Acceleration f(t) 3ii

A Level

A cyclist travels along a straight road. Her velocity $v\,\mathrm{m\,s^{-1}}$, at time t seconds after starting from a point O, is given by

$$v = 2 \text{ for } 0 \leq t \leq 10$$

$$v = 0.03t^2 - 0.3t + 2 \text{ for } t \geq 10$$

Part A Displacement at $t = 10$

Find the displacement of the cyclist from O when $t = 10\,\mathrm{s}$.

Part B Expression for displacement

Find an expression for the displacement of the cyclist from O as a function of time for $t \geq 10\,\mathrm{s}$. Give your answer using fractions, not decimals.

The following symbols may be useful: `t`

Part C Time

Find the time when the acceleration of the cyclist is $0.6\,\mathrm{m\,s^{-2}}$.

Part D Displacement

Find the displacement of the cyclist from O when her acceleration is $0.6\,\mathrm{m\,s^{-2}}$.

Used with permission from UCLES, A Level, June 2006, OCR M1, Question 4

Calculus and Vectors 1ii

A Level

A particle P of mass $0.2\,\mathrm{kg}$ moves on a smooth horizontal plane. Initially it is projected with velocity $0.8\,\mathrm{m\,s^{-1}}$ from a fixed point O towards another fixed point A. At time $t\,\mathrm{s}$ after projection, P is $x\,\mathrm{m}$ from O and is moving with velocity $v\,\mathrm{m\,s^{-1}}$, with the direction OA being positive. A force of $(1.5t - 1)\,\mathrm{N}$ acts on P in the direction parallel to OA.

Part A Expression for v

Find an expression for v in terms of t.

(Use fractions rather than decimals when entering your answer.)

The following symbols may be useful: `t, v`

Part B Time when $v = 0.8\,\mathrm{m\,s^{-1}}$

Find the time (in seconds) when the velocity of P is next $0.8\,\mathrm{m\,s^{-1}}$.

Give your answer to 3 sf.

Part C Times through O

Find the first time when P subsequently passes through O.

Find the second time when P subsequently passes through O.

Part D Distance in third second

Find the distance P travels in the third second of its motion.

Gameboard:

