Transformations of Graphs 2ii

A Level

Part A Sketch the curve: $\frac{1}{x}$

Sketch the curve $y = \frac{1}{x}$. Check your answer by answering the question below.

Does the curve have any lines of mirror symmetry? If yes, give an example.

- Yes, the line $y = x$
- No
- Yes, the y-axis
- Yes, the line $y = -x$
- Yes, the x-axis

Part B Sketch the curve: x^4

Sketch the curve $y = x^4$. Check your answer by answering the question below.

Does the curve have any lines of mirror symmetry? If yes, give an example.

- Yes, the x-axis
- Yes, the y-axis
- No
- Yes, the line $y = -x$
- Yes, the line $y = x$

Part C Transformation

Which TWO of the following describe a single transformation that maps the curve $y = x^3$ onto the curve $y = 8x^3$? Choose one of the two correct answers.

- A stretch of scale factor $\frac{1}{8}$ parallel to the x axis.
- A translation $+8$ units parallel to the y axis.
- A stretch of scale factor $\frac{1}{2}$ parallel to the x axis.
- A stretch of scale factor 8 parallel to the x axis.
- A stretch of scale factor 8 parallel to the y axis
- A stretch of scale factor $\frac{1}{8}$ parallel to the y axis.

Part D Sketch the curve: $-\frac{1}{x}$

Sketch the curve $y = -\frac{1}{x}$. Check your answer by answering the question below.

Does the have have any rotational symmetry about the origin?

- Yes, of order 4
- Yes, of order 2
- No
- Yes, of order 8

Part E State the equation

The curve $y = -\frac{1}{x}$ is translated by $+2$ units parallel to the x-axis in the positive direction. State the equation of the transformed curve.

The following symbols may be useful: `x, y`

Part F Transformation

Which TWO of the following describe a single transformation that maps the curve $y = -\frac{1}{x}$ onto the curve $y = -\frac{1}{3x}$? Choose either one of the correct answers.

- A stretch of scale factor 3 parallel to the x axis.
- A translation by $+3$ units parallel to the x axis.
- A stretch of scale factor $\frac{1}{3}$ parallel to the y axis.
- A stretch of scale factor $\frac{1}{3}$ parallel to the x axis.
- A stretch of scale factor 3 parallel to the y axis.

Used with permission from UCLES, A level, June 2007, Paper 4721 Question 2 and June 2015, Paper 4721, Question 2.

