Probabilities: Households

A Level P P P

The people living in 3 households are classified as children (C), parents (P) or grandparents (G). The numbers living in each house are shown in the table below.

House number 1	House number 2	House number 3
$4C, 1P, 2G$	$2C, 2P, 3G$	$1C, 1G$

Part A Scenario 1 - a grandparent

All the people in all 3 houses meet for a party. One person at the party is chosen at random. Calculate the probability of choosing a grandparent.

Part B Scenario 2 - a grandparent

A house is chosen at random. Then a person in that house is chosen at random. Using a tree diagram, or otherwise, calculate the probability that the person chosen is a grandparent.

Part C Scenario 2 - a parent

Given that the person chosen by the method in Part B is a grandparent, calculate the probability that there is also a parent living in the house.

Used with permission from UCLES, A level, June 2003, CIE, Paper 6 Probability & Statistics 1 (S1), Question 6

Probabilities: Mailing

A Level

A magazine mails a large number of households with an offer. Each household may answer "Yes" or "No" or may not reply at all. A second mailing is sent only to those households who have answered "Yes" or "No". Again each household may answer "Yes" or "No" or not reply at all. The proportions of households which reply are shown in the partially completed tree diagram.

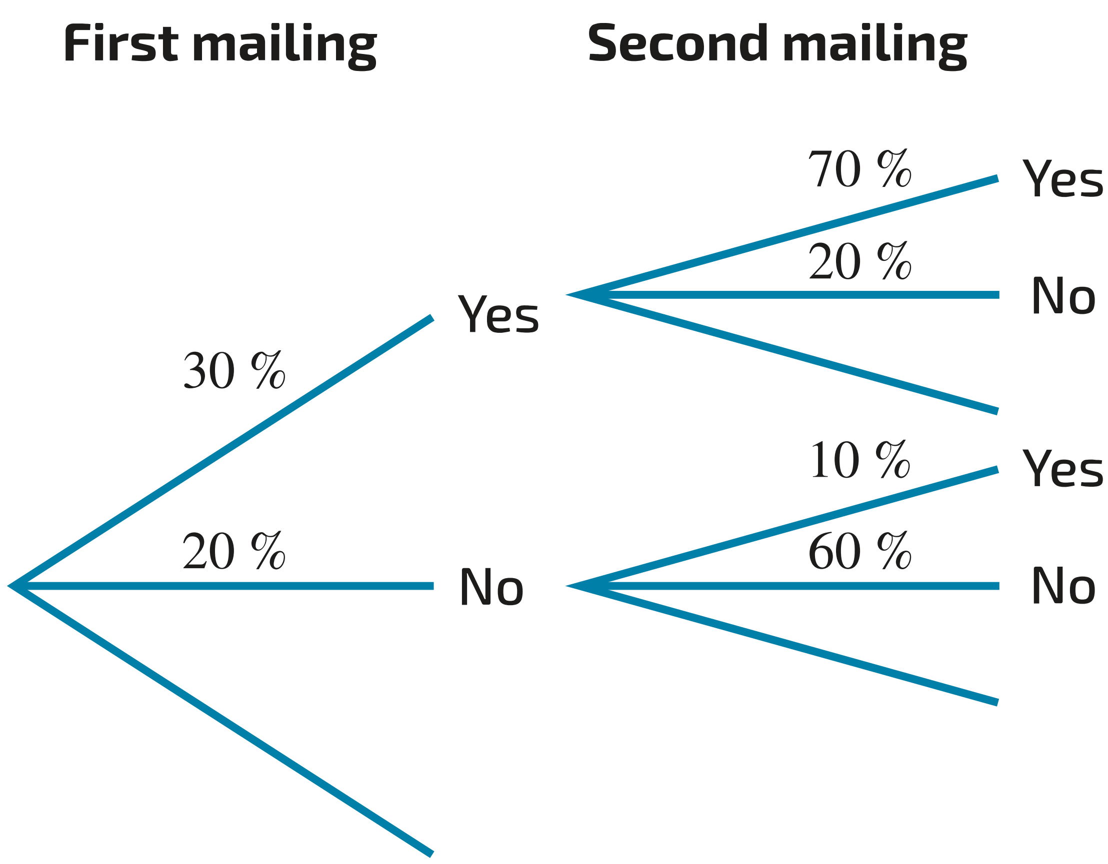

Figure 1: A tree diagram for the two mailings.

