Probabilities: Households

A Level

P P P

The people living in 3 households are classified as children (C), parents (P) or grandparents (G). The numbers living in each house are shown in the table below.

House number 1	House number 2	House number 3
$4C, 1P, 2G$	$2C, 2P, 3G$	$1C, 1G$

Part A Scenario 1 - a grandparent

All the people in all 3 houses meet for a party. One person at the party is chosen at random. Calculate the probability of choosing a grandparent.

Part B Scenario 2 - a grandparent

A house is chosen at random. Then a person in that house is chosen at random. Using a tree diagram, or otherwise, calculate the probability that the person chosen is a grandparent.

Part C Scenario 2 - a parent

Given that the person chosen by the method in Part B is a grandparent, calculate the probability that there is also a parent living in the house.

Probabilities: Mailing

A Level
P P P

A magazine mails a large number of households with an offer. Each household may answer "Yes" or "No" or may not reply at all. A second mailing is sent only to those households who have answered "Yes" or "No". Again each household may answer "Yes" or "No" or not reply at all. The proportions of households which reply are shown in the partially completed tree diagram.

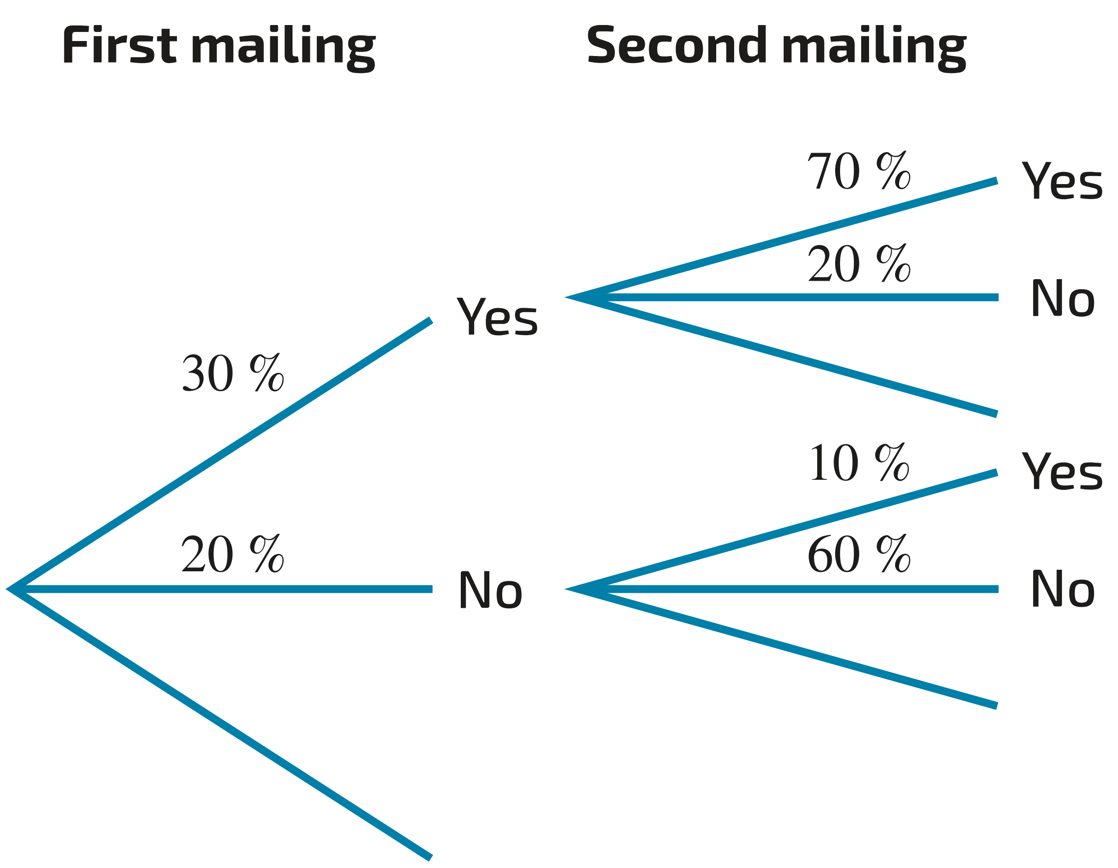

Figure 1: A tree diagram for the two mailings.

