Functions and Algebra 5ii

The function f is defined by

$$f(x) = \frac{1}{\sqrt{x}} + 2, \quad x > 0.$$

The function g is defined for all real values of x by

$$g(x) = 10 - (x + 3)^2.$$

Part A   Range of f

State the range of $f(x)$ as an inequality.

The following symbols may be useful: `<, <=, >, >=, f(x), x, y`

Part B   Inverse of f

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, x`

Part C   Range of g

State the range of $g(x)$ as an inequality.

The following symbols may be useful: `<, <=, >, >=, g(x), x, y`

Find the value of $g\big(g(-1)\big)$.

Inverse Quadratic Function

Figure 1 shows the graph of $y = f(x)$, where

$$f(x) = 2 - x^2, \quad x \leq 0$$
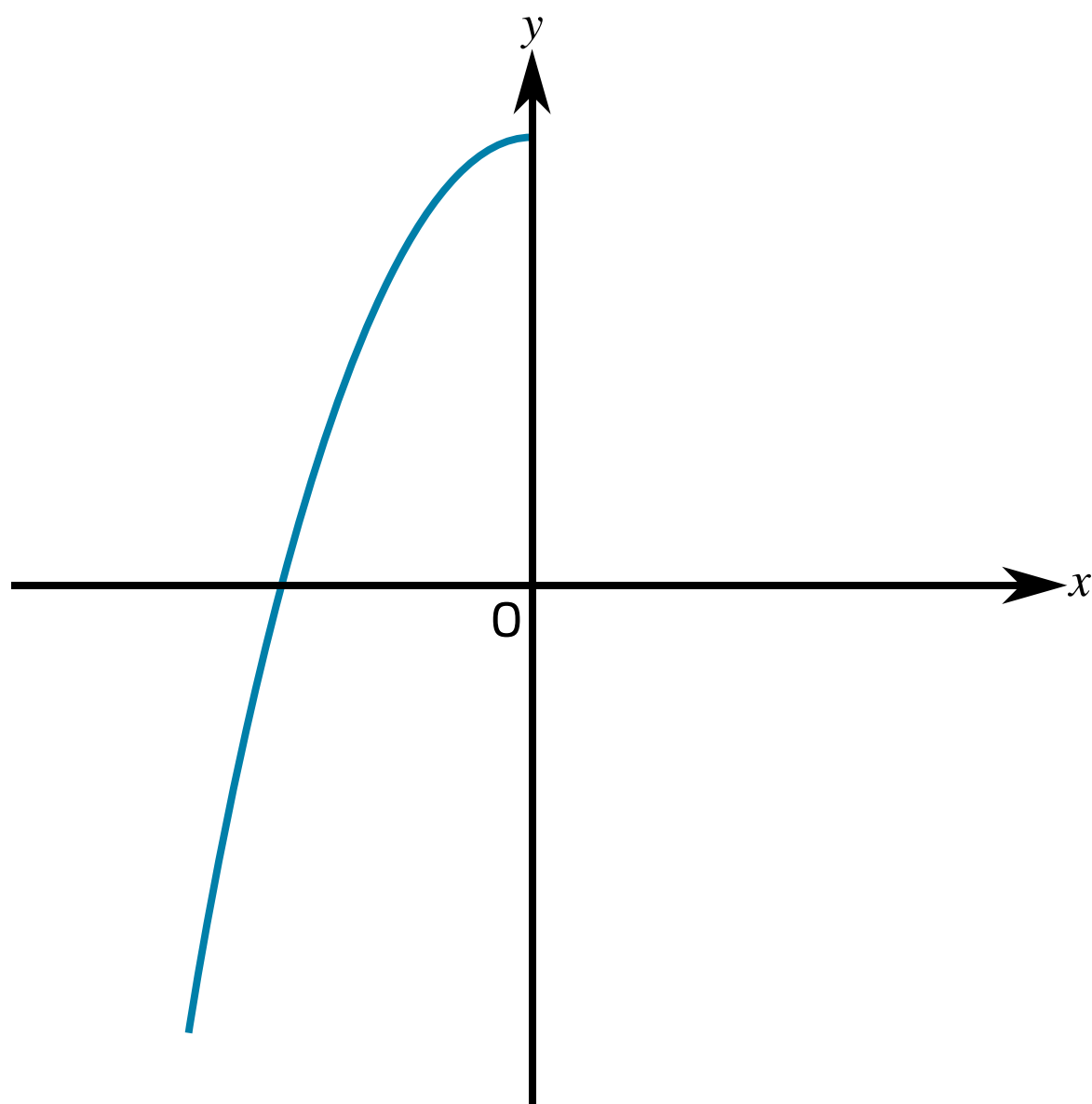
Figure 1: The graph of $y = f(x)$, for $x \leq 0$.

