



Physics. *You work it out.*

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## Linear regression 3.2

A Level Further A  

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| c | c | c | c | c | c |
|---|---|---|---|---|---|

An experiment is carried out to measure the resistance  $R$  of a semiconductor as a function of absolute temperature  $T$ . Theory suggests that above a certain temperature

$$R = R_0 e^{\frac{b}{T}}$$

where  $R_0$  and  $b$  are constants.

### Part A Rearrange the equation

By taking the natural logarithms of both sides of the equation show that it can be written

$$\ln R = a + f(T)$$

where  $a$  is a constant and  $f(T)$  is a function of  $T$ . Find expressions for  $a$  and  $f(T)$ .

Find an expression for  $a$ .

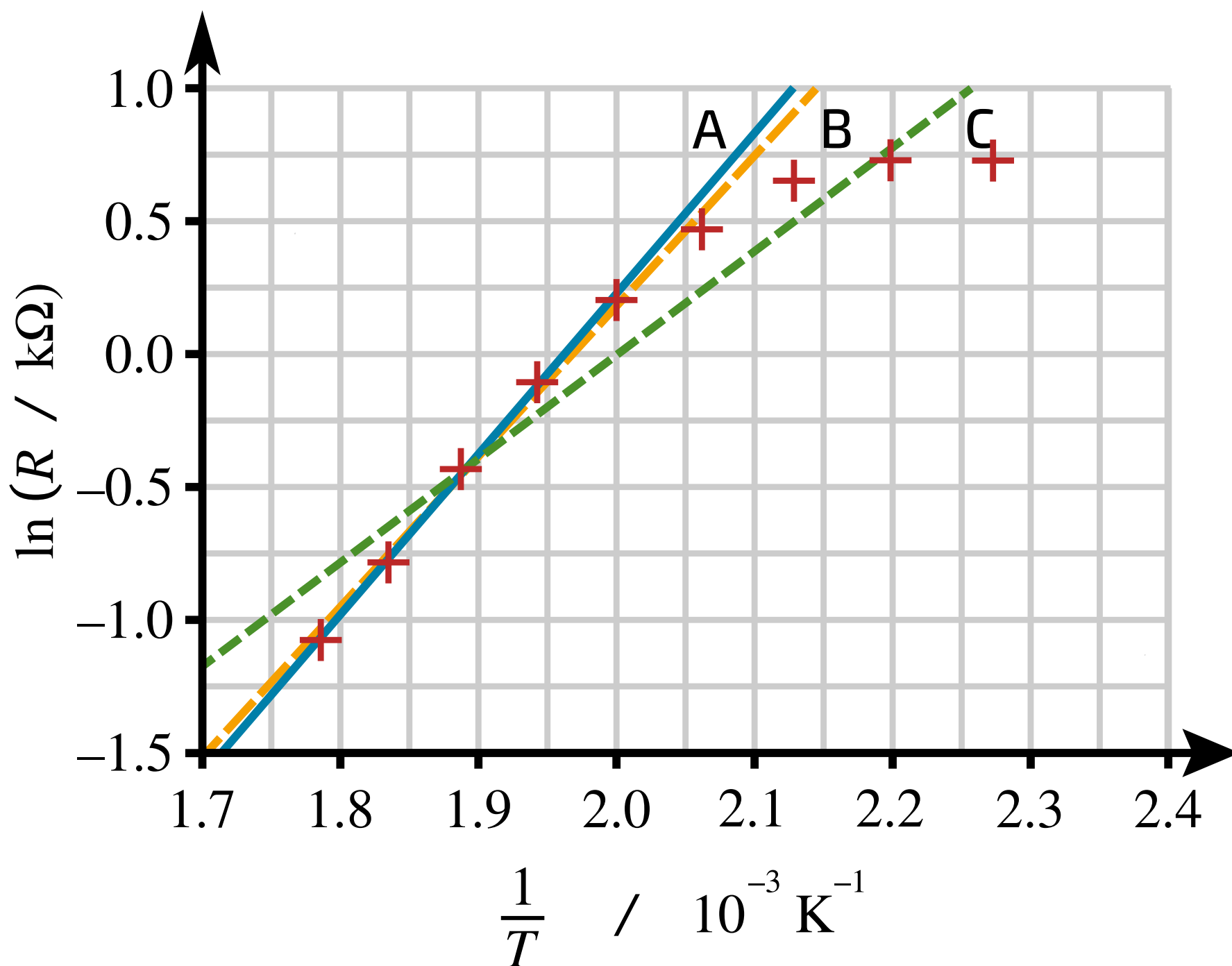
The following symbols may be useful:  $R_0$ ,  $T$ ,  $a$ ,  $b$ ,  $e$ ,  $f(T)$

