Powers Using Chain Rule 1

A Level P P P Further A P P P

Part A Differentiate $w = (4s + 3)^3$

Find $\frac{\mathrm{d}w}{\mathrm{d}s}$ if $w = (4s + 3)^3$.

The following symbols may be useful: `s`

Part B First derivative of $z = (b - aw)^4$

Find $\frac{\mathrm{d}z}{\mathrm{d}w}$ when $z = (b - aw)^4$, where a and b are constants.

The following symbols may be useful: `a, b, w`

Part C Second derivative of $z = (b - aw)^4$

Find $\frac{\mathrm{d}^2z}{\mathrm{d}w^2}$ when $z = (b - aw)^4$, where a and b are constants.

The following symbols may be useful: `a, b, w`

Created for isaacphysics.org by Julia Riley

Differentiating Exponentials 1

A Level P P P Further A P P P

Part A Differentiate $\beta \mathrm{e}^{-\alpha t}$

Differentiate $\beta \mathrm{e}^{-\alpha t}$ with respect to t, where α and β are constants.

The following symbols may be useful: `alpha, beta, e, t`

Part B Differentiate $C \mathrm{e}^{\beta m} + D$

Differentiate $C \mathrm{e}^{\beta m} + D$ with respect to m, where β, C and D are constants.

The following symbols may be useful: `C, D, beta, e, m`

Created for isaacphysics.org by Julia Riley

Gameboard:

STEM SMART Single Maths 32 - The Chain Rule

Differentiating Trig Functions 2

A Level P P P Further A P P P

Part A Differentiate $s = r\sin(\alpha\theta)$

Find $\dfrac{\mathrm{d}s}{\mathrm{d}\theta}$ if $s = r\sin(\alpha\theta)$ and r and α are constants.

The following symbols may be useful: `alpha, r, theta`

Part B Differentiate $q = l\cos(\alpha - 2\beta\theta)$

Find $\dfrac{\mathrm{d}q}{\mathrm{d}\theta}$ if $q = l\cos(\alpha - 2\beta\theta)$ and l, α and β are constants.

The following symbols may be useful: `alpha, beta, l, theta`

Created for isaacphysics.org by Julia Riley

Gameboard:

STEM SMART Single Maths 32 - The Chain Rule

Differentiating Natural Logs

A Level P P P Further A P P P

Part A Differentiate $u = \ln(2v + 3)$

Find $\frac{\mathrm{d}u}{\mathrm{d}v}$ if $u = \ln(2v + 3)$.

The following symbols may be useful: v

Part B Stationary point of $p = 2\ln(2q) - 3q$

Find the coordinates and nature of the stationary point of the function $p = 2\ln(2q) - 3q$.

Find the q coordinate of the stationary point.

The following symbols may be useful: q

Find the p coordinate of the stationary point.

The following symbols may be useful: p

Determine the nature of the stationary point.

- Minimum
- Maximum

Differentiation: Chain Rule 1ii

A Level

The volume, $V\,\mathrm{m}^3$, of liquid in a container is given by

$$V = \left(3h^2 + 4\right)^{\frac{3}{2}} - 8$$

where $h\,\mathrm{m}$ is the depth of the liquid.

Part A Rate of Change (a)

Find the value of $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 0.6$, giving your answer to four significant figures.

Part B Rate of Change (b)

Liquid is leaking from the container. It is observed that, when the depth of the liquid is $0.6\,\mathrm{m}$, the depth is decreasing at a rate of $0.015\,\mathrm{m}$ per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is $0.6\,\mathrm{m}$. Answer to four significant figures.

