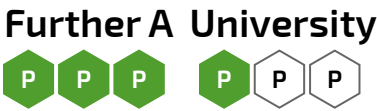


Integrating Factors 2

Pre-Uni Maths for Sciences L2.2



Find the general solution of the differential equation

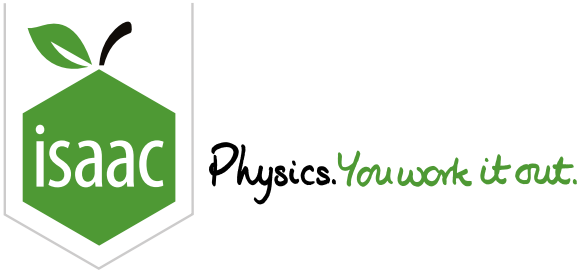
$$x \frac{dy}{dx} + (a + x)y = e^{-x}.$$

Find the general solution for y as a function of x .

The following symbols may be useful: a , c , e , k , x , y

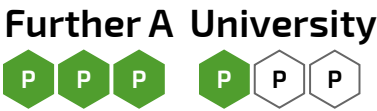
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RC Circuit (Integrating Factors)

Pre-Uni Maths for Sciences L2.5



A circuit consists of a capacitor C , a resistor R and a switch in series with a battery of emf V_0 . The switch is initially open and the capacitor is uncharged. At $t = 0$ the switch is closed. The equation for the charge q on the capacitor as a function of time t after the switch is closed is

$$R\frac{dq}{dt} + \frac{q}{C} = V_0.$$

Find how the charge on the capacitor varies with time t given that $q = 0$ at $t = 0$.

Find the equation for the charge q on the capacitor as a function of time t .

The following symbols may be useful: C , R , V_0 , e , q , t

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2nd Order Homogeneous 1

Further A



The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 15 = 0$$

Find the solution of the equation given that when $x = 0$, $y = 5$ and $\frac{dy}{dx} = -1$.

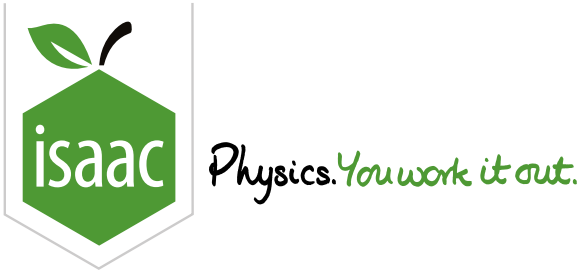
The following symbols may be useful: e , x , y

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2nd Order Homogeneous 2

Further A

P

P

P

The variables z and t satisfy the differential equation

$$4\frac{d^2z}{dt^2} + 12\frac{dz}{dt} + 9 = 0$$

Find the solution of the equation given that when $t = 0$, $z = 4$ and $\frac{dz}{dt} = 1$.

The following symbols may be useful: e, t, z

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Mass on Spring (2nd Order)

Pre-Uni Maths for Sciences L3.6



A mass m on a spring is subjected to a damping force. The equation describing its displacement x from its equilibrium position as a function of time t is

$$m \frac{d^2x}{dt^2} = -kx - b \frac{dx}{dt},$$

where $-kx$ is the force from the spring and $-b \frac{dx}{dt}$ is the force due to damping. The damping coefficient b is related to the spring constant k by $k = \frac{4b^2}{25m}$. Find an expression for the subsequent motion of the mass given that $x = 0$ and $\frac{dx}{dt} = V$ at $t = 0$.

Find the equation describing the subsequent motion of the mass given that $x = 0$ and $\frac{dx}{dt} = V$ at $t = 0$. Give your answer in terms of the constants b , m , and V .

The following symbols may be useful: v , b , e , m , t , x

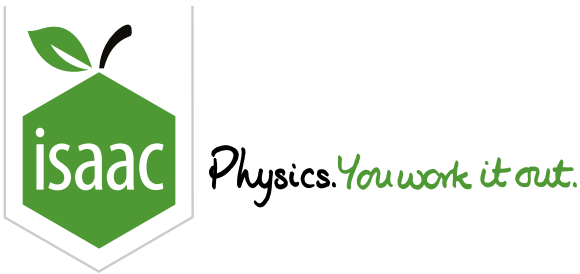
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Damped Pendulum (2nd Order)



Pre-Uni Maths for Sciences L3.3

The equation describing the displacement x of the bob of a damped pendulum from its equilibrium position is given by

$$\frac{d^2x}{dt^2} = -\omega_0^2 x - 2\gamma \frac{dx}{dt}$$

where ω_0 is the angular frequency of undamped oscillations of the pendulum and γ is related to the damping. Assuming $\omega_0 > \gamma$ find an equation for x at time t given that $x = X$ and $\frac{dx}{dt} = 0$ at $t = 0$. (You will find it helpful to define a new constant ω_1 such that $\omega_1^2 = \omega_0^2 - \gamma^2$.)

