Transformations of Graphs 3ii

A Level

The graph of $y = f(x)$ for $-2 \leq x \leq 2$ is shown in **Figure 1**.

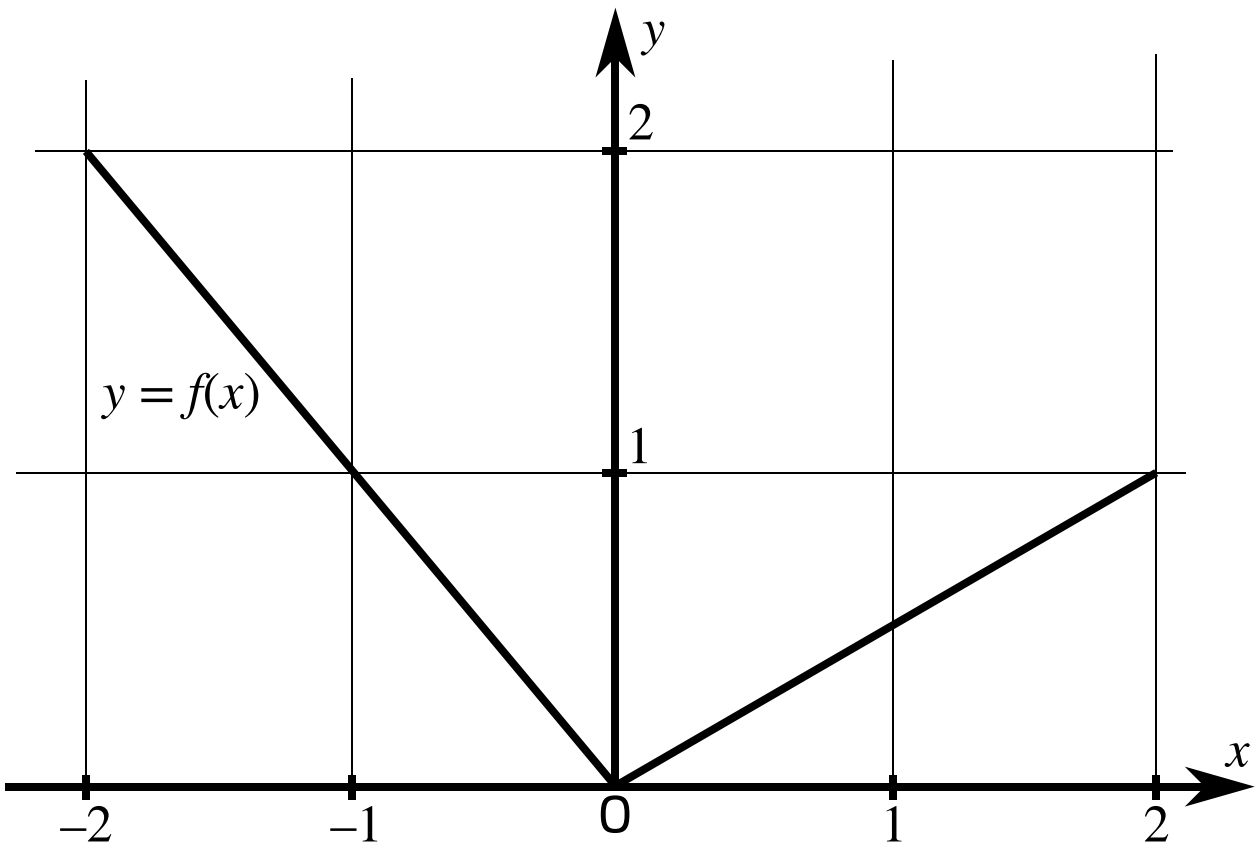


Figure 1: The graph of $y = f(x)$ for $-2 \leq x \leq 2$.

Part A Sketch $y = f(-x)$

Sketch the curve $y = f(-x)$ for $-2 \leq x \leq 2$.

What is the y-value of the curve $y = f(-x)$ when $x = 1$?

