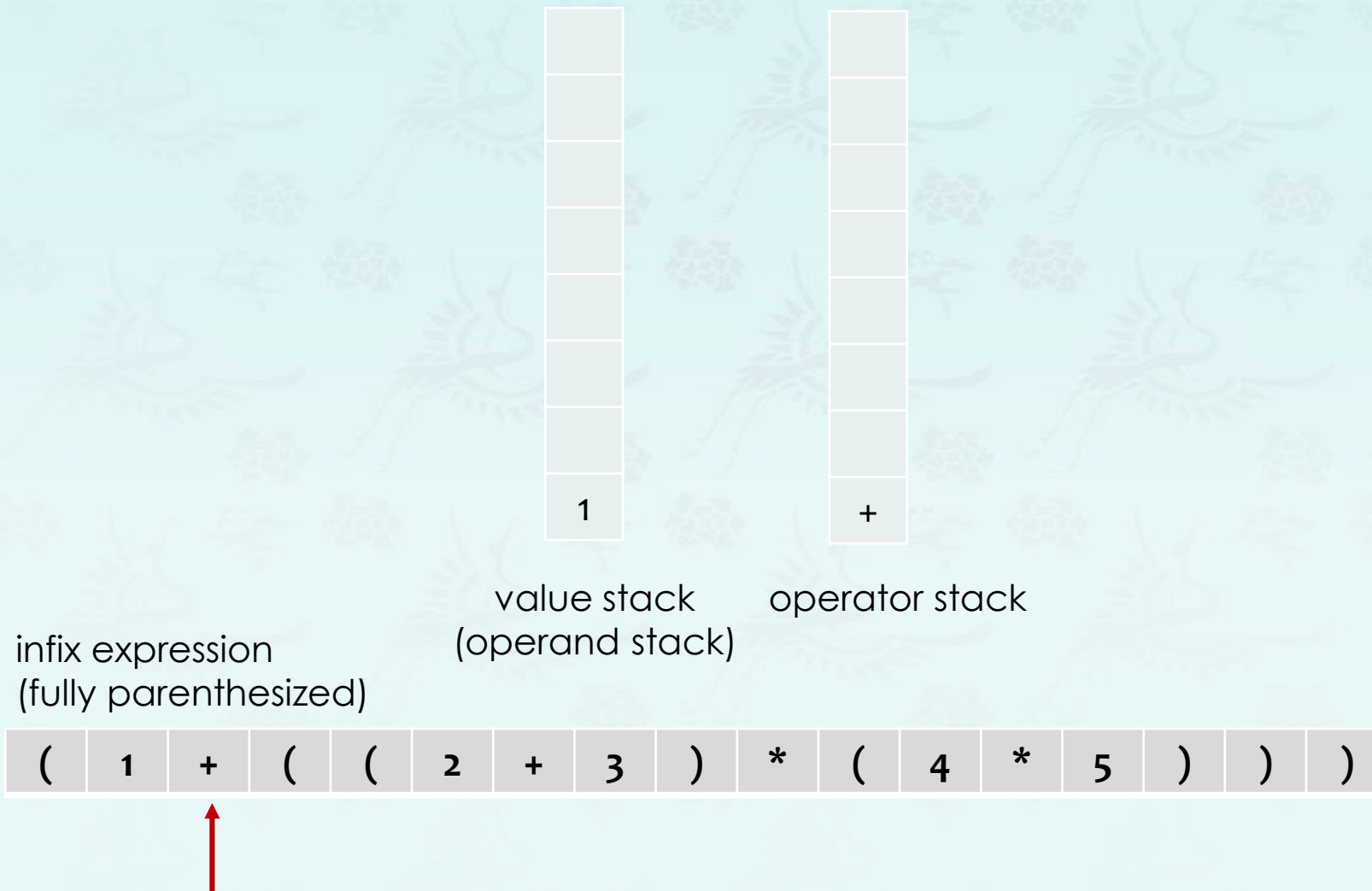
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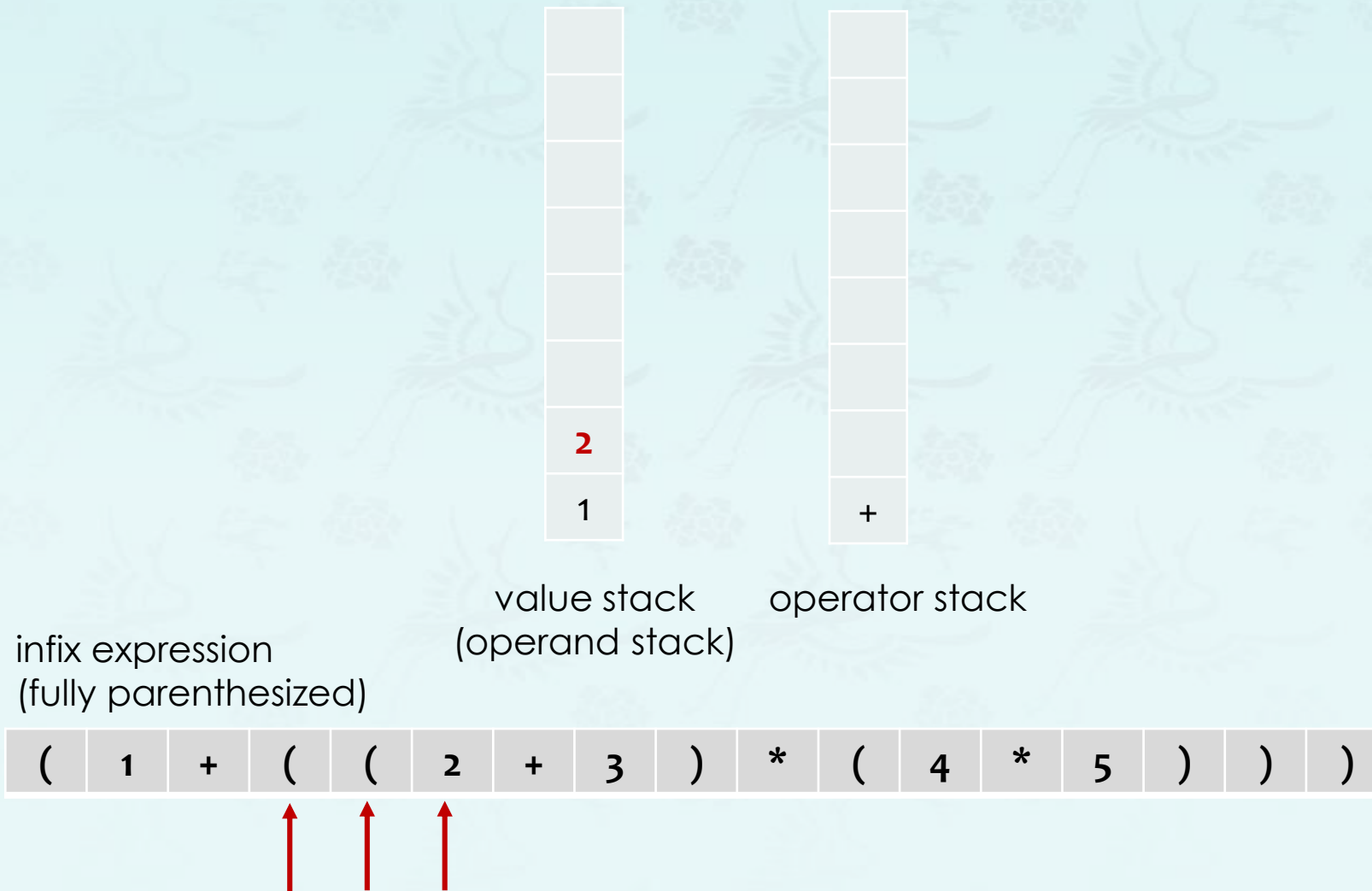
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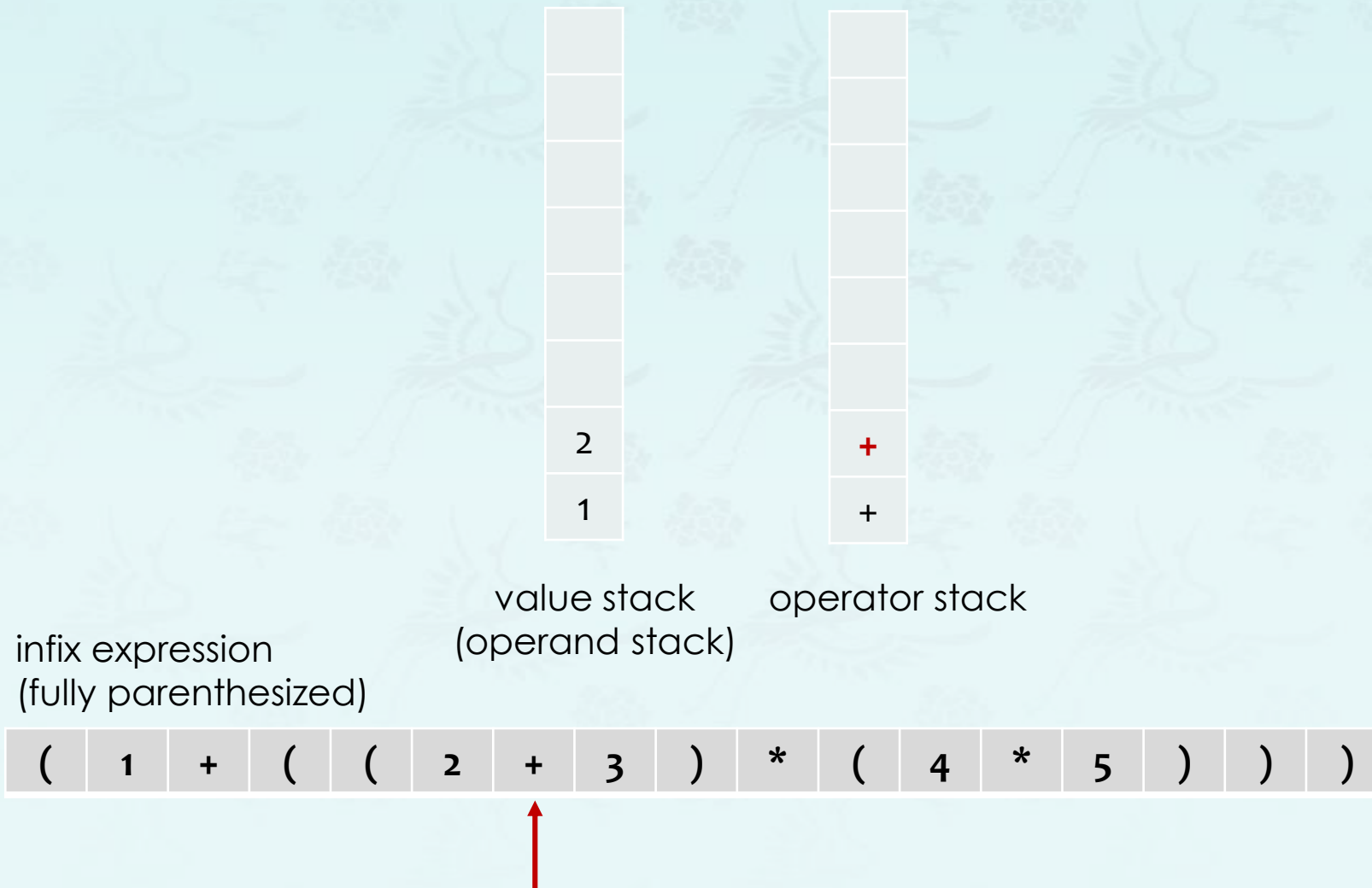
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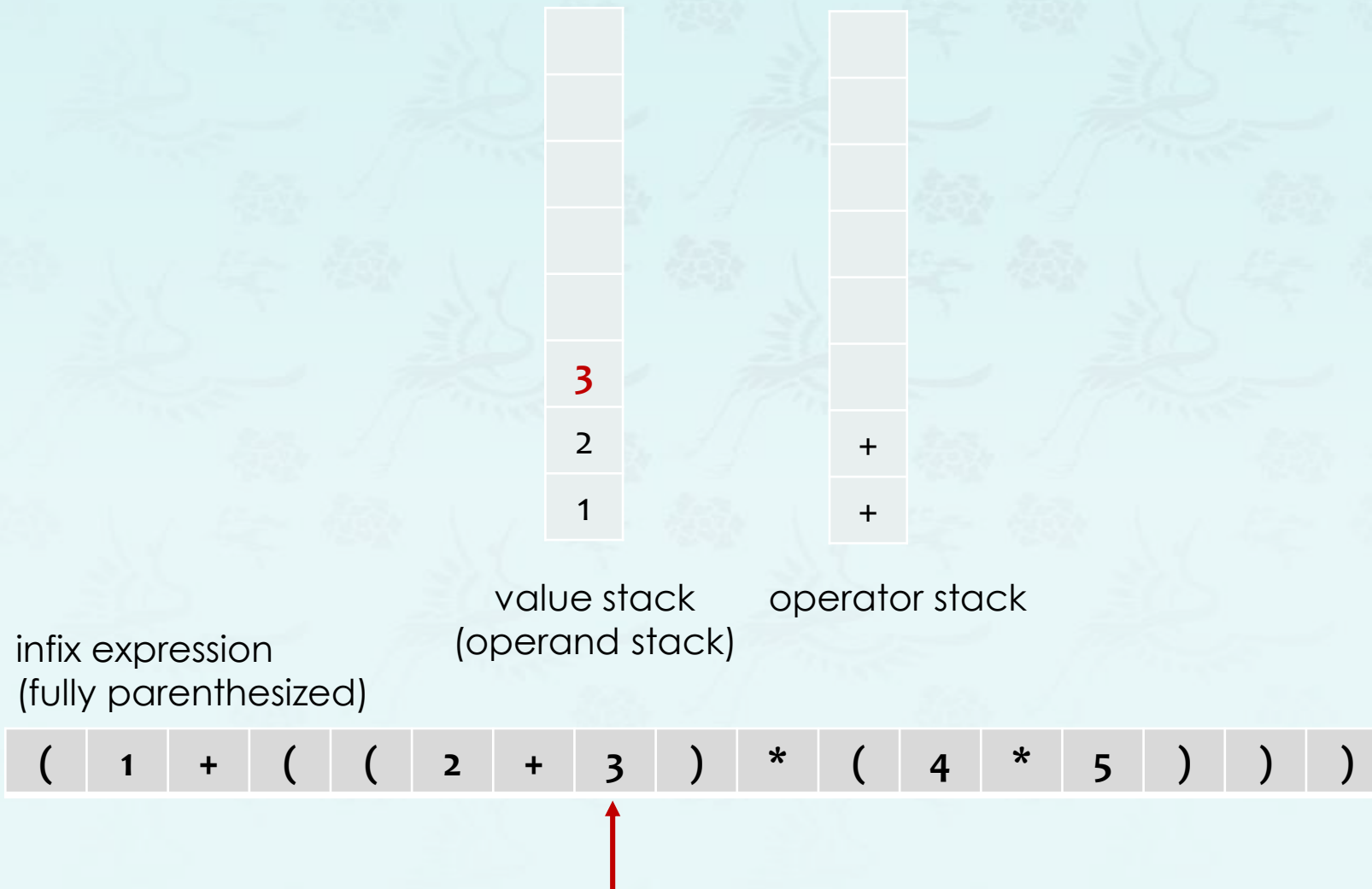
