Radians-problems involving area 5ii

A Level

Figure 1 shows a sector OAB of a circle, centre O and radius $8\,\text{cm}$. The angle AOB is $46\,^\circ$.

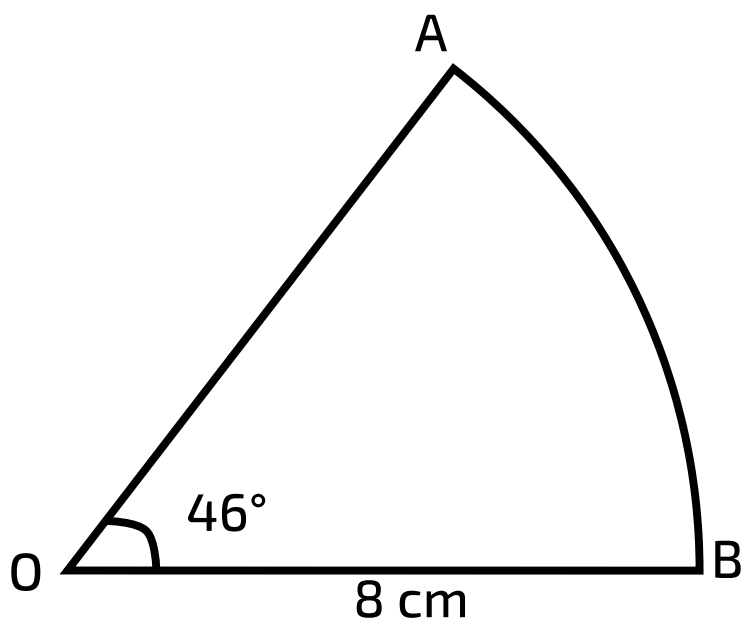

Figure 1: Sector AOB.

Part A Convert angle to radians

Express $46\,^\circ$ in radians, correct to 3 significant figures.

Part B Arc length

Find the length of the arc AB.

Part C Area of sector

Find the area of the sector OAB.

Radians-problems involving area 2ii

A Level

Figure 1 shows two congruent triangles, BCD and BAE, where ABC is a straight line. In triangle BCD, $BD = 8\,\text{cm}$, $CD = 11\,\text{cm}$ and angle $CBD = 65\,°$. The points E and D are joined by an arc of a circle with centre B and radius $8\,\text{cm}$.

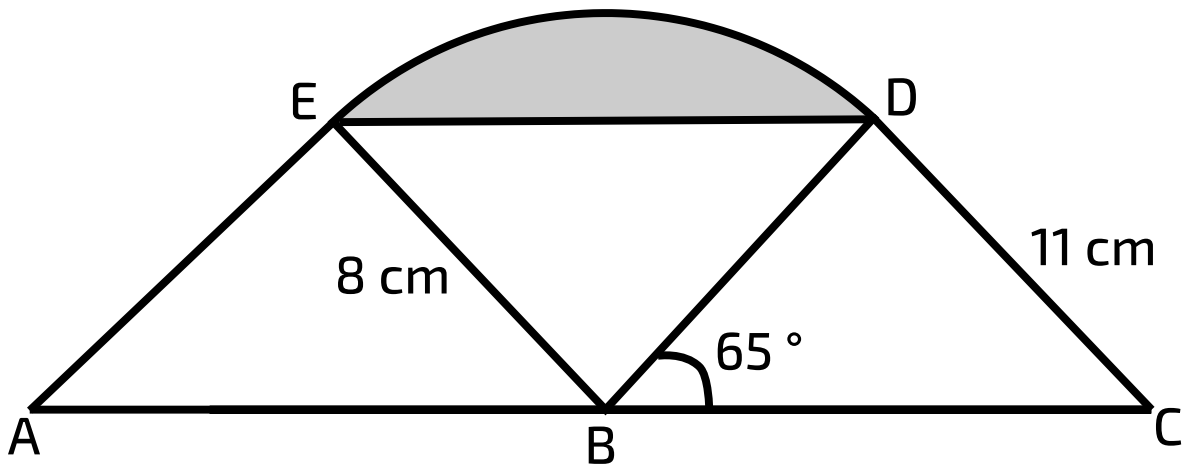

Figure 1: Diagram of the triangles.

Part A Angle BCD

Find angle BCD. Give your answer in radians, correct to 3 significant figures.

Part B Angle EBD

Find the angle EBD, giving your answer in radians correct to 3 significant figures.

