isaac You work it out.

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Complex Numbers: Manipulations 3i

Subject & topics: Maths **Stage & difficulty:** Further A P1

The complex number $2 + i$ is denoted by z, and the complex conjugate of z is denoted by z^*.

Part A

z^2

Express z^2 in the form $x + iy$, where x and y are exact real numbers.

The following symbols may be useful: `i`

Part B

$4z - z^2$

Express $4z - z^2$ in the form $x + iy$, where x and y are exact real numbers.

The following symbols may be useful: `i`

Part C

zz^*

Express zz^* in the form $x + iy$, where x and y are exact real numbers.

The following symbols may be useful: `i`

Part D

$\frac{z+1}{z-1}$

Express $\frac{z+1}{z-1}$ in the form $x + iy$, where x and y are exact real numbers.

The following symbols may be useful: `i`

Adapted with permission from UCLES, A Level, OCR FP1 Specimen paper, Paper 4725, Question 3.

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STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Complex Numbers: Manipulations 1i

Subject & topics: Maths **Stage & difficulty:** Further A P1

The complex number z has modulus $2\sqrt{3}$ and argument $-\frac{\pi}{3}$.

Part A

z

Find z in the form $z = x + iy$, where x and y are are exact real numbers.

The following symbols may be useful: `i, z`

Part B

$\frac{1}{(z^*-5i)^2}$

Find $\frac{1}{(z^*-5i)^2}$ in the form $x + iy$, where x and y are exact real numbers.

The following symbols may be useful: `i`

Adapted with permission from UCLES, A Level, June 2016, Paper 4725, Question 2.

Question deck:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

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STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Complex Numbers: x+iy and Euler 3i

Subject & topics: Maths **Stage & difficulty:** Further A P1

The complex number z satisfies the equation

$$z + 2iz^* = 12 + 9i$$

Part A

z

Find z in the form $z = x + iy$.

The following symbols may be useful: `i, z`

Part B

Modulus-Argument

z can also be expressed in the form

$$z = r\left(\cos\theta + i\sin\theta\right)$$

Find the values of r and θ to 3 significant figures.

$r =$ ____

$\theta =$ ____

Adapted with permission from UCLES, A Level, Jan 2010, Paper 4725, Question 3.

Question deck:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

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STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Complex Numbers: Equations to Quartics 1ii

Subject & topics: Maths **Stage & difficulty:** Further A P1

Part A

Square roots

The square roots of the complex number $5 + 12i$ can be expressed in the form $x + iy$.

Give the square root with positive x and positive y.

The following symbols may be useful: `i`

Give the square root with negative x and negative y.

The following symbols may be useful: `i`

Part B

$(3 - 2i)^2$

Find $(3 - 2i)^2$ in the form $x + iy$ where x and y are exact.

The following symbols may be useful: `i`

Part C

Roots of quartic

The answers to the previous parts can be used to solve the quartic

$$z^4 - 10z^2 + 169 = 0$$

The roots to the quartic can be expressed in the form $x + iy$.

Give the root with positive x and positive y.

The following symbols may be useful: `i`

Give the root with positive x and negative y.

The following symbols may be useful: `i`

Give the root with negative x and positive y.

The following symbols may be useful: `i`

Give the root with negative x and negative y.

The following symbols may be useful: `i`

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 9.

Question deck:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

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STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Applying Complex Numbers 2ii

Subject & topics: Maths **Stage & difficulty:** Further A P1

One root of the cubic equation $z^3 + bz^2 + cz - 15 = 0$, where b and c are real constants, is the complex number $2 + i$.

Part A

Complex root

Find the other complex root in the form $x + iy$.

The following symbols may be useful: `i`

Part B

Real root

Find the real root.

Part C

b

Find b.

The following symbols may be useful: `b`

Part D

c

Find c.

The following symbols may be useful: c

Adapted with permission from Sally Waugh.

Question deck:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

isaac You work it out.

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Complex Numbers: Equations to Quartics 1i

Subject & topics: Maths **Stage & difficulty:** Further A P1

One root of the quadratic equation $z^2 + az + b = 0$, where a and b are real, is $16 - 30i$.

Part A
Other root

Give the other root in the form $x + iy$.

The following symbols may be useful: `i`

Part B
a and b

Find the values of a and b.

$a =$ ☐

$b =$ ☐

Part C

Quartic

The quartic equation $z^4 + az^2 + b = 0$ has roots in the form $x + iy$.

Give the root with positive x and positive y.

The following symbols may be useful: `i`

Give the root with positive x and negative y.

The following symbols may be useful: `i`

Give the root with negative x and positive y.

The following symbols may be useful: `i`

Give the root with negative x and negative y.

The following symbols may be useful: `i`

Adapted with permission from UCLES, A Level, June 2011, Paper 4725, Question 9.

