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Question

Sigma Notation in Statistics

A-level Maths Topic Summaries - Statistics

Subject & topics: Maths | Statistics | Data Analysis **Stage & difficulty:** A Level P2

Fill in the blanks to complete these notes about sigma notation in statistics.

The symbol Σ is a Greek upper case sigma. **Sigma notation** uses Σ to write sums of values in a compact form. Let us suppose that we have a data set containing n values: $x_1, x_2, x_3, ..., x_n$. Using sigma notation we write the sum of these values and the sum of the squares of these values as follows.

$$x_1 + x_2 + x_3 + ... + x_n = \square\, x_{\square}$$

$$x_1^2 + x_2^2 + x_3^2 + ... + x_n^2 = \sum_{\square}^{n} x_{\square}$$

In statistics the **index of summation** i can often be assumed to run from 1 to n. When this is the case, the labels are often dropped to keep the notation simple. You may see the sums above written as $\sum_i x_i$ and $\sum_i x_i^2$, or $\sum x_i$ and $\sum x_i^2$, or most simply as $\sum x$ and $\sum x^2$. Using sigma notation, we can write the formulae for the mean $\overline{x}$ and standard deviation σ as follows.

$$\overline{x} = \frac{\square}{n}$$

$$\sigma = \sqrt{\frac{\square}{n} - \overline{x}^2}$$

An important point is that $\sum x_i^2$ is not the same as $\left(\sum x_i\right)^2$. The reason for this is that $x_1^2 + x_2^2 + ... + x_n^2$ is not the same as $(x_1 + x_2 + ... + x_n)^2$.

Items:

2 i $i = 1$ $\sum_{i=1}^{n}$ $\sum x$ $\sum x^2$

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Question

Data Analysis 3.8

Subject & topics: Maths | Statistics | Data Analysis **Stage & difficulty:** A Level P1

Consider the following data set:

$3.91, 4.29, 3.75, 4.28, 3.68, 4.13, 3.61, 4.19, 4.18, 4.01.$

Find the median, interquartile range, mean and standard deviation of the data set.

Part A

The median

Find the median of the data set.

Part B

The interquartile range

Find the interquartile range of the data set.

Part C

The mean

Find the mean of the data set. Give your answer to 3 sf.

Part D

The standard deviation

Find the standard deviation of the data set. Give your answer to 3 sf.

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Question deck:

STEM SMART Single Maths 6 - Data Analysis

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Question

Summary Statistics 11

Essential GCSE Maths 55.11

Subject & topics: Maths | Statistics | Data Analysis **Stage & difficulty:** GCSE C2, A Level P1

A technician is given a list of measurements in cm, correct to the nearest $0.1\,\mathrm{cm}$. He is told that the mean of the values is $3.3\,\mathrm{cm}$, but when he checks the calculation he finds a different value. Here is the list:

$$3.6, 3.4, 3.2, 2.9, 3.8, 3.4, 3.6, 3.2, 3.3, 3.6$$

Part A

What is the mean of the values?

What is the mean of these values?

Part B

What is the value of the missing numbers?

To find the source of the discrepancy, the technician checks the list he was given against the original data for the experiment, and finds two identical numbers are missing. What is the value of these numbers?

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Question

Data Analysis 3.1

Subject & topics: Maths | Statistics | Data Analysis **Stage & difficulty:** A Level P1

Nine measurements were made of the time taken by a pendulum to perform six swings. The mean of the values was $10.240\,\text{s}$ with a standard deviation of $0.073\,\text{s}$. A tenth measurement was included changing the mean to $10.253\,\text{s}$. Find (a) the value of the tenth measurement and (b) the new value of the standard deviation.

Part A

The value of the tenth measurement

Find the value of the tenth measurement; give your answer to 3 decimal places.

Part B

The new value of the standard deviation

Find the value of the new standard deviation; give your answer to 2 sf.

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Question

Grouped Data and Diagrams 5

Essential GCSE Maths 56.5

Subject & topics: Maths | Statistics | Data Analysis **Stage & difficulty:** GCSE P3, A Level P1

x	Frequency
$0 \leqslant x < 5$	4
$5 \leqslant x < 10$	5
$10 \leqslant x < 20$	13
$20 \leqslant x < 30$	25
$30 \leqslant x < 35$	6

Part A

Construct a histogram

Construct a histogram for these data.

Choose the figure which is drawn correctly.

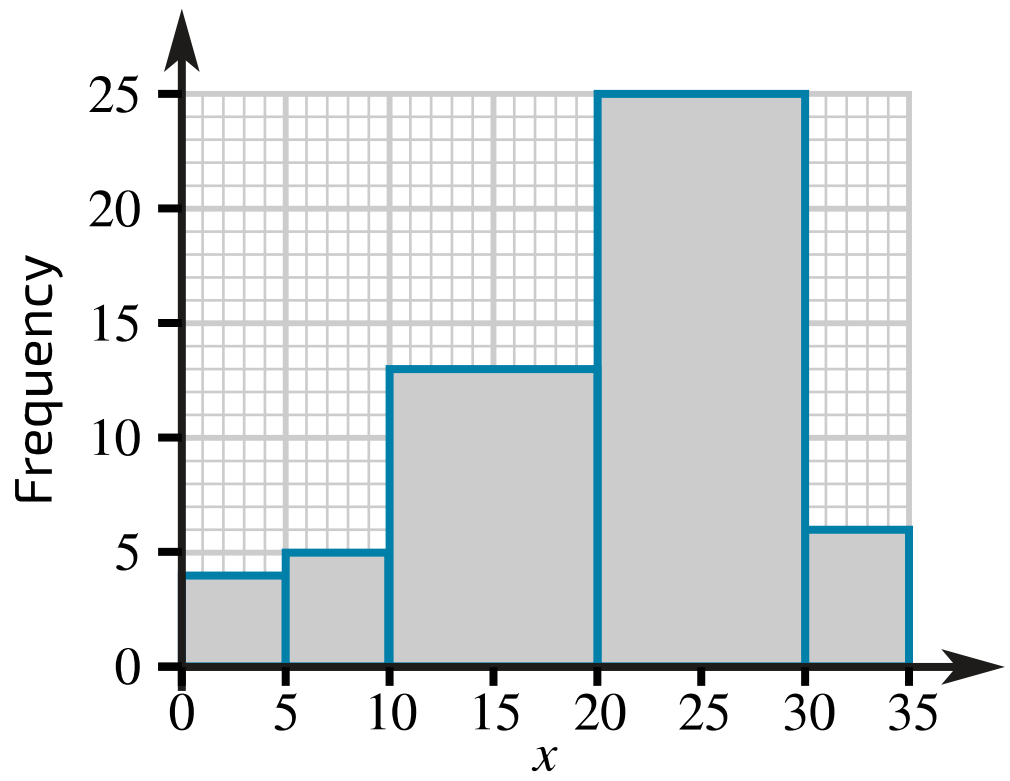

Figure 1: Option A.

