Electricity Explored

Terms and symbols

V_A	Potential at point A = electrostatic potential energy per unit charge relative to reference point
$V_{AB} = V_A - V_B$	Potential difference between A and B
I_{AB}	Conventional current flow (charge per unit time) from A to B
I_A	External current entering the circuit at point A
$R_{AB} = V_{AB} / I_{AB}$	The resistance of the direct connection between A and B
$G_{AB} = 1/R_{AB}$	The conductance of the direct connection between A and B

Circuit Laws

Current	Total current flowing into a point = total current leaving it
Potential	There is a unique value of potential at each point in a circuit.

Equations

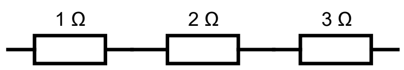
$V_{AB} = R_{AB} I_{AB}$ $\quad I_{AB} = G_{AB} V_{AB}$ $\quad$ Electric field strength $E = V_{AB} \div L_{AB}$ $\quad F = ma = qE$

Momentum $p = mv = \frac{h}{\lambda}$ $\quad$ K.E. $= \frac{1}{2}mv^2$ $\quad h = 6.63 \times 10^{-34}$ Js $\quad k_B = 1.38 \times 10^{-23}$ J/K

Electron charge: $e = 1.60 \times 10^{-19}$ C, mass $m_e = 9.11 \times 10^{-31}$ kg

Series

$V = IR$, current same for all, so $V \propto R$

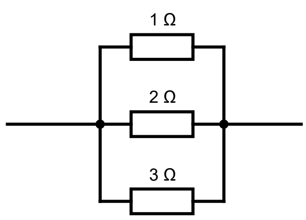


Total resistance = 6 Ω

Left resistor has 1/6 of p.d.
2nd resistor has 2/6 of p.d.
3rd resistor has 3/6 of p.d.

Parallel

$I = VG$, p.d. same for all, so $I \propto G$



1/1 S = 6/6 S

1/2 S = 3/6 S

1/3 S = 2/6 S

Total conductance = 11/6 S

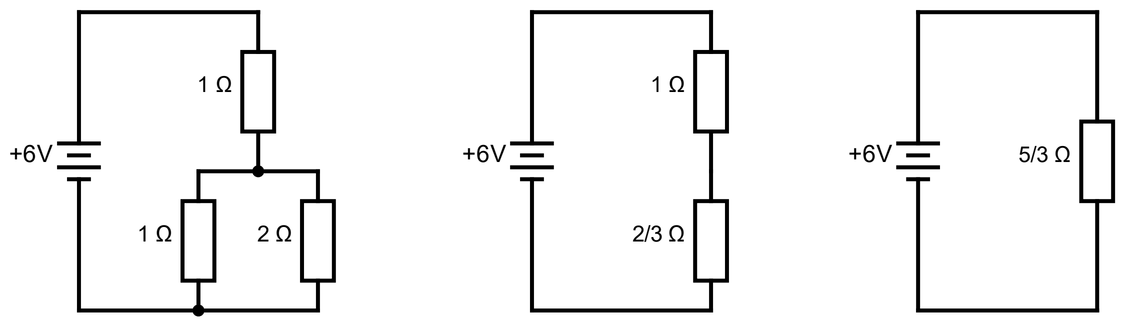
Top resistor carries 6/11 of current
2nd resistor carries 3/11 of current
3rd resistor carries 2/11 of current

Potential and Current – analogies

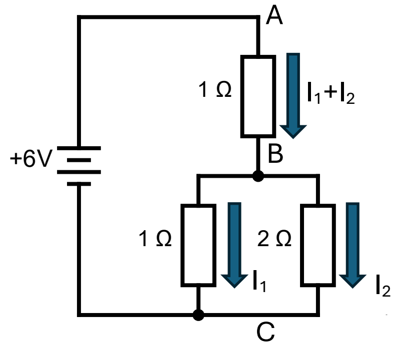
Potential	Current
Height (roller coaster)	Trains passing each hour
Pressure (of hydraulic fluid in pipe)	Flow rate of fluid (volume per unit time)
Temperature (of point in thermal system)	Heat flow rate (energy per unit time)

