Area Between Two Curves 3i

A Level

Figure 1 shows parts of the curves $y = 11 - x - 2x^2$ and $y = \frac{8}{x^3}$.

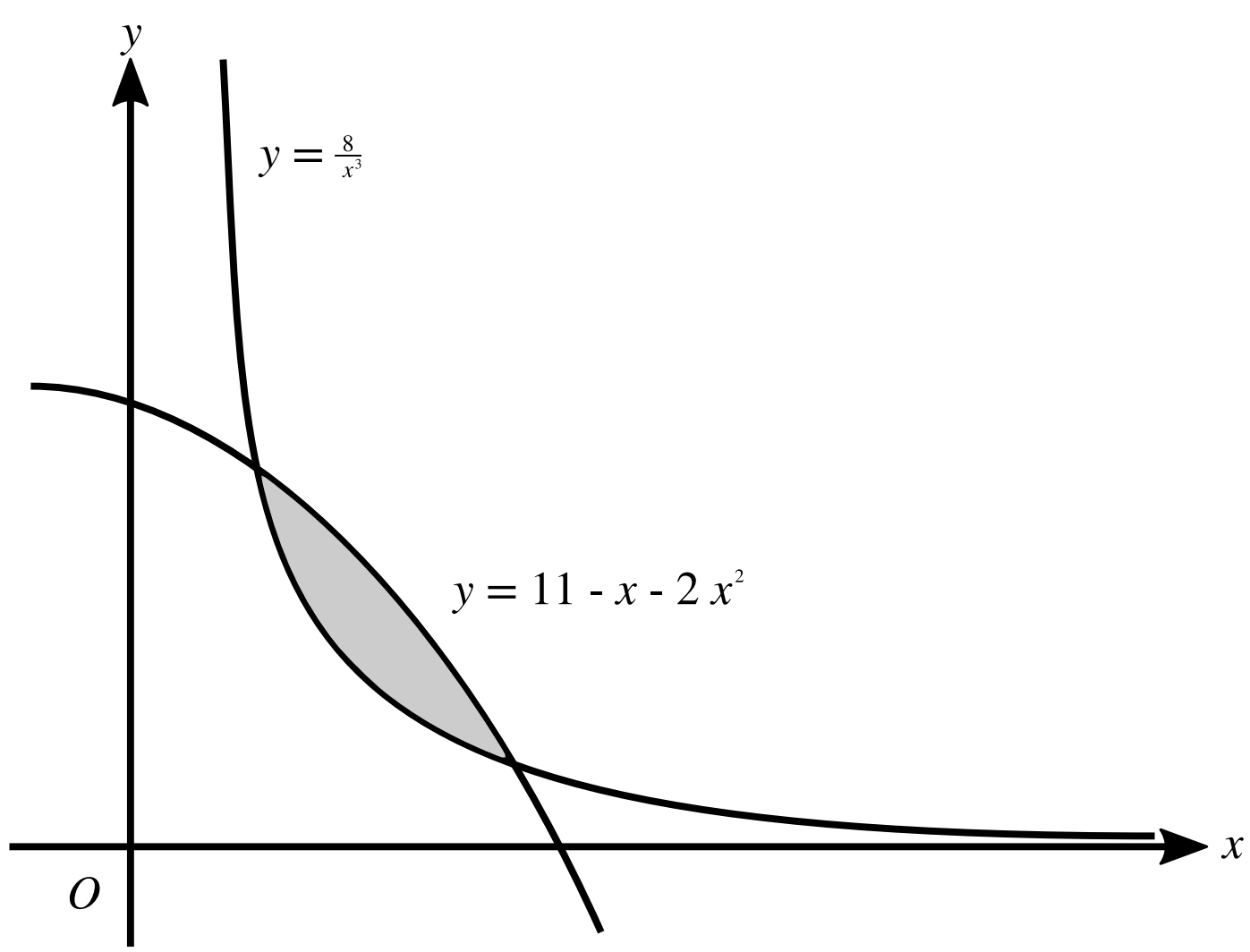

Figure 1: Parts of the curves $y = 11 - x - 2x^2$ and $y = \frac{8}{x^3}$.

Use integration to find the exact area of the shaded region enclosed between the two curves, given that the curves intersect at $(1, 8)$ and $(2, 1)$.

Modelling - Advanced 1ii

A Level

A forest is burning so that, t hours after the start of the fire, the area burnt is A hectares. It is given that, at any instant, the rate at which this area is increasing is proportional to A^2.

Part A Differential equation

Write down a differential equation which models this situation. Your answer should include an unknown constant k.

The following symbols may be useful: `A, Derivative(A, t), k, t`

Part B Time taken to burn an area

After 1 hour, $1000\,\text{hectares}$ have been burnt; after 2 hours, $2000\,\text{hectares}$ have been burnt. After how many hours have $3000\,\text{hectares}$ have been burnt? Give your answer as an exact fraction.