Transformations of Graphs 3ii

A Level

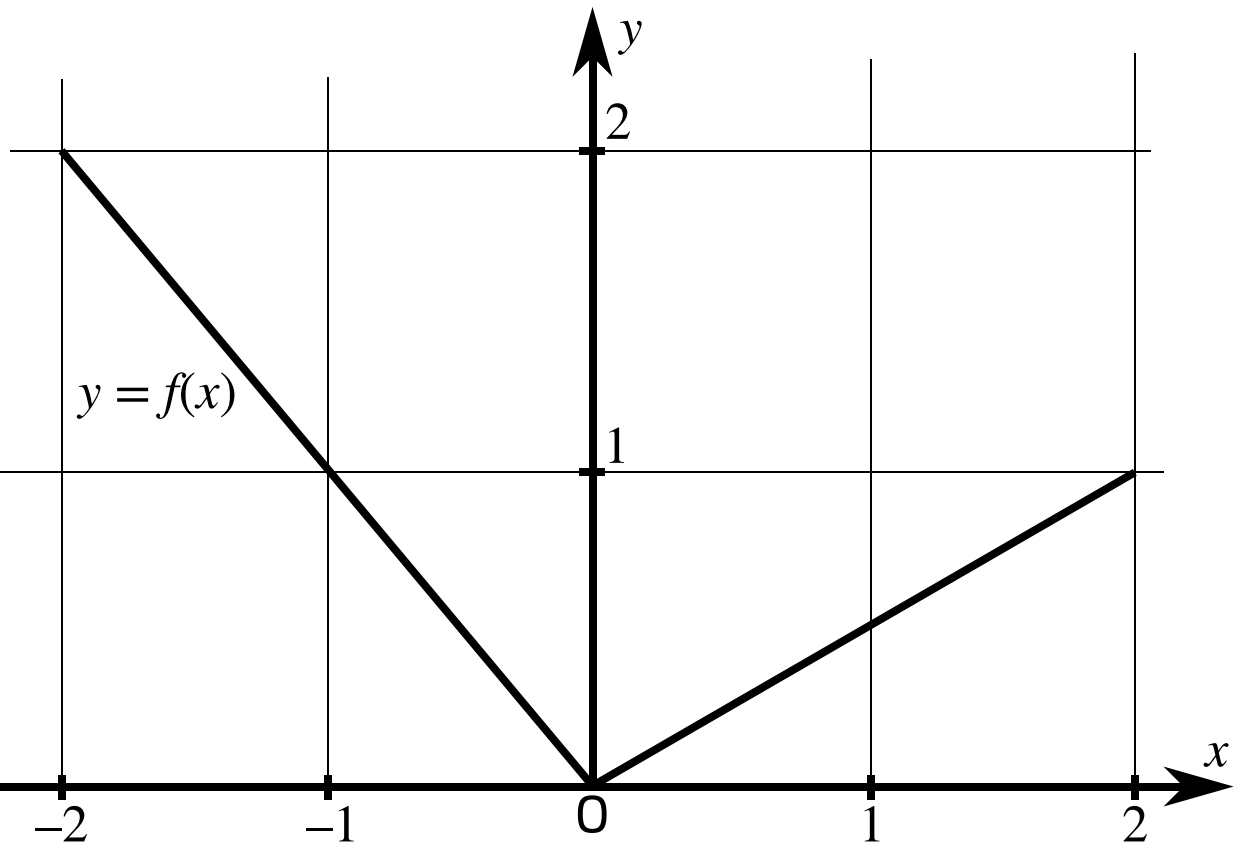

Figure 1: The graph of $y = f(x)$ for $-2 \leq x \leq 2$ is shown to the left.

Part A Sketch $y = f(-x)$

Sketch the curve $y = f(-x)$ for $-2 \leq x \leq 2$.

What is the y-value of the curve $y = f(-x)$ when $x = 1$?

The following symbols may be useful: y

Part B Sketch $y = f(-x) + 2$

Sketch the curve $y = f(-x) + 2$ for $-2 \leq x \leq 2$.

What is the y-value of the curve $y = f(-x) + 2$ when $x = -2$?

The following symbols may be useful: y

Part C Sketch $y = -\frac{1}{x^2}$

Sketch the curve $y = -\frac{1}{x^2}$.

For large negative values of x, the curve $y = -\frac{1}{x^2}$ becomes asymptotic to the horizontal line with which y-value?

The following symbols may be useful: y

Part D Sketch $y = 3 - \frac{1}{x^2}$

Sketch the curve $y = 3 - \frac{1}{x^2}$.

For large negative values of x, the curve $y = 3 - \frac{1}{x^2}$ becomes asymptotic to the horizontal line with which y-value?

The following symbols may be useful: y

Part E State the equation

The curve $y = -\frac{1}{x^2}$ is stretched parallel to the y-axis by scale factor 2. State the equation of the transformed curve.

The following symbols may be useful: x, y

Gameboard:

STEM SMART Single Maths 18 - Graph Transformations and Circles

Transformations of Graphs 1i

A Level

Part A Sketch y

Find the roots of the curve $y = x^2(3 - x)$ and sketch it. You can check your sketch after entering your answer.

Give the value of the root at which y has a minimum.

The following symbols may be useful: `x`

Part B Translate y

The curve $y = x^2(3 - x)$ is translated by two units in the positive direction parallel to the x axis.

State the equation of the curve after this transformation.

The following symbols may be useful: `x, y`

Part C Find transformation of y

Which of these describes the transformation of the curve $y = x^2(3 - x)$ to $y = \frac{1}{2}x^2(3 - x)$?

- A stretch of scale factor $\frac{1}{2}$ parallel to the y-axis.
- A stretch of scale factor 2 parallel to the y-axis.
- A stretch of scale factor 2 parallel to the x-axis.
- A stretch of scale factor $\frac{1}{2}$ parallel to the x-axis.

Part D Vertical translation of $f(x)$

The curve $y = f(x)$ passes through the point P with coordinates $(2, 5)$.