How to solve an electric circuit

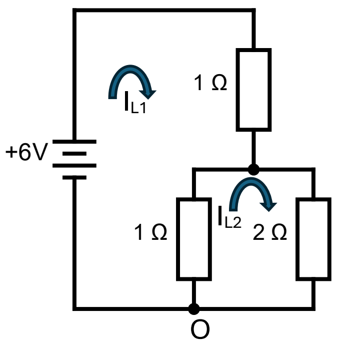
1 Combine then separate resistances



2 Raw algebra



3 Loop currents



From O, clockwise

$$6 - 1 \times I_{L1} - 1 \times (I_{L1} - I_{L2}) = 0$$

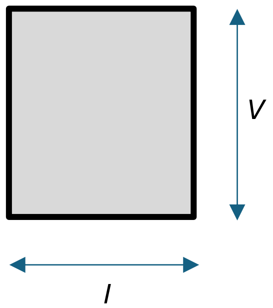
$$-1 \times (I_{L2} - I_{L1}) - 2 \times I_{L2} = 0$$

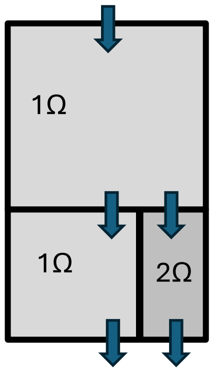
4 AVOW diagram: components are rectangles

Area =

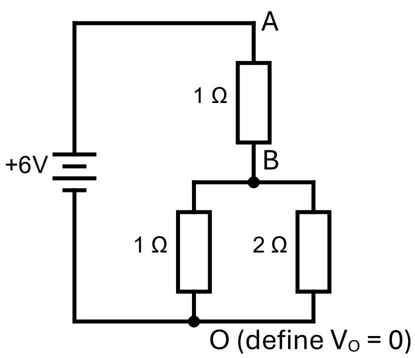
Gradient of diagonal =

Circuit laws mean that…





5 Matrix



$I_A = I_{AB}$ $\quad$ $I_A = G_{AB} (V_A - V_B) = 1 \times (V_A - V_B)$

$I_B = I_{BA} + I_{BO}$ $\quad$ $0 = G_{BA} (V_B - V_A) + G_{BO} (V_B - V_O)$

$0 = 1 \times (V_B - V_A) + (3/2) \times V_B$

$$\begin{pmatrix} I_A \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 5/2 \end{pmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix} \text{ so } \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \frac{1}{3/2} \begin{pmatrix} 5/2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} I_A \\ 0 \end{pmatrix}$$

In general: (this matrix for 4 points O, A, B, C)

$$\begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix} = \begin{pmatrix} G_{AB} + G_{AC} + G_{AO} & -G_{AB} & -G_{AC} \\ -G_{AB} & G_{BA} + G_{BC} + G_{BO} & -G_{BC} \\ -G_{AC} & -G_{BC} & G_{CA} + G_{CB} + G_{CO} \end{pmatrix} \begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix}$$

Kirchhoff's Laws 5

GCSE P P P A Level P P P

Essential Pre-Uni Physics C4.5

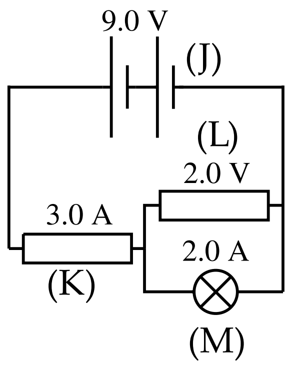


Figure 1: Circuit diagram

Part A Current in (J)

What is the current in (J)?

Part B Voltage across (K)

What is the voltage across (K)?

Part C Current in (L)

What is the current in (L)?

Home Gameboard Physics Electricity Resistors Potential Dividers 4

Potential Dividers 4

Essential Pre-Uni Physics C5.4

GCSE P P P A Level P P P

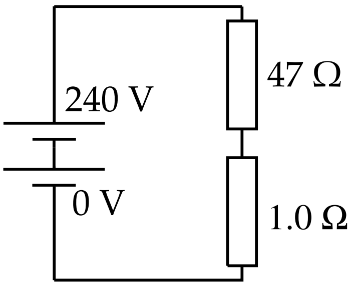


Figure 1: Circuit diagram

What is the voltage across the lower resistor in the circuit?

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Current Division 2

A Level

Linking Concepts in Pre-Uni Physics 9.2

A $9.0\,\Omega$ resistor is connected in parallel with a $81\,\Omega$ resistor. What fraction of the total current flows through the $81\,\Omega$ resistor?

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A Power Problem

GCSE A Level

The circuit shown in the figure below is made up of a battery connected to a set of resistors with different values of resistance. However, if a power of over $2.00\ \mathrm{W}$ is dissipated in one of these resistors, that resistor will fail.

