Stationary Points 1

A Level

Pre-Uni Maths for Sciences J2.1

Find the coordinates, nature and number of the stationary points of the following functions.

Part A $y = 2x^3 - 24x - 5$

$$y = 2x^3 - 24x - 5$$

Give the stationary points in order of increasing x-coordinate.

- There is a [ ] at ([ ] , [ ])
- There is a [ ] at ([ ] , [ ])

Part B $y = 2x^3 - 5x^2 + 4x + 6$

$$y = 2x^3 - 5x^2 + 4x + 6$$

Give the stationary points in order of increasing x-coordinate. Enter fractions as improper fractions in their simplest form (e.g. `4/3`).

- There is a [ ] at ([ ] , [ ])
- There is a [ ] at ([ ] , [ ])

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Maxima and Minima: Problems 2ii

A Level

A curve has equation $y = 3x^3 - 7x + \frac{2}{x}$

Part A Verify stationary point

Fill in the blanks to verify that the curve $y = 3x^3 - 7x + \frac{2}{x}$ has a stationary point when $x = 1$.

At a stationary point the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ is [ ].

For the curve, the first derivative is

$$\frac{\mathrm{d}y}{\mathrm{d}x} = [\ ]x^2 + [\ ] + [\ ]x^{-2}$$

Hence, when $x = 1$,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = [\ ]$$

Therefore, the curve has a stationary point when $x = 1$.

Part B Nature of stationary point

Fill in the blanks to determine the nature of the stationary point when $x = 1$.

The second derivative of $y = 3x^3 - 7x + \frac{2}{x}$ is:

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \square\, x + \square\, x^{-3}$$

Hence, when $x = 1$,

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \square$$

Therefore, the stationary point when $x = 1$ is (maximum / minimum / point of inflection):

$\square$

Part C Tangent to curve

The tangent to the curve at this stationary point meets the y-axis at the point Q. Find the y-coordinate of Q.

Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Stationary Points 1ii

A Level

The curve $y = x^3 - kx^2 + x - 3$ has two stationary points.

Part A Differentiate

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `k, x`

Part B Find k

Given that there is a stationary point when $x = 1$, find the value of k.

The following symbols may be useful: `k`

Part C Differentiate twice

Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

The following symbols may be useful: x

Hence determine whether the stationary point is a minimum or a maximum.

- Minimum
- Maximum

Part D Find coordinate

Find the x-coordinate of the other stationary point.

The following symbols may be useful: x

Used with permission from UCLES, A Level, June 2009, Paper 4721, Question 10.

Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Stationary Points 2ii

A Level

P P P

Part A Find coordinate

Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 + 4$.

Enter the x and y coordinates of the stationary point with the greatest x coordinate. If a value is not a whole number, enter the value as a decimal.

(____ , ____)

Part B Stationary point

Determine whether the stationary point whose coordinates you entered is a maximum point or a minimum point.

- Maximum
- Minimum
- Inconclusive

Part C Range of x

For what range of values of x does $x^3 - 3x^2 + 4$ decrease as x increases?

Construct your answer from the items below.

Items:

$<$ $>$ x $< x <$ $\leq x \leq$ $< x$ **or** $x <$ $\leq x$ **or** $x \leq$ $\leq$ $\geq$ -2 -1 0 1 2 3 4

Used with permission from UCLES, A level, January 2006, Paper 4721, Question 6

Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Maxima and Minima: Problems 1ii

A Level

Figure 1 shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is x metres, and the area of the enclosure is $A\,\mathrm{m}^2$.

Figure 1: The rectangular enclosure.

Part A Express as equation

Show that A can be expressed in the form $px - qx^2$, and find this expression.

The following symbols may be useful: `A, x`

Part B Use differentiation

Use differentiation to find the maximum value of the area of the enclosure, $A\,\mathrm{m}^2$.

Enter your value of A:

The following symbols may be useful: `A`

Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Maxima and Minima: Problems 1i

A Level

P P P

A cuboid has a volume of exactly $8\,\mathrm{m}^3$. The base of the cuboid is a square with side length x metres. The surface area of the cuboid is $A\,\mathrm{m}^2$.

Part A Find expression for A

Show that A can be expressed in the form $ax^2 + \frac{b}{x}$, where a and b are constants, and find this expression.

The following symbols may be useful: x

Part B Find $\frac{\mathrm{d}A}{\mathrm{d}x}$

Find $\frac{\mathrm{d}A}{\mathrm{d}x}$.

The following symbols may be useful: x

Part C Find minimum

Find the value of x which gives the smallest surface area of the cuboid.

The following symbols may be useful: x

Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Physics. You work it out.

Differentiating Powers 7

A Level

Pre-Uni Maths for Sciences J2.3

A quadratic function has the form $y = a + bx + cx^2$ where a, b and c are constants. It has a stationary point at $(2, 2)$ and, at $x = 1$, the tangent to the curve has a gradient of -2. Find the values of a, b and c. (In practice, with the information given, you will need to find b and c before you can find a.)

Drag and drop the correct values into the equation below.

$$y = \square + \square\, x + \square\, x^2$$

Items:

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

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Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Stationary Points 3

A Level

C C C

Pre-Uni Maths for Sciences J2.5

Part A Maximum height of a projectile

A particle is fired upwards into the air with a initial speed w and moves subsequently under the influence of gravity with an acceleration g downwards, such that its height h at time t is given by $h = wt - \frac{1}{2}gt^2$, where w and g are constants. Find an expression for its maximum height above its initial position.

The following symbols may be useful: `g, h, w`

Part B Potential energy of two molecules

The potential energy of two molecules separated by a distance r is given by

$$U = U_0 \left(\left(\frac{a}{r} \right)^{12} - 2 \left(\frac{a}{r} \right)^{6} \right)$$

where U_0 and a are positive constants. The equilibrium separation of the two molecules occurs when the potential energy is a minimum.

Find an expression for the equilibrium separation of the molecules.

The following symbols may be useful: `U, U_0, a, r`

Find an expression for the potential energy when the molecules are at their equilibrium separation.

The following symbols may be useful: `U, U_0, a, r`
