# Essential GCSE Maths 54.1

GCSE C C C A Level P P P

A hospital has a total of $9760$ employees. They are divided into $4$ categories:

| Doctors | Nurses | Scientists | Administrators |
|---|---|---|---|
| 1240 | 5020 | 1080 | ? |

## Part A Number of administrators

How many administrators work for the hospital?

## Part B Number of employees surveyed

A stratified sample is required for a survey of employees' opinions. $5\%$ of the employees will be surveyed.

How many employees will be included in the survey?

## Part C Numbers of people from each category

A stratified sample is required for a survey of employees' opinions. $5\%$ of the employees will be surveyed.

Work out how many people must be randomly selected from each of the four employee categories to give a stratified sample.

How many doctors?

How many nurses?

How many scientists?

How many administrators?

# Woodland Survey

A Level

P P P

A conservationist wishes to collect data on the number of trees in a woodland nature reserve that have been parasitised by mistletoe. The reserve consists of birch, aspen and oak trees.

## Part A Conducting a census

Why would it be impractical to conduct a census to collect data from every tree in the woodland?

- A sampling frame would be needed.
- The population would need to be classified into distinct strata.
- It would be very time-consuming and expensive.
- The act of collecting data from a tree would destroy it.
- Collecting this data might spread the mistletoe

## Part B Type of sample

The conservationist wishes to determine if any particular species of tree is more likely to be parasitised by mistletoe. To do so, they decide to survey the first $10$ birches, $10$ aspen and $10$ oak trees that they come across in the reserve.

What type of sample are they collecting?

## Part C Disadvantage of sampling

What is a disadvantage of the type of sampling described in Part B?

- It is very time-consuming and expensive.
- This non-random sampling could introduce bias.
- A sampling frame is needed in order to select the sample.

## Part D Species data type

What type of data is the species of a tree?

## Part E Number of trees data type

What type of data is the number of trees that have been parasitised by mistletoe?

- Discrete quantitative data.
- Continuous quantitative data.
- Discrete qualitative data.
- Continuous qualitative data.

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# Researcher Survey

A Level

A research institute wishes to collect data from its employees on the methods by which they travel to work. The institute has an alphabetised list of its $450$ employees.

## Part A Systematic sample

Describe how they could take a systematic sample of size $30$.

Assign numbers from $1$ to [ ] to the employees on the alphabetised list. Calculate $450 \div$ [ ] $=$ [ ]. Generate a random number between $1$ and [ ]. Select the employee corresponding to that number and then select every [ ] th employee on the list after that.

Items:

$30$ $450$ $15$

## Part B Alternative method

One researcher suggests that it would be much easier to collect a sample by speaking to the first $30$ employees in the canteen.

What type of sampling would this be?

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Give one disadvantage of this type of sampling.

- This method is likely to introduce bias towards employees who use the canteen.
- This method would be time-consuming to carry out.
- This method requires the use of a sampling frame.
- Increasing the size of the sample can be very expensive.
- The sample is unlikely to be representative of the different groups among the employees.

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## Part C Stratified sample

The research institute also wishes to gather data from its research staff as to which new pieces of lab equipment will be required over the next year. The research staff within the institute consist of $40$ geneticists, $25$ ecologists and $55$ epidemiologists. It is thought that the different types of staff will have different requirements for lab equipment.

Describe how the institute could collect a stratified sample of size $20$ from its research staff.

There are a total of $120$ research staff. The sample is $\square \div 120 = \square$ of the research staff.

The institute should survey $40 \times \square \approx \square$ geneticists, $25 \times \square \approx \square$ ecologists and $55 \times \square \approx \square$ epidemiologists. The staff should be selected randomly from each group, by generating random numbers from $1$ to $\square$ for geneticists, $1$ to $\square$ for ecologists and $1$ to $\square$ for epidemiologists and selecting the corresponding members of staff, discarding and replacing any duplicate numbers within each group.

Items:

$\frac{1}{3}$ $5$ $40$ $9$ $20$ $\frac{5}{24}$ $\frac{1}{6}$ $7$ $25$ $10$ $6$ $4$ $55$ $\frac{11}{24}$

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# Hypothesis Testing: Meeples

A Level

A manufacturer of meeples (a type of playing piece used in many board games) finds that, on average, $10\%$ of the meeples they produce are defective. In order to improve the production of their meeples, they purchase new equipment to reduce the number of defects. In order to test the new equipment, a member of their quality assurance team collects a random sample of $80$ meeples produced using the new equipment and finds that $3$ of them have defects. Test at the $5\%$ significance level whether the new equipment has reduced the number of defects.

