

STEM SMART Single Maths 25 - Defining Functions

Functions and Algebra 5ii

Subject & topics

Maths

Status

Not started

Stage & difficulty

A Level Challenge 1

The function f is defined by

$$f(x) = \frac{1}{\sqrt{x}} + 2, \quad x > 0.$$

The function g is defined for all real values of x by

$$g(x) = 10 - (x + 3)^2.$$

Part A

Range of f

State the range of $f(x)$ as an inequality.

The following symbols may be useful: <, <=, >, >=, f(x), x, y

Part B**Inverse of f**

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: f , x

Part C**Range of g**

State the range of $g(x)$ as an inequality.

The following symbols may be useful: $<$, \leq , $>$, \geq , $g(x)$, x , y

Part D**Compound function of g**

Find the value of $g(g(-1))$.

Used with permission from UCLES A-level Maths papers, 2003-2017.



Inverse Quadratic Function

Subject & topics

Maths | Functions | Graph Sketching

Status

Not started

Stage & difficulty

A Level Practice 2

Figure 1 shows the graph of $y = f(x)$, where

$$f(x) = 2 - x^2, \quad x \leq 0$$

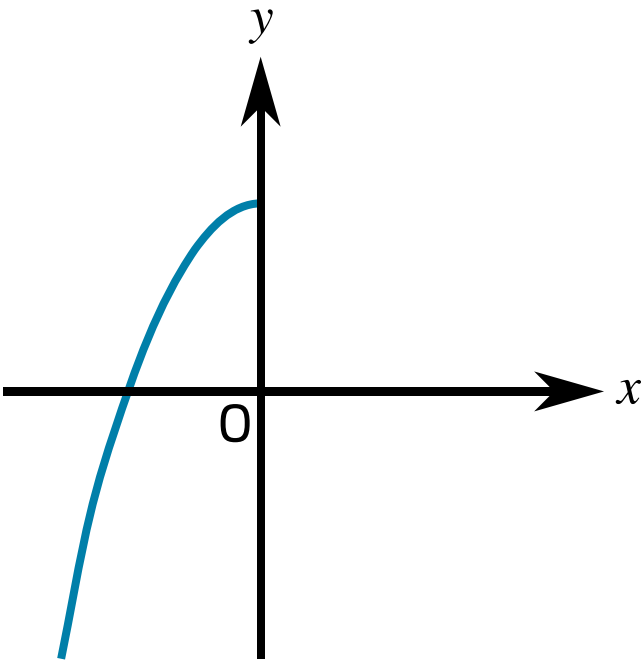


Figure 1: The graph of $y = f(x)$, for $x \leq 0$.

Part A

$f^2(-3)$

Evaluate $f^2(-3)$.

Part B

$f^{-1}(x)$

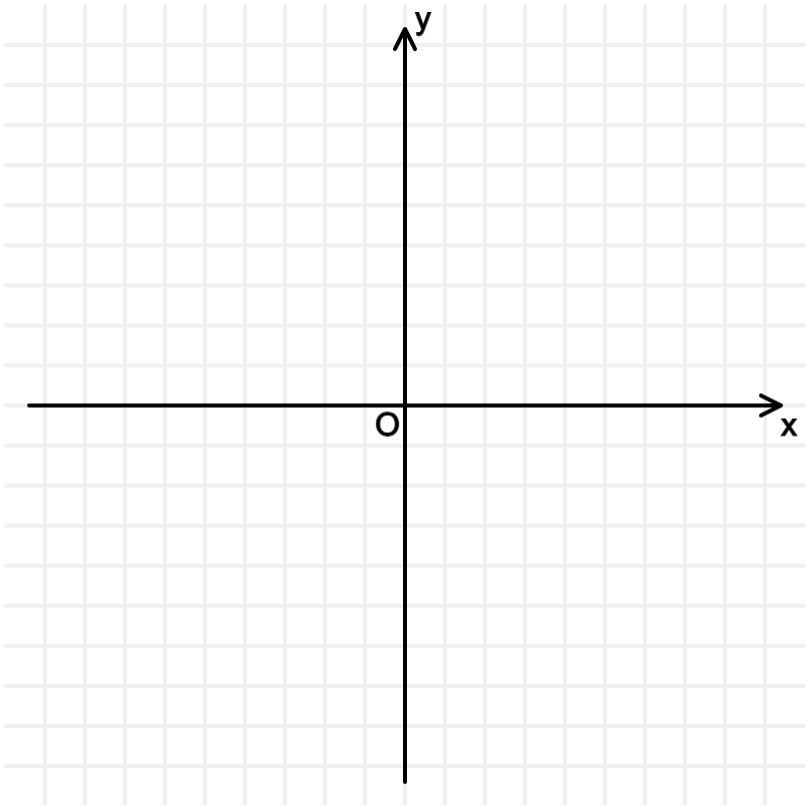
Find an expression for $f^{-1}(x)$.