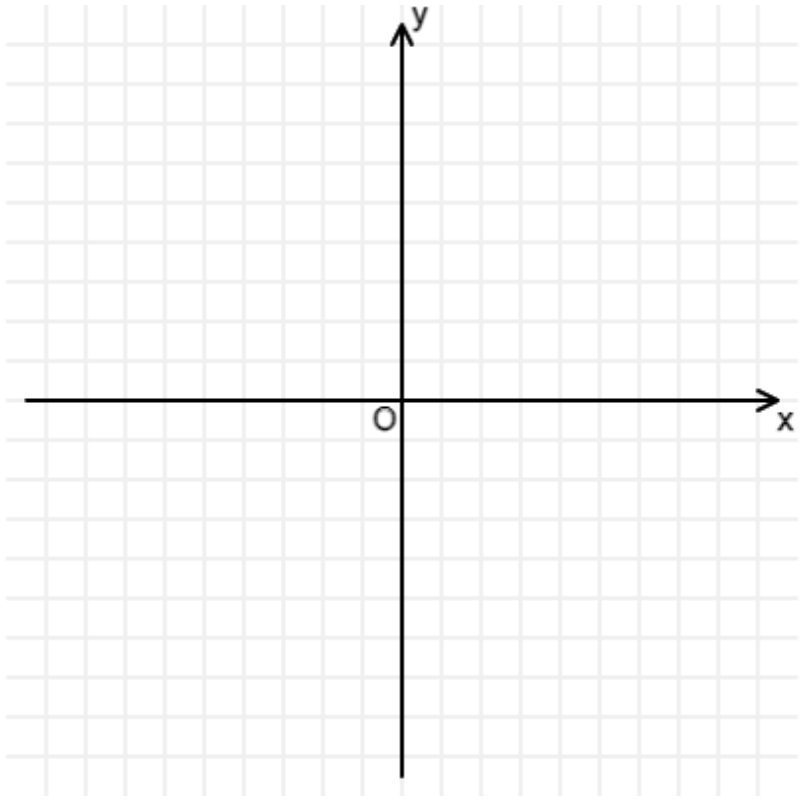**Part A** $\quad f^2(-3)$

Evaluate $f^2(-3)$.

Part B $f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, x, y`

Part C   **Graph of $f^{-1}(x)$**

Sketch the graph of $y = f^{-1}(x)$.

