isaac You work it out.

Question

Friction

A-level Maths Topic Summaries - Mechanics

Subject & topics: Mechanics | Dynamics **Stage & difficulty:** A Level P1

Fill in the blanks to complete these notes about friction.

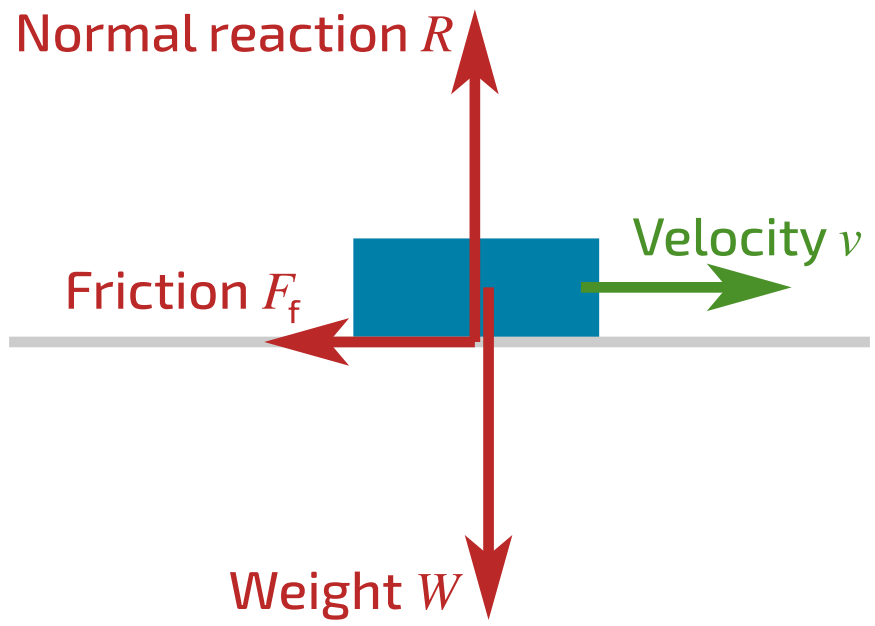

Figure 1: The forces on a sliding block.

Friction is a [ ] force. The amount of friction between two surfaces depends on the [ ] from which the surfaces are made and [ ] the surfaces are pushed together.

Friction is a force, so it has both size and direction. Friction always [ ] the motion of an object.

The amount of friction between two objects is given by

$$F_f \leq \boxed{}$$

In this inequality

- μ is a constant called the [ ]. Its value depends on the materials from which the surfaces in contact are made.
- R is the [ ] force between the objects in contact.

If two objects are moving past one another, the friction between them will always be at its [ ] value.

Items:

coefficient of friction | contact | how hard | materials | maximum | μR | normal reaction | opposes

Created for isaacscience.org by Jonathan Waugh

Question

Motion on Inclined Planes

A-level Maths Topic Summaries - Mechanics

Subject & topics: Mechanics | Dynamics **Stage & difficulty:** A Level P3

Fill in the boxes to complete the notes about motion on inclined planes.

Part A

Motion down a plane

The easiest way to analyse the motion of an object that is sliding down an inclined plane is to **resolve perpendicular to the plane** and **resolve parallel to the plane**. We also use the equation for maximum friction. We end up with **three** equations.

Figure 1 shows the forces on an object of mass m that is sliding down a plane inclined at an angle θ to the horizontal. We choose the positive direction to be down the slope when we resolve parallel to the plane.

