Continuous Random Variables 1

Further A

A continuous random variable X_1 has a probability density function given by

$$f(x) = \begin{cases} kx & 0 \leq x \leq 2, \\ 0 & \text{otherwise}, \end{cases}$$

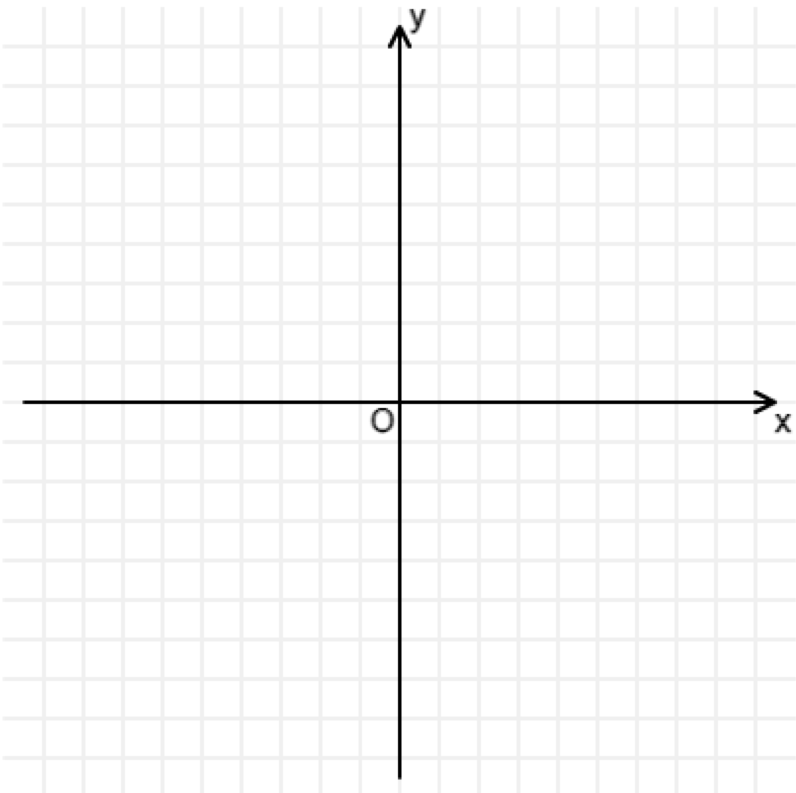
where k is constant.

Part A Find k

Find the value of k.

Part B Sketch $y = f(x)$

Sketch the graph of $y = f(x)$, sketch only where $f(x)$ is nonzero.



Part C Find $\mathrm{E}(X_1)$

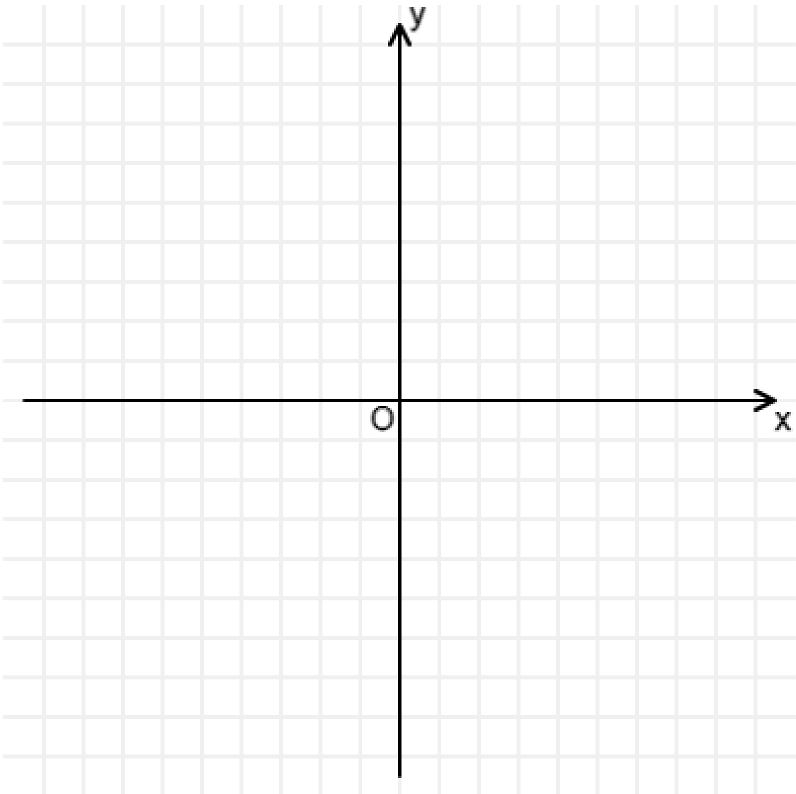
Find the expectation of X_1.