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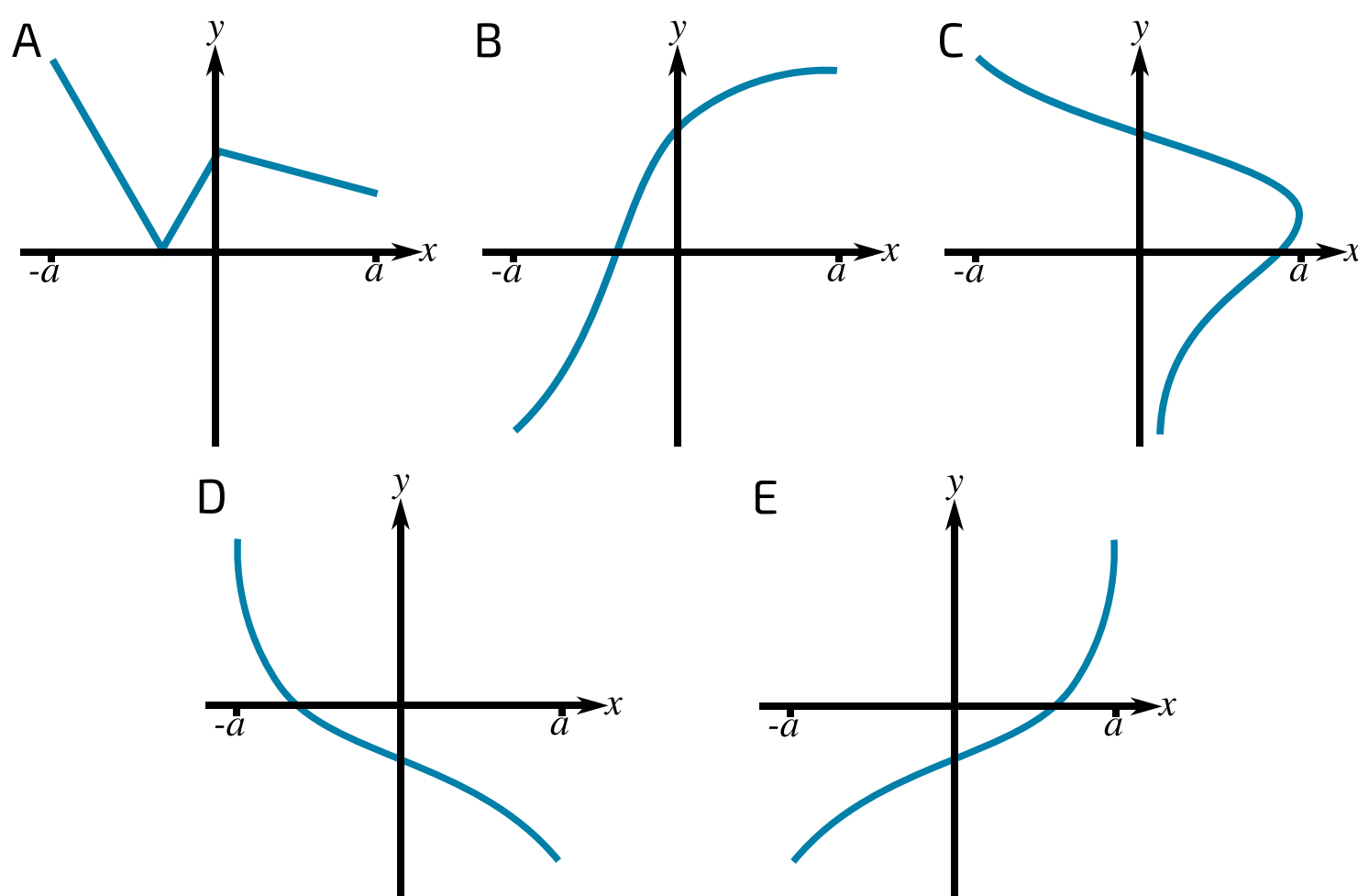
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STEM SMART Single Maths 24 - Defining Functions

Function Types and Inverses

Figure 1 shows five different graphs, A, B, C, D and E, each for values of x such that $-a \leq x \leq a$ where a is a constant.

Figure 1: The set of five graphs, labelled A, B, C, D and E

Part A   Function

Which diagram does not show the graph of a function?

- ◯ A
- ◯ B
- ◯ C
- ◯ D
- ◯ E

Which diagram shows the graph of a function that is not one-to-one?