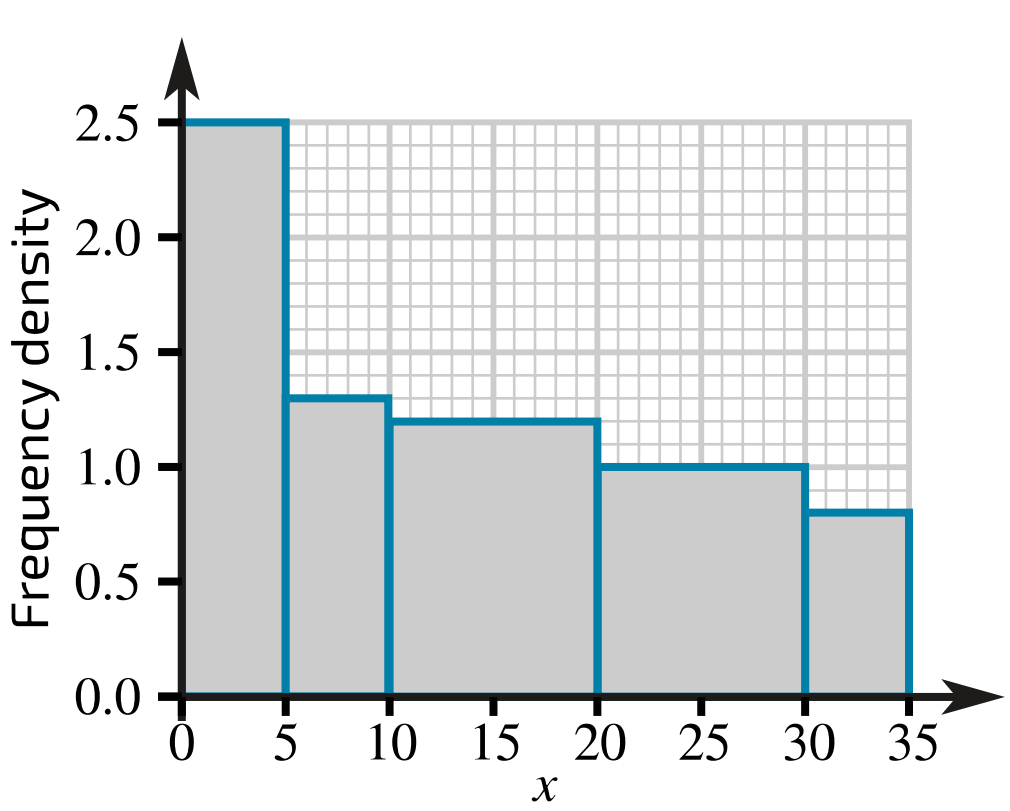

Figure 2: Option B.

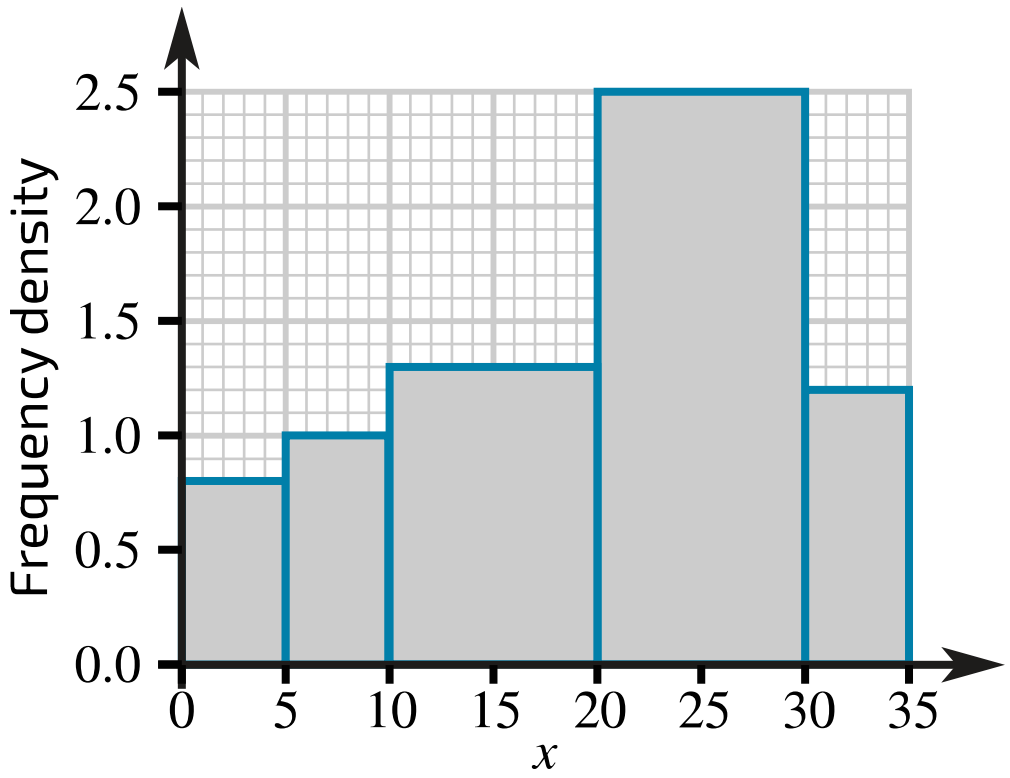

Figure 3: Option C.

- Option A
- Option B
- Option C

Part B

What is the modal class?

What is the modal class?

- $10 \leqslant x < 20$
- $0 \leqslant x < 5$
- $5 \leqslant x < 10$
- $20 \leqslant x < 30$
- $30 \leqslant x < 35$

Part C

Estimate the mean value of x

Estimate the mean value of x for these data to 3 sf.

Part D

Construct a cumulative frequency diagram

Construct a cumulative frequency diagram for these data.

Choose the figure which is drawn correctly.

