heap

- *complete binary tree (review)*
- *heap and priority queues (Chapter 9)*
- *binary heap and min-heap*
- *max-heap demo*
- *max-heap coding*
- *heap sort (Chapter 7)*

heap coding: heap.h

Heap ADT: A **one dimensional array** is used to simplify parent and child calculations.

```
struct Heap {
  int *nodes;          // an array of nodes
  int capacity;         // array size of node or key, item
  int N;                // the number of nodes in the heap
  bool (*comp)(Heap*, int, int);
  Heap(int capa = 2) {
    capacity = capa;
    nodes = new int[capacity];
    N = 0;
    comp = nullptr;
  };
  ~Heap() {};
};
using heap = Heap*;
```

heap coding: heap.h

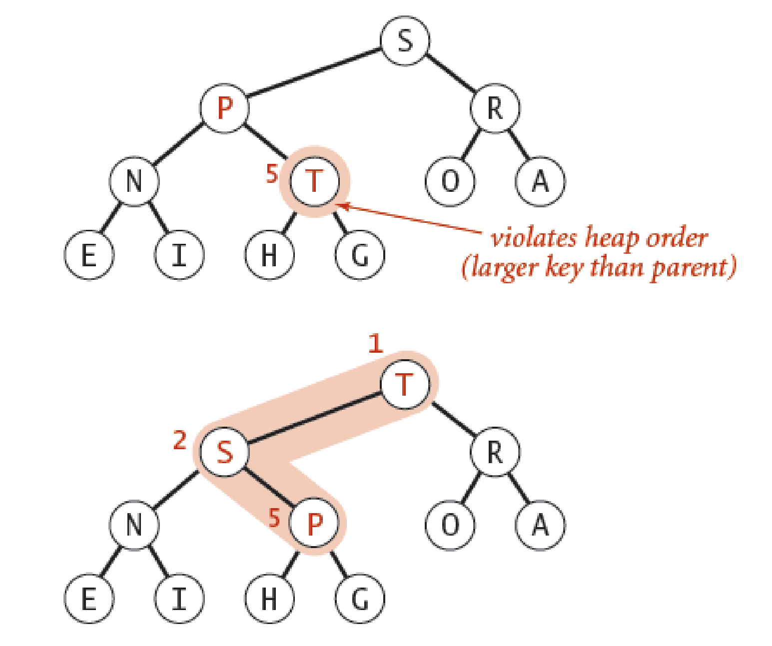
```
void clear(heap hp);                 // deallocate heap
int size(heap hp);                   // return nodes in heap currently
int level(int n);                    // return level based on num of nodes
int capacity(heap hp);               // return its capacity (array size)
int reserve(heap hp, int capa);      // reserve the array size (= capacity) node의 크기를 재조정
int full(heap hp);                   // return true/false  full = true? reserve : continue
int empty(heap hp);                  // return true/false
void grow(heap hp, int key);         // add a new key  parent가 child 보다 크거나 같다 (같을 수 있음)
void trim(heap hp);                  // delete a queue
int heapify(heap hp);                // convert a complete BT into a heap

// helper functions to support grow/trim functions
int less(heap hp, int i, int j);     // used in max heap
int more(heap hp, int i, int j);     // used in min heap
void swim(heap hp, int k);           // bubble up
void sink(heap hp, int k);           // tickle down
// helper functions to check heap invariant
int heapOrdered(heap hp);           // is heap[1..N] a heap?
```

Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until **heap order** restored.

This is a maxheap example.

swim up
or
sink down

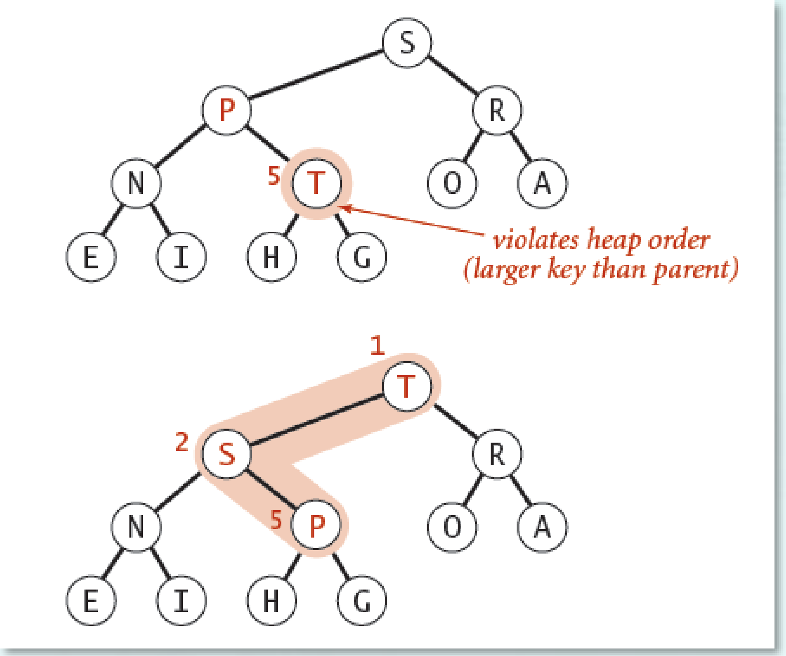


Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

guess a name?

```
bool       (heap h, int p, int c) {
    return h->nodes[p] < h->nodes[c];
}
```



heap coding

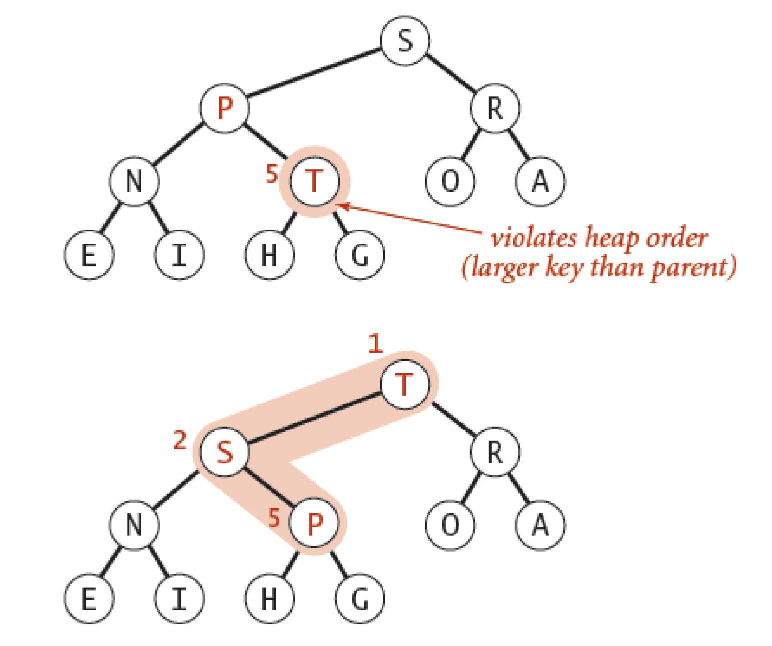
Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

guess a name?

```
bool      (heap h, int p, int c) {
    return h->nodes[p] < h->nodes[c];
}
```

```
void      (heap h, int p, int c) {
    int item = h->nodes[p];
    h->nodes[p] = h->nodes[c];
    h->nodes[c] = item;
}
```

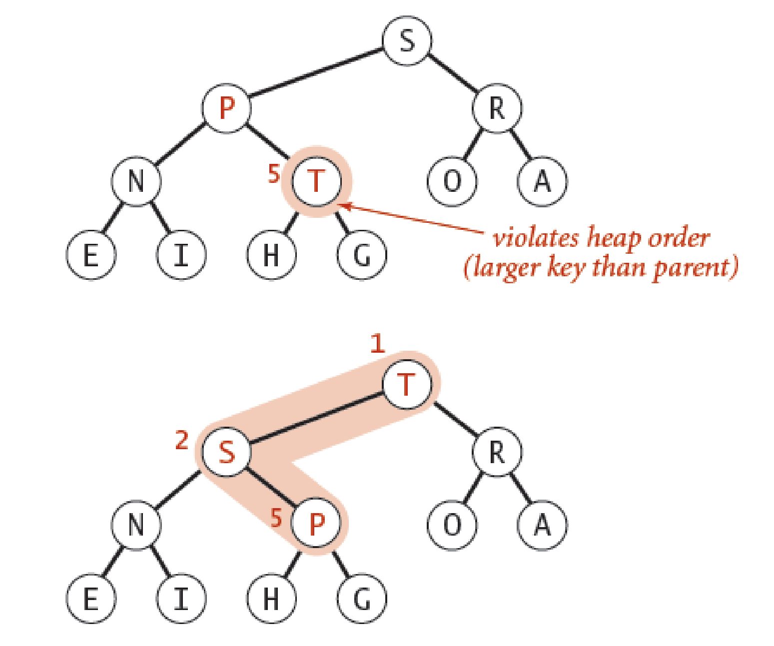


Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

```
bool less(heap h, int p, int c) {
    return h->nodes[p] < h->nodes[c];
}
```

```
void swap(heap h, int p, int c) {
    int item = h->nodes[p];
    h->nodes[p] = h->nodes[c];
    h->nodes[c] = item;
}
```



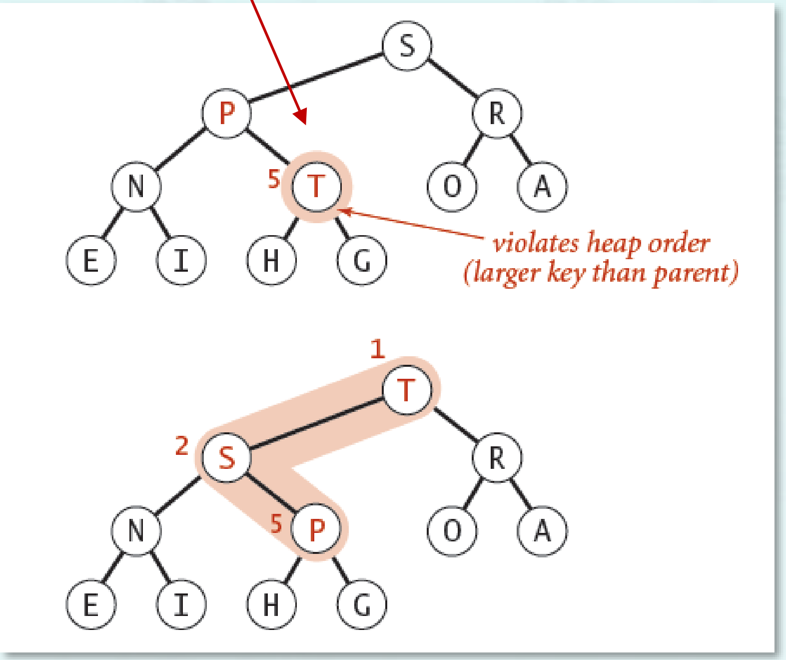
Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

guess a name?

```
void        (heap h, int k)
{
  while (not reached at root &&
             k's parent key < k's key)
  {

  }
}
```

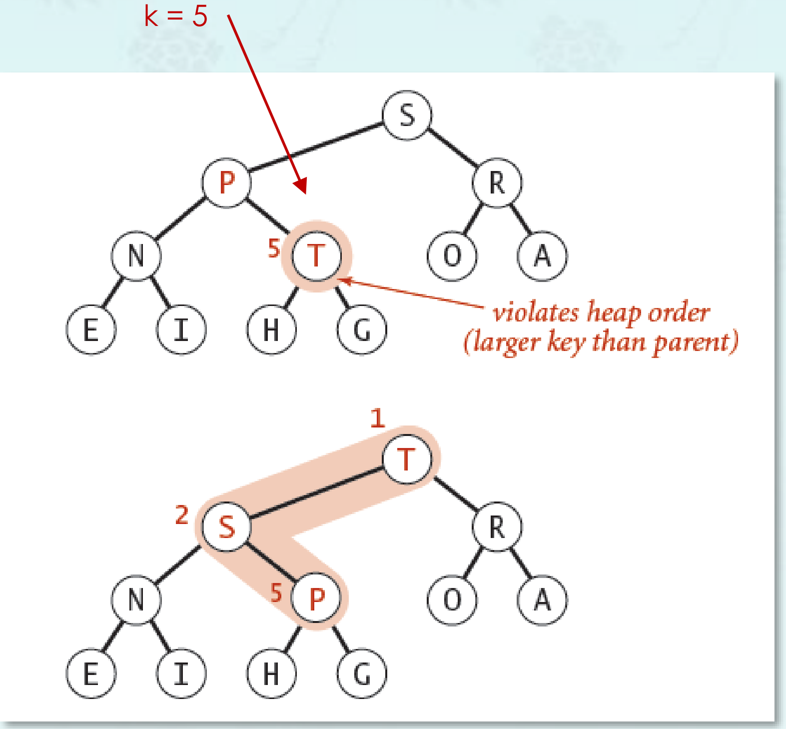
k = 5



Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

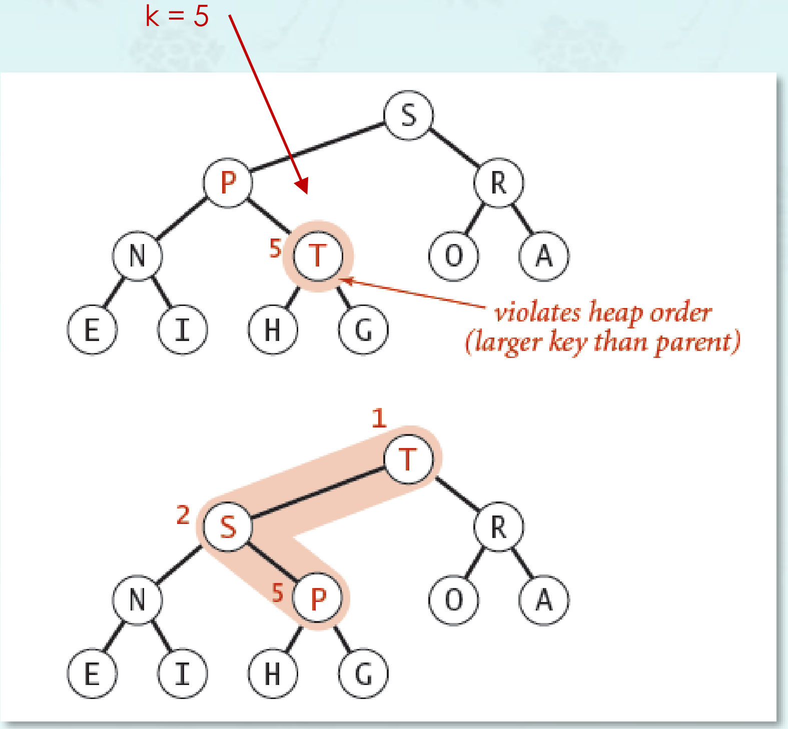
```
void swim(heap h, int k)
{
  while (not reached at root &&
         k's parent key < k's key)
  {

  }
}
```



Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

```
void swim(heap h, int k)
{
  while (not reached at root &&
          k's parent key < k's key)
  {
        swap k and its parent
        go for the next
  }
}
```



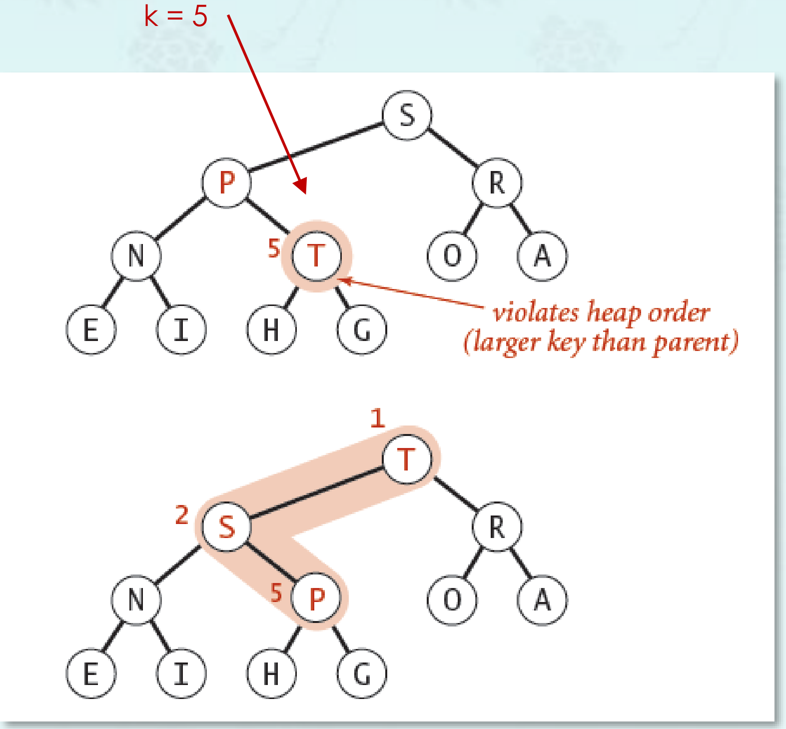
Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

not reached at root

parent of k

```
void swim(heap h, int k)
{
  while (
  {

  }
}
```



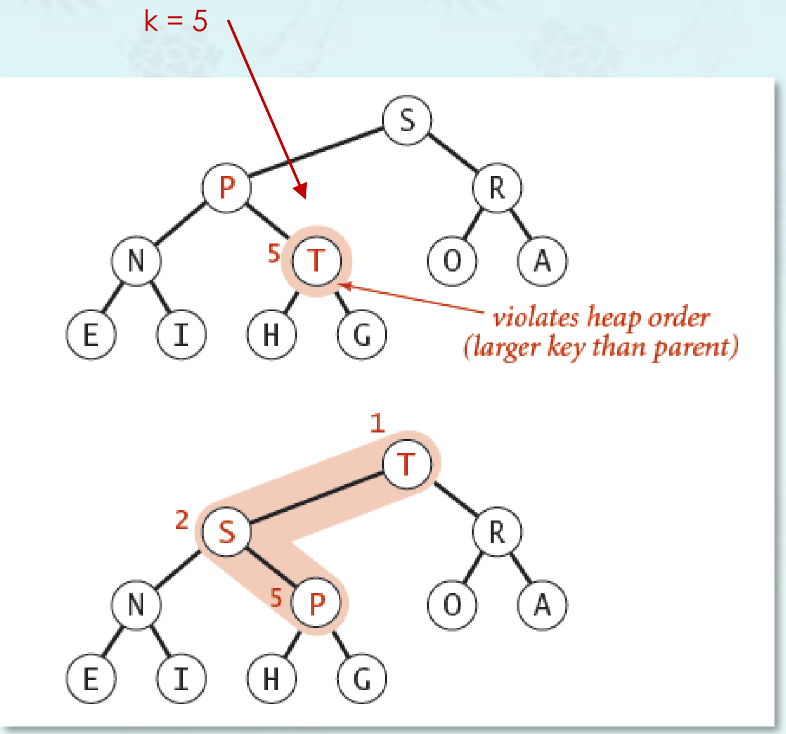
Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

not reached at root

parent(k/2) is less its child(k)

```
void swim(heap h, int k)
{
  while (k > 1 &&                    )
  {

  }
}
```