Part A The first mailing - no reply

For a randomly chosen household on the magazine's initial mailing list, find the probability that the household does not reply to the first mailing. Give your answer as a percentage.

Part B The second mailing - a reply of "No"

For a randomly chosen household on the magazine's initial mailing list, find the probability that the household answers "No" to the second mailing. Give your answer as a percentage.

Part C The first mailing - a reply of "No"

For a randomly chosen household on the magazine's initial mailing list, find the probability that the household answers "No" to the first mailing, given that it answers "No" to the second mailing. Give your answer as a fraction.

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Normal Distribution 1

Further A

The random variable Y is normally distributed with mean μ and variance σ^2, where μ and σ are integers.

It is found that P$(Y > 150.0) = 0.0228$ and P$(Y > 143.0) = 0.9332$. Find the values of μ and σ.

Part A Value of μ

Enter the value of μ.

Part B Value of σ

Enter the value of σ.

Adapted with permission from UCLES, A level, Jun 2015, OCR, S2, Question 1

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Physics. You work it out.

Home Gameboard Maths Statistics Probability Normal Distribution 2

Normal Distribution 2

A Level

P P P

Each of the three parts of this question is a different type of normal distribution problem.

Part A Employee salary

The annual salaries of employees in a company have mean $£30\,000$ and standard deviation $£12\,000$. Assuming a normal distribution, calculate the probability that the salary of one randomly chosen employee lies between $£20\,000$ and $£24\,000$.

Part B Sample size

The continuous random variable Y has the distribution $\mathrm{N}(23.0, 5.00^2)$. The mean of n observations of Y is denoted by $\overline{Y}$. It is given that $\mathrm{P}(\overline{Y} > 23.625) = 0.0228$. Find the value of n.

Part C $\mathrm{P}(G > 65)$

The random variable G has a normal distribution. It is known that

$$\mathrm{P}(G < 56.2) = \mathrm{P}(G > 63.8) = 0.100$$

Find $\mathrm{P}(G > 65.0)$.

Adapted with permission from UCLES, A level, OCR, S2, Jun 2008 Q2, Jan 2009 Q.2, Jan 2012 Q. 3

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Probability 5.7

A Level Further A

P P P P P P

Ball bearings produced by a machine have diameters which have a normal distribution with mean $6.50\,\mathrm{mm}$ and standard deviation $0.07\,\mathrm{mm}$.

Part A Number of ball bearings rejected

A random sample of 1000 of these ball bearings is selected. Find how many will be rejected if the tolerance required is in the range $6.40 - 6.60\,\mathrm{mm}$.

Part B Selection using a grid

The ball bearings are passed over a grid and those with diameters $< 6.37\,\mathrm{mm}$ fall through this grid. A sample of 1000 bearings is selected from those that have passed over the grid.

Find how many bearings will be rejected if the tolerance required is in the range $6.40 - 6.60\,\mathrm{mm}$.

Find how many of these will have diameters which exceed the tolerance limit (i.e. have diameters $> 6.60\,\mathrm{mm}$).

Given that 1000 bearings have passed over the grid, how many were there to start with?

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Correlation Hypothesis Testing 2

A Level P P P

A town planner believes that on summer weekday afternoons the amount of traffic into the centre of their town is higher when the temperature is higher. They want to test this hypothesis at the 1% significance level.

Every weekday (Monday to Friday) for six weeks they monitor the traffic on the main roads into town, and record the mean afternoon temperature, and they find that the correlation coefficient is -0.4517.

Part A Initial conclusion

Without doing any calculations, which of these statements can the town planner make? Choose all that apply.

- [ ] The correlation coefficient of their sample is negative, so there is a negative correlation between the amount of traffic on summer afternoons and temperature.
- [ ] The correlation coefficient is negative, so there is definitely no positive correlation between the amount of traffic and temperature.
- [ ] The correlation coefficient is negative, so there is no evidence that the amount of traffic is positively correlated with temperature.
- [ ] There is no evidence that the amount of traffic and afternoon temperature are correlated.
- [ ] The correlation coefficient is negative. There may be a negative correlation between the amount of traffic and afternoon temperature.