Question deck:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

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STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Argand Diagrams and Simple Loci 2i

Subject & topics: Maths **Stage & difficulty:** Further A P1

The complex number a is denoted by $1 + i\sqrt{3}$.

Part A

Modulus and argument

Find $|a|$ and $\arg a$ in exact form.

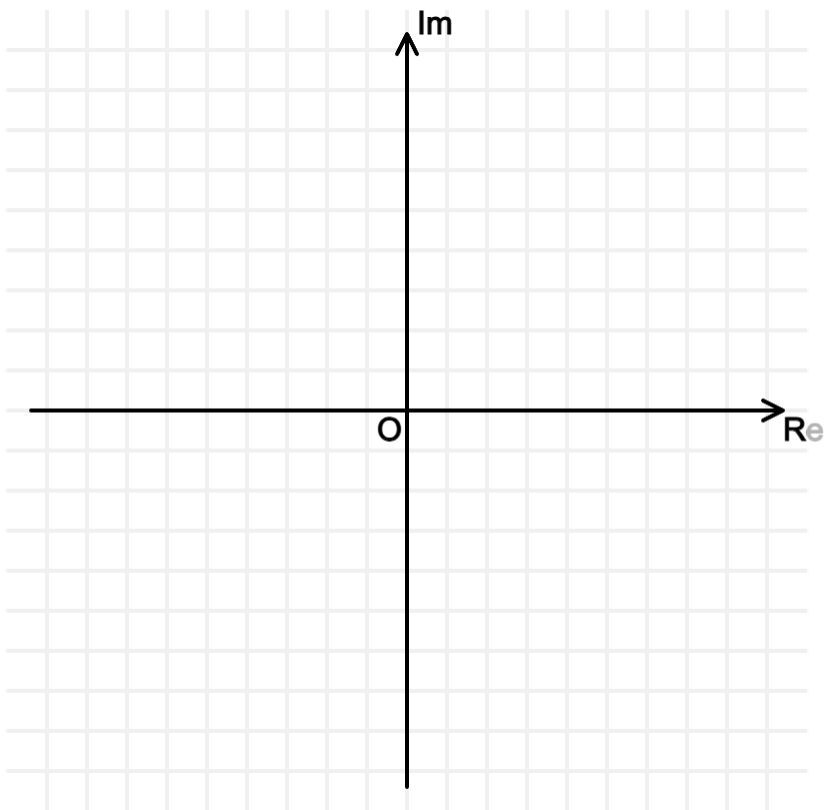
$$|a| = \square$$

$$\arg a = \square\,\pi$$

Part B

Loci

Sketch the loci given by $|z-a|=|a|$ and $\arg(z-a)=\frac{1}{2}\pi$ on a single Argand diagram.



Adapted with permission from UCLES, A Level, June 2010, Paper 4725, Question 5.

Question deck:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

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STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Argand Diagrams: Using Loci 2i

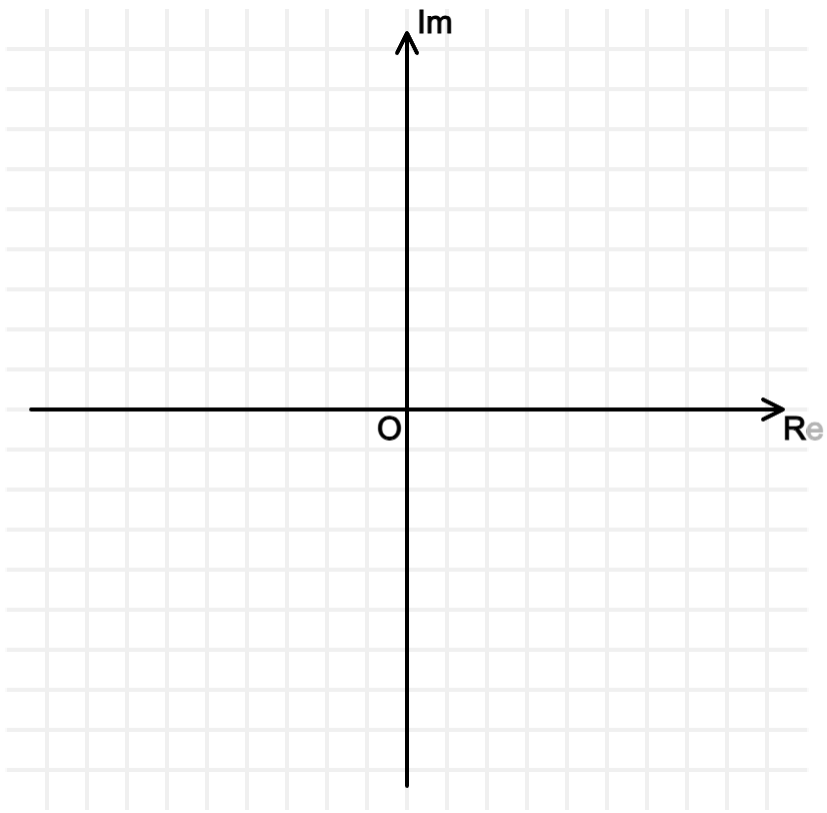
Subject & topics: Maths **Stage & difficulty:** Further A P1

The loci C_1 and C_2 are given by $|z| = |z - 4i|$ and $\arg z = \frac{\pi}{6}$ respectively.