Part C Area of shaded segment

Hence find the area (in cm^2) of the shaded segment bounded by the chord ED and the arc ED, giving your answer correct to 3 significant figures.

Gameboard:
STEM SMART Single Maths 29 - Radians & Small Angle Approximations

Radians-problems involving area 1ii

A Level

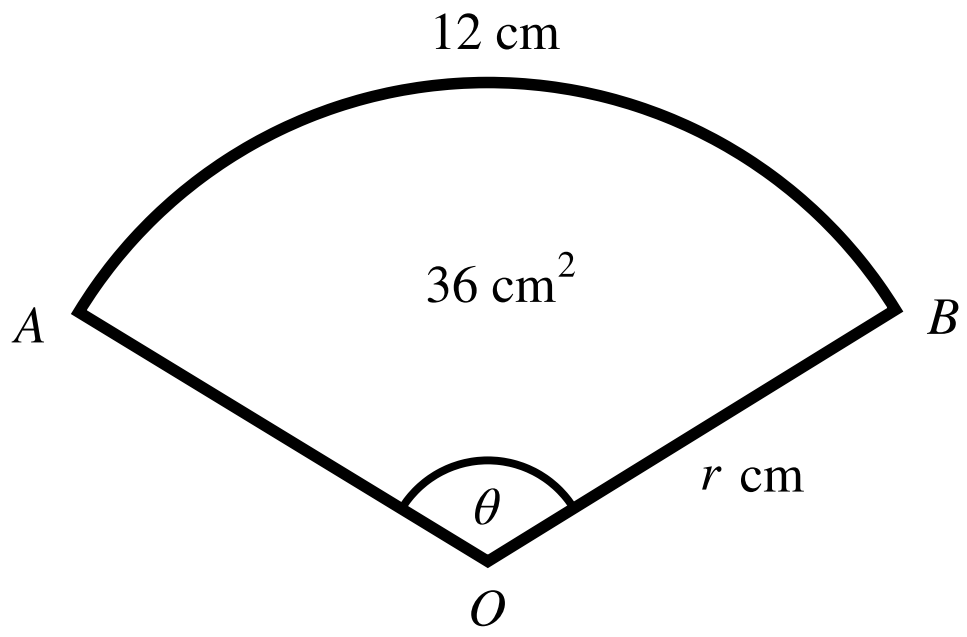

Figure 1: The sector OAB.

A sector OAB of a circle of radius $r\,\mathrm{cm}$ has angle $\theta\,\mathrm{radians}$. The length of the arc of the sector is $12\,\mathrm{cm}$ and the area of the sector is $36\,\mathrm{cm}^2$ (see **Figure 1**).

Part A First equation

By considering the length of the arc of the sector, write down an equation involving r and θ, where one side of the equation is a numerical constant.

The following symbols may be useful: `r, theta`

Part B Second equation

By considering the area of the sector, write down another equation involving r and θ, where one side of the equation is a numerical constant.

The following symbols may be useful: `r, theta`

Part C Values of r and θ

Hence show that $r = 6\,\text{cm}$ and find the value of θ.

Part D Area of segment

Find the area of the segment bounded by the arc AB and the chord AB. Give your answer to 3 s.f.

Gameboard:

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Radians and Trig Functions 2i

A Level

Figure 1 shows part of the curve $y = \cos 2x$, where x is in radians. The point A is the minimum point of this part of the curve.

