Complex Numbers: Manipulations 3i

Further A
P P P

The complex number $2+i$ is denoted by z, and the complex conjugate of z is denoted by z^*.

Part A z^2

Express z^2 in the form $x+iy$, where x and y are exact real numbers.

The following symbols may be useful: `i`

Part B $4z-z^2$

Express $4z-z^2$ in the form $x+iy$, where x and y are exact real numbers.

The following symbols may be useful: `i`

Part C zz^*

Express zz^* in the form $x+iy$, where x and y are exact real numbers.

The following symbols may be useful: `i`

Part D $\frac{z+1}{z-1}$

Express $\frac{z+1}{z-1}$ in the form $x+iy$, where x and y are exact real numbers.

The following symbols may be useful: `i`

Adapted with permission from UCLES, A Level, OCR FP1 Specimen paper, Paper 4725, Question 3.

Home Gameboard Maths Complex Numbers: Manipulations 1i

Complex Numbers: Manipulations 1i

Further A

P P P

The complex number z has modulus $2\sqrt{3}$ and argument -$\frac{\pi}{3}$.

Part A z

Find z in the form $z = x + iy$, where x and y are are exact real numbers.

The following symbols may be useful: `i`, `z`

Part B $\frac{1}{(z^{*}-5i)^{2}}$

Find $\frac{1}{(z^{*}-5i)^{2}}$ in the form $x + iy$, where x and y are exact real numbers.

The following symbols may be useful: `i`

Adapted with permission from UCLES, A Level, June 2016, Paper 4725, Question 2.

Gameboard:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Complex Numbers: x+iy and Euler 3i

Further A
P P P

The complex number z satisfies the equation

$$z + 2iz^{*} = 12 + 9i$$

Part A z

Find z in the form $z = x + iy$.

The following symbols may be useful: `i`, `z`

Part B Modulus-Argument

z can also be expressed in the form

$$z = r\left(\cos\theta + i\sin\theta\right)$$

Find r.

The following symbols may be useful: `r`

Find θ to 3 significant figures in radians.

Adapted with permission from UCLES, A Level, Jan 2010, Paper 4725, Question 3.

Gameboard:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Complex Numbers: Equations to Quartics 1ii

Further A

P P P

Part A Square roots

The square roots of the complex number $5 + 12i$ can be expressed in the form $x + iy$.

Give the square root with positive x and positive y.

The following symbols may be useful: `i`

Give the square root with negative x and negative y.

The following symbols may be useful: `i`

Part B $(3 - 2i)^2$

Find $(3 - 2i)^2$ in the form $x + iy$ where x and y are exact.

The following symbols may be useful: `i`

Part C Roots of quartic

The answers to the previous parts can be used to solve the quartic

$$z^4 - 10z^2 + 169 = 0$$

The roots to the quartic can be expressed in the form $x + iy$.

Give the root with positive x and positive y.

The following symbols may be useful: `i`

Give the root with positive x and negative y.

The following symbols may be useful: `i`

Give the root with negative x and positive y.

The following symbols may be useful: `i`

Give the root with negative x and negative y.

The following symbols may be useful: `i`

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 9.

Gameboard:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Applying Complex Numbers 2ii

Further A

P P P

One root of the cubic equation $x^3 + bx^2 + cx - 15 = 0$, where b and c are real, is the complex number $2 + i$.

Part A Complex root

Find the other complex root in the form $x + iy$.

The following symbols may be useful: `i`

Part B Real root

Find the real root.

Part C b

Find b.

The following symbols may be useful: `b`

Part D c

Find c.

The following symbols may be useful: c

Adapted with permission from Sally Waugh.

Gameboard:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Isaac Physics. You work it out.

Complex Numbers: Equations to Quartics 1i

Further A P P P

One root of the quadratic equation $z^2 + az + b = 0$, where a and b are real, is $16 - 30i$.

Part A Other root

Give the other root in the form $x + iy$.

The following symbols may be useful: `i`

Part B a and b

Find the value of a

The following symbols may be useful: `a`

Find the value of b.

The following symbols may be useful: `b`

Part C Quartic

The quartic equation $z^4 + az^2 + b = 0$ has roots in the form $x + iy$.

Give the root with positive x and positive y.

The following symbols may be useful: `i`

Give the root with positive x and negative y.

The following symbols may be useful: `i`

Give the root with negative x and positive y.

The following symbols may be useful: `i`

Give the root with negative x and negative y.

The following symbols may be useful: `i`

Adapted with permission from UCLES, A Level, June 2011, Paper 4725, Question 9.

Gameboard:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Argand Diagrams and Simple Loci 2i

Further A

P P P

The complex number a is denoted by $1+i\sqrt{3}$.

Part A a

Find the value of $|a|$.

Find $\arg a$ in exact form.

The following symbols may be useful: `pi`

Part B Loci

Sketch the loci given by $|z-a|=|a|$ and $\arg(z-a)=\frac{1}{2}\pi$ on a single Argand diagram.

Easier question?

Adapted with permission from UCLES, A Level, June 2010, Paper 4725, Question 5.

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STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Argand Diagrams: Using Loci 2i

Further A

P P P

The loci C_1 and C_2 are given by

$$|z| = |z - 4i|$$

and

$$\arg z = \frac{\pi}{6}$$

respectively.

Part A Loci of C_1 and C_2

Sketch the loci of C_1 and C_2 on a single Argand diagram.

Easier question?

Part B Intersection

Hence find, in the form $x + iy$, the complex number represented by the point of intersection of C_1 and C_2. Give your answer in exact form.

The following symbols may be useful: `i`

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 6.

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STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Argand Diagrams: Solving Inequalities 1ii

Further A P P P

The loci L_1 and L_2 are given by

$$|z| = 2$$

and

$$\arg(z - 3 - i) = \pi$$

respectively.

Part A Equation of L_1

By writing z in the form $x + iy$, express the equation for L_1 in Cartesian form, simplifying your answer as far as possible.

The following symbols may be useful: x, y

Part B Loci

Sketch L_1 and L_2 on a single Argand diagram.

More practice questions?

Part C Inequalities

Indicate, by shading, the region of the Argand diagram for which

$$|z| \leq 2 \text{ and } 0 \leq \arg(z-3-i) \leq \pi$$

More practice questions?

Adapted with permission from UCLES, A Level, June 2018, Paper 4725, Question 4.

Gameboard:

STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams

Argand Diagrams: Solving Inequalities 4ii

Further A

P P P

The loci L_1 and L_2 are given by

$$|z - 3 + 4i| = 5$$

and

$$|z| = |z - 6|$$

respectively.

Part A Equation of L_1

Give the equation of L_1 in the form $(x-a)^2 + (y-b)^2 = c^2$.

The following symbols may be useful: x, y

Part B Inequalities

Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 + 4i| \leq 5 \text{ and } |z| \geq |z - 6|$$

More practice questions?

Adapted with permission from UCLES, A Level, June 2018, Paper 4725, Question 4.