Gameboard:

STEM SMART Single Maths 50 - Synoptic Revision 1

Functions and Algebra 1i

A Level

The functions f and g are defined by

$$f(x) = 2\sin x \text{ for } -\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi,$$

$$g(x) = 4 - 2x^2 \text{ for } x \in \mathbb{R}.$$

Part A Range of f

Find the range of f.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $y < a$
- $y \leq a$
- $y > a$
- $y \geq a$
- $a < y < b$
- $a \leq y \leq b$
- $y < a$ or $y > b$
- $y \leq a$ or $y \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

Part B Range of g

Find the range of g.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $y < a$
- $y \leq a$
- $y > a$
- $y \geq a$
- $a < y < b$
- $a \leq y \leq b$
- $y < a$ or $y > b$
- $y \leq a$ or $y \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

Part C Function of a function

Give the value of $g\big(f(0.5)\big)$ correct to 3 significant figures.

Explain why $f\big(g(0.5)\big)$ is not defined.

Easier question?

Part D $f^{-1}\big(g(x)\big)$

Find the set of values of x for which $f^{-1}\big(g(x)\big)$ is defined.

Give the largest value of x for which $f^{-1}\big(g(x)\big)$ is defined in the form $x < a$ or $x \leq a$.

The following symbols may be useful: `<, <=, >, >=, x`

Give the lowest value of x for which $f^{-1}\big(g(x)\big)$ is defined in the form $x > a$ or $x \geq a$.

The following symbols may be useful: `<, <=, >, >=, x`

Give either of the centre bounds for the domain of $f^{-1}\big(g(x)\big)$.

The following symbols may be useful: `, <, <=, >, >=, x`

Gameboard:
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Newton-Raphson Method 1i

A Level

It is given that the equation $x^4 - 2x - 1 = 0$ has only one positive root α, and that $1.3 < \alpha < 1.5$.

Part A Iteration

Figure 1 shows a sketch of $y = x$ and $y = \sqrt[4]{2x+1}$ for $x \geq 0$. Use the iteration $x_{n+1} = \sqrt[4]{2x_n+1}$ with $x_1 = 1.35$ to find x_2 and x_3, correct to 5 significant figures. On the copy of the diagram show how the iteration converges to α.