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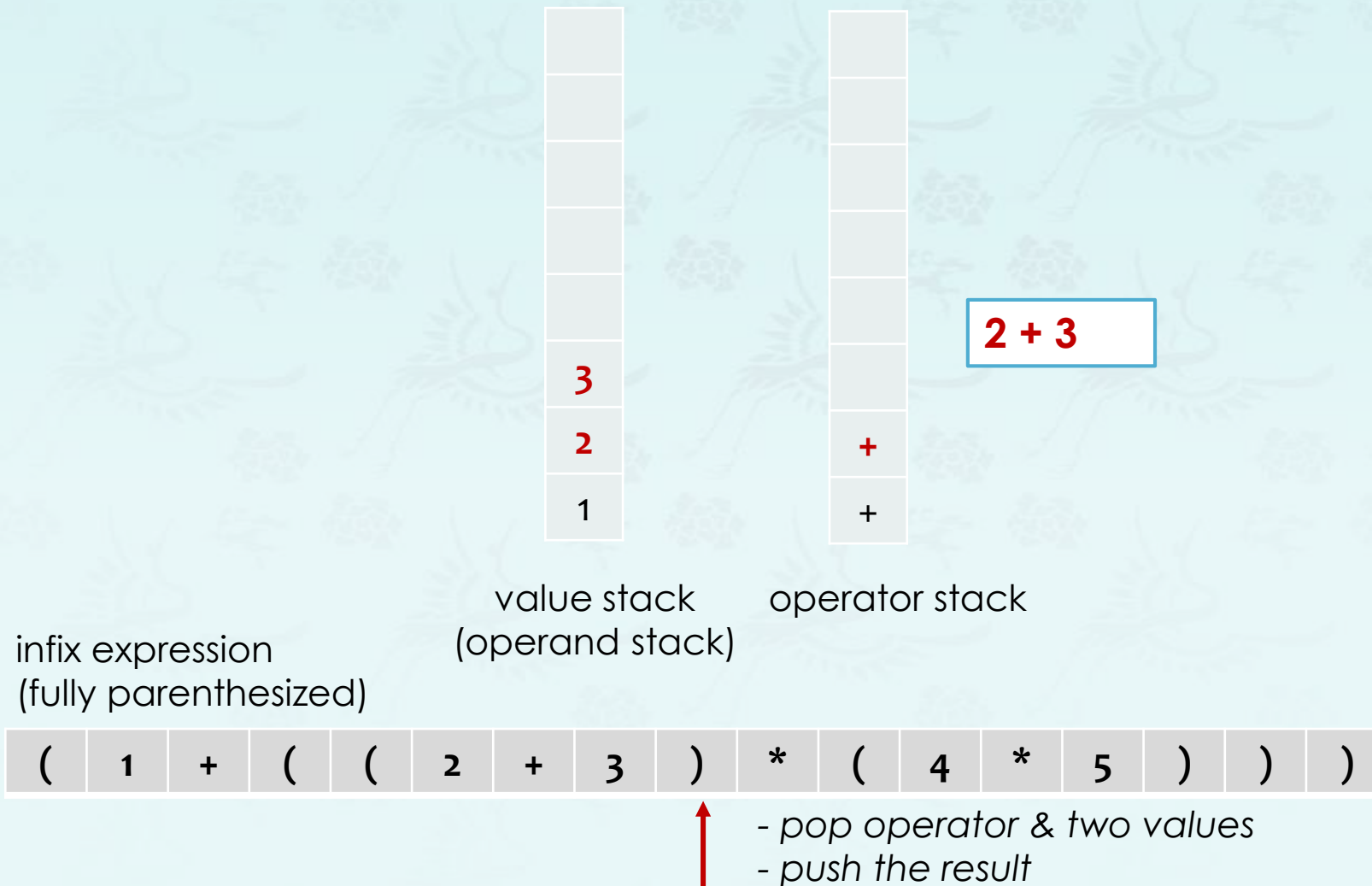
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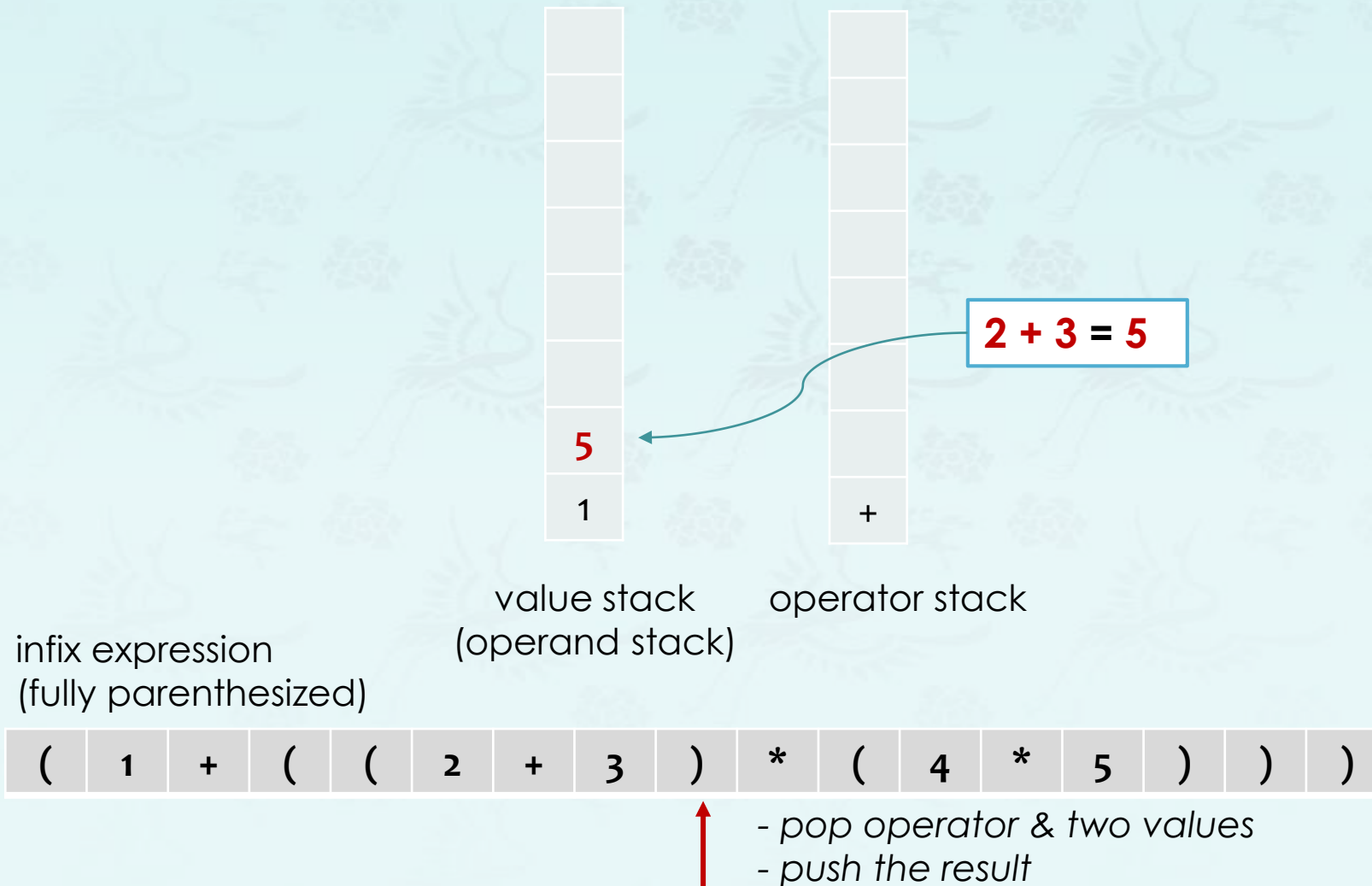
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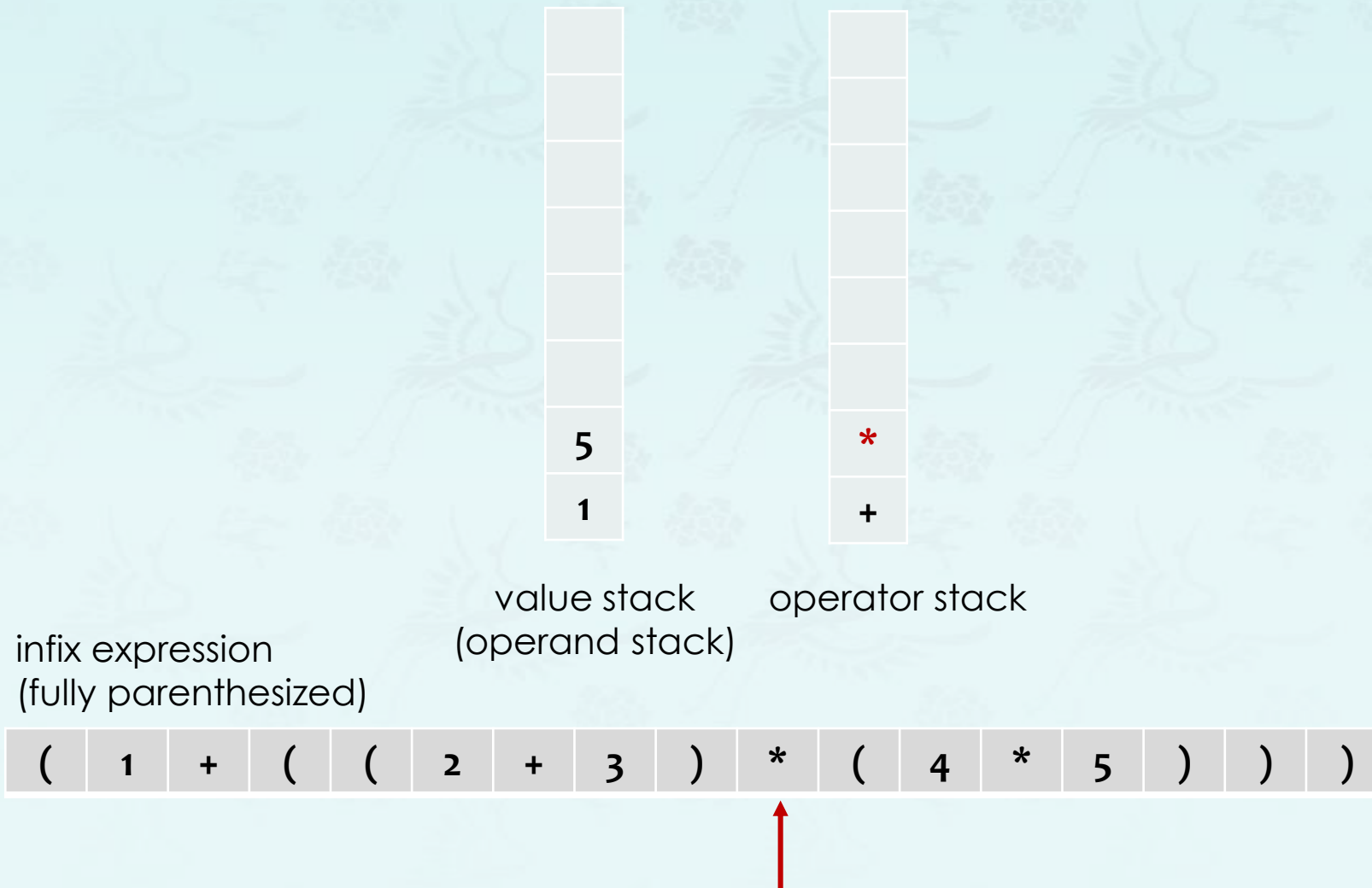
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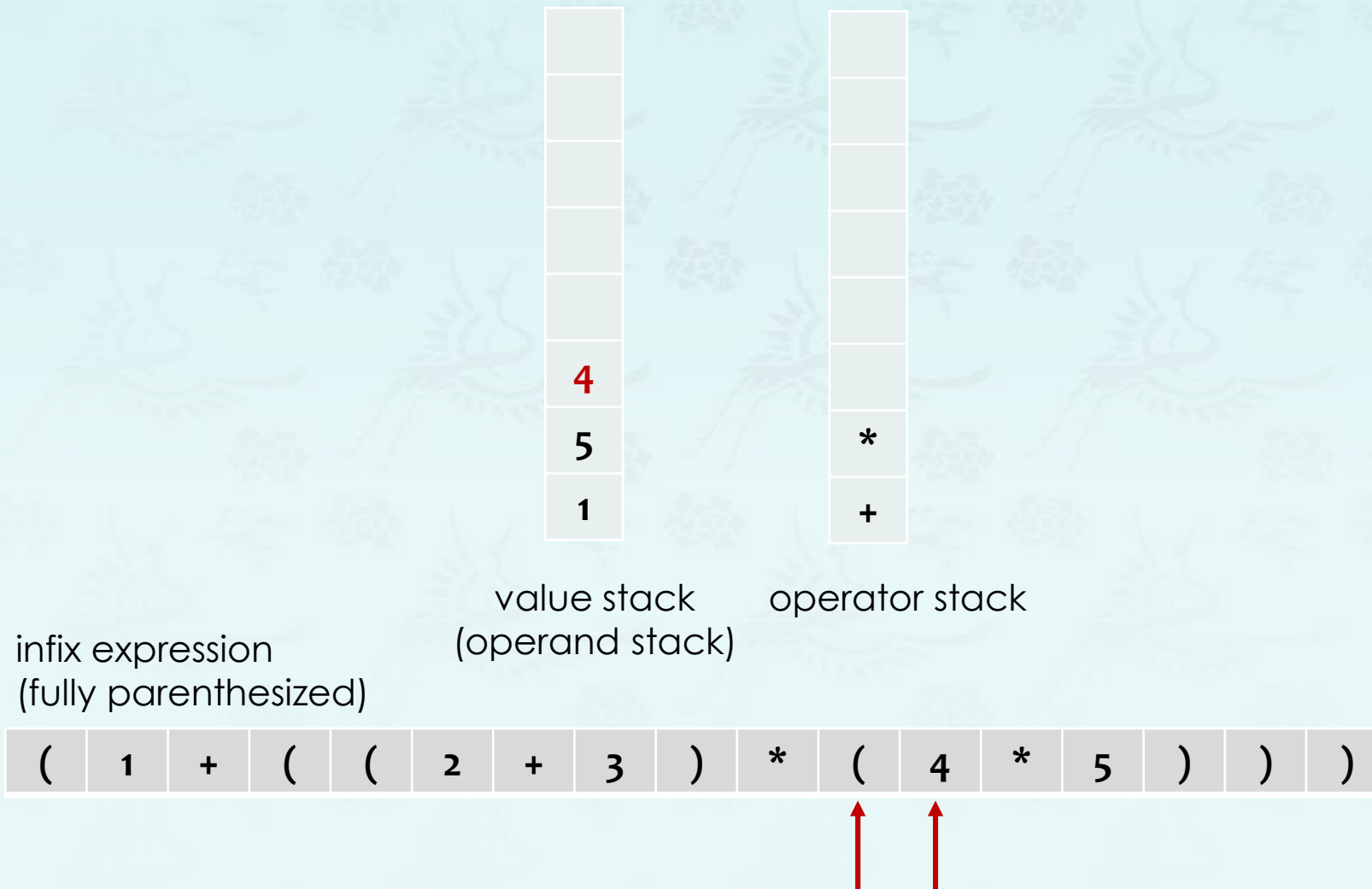


