



Physics. *You work it out.*

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Probability 5.5

A Level Further A
P P P P P P

Assuming they all follow a normal distribution find the following probabilities.

Part A $X \sim N(10, 4)$

If $X \sim N(10, 4)$ find:

$P(X \leq 9)$. Give your answer to 3 sf.

$P(X \geq 15)$. Give your answer to 3 sf.

Part B $X \sim N(15, 0.3)$

If $X \sim N(15, 0.3)$ find:

$P(X < 14)$. Give your answer to 3 sf.

$P(14 < X < 16)$. Give your answer to 3 sf.

Part C S normally distributed

Assuming that the variable S follows a normal distribution with mean 40 and variance 16 find:

$P(S > 44)$. Give your answer to 3 sf.

$P(|S - 40| < 5)$. Give your answer to 3 sf.

Part D Points of inflection

The points of inflection of a normal distribution curve occur at -2 and 6 .

Deduce the mean of the distribution.

Deduce the variance of the distribution.

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Probability 5.6

A Level

Further A

P

P

P

P

P

P

Assuming they all follow a normal distribution find the following.

Part A $X \sim N(18, 9)$

Consider the normal distribution $X \sim N(18, 9)$.

Find the Z -score of the value $x = 12$.

Find the Z -score of the value $x = 21$.

Part B $X \sim N(90, 100)$

Consider the normal distribution $X \sim N(90, 100)$.

Find the value of X that is 3 standard deviations above the mean.

If $x = 0$, find how many standard deviations it is from the mean. (If $x = 0$ is above the mean, enter a positive number; if $x = 0$ is below the mean, enter a negative number.)

Part C $X \sim N(\mu, 16)$

Consider the normal distribution $X \sim N(\mu, 16)$. When $x = 17$, it is 2.5 standard deviations below the mean μ ; find μ .

Part D $X \sim N(100, \sigma^2)$

Consider the normal distribution $X \sim N(100, \sigma^2)$. When $x = 82$, it is 3 standard deviations below the mean; find σ^2 .

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Probability 5.8

A Level Further A
P P P P P P

The number of gamma-rays emitted by a radioactive sample in a given time period follows a normal distribution and has a mean of 500.8 and standard deviation of 7.1. Find the following.

Part A 30% of the measurements $< p$

30% of the measurements have a value less than p ; find the value of p . Give your answer to 4 sf.

Part B 15% of the measurements $> q$

15% of the measurements have values greater than q ; find the value of q . Give your answer to 4 sf.

Part C Six successive measurements

Six measurements are made successively.

Find the probability that none of the 6 measurements are greater than 515. Give your answer to 3 sf.

Find the probability that exactly 4 of the 6 are less than 510. Give your answer to 3 sf.

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Probability 5.9

A Level Further A
P P P P P P

Answer the following questions. You may assume in all cases that the values of the quantities follow a normal distribution.

Part A A batch of batteries

A batch of batteries has a mean voltage of 1.50 V and standard deviation σ . It is found that 20% have a voltage less than 1.47 V . Deduce the value of σ , giving your answer to 3 sf.

Part B Falling times

A number of students measure the time it takes for an object to fall a certain distance. The distribution of times has a standard deviation of 0.095 s . It is found that 10% of the measurements exceed 5.80 s . Find the mean time taken, giving your answer to 3 sf.

Part C A batch of lenses

An experimenter has a batch of 150 lenses. They find that 10 of them have focal lengths less than 14.7 cm and 6 have focal lengths greater than 15.6 cm .

Find the mean focal length giving your answer to 3 sf.

Find the variance of the focal lengths, giving your answer to 3 sf.

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Probability 5.7

A Level Further A
P P P P P P

Ball bearings produced by a machine have diameters which have a normal distribution with mean 6.50 mm and standard deviation 0.07 mm .

Part A Number of ball bearings rejected

A random sample of 1000 of these ball bearings is selected. Find how many will be rejected if the tolerance required is in the range $6.40 - 6.60 \text{ mm}$.

Part B Selection using a grid

The ball bearings are passed over a grid and those with diameters $< 6.37 \text{ mm}$ fall through this grid. A sample of 1000 bearings is selected from those that have passed over the grid.

Find how many bearings will be rejected if the tolerance required is in the range $6.40 - 6.60 \text{ mm}$.

Find how many of these will have diameters which exceed the tolerance limit (i.e. have diameters $> 6.60 \text{ mm}$).

Given that 1000 bearings have passed over the grid, how many were there to start with?

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Data Analysis 5.4

A Level



The distribution of the velocities of gas molecules in one dimension is given by a normal distribution with mean zero and variance which is proportional to the temperature of the gas.

