

Question

Centre of Mass: Lamina 1

Subject & topics: Physics | Mechanics | Statics Stage & difficulty: Further A P2

A uniform square lamina ABCD of side 6 cm has a semicircular piece, with AB as diameter, removed (see Figure 1).

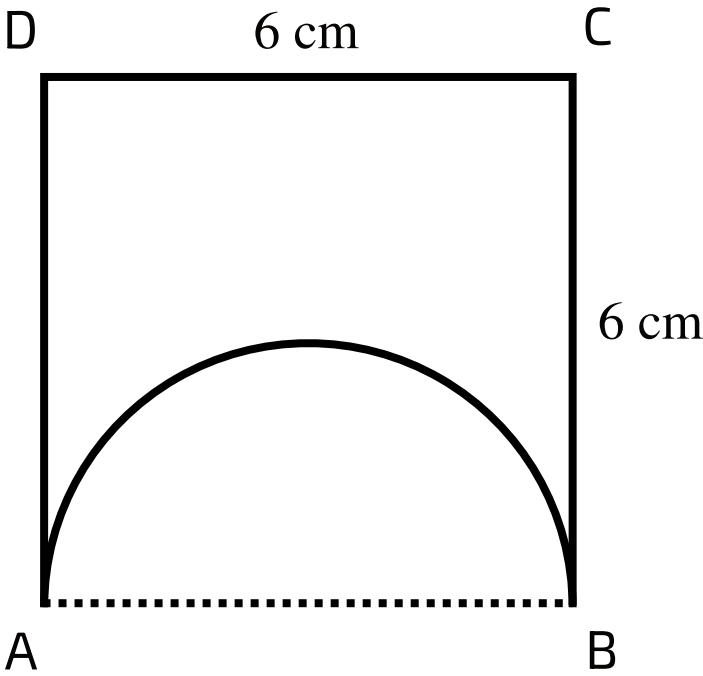


Figure 1: The lamina described in the question.

Part A

Position of centre of mass

Find the distance of the centre of mass of the remaining shape from CD. Give your answer to 3 sf.

Part B

Angle to the vertical

The remaining shape is suspended from a fixed point by a string attached at C and hangs in equilibrium.

Find the acute angle between CD and the vertical. Give your answer to 3 sf.

Used with permission from UCLES A Level, January 2013, Paper 4729, Question 4.



Question

Double Decker Bus

Subject & topics: Physics | Mechanics | Statics

Stage & difficulty: A Level C2



This problem involves centre of mass calculations and tipping points, which are not covered in some Physics A Levels. For more information please check with your teacher.

A double-decker bus of mass $M = 12000 \text{ kg}$ has a width at its base of $w = 2.00 \text{ m}$. Its ~~centre of mass~~ when empty is a height $y_0 = 0.900 \text{ m}$ above the ground and equidistant between the two sides of the bus. Thirty-two passengers, each of mass $m = 75.0 \text{ kg}$, are seated on each deck, symmetrically placed from side to side. The centre of mass of those on the lower deck is at a height $y_1 = 1.10 \text{ m}$ above the ground and that of those on the upper deck is at a height $y_2 = 2.80 \text{ m}$ above the ground.

Part A

Angle of tilt

Find the maximum angle of tilt θ (the angle that the base of the bus makes with the horizontal) at which the bus would not topple.

Part B

Angle of tilt to the left

If all the passengers on the right-hand side of the bus get off, leaving those on the left-hand side, the centre of mass of the remaining passengers is a horizontal distance $x_0 = 0.400 \text{ m}$ from the left-hand side of the bus.

Find the maximum angle α (the angle the base of the bus makes with the horizontal) by which the bus can now tilt to the left without toppling.

Part C

Angle of tilt to the right

What is the angle β (the angle the base of the bus makes to the horizontal) by which the bus must now tilt to the right in order to topple?

Adapted with permission from UCLES, A Level Applied Maths, June 1974, Paper I, Question 5.

Question deck:

STEM SMART Double Maths 45 - Centres of Mass



Question

Centre of Mass: Arc on a Pivot

Subject & topics: Physics | Mechanics | Statics

Stage & difficulty: Further A P2

A uniform semicircular arc ACB is freely pivoted at A . The arc has mass 0.3 kg and is held in equilibrium by a force of magnitude P applied at B . The line of action of this force lies in the same plane as the arc, and is perpendicular to AB . The diameter AB has length 4 cm and makes an angle θ with the downward vertical (see **Figure 1**).