Find  $f(T)$ .

The following symbols may be useful:  $R_0$ ,  $T$ ,  $a$ ,  $b$ ,  $e$ ,  $f(T)$

## Part B Lines of best fit

In **Figure 1**,  $\ln R$ , where  $R$  is in  $\text{k}\Omega$ , is plotted as a function of  $\frac{1}{T}$ , where  $\frac{1}{T}$  is in  $10^{-3} \text{ K}^{-1}$ . Thus if  $R = 1000 \Omega$ , then  $\ln R = \ln(1 \text{ k}\Omega) = 0$  and, if  $T = 500 \text{ K}$ , then  $\frac{1}{T} = 0.002 \text{ K}^{-1} = 2 \times 10^{-3} \text{ K}^{-1}$ .



**Figure 1:** A plot of  $\ln R$  against  $\frac{1}{T}$ ; three best fit lines, A (blue, solid), B (yellow, long-dashed) and C (green, short-dashed) fitted to different ranges of the data, are shown.

Three best fit lines A, B and C are fitted to different ranges of the data as shown in **Figure 1**.

The parameters of the three fitted lines are:

|       |               |             |             |               |
|-------|---------------|-------------|-------------|---------------|
| I :   | $a = -11.197$ | $b = 5.686$ | $r = 0.997$ | $r^2 = 0.994$ |
| II :  | $a = -11.857$ | $b = 6.042$ | $r = 0.999$ | $r^2 = 0.998$ |
| III : | $a = -7.816$  | $b = 3.906$ | $r = 0.955$ | $r^2 = 0.913$ |

where  $a$  and  $b$  are as defined in Part A and the initial equation, and  $b$  has units of  $10^3 \text{ K}$ .

Match the lines to the parameters. Give your answer in the order I, II, III; for example, if you think I  $\equiv$  A, II  $\equiv$  B and III  $\equiv$  C, type A,B,C with no spaces.

---

### Part C Deductions from the graphs

Theory suggests that above a certain temperature

$$R = R_0 e^{\frac{b}{T}}$$

where  $R_0$  and  $b$  are constants.

Using the information from the graphs in part B, suggest, to 1 sf, the temperature above which the theory is valid.

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### Part D Estimate the energy gap

According to the theory the energy gap between the insulating and the conducting energy bands in a semiconductor is  $E_g = 2kb$ , where  $k$  is the Boltzmann constant ( $k = 1.4 \times 10^{-23} \text{ J K}^{-1}$ ). Select the line of best fit from Part B which best fits the theory and deduce the value of  $b$ ; hence estimate  $E_g$ .

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### Part E Resistance when $T = 520 \text{ K}$

Use the equation of the line of best fit from Part B to deduce the resistance at 520 K.

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### Part F Resistance when $T = 450 \text{ K}$

By looking at the graph in Part B, deduce the resistance when  $T = 450 \text{ K}$  giving your answer to 1 significant figure.