At room temperature, $T_1 = 300 \text{ K}$, 20% of the molecules have speeds greater than 370 m s^{-1} . At a higher temperature, T_2 , 30% of the molecules have speeds greater than 370 m s^{-1} . Answer the following questions and hence deduce the value of T_2 .

Part A σ_1 at T_1

At room temperature, $T_1 = 300 \text{ K}$, 20% of the molecules have speeds greater than 370 m s^{-1} ; find the value of the standard deviation in the velocities σ_1 . Give your answer to 2 sf.

Part B σ_2 at T_2

At T_2 , 30% of the molecules have speeds greater than 370 m s^{-1} ; find the value of the standard deviation in the velocities σ_2 . Give your answer to 2 sf.

Part C The value of T_2

Use your answers to parts A and B and the information given above to deduce the temperature T_2 . Give your answer to 2 sf.

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Data Analysis 5.9

A Level



To indicate that an observation X comes from a normal distribution with mean μ and standard deviation σ the following notation is used

$$X \sim N(\mu, \sigma^2).$$

If we look at a sample of n independent observations of X and calculate the mean \bar{X}_n , then \bar{X}_n also comes from a normal distribution where

$$\bar{X}_n \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

i.e. the standard deviation for the distribution of the means is σ/\sqrt{n} .

Using this notation answer the following questions.

Part A $X \sim N(0, 1)$

An observation X comes from a normal distribution $X \sim N(0, 1)$. (i) Write down the expression for the distribution of sample means \bar{X}_5 . Find the probability that (ii) $X > 0.8$ and (iii) $\bar{X}_5 > 0.8$.

(i) The distribution of the sample means is given by $\bar{X}_5 \sim N(0, a)$. What is the value of a ?

(ii) Find the probability that $X > 0.8$. Give your answer to 3 sf.

(iii) Find the probability that $\bar{X}_5 > 0.8$. Give your answer to 2 sf.

Part B $X \sim N(10, 4)$

An observation X comes from a normal distribution $X \sim N(10, 4)$. (i) Write down the expression for the distribution of sample means \bar{X}_{10} . Find the probability that (ii) $X < 11$ and (iii) $\bar{X}_{10} < 11$.

(i) The distribution of the sample means is given by $\bar{X}_{10} \sim N(10, b)$. What is the value of b ?

(ii) Find the probability that $X < 11$. Give your answer to 3 sf.

(iii) Find the probability that $\bar{X}_{10} < 11$. Give your answer to 3 sf.

Part C $X \sim N(8, 100)$

An observation X comes from a normal distribution $X \sim N(8, 100)$. (i) Write down the expression for the distribution of sample means \bar{X}_4 . (ii) Find the probability that $-5 < \bar{X}_4 < 13$.

The distribution of the sample means is given by $\bar{X}_4 \sim N(8, c)$. What is the value of c ?

Find the probability that $-5 < \bar{X}_4 < 13$. Give your answer to 3 sf.

Part D $X \sim N(-10, 3)$

An observation X comes from a normal distribution $X \sim N(-10, 3)$. (i) Write down the expression for the distribution of sample means \bar{X}_6 . (ii) Find the probability that $-10 < \bar{X}_6 < -8.5$.

(i) The distribution of the sample means is given by $\bar{X}_6 \sim N(-10, d)$. What is the value of d ?

(ii) Find the probability that $-10 < \bar{X}_6 < -8.5$. Give your answer to 3 sf.

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Physics. *You work it out.*

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Hypothesis Testing: Normal Distribution 1

A Level



A species of chicken is known to produce eggs which have a mass that follows a normal distribution with a mean mass of 55.0 g and a variance of 5.00 g^2 .

A chicken-keeper has 1000 birds. Each bird lays one egg per day, and on one particular day the chicken-keeper collects the eggs from 125 birds and finds a mean mass of 54.0 g. The chicken-keeper wants to know whether their birds produce eggs with a below average mass.

Test, at the 5% significance level, whether this is the case.

Part A Assumptions

In order to carry out a hypothesis test, which two of the following options do you need to assume?

- ☐ The masses of the eggs produced by the chicken-keeper's birds follow a binomial distribution.
- ☐ The masses of the eggs produced by the chicken-keeper's birds are normally distributed.
- ☐ The variance of the masses of the eggs produced by the chicken-keeper's birds is 1000 times than that of the population as the chicken keeper has 1000 birds.
- ☐ The variance of the masses of the eggs produced by the chicken-keeper's birds is the same as that of the whole chicken population.

Part B The null and alternative hypotheses

Drag and drop into the spaces below to define the variables and state the null and alternative hypotheses for this test.

Let M be the mass of an egg produced by the chicken-keeper's birds, and let μ be the mean mass of the eggs produced by the chicken-keeper's birds. Then $M \sim$ (μ ,).

