Differentiation: Implicit 3ii

A Level

Part A Derivative

For the curve $2x^2 + xy + y^2 = 14$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

The following symbols may be useful: `Derivative(y, x)`, `ln()`, `log()`, `x`, `y`

Part B Stationary points

Find the coordinates of the points at which the tangents to the curve $2x^2 + xy + y^2 = 14$ are parallel to the x -axis.

(,)

Differentiation: Implicit 5i

A Level

P P P

Part A Derivative

Given that $y \sin 2x + \frac{1}{x} + y^2 = 5$, find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

The following symbols may be useful: `Derivative(y, x)`, `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `tan()`, `x`, `y`

Part B Gradient

Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point $(1, 2)$.

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Differentiation: Implicit 3i

A Level

The equation of a curve is $xy^2 = 2x + 3y$.

Part A Implicit Differentiation

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

The following symbols may be useful: `Derivative(y, x), x, y`

Part B Tangents

Give the number of tangents to this curve which are parallel to the y-axis.

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Sketching a Parametric Curve

A Level

A curve has parametric equations $x = 1 - \cos t$, $y = \sin t \sin 2t$, for $0 \leq t \leq \pi$.

Part A x-axis

Find the coordinates of the points where the curve meets the x-axis.

(,)

Part B Derivative

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

The following symbols may be useful: `Derivative(y, x)`, `arccos()`, `arccosec()`, `arccot()`, `arcsec()`, `arcsin()`, `arctan()`, `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `t`, `tan()`, `x`, `y`

Part C Stationary points

Hence find the coordinates of the stationary points. Give your answer to 3 significant figures.

(,)

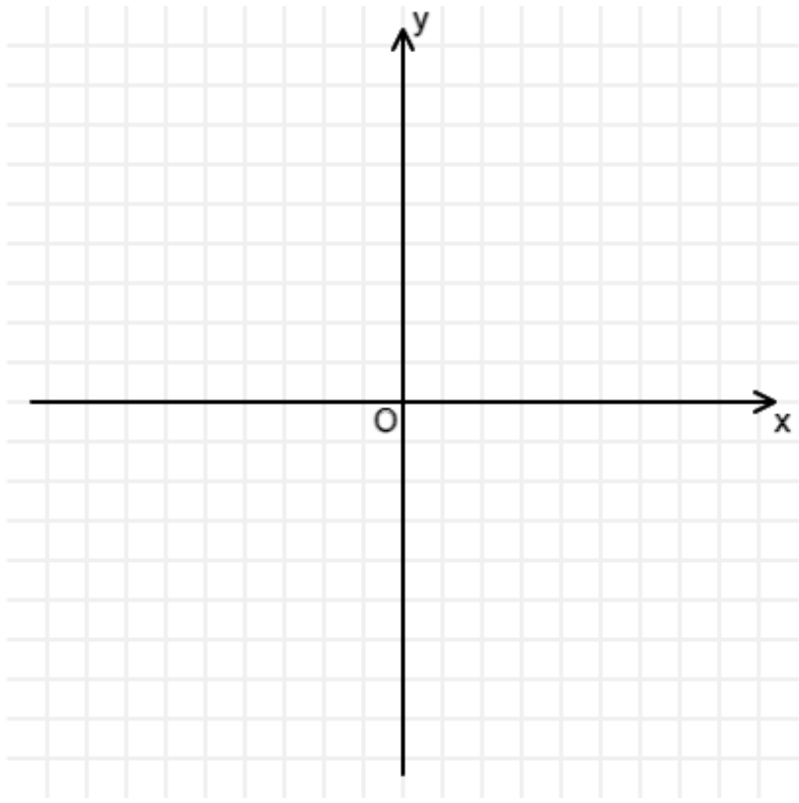
Part D Cartesian equation

Find the cartesian equation of the curve. Give your answer in the form $y = f(x)$, where $f(x)$ is a polynomial.

The following symbols may be useful: `x, y`

Part E Sketch

Sketch the curve.



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Parametric Equations 2i

A Level

A curve has parametric equations

$$x = \frac{1}{t+1}, y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

Part A Value of t

Show that the value of t at one of these points is -2 and find the value of t at the other point.

The following symbols may be useful: `t`

Part B Normal

Find the equation of the normal to the curve at the point for which $t = -2$, giving your answer in the form $y = f(x)$.

The following symbols may be useful: `x, y`

Part C Value of t

Find the value of t at the point where this normal meets the curve again.

The following symbols may be useful: `t`

Part D Cartesian Equation

Find a cartesian equation of the curve, giving your answer in the form $y = f(x)$.

The following symbols may be useful: `x, y`

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Parametric Equations 3i

A Level

The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, y = 4\sin\theta$$

and part of its graph is shown in **Figure 1**.

