

Gameboard

Maths

Matrices: nxm Rules 2i

# Matrices: nxm Rules 2i



The matrices  ${f A}$ ,  ${f B}$  and  ${f C}$  are given by  ${f A}=\begin{pmatrix}1&-4\end{pmatrix}$ ,  ${f B}=\begin{pmatrix}5\\3\end{pmatrix}$  and  ${f C}=\begin{pmatrix}3&0\\-2&2\end{pmatrix}$ 

### Part A AB

The matrix  $\mathbf{AB}$  can be written as the  $1 \times 1$  matrix a.

Find a.

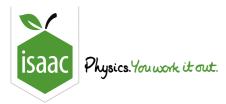
The following symbols may be useful: a

## Part B $\mathbf{BA} - 4\mathbf{C}$

Give the first row of the matrix given by  $\mathbf{BA} - 4\mathbf{C}$  in the form x y with a single space between x and y.

Give the second row of the matrix given by  $\mathbf{BA} - 4\mathbf{C}$  in the form  $x\,y$  with no spaces at the beginning or end.

Adapted with permission from UCLES, A Level, June 2010, Paper 4725, Question 2.



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Maths

2x2 Operations 2ii

# 2x2 Operations 2ii



The matrices  ${f A}$  and  ${f B}$  are given by  ${f A}=egin{pmatrix} 2 & 1 \ 3 & 2 \end{pmatrix}$  and  ${f B}=egin{pmatrix} a & -1 \ -3 & -2 \end{pmatrix}$ .

Part A a

a satisfies the equation  $2\mathbf{A}+\mathbf{B}=egin{pmatrix}1&1\3&2\end{pmatrix}$  .

Find the value of a.

The following symbols may be useful: a

### Part B Alternate value of a

Now take a to satisfy the equation  $\mathbf{AB} = \begin{pmatrix} 7 & -4 \\ 9 & -7 \end{pmatrix}$ .

Find the value of a.

The following symbols may be useful: a

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Maths

2x2 Determinants and Inverses 1ii

# 2x2 Determinants and Inverses 1ii



The matrices  ${f A}$  and  ${f B}$  are given by  ${f A}=\begin{pmatrix}2&1\\-4&5\end{pmatrix}$  and  ${f B}=\begin{pmatrix}3&1\\2&3\end{pmatrix}$ .  ${f I}$  denotes the  $2\times 2$  identity matrix.

Part A 
$$4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$$

Give the first row of the matrix given by  $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$  in the form x y with a single space between x and y.

Give the second row of the matrix given by  $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$  in the form x y with a single space between x and y.

## Part B $\mathbf{A}^{-1}$

$$\mathbf{A}^{-1}$$
 can be written in the form  $\mathbf{A}^{-1} = egin{pmatrix} lpha & eta \\ \gamma & \delta \end{pmatrix}$  .

Find  $\alpha + \beta + \gamma + \delta$  in exact form.

Part C 
$$\left(\mathbf{A}\mathbf{B}^{-1}\right)^{-1}$$

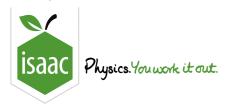
$$\left(\mathbf{A}\mathbf{B}^{-1}\right)^{-1}$$
 can be written in the form  $\left(\mathbf{A}\mathbf{B}^{-1}\right)^{-1}=egin{pmatrix} lpha & eta \\ \gamma & \delta \end{pmatrix}$  .

Find  $\alpha + \beta + \gamma + \delta$  in exact form.

Adapted with permission from UCLES, A Level, Jan 2014, Paper 4725, Question 3.

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Matrices: 3x3 Determinants and Inverses 1i

# Matrices: 3x3 Determinants and Inverses 1i



The matrix  $\mathbf{A}$  is given by  $\mathbf{A}=\begin{pmatrix}a&8&10\\2&1&2\\4&3&6\end{pmatrix}$ . The matrix  $\mathbf{B}$  is such that  $\mathbf{A}\mathbf{B}=\begin{pmatrix}a&6&1\\1&1&0\\1&3&0\end{pmatrix}$ .

Part A  $\det \mathbf{AB}$ 

Find  $\det \mathbf{AB}$ .

The following symbols may be useful: a

Part B  $(AB)^{-1}$ 

Give the first row of  $(\mathbf{AB})^{-1}$  in the form  $x\ y\ z$  with a space between  $x,\ y$  and  $z.\ x,\ y$  and z are in exact form.

Give the second row of  $(\mathbf{AB})^{-1}$  in the form  $x\ y\ z$  with a space between  $x,\ y$  and  $z.\ x,\ y$  and z are in exact form.

Give the third row of  $(\mathbf{AB})^{-1}$  in the form  $x\ y\ z$  with a space between  $x,\ y$  and  $z.\ x,\ y$  and z are in exact form.