The following symbols may be useful: f, x, y

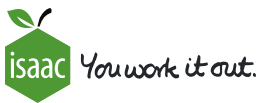
Part C

Graph of $f^{-1}(x)$

Sketch the graph of $y = f^{-1}(x)$.



Used with permission from UCLES A-level Maths papers, 2003-2017.



Function Types and Inverses

Subject & topics		
Maths Functions General Functions		
Status		Stage & difficulty
Not started		A Level Challenge 1 <div><div></div><div></div><div></div></div>

Figure 1 shows five different graphs, A, B, C, D and E, each for values of x such that $-a \leq x \leq a$ where a is a constant.

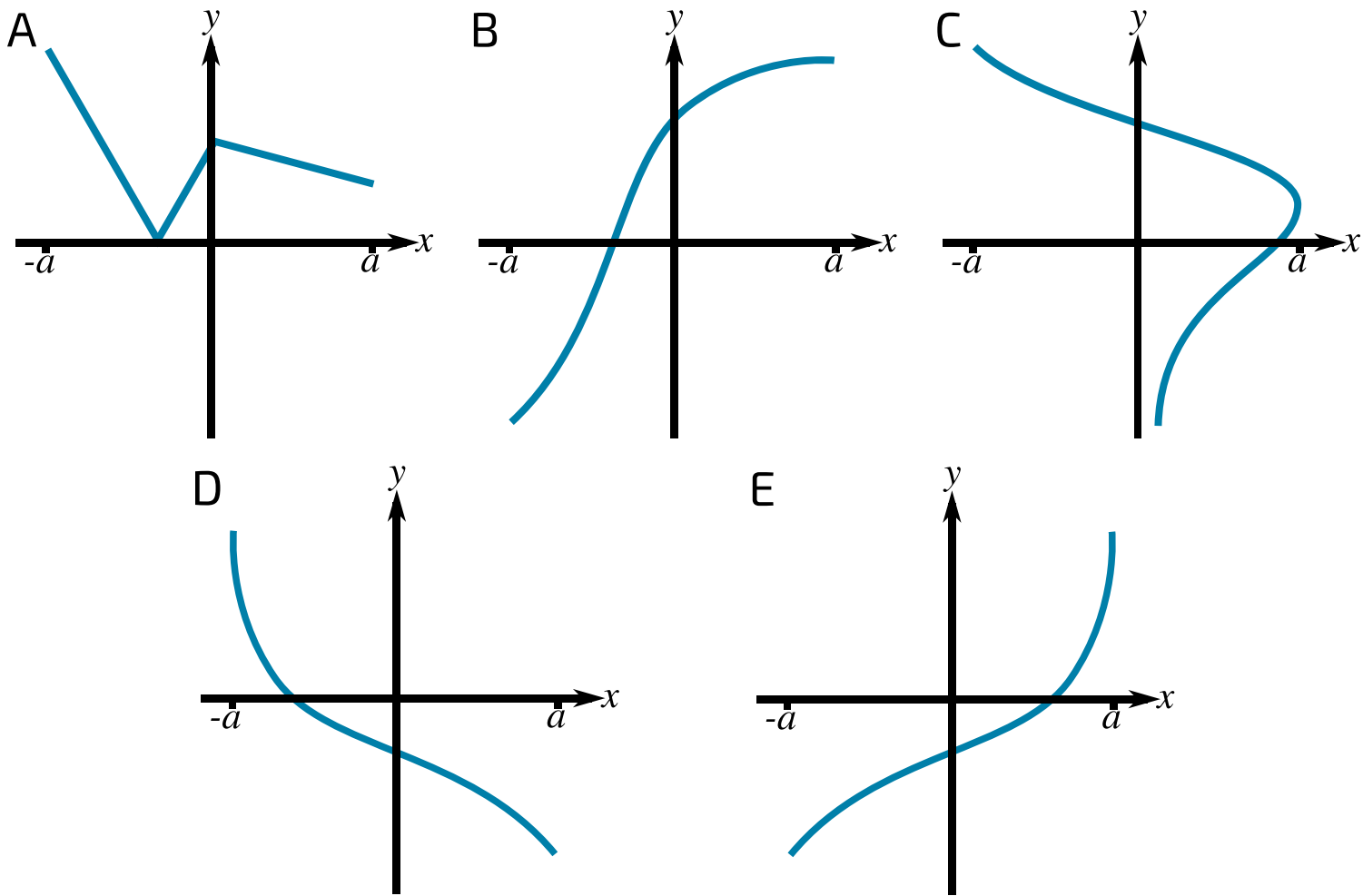


Figure 1: The set of five graphs, labelled A, B, C, D and E

Part A

Function

Which diagram does not show the graph of a function?

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E

Part B

One-to-one Function

Which diagram shows the graph of a function that is not one-to-one?

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E

