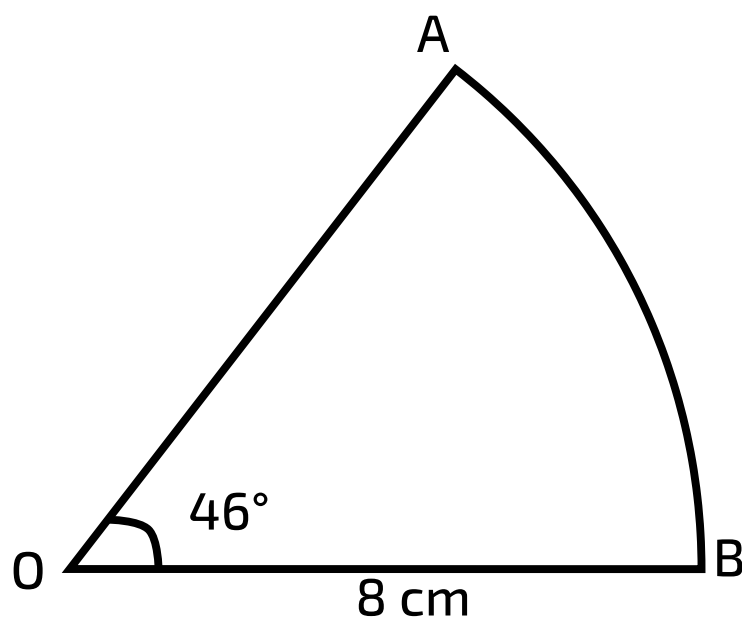# Radians-problems involving area 5ii

Figure 1 shows a sector OAB of a circle, centre O and radius $8\,\mathrm{cm}$. The angle AOB is $46\,^\circ$.

Figure 1: Sector AOB.

Part A   Convert angle to radians

Express $46\,^\circ$ in radians, correct to 3 significant figures.

Part B   Arc length

Find the length of the arc AB.

Part C   Area of sector

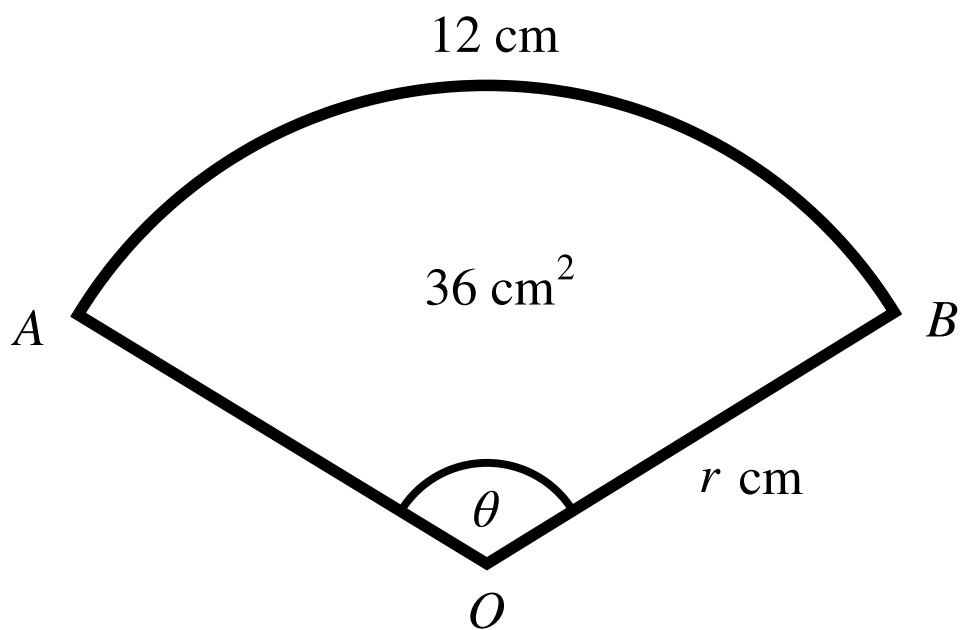Find the area of the sector OAB.

Radians-problems involving area 1ii

A Level

P P P

Figure 1: The sector OAB.

A sector OAB of a circle of radius r cm has angle θ radians. The length of the arc of the sector is $12\,\mathrm{cm}$ and the area of the sector is $36\,\mathrm{cm}^2$ (see **Figure 1**).

Part A   First equation

By considering the length of the arc of the sector, write down an equation involving r and θ, where one side of the equation is a numerical constant.

The following symbols may be useful: `r, theta`

Part B   Second equation

By considering the area of the sector, write down another equation involving r and θ, where one side of the equation is a numerical constant.

