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STEM SMART Double Maths 36 - Vector Lines, Products & Planes

Angles Between a 3D Vector and the Axes

Subject & topics: Maths | Geometry | Vectors **Stage & difficulty:** A Level P3

Find the angles between the vector $\underline{\boldsymbol{i}} + 2\underline{\boldsymbol{j}} + 3\underline{\boldsymbol{k}}$ and the x-, y- and z-coordinate axes.

Part A

Angle with x-axis

What is the angle in degrees between the vector and the x-axis? Give your answer to 3 sf.

Part B

Angle with y-axis

What is the angle in degrees between the vector and the y-axis? Give your answer to 3 sf.

Part C

Angle with z-axis

What is the angle in degrees between the vector and the z-axis? Give your answer to 3 sf.

Adapted for Isaac Physics from NST IA Biology preparation work

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Scalar Product 1

Pre-Uni Maths for Sciences I2.3

Subject & topics: Maths | Geometry | Vectors **Stage & difficulty:** Further A P1

Find the scalar product $\underline{\boldsymbol{a}} \cdot \underline{\boldsymbol{b}}$, where $\underline{\boldsymbol{a}} = \underline{\boldsymbol{i}} + 2\underline{\boldsymbol{j}} + 4\underline{\boldsymbol{k}}$ and $\underline{\boldsymbol{b}} = 2\underline{\boldsymbol{i}} - 3\underline{\boldsymbol{j}} + \underline{\boldsymbol{k}}$. Hence, deduce the angle between $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$. Give your answer to 3 sf.

Created for isaacphysics.org by Julia Riley.

