Pascal's Resistors

A Level

The diagram below shows an infinite array of resistors. Each resistor in the array has a resistance of the reciprocal of the number for pascals triangle for that position.

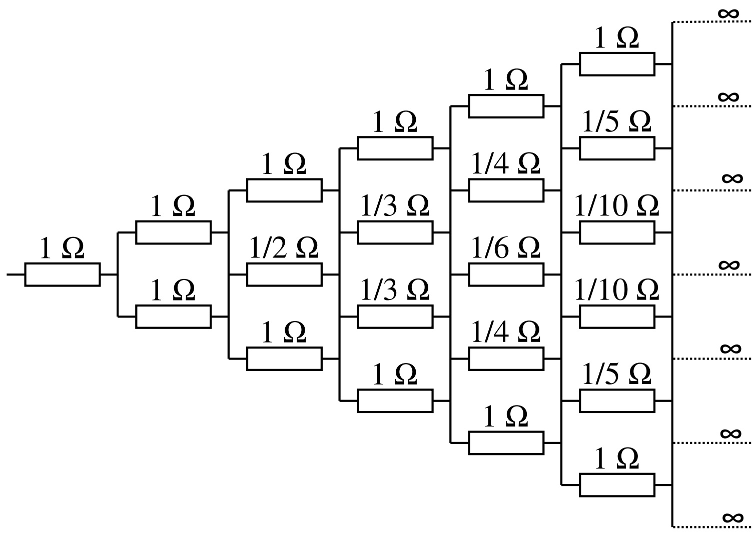

Figure 1: Circuit diagram showing an infinite array of resistors.

Find the equivalent resistance of the array, for a current flowing through the $1\,\Omega$ resistor at the left hand end through to the right hand end. Answer to 3 significant figures.

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Geometric Series 3

A Level C C C Further A C C C

A bouncing ball loses the same fraction of its energy every time it bounces i.e. when dropped from an initial height h, after its first bounce it rises to a height αh, after the second bounce $\alpha^2 h$ and so on (α is a number less than 1). Find an expression for the total distance travelled.

The following symbols may be useful: `alpha, h`

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Gameboard:
STEM SMART Physics 35 - Using Series

Integrating Powers 4

A Level C C C Further A C C C

Part A Integrate $\frac{A}{r^7} - \frac{B}{r^{13}}$

Find $\int_a^\infty \left(\frac{A}{r^7} - \frac{B}{r^{13}}\right) \mathrm{d}r$.

(The force between, for example, two atoms of an inert gas, a distance r apart is given by $\left(\frac{A}{r^7} - \frac{B}{r^{13}}\right)$, where A and B are (negative) constants; the first term is the attractive force between them (the van der Waals interaction, due to their fluctuating induced dipoles) and the second is the repulsive force due to the overlap of their electron shells. The integral describes the potential energy of such a system i.e. the work done bringing one atom from infinity to within a distance a of the other atom.)

Find $\int_a^\infty \left(\frac{A}{r^7} - \frac{B}{r^{13}}\right) \mathrm{d}r$.

The following symbols may be useful: `A`, `B`, `a`

Part B Integrate $\frac{C}{x^2} + D$

Find $\int_{x_1}^{x_2} \left(\frac{C}{x^2} + D\right) \mathrm{d}x$.

(The function $\left(\frac{C}{x^2} + D\right)$, where C and D are constants, could describe the component of an electric field in the x-direction due to a combination of the field due to a point charge at the origin and a uniform field in the x-direction. The integral is then the potential difference between two points x_1 and x_2 on the x-axis.)

Find $\int_{x_1}^{x_2} \left(\frac{C}{x^2} + D\right) \mathrm{d}x$.

The following symbols may be useful: `C`, `D`, `x_1`, `x_2`

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STEM SMART Physics 35 - Using Series

Physics. You work it out.

Apparent Magnitudes

A Level C C C Further A C C C

The apparent magnitude m of an astronomical object describes on a logarithmic scale how bright an object appears to an observer. It is related to its actual brightness or energy flux F (i.e. the energy arriving at the Earth per unit area per second) in the following way. Consider two objects with magnitudes m_1 and m_2 and brightnesses F_1 and F_2; the relationship between these quantities is

$$\frac{F_1}{F_2} = 100^{\frac{m_2-m_1}{5}}.$$

Part A Sun and Moon

The magnitude of the Sun is -26.8 and it is a factor of 4.80×10^5 brighter than the full Moon. Find the magnitude of the full Moon.

Part B Supernova 1987A

Supernova 1987A was discovered in the nearby dwarf galaxy the Large Magellanic Cloud and, with a magnitude of $+2.9$, it was visible with the naked eye. It was subsequently discovered that its progenitor was a blue supergiant with a magnitude of $+12.2$. Find the ratio of the brightness of Supernova 1987A to that of its progenitor (give your answer to 2 sig figs).

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Energy Decay

A Level C C C Further A P P P

A steel bar is tapped on one end and the resulting pulse of energy travels backwards and forwards along the bar. A very small fraction α of its energy is lost on each reflection so that after n reflections the fraction of its initial energy left is $(1-\alpha)^n$. It takes a time τ to travel from one end of the bar to the other.

Part A Time for energy to halve

Find an expression for the time it takes for the energy in the pulse to halve.

Use either $\log_{10}$, or the natural log, $\ln$. When you are entering your answer, note that $\log_{10} a$ can be written using `log(a,10)`.

The following symbols may be useful: `alpha`, `ln()`, `log()`, `tau`

Part B Time for energy to fall by factor of 100

Find an expression for the time it takes for the energy in the pulse to fall by a factor of 100.

Use either $\log_{10}$, or the natural log, $\ln$. When you are entering your answer, note that $\log_{10} a$ can be written using `log(a,10)`.