Promotion in a heap: swim

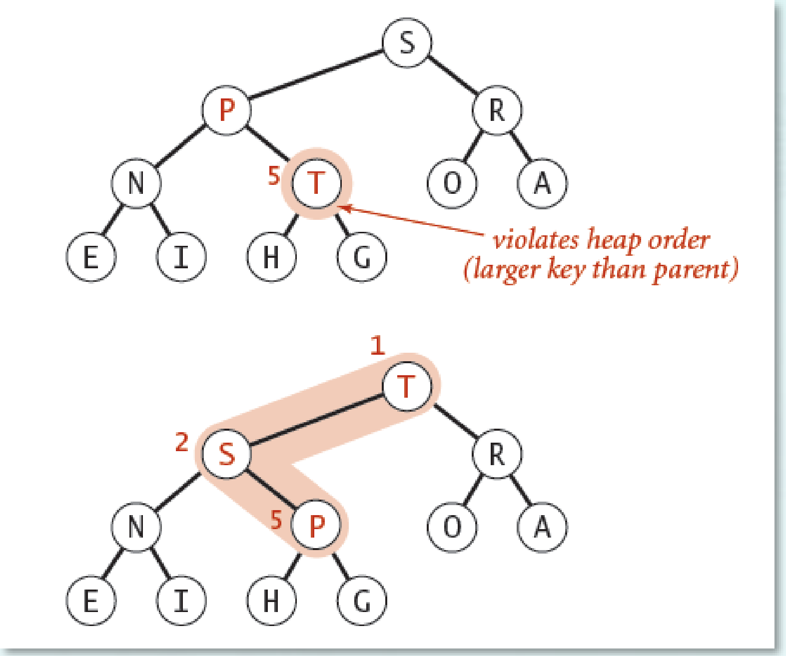
- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

not reached at root

parent(k/2) is less its child(k)

```
void swim(heap h, int k)
{
  while (k > 1 && less(h, k / 2, k))
  {

  }
}
```

swap parent(k/2) and its child(k)



Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

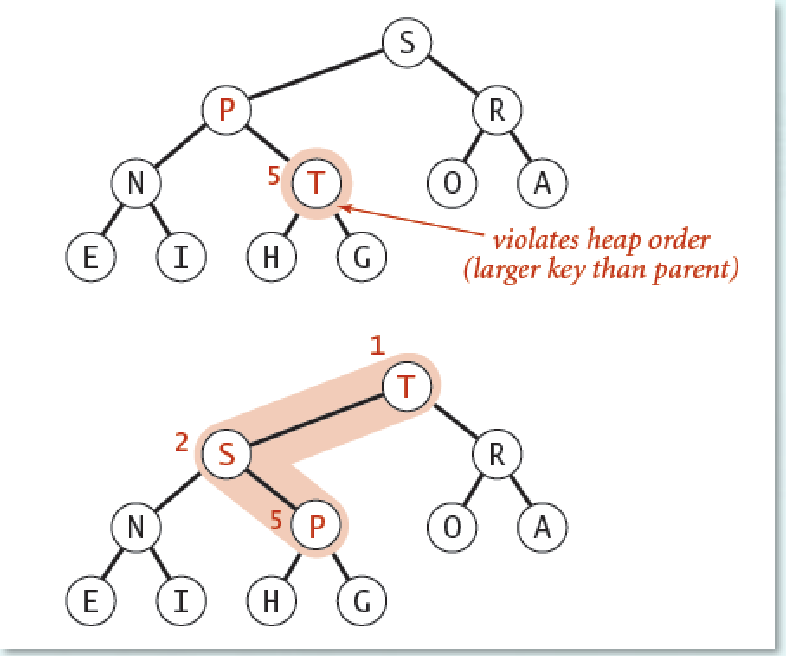
not reached at root

parent(k/2) is less its child(k)

```
void swim(heap h, int k)
{
  while (k > 1 && less(h, k / 2, k))
  {
    swap(h, k / 2, k);

  }
}
```

swap parent(k/2) and its child(k)

move up one level



Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

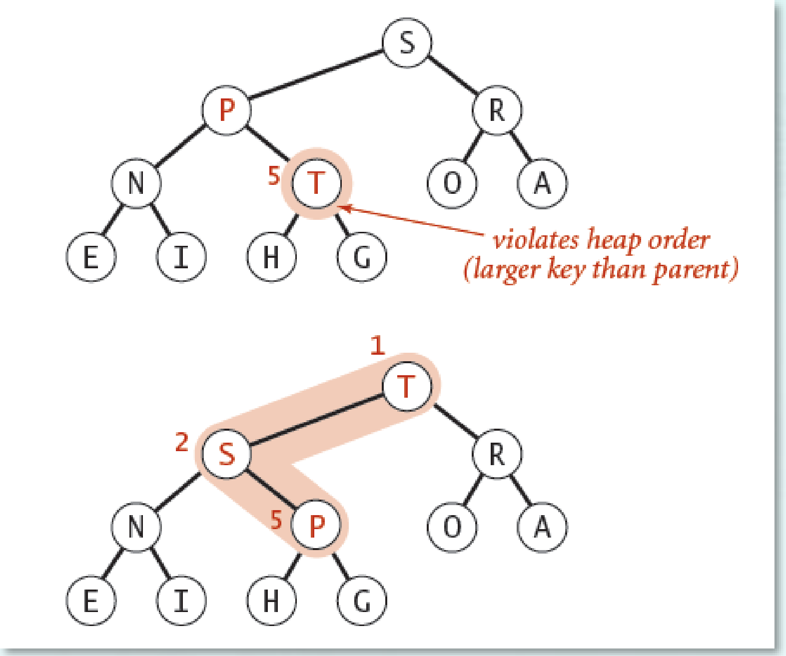
not reached at root

parent(k/2) is less its child(k)

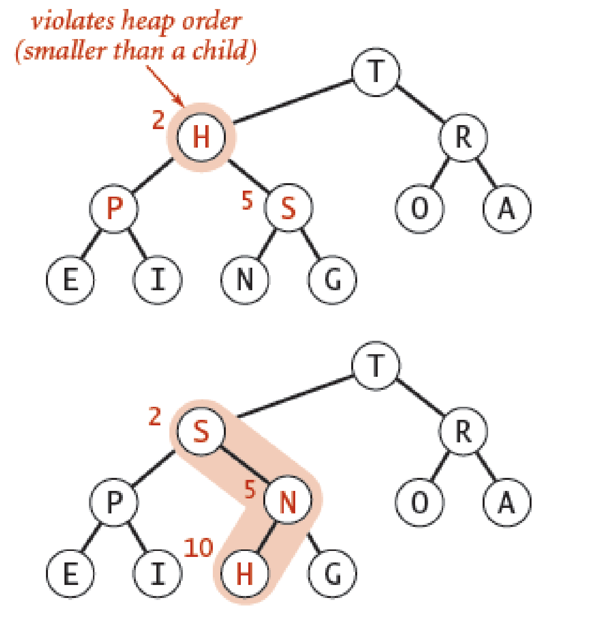
```
void swim(heap h, int k)
{
  while (k > 1 && less(h, k / 2, k))
  {
    swap(h, k / 2, k);
    k = k / 2;
  }
}
```

swap parent(k/2) and its child(k)

move up one level



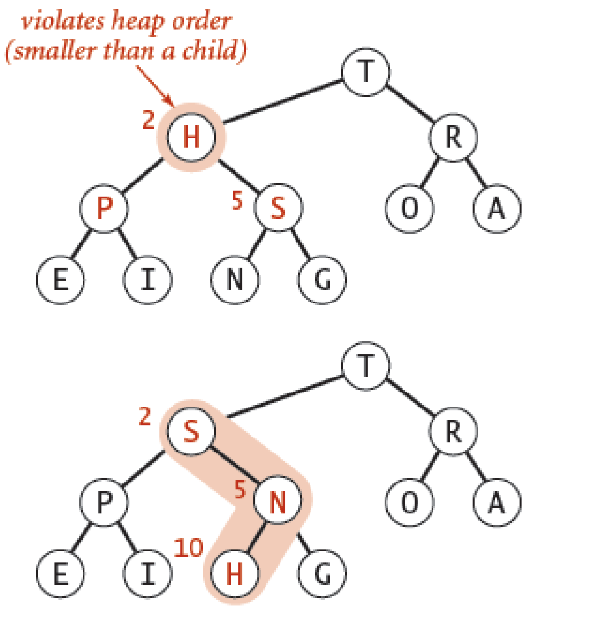
swim up
or
sink down



Top-down reheapify (sink)

Demotion in a heap: sink 둘중에 더 큰 것이 올라가야한다.

- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation: Why not smaller child?
  - Swap key in parent with key in **larger** child (of two)
  - Repeat until heap order restored.



Top-down reheapify (sink)

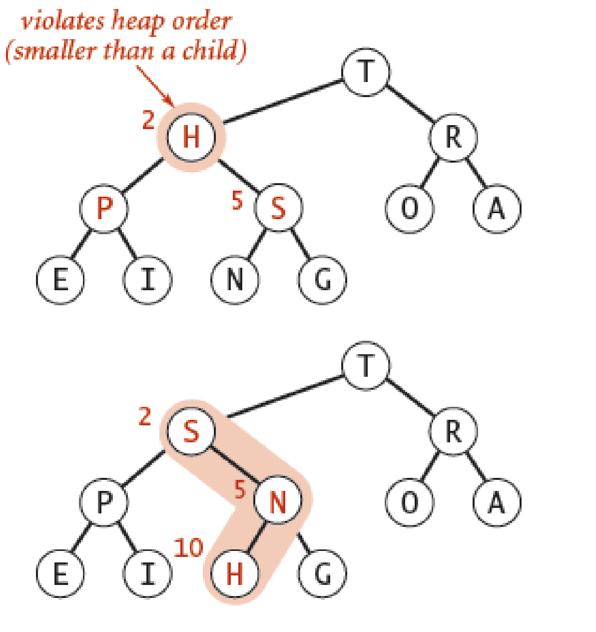
This is a maxheap example.

Demotion in a heap: sink

- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation: Why not smaller child?
  - Swap key in parent with key in **larger** child (of two)
  - Repeat until heap order restored.

```
void sink(heap h, int k)
{
   while (k's child not reached the last)
   {
      find the larger child of k, let it be j. (j = 5)

      if k's key is not less than j's key, break;
      swap k and j since k's key > j's key
      set k to the next node wh
   }
}
```

k = 2



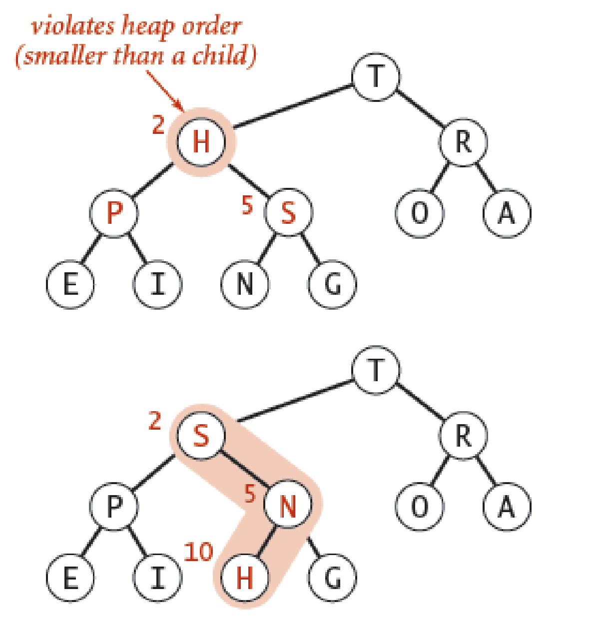
Top-down reheapify (sink)

Demotion in a heap: sink

- Parent’s key becomes **smaller** than one (or both) of its children’s.
- To eliminate the violation: Why not smaller child?
  - Swap key in parent with key in **larger** child (of two)
  - Repeat until heap order restored.

```
void sink(heap h, int k)
{
   while (k's child not reached the last)
   {
      find the larger child of k, let it be j. (j = 5)

      if k's key is not less than j's key, break;
      swap k and j since k's key > j's key
      set k to the next node which is j.
   }
}
```

k = 2

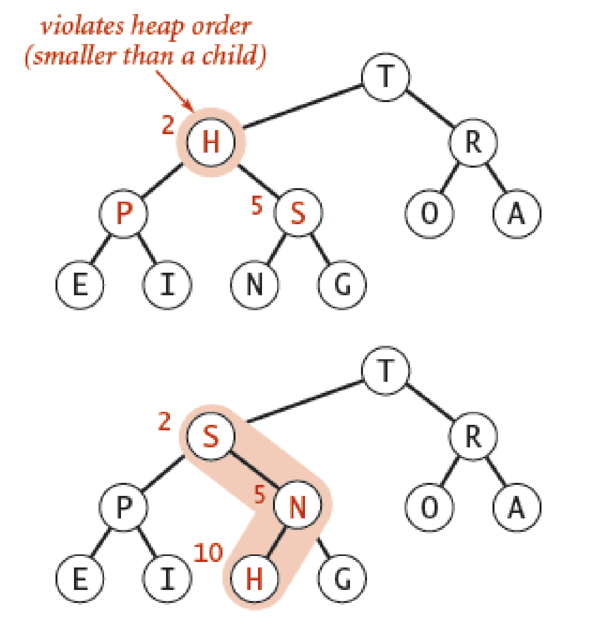


Top-down reheapify (sink)

Demotion in a heap: sink

- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation: Why not smaller child?
  - Swap key in parent with key in **larger** child
  - Repeat until heap order restored.

```
void sink(heap h, int k)
{
  while (k's child not reached the last)
  {
  find the larger child of k, let it be j. (j = 5)

}
```



Top-down reheapify (sink)

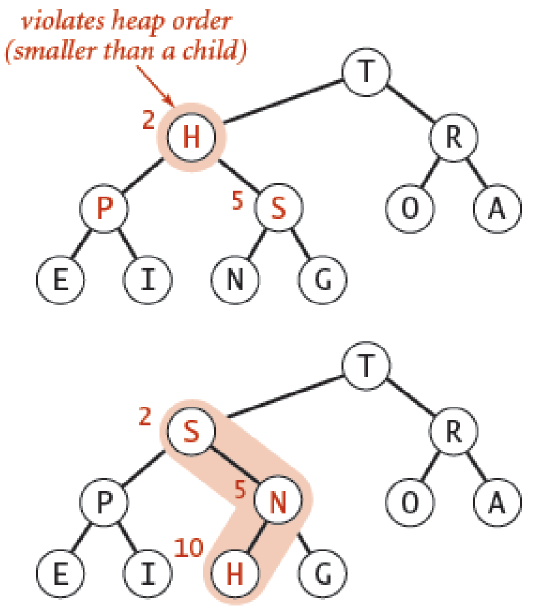
Demotion in a heap: sink

- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation:
  - Swap key in parent with key in **larger** child
  - Repeat until heap order restored.

