Matrices: nxm Rules 2i

Further A

The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$

Part A $\mathbf{AB}$

The matrix $\mathbf{AB}$ can be written as the 1×1 matrix a.

Find a.

The following symbols may be useful: a

Part B $\mathbf{BA} - 4\mathbf{C}$

Give the first row of the matrix given by $\mathbf{BA} - 4\mathbf{C}$ in the form $x\ y$ with a single space between x and y.

Give the second row of the matrix given by $\mathbf{BA} - 4\mathbf{C}$ in the form $x\ y$ with no spaces at the beginning or end.

Adapted with permission from UCLES, A Level, June 2010, Paper 4725, Question 2.

2x2 Operations 2ii

Further A
P P P

The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} a & -1 \\ -3 & -2 \end{pmatrix}$.

Part A a

a satisfies the equation $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.

Find the value of a.

The following symbols may be useful: a

Part B Alternate value of a

Now take a to satisfy the equation $\mathbf{AB} = \begin{pmatrix} 7 & -4 \\ 9 & -7 \end{pmatrix}$.

Find the value of a.

The following symbols may be useful: a

Adapted with permission from UCLES, A Level, June 2007, Paper 4725, Question 1.

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2x2 Determinants and Inverses 1ii

Further A

The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -4 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}$. $\mathbf{I}$ denotes the 2×2 identity matrix.

Part A $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$

Give the first row of the matrix given by $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$ in the form $x\ y$ with a single space between x and y.

Give the second row of the matrix given by $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$ in the form $x\ y$ with a single space between x and y.

Part B $\mathbf{A}^{-1}$

$\mathbf{A}^{-1}$ can be written in the form $\mathbf{A}^{-1} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$.

Find $\alpha + \beta + \gamma + \delta$ in exact form.

Part C $\left(\mathbf{AB}^{-1}\right)^{-1}$

$\left(\mathbf{AB}^{-1}\right)^{-1}$ can be written in the form $\left(\mathbf{AB}^{-1}\right)^{-1} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$.

Find $\alpha + \beta + \gamma + \delta$ in exact form.

Adapted with permission from UCLES, A Level, Jan 2014, Paper 4725, Question 3.

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Physics. You work it out.

Home Gameboard Maths Matrices: 3x3 Determinants and Inverses 1i

Matrices: 3x3 Determinants and Inverses 1i

Further A P P P

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$. The matrix $\mathbf{B}$ is such that $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$.

Part A $\det \mathbf{AB}$

Find $\det \mathbf{AB}$.

The following symbols may be useful: a

Part B $(\mathbf{AB})^{-1}$

Give the first row of $(\mathbf{AB})^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Give the second row of $(\mathbf{AB})^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Give the third row of $(\mathbf{AB})^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Part C $\mathbf{B}^{-1}$

Give the first row of $\mathbf{B}^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Give the second row of $\mathbf{B}^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Give the third row of $\mathbf{B}^{-1}$ in the form $x\ y\ z$ with a space between x, y and z. x, y and z are in exact form.

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 10.

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Physics. You work it out.

Home Gameboard Maths 3 Simultaneous Equations 3i

3 Simultaneous Equations 3i

Further A

P P P

The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

Part A a

Find the value of a in exact form, given that $\mathbf{B}$ is singular.

The following symbols may be useful: a

Part B $\mathbf{B}^{-1}$

$\mathbf{B}^{-1}$ can be written in the form $\mathbf{B}^{-1} = \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \zeta \\ \eta & \theta & \iota \end{pmatrix}$. You are given that $\mathbf{B}$ is non-singular.

Give an expression for $\alpha - \beta + \gamma - \delta + \epsilon - \zeta + \eta - \theta + \iota$ in terms of a.

The following symbols may be useful: a

Part C Simultaneous equations

x, y and z satisfy the following simultaneous equations

$$-x + y + 3z = 1$$

$$2x + y - z = 4$$

$$y + 2z = -1$$

Use matrix methods to solve this question only.

Find x in exact form.

The following symbols may be useful: x

Find y in exact form.

The following symbols may be useful: y

Find z in exact form.

The following symbols may be useful: z

Adapted with permission from UCLES, A Level, June 2005, Paper 4725, Question 7.

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Matrices - Intersecting Lines

Further A P P P University P P P

Two lines are described by

$$3x - 4y - 1 = 0$$
$$2x + py - 10 = 0.$$

where p is a constant. Use matrix notation to find the coordinates of the point of intersection of these two lines.

Part A Write in matrix form

Write these equations in matrix form $\mathbf{Ax} = \mathbf{b}$.

If the matrix $\mathbf{A}$ is written in the form

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

give the values of these matrix elements.

Give the value of a_{11}.

Give the value of a_{12}.

Give the value of a_{21}.

Give the value of a_{22}.

The following symbols may be useful: p

Part B Condition for no intersection

Use the matrix to find the value of p for which the lines do not intersect. Give your answer as an improper fraction.

The following symbols may be useful: p

Part C The inverse matrix

Find $\mathbf{A}^{-1}$, the inverse of $\mathbf{A}$.

If the matrix $\mathbf{A}^{-1}$ is written in the form

$$\mathbf{A}^{-1} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$$

give the values of these matrix elements

Give an expression for α_{11}.