- ○ A
- ○ B
- ○ C
- ○ D
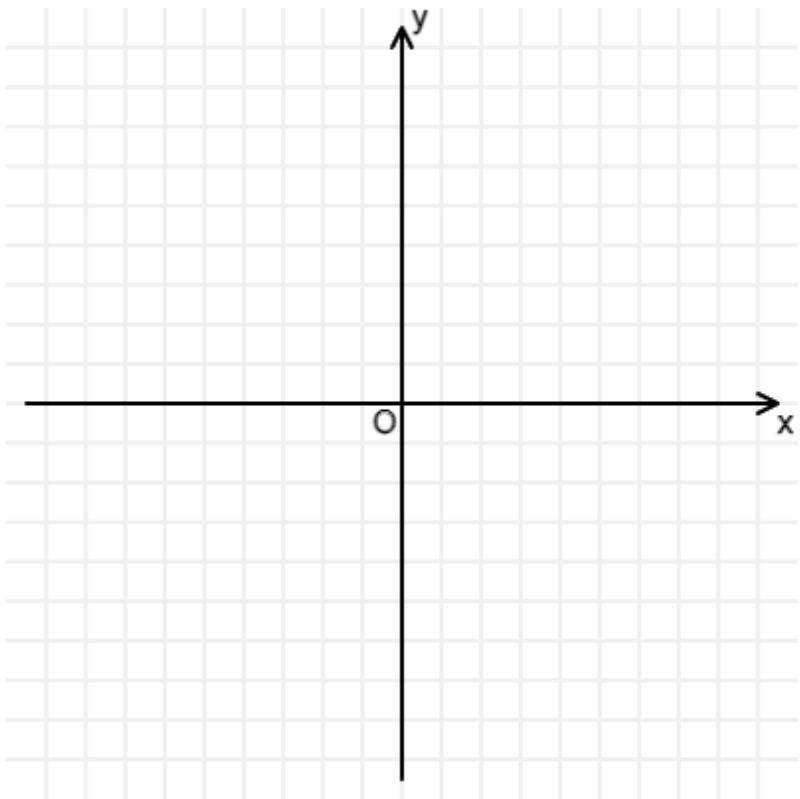- ○ E

Part C   Inverses

It is given that two of the diagrams illustrate functions that are inverses of each other. Identify one of these two diagrams.

- ○ A
- ○ B
- ○ C
- ○ D
- ○ E

Part D   Sketch

The graph in E has equation $y = f(x)$. Sketch the graph of $y = \left| f(x) \right|$.

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.

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Functions and Algebra 3i

The functions f and g are defined for all real values of x by

$$f(x) = \left| 2x + a \right| + 3a \quad \text{and} \quad g(x) = 5x - 4a,$$

where a is a positive constant.

Part A   Range

Find the range of $f(x)$.

Fill in the inequality below.

☐ ☐ ☐

Items:

| $f(x)$ | $<$ | $\leq$ | $>$ | $\geq$ | $< f(x) <$ | $\leq f(x) \leq$ | $< f(x)$ **or** $f(x) <$ | $\leq f(x)$ **or** $f(x) \leq$ | $\frac{a}{3}$ | $\frac{a}{2}$ | a |

| $2a$ | $3a$ | $4a$ | 0 | $-\frac{a}{3}$ | $-\frac{a}{2}$ | $-a$ | $-2a$ |

Part B   Inverse function of $f(x)$

Fill in the blanks to explain why the function $f(x)$ has no inverse.

The function $f(x)$ is not [   ]. For example, $f(0) = 4a$ and $f($ [   ] $)$ also equals $4a$. Hence, $f(x)$ has no inverse.

Items:

| many-to-many | many-to-one | $-2a$ | $-a$ | $2a$ | a | one-to-one | one-to-many |

Part C   Inverse function of $g(x)$

Find an expression for $g^{-1}(x)$.

The following symbols may be useful: a, x

Part D   Solve for x

Solve for x the equation $g\big(f(x)\big) = 31a$.

Give the value of x furthest from 0.

The following symbols may be useful: a, x

Give the value of x that is closest to 0.

The following symbols may be useful: a, x

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Modulus 3ii

Solve the inequality $\left| 2x - 5 \right| > \left| x + 1 \right|$.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