Let the random variable $X$ be the number of meeples with defects. Then $X \sim \mathrm{B}(\square, p)$.

The null and alternative hypotheses are:

$\mathrm{H}_0 : p \square \square$ $\mathrm{H}_1 : p \square \square$

The test statistic, $\mathrm{P}(X \leq \square) = \square$.

Comparing this to the significance level, we find that $\mathrm{P}(X \leq \square) \square \square$.

Therefore we $\square$ $\mathrm{H}_0$ at the $5\%$ level. There $\square$ evidence to suggest that the new equipment has reduced the number of meeples with defects.

Items:

$>$ $<$ $=$ $\neq$ **reject** **do not reject** **is** **is insufficient** $0.1$ $0.05$ $0.025$ $3$ $5$ $8$ $10$

$70$ $72$ $80$ $88$ $90$ $0.0107$ $0.0246$ $0.0353$ $0.0527$ $0.0880$

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# Hypothesis Testing 3.1

A Level P P P Further A P P P

An experiment has two possible outcomes, A and B. An experimenter investigates the probabilities with which the two outcomes occur by carrying out the experiment a large number of times; $X$, the number of times the experimenter gets outcome A, is recorded. Answer the following questions.

## Part A Outcomes equally likely

In a previous set of measurements the two outcomes of the experiment (A and B) have been found to be equally likely. An experimenter wishes to test, at the $5\%$ significance level, the null hypothesis that the two outcomes are equally likely against the alternative hypothesis that the two outcomes are not equally likely. The experiment is carried out $50$ times and $X$, the number of times the experimenter gets outcome A, is recorded.

Find the lower bound $X_l$ of the acceptance region.

Find the upper bound $X_u$ of the acceptance region.

## Part B Slight difference in probability of outcomes

In a previous set of measurements the two outcomes of the experiment (A and B) have been found to be equally likely. The experiment is carried out $50$ times and the experimenter gets outcome A $34$ times i.e. $X = 34$. By answering the following three questions use the data to test, at the $1\%$ significance level, the hypothesis that the outcomes are not equally likely.

Assuming that the null hypothesis is correct, find $P_p$, the probability that $X = 34$. Give your answer to $2$ s.f.

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Assuming that the null hypothesis is correct, find $P_c$, the probability that $X \geq 34$. Give your answer to $2$ s.f.

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You were asked to use the data above to test, at the $1\%$ significance level, the hypothesis that the outcomes are not equally likely. Using the appropriate answer from earlier is there sufficient evidence to reject, at this level, the null hypothesis that the outcomes are equally likely? Which of the following arguments is correct in this regard?

- $P_c < 0.005$ so reject the null hypothesis
- $P_c < 0.01$ so reject the null hypothesis
- $P_c > 0.005$ so there is not sufficient evidence to reject the null hypothesis
- $P_p > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_c > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_p < 0.005$ so reject the null hypothesis
- $P_p > 0.005$ so there is not sufficient evidence to reject the null hypothesis
- $P_p < 0.01$ so reject the null hypothesis

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## Part C Probability of A greater than that of B

In a previous set of measurements the two outcomes of the experiment (A and B) have been found to be equally likely. As described in Part B the experiment is carried out $50$ times and the experimenter gets outcome A $34$ times i.e. $X = 34$. Test, at the $1\%$ significance level, the hypothesis that the probability of getting the outcome A is greater than that of getting the outcome B.

Is there sufficient evidence to reject, at this level, the null hypothesis that the outcomes are equally likely? Which of the following arguments is correct in this regard?

- $P_p < 0.01$ so reject the null hypothesis
- $P_p > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_c > 0.01$ so there is not sufficient evidence to reject the null hypothesis
- $P_c > 0.005$ so there is not sufficient evidence to reject the null hypothesis
- $P_c < 0.01$ so reject the null hypothesis
- $P_c < 0.005$ so reject the null hypothesis
- $P_p > 0.005$ so there is not sufficient evidence to reject the null hypothesis
- $P_p < 0.005$ so reject the null hypothesis

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# Hypothesis Testing 3.2

A Level P P P Further A P P P

An experimenter uses a piece of test equipment to make measurements of samples she has manufactured. There is a $10\%$ probability that the equipment will fail when she makes a measurement. She makes some changes to the equipment which she hopes will increase its reliability. Answer the following questions about how she will test whether her changes have increased the reliability.