The following symbols may be useful: y

Part B **Sketch** $y = f(-x) + 2$

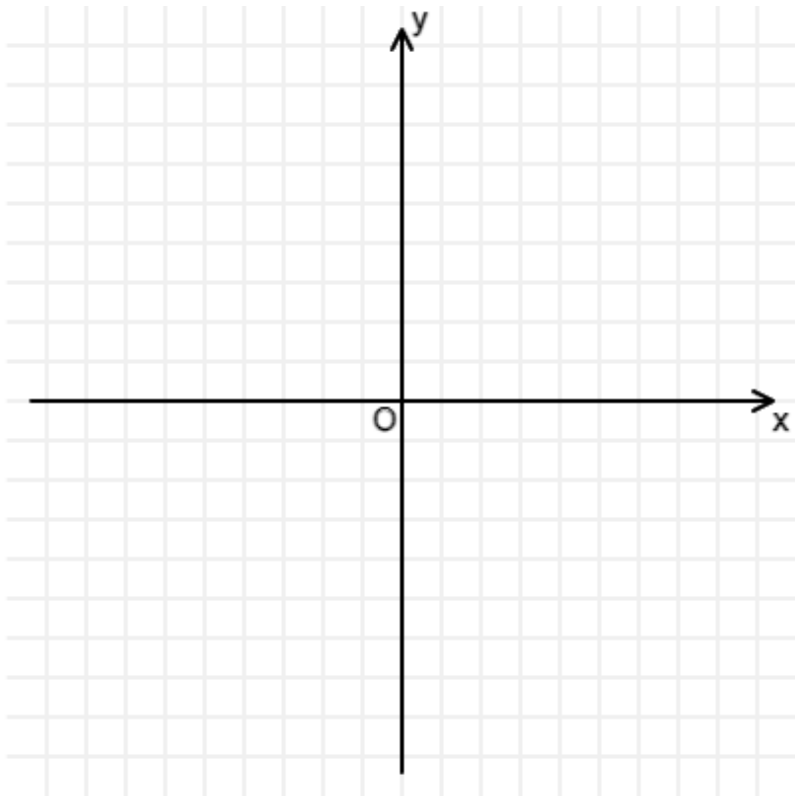
Sketch the curve $y = f(-x) + 2$ for $-2 \leq x \leq 2$.

What is the y-value of the curve $y = f(-x) + 2$ when $x = -2$?

The following symbols may be useful: y

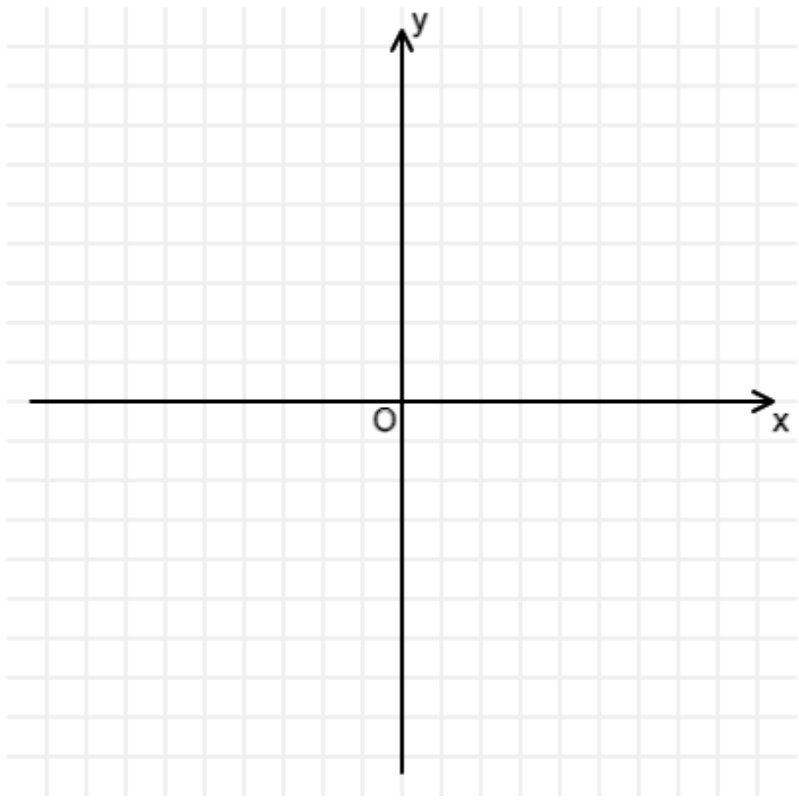
Part C **Sketch** $y = -\frac{1}{x^2}$

Sketch the curve $y = -\frac{1}{x^2}$.



Part D Sketch $y = 3 - \frac{1}{x^2}$

Sketch the curve $y = 3 - \frac{1}{x^2}$.



Part E State the equation

The curve $y = -\frac{1}{x^2}$ is stretched parallel to the y-axis by scale factor 2. State the equation of the transformed curve.

The following symbols may be useful: `x, y`

Used with permission from UCLES, A level, January 2012, Paper 4721, Question 2 and June 2010, Paper 4721, Question 2.

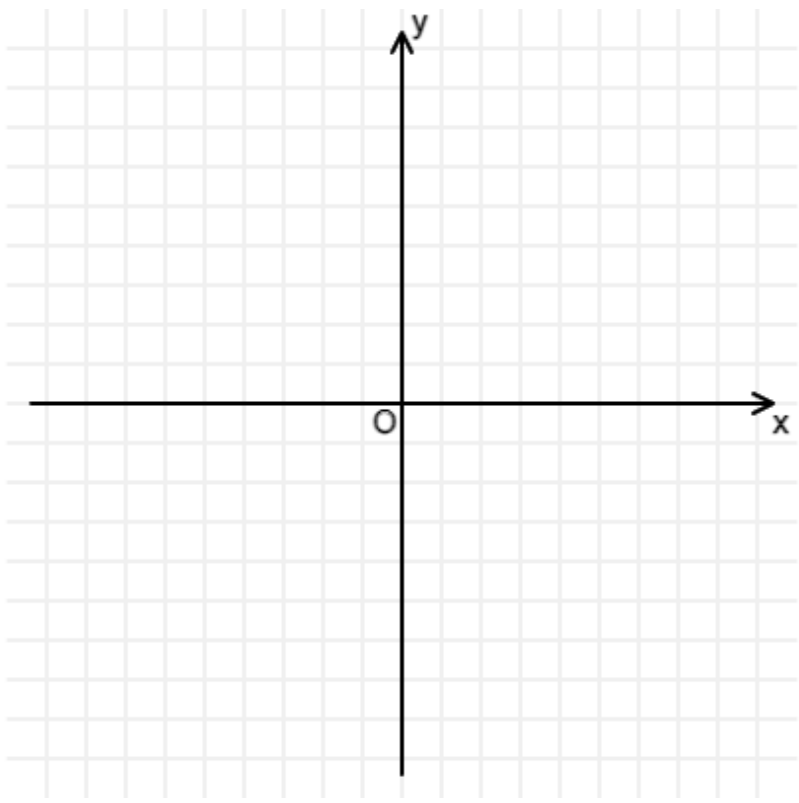
All materials on this site are licensed under the **Creative Commons license**, unless stated otherwise.