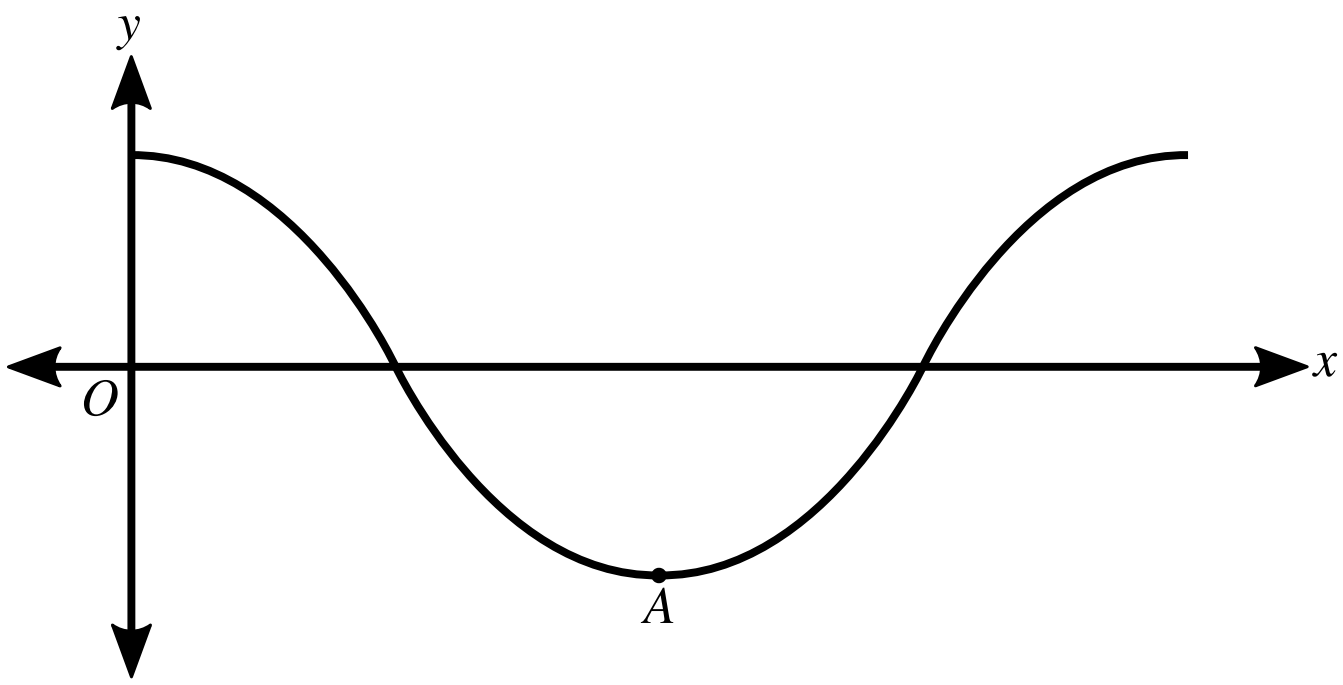


Figure 1: The graph of $y = \cos 2x$.

Part A Period

State the period of $y = \cos 2x$.

The following symbols may be useful: `pi, t`

Part B Coordinates of A

What is the x coordinate of A?

The following symbols may be useful: `pi, x`

What is the y-coordinate of A?

The following symbols may be useful: `pi, y`

Part C The inequality $\cos 2x \leqslant \frac{1}{2}$

Solve the inequality $\cos 2x \leqslant \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$, giving your answer as a range of angles x.

Give the exact lower bound, in the form $x > a$ or $x \geq a$.

The following symbols may be useful: `<, <=, >, >=, pi, x`

Give the exact upper bound, in the form $x < b$ or $x \leq b$.

The following symbols may be useful: `<, <=, >, >=, pi, x`

Gameboard:
STEM SMART Single Maths 29 - Radians & Small Angle Approximations

Radians and Trig Functions 2ii

A Level

This question is about solving the equation $2\cos x = \tan 2x$ for $0 \leq x \leq \pi$.

Part A The equation $2\cos x = \tan 2x$

Write down the exact values of $\cos\frac{\pi}{6}$ and $\tan\frac{\pi}{3}$ (where the angles are in radians).

- $\cos\frac{\pi}{6} =$ [ ]
- $\tan\frac{\pi}{3} =$ [ ]

To verify that $x = \frac{\pi}{6}$ is a solution of the equation $2\cos x = \tan 2x$, consider the two sides of the equation separately:

- When $x = \frac{\pi}{6}$, $2\cos x =$ [ ].
- When $x = \frac{\pi}{6}$, $\tan 2x =$ [ ].

The left hand side and right hand side are equal when $x = \frac{\pi}{6}$. Hence, $x = \frac{\pi}{6}$ is a solution of the equation $2\cos x = \tan 2x$.