Dijkstra's two-stack algorithm

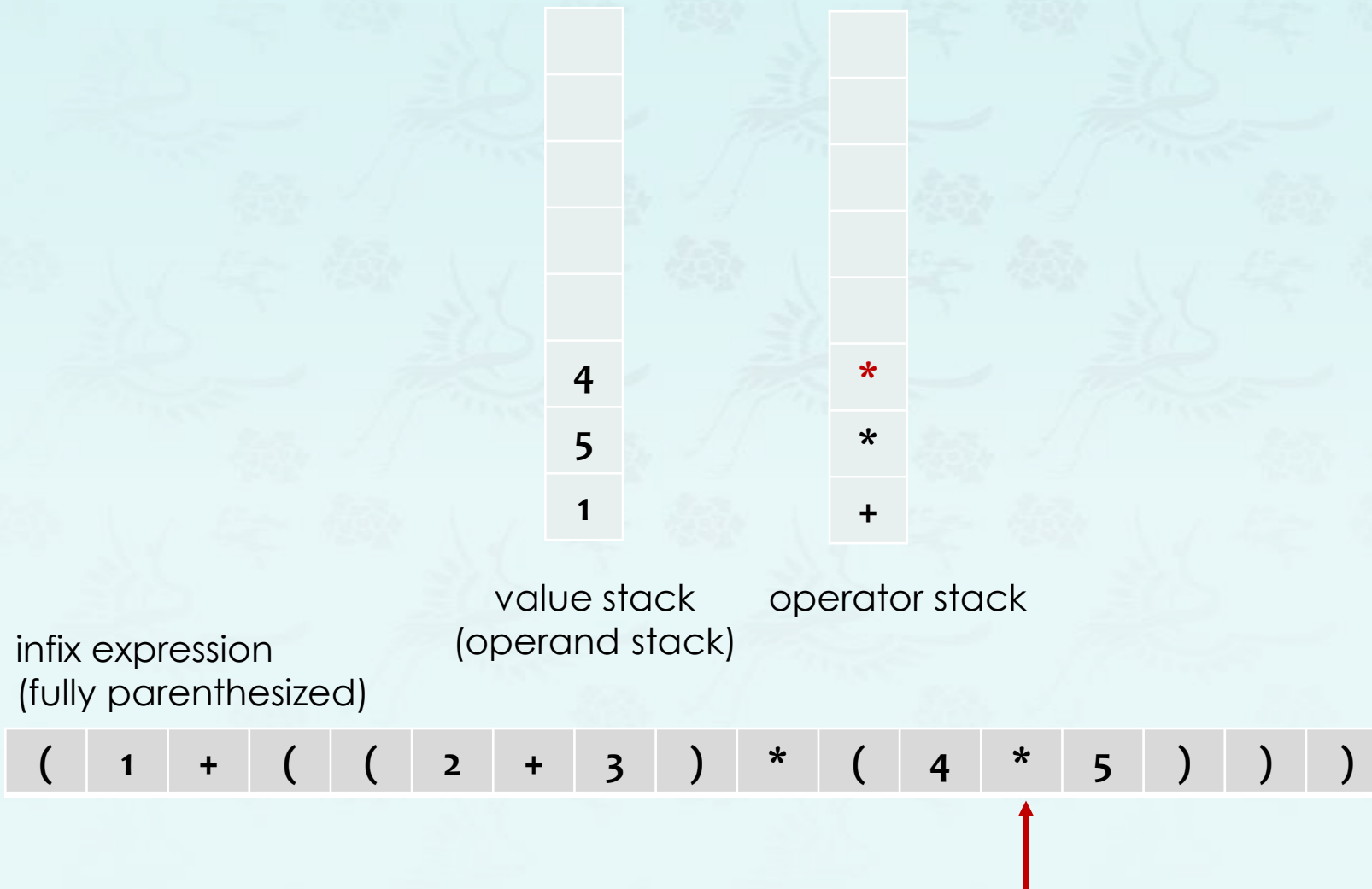
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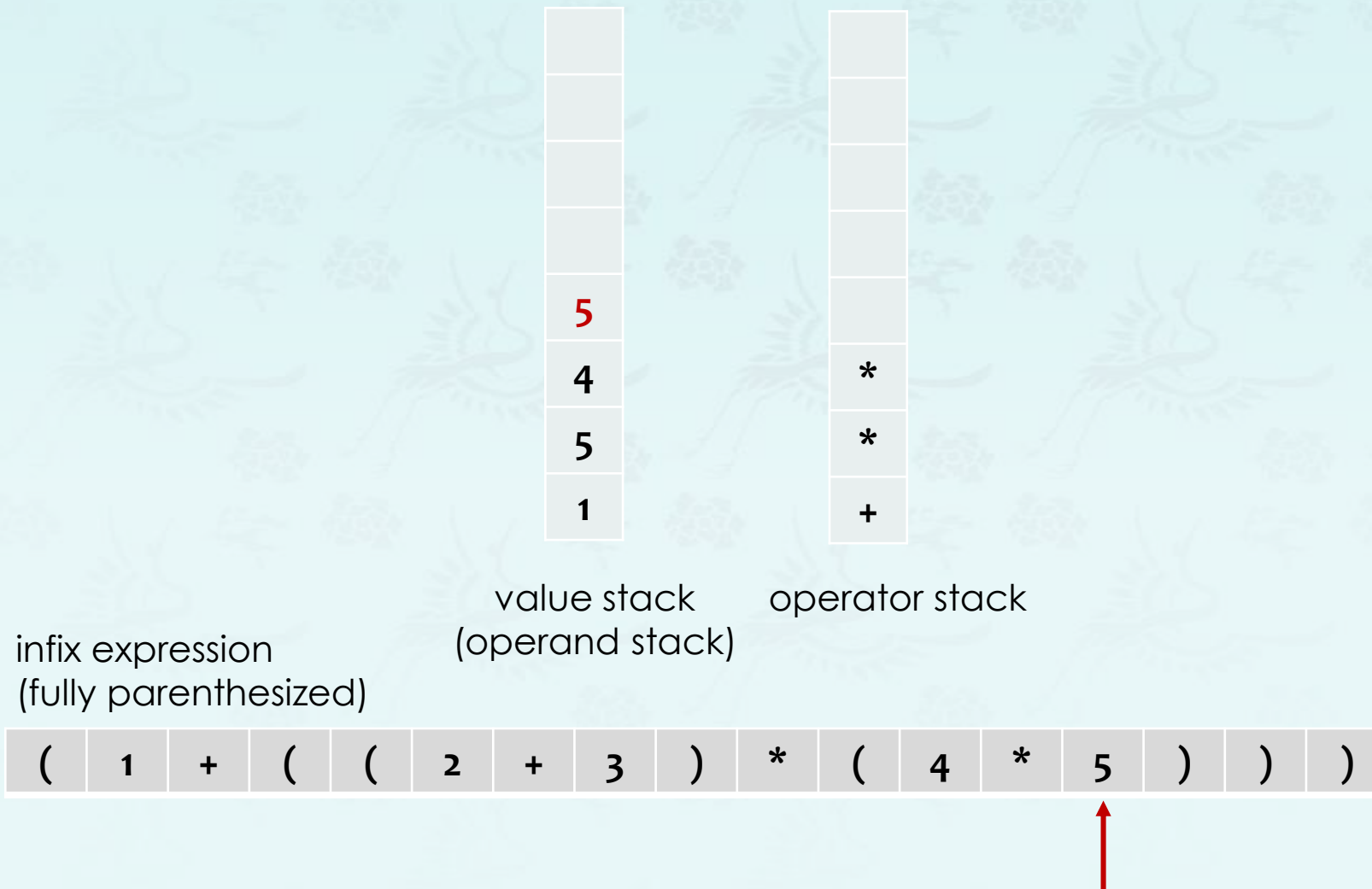
Dijkstra's two-stack algorithm