State the coordinates of the point corresponding to P on the curve $y = f(x) + 2$. Enter the x and y coordinates below.

Enter the x coordinate:

The following symbols may be useful: x

Enter the y coordinate:

The following symbols may be useful: y

Part E Lateral stretching of $f(x)$

The curve $y = f(x)$ passes through the point P with coordinates $(2, 5)$.

State the coordinates of the point corresponding to P on the curve $y = f(2x)$. Enter the x and y coordinates below.

Enter the x cooordinate:

The following symbols may be useful: x

Enter the y coordinate:

The following symbols may be useful: y

Part F Find transformation of $f(x)$

Which of the following describes the single transformation that maps the curve $y = f(x)$ onto $y = f(x + 4)$?

- A translation of 4 units parallel to the x-axis.
- A translation of 4 units parallel to the y-axis.
- A translation of -4 units parallel to the y-axis.
- A translation of -4 units parallel to the x-axis.

Used with permission from UCLES, A level, June 2016, Paper 4721, Question 7 and June 2014, Paper 4721, Question 4.

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Lateral and Vertical Translations

A Level

Investigate the transformations of the following functions.

Part A Lateral translation

Consider the function $f(x) = x^2 + 2x + 1$. The function $g(x) = f(x - a)$, where a is a constant. If $g(1) = 9$ find the value of a, given that it is positive.

The following symbols may be useful: a

Part B Vertical translation

Consider the function $r(u) = \dfrac{2}{u - 2}$. The function $s(u) = r(u) + b$, where b is a constant. If $s(0) = 1$, find the value of b.

The following symbols may be useful: b

Part C Lateral and vertical translation

Consider the function $p(r) = \dfrac{1}{r}$. The function $q(r) = p(r - c) + d$, where c and d are constants. If $q(0) = 1$ and $q(2) = 3$, find the values of c and d.

Find the value of c.

The following symbols may be useful: `c`

Find the value of d.

The following symbols may be useful: `d`

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Reflection and Symmetry

A Level

The following questions ask you to deduce the symmetry properties of a number of functions. There are three choices:

- even - a function for which $f(x) = f(-x)$ which is also described as being symmetric about the vertical axis,
- odd - a function for which $f(x) = -f(-x)$ which is also described as being antisymmetric about the vertical axis (or symmetric about zero),
- neither even nor odd.

For more details see the section on Symmetry in
Graph interpreting - Level 2: More powers of x and general polynomials

Where relevant you may assume that a and b are non-zero constants.

Part A Even functions

In one of the following lists of functions all the functions are even. Pick the correct option from the choices below.

- ax^2, $ax^2 + b$, $ax^2 + bx^4$, $\frac{a}{x^2} + bx^2$, $(x - a)(x + a)$, $a\cos x$
- ax^2, $a(x + b)^2$, $x^2(a + bx)$, $\frac{a}{x^2} + bx^2$, $(x - a)(x + a)$, $a\sin x$
- ax^2, $a(x + b)^2$, $ax^2 + bx^4$, $\frac{a}{x^2} + b$, $(x - a)(x + b)$ $(a \neq b)$, $a\sin x$
- ax^2, $ax^2 + b$, $x^2(a + bx)$, $\frac{a}{x^2} + b$, $(x - a)(x + a)$, $a\cos x$
- ax^2, $ax^2 + b$, $ax^2 + bx^4$, $\frac{a}{x^2} + b$, $(x - a)(x + b)$ $(a \neq b)$, $a\sin x$
- ax^2, $a(x + b)^2$, $x^2(a + bx)$, $\frac{a}{x^2} + bx^2$, $(x - a)(x + b)$ $(a \neq b)$, $a\cos x$

Part B Odd functions

In one of the following lists of functions all the functions are odd. Pick the correct option from the choices below.