The following symbols may be useful: `r, theta`

Part C   Values of r and θ

Hence show that $r = 6 \, \mathrm{cm}$ and find the value of θ.

Part D   Area of segment

Find the area of the segment bounded by the arc AB and the chord AB. Answer to 3 sf.

Radians and Trig Functions 1i

A curve has equation $y = \sin(ax)$, where a is a positive constant and x is in radians.

Part A   Period

State the period of $y = \sin(ax)$, giving your answer in an exact form in terms of a.

The following symbols may be useful: `a, pi, t`

Part B $\sin(ax) = k$

Given that $x = \frac{1}{5}\pi$ and $x = \frac{2}{5}\pi$ are the two smallest positive solutions of $\sin(ax) = k$, where k is a positive constant, find the values of a and k.

Find the value of a.

The following symbols may be useful: `a`

Find the value of k.

The following symbols may be useful: `k`

Part C $\sin(ax) = \sqrt{3}\cos(ax)$

Given instead that $\sin(ax) = \sqrt{3}\cos(ax)$, find the two smallest positive solutions for x, giving your answers in an exact form in terms of a.

Give the smallest positive solution.

The following symbols may be useful: `a, pi, x`

Give the second smallest positive solution.

The following symbols may be useful: `a, pi, x`

Gameboard:

STEM SMART Double Maths 24 - Radians, Trigonometric Reciprocals & t-Formulae

Small Angle Approximations 3ii

A Level

P P P

Part A   Expression

Use the small angle approximation to write an approximate expression for $(\cos\theta + 5)(1 + \sin(2\theta))$ in powers of θ up to θ^2.

The following symbols may be useful: `theta`

Part B   Approximation

Use your answer from the above part to work out an approximate value for $(\cos\theta + 5)(1 + \sin(2\theta))$ when $\theta = 0.075$. Give your answer to 3 significant figures.

Gameboard:

STEM SMART Double Maths 24 - Radians, Trigonometric Reciprocals & t-Formulae

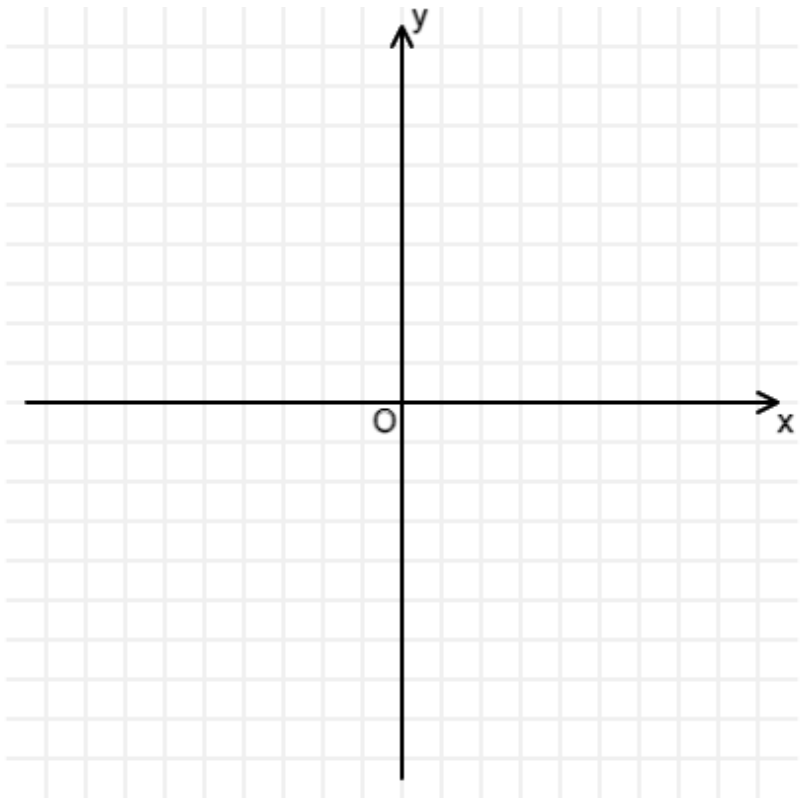# Sketching Reciprocal Trigonometric Functions