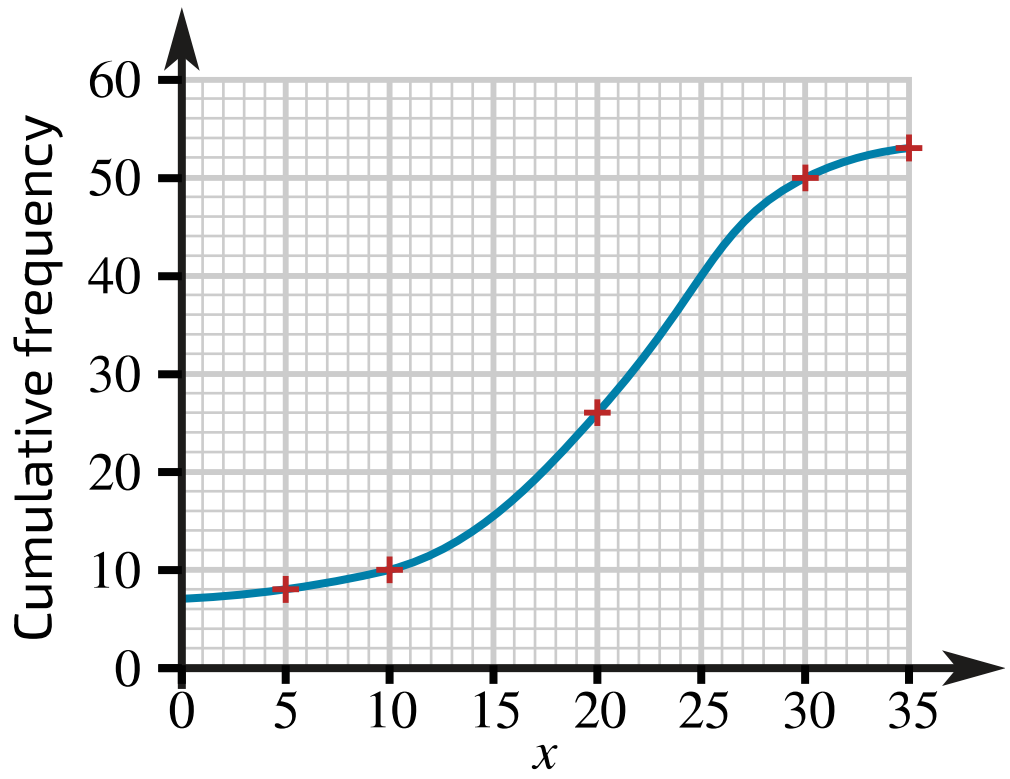

Figure 4: Option A.

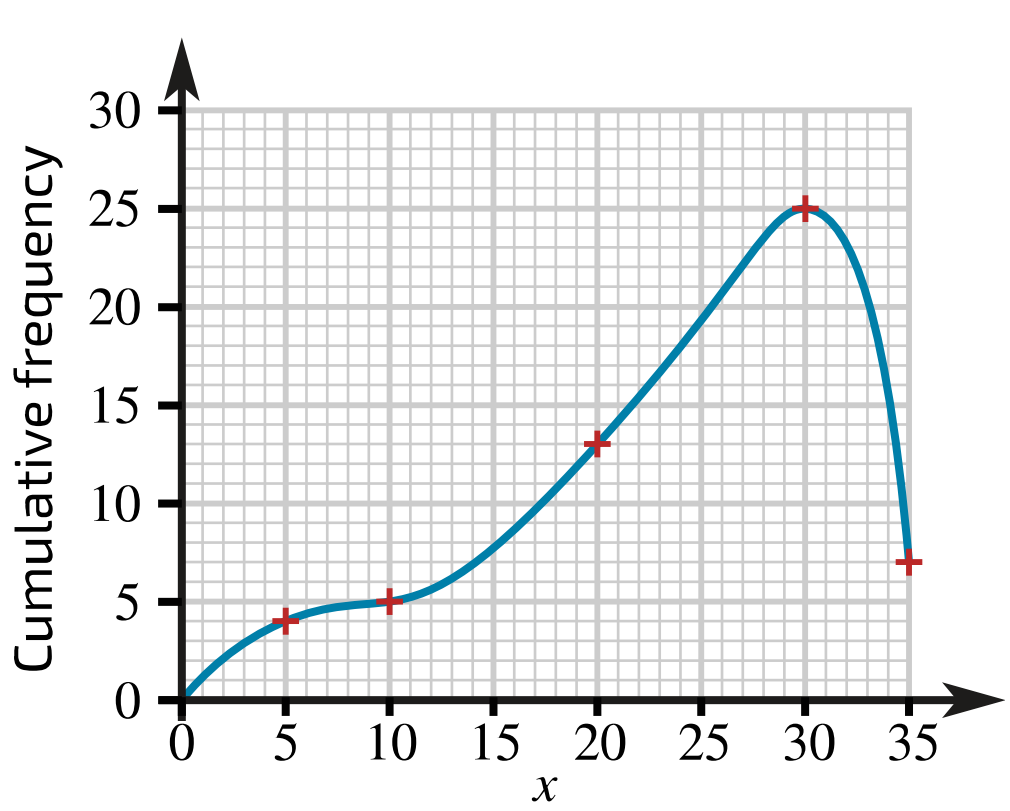

Figure 5: Option B.

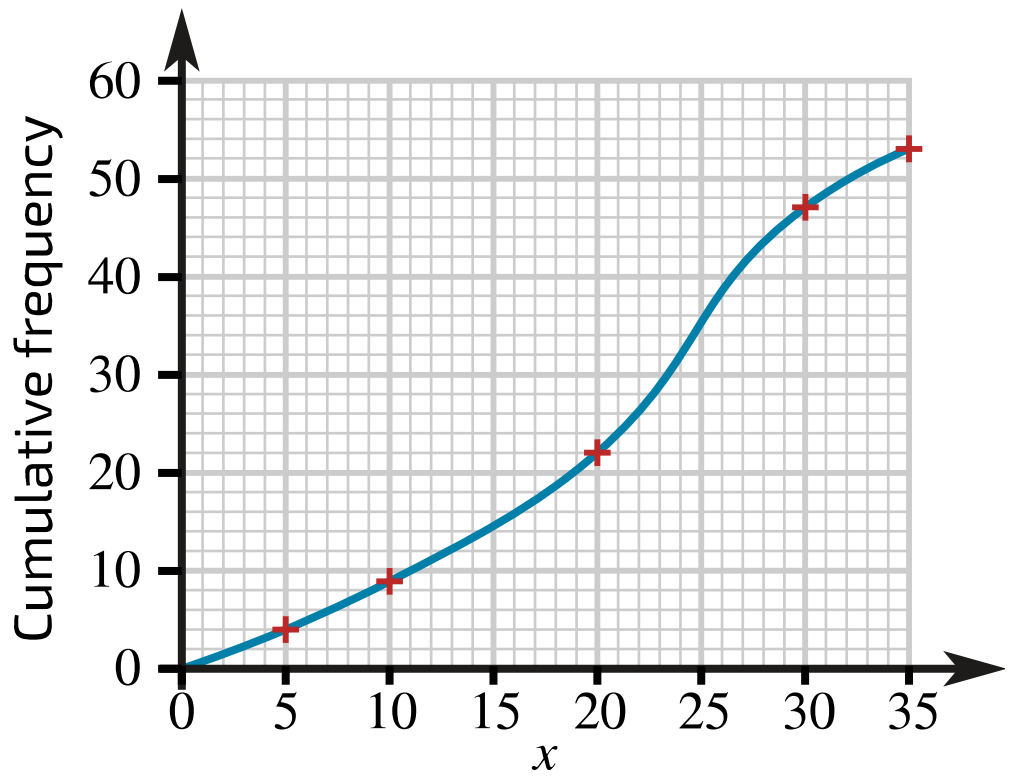

Figure 6: Option C.

- Option A
- Option B
- Option C

Question deck:

STEM SMART Single Maths 6 - Data Analysis

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Question

Grouped Data and Diagrams 7

Essential GCSE Maths 56.7

Subject & topics: Maths | Statistics | Data Analysis **Stage & difficulty:** GCSE C3, A Level P1

The histogram below summarises the total annual payments (including expenses) made to employees in a company.

Figure 1: A histogram summarises the total annual payment made to employees.

Part A

Make a frequency table

Let the variable for pay be p. Make a frequency table for the data in the histogram.

