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<u>Home</u>   **Gameboard**   Maths   Differentiation: Products 3ii

Differentiation: Products 3ii

A Level

Find the exact value of the x-coordinate of the stationary point of the curve $y = x \ln x$.

The following symbols may be useful: `e, x`

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Differentiation: Quotients 3ii

A Level

P P P

Part A   Differentiate

A curve has equation $y = \frac{2x+1}{3x-1}$. Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part B   Tangent

Hence find the equation of the tangent to this curve at the point $(1, \frac{3}{2})$, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

The following symbols may be useful: `x, y`

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Area Between Two Curves 1ii

A Level

P P P

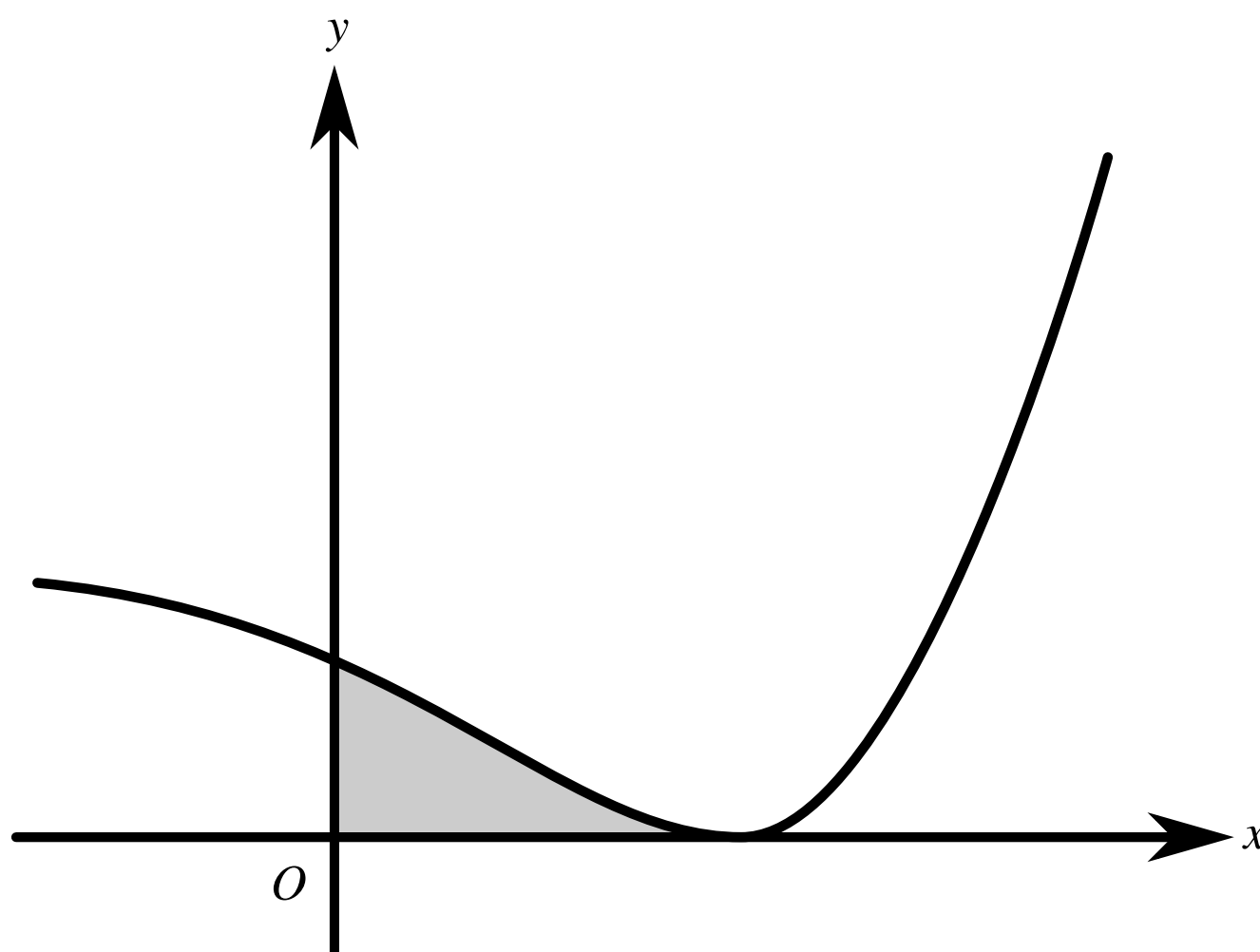**Figure 1** shows the curve $y = e^{3x} - 6e^{2x} + 32$.

