Physics. You work it out.

Vectors: Positions and Problems 1ii

A Level

P P P

It is given that $\underline{\boldsymbol{a}} = 3\underline{\boldsymbol{i}} + 4\underline{\boldsymbol{j}}$, $\underline{\boldsymbol{b}} = -5\underline{\boldsymbol{i}} + 2\underline{\boldsymbol{j}}$, $\underline{\boldsymbol{c}} = 7\underline{\boldsymbol{i}} - 3\underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{d}} = 3\underline{\boldsymbol{a}} - 2\underline{\boldsymbol{b}} + \underline{\boldsymbol{c}}$.

Part A Vector $\underline{\boldsymbol{d}}$

Write down vector $\underline{\boldsymbol{d}}$ in terms of $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$.

$$\underline{\boldsymbol{d}} = \square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}}$$

Part B Magnitude of $\underline{\boldsymbol{d}}$

Find the magnitude of $\underline{\boldsymbol{d}}$. Give your answer to 3 significant figures.

Part C Direction of $\underline{\boldsymbol{d}}$

Find the direction of vector $\underline{\boldsymbol{d}}$, relative to $\underline{\boldsymbol{i}}$. If the direction is anticlockwise from $\underline{\boldsymbol{i}}$ give a positive angle, if it is clockwise from $\underline{\boldsymbol{i}}$ give a negative angle. Give your answer in degrees to 3 significant figures.

Created by Sally Waugh for Isaac Physics

Vectors: Diagrams and Proof 2ii

A Level

Part A Resultant vector

Find the resultant of the vectors $4\underline{\boldsymbol{i}} - \underline{\boldsymbol{j}}$ and $-2\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}}$.

$\square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}}$

Part B $\overrightarrow{\text{MN}}$

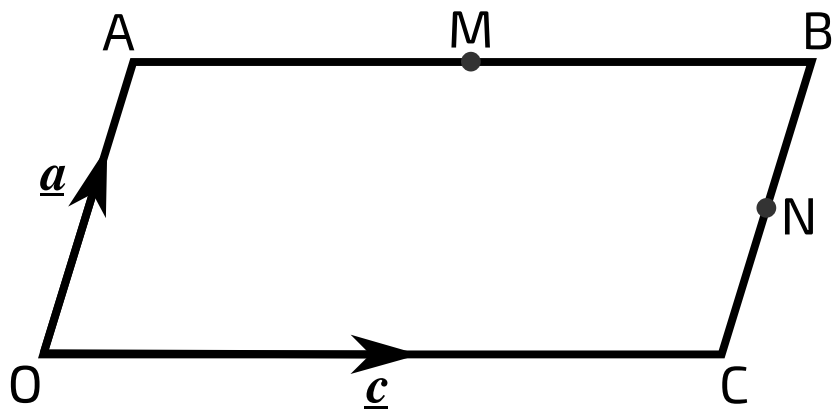

Figure 1: A parallelogram OABC.

In **Figure 1**, OABC is a parallelogram. M is the midpoint of AB. N is the midpoint of BC. $\overrightarrow{\text{OA}} = \underline{\boldsymbol{a}}$ and $\overrightarrow{\text{OC}} = \underline{\boldsymbol{c}}$.

Find $\overrightarrow{\text{MN}}$ in terms of $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{c}}$.

The following symbols may be useful: `a, c`

Adapted with permission from UCLES, A Level, 2012

Gameboard:

STEM SMART Double Maths 4 - Vectors

Ferry and Current

A Level P P P Further A P P P

A ferry is to cross the Sound of Islay from Port Askaig on Islay to Feolin on Jura which is $0.950\,\mathrm{km}$ due east of Port Askaig. The tidal current in the Sound of Islay is strong and the water is flowing at $3.50\,\mathrm{m\,s^{-1}}$ in a northerly direction. The ferry travels at a speed of $5.00\,\mathrm{m\,s^{-1}}$ relative to the water.

Part A Which direction?

In what direction should the ferry set out? Give your answer as a bearing.

Part B How long to cross

How long, in seconds, will the ferry take to make the crossing?

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Vectors: Position, Distance and Problems 2i

A Level

A man drives his friend to a station, A, to catch a train. When the man and his friend arrive at station A, the train is ready to leave and the friend cannot catch the train.

The man knows the train goes to another station D on a journey which can be modelled as a straight line. The train travels at an average speed of $30\,\mathrm{mph}$ and arrives at station D $22\,\mathrm{minutes}$ after leaving station A.