The following symbols may be useful: `alpha`, `ln()`, `log()`, `tau`

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7.3.8. A Bouncing Ball

A Level C C C Further A C C C

The height of a bouncing ball can be expressed as a geometric progression. Quantities changing geometrically are in fact exponential processes: they change in direct proportion to themselves. Algebraic manipulation exposes the natural exponential form. A ball is dropped from an initial height h_0 on to a table such that the height of the bounce is $h_1 = \alpha h_0$. Draw a diagram showing several bounces.

Part A Height (n)

Using the properties of powers, exponentials and logarithms, show that the height of the nth bounce as a function of n is $h_n = h_0 e^{cn}$ and give an expression for the constant c

The following symbols may be useful: `alpha, c, ln()`

Given $\alpha < 1$, give the range of c (and hence reassure yourself that h_n is a decaying function of n).

The following symbols may be useful: `<, <=, >, >=, c`

Part B Distance of travel

How far does the ball travel from the start, H, before it stops bouncing, in the ideal case?

The following symbols may be useful: `H, alpha, h_0`

Part C In practice

Why might this distance not be attained in practice?

Part D Time between bounces

Find the time between the nth and $(n+1)$th bounces.

The following symbols may be useful: `alpha, g, h_0, h_n, ln(), log(), n, t_n`

Show that it can be written as $2t_0e^{-\beta n}$ and give expressions for t_0 and β.

t_0

The following symbols may be useful: `, alpha, g, h_0, h_n, ln(), ln(), log(), n, t_0, t_n`

β (in terms of α)

The following symbols may be useful: `alpha, beta, g, h_0, ln(), ln(), log(), n, n`

Part E Time spent bouncing

Find the total time, T, spent bouncing in the ideal case. (give answer in terms of α)

The following symbols may be useful: `T, alpha, g, h_0, n, t_0`

How many bounces does the ball execute in this time?

Part F Frequency of bounce

Calculate the frequency of bouncing heard as a function of bounce number n.

(give answer in terms of α and t_0)

The following symbols may be useful: `alpha, beta, f_n, g, h_0, n, t_0`

Calculate the frequency of bouncing heard as a function of time T_n at the nth bounce and t_0.

The following symbols may be useful: `T_n, alpha, beta, f_n, g, h_0, n, t_0`

Part G Divergence

Discuss any divergences that arise!

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7.3.4 Money!

A Level C C C Further A C C C

A sum of money m is invested and earns interest at a rate p. [Note: p is a rate, that is “per unit of time”.]

Part A Compounding annually

If $p = 10\%$ per year, and the interest is calculated and compounded annually, by what multiple has your money increased after 10 years? [“Compounded” means the interest is added to your capital, whereupon interest is paid on the enhanced sum.]

Part B Compounding continuously

For continuously compounded interest at a rate $p\,\%$, the sum of money m increases by $\mathrm{d}m = \frac{mp}{100}\mathrm{d}t$ in time $\mathrm{d}t$. What are the units on each side of the expression?

- year^{-1}
- $£\ \mathrm{year}^{-1}$
- $\mathrm{dimensionless}$
- $£$

Part C Money(t)

Find an expression for $m(t)$ if you start with m_0 at time $t = 0$.

The following symbols may be useful: `e, m, m_0, p, t`

Part D Ten years

How much is your money worth after $10\,\mathrm{years}$ of being continuously compounded at rate $p = 10\%$ per year? Compare your answer to that in part a).

The following symbols may be useful: `e, m, m_0, p, t`

Part E The Charity Commissioners

If m is the capital of a charity, payments are made at a rate q, and interest is continuously compounded at a rate of $p\,\%$, show that the equation for m is now

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{p}{100}m - q$$

If the Charity Commissioners demand that capital does not decrease, what is the maximum rate of payments q allowed?

The following symbols may be useful: `m, p, q, t`

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Gameboard:

STEM SMART Physics 35 - Using Series

Essential Pre-Uni Physics K2.7

A Level

P P P

It is advisable to have completed section J3 before beginning the questions in section K2.

It is said to be safe to view the Sun through a filter if it only lets 10^{-5} of the light through. Suppose you have some material which lets 2.0% of the light through. How many sheets do you need to put together back-to-back before you can safely look through it at the Sun? NB - Never make your own filter for viewing the Sun in this way - most filters bleach with very high intensities and aren't designed with eye protection in mind, so the quality is not good enough for a device which is to prevent blindness. Give your answer to 1 significant figure.

Gameboard:

STEM SMART Physics 35 - Using Series

Physics. You work it out.

Home Gameboard Physics Waves & Particles Nuclear Essential Pre-Uni Physics K2.9

Essential Pre-Uni Physics K2.9

A Level

It is advisable to have completed section J3 before beginning the questions in section K2.

The thickness of lead needed to stop half of the neutrinos in a beam is about $3000\,\mathrm{lightyears}$ (which you may take as $3.0 \times 10^{19}\,\mathrm{m}$).

Calculate the fraction of neutrinos which would be stopped by $100\,\mathrm{m}$ of water assuming that the attenuation coefficients for water and lead are about the same (which they're not). You may need to use the approximation $\mathrm{e}^x \approx 1 + x$ provided x has a small magnitude.

Gameboard:

STEM SMART Physics 35 - Using Series

Essential Pre-Uni Physics K2.10

A Level

P P P

It is advisable to have completed section J3 before beginning the questions in section K2.

You start with a credit card debt of £150. For each month in which you don't pay it off, the debt increases by 3.0%. Assuming you pay nothing for $3.0\,\mathrm{years}$, and then want to settle the debt in one go, how much would you have to pay in pounds? Give your answer to 3 significant figures.