



Physics. *You work it out.*

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Vectors: Diagrams and Proof 2ii

A Level
P P P

Part A Resultant vector

Find the resultant of the vectors $4\mathbf{i} - \mathbf{j}$ and $-2\mathbf{i} + 5\mathbf{j}$.

The following symbols may be useful: \mathbf{i} , \mathbf{j}

Part B \vec{MN}

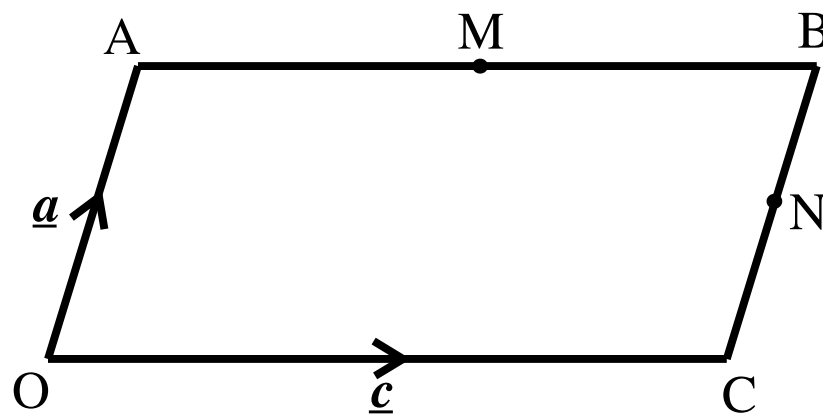


Figure 1: A parallelogram $OABC$.

In **Figure 1**, $OABC$ is a parallelogram. M is the midpoint of \vec{AB} . N is the midpoint of \vec{BC} . $\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$.

Find \vec{MN} in terms of \mathbf{a} and \mathbf{c} .

The following symbols may be useful: \mathbf{a} , \mathbf{c}

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Gameboard:

Pure Maths Practice: Vectors - Diagrams and Proof

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Vectors: Diagrams and Proof 1ii

A Level


Part A $2\underline{s} - \underline{t}$

Given that $\underline{s} = 3\underline{i} + 4\underline{j}$ and $\underline{t} = 6\underline{i} - \underline{j}$, find $2\underline{s} - \underline{t}$.

The following symbols may be useful: \underline{i} , \underline{j}

Part B In terms of \underline{p}

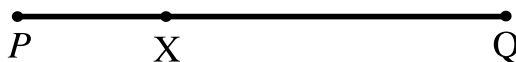


Figure 1: Three points P , X and Q .

Figure 1 shows three points P , X and Q such that $\vec{XQ} = 3\vec{PX}$.

Given that $\vec{PX} = \underline{p}$, find \vec{XQ} in terms of \underline{p}

The following symbols may be useful: \underline{p}

Given that $\vec{PX} = \underline{p}$, find \vec{QP} in terms of \underline{p}

The following symbols may be useful: \underline{p}



Part C Proving $AMCN$ is a parallelogram

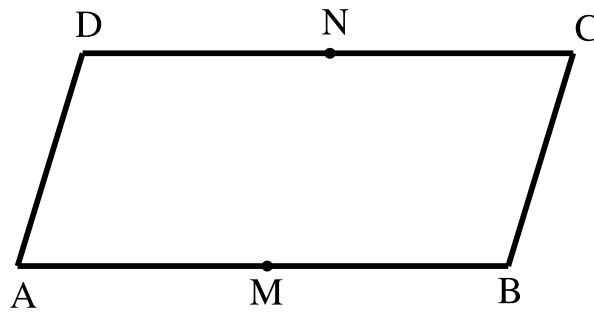


Figure 2: $ABCD$ is a parallelogram

In Figure 2 $ABCD$ is a parallelogram. M and N are the mid-points of AB and DC . $\vec{AB} = \underline{a}$ and $\vec{AD} = \underline{b}$. Use a vector method to prove that $AMCN$ is also a parallelogram.

Choose four items from the left and put them into order on the right to create a proof.