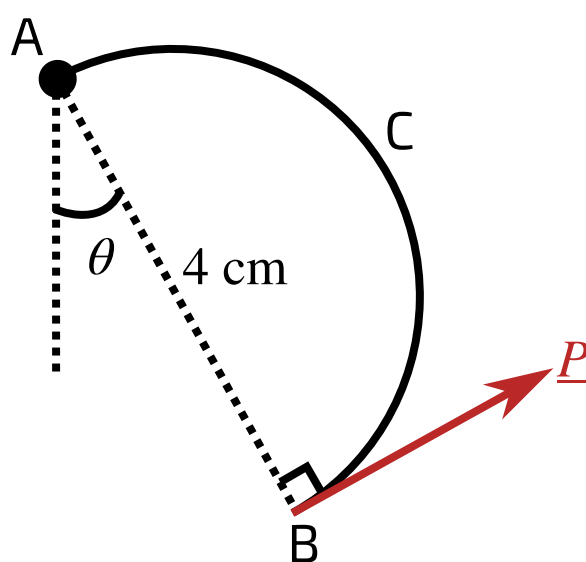


Figure 1: The semicircular arc ACB .

Part A

Force at A

Given that $\theta = 0^\circ$, find the magnitude of the force acting on the arc at A . Give your answer to 3 sf.

Part B

Perpendicular force

Given instead that $\theta = 30^\circ$, find the value of P . Give your answer to 3 sf.

Used with permission from UCLES A Level, June 2013, Paper 4729, Question 3.

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STEM SMART Double Maths 45 - Centres of Mass



Question

Centre of Mass: Prism Topple

Subject & topics: Physics | Mechanics | Statics

Stage & difficulty: Further A C2

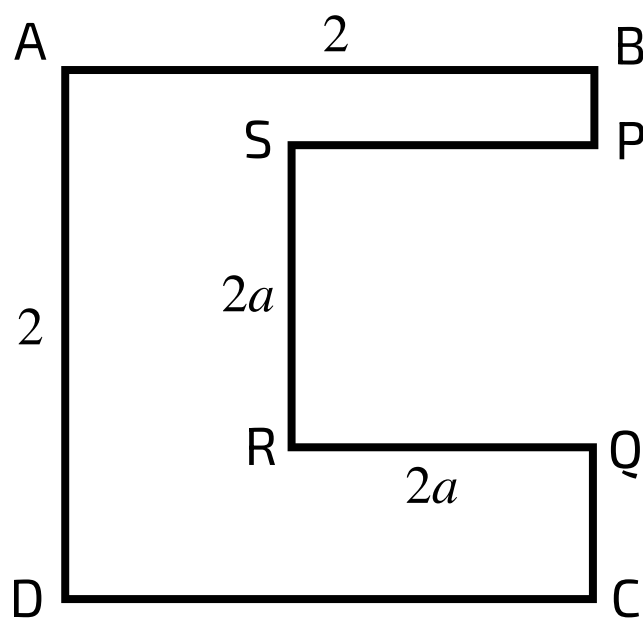


Figure 1: Cross-section of the prism described in the question.

Figure 1 shows the cross-section through the centre of mass of a uniform solid prism. The cross-section is a square ABCD of side 2 with a square PQRS of side $2a$ removed.

It is given that the centre of mass of the prism lies on the line through R and S.

Part A

Value of a

Find the exact value of a .