- $ax, \frac{a}{x}, \frac{a}{x} + bx^3, x(a + bx^2), x^{1/3}, a\sin x$
- $ax, \frac{a}{x}, \frac{a}{x} + bx^3, x^2(a + bx), (x + a)^{1/3}, a\tan x$
- $ax, \frac{a}{x} + b, \frac{a}{x} + bx^3, x^2(a + bx), (x + a)^{1/3}, a\tan x$
- $ax, \frac{a}{x} + b, \frac{a}{x} + \frac{b}{x^3}, x(a + bx^2), (x + a)^{1/3}, a\sin x$
- $ax, \frac{a}{x}, \frac{a}{x} + \frac{b}{x^3}, x^2(a + bx), x^{1/3}, a\sin x$
- $ax, \frac{a}{x} + b, \frac{a}{x} + \frac{b}{x^3}, x(a + bx^2), x^{1/3}, a\tan x$

Part C Neither odd nor even functions

In one of the following lists of functions all the functions are neither odd nor even. Pick the correct option from the choices below.

- $ax - b, x(ax^2 + b), (x - a)(x + a)^2, a\left(\frac{1}{x^2} - \frac{1}{b^2}\right), a(b - x)^{1/2}, \cos x + \sin x$
- $ax - b, x(ax^2 + b), (x - a)(x + a), \frac{a}{(x - b)^2}, a(b - x)^{1/2}, a\tan(x + 45°)$
- $ax - b, x(ax^2 + b), (x - a)(x + a), \frac{a}{(x - b)^2}, ax^{1/2}, \cos x + \sin x$
- $ax - b, x^2(ax + b), (x - a)(x + a), a\left(\frac{1}{x^2} - \frac{1}{b^2}\right), ax^{1/2}, a\tan(x + 45°)$
- $ax - b, x^2(ax + b), (x - a)(x + a)^2, a\left(\frac{1}{x^2} - \frac{1}{b^2}\right), a(b - x)^{1/2}, \cos x + \sin x$
- $ax - b, x^2(ax + b), (x - a)(x + a)^2, \frac{a}{(x - b)^2}, ax^{1/2}, a\tan(x + 45°)$

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Gameboard:
STEM SMART Single Maths 18 - Graph Transformations and Circles

Circles 1ii

A Level

P P P

The circle with equation $x^2 + y^2 - 6x - k = 0$ has radius 4.

The points $A(3, a)$ and $B(-1, 0)$ lie on the circumference of the circle, with $a > 0$.

Part A Centre

By completing the square for x and y find the coordinates of the centre of the circle. Enter the x and y coordinates below.

Enter the x-coordinate:

The following symbols may be useful: x

Enter the y coordinate:

The following symbols may be useful: y

Part B Value of k

Find the value of k.

The following symbols may be useful: k

Part C Length AB

Calculate the length of AB, giving your answer in simplified surd form.

Part D Equation

Find the equation of the line AB. Give your answer in the form $y = mx + c$.

The following symbols may be useful: `x, y`

Used with permission from UCLES, A level, June 2007, Paper 4721, Question 9.

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Circles 3ii

A Level

A circle has centre $(3, 1)$ and radius 5, and a line has equation $y = 2x$.

Part A Circle equation

Write down the equation of the circle.

The following symbols may be useful: `x, y`

Part B Intersection point

Find the coordinates of the point of intersection of the line and the circle with the largest x value.

Give the x-coordinate.

The following symbols may be useful: `x`

Give the y-coordinate.

The following symbols may be useful: `y`

Part C Point on the line

Find the coordinates of the point on the line which is closest to the centre of the circle.

Give the x-coordinate.

The following symbols may be useful: x

Give the y-coordinate.

The following symbols may be useful: y

Used with permission from UCLES, A level, January 2010, Paper 4721, Question 8.