Available items

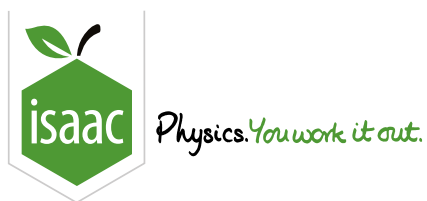
1. A parallelogram has two pairs of sides which are parallel and of equal length.
1. A parallelogram has two pairs of sides which are parallel. All sides of a parallelogram are the same length.
2. $\vec{MB} = \vec{NC} = \frac{1}{2}\underline{a}$. Therefore \vec{MB} is parallel to \vec{NC} and has the same length.
2. $\vec{AM} = \vec{NC} = \frac{1}{2}\underline{a}$. Therefore \vec{AM} is parallel to \vec{NC} and has the same length.
3. $\vec{AN} = \vec{MC} = \frac{1}{2}\underline{a} + \underline{b}$. Therefore \vec{AN} is parallel to \vec{MC} and has the same length.
3. $\vec{AD} = \vec{BC} = \underline{b}$. Therefore \vec{AD} is parallel to \vec{BC} and has the same length.
4. $AMCN$ has two pairs of sides which are parallel and of equal length. Hence, $AMCN$ is a parallelogram.
4. $AMCN$ has four sides which are parallel and of equal length. Hence, $AMCN$ is a parallelogram.

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Gameboard:

STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D

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Vectors: Diagrams and Proof 2i

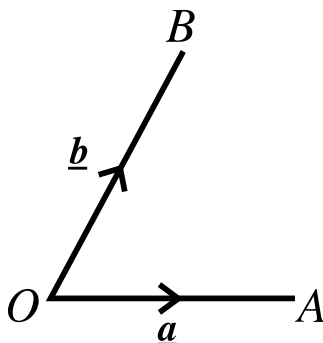


Figure 1: Points A and B and their position vectors with respect to the origin O .

In **Figure 1**, the points A and B have position vectors \underline{a} and \underline{b} with respect to the origin O .

Part A Sketch

Make a sketch of the diagram, and mark on the points C , D and E such that $\vec{OC} = 2\vec{a}$, $\vec{OD} = 2\vec{a} + \vec{b}$ and $\vec{OE} = \frac{1}{3}\vec{OD}$.

Which of the sketches below correctly shows this information?