The man knows a road route from station A to station D that can be modelled as three straight roads, going via road junctions, B and C, such that

$$\overrightarrow{\mathrm{AB}} = 4\underline{\boldsymbol{i}} + 3\underline{\boldsymbol{j}} \quad \overrightarrow{\mathrm{BC}} = 2\underline{\boldsymbol{j}}, \quad \overrightarrow{\mathrm{CD}} = -4\underline{\boldsymbol{i}} + 6\underline{\boldsymbol{j}}$$

where the numbers are distances in miles.

The man can drive with an average road speed of $45\,\mathrm{mph}$. Can the man drive his friend from station A to station D in time for his friend to catch the train from station D? Show clearly how you arrive at your answer.

How many minutes spare does the friend have or by how many minutes has he missed it by? If he missed the train, give a negative number of minutes.

Written for Isaac Physics by Sally Waugh

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Vectors: Position, Distance and Problems 1i

A Level

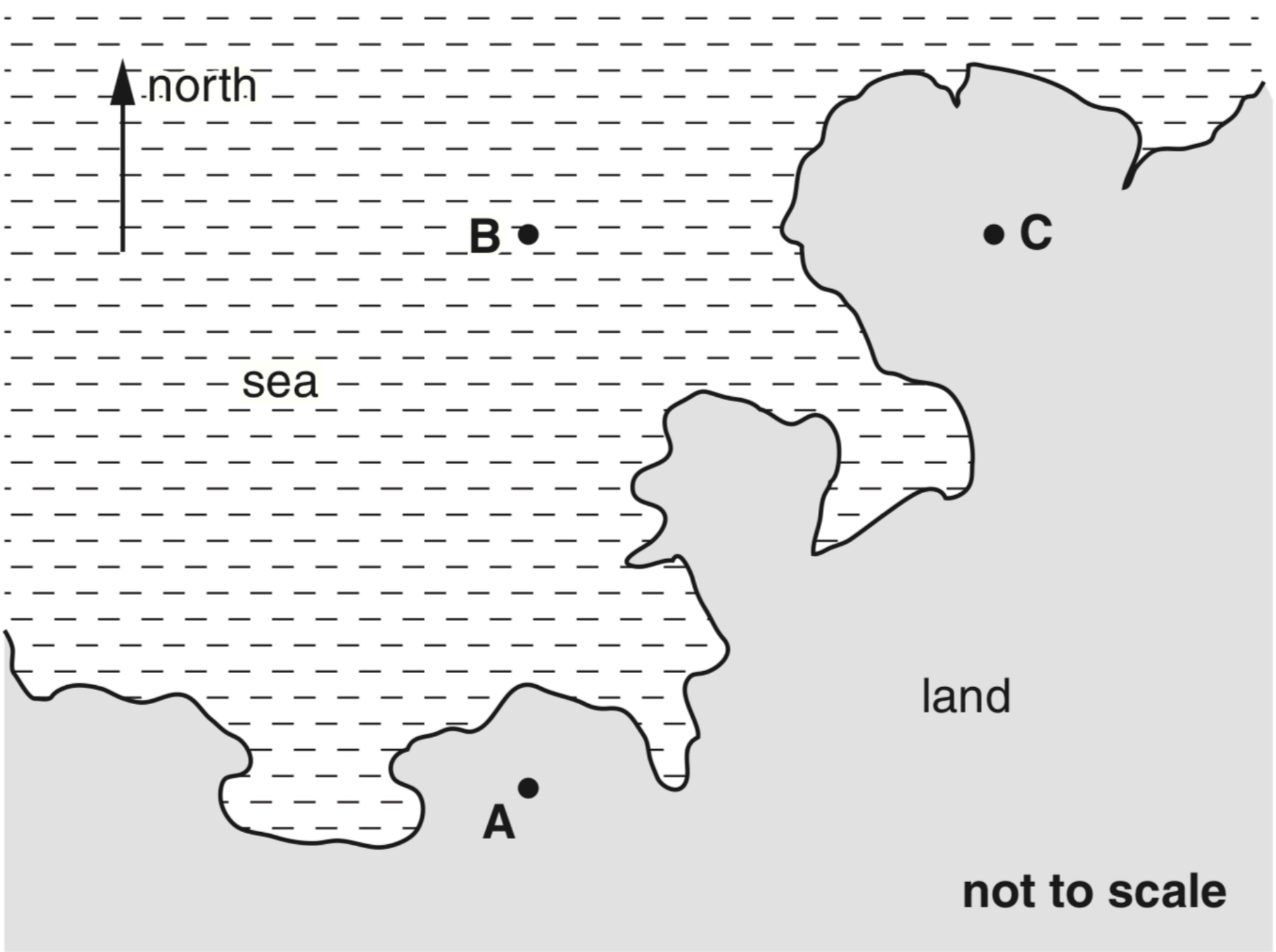

Figure 1: Map of a coastline showing A and C on land, and B in the sea.

