Physics. You work it out.

Differentiation from First Principles 2

A Level

P P P

Pre-Uni Maths for Sciences J3.3 & J3.4

Differentiating a function $f(x)$ from first principles involves taking a limit. The derivative of $f(x)$ is given by the expression

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Part A Differentiate $4x^2 + 2x + 7$ from first principles

Differentiate $f(x) = 4x^2 + 2x + 7$ from first principles. Drag and drop options into the spaces below.

$f(x+h) = 4(x+h)^2 + 2(x+h) + 7$. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(4(x+h)^2 + 2(x+h) + 7) - (4x^2 + 2x + 7)}{h}.$$

Next, expanding the brackets in the numerator and simplifying gives

$$f'(x) = \lim_{h \to 0} \frac{\boxed{} - (4x^2 + 2x + 7)}{h}$$

$$f'(x) = \lim_{h \to 0} (\boxed{} + (\boxed{})h).$$

Finally, take the limit. As $h \to 0$, the terms containing h tend to 0. Therefore,

$$f'(x) = \boxed{}.$$

Items:

$8x + 4$ | $4x^2 + 2x + 7 + 8hx + 2h + 4h^2$ | $8x + 2$ | $4x^2 + 2x + 7 + 4hx + 2h + 4h^2$ | 7 | $4x^2 + 4h^2$ | 4

Part B Differentiate $ax^2 + bx + c$ from first principles

Differentiate $f(x) = ax^2 + bx + c$, where a, b and c are constants, from first principles.

$f(x+h) = a(x+h)^2 + b(x+h) + c$. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(a(x+h)^2 + b(x+h) + c) - (ax^2 + bx + c)}{h}.$$

Next, expanding the brackets in the numerator and simplifying gives

$$f'(x) = \lim_{h \to 0} \frac{\boxed{} + (\boxed{})h + (\boxed{})h^2}{h}$$

$$f'(x) = \lim_{h \to 0} (\boxed{} + (\boxed{})h).$$

Finally, take the limit. As $h \to 0$, the terms containing h tend to 0. Therefore,

$$f'(x) = \boxed{}.$$

Items:

$\boxed{ax^2 + 2ahx + ah^2}$ $\boxed{2ax + b}$ $\boxed{0}$ $\boxed{b + ah}$ $\boxed{ab}$ $\boxed{a}$ $\boxed{1}$ $\boxed{2a}$

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Home Gameboard Maths Calculus

Calculus

A Level
C C C

Part A Integrating a factorised expression

Find $\int (x^2+9)(x-4)\mathrm{d}x$.

The following symbols may be useful: `C, c, k, x`

Part B Differentiation

A curve has the equation $y = \frac{1}{3}x^3 - 9x$.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), x, y`

Part C Stationary points

Find the coordinates of the stationary points of the curve $y = \frac{1}{3}x^3 - 9x$. Enter the x and y coordinates of the stationary point with the largest x coordinate.

(___ , ___)

Part D Nature of stationary point

Determine the nature of the stationary point with the largest x-coordinate.

- Minimum
- Maximum
- Neither/Inconclusive

Part E Tangent to the curve

Given that $24x + 3y + 2 = 0$ is the equation of the tangent to the curve $y = \frac{1}{3}x^3 - 9x$ at the point (p, q), find the values of p and q.

(i) Enter value of p:

The following symbols may be useful: p

(ii) Enter value of q:

The following symbols may be useful: q

Part F Normal to the curve

Find the equation of the normal to the curve $y = \frac{1}{3}x^3 - 9x$ at the point (p, q) you found in Part E.

Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers

The following symbols may be useful: x, y

Modified by Sally Waugh with permission from UCLES, A Level, June 2005, Paper 4721, Question 10.

Gameboard:

STEM SMART Double Maths 16 - Calculus Revision

Minimisation: Surface Area

A Level

P P P

Figure 1 shows a solid shape, which is made out of a cuboid and two half-cylinders.

