

# Partial Fractions 1

Pre-Uni Maths for Sciences A5.1

A Level  
P P P

The function  $\frac{2x - 1}{(3x - 2)(x - 1)}$  can be written as  $\frac{A}{3x - 2} + \frac{B}{x - 1}$ . Find  $A$  and  $B$ .

Part A Find  $A$

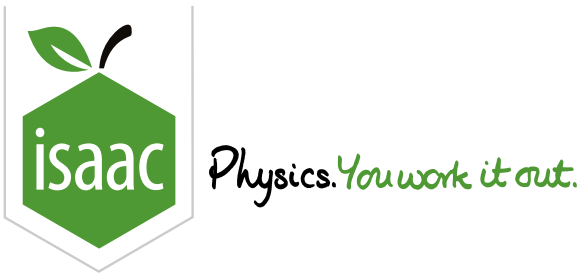
Find the constant  $A$ .

Part B Find  $B$

Find the constant  $B$ .

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# Partial Fractions 2

Pre-Uni Maths for Sciences A5.2

A Level  
P P P

The function  $\frac{w + 2}{(w - 1)(w + 1)(2w + 1)}$  can be written as  $\frac{A}{(w - 1)} + \frac{B}{(w + 1)} + \frac{C}{(2w + 1)}$ . Using the substitution method find the constants  $A$ ,  $B$  and  $C$ .

Part A   Find  $A$

Find the constant  $A$ .

The following symbols may be useful: A

Part B   Find  $B$

Find the constant  $B$ .

The following symbols may be useful: B

Part C   Find  $C$

Find the constant  $C$ .

The following symbols may be useful: c

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Physics. *You work it out.*

# Partial Fractions 3

A Level  
P P P

The function  $\frac{24t^2 + 31t + 2}{(2t + 1)^2(t + 3)}$  can be written as  $\frac{A}{(2t + 1)^2} + \frac{B}{(2t + 1)} + \frac{C}{(t + 3)}$ . Find the constants  $A$ ,  $B$  and  $C$ .

Part A   Find  $A$

Find the constant  $A$ .

The following symbols may be useful: A

Part B   Find  $B$

Find the constant  $B$ .

The following symbols may be useful: B

Part C   Find  $C$

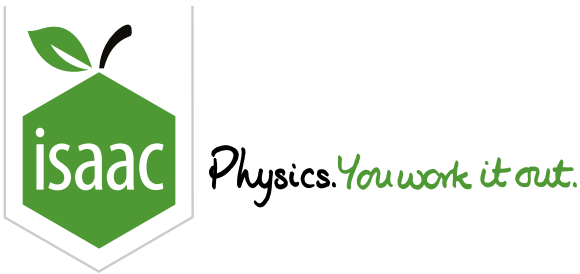
Find the constant  $C$ .

The following symbols may be useful: c

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# Partial Fractions 4

A Level  
P P P

The function  $\frac{8a^2}{(x-a)(x+a)^2}$ , where  $a$  is a constant, can be written as  $\frac{A}{(x+a)^2} + \frac{B}{x+a} + \frac{C}{x-a}$ . Find the constants  $A$ ,  $B$  and  $C$ .

Part A   Find  $A$

Find the constant  $A$ .

The following symbols may be useful:  $A$ ,  $a$

Part B   Find  $B$

Find the constant  $B$ .

The following symbols may be useful:  $B$ ,  $a$

Part C   Find  $C$

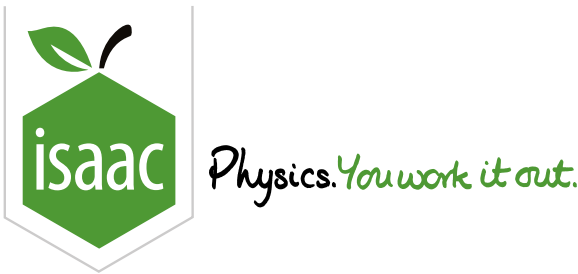
Find the constant  $C$ .