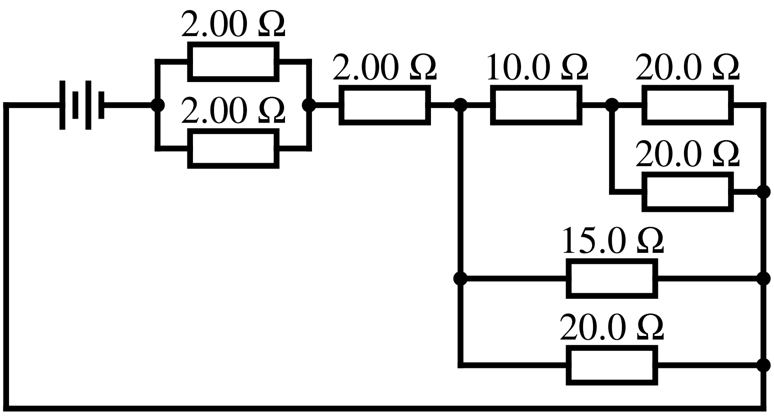

Figure 1: Circuit diagram showing a resistor network with the resistance values on the resistors.

What is the maximum voltage of the battery that can be used without any of the resistors failing?

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Current Between Resistors

GCSE C C C A Level C C C

The figure below shows a circuit diagram with an ideal ammeter connecting two branches of resistors.

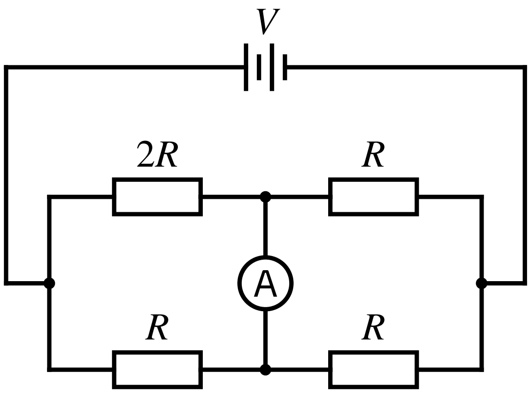

Figure 1: Circuit diagram showing two branches of resistors connected by an ideal ammeter.

What is the reading on the ammeter?

The following symbols may be useful: `R, V`

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Seven Resistors

GCSE C C C A Level C C C

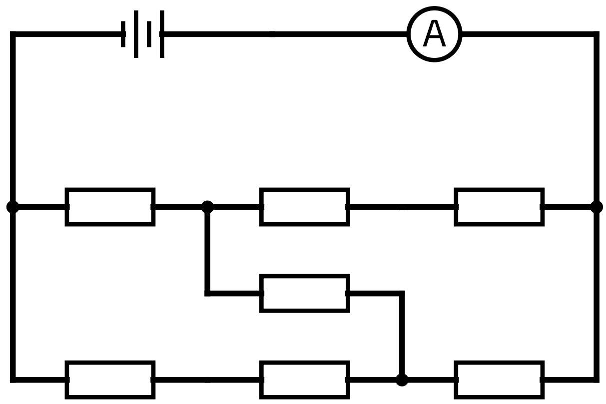

Figure 1: A circuit diagram which uses seven identical resistors.

The circuit shown above comprises seven identical resistors. The current flowing through the ammeter is $5.0\ \mathrm{A}$. What is the current flowing through the central resistor?

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Ohm's Law Unpacked

Further A

C C C

In this question, you will use ideas about electric current and electric fields to understand how quantities like resistance depend on fundamental properties of a metal.

We start with an assumption: that electrons in a metal accelerate as a result of the potential difference provided by the battery, stop as a result of a collision, and are then re-accelerated in a cycle which repeats. The average time between collisions is called the **relaxation time**.

Part A Electron Acceleration

An electron in a wire of length L has mass m and charge e. The wire is used in a circuit, and the potential difference between the two ends of the wire is V. Write an expression for the acceleration of the electron.

The following symbols may be useful: `L, V, e, m`

Part B Accelerated Motion

The electron accelerates from rest. What is its speed after time t?

The following symbols may be useful: `L, V, e, m, t`

How far will the electron move during this time?

The following symbols may be useful: `L, V, e, m, t`

Part C The Drude Model

In practice, the electrons do not all collide after the same amount of time. A useful model of the situation is to assume that the probability of a collision at time t after the previous collision is proportional to $e^{-t/\tau}$ where τ is the average time between collisions. The time τ is called the relaxation time.