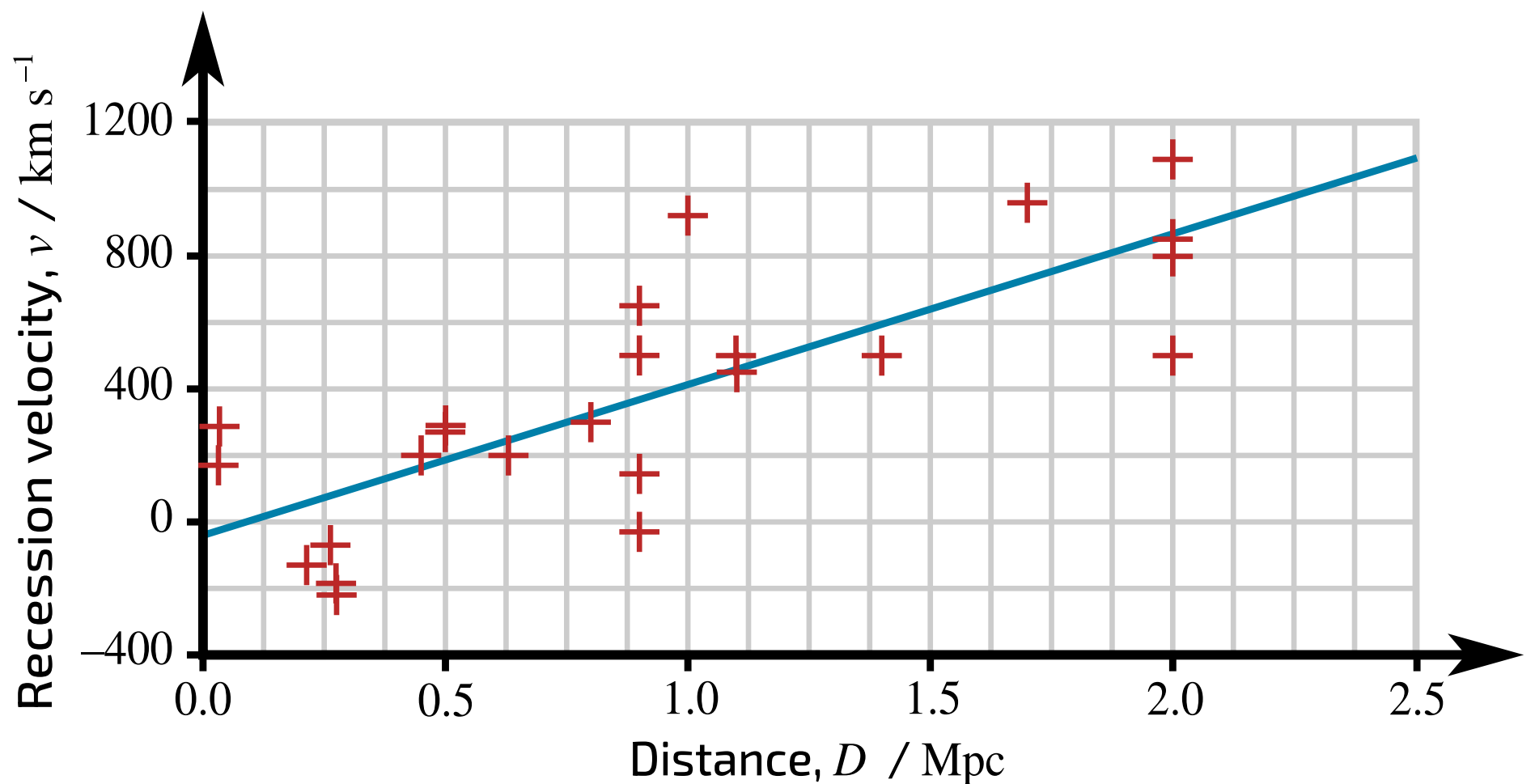
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## Linear regression 3.3

A Level Further A  
C C C C C C

A graph of Hubble's original data relating the recession velocity  $v$  of a galaxy to its distance  $D$  from us is shown in **Figure 1**; the velocity  $v$  is in  $\text{km s}^{-1}$  and the distance  $D$  is in Mpc. (Distances in astronomy are often measured in parsecs (abbreviated to pc), where  $1 \text{ pc} = 3.26 \text{ light-years} = 3.09 \times 10^{16} \text{ m}$  and  $1 \text{ Mpc} = 10^6 \text{ pc}$ .)