Part C

Inverses

It is given that two of the diagrams illustrate functions that are inverses of each other. Identify one of these two diagrams.

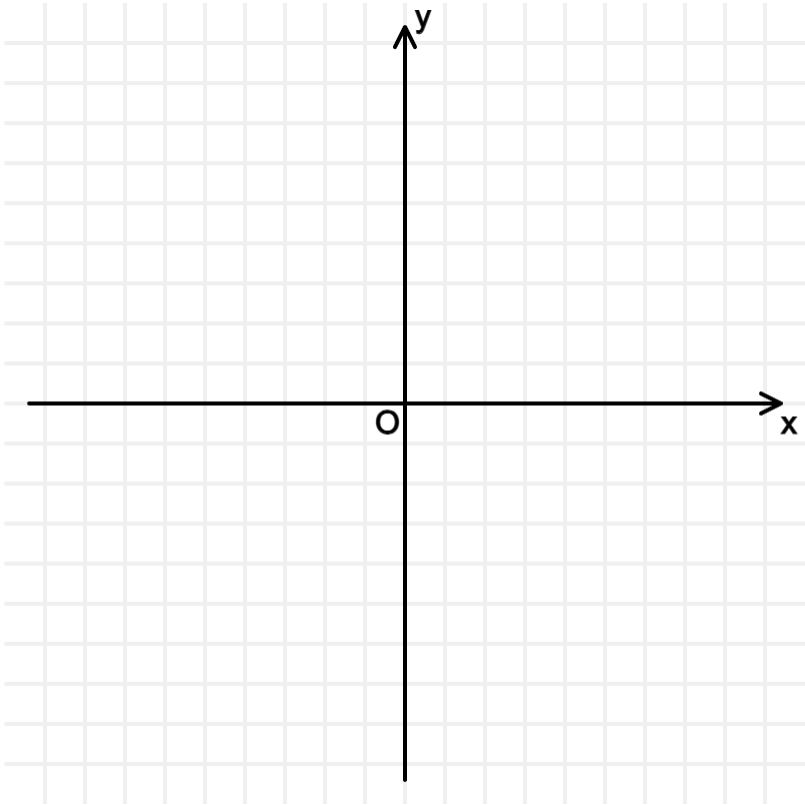
- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E

Part D

Sketch

The graph in E has equation $y = f(x)$. Sketch the graph of $y = |f(x)|$.

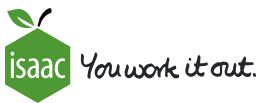
To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.



Used with permission from UCLES A-level Maths papers, 2003-2017.

Question deck:

STEM SMART Single Maths 25 - Defining Functions



STEM SMART Single Maths 25 - Defining Functions

Functions and Algebra 3i

Subject & topics

Maths

Status

Not started

Stage & difficulty

A Level Practice 2

The functions f and g are defined for all real values of x by

$$f(x) = |2x + a| + 3a \quad \text{and} \quad g(x) = 5x - 4a,$$

where a is a positive constant.

