Differentiation from First Principles 1

A Level

To differentiate a function $f(x)$ from first principles involves taking a limit. The derivative of $f(x)$ is given by the expression

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Part A Differentiate x^3 from first principles

Differentiate x^3 from first principles. Drag and drop options into the spaces below.

In this question $f(x) = x^3$. Therefore, $f(x+h) =$ [ ]. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\square - x^3}{h}.$$

Next, expand the brackets in the numerator and simplify:

$$f'(x) = \lim_{h \to 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\square}{h} = \lim_{h \to 0} \square.$$

Finally, take the limit. As $h \to 0$, the term containing x^2 is unchanged (because it does not depend on h), but the terms containing xh and h^2 tend to 0. Therefore,

$$f'(x) = \square.$$

Items:

$3x$ | $x^2h + xh^2 + h^3$ | $3x^2$ | $3x^2 + 3xh + h^2$ | $x^2 + xh$ | x^3h^3 | $(x+h)^3$ | $3x^2h + 3xh^2 + h^3$ | $2x^2h + 2xh^2 + h^3$

Part B Differentiate $2x^3 + 5$ from first principles

Differentiate x^3 from first principles. Drag and drop options into the spaces below.

In this question $f(x) = 2x^3 + 5$. Therefore, $f(x+h) =$ [ ]. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[\quad] - (2x^3 + 5)}{h}.$$

Next, just as in part A, expand the brackets in the numerator. After simplification, this produces:

$$f'(x) = \lim_{h \to 0} [\quad].$$

Finally, take the limit. As $h \to 0$, the term containing x^2 is unchanged (because it does not depend on h), but the terms containing xh and h^2 tend to 0. Therefore,

$$f'(x) = [\quad].$$

Items:

$2x^3 + 5h$ | $6x^2$ | $6x^2 + 5$ | $2(x+h)^3 + 5$ | $6x^2 + 6xh + 2h^2 + 5$ | $2x^3h^3 + 5$ | $6x^2 + 6xh + 2h^2$

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Physics. You work it out.

Differentiating Powers 1

A Level Further A

Part A Differentiate $y = x^4$

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x^4$.

The following symbols may be useful: x

Part B Differentiate $x = t^2$

Find the gradient of the curve $x = t^2$ at the points $t = 0$, $t = 3$ and $t = -3$.

Find the gradient at $t = 0$.

Find the gradient at $t = 3$.

Find the gradient at $t = -3$.

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Differentiating Powers 4

A Level Further A

Part A Gradient of curve $t = 4s^{-\frac{3}{4}}$

Find the gradient of the curve $t = 4s^{-\frac{3}{4}}$ at the point $s = 16$.

Part B First derivative of $x = bt^{\frac{3}{2}}$

Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ if $x = bt^{\frac{3}{2}}$.

The following symbols may be useful: `b, t`

Part C Second derivative of $x = bt^{\frac{3}{2}}$

Find $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ if $x = bt^{\frac{3}{2}}$.

The following symbols may be useful: `b, t`

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Differentiation (powers of x) 3ii

A Level

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in each of the following cases.

Part A Algebraic fraction

$$y = \frac{(3x)^2 \times x^4}{x}.$$

The following symbols may be useful: x

Part B Cube root

$$y = \sqrt[3]{x}.$$

The following symbols may be useful: x

Part C Reciprocal

$$y = \frac{1}{2x^3}.$$

The following symbols may be useful: x

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Differentiation (powers of x) 1ii

A Level

It is given that $y = 6x^3 + \frac{4}{\sqrt{x}} + 5x$.

Part A Find derivative

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: x

Part B Find second derivative

Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

The following symbols may be useful: x

Used with permission from UCLES, A level, June 2014, Paper 4721, Question 6.

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Differentiating Sums and Differences 1

A Level Further A

Part A Differentiate $ax^3 + \frac{b}{x} + c$

Differentiate $ax^3 + \frac{b}{x} + c$ with respect to x (a, b and c are constants).

The following symbols may be useful: `a, b, c, x`

Part B Differentiate $(2m + 3)(m - 1)$

Differentiate $(2m + 3)(m - 1)$ with respect to m.

The following symbols may be useful: `m`

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Physics. You work it out.

Gradient Function: Tangents and Normals 1ii

A Level

Part A Equation of tangent

Find the equation of the tangent to the curve $y = 7 + 6x - x^2$ at the point P where $x = 5$, giving your answer in the form $ax + by + c = 0$.

The following symbols may be useful: `x, y`

Part B Mid-point coordinate

This tangent meets the x-axis at Q. Find the x-coordinate of the mid-point of PQ.

Used with permission from UCLES, A level, January 2010, Paper 4721, Question 6

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Gradient Function: Tangents and Normals 1i

A Level

A curve has equation $y = x^2 + x$.

Part A Gradient

Find the gradient of the curve at the point where $x = 2$.

Part B Normal

Find the equation of the normal to the curve at the point for which $x = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

The following symbols may be useful: `x, y`

Part C Find k

Find the smallest value of k for which the line $y = kx - 4$ is a tangent to the curve.

The following symbols may be useful: `k`

Gameboard:

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Differentiating Powers 3

A Level C C C Further A P P P

Part A Derivative of $v = Bu^{-3}$

Find $\frac{\mathrm{d}v}{\mathrm{d}u}$ if $v = Bu^{-3}$, where B is a constant.

The following symbols may be useful: `B, u`

Part B Force if potential $V = \frac{q^2}{4\pi\epsilon_0 r}$

The electrostatic potential energy V of two equal charges q a distance r apart is given by $V = \frac{q^2}{4\pi\epsilon_0 r}$, where ϵ_0 and q are constants. The force between the two charges is given by $-\frac{\mathrm{d}V}{\mathrm{d}r}$; find an expression for this force.

The following symbols may be useful: `epsilon_0, pi, q, r`

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