**Figure 1:** A graph of Hubble's original data relating the recession velocity  $v$  of a galaxy to its distance  $D$  from us. The regression line of best fit is shown.

The equation describing the best fit to the data is of the form  $v = a + bD$  and has the following parameters

$$a = -40.8 \quad b = 454.2 \quad r = 0.790 \quad r^2 = 0.623.$$

## Part A The units of $a$

---

What are the units of  $a$ ?

- ☐  $\text{km s}^{-1}/\text{Mpc}$
  - ☐  $\text{km s}^{-1} \text{Mpc}^{-1}$
  - ☐  $\text{km s}^{-1}$
  - ☐  $\text{Mpc s km}^{-1}$
  - ☐  $\text{Mpc}/\text{km s}^{-1}$
  - ☐  $\text{s km}^{-1}$
- 

## Part B The units of $b$

---

What are the units of  $b$ ? (The quantity  $b$  is called the Hubble constant and is usually written  $H_0$ ).

- ☐  $\text{Mpc}/\text{km s}^{-1}$
  - ☐  $\text{Mpc km s}^{-1}$
  - ☐  $\text{km s}^{-1}/\text{Mpc}$
  - ☐  $\text{Mpc s}/\text{km}$
  - ☐  $\text{km s}^{-1} \text{Mpc}^{-1}$
  - ☐  $\text{Mpc}$
- 

## Part C Recession velocity

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Using the best fit equation above estimate the recession velocity of a galaxy at a distance of  $6.0 \times 10^6$  light years; give your answer to 2 s.f.

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## Part D The age of the Universe using the original data

Nowadays the value of the Hubble constant is known to be close to 70 in the same units as  $b$ . (It is significantly smaller than that originally determined by Hubble because of a calibration error in Hubble's original data.) The equation describing the relationship between  $v$  and  $D$  in the same units as above is therefore

$$v = 70D.$$

It is straightforward to show that the age of the Universe is given by  $\frac{1}{H_0}$ , where  $H_0$  is the Hubble constant.

Find the age of the Universe using the value of  $b$  estimated from Hubble's original data above. Give your answer in years and to 2 s.f.

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## Part E The age of the Universe using current data

Find the age of the Universe using the current value of  $H_0 = 70$  (in the same units as  $b$ ). Give your answer in years and to 2 s.f.

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# Correlation Hypothesis Testing 1

A Level



In each part, carry out a hypothesis test for the requested type of correlation at the stated significance level.

## Part A Positive correlation

A sample of size  $n = 17$  has a correlation coefficient of  $r = 0.601$ . Test at the 1% significance level whether the population from which the sample was taken has positive correlation, then fill in the blanks below.

The null hypothesis is that the population has no correlation. The alternative hypothesis is that the population exhibits positive correlation.

$$H_0 : \rho = 0$$

For a one-tailed test, the critical value of the correlation coefficient for a sample of size 17 is  at the 1% significance level.

The correlation coefficient for the sample is 0.601. This is  than the critical value. Hence, we  the null hypothesis. There is evidence that the population exhibits positive correlation.

Items:

0.6055

larger

equal to

$H_1 : \rho \neq 0$

0.5577

$H_1 : \rho > 0$

$H_1 : \rho < 0$

smaller

reject

0.5742

do not reject

Part B    Negative correlation

A researcher believes that times to run the 400 m at a particular track are slower if there has been a larger amount of rainfall earlier in the day. The researcher times one particular athlete at the same time every day on ten different autumn days, recording the depth of rainfall (in mm) beforehand. The researcher calculates that the correlation coefficient is  $-0.713$ . Test at the 5% significance level whether an athlete's time and the amount of rainfall are indeed negatively correlated at this track.