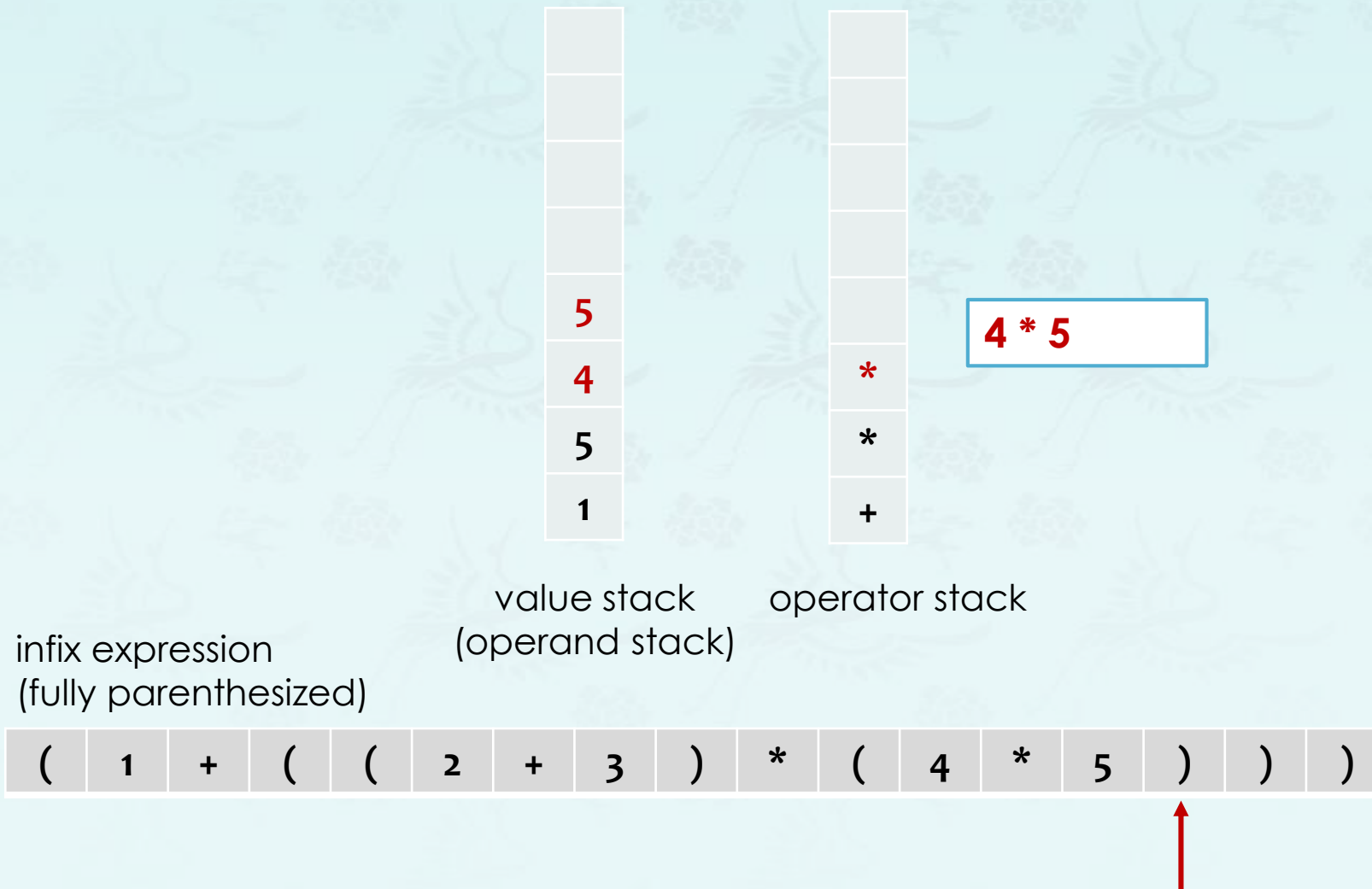
Dijkstra's two-stack algorithm



Dijkstra's two-stack algorithm

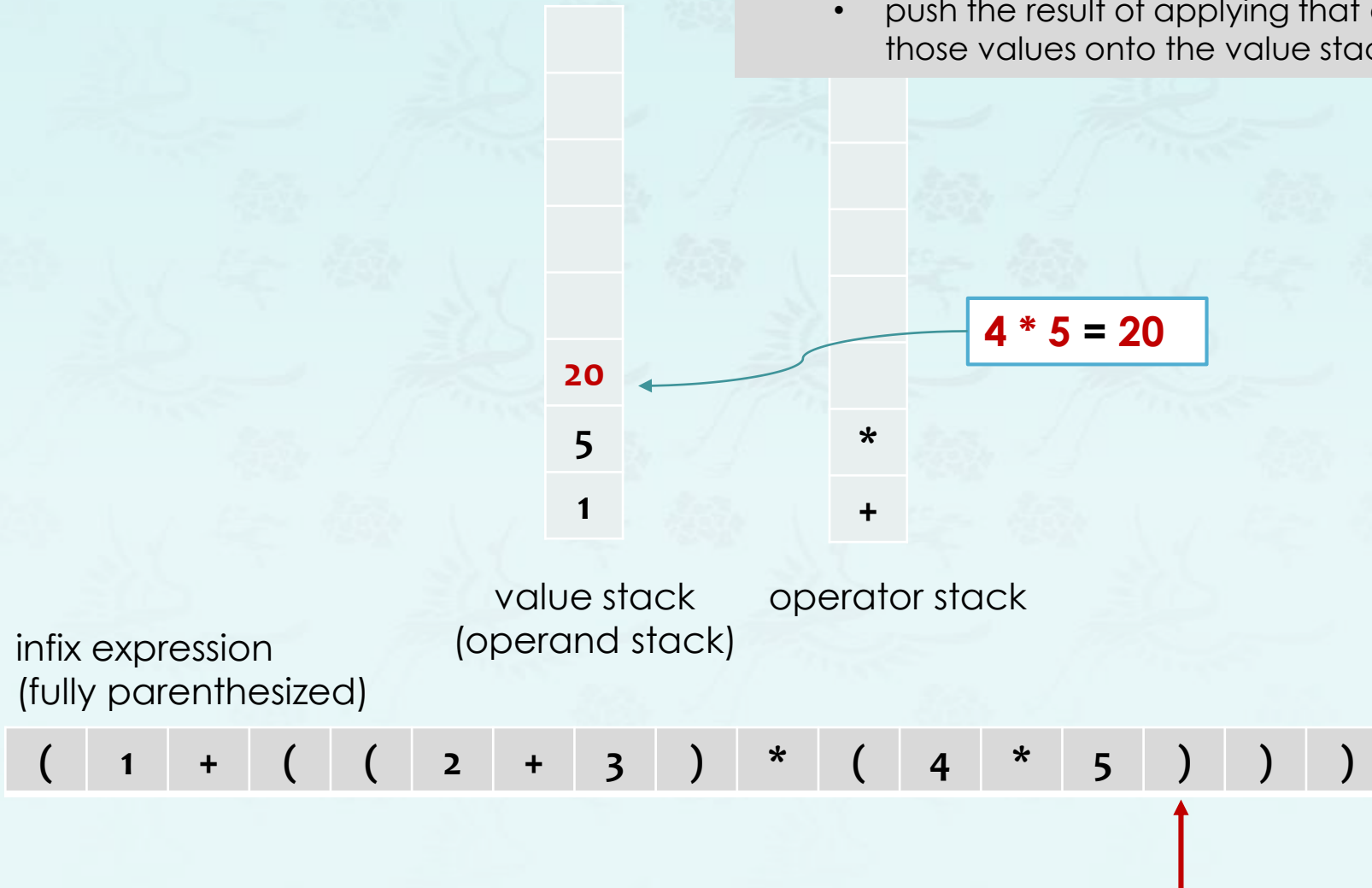


Dijkstra's two-stack algorithm



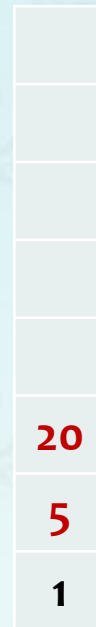
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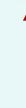
5 * 20

value stack
(operand stack)

operator stack

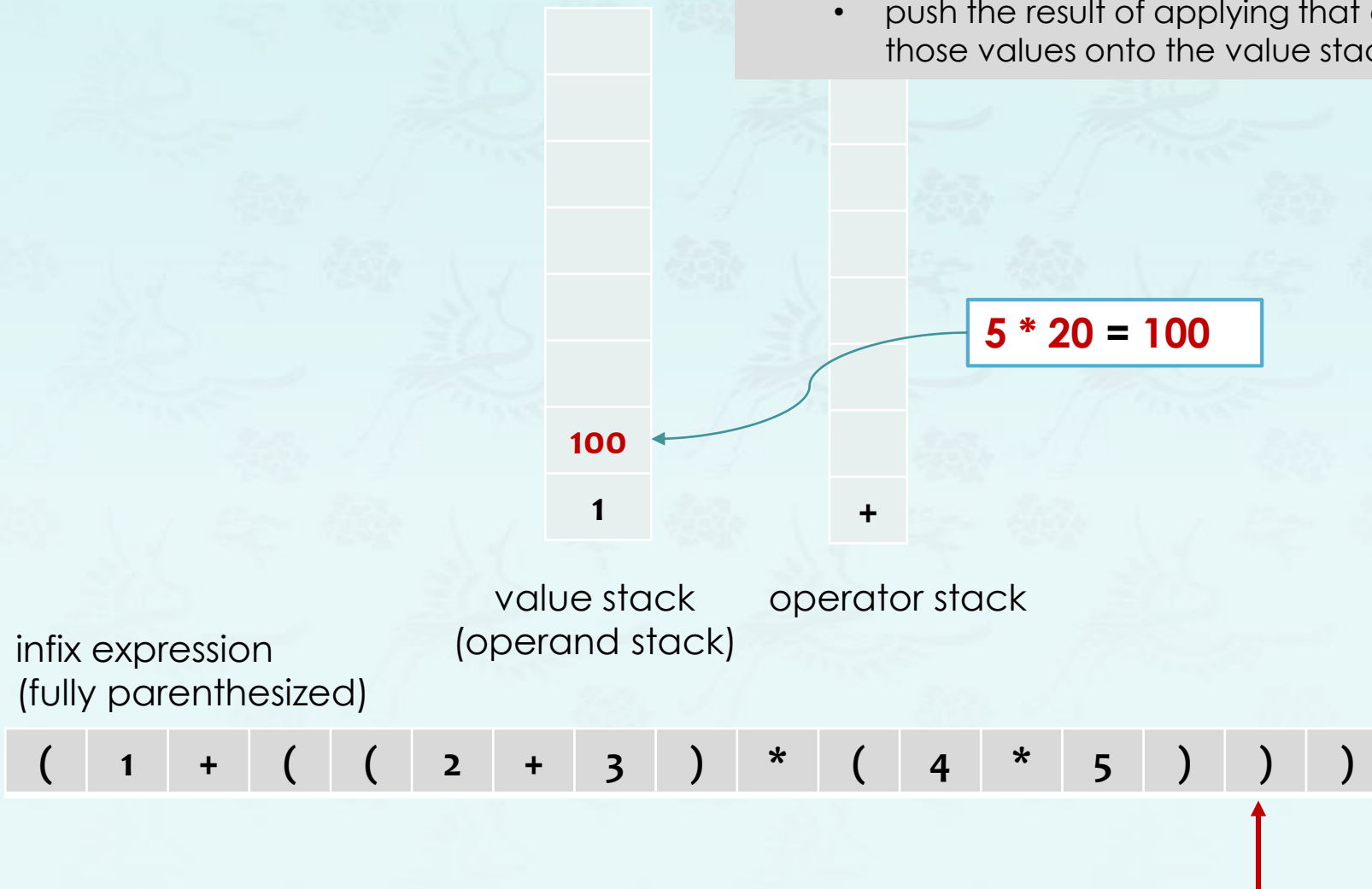
infix expression
(fully parenthesized)

(1 + ((2 + 3) * (4 * 5)))



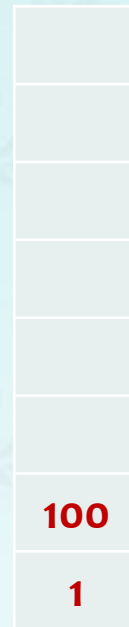
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1 + 100

value stack
(operand stack)

operator stack

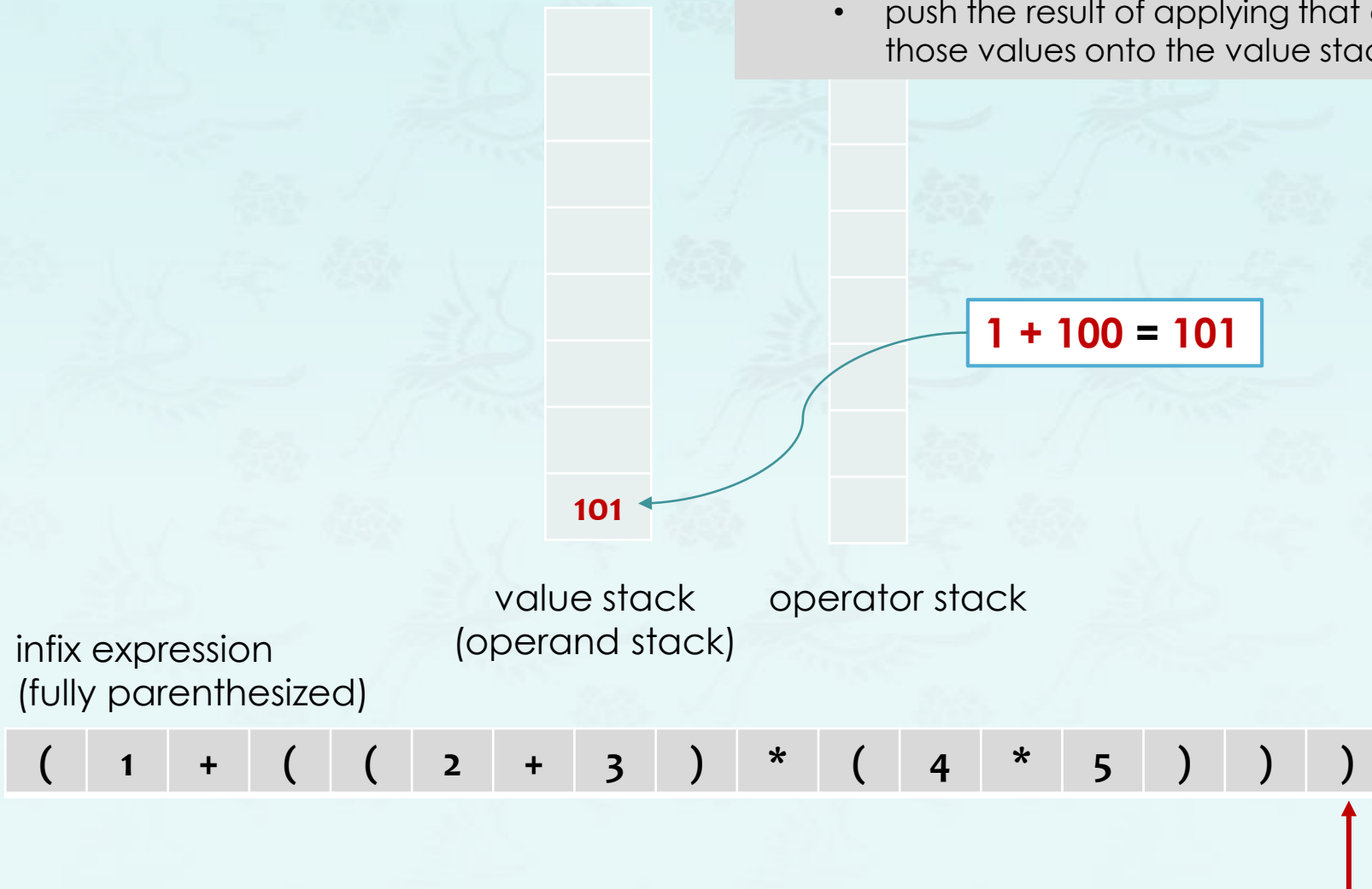
infix expression
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(1 + ((2 + 3) * (4 * 5)))