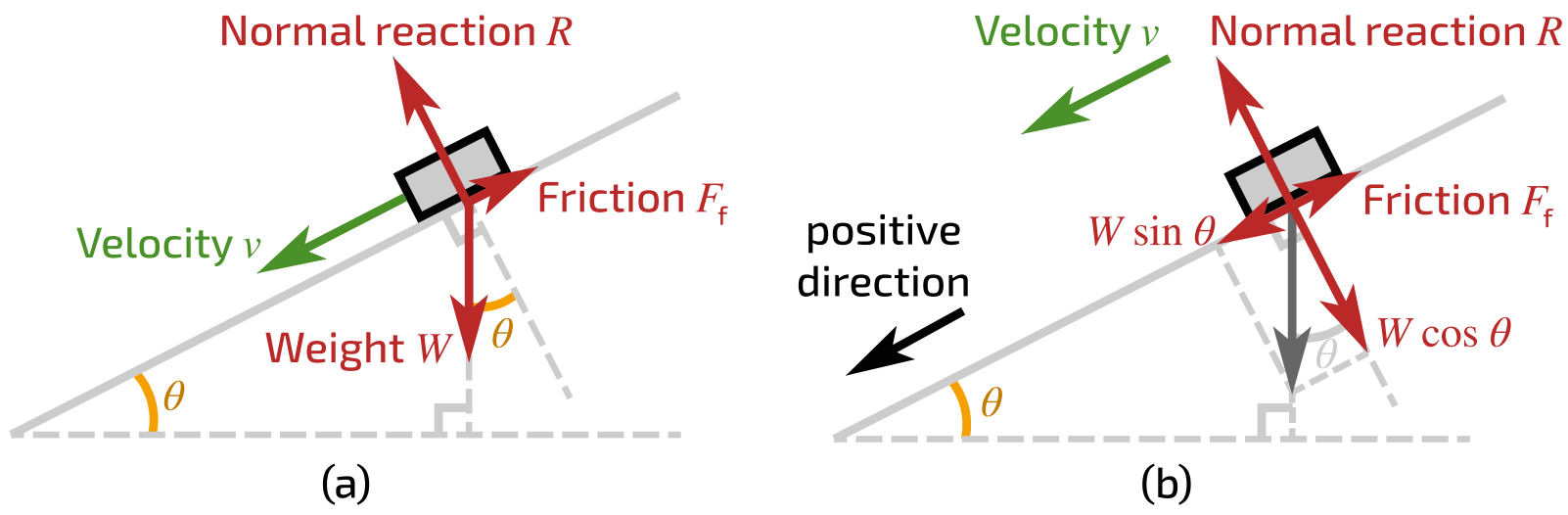

Figure 1: The forces on an object sliding down an inclined plane. In (b) the weight has been resolved into components.

We start by by resolving the weight. The component of the weight down the plane is [ ], and the component of the weight perpendicular to the plane is [ ].

Resolving perpendicular to the plane:

There are two forces on the object that are perpendicular to the plane. These are the [ ] R and the component of the weight perpendicular to the plane. The object does not sink into the plane or jump off the plane, so these forces must be balanced. Therefore,

$$R = \boxed{} \qquad (1)$$

Resolving parallel to the plane:

There are two forces on the object that are parallel to the plane. These are the friction force F_f, which is directed [ ] the plane, and the component of the weight down the plane. The resultant force **down** the plane is therefore $F_{res} =$ [ ] $- F_f$. Newton's second law tells us that the resultant force down the plane is related to the acceleration down the plane a by $F_{res} = ma$. Therefore,

$$ma = \boxed{} - F_f \qquad (2)$$

Friction:

For an object that is moving, the friction force must be at its maximum value. Hence, the friction force and the normal reaction are related by the equation

$$F_f = \boxed{} \qquad (3)$$

Items:

$mg\cos\theta$ $mg\sin\theta$ μR down normal reaction up

Part B

Motion up a plane

Analysing the motion of an object that is sliding up an inclined plane is similar to analysing the motion of an object sliding down a plane. As before, we end up with **three** equations. The equations that come from **resolving perpendicular to the plane** and maximum friction are the same. However, the equation from **resolving parallel to the plane** contains a change of sign.