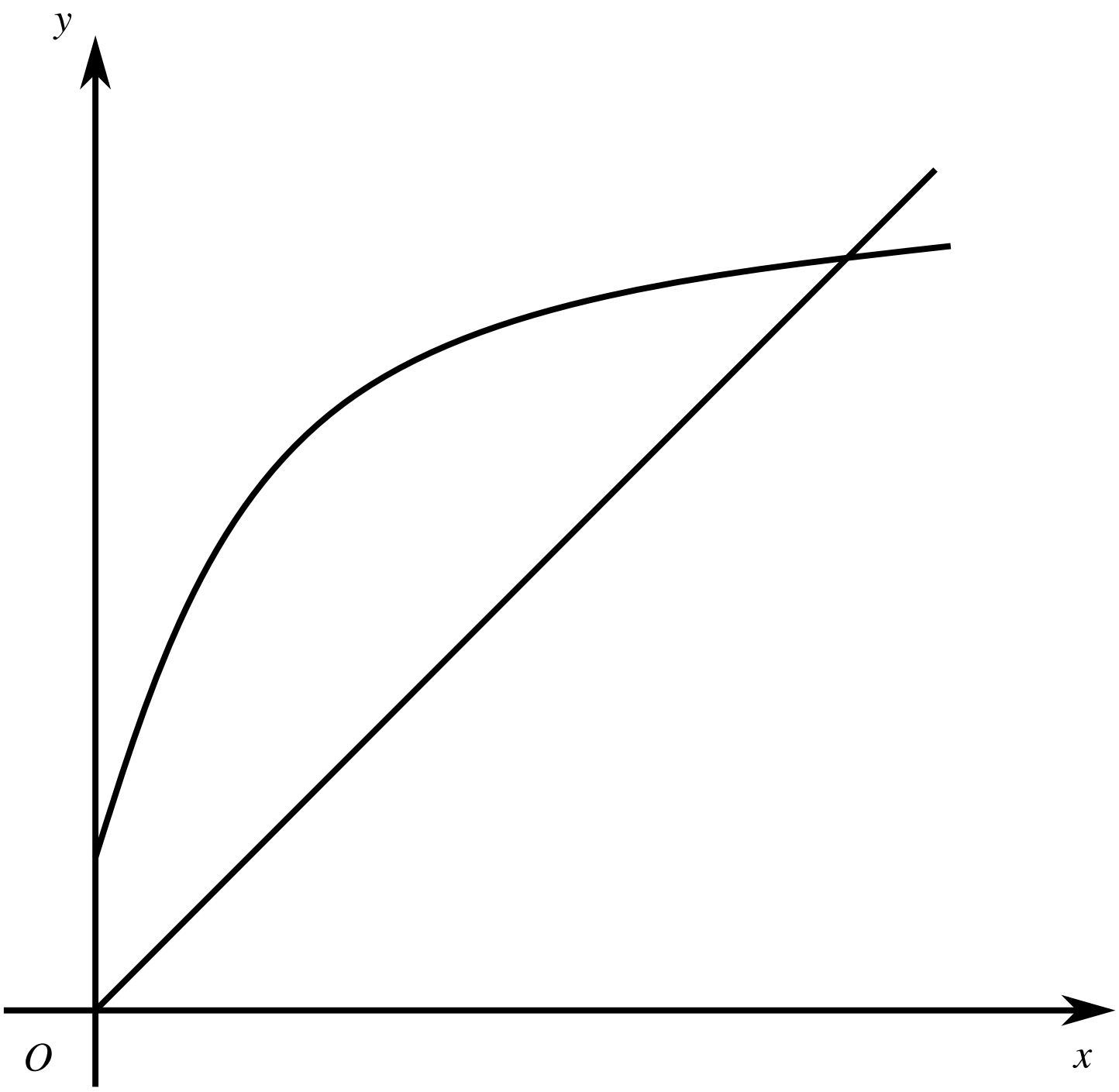

Figure 1: $y = x$ and $y = \sqrt[4]{2x+1}$

Find x_2 correct to 5 significant figures.

Find x_3 correct to 5 significant figures.

On a copy of the diagram show how the iteration converges to α.

Easier question?

Part B Sketch

For the same equation, the iteration $x_{n+1} = \frac{1}{2}(x_n^4 - 1)$ with $x_1 = 1.35$ gives $x_2 = 1.1608$ and $x_3 = 0.4071$, correct to 4 decimal places. Draw a sketch of $y = x$ and $y = \frac{1}{2}(x^4 - 1)$ for $x \geq 0$ and show how this iteration does not converge to α.

Easier question?

Part C Root

Find the positive root of the equation $x^4 - 2x - 1 = 0$ by using the Newton-Raphson method with $x_1 = 1.35$, giving the root correct to 5 significant figures.

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Integration by Substitution 3ii

A Level

By first using the substitution $t = \sqrt{x+1}$, find $\int \mathrm{e}^{2\sqrt{x+1}}\mathrm{d}x$.

The following symbols may be useful: `I, c, e, x`

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Pulley with Three Masses

A Level

C C C

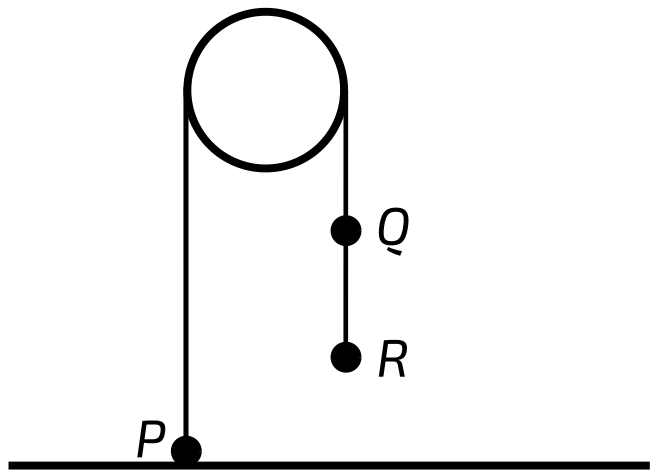

Figure 1: Particles P, Q, and R attached along a string that passes over a pulley.

Particles P and Q, of masses $m\,\mathrm{kg}$ and $0.05\,\mathrm{kg}$ respectively, are attached to the ends of a light inextensible string which passes over a smooth pulley. Q is attached to a particle R of mass $0.45\,\mathrm{kg}$ by a light inextensible string. The strings are taut, and the portions of the strings not in contact with the pulley are vertical. P is in contact with a horizontal surface when the particles are released from rest. The tension in the string QR is $2.52\,\mathrm{N}$ during the descent of R.

Part A Light string

The string is described as light. Explain how this modelling assumption affects calculations when **treating the whole system as a single particle**.

Easier question?

The string is described as light. Explain how this modelling assumption affects calculations when **finding the tension in a vertical string**.

Easier question?

Part B Acceleration of R

Find the acceleration of R during its descent. Give your answer to 2 significant figures.

Part C Tension in PQ

By considering the motion of Q, calculate the tension in the string PQ during the descent of R. Give your answer to 2 significant figures.

Part D Finding m

Find the value of m. Give your answer to 2 significant figures.

Part E Max height of P

R strikes the surface $0.5\,\mathrm{s}$ after release and does not rebound. During their subsequent motion, P does not reach the pulley and Q does not reach the surface.

Calculate the greatest height of P above the surface. Give your answer to 2 significant figures.

Adapted with permission from UCLES, A Level Maths, January 2012, OCR M1, Question 7