Part A

Range

Find the range of $f(x)$.

Fill in the inequality below.

Items:

$f(x)$

$<$

\leq

$>$

\geq

$< f(x) <$

$\leq f(x) \leq$

$< f(x) \text{ or } f(x) <$

$\leq f(x) \text{ or } f(x) \leq$

$\frac{a}{3}$

$\frac{a}{2}$

a

$2a$

$3a$

$4a$

0

$-\frac{a}{3}$

$-\frac{a}{2}$

$-a$

$-2a$

Part B

Inverse function of $f(x)$

Fill in the blanks to explain why the function $f(x)$ has no inverse.

The function $f(x)$ is not . For example, $f(0) = 4a$ and $f(\text{})$ also equals $4a$. Hence, $f(x)$ has no inverse.

Items:

-
-
-
-
-
-
-
-

Part C

Inverse function of $g(x)$

Find an expression for $g^{-1}(x)$.

The following symbols may be useful: a , x

Part D

Solve for x

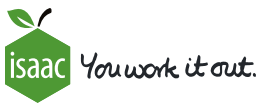
Solve for x the equation $g(f(x)) = 31a$. Give your solutions in ascending order.

- $x = \text{} a$
- $x = \text{} a$

Used with permission from UCLES A-level Maths papers, 2003-2017.

Question deck:

STEM SMART Single Maths 25 - Defining Functions



STEM SMART Single Maths 25 - Defining Functions

Modulus 3ii

Subject & topics

Maths

Status

Not started

Stage & difficulty

A Level Practice 1

Solve the inequality $|2x - 5| > |x + 1|$.

Construct your answer from the items below.

Items:

x

$<$

\leq

$>$

\geq

$< x <$

$\leq x \leq$

$> x \text{ or } x >$

$\geq x \text{ or } x \geq$

-6

-4

-2

$-\frac{3}{2}$

$-\frac{4}{3}$

-1

$-\frac{1}{2}$

0

$\frac{1}{2}$

1

$\frac{4}{3}$

$\frac{3}{2}$

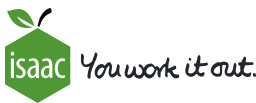
2

4

6

Used with permission from UCLES A-level Maths papers, 2003-2017.

Question deck:
STEM SMART Single Maths 25 - Defining Functions



STEM SMART Single Maths 25 - Defining Functions

Modulus Functions 2

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

A Level Practice 2
Further A Practice 1

Part A

Divergence of $y = \left|\frac{1}{x}\right|$

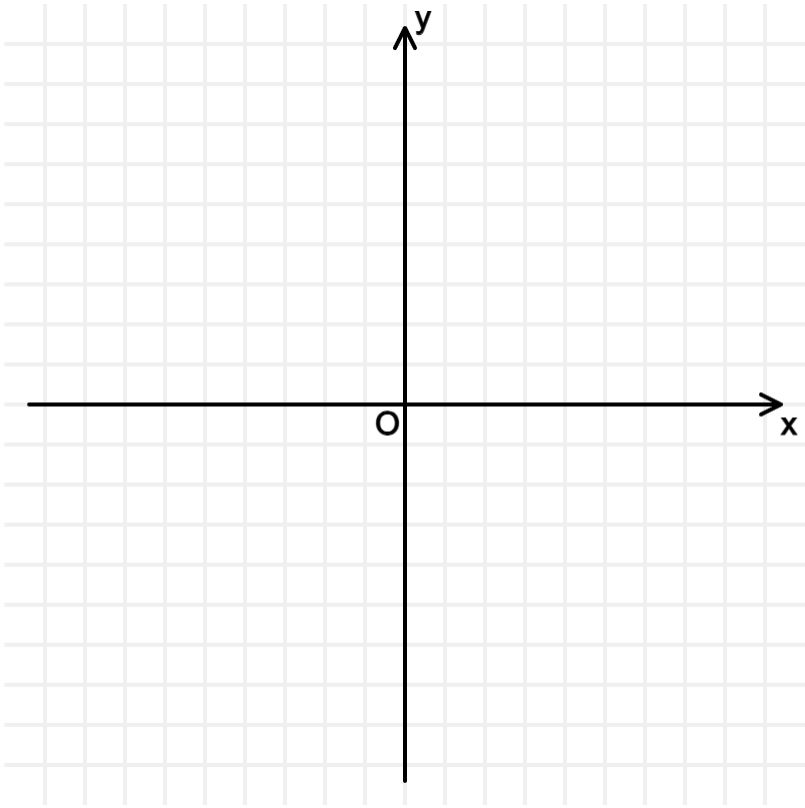
Does the function $y = \left|\frac{1}{x}\right|$ diverge anywhere? Where?