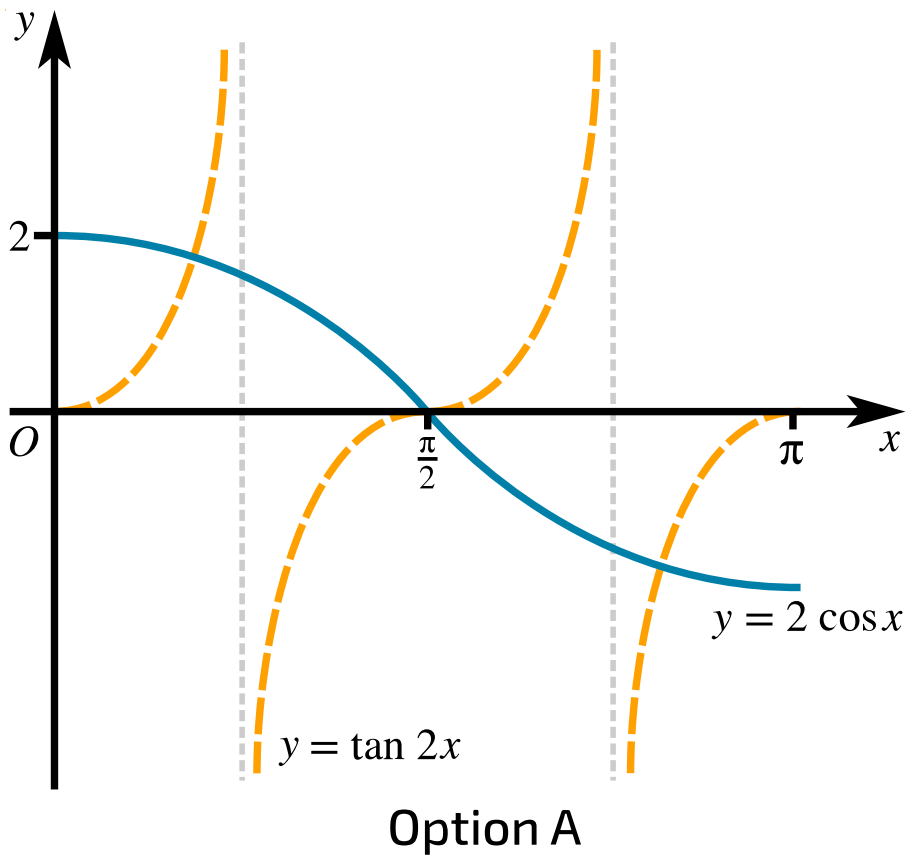
Items:

2 3 $\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{2}}$ $\frac{\sqrt{3}}{2}$ $\sqrt{3}$ 1 $\frac{1}{2}$

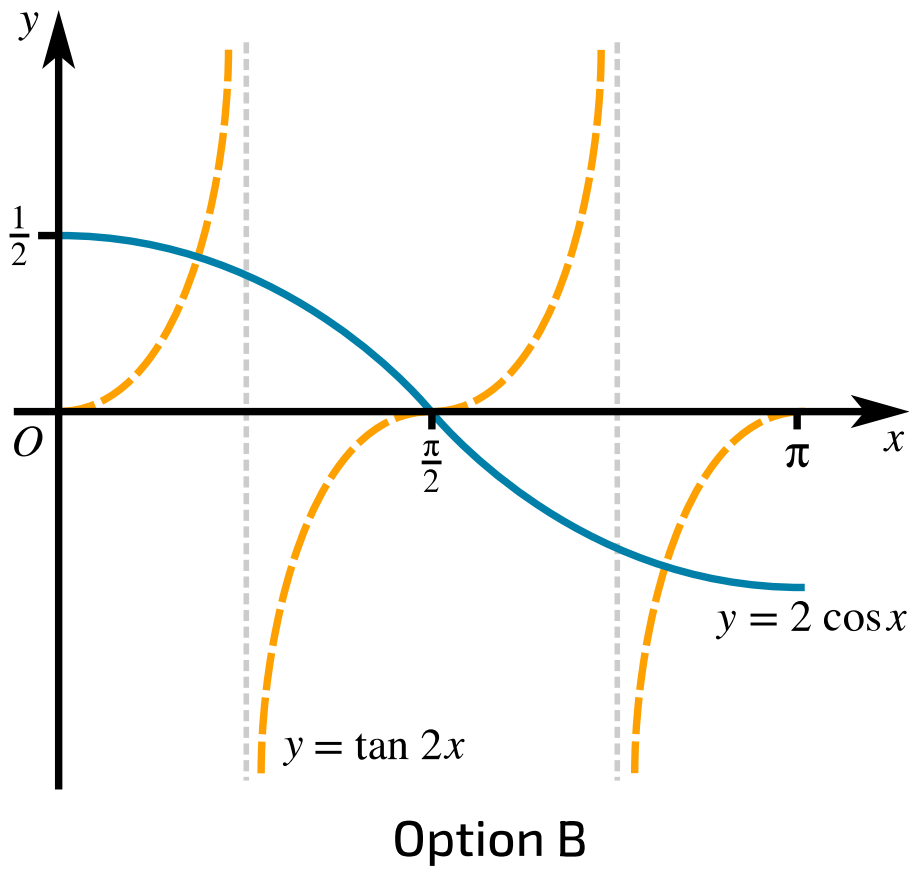
Part B Sketch

Sketch, on a single diagram, the graphs of $y = 2\cos x$ and $y = \tan 2x$, for x (radians) such that $0 \leqslant x \leqslant \pi$.