Transformations of Graphs 1i

A Level

Part A Sketch y

Find the roots of the curve $y = x^2(3 - x)$ and sketch it.



Part B Translate y

The curve $y = x^2(3 - x)$ is translated by two units in the positive direction parallel to the x axis.

State the equation of the curve after this transformation.

The following symbols may be useful: `x`, `y`

Part C Find transformation of y

Which of these describes the transformation of the curve $y = x^2(3 - x)$ to $y = \frac{1}{2}x^2(3 - x)$?

- A stretch of scale factor 2 parallel to the x-axis.
- A stretch of scale factor $\frac{1}{2}$ parallel to the x-axis.
- A stretch of scale factor 2 parallel to the y-axis.
- A stretch of scale factor $\frac{1}{2}$ parallel to the y-axis.

Part D Vertical translation of $f(x)$

The curve $y = f(x)$ passes through the point P with coordinates $(2, 5)$.

State the coordinates of the point corresponding to P on the curve $y = f(x) + 2$.

(,)

Part E Lateral stretching of $f(x)$

The curve $y = f(x)$ passes through the point P with coordinates $(2, 5)$.

State the coordinates of the point corresponding to P on the curve $y = f(2x)$.

(,)

Part F Find transformation of $f(x)$

Which of the following describes the single transformation that maps the curve $y = f(x)$ onto $y = f(x + 4)$?

- A translation of -4 units parallel to the y-axis.
- A translation of 4 units parallel to the x-axis.
- A translation of -4 units parallel to the x-axis.
- A translation of 4 units parallel to the y-axis.

Used with permission from UCLES, A level, June 2016, Paper 4721, Question 7 and June 2014, Paper 4721, Question 4.

Gameboard:

STEM SMART Double Maths 18 - Transformations and Circles

All materials on this site are licensed under the **Creative Commons license**, unless stated otherwise.

Reflection and Symmetry

A Level

C C C

Pre-Uni Maths for Sciences E2.10

The following questions ask you to deduce the symmetry properties of a number of functions. There are three choices:

- even - a function for which $f(x) = f(-x)$ which is also described as being symmetric about the vertical axis,
- odd - a function for which $f(x) = -f(-x)$ which is also described as being antisymmetric about the vertical axis (or symmetric about zero),
- neither even nor odd.

Where relevant you may assume that a and b are non-zero constants.

Part A Even functions

Decide which of the following functions are even.

- [ ] $(x-a)(x+a)$
- [ ] $\frac{a}{x^2}+bx^2$
- [ ] $(x-a)(x+b)\ (a \neq b)$
- [ ] $x^2(a+bx)$
- [ ] ax^2+bx^4
- [ ] ax^2+b
- [ ] $a(x+b)^2$
- [ ] $a\cos x$
- [ ] $\frac{a}{x^2}+b$
- [ ] ax^2
- [ ] $a\sin x$

Part B Odd functions

Decide which of the following functions are odd.

- [ ] $a \tan x$
- [ ] $(x+a)^{\frac{1}{3}}$
- [ ] $x(a+bx^2)$
- [ ] $x^{\frac{1}{3}}$
- [ ] $\frac{a}{x}+bx^3$
- [ ] ax
- [ ] $a \sin x$
- [ ] $\frac{a}{x}$
- [ ] $\frac{a}{x}+b$
- [ ] $x^2(a+bx)$
- [ ] $\frac{a}{x}+\frac{b}{x^3}$

Part C Neither odd nor even functions

Decide which of the following functions are neither odd nor even.

- [ ] $ax - b$
- [ ] $a\tan(x + 45^\circ)$
- [ ] $ax^{\frac{1}{2}}$
- [ ] $(x - a)(x + a)$
- [ ] $(x - a)(x + a)^2$
- [ ] $a\left(\frac{1}{x^2} - \frac{1}{b^2}\right)$
- [ ] $a(b - x)^{\frac{1}{2}}$
- [ ] $x(ax^2 + b)$
- [ ] $\cos x + \sin x$
- [ ] $\frac{a}{(x - b)^2}$
- [ ] $x^2(ax + b)$

Created for isaacphysics.org by Julia Riley

Gameboard:
STEM SMART Double Maths 18 - Transformations and Circles

All materials on this site are licensed under the **Creative Commons license**, unless stated otherwise.