The following symbols may be useful: x

Part B

Graph of $y = \left|\frac{1}{x}\right|$

Sketch the graph of $y = \left|\frac{1}{x}\right|$.



Part C

Divergence of $y = \left|\frac{1}{x^2-4}\right|$

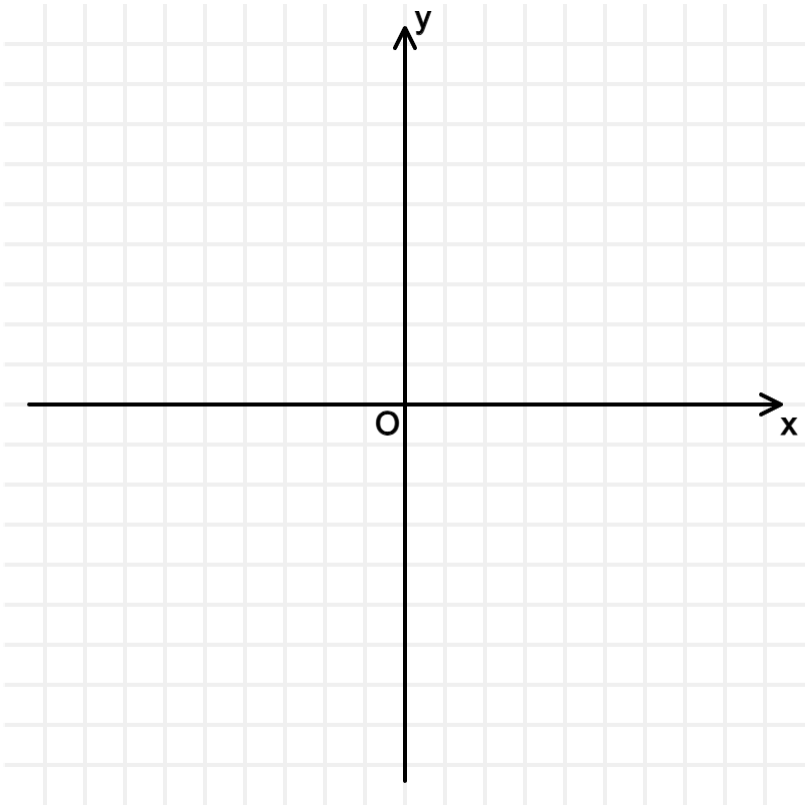
Does the function $y = \left|\frac{1}{x^2-4}\right|$ diverge anywhere? Where?

The following symbols may be useful: x , \pm

Part D

Graph of $y = \left| \frac{1}{x^2 - 4} \right|$

Sketch the graph of $y = \left| \frac{1}{x^2 - 4} \right|$.

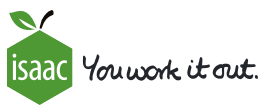


Part E

Solve equation graphically

Solve the equation $|x| = \left| \frac{1}{x} \right|$ graphically and give the solution as a single expression.

The following symbols may be useful: x , \pm



Curve Sketching and Combined Transformations 3i

Subject & topics		
Maths		
Status		Stage & difficulty
Not started		A Level Practice 2



The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x)$$

where k is a positive constant. **Figure 1** shows the curve with equation $y = f(x)$.