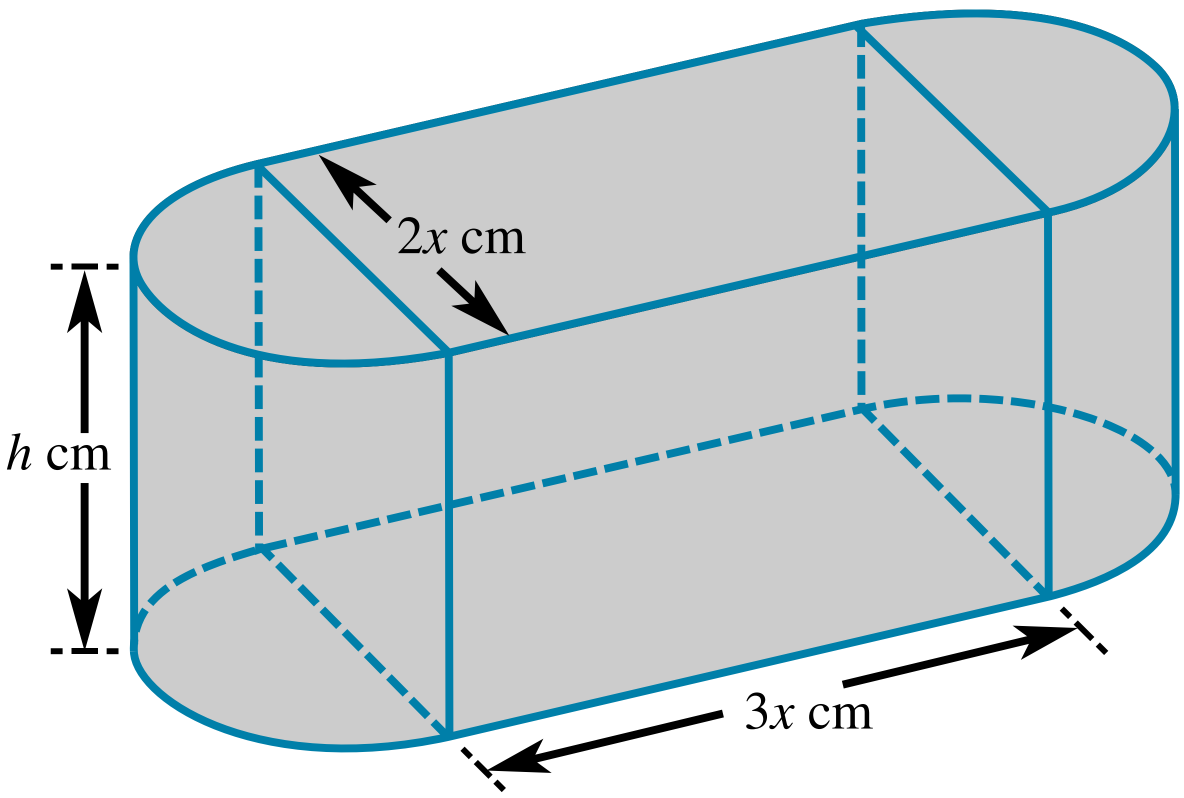

Figure 1: A solid shape made out of a cuboid and two half-cylinders.

Part A Expression for surface area

Find an expression for the surface area of the shape S in terms of π, x and h.

The following symbols may be useful: `S`, `h`, `pi`, `x`

Part B Expression for volume

Find an expression for the volume of the shape V in terms of π, x and h.

The following symbols may be useful: `V`, `h`, `pi`, `x`

Part C Minimum surface area

If the volume of the shape is $32\,000\,\mathrm{cm}^3$, find the value of x for which the surface area is a minimum. Give your answer to 3 significant figures.

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Gameboard:

STEM SMART Double Maths 16 - Calculus Revision

Differentiation and Gradients: Beyond Polynomials 4ii

A Level

P P P

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in each of the following cases.