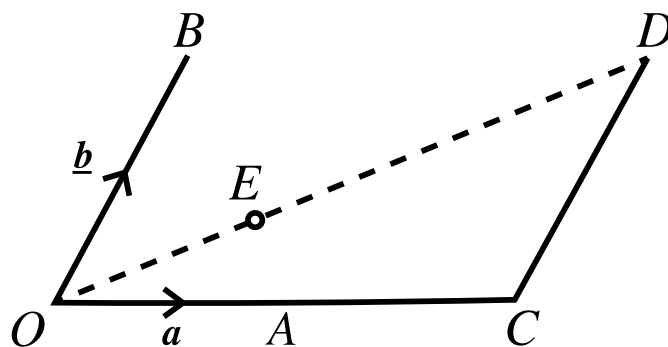


Figure 2: Option A

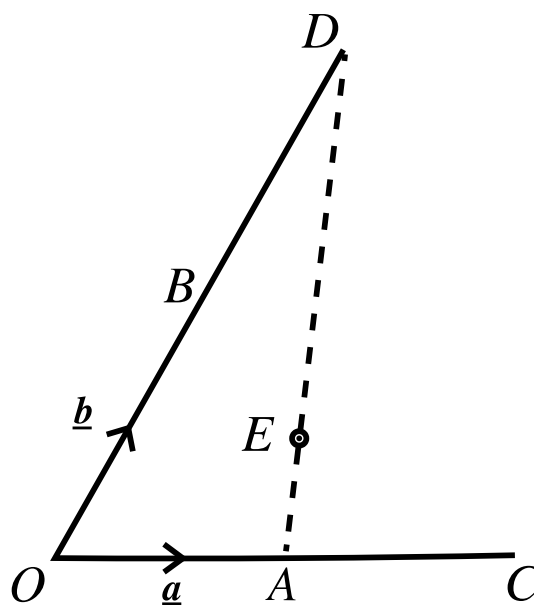


Figure 3: Option B

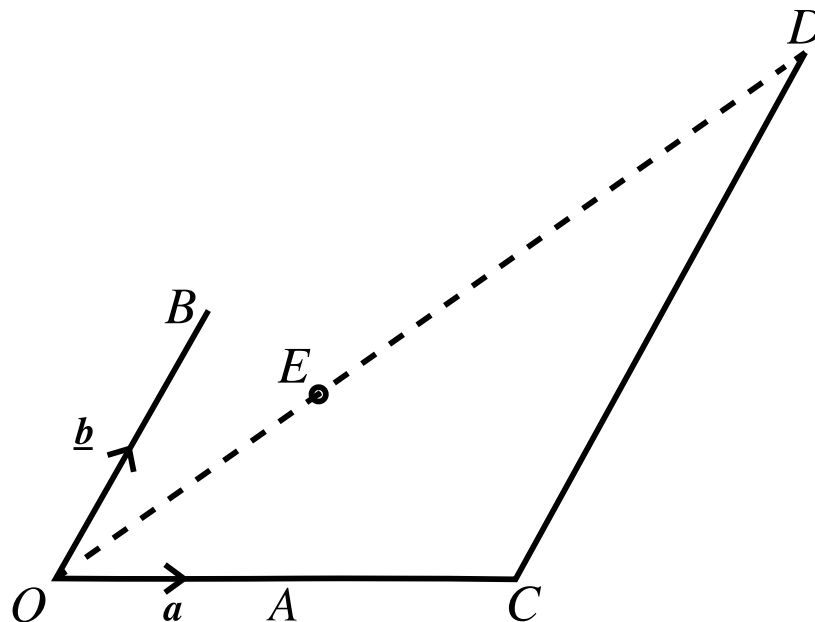


Figure 4: Option C

- ☐ Option A
- ☐ Option B
- ☐ Option C

Part B Position vector of E

Give the position vector of E with respect to A .

The following symbols may be useful: a , b

Part C Proof

Hence prove that E lies on the line joining A and B .

Choose three items from the choices on the left and put them into order on the right to create a proof.

Available items

1. Point E lies on the line joining A and B if the vector \vec{OE} is a scalar multiple of the vector \vec{OB} , i.e. $\vec{OE} = k\vec{OB}$.

1. Point E lies on the line joining A and B if the vector \vec{AE} is a scalar multiple of the vector \vec{AB} , i.e. $\vec{AE} = k\vec{AB}$.

2. $\vec{AE} = \frac{1}{3}(\underline{b} - \underline{a})$ and $\vec{AB} = \underline{b} - \underline{a}$. Therefore, $\vec{AE} = \frac{1}{3}\vec{AB}$.

2. $\vec{AE} = \underline{b} - \underline{a}$ and $\vec{AB} = \frac{1}{3}(\underline{b} - \underline{a})$. Therefore, $\vec{AE} = 3\vec{AB}$.

3. \vec{OE} is a scalar multiple of \vec{OD} . Hence, E lies on the line joining O and D .

3. \vec{AE} is a scalar multiple of \vec{AB} . Hence, E lies on the line joining A and B .

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3D Vectors 2ii

A Level



The points A and B have position vectors \underline{a} and \underline{b} relative to an origin O , where $\underline{a} = 4\underline{i} + 3\underline{j} - 2\underline{k}$ and $\underline{b} = -7\underline{i} + 5\underline{j} + 4\underline{k}$.

Part A Length AB

Find the length of \vec{AB} . Give your answer as an exact surd.

Part B Unit vector

Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. Give your answer using ijk notation.

The following symbols may be useful: \underline{i} , \underline{j} , \underline{k}

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Gameboard:

Mechanics Practice: 3D Vectors

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3D Vectors 1ii

A Level



$ABCD$ is a parallelogram. The position vectors of A , B and C are given respectively by

$$\underline{a} = 2\underline{i} + \underline{j} + 3\underline{k} \qquad \underline{b} = 3\underline{i} - 2\underline{j} \qquad \underline{c} = \underline{i} - \underline{j} - 2\underline{k}$$

Part A Position of D

Find the position vector of D . Give your answer using ijk notation.

The following symbols may be useful: \underline{i} , \underline{j} , \underline{k}

Part B Unit vector

Find the unit vector in the direction \vec{OD} where O is the fixed origin. Give your answer using ijk notation.

