Vectors: Diagrams and Proof 2ii

A Level

Part A Resultant vector

Find the resultant of the vectors $4\underline{\boldsymbol{i}} - \underline{\boldsymbol{j}}$ and $-2\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}}$.

The following symbols may be useful: `i, j`

Part B $\vec{MN}$

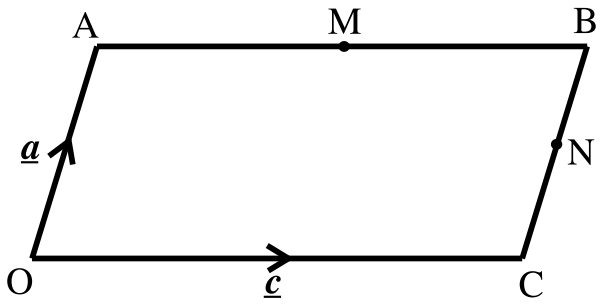

Figure 1: A parallelogram $OABC$.

In **Figure 1**, $OABC$ is a parallelogram. M is the midpoint of $\vec{AB}$. N is the midpoint of $\vec{BC}$. $\vec{OA} = \underline{\boldsymbol{a}}$ and $\vec{OC} = \underline{\boldsymbol{c}}$.

Find $\vec{MN}$ in terms of $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{c}}$.

The following symbols may be useful: `a, c`

Adapted with permission from UCLES, A Level, 2012

Physics. You work it out.

Vectors: Diagrams and Proof 1ii

A Level

Part A $2\underline{s} - \underline{t}$

Given that $\underline{s} = 3\underline{i} + 4\underline{j}$ and $\underline{t} = 6\underline{i} - \underline{j}$, find $2\underline{s} - \underline{t}$.

The following symbols may be useful: `i, j`

Part B In terms of $\underline{p}$

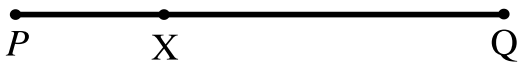

Figure 1: Three points P, X and Q.

Figure 1 shows three points P, X and Q such that $\vec{XQ} = 3\vec{PX}$.

Given that $\vec{PX} = \underline{p}$, find $\vec{XQ}$ in terms of $\underline{p}$

The following symbols may be useful: `p`

Given that $\vec{PX} = \underline{p}$, find $\vec{QP}$ in terms of $\underline{p}$

The following symbols may be useful: `p`

Part C Proving $AMCN$ is a parallelogram

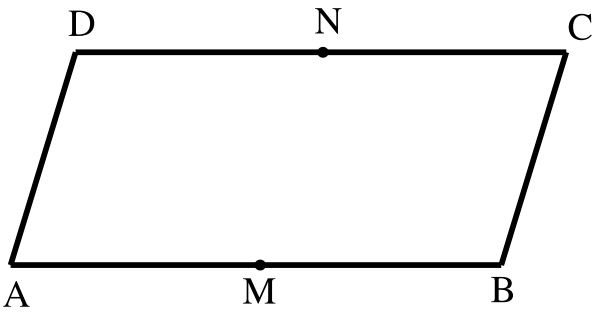

Figure 2: $ABCD$ is a parallelogram

In **Figure 2** $ABCD$ is a parallelogram. M and N are the mid-points of AB and DC. $\vec{AB} = \underline{\boldsymbol{a}}$ and $\vec{AD} = \underline{\boldsymbol{b}}$. Use a vector method to prove that $AMCN$ is also a parallelogram.

Choose four items from the left and put them into order on the right to create a proof.

Available items

1. A parallelogram has two pairs of sides which are parallel and of equal length.

1. A parallelogram has two pairs of sides which are parallel. All sides of a parallelogram are the same length.

2. $\vec{AM} = \vec{NC} = \frac{1}{2}\underline{\boldsymbol{a}}$. Therefore $\vec{AM}$ is parallel to $\vec{NC}$ and has the same length.

2. $\vec{MB} = \vec{NC} = \frac{1}{2}\underline{\boldsymbol{a}}$. Therefore $\vec{MB}$ is parallel to $\vec{NC}$ and has the same length.

3. $\vec{AD} = \vec{BC} = \underline{\boldsymbol{b}}$. Therefore $\vec{AD}$ is parallel to $\vec{BC}$ and has the same length.

3. $\vec{AN} = \vec{MC} = \frac{1}{2}\underline{\boldsymbol{a}} + \underline{\boldsymbol{b}}$. Therefore $\vec{AN}$ is parallel to $\vec{MC}$ and has the same length.