Available items

1. The null hypothesis is that there is no correlation. The alternative hypothesis is there is negative correlation.

1. The null hypothesis is that there is negative correlation. The alternative hypothesis is that there is no correlation.

2.  $H_0 : \rho = 0$                        $H_1 : \rho > 0$

2.  $H_0 : \rho = 0$                        $H_1 : \rho < 0$

3. For a one-tailed test, the critical value of the correlation coefficient for a sample of size 5 is 0.8054 at the 10% significance level.

3. For a one-tailed test, the critical value of the correlation coefficient for a sample of size 10 is 0.5494 at the 5% significance level.

4. The correlation coefficient for the sample is  $-0.713$ . This is negative, and has a magnitude greater than the critical value ( $|-0.713| > 0.5494$ ).

4. The correlation coefficient for the sample is  $-0.713$ , and  $-0.713 < 0.5494$ .

5. Hence, we do not reject the null hypothesis. There is not significant evidence for negative correlation between an athlete's time and the amount of rainfall.

5. Hence, we reject the null hypothesis. There is evidence that an athlete's time and the amount of rainfall have negative correlation.



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## Part C    Any (linear) correlation

An author wonders whether the amount of time their cat sits next to them is correlated with the number of words they write during the day. Over fifty-three days, the author records the number of words they write and for how long the cat sits nearby, and finds  $r = 0.3300$ . Test the data at the 1% significance level.

Choose from the options below to construct a complete hypothesis test.

- ☐ This question is looking for correlation in either direction. A two-tailed test is needed.

$$H_0 : \rho = 0 \qquad H_1 : \rho \neq 0$$

- ☐ This question is looking for positive correlation. A one-tailed test is needed.

$$H_0 : \rho = 0 \qquad H_1 : \rho > 0$$

- ☐ For a one-tailed test, the critical value of the correlation coefficient for a sample of size 53 is 0.3188 at the 1% significance level.

- ☐ For a two-tailed test, the critical value of the correlation coefficient for a sample of size 53 is 0.3509 at the 1% significance level.

- ☐ The correlation coefficient for the sample is 0.3300, and  $0.3300 < 0.3509$ .

- ☐ The correlation coefficient for the sample is 0.3300, and  $0.3300 > 0.3188$ .

- ☐ Hence, we do not reject the null hypothesis. There is not significant evidence that the number of words the author writes is correlated with the amount of time their cat sits near them.

- ☐ Hence, we reject the null hypothesis. There is evidence that the number of words the author writes is correlated with the amount of time their cat sits near them.

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# Correlation Hypothesis Testing 2

A Level



A town planner believes that on summer weekday afternoons the amount of traffic into the centre of their town is higher when the temperature is higher. They want to test this hypothesis at the 1% significance level.

Every weekday (Monday to Friday) for six weeks they monitor the traffic on the main roads into town, and record the mean afternoon temperature, and they find that the correlation coefficient is  $-0.4517$ .

## Part A Initial conclusion

Without doing any calculations, which of these statements can the town planner make? Choose all that apply.

- ☐ There is no evidence that the amount of traffic and afternoon temperature are correlated.
- ☐ The correlation coefficient is negative. There may be a negative correlation between the amount of traffic and afternoon temperature.
- ☐ The correlation coefficient is negative, so there is no evidence that the amount of traffic is positively correlated with temperature.
- ☐ The correlation coefficient is negative, so there is definitely no positive correlation between the amount of traffic and temperature.
- ☐ The correlation coefficient of their sample is negative, so there is a negative correlation between the amount of traffic on summer afternoons and temperature.

Part B    Choosing a hypothesis test

Using the given data, which of the following hypothesis tests would it be most useful for the town planner to carry out?