Part B Choosing a hypothesis test

Using the given data, which of the following hypothesis tests would it be most useful for the town planner to carry out?

- A hypothesis test to see if the amount of traffic and afternoon temperature are negatively correlated at the 1% significance level.
- A hypothesis test to see if the amount of traffic and afternoon temperature are negatively correlated at the 20% significance level.
- A hypothesis test to see if the amount of traffic and afternoon temperature are negatively correlated at the 50% significance level.

Part C Null and alternative hypotheses

The town planner carries out the most useful test listed in part B.

Drag and drop symbols into the spaces below to state the null and alternative hypotheses for this test.

H_0: [ ] [ ] [ ]

H_1: [ ] [ ] [ ]

Items:

ρ r $>$ $=$ $<$ 0 1

Part D Carrying out the test

Carry out the hypothesis test, and make a conclusion. Then fill in the blanks below.

The critical value of the correlation coefficient is ______.

Comparing the town planner's value to the critical value gives ______.

Therefore, ______ the null hypothesis. There ______ significant evidence that there is a ______ correlation between the amount of traffic in summer and afternoon temperature.

Items:

reject **no** **may be** **is** 0.4226 0.8822 $|-0.4517| < 0.9343$ $|-0.4517| < 0.8822$ 0.9343 **do not reject** **negative** **positive** **is not** $|-0.4517| > 0.4226$

Created for Isaac Physics by Jonathan Waugh

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Hypothesis Testing: Normal Distribution 2

A Level

Using a single black ink cartridge, a specific type of large printer is known to print a mean of $10\,750$ pages of black and white text, with a variance of $90\,000\,\mathrm{pages}^2$.

The company that makes the printer changes some of its software in an attempt to make the use of ink by their printers more efficient. They perform 80 tests on printers using the updated software and calculate a mean of $10\,834$ pages per ink cartridge.

Test at the 2% significance level whether the new software has improved the ink efficiency.

Part A Assumptions

Which of the following options do you need to assume in order to perform a hypothesis test? Select all that apply.

- [ ] That the variance in the number of pages printed in the sample of 80 tests is much smaller than the variance when using the old software, because 80 is much smaller than the number of printers using the old software.
- [ ] That the variance of the sample obeys a Poisson distribution.
- [ ] That the number of pages printed using one ink tank has a normal distribution.
- [ ] That the variance in the number of pages printed using the new software is the same as the variance when using the old software.

Part B Carrying out the test

Fill in the blanks to complete the description of the hypothesis test.

Let X be the number of pages printed using one cartridge with the new software. Then $X \sim \mathrm{N}(\mu, \boxed{})$, where μ is the mean.

The null hypothesis is that the new software does not improve ink efficiency, and the mean number of pages per cartridge is the same as before. The alternative hypothesis is that the new software improves ink efficiency.

$$\mathrm{H}_0 : \mu = 10750 \qquad \mathrm{H}_1 : \boxed{}$$

Let $\overline{X}$ be the mean number of pages printed using one cartridge with the new software. Assuming that the null hypothesis is true, then $\overline{X} \sim \mathrm{N}(10\,750, \frac{\boxed{}}{80})$.

Using the z-statistic $z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}}$, the p-value for a sample mean of 10834 pages is found to be

$$p = \mathrm{P}(\overline{X} \geq 10\,834) = \mathrm{P}(z \geq \frac{10\,834 - 10\,750}{\sqrt{\frac{\boxed{}}{80}}}) = \boxed{}$$

For a one-tailed test at the 2% significance level, p-values of less than 0.02 are in the critical (rejection) region. The calculated p value for the sample is $\boxed{}$ 0.02.

Therefore, $\boxed{}$ the null hypothesis. There $\boxed{}$ significant evidence that the new software improves ink efficiency.

Items:

is | 300 | **less than** | $\mu > 10\,750$ | $90\,000$ | 0.3897 | **is not** | $\mu < 10\,750$ | **greater than** | **do not reject** | 0.006133 | **reject**

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