When this is taken into account, the average distance travelled between collisions is given by

$$\frac{eV\tau^2}{mL}.$$

Using this assumption, what is the average speed of the electrons? Give your answer in terms of e, V, m, L and the average time between collisions τ.

The following symbols may be useful: `L, V, e, m, tau`

Part D Current

Current I is given by the equation

$$I = Anev$$

where A is the cross sectional area of the conductor, n is the number of charge carriers (for example, electrons) per unit volume, e is the charge of the charge carrier and v is the drift (average) velocity of the charge carriers.

For a typical metal, $n \approx 10^{29}\,\mathrm{m}^{-3}$.

Use your answer for the drift velocity in the last question to write an expression for the current I.

The following symbols may be useful: `A, L, V, e, m, n, tau`

For metals held at a steady temperature, the voltage is proportional to the current. What does this equation tell us about the relaxation time τ as the voltage increases?

- As the voltage and current increase, the relaxation time gets longer.
- As the voltage and current increase, the relaxation time gets shorter.
- As the voltage and current increase, the relaxation time stays the same.

Part E Resistance

Write an expression for the resistance R of the wire using your expression for the current in the last question.

The following symbols may be useful: `A, L, e, m, n, tau`

Part F An intrinsic quantity

Your expression for the resistance R is of the form

$$R = \frac{kL}{A}$$

where k depends only on the material of which the conductor is made (not its size or shape). What is the name of the quantity k? (Please note that this quantity is not usually given the letter k.)

Write an expression for the quantity k using your expression for the resistance R.

The following symbols may be useful: `e, m, n, tau`

Part G Relaxation Time

For copper, $k = 1.77 \times 10^{-8}\,\Omega\,\mathrm{m}$ and $n = 8.49 \times 10^{28}\,\mathrm{m}^{-3}$. Work out the relaxation time τ using your expression for k.

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Fermi Velocity

Further A

C C C

In this question you will find out the speed of the electrons which are responsible for energy transfer in a circuit. You may be surprised how quickly they travel! You will use ideas from your understanding of standing waves and quantum physics as well as motion.

Part A One Dimensional Standing Wave

A wave bounces back and forth in a one-dimensional region of length L. Assuming that it can not leave the region, the amplitude of the oscillation must always be zero at the place where $x = 0$ and the place where $x = L$. What does this mean?

- Adjacent nodes must be distance $2L$ apart.
- Adjacent nodes must be a distance $\frac{2L}{n_x}$ apart where n_x is an integer.
- Adjacent nodes of the wave must be a distance L apart.
- Adjacent nodes must be a distance $\frac{L}{n_x}$ apart where n_x is an integer.

Which of the following equations correctly states the allowed wavelengths of the wave? In these expressions n_x can take any positive integer value.

- $\lambda = \frac{n_x}{L}$
- $\lambda = \frac{Ln_x}{2}$
- $\lambda = 2Ln_x$
- $\lambda = \frac{L}{n_x}$
- $\lambda = \frac{L}{2n_x}$
- $\lambda = \frac{2L}{n_x}$
- $\lambda = Ln_x$
- $\lambda = \frac{n_x}{2L}$

Part B One Dimensional Electron Wave

Use your answer to the previous question to write an expression for the allowed values (magnitudes) for the momentum of an electron, if its wave needs to have a node at $x = 0$ and $x = L$. Give your answer in terms of the Planck constant h, the length L, and use n_x to represent a positive integer.

The following symbols may be useful: `L, h, n_x`

Write an expression for the allowed values for the speed of the electron. Give your answer in terms of the Planck constant h, the length L, the electron mass m, and use n_x to represent a positive integer.

The following symbols may be useful: `L, h, m, n_x`

Part C Three Dimensional Electron Wave

We now think about free electrons in a metal cube with length L, whose sides have been aligned with the x, y and z axes of our co-ordinates. Your answer to the previous question gives you the allowed values for the magnitude of the x-component of the velocity of an electron in the cube.

Use your answer to the previous question to write an expression for the magnitudes of the allowed y velocities of the electrons in the cube. Give your answer in terms of the Planck constant h, the length L, the electron mass m, and use n_y to represent a positive integer.

The following symbols may be useful: `L, h, m, n_y`

Using your previous answers, write an expression for the allowed kinetic energies of the electron in the cube. Give your answer in terms of h, L, the electron mass m and the three integers n_x, n_y and n_z.