- ☐ A hypothesis test to see if the amount of traffic and afternoon temperature are negatively correlated at the 1% significance level.
- ☐ A hypothesis test to see if the amount of traffic and afternoon temperature are negatively correlated at the 20% significance level.
- ☐ A hypothesis test to see if the amount of traffic and afternoon temperature are negatively correlated at the 50% significance level.

Part C    Null and alternative hypotheses

The town planner carries out the most useful test listed in part B.

Drag and drop symbols into the spaces below to state the null and alternative hypotheses for this test, where  $\rho$  represents the population correlation coefficient and  $r$  represents the sample correlation coefficient.

H<sub>0</sub>:

H<sub>1</sub>:

Items:

- 1

$\rho$

$r$

<

=

0

>

Part D Carrying out the test

Carry out the hypothesis test, and make a conclusion. Then fill in the blanks below.

The critical value of the correlation coefficient is .

Comparing the town planner's value to the critical value gives .

Therefore,  the null hypothesis. There  significant evidence that there is a  correlation between the amount of traffic in summer and afternoon temperature.

Items:

positive

do not reject

$| - 0.4517 | < 0.9343$

0.4226

negative

$| - 0.4517 | > 0.4226$

is

is not

0.9343

reject

$| - 0.4517 | < 0.8822$

0.8822

no

may be



# Spearman's Rank Test 1

Further A

P

P

P

In an examination consisting of 4 separate subjects, a group of 10 students obtained the following marks. The overall mark was out of 375, and the Physics and Chemistry marks were each out of 100.

| Student   | A   | B   | C   | D   | E   | F   | G   | H   | I   | J   |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Overall   | 274 | 255 | 246 | 245 | 229 | 228 | 219 | 213 | 205 | 176 |
| Physics   | 76  | 77  | 67  | 65  | 58  | 60  | 52  | 63  | 47  | 45  |
| Chemistry | 82  | —   | 65  | 67  | —   | 64  | 68  | 54  | 51  | 38  |

Find Spearman's rank correlation coefficients between each pair of data to test whether there is evidence of a positive correlation between them.

## Part A   Overall and Physics marks

Find Spearman's rank correlation coefficient  $r_s$  for the relationship between the overall mark and the Physics mark, giving your answer to 3 s.f.

---

## Part B Overall and Physics significance

Test at the 1% significance level whether there is a positive association between the overall mark and that obtained in Physics.

Find the appropriate critical value for Spearman's rank correlation coefficient, giving your answer to 3 s.f.

---

What do you conclude about whether there is a positive association at the 1% significance level between the overall mark and that obtained in Physics?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Spearman's rank correlation coefficient is  the calculated value; the null hypothesis that there is  association between the two values can therefore be  at the 1% level and there is  evidence for a positive association at this level.

Items:

less than

negative

accepted

greater than

significant

positive

rejected

no

equal to

---

---

## Part C Overall and Chemistry marks

Find Spearman's rank correlation coefficient  $r_s$  for the relationship between the overall mark and the Chemistry mark, giving your answer to 3 s.f.

---

## Part D Overall and Chemistry significance

Test at the 1% significance level whether there is a positive association between the overall mark and that obtained in Chemistry.

Find the appropriate critical value for Spearman's rank correlation coefficient, giving your answer to 3 s.f.

---

What do you conclude about whether there is a positive association at the 1% significance level between the overall mark and that obtained in Chemistry?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Spearman's rank correlation coefficient is  the calculated value; the null hypothesis that there is  association between the two values cannot therefore be  at the 1% level, and there is no  evidence for a positive association at this level.

Items:

greater than

rejected

accepted

negative

significant

no

less than

equal to

positive

---

## Part E Physics and Chemistry marks

Find Spearman's rank correlation coefficient  $r_s$  for the relationship between the Physics mark and the Chemistry mark, giving your answer to 3 s.f.