H_0 :

H_1 :

Items:

μ

B

=

>

Po

<

54.0 g

N

55.0 g

5.00

Part C The distribution of the sample means

Fill in the blanks below to complete the description of the distribution of the sample means.

The masses of the eggs produced by the chicken-keeper's birds have a distribution. Therefore, if samples of the these eggs are taken, the mean values of these samples, \bar{M} , have a distribution.

The masses of the eggs produced by the chicken-keeper's birds have a variance of . For samples containing 125 eggs, the variance of the sample is therefore .

Hence, under the assumption that the null hypothesis is true, the sample distribution is $\bar{M} \sim$ (,).

Items:

55.0

5.00 g^2

0.04

B

$\frac{\sigma^2}{n} = \frac{5.00}{1000} = 0.005 \text{ g}^2$

0.005

N

$\frac{\sigma^2}{n} = \frac{5.00}{125} = 0.04 \text{ g}^2$

normal

binomial

Part D Carrying out the test

Choose three options and put them into order to complete the hypothesis test.

Available items

$z = \frac{\bar{M} - \mu}{\sqrt{\sigma^2/n}}$. Hence, the boundary of the critical region is at $\bar{M} = 55.0 + 0.2 \times 1.645 = 55.33$. The critical region is therefore $\bar{M} < 55.33$.

We need to carry out a one-tailed test at the 5% significance level. Using the inverse normal distribution, the z -value at the upper boundary of the critical (rejection) region is $z = \Phi^{-1}(0.05) = -1.645$.

We need to carry out a one-tailed test at the 5% significance level. Using the inverse normal distribution, the z -value at the upper boundary of the critical (rejection) region is $z = \Phi^{-1}(0.95) = 1.645$.

$z = \frac{\bar{M} - \mu}{\sqrt{\sigma^2/n}}$. Hence, the boundary of the critical region is at $\bar{M} = 55.0 + 0.2 \times -1.645 = 54.67$. The critical region is therefore $\bar{M} < 54.67$.

Therefore, as the value of the sample mean (54.0 g) lies in the acceptance region, we do not reject the null hypothesis. There is no significant evidence that the chicken-keeper's birds produce eggs with a below average mass.

Therefore, as the value of the sample mean (54.0 g) lies in the critical region, we reject the null hypothesis. There is evidence that the chicken-keeper's birds produce eggs with a below average mass.



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Hypothesis Testing: Normal Distribution 2

A Level



Using a single black ink cartridge, a specific type of large printer is known to print a mean of 10 750 pages of black and white text, with a variance of 90 000 pages².

The company that makes the printer changes some of its software in an attempt to make the use of ink by their printers more efficient. They perform 80 tests on printers using the updated software and calculate a mean of 10 834 pages per ink cartridge.

Test at the 2% significance level whether the new software has improved the ink efficiency.

Part A Assumptions

Which of the following options do you need to assume in order to perform a hypothesis test? Select all that apply.

- ☐ That the variance in the number of pages printed in the sample of 80 tests is much smaller than the variance when using the old software, because 80 is much smaller than the number of printers using the old software.
- ☐ That the variance of the sample obeys a Poisson distribution.
- ☐ That the number of pages printed using one ink tank has a normal distribution.
- ☐ That the variance in the number of pages printed using the new software is the same as the variance when using the old software.

Part B Carrying out the test

Fill in the blanks to complete the description of the hypothesis test.

Let X be the number of pages printed using one cartridge with the new software. Then $X \sim N(\mu, \text{[]})$, where μ is the mean.

The null hypothesis is that the new software does not improve ink efficiency, and the mean number of pages per cartridge is the same as before. The alternative hypothesis is that the new software improves ink efficiency.

$$H_0 : \mu = 10750 \quad H_1 : \text{[]}$$

Let \bar{X} be the mean number of pages printed using one cartridge with the new software. Assuming that the null hypothesis is true, then $\bar{X} \sim N(10\,750, \text{[]})$.

Using the z -statistic $z = \frac{\bar{X} - \mu}{\sqrt{\sigma^2/n}}$, the p -value for a sample mean of 10834 pages is found to be

$$p = P(\bar{X} \geq 10\,384) = P(z \geq \frac{10\,834 - 10\,750}{\sqrt{\text{[]}/80}}) = \text{[]}$$

For a one-tailed test at the 2% significance level, p -values of less than 0.02 are in the critical (rejection) region. The calculated p value for the sample is [] 0.02.

Therefore, [] the null hypothesis. There [] significant evidence that the new software improves ink efficiency.

Items:

is

300

less than

$\mu > 10\,750$

90 000

0.3897

is not

$\mu < 10\,750$

greater than

do not reject

0.006133

reject

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