Fixed or Flexible?
Wage Setting in Search Equilibrium

Tore Ellingsen* Åsa Rosén**

Working Papers in Economics and Finance No 185
August 1997

Abstract

Why do some vacancies offer a posted wage whereas others offer a negotiable wage? The paper endogenizes the choice of wage policy in a search model with heterogeneous workers. In particular, we characterize the circumstances under which there exist an equilibrium where all firms negotiate wages. Generally, we find that a tight labor market favors bargaining over posting, as does large worker heterogeneity. In the equilibrium of our model, labor markets are tighter when workers are more productive, suggesting a reason why wages are more often negotiated for highly paid jobs.

JEL classification: J31, J41.
Keywords: Search, Wage offers, Bargaining, Posting.

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*Address: Department of Economics, Stockholm School of Economics, Box 6501, S—113 83 Stockholm, Sweden. Email: gte@hhs.se.

**Address: Department of Economics, Uppsala University, Box 513, S—15120 Uppsala, Sweden. Email: asa.rosen@nek.uu.se.

Thanks to Peter Fredriksson and Espen Moen for useful comments on an earlier draft. We are grateful to the Swedish Council for Research in the Humanities and Social Sciences (both authors) and to Jan Wallander och Tom Hedelius' Stiftelse (Rosén) for financial support.

1 Introduction

The ways in which workers are recruited vary widely across firms and across occupations. In this paper we shall discuss one of the most striking differences in recruitment policies; whether wages are posted by the firm or negotiated with each applicant individually. Whereas posted wages appear to be common for blue collar and for low level white collar workers, many well–paid jobs are advertized with a negotiable level of pay.

In this paper we will argue that search costs could play an important role in shaping firms' wage policies. Because of the search friction, firms face the following trade–off when they choose between the two modes: By negotiating wages, the level of pay can be adapted to the skills of the applicant, which might be an advantage when applicants are heterogeneous. In particular, no profitable employment opportunities are foregone. The drawback for the firm is that the applicant may be able to extract a considerable part of the surplus through the negotiation. By posting a wage, the firm gives up some of the flexibility, and may have to forego some profitable trades, but may be able to extract a larger fraction of the surplus. The paper's twin objectives are to make this informal intuition more precise and to show how the trade–off between posting and bargaining is affected by changes in a number of empirically measurable variables.

The central results of the paper concern the conditions under which it is an equilibrium that all firms negotiate wages despite being able to post wages. One prediction is that bargaining should be more common in tight labor markets, i.e. when the equilibrium unemployment/vacancy ratio is low. More exactly, we show that an equilibrium in which all firms bargain is more likely when

- it is cheap to keep open vacancies,
- separation rates are low,
- matching is faster (for any given vacancy/unemployment rate),
- workers are more productive.

The reason why tightness is important is that it determines workers' reservation wages, and we show that an increase in reservation wages hurts the firm more if it posts wages than if it bargains. Very roughly, an increase in reservation wages hurt "posters" more than "hagglers" because a posted wage has to increase by the full amount of the increase in reservation wage, whereas a haggler will only have to pay its recruits a fraction of the increase in their reservation wage.

We also find that worker heterogeneity matters for wage policies. There is a precise sense in which it is true that more worker heterogeneity favors bargaining over posting.

Besides developing a set of testable hypotheses, the paper extends search theory in what is hopefully a fruitful direction. Existing models of bilateral search in the labor market treat the wage determination mechanism as exogenous. Traditionally, it was assumed that the employers post wages, as e.g. in the work of Mortensen (1970), whereas much recent work follows Diamond (1982) and Mortensen (1982) in assuming that employers bargain with applicants. The most closely related literature is a number of papers which take wage posting for granted, but studies in detail the trade–offs involved in choosing an appropriate wage level. Examples include Albrecht and Axell (1984), Lang (1991), Montgomery (1991), and Sattinger (1991). It is well known that many of the qualitative results of earlier work are sensitive to the assumptions regarding wage determination, reflecting a somewhat artificial dichotomy in the search literature between results which hold under posting and results which hold under bargaining – witness for example Diamond (1987). Our analysis may help to identify the circumstances in which each assumption is appropriate, – or at least guide empirical work with this objective.

While there is no similar study in the labor market literature, there exists a small literature on bargaining versus price posting in product markets with search frictions. Notably, Bester (1994) presents a model in which trade is conducted via bargaining if search costs are sufficiently low, a result which appears similar to ours. However, in his model it is crucial that there is an exogenous cost associated with posting a price. Otherwise, all firms post prices in equilibrium. In our model, there is no such exogenous difference in the cost of wage policies. One explanation why we nevertheless find a bargaining equilibrium is that we allow workers to be heterogeneous, whereas Bester assumes that all buyers are identical (workers in our model and buyers in Bester's model are similar in that they take the pricing institution as given).[1]

The paper is organized as follows. Section 2 describes the model and derives some preliminary results. Section 3 provides our main characterization results. Existence of equilibria is discussed in Section 4. Section 5 contains a brief assessment of the model's limitations and shortcomings, and Section 6 concludes.

[1]Spier (1990), also asking whether sellers will be hagglers or posters, does allow for buyer heterogeneity. However, there is little overlap between her analysis and ours.

2 The Model

The analysis is set within a standard bilateral search model, a detailed justification of which can be found e.g. in Pissarides (1990). The main departures from the standard model are that we allow workers to be heterogeneous, and firms to choose a wage policy. We will think of our model as depicting the behavior of workers and firms within a narrowly defined labor market, possibly that of a certain profession in a certain region. This will sometimes be important in evaluating the appropriateness of the model's assumptions.