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Vectors: Geometry 1i

Subject & topics: Maths **Stage & difficulty:** Further A P2

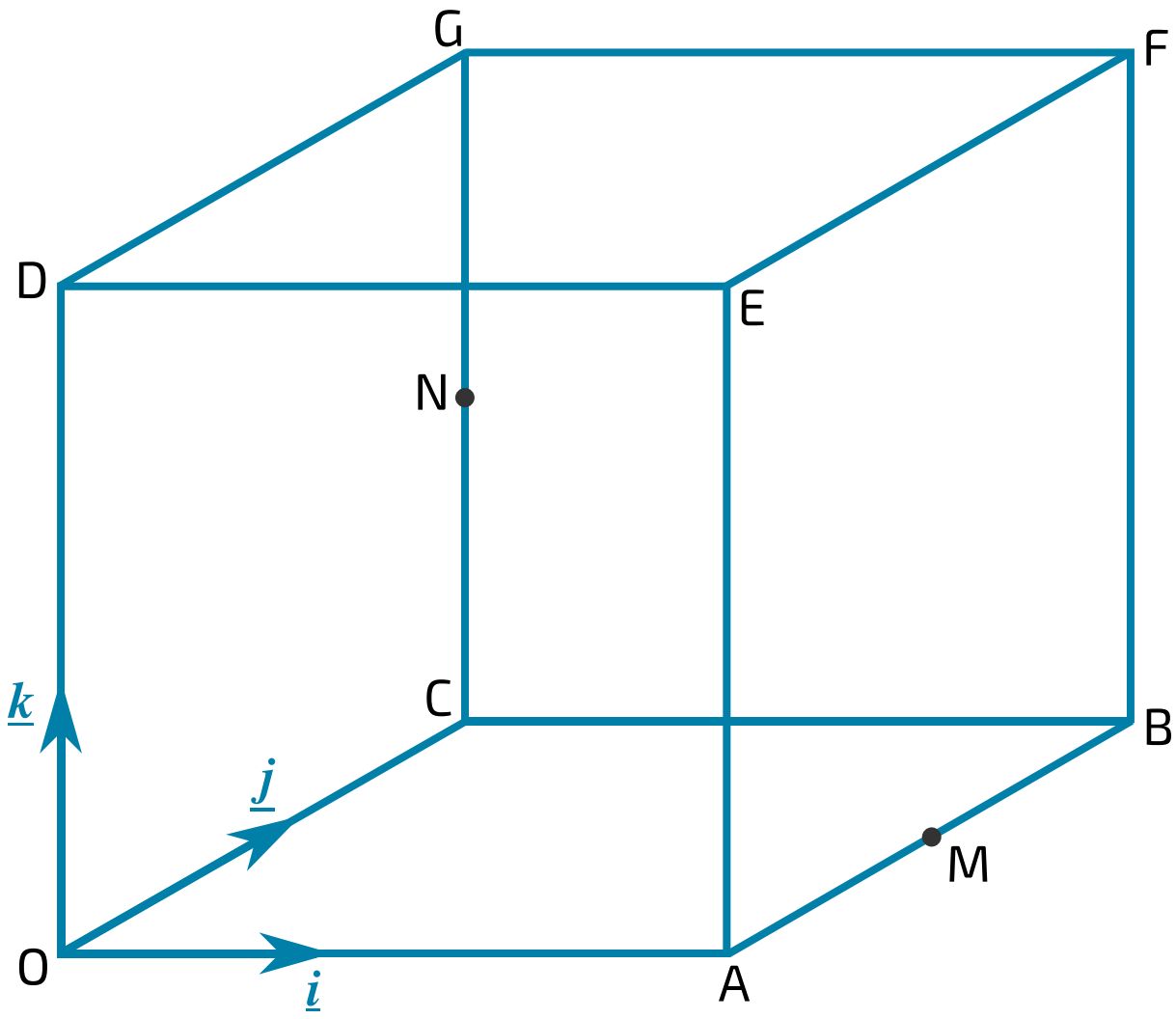

Figure 1: The cube OABCDEFG with side length 2 units.

The diagram shows a cube OABCDEFG with sides of length 2 units. Unit vectors $\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{k}}$ are directed along OA, OC and OD respectively. The midpoint of AB is M and the midpoint of CG is N.

The point P on the line MN is such that $\overrightarrow{\text{MP}} = 2\overrightarrow{\text{PN}}$.

Part A

P

Give the coordinates of P in the form

$$\overrightarrow{OP} = \frac{1}{3}(a\underline{\boldsymbol{i}} + b\underline{\boldsymbol{j}} + c\underline{\boldsymbol{k}})$$

$$\overrightarrow{OP} = \frac{1}{3} \quad \square \ \underline{\boldsymbol{i}} + \square \ \underline{\boldsymbol{j}} + \square \ \underline{\boldsymbol{k}}$$

Part B

Acute angle

Find the acute angle between OP and MN to 3 significant figures.

Part C

Intersection

To say a straight line XY is "produced" means that the line continues on beyond the second point stated, Y. For example, the line "OP produced" starts at O, goes from O to P, and then continues on in a straight line beyond P.

The lines "OP produced" and "EF produced" intersect.

Find the coordinates of the point of intersection. Give your answer in the form (x, y, z).

$$(\square , \square , \square)$$

Adapted with permission from UCLES, A Level, June 2004, Paper 2633, Question 6.

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Vectors: Intersection or Skew 2i

Subject & topics: Maths **Stage & difficulty:** Further A P2

The lines l_1 and l_2 have the equations

$$\underline{\boldsymbol{r}} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + s \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$$

and

$$\underline{\boldsymbol{r}} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

respectively.

Part A

Do they meet?

Do l_1 and l_2 intersect?

- They are parallel lines.
- They intersect at a point.
- They are skew lines.

Part B

Acute angle

Find the acute angle between l_1 and l_2 to 3 significant figures, in degrees.

Part C

a

One of the numbers in the equation of line l_1 is changed so that the equation becomes

$$\underline{\boldsymbol{r}} = \begin{pmatrix} 3 \\ 0 \\ a \end{pmatrix} + s \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$$

l_1 and l_2 now intersect for some constant a.