Circles 1ii

A Level

The circle with equation $x^2 + y^2 - 6x - k = 0$ has radius 4.

The points A$(3, a)$ and B$(-1, 0)$ lie on the circumference of the circle, with $a > 0$.

Part A Centre

By completing the square for x and y find the coordinates of the centre of the circle.

(☐ , ☐)

Part B Value of k

Find the value of k.

The following symbols may be useful: k

Part C Length AB

Calculate the length of AB, giving your answer in simplified surd form.

Part D Equation

Find the equation of the line AB. Give your answer in the form $y = mx + c$.

The following symbols may be useful: `x, y`

Used with permission from UCLES, A level, June 2007, Paper 4721, Question 9.

Gameboard:
STEM SMART Double Maths 18 - Transformations and Circles

All materials on this site are licensed under the **Creative Commons license**, unless stated otherwise.

Circles 3ii

A Level

A circle has centre $(3, 1)$ and radius 5, and a line has equation $y = 2x$.

Part A Circle equation

Write down the equation of the circle.

The following symbols may be useful: `x`, `y`

Part B Intersection points

Find the coordinates of the points of intersection of the line and the circle.

(☐ , ☐)

Part C Point on the line

Find the coordinates of the point on the line which is closest to the centre of the circle.

(☐ , ☐)

Used with permission from UCLES, A level, January 2010, Paper 4721, Question 8.

Circles 2i

A Level

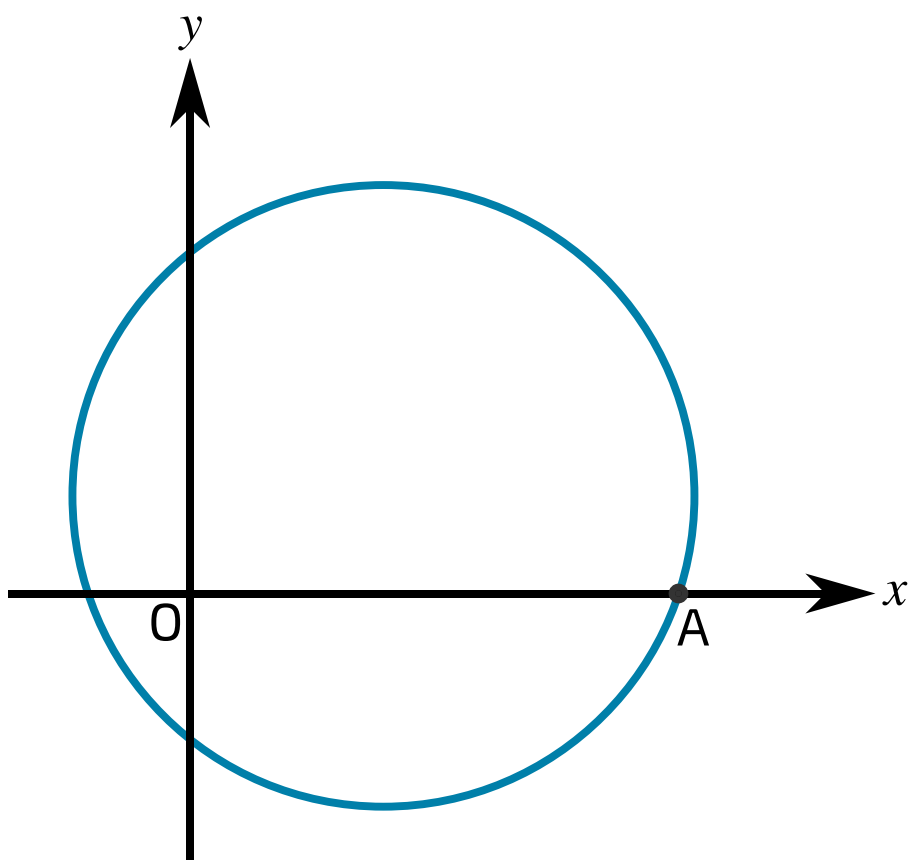


Figure 1: The circle with equation $x^2 + y^2 - 8x - 6y - 20 = 0$.

Figure 1 shows the circle with equation $x^2 + y^2 - 8x - 6y - 20 = 0$. The circle crosses the positive x axis at point A.

Part A Find centre

By completing the square for x and y find the coordinates of the centre of the circle.

(☐ , ☐)

Part B Find radius

Find the radius of the circle.

Part C Tangent to the circle at A

Find the equation of the tangent to the circle at A. Give your answer in the form $y = mx + c$.

The following symbols may be useful: `x, y`

Part D Another tangent to the circle

A second tangent to the circle is parallel to the tangent at A. Find the equation of this second tangent in the form $y = mx + c$.

The following symbols may be useful: `x, y`

Part E Find a radius

Another circle has its centre at the origin O and radius r. This circle lies wholly inside the first circle. Find the set of possible values of r. Give your answer as an inequality.