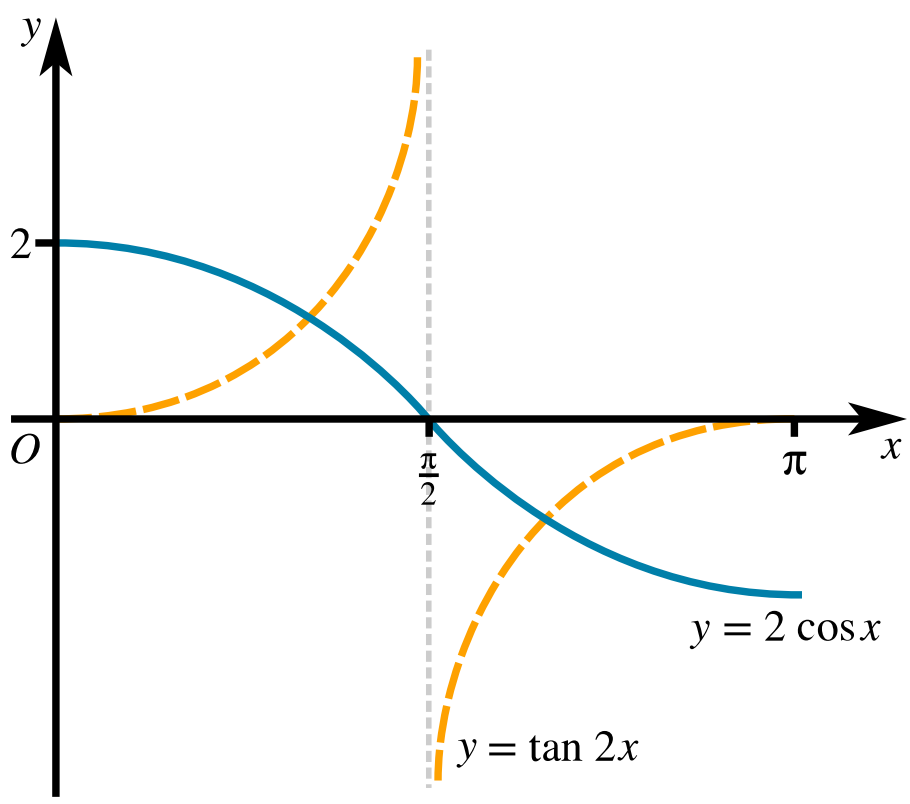
Choose the correct graph from the three options below.



Option A



Option B



Option C

Figure 1: Options A, B and C.

- A
- B
- C

Part C Other solutions

Hence state, in terms of π, the two other values of x between 0 and π satisfying the equation $2\cos x = \tan 2x$.

Give the exact value of the root with the smaller value of x.

The following symbols may be useful: `pi, x`

Give the exact value of the root with the larger value of x.

The following symbols may be useful: `pi, x`

Gameboard:

STEM SMART Single Maths 29 - Radians & Small Angle Approximations

Radians and Trig Functions 1i

A Level

A curve has equation $y = \sin(ax)$, where a is a positive constant and x is in radians.

Part A Period

State the period of $y = \sin(ax)$, giving your answer in an exact form in terms of a.

The following symbols may be useful: `a, pi, t`

Part B $\sin(ax) = k$

Given that $x = \frac{1}{5}\pi$ and $x = \frac{2}{5}\pi$ are the two smallest positive solutions of $\sin(ax) = k$, where k is a positive constant, find the values of a and k.

Find the value of a.

The following symbols may be useful: `a`

Find the value of k.

The following symbols may be useful: `k`

Part C $\sin(ax) = \sqrt{3}\cos(ax)$

Given instead that $\sin(ax) = \sqrt{3}\cos(ax)$, find the two smallest positive solutions for x, giving your answers in an exact form in terms of a.

Give the smallest positive solution.

The following symbols may be useful: `a, pi, x`

Give the second smallest positive solution.

The following symbols may be useful: `a, pi, x`

Gameboard:

STEM SMART Single Maths 29 - Radians & Small Angle Approximations

Small Angle Approximations 1ii

A Level

P P P

$$f(x) = \frac{\sin x + \tan(2x)}{\tan x + 2}$$

Part A Small angle approximation

Use the small angle approximation to write an approximate expression to second order for $f(x)$, valid when x is small.