Why not smaller child?

```
void sink(heap h, int k)
{
   while (2 * k <= h->N)
   {
     find the larger child of k, let it be j. (j = 5)

   }
}
```

children of node **k** are **2k** and **2k+1**



Top-down reheapify (sink)

Demotion in a heap: sink

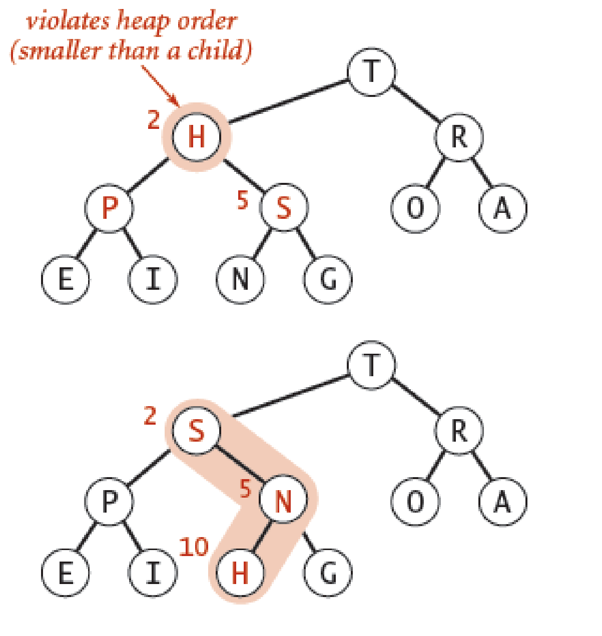
- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation: Why not smaller child?
  - Swap key in parent with key in **larger** child
  - Repeat until heap order restored.

```
void sink(heap h, int k)
{
   while (2 * k <= h->N)
   {
      int j = 2 * k;
```

children of node **k** are **2k** and **2k+1**

find the larger child of k, let it be j. (j = 5)

```
}
```



Top-down reheapify (sink)

Demotion in a heap: sink

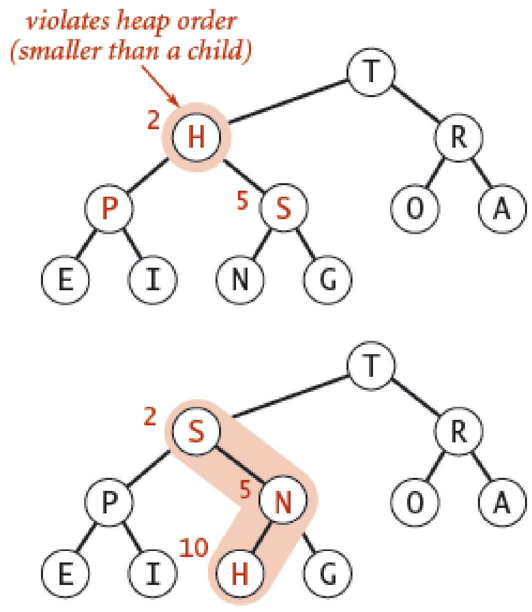
- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation:
  - Swap key in parent with key in **larger** child
  - Repeat until heap order restored.

Why not smaller child?

```
void sink(heap h, int k)
{
   while (2 * k <= h->N)
   {
      int j = 2 * k;

      if (j < h->N && less(h, j, j + 1)) j++;
```

```
}
```

children of node **k** are **2k** and **2k+1**



Top-down reheapify (sink)

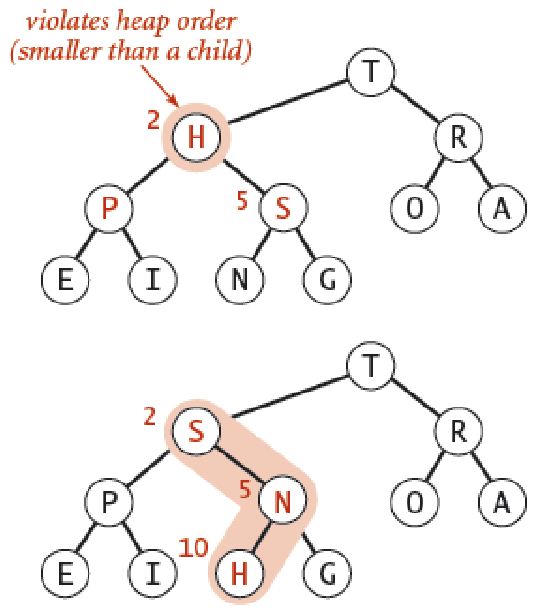
Demotion in a heap: sink

- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation: Why not smaller child?
  - Swap key in parent with key in **larger** child
  - Repeat until heap order restored.

```
void sink(heap h, int k)
{
  while (2 * k <= h->N)
  {
    int j = 2 * k;

    if (j < h->N && less(h, j, j + 1)) j++;
}
```

children of node **k** are **2k** and **2k+1**

if k's key is not less than j's key, break;
swap k and j since k's key > j's key
set k to the next node (which is j.)



Top-down reheapify (sink)

Demotion in a heap: sink

- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation:
  - Swap key in parent with key in **larger** child
  - Repeat until heap order restored.

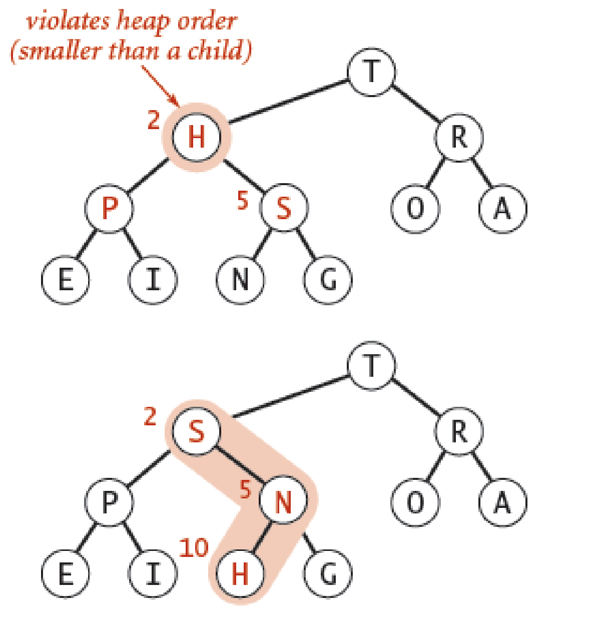
Why not smaller child?

```
void sink(heap h, int k)
{
  while (2 * k <= h->N)
  {
    int j = 2 * k;

    if (j < h->N && less(h, j, j + 1)) j++;
    if (!less(h, k, j)) break;
```

children of node **k** are **2k** and **2k+1**

swap k and j since k's key > j's key
set k to the next node (which is j.)

```
}
```



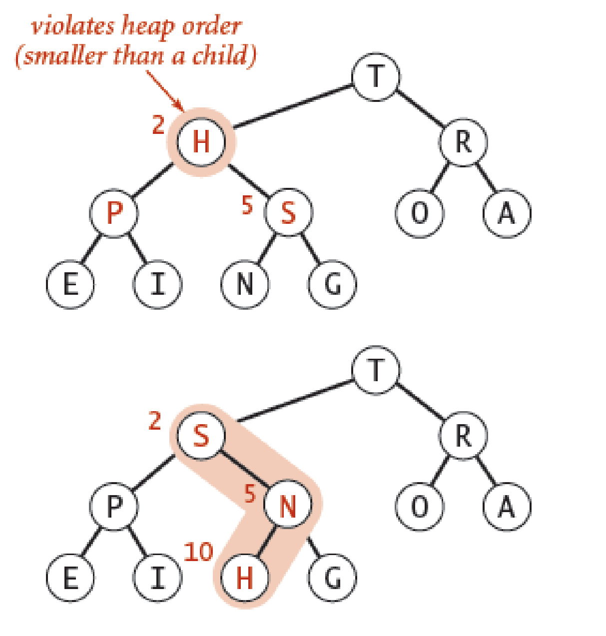
Top-down reheapify (sink)

Demotion in a heap: sink

- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation: Why not smaller child?
  - Swap key in parent with key in **larger** child
  - Repeat until heap order restored.

```
void sink(heap h, int k)
{
  while (2 * k <= h->N)
  {
    int j = 2 * k;

    if (j < h->N && less(h, j, j + 1)) j++;
    if (!less(h, k, j)) break;
    swap(h, k, j);
    set k to the next node (which is j.)
  }
}
```

children of node **k** are **2k** and **2k+1**



Top-down reheapify (sink)

Demotion in a heap: sink

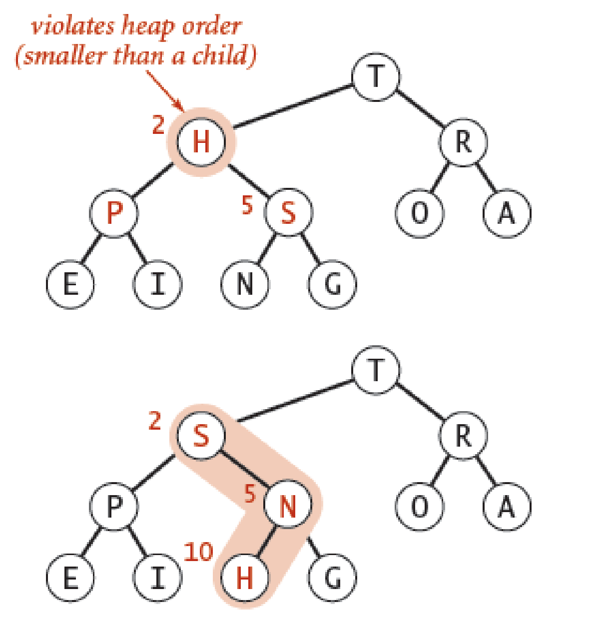
- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation:
  - Swap key in parent with key in **larger** child
  - Repeat until heap order restored.

Why not smaller child?

```
void sink(heap h, int k)
{
   while (2 * k <= h->N)
   {
      int j = 2 * k;

      if (j < h->N && less(h, j, j + 1)) j++;
      if (!less(h, k, j)) break;
      swap(h, k, j);
      k = j;
   }
}
```

children of node **k** are **2k** and **2k+1**

max heap일 때는 less
min heap일 때는 more을 사용하게 된다.
pset에서는 function pointer를 사용해서
공통으로 사용할 수 있게 하려고 한다.



Top-down reheapify (sink)

heap coding

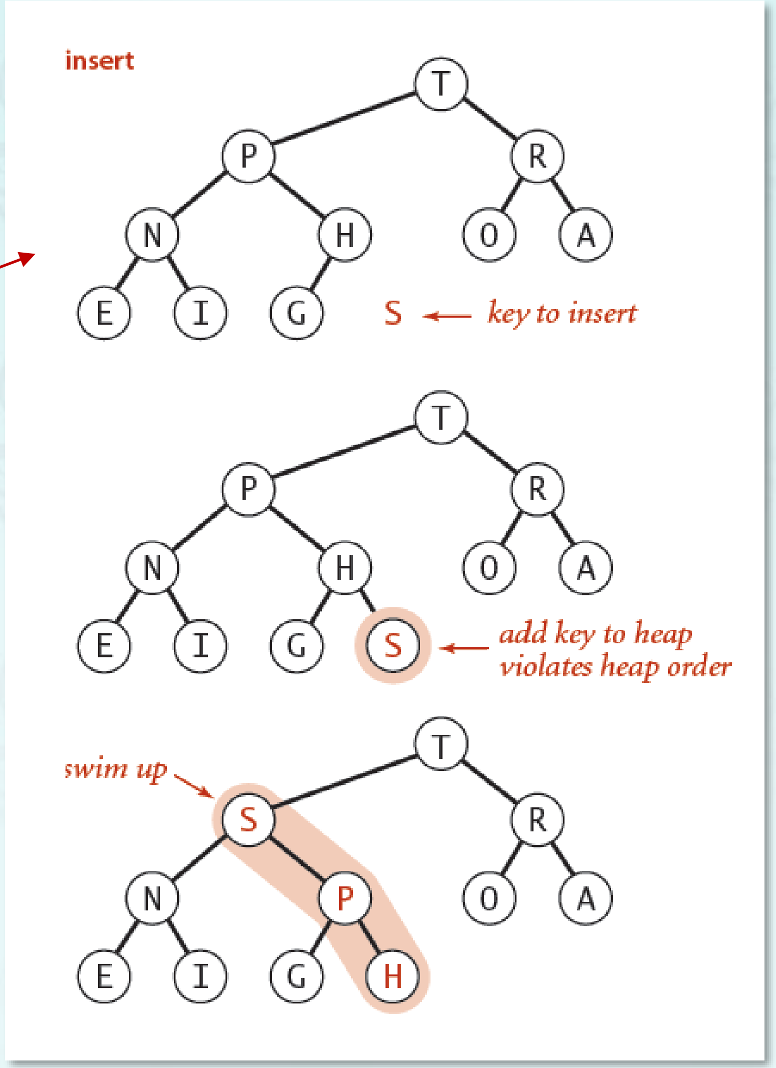
- Insert: Add node at end, then **swim** it up.
  - Cost: At most 1 + log N compares.

What is N now?

Insert

Step 1

Step 2



heap coding

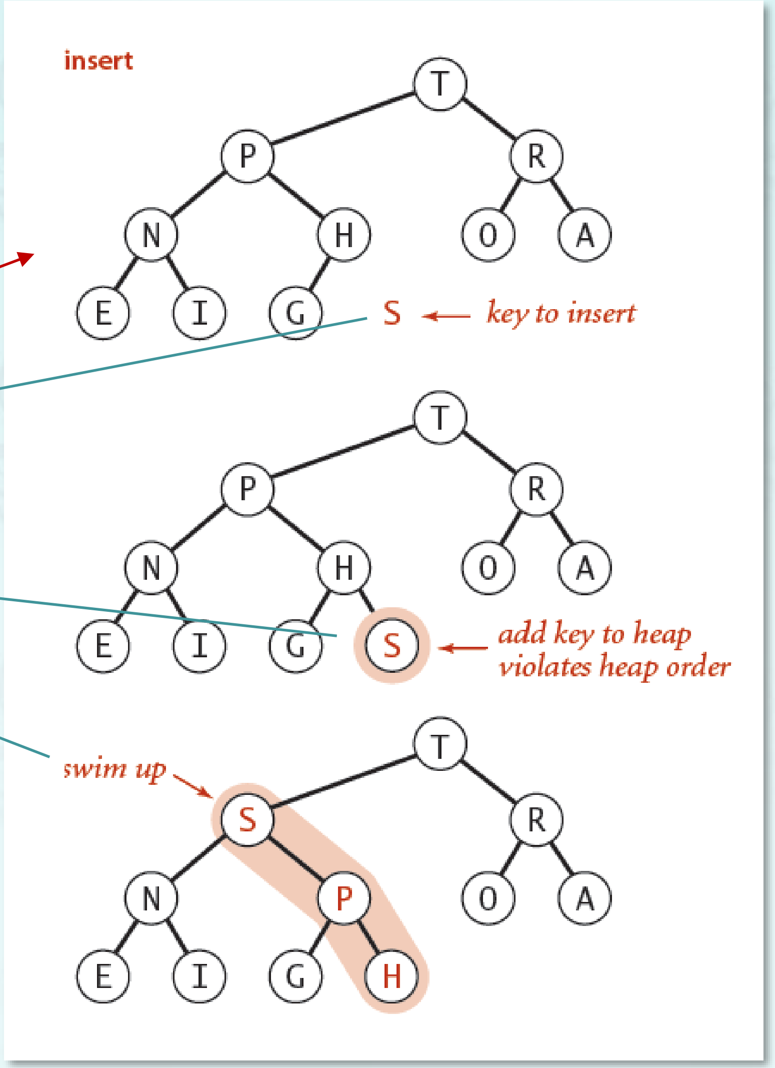
Insertion in a heap:

- Insert: Add node at end, then **swim** it up.
  - Cost: At most 1 + log N compares.