There are n heterogeneous and infinitely lived workers. Worker skills are general; i.e. each worker has a productivity $x > 0$ in any job. (This is one assumption which one might like to relax in a study of the labor market as a whole.) The distribution of skills in the population is given by the cumulative distribution function $G(x)$, which is assumed to have no mass points and bounded support $[\underline{x}, \overline{x}]$, with $\underline{x} \geq 0$. Let $g(x)$ be the associated density function. The number of jobs, m, is going to be endogenously determined by a zero profit condition. We assume that each firm is sufficiently small to take the number of unemployed workers and their reservation wages as given (hence we avoid the possibility of strategic wage setting).

The model is set in continuous time. We are looking for steady state equilibria only, so we do not index variables by time. The cost of keeping an open vacancy is $c \in (0, \overline{x})$ per unit of time. Both workers and firms discount future payments at a rate of interest r per unit of time. Let v denote the number of vacant jobs and u the number of unemployed workers. Employed workers are separated from their jobs at an exogenous rate $s \in (0, 1)$. Only unemployed workers look for jobs. We assume random pairwise matching between unemployed workers and vacant jobs. Hence, matching is a Poisson process. The rate at which a vacant job is observed by some unemployed worker is denoted $q(v/u)$. As usual, labor market tightness is defined as $\theta = v/u$, and we refer to $q(\cdot)$ as the matching function. Clearly, an unemployed worker observes a vacancy at rate $\phi(\theta) = \theta q(\theta)$. Firms know the distribution of skills in the population, and they can perfectly observe the skill of any applicant during the job interview.

We first describe the behavior of firms. With any vacant job comes a wage policy. The policy is either a fixed wage, $w \in \mathcal{R}_+$, or it is a negotiated wage. Denote the set of possible wage policies $\mathcal{P} = \mathcal{R}_+ \cup \{b\}$, where the b stands for "bargaining". Whereas a firm is allowed to post any wage, we rule out wage offers which are conditional upon the worker's productivity. This modelling choice is justified as follows. When employers post conditional wage offers, the conditions are typically fairly rough; taking account of age, education possibly some broad measure of work experience. It is likely that there is considerably worker heterogeneity which is not picked up by these variables, and our model should be interpreted as modelling how firms behave given this residual heterogeneity.

Although we make no attempt to fully explain why workers cannot post wages, it appears quite reasonable that firms have greater opportunities to commit to a wage policy than workers have, either through delegation of responsibility or through reputation. After all, firms tend to have many employees, whereas workers tend only to have one employer. (We do not model the number of jobs in each firm here, but implicitly we assume that this number is small relative to the total number of jobs.)

Let $F_i(x)$ denote the net present value to the firm of a job which is occupied by a worker of productivity x given that the firm has wage policy $i \in \mathcal{P}$. Let p_i denote the probability that a vacancy with policy i is filled given that an applicant turns up (of course, p_i is endogenous, and will be derived below). In flow terms, we can then write the net present value of opening a vacancy with policy i as

$$rV_i = -c + q(\theta)p_i E[F_i(x) - V_i], \tag{1}$$

where E is the expectations operator. In flow terms, the value of an occupied job is

$$rF_i(x) = x - w_i(x) + s(V_i - F_i(x)), \tag{2}$$

which can be rearranged to yield

$$F_i(x) = \frac{x - w_i(x) + sV_i}{r+s}.$$

Inserting this expression back into equation (1) and solving, the flow value of a vacant job with wage policy i can be written

$$rV_i = \frac{-(r+s)c + q(\theta)p_i E[x - w_i(x)]}{r + s + p_i q(\theta)}. \tag{3}$$

In this paper, we are merely concerned with the question of whether there are equilibria in which all firms negotiate wages. In other words, we ask whether there are steady states such that $rV_b = 0$ and $rV_w \leq 0$ for all $w \in \mathcal{R}_+$. This makes it quite easy to describe the behavior of workers. Let $U(x)$ be an unemployed worker's present discounted utility, and let $M(x)$ be an employed worker's present discounted utility, given that all firms negotiate wages. Finally, let $y(x)$ be the probability that a worker of type x is offered a job given that he observes one. For simplicity, suppose that a worker earns nothing when unemployed. In flow terms, the utility of an unemployed worker is

$$rU(x) = \phi y(x)(M(x) - U(x)). \tag{4}$$

In this model, $rU(x)$ is also the reservation wage of a worker of productivity x. Analogously, the flow utility of being employed is

$$rM(x) = w_b(x) + s(U(x) - M(x)), \tag{5}$$

which implies

$$M(x) = \frac{w_b(x) + sU(x)}{r + s}.$$

Inserting into equation (4) and solving, we have

$$rU(x) = \frac{\phi y(x) w_b(x)}{r + s + \phi y(x)}. \tag{6}$$

As usual, we assume that the negotiated wage is set according to the Nash bargaining solution, i.e.

$$w_b(x) = \arg\max_{\hat{w}(x)} (M(x) - U(x))^{\beta} (F_b(x) - V_b)^{1-\beta}.$$

From the first–order condition, inserting the above expression for $M(x)$ and $F_b(x)$, we have

$$w_b(x) = \beta(x - rV_b) + (1 - \beta) rU(x). \tag{7}$$

Free–entry equilibria in which all firms bargain have a particularly simple structure, because all matches lead to employment. Or more precisely:

Lemma 1 *In any free–entry equilibrium in which all firms bargain,* $y(x) = p_b = 1$.