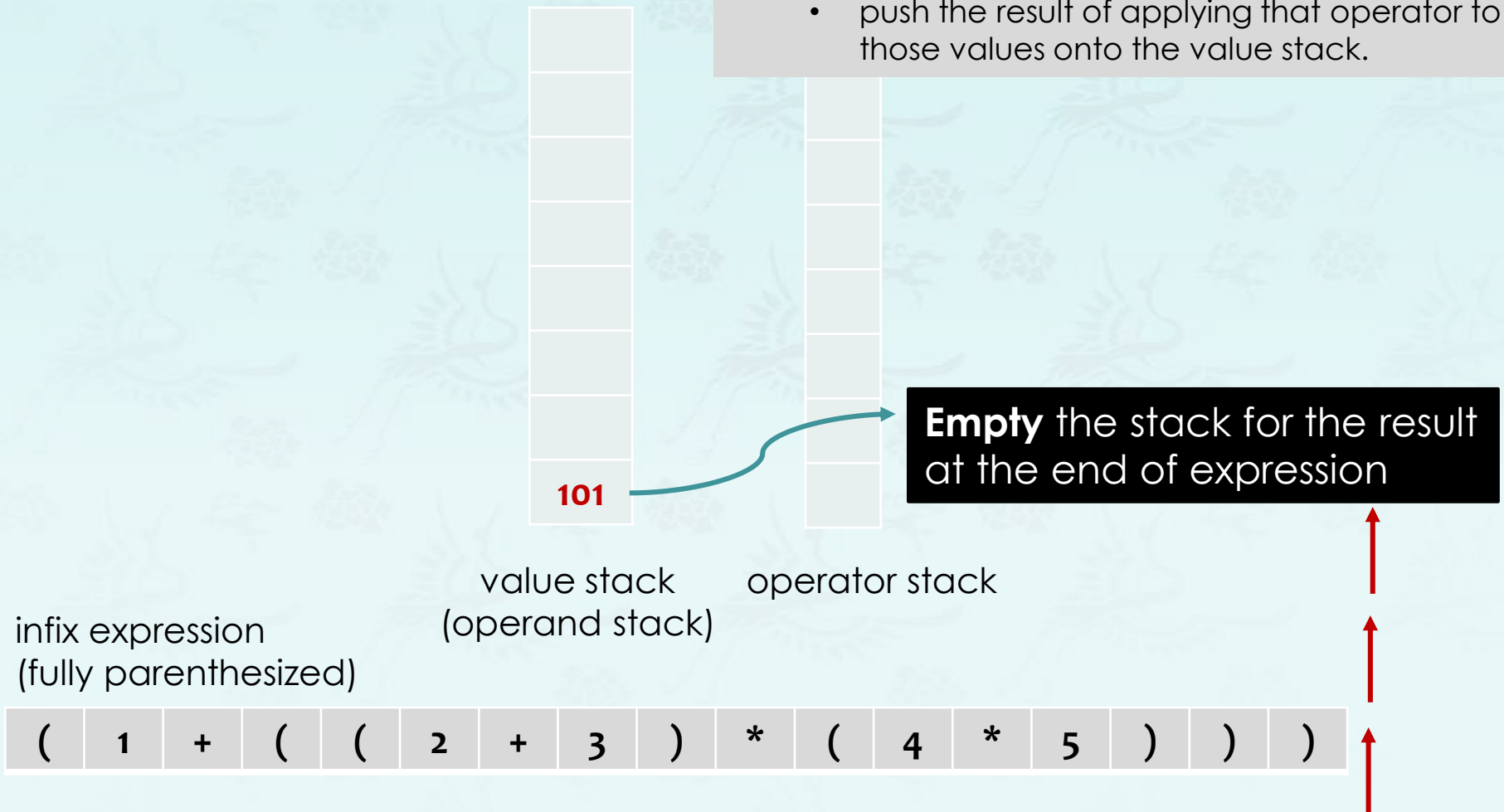
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Dijkstra's two-stack algorithm

```
public class ArithmeticExpression {
    public static void main(String[] args) {
        Stack<Character> ops = new Stack<Character>();
        Stack<Double> vals = new Stack<Double>();
        String e = JOptionPane.showInputDialog(null,
            "Enter an expression", "Stack application", JOptionPane.QUESTION_MESSAGE);
        if (e == null) return;          // Check "Cancel"

        for (int i = 0; i < e.length(); i++) {
            Character c = e.charAt(i);
            if (c.equals(' ') || c.equals('(')) ;
            else if (c == '+') ops.push(c);
            else if (c == '*') ops.push(c);
            else if (c == ')') {
                Character op = ops.pop();
                if (op.equals('+')) vals.push(vals.pop() + vals.pop());
                else if (op.equals('*')) vals.push(vals.pop() * vals.pop());
            }
            else {
                String s = "" + c;
                vals.push(Double.parseDouble(s));
            }
        }
        JOptionPane.showMessageDialog(null, e + " = " + vals.pop());
    }
}
```


Arithmetic expression evaluation

Q: How does it work?

A: When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

$$(1 + ((2 + 3) * (4 * 5)))$$

as if the original input were:

$$(1 + (5 * (4 * 5)))$$

Repeating the argument:

$$\begin{array}{l} (1 + (5 * 20)) \\ (1 + 100) \\ 101 \end{array}$$

Extensions: More ops, precedence order, associativity.

Arithmetic expression evaluation

Observation 1. Dijkstra's two-stack algorithm computes the same value if the operator occurs **after** the two values.

$$(1 + ((2 + 3) * (4 * 5)))$$
$$(1 ((2 3 +) (4 5 *) *) +)$$

Observation 2.

Arithmetic expression evaluation

Observation 1. Dijkstra's two-stack algorithm computes the same value if the operator occurs **after** the two values.

$(1 + ((2 + 3) * (4 * 5)))$

$(1 ((2 3 +) (4 5 *) *) +)$

Observation 2. All of the parentheses are redundant!

Arithmetic expression evaluation

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$(1 + ((2 + 3) * (4 * 5)))$

$(1 ((2 3 +) (4 5 *) *) +)$

Observation 2. All of the parentheses are redundant!

$1\ 2\ 3\ +\ 4\ 5\ *\ * +$

Bottom line: Postfix or “reverse Polish” notation.

Applications: Postscript, calculators, JVM,

Arithmetic expression evaluation

infix	postfix
$2 + 3 * 4$	$2\ 3\ 4\ *\ +$
$a * b + 5$	$a\ b\ *\ 5\ +$
$(1 + 2) * 7$	
$a * b / c$	
$(a / (b - c + d)) * (e - a) * c$	
$a / b - c + d * e - a * c$	

Infix and postfix notation

Arithmetic expression evaluation

infix	postfix
$2 + 3 * 4$	$2\ 3\ 4\ *\ +$
$a * b + 5$	$a\ b\ *\ 5\ +$
$(1 + 2) * 7$	$1\ 2\ +\ 7\ *$
$a * b / c$	$a\ b\ *\ c\ /$
$(a / (b - c + d)) * (e - a) * c$	$a\ b\ c\ -\ d\ +\ /\ e\ a\ -\ *\ c\ *$
$a / b - c + d * e - a * c$	$a\ b\ /\ c\ -\ d\ e\ *\ +\ a\ c\ *\ -$

Infix and postfix notation

Arithmetic expression evaluation

infix	postfix
2 + 3 * 4	2 3 4 * +
a * b + 5	a b * 5 +
	1 2 + 7 *
	a b * c /
	a b c - d + / e a - * c *
	a b / c - d e * + a c * -

Infix and postfix notation

Arithmetic expression evaluation

Goal: Evaluate postfix expressions.

$a\ b\ c\ -\ d\ +\ /\ e\ a\ -\ *\ c\ *$ \Rightarrow $(a / ((b - c) + d)) * (e - a) * c$

a	b	c	-	d	+	/	e	a	-	*	c	*
---	---	---	---	---	---	---	---	---	---	---	---	---

Arithmetic expression evaluation

Goal: Evaluate postfix expressions.

a b c - d + / e a - * c *



(a / ((b - c) + d)) * (e - a) * c

value stack
(operand stack)

a

a	b	c	-	d	+	/	e	a	-	*	c	*
---	---	---	---	---	---	---	---	---	---	---	---	---



*push the operands
until an operator comes up.*

Arithmetic expression evaluation

Goal: Evaluate postfix expressions.

a b c - d + / e a - * c *



(a / ((b - c) + d)) * (e - a) * c

value stack
(operand stack)

b

a

a	b	c	-	d	+	/	e	a	-	*	c	*
---	---	---	---	---	---	---	---	---	---	---	---	---



Arithmetic expression evaluation

Goal: Evaluate postfix expressions.

a b c - d + / e a - * c *



(a / ((b - c) + d)) * (e - a) * c

value stack
(operand stack)

c
b
a

a	b	c	-	d	+	/	e	a	-	*	c	*
---	---	---	---	---	---	---	---	---	---	---	---	---



Arithmetic expression evaluation

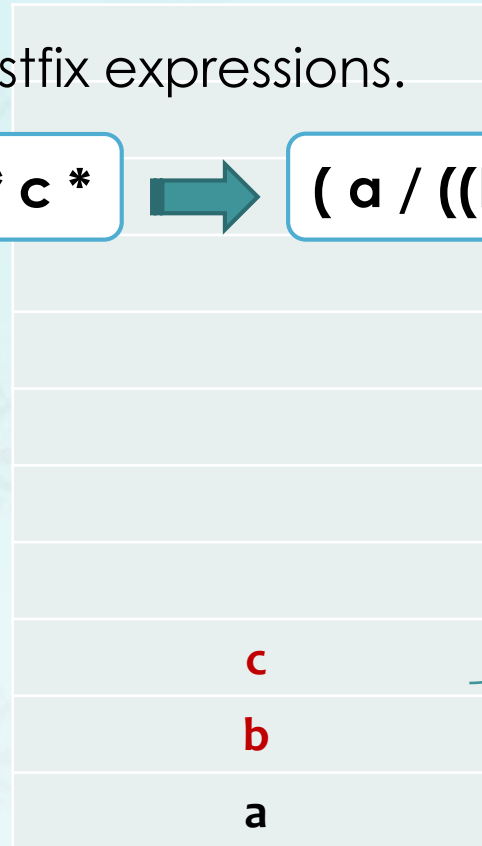
Goal: Evaluate postfix expressions.

a b c - d + / e a - * c *



(a / ((b - c) + d)) * (e - a) * c

value stack
(operand stack)