Figure 2 shows the forces on an object of mass m that is sliding up a plane inclined at an angle θ to the horizontal. We choose the positive direction to be up the slope when we resolve parallel to the plane.

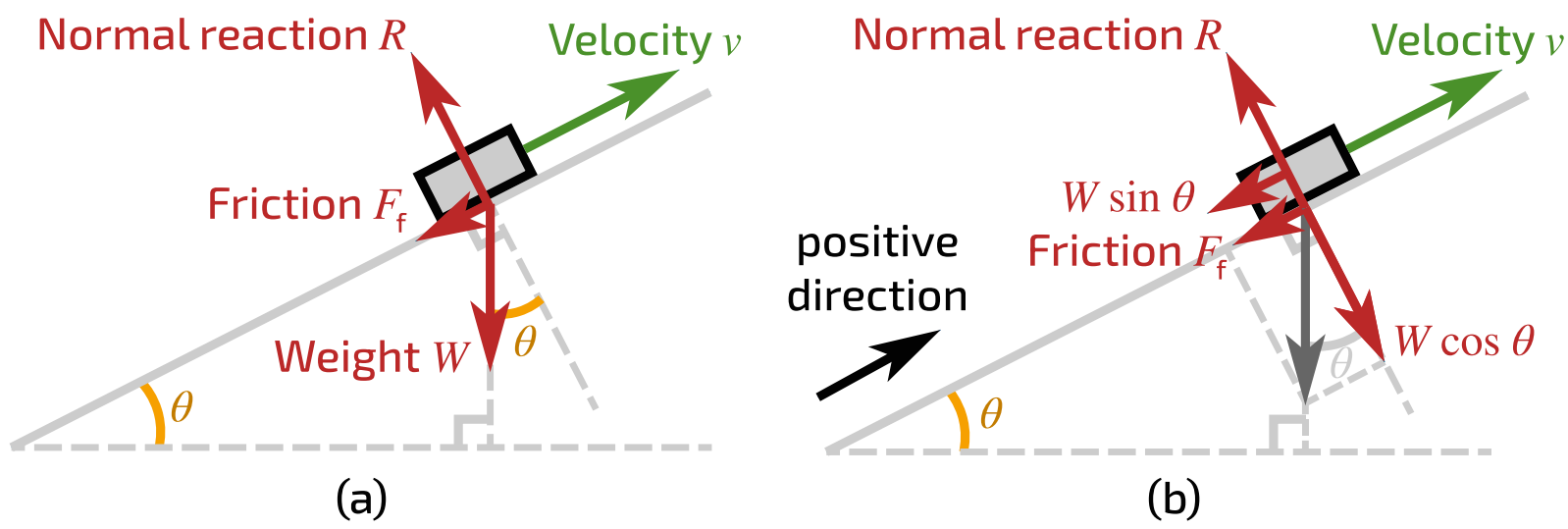

Figure 2: The forces on an object sliding up an inclined plane. In (b) the weight has been resolved into components.

The analysis of an object sliding up an inclined plane is similar to the analysis of an object sliding down a plane. The main difference is that the [ ] force is in the opposite direction as this force always [ ] the motion of an object.

Resolving perpendicular to the plane:

There are two forces on the object that are perpendicular to the plane. These are the [ ] R and the component of the weight perpendicular to the plane. The object does not sink into the plane or jump off the plane, so these forces must be balanced. Therefore,

$$R = \boxed{} \qquad (1)$$

Resolving parallel to the plane:

There are two forces on the object that are parallel to the plane. These are the friction force F_f, which is directed [ ] the plane, and the component of the weight down the plane. The resultant force **up** the plane is therefore $F_{res} =$ [ ] $- F_f$. Newton's second law tells us that the resultant force down the plane is related to the acceleration up the plane a by $F_{res} = ma$. Therefore,

$$ma = \boxed{} - F_f \qquad (2)$$

Friction:

For an object that is moving, the friction force must be at its maximum value. Hence, the friction force and the normal reaction are related by the equation

$$F_f = \boxed{} \qquad (3)$$

Items:

friction | opposes | $mg\cos\theta$ | $mg\sin\theta$ | $-mg\sin\theta$ | μR | down | normal reaction | up

Created for isaacscience.org by Jonathan Waugh

Question deck:

STEM SMART Single Maths 42 - Forces & Friction

Question

Resolving Forces 2i

Subject & topics: Maths **Stage & difficulty:** A Level P2

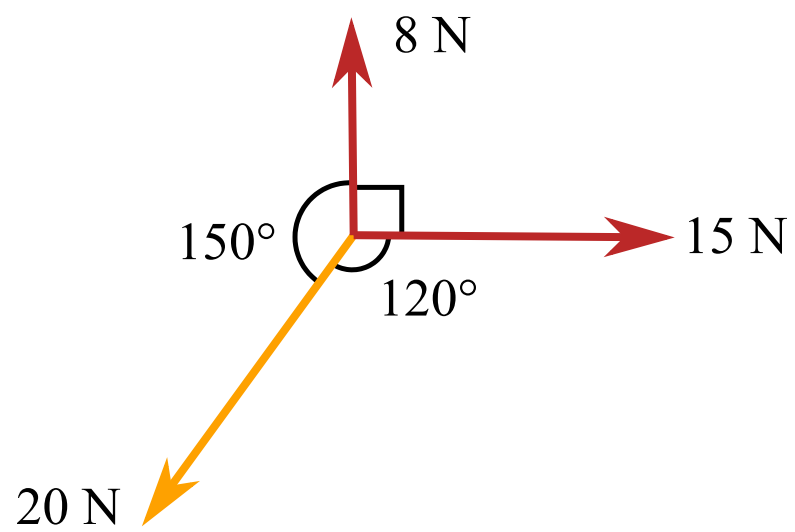

Figure 1: Three horizontal forces of magnitudes $8\,\mathrm{N}$, $15\,\mathrm{N}$ and $20\,\mathrm{N}$ acting at a point

Three horizontal forces of magnitudes $8\,\mathrm{N}$, $15\,\mathrm{N}$ and $20\,\mathrm{N}$ act at a point. The $8\,\mathrm{N}$ and $15\,\mathrm{N}$ forces are at right angles. The $20\,\mathrm{N}$ force takes an angle of 150° with the $8\,\mathrm{N}$ force and an angle of 120° with the $15\,\mathrm{N}$ force.

Part A

Resultant force

Calculate the component of the resultant of the three forces that is along the direction of the $15\,\mathrm{N}$ force.

Calculate the component of the resultant of the three forces that is along the direction of the $8\,\mathrm{N}$ force. Give your answer to 3 significant figures.

Part B

Magnitude and angle

Calculate the magnitude of the resultant force to 3 significant figures.

Calculate the angle it makes with the direction of the $8\,\mathrm{N}$ force to 3 significant figures.

Part C

Greatest and least

The directions in which the three horizontal forces act can be altered.

State the greatest possible magnitude of the resultant force to 2 significant figures.

State the least possible magnitude of the resultant force.

Used with permission from UCLES, A Level Maths, January 2012, OCR M1, Question 4

Question deck:

STEM SMART Single Maths 42 - Forces & Friction

isaac You work it out.

Question

Resolving Forces 1i

Subject & topics: Maths **Stage & difficulty:** A Level P2

Three coplanar forces of magnitudes $5.00\,\mathrm{N}$, $8.00\,\mathrm{N}$ and $8.00\,\mathrm{N}$ act at the origin O of rectangular coordinate axes. The directions of the forces are as shown in **Figure 1**.