Proof: The necessary and sufficient condition for $p_b = y(x) = 1$ is that (i) $w_b(x) \geq rU(x)$ (any worker accepts to work for the wage $w_b(x)$) and that (ii) $x - w_b(x) \geq 0$ (the firm accepts to hire any worker at $w_b(x)$). Since $rV_b = 0$, we have from (7) that

$$w_b(x) = \beta x + (1 - \beta) rU(x). \tag{8}$$

Inserting into (6) we then have

$$rU(x) = \frac{\beta \phi y(x) x}{r + s + \beta \phi y(x)},$$

which implies that $rU(x) < x$. Both (i) and (ii) then follow from this fact in conjunction with (8). ■

The intuition is simple. A worker's reservation wage cannot exceed his productivity (because then he would never be employed, in which case his reservation wage would have to be zero). Indeed, due to the search friction, the reservation wage must be strictly smaller than the productivity. Since the firms' value of waiting is zero, there is thus always a surplus to be shared.

Using the free–entry requirement that $rV_b = 0$, equations (6) and (7) together with Lemma 1 imply that the reservation wage can be written

$$rU(x) = x/\gamma \tag{9}$$

where

$$\gamma = \frac{r + s + \beta\phi}{\beta\phi}. \tag{10}$$

It follows that the wage is

$$\begin{aligned} w_b(x) &= \beta x + (1-\beta) rU(x) \\ &= \left(\beta + \frac{1-\beta}{\gamma}\right) x. \end{aligned} \tag{11}$$

When we now insert our expression for the wage, $w_b(x)$, into (3) we obtain the free–entry condition

$$L(\theta) = 0, \tag{12}$$

where

$$L = -(r+s)c + (1-\beta)q(\theta)(1 - 1/\gamma)E[x].$$

Our analysis so far has built on the assumption that we are in a steady state. To make sure that there is a steady state, we make the following quite weak assumptions:

Assumption 1 *The matching function satisfies the conditions (i)* $q'(\theta) < 0$ *for all* θ*, (ii)* $q(0) > (c(r+s))/((1-\beta)E[x])$*, (iii)* $\lim_{\theta\to\infty} q(\theta) = 0$*, (iv)* $\phi'(\theta) > 0$ *for all* θ*.*

(The requirement (ii), which looks non–standard at first, simply expresses the condition that $rV_b \geq 0$ at $\theta = 0$, i.e. that the economy is productive.) Let us now explicitly impose the steady state conditions. For the unemployment to be constant, we must have that the number of separations equals the number of new matches,

$$s(n - u) = \phi u,$$

or equivalently,

$$u = \frac{ns}{s + \theta q(\theta)}. \tag{13}$$

For the number of vacancies to be constant, we need

$$s(m - v) = vq(\theta),$$

or equivalently

$$v = \frac{ms}{s + q(\theta)}. \tag{14}$$

These are two equations in three unknowns: m, v, u. The final equation is the firms' zero profit condition under bargaining, (12). It is straightforward to show that this equation delivers a solution for θ and hence for $q(\theta)$: Since $\lim_{\theta\to\infty} q(\theta) = 0$, we see that L must be negative for θ large enough. To show that L can be positive for some θ, consider the case of $\theta = 0$. Then the sign of L is given by the sign of

$$-c(r + s) + q(0)(1 - \beta)E[x],$$

which is positive by Assumption 1. Since L is a continuous function of θ, it follows by the intermediate value theorem that there is some θ such that $L = 0$. Indeed, the solution is unique. The sign of $dV_b/d\theta$ at $L = 0$ is the same as the sign of

$$\frac{\partial(q(\theta)(1 - 1/\gamma))}{\partial\theta} = q'(\theta)(1 - 1/\gamma) - q(\theta)(\gamma - 1)\frac{\phi'(\theta)}{\phi(\theta)\gamma^2} < 0 \tag{15}$$

Since the equilibrium value of θ is unique and $q(\theta)$ is strictly decreasing, we can plug this value into (13) to get a unique solution for the unemployment rate u, which in turn gives us v. Having determined u and v, we get a unique solution for m from equation (14). To conclude, under the assumption that all firms negotiate wages, there is exactly one steady state. Notice here that the properties of a bargaining equilibrium, in particular the equilibrium labor market tightness, depend on the workers' average productivity, $E[x]$, but not on other properties of the skill distribution.

For it to be an equilibrium that all firms negotiate wages, we must make sure that no firm can earn a positive profit by offering a fixed wage. Given that all other firms haggle, a poster can be shown to solve the problem

$$\max_{w,l} rV_w = \frac{-(r+s)c + q(\theta)\int_l^{\gamma w}(x - w)g(x)\,dx}{r + s + q(\theta)(G(\gamma w) - G(l))}$$

subject to the constraints $l \geq \underline{x}$ and $\gamma w \leq \overline{x}$. To see why the objective function

rV_w has the specified form, consider the expression (3) and recall from (9) that the reservation wage of a worker with productivity x is x/γ. Hence, the best worker who accepts the wage w has productivity γw. Moreover, we can be sure that all workers with lower productivities accept the wage w, because $\partial rU(x)/\partial x < 1$. We think of l as a hiring standard. Applicants with higher productivity than l are accepted, other workers are rejected for the job.