The following symbols may be useful: `<, <=, >, >=, r`

Used with permission from UCLES, A level, June 2016, Paper 4721, Question 10.

Gameboard:
STEM SMART Double Maths 18 - Transformations and Circles

Transformations and Area 2i

Further A

Part A Enlargement

The matrix $\mathbf{A}$ represents an enlargement, centre $(0, 0)$, with scale factor $\sqrt{2}$.

Complete the matrix $\mathbf{A}$ using the items below.

$$\mathbf{A} = \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$$

Items:

-2 $-\sqrt{2}$ -1 $-\frac{\sqrt{2}}{2}$ 0 $\frac{\sqrt{2}}{2}$ 1 $\sqrt{2}$ 2

Part B Matrix $\mathbf{B}$

The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$.

Which of the following transformations is represented by $\mathbf{B}$?

- Rotation, about the origin, $45°$ clockwise.
- Stretch, scale factor $\frac{\sqrt{2}}{2}$ parallel to the y axis.
- Reflection in the line $y = \frac{x}{\sqrt{2}}$
- Enlargement, centre $(0, 0)$, scale factor $\frac{1}{\sqrt{2}}$.

Part C Successive transformations

$\mathbf{C}$ is given by $\mathbf{C} = \mathbf{AB}$. Find $\mathbf{C}$.

$$\mathbf{C} = \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$$

Items:

-2 $-\sqrt{2}$ -1 $-\frac{\sqrt{2}}{2}$ $-\frac{1}{2}$ 0 $\frac{1}{2}$ $\frac{\sqrt{2}}{2}$ 1 $\sqrt{2}$ 2

Part D Transformed area

Find the area of the image of the unit square under the transformation represented by $\mathbf{C}$.

Part E Unit square

Which of the figures below shows the unit square and its image under the transformation represented by $\mathbf{C}$?