Part A

Loci of C_1 and C_2

Sketch the loci C_1 and C_2 on a single Argand diagram.



Part B

Intersection

Hence find, in the form $x + iy$, the complex number represented by the point of intersection of C_1 and C_2. Give your answer in exact form.

The following symbols may be useful: `i`

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 6.

Question deck:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

isaac You work it out.

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Argand Diagrams: Solving Inequalities 1ii

Subject & topics: Maths **Stage & difficulty:** Further A P1

The loci L_1 and L_2 are given by $|z| = 2$ and $\arg(z - 3 - i) = \pi$ respectively.

Part A

Equation of L_1

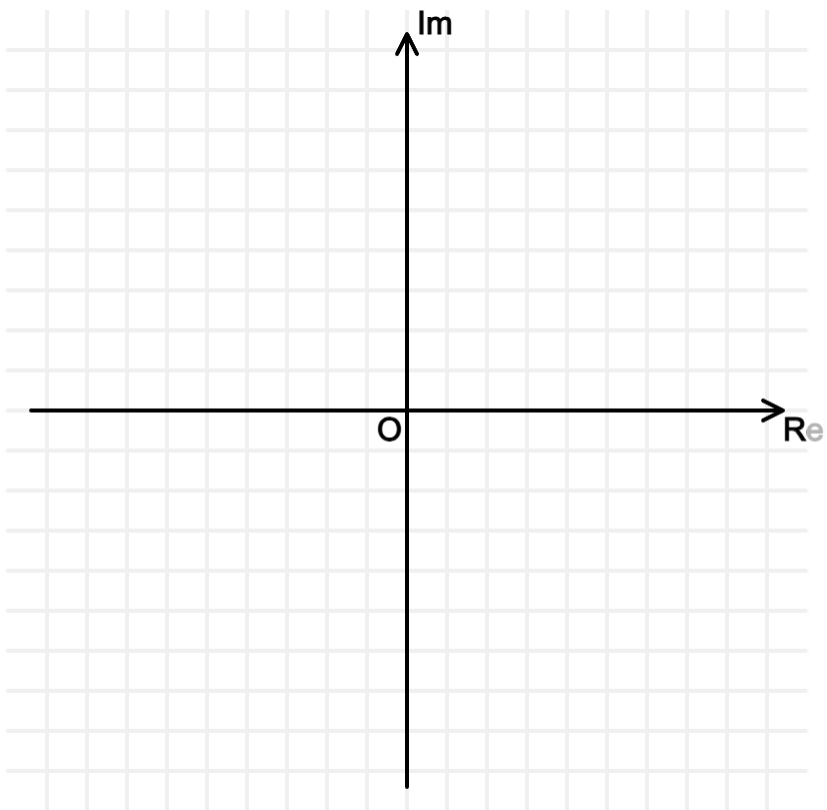
By writing z in the form $x + iy$, express the equation for L_1 in Cartesian form, simplifying your answer as far as possible.

The following symbols may be useful: `x`, `y`

Part B

Loci

Sketch L_1 and L_2 on a single Argand diagram.