The following symbols may be useful: a

Part B

Position of centre of mass

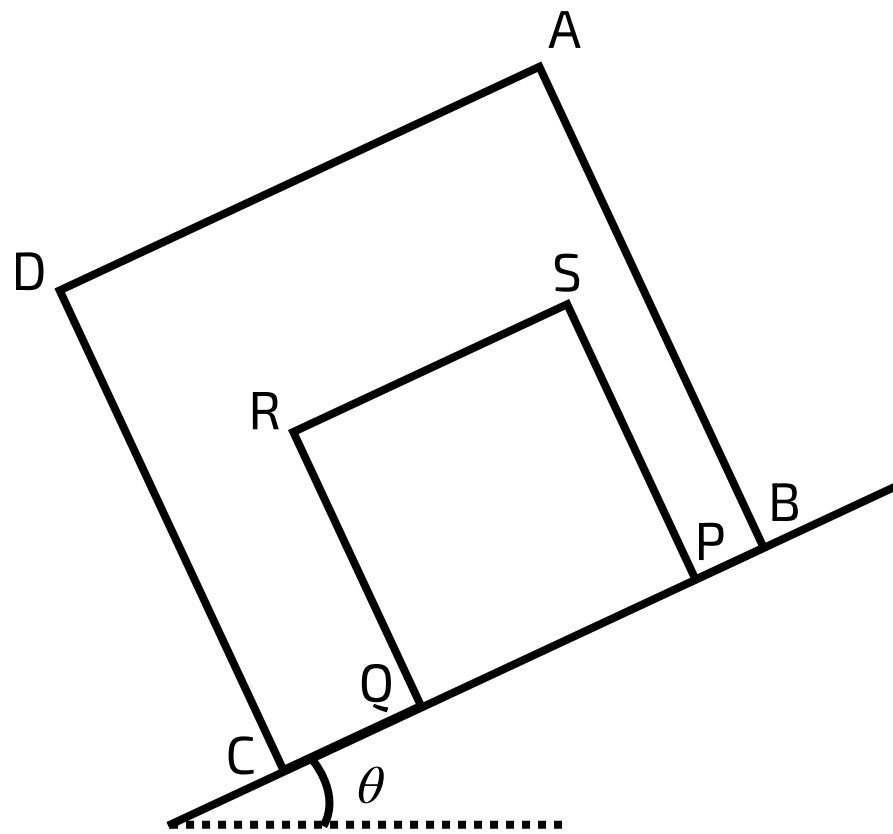


Figure 2: The prism on an inclined plane.

The prism is now placed on an inclined plane which makes an angle θ with the horizontal. BC lies along a line of greatest slope with B higher than C, as shown in **Figure 2**. The plane is slowly tilted until the prism topples, without slipping, when $\theta = 30^\circ$.

Find the exact distance of the centre of mass of the prism from CD.

Adapted with permission from OCR A Level, June 2018, Paper 4729, Question 7.

Question deck:

STEM SMART Double Maths 45 - Centres of Mass



Question

Centre of Mass: Prism Topple 2

Subject & topics: Physics | Mechanics | Statics

Stage & difficulty: Further A C2

Figure 1 shows the cross-section through the centre of mass of a uniform solid prism. The cross-section is a trapezium ABCD with AB and CD perpendicular to AD. The lengths of AB and AD are each 5 cm and the length of CD is $(a + 5)$ cm.

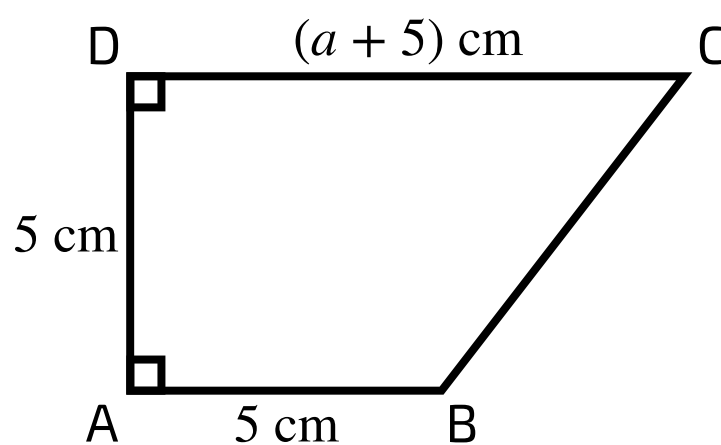


Figure 1: The cross-section of the prism is a trapezium ABCD.

Part A

Distance from AD

Find the distance of the centre of mass of the prism from AD in terms of a . Give your answer as an algebraic fraction $\frac{f(a)}{g(a)}$ where $f(a)$ is a quadratic expression and $g(a)$ is a linear expression.

The following symbols may be useful: a

Part B**Find a**

The prism is placed with the face containing **AB** in contact with a horizontal surface.