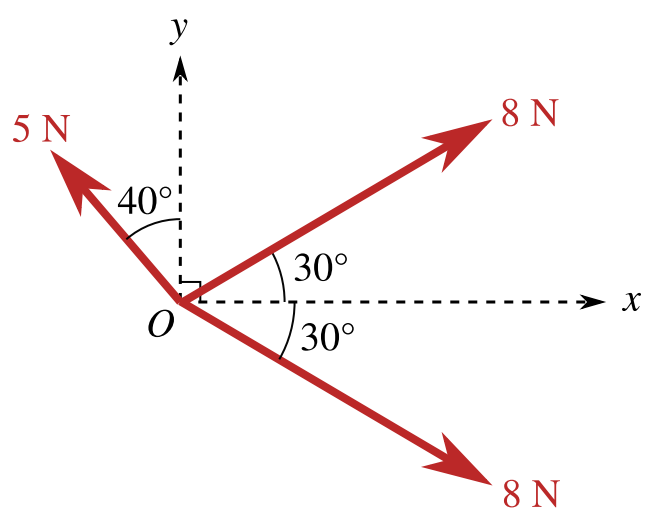

Figure 1: The magnitudes and directions of the three forces.

Part A
The x component

Find the component of the resultant of the three forces in the x-direction.

Part B
The y component

Find the component of the resultant of the three forces in the y-direction.

Part C

The magnitude

Find the magnitude of the resultant.

Part D

The direction

Find the direction of the resultant, as an anti-clockwise angle from the positive x-axis. Give your answer in degrees to 3 significant figures

Question deck:

STEM SMART Single Maths 42 - Forces & Friction

isaac You work it out.

Question

Friction 2ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

A block B of mass $0.8\,\mathrm{kg}$ is pulled across a horizontal surface by a force of $6\,\mathrm{N}$ inclined at an angle of 60° to the upward vertical. The coefficient of friction between the block and the surface is 0.2.

Part A

Vertical component

Calculate the vertical component of the force exerted on B by the surface. Give your answer to 3 significant figures.

Part B

Acceleration of B

Calculate the acceleration of B correct to 3 significant figures.

Part C

Time taken for B

The $6\,\mathrm{N}$ force is removed when B has speed $4.9\,\mathrm{m\,s^{-1}}$.

Calculate the time taken for B to decelerate from a speed of $4.9\,\mathrm{m\,s^{-1}}$ to rest. Give your answer correct to 2 significant figures.

Question deck:

STEM SMART Single Maths 42 - Forces & Friction

isaac You work it out.

Question

General Contact Force 1ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

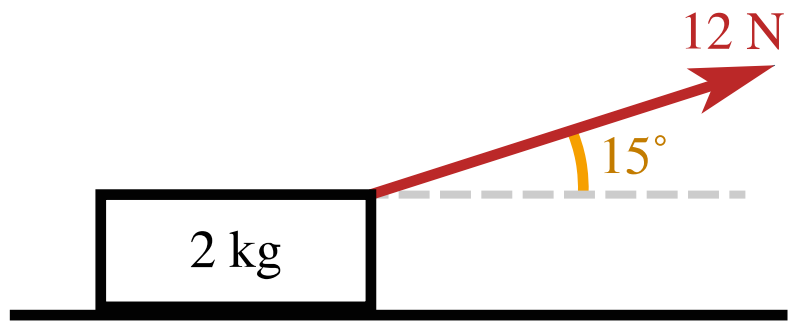

Figure 1: A block resting on a rough horizontal plane acted on by a force of $12\,\mathrm{N}$ at an angle of 15° to the horizontal.

A block of mass $2\,\mathrm{kg}$ is at rest on a rough horizontal plane, acted on by a force of magnitude $12\,\mathrm{N}$ at an angle of 15° upwards from the horizontal.

Part A

Frictional component

Find the frictional component of the contact force exerted on the block by the plane. Give your answer to 3 significant figures.

Part B

Magnitude of normal component

Find the magnitude of the normal component of the contact force exerted on the block by the plane, correct to 3 significant figures.