A Level

P P P

Part A   **Sketch** $2\sec x + 2$

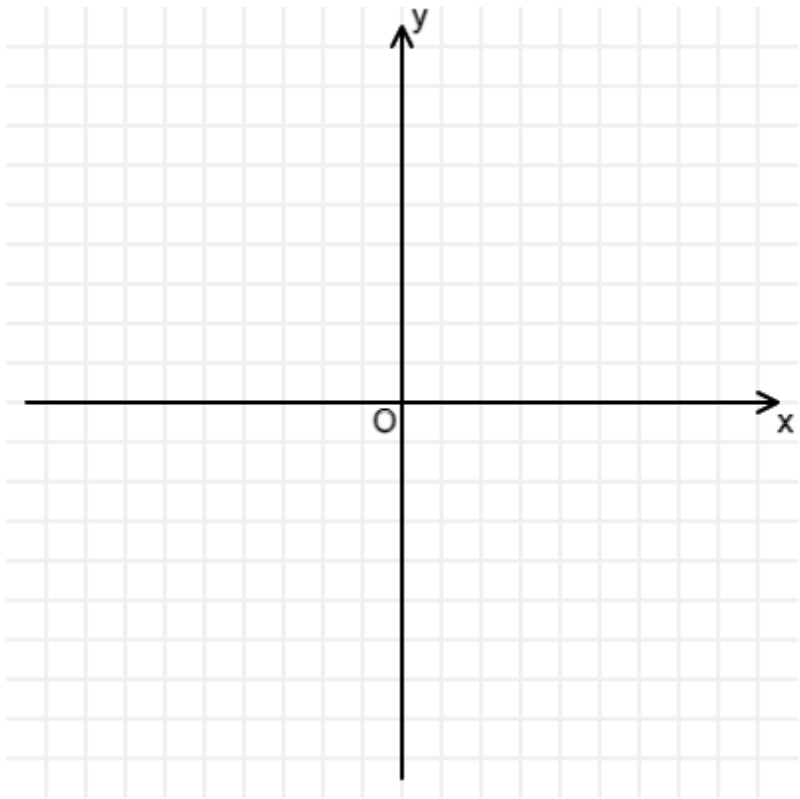Sketch the graph of $y = 2\sec x + 2$ in the interval $0° \le x \le 360°$.
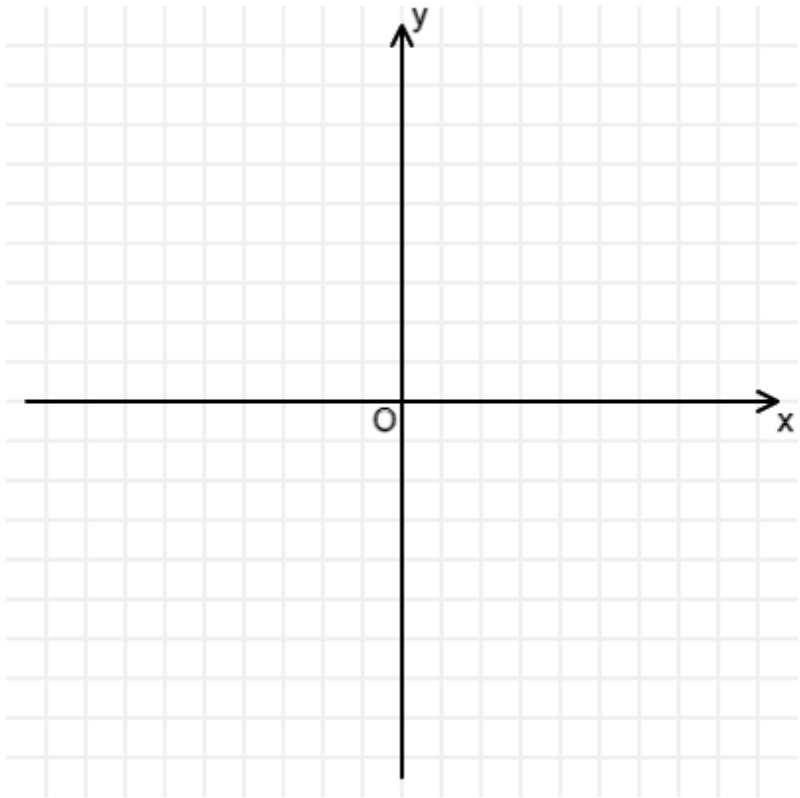
Part B   **Sketch** $\cot\left(x + \frac{\pi}{4}\right) - 1$

Sketch the graph of $y = \cot\left(x + \frac{\pi}{4}\right) - 1$ in the interval $0 \le x \le 2\pi$.

Part C   **Sketch** $-\csc\frac{x}{2} - 3$

Sketch the graph of $y = -\csc\frac{x}{2} - 3$ in the interval $-2\pi \le x \le 2\pi$.