The following symbols may be useful: \underline{i} , \underline{j} , \underline{k}

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3D Vectors 2i

A Level



$ABCD$ is a quadrilateral. You are given four pieces of information:

- Relative to a fixed origin O , the position vector of A is $2\underline{i} + 5\underline{j} + 8\underline{k}$.
- Relative to a fixed origin O , the position vector of B is $5\underline{i} + 9\underline{j} + 8\underline{k}$.
- The vector $\vec{BC} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$.
- The vector $\vec{BD} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$.

Part A Finding AB

Find the vector \vec{AB} . Give your answer in the form (x,y,z) with the commas and without the spaces.

Part B Finding CD

Find the vector \vec{CD} . Give your answer in the form (x,y,z) with the commas and without the spaces.

Part C Finding AD

Find the vector \vec{AD} . Give your answer in the form (x,y,z) with the commas and without the spaces.

Part D Type of quadrilateral

The shape $ABCD$ lies in a plane. What type of quadrilateral is $ABCD$?

- ☐ An irregular quadrilateral
 - ☐ A square
 - ☐ A rhombus
 - ☐ A rectangle
 - ☐ A kite
 - ☐ A trapezium
 - ☐ A parallelogram
-

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Vectors in 3D

A Level



Given that $\underline{a} = 6\underline{i} + (p - 10)\underline{j} + (3p - 5)\underline{k}$, and that $|\underline{a}| = 11$, find the possible values of p .

Part A Smaller value of p

Enter the smaller value of p :

Part B Larger value of p

Enter the larger value of p :

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Angles Between a 3D Vector and the Axes

A Level



Find the angles between the vector $\underline{i} + 2\underline{j} + 3\underline{k}$ and the x , y and z coordinate axes.

Part A Angle with x axis

What is the angle in degrees between the vector and the x axis? Give your answer to 3s.f.

Part B Angle with y axis

What is the angle in degrees between the vector and the y axis? Give your answer to 3s.f.

Part C Angle with z axis

What is the angle in degrees between the vector and the z axis? Give your answer to 3s.f.

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Manipulating Vectors in 3D

A Level



Pre-Uni Maths for Science I1.10

A vector $\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$ has a length of 4 units. For the following three cases, find u_x , u_y and u_z and answer the questions.

Part A Case 1

The vector lies in the (x, y) -plane, makes an angle of 30° with the x -direction and u_y is positive.

What is u_x ? Give your answer to 3.s.f

Part B Case 2

The vector has $u_x = u_y = 2$ and u_z is negative.

What is u_z ? Give your answer to 3.s.f

Part C Case 3

The vector is such that $u_z = 1$, $u_y = 2u_x$ and u_y is positive.

What is u_y ? Give your answer to 3.s.f

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3D Vectors 3ii

A Level



Two points A and B have position vectors $3\underline{i} - \underline{j} + 2\underline{k}$ and $2\underline{j} + 3\underline{k}$ respectively.

Part A Vector AB

Find the vector \vec{AB} . Give your answer using ijk notation.

The following symbols may be useful: \underline{i} , \underline{j} , \underline{k}

Hence find the length of \vec{AB} . Give your answer as an exact surd.

Part B Intersection

Show that the line through A and B does not intersect the line through the origin parallel to the vector i .

One way to prove this is to use proof by contradiction. Fill in the blanks to complete the proof below.

Opening statement:

The line through the origin parallel to the vector i is the x -axis. On the x -axis, $y = z =$. If the line through A and B intersects the x -axis, then there is a value of λ such that

$$\vec{OA} + \lambda \vec{AB} = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

where μ is the value of x where the line intersects the x -axis.

Calculations:

Putting in expressions for \vec{OA} and \vec{AB} ,

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \lambda \text{ } = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

This gives three equations, one for each of the x , y and z components:

$$3 - 3\lambda = \mu, \quad -1 + 3\lambda = 0 \quad \text{and} \quad 2 + \lambda = 0$$

The second of these equations re-arranges to $\lambda =$, but the third equation rearranges to $\lambda =$. Hence, these equations are inconsistent and we have reached a contradiction.

Conclusion:

There is no point on the line through A and B for which y and z are both zero, so this line does not intersect the x -axis, and hence this line does not intersect the line through the origin parallel to the vector i .

Items:

-3

-2

-1

0

1

2

3

$\frac{1}{3}$

$\frac{2}{3}$

$\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$