---

---

## Part F    Physics and Chemistry significance

Test at the 1% significance level whether there is a positive association between the Physics mark and that obtained in Chemistry.

Find the appropriate critical value for Spearman's rank correlation coefficient, giving your answer to 3 s.f.

---

What do you conclude about whether there is a positive association at the 1% significance level between the mark obtained in Physics and that obtained in Chemistry?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Spearman's rank correlation coefficient is  the calculated value; the null hypothesis that there is  association between the two values cannot therefore be  at the 1% level, and there is no  evidence for a positive association at this level.

Items:

no

greater than

accepted

rejected

negative

positive

significant

less than

equal to

---

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# Using Pearson's PMCC 1

Further A



Find Pearson's product moment correlation coefficient for the following sets of statistics.

## Part A Data set 1

Find the correlation coefficient for the following set of statistics, giving your answer to 3 s.f.

$$\Sigma x = 275, \Sigma x^2 = 10781, \Sigma y = 251, \Sigma y^2 = 8407, \Sigma xy = 7842, n = 10$$

## Part B Data set 2

Find the correlation coefficient for the following set of statistics, giving your answer to 3 s.f.

$$\Sigma x = 907, \Sigma x^2 = 105944, \Sigma y = 289, \Sigma y^2 = 10916, \Sigma xy = 14929, n = 12$$

## Part C Data set 3

Find the correlation coefficient for the following set of statistics, giving your answer to 3 s.f.

$$\Sigma(x - \bar{x})^2 = 1592, \Sigma(y - \bar{y})^2 = 2473, \Sigma(x - \bar{x})(y - \bar{y}) = -1280, n = 10$$



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## Using Pearson's PMCC 2

Further A



An electrical device has been designed and is being tested. The summary statistics for the relationship between the input and output voltages ( $V_i$  and  $V_o$  respectively) from the device are as follows.

$$\Sigma V_i = 302, \Sigma V_i^2 = 15532, \Sigma V_o = 228, \Sigma V_o^2 = 10980, \Sigma V_i V_o = 12296, n = 6$$

### Part A The correlation coefficient

Find Pearson's product moment correlation coefficient for the set of statistics, giving your answer to 3 s.f.

## Part B Significance of the correlation between input and output

Test at the 1% significance level whether there is a correlation between the input and output voltages.

Find the appropriate critical value for Pearson's product moment correlation coefficient, giving your answer to 3 s.f.

---

What do you conclude about whether there is a correlation at the 1% significance level between the input and output voltages?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Pearson's product moment correlation coefficient is   
the calculated value; the null hypothesis that there is  correlation between the two  
values can therefore be  at the 1% level and there is  evidence for a  
correlation at this level.

Items:

negative

greater than

no

rejected

equal to

positive

accepted

less than

significant

---

## Part C The regression line

Calculate the equation of the regression line  $V_o = a + bV_i$  relating the output voltage ( $V_o$ ) to the input voltage ( $V_i$ ).

Find  $b$  giving your answer to 3 s.f.

---

Find  $a$ , giving your answer to 3 s.f.

---



# Correlation - the Photoelectric Effect

A Level

Further A

C C C P P P

Millikan carried out experiments to test the photoelectric effect equation that Einstein proposed in 1905. He measured what was called the stopping potential  $V$  as a function of the frequency  $f$  of light falling on a photocathode. One set of data he obtained is as follows.

|                                      |      |      |      |      |      |
|--------------------------------------|------|------|------|------|------|
| Frequency $f$ ( $10^{14}\text{Hz}$ ) | 5.49 | 6.91 | 7.41 | 8.22 | 9.58 |
| Voltage $V$ (V)                      | 0.46 | 1.04 | 1.16 | 1.61 | 2.19 |

## Part A   The correlation coefficient

Find Pearson's product moment correlation coefficient for these data, giving your answer to 3 s.f.

## Part B   The equation of the regression line

Calculate the equation of the regression line  $V = a + bf$  relating the stopping potential  $V$  to the frequency  $f$ .