## Part A Carrying out $30$ measurements

The experimenter would like to test the hypothesis, at the $2\%$ level of significance, that her changes have increased the reliability of her equipment. She decides to make $30$ measurements and there are no failures.

Assuming that the failure rate is unchanged find the probability that there are no failures. Give your answer to $2$ s.f.

Deduce what the experimenter can conclude about the changes she has made.

- There is insufficient evidence at the $2\%$ level to reject the null hypothesis that the changes have had no effect; therefore the changes have not improved the reliability of the test equipment.
- The changes have not improved the reliability of the test equipment.
- Given the original failure rate she would have expected about $3$ failures, so there is sufficient evidence to support her hypothesis
- There is sufficient evidence at the $2\%$ level to reject the null hypothesis that the changes have had no effect; at the $2\%$ level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- There is insufficient evidence at the $2\%$ level to reject the null hypothesis that the changes have had no effect; there is insufficient evidence to indicate that the changes have improved the reliability of the test equipment.

## Part B Carrying out more measurements

The experimenter realises that she needs more evidence to test, at the $2\%$ level of significance, whether or not her changes have increased the reliability of her equipment. She decides to make more measurements. Assuming that she still gets no failures, how many measurements will she have to make in total to provide support for her hypothesis at the $2\%$ level of significance?

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## Part C Carrying out $60$ measurements

The experimenter makes $60$ measurements in all and gets one failure.

Assuming that the failure rate is unchanged find the probability that she will get one failure or no failures. Give your answer to $2$ s.f.

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Hence deduce what the experimenter can conclude about the changes she has made.

- There is insufficient evidence at the $2\%$ level to reject the null hypothesis that the changes have had no effect; at the $2\%$ level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- There is sufficient evidence at the $2\%$ level to reject the null hypothesis that the changes have had no effect; at the $2\%$ level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- The number of failures is less than the $6$ expected so the null hypothesis can be rejected; the reliability of the test equipment has therefore improved.
- There is sufficient evidence at the $2\%$ level to reject the null hypothesis that the changes have had no effect; at the $2\%$ level there is insufficient evidence to support the hypothesis that the changes have improved the reliability of the test equipment
- There is insufficient evidence at the $2\%$ level to reject the null hypothesis that the changes have had no effect; at the $2\%$ level there is insufficient evidence to support the hypothesis that the changes have improved the reliability of the test equipment.

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# Hypothesis Testing 3.3

A Level P P P Further A P P P

According to one theory the probability that a particular result will be obtained in an experiment is $0.6$ whereas according to another theory the probability is $0.7$. The experiment is carried out $100$ times and the particular result is obtained $76$ times.

## Part A Testing $p = 0.6$

Consider the first theory which suggests that $p = 0.6$; assume the null hypothesis is that $p = 0.6$ and the alternative hypothesis is that $p > 0.6$. Find the critical region for the test (in this case if $X$ is the number of times the particular result is obtained the critical region has one part with $X \geq X_h$). If the theory is to be tested at the $2\%$ level of confidence, deduce $X_h$.

## Part B Testing $p = 0.7$

Now consider the second theory which suggests that $p = 0.7$; assume the null hypothesis is that $p = 0.7$ and the alternative hypothesis is that $p \neq 0.7$. Find the critical region for the test (in this case if $X$ is the number of times the particular result is obtained the critical region has two parts $X \leq X_l$ and $X \geq X_h$). If the theory is to be tested at the $2\%$ level of confidence, deduce $X_l$ and $X_h$.

Find $X_l$.

Find $X_h$

## Part C Conclusions from the tests

What can be concluded at the $2\%$ level from the tests?

- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; there is sufficient evidence to reject the hypothesis that $p = 0.7$ and to support the theory that $p \neq 0.7$.
- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; there is insufficient evidence to reject the hypothesis that $p = 0.7$.
- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; the hypothesis that $p = 0.7$ is correct.
- There is insufficient evidence to reject the hypothesis that $p = 0.6$; there is insufficient evidence to reject the hypothesis that $p = 0.7$.
- There is insufficient evidence to reject the hypothesis that $p = 0.6$; there is sufficient evidence to reject the hypothesis that $p = 0.7$ and to support the theory that $p \neq 0.7$.
- The hypotheses that $p > 0.6$ and that $p = 0.7$ are correct.

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