Partial Fractions 1

A Level P P P Further A P P P

The function $\dfrac{2x-1}{(3x-2)(x-1)}$ can be written as $\dfrac{A}{3x-2}+\dfrac{B}{x-1}$. Find A and B.

Part A Find A

Find the constant A.

Part B Find B

Find the constant B.

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Partial Fractions 2

A Level P P P Further A P P P

The function $\frac{w+2}{(w-1)(w+1)(2w+1)}$ can be written as $\frac{A}{(w-1)} + \frac{B}{(w+1)} + \frac{C}{(2w+1)}$. Using the substitution method find the constants A, B and C.

Part A Find A

Find the constant A.

The following symbols may be useful: A

Part B Find B

Find the constant B.

The following symbols may be useful: B

Part C Find C

Find the constant C.

The following symbols may be useful: C

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Partial Fractions 3

A Level

The function $\frac{24t^2+31t+2}{(2t+1)^2(t+3)}$ can be written as $\frac{A}{(2t+1)^2}+\frac{B}{(2t+1)}+\frac{C}{(t+3)}$. Find the constants A, B and C.

Part A Find A

Find the constant A.

The following symbols may be useful: A

Part B Find B

Find the constant B.

The following symbols may be useful: B

Part C Find C

Find the constant C.

The following symbols may be useful: C

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Partial Fractions 4

A Level

P P P

The function $\frac{8a^2}{(x-a)(x+a)^2}$, where a is a constant, can be written as $\frac{A}{(x+a)^2}+\frac{B}{x+a}+\frac{C}{x-a}$. Find the constants A, B and C.

Part A Find A

Find the constant A.

The following symbols may be useful: `A`, `a`

Part B Find B

Find the constant B.

The following symbols may be useful: `B`, `a`

Part C Find C

Find the constant C.

The following symbols may be useful: `C`, `a`

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Improper Partial Fractions 1

A Level

P P P

Express $\dfrac{-6x^3+15x^2+x-11}{2x^2-5x-3}$ as partial fractions.

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Proof and Odd Perfect Numbers

A Level

The **proper divisors** of a number are those factors which are not equal to the number itself. For the number 6,

- The divisors of 6 are 1, 2, 3 and 6.
- The proper divisors of 6 are 1, 2 and 3.

The number 6 is an example of a **perfect number**. A perfect number is a number for which the sum of its proper divisors is equal to the number itself. For the number 6,

$$1+2+3=6$$

In this question you will use proof by contradiction to show that an odd perfect number cannot be a square number.

Assumption:

We will assume that there is an odd perfect number, n, that is also a square number. Then $n=m^2$, where m is an integer.

Part A Reasoning: odd and even factors

- An even number multiplied by an even number is always an [ ] number.
- An even number multiplied by an odd number is always an [ ] number.
- An odd number multiplied by an odd number is always an [ ] number.

Therefore, as n is an [ ] number, the factors of n can only be [ ] numbers.

Items:

odd even

Part B Reasoning: sum of proper divisors

As $n = m^2$, m is a factor of n.

Consider another factor of n. Call this factor p. As p is a factor of n, $q =$ ____ is also a factor of n. As $n = m^2$, $q = \frac{m^2}{p}$. Hence,

- If $p < m$, q ____ m.
- If $p > m$, q ____ m.

Therefore, with the exception of m, the factors of n occur in pairs. One factor in the pair is smaller than m, and the other factor is larger than m. Including m, the total number of factors of n is therefore an ____ number.

For any value of n, one of the factor pairs is 1 and n. The number of proper divisors (factors other than n itself) is therefore an ____ number. As we have shown in part A that all of the factors of n are ____ numbers, the sum of the proper divisors of n is therefore an ____ number.

Items:

odd | $\frac{p}{n}$ | $<$ | $>$ | $\frac{n}{p}$ | even

Part C Conclusion

Our starting assumption was that n is an odd perfect number and also a square number.

The definition of a perfect number means that the sum of the proper divisors of n is equal to ____. The sum of the proper divisors must therefore be an ____ number.

However, in part B we have shown that if n is an odd number which is also a square number, the sum of the proper divisors has to be an ____ number.

Therefore, we have arrived at a contradiction. We conclude that there are no odd perfect numbers that are also square numbers.

Items:

$2n$ | even | n | odd | n^2

Proof Applied to Surface Areas

A Level

Consider a sphere with a radius $r\,\mathrm{cm}$, where r is a rational number. Using proof by contradiction, show that the side length of a cube with the same surface area cannot also be a rational number of cm.

Once you have worked out a proof yourself, consider the model proof below. Drag and drop answers into the blank spaces to complete the proof. You may use the same answer more than once.

Assumption:

Consider a sphere of radius $r\,\mathrm{cm}$, where r is a rational number. Let the side length of a cube with the same surface area as the sphere be $a\,\mathrm{cm}$. Assume that a is a rational number, in which case $a = \frac{b}{c}$, where b and c are integers with no common factor.

Reasoning:

The surface area of the sphere is ______. Because r is a rational number, $r = \frac{p}{q}$, where p and q are integers with no common factor. Hence, the surface area of the sphere may be written as ______.

The surface area of the cube is ______. Using $a = \frac{b}{c}$, the surface area may be written as ______.