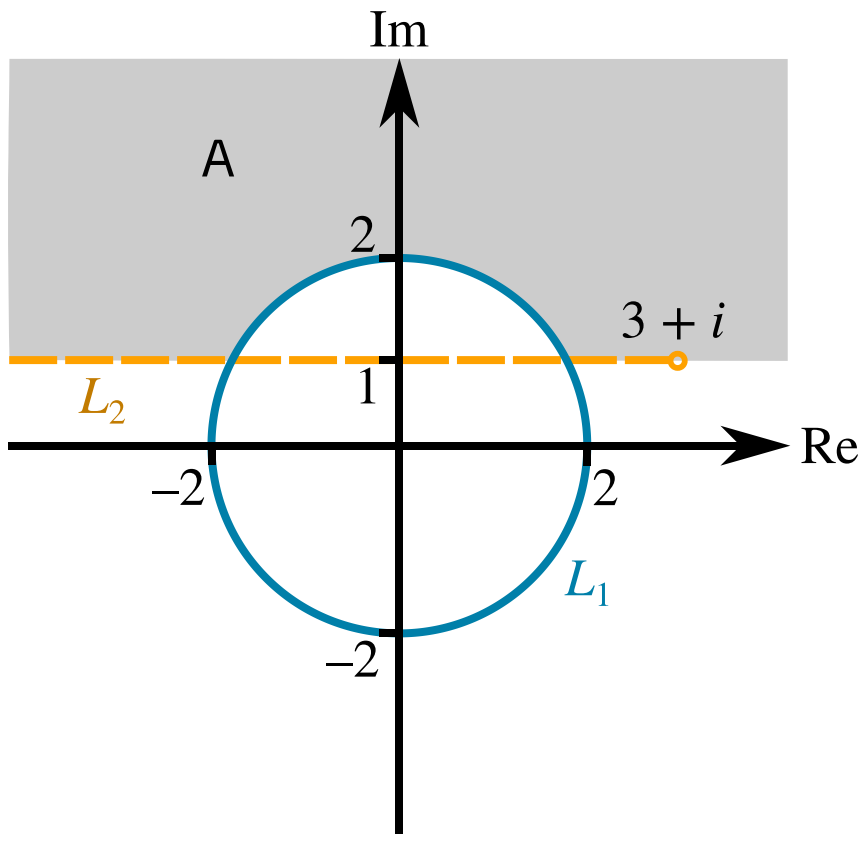


Part C

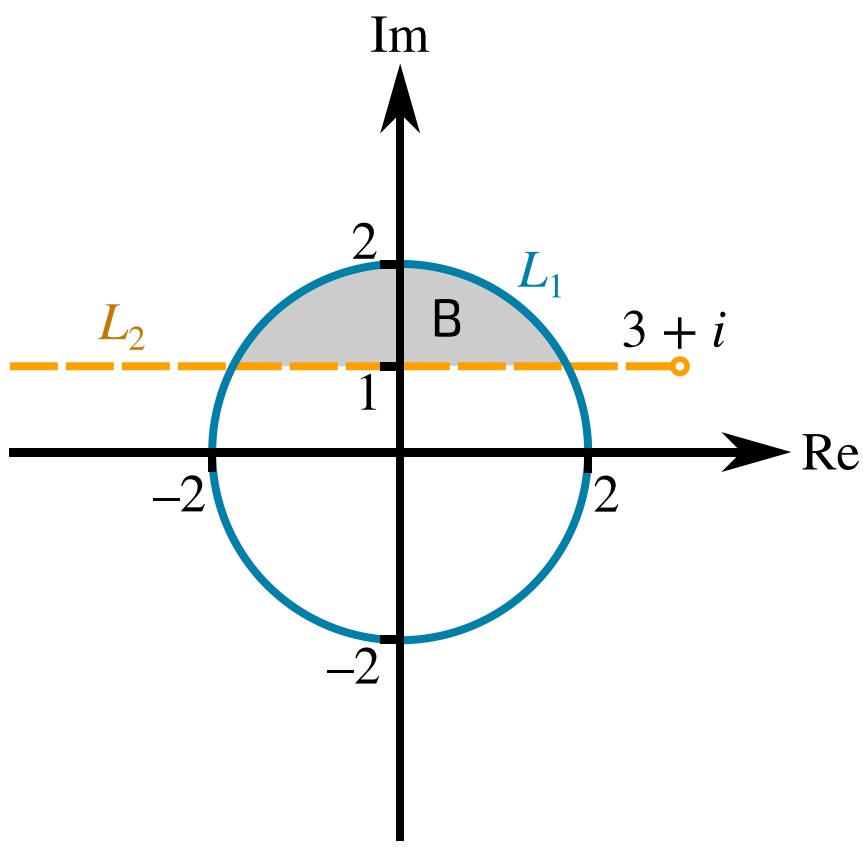
Inequalities

Select from the images below the shaded region of the Argand diagram for which

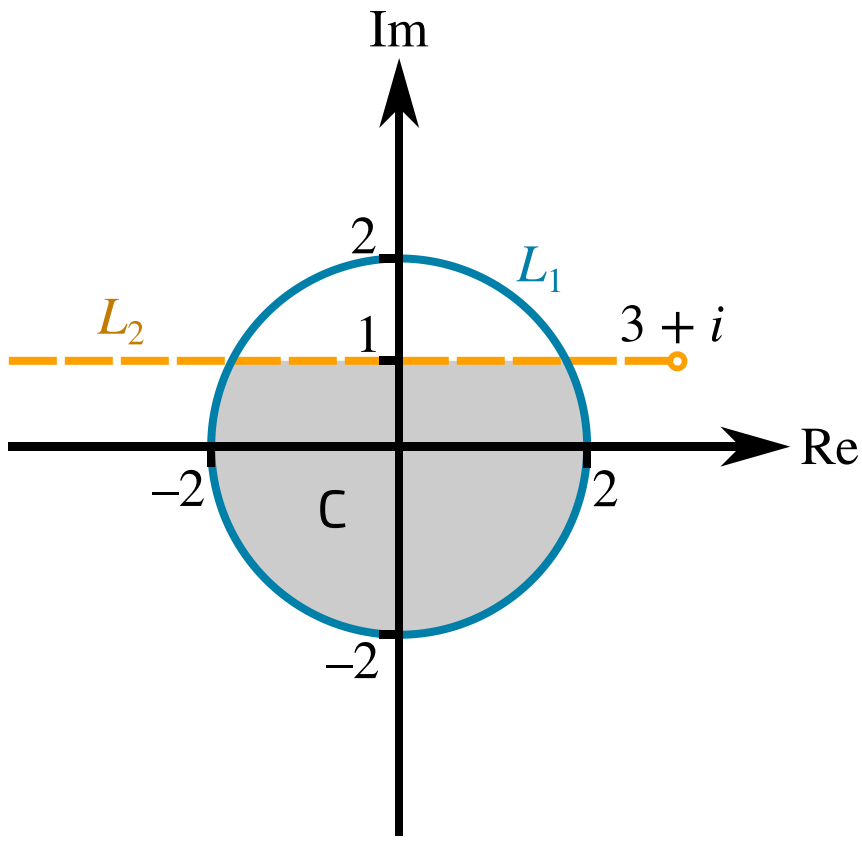
$$|z| \leq 2 \quad \text{and} \quad 0 \leq \arg(z - 3 - i) \leq \pi$$