A partially complete frequency table is given below. Fill in the missing values.

Pay in $£1\,000$, p	Frequency
$0 \leq p < 5$	10
$5 \leq p < 10$	
$10 \leq p < 20$	
$20 \leq p < 30$	330
$30 \leq p < 40$	130
$40 \leq p < 45$	

Part B

What is the frequency density of the new class?

An extra class is added for $45 \leq p < 60$. This class has a frequency of 15. What is the frequency density of this class?

Part C

Estimate the mean pay

Calculate an estimate of the mean amount paid out to an employee, including the extra class from part B. Give your answer in thousands of pounds to 3 sf (e.g. $£32,460$ would be entered as 32.5).

Question deck:

STEM SMART Single Maths 6 - Data Analysis

Question

Grouped Data and Diagrams 8

Essential GCSE Maths 56.8

Subject & topics: Maths | Statistics | Data Analysis **Stage & difficulty:** GCSE C3, A Level P1

A company conducts plant growth trials of two varieties of chilli pepper, A and B. The graph shows cumulative frequency plots for the heights of both types of pepper after 13 weeks.

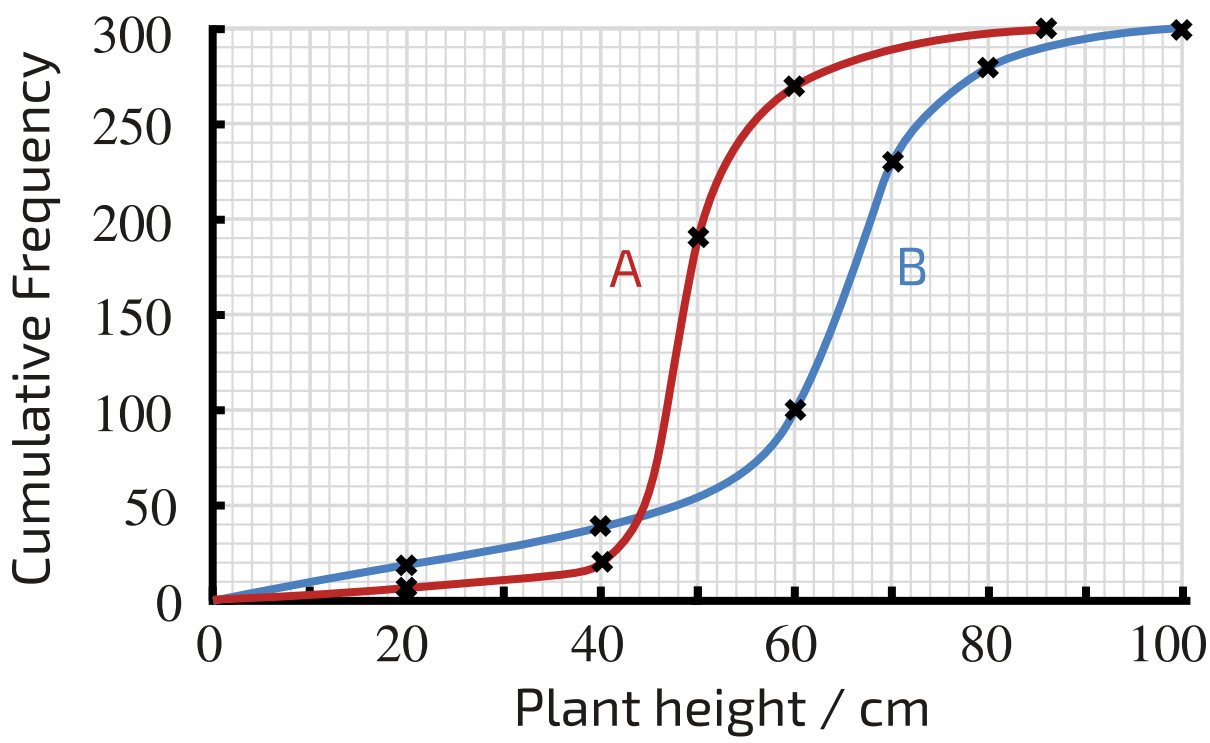

Figure 1: A cumulative frequency plot for the height of two types of chilli pepper.

Part A

Create box plots

Create box plots for both varieties of pepper.

Choose the figure which is drawn correctly.

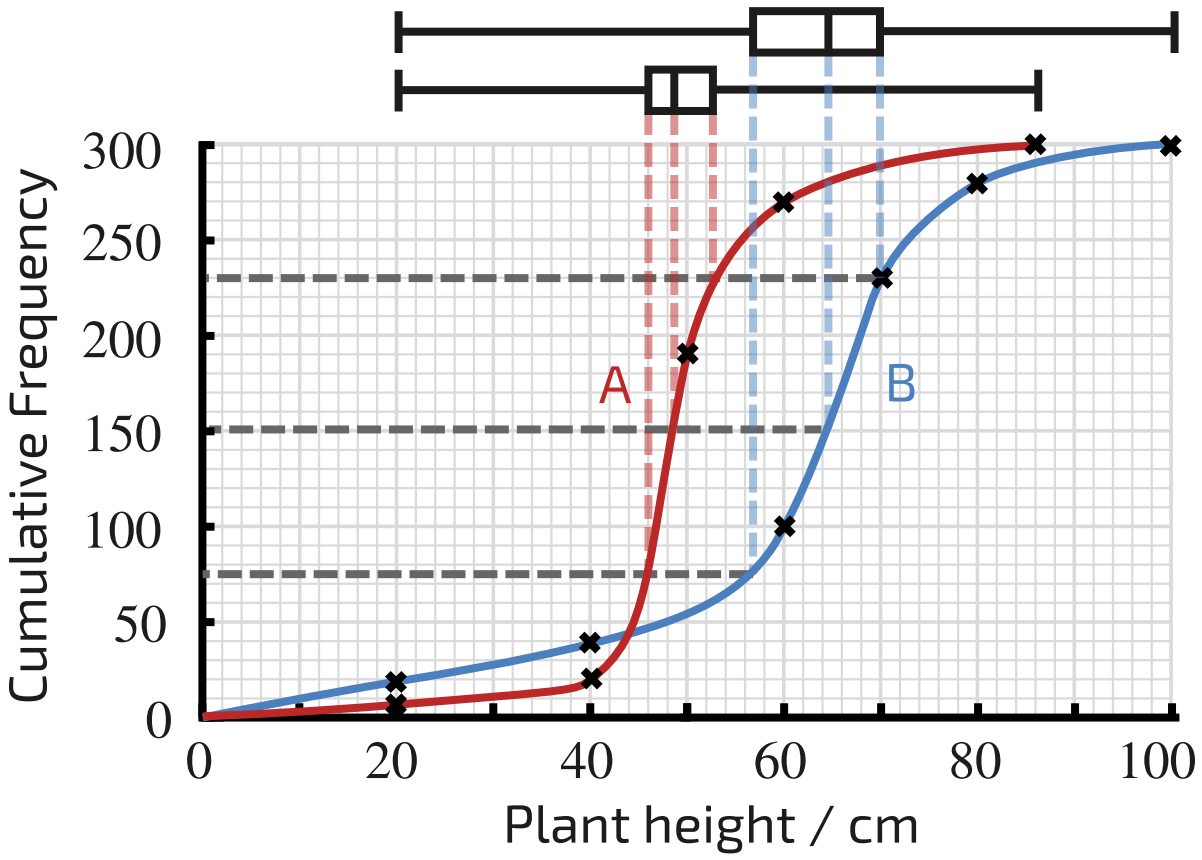

Figure 2: Option A.