Figure 1: The curve $y = e^{3x} - 6e^{2x} + 32$.

Part A x-coordinate

Give the exact x-coordinate of the minimum point and verify that the y-coordinate of the minimum point is 0.

The following symbols may be useful: x

Part B   Area of shaded region

Find the exact area of the shaded region enclosed by the curve and the coordinate axes.
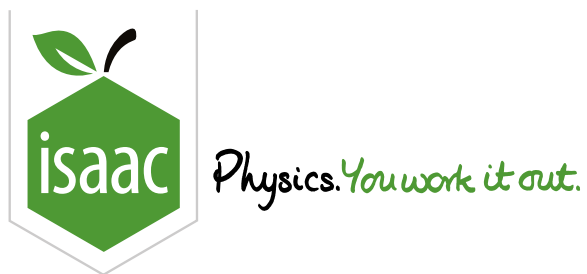
The following symbols may be useful: `ln(), log()`

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Home   Gameboard   Maths   Integration by Substitution 3i

Integration by Substitution 3i

A Level

Part A   Substitution

Find the expression that appears to the right of the integral sign after the substitution $u = e^x + 1$ has been applied to $\displaystyle\int \frac{e^{2x}}{e^x + 1}\, dx$. Include du in your answer.

The following symbols may be useful: `du, u`

Part B   Integral

Hence find the exact value of $\displaystyle\int_0^1 \frac{e^{2x}}{e^x + 1}\, dx$.

The following symbols may be useful: `e`

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Home   Gameboard   Maths   Differentiation: Chain Rule 4i

Differentiation: Chain Rule 4i

A Level
P P P

Earth is being added to a pile so that, when the height of the pile is h metres, its volume is V cubic metres, where

$$V = (h^6 + 16)^{\frac{1}{2}} - 4.$$

Part A   Rate of change of volume

Find the value of $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 2$, to three significant figures.
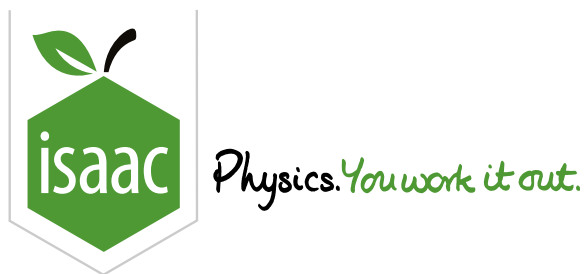
Part B   Rate of change of height

The volume of the pile is increasing at a constant rate of 8 cubic metres per hour. Find the rate in metres per hour, at which the height of the pile is increasing at the instant when $h = 2$. Give your answer correct to 2 significant figures.

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Parametric Equations 4i

A Level

P P P

A curve has parametric equations

$$x = 2\sin t, \qquad y = \cos 2t + 2\sin t$$

for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

Part A   Derivative

Find $\frac{dy}{dx}$ as a function of t.

The following symbols may be useful: `Derivative(y, x), cos(), cosec(), cot(), sec(), sin(), t, tan(), x, y`

Part B   Coordinates

Find the x-coordinate of the stationary point.

The following symbols may be useful: `x`

Find the y-coordinate of the stationary point.

The following symbols may be useful: `y`

Part C   Equation

Find the Cartesian equation of the curve.