Dart	_	$\mathbf{P}^{-1}$
Part		

Give the first row of  $\mathbf{B}^{-1}$  in the form x y z with a space between x, y and z. x, y and z are in exact form.

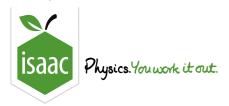
Give the second row of  $\mathbf{B}^{-1}$  in the form  $x\ y\ z$  with a space between  $x,\ y$  and  $z.\ x,\ y$  and z are in exact form.

Give the third row of  $\mathbf{B}^{-1}$  in the form  $x\ y\ z$  with a space between x, y and z. x, y and z are in exact form.

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 10.

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Maths

3 Simultaneous Equations 3i

# 3 Simultaneous Equations 3i



The matrix 
$${f B}$$
 is given by  ${f B}=egin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$  .

#### Part A a

Find the value of a in exact form, given that  $\mathbf{B}$  is singular.

The following symbols may be useful: a

Part B 
$$\mathbf{B}^{-1}$$

$$\mathbf{B}^{-1}$$
 can be written in the form  $\mathbf{B}^{-1} = \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \zeta \\ \eta & \theta & \iota \end{pmatrix}$ . You are given that  $\mathbf{B}$  is non-singular.

Give an expression for  $\alpha-\beta+\gamma-\delta+\epsilon-\zeta+\eta-\theta+\iota$  in terms of a.

The following symbols may be useful: a

### Part C Simultaneous equations

x, y and z satisfy the following simultaneous equations

$$-x + y + 3z = 1$$

$$2x + y - z = 4$$

$$y + 2z = -1$$

Use matrix methods to solve this question only.

Find x in exact form.

The following symbols may be useful: x

Find y in exact form.

The following symbols may be useful: y

Find z in exact form.

The following symbols may be useful: z

Adapted with permission from UCLES, A Level, June 2005, Paper 4725, Question 7.

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# **Matrices - Intersecting Lines**



Two lines are described by

$$3x - 4y - 1 = 0$$
  
 $2x + py - 10 = 0$ .

where p is a constant. Use matrix notation to find the coordinates of the point of intersection of these two lines.

#### Part A Write in matrix form

Write these equations in matrix form  $\mathbf{A}\mathbf{x} = \mathbf{b}$ . If the matrix A is written in the form  $\mathbf{A} = egin{pmatrix} a_{11} & a_{12} \ a_{21} & a_{22} \end{pmatrix}$ give the values of these matrix elements. Give the value of  $a_{11}$ . Give the value of  $a_{12}$ . Give the value of  $a_{21}$ . Give the value of  $a_{22}$ . The following symbols may be useful: p Part B Condition for no intersection Use the matrix to find the value of p for which the lines do not intersect. Give your answer as an improper fraction. The following symbols may be useful: p

#### Part C The inverse matrix

Find  $\mathbf{A}^{-1}$ , the inverse of  $\mathbf{A}$ .

If the matrix  $\mathbf{A}^{-1}$  is written in the form

$$\mathbf{A}^{-1} = egin{pmatrix} lpha_{11} & lpha_{12} \ lpha_{21} & lpha_{22} \end{pmatrix}$$

give the values of these matrix elements

Give an expression for  $\alpha_{11}$ .

The following symbols may be useful: p

Give an expression for  $\alpha_{12}$ .

The following symbols may be useful: p

Give an expression for  $\alpha_{21}$ .

The following symbols may be useful: p

Give an expression for  $\alpha_{22}$ .

The following symbols may be useful: p

Part D	Components of	point of in	itersection
--------	---------------	-------------	-------------

Using ${f A}^{-1}$ obtain expressions for the $x$ and $y$ components for the point of intersection.		
Give an expression for the $x$ -component of the point of intersection.		
The following symbols may be useful: p		
Give an expression for the $y$ -component of the point of intersection.		
The following symbols may be useful: p		

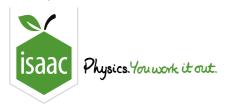
# Part E A value for p

If the y-component of the point of intersection is equal to 2, find the value of p.

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Algebra

Matrices - Linear Equations 2

# **Matrices - Linear Equations 2**



Use matrix notation to solve the following set of three equations for x, y and z:

$$x + cy = c$$
$$x - y + 3z = -c$$
$$2x - 2y - z = 2.$$

### Part A Determinant of the matrix

Write these equations in matrix form  $\mathbf{R}\mathbf{x} = \mathbf{p}$ . Hence deduce the determinant of  $\mathbf{R}$  and find the value of c for which there is no unique solution.

Find the determinant of  $\mathbf{R}$ .

The following symbols may be useful:  $\ensuremath{c}$ 

Deduce the value of c for which there is no unique solution.

#### Part B The inverse matrix

Find the inverse matrix  $\mathbf{R}^{-1}$ .