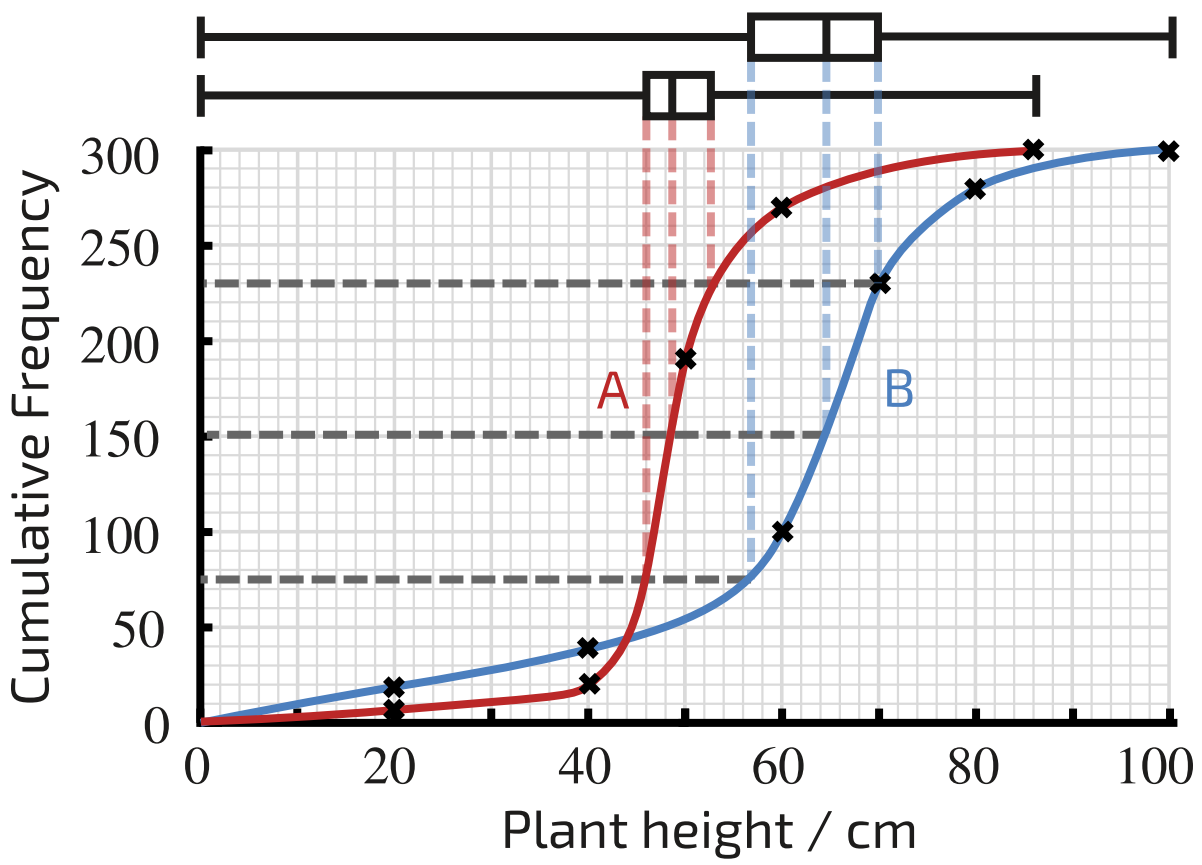

Figure 3: Option B.

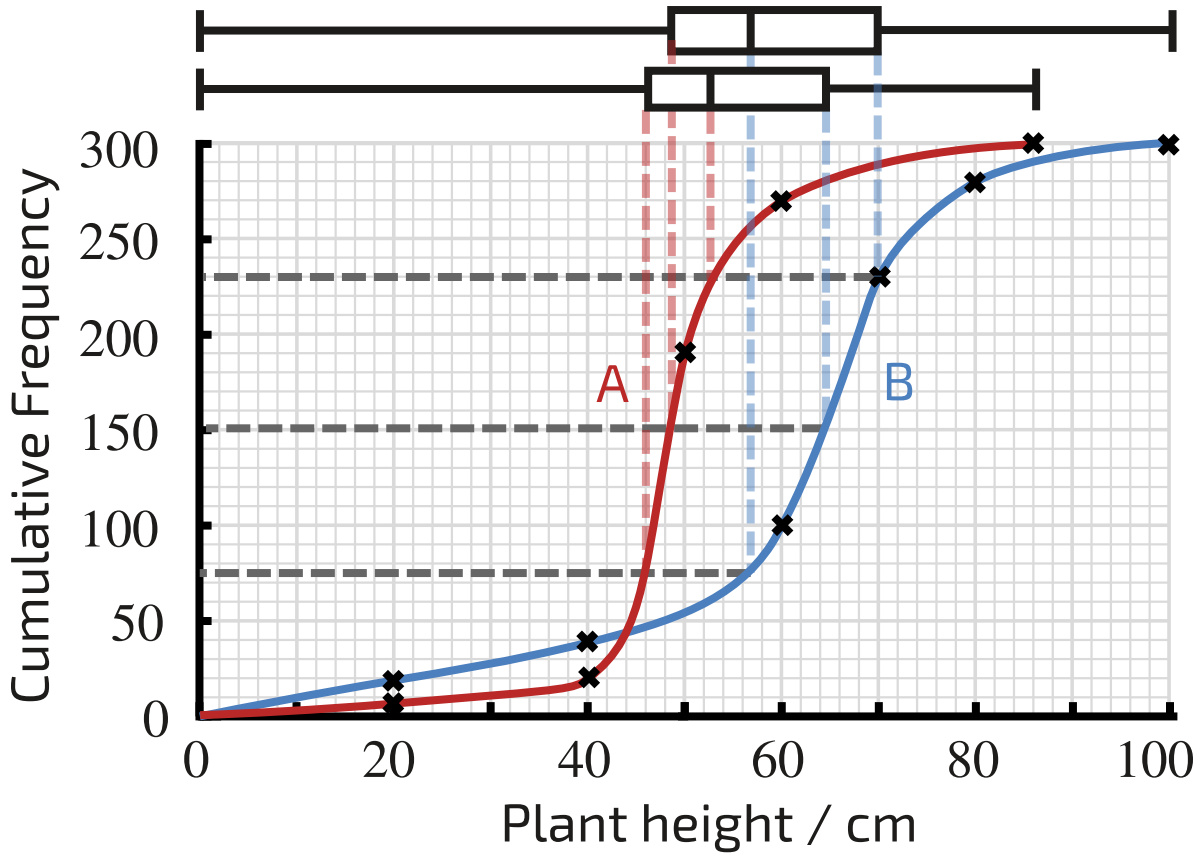

Figure 4: Option C.