Part A The first mailing - no reply

For a randomly chosen household on the magazine's initial mailing list, find the probability that the household does not reply to the first mailing. Give your answer as a percentage.

Part B The second mailing - a reply of "No"

For a randomly chosen household on the magazine's initial mailing list, find the probability that the household answers "No" to the second mailing. Give your answer as a percentage.

Part C The first mailing - a reply of "No"

For a randomly chosen household on the magazine's initial mailing list, find the probability that the household answers "No" to the first mailing, given that it answers "No" to the second mailing. Give your answer as a fraction.

Used with permission from UCLES, A level, Nov 2000, OCR, Paper 4, Question 2

Gameboard:

STEM SMART Single Maths 50 - Statistics Revision

Normal Distribution 2

A Level

P P P

Each of the three parts of this question is a different type of normal distribution problem.

Part A Employee salary

The annual salaries of employees in a company have mean $£30\,000$ and standard deviation $£12\,000$. Assuming a normal distribution, calculate the probability that the salary of one randomly chosen employee lies between $£20\,000$ and $£24\,000$.

Part B Sample size

The continuous random variable Y has the distribution $\mathrm{N}(23.0, 5.00^2)$. The mean of n observations of Y is denoted by $\overline{Y}$. It is given that $\mathrm{P}(\overline{Y} > 23.625) = 0.0228$. Find the value of n.

Part C $\mathrm{P}(G > 65)$

The random variable G has a normal distribution. It is known that

$$\mathrm{P}(G < 56.2) = \mathrm{P}(G > 63.8) = 0.100$$

Find $\mathrm{P}(G > 65.0)$.

Adapted with permission from UCLES, A level, OCR, S2, Jun 2008 Q2, Jan 2009 Q.2, Jan 2012 Q. 3

Gameboard:

STEM SMART Single Maths 50 - Statistics Revision

Normal Distribution 1

Further A

P P P

The random variable Y is normally distributed with mean μ and variance σ^2, where μ and σ are integers.

It is found that P$(Y > 150.0) = 0.0228$ and P$(Y > 143.0) = 0.9332$. Find the values of μ and σ.

Part A Value of μ

Enter the value of μ.

Part B Value of σ

Enter the value of σ.

Adapted with permission from UCLES, A level, Jun 2015, OCR, S2, Question 1

Gameboard:
STEM SMART Single Maths 50 - Statistics Revision

Physics. You work it out.

Home Gameboard Maths Statistics Hypothesis Tests Hypothesis Testing: Normal Distribution 3

Hypothesis Testing: Normal Distribution 3

A Level
P P P

Average Speed Cameras (ASC) are a way of encouraging motorists to reduce their speed. Modern systems use automated numberplate recognition to monitor how long it takes a car to travel between two points, from which the average speed of the car can be worked out.

A particular stretch of road is known to be an accident blackspot, and an ASC system is introduced with the aim of improving safety. Before the cameras are installed, the speed of the cars is monitored over a long period of time and it is found to obey a normal distribution with a mean of $75.0\,\mathrm{mph}$ and a standard deviation of $3.00\,\mathrm{mph}$. A few weeks after the cameras are installed the speed of the cars is monitored again. A sample of 400 cars is taken, and their speed is found to be normally distributed with a mean of $72.0\,\mathrm{mph}$.

Part A Null and alternative hypotheses

A hypothesis test is carried out to determine whether the cameras have been effective in reducing driver mean speed. Let μ represent the population mean speed. Drag and drop items into the boxes below to state the null and alternative hypotheses for the test.

H_0: μ ☐ ☐

H_1: μ ☐ ☐

Items:

μ | $\overline{X}$ | $<$ | $\neq$ | $=$ | $>$ | 0.00 | 72.0 | 75.0

Part B Probability distributions

Drag and drop answers into the boxes below to describe the probability distributions involved in the hypothesis test. You may assume that the standard deviation of the speeds of the cars after the cameras are installed is the same as before the cameras were installed, $3.00\,\mathrm{mph}$.