The following symbols may be useful:  $C$ ,  $a$

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# Improper Partial Fractions 1

Pre-Uni Maths for Sciences A5.3

A Level  
P P P

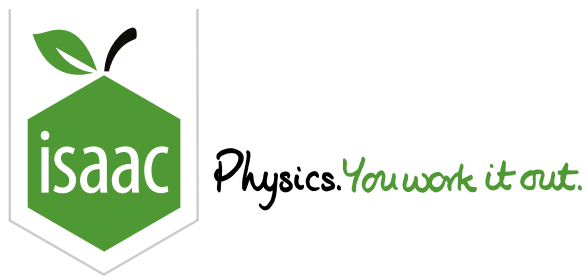
Express  $\frac{-6x^3 + 15x^2 + x - 11}{2x^2 - 5x - 3}$  as partial fractions.

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# Proof and Odd Perfect Numbers

A Level



The **proper divisors** of a number are those factors which are not equal to the number itself. For the number 6,

- The divisors of 6 are 1, 2, 3 and 6.
- The proper divisors of 6 are 1, 2 and 3.

The number 6 is an example of a **perfect number**. A perfect number is a number for which the sum of its proper divisors is equal to the number itself. For the number 6,

$$1 + 2 + 3 = 6$$

In this question you will use proof by contradiction to show that an odd perfect number cannot be a square number.

## Assumption:

We will assume that there is an odd perfect number,  $n$ , that is also a square number. Then  $n = m^2$ , where  $m$  is an integer.

### Part A Reasoning: odd and even factors

- An even number multiplied by an even number is always an  number.
- An even number multiplied by an odd number is always an  number.
- An odd number multiplied by an odd number is always an  number.

Therefore, as  $n$  is an  number, the factors of  $n$  can only be  numbers.

Items:

Part B Reasoning: sum of proper divisors

As  $n = m^2$ ,  $m$  is a factor of  $n$ .

Consider another factor of  $n$ . Call this factor  $p$ . As  $p$  is a factor of  $n$ ,  $q =$  is also a factor of  $n$ . As

$n = m^2$ ,  $q = \frac{m^2}{p}$ . Hence,

- If  $p < m$ ,  $q$    $m$ .
- If  $p > m$ ,  $q$    $m$ .

Therefore, with the exception of  $m$ , the factors of  $n$  occur in pairs. One factor in the pair is smaller than  $m$ , and the other factor is larger than  $m$ . Including  $m$ , the total number of factors of  $n$  is therefore an  number.

For any value of  $n$ , one of the factor pairs is 1 and  $n$ . The number of proper divisors (factors other than  $n$  itself) is therefore an  number. As we have shown in part A that all of the factors of  $n$  are  numbers, the sum of the proper divisors of  $n$  is therefore an  number.

Items:

$\frac{n}{p}$

$\frac{p}{n}$

odd

>

even

<

Part C Conclusion

Our starting assumption was that  $n$  is an odd perfect number and also a square number.

The definition of a perfect number means that the sum of the proper divisors of  $n$  is equal to . The sum of the proper divisors must therefore be an  number.

However, in part B we have shown that if  $n$  is an odd number which is also a square number, the sum of the proper divisors has to be an  number.

Therefore, we have arrived at a contradiction. We conclude that there are no odd perfect numbers that are also square numbers.

Items:

$2n$

$n$

even

odd

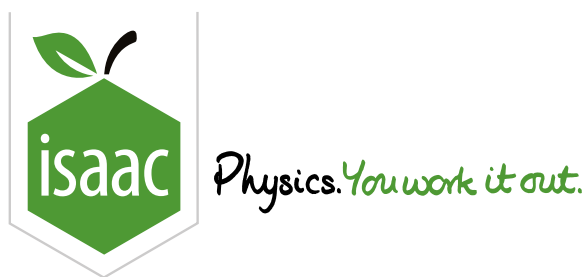
$n^2$

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# Proof Applied to Surface Areas

A Level



Consider a sphere with a radius  $r$  cm, where  $r$  is a rational number. Using proof by contradiction, show that the side length of a cube with the same surface area cannot also be a rational number of cm.