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Advanced Trig Identities 2ii

Part A $2\tan^2\theta - \dfrac{1}{\cos\theta}$

Express $2\tan^2\theta - \dfrac{1}{\cos\theta}$ in terms of $\sec\theta$.

The following symbols may be useful: `sec()`, `theta`

Part B   **Solve**

Hence solve, for $0° < \theta < 360°$, the equation

$$2\tan^2\theta - \frac{1}{\cos\theta} = 4$$

Give the smallest solution to three significant figures.

Give the second smallest solution to four significant figures.

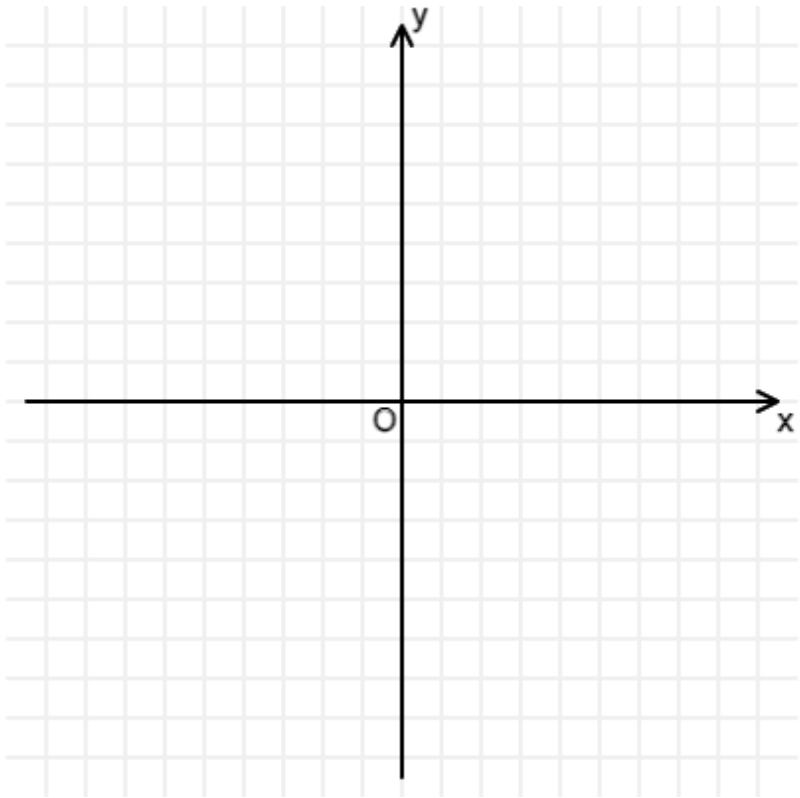Give the second largest solution to four significant figures.

Give the largest solution to three significant figures.