The following symbols may be useful: x, y

Part D   Range

Find the range of values x can take.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- ⚪ $x < a$
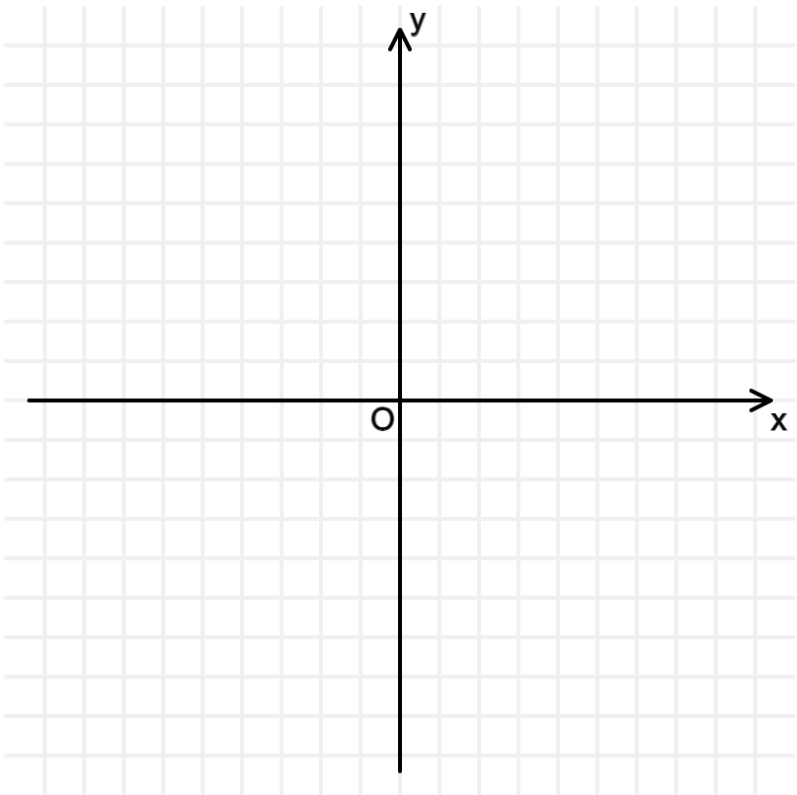
- ⚪ $x \leq a$

- ⚪ $x > a$

- ⚪ $x \geq a$

- ⚪ $a < x < b$

- ⚪ $a \leq x \leq b$

- ⚪ $x < a$ or $x > b$

- ⚪ $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

Part E   Sketch

Hence sketch the curve.

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Modelling – Advanced 2ii

A Level

At time t seconds, the radius of a spherical balloon is $r\,\mathrm{cm}$. The balloon is being inflated so that the rate of increase of its radius is inversely proportional to the square root of its radius. When $t = 5$, $r = 9$ and, at this instant, the radius is increasing at $1.08\,\mathrm{cm\,s^{-1}}$.

Part A   Differential equation

Write down a differential equation to model this situation. Your answer should include a constant k, whose value you do not need to determine yet.

The following symbols may be useful: `Derivative(r, t), k, r, t`

Part B   Solution

Solve the differential equation to express r in terms of t. Your answer should include some numerical constants, which should be converted to exact fractions.

The following symbols may be useful: `r, t`

Part C   Initial condition

How much air (in $\mathrm{cm^3}$) is in the balloon initially? Write your answer as an exact expression.

The following symbols may be useful: `pi`

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Differentiation: Implicit 4i

A Level

Find the equation of the normal to the curve $x^3 + 2x^2y = y^3 + 15$ at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.
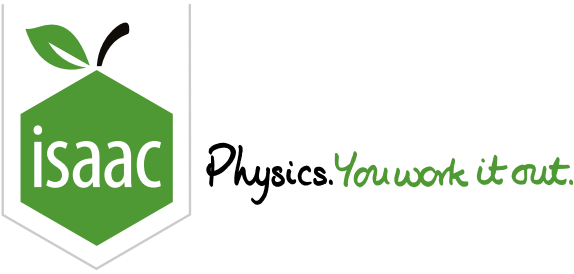
The following symbols may be useful: x, y

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Integration by Parts 3i

A Level

Find $\displaystyle\int_0^\pi \left(x^2 + 5x + 7\right) \sin x \, \mathrm{d}x$.

The following symbols may be useful: `pi`

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