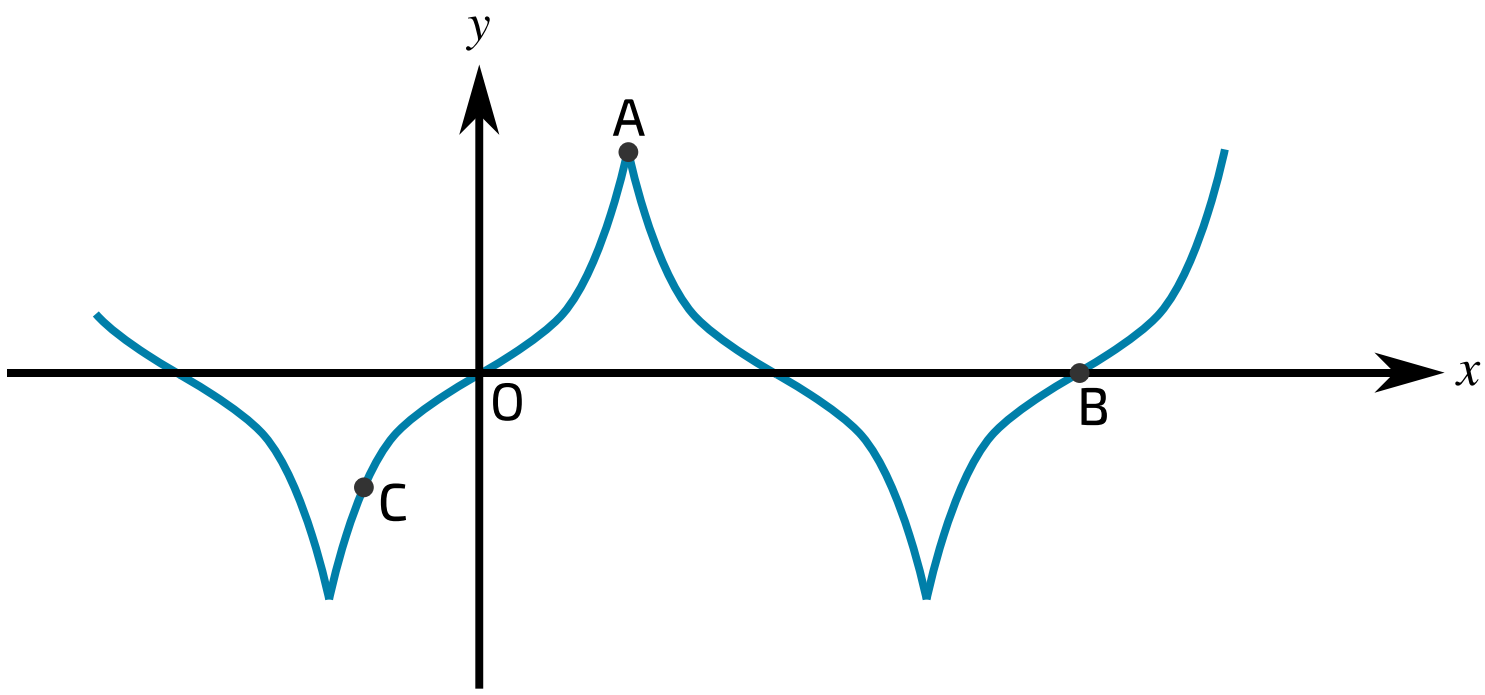

Figure 1: A sketch of the curve.

Part A Value of θ at A

Find the value of θ at A.

The following symbols may be useful: `pi, theta`

Part B Value of θ at B

Find the value of θ at B.

The following symbols may be useful: `pi, theta`

Part C Derivative

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ.

The following symbols may be useful: `Derivative(y, x), arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), sec(), sin(), tan(), theta, x, y`

Part D Coordinates

At the point C on the curve the gradient is 2. Find the coordinates of C, giving your answer to 3 significant figures.

(,)

Part E Nature of Origin

Point O is at the origin. State the nature of point O, justifying your answer by reference to suitable values of $\frac{\mathrm{d}y}{\mathrm{d}x}$.

At O, we find that $\theta =$ [ ] and $\frac{\mathrm{d}y}{\mathrm{d}x} =$ [ ]. Hence, O is not a stationary point.

When $\theta = 0.1$, we find that $x =$ [ ] (2 sf) and $\frac{\mathrm{d}y}{\mathrm{d}x} =$ [ ] (4 sf).

When $\theta = -0.1$, we find that $x =$ [ ] (2 sf) and $\frac{\mathrm{d}y}{\mathrm{d}x} =$ [ ] (4 sf).

Since $\frac{\mathrm{d}y}{\mathrm{d}x}$ is [ ] on both sides of O than it is at O, O must be a non-stationary [ ].

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Parametric Integration 1

A Level

The curve C has parametric equations

$$x = 2t^2 - 3 \qquad y = t(4 - t^2)$$

The curve crosses the x-axis at the points A and B and the region R is enclosed by the loop of the curve, as shown in **Figure 1**.