An aircraft flies due north from A for a distance of $360\,\mathrm{km}$ $\left(3.6 \times 10^5\,\mathrm{m}\right)$ to point B. Its average speed between A and B is $170\,\mathrm{m\,s^{-1}}$. At B the aircraft is forced to change course and flies due east for a distance of $100\,\mathrm{km}$ to arrive at C.

Part A Journey time

Calculate the time of the journey from A to B.

Part B A vector triangle

Draw a labelled displacement triangle to represent the aircraft's journey.

Easier question?

Part C Distance A to C

Use the displacement triangle to determine the magnitude of the displacement in km of the aircraft at C from A.

Part D Bearing

Find the bearing of C from A.

Modified with permission from UCLES, OCR Physics A, June 2009, Question 1 part B.

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STEM SMART Double Maths 4 - Vectors

Manipulating Vectors in 3D

A Level

C C C

Pre-Uni Maths for Sciences I1.10

A vector $\underline{\boldsymbol{u}} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$ has a length of 4.00 units.

Part A Case 1

If $\underline{\boldsymbol{u}}$ lies in the (x, y)-plane, makes an angle of 30° with the x-direction and u_y is positive, find u_x.

Give your answer to 3 sf.

Part B Case 2

If $u_x = u_y = 2$ and u_z is negative, find u_z.

Give your answer to 3 sf.

Part C Case 3

If $u_z = 1$, $u_y = 2u_x$ and u_y is positive, find u_y.

Give your answer to 3 sf.

Created for isaacphysics.org by Julia Riley.

Gameboard:

STEM SMART Double Maths 4 - Vectors

3D Vectors 1ii

A Level

ABCD is a parallelogram. The position vectors of A, B and C are given respectively by

$$\underline{a} = 2\underline{i} + \underline{j} + 3\underline{k} \qquad \underline{b} = 3\underline{i} - 2\underline{j} \qquad \underline{c} = \underline{i} - \underline{j} - 2\underline{k}$$

Part A Position of D

Find the position vector of D. Give your answer using ijk notation.

The following symbols may be useful: `i, j, k`

Part B Unit vector

Find the unit vector in the direction $\overrightarrow{OD}$ where O is the fixed origin. Give your answer using ijk notation.

The following symbols may be useful: `i, j, k`

Adapted with permission from UCLES, A Level, June 2005, OCR C4, Question 5

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STEM SMART Double Maths 4 - Vectors

3D Vectors 1i

A Level

The points A, B and C are three of the vertices of a parallelogram ABCD. In this question the vertices are **NOT** necessarily labelled in order.

The point P lies at the location $8\underline{\boldsymbol{i}} + 8\underline{\boldsymbol{j}} + 5\underline{\boldsymbol{k}}$, relative to the fixed origin O. Relative to P, the locations of A, B and C are

$$\overrightarrow{\text{PA}} = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}, \qquad \overrightarrow{\text{PB}} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \qquad \overrightarrow{\text{PC}} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

Part A Three locations

Find the three possible locations for D, the fourth point of the parallelogram.

Location with lowest x component:

$$\square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}}$$

Location with middle x component:

$$\square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}}$$

Location with highest x component:

$$\square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}}$$

Part B One location

You are given the extra information that point D lies on a sphere of radius $\sqrt{29}$ centred on P.

What is the location of D?

$$\overrightarrow{OD} = \square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}}$$

Created for isaacphysics.org by Jonathan Waugh

Gameboard:

STEM SMART Double Maths 4 - Vectors

The Harbour Entrance

A Level

C C C

The entrance to a harbour is a channel of length a which runs between two sandbanks a distance b apart. The banks and the channel can be assumed to be rectangular. On this particular day, there is a current of constant speed v flowing from one sandbank to the other. A yacht travelling at a constant speed relative to the water of u wants to enter the harbour.

Find the least value of u needed for the yacht to safely enter the harbour.

The following symbols may be useful: `a, b, u, v`

Used with permission from UCLES, A Level Further Maths, Syllabus C, June 1989, Special Paper, Question 7.