What is N now?

```
void insert(heap h, int key)
{

}
```

```
struct Heap {
  int *nodes;         // an array of nodes
  int capacity;       // array size of node or key, item
  int N;              // the number of nodes in the heap
  //
};
using heap = *Heap;
```



heap coding

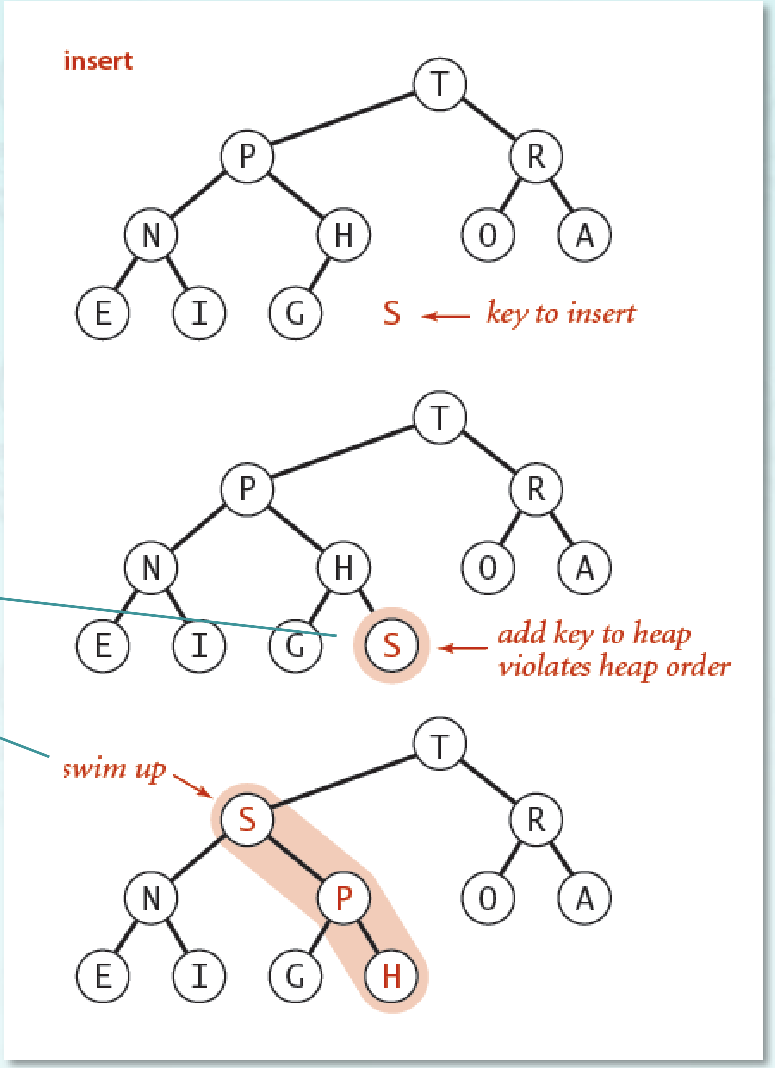
Insertion in a heap:

- Insert: Add node at end, then **swim** it up.
  - Cost: At most 1 + log N compares.

```
void insert(heap h, int key)
{
  h->nodes[++h->N] = key;
  
}
```

```
struct Heap {
  int *nodes;        // an array of nodes
  int capacity;      // array size of node or key, item
  int N;             // the number of nodes in the heap
  //
};
using heap = *Heap;
```

```
void swim(heap h, int k)
void sink(heap h, int k)
```



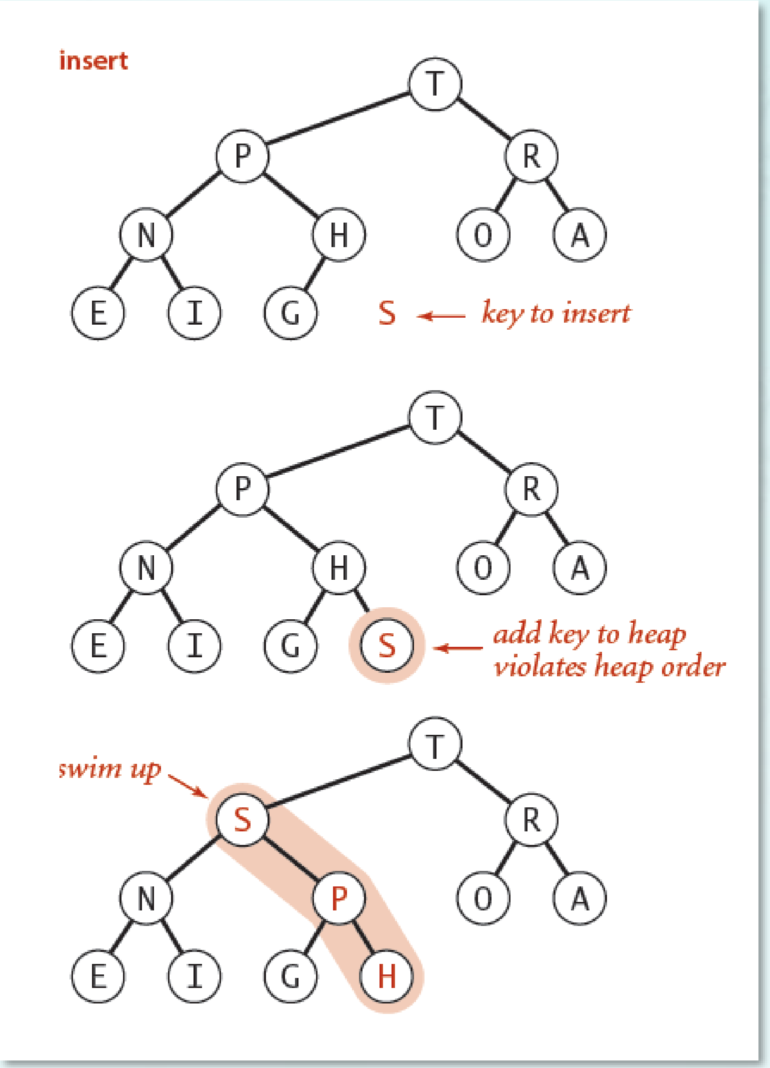
heap coding

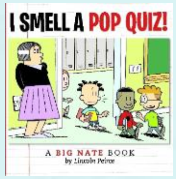
Insertion in a heap:

- Insert: Add node at end, then **swim** it up.
  - Cost: At most 1 + log N compares.

```
void insert(heap h, int key)
{
   h->nodes[++h->N] = key;
   swim(h, h->N);
}
```

```
struct Heap {
  int *nodes;         // an array of nodes
  int capacity;       // array size of node or key, item
  int N;              // the number of nodes in the heap
  //
};
using heap = *Heap;
```



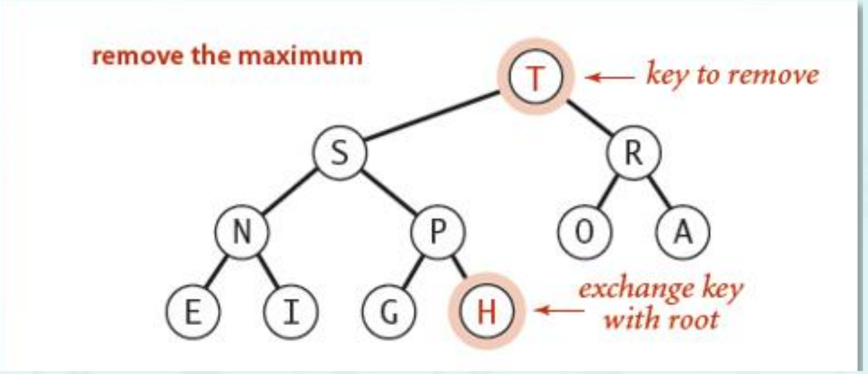


heap coding

(1) Delete the root (max or min) in a heap:

(2) How many times do comparisons occur for n nodes ? (select one):

n, 2n, n^2, 2 log n, n log n, log n




```
void delete(heap h) {


}
```

← **2 or 3 lines of code**

```
void swim(heap h, int k)
void sink(heap h, int k)
```

```
bool less(heap h, int p, int c)
```

```
void swap(heap h, int p, int c)
```

heap coding

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most 2 log N compares.

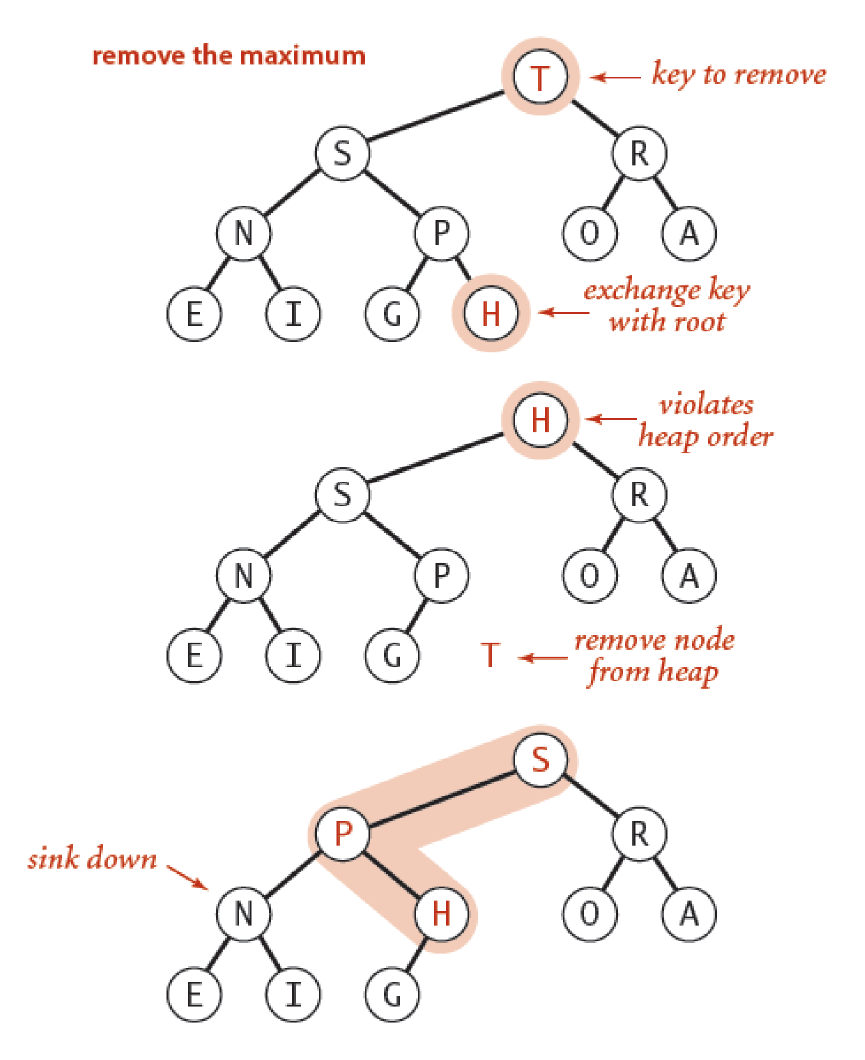
```
void delete(heap h) {


}
```

```
    swap(h, ..., ... );
    sink(h, ...);
}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
```

```
bool less(heap h, int p, int c)
```

```
void swap(heap h, int p, int c)
```



heap coding

Delete the root (max or min) in a heap:

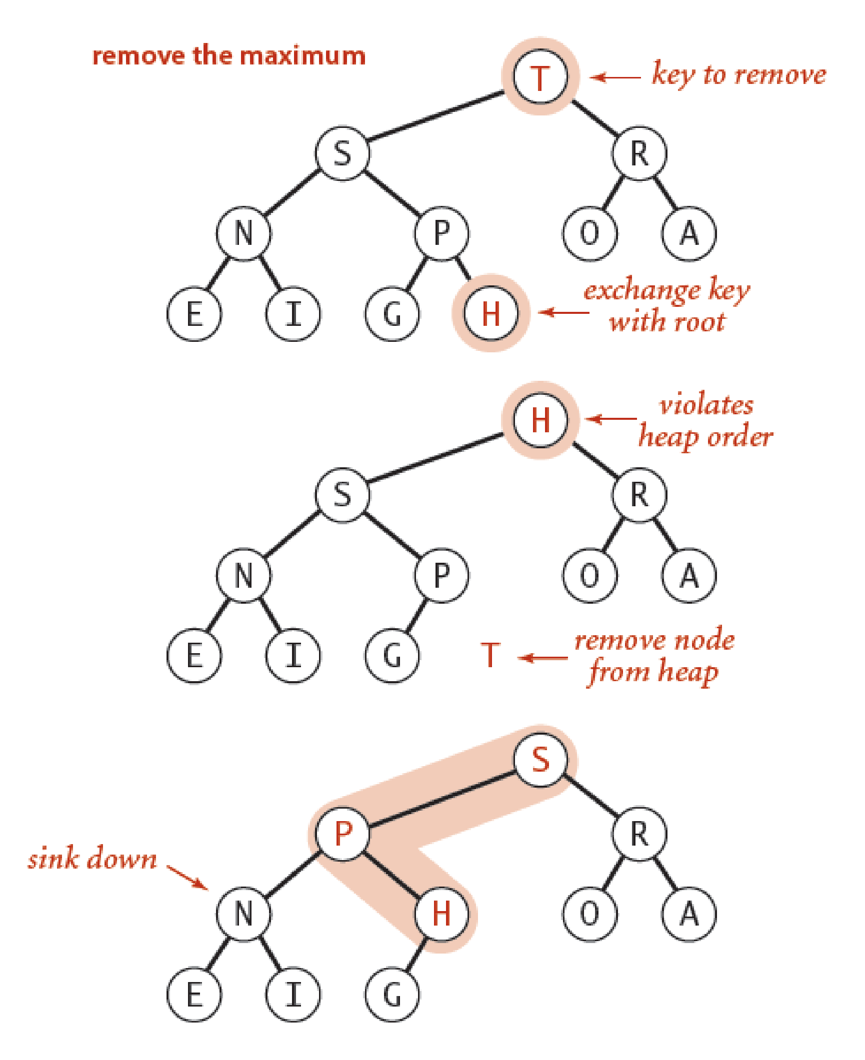
- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most 2 log N compares.

```
void delete(heap h) {

}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
```

```
bool less(heap h, int p, int c)
```

```
void swap(heap h, int p, int c)
```



heap coding

Delete the root (max or min) in a heap:

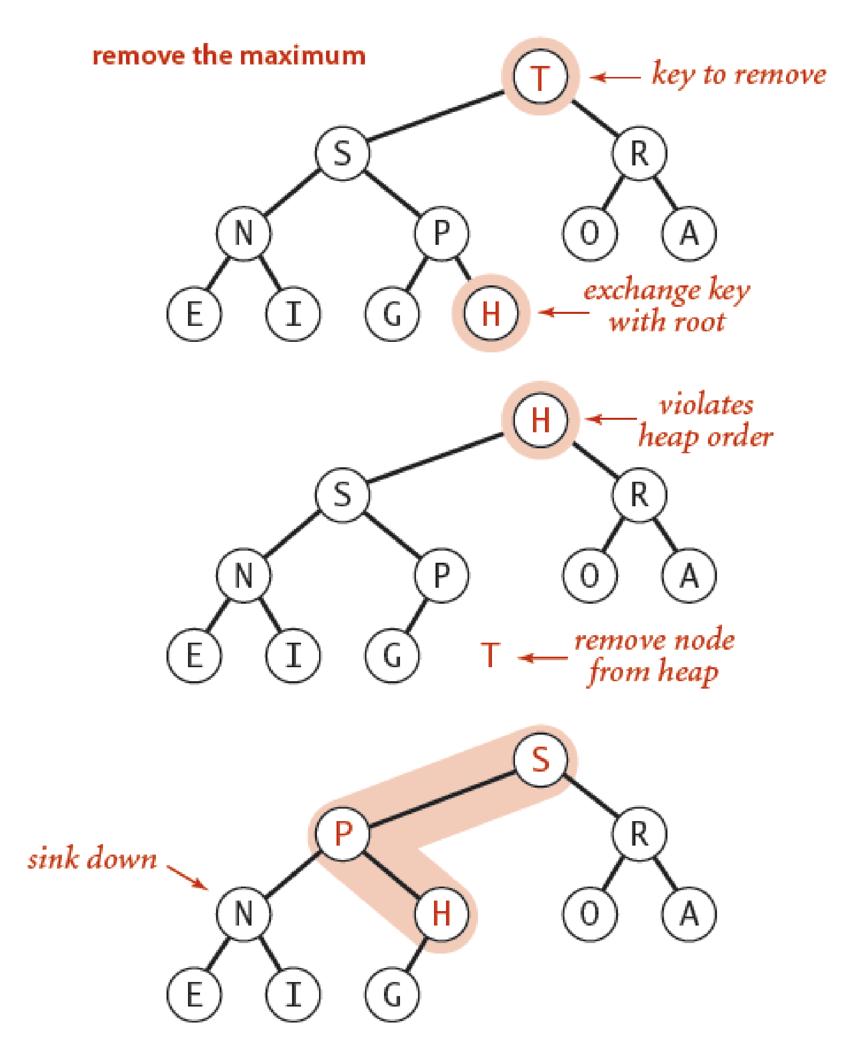
- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most 2 log N compares.

```
void delete(heap h) {
    swap(h, 1, h->N--);

}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
```

```
bool less(heap h, int p, int c)
```