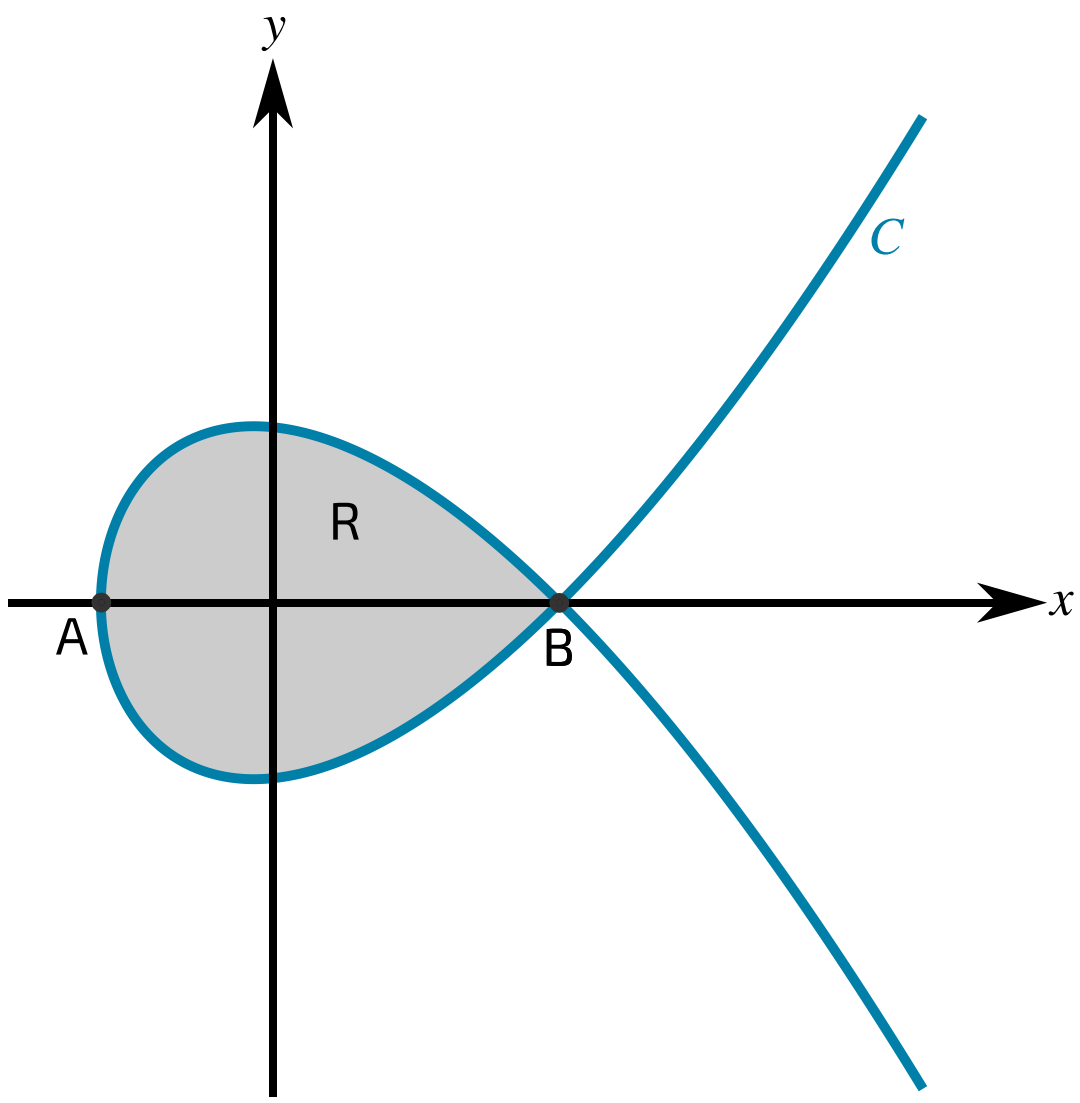

Figure 1: A graph of the curve C.

Part A Point A

Find the x-coordinate of the point A.

Part B Point B

Find the x-coordinate of the point B.

Part C Area of R

The region R is enclosed by the loop of the curve, as shown in **Figure 1**. Find the exact value of the area of R .

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Partial Fractions 1i

A Level

P P P

Part A Partial Fractions

Express $\frac{2+x^2}{(1+2x)(1-x)^2}$ in the form $\frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$.

The following symbols may be useful: x

Part B Integration

Hence find $\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2}\,\mathrm{d}x$ in exact form.

The following symbols may be useful: , _logs

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Integration with Partial Fractions 4

Further A

Pre-Uni Maths for Sciences K5.4

Part A Find A, B and C

Write the function $\dfrac{2z^2 - z - 3}{(z+2)(z^2 - 2z - 1)}$ in the form $\dfrac{A}{z+2} + \dfrac{B + Cz}{z^2 - 2z - 1}$.

Drag and drop the correct values in the expression below.

$$\frac{\square}{z+2} + \frac{\square + \square z}{z^2 - 2z - 1}$$

Items:

-5 -4 -3 -2 -1 0 1 2 3 4 5

Part B Integrate

Hence find $\displaystyle\int_1^2 \frac{2z^2 - z - 3}{(z+2)(z^2 - 2z - 1)}\,\mathrm{d}z$.

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