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Triangular Flying Course

A Level

C C C

Two identical aeroplanes, P_1 and P_2, can both complete a course in the form of an equilateral triangle ABC in time $3T$ at a speed V when there is no wind. They then fly over the same course in a wind of velocity kV parallel to the side AB and blowing from B to A, where $k < 1$.

With both planes starting at A, the pilot of P_1 thinks that he can complete the course faster by initially flying along AC (completing the course ACBA). However, the pilot of P_2 thinks that it will be faster to first travel along AB (completing the course ABCA). This is shown in **Figure 1**.

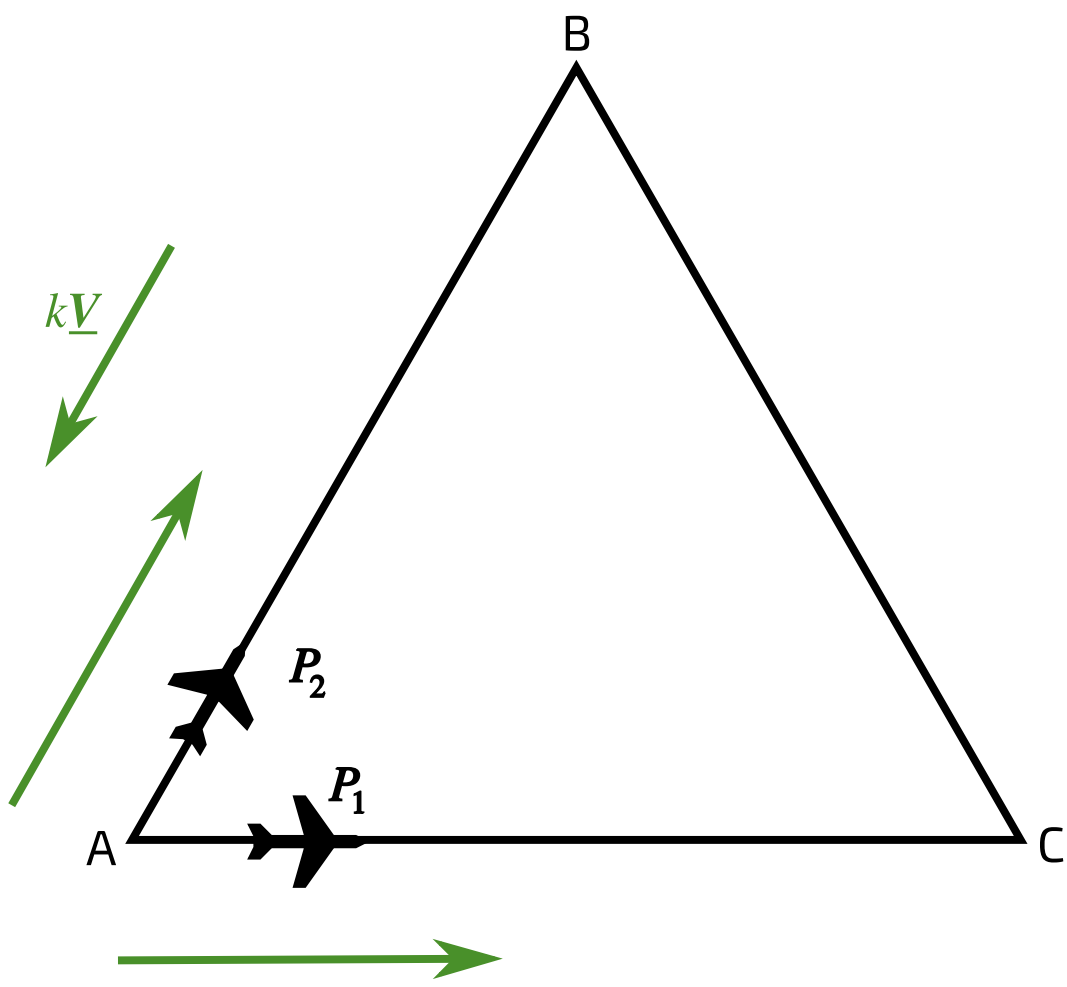

Figure 1: Pilot P_1 flies along AC first. Pilot P_2 flies along AB first.

Find an expression for the difference between the times taken by each pilot to complete the course.

The following symbols may be useful: `T, V, k`

Adapted with permission from UCLES, Higher School Certificate Applied Mathematics, June 1936, Question 6.