The following symbols may be useful: `f, x`

Part B Estimation

Use your expression to estimate the value of $f(0.1)$ to 4 significant figures.

Part C Percentage error

What is the percentage error in this estimate? Give your answer to 3 significant figures.

Gameboard:

STEM SMART Single Maths 29 - Radians & Small Angle Approximations

Small Angle Approximations 1i

A Level

The small angle approximation is used when measuring distances in astronomy.

The two stars Alpha Centauri A and Alpha Centauri B are in a binary pair (they orbit one another). The distance between them is an average of 11 Astronomical Units, and they are an average of 4.4 light years from Earth.

$1\,\mathrm{AU} = 1\,\mathrm{Astronomical\ Unit} = 149\,597\,870\,700\,\mathrm{m}$
$1\,\mathrm{ly} = 1\,\mathrm{Light\ Year} = 9.4607 \times 10^{15}\,\mathrm{m}$

Assume that a telescope is pointing straight at Alpha Centauri A with the geometry shown in **Figure 1**.

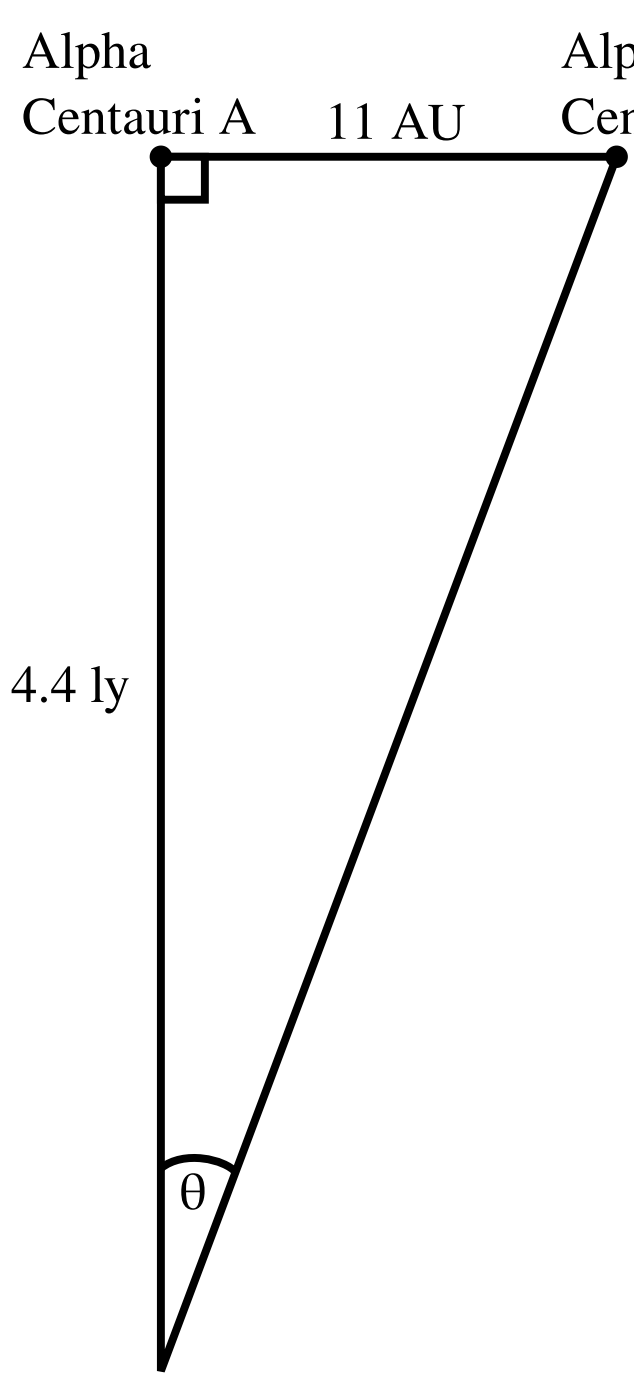


Figure 1: A telescope pointing straight at Alpha Centauri A

Use the small angle approximation to estimate θ, the angular separation between the stars as seen by the telescope. Give your answer to 2 significant figures.

Part A Radians

Give the answer in radians.

Part B Degrees

Give the answer in degrees.

Part C Arc Seconds

Give the answer in Arc Seconds. (Where 1 arc second is one $\left(\frac{1}{3600}\right)^{\text{th}}$ of a degree.)