Find a.

The following symbols may be useful: a

Adapted with permission from UCLES, A Level, Jan 2011, Paper 4724, Question 6.

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Vectors: Perpendiculars 1i

Subject & topics: Maths **Stage & difficulty:** Further A P2

A straight line is given by the equation $\underline{\boldsymbol{r}} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$. O is the origin.

Part A

Acute angle

The point P on the line is given by $t = 1$.

Calculate the acute angle between OP and the line. Give your answer to 3 significant figures.

Part B

Q

The point Q on the line is located such that OQ is perpendicular to the line. Find the position vector of Q.

$\overrightarrow{OQ} = \frac{1}{3}($ ☐ $\underline{\boldsymbol{i}} +$ ☐ $\underline{\boldsymbol{j}} +$ ☐ $\underline{\boldsymbol{k}}$ $)$

Part C

OQ

Find the length of OQ in exact form.

Adapted with permission from UCLES, A Level, Jan 2010, Paper 4724, Question 9.

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STEM SMART Double Maths 36 - Vector Lines, Products & Planes

Vectors: Lines and Planes 1ii

Subject & topics: Maths **Stage & difficulty:** Further A P2

Two intersecting lines, lying in a plane p , have equations:

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{-3} \quad \text{and} \quad \frac{x-1}{-1} = \frac{y-3}{2} = \frac{z-4}{4}.$$

Part A

Finding the equation of p

Obtain the equation of p in the form $2x + by + z = d$.

The following symbols may be useful: `x, y, z`

Part B

Distance between p and q

Plane q has equation $2x - y + z = 21$. Find the perpendicular distance between p and q.

Adapted with permission from UCLES, A Level, January 2011, Paper 4727, Question 2.

Question deck:

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Vectors: Intersecting Planes 3i

Subject & topics: Maths **Stage & difficulty:** Further A P3

The plane Π_1 has equation $\underline{\boldsymbol{r}} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix}$

Part A
Equation of Π_1

Express the equation of Π_1 in the form $\underline{\boldsymbol{r}} \cdot \underline{\boldsymbol{n}} = p$ where:

$$\underline{\boldsymbol{n}} = \underline{\boldsymbol{i}} + a_y \underline{\boldsymbol{j}} + a_z \underline{\boldsymbol{k}}$$

What is $\underline{\boldsymbol{n}}$? Write your answer in the form $\underline{\boldsymbol{n}} = \underline{\boldsymbol{i}} + a_y \underline{\boldsymbol{j}} + a_z \underline{\boldsymbol{k}}$.

$$\underline{\boldsymbol{n}} = \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}}$$

What is p?

$$p = \square$$

Part B
Intersection of Π_1 and Π_2

The plane Π_2 has equation $\underline{\boldsymbol{r}} \cdot \begin{pmatrix} 7 \\ 17 \\ -3 \end{pmatrix} = 21.$

Find an equation of the line of intersection of Π_1 and Π_2.

Give your answer in the form $\underline{\boldsymbol{r}} = (3\underline{\boldsymbol{i}} + a_y \underline{\boldsymbol{j}} + a_z \underline{\boldsymbol{k}}) + t(2\underline{\boldsymbol{i}} + b_y \underline{\boldsymbol{j}} + b_z \underline{\boldsymbol{k}})$.

$$\underline{\boldsymbol{r}} = (3\underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}}) + t\,(2\underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}})$$

Adapted with permission from UCLES, A Level, June 2019, Paper 4727, Question 6.

Question deck:

STEM SMART Double Maths 36 - Vector Lines, Products & Planes

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STEM SMART Double Maths 36 - Vector Lines, Products & Planes

Vectors: Angles and Distances 1i

Subject & topics: Maths **Stage & difficulty:** Further A P2

The plane Π has equation $x + 2y - 2z = 5$.

The line l has equation $\frac{x-1}{2} = \frac{y+1}{5} = \frac{z-2}{1}$.