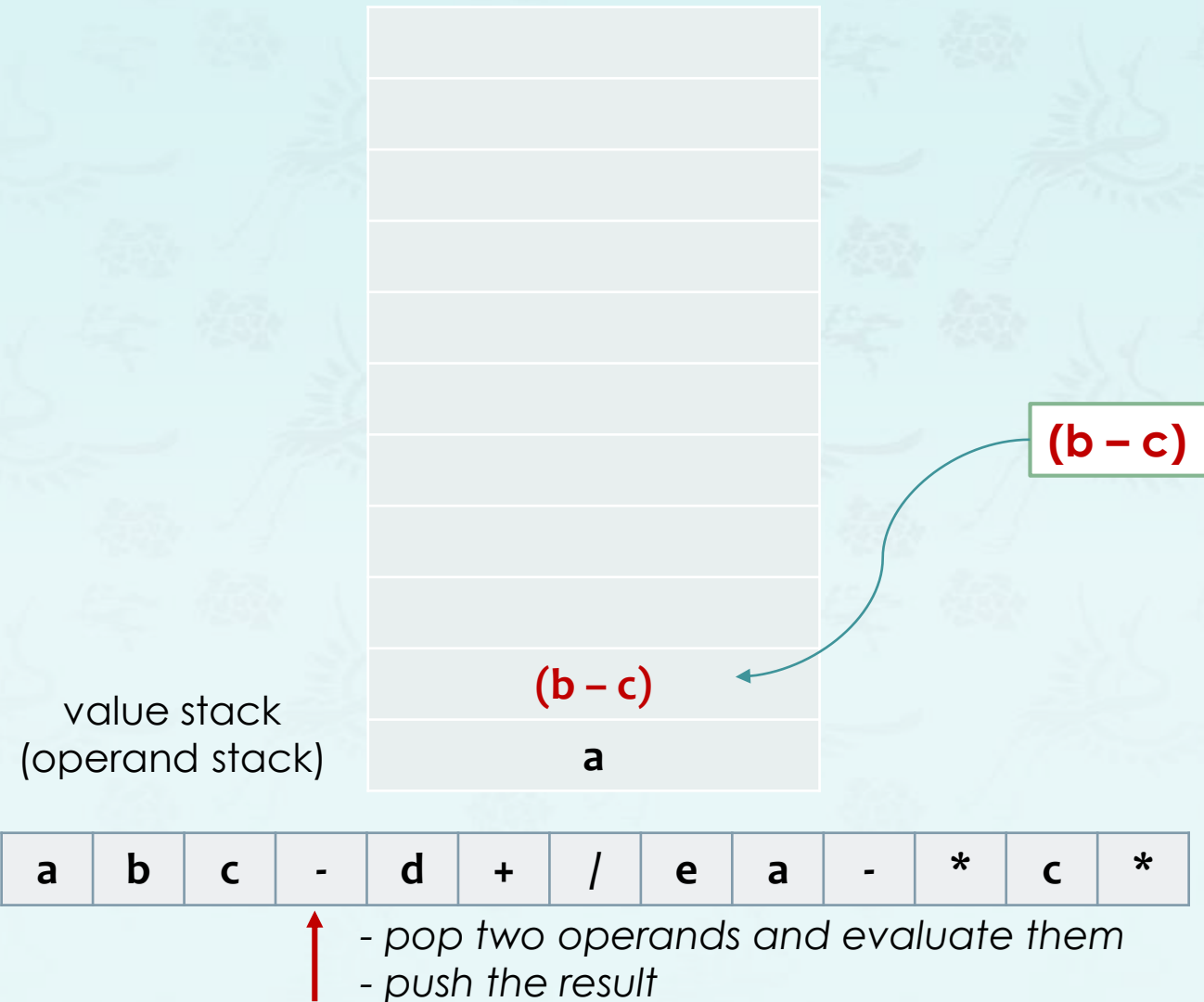
(b - c)

a	b	c	-	d	+	/	e	a	-	*	c	*
---	---	---	---	---	---	---	---	---	---	---	---	---



- pop two operands and evaluate them
- push the result

Arithmetic expression evaluation



Arithmetic expression evaluation

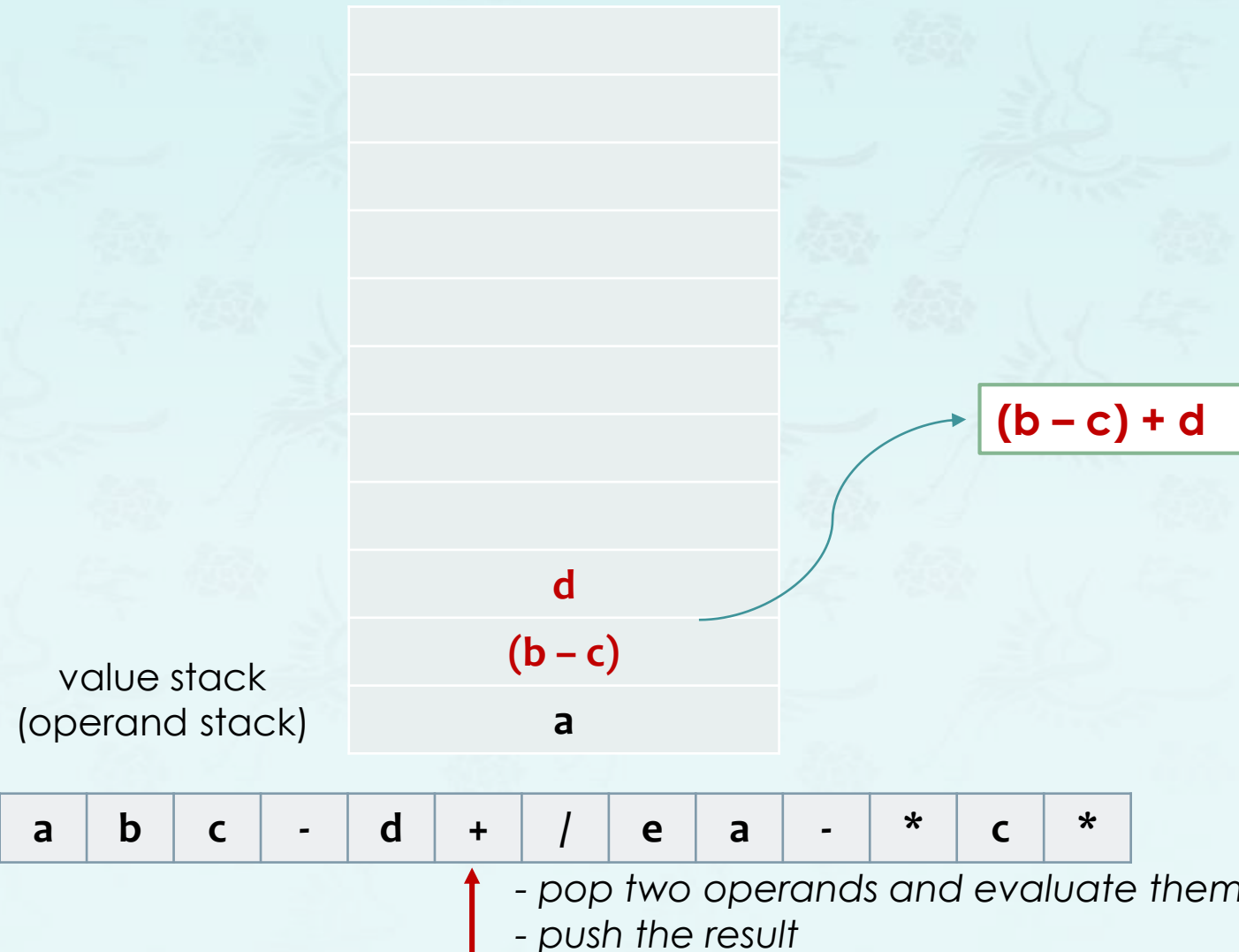
value stack
(operand stack)



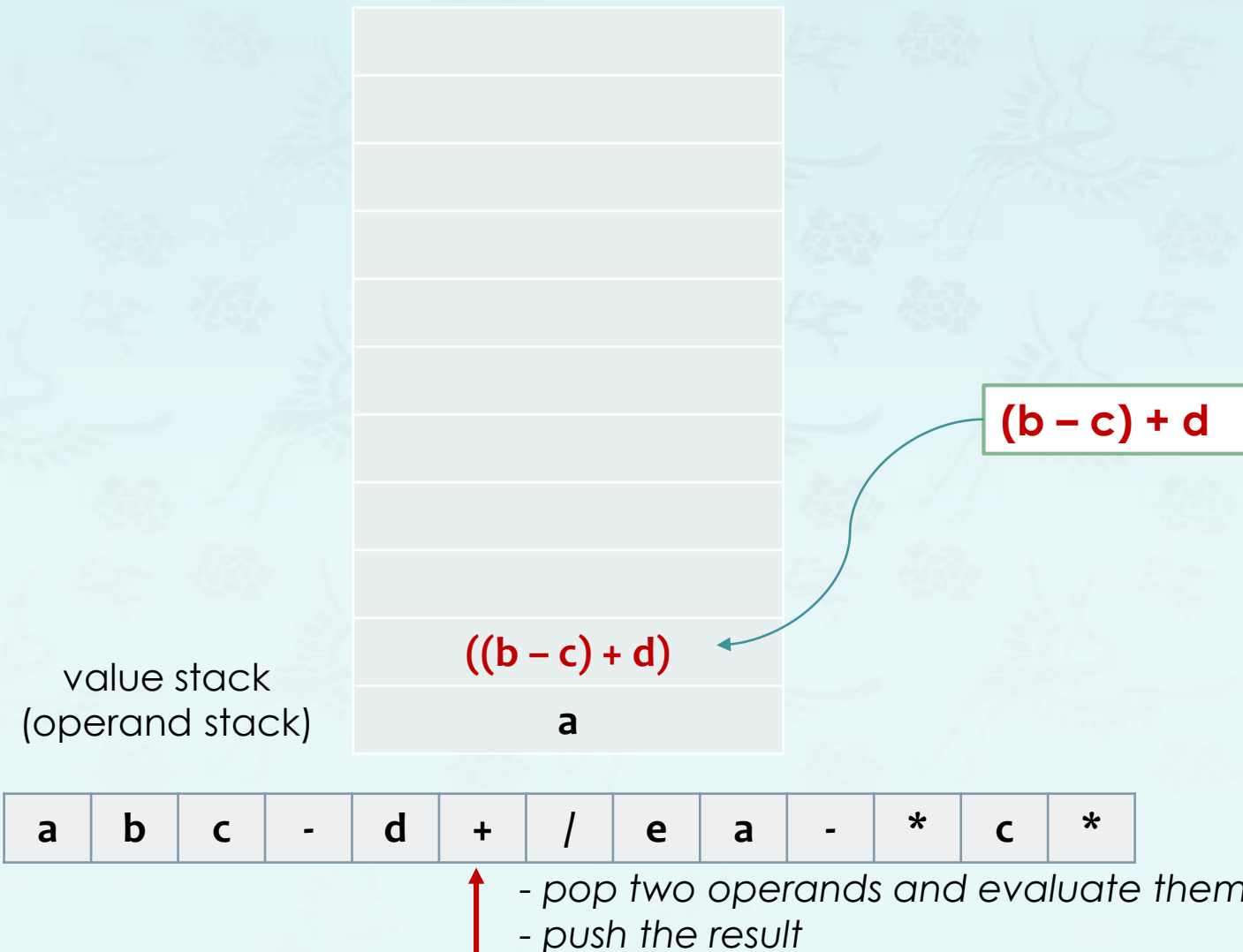
a	b	c	-	d	+	/	e	a	-	*	c	*
---	---	---	---	---	---	---	---	---	---	---	---	---



