Hypothesis Testing: Meeples

A Level

A manufacturer of meeples (a type of playing piece used in many board games) finds that, on average, 10% of the meeples they produce are defective. In order to improve the production of their meeples, they purchase new equipment to reduce the number of defects. In order to test the new equipment, a member of their quality assurance team collects a random sample of 80 meeples produced using the new equipment and find that 3 of them have defects. Test at the 5% significance level whether the new equipment has reduced the number of defects.

Let the random variable X be the number of meeples with defects. Then $X \sim \mathrm{B}($ ____ $, p)$.

The null and alternative hypotheses are:

$\mathrm{H}_0 : p$ ____ ____ $\quad\quad$ $\mathrm{H}_1 : p$ ____ ____

The test statistic, $\mathrm{P}(X \leq$ ____ $) =$ ____.

Comparing this to the significance level, we find that $\mathrm{P}(X \leq$ ____ $)$ ____ ____.

Therefore we ____ H_0 at the 5% level. There ____ evidence to suggest that the new equipment has reduced the number of meeples with defects.

Items:

$>$	$<$	$=$	$\neq$	**reject**	**do not reject**	**is**	**is insufficient**	0.1	0.05	0.025	3	5	8	10	70	72

80	88	90	0.0107	0.0246	0.0353	0.0527	0.0880

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Hypothesis Testing 3.1

A Level Further A

An experiment has two possible outcomes, A and B. An experimenter investigates the probabilities with which the two outcomes occur by carrying out the experiment a large number of times; X, the number of times the experimenter gets outcome A, is recorded. Answer the following questions.

Part A Outcomes equally likely

In a previous set of measurements the two outcomes of the experiment (A and B) have been found to be equally likely. An experimenter wishes to test, at the 5% significance level, the null hypothesis that the two outcomes are equally likely against the alternative hypothesis that the two outcomes are not equally likely. The experiment is carried out 50 times and X, the number of times the experimenter gets outcome A, is recorded.

Find the lower bound X_l of the acceptance region.

Find the upper bound X_u of the acceptance region.

Part B Slight difference in probability of outcomes

In a previous set of measurements the two outcomes of the experiment (A and B) have been found to be equally likely. The experiment is carried out 50 times and the experimenter gets outcome A 34 times i.e. $X = 34$. By answering the following three questions use the data to test, at the 1% significance level, the hypothesis that the outcomes are not equally likely.

Assuming that the null hypothesis is correct, find P_p, the probability that $X = 34$. Give your answer to 2 s.f.

Assuming that the null hypothesis is correct, find P_c, the probability that $X \geq 34$. Give your answer to 2 s.f.

You were asked to use the data above to test, at the 1% significance level, the hypothesis that the outcomes are not equally likely. Using the appropriate answer from earlier is there sufficient evidence to reject, at this level, the null hypothesis that the outcomes are equally likely? Which of the following arguments is correct in this regard?

- $P_p > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_c < 0.01$ so reject the null hypothesis
- $P_p < 0.005$ so reject the null hypothesis
- $P_c < 0.005$ so reject the null hypothesis
- $P_p < 0.01$ so reject the null hypothesis
- $P_c > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_c > 0.005$ so there is not sufficient evidence to reject the null hypothesis
- $P_p > 0.005$ so there is not sufficient evidence to reject the null hypothesis

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Part C **Probability of A greater than that of B**

In a previous set of measurements the two outcomes of the experiment (A and B) have been found to be equally likely. As described in Part B the experiment is carried out 50 times and the experimenter gets outcome A 34 times i.e. $X = 34$. Test, at the 1% significance level, the hypothesis that the probability of getting the outcome A is greater than that of getting the outcome B.

Is there sufficient evidence to reject, at this level, the null hypothesis that the outcomes are equally likely? Which of the following arguments is correct in this regard?

- $P_c < 0.005$ so reject the null hypothesis
- $P_p > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_p < 0.01$ so reject the null hypothesis
- $P_c < 0.01$ so reject the null hypothesis
- $P_p < 0.005$ so reject the null hypothesis
- $P_c > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_p > 0.005$ so there is not sufficient evidence to reject the null hypothesis
- $P_c > 0.005$ so there is not sufficient evidence to reject the null hypothesis

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Gameboard:

STEM SMART Single Maths 20 - Binomial Hypothesis Tests & Combinatorics

Hypothesis Testing 3.2

A Level P P P Further A P P P

An experimenter uses a piece of test equipment to make measurements of samples she has manufactured. There is a 10% probability that the equipment will fail when she makes a measurement. She makes some changes to the equipment which she hopes will increase its reliability. Answer the following questions about how she will test whether her changes have increased the reliability.

Part A Carrying out 30 measurements

The experimenter would like to test the hypothesis, at the 2% level of significance, that her changes have increased the reliability of her equipment. She decides to make 30 measurements and there are no failures.