- Option A
- Option B
- Option C

Part B

Which variety produced fewer failures?

The company defines failures as plants which do not reach $40\,\mathrm{cm}$ in height. Which variety produced fewer failures?

- Type A
- Type B

Part C

For which plant was the spread of plant heights greater?

For which plant was the spread of plant heights greater? Explain your answer.

- Type A
- Type B

Question deck:

STEM SMART Single Maths 6 - Data Analysis

isaac You work it out.

Question

Correlation 4

Essential GCSE Maths 57.4

Subject & topics: Maths | Statistics | Data Analysis **Stage & difficulty:** GCSE C2, A Level P1

Look at the following sketches and work out which equation describes the line of best fit.

Part A

First line of best fit

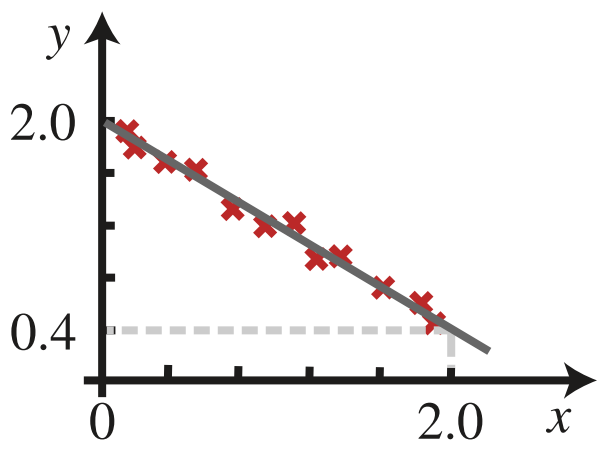

Figure 1: A graph of some data points and a line of best fit.

- $y = -1.8x + 4.0$
- $y = -0.8x + 2.0$
- $y = 0.8x - 2.0$

Part B

Second line of best fit

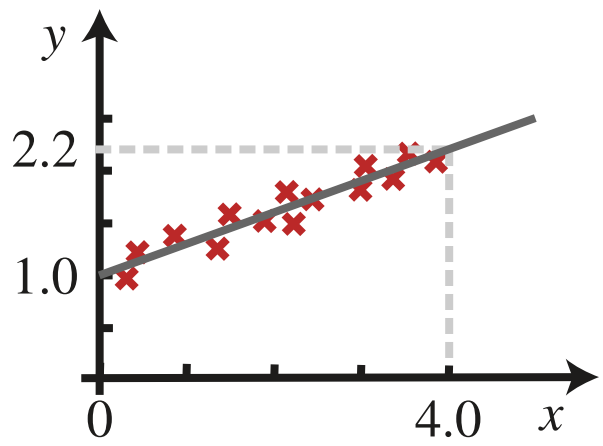

Figure 2: A graph of some data points and a line of best fit.

- $y = 0.4x - 1.5$
- $y = 0.4x + 1.5$
- $y = 0.3x + 1.0$

Part C

Third line of best fit

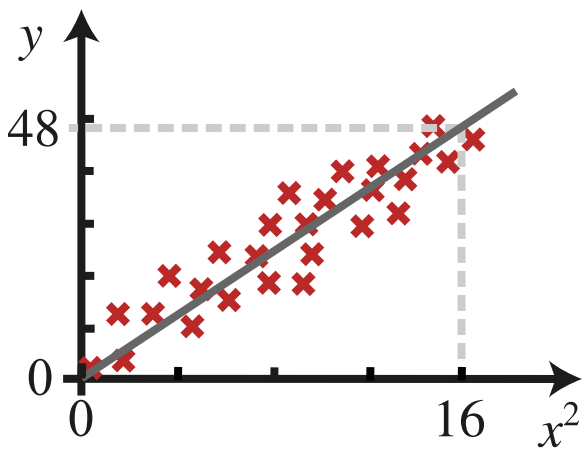

Figure 1: A graph of some data points and a line of best fit.

- $y = 3.0x^2$
- $y = 3.0x$
- $y = (3.0x)^2$

Question deck:

STEM SMART Single Maths 6 - Data Analysis

isaac You work it out.

Question

Correlation 5

Essential GCSE Maths 57.5

Subject & topics: Maths | Statistics | Data Analysis **Stage & difficulty:** GCSE C2, A Level P1

The nuclei of atoms contain protons and neutrons. In this question p is the number of protons and n is the number of neutrons.

Part A

Graph of n against p: light nuclei

Plot a graph with p on the x-axis and n on the y-axis for the following selected light nuclei. Then choose which of the options below is the best.

Element	He	Be	C	N	F	Mg	Cl	Ca
p	2	4	6	7	9	12	17	20
n	2	5	6	7	10	12	18	20

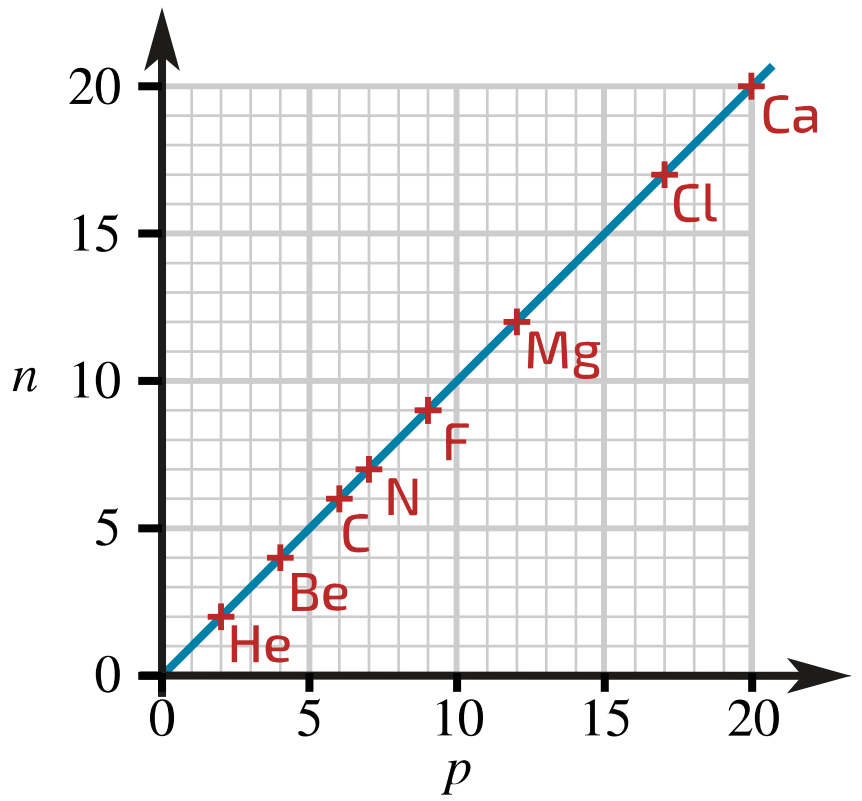

Figure 1: Option A.