4. $AMCN$ has two pairs of sides which are parallel and of equal length. Hence, $AMCN$ is a parallelogram.

4. $AMCN$ has four sides which are parallel and of equal length. Hence, $AMCN$ is a parallelogram.

Adapted with permission from UCLES, A Level, 2000

Gameboard:
STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D

Vectors: Diagrams and Proof 2i

A Level

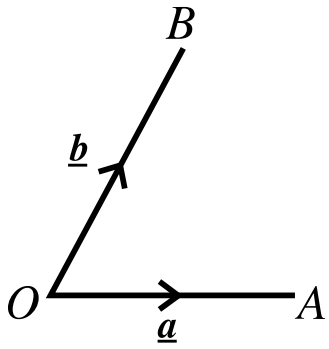

Figure 1: Points A and B and their position vectors with respect to the origin O.

In **Figure 1**, the points A and B have position vectors $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ with respect to the origin O.

Part A Sketch

Make a sketch of the diagram, and mark on the points C, D and E such that $\vec{OC} = 2\underline{\boldsymbol{a}}$, $\vec{OD} = 2\underline{\boldsymbol{a}} + \underline{\boldsymbol{b}}$ and $\vec{OE} = \frac{1}{3}\vec{OD}$.

Which of the sketches below correctly shows this information?

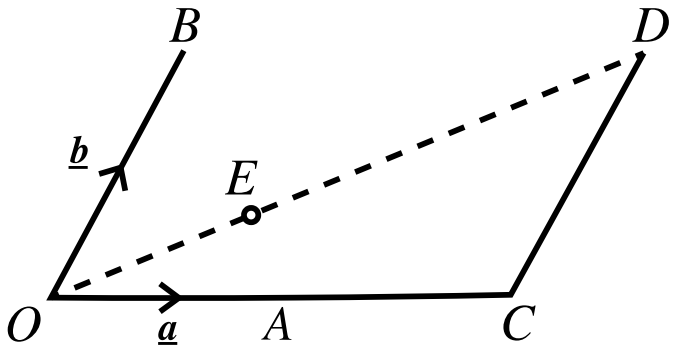

Figure 2: Option A

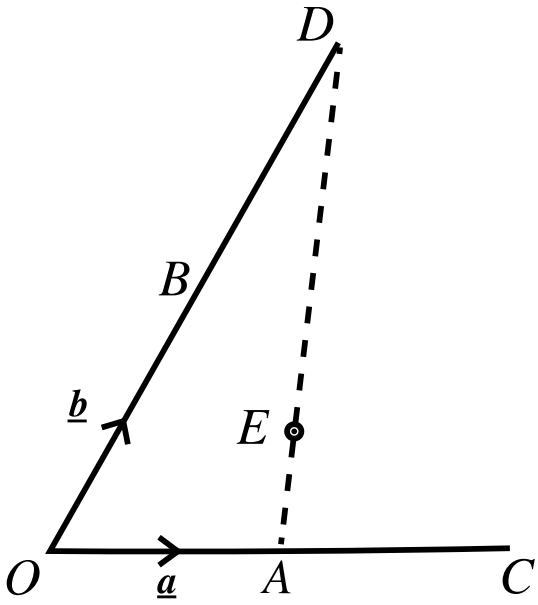

Figure 3: Option B

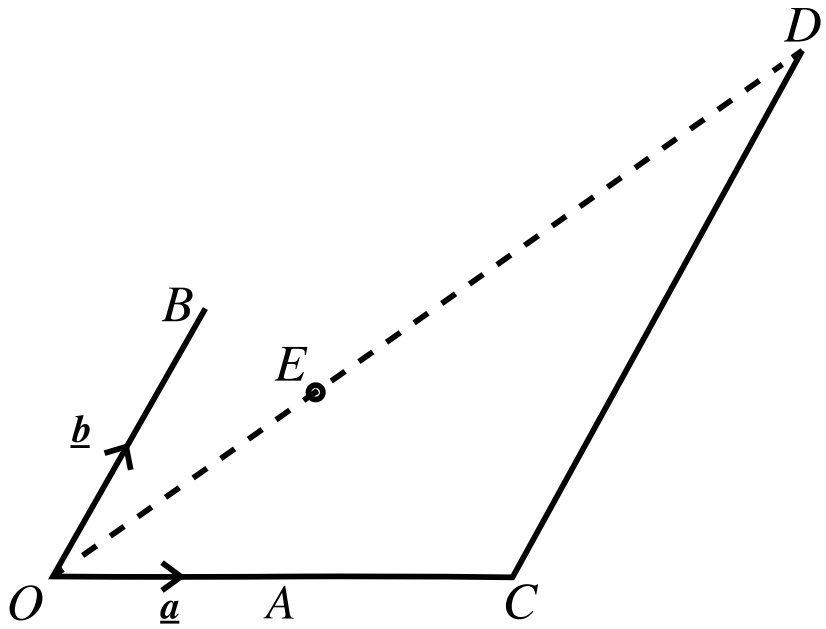

Figure 4: Option C

- Option A
- Option B
- Option C

Part B Position vector of E

Give the position vector of E with respect to A.