Find an equation for x at time t given that $x = X$ and $\frac{dx}{dt} = 0$ at $t = 0$.

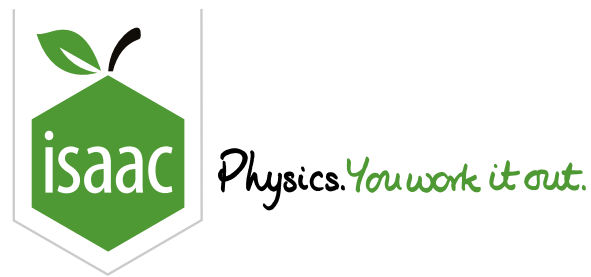
The following symbols may be useful: x , e , γ , ω_1 , t , x

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Second Order Differential Equation 2

Pre-Uni Maths for Sciences L3.2



Find the solution of the equation

$$\frac{d^2p}{dq^2} - 4\frac{dp}{dq} + 3p = 3q - 1$$

given that $p = 2$ and $\frac{dp}{dq} = -1$ when $q = 0$.

Find the solution of the equation.

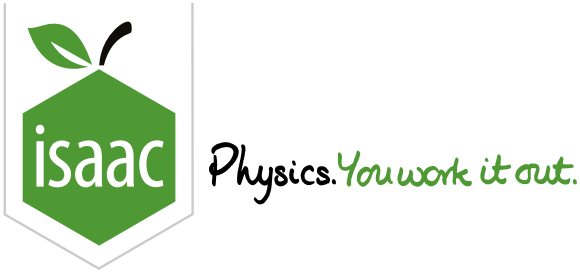
The following symbols may be useful: e , p , q

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Forced Oscillator (2nd Order)

Pre-Uni Maths for Sciences L3.4



The equation of motion of a forced oscillator is given by

$$\frac{d^2z}{dt^2} + \omega_0^2 z = Z_0 \sin(\omega_1 t)$$

Given that $\omega_0 \neq \omega_1$ find the solution for z given that $z = 0$ and $\frac{dz}{dt} = 0$ at $t = 0$.

Find the solution for z given that $z = 0$ and $\frac{dz}{dt} = 0$ at $t = 0$.

The following symbols may be useful: z_0 , ω_0 , ω_1 , t , z

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Differential Equations: General Applications 2i

Further A



During an industrial process substance X is converted into substance Z . Some of the substance X goes through an intermediate phase, and is converted into substance Y , before being converted into substance Z . The situation is modelled by

$$\frac{dy}{dt} = 0.3x - 0.2y \quad \text{and} \quad \frac{dz}{dt} = 0.2y + 0.1x$$

where x , y and z are the amounts in kg of X , Y and Z at time t hours after the process starts.

Initially there is 10 kg of substance X and nothing of substances Y and Z . The amount of substance X decreases exponentially. The initial rate of decrease is 4 kg hour^{-1} .

Part A Expression for x

Find an expression for x .

The following symbols may be useful: e , t

Part B $\frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt}$

Show that $\frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} = k$ where k is a constant.

State the value of k .

Comment on this result in the context of the industrial process.

- ☐ The total amount of all three substances increases throughout the process.
- ☐ The total amount of all three substances is constant throughout the process.
- ☐ The total amount of all three substances is zero throughout the process.
- ☐ The total amount of all three substances decreases throughout the process.

Part C Expression for y

Find an expression for y in terms of t .

The following symbols may be useful: e , t

Part D Maximum amount of Y

Determine the maximum amount of substance Y present during the process.

Part E Time to produce 9 kg of substance Z

How long does it take to produce 9 kg of substance Z ? Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, Sample Paper 2017, Paper Y541, Question 11.

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