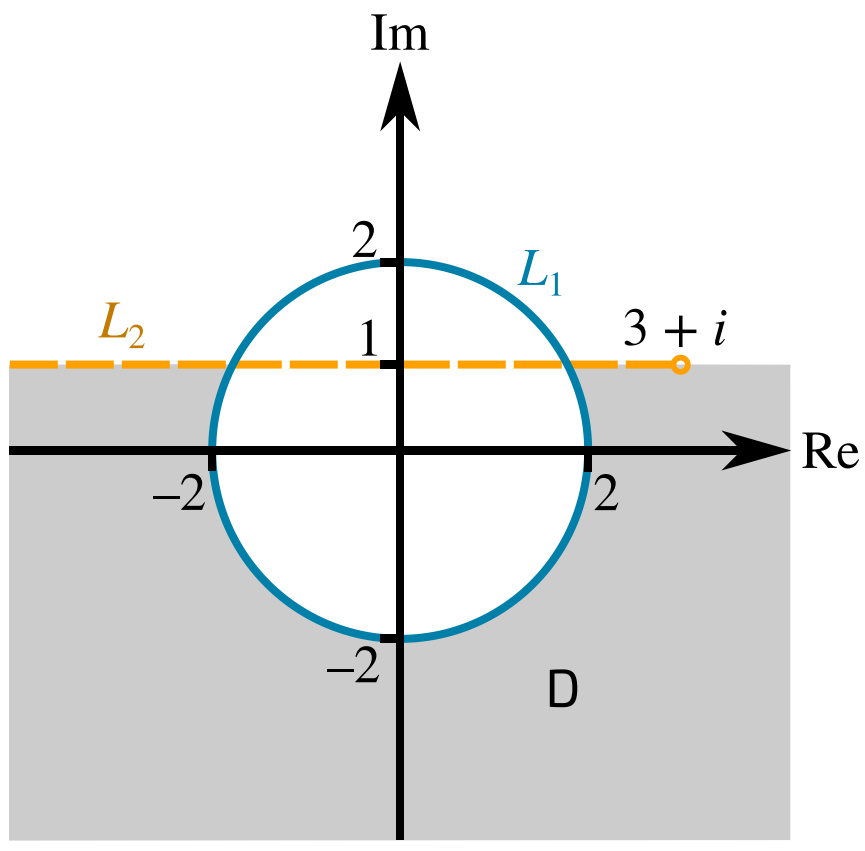
Region A.



Region B.



Region C.



Region D.

- Region A.
- Region B.
- Region C.
- Region D.

Adapted with permission from UCLES, A Level, June 2018, Paper 4725, Question 4.

Question deck:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Argand Diagrams: Solving Inequalities 4ii

Subject & topics: Maths **Stage & difficulty:** Further A P1

The loci L_1 and L_2 are given by $|z-3+4i|=5$ and $|z|=|z-6|$ respectively.