Part A Derivative 1

$y = x^3 e^{2x}$

The following symbols may be useful: `Derivative(y, x)`, `e`, `ln()`, `log()`, `x`, `y`

Part B Derivative 2

$y = \ln\left(3 + 2x^2\right)$

The following symbols may be useful: `Derivative(y, x)`, `e`, `ln()`, `log()`, `x`, `y`

Part C Derivative 3

$y = \frac{x}{2x+1}$

The following symbols may be useful: `Derivative(y, x)`, `e`, `ln()`, `log()`, `x`, `y`

Gameboard:

STEM SMART Double Maths 16 - Calculus Revision

Differentiating Trig Functions 2

A Level

P P P

Pre-Uni Maths for Sciences J4.2

Part A Differentiate $s = r\sin(\alpha\theta)$

Find $\frac{\mathrm{d}s}{\mathrm{d}\theta}$ if $s = r\sin(\alpha\theta)$, where r and α are constants.

The following symbols may be useful: `alpha`, `cos()`, `r`, `sin()`, `tan()`, `theta`

Part B Differentiate $q = l\cos(\alpha - 2\beta\theta)$

Find $\frac{\mathrm{d}q}{\mathrm{d}\theta}$ if $q = l\cos(\alpha - 2\beta\theta)$, where l, α and β are constants.

The following symbols may be useful: `alpha`, `beta`, `cos()`, `l`, `sin()`, `tan()`, `theta`

Created for isaacphysics.org by Julia Riley

Gameboard:

STEM SMART Double Maths 16 - Calculus Revision

Physics. You work it out.

Home Gameboard Maths Differentiation: Chain Rule 4ii

Differentiation: Chain Rule 4ii

A Level
P P P

The volume, V cubic metres, of water in a reservoir is given by

$$V = 3(2+\sqrt{h})^6 - 192,$$

where h metres is the depth of the water. Water is flowing into the reservoir at a constant rate of 150 cubic metres per hour. Find the rate at which the depth of water is increasing at the instant when the depth is 1.4 metres, to three significant figures, in metres per hour.

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Gameboard:
STEM SMART Double Maths 16 - Calculus Revision