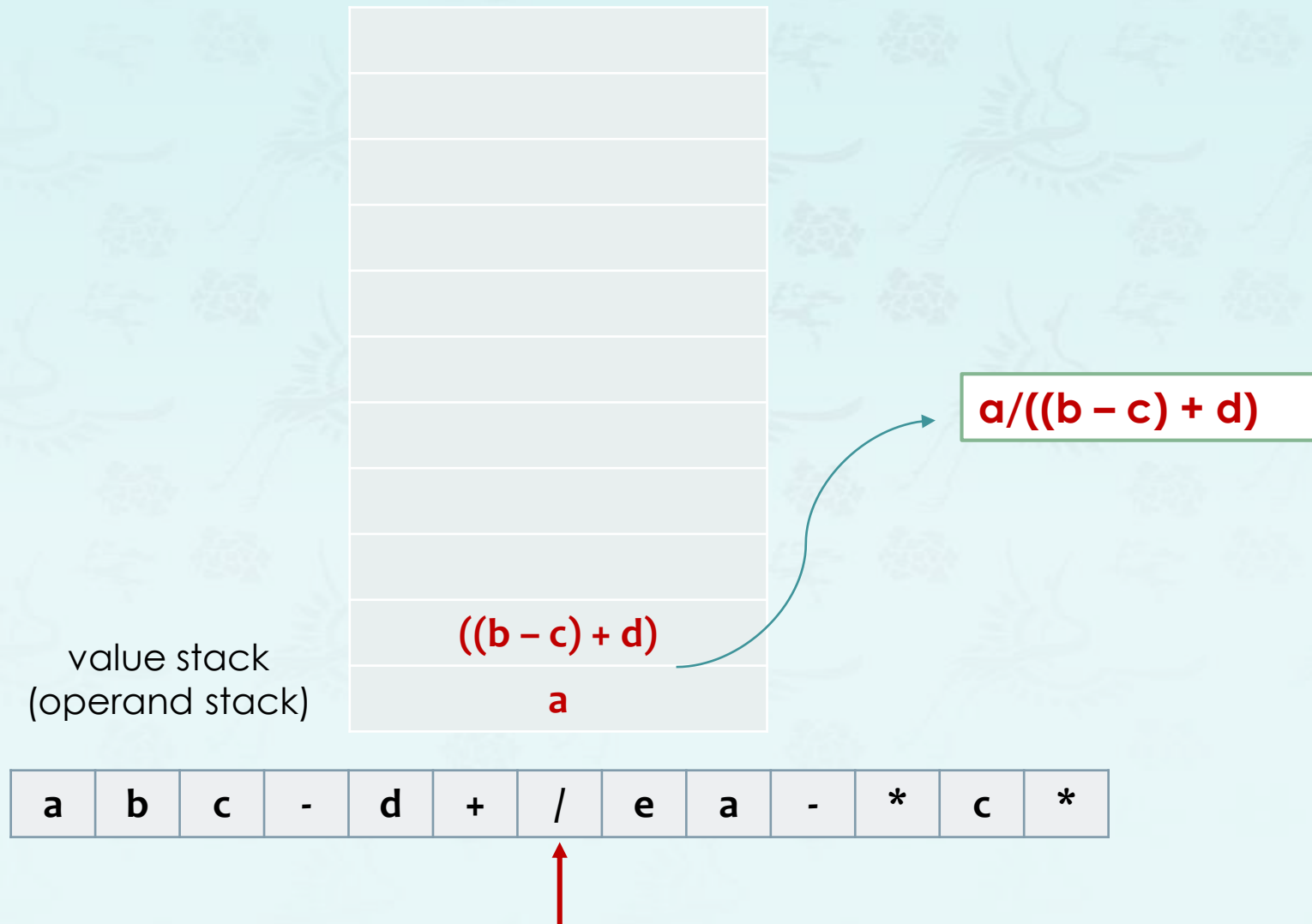
Arithmetic expression evaluation



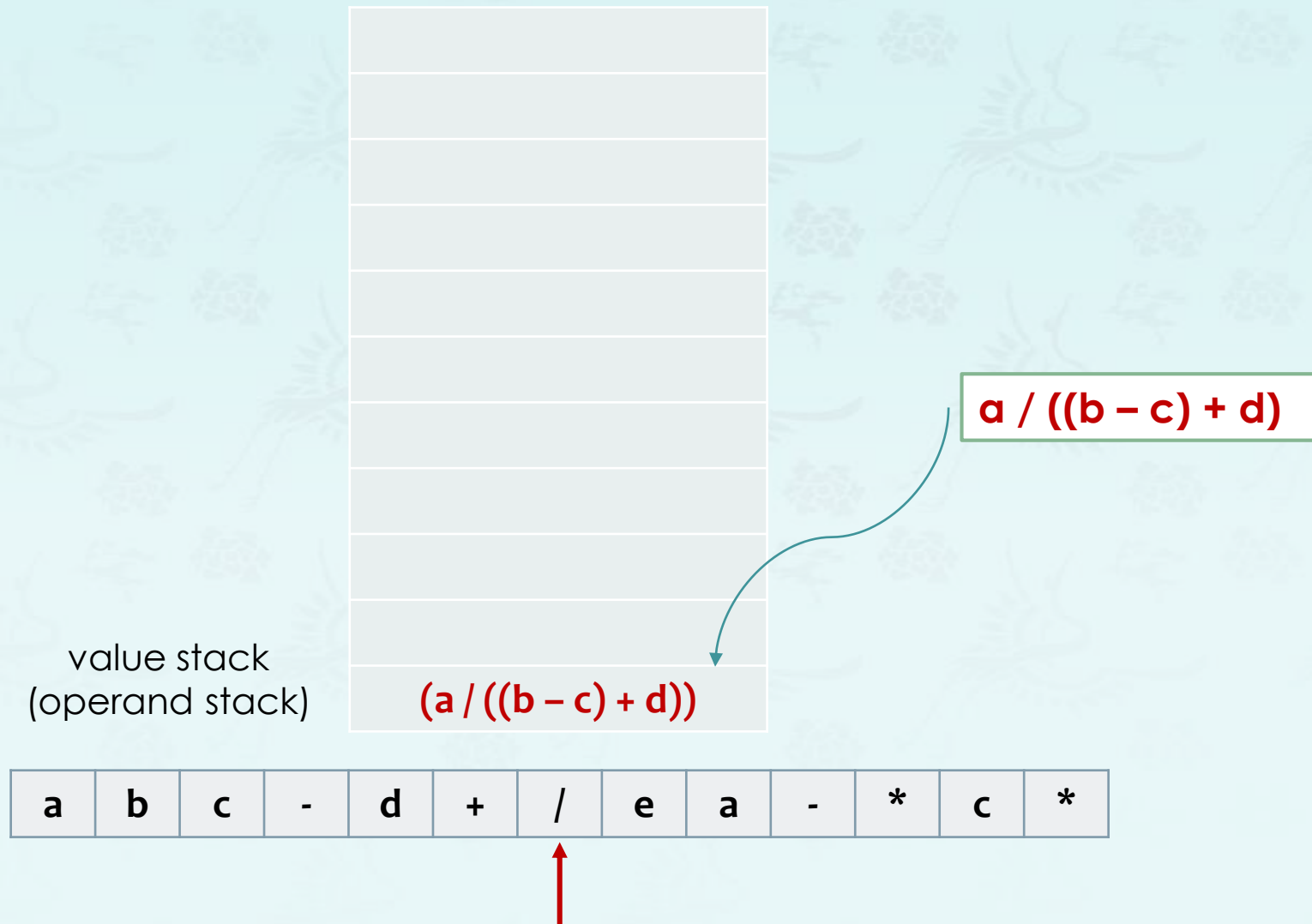
Arithmetic expression evaluation



Arithmetic expression evaluation

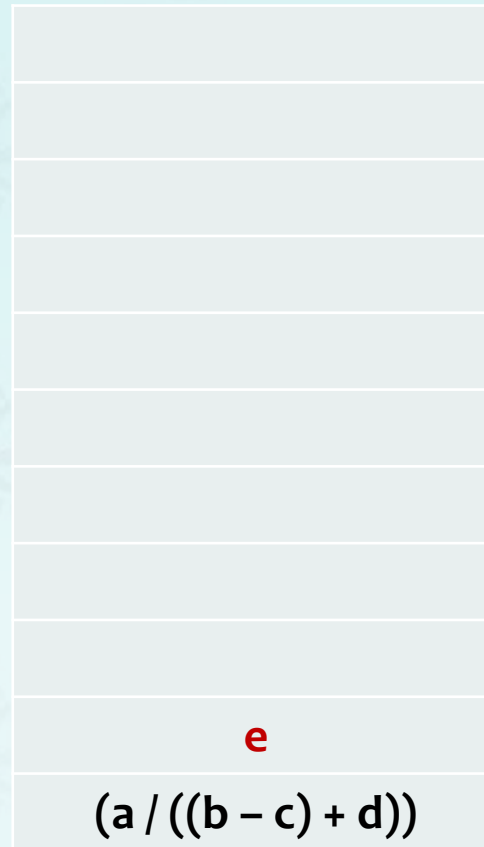


Arithmetic expression evaluation



Arithmetic expression evaluation

value stack
(operand stack)

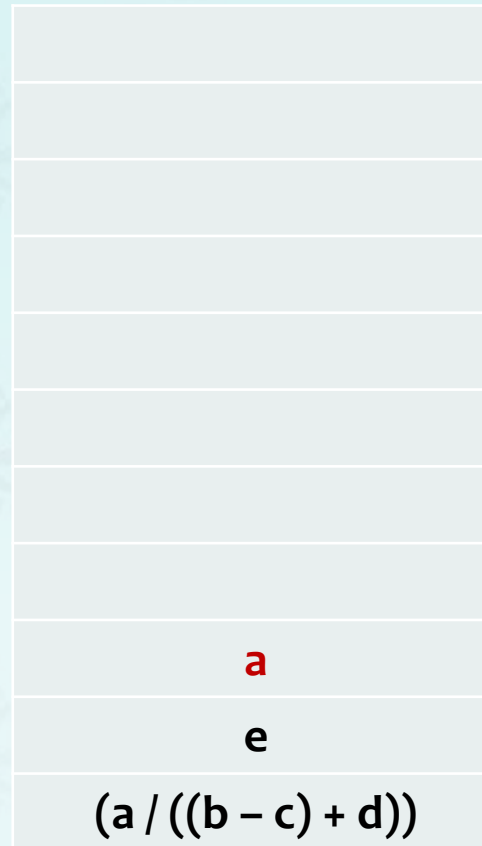


a	b	c	-	d	+	/	e	a	-	*	c	*
---	---	---	---	---	---	---	---	---	---	---	---	---



Arithmetic expression evaluation

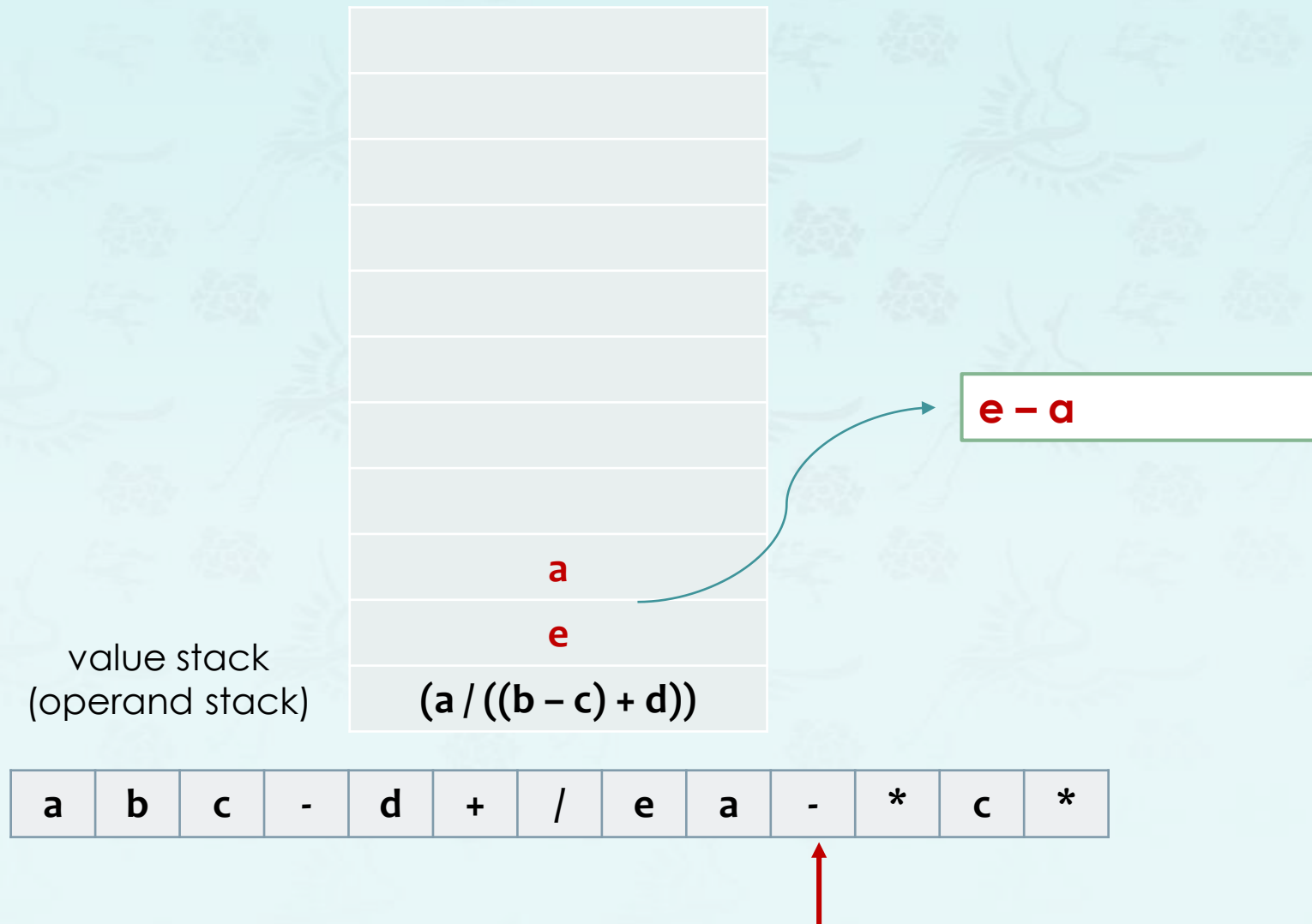
value stack
(operand stack)



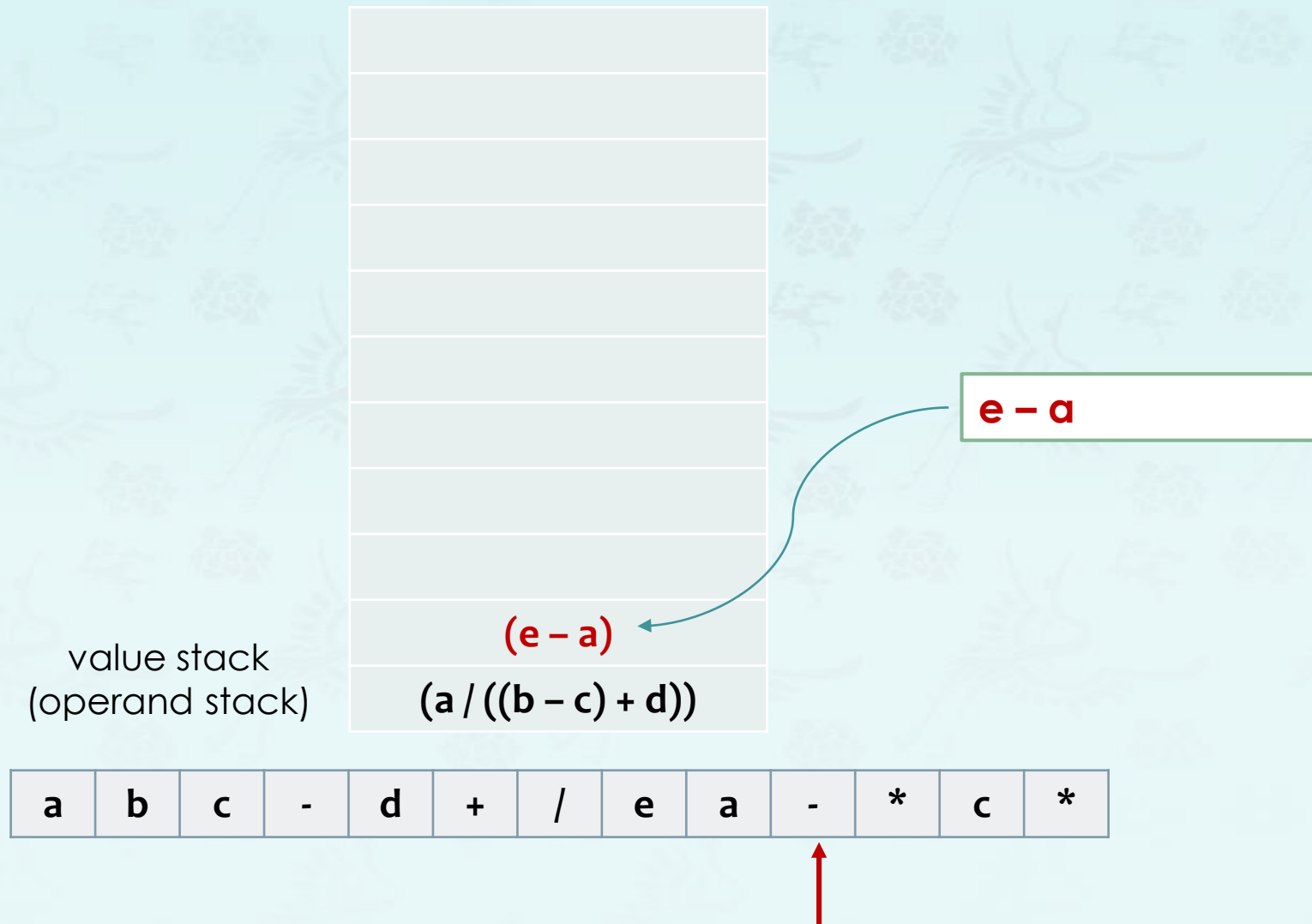
a	b	c	-	d	+	/	e	a	-	*	c	*
---	---	---	---	---	---	---	---	---	---	---	---	---