○ $x < a$

○ $x \leq a$

○ $x > a$

○ $x \geq a$

○ $a < x < b$

○ $a \leq x \leq b$

○ $x < a$ or $x > b$

○ $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

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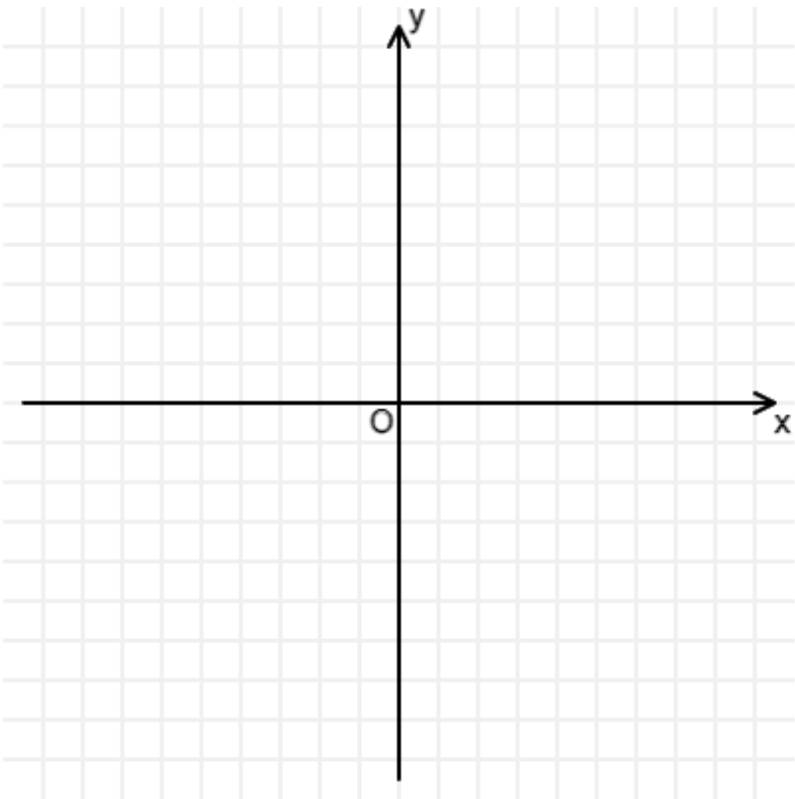# Modulus Functions 2

A Level   Further A

P P P P P P

Part A   Divergence of $y = \left| \frac{1}{x} \right|$

Does the function $y = \left| \dfrac{1}{x} \right|$ diverge anywhere? Where?

The following symbols may be useful: x

Part B   Graph of $y = \left| \frac{1}{x} \right|$

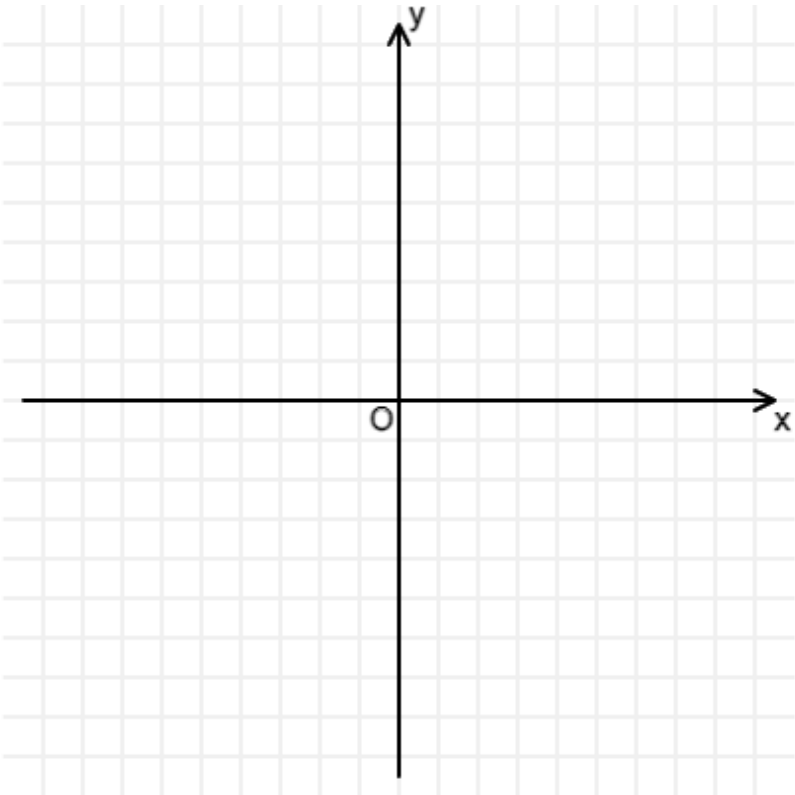Sketch the graph of $y = \left| \dfrac{1}{x} \right|$.

Part C Divergence of $y = \left| \frac{1}{x^2-4} \right|$

Does the function $y = \left| \dfrac{1}{x^2 - 4} \right|$ diverge anywhere? Where?

The following symbols may be useful: x, ±

Part D Graph of $y = \left| \frac{1}{x^2-4} \right|$

Sketch the graph of $y = \left| \dfrac{1}{x^2 - 4} \right|$.