Area Between Two Curves 4ii

A Level

P P P

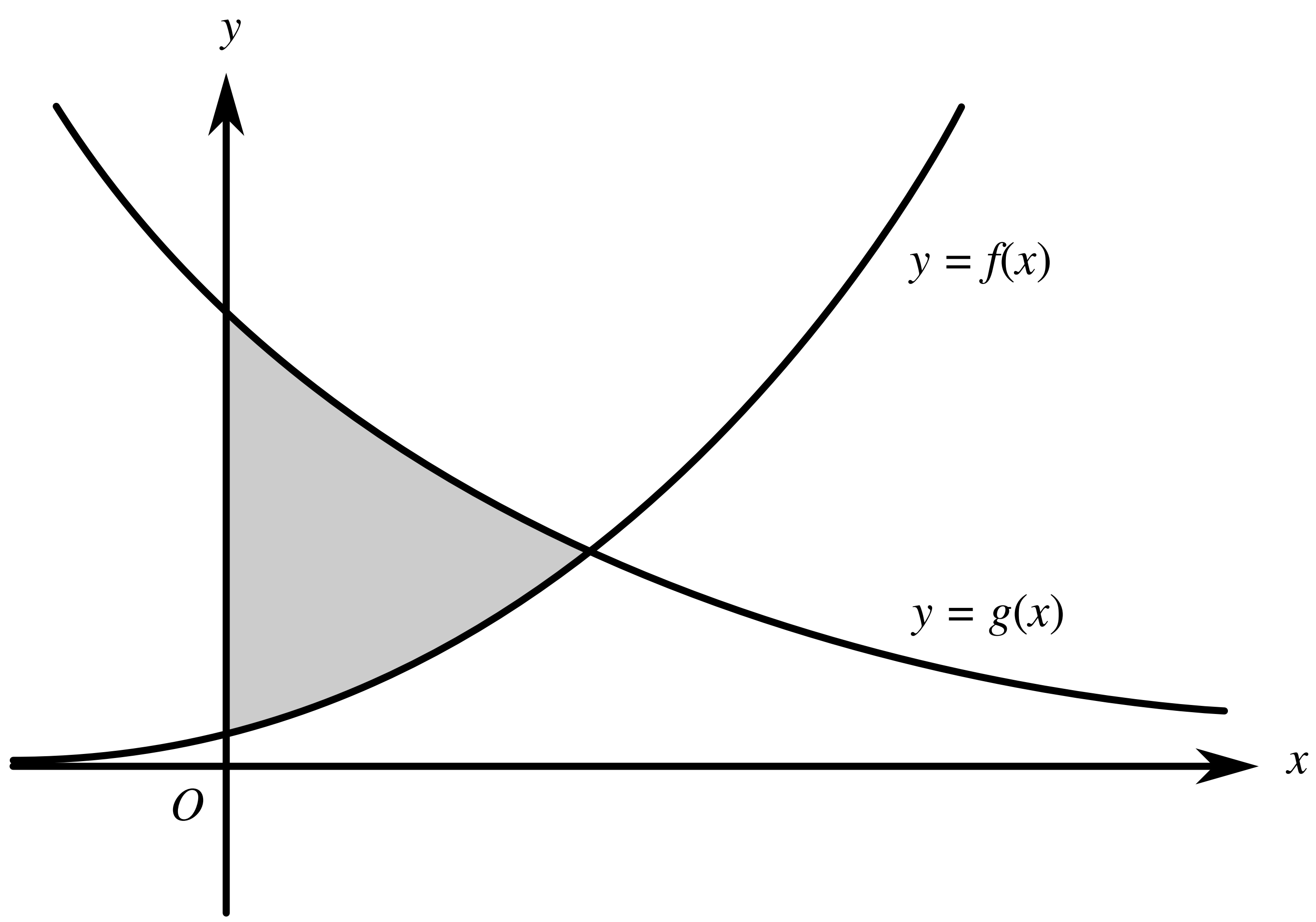

Figure 1: The curves $y = f(x)$ and $y = g(x)$.

Figure 1 shows the curves $y = f(x)$ and $y = g(x)$, where

$$f(x) = \mathrm{e}^{2x}$$

$$g(x) = 8\mathrm{e}^{-x}$$

The shaded region is bounded by the curves and the y-axis.

Part A x-coordinate

Find the exact x-coordinate of the point of intersection of the curves.

The following symbols may be useful: x

Part B Integral

Find the exact area of the shaded region. Give your answer as a fraction.

Gameboard:

STEM SMART Double Maths 16 - Calculus Revision

Physics. You work it out.

Home Gameboard Maths Integration by Substitution 3i

Integration by Substitution 3i

A Level
P P P

Part A Substitution

Find the expression that appears to the right of the integral sign after the substitution $u = \mathrm{e}^x + 1$ has been applied to $\displaystyle\int \frac{\mathrm{e}^{2x}}{\mathrm{e}^x + 1}\,\mathrm{d}x$. Include $\mathrm{d}u$ in your answer.

The following symbols may be useful: `du`, `u`

Part B Integral

Hence find the exact value of $\displaystyle\int_0^1 \frac{\mathrm{e}^{2x}}{\mathrm{e}^x + 1}\,\mathrm{d}x$.

The following symbols may be useful: `e`

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Gameboard:
STEM SMART Double Maths 16 - Calculus Revision

Integration by Parts 2ii

A Level

P P P

Find the exact value of $\int_1^8 \frac{1}{\sqrt[3]{x}} \ln(x) \mathrm{d}x$, giving your answer in the form $A\ln(2) + B$, where A and B are constants to be found.