Part A

Equation of L_1

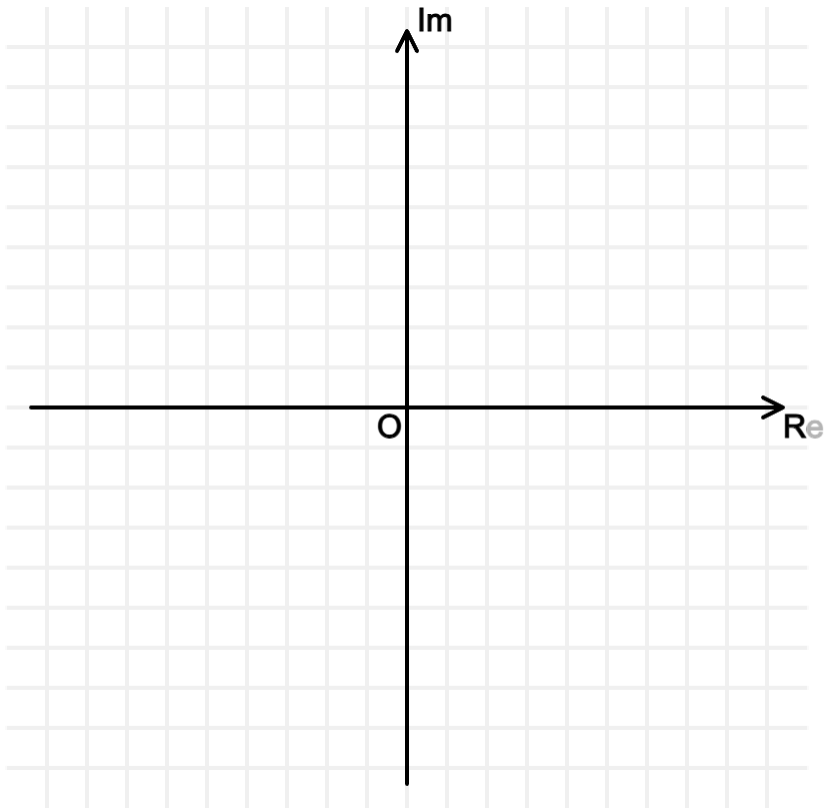
Give the equation of L_1 in the form $(x-a)^2+(y-b)^2=c^2$.

The following symbols may be useful: x, y

Part B

Loci

Sketch the loci L_1 and L_2 on a single Argand diagram.