Part C

Coefficient of friction

It is given that the block is on the point of sliding.

Find the coefficient of friction between the block and the plane.

Part D

Acceleration of block

The force of magnitude $12\,\mathrm{N}$ is now replaced by a horizontal force of magnitude $20\,\mathrm{N}$. The block starts to move.

Assuming that the frictional force due to the rough plane has the same coefficient of friction as that found in Part C, find the acceleration of the block correct to 3 significant figures.

Question deck:

STEM SMART Single Maths 42 - Forces & Friction

isaac You work it out.

Question

Motion on Inclined Planes 1i

Subject & topics: Maths **Stage & difficulty:** A Level P2

A and B are two points on a line of greatest slope of a plane inclined at $45\,^{\circ}$ to the horizontal and AB $= 2.0\,\mathrm{m}$. A particle P of mass $0.40\,\mathrm{kg}$ is projected from A towards B with speed $5.0\,\mathrm{m\,s^{-1}}$. The coefficient of friction between the plane and P is 0.20.

Part A
Speed of P

Given that the level of A is above the level of B, calculate the speed of P when it passes through the point B.

Part B
Time taken by P

What is the time taken for P to travel from A to B?

Part C

P reaching B

Given instead that the level of A is below that of B, will P reach B?

- Yes, it reaches B with a speed of $2\,\mathrm{m\,s^{-1}}$.
- No, it does not reach B. It only travels $0.5\,\mathrm{m}$ up the slope.
- No, it does does not reach B. It only travels $1.5\,\mathrm{m}$ up the incline.
- Yes, it just reaches B, but travels no further.

Used with permission from UCLES, A Level Maths, January 2013, OCR M1, Question 7

Question deck:

STEM SMART Single Maths 42 - Forces & Friction

isaac You work it out.

Question

General Contact Force 1i

Subject & topics: Maths **Stage & difficulty:** A Level P2

A particle P of mass $0.6\,\mathrm{kg}$ is projected up a line of greatest slope of a plane inclined at 30° to the horizontal. P moves with deceleration $10\,\mathrm{m\,s^{-2}}$ and comes to rest before reaching the top of the plane.

Part A
Frictional force

Calculate the frictional force acting on P to 3 significant figures.

Part B
Finding μ

Calculate the coefficient of friction between P and the plane to 3 significant figures.

Part C

P in motion

For when P is **in motion**, find the magnitude of the contact force exerted on P by the plane. Give your answer to 3 significant figures.

For when P is **in motion**, find the angle between the contact force and the upward direction of the line of greatest slope. Give your answer to 3 significant figures.

Part D

P at rest

For when P is **at rest**, find the magnitude of the contact force exerted on P by the plane. Give your answer to 3 significant figures.

For when P is **at rest**, find the angle between the contact force and the upward direction of the line of greatest slope. Give your answer to 2 significant figures.

Adapted with permission from UCLES, A Level, January 2011, OCR M1, Question 7

Question deck:

STEM SMART Single Maths 42 - Forces & Friction

isaac You work it out.

Question

Friction on Inclined Planes 1i

Subject & topics: Maths **Stage & difficulty:** A Level P2

A board is fixed so that it makes an angle of 11° with the horizontal. A block of mass $0.2\,\mathrm{kg}$ is placed on the board and then set in motion with an initial speed of $2\,\mathrm{m\,s^{-1}}$ down a line of greatest slope of the board. The block comes to rest in $4\,\mathrm{s}$. The coefficient of friction between the block and the board is μ.

Part A

Deceleration

Find the deceleration of the block.

Part B

Frictional force

Find the frictional force on the block while the block is in motion. Give your answer to 3 significant figures.

Part C

Finding μ

Find the value of μ to 3 significant figures.

Part D

Find α when the block starts to slide

With the block at rest on the board, the inclination of the board is gradually increased. The angle that the board makes with the horizontal is α.