The first–order condition for an optimal hiring standard, $l^*(w)$, is

$$\frac{q(\theta)g(l^*)}{r+s+q(\theta)(G(\gamma w)-G(l^*))}(l^*-w-rV_w) \leq 0, \text{ with equality if } l^* > \underline{x}. \quad (16)$$

It follows that

$$l^*(w) = \max\{\underline{x}, w + rV_w\}. \quad (17)$$

Insert $l^*(w)$ back into the objective function. After a manipulation, the first–order condition for an optimal posted wage can be written

$$\begin{aligned} &\gamma g(\gamma w^*)[\gamma w^* - w^* - rV_{w^*}] - [G(\gamma w^*) - G(l^*(w^*))] \\ &- \frac{\partial l^*(w^*)}{\partial w} g(l^*(w^*))[-rV_{w^*} + l^*(w^*) - w^*] \geq 0, \text{ with equality if } \gamma w^* < \overline{x}. \end{aligned} \quad (18)$$

If there is a corner solution, $w^* = \overline{x}/\gamma$. Now, either $\partial l^*(w^*)/\partial w = 0$, in which case $l^*(w^*) = \underline{x}$, or $\partial l^*(w^*)/\partial w = \partial(rV_{w^*})/\partial w + 1$, in which case $l^*(w^*) = rV_{w^*} + w^*$. Thus, in either case the last term of equation (18) is zero, and the first–order condition simplifies to

$$\gamma g(\gamma w^*)[\gamma w^* - w^* - rV_{w^*}] - [G(\gamma w^*) - G(l^*(w^*))] \geq 0. \quad (19)$$

This condition is easy to interpret. If the solution is interior, it says that the marginal benefit of increasing the wage, which is the firm's profit from hiring a worker who rejects the original wage (but accepts the new wage) multiplied by the probability that a worker is of this type, is equal to the marginal cost of increasing the wage, which is the cost of paying any acceptable worker a higher wage.

Let us now study in more detail the condition that it is not profitable to offer a posted wage if all other bargains, i.e. the condition that $rV_{w^*} \leq 0$. When $rV_b = 0$ we have from (12) that

$$(r+s)c = (1-\beta)q(\theta)(1-1/\gamma)E[x].$$

Thus, we can write

$$rV_{w^*}(l^*) = \frac{-(1-\beta)q(\theta)(1-1/\gamma)E[x] + q(\theta)\int_{l^*(w^*)}^{\gamma w^*}(x-w^*)g(x)\,dx}{r+s+q(\theta)[G(\gamma w^*) - G(l^*(w^*))]}. \qquad (20)$$

Let us define

$$B = (1-\beta)(1-1/\gamma)E[x] - \int_{l^*}^{\gamma w^*}(x-w^*)g(x)\,dx. \qquad (21)$$

Since the denominator of (20) is positive, we see that $rV_{w^*}(l^*) \leq 0$ only if B is non–negative. For future reference we state the result as follows.

Proposition 1 *There exists an equilibrium in which all firms bargain over wages if and only if* $B \geq 0$.

The two questions we want to answer are: (i) Are there parameters such that bargaining is an equilibrium and parameters such that it is not? (ii) If so, what makes bargaining a more likely policy? For ease of exposition, we present the answers in reverse order.

3 When is Bargaining More Likely?

Let us now turn to the central issue; which factors make it more likely that all firms bargain? We know from Proposition 1, that such an equilibrium exists if and only if B is non–negative. Thus, if B is increasing in one of the model's parameters around any point where $B = 0$, an increase in that parameter makes bargaining more likely.

Let us first establish a few preliminary results. Notice that w^* is a minimizer of B around $B = 0$. Thus, the term $(\partial B/\partial w)(\partial w/\partial z)$ is zero for any parameter z. For all comparative static exercises, the term $\partial B/\partial\gamma$ is crucial. Thus, we start by deriving the sign of this expression.

Lemma 2 $\partial B/\partial\gamma|_{B=0} < 0$.

PROOF: In proving this result, it is convenient to treat the case of a boundary solution for w^* separately. If w^* is interior, we have

$$\begin{aligned}\frac{\partial B}{\partial\gamma}\,|_{B=0} &= \frac{(1-\beta)E[x]}{\gamma^2} - g(\gamma w^*)(w^*)^2(\gamma-1) \\ &= \frac{1}{\gamma^2}[(1-\beta)E[x] - [G(\gamma w^*) - G(l^*)]\gamma w^*]\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\gamma}\left[\frac{1}{\gamma-1}\int_{l^*}^{\gamma w^*}(x-w^*)g(x)\,dx - [G(\gamma w^*)-G(l^*)]w^*\right] \\
&= \frac{G(\gamma w^*)-G(l^*)}{\gamma(\gamma-1)}\left[\frac{1}{G(\gamma w^*)-G(l^*)}\int_{l^*}^{\gamma w^*} xg(x)\,dx - \gamma w^*\right] \\
&< 0,
\end{aligned}
$$

where the second equality uses the first–order condition (19), the third equality uses the fact that we evaluate the expression at a point where $B=0$, and the fourth equality is a consequence of some straightforward manipulations. If instead we have a boundary solution, $w^* = \overline{x}/\gamma$, we have that

$$
B = (1-\beta)(1-1/\gamma)E[x] - \int_{l^*}^{\overline{x}}(x-\overline{x}/\gamma)g(x)\,dx,
$$