If the matrix  $\mathbf{R}^{-1}$  is written in the form

$$\mathbf{R}^{-1} = egin{pmatrix} 
ho_{11} & 
ho_{12} & 
ho_{13} \ 
ho_{21} & 
ho_{22} & 
ho_{23} \ 
ho_{31} & 
ho_{32} & 
ho_{33} \end{pmatrix}$$

give expressions for the elements of  ${\bf R}^{-1}$  on the leading diagonal i.e.  $\rho_{11}$ ,  $\rho_{22}$  and  $\rho_{33}$ .

Give an expression for  $\rho_{11}$ 

The following symbols may be useful: c

Give an expression for  $\rho_{22}$ 

The following symbols may be useful: c

Give an expression for  $\rho_{33}$ .

The following symbols may be useful: c

Part C	Solution to the set of equations if $c=1$
Using ${f R}$	$^{-1}$ , find the solutions for $x,y$ and $z$ if $c=1.$
Find the	value of $x$ .
Find the	value of $y$ .
Find the	value of $z$ .
, ma are	value of 2.

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Home Gameboard Maths Algebra Matrices - Linear Equations 3

# **Matrices - Linear Equations 3**



A system consists of three masses  $m_1$ ,  $m_2$  and  $m_3$  in a line; they each have the same mass m. The mass  $m_2$  is in the centre and connected by springs of spring constant k to  $m_1$  on the left and  $m_3$  on the right. The masses are all performing simple harmonic motion at the same angular frequency  $\omega$  such that their equations of motion are

$$-kx_1+kx_2=-m\omega^2x_1 \ kx_1-2kx_2+kx_3=-m\omega^2x_2 \ kx_2-kx_3=-m\omega^2x_3.$$

where  $x_1$ ,  $x_2$  and  $x_3$  are the displacements of  $m_1$ ,  $m_2$  and  $m_3$  respectively.

These equations can be written in matrix form

$$\mathbf{A}\mathbf{x} = -m\omega^2\mathbf{x}$$

$$= -m\omega^2\mathbf{I}\mathbf{x}$$

$$\Rightarrow (\mathbf{A} + m\omega^2\mathbf{I})\mathbf{x} = 0$$

A matrix equation of this sort only has solutions if  $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$ . Use this to find the possible values of  $\omega^2$ . For each value of  $\omega$  find the relationship between  $x_1$ ,  $x_2$  and  $x_3$ .

#### Part A The matrix A

If the matrix A is written in the form

$$\mathbf{A} = egin{pmatrix} a_{11} & a_{12} & a_{13} \ a_{21} & a_{22} & a_{23} \ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

deduce the expressions for the following elements of  ${\bf A}$ .

Give the expression for  $a_{11}$ .

The following symbols may be useful: k,  $\ m$ 

Give the expression for  $a_{21}$ .

The following symbols may be useful: k,  $\ m$ 

Give the expression for  $a_{22}$ .

The following symbols may be useful: k, m

Give the expression for  $a_{31}$ .

The following symbols may be useful: k,  $\ m$ 

# Part B The possible values of $\omega^2$

Write down the matrix  $\mathbf{A} + m\omega^2\mathbf{I}$ . Using the fact that non-zero solutions to the equation  $(\mathbf{A} + m\omega^2\mathbf{I})\mathbf{x} = 0$  require that  $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$ , deduce the three values of  $\omega^2$ . The three values,  $\omega_1^2$ ,  $\omega_2^2$  and  $\omega_3^2$ , are such that  $\omega_1^2 < \omega_2^2 < \omega_3^2$ .

Give an expression for the 11 component (i.e. the component in row 1, column 1) of  ${f A}+m\omega^2{f I}$ .

The following symbols may be useful: k, m, omega

Find an expression for  $\omega_1^2$ .

The following symbols may be useful: k, m

Find an expression for  $\omega_2^2$ .

Find an expression for  $\omega_3^2$ .

The following symbols may be useful: k, m

## Part C The relationship between $x_1$ , $x_2$ and $x_3$

Since the determinant of the matrix is zero there are no unique solutions to the set of three equations; however, for each value of  $\omega^2$ ,  $x_1$ ,  $x_2$  and  $x_3$  have a fixed relationship to each other. On the assumption that  $x_1=1$ , find  $x_2$  and  $x_3$  for each of the three frequencies deduced in Part B. Give your answers using the format 1,a,b with no spaces, where  $x_1=1$ ,  $x_2=a$  and  $x_3=b$ .

Given that  $x_1 = 1$ , find  $x_2$  and  $x_3$  for  $\omega_1^2$ .

Given that  $x_1=1$ , find  $x_2$  and  $x_3$  for  $\omega_2^2$ .

Given that  $x_1=1$ , find  $x_2$  and  $x_3$  for  $\omega_3^2$ .

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