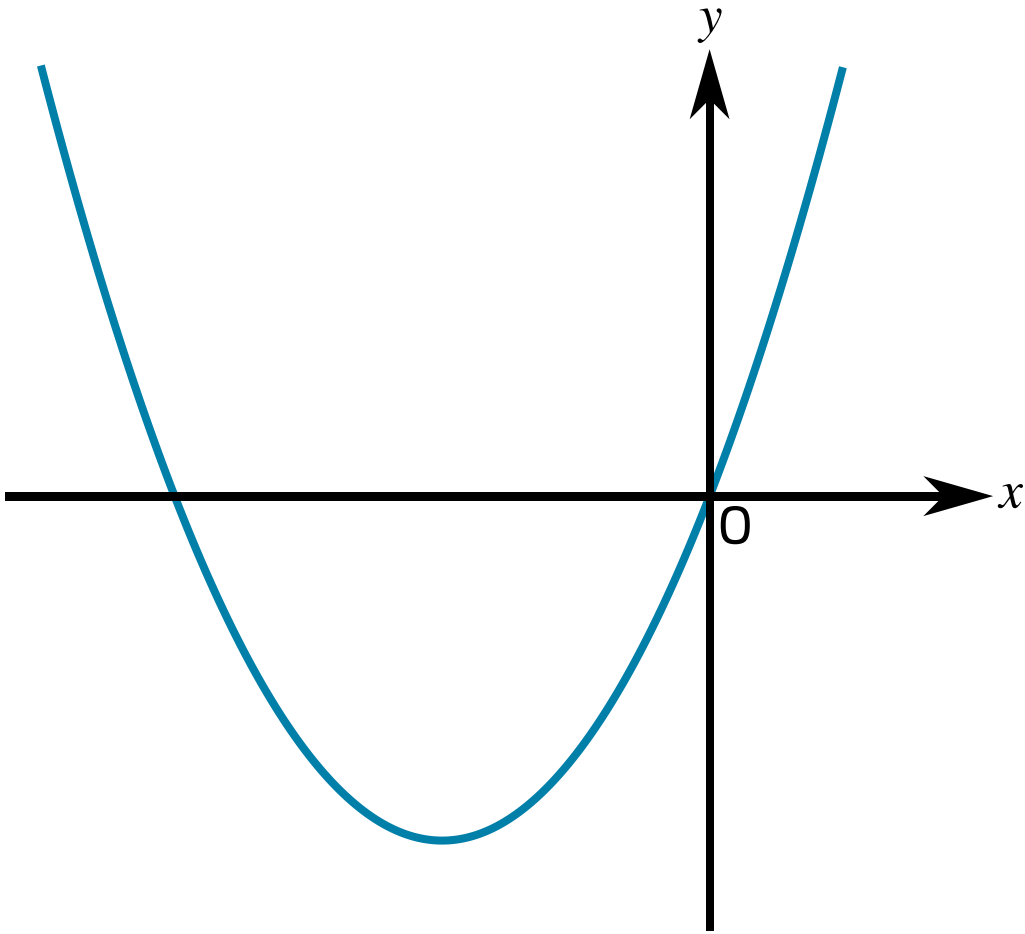


Figure 1: The graph of $y = f(x)$

Part A

Transformations

Give a sequence of transformations that will transform the curve $y = x^2$ to the curve $y = f(x)$.

Available items

- Translate the curve 2 units in the negative y direction.
- Stretch the curve in the x direction by a factor of k .
- Translate the curve 4 units in the positive y direction.
- Translate the curve 2 units in the positive y direction.
- Translate the curve 4 units in the negative y direction.
- Translate the curve 2 units in the positive x direction.
- Stretch the curve in the x direction by a factor of $\frac{1}{k}$.
- Translate the curve 4 units in the negative x direction.
- Translate the curve 2 units in the negative x direction.
- Stretch the curve in the y direction by a factor of $\frac{1}{k}$.
- Translate the curve 4 units in the positive x direction.
- Stretch the curve in the y direction by a factor of k .

Part B

Range

Find the range of $f(x)$ as a single inequality in terms of k .

The following symbols may be useful: $<$, $<=$, $>$, $>=$, $f(x)$, k , x , y

Part C

Find k

It is given that there are three distinct values of x which satisfy the equation $|f(x)| = 20$.

Find the value of k .