and hence

$$
\begin{aligned}
\frac{\partial B}{\partial \gamma}\,|_{B=0} &= \frac{1}{\gamma^2}(1-\beta)E[x] - \frac{\overline{x}}{\gamma^2}[G(\overline{x})-G(l^*)] \\
&= \frac{1}{\gamma^2}\left[\frac{\gamma}{\gamma-1}\int_{l^*}^{\overline{x}}(x-\overline{x}/\gamma)g(x)\,dx - [G(\overline{x})-G(l^*)]\overline{x}\right] \\
&= \frac{[G(\overline{x})-G(l^*)]}{\gamma(\gamma-1)}\left[\frac{1}{G(\overline{x})-G(l^*)}\int_{l^*}^{\overline{x}} xg(x)\,dx - \overline{x}\right], \\
&< 0,
\end{aligned}
$$

where the second equality follows from $B=0$ and the third from simple manipulations. ■

Thus, as reservation wages increase (γ goes down), bargaining becomes more likely. Higher reservation wages mean that any given fixed wage attracts fewer workers. The result shows that the cost to a fixed wage firm of losing its best potential applicants is greater than the cost to a flexible wage firm of having to pay its workers more. To get some rough intuition for the result, think of a poster as optimizing with respect to the recruiting interval rather than the wage itself. While increase in reservation wages might lead a poster to change the interval of skill it recruits from, this yields only a second–order effect on profits. The first–order effect comes from the fact that the posted wage has to increase by the full amount of the increase in the reservation wage of the best worker who accepts the original posted wage. A haggler on the other hand, will only have to pay its recruits a fraction of the increase in their reservation wage.[2]

[2]This argument is incomplete, however, because the probability of employing a worker is

Another term which plays a crucial role is $\partial\gamma/\partial\theta$, which we can easily see is negative.

Lemma 3 $\partial\gamma/\partial\theta < 0$.

PROOF: Note that γ can be written

$$\gamma = 1 + \frac{r+s}{\beta\phi}.$$

Hence,

$$\frac{\partial\gamma}{\partial\theta} = \frac{-(r+s)}{\beta\phi^2}\frac{\partial\phi}{\partial\theta} = \frac{(1-\gamma)}{\phi}\frac{\partial\phi}{\partial\theta} < 0,$$

where the inequality follows from Assumption 1 and the fact that $\gamma > 1$. ■

Intuitively, when the labor market tightness θ increases, it is easier for workers to find jobs. Hence, ceteris paribus, workers' reservation wages increase, which is equivalent to saying that γ goes down.

Finally, we shall make repeated use of the inequality in (15).

Lemma 4

$$\frac{\partial(q(\theta)(1-1/\gamma))}{\partial\theta} < 0.$$

We are now ready to prove our main comparative static results.

Proposition 2 *The more costly it is to keep a vacancy open, the less likely it is that all firms negotiate wages, i.e.* $dB/dc|_{B=0} < 0$.

PROOF: From equation (21) we have that

$$\frac{dB}{dc}\mid_{B=0} = \frac{\partial B}{\partial\gamma}\frac{\partial\gamma}{\partial\theta}\frac{d\theta}{dc}\mid_{B=0}.$$

By Lemmas 2 and 3 this expression has the same sign as $d\theta/dc$. Differentiation of (the numerator in) (12) yields

$$\frac{d\theta}{dc} = \frac{r+s}{(1-\beta)E[x]\dfrac{\partial(q(\theta)(1-1/\gamma))}{\partial\theta}},$$

which is negative by Lemma 4. ■

The explanation is that when c increases, the equilibrium vacancy/unemployment rate (θ) decreases, which in turn lowers workers' reservation wages. And, as we have explained above, lower reservation wages favor posted wages.

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not the same for a poster as for a haggler.

Next, let us consider the impact of the discount and separation rates.

Proposition 3 *The higher are the discount rate r and the separation rate s, the less likely it is that all firms negotiate wages, i. e. $dB/dr|_{B=0} < 0$ and $dB/ds|_{B=0} < 0$.*

Proof: From equation (21) we have that

$$\frac{dB}{dr}\,|_{B=0} = \frac{\partial B}{\partial \gamma}\left(\frac{\partial \gamma}{\partial r} + \frac{\partial \gamma}{\partial \theta}\frac{d\theta}{dr}\right)|_{B=0}\,.$$

We know that $\partial B/\partial\gamma|_{B=0} > 0$ (Lemma 2) and that $\partial\gamma/\partial\theta > 0$ (Lemma 3). By equation (10), $\partial\gamma/\partial r = 1/(\beta\phi) > 0$, so it is sufficient to show that $d\theta/dr$ is negative. Differentiation of equation (12) yields

$$\frac{d\theta}{dr} = \frac{c - \dfrac{(1-\beta)q(\theta)E[x]}{\gamma^2}\dfrac{\partial\gamma}{\partial r}}{(1-\beta)E[x]\dfrac{\partial(q(\theta)(1-1/\gamma))}{\partial\theta}}.$$

The denominator is negative by Lemma 4. To see that the numerator is positive, note that by equation (12) we have

$$c = (1-\beta)(1-1/\gamma)q(\theta)E[x]/(r+s).$$

Hence, the numerator is

$$(1-\beta)(\gamma-1)^2 q(\theta)E[x]/((r+s)\gamma^2) > 0,$$

which completes the proof that $dB/dr|_{B=0} < 0$. Since r and s enter B symmetrically, the proof is identical for $dB/ds|_{B=0} < 0$. ■

Again, the reason is straightforward. First, there is a direct negative influence on reservation wages, due to the fact that waiting for future jobs becomes less attractive with more discounting and higher separation rates. There is also an indirect effect on the equilibrium reservation wage through the vacancy/unemployment rate θ. An increase in r or s reduces this rate, because ceteris paribus the discounted expected revenue from opening a vacancy is smaller. Here too, the effect is to lower reservation wages. And, to repeat, lower reservation wages favor wage posting over negotiated wages.