Assuming that the failure rate is unchanged find the probability that there are no failures. Give your answer to 2 s.f.

Deduce what the experimenter can conclude about the changes she has made.

- The changes have not improved the reliability of the test equipment.
- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; therefore the changes have not improved the reliability of the test equipment.
- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; there is insufficient evidence to indicate that the changes have improved the reliability of the test equipment.
- Given the original failure rate she would have expected about 3 failures, so there is sufficient evidence to support her hypothesis
- There is sufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.

Part B Carrying out more measurements

The experimenter realises that she needs more evidence to test, at the 2% level of significance, whether or not her changes have increased the reliability of her equipment. She decides to make more measurements. Assuming that she still gets no failures, how many measurements will she have to make in total to provide support for her hypothesis at the 2% level of significance?

Part C Carrying out 60 measurements

The experimenter makes 60 measurements in all and gets one failure.

Assuming that the failure rate is unchanged find the probability that she will get one failure or no failures. Give your answer to 2 s.f.

Hence deduce what the experimenter can conclude about the changes she has made.

- The number of failures is less than the 6 expected so the null hypothesis can be rejected; the reliability of the test equipment has therefore improved.
- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is insufficient evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- There is sufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is insufficient evidence to support the hypothesis that the changes have improved the reliability of the test equipment
- There is sufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.

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Hypothesis Testing 3.3

A Level Further A

According to one theory the probability that a particular result will be obtained in an experiment is 0.6 whereas according to another theory the probability is 0.7. The experiment is carried out 100 times and the particular result is obtained 76 times.

Part A Testing $p = 0.6$

Consider the first theory which suggests that $p = 0.6$; assume the null hypothesis is that $p = 0.6$ and the alternative hypothesis is that $p > 0.6$. Find the critical region for the test (in this case if X is the number of times the particular result is obtained the critical region has one part with $X \geq X_h$). If the theory is to be tested at the 2% level of confidence, deduce X_h.

Part B Testing $p = 0.7$

Now consider the second theory which suggests that $p = 0.7$; assume the null hypothesis is that $p = 0.7$ and the alternative hypothesis is that $p \neq 0.7$. Find the critical region for the test (in this case if X is the number of times the particular result is obtained the critical region has two parts $X \leq X_l$ and $X \geq X_h$). If the theory is to be tested at the 2% level of confidence, deduce X_l and X_h.

Find X_l.

Find X_h

Part C Conclusions from the tests

What can be concluded at the 2% level from the tests?

- The hypotheses that $p > 0.6$ and that $p = 0.7$ are correct.
- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; there is sufficient evidence to reject the hypothesis that $p = 0.7$ and to support the theory that $p \neq 0.7$.
- There is insufficient evidence to reject the hypothesis that $p = 0.6$; there is sufficient evidence to reject the hypothesis that $p = 0.7$ and to support the theory that $p \neq 0.7$.
- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; there is insufficient evidence to reject the hypothesis that $p = 0.7$.
- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; the hypothesis that $p = 0.7$ is correct.
- There is insufficient evidence to reject the hypothesis that $p = 0.6$; there is insufficient evidence to reject the hypothesis that $p = 0.7$.

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Permutations and Combinations 1

A Level

This question is about the number of possible orders when rearranging the letters of the word NEVER.

Part A Distinct permutations

The five letters of the word NEVER are arranged in a random order in a straight line.

How many different orders of the letters are possible?

Part B Orders with adjacent Es

The five letters of the word NEVER are arranged in random order in a straight line.

In how many of the possible orders are the two E's next to each other?

Part C Orders with one E in the first two letters

The five letters of the word NEVER are arranged in random order in a straight line.

Find the probability that the first two letters in the order include exactly one letter E.

Used with permission from UCLES, A level, January 2010, Paper 4732, Question 8

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Permutations and Combinations 2

A Level

This question is about arrangements of cards with numbers on them.

Part A Permutations of cards

The diagram shows 7 cards, each with a digit printed on it. The digits form a 7-digit number.

Figure 1

Seven cards with numbers printed on them.

How many different 7-digit numbers can be formed using these cards?

Part B Combinations of cards

The diagram below shows 5 white cards and 10 grey cards, each with a letter printed on it.