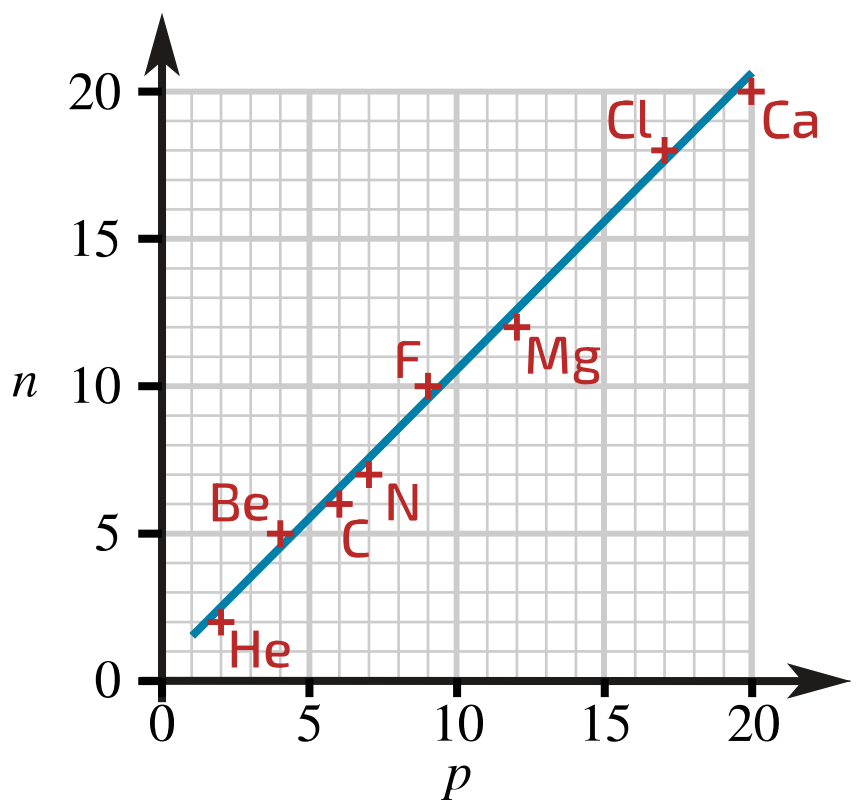

Figure 2: Option B.

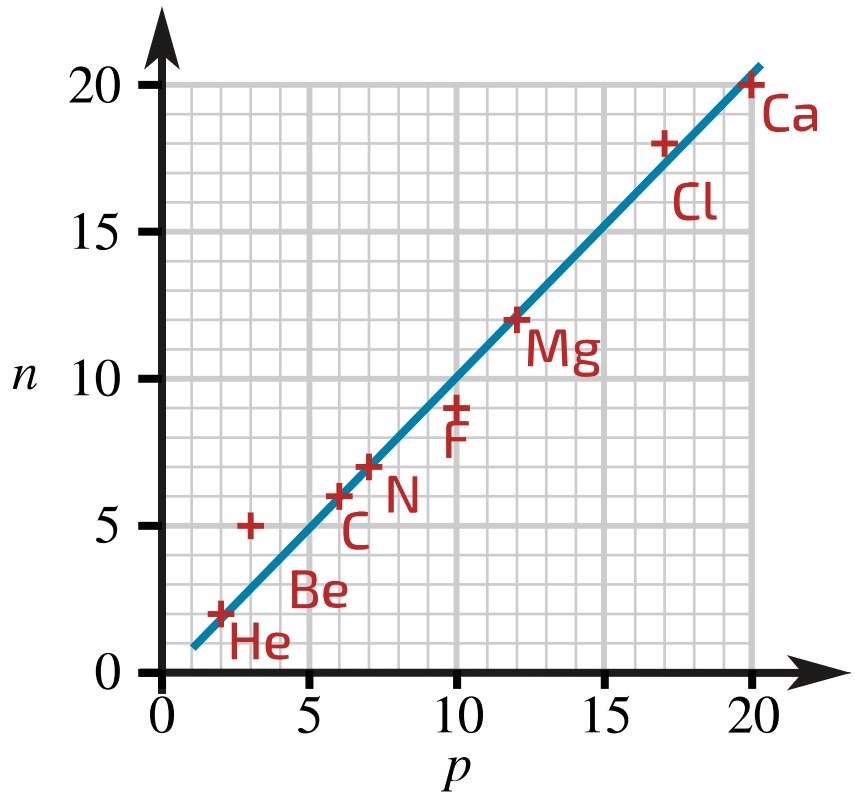

Figure 3: Option C.

- Option A
- Option B
- Option C

Part B

Type of correlation: light nuclei

What sort of correlation do you see?

- No correlation
- Negative linear correlation
- Correlation that is not linear
- Positive linear correlation

Part C

Conclusions from graph: light nuclei

What can you conclude from the graph about the value of the ratio $n : p$ for the nuclei given?

- There is no relation between the number of protons (p) and the number of neutrons (n).
- The number of neutrons is roughly the same as the number of protons. For these light nuclei $n : p \approx 1 : 1$
- The number of neutrons is generally larger than the number of protons. $n > p$.
- The number of neutrons is generally smaller than the number of protons. $n < p$.

Part D

Graph of n against p: heavy nuclei

Plot a graph with p on the x-axis and n on the y-axis for the following selected heavier nuclei, and then choose which of the options below is the best.

Element	Pd	Cs	Pr	Tb	W	Pt	Au	Pb
p	46	55	59	65	74	78	79	82
$p+n$	106	133	141	159	184	195	197	207