So far, we have kept the matching technology constant. An interesting question is what happens when the matching becomes more efficient. Let us therefore write $q(\theta, a)$, where a is a shift parameter and $\partial q/\partial a > 0$. Thus, $\phi(\theta, a) = \theta q(\theta, a)$. As it turns out, better matching favors bargaining over posting.

Proposition 4 *The better is the matching technology, the more likely it is that all firms negotiate wages, i.e.* $dB/da|_{B=0} > 0$.

PROOF: From equation (21) we see that we can write

$$\frac{dB}{da}\,|_{B=0}=\frac{\partial B}{\partial \gamma}\frac{\partial \gamma}{\partial \phi}\left(\frac{\partial \phi}{\partial a}+\frac{\partial \phi}{\partial \theta}\frac{d\theta}{da}\right)|_{B=0}\ .$$

We know that $\partial B/\partial\gamma|_{B=0} > 0$ (Lemma 2) and that $\partial\gamma/\partial\phi < 0$. Furthermore, by Assumption 1, $\partial\phi/\partial\theta > 0$. Since $\partial q/\partial a > 0$, it follows that $\partial\phi/\partial a > 0$. Thus, it remains to sign the term $d\theta/da$. Differentiation of equation (12) yields

$$\frac{d\theta}{da}=\frac{-\left(1-\dfrac{1}{\gamma}\right)\dfrac{\partial q}{\partial a}-\dfrac{q}{\gamma^2}\dfrac{\partial \gamma}{\partial a}}{\dfrac{\partial(q(1-1/\gamma))}{\partial\theta}}.$$

The denominator is negative by Lemma 4. To determine the sign of the numerator, we use the facts

$$\begin{aligned}\frac{\partial q}{\partial a} &= \frac{1}{\theta}\frac{\partial \phi}{\partial a},\\ \frac{q}{\phi} &= \frac{1}{\theta},\\ \frac{\partial \gamma}{\partial a} &= \frac{1-\gamma}{\phi}\frac{\partial \phi}{\partial a},\end{aligned}$$

which follow directly from the definitions of q, ϕ, and γ. After some manipulation the numerator becomes

$$\frac{-(\gamma-1)^2}{\gamma^2\theta}\frac{\partial\phi}{\partial a} < 0,$$

and hence $d\theta/da > 0$. ■

What happens here is that an improvement of the matching technology serves to increase the equilibrium rate at which unemployed workers observe vacancies – both directly and indirectly through an increase in the equilibrium tightness θ. This raises the workers' reservation wage and thus improves the profitability of bargaining relative to posting.

Given the observation that bargaining appears to be more frequent for highly paid jobs, it is natural to investigate how B is affected by a general improvement of workers' productivity. Let workers' true productivity be denoted kx. Increasing k keeps relative productivities the same, but makes average productivity higher.

Proposition 5 *The higher is the workers' average productivity, the more likely it is that all firms negotiate wages, i. e.* $dB/dk|_{B=0} > 0$.

PROOF: The proof is somewhat long, so we only report the case in which w^* is interior. Since a worker of type x now has productivity kx, the analog to equation (6) is $rU(kx) = kx/\gamma$. Hence, the best worker who accepts a wage w is of type $x = \gamma w/k$. The optimal wage is determined exactly the same way as before, the more general first–order condition being

$$\gamma g(\gamma w^*/k)[(\gamma - 1)w^* - rV_{w^*}]/k - [G(\gamma w^*/k) - G(l^*(w^*))] \geq 0,$$

which reduces to (19) when $k = 1$.

The general expression for B is

$$B = (1-\beta)(1-1/\gamma)kE[x] - \int_{l^*}^{\gamma w^*/k} (kx - w^*)g(x)\, dx.$$

The total effect of a change in k on B is

$$\frac{dB}{dk}\,|_{B=0} = \left(\frac{\partial B}{\partial \gamma} \frac{\partial \gamma}{\partial \theta} \frac{d\theta}{dk} + \frac{\partial B}{\partial k} \right) |_{B=0}\ .$$

Lemma 2 can be shown to go through for any $k > 0$, so $\partial B/\partial\gamma > 0$. By Lemma 3, $\partial\gamma/\partial\theta < 0$. To find $d\theta/dk$, notice that the general version of equation (12) yields

$$(1-\beta)q(\theta)(1-1/\gamma)kE[x] = (r+s)c.$$

Thus,

$$\frac{d\theta}{dk} = \frac{-q(\theta)(1-1/\gamma)}{k\dfrac{\partial(q(\theta)(1-1/\gamma))}{\partial\theta}}.$$