Find the greatest value of a for which the prism does not topple. Give your answer in the form $p\sqrt{q}$, where p and q are integers.

The following symbols may be useful: a

Part C**Find θ**

The prism is now placed on an inclined plane which makes an angle of θ with the horizontal. **AB** lies along a line of greatest slope with B higher than A.

Using the value for a found in part B, and assuming the prism does not slip down the plane, find the greatest value of θ for which the prism does not topple.

Adapted with permission from OCR A Level, June 2012, Paper 4729, Question 7.

Question deck:

STEM SMART Double Maths 45 - Centres of Mass



Question

Centre of Mass: Cones on Strings

Subject & topics: Physics | Mechanics | Statics

Stage & difficulty: Further A P2

A **solid uniform cone** has height 8 cm, base radius 5 cm and mass 4 kg.

A **uniform conical shell** has height 10 cm, base radius 5 cm and mass 0.4 kg.

The two shapes are joined together so that the circumferences of their circular bases coincide.

Part A

Position of centre of mass

Find the distance of the centre of mass of the shape from the common circular base. Give your answer to 3 sf.

Part B

String tensions

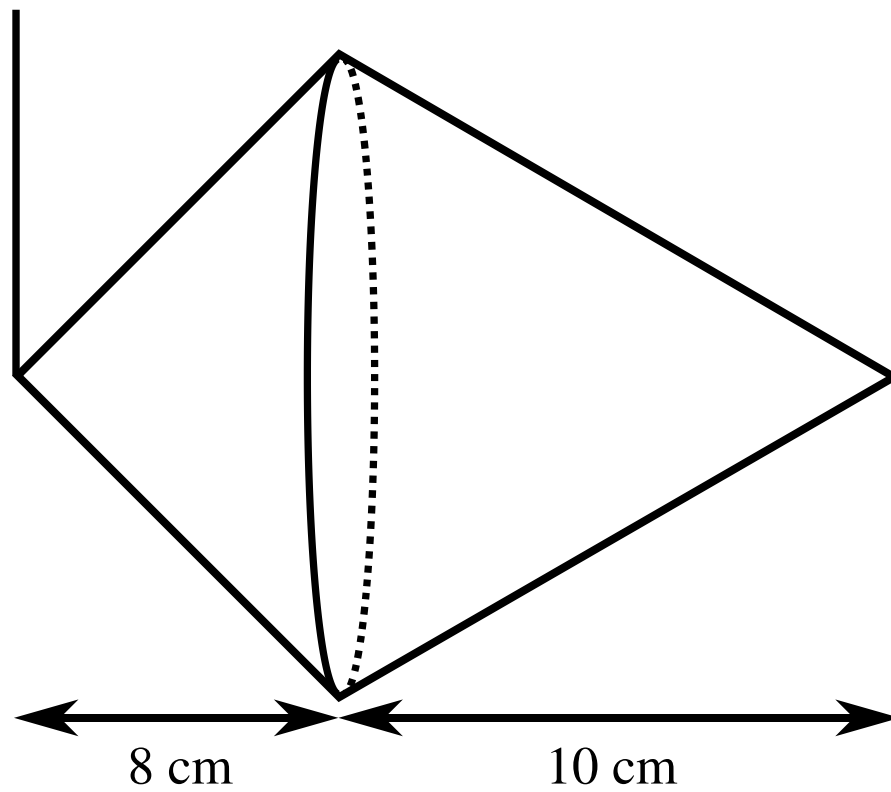


Figure 1: The situation described in the question.

The object is suspended with a string attached to the vertex of the cone and another string attached to the vertex of the conical shell. The object is in equilibrium with the strings vertical and the axis of symmetry of the object horizontal, as shown in **Figure 1**.

Find the tension in the string attached to the **conical shell**. Give your answer to 3 sf.

Find the tension in the string attached to the **solid cone**. Give your answer to 3 sf.

Used with permission from OCR A Level, June 2013, Paper 4729, Question 4.

Question deck:

STEM SMART Double Maths 45 - Centres of Mass



Question

Centre of Mass by Integration

Subject & topics: Physics | Mechanics | Statics

Stage & difficulty: Further A P3

The region bounded by the x -axis, the lines $x = 1$ and $x = 2$, and the curve $y = kx^2$, where k is a positive constant, is occupied by a uniform lamina.