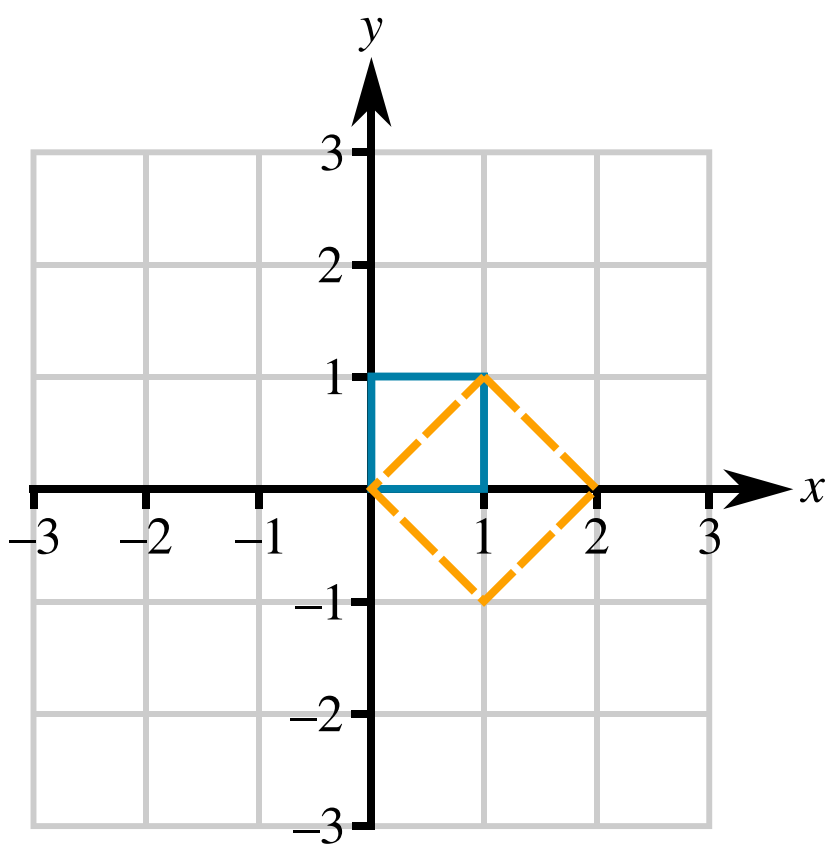


Figure 1: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

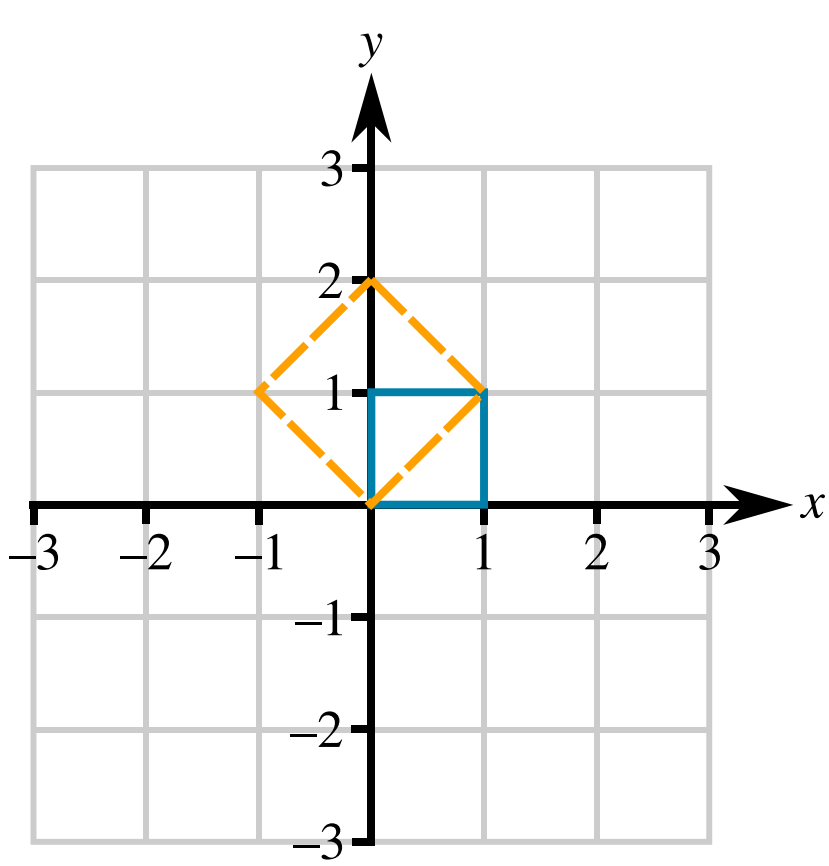


Figure 2: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

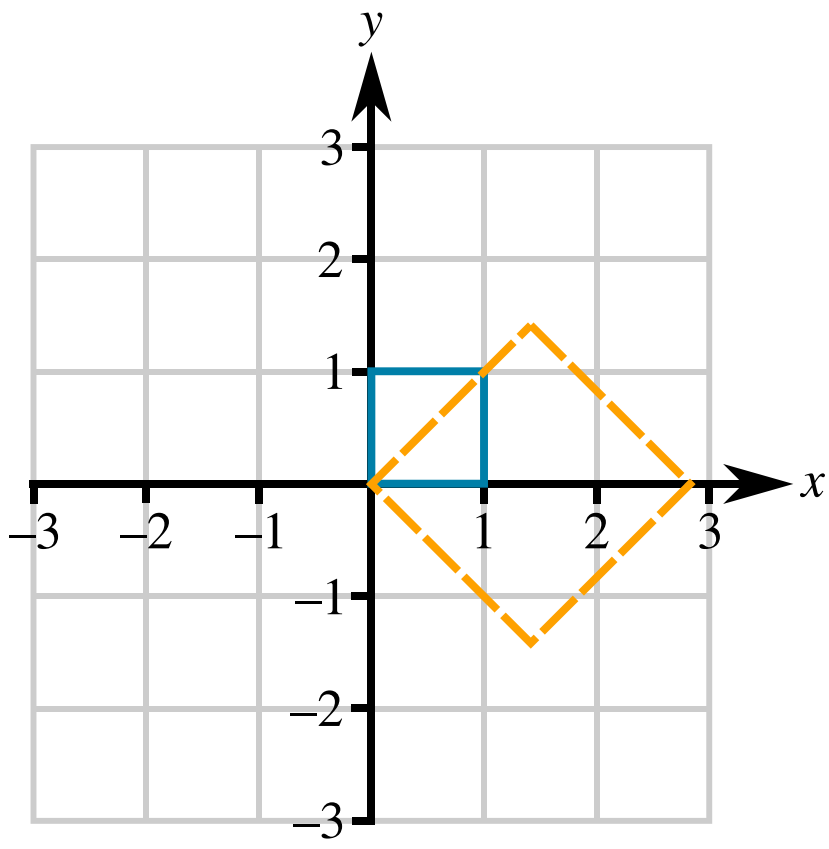


Figure 3: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

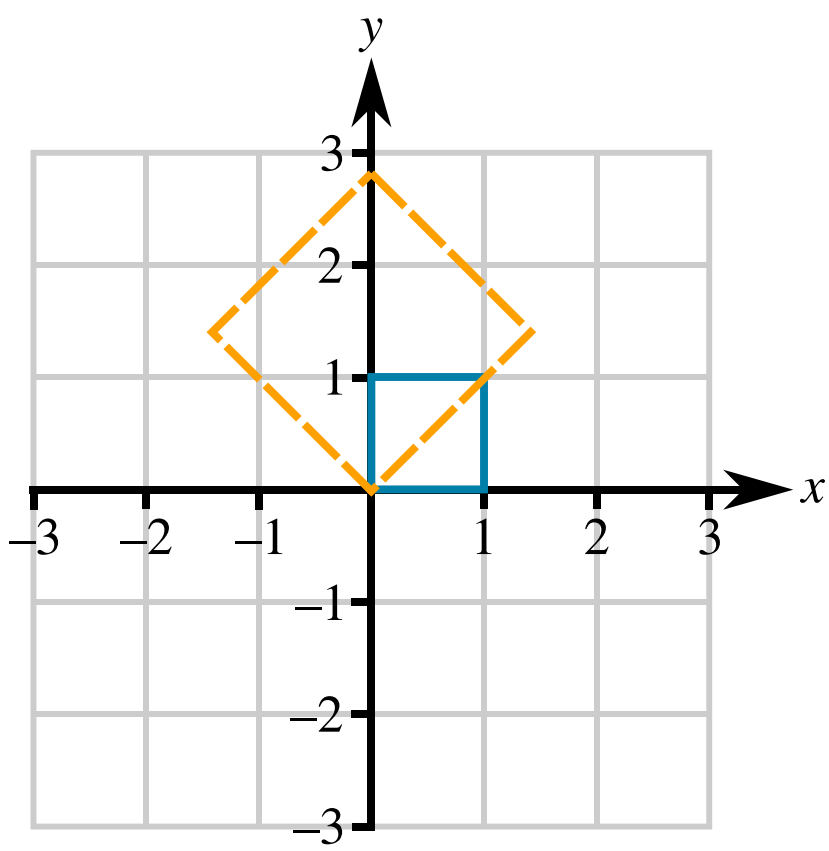


Figure 4: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

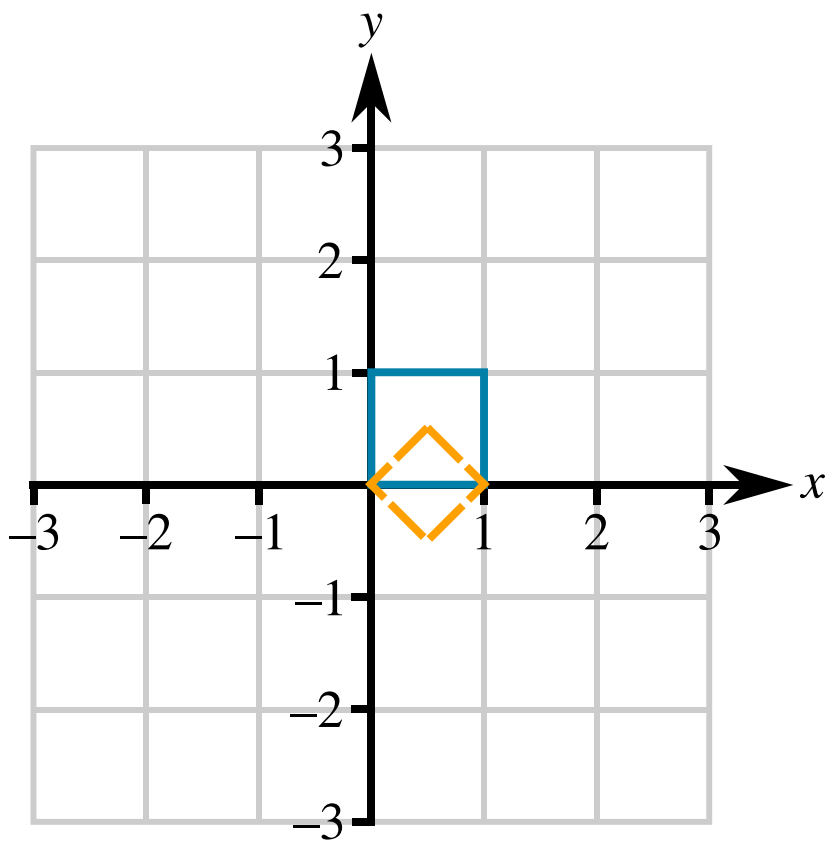


Figure 5: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

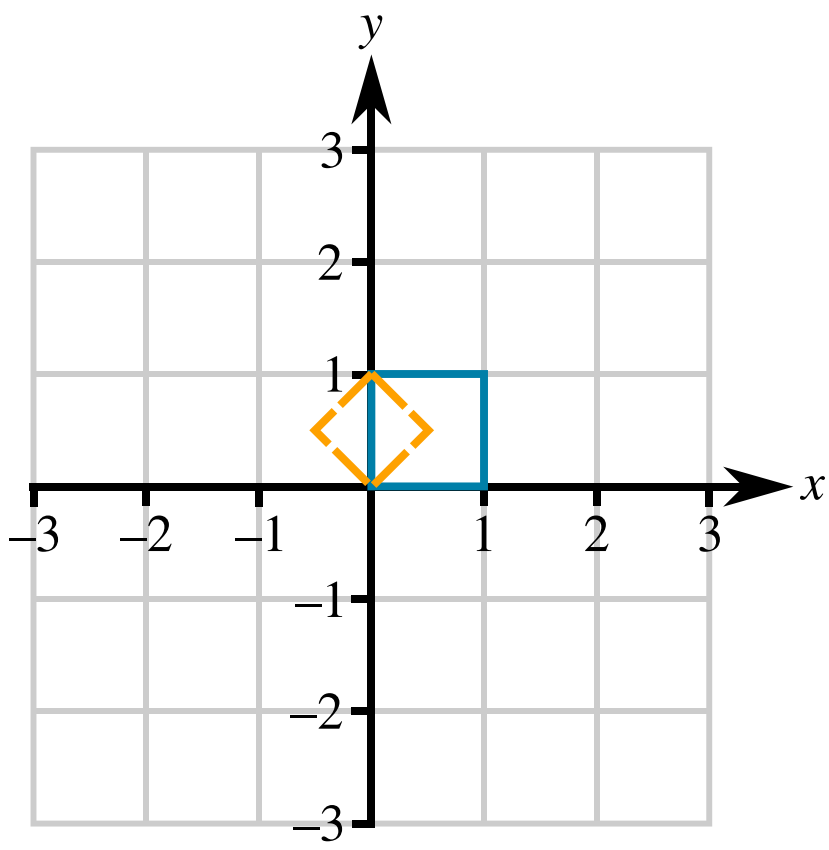


Figure 6: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

- Figure 1
- Figure 2
- Figure 3
- Figure 4
- Figure 5
- Figure 6

Transformations - Successive 3i

Further A

The diagram in **Figure 1** shows the unit square OABC, and its image OAB′C′ after a transformation.

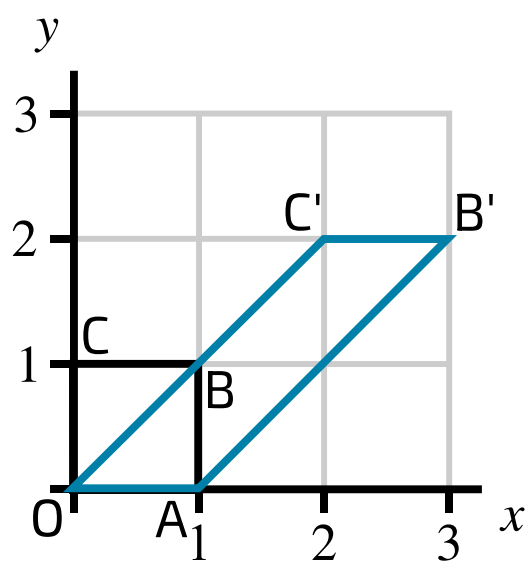


Figure 1: The unit square is shown in black, and the image after transformation is shown in blue.

Part A Matrix $\mathbf{X}$

Find the matrix, $\mathbf{X}$, for this transformation.

$$\mathbf{X} = \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$$

Items:

0 1 2 3

Part B Transformations P & Q