STEM SMART Double Maths 22 - Vector Equations of Motion

Kinematics & Calculus

A Level

C C C

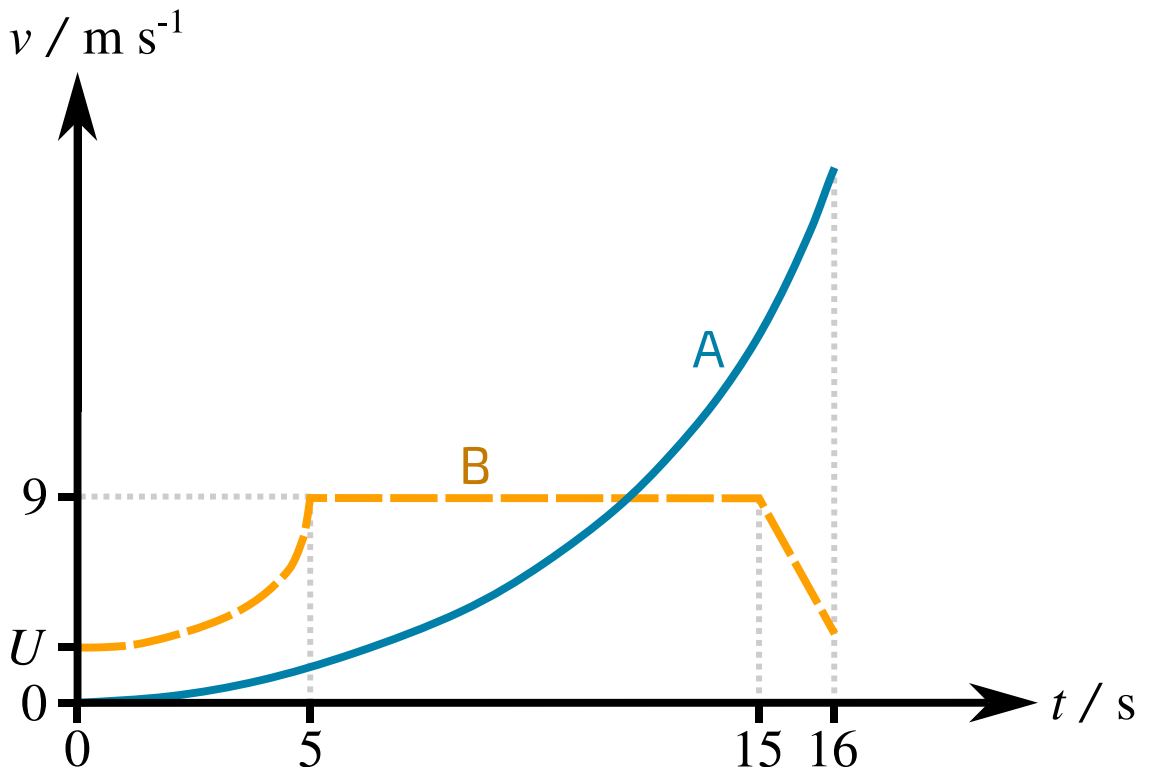

Figure 1: Velocity-time graph of the motion of two particles A and B along the same straight line.

The diagram shows the (t, v) graphs for two particles A and B which move on the same straight line. The units of v and t are $\mathrm{m\,s^{-1}}$ and s respectively. Both particles are at the point S on the line when $t = 0$. The particle A is initially at rest, and moves with acceleration $0.18t\,\mathrm{m\,s^{-2}}$ until the two particles collide when $t = 16\,\mathrm{s}$. The initial velocity of B is $U\,\mathrm{m\,s^{-1}}$ and B has variable acceleration for the first five seconds of its motion. For the next ten seconds of its motion B has a constant velocity of $9\,\mathrm{m\,s^{-1}}$; finally B moves with constant deceleration for one second before it collides with A.

Part A t for same velocity

Calculate the value of t at which the two particles have the same velocity.

Part B Calculate U

For $0 \leq t \leq 5$ the distance of B from S is $(Ut + 0.08t^3)\,\mathrm{m}$.

Calculate U.

Part C Distance from S

Calculate how far B is from S when $t = 5\,\mathrm{s}$.

Part D v_B when $t = 16\,\mathrm{s}$

Calculate the velocity of B when $t = 16\,\mathrm{s}$.

Gameboard:

STEM SMART Double Maths 22 - Vector Equations of Motion

Vectors & Calculus 2i

A Level

A projectile has velocity $\begin{pmatrix} A \\ 5 - gt \end{pmatrix} \mathrm{m\,s^{-1}}$, where A is a constant.

Part A Displacement

The particle is at $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$ when $t = 0$.

Find an expression for the x-component of the particle's displacement, in metres, as a function of t.

The following symbols may be useful: `A, g, t`

Find an expression for the y-component of the particle's displacement, in metres, as a function of t.

The following symbols may be useful: `A, g, t`

Part B Force

Find an expression for the force on the particle, given that it has mass $m\,\mathrm{kg}$. Give your answer in the form $a\underline{\boldsymbol{i}} + b\underline{\boldsymbol{j}}$ where $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$ are unit vectors in the x and y directions respectively.

The following symbols may be useful: `A, g, i, j, m`

Part C Value of A

The projectile hits a target at the coordinates $\begin{pmatrix} 20 \\ 0 \end{pmatrix}$.

What is the value of A? Give your answer to 2 significant figures. In your calculation, use the approximation $g \approx 10\,\mathrm{m\,s^{-2}}$ and assume that the target is hit at $t > 0$.

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Gameboard:
STEM SMART Double Maths 22 - Vector Equations of Motion

Vectors & Calculus 1i

A Level

A planet moves through space. The force on the planet is given by

$$\underline{\boldsymbol{F}} = \begin{pmatrix} -mAB^2 \cos Bt \\ -mAB^2 \sin Bt \end{pmatrix}$$

where A and B are numerical constants and m is the mass of the planet.

Part A Velocity

Given that the velocity of the planet when $t = 0$ is $\begin{pmatrix} 0 \\ AB \end{pmatrix}$.

Find an expression for the x-component of the velocity of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Find an expression for the y-component of the velocity of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part B Displacement

Given that the displacement of the planet when $t = 0$ is $\begin{pmatrix} A \\ 0 \end{pmatrix}$.

Find an expression for the x-component of the displacement of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Find an expression for the y-component of the displacement of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part C Modulus

Find an expression for the modulus of the displacement. Simplify your answer as far as possible.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part D Shape of path

What is the shape of the path that the planet follows?