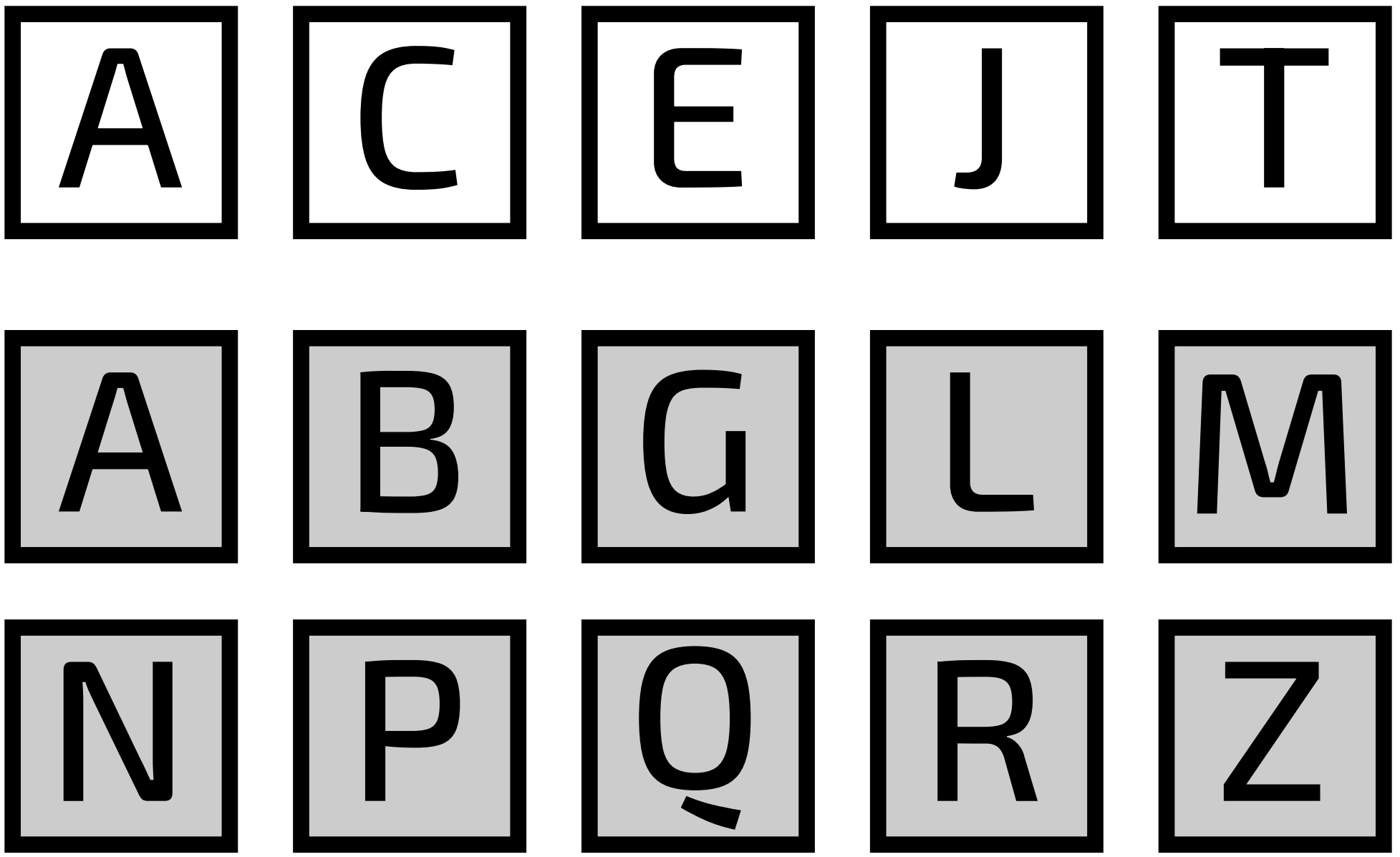

Figure 2: White and grey cards with letters printed on them.

From these cards, 3 white and 4 grey cards are selected at random **without** regard to order.

How many selections of cards are possible?

Part C The probability of drawing a single A

The diagram below shows 5 white cards and 10 grey cards, each with a letter printed on it.