Sketching Inverse Trigonometric Functions

A Level
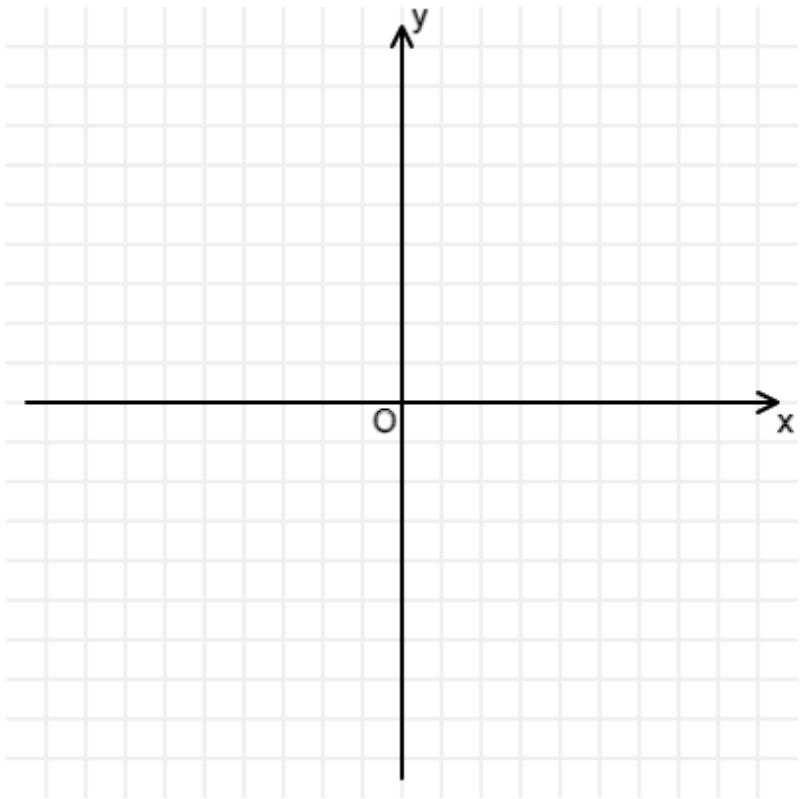
Part A   **Sketch** $\arcsin \frac{x}{3} + \frac{\pi}{2}$
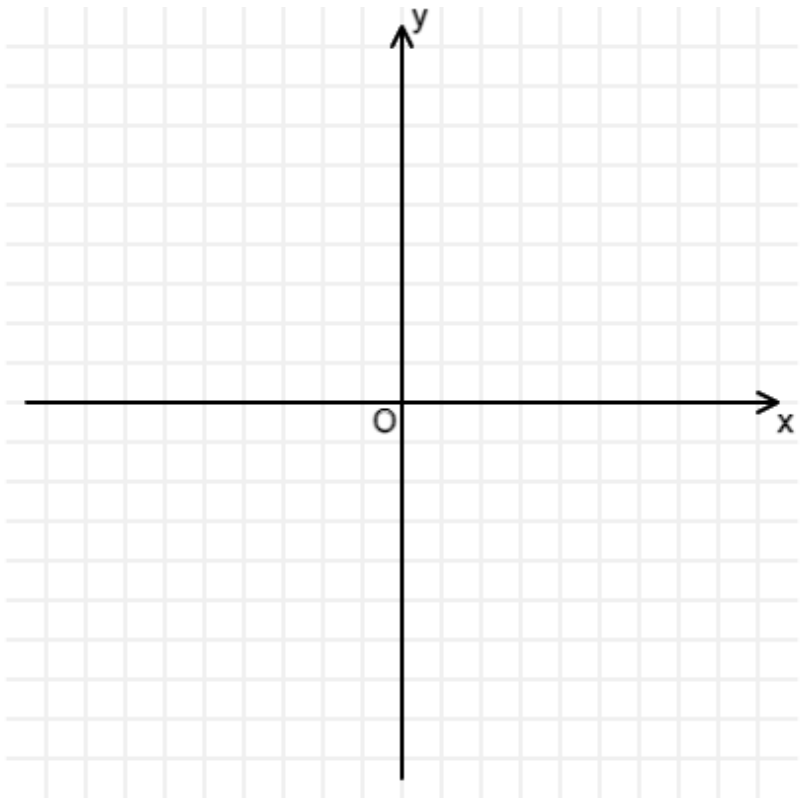
Sketch the graph of $y = \arcsin \frac{x}{3} + \frac{\pi}{2}$.

Part B   **Sketch** $2 \arctan(x-1)$

Sketch the graph of $y = 2\arctan(x-1)$.

Part C   **Sketch** $\arccos\left(-\frac{x}{2} + \frac{1}{2}\right) - \frac{\pi}{3}$

Sketch the graph of $y = \arccos\left(-\frac{x}{2} + \frac{1}{2}\right) - \frac{\pi}{3}$.

Functions: Reciprocal Trig 1i

Part A   Sketch

Sketch the graph of $y = \operatorname{cosec} x$ for $0 < x < 4\pi$.

> **Easier question?**

Part B β in terms of α

It is given that $\operatorname{cosec} \alpha = \operatorname{cosec} \beta$, where $\frac{1}{2}\pi < \alpha < \pi$ and $2\pi < \beta < \frac{5}{2}\pi$. By using your sketch, or otherwise, express β in terms of α.

The following symbols may be useful: `alpha, beta, pi`

Part C   Double angle $\tan$ properties

Write down the identity giving $\tan 2\theta$ in terms of $\tan \theta$.

The following symbols may be useful: `cos(), sin(), tan(), theta`

Part D $\tan\phi\cot 2\phi\tan 4\phi$

Given that $\cot\phi = 4$, find the exact value of $\tan\phi\cot 2\phi\tan 4\phi$.

The following symbols may be useful: `phi`

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t-Formulae Substitution

Further A

Part A   Substitution

Using the substitution $t = \tan \frac{\theta}{2}$, write the equation $2\cos\theta - 5\sin\theta = 2 - 5\tan\theta$ in the form $f(t) = 0$ where $f(t)$ is a polynomial with integer coefficients and degree 4.

The following symbols may be useful: `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `t`, `tan()`

Part B   Solutions

Hence find all the solutions to the equation $2\cos\theta - 5\sin\theta = 2 - 5\tan\theta$ in the range $0 \leq \theta < 2\pi$.

Give the smallest solution.

Give the second smallest solution to 3 significant figures.

Give the largest solution to 3 significant figures.

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