The surface area of the sphere and the cube are equal. Hence, $4\pi\left(\frac{p}{q}\right)^2 = 6\left(\frac{b}{c}\right)^2$. Re-arranging this equation to give an expression for ______ produces ______.

As b, c, p and q are all integers, ______ must be ______ number. However, π is not ______ number.

Conclusion:

The assumption that a is rational has resulted in a contradiction. Hence, the assumption cannot be true. Therefore, the side length of a cube with the same surface area as a sphere of radius $r\,\mathrm{cm}$, where r is a rational number, cannot be a rational number of cm.

Items:

$4\pi\left(\frac{p}{q}\right)^2$ | $\pi = \frac{3b^2p^2}{2c^2q^2}$ | $6\left(\frac{b}{c}\right)^2$ | $\pi = \frac{3b^2q^2}{2c^2p^2}$ | a real | an irrational | a^3 | π | $4\pi r^2$ | $6a^2$ | $\frac{3b^2q^2}{2c^2p^2}$ | a rational

Partial Fractions Applied to Other Functions

A Level

Express the following functions in partial fraction form.

Part A A trigonometric function

Express the function $\dfrac{\cos y}{(\cos y+1)(2\cos y+1)}$ in the form $\dfrac{A}{\cos y+1}+\dfrac{B}{2\cos y+1}$, where A and B are constants.

The following symbols may be useful: `cos()`, `sin()`, `tan()`, `y`

Part B An exponential function

Express the function $\dfrac{\mathrm{e}^{2x}+5}{(\mathrm{e}^{x}-1)(\mathrm{e}^{x}-2)(\mathrm{e}^{x}-3)}$ in the form $\dfrac{A}{\mathrm{e}^{x}-1}+\dfrac{B}{\mathrm{e}^{x}-2}+\dfrac{C}{\mathrm{e}^{x}-3}$, where A, B and C are constants.

The following symbols may be useful: `e`, `x`

Part C A logarithmic function

Express the function $\dfrac{5\ln z+20}{(\ln z)^2+\ln(z^2)+1}$ in the form $\dfrac{A}{(\ln z+1)^2}+\dfrac{B}{\ln z+1}$, where A and B are constants.

The following symbols may be useful: `ln()`, `log()`, `z`

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Force From Electric Dipole

A Level

An electric dipole consists of two charges that are equal in size but opposite in sign, with a separation between them. The diagram below shows an electric dipole PQ. P has charge $-q$ and Q has charge $+q$, and the separation between P and Q is $2a$. Another charge, S, is near to the dipole. S is in line with the axis of the dipole and a distance r from the dipole's centre.

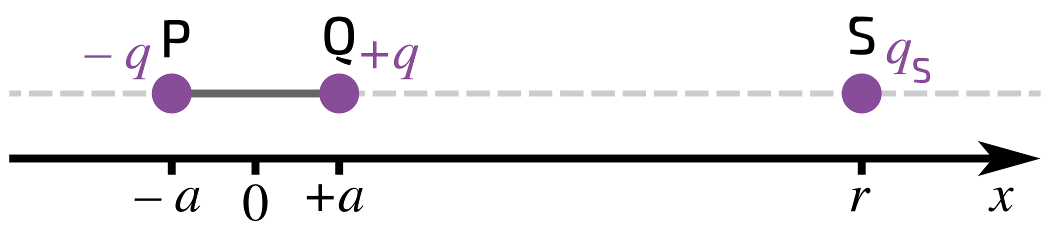

Figure 1: An electric dipole PQ and a charge S.

The resultant force on charge S is the sum of the force on S from P and the force on S from Q. For a particular value of q_S, the resultant force is given by the expression

$$F_{\text{res}} = \frac{-3q^2}{4\pi\varepsilon_0}\frac{ar}{(r^2-a^2)^2}$$

where ε_0 is a constant.

Part A Splitting into terms - A

In general, a rational function with a denominator of $4\pi\varepsilon_0(r^2-a^2)^2$ would produce four terms when written in terms of partial fractions:

$$\frac{A}{4\pi\varepsilon_0(r+a)^2}+\frac{B}{4\pi\varepsilon_0(r-a)^2}+\frac{C}{4\pi\varepsilon_0(r+a)}+\frac{D}{4\pi\varepsilon_0(r-a)}$$

However, if the expression for F_{res} is written in terms of partial fractions, it turns out that two of the coefficients (C and D) are both 0.

Write the expression for F_{res} in the form $\frac{A}{4\pi\varepsilon_0(r+a)^2}+\frac{B}{4\pi\varepsilon_0(r-a)^2}$, where A and B are constants which depend on q.

Enter your expression for A.

The following symbols may be useful: `A, a, pi, q, varepsilon_0`

Part B Splitting into terms - B

Write the expression for F_{res} in the form $\frac{A}{4\pi\varepsilon_0(r+a)^2} + \frac{B}{4\pi\varepsilon_0(r-a)^2}$, where A and B are constants which depend on q.

Enter your expression for B.

The following symbols may be useful: `B, a, pi, q, varepsilon_0`

Part C Finding q_S

The force between two particles with electric charges q_1 and q_2 separated by a distance d is given by

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 d^2}$$

where ε_0 is a constant.

Using your answers to parts A and B, or otherwise, find an expression for the charge on S, q_S, in terms of q.

The following symbols may be useful: `a, pi, q, q_S, varepsilon_0`

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