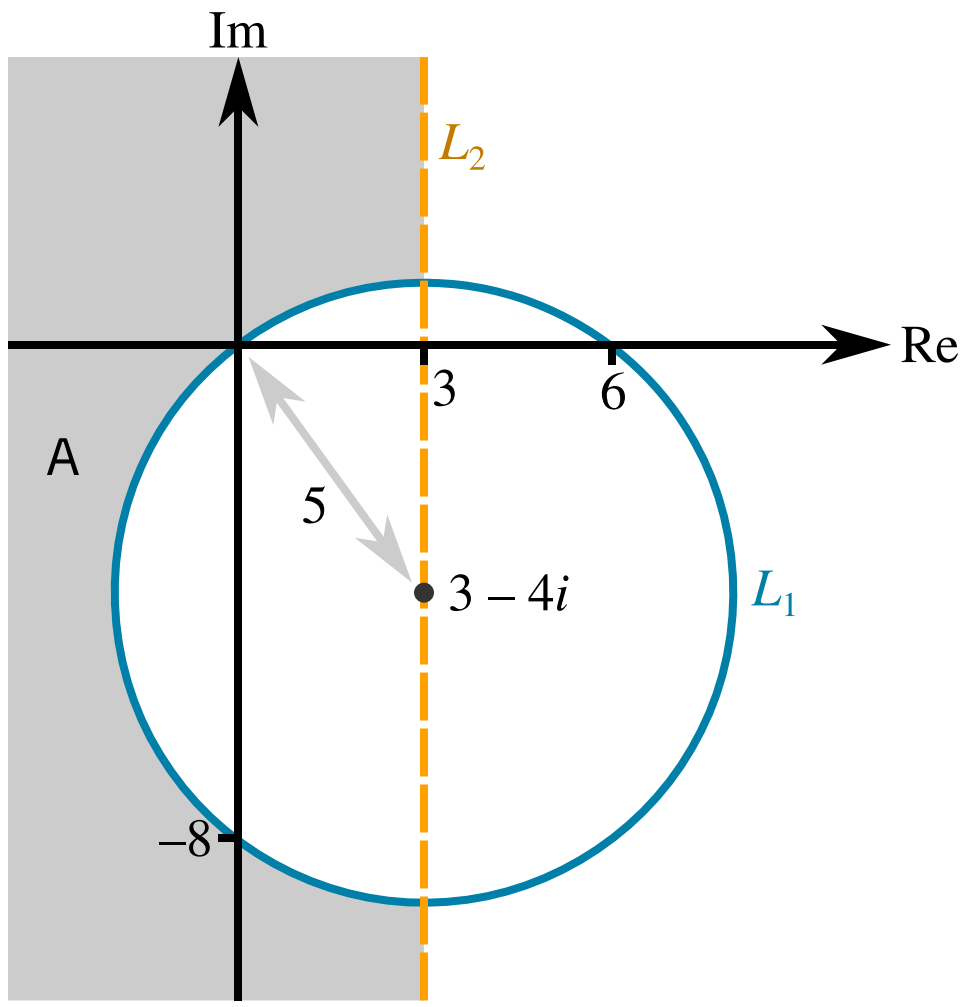


Part C

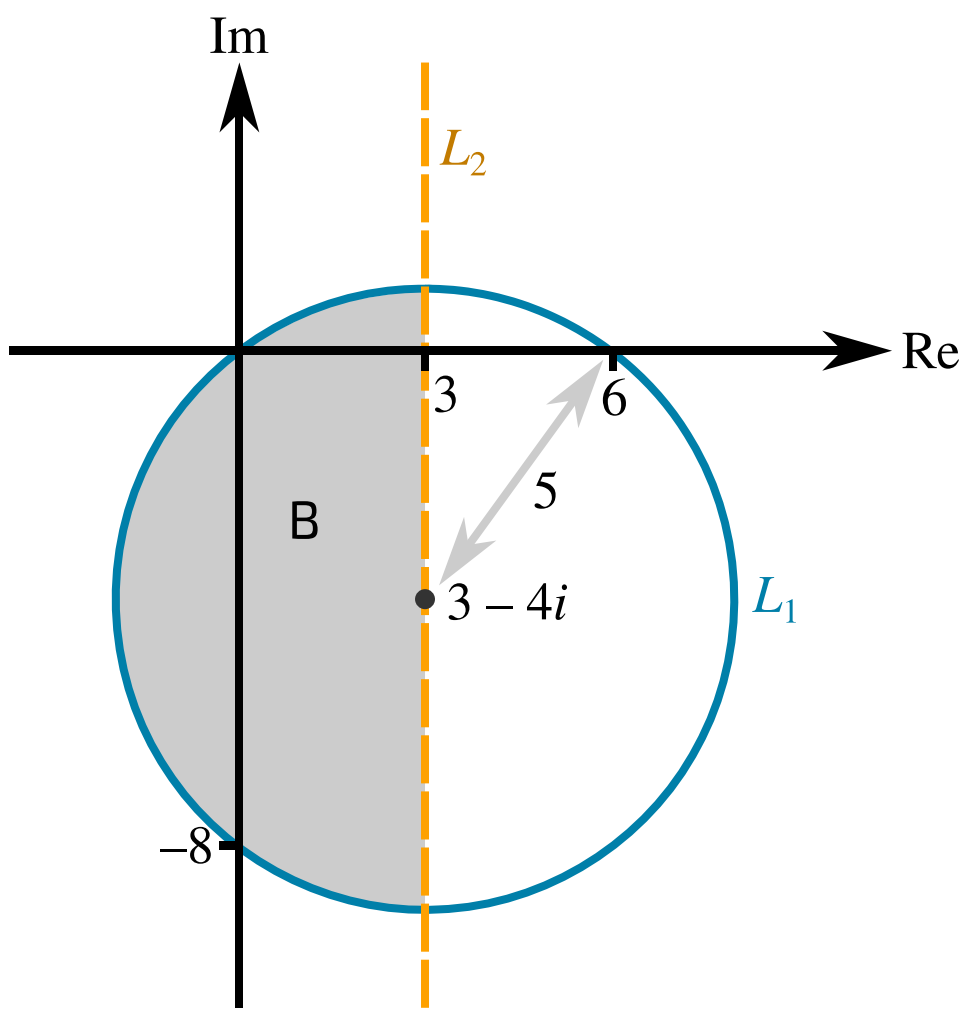
Inequalities

Select from the images below the shaded region of the Argand diagram for which

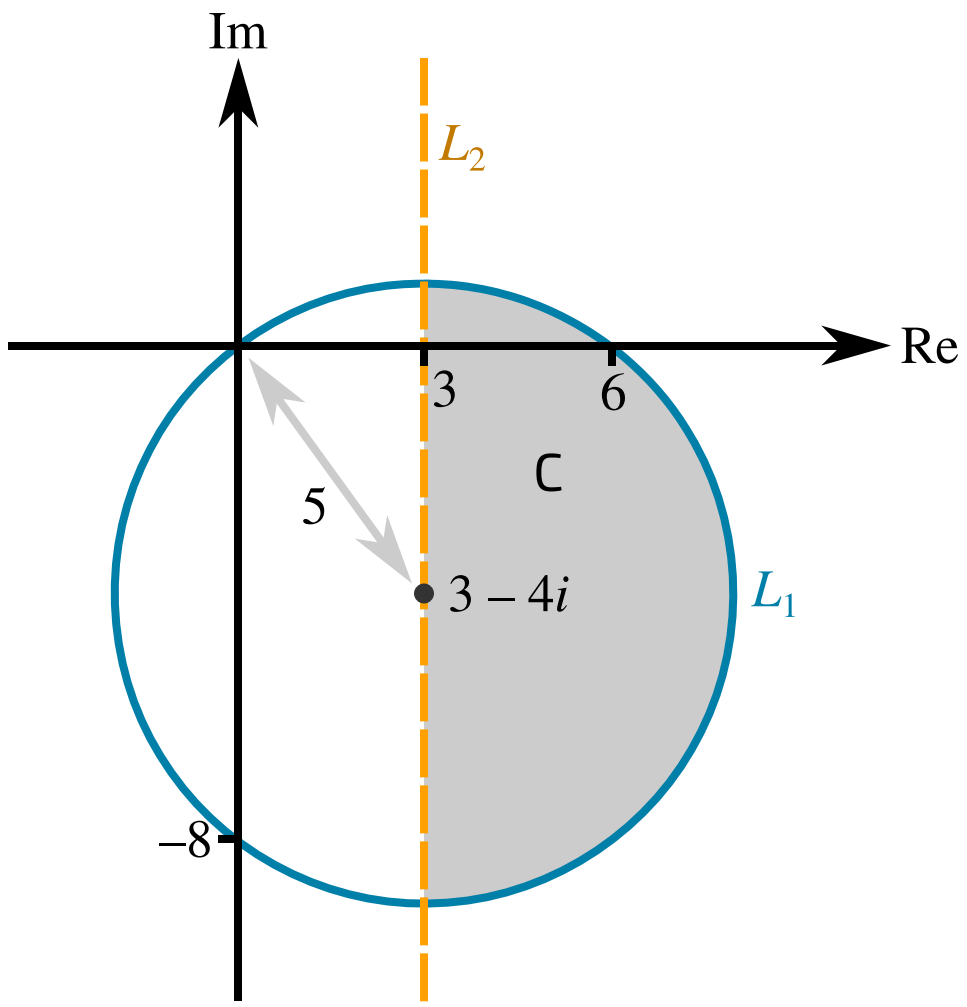
$$|z - 3 + 4i| \leq 5 \quad \text{and} \quad |z| \geq |z - 6|$$