Part D Find $\mathrm{Var}(X_1)$

Find the variance of X_1.

Part E Sketch $y = f(x-1)$

Sketch the graph of $y = f(x-1)$, sketch only where $f(x-1)$ is nonzero.



Part F Find $\mathrm{E}(X_2)$

The continuous random variable X_2 has probability distribution function $f(x-1)$ for all x.

Find the expectation of X_2.

Part G Find $\mathrm{Var}(X_2)$

Find the variance of X_2.

Adapted with permission from UCLES, A Level, January 2008, Paper 4733, Question 7

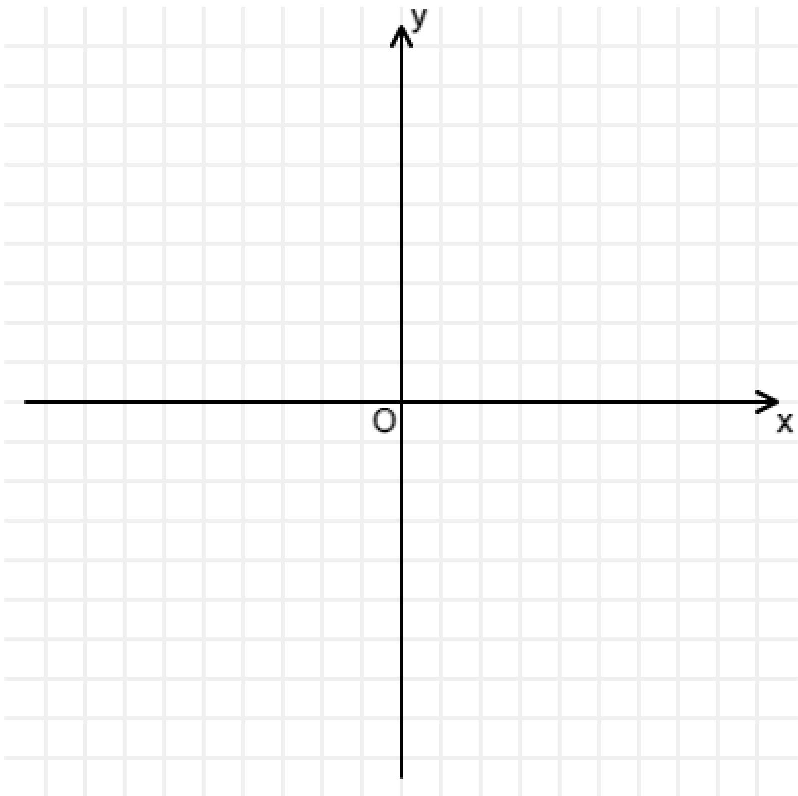
Continuous Random Variables 3

Further A

The continuous random variable $T \sim \mathrm{U}[5.0, 11.0]$.

Part A Sketch the probability density function

Sketch the graph of the probability density function of T. Sketch only the nonzero parts of the probability density function.



Part B Find $\mathrm{E}(T)$

Find the expectation of T.

Part C Find $\mathrm{Var}(T)$

Find the variance of T.

Part D Sample of T

A random sample of 48 observations of T is obtained. Find the probability that the mean of the sample is greater than 8.3. Give your answer to 3 significant figures.

Part E Approximation

Explain whether or not the probability found in part D is an approximation.

- Since T is a continuous random variable, the central limit theorem will give an exact distribution for the distribution of sample means.
- Since the distribution of T is not normal, the central limit theorem only gives an approximate distribution for the distribution of sample means.
- Since the distribution of T is not normal, we can't use the central limit theorem, so the distribution of sample means is only approximately normally distributed.
- Since the distribution of T is not normal, we must use the central limit theorem, which always gives an exact distribution for the distribution of sample means.