Part A

Intersection of l and Π

Find the coordinates of the point of intersection of l with the plane Π.

If a value is not a whole number, enter the value as a decimal.

(☐ , ☐ , ☐)

Part B

Angle between l and Π

Find the acute angle between l and Π. Give your answer to 3 significant figures.

Part C

Points on l Equidistant From Π

Find the position vector of the two points on the line l for which the shortest distance from the line to the plane Π is 2.

Give your answer in the form:

$$(a_x\underline{\boldsymbol{i}} + a_y\underline{\boldsymbol{j}} + a_z\underline{\boldsymbol{k}}) \pm (b_x\underline{\boldsymbol{i}} + b_y\underline{\boldsymbol{j}} + b_z\underline{\boldsymbol{k}})$$

If a value is not a whole number, enter the value as a decimal.

$$(\square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}}) \pm (\square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}})$$

Adapted with permission from UCLES, A Level, June 2013, Paper 4727, Question 6.

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STEM SMART Double Maths 36 - Vector Lines, Products & Planes

Vectors: Angles and Distances 3ii

Subject & topics: Maths **Stage & difficulty:** Further A P3

Part A

Distance between two lines

Find the shortest distance between the lines $\underline{\boldsymbol{r}} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\underline{\boldsymbol{r}} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$.

Part B

Distance from a point to a plane

Find the shortest distance from the point $(3, -1, -2)$ to the plane with equation $x - 2y + 4z = 11$.

Part C

Equation of a plane

Find a cartesian equation of the plane which passes through the point $(3, -1, -2)$ and is parallel to the plane $x - 2y + 4z = 11$.

Give your answer in the form $x + by + cz = d$.

The following symbols may be useful: `x, y, z`

Adapted with permission from UCLES, A Level, June 2018, Paper 4727, Question 2.

Question deck:

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STEM SMART Double Maths 36 - Vector Lines, Products & Planes

Vector Area

Pre-Uni Maths for Sciences I3.3

Subject & topics: Maths | Geometry | Vectors **Stage & difficulty:** Further A P3

The vector $\underline{\boldsymbol{c}} = \underline{\boldsymbol{a}} \times \underline{\boldsymbol{b}}$, where $\underline{\boldsymbol{a}} = 4\underline{\boldsymbol{i}} + \underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{b}} = \underline{\boldsymbol{i}} + 4\underline{\boldsymbol{j}}$.

Part A

Find $\underline{\boldsymbol{c}}$

Find the vector $\underline{\boldsymbol{c}}$.

Drag and drop the correct values below.

$$\underline{\boldsymbol{c}} = \square\,\underline{\boldsymbol{i}} + \square\,\underline{\boldsymbol{j}} + \square\,\underline{\boldsymbol{k}}$$

Items:

-15 -10 -5 -1 0 1 5 10 15

Part B

Vector area of a parallelogram 1

Hence find the area of the parallelogram with sides $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$.

Part C

Find ϕ

Find the angle between the vectors $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$. Give your answer to the nearest degree.

Part D

Vector area of a parallelogram 2

The vector area $\underline{\boldsymbol{A}}$ of the parallelogram with sides $\underline{\boldsymbol{p}} = \underline{\boldsymbol{i}} + \underline{\boldsymbol{j}} + \underline{\boldsymbol{k}}$ and $\underline{\boldsymbol{q}} = 2\underline{\boldsymbol{i}} + 3\underline{\boldsymbol{j}}$ is equal to $\underline{\boldsymbol{p}} \times \underline{\boldsymbol{q}}$.

Find the vector $\underline{\boldsymbol{A}}$.

Drag and drop the correct values below.

$$\underline{\boldsymbol{A}} = \square\,\underline{\boldsymbol{i}} + \square\,\underline{\boldsymbol{j}} + \square\,\underline{\boldsymbol{k}}$$

Items:

-4 -3 -2 -1 0 1 2 3 4

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