Exponentials and Logs

A Level

C C C

Part A Sketching

Consider the curve $y = 6 \times 5^x$, sketch it and find the value of the y intercept of the curve.

What is the value of the y intercept of the curve?

The following symbols may be useful: y

Part B Find x-coordinate

The point P on the curve $y = 9^x$ has y-coordinate equal to 150. Use logarithms to find the x-coordinate of P.

Give the x-coordinate of P to 3 significant figures.

Part C New x-coordinate

The curves $y = 6 \times 5^x$ and $y = 9^x$ intersect at the point Q.

Find the exact value of the x-coordinate at point Q, giving any logarithms in base three $(\log_3)$.

When you are entering your answer, note that $\log_a b$ can be written using `log(b,a)`.

The following symbols may be useful: `log(), x`

Solving Equations & Logs 2i

A Level

Part A Solve equation

Use logarithms to solve the equation $2^{n-3} = 18000$, giving your answer to 3 significant figures.

Part B Simultaneous equations

Solve the simultaneous equations $\log_2 x + \log_2 y = 8$ and $\log_2(\frac{x^2}{y}) = 7$

State the value of x.

The following symbols may be useful: x

State the value of y.

The following symbols may be useful: y

Used with permission from UCLES, A Level Maths, June 2015, OCR C2, Question 8

Gameboard:
STEM SMART Double Maths 14 - Logarithms, Lines, Matrices & Roots of Polynomials Revision

Physics. You work it out.

Logarithmic Plots 4

A Level

P P P

A student used a graph of $\ln y$ against x to discover that $y = e^{2x+5}$.

What were the gradient and intercept of the graph?

Part A Find the gradient

What was the gradient of the graph?

Part B Find the intercept

What was the intercept of the graph?

Adapted for Isaac Physics from NST IA Biology preparation work

Gameboard:

STEM SMART Double Maths 14 - Logarithms, Lines, Matrices & Roots of Polynomials Revision

Straight lines: gradients and normals 1i

A Level

Part A Gradient of line

Find the gradient of the line l_1 which has equation $4x - 3y + 5 = 0$.

Part B Perpendicular line

Find the equation of the line l_2, which passes through the point $(1, 2)$ and is perpendicular to the line l_1, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

The following symbols may be useful: `x, y`

Part C Midpoint

The line l_1 crosses the x-axis at P and the line l_2 crosses the y-axis at Q. Find the coordinates of the midpoint of PQ.

Enter the x-coordinate:

The following symbols may be useful: `x, y`

Enter the y-coordinate:

The following symbols may be useful: `x, y`

Part D Length of PQ

Find the length of PQ.

Used with permission from UCLES, A level, June 2005, Paper 4721, Question 9

Gameboard:

STEM SMART Double Maths 14 - Logarithms, Lines,

Matrices & Roots of Polynomials Revision

Algebra and Roots: Cubics with Substitution 1ii

Further A

The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

Part A Roots

Give the value of $\alpha + \beta + \gamma$.

The following symbols may be useful: k

Give the value of $\alpha\beta + \beta\gamma + \gamma\alpha$.

Part B $\alpha^2 + \beta^2 + \gamma^2$

Hence, find the value of $\alpha^2 + \beta^2 + \gamma^2$.

Part C Substitution

Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u in the form $au^3 + bu^2 + cu + d = 0$ where a, b, c and d are integers.

The following symbols may be useful: u

Part D $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

Adapted with permission from UCLES, A Level, June 2007, Paper 4725, Question 6.

Gameboard:

STEM SMART Double Maths 14 - Logarithms, Lines, Matrices & Roots of Polynomials Revision

Roots of Polynomials

A Level

C C C

This question is about manipulation of the roots of two polynomials.

$$x^2 + kx + 2k = 0$$

has the roots α and β, while

$$x^3 + 4x + 3 = 0$$

has the roots α', β' and γ'. Take $k \neq 0$.