Part A

Find the x -coordinate

Find the exact x -coordinate of the centre of mass of the lamina.

The following symbols may be useful: x

Part B

Find k

Given that the x - and y - coordinates of the centre of mass of the lamina are equal, find the exact value of k .

The following symbols may be useful: k

Used with permission from UCLES, A Level, June 2015, Paper 4731, Question 2

Question deck:

STEM SMART Double Maths 45 - Centres of Mass



Question

Solid of Revolution Centre of Mass

Subject & topics: Physics | Mechanics | Statics

Stage & difficulty: Further A P3

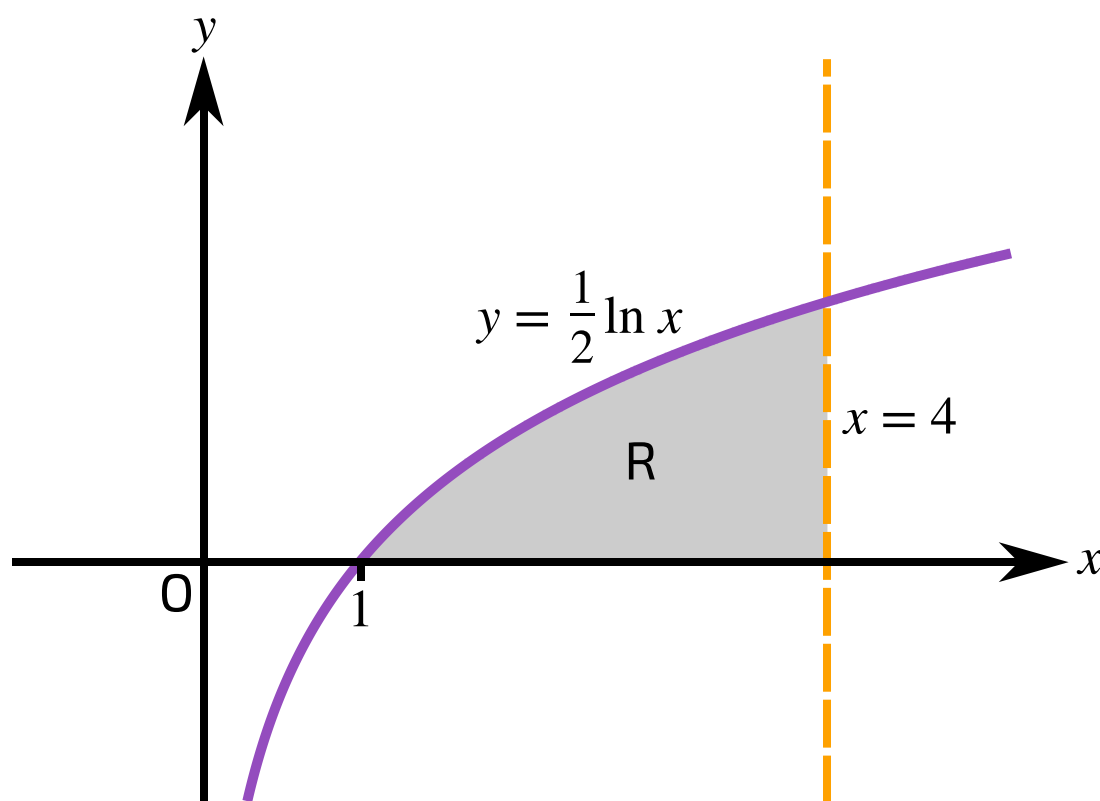


Figure 1: The region R.

Figure 1 shows the curve with equation $y = \frac{1}{2} \ln x$. The region R, shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 4$. A uniform solid of revolution is formed by rotating R completely about the y -axis to form a solid of volume V .

Part A

Volume of revolution

Find the exact volume V of the solid of revolution. Give your answer in the form $V = \frac{\pi}{4} (a \ln 2 + b)$, where a and b are integers.

The following symbols may be useful: v , a , b , $\ln()$, π

Part B

Centre of mass

Find the exact y -coordinate of the centre of mass of the solid.

The following symbols may be useful: $\ln()$, y

Used with permission from UCLES, A Level, June 2017, Paper 4731, Question 4