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STEM SMART Double Maths 22 - Vector Equations of Motion

Vector Equations of Motion 1

A Level

This question looks at three different uses of calculus in vector problems.

Part A Integrating to find particle displacement

A particle moves in the x-y plane with velocity $\underline{\boldsymbol{v}} = \begin{pmatrix} 2t\mathrm{e}^{-2t^2} \\ 3t\mathrm{e}^{-4t^2} \end{pmatrix}$. Find an expression for the displacement of the particle at time t, given that the particle is at the origin when $t = 0$.

Enter an expression for the x-component of the displacement.

The following symbols may be useful: `e, t`

Enter an expression for the y-component of the displacement.

The following symbols may be useful: `t`

Part B Finding a maximum speed

At a time $t\,\mathrm{s}$ a particle moves in the x-y plane with velocity $\underline{\boldsymbol{v}} = \begin{pmatrix} 2t\mathrm{e}^{-2t^2} \\ 3 \end{pmatrix} \mathrm{m\,s^{-1}}$. What is the maximum speed of the particle? Give your answer as an expression in terms of e.

The following symbols may be useful: `e`

Part C Distance of closest approach to the origin

The displacement of a particle is given by the expression $\underline{\boldsymbol{s}} = \begin{pmatrix} \mathrm{e}^{3t} \\ \mathrm{e}^{6t} - 5 \end{pmatrix}$. Find the shortest distance between the particle and the origin during the particle's motion. Give your answer in the form $\frac{\sqrt{a}}{2}$.

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STEM SMART Double Maths 22 - Vector Equations of Motion

Projectiles: Trajectories 3i

A Level

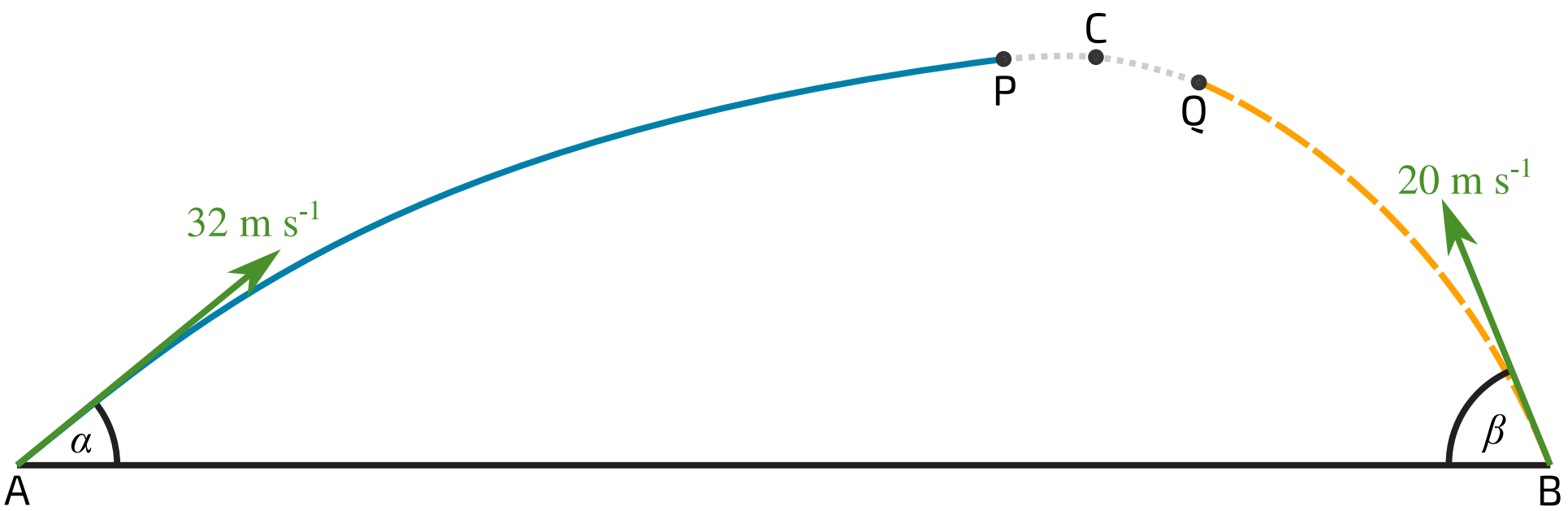

Figure 1: The trajectories of particles P and Q.