Part A Roots of the quadratic

Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

The following symbols may be useful: `k, x`

Part B Substitution

Starting from the cubic equation above, use the substitution $x = \sqrt{u}$ to obtain a cubic equation in u.

The following symbols may be useful: `u`

Part C Roots of the cubic

Find an expression for $\alpha'^4 + \beta'^4 + \gamma'^4 + \alpha'\beta'\gamma'$.

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 8 and June 2015, Paper 4725, Question 10.

Gameboard:
STEM SMART Double Maths 14 - Logarithms, Lines,
Matrices & Roots of Polynomials Revision

Matrices: nxm Rules 1i

Further A

P P P

The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by $\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 2 & 4 & -1 \end{pmatrix}$

Part A $\mathbf{A} - 4\mathbf{B}$

The result of $\mathbf{A} - 4\mathbf{B}$ can be written in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Find x.

The following symbols may be useful: x

Find y.

The following symbols may be useful: y

Find z.

The following symbols may be useful: z

Part B $\mathbf{BC}$

Give the first row of the matrix given by $\mathbf{BC}$ in the form $x\ y\ z$ with a single space between x, y and z.

Give the second row of the matrix given by $\mathbf{BC}$ in the form $x\ y\ z$ with a single space between x, y and z.

Give the final row of the matrix given by $\mathbf{BC}$ in the form $x\ y\ z$ with a single space between x, y and z.

Part C $\mathbf{CA}$

In contrast to the previous part of the question, the matrix $\mathbf{CA}$ is a 1×1 matrix a.

Find a.

The following symbols may be useful: a

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 5.

Gameboard:

STEM SMART Double Maths 14 - Logarithms, Lines,

Matrices & Roots of Polynomials Revision

Matrices

Further A

C C C

This question will look at 4 matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ given by

$$\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & -3 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 4 & 6 \\ 3 & -5 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} a & 3 \\ -2 & 1 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & b \end{pmatrix}$$

where $b \neq 1$.

Part A Matrix equation

p and q satisfy the equation $p\mathbf{A} + q\mathbf{B} = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix.

Find the constant p.

The following symbols may be useful: p

Find the constant q.

The following symbols may be useful: q

Part B A singularity!

Given that $\mathbf{C}$ is singular, find the value of a.

The following symbols may be useful: `a`

Part C $\mathbf{C}^{-1}$

Given that $\mathbf{C}$ is non singular, its inverse can be written in the form $\mathbf{C}^{-1} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$.

Give an expression for $\alpha + \beta + \gamma + \delta$ as a single fraction.

The following symbols may be useful: `a`

Part D Simultaneous equations

Using the previous part, solve the following simultaneous equations

$$ax + 3y = 1$$

$$-2x + y = -1$$

Give an expression for x as a single fraction.

The following symbols may be useful: `a, x`

Give an expression for y as a single fraction.

The following symbols may be useful: `a, y`

Part E $\mathbf{D}^{-1}$

Find the determinant of $\mathbf{D}$ in terms of b.

The following symbols may be useful: b

$\mathbf{D}^{-1}$ can be written in the form

$$\begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \zeta \\ \eta & \theta & \iota \end{pmatrix}.$$

Find $\alpha - \beta + \gamma - \delta + \epsilon - \zeta + \eta - \theta + \iota$ as a single fraction.

The following symbols may be useful: b

Part F More simultaneous equations

Using the answer to the previous part, or otherwise, solve the following set of simultaneous equations

$$2x - y + z = 1$$

$$3y + z = 2$$

$$x + y + bz = 2$$

Give an expression for x in terms of b as a single fraction.

The following symbols may be useful: `b, x`

Give an expression for y in terms of b as a single fraction.

The following symbols may be useful: `b, y`

Give an expression for z in terms of b as a single fraction.

The following symbols may be useful: `b, z`

Adapted with permission from UCLES, A Level, June 2012, Paper 4725, Question 2, UCLES, A Level, June 2008, Paper 4725, Question 7 and UCLES, A Level, June 2010, Paper 4725, Question 9