Find  $b$ , giving your answer to 4 s.f.

Find  $a$ , giving your answer to 4 s.f.

---

## Part C    The Planck constant and work function

Einstein's equation is commonly written in the form

$$eV = hf - \phi$$

where  $e$  is the charge on an electron ( $1.6 \times 10^{-19}$  C),  $h$  is the Planck constant and  $\phi$  is called the work function of the material used for the photocathode (in this case sodium). Given  $V$  is in volts and the frequency is in units of  $10^{14}$  Hz, deduce from your answers in Part B a value for  $h$  in J s and  $\phi$  in J.

Find the value of the Planck constant  $h$ , giving your answer to 2 s.f.

---

Find the value of the work function  $\phi$ , giving your answer to 2 s.f.

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# Correlation - Attenuation of Gamma Rays

A Level

C

C

C

Further A

P

P

P

An experiment to investigate the attenuation of gamma rays by lead was carried out by measuring the count rate as a function of the number of lead sheets placed in front of a gamma ray source. The following results were obtained.

|                       |       |      |      |     |     |    |    |
|-----------------------|-------|------|------|-----|-----|----|----|
| Number of lead sheets | 0     | 1    | 2    | 3   | 4   | 5  | 6  |
| Gamma ray count rate  | 13106 | 4301 | 1469 | 474 | 163 | 63 | 17 |

## Part A Rank correlation coefficient

Write down Spearman's rank correlation coefficient for these data.

## Part B Linear product moment correlation coefficient

Find the Pearson product moment correlation coefficient between the two sets of data, giving your answer to 4 s.f.

---

### Part C    Significance of the linear product moment correlation coefficient

Test at the 1% significance level whether there is a negative correlation between the count rate and the number of lead sheets.

Find the appropriate critical value for Pearson's product moment correlation coefficient, giving your answer to 4 s.f.

---

What do you conclude about whether there is a negative correlation at the 1% significance level between the count rate and the number of lead sheets?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Pearson's product moment correlation coefficient is   
the modulus of the calculated value; the  hypothesis cannot be rejected so there is  
 evidence for a correlation at the 1% level.

Items:

accepted

equal to

null

greater than

no significant

less than

alternative

sufficient

rejected

---

---

### Part D    Logarithmic product moment correlation coefficient

Consider now the relationship between the natural log of the counts and the number of lead sheets.

Find the Pearson product moment correlation coefficient between the natural log of the count rate and the number of lead sheets, giving your answer to 4 s.f.

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## Part E Significance of the logarithmic product moment correlation coefficient

Test at the 1% significance level whether there is a negative correlation between the natural log of the count rate and the number of lead sheets.

Find the appropriate critical value for Pearson's product moment correlation coefficient, giving your answer to 4 s.f.

---

What do you conclude about whether there is a negative correlation at the 1% significance level between the natural log of the count rate and the number of lead sheets?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Pearson's product moment correlation coefficient is   
the modulus of the calculated value; the  hypothesis can be rejected at the 1% level and  
there is  evidence for a negative correlation at this level.

Items:

rejected

significant

accepted

alternative

equal to

less than

no sufficient

greater than

null

---

---

## Part F The equation of the regression line

Calculate the equation of the regression line

$$\ln N = a + bm$$

where  $N$  is the gamma ray count and  $m$  is the number of lead sheets.

Find the value of  $b$ , giving your answer to 4 s.f.

---

Find the value of  $a$ , giving your answer to 4 s.f.

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## Part G    Attenuation constant

Theory predicts that the relationship expected between the number of counts  $N$  and the number of lead sheets  $m$  is given by

$$N = N_0 e^{-\alpha m}$$

where  $\alpha$  is the attenuation constant and  $N_0$  is a constant.

Deduce the value of  $\alpha$ , giving your answer to 4 s.f.

---

Find the value of  $N_0$ , giving your answer to 4 s.f.

---