Let X be the speed of a car. If the null hypothesis is true, the speed of cars in the population is given by a normal distribution with a mean of [ ] and a variance of [ ]. Hence,

$$X \sim \mathrm{N}(\quad,\quad)$$

As X is normally distributed, the mean values of samples of X with size n will also be normally distributed. Let $\overline{X}$ be the mean speed of a sample of 400 cars. Then the distribution of $\overline{X}$ is

$$\overline{X} \sim \mathrm{N}(\quad,\quad)$$

Items:

0.00 0.00750 0.0225 0.150 0.450 1.73 3.00 9.00 72.0 75.0

Part C Carrying out the test

Fill in the blanks to complete the description of the hypothesis test.

Drag and drop items into the boxes below to carry out the hypothesis test at the 1% significance level.

The hypothesis test is a [ ]-tail test. At the 1% significance level, the critical region is $\overline{X}$ [ ] [ ].

The mean speed of the sample of 400 cars was found to be $72.0\,\mathrm{mph}$. This value is in the [ ].

Therefore, [ ] the null hypothesis. There [ ] significant evidence that the mean speed of drivers is lower after the cameras were installed.

Items:

one **two** **three** $<$ $\neq$ $=$ $>$ 74.65 74.95 75.05 75.35 **acceptance region** **critical region** **reject** **do not reject** **is** **is not**

Created for isaacphysics.org by Jonathan Waugh

Gameboard:

STEM SMART Single Maths 50 - Statistics Revision

Correlation Hypothesis Testing 3

A Level

The owner of an online shop believes that the number of new customers each week increases with the amount spent on advertising. They monitor their sales over 20 weeks, and find that the Pearson's product moment correlation coefficient between the number of new customers per week and the amount spent on advertising is 0.4010.

Part A Null and alternative hypotheses

The owner of the shop wants to test whether there is evidence that the number of new customers each week increases with the amount spent on advertising. Drag and drop symbols into the spaces below to state the null and alternative hypotheses for this test.

(ρ represents the population correlation coefficient, and r represents the sample correlation coefficient.)

H_0: [ ] [ ] [ ]

H_1: [ ] [ ] [ ]

Items:

ρ r $<$ $=$ $\neq$ $>$ 0 1

Part B Carrying out the test

Carry out the hypothesis test at the 5% significance level, and make a conclusion. Then fill in the blanks below.

The test is a [ ]-tailed test. At the 5% significance level, the critical value of the correlation coefficient is [ ].

Comparing the shop owner's value to the critical value gives [ ].

Therefore, [ ] the null hypothesis. There [ ] significant evidence that there is a [ ] correlation between the number of new customers each week and the amount spent on advertising.

Items:

one **two** **three** 0.3783 0.4438 0.9877 0.9969 $0.4010 > 0.3783$ $0.4010 < 0.4438$ $0.4010 < 0.9877$ **reject** **do not reject** **is** **may be** **is not** **negative** **positive** **no**

Part C Types of error

When making a conclusion from a hypothesis test, two types of error are possible:

- **Type I** - A null hypothesis about the population that is true is rejected.
- **Type II** - A null hypothesis about the population that is false is not rejected.

Making one of these types of error is always a possibility. For the conclusion in part B, what type of error might the shop owner have made?

- ○ Type I
- ○ Type II

Created for isaacphysics.org by Jonathan Waugh

Gameboard:

STEM SMART Single Maths 50 - Statistics Revision

Probability 5.7

A Level Further A

P P P P P P

Ball bearings produced by a machine have diameters which have a normal distribution with mean $6.50\,\mathrm{mm}$ and standard deviation $0.07\,\mathrm{mm}$.

Part A Number of ball bearings rejected

A random sample of 1000 of these ball bearings is selected. Find how many will be rejected if the tolerance required is in the range $6.40 - 6.60\,\mathrm{mm}$.

Part B Selection using a grid

The ball bearings are passed over a grid and those with diameters $< 6.37\,\mathrm{mm}$ fall through this grid. A sample of 1000 bearings is selected from those that have passed over the grid.

Find how many bearings will be rejected if the tolerance required is in the range $6.40 - 6.60\,\mathrm{mm}$.

Find how many of these will have diameters which exceed the tolerance limit (i.e. have diameters $> 6.60\,\mathrm{mm}$).

Given that 1000 bearings have passed over the grid, how many were there to start with?

Created by for isaacphysics.org by Julia Riley