The transformation represented by $\mathbf{X}$ is equivalent to a transformation P followed by a transformation Q, which can be represented by the matrices $\mathbf{P}$ and $\mathbf{Q}$.

Fill in the gaps below to describe a pair of possible transformations P and Q, and find the matrices $\mathbf{P}$ and $\mathbf{Q}$ that represent them.

- P is a [ ], and is represented by $\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$.
- Q is a [ ], and is represented by $\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$.

Now instead find the matrix that represents transformation Q followed by transformation P.

- Q followed by P is represented by $\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$.

Items:

0	1	2	3	4	**stretch**	**shear**	**rotation**

Adapted with permission from UCLES, A Level, Jan 2013, Paper 4725, Question 6.

Gameboard:

STEM SMART Double Maths 18 - Transformations and Circles

All materials on this site are licensed under the **Creative Commons license**, unless stated otherwise.

Matrices - Transformations 1

Further A University

$\mathbf{P}$ and $\mathbf{Q}$ are 3×3 matrices which carry out a reflection in the plane $y = 0$ and a rotation about the x-axis, respectively. The matrix $\mathbf{R} = \mathbf{QP}$.

Part A Reflection in the $y = 0$ plane

The 3×3 matrix $\mathbf{P}$ carries out a reflection in the plane $y = 0$.