Find α when the block starts to slide. Give your answer to 3 significant figures.

Part E

Find α when the block accelerates at $g(1-\mu)\cos\alpha$

Find α when the block is moving with acceleration of $g(1-\mu)\cos\alpha$.

Question deck:

STEM SMART Single Maths 42 - Forces & Friction

isaac You work it out.

Question

Friction on Inclined Planes 3i

Subject & topics: Maths **Stage & difficulty:** A Level P2

A sledge of mass $25\,\mathrm{kg}$ is on a plane inclined at 30° to the horizontal. The coefficient of friction between the sledge and the plane is 0.2.

Part A

Tension in cable

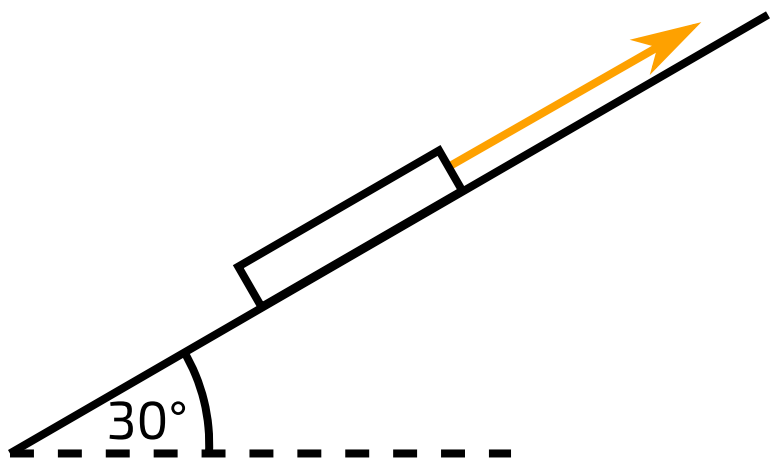

Figure 1: A sledge being pulled up an inclined plane.

The sledge is pulled up the plane, with constant acceleration, by means of a light cable which is parallel to a line of greatest slope. The sledge starts from rest and acquires a speed of $0.8\,\mathrm{m\,s^{-1}}$ after being pulled for $10\,\mathrm{s}$.

Ignoring air resistance, find the tension in the cable correct to 3 significant figures.

Part B

Least value of P

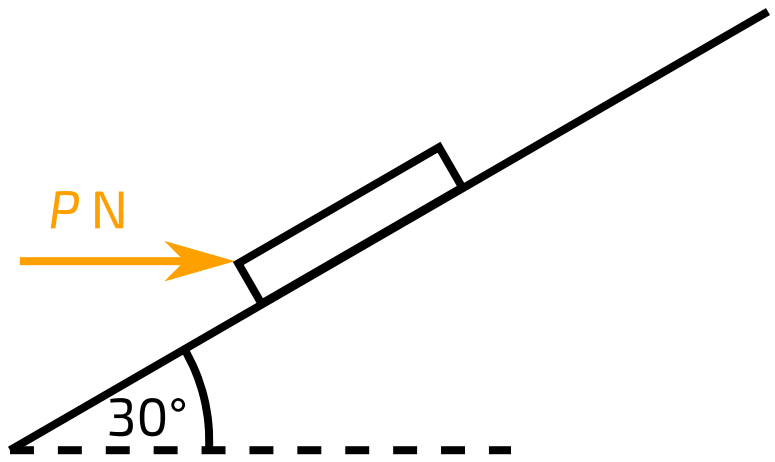

Figure 2: A sledge held at rest by a horizontal force on an inclined plane.

On a subsequent occasion the cable is not in use and two people of total mass $150\,\mathrm{kg}$ are seated in the sledge. The sledge is held at rest by a horizontal force of magnitude P newtons, as shown in **Figure 2**.

Find the least value of P which will prevent the sledge from sliding down the plane. Give your answer correct to 3 significant figures.