The following symbols may be useful: `a, b`

Part C Proof

Hence prove that E lies on the line joining A and B.

Choose three items from the choices on the left and put them into order on the right to create a proof.

Available items

1. Point E lies on the line joining A and B if the vector $\vec{OE}$ is a scalar multiple of the vector $\vec{OB}$, i.e. $\vec{OE} = k\vec{OB}$.

1. Point E lies on the line joining A and B if the vector $\vec{AE}$ is a scalar multiple of the vector $\vec{AB}$, i.e. $\vec{AE} = k\vec{AB}$.

2. $\vec{AE} = \frac{1}{3}(\underline{\boldsymbol{b}} - \underline{\boldsymbol{a}})$ and $\vec{AB} = \underline{\boldsymbol{b}} - \underline{\boldsymbol{a}}$. Therefore, $\vec{AE} = \frac{1}{3}\vec{AB}$.

2. $\vec{AE} = \underline{\boldsymbol{b}} - \underline{\boldsymbol{a}}$ and $\vec{AB} = \frac{1}{3}(\underline{\boldsymbol{b}} - \underline{\boldsymbol{a}})$. Therefore, $\vec{AE} = 3\vec{AB}$.

3. $\vec{OE}$ is a scalar multiple of $\vec{OD}$. Hence, E lies on the line joining O and D.

3. $\vec{AE}$ is a scalar multiple of $\vec{AB}$. Hence, E lies on the line joining A and B.

Adapted with permission from UCLES, A Level, Specimen Paper

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Physics. You work it out.

3D Vectors 2ii

A Level

The points A and B have position vectors $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ relative to an origin O, where $\underline{\boldsymbol{a}} = 4\underline{\boldsymbol{i}} + 3\underline{\boldsymbol{j}} - 2\underline{\boldsymbol{k}}$ and $\underline{\boldsymbol{b}} = -7\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}} + 4\underline{\boldsymbol{k}}$.

Part A Length AB

Find the length of $\vec{AB}$. Give your answer as an exact surd.

Part B Unit vector

Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. Give your answer using ijk notation.

The following symbols may be useful: `i, j, k`

Adapted with permission from UCLES, A Level, January 2007 & June 2011, OCR C4

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Physics. You work it out.

3D Vectors 1ii

A Level

$ABCD$ is a parallelogram. The position vectors of A, B and C are given respectively by

$$\underline{\boldsymbol{a}} = 2\underline{\boldsymbol{i}} + \underline{\boldsymbol{j}} + 3\underline{\boldsymbol{k}} \qquad \underline{\boldsymbol{b}} = 3\underline{\boldsymbol{i}} - 2\underline{\boldsymbol{j}} \qquad \underline{\boldsymbol{c}} = \underline{\boldsymbol{i}} - \underline{\boldsymbol{j}} - 2\underline{\boldsymbol{k}}$$

Part A Position of D

Find the position vector of D. Give your answer using ijk notation.

The following symbols may be useful: `i, j, k`

Part B Unit vector

Find the unit vector in the direction $\vec{OD}$ where O is the fixed origin. Give your answer using ijk notation.

The following symbols may be useful: `i, j, k`

Adapted with permission from UCLES, A Level, June 2005, OCR C4, Question 5

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Physics. You work it out.

3D Vectors 2i

A Level

$ABCD$ is a quadrilateral. You are given four pieces of information:

- Relative to a fixed origin O, the position vector of A is $2\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}} + 8\underline{\boldsymbol{k}}$.
- Relative to a fixed origin O, the position vector of B is $5\underline{\boldsymbol{i}} + 9\underline{\boldsymbol{j}} + 8\underline{\boldsymbol{k}}$.
- The vector $\vec{BC} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$.
- The vector $\vec{BD} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$.

Part A Finding AB

Find the vector $\vec{AB}$. Give your answer in the form `(x,y,z)` with the commas and without the spaces.