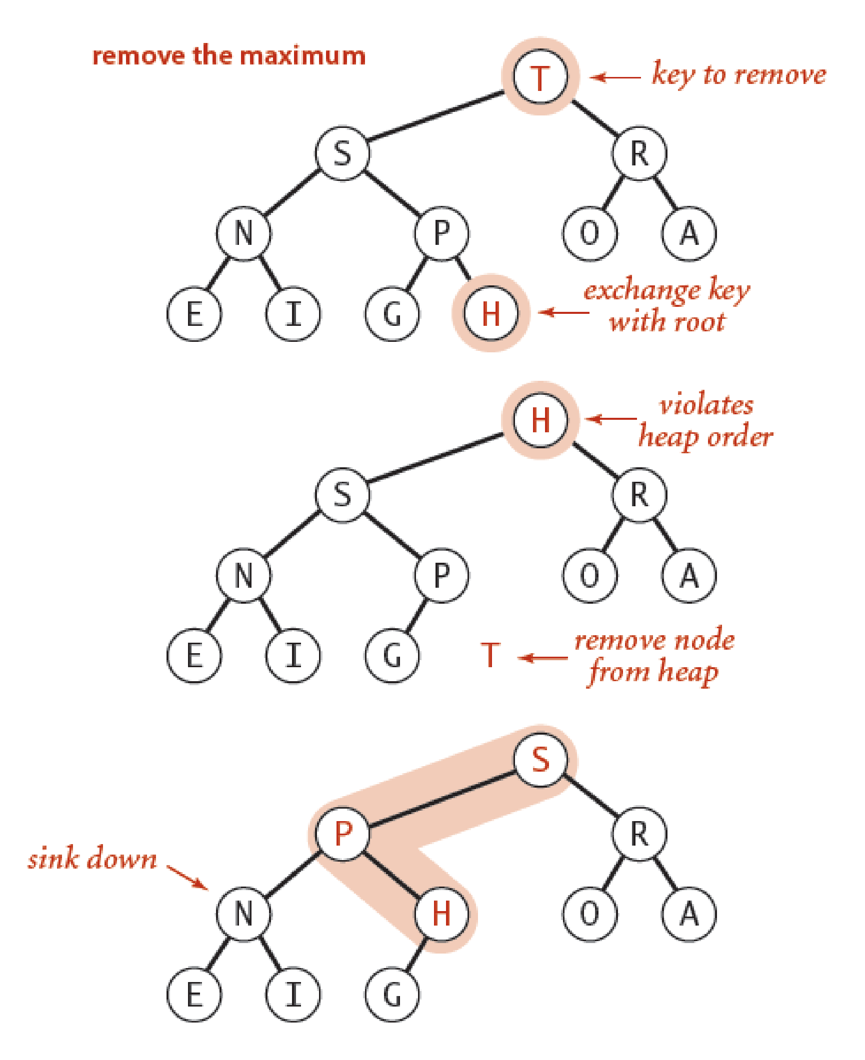
```
void swap(heap h, int p, int c)
```



heap coding

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most 2 log N compares.




```
void delete(heap h) {
    swap(h, 1, h->N--);
    sink(h, 1);
}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
```

```
bool less(heap h, int p, int c)
```

```
void swap(heap h, int p, int c)
```

Heap Coding

```
void clear(heap hp);                    // deallocate heap
int size(heap hp);                      // return nodes in heap currently
int level(int n);                       // return level based on num of nodes
int capacity(heap hp);                   // return its capacity (array size)
int reserve(heap hp, int capa);         // reserve the array size (= capacity)
int full(heap hp);                      // return true/false
int empty(heap hp);                     // return true/false
void grow(heap hp, int key);            // add a new key
void trim(heap hp);                     // delete a queue
int heapify(heap hp);                   // convert a complete BT into a heap

// helper functions to support grow/trim functions
int less(heap hp, int i, int j);        // used in max heap
int more(heap hp, int i, int j);        // used in min heap
void swim(heap hp, int k);              // bubble up
void sink(heap hp, int k);              // tickle down
// helper functions to check heap invariant
int heapOrdered(heap hp);              // is heap[1..N] a heap?
```

Heap Coding

```
// return the number of items in heap
int size(heap hp) {
    return heap->N;
}
```

```
// Is this heap empty?
int empty(heap hp) {
    return (heap->N == 0) ? true : false;
}
```

```
// Is this heap full?
int full(heap hp) {
    return (heap->N == heap->capacity - 1) ? true : false;
}
```

Heap Coding

```
int less(heap hp, int i, int j) {
    return heap->nodes[i] < heap->nodes[j];
}
```

```
void swap(heap hp, int i, int j) {
    int t = heap->nodes[i];
    heap->nodes[i] = heap->nodes[j];
    heap->nodes[j] = t;
}
```

```
void swim(heap hp, int k) {

}
```

```
void sink(heap hp, int k) {

}
```

Heap Coding

```
void grow(heap hp, int key) {

    cout << "YOUR CODE HERE\n";

    // add key @ ++heap->N

    // swim up @ heap->N

}
```

```
void trim(heap hp) {
    if (empty(heap)) return;

    cout << "YOUR CODE HERE\n";

}
```

Heap Coding

newCBT()	with a given array, instantiate a new complete binary tree its result is neither maxheap nor minheap.
heapify()	make a complete binary tree into a max/minheap
heapsort()	use max/min-heap to sort elements in heap
heapprint()	build a binary tree from heap/CBT for display purpose only

newCBT() – convert an int array to CBT

```
// instantiates a CBT with given data and its size.
heap newCBT(int *a, int n) {
    int capa = ?

    heap p = new Heap{ capa };

    p->N = n;
    for (int i = 0; i < n; i++)
        p->nodes[i + 1] = a[i];
    return p;
}
```

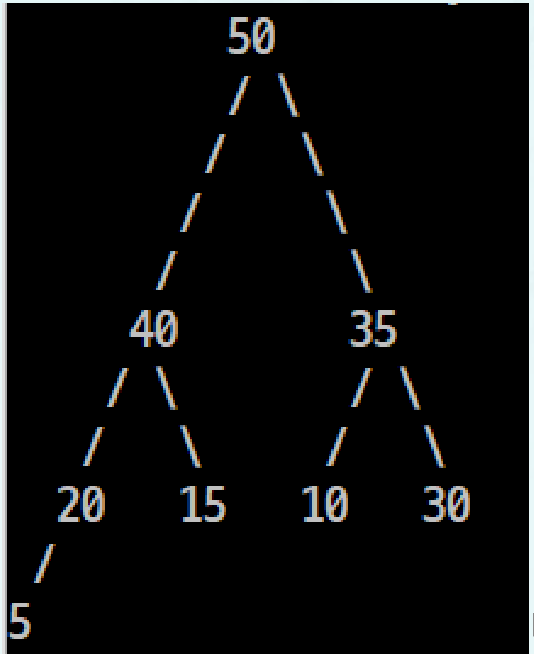
Priority queue

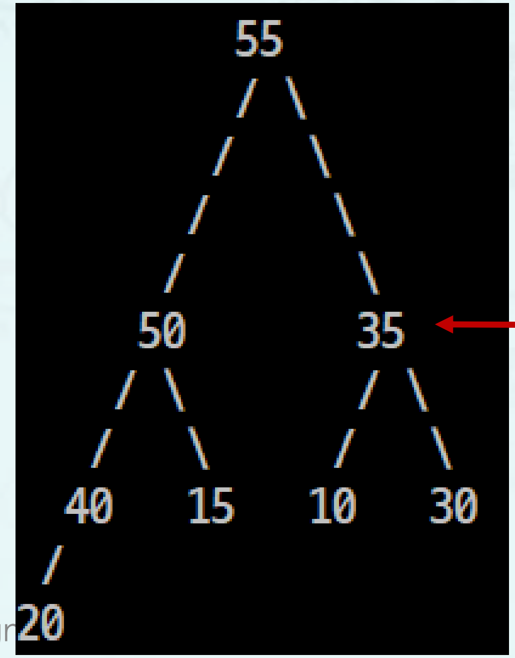
It is like a regular queue or stack data structure, but where additionally **each element has a "priority" associated with it.** In a priority queue, an element with high priority is served before an element with low priority.

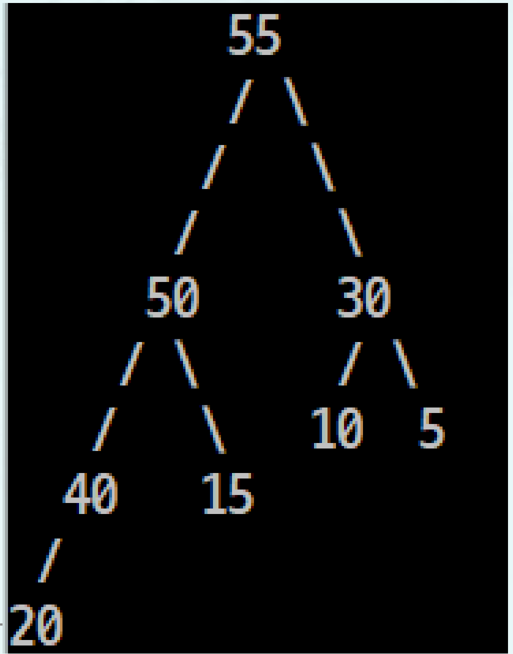
- **"trim"** removes the root which has the highest priority
- **"priority queue"** lets user modify the priority (or value) of an element) and move it to the position based on its new priority in the queue.

For example:

- If you **change 5 to 55**, it will go up to the root and 20 is placed at the bottom.
- If you **change 35 to 5**, 30 will go up where 35 is, then 5 goes down to the right corner.







grow() - inserts a new key to the max-heap or min-heap.

```
growN(heap p, int key)
1. if full(p), invoke reserve() to double the size of nodes[]. Use p->capacity * 2.
2. add the key to nodes[]. The index of nodes[] must be ++p->N.
3. swim up to maintain heap invariant.
```

```
void grow(heap p, int key) {

  if (full(p)) ...
  p->nodes ...
  swim...

  return;
}
```


Prof. Youngsup Kim, idebtor@gmail.com, Data Structures, CSEE Dept., Handong Global University

growN() & trimN()

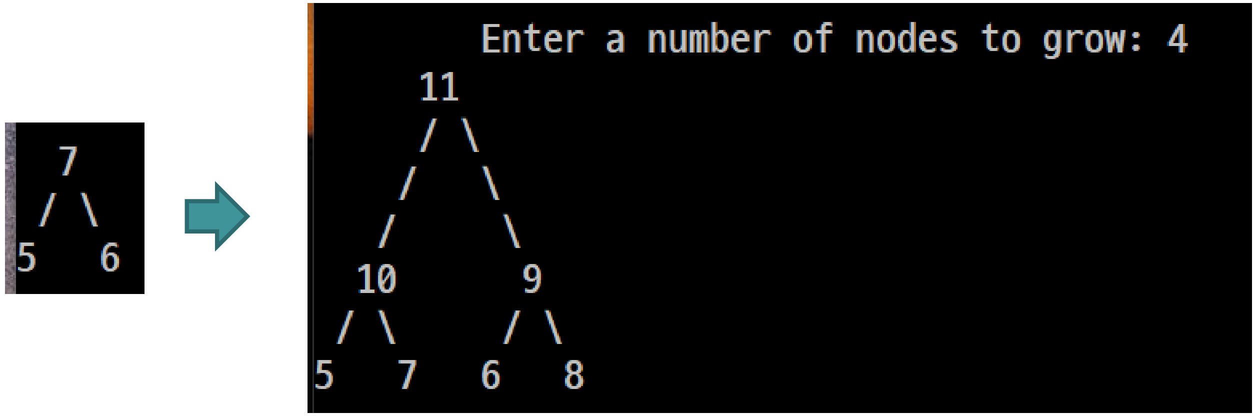
```
growN()
1. Find the max key(max) in heap or CBT.
2. Set a function pointer to the function to insert a node.
3. Allocate a Key type array such as keys to store random keys.
4. Invoke randomN() function to generate keys in the range [(max + 1)..(max + count)]
5. Invoke the function to insert keys in keys[], but one key at a time.
6. Print the heap if DEBUG is defined whenever a node is inserted.
7. Don't forget freeing the array of keys you allocated in Step 3.
```

```
void growN(heap p, int count, bool heapOrdered) {
  int max = empty(p) ? 1 : maximum(p) + 1;
  void(*insertFunc)(heap h, int key) = heapOrdered ? grow : growCBT;




                                             // insertFunc(p, keys[i]);




}
```

growN() & trimN()

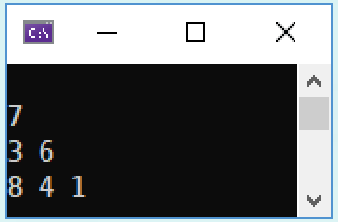
```
growN()
1. Find the max key(max) in heap or CBT.
2. Set a function pointer to the function to insert a node.
3. Allocate a Key type array such as keys to store random keys.
4. Invoke randomN() function to generate keys in the range [(max + 1)..(max + count)]
5. Invoke the function to insert keys in keys[], but one key at a time.
6. Print the heap if DEBUG is defined whenever a node is inserted.
7. Don't forget freeing the array of keys you allocated in Step 3.
```

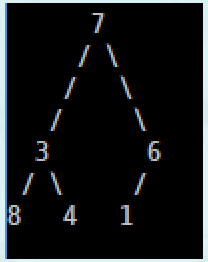
```
void growN(heap p, int count, bool heapOrdered) {
  int max = empty(p) ? 1 : maximum(p) + 1;
  void(*insertFunc)(heap h, int key) = heapOrdered ? grow : growCBT;
```




```
}
```

Heapprint(): build a tree from CBT






```
// print a heap using treeprint() - must build a tree to call treeprint()
void heapprint(heap p) {
        if (empty(p)) return;
#if 0
        tree root = buildBT(p->nodes, 1, size(p));   // using recursion
#else
        tree root = buildBT(p);                      // using queue
#endif
        treeprint(root);
        clear(root);
}
```

heapprint() – build a binary tree from heap/CBT using recursion

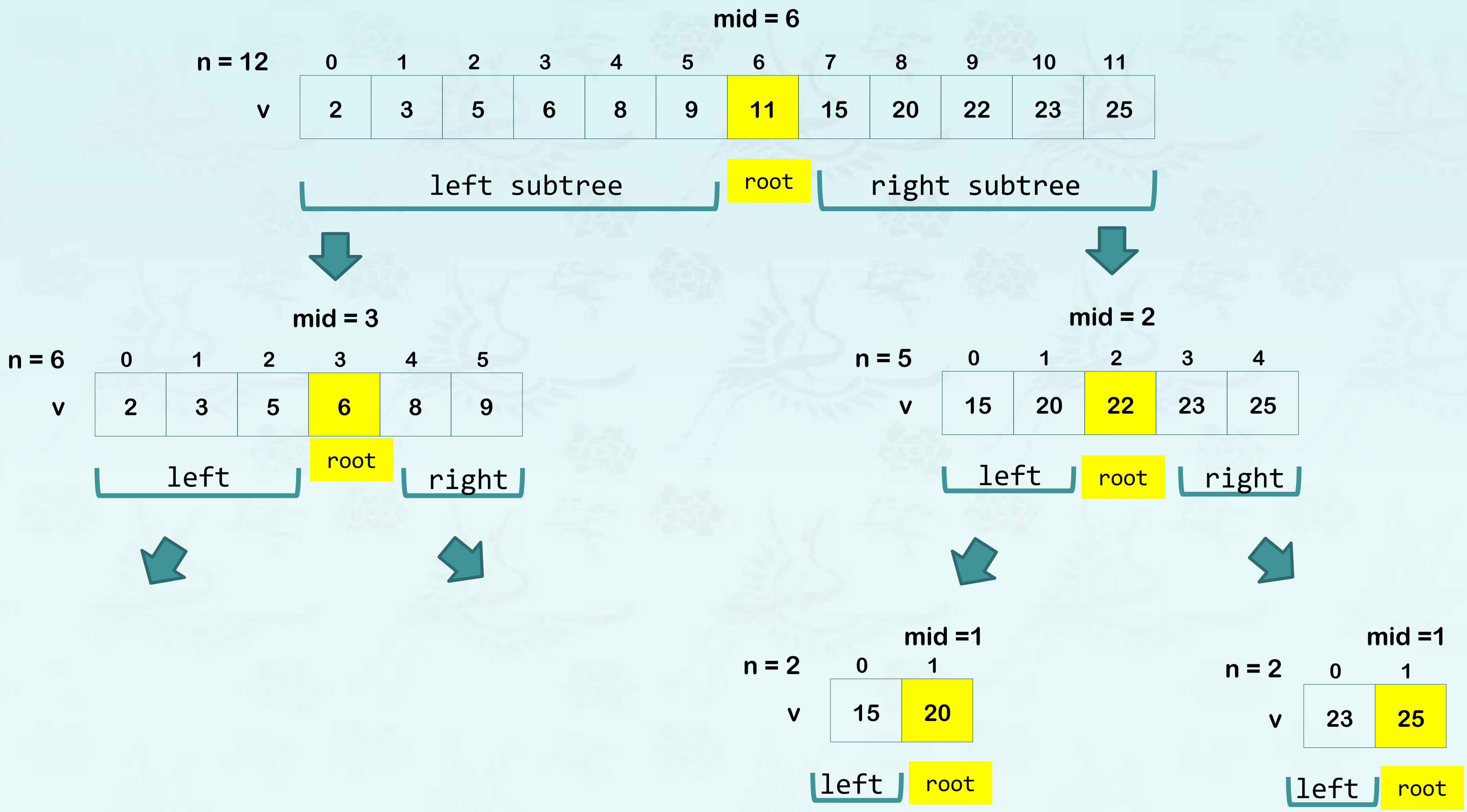
Building AVL tree from BST in O(n) – Review

mid = 6

n = 12	0	1	2	3	4	5	6	7	8	9	10	11
v	2	3	5	6	8	9	11	15	20	22	23	25

left subtree — root — right subtree

Building AVL tree from BST in O(n) – Review



Building AVL tree from BST in O(n) – Review

```
// rebuilds an AVL tree with a list of keys sorted.
// v – an array of keys sorted, n – the array size

tree buildAVL(int* v, int n) {
  if (n <= 0) return nullptr;

  int mid = n / 2;
  tree root = new TreeNode(v[mid]);
  root->left  = buildAVL(___________________________);
  root->right = buildAVL(___________________________);
  return root;
}
```

```
tree rebalanceTree(tree root) {
  if (root == nullptr) return nullptr;
  vector<int> v;        // get keys sorted
  inorder(root, v);     // O(n)
  clear(root);

  tree new_root = buildAVL(v.data(), v.size()); // O(n)
  return new_root;
}
```

heapprint() – build a binary tree from heap/CBT using recursion

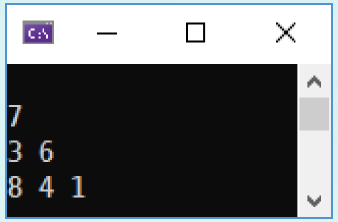
Create a recursive function that creates a binary tree from an int array. This function takes an int array, starting index, and size of the array and returns the root as shown below:

`tree buildBT(int *nodes, int i, int n) {`

1. If **`i > n`**, return nulltptr – terminate condition
2. Create the **tree (root) node** with **nodes[i]**).
   A. Invoke buildBT() for all its left children (or **`i * 2`**).
      Set its return to the left child of the root.
   B. Invoke buildBT() for all its right children (or **`i * 2 + 1`**).
      Set its return to the right child of the root.
3. return root

}

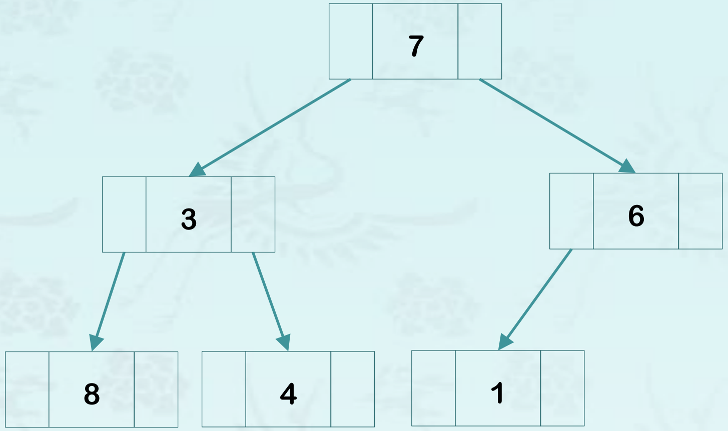
heapprint() – build a binary tree from heap/CBT using recursion

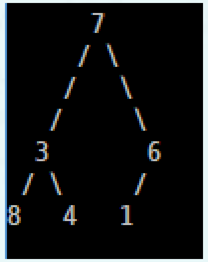


0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

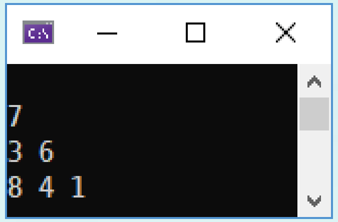
hp->nodes[]   hp->N

```
tree buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = ___________________________
  root->left  = buildBT_____________________
  root->right = buildBT_____________________
  return  root;
}
```





heapprint() – build a binary tree from heap/CBT using recursion



0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

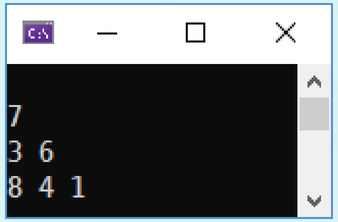
hp->nodes[]   hp->N

```
tree buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root =
  root->left  =
  root->right =
  return  root;
}
```

```
void heapprint(heap p) {
  if (empty(p)) return;
  tree root = buildBT(p->nodes, 1, size(p));
  treeprint(root);
}
```

```
tree buildBT(*nodes, i=1, n=6) { }
```

heapprint() – build a binary tree from heap/CBT using recursion



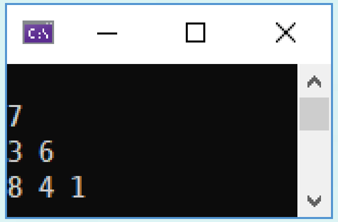
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[] hp->N

```
tree buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root =
  root->left  =
  root->right =
  return  root;
}
```

```
tree buildBT(*nodes, i=1, n=6) { }
```

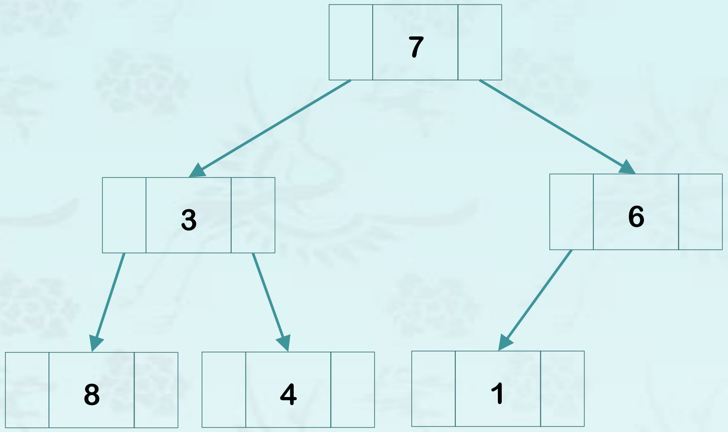
heapprint() – build a binary tree from heap/CBT using recursion

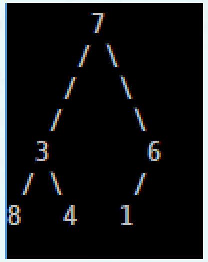


0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

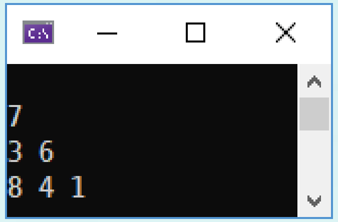
hp->nodes[] hp->N

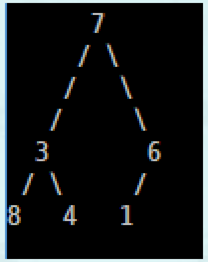
```
tree buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root =
  root->left  =
  root->right =
  return  root;
}
```





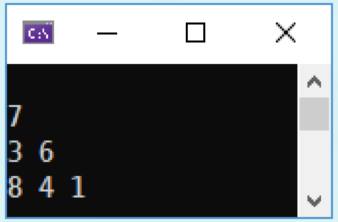
Heapprint(): build a tree from CBT

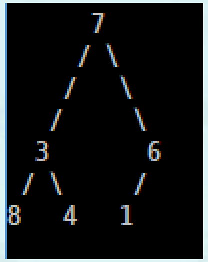





```
// print a heap using treeprint() - must build a tree to call treeprint()
void heapprint(heap p) {
        if (empty(p)) return;

#if 0
        tree root = buildBT(p->nodes, 1, size(p));   // using recursion
#else
        tree root = buildBT(p);                      // using queue
#endif

        treeprint(root);
        clear(root);
}
```

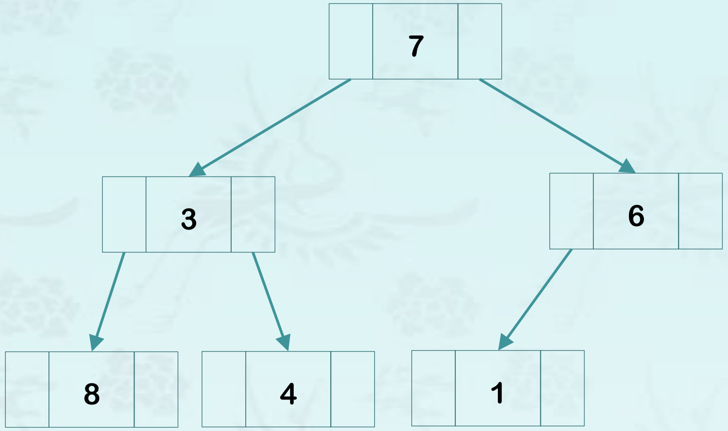
heapprint() – build a binary tree from heap/CBT using **queue**



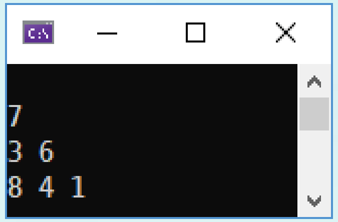


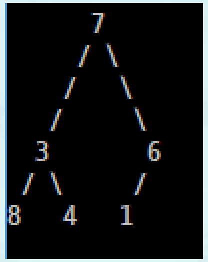
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

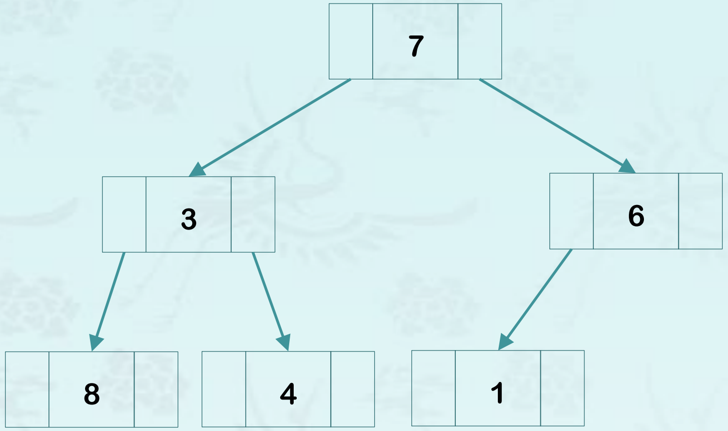
`hp->nodes[]` `hp->N`



heapprint() – build a binary tree from heap/CBT using queue







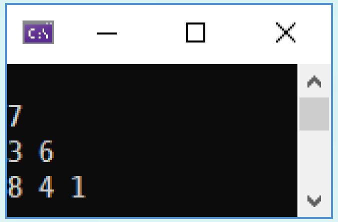
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

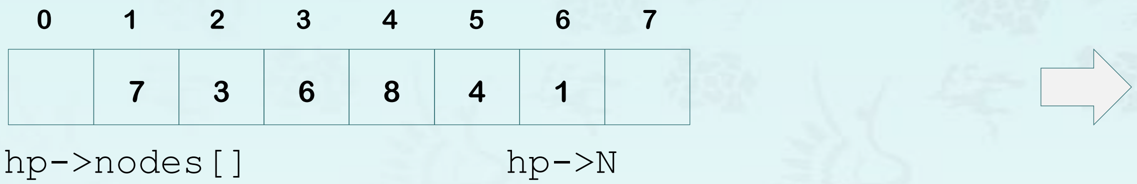
hp->nodes[]   hp->N

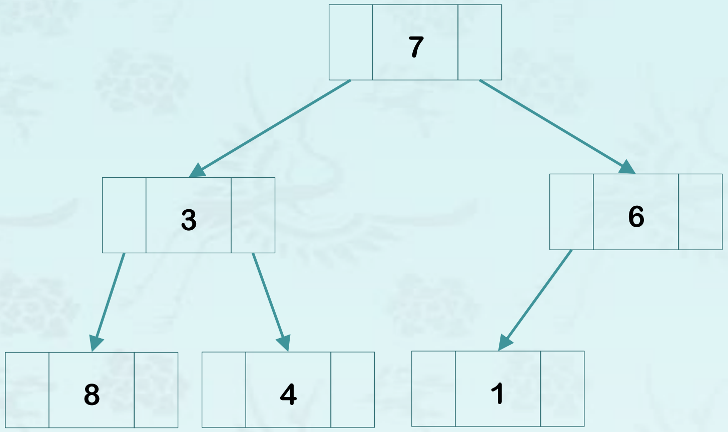
```
struct Heap {
  int *nodes;
  int capacity;
  int N;
  bool (*comp)(Heap*, int, int);
  Heap(int capa = 2) {
    capacity = capa;
    nodes = new int[capacity];
    N = 0;
    comp = nullptr;
  };
  ~Heap() {};
};
using heap = Heap*;
```

```
struct TreeNode {
  int key;
  TreeNode  *left;
  TreeNode  *right;
  TreeNode(int k, TreeNode* l, TreeNode* r) {
           key = k;   left = l;  right = r;
  }
  TreeNode(int k) : key(k), left(nullptr), right(nullptr) {}
  ~TreeNode() {}
};
using tree = TreeNode*;
```

heapprint() – build a binary tree from heap/CBT using queue

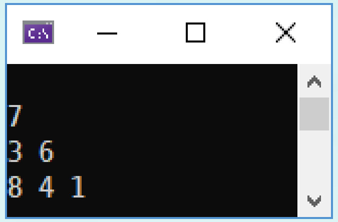


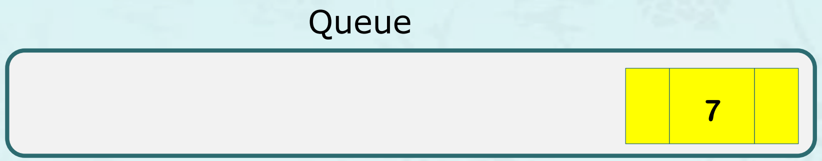




1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
         set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
         set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue



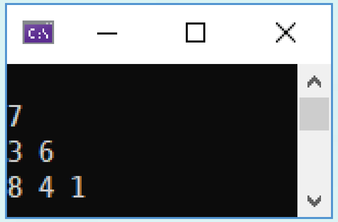


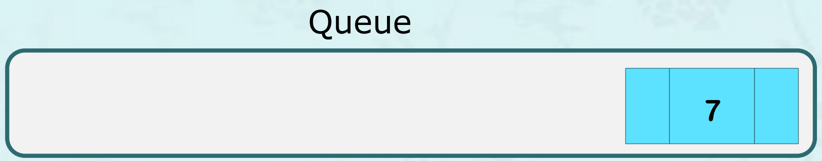
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

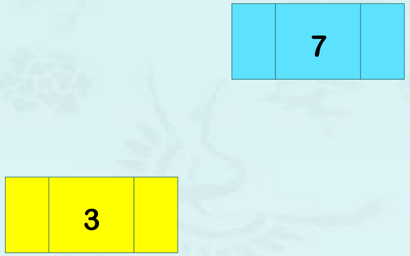
hp->nodes[] hp->N

1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue





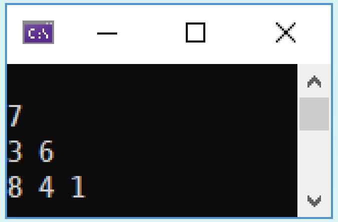


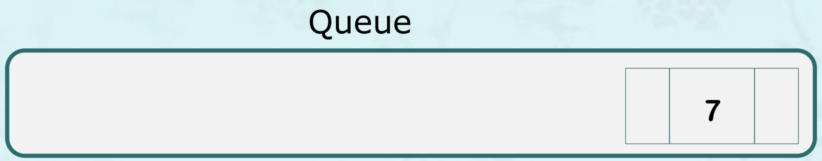
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

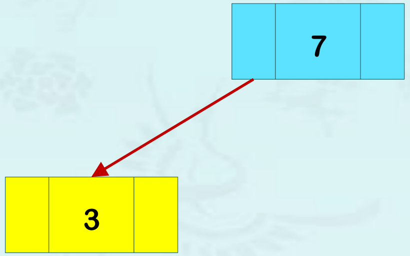
`hp->nodes[]` `hp->N`

1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue





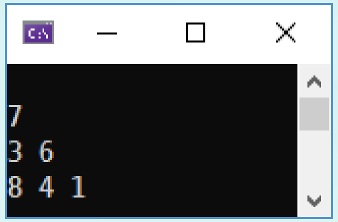


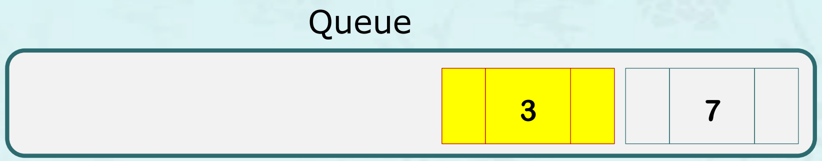
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

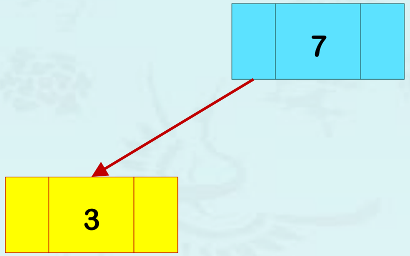
hp->nodes[]   hp->N

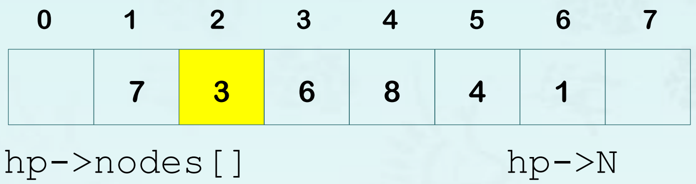
1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
   set the new node to the left of the tree node.
   else if the right of this tree node doesn't exist,
   set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue



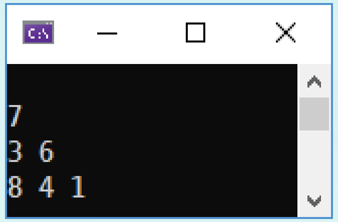


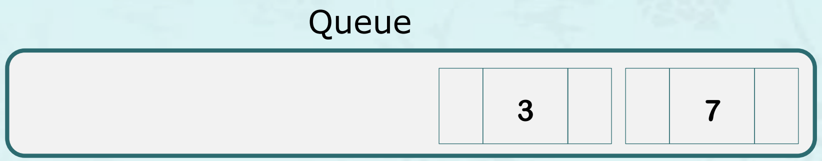


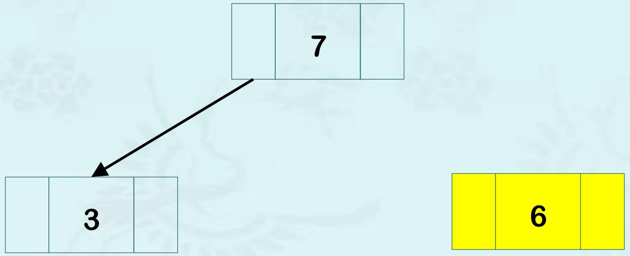


1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   - A. Make a **new node from nodes[].**
   - B. Get a **tree node** in the queue.
   - C. If the left of the tree node doesn't exist,
     set the new node to the left of the tree node.
     else if the right of this tree node doesn't exist,
     set the new node to the right of the tree node.
   - D. If this tree node is full, pop (or dequeu) it.
   - **E. enqueue the new node (to add children later if any).**
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue





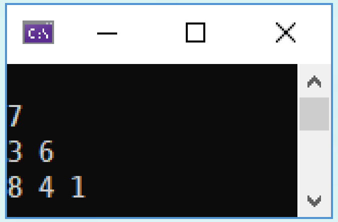


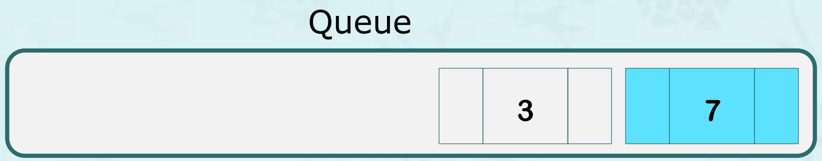
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

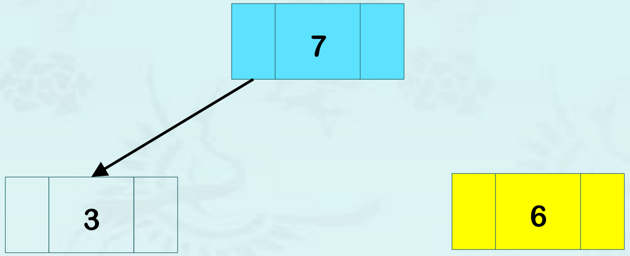
hp->nodes[]   hp->N

1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue





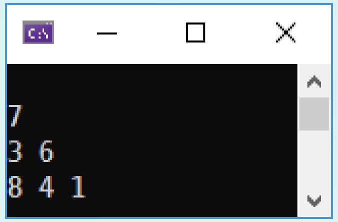


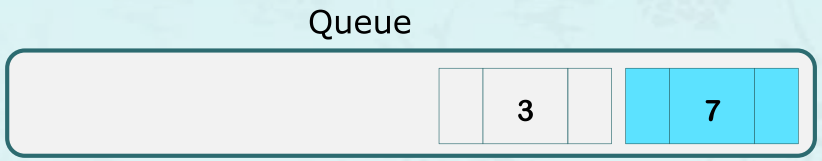
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

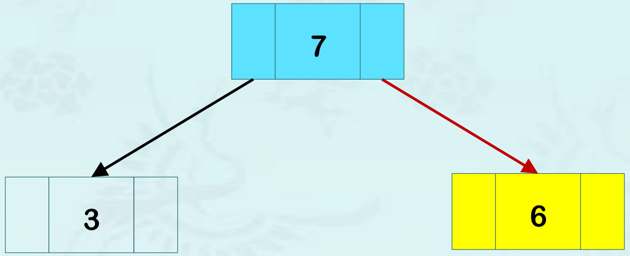
hp->nodes[]　　　　　hp->N

1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue





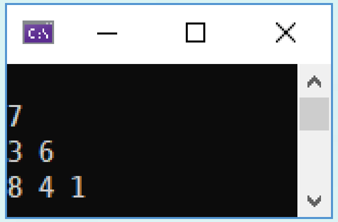


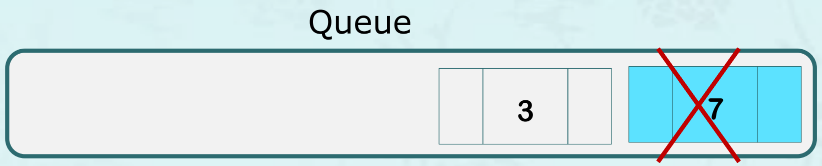
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

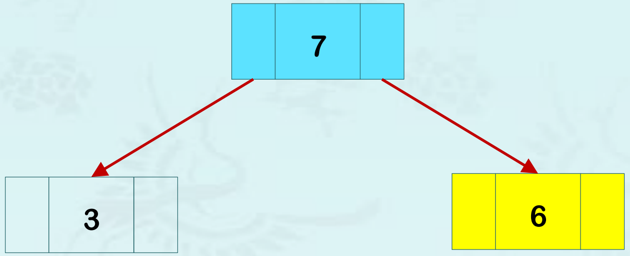
hp->nodes[] hp->N

1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue





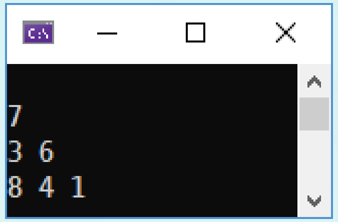


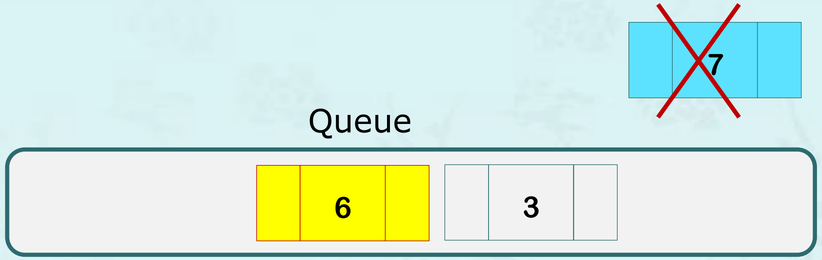
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

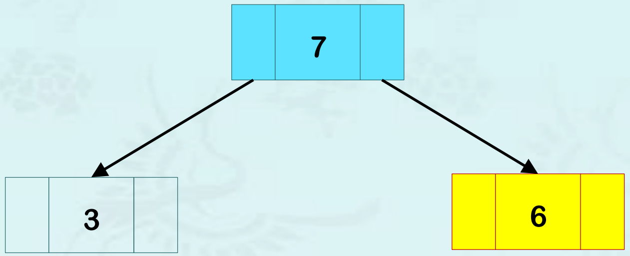
`hp->nodes[]` `hp->N`

1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   - A. Make a **new node from nodes[].**
   - B. Get a **tree node** in the queue.
   - C. If the left of the tree node doesn't exist,
     set the new node to the left of the tree node.
     else if the right of this tree node doesn't exist,
     set the new node to the right of the tree node.
   - **D. If this tree node is full, pop (or dequeu) it.**
   - E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue





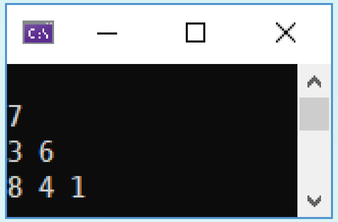


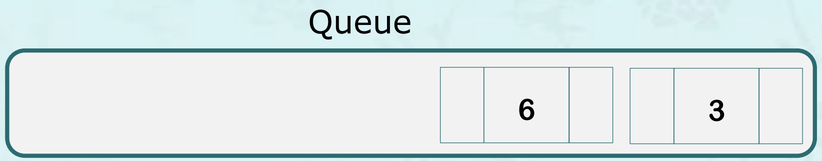
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

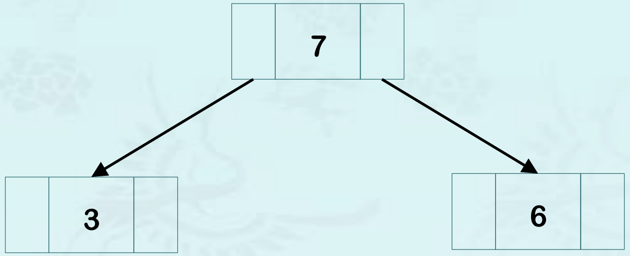
hp->nodes[] hp->N

1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue

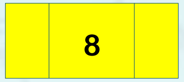






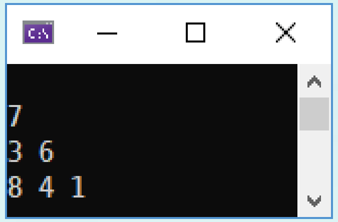
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

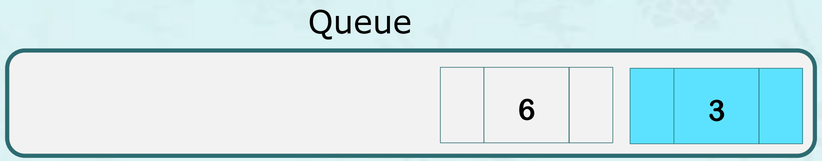
hp->nodes[] hp->N

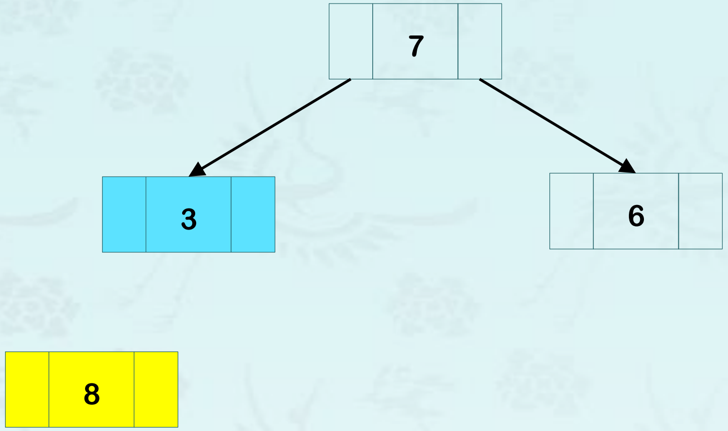


1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue





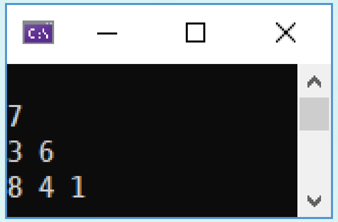


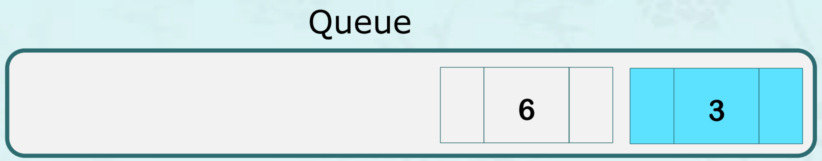
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

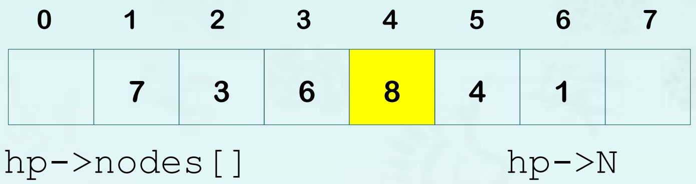
`hp->nodes[]` `hp->N`

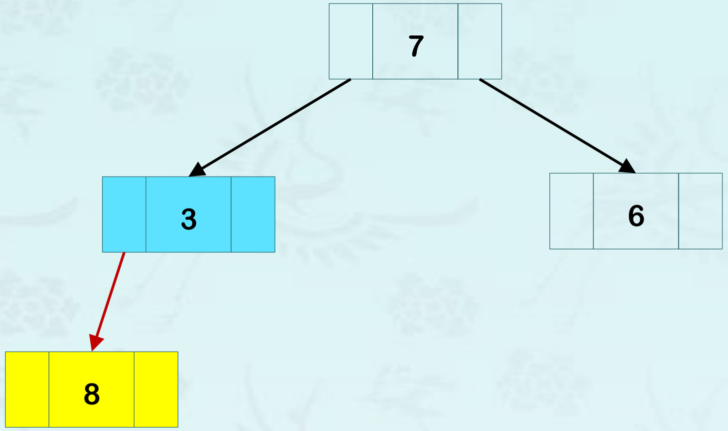
1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   - A. Make a **new node from nodes[].**
   - B. Get a **tree node** in the queue.
   - C. If the left of the tree node doesn't exist,
     set the new node to the left of the tree node.
     else if the right of this tree node doesn't exist,
     set the new node to the right of the tree node.
   - D. If this tree node is full, pop (or dequeu) it.
   - E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue



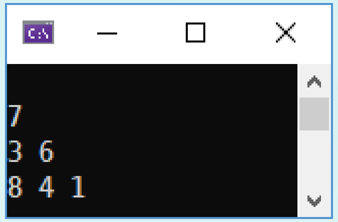


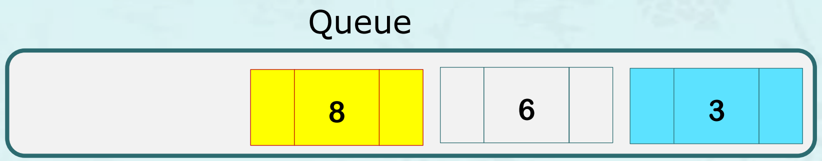




1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   - A. Make a **new node from nodes[].**
   - B. Get a **tree node** in the queue.
   - C. **If the left of the tree node doesn't exist,**
     set the new node to the left of the tree node.
     else if the right of this tree node doesn't exist,
     set the new node to the right of the tree node.
   - D. If this tree node is full, pop (or dequeu) it.
   - E. enqueue the new node (to add children later if any).
4. treeprint(root)

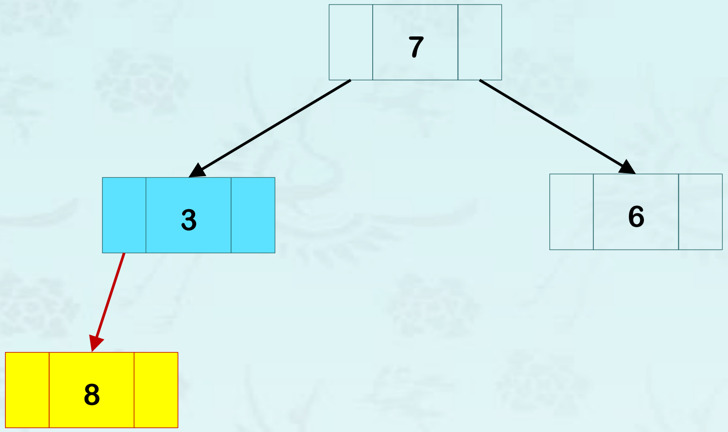
heapprint() – build a binary tree from heap/CBT using queue





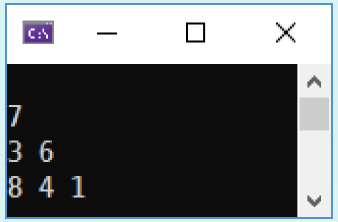
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

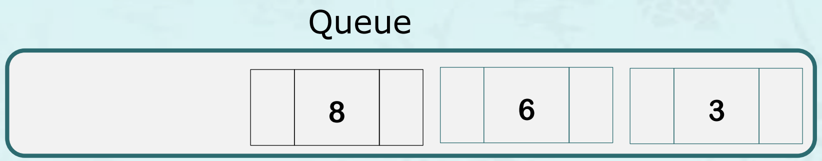
hp->nodes[]　　　　hp->N

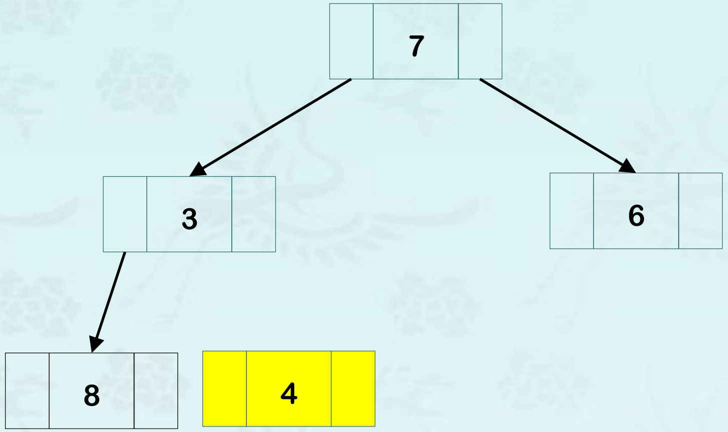


1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   - A. Make a **new node from nodes[].**
   - B. Get a **tree node** in the queue.
   - C. If the left of the tree node doesn't exist,
     set the new node to the left of the tree node.
     else if the right of this tree node doesn't exist,
     set the new node to the right of the tree node.
   - D. If this tree node is full, pop (or dequeu) it.
   - **E. enqueue the new node (to add children later if any).**
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue

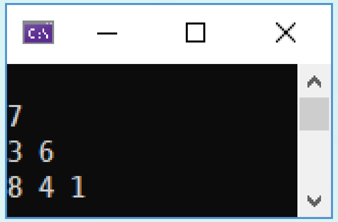


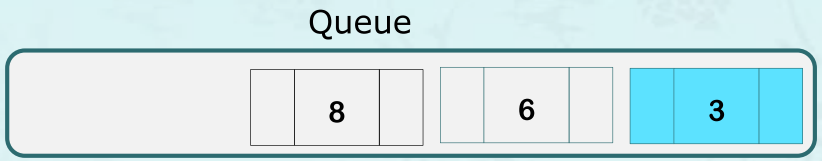


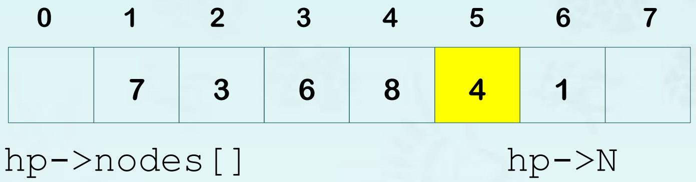


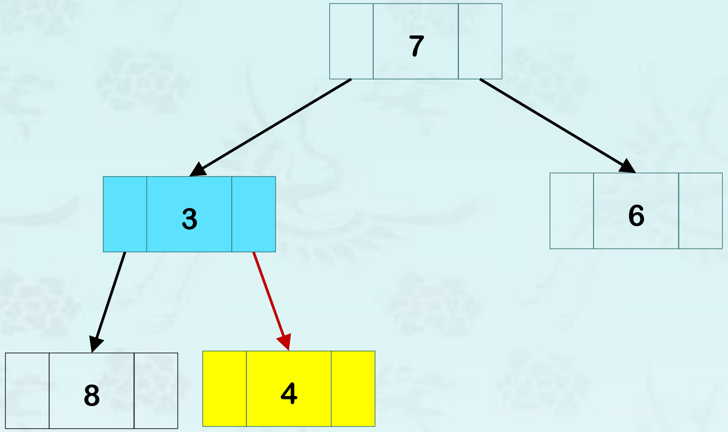
1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue



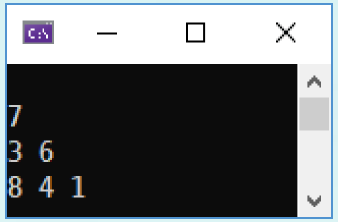


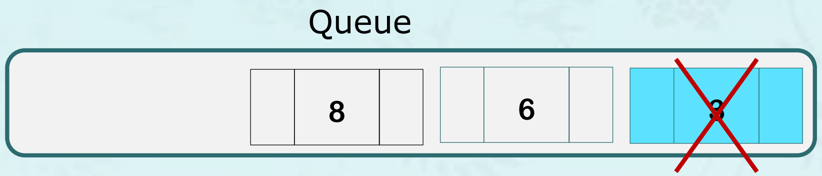


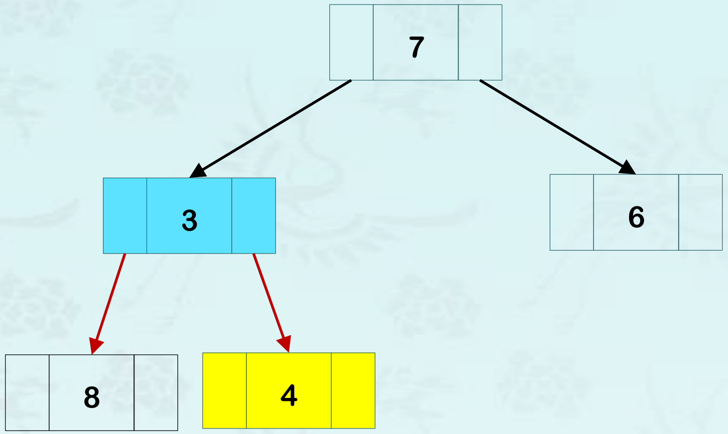


1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue

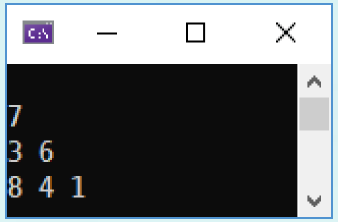


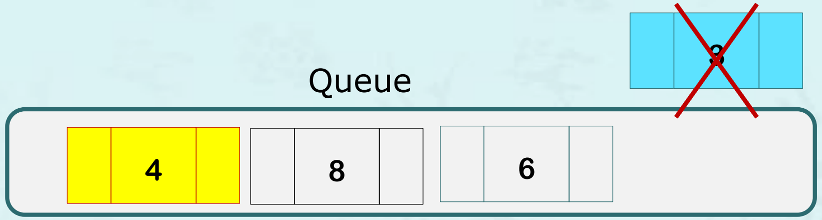


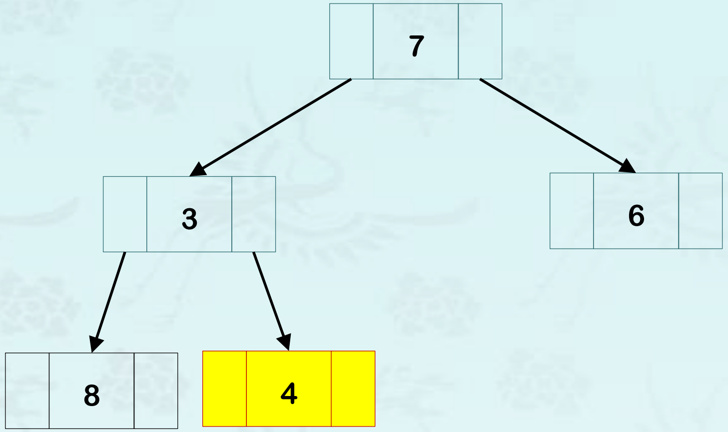


1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   - A. Make a **new node from nodes[].**
   - B. Get a **tree node** in the queue.
   - C. If the left of the tree node doesn't exist,
     set the new node to the left of the tree node.
     else if the right of this tree node doesn't exist,
     set the new node to the right of the tree node.
   - **D. If this tree node is full, pop (or dequeu) it.**
   - E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue





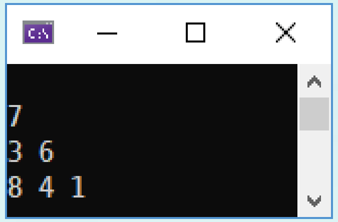


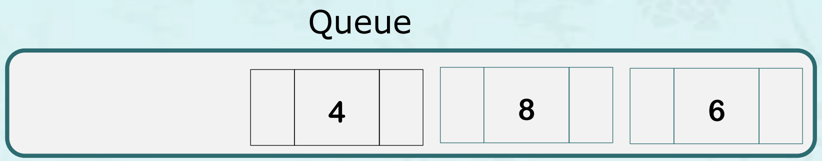
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

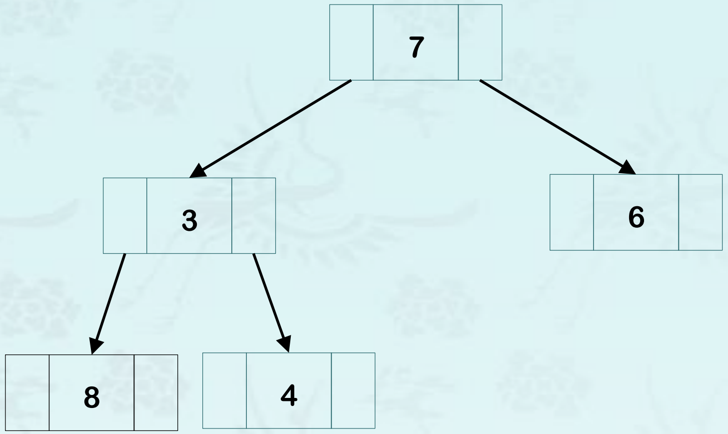
hp->nodes[]　　　　hp->N

1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   - A. Make a **new node from nodes[].**
   - B. Get a **tree node** in the queue.
   - C. If the left of the tree node doesn't exist,
     set the new node to the left of the tree node.
     else if the right of this tree node doesn't exist,
     set the new node to the right of the tree node.
   - D. If this tree node is full, pop (or dequeu) it.
   - E. **enqueue the new node (to add children later if any).**
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue

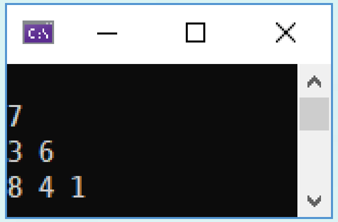


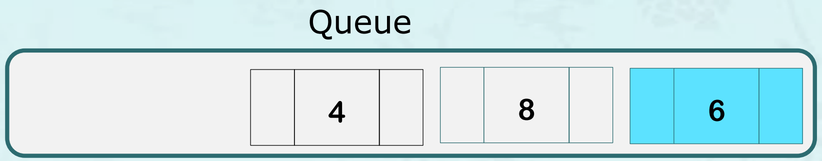


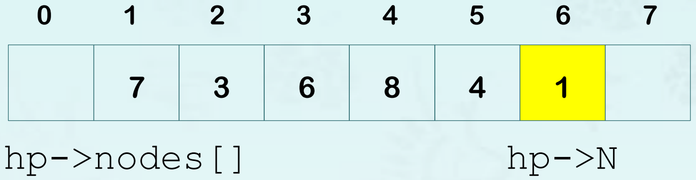


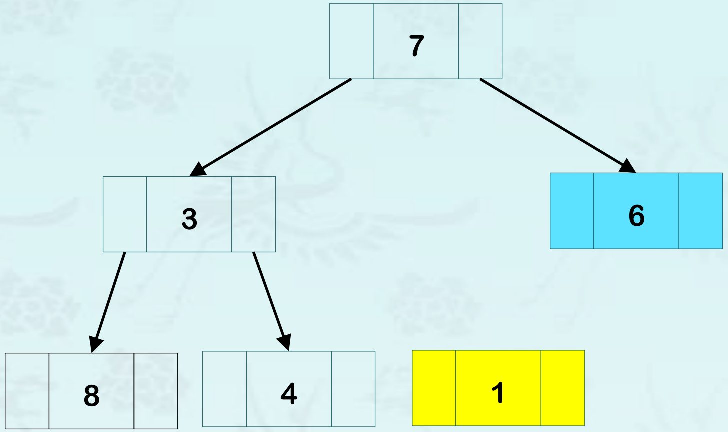
1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue



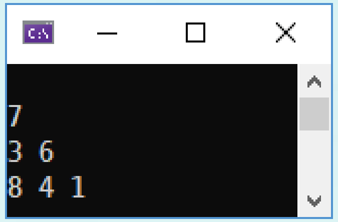


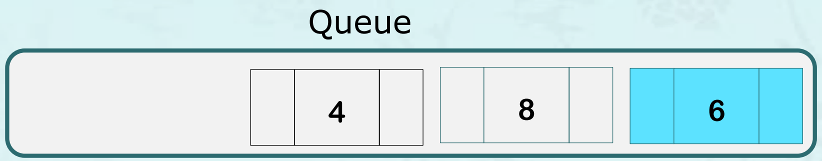


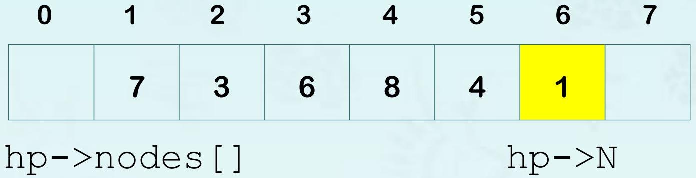


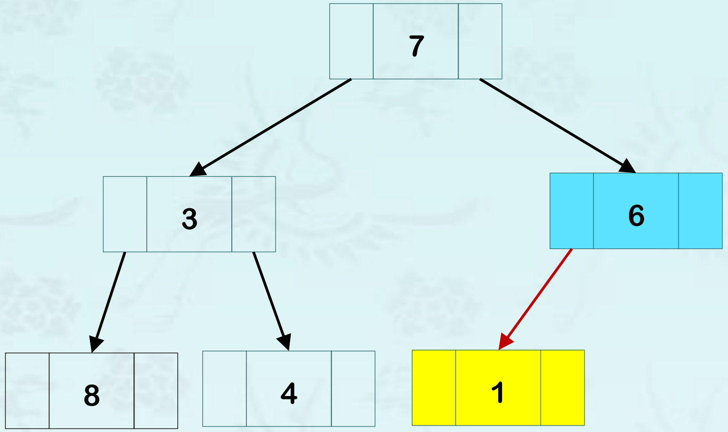
1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   - A. Make a **new node from nodes[].**
   - B. Get a **tree node** in the queue.
   - C. If the left of the tree node doesn't exist,
     set the new node to the left of the tree node.
     else if the right of this tree node doesn't exist,
     set the new node to the right of the tree node.
   - D. If this tree node is full, pop (or dequeu) it.
   - E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue



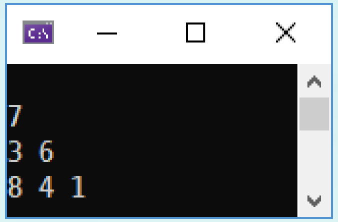


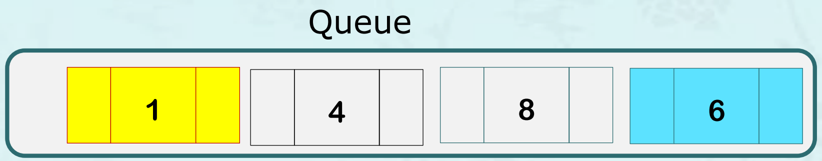




1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. **If the left of the tree node doesn't exist,**
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

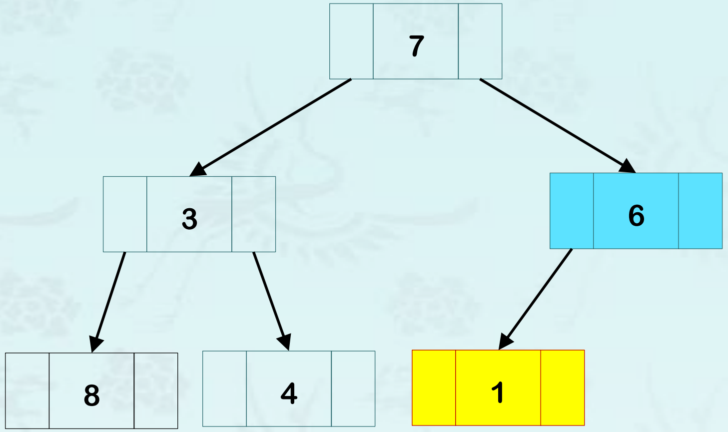
heapprint() – build a binary tree from heap/CBT using **queue**





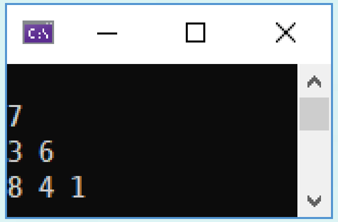
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

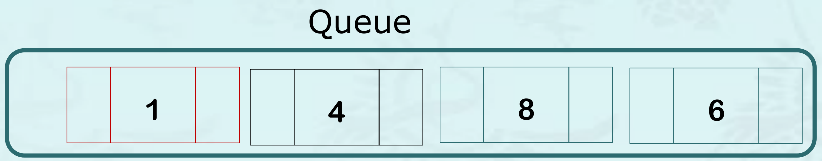
hp->nodes[]   hp->N

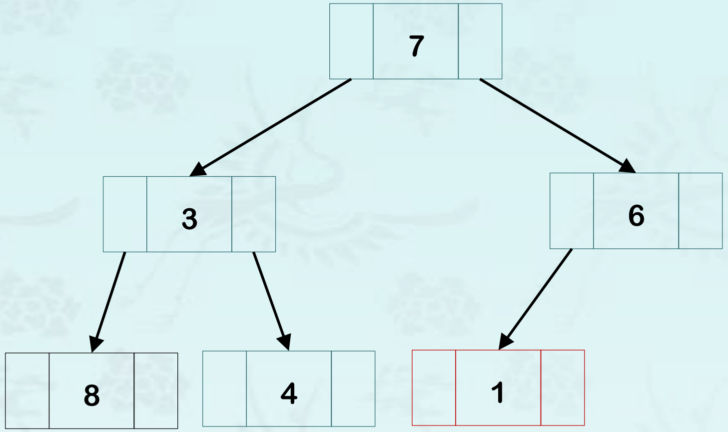


1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
      else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heapprint() – build a binary tree from heap/CBT using queue







1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. **Loop through from the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
   B. Get a **tree node** in the queue.
   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
   else if the right of this tree node doesn't exist,
      set the new node to the right of the tree node.
   D. If this tree node is full, pop (or dequeu) it.
   E. enqueue the new node (to add children later if any).
4. treeprint(root)

heap

- *complete binary tree (review)*
- *heap and priority queues (Chapter 9)*
- *binary heap and min-heap*
- *max-heap demo*
- *max-heap coding*
- *heapsort (Chapter 7)*