Confidence Intervals - Scales

Further A

A set of bathroom scales is known to operate with an error which is normally distributed. One morning a man weighs himself 4 times. The 4 values for his mass, in kg, which can be considered to be a random sample are as follows:

$$62.6,\ 62.8,\ 62.1,\ 62.5$$

Part A Confidence interval

Find a 95% confidence interval for his mass.

Give the lower confidence limit to 3 s.f.

Give the upper confidence limit to 3 s.f.

Part B Width of a confidence interval

Based on these results, a $y\%$ confidence interval has a width of 0.4822. Find y.

Used with permission from UCLES, A Level, June 2015, Paper 4734/01, Question 4

t-tests: Chemicals

Further A

P P P

The senior technician of a university science department is considering using a new supplier of chemicals for student laboratory experiments. These chemicals do not need to be of a very high grade of purity, but must be sufficiently pure that the experiments can be conducted efficiently and safely.

The new supplier states that the average level of impurity in the chemicals it supplies is no more than 4%. The senior technician carefully measures the impurity in a random sample of eight containers of these chemicals and finds the percentages to be as follows.

$$3.9 \quad 4.5 \quad 4.7 \quad 4.3 \quad 4.9 \quad 3.4 \quad 4.5 \quad 5.0$$

Part A Null and alternative hypotheses

State the appropriate null and alternative hypotheses for the usual t-test for examining whether the supplier is meeting the stated standard.

$$\mathrm{H}_0 : \square \; \square \; 4 \qquad \mathrm{H}_1 : \square \; \square \; 4$$

Items:

$>$ s $<$ σ $\neq$ μ $=$ $\overline{x}$

Part B $\mathrm{N}(0, 1)$ distribution

Explain why the corresponding test based on the $\mathrm{N}(0, 1)$ distribution cannot be used. Select all that apply.

- [ ] The true value of μ is unknown.
- [ ] The sample is small.
- [ ] The sample is large.
- [ ] The true value of σ^2 is unknown.

Part C Necessary condition

What condition is necessary for the correct use of the t-test?

- The underlying distribution is uniform.
- The underlying distribution is unknown.
- The underlying distribution is normal.

Part D t-test

Carry out the t-test, using a 5% significance level. Fill in the gaps below.

The test statistic, $t =$ ______. The critical value at the 5% level is $t_{\text{crit}} =$ ______.

Comparing these, we find that t ______.

Therefore we ______ H_0 at the 5% level. There ______ evidence to suggest that the average level of impurity in the chemicals is ______ the supplier stated.

Items:

higher than | 1.860 | 0.606 | **lower than** | 1.895 | $> t_{\text{crit}}$ | $< t_{\text{crit}}$ | 2.274 | **do not reject**

is insufficient | 2.365 | **is** | 2.306 | **not as** | $\neq t_{\text{crit}}$ | 1.990 | 2.127 | **reject**

Part E Confidence interval

Provide a two-sided 95% confidence interval for the true mean percentage of impurity in the supplier's chemicals.

Give the lower bound of the interval. Give your answer to 3 s.f.

Give the upper bound of the interval. Give your answer to 3 s.f.

t-test: Geography Fieldwork

Further A

A group of students were tested in Geography before and after a fieldwork course. The marks of 10 randomly selected students are shown in the table.

Student	A	B	C	D	E	F	G	H	I	J
Mark before fieldwork	19	84	84	99	59	19	29	49	54	69
Mark after fieldwork	23	98	83	88	68	33	28	53	58	88

Part A t-test

Use a suitable t-test, at the 5% level of significance, to test whether the students' performance has improved. Fill in the gaps below.

The null and alternative hypotheses are:

$$\mathrm{H}_0 : \mu_2 \; \square \; \mu_1 \qquad \mathrm{H}_1 : \mu_2 \; \square \; \mu_1$$

The test statistic, $t =$ ______. The critical value is $t_{\text{crit}} =$ ______.

Comparing these, we find that t ______ t_{crit}.

Therefore we ______ H_0 at the 5% level. There ______ evidence to suggest that the fieldwork has improved the students' performance.

Items:

is insufficient | 2.262 | 2.228 | 1.829 | **reject** | 1.969 | $<$ | **do not reject** | 1.833 | $>$ | $=$ | 2.075 | 1.812 | **is**

Part B Assumption

Which of these is a necessary assumption in applying the test?