Complete the matrix $\mathbf{P}$ using the items below.

$$\mathbf{P} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

Items:

-3 -2 -1 0 1 2 3

Part B Rotation about the x-axis

The 3×3 matrix $\mathbf{Q}$ carries out an anticlockwise rotation about the x-axis through an angle A.

Complete the matrix $\mathbf{Q}$ using the items below.

$$\mathbf{Q} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

Items:

-1 | 0 | 1 | $-\sin A$ | $\sin A$ | $-\cos A$ | $\cos A$ | $-\tan A$ | $\tan A$

Part C Reflection followed by rotation

Find the 3×3 matrix $\mathbf{R} = \mathbf{QP}$ which carries out a reflection in the plane $y = 0$ followed by a rotation anticlockwise about the x-axis through an angle A.

$$\mathbf{R} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

Items:

-1 | 0 | 1 | $-\sin A$ | $\sin A$ | $-\cos A$ | $\cos A$ | $-\tan A$ | $\tan A$

Part D Reflection in the $z = 0$ plane

If the value of A is such that the 3×3 matrix $\mathbf{R}$ matrix represents a reflection in the plane $z = 0$, find the angle A. Assume $0 \leq A < 2\pi$ and give your answer in radians.

The following symbols may be useful: `A, pi`

Created for isaacphysics.org by Julia Riley

All materials on this site are licensed under the **Creative Commons license**, unless stated otherwise.