The denominator is negative by Lemma 4, so $d\theta/dk > 0$. It remains to determine the sign of the direct effect $\partial B/\partial k$. Now, $kl^*(w)$ is the productivity of the worst acceptable worker, so we know that $(kl^*(w^*) - w^*)\partial l^*/\partial k = 0$. Using this fact, we have

$$\begin{aligned}
\frac{\partial B}{\partial k}\,|_{B=0} &= (1-\beta)(1-1/\gamma)E[x] + \frac{\gamma w^*}{k^2} g(\gamma w^*/k)(\gamma w^* - w^*) - \int_{l^*}^{\gamma w^*/k} xg(x)\, dx \\
&= (1-\beta)(1-1/\gamma)E[x] + (G(\gamma w^*/k) - G(l^*))w^*/k - \int_{l^*}^{\gamma w^*/k} xg(x)\, dx \\
&= (1-\beta)(1-1/\gamma)E[x] - \int_{l^*}^{\gamma w^*/k} (x - w^*/k)g(x)\, dx
\end{aligned}$$

$$= \quad B/k = 0,$$

where the second equality follows from the first–order condition determining w^*, and the third equality is the result of a simple manipulation. This completes the proof that $dB/dk|_{B=0} > 0$. ∎

The intuition for the result, which is revealed by the proof, is that the direct effect of a change in k is neutral. This is understandable; for a given heterogeneity of skills and for a given labor market tightness, the expected surplus from a match is proportional to k for both parties. However, as the absolute level of skill increases, it pays to open additional vacancies. The equilibrium labor market tightness increases, and as we have seen above bargaining then becomes the better strategy.[3]

Finally, let us consider the effect of changing the heterogeneity of worker skills, $G(x)$. Perhaps the most natural exercise is to look at a mean preserving increase in spread. One might guess that any such increase in spread would favor bargaining over posting. After all, an advantage of bargaining is that the wage adjusts according to the applicant's productivity, whereas with a posted wage any acceptable applicant gets the same wage. Flexibility thus appears more profitable with greater heterogeneity. However, this intuition is incomplete. A mean preserving increase in spread applies a *global* measure of dispersion, whereas the dispersion which is of relevance to a poster is the *local* shape of the density function of productivity in a certain interval above the optimal posted wage. To illustrate the importance of this distinction, let us study two simple examples. In both examples, we start from a uniform distribution of worker skills. In the first example, we change the dispersion of skill but maintain the uniform distribution. Uniform skill distributions with the same mean are characterized by the equation $\overline{x}+\underline{x} = \kappa$. Hence, the density function can be written

$$g(x) = \frac{1}{\overline{x} - \underline{x}} = \frac{1}{2\overline{x} - \kappa}.$$

From equation (19), we see that the optimal wage is the corner solution, $w^* =$

[3]Here we would like to stress the interpretation of our model as a narrowly defined labor market. In particular, the result should not be taken to imply that average unemployment would go down in an economy which experiences technological progress. When all sectors become more productive, wages go up, and it is likely that the costs of opening a vacancy goes up too.

$\overline{x}/\gamma$. The effect on B of an increase in the dispersion is thus

$$\frac{dB}{d\overline{x}} = -\frac{\partial}{\partial \overline{x}} \int_{\overline{x}/\gamma}^{\overline{x}} \frac{x - \overline{x}/\gamma}{2\overline{x} - \kappa}\, dx = \frac{(\gamma - 1)^2 (\kappa - \overline{x})\overline{x}}{\gamma^2 (\kappa - 2\overline{x})^2}, \tag{22}$$

which is positive due to the fact that $\kappa - \overline{x} = \underline{x} > 0$. The example confirms the intuition that more heterogeneity favors bargaining. However, we can easily find a mean preserving increase in spread which does not favor bargaining. Starting from a uniform skill distribution, take some mass from around the center of the distribution and allocate it uniformly across two symmetric intervals at the ends, without changing the support. Since the optimal posted price remains $\overline{x}$ even for the new non–uniform distribution, and since there are now more highly productive workers, B must be reduced. Hence, it is the density of workers who are employed at the posted wage which matters, not the spread of the whole distribution.

4 Existence

The above characterization results are only interesting if there are indeed "realistic" parameter configurations such that $B = 0$. Let us first show that B can be negative, in which case a posted wage dominates bargaining. This occurs, for example, when worker skills are very homogeneous.

Proposition 6 *If the support of the skill distribution, $[\underline{x}, \overline{x}]$, is sufficiently small, then there is no equilibrium in which all firms bargain.*

Proof: Define μ as the mean skill. In the limit, as the support shrinks, we have

$$\lim_{\underline{x}\to\mu,\, \overline{x}\to\mu} \int_{l^*}^{\gamma w^*} (x - w^*) g(x)\, dx = \mu - \mu/\gamma.$$

Thus,

$$\lim_{\underline{x}\to\mu,\, \overline{x}\to\mu} B = (1-\beta)(1 - 1/\gamma)\mu - (1 - 1/\gamma)\mu < 0.$$

■

The explanation is simple. A bargaining equilibrium has the property that when there is little variation in skill, there is little variation in reservation wages. Thus, by offering a fixed wage equal to the reservation wage of the best worker, a poster can extract virtually all gains from trade while recruiting from the whole skill distribution. A haggler, on the other hand, always gives away a fraction β of the gains from trade. Note how this result concerning the *support* of the skill distribution complements our result concerning the *density* of the distribution. In both cases, increased heterogeneity favors bargaining over posting.

The slightly harder part is to show that there are parameters such that it is not profitable to deviate from bargaining to a posted wage. An investigation of this issue necessitates a discussion of parameter values, and we warn that our model is perhaps slightly too stylized to make the exercise fully convincing. In particular, we have assumed that unemployment benefits are zero. Notice that with positive unemployment benefits, reservation would be higher and bargaining would thus become more likely.