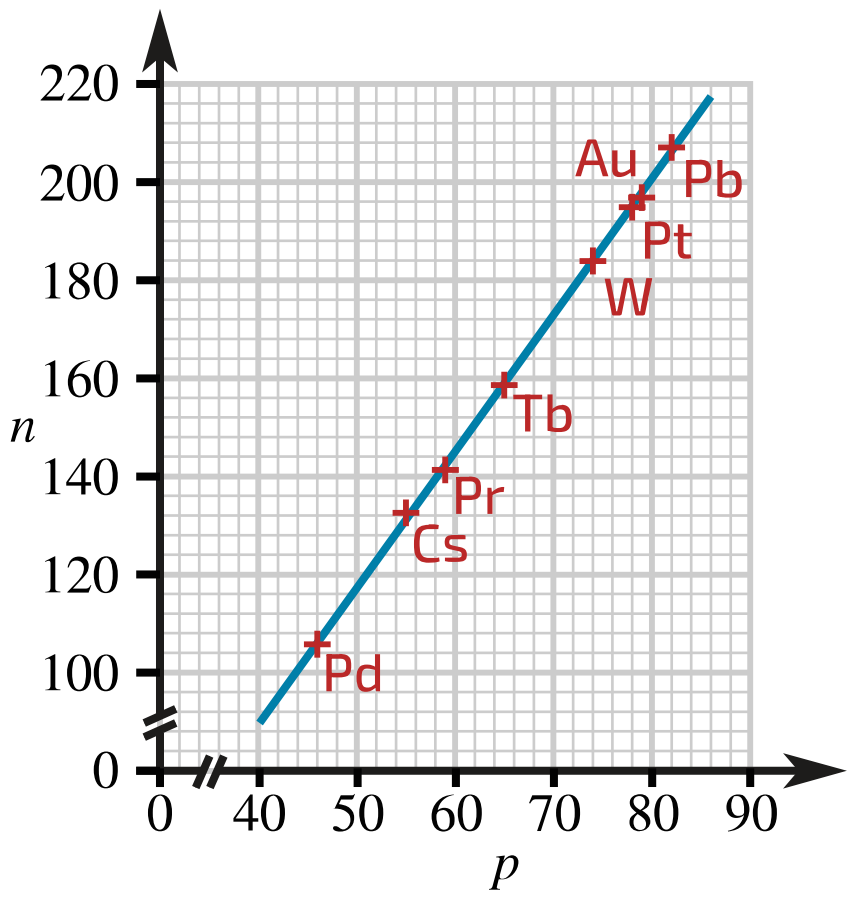

Figure 4: Option A.

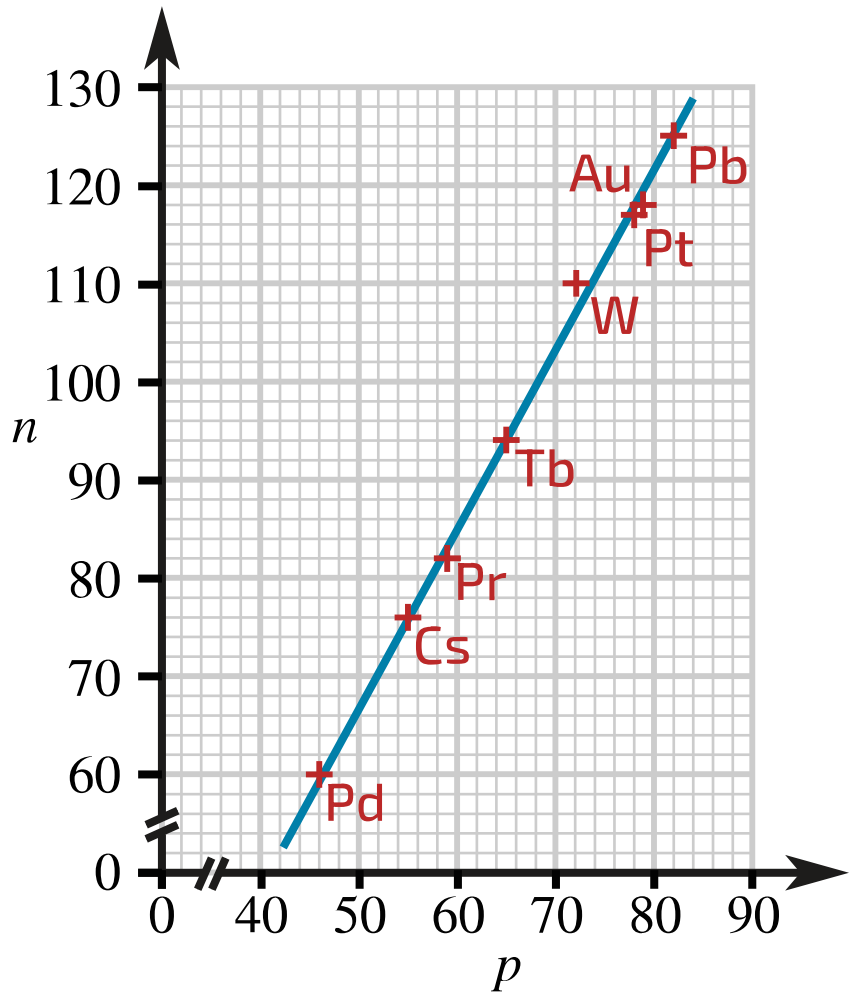

Figure 5: Option B.

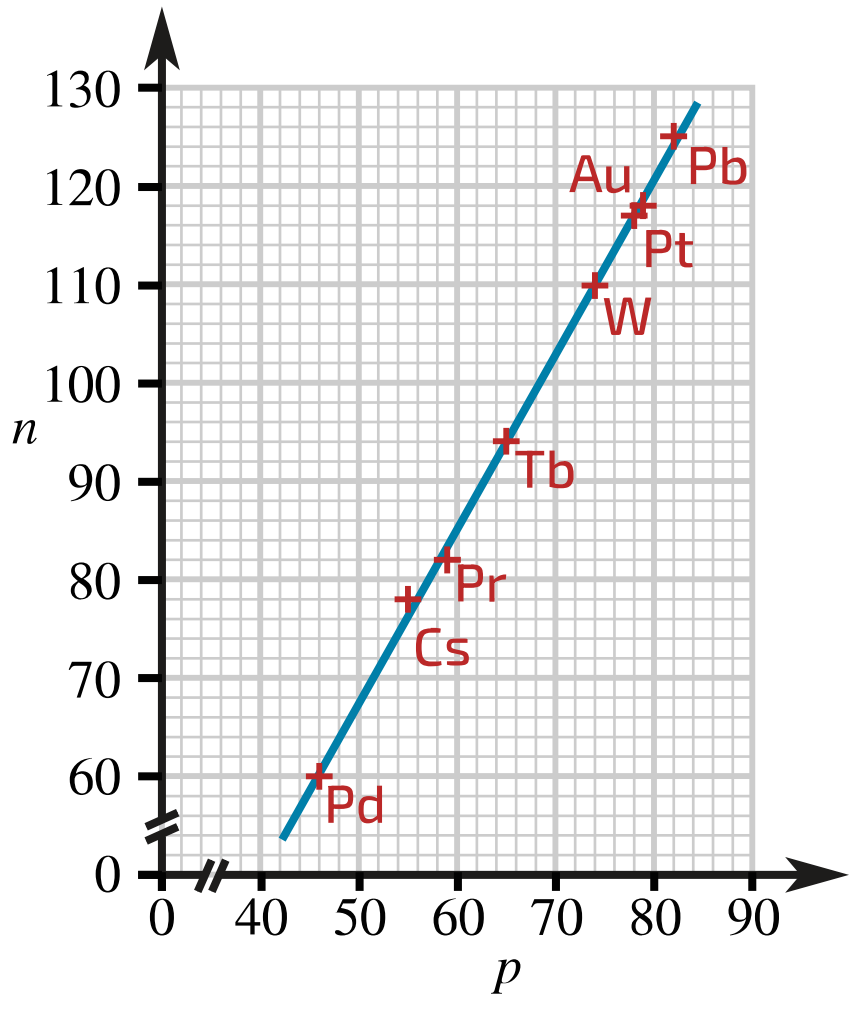

Figure 6: Option C.

- Option A
- Option B
- Option C

Part E

Type of correlation: heavy nuclei

What sort of correlation do you see?

- Negative linear correlation
- Correlation that is not linear
- Positive linear correlation
- No correlation

Part F

Line of best fit: heavy nuclei

Find the gradient.