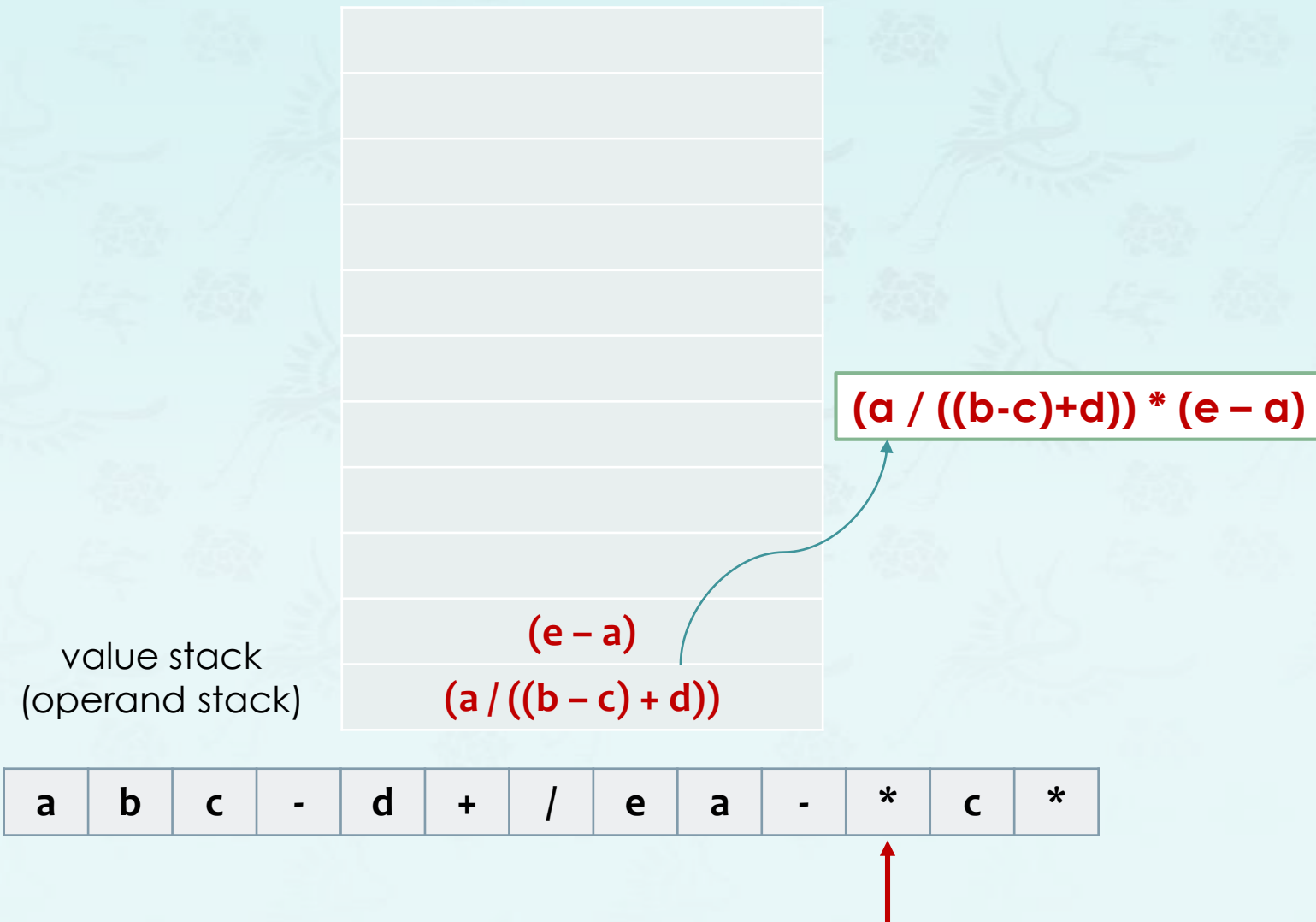
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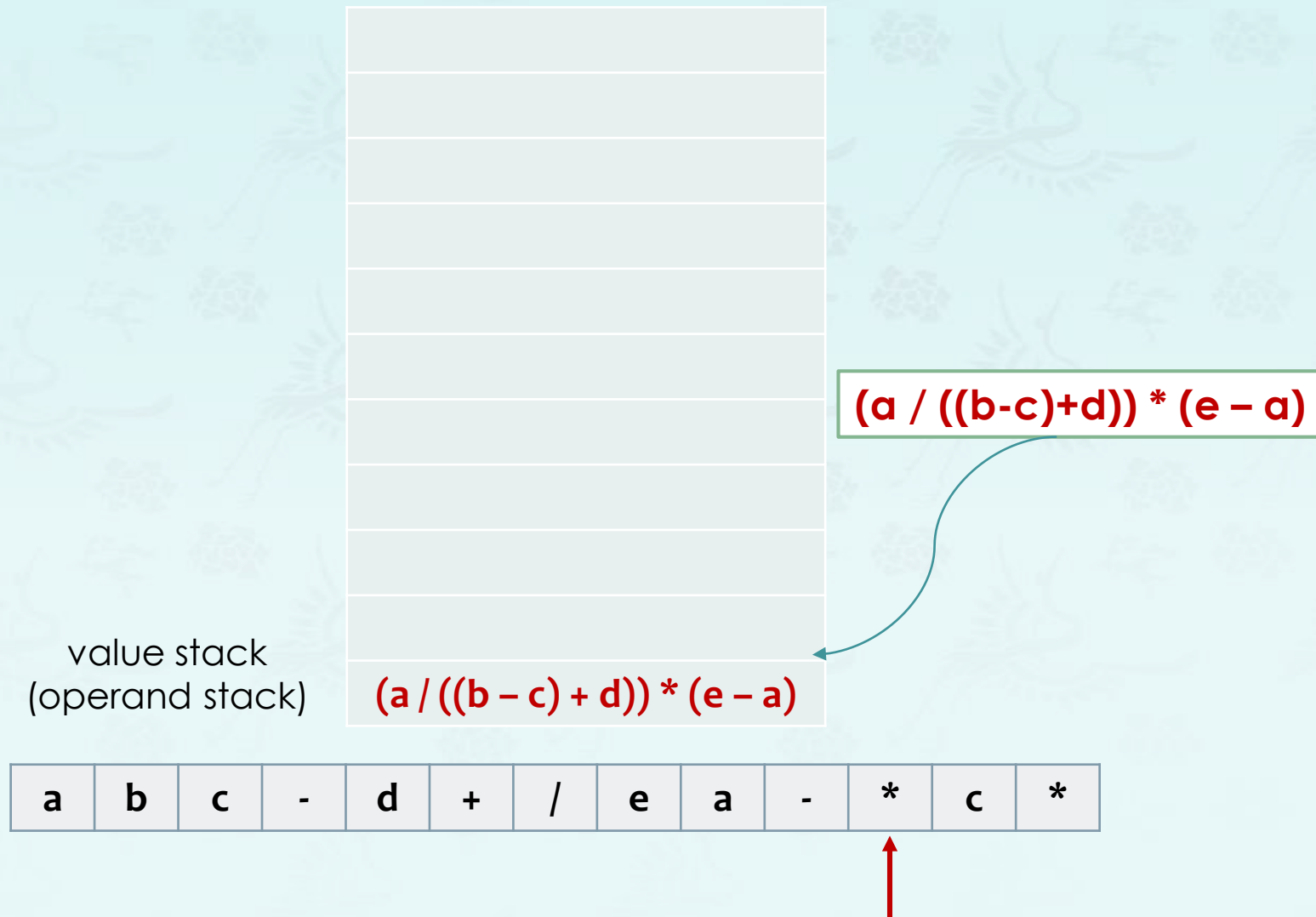
Arithmetic expression evaluation



Arithmetic expression evaluation

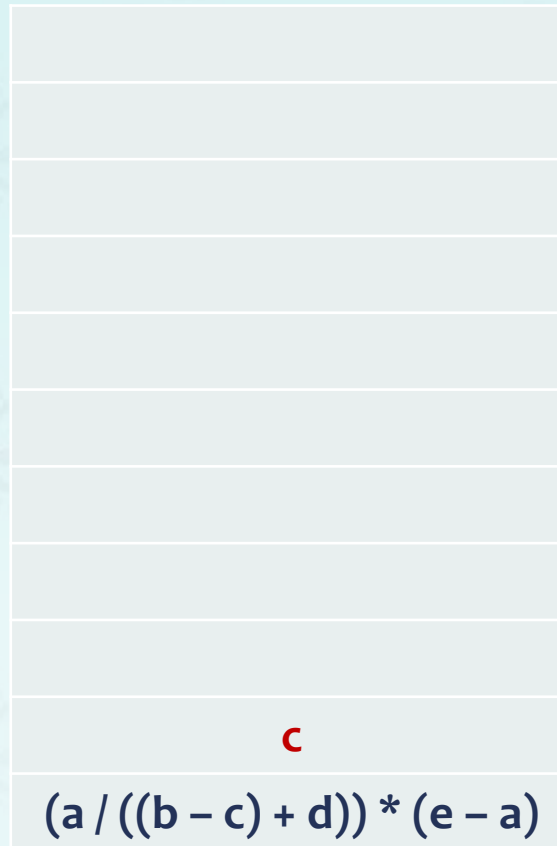


Arithmetic expression evaluation



Arithmetic expression evaluation

value stack
(operand stack)



a	b	c	-	d	+	/	e	a	-	*	c	*
---	---	---	---	---	---	---	---	---	---	---	---	---



Arithmetic expression evaluation

Goal: Evaluate postfix expressions.

a b c - d + / e a - * c *



(a / ((b - c) + d)) * (e - a) * c

value stack
(operand stack)

(a / ((b - c) + d)) * (e - a)

c

(a / ((b - c) + d)) * (e - a) * **c**

a	b	c	-	d	+	/	e	a	-	*	c	*
---	---	---	---	---	---	---	---	---	---	---	---	---



Arithmetic expression evaluation - Infix to Postfix Conversion

We use a stack.

1. When an **operand** is read, output it.
2. When an **operator** is read,
 - **Pop** until the top of the stack has an element of **lower** precedence.
 - Then **push** it.
3. When) is found, pop until we find the matching (.
4. (has the lowest precedence when in the stack but has the highest precedence when in the input.
5. When we reach the end of input, pop until the stack is empty.

Arithmetic expression evaluation - Infix to Postfix Conversion

Example 1:

infix: $3+4*5/6$

in	stack(bottom to top)	postfix
3		
+		
4		
*		
5		
/		
6		

Arithmetic expression evaluation - Infix to Postfix Conversion

Example 2:

infix: $(1+3)*(4-2)/(5+7)$

in	stack (bottom to top)	postfix
((
1		1
+	(+	
3		1 3
)		1 3 +
*	*	
(* (
4		1 3 + 4
-	* (-	
2		1 3 + 4 2
)	*	1 3 + 4 2 -
/	/	1 3 + 4 2 - *

in	stack	postfix
(/ (1 3 + 4 2 - *
5		1 3 + 4 2 - * 5
+	/ (+	
7		1 3 + 4 2 - * 5 7
)		1 3 + 4 2 - * 5 7 +
		1 3 + 4 2 - * 5 7 + /
	- Operands are output immediately	
	- Stack operators until right parens	
	- Unstack until left parens	
	Delete left parens	
	- In general, higher precedence ops must be output before lower one.)	

Arithmetic expression evaluation - Infix to Postfix Conversion

Example 3:

infix: $a - (b + c * d) / e$

in	stack(bottom to top)	postfix
a		
-		
(
b		
+		
c		
*		
d		
)		
/		
e		

- Operands are output immediately
- Stack operators until right parens
- Unstack until left parens
Delete left parens
- In general, higher precedence operator must be output before lower one.)

Arithmetic expression evaluation - Infix to Postfix Conversion

Example 3:

infix: $A * (B + C * D) + E$

	in	stack(bottom to top)	postfix
1	A		
2	*		
3	(
4	B		
5	+		
6	C		
7	*		
8	D		
9)		
10	+		
11	E		
12			

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Stack and Queue

Data Structures
C++ for C Coders

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*applications - **infix to postfix***