A particle P is projected with speed $32\,\mathrm{m\,s^{-1}}$ at an angle of elevation α, where $\sin\alpha = \frac{3}{5}$, from a point A on horizontal ground. At the same instant a particle Q is projected with speed $20\,\mathrm{m\,s^{-1}}$ at an angle of elevation β, where $\sin\beta = \frac{24}{25}$, from a point B on the same horizontal ground. The particles move freely under gravity in the same vertical plane and collide with each other at the point C at the instant when they are travelling horizontally.

Part A Velocities of P and Q

Express the initial velocities of P and Q in vector form using the unit vectors $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$, where $\underline{\boldsymbol{i}}$ is a unit vector in the direction of $\overrightarrow{AB}$ and $\underline{\boldsymbol{j}}$ is a unit vector vertically upwards.

$$\underline{\boldsymbol{u}}_{\mathrm{P}} = \tfrac{1}{5}(\ \square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}}\ )\ \mathrm{ms}^{-1}$$

$$\underline{\boldsymbol{u}}_{\mathrm{Q}} = \tfrac{1}{5}(\ \square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}}\ )\ \mathrm{ms}^{-1}$$

Part B Height of C

Calculate the height of C above the ground. Give your answer to 3 significant figures.

Part C Time in air

Find the time, t, between projection and collision. Give your answer to 3 significant figures.

Part D Distance AB

Calculate the distance AB. Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, June 2016, OCR M2, Question 7

Gameboard:

STEM SMART Double Maths 22 - Vector Equations of Motion

Ships at Sea

A Level

C C C

Part A Unit vector

Find the unit vector in the same direction as $\underline{\boldsymbol{p}} = 6\underline{\boldsymbol{i}} — 3\underline{\boldsymbol{j}} + 2\underline{\boldsymbol{k}}$ in terms of the unit vectors $\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{k}}$.

The following symbols may be useful: `i, j, k, p`

Hence, find a vector of magnitude 4 parallel to $\underline{\boldsymbol{p}}$.

The following symbols may be useful: `i, j, k, p`

Part B S and T

In an experiment two ships, S and T, move in a calm sea.

In a simple model, S and T are treated as objects whose dimensions are negligible. Which single word describes an object for which such a modelling assumption is made?

Part C Position of S

S starts from the origin and moves with constant velocity $\underline{\boldsymbol{v}}_1 = 4\underline{\boldsymbol{i}} — 3\underline{\boldsymbol{j}}$, where $\underline{\boldsymbol{v}}_1$ is measured in $\mathrm{km\,h^{-1}}$.

Write down the position vector of S in terms of t using the unit vectors $\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{k}}$.

The following symbols may be useful: `i, j, k, t`

Part D Calm sea assumption

In the experiment the two ships, S and T, move in a calm sea.

Using the information given, justify limiting the velocity vectors of S and T to two dimensions.

The sea is described as calm so we assume the surface to be a [ ] so we only require [ ] perpendicular directions to describe the motion rather than [ ].

Items:

point **line** **plane** **space** 1 2 3

Part E Do they meet?

T starts from the point with position vector $(3\underline{\boldsymbol{i}} — 5\underline{\boldsymbol{j}})$ and moves with constant velocity $\underline{\boldsymbol{v}}_2 = (\underline{\boldsymbol{i}} + 4\underline{\boldsymbol{j}})$ where $\underline{\boldsymbol{v}}_2$ is measured in $\mathrm{km\,h^{-1}}$.

Do S and T ever meet? If so, find the time in seconds. If not, enter in 0.

Part F Minimum distance

Find the minimum distance between S and T. Give your answer using exact fractions and surds.

Part G Finding y

A second experiment is undertaken. The behavior of S is unchanged. T starts from the same point as before, but this time it is the intention that S and T meet. The required constant velocity to set for T is of the form $(\underline{\boldsymbol{i}} + y\underline{\boldsymbol{j}})$.

Find the value of y.

Part H Percentage accuracy

The ships are each approximately $40\,\mathrm{m}$ long. Estimate the maximum percentage error in the calculation of the time it takes for the ships to meet due to using the assumption described in Part B. Give your answer to 1 significant figure.

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