Once you have worked out a proof yourself, consider the model proof below. Drag and drop answers into the blank spaces to complete the proof. You may use the same answer more than once.

## Assumption:

Consider a sphere of radius  $r$  cm, where  $r$  is a rational number. Let the side length of a cube with the same surface area as the sphere be  $a$  cm. Assume that  $a$  is a rational number, in which case  $a = \frac{b}{c}$ , where  $b$  and  $c$  are integers with no common factor.

## Reasoning:

The surface area of the sphere is . Because  $r$  is a rational number,  $r = \frac{p}{q}$ , where  $p$  and  $q$  are integers with no common factor. Hence, the surface area of the sphere may be written as .

The surface area of the cube is . Using  $a = \frac{b}{c}$ , the surface area may be written as .

The surface area of the sphere and the cube are equal. Hence,  $4\pi \left(\frac{p}{q}\right)^2 = 6 \left(\frac{b}{c}\right)^2$ . Re-arranging this equation to give an expression for  produces .

As  $b$ ,  $c$ ,  $p$  and  $q$  are all integers,  must be  number. However,  $\pi$  is not  number.

## Conclusion:

The assumption that  $a$  is rational has resulted in a contradiction. Hence, the assumption cannot be true.

Therefore, the side length of a cube with the same surface area as a sphere of radius  $r$  cm, where  $r$  is a rational number, cannot be a rational number of cm.

Items:

$4\pi r^2$

$\frac{3b^2q^2}{2c^2p^2}$

an irrational

$a^3$

a rational

$\pi = \frac{3b^2p^2}{2c^2q^2}$

$4\pi \left(\frac{p}{q}\right)^2$

$\pi$

$\pi = \frac{3b^2q^2}{2c^2p^2}$

a real

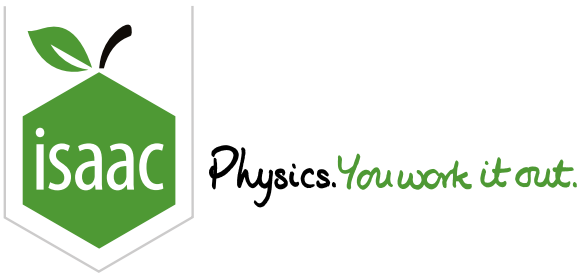
$6\left(\frac{b}{c}\right)^2$

$6a^2$

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# Partial Fractions Applied to Other Functions

A Level  
C C C

Pre-Uni Maths for Sciences A5.5, A5.6 & A5.7

Express the following functions in partial fraction form.

**Part A**   A trigonometric function

Express the function  $\frac{\cos y}{(\cos y + 1)(2 \cos y + 1)}$  in the form  $\frac{A}{\cos y + 1} + \frac{B}{2 \cos y + 1}$ , where  $A$  and  $B$  are constants.

The following symbols may be useful:  $\cos()$ ,  $\sin()$ ,  $\tan()$ ,  $y$

**Part B**   An exponential function

Express the function  $\frac{e^{2x} + 5}{(e^x - 1)(e^x - 2)(e^x - 3)}$  in the form  $\frac{A}{e^x - 1} + \frac{B}{e^x - 2} + \frac{C}{e^x - 3}$ , where  $A$ ,  $B$  and  $C$  are constants.

The following symbols may be useful:  $e$ ,  $x$

Part C    A logarithmic function

Express the function  $\frac{5 \ln z + 20}{(\ln z)^2 + \ln(z^2) + 1}$  in the form  $\frac{A}{(\ln z + 1)^2} + \frac{B}{\ln z + 1}$ , where  $A$  and  $B$  are constants.

The following symbols may be useful:  $\ln()$ ,  $\log()$ ,  $z$

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