Part B Finding CD

Find the vector $\vec{CD}$. Give your answer in the form `(x,y,z)` with the commas and without the spaces.

Part C Finding AD

Find the vector $\vec{AD}$. Give your answer in the form `(x,y,z)` with the commas and without the spaces.

Part D Type of quadrilateral

The shape $ABCD$ lies in a plane. What type of quadrilateral is $ABCD$?

- A square
- A kite
- A rectangle
- An irregular quadrilateral
- A parallelogram
- A rhombus
- A trapezium

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Physics. You work it out.

Vectors in 3D

A Level

Given that $\underline{\boldsymbol{a}} = 6\underline{\boldsymbol{i}} + (p-10)\underline{\boldsymbol{j}} + (3p-5)\underline{\boldsymbol{k}}$, and that $|\underline{\boldsymbol{a}}| = 11$, find the possible values of p.

Part A Smaller value of p

Enter the smaller value of p:

Part B Larger value of p

Enter the larger value of p:

Adapted for Isaac Physics from NST IA Biology preparation work

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Physics. You work it out.

Angles Between a 3D Vector and the Axes

A Level

Find the angles between the vector $\underline{\boldsymbol{i}} + 2\underline{\boldsymbol{j}} + 3\underline{\boldsymbol{k}}$ and the x, y and z coordinate axes.

Part A Angle with x axis

What is the angle in degrees between the vector and the x axis? Give your answer to 3 s.f.

Part B Angle with y axis

What is the angle in degrees between the vector and the y axis? Give your answer to 3 s.f.

Part C Angle with z axis

What is the angle in degrees between the vector and the z axis? Give your answer to 3 s.f.

Adapted for Isaac Physics from NST IA Biology preparation work

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Manipulating Vectors in 3D

Pre-Uni Maths for Science I1.10

A Level

C C C

A vector $\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$ has a length of 4 units. For the following three cases, find u_x, u_y and u_z and answer the questions.

Part A Case 1

The vector lies in the (x, y)-plane, makes an angle of 30° with the x-direction and u_y is positive.

What is u_x? Give your answer to 3.s.f

Part B Case 2

The vector has $u_x = u_y = 2$ and u_z is negative.

What is u_z? Give your answer to 3.s.f

Part C Case 3

The vector is such that $u_z = 1$, $u_y = 2u_x$ and u_y is positive.

What is u_y? Give your answer to 3.s.f

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3D Vectors 3ii

A Level

Two points A and B have position vectors $3\underline{\boldsymbol{i}} - \underline{\boldsymbol{j}} + 2\underline{\boldsymbol{k}}$ and $2\underline{\boldsymbol{j}} + 3\underline{\boldsymbol{k}}$ respectively.

Part A Vector AB

Find the vector $\vec{AB}$. Give your answer using ijk notation.

The following symbols may be useful: `i, j, k`

Hence find the length of $\vec{AB}$. Give your answer as an exact surd.

Part B Intersection

Show that the line through A and B does not intersect the line through the origin parallel to the vector i.

One way to prove this is to use proof by contradiction. Fill in the blanks to complete the proof below.

Opening statement:

The line through the origin parallel to the vector i is the x-axis. On the x-axis, $y = z =$ [ ]. If the line through A and B intersects the x-axis, then there is a value of λ such that

$$\vec{OA} + \lambda\vec{AB} = \begin{pmatrix}\mu\\0\\0\end{pmatrix}$$

where μ is the value of x where the line intersects the x-axis.

Calculations:

Putting in expressions for $\vec{OA}$ and $\vec{AB}$,

$$\begin{pmatrix}3\\-1\\2\end{pmatrix} + \lambda \boxed{} = \begin{pmatrix}\mu\\0\\0\end{pmatrix}$$

This gives three equations, one for each of the x, y and z components:

$$3 - 3\lambda = \mu, \quad -1 + 3\lambda = 0 \quad \text{and} \quad 2 + \lambda = 0$$

The second of these equations re-arranges to $\lambda =$ [ ], but the third equation rearranges to $\lambda =$ [ ]. Hence, these equations are inconsistent and we have reached a contradiction.

Conclusion:

There is no point on the line through A and B for which y and z are both zero, so this line does not intersect the x-axis, and hence this line does not intersect the line through the origin parallel to the vector i.

Items:

-3 -2 -1 0 1 2 3 $\frac{1}{3}$ $\frac{2}{3}$ $\begin{pmatrix}-3\\3\\1\end{pmatrix}$ $\begin{pmatrix}3\\-3\\1\end{pmatrix}$