Differentiation: Chain Rule 2ii

A Level

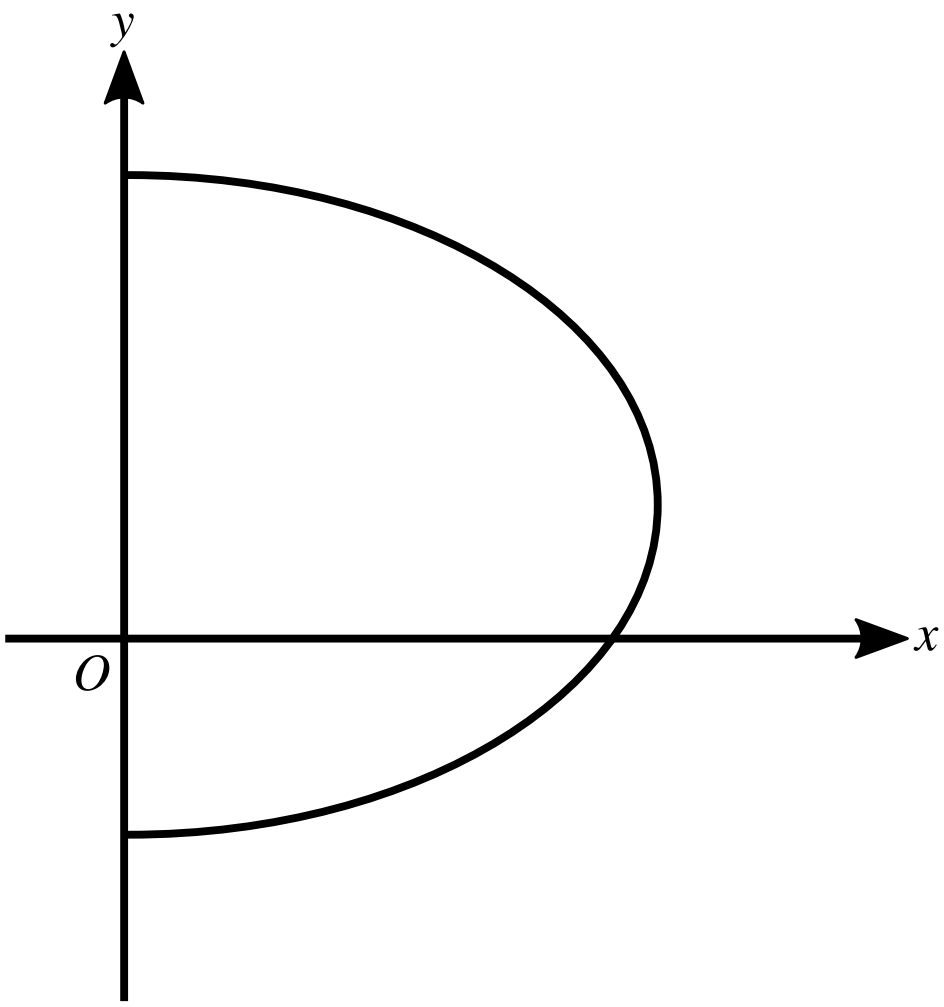

Figure 1: The curve with equation $x = \left(37 + 10y - 2y^2\right)^{\frac{1}{2}}$.

Figure 1 shows the curve with equation $x = \left(37 + 10y - 2y^2\right)^{\frac{1}{2}}$.

Part A Differentiate

Find an expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

The following symbols may be useful: `Derivative(x, y), x, y`

Part B Tangent

Hence find the equation of the tangent to the curve at the point $(7, 3)$, giving your answer in the form $y = mx + c$.

The following symbols may be useful: `x, y`

Gameboard:

STEM SMART Single Maths 32 - The Chain Rule

Differentiation: Chain Rule 3ii

A Level

P P P

Part A Derivatives

Given that $x = (4t + 9)^{\frac{1}{2}}$ and $y = 6\mathrm{e}^{\frac{1}{2}x+1}$, find expressions for $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Give the expression for $\frac{\mathrm{d}x}{\mathrm{d}t}$.

The following symbols may be useful: `Derivative(x, t), t, x`

Give the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), e, ln(), log(), x, y`

Part B $\frac{\mathrm{d}y}{\mathrm{d}t}$

Hence find the value of $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $t = 4$, giving your answer correct to three significant figures.

Gameboard:

STEM SMART Single Maths 32 - The Chain Rule

Differentiating Exponentials 3

A Level P P P Further A P P P

Part A Tangent to $y = \mathrm{e}^{2x} - \mathrm{e}^{-2x}$

Find the equation of the tangent to the curve $y = \mathrm{e}^{2x} - \mathrm{e}^{-2x}$ at the point $x = \frac{1}{2}$.

The following symbols may be useful: `e, x, y`

Part B Stationary point of $u = 2\mathrm{e}^{3v} - 3v$

Find the coordinates and nature of the stationary point of the function $u = 2\mathrm{e}^{3v} - 3v$.

Find the v coordinate of the stationary point.

The following symbols may be useful: `v`

Find the u coordinate of the stationary point.

The following symbols may be useful: `u`

Determine the nature of the stationary point.

- Maximum
- Minimum

Chain Rule 2

A Level P P P Further A P P P

Part A Differentiate $E = B\sin^2(\omega t)$.

Find $\dfrac{\mathrm{d}E}{\mathrm{d}t}$ if $E = B\sin^2(\omega t)$, where B and ω are constants.

The following symbols may be useful: `B, E, omega, t`

Part B Differentiate $y = \mathrm{e}^{-\frac{x^2}{2\sigma^2}}$

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \mathrm{e}^{-\frac{x^2}{2\sigma^2}}$, where σ is a constant.

The following symbols may be useful: `e, sigma, x`

Created for isaacphysics.org by Julia Riley

Gameboard:

STEM SMART Single Maths 32 - The Chain Rule

Further Derivatives of Exponentials and Logarithms

A Level P P P Further A P P P

This question uses the chain rule to find the derivatives of several functions involving exponentials and logarithms.

Part A Rewriting a^x

Use the rules for exponentials and logarithms to write $y = a^x$, where a is a positive constant, in the form $y = e^{bx}$, where b is a constant. Enter an expression for b in terms of a.

The following symbols may be useful: `a, ln(), log()`

Part B Differentiating a^x

Using your answer to part A, use the chain rule to find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for the function $y = a^x$. Give your answer in the form $f(a)a^x$, where $f(a)$ is a function of a to be determined.

The following symbols may be useful: `a, ln(), log()`

Part C Differentiating $\log_a(x)$

Use the chain rule to find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for the function $y = \log_a(x)$.

The following symbols may be useful: `a, ln(), log()`

Part D **Differentiate** $\mathrm{e}^{\mathrm{e}^x}$

Use the chain rule to find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for the function $y = \mathrm{e}^{\mathrm{e}^x}$.

The following symbols may be useful: `e, x`

Part E **Differentiate** $\ln(\ln(x))$

Use the chain rule to find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for the function $y = \ln\left(\ln x\right)$.

The following symbols may be useful: `ln(), log(), x`

Created for Isaac Physics by Jonathan Waugh