The following symbols may be useful: p

Give an expression for α_{12}.

The following symbols may be useful: p

Give an expression for α_{21}.

The following symbols may be useful: p

Give an expression for α_{22}.

The following symbols may be useful: p

Part D Components of point of intersection

Using $\mathbf{A}^{-1}$ obtain expressions for the x and y components for the point of intersection.

Give an expression for the x-component of the point of intersection.

The following symbols may be useful: p

Give an expression for the y-component of the point of intersection.

The following symbols may be useful: p

Part E A value for p

If the y-component of the point of intersection is equal to 2, find the value of p.

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Physics. You work it out.

Home Gameboard Maths Algebra Matrices Matrices - Linear Equations 2

Matrices - Linear Equations 2

Further A University

Use matrix notation to solve the following set of three equations for x, y and z:

$$x + cy = c$$
$$x - y + 3z = -c$$
$$2x - 2y - z = 2.$$

Part A Determinant of the matrix

Write these equations in matrix form $\mathbf{Rx} = \mathbf{p}$. Hence deduce the determinant of $\mathbf{R}$ and find the value of c for which there is no unique solution.

Find the determinant of $\mathbf{R}$.

The following symbols may be useful: c

Deduce the value of c for which there is no unique solution.

Part B The inverse matrix

Find the inverse matrix $\mathbf{R}^{-1}$.

If the matrix $\mathbf{R}^{-1}$ is written in the form

$$\mathbf{R}^{-1} = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{pmatrix}$$

give expressions for the elements of $\mathbf{R}^{-1}$ on the leading diagonal i.e. ρ_{11}, ρ_{22} and ρ_{33}.

Give an expression for ρ_{11}

The following symbols may be useful: c

Give an expression for ρ_{22}

The following symbols may be useful: c

Give an expression for ρ_{33}.

The following symbols may be useful: c

Part C Solution to the set of equations if $c = 1$

Using $\mathbf{R}^{-1}$, find the solutions for x, y and z if $c = 1$.

Find the value of x.

Find the value of y.

Find the value of z.

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Matrices - Linear Equations 3

Further A C C C University P P P

A system consists of three masses m_1, m_2 and m_3 in a line; they each have the same mass m. The mass m_2 is in the centre and connected by springs of spring constant k to m_1 on the left and m_3 on the right. The masses are all performing simple harmonic motion at the same angular frequency ω such that their equations of motion are

$$\begin{aligned} -kx_1 + kx_2 &= -m\omega^2 x_1 \\ kx_1 - 2kx_2 + kx_3 &= -m\omega^2 x_2 \\ kx_2 - kx_3 &= -m\omega^2 x_3. \end{aligned}$$

where x_1, x_2 and x_3 are the displacements of m_1, m_2 and m_3 respectively.

These equations can be written in matrix form

$$\begin{aligned} \mathbf{Ax} &= -m\omega^2\mathbf{x} \\ &= -m\omega^2\mathbf{Ix} \\ \Rightarrow (\mathbf{A} + m\omega^2\mathbf{I})\mathbf{x} &= 0 \end{aligned}$$

A matrix equation of this sort only has solutions if $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$. Use this to find the possible values of ω^2. For each value of ω find the relationship between x_1, x_2 and x_3.

Part A The matrix $\mathbf{A}$

If the matrix $\mathbf{A}$ is written in the form

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

deduce the expressions for the following elements of $\mathbf{A}$.

Give the expression for a_{11}.

The following symbols may be useful: `k`, `m`

Give the expression for a_{21}.

The following symbols may be useful: `k`, `m`

Give the expression for a_{22}.

The following symbols may be useful: `k`, `m`

Give the expression for a_{31}.

The following symbols may be useful: `k`, `m`

Part B The possible values of ω^2

Write down the matrix $\mathbf{A} + m\omega^2\mathbf{I}$. Using the fact that non-zero solutions to the equation $(\mathbf{A} + m\omega^2\mathbf{I})\mathbf{x} = 0$ require that $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$, deduce the three values of ω^2. The three values, ω_1^2, ω_2^2 and ω_3^2, are such that $\omega_1^2 < \omega_2^2 < \omega_3^2$.

Give an expression for the 11 component (i.e. the component in row 1, column 1) of $\mathbf{A} + m\omega^2\mathbf{I}$.

The following symbols may be useful: `k`, `m`, `omega`

Find an expression for ω_1^2.

The following symbols may be useful: `k`, `m`

Find an expression for ω_2^2.

Find an expression for ω_3^2.

The following symbols may be useful: `k`, `m`

Part C The relationship between x_1, x_2 and x_3

Since the determinant of the matrix is zero there are no unique solutions to the set of three equations; however, for each value of ω^2, x_1, x_2 and x_3 have a fixed relationship to each other. On the assumption that $x_1 = 1$, find x_2 and x_3 for each of the three frequencies deduced in Part B. Give your answers using the format 1,a,b with no spaces, where $x_1 = 1$, $x_2 = a$ and $x_3 = b$.

Given that $x_1 = 1$, find x_2 and x_3 for ω_1^2.

Given that $x_1 = 1$, find x_2 and x_3 for ω_2^2.

Given that $x_1 = 1$, find x_2 and x_3 for ω_3^2.

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