Part E Solve equation graphically

Solve the equation $|x| = \left| \dfrac{1}{x} \right|$ graphically and give the solution as a single expression.

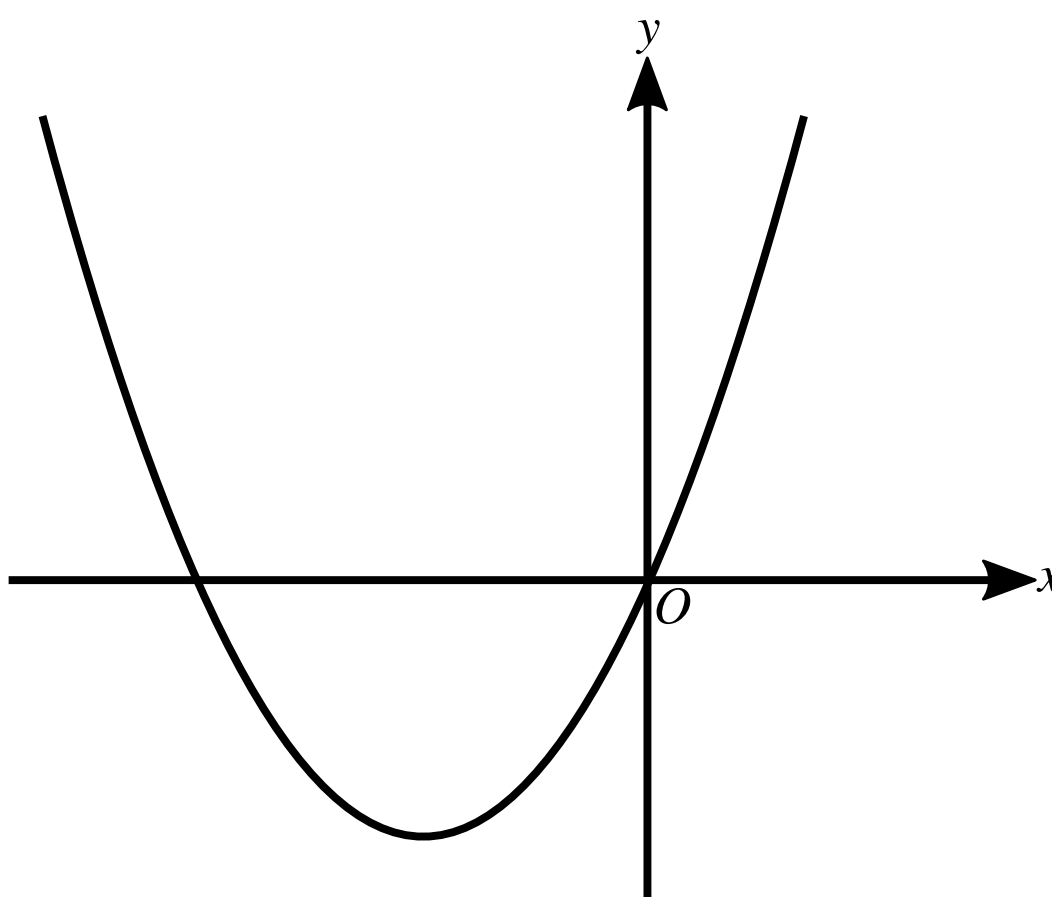The following symbols may be useful: x, ±

Curve Sketching and Combined Transformations 3i

The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x)$$

where k is a positive constant. **Figure 1** shows the curve with equation $y = f(x)$.

Figure 1: The graph of $y = f(x)$

Part A   Transformations

The curve $y = x^2$ can be transformed to the curve $y = f(x)$ by the following sequence of transformations

a translation parallel to the x-axis,

a translation parallel to the y-axis,

a stretch.

Give details, in terms of k where appropriate, of these transformations.

Give the number of units that the curve is translated in the x direction, assuming right to be positive.

The following symbols may be useful: k

Give the number of units that the curve is translated in the y direction, assuming up to be positive.

The following symbols may be useful: k

Give the stretch factor of the transformation.

The following symbols may be useful: k

Part B   Range

Find the range of $f(x)$ as a single inequality in terms of k.