The following symbols may be useful: `L, h, m, n_x, n_y, n_z`

Part D Velocity Space

We now plot the allowed velocities of the electrons as dots in three dimensions, where the axes are labelled with the x, y and z components of the velocity. Each dot corresponds to a different choice of the positive integers n_x, n_y and n_z.

When we look at a three dimensional diagram where the axes are the components of velocity, we say we are looking at the situation in **velocity space**.

Our diagram is different in that we are plotting the magnitudes of the velocities in the three dimensions - we will call our plot a diagram in **magnitude-of-velocity space**.

The points form a cubic grid in the first octant (that is the part of the diagram where n_x, n_y and n_z are all positive).

How far apart are adjacent dots in velocity space? Please note that your answer is not a distance, but a difference between allowed velocity magnitudes.

The following symbols may be useful: `L, h, m`

If you join adjacent dots with lines parallel to the x, y and z axes, you split velocity space up into lots of cubes.

Give an expression for the 'volume' of each of these cubes in velocity space.

The following symbols may be useful: `L, h, m`

Now express your answer to the 'volume' of the cubes in terms of the actual volume V of the metal cube containing the electrons rather than the cube length L.

The following symbols may be useful: `V, h, m`

Part E Electron Counting

Electrons are part of a family of particles called fermions, which all have a special property: no two of them may be in the same quantum state at the same time. If electrons did not have a magnetic moment, then this means that two (or more) electrons could not have the same values for all of n_x, n_y and n_z at the same time. Or, to put it another way, each dot in our magnitude-of-velocity space digram could either hold one electron or none.

In practice electrons have a magnetic moment which can point in one of two directions relative to an external magnetic field. This means that two electrons **can** be in the same place in velocity space at the same time - one with each of the two allowed orientations of the magnetic moment. However you can't have more than two electrons sharing the same set of values of n_x, n_y and n_z.

Under normal circumstances, the electrons in the metal will occupy the lowest energy positions in velocity space (that is the dots nearest to the origin) such that there are two electrons on each 'dot'.

Our metal cube contains N free electrons which need to be put on the 'dots' in our velocity space diagram to minimise their total energy.

How much 'volume' in our velocity space diagram is taken up by each electron?

The following symbols may be useful: `V, h, m`

How much 'volume' in velocity space is needed to hold N electrons if they are in the minimum energy arrangement?

The following symbols may be useful: `N, V, h, m`

Write an expression for the 'volume' in velocity space needed to hold N electrons in terms of the number density of free electrons in the metal $n = \frac{N}{V}$.

Please note that the number density n is a property of the metal and has nothing to do with the n_x, n_y and n_z used earlier in the question.

The following symbols may be useful: `h, m, n`

Part F Fermi Surface

Assuming that the N free electrons in our metal cube occupy the allowed locations in our magnitude-of-velocity space to minimise the total kinetic energy, which of the following statements best describes the region they occupy?

- A sphere with the origin just touching its outer surface.
- A cylinder with the centre of one straight face at the origin.
- A cube centred on the origin.
- A sphere, centred on the origin.
- A cube with one of its corners at the origin.
- The part of a sphere, centred on the origin, which is in the region where x, y and z are all positive.

Part G Fermi Velocity

Use your previous answers to work out the Fermi velocity v_{F} of the electrons in this metal. The Fermi velocity is the speed of the fastest electrons if they occupy the allowed positions in our magnitude-of-velocity space to minimise the total energy.

The following symbols may be useful: `h, m, n, pi`

Evaluate the Fermi velocity v_{F} for free electrons in copper, where $n = 8.49 \times 10^{28}\,\mathrm{m}^{-3}$.

The Fermi energy E_{F} is the kinetic energy of an electron travelling at the Fermi velocity. Evaluate the Fermi energy for copper in joules.

Part H Room Temperature

In practice, the electrons could only be expected to occupy the lowest energy positions in velocity space at a very low temperature. At room temperature T, electrons with energies within about $k_{\mathrm{B}}T$ of the Fermi energy E_{F} are able to jump to energies outside the Fermi surface in response to collisions with ion cores in the metal.

Evaluate $\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}$ for electrons at $20^{\circ}\mathrm{C}$ to two significant figures.

Using this information, what can you conclude about the free electrons in the metal which are able to engage in collisions (and thereby change velocity)?

- They are all moving at exactly the Fermi velocity.
- They are moving very slowly - with a kinetic energy of about one hundredth of the Fermi Energy.
- To within two significant figures, they are all moving at the Fermi velocity.
- They are moving with a range of speeds varying from zero all the way up to the Fermi energy (and a little bit beyond).

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