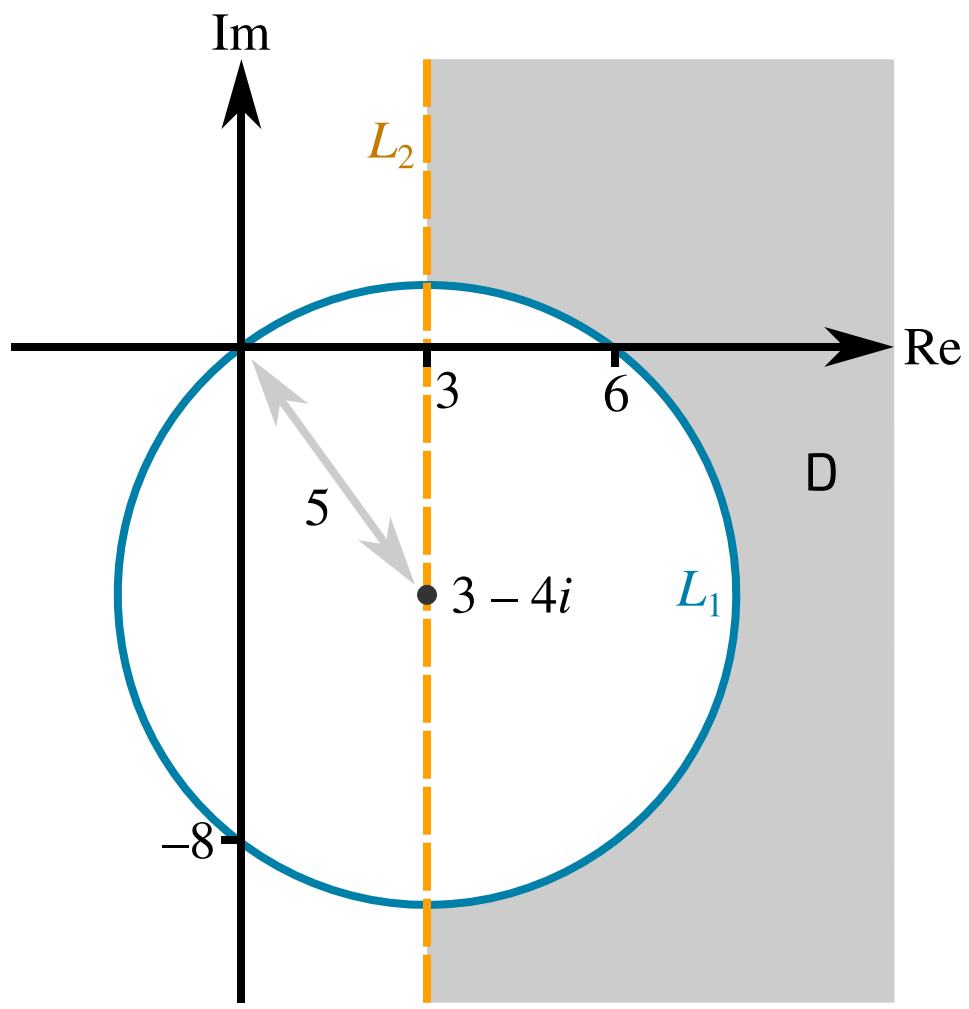
Region A.



Region B.



Region C.



Region D.

- Region A.
- Region B.
- Region C.
- Region D.

Adapted with permission from UCLES, A Level, June 2018, Paper 4725, Question 4.