With this caveat, let us now look at a specific example. Of course, as Proposition 6 shows, the first requirement is that the skill dispersion is not too small. To be concrete, we shall consider a uniform skill distribution on an interval $[0, 2\mu]$. Note that, for a uniform distribution, this has the maximum dispersion consistent with mean μ and non–negative productivities. The density of the skill distribution is then $1/(2\mu)$. Above we showed that the optimal fixed wage in this case is $w^* = 2\mu/\gamma$. Since $w^* > \underline{x} = 0$, we have

$$\begin{aligned} B &= (1-\beta)(1-1/\gamma)\mu - \int_{2\mu/\gamma}^{2\mu} (x - 2\mu/\gamma)/(2\mu)\, dx \\ &= \frac{(1-\beta)(\gamma-1)\mu}{\gamma} - \frac{(\gamma-1)^2\mu}{\gamma^2}. \end{aligned} \tag{23}$$

Thus, in this case B is positive if and only if $\beta < 1/\gamma$ or equivalently (using (9), if $rU(x) > \beta x$. In words, the reservation wage should exceed β times the worker's productivity. If for example $\beta = 1/2$, which is the most common assumption, there is a bargaining equilibrium if all workers find it optimal to reject any offer to work for less than half of what they is worth. This is surely a plausible condition. A more explicit version of the condition $\beta < 1/\gamma$ is obtained by using equation (10) to get

$$\beta < 1 - \frac{r+s}{\phi}.$$

Again, the condition holds for reasonable parameter values: Even if some workers are credit constrained, the yearly interest rate should probably not exceed $r = 0.2$. The average duration of an employment spell is $1/s$. Since in most countries the average employment relationship exceeds four years, we assume that $s \leq 0.25$. Finally, the average duration of an unemployment spell, $1/\phi$, is well below one year in many countries (for more detailed figures on employment and unemployment spells, see e.g. Layard, Nickell and Jackman (1991)). And even in those countries where it is not, the long spells are mostly due to workers who have all but withdrawn from the labor market and therefore do not affect the opening of new vacancies. (These long–term unemployed should therefore not count in our model's measure of u.) Thus, when skill dispersion is large, even in what is otherwise a "worst case" scenario, the critical condition amounts to $\beta < 0.55$, which is entirely palatable.

We conclude that an equilibrium in which all firms bargain may or may not exist, even when the parameters are constrained to lie in some reasonable range.

5 Limitations

Given that our modelling choices are accepted, the analysis is nonetheless somewhat incomplete. We have shown that there cannot be more than one equilibrium in which all firms bargain, but there may be different kinds of equilibria. First, there is always a degenerate equilibrium in which all firms post a wage of zero (see Ellingsen and Rosén (1994)). And even if it may be possible to reformulate the model in such a way that the pure posting equilibrium disappears (for example by assuming that some firms cannot credibly commit to posted wages), we also know that bargaining and posting may co–exist for some parameter values. Indeed, we know from numerical examples reported in Ellingsen and Rosén (1994) that there are parameter values such that there are several of these mixed equilibria. Since we have been unable to prove that the problem is supermodular, our comparative static results may not generalize to cover sets of mixed equilibria.

Also, our analysis is limited by its assumptions. For example, search is random and there is at most one applicant at the time. Realistically, there are often many workers applying for a job, and the choice of wage policy probably affect the number of workers who observe the vacancies of a particular firm. The reader may also object to the way in which we model wage posting. How realistic is it that the firm will never listen to a worker who does not meet the firm's hiring standard, but offers to work for a lower wage? And will the firm never try to raise its wage offer in order to make the job attractive to an applicant whose reservation wage is above the posted wage? These questions are theoretically tricky, and some of them have a general interest beyond the problem at hand. For convenience as well as comparability with earlier work, we have chosen to stay close to the existing literature as possible.

One quite straightforward extension of our work would be to consider a policy of maximum wages, whereby the firm allows itself to bargain with relatively lowly skilled workers while sufficiently skilled applicants are offered the maximum wage. It is easy to see that such a policy, if credible, dominates posting. In general, however, it does not dominate bargaining, and we conjecture that our main insights would remain valid if this policy is allowed.

While extending the theoretical analysis may be a worthwhile exercise, we nonetheless think that empirical work has priority at the moment. We are not aware of a single systematic study where the choice between posting and bargaining has been linked to properties of the firms, the workers, or to market conditions.

6 Conclusion

Casual observation indicate that firms can chose whether to post wages or bargain with individual applicants. We have shown that it is possible to endogenize this choice within a standard search–matching model. In particular, we have identified conditions under which there is an equilibrium in which all firms chose to bargain.

A rough summary of our results would be that efficient matching, high average productivity and large worker heterogeneity make bargaining more likely. This seems to fit the observation that firms are more often willing to negotiate wages for top jobs than for blue collar workers and clerical staff. At the top, workers are highly skilled, and differences in productivity can be very substantial. At the bottom, people have lower skills, and tasks are simpler – often making workers more homogeneous as seen from the employers' perspective.

We think there is ample scope for empirical work in this area. The model's key parameters vary considerably across occupations, and possibly also over the business cycle. If it does turn out that our model is consistent with the facts, it would be an indication that search frictions affect not only the level of wages, but also the method of pay.

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