Adapted with permission from UCLES, A Level, January 2010, Paper 4733, Question 7

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Continuous Random Variables 4

Further A

The cumulative distribution function of the continuous random variable, X, is given by

$$F(x) = \begin{cases} 0 & x < 1, \\ \frac{2x-2}{x+3} & 1 \leq x \leq 5, \\ 1 & x > 5. \end{cases}$$

Given that $Y = 2X - 3$, find the probability density function of Y.

The following symbols may be useful: y

Adapted with permission from UCLES, A Level, January 2018, Paper 4734/01, Question 3

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Continuous Random Variables 5

Further A

The continuous random variable X has probability density function

$$f(x)=\begin{cases}k\cos x & 0\leqslant x<\frac{\pi}{4},\\ k\sin x & \frac{\pi}{4}\leqslant x\leqslant\frac{\pi}{2},\\ 0 & \text{otherwise.}\end{cases}$$

Part A Value of k

Find the exact value of k.

Part B $\mathrm{P}(X\leq 1)$

Find the value of $\mathrm{P}(X\leq 1)$. Give your answer to 3 s.f.

Part C Upper quartile

Find the upper quartile of X. Give your answer to 3 s.f.

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Continuous Random Variables 2

Further A
P P P

The continuous random variable X has the probability density function

$$f(x) = \begin{cases} \dfrac{1}{2\sqrt{x}} & 1 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

Part A Expectation of X

Find $\mathrm{E}(X)$. Give your answer as a fraction.

Part B Median of X

Find the median of X. Give your answer as a fraction.

Part C Variance of Y

The continuous random variable Y has the probability density function

$$g(y) = \begin{cases} \dfrac{1.5}{y^{2.5}} & y \geqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Given $\mathrm{E}(Y) = 3$, what can you say about $\mathrm{Var}(Y)$? Fill in the gaps below.

We know that $\mathrm{Var}(Y) =$ ____ ____ $\mathrm{d}y - (\mathrm{E}(Y))^2$.

This gives $\mathrm{Var}(Y) = 3[$ ____ ____ $- 9$.

By substituting the limits, we find that $\mathrm{Var}(Y)$ is ____.

Items:

zero | infinite | negative | imaginary | $\int_{-\infty}^{\infty}$ | $]_{-\infty}^{\infty}$ | $\int_{0}^{\infty}$ | $]_{0}^{\infty}$ | $\int_{1}^{\infty}$ | $]_{1}^{\infty}$ | $\int_{0}^{1}$ | $]_{0}^{1}$ | $y\,g(y)$ | $y^2\,g(y)$ | $g(y)$ | $y^{0.5}$ | $y^{-0.5}$

Adapted with permission from UCLES, A Level, January 2012, Paper 4733, Question 7

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Combining Variables: Laminate

Further A

A laminate consists of 4 layers of material C and 3 layers of material D. The thickness of a layer of material C has a normal distribution with mean $1\,\mathrm{mm}$ and standard deviation $0.1\,\mathrm{mm}$, and the thickness of a layer of material D has a normal distribution with mean $8\,\mathrm{mm}$ and standard deviation $0.2\,\mathrm{mm}$. The layers are independent of one another.

Part A Mean

Find the mean of the total thickness of the laminate.

Part B Variance

Find the variance of the total thickness of the laminate.

Part C 1% of the laminates

What total thickness is exceeded by 1% of the laminates? Give your answer to 3 s.f.

Used with permission from UCLES, A Level, June 2015, Paper 4734/01, Question 1

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Combining Normal Variables

Further A

P P P

The independent random variables X and Y have distributions $\mathrm{N}(30, \sigma^2)$ and $\mathrm{N}(20, \sigma^2)$ respectively. The random variable $aX + bY$, where a and b are constants, has the distribution $\mathrm{N}(410, 130\sigma^2)$.

Part A Values of a and b

It is given that a and b are integers.

Find the value of a.

Find the value of b.

Part B Value of σ^2

Given that P$(X > Y) = 0.966$, find σ^2. Give your answer to 3 s.f.

Used with permission from UCLES, A Level, June 2016, Paper 4734/01, Question 5