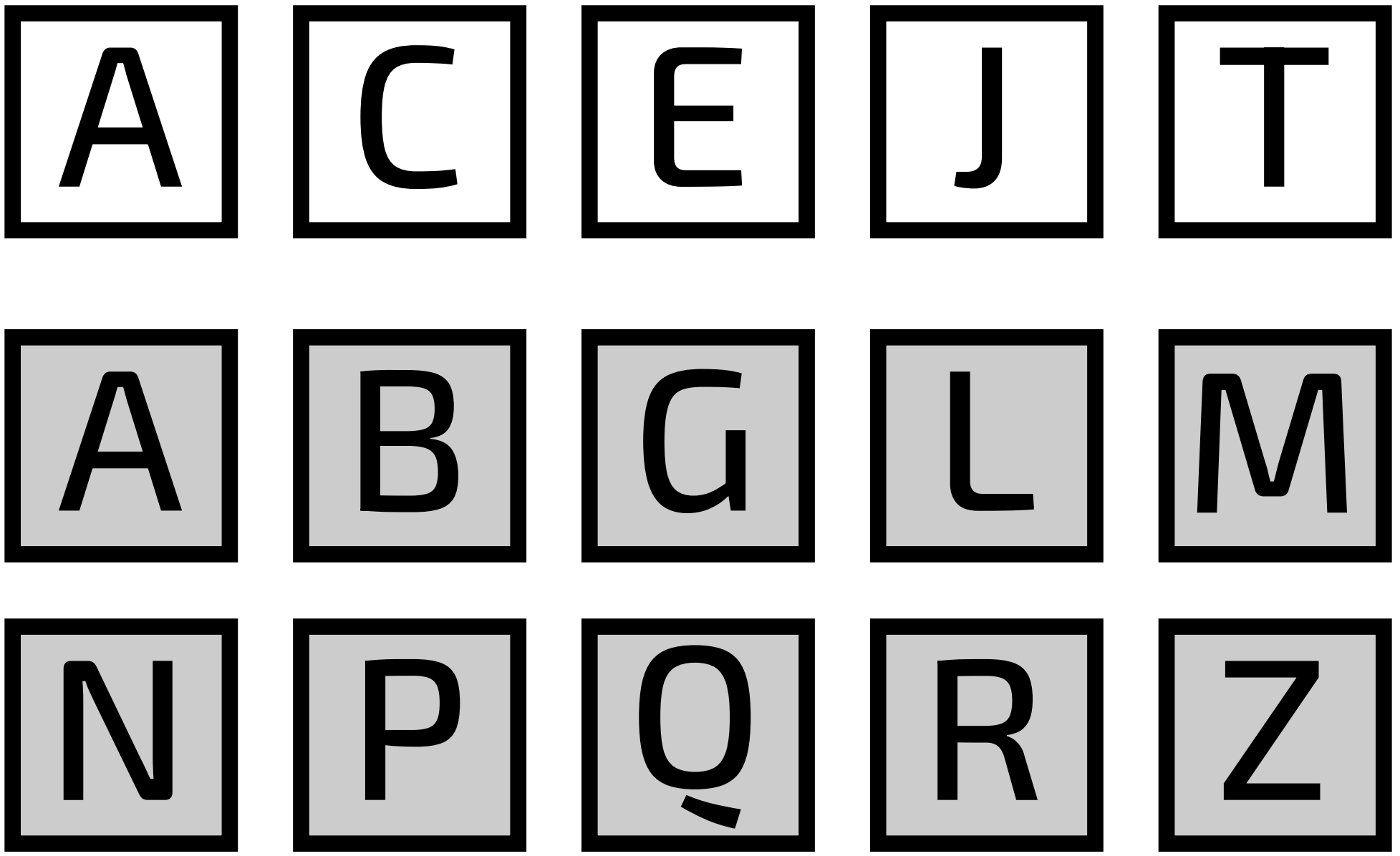

Figure 3: White and grey cards with letters printed on them.

From these cards, 3 white and 4 grey cards are selected at random **without** regard to order.

Find the probability that the seven cards include exactly one card showing the letter A.

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Gameboard:

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Permutations and Combinations 3

A Level

This question is about the number of possible orders of 7 students sitting on a bench.

Part A Number of permutations

A group of 7 students sit in a random order on a bench. Find the number of orders in which they can sit.

Part B Permutations where two students must be adjacent

A group of 7 students sit in a random order on a bench. The 7 students include Tom and Jerry. Find the probability that Tom and Jerry sit next to each other.

Part C Permutations where no boys are adjacent

A group of 7 students sit in random order on a bench. The students consist of 3 girls and 4 boys. Find the probability that no two boys sit next to each other.

Part D Permutations where all girls are adjacent

A group of 7 students sit in a random order on a bench. The students consist of 3 girls and 4 boys. Find the probability that all three girls sit next to each other.

Permutations and Combinations 4

A Level

This question is about students taking numbered cards out of a bag to make 4-digit numbers, and finding the probabilities of the results they might get.

Part A How many 4-digit numbers can be made?

A bag contains 9 discs numbered 1, 2, 3, 4, 5, 6, 7, 8, 9. Andrea chooses 4 discs at random, without replacement, and places them in a row.

How many different 4-digit numbers can be made?

Part B How many odd 4-digit numbers?

A bag contains 9 discs numbered 1, 2, 3, 4, 5, 6, 7, 8, 9. Andrea chooses 4 discs at random, without replacement, and places them in a row.

How many different **odd** 4-digit numbers can be made?

Part C Numbers with 3 odd digits

A bag contains 9 discs numbered 1, 2, 3, 4, 5, 6, 7, 8, 9. Martin chooses 4 discs at random, without replacement.

Find the probability that the 4 digits include at least 3 odd digits.

Part D Digits that add up to 28

A bag contains 9 discs numbered 1, 2, 3, 4, 5, 6, 7, 8, 9. Martin chooses 4 discs at random, without replacement.

Find the probability that the 4 digits add up to 28.