The following symbols may be useful: <, <=, >, >=, f(x), k, x, y

Part C $\left|f(x)\right| = 20$

It is given that there are three distinct values of x which satisfy the equation $\left|f(x)\right| = 20$. Find the value of k and determine exactly the three values of x which satisfy the equation in this case.

State the value of k.

The following symbols may be useful: k

Give the rational value of x which satisfies this equation.

The following symbols may be useful: x

Give one of the irrational solutions for x in its simplest exact form.

The following symbols may be useful: x

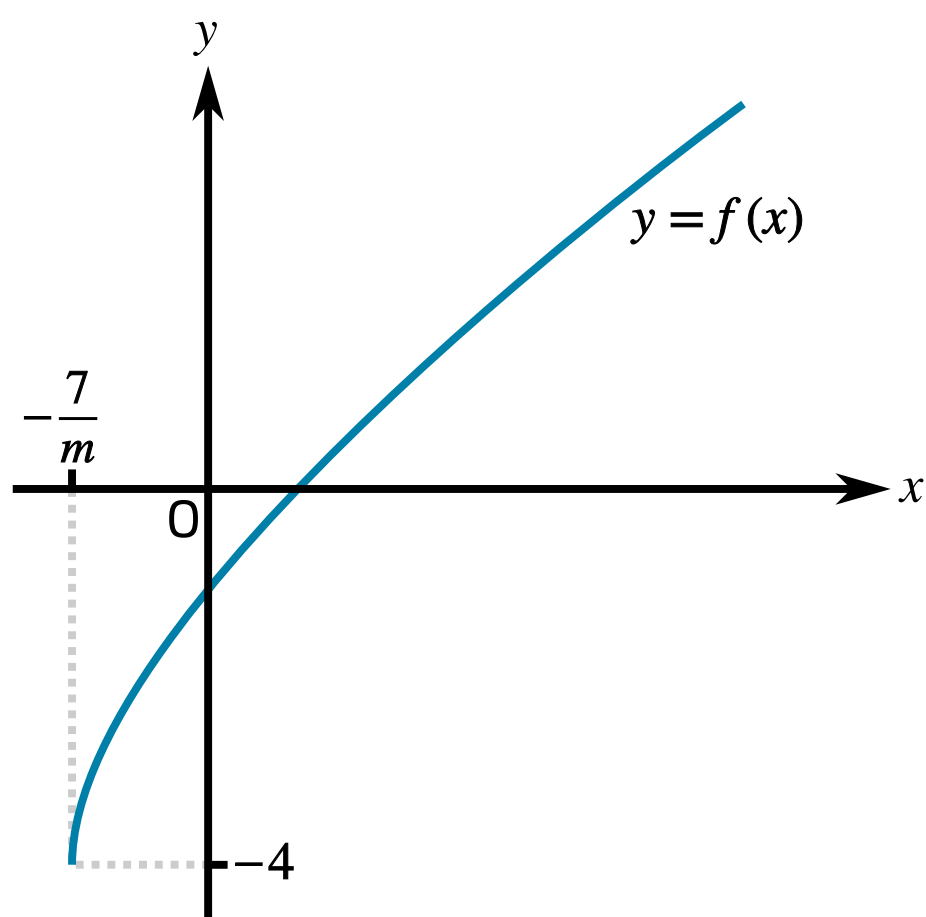Used with permission from UCLES A-level Maths papers, 2003-2017.

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Combined Transformations

A Level

The function f is defined by $f(x) = \sqrt{mx + 7} - 4$, where $x \geqslant -\frac{7}{m}$ and m is a positive constant. **Figure 1** shows the curve $y = f(x)$.

Figure 1: The curve $y = f(x)$

Part A   Translation of the curve $y = \sqrt{x}$

A sequence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations.

Available items

Translate the curve 4 units in the positive y direction.

Translate the curve 4 units in the negative y direction.

Stretch the curve in the x direction by a factor of $\frac{1}{m}$.

Translate the curve 7 units in the positive x direction.

Stretch the curve in the y direction by a factor of $\frac{1}{m}$.

Stretch the curve in the x direction by a factor of m.

Translate the curve 7 units in the negative y direction.

Translate the curve 4 units in the negative x direction.

Translate the curve 7 units in the negative x direction.

Part B $f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: f, m, x

Part C   Values