Part D

Solve $|f(x)| = 20$

Using the value of k from part C, find the three distinct values of x which satisfy the equation $|f(x)| = 20$. Give any irrational values to 3 sf.

(

)

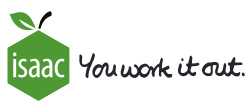
(

)

(

)

Used with permission from UCLES A-level Maths papers, 2003-2017.



Combined Transformations

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

A Level Practice 2

The function f is defined by $f(x) = \sqrt{mx + 7} - 4$, where $x \geqslant -\frac{7}{m}$ and m is a positive constant. **Figure 1** shows the curve $y = f(x)$.

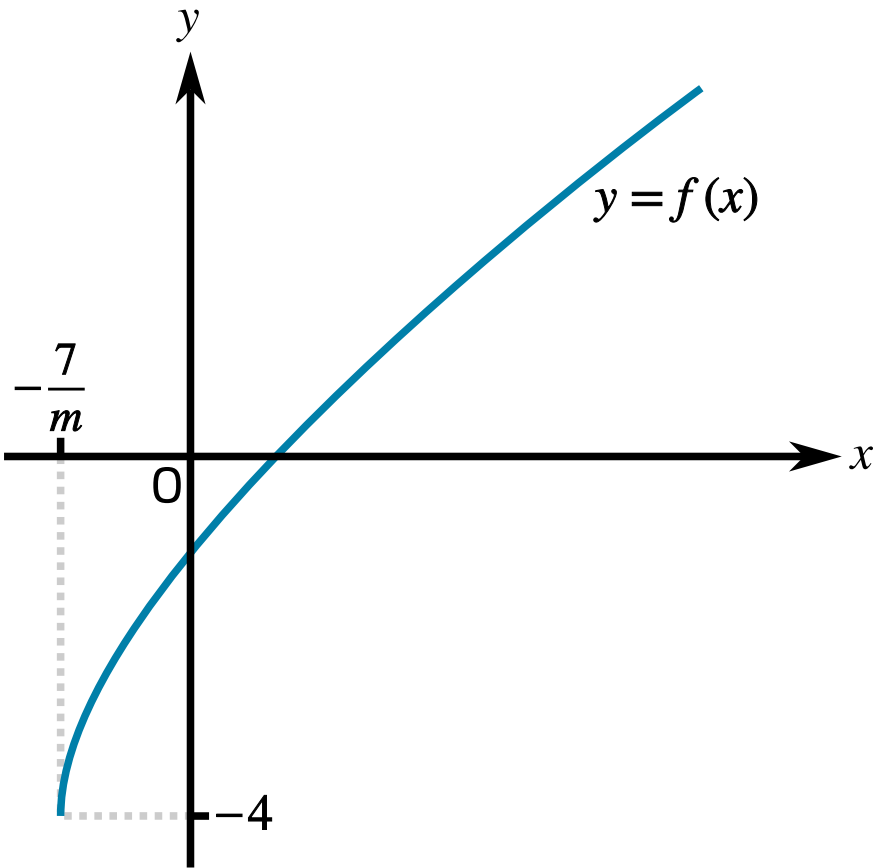


Figure 1: The curve $y = f(x)$

Part A

Translation of the curve $y = \sqrt{x}$

ence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations.

ible items

atch the curve in the x direction by a factor of m .

nslate the curve 7 units in the negative y direction.

atch the curve in the y direction by a factor of $\frac{1}{m}$.

nslate the curve 4 units in the positive y direction.

nslate the curve 4 units in the negative x direction.

atch the curve in the x direction by a factor of $\frac{1}{m}$.

nslate the curve 7 units in the negative x direction.

nslate the curve 7 units in the positive x direction.

nslate the curve 4 units in the negative y direction.

Part B

$f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: f, m, x

Part C

Values of m

It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Thus it can be deduced that neither curve meets the line $y = x$. Hence determine the possible values of m .

Construct your answer from the items below.

Items:

m $<$ \leq $>$ \geq $< m <$ $\leq m \leq$ $> m \text{ or } m >$ $\geq m \text{ or } m \geq$ -28 -14 -8 -7 -4 -2

-1 0 1 2 4 7 8 14 28

Used with permission from UCLES A-level Maths papers, 2003-2017.