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Circles 2i

A Level

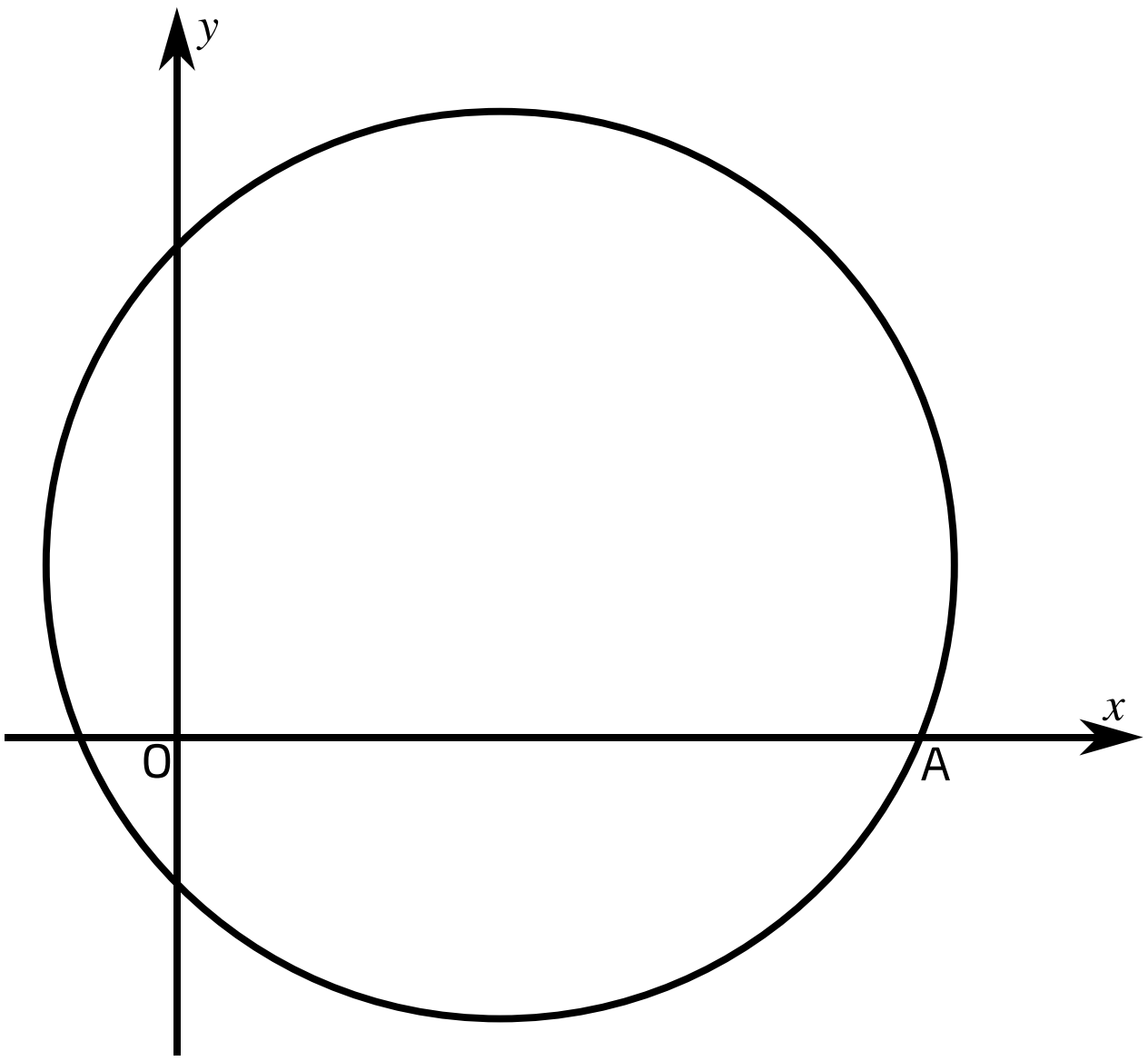

Figure 1: The circle with equation $x^2 + y^2 - 8x - 6y - 20 = 0$.

Figure 1 shows the circle with equation $x^2 + y^2 - 8x - 6y - 20 = 0$. The circle crosses the positive x axis at point A.

Part A Find C

By completing the square for x and y find the coordinates of the centre of the circle. Enter the x and y coordinates below.

Enter the x coordinate:

The following symbols may be useful: x

Enter the y coordinate:

The following symbols may be useful: y

Part B Find radius

Find the radius of the circle.

Part C Tangent to the circle 1

Find the equation of the tangent to the circle at A. Give your answer in the form $y = mx + c$.

The following symbols may be useful: x, y

Part D Tangent to the circle 2

A second tangent to the circle is parallel to the tangent at A. Find the equation of this second tangent in the form $y = mx + c$.

The following symbols may be useful: `x, y`

Part E Find a radius

Another circle has its centre at the origin O and radius r. This circle lies wholly inside the first circle. Find the set of possible values of r. Give your answer as an inequality.

The following symbols may be useful: `<, <=, >, >=, r`