- The differences are normally distributed.
- The differences have a positive mean.
- The differences are uniformly distributed.

Used with permission from UCLES, A Level, June 2016, Paper 4734/01, Question 4

Gameboard:

STEM SMART Double Maths 47 - t-tests & Quality of Tests

Quality of Tests: Accidents

Further A

The number of accidents per month at a certain road junction has a Poisson distribution with mean 4.8. A new road sign is introduced warning drivers of the danger ahead, and in a subsequent month 2 accidents occurred.

Part A Hypothesis test

A hypothesis test at the 10% level is used to determine whether there were fewer accidents after the new road sign was introduced.

Find the critical region for this test and carry out the test. Fill in the gaps below.

Let the random variable X be the number of accidents per month at the road junction. Then $X \sim \mathrm{Po}(\lambda)$.

The null and alternative hypotheses are:

$$\mathrm{H}_0 : \lambda \ \square \ \square \qquad \mathrm{H}_1 : \lambda \ \square \ \square$$

The critical region is $\square \leq X \leq \square$.

$\square$ is $\square$ the critical region.

Therefore we $\square$ H_0. There $\square$ evidence to suggest that the road sign has decreased accidents.

Items:

$<$ | 4 | $=$ | 5 | 3 | 2.8 | 1 | **do not reject** | 2 | 4.8 | **is insufficient** | **reject** | **is** | **within** | **not in** | 0 | $>$

Part B Type I error

Find the probability of a Type I error. Give your answer to 3 s.f.

Quality of Tests: Plant Research

Further A

In a research laboratory where plants are studied, the probability of a certain type of plant surviving was 0.35. The laboratory manager changed the growing conditions and wished to test whether the probability of a plant surviving had increased.

Part A Plant sample

The plants were grown in rows, and when the manager requested a random sample of 8 plants to be taken, the technician took all 8 plants from the front row. Explain what was wrong with the technician's sample.

- The sample should have been taken from the back row instead.
- The sample was not random.
- The sample was too large.

Part B Hypothesis test

A suitable sample of 8 plants was taken and 4 of these 8 plants survived.

Using a 5% significance level, find the critical region and carry out the test. Fill in the gaps below.

Let the random variable X be the number of plants that survived. Then $X \sim \mathrm{B}($ ____ $, p)$.

The null and alternative hypotheses are:

$\mathrm{H}_0 : p$ ____ ____ $\mathrm{H}_1 : p$ ____ ____

The critical region is ____ $\leq X \leq$ ____.

____ is ____ the critical region.

Therefore we ____ H_0. There ____ evidence to suggest that the probability of a plant surviving has improved.

Items:

$>$ | $<$ | $=$ | **reject** | **do not reject** | **is** | **is insufficient** | **within** | **not in** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 0.35 | 0.45 | 0.65

Part C Type II error meaning

State the meaning of a Type II error in the context of the test in Part B.

- Saying that there is no improvement, when there is.
- Saying that there is improvement, when there isn't.
- Saying that there is no improvement, when there isn't.
- Saying that there is improvement, when there is.

Part D Type II error probability

Find the probability of a Type II error for the test in Part B if the probability of a plant surviving is now 0.4. Give your answer to 3 s.f.

Part E Power

Hence calculate the power of the test when the probability of a plant surviving is 0.4. Give your answer to 2 s.f.

Used with permission from UCLES, A Level, November 2004, Paper 9709/07/O/N/04, Question 7

Gameboard:

STEM SMART Double Maths 47 - t-tests & Quality of Tests

Quality of Tests: Cash Withdrawal

Further A

P P P

Over a long period of time it is found that the time spent at cash withdrawal points follows a normal distribution with mean 2.1 minutes and standard deviation 0.9 minutes.

A new system is tried out, to speed up the procedure. The null hypothesis is that the mean time spent is the same under the new system as previously. It is decided to reject the null hypothesis and accept that the new system is quicker if the mean withdrawal time from a random sample of 20 cash withdrawals is less than 1.7 minutes.

Assume that, for the new system, the standard deviation is still 0.9 minutes, and the time spent still follows a normal distribution.

Part A Size

Calculate the size of the test. Give your answer to 3 s.f.

Part B Power

If the mean withdrawal time under the new system is actually 1.5 minutes, calculate the power of the test. Give your answer to 3 s.f.

Used with permission from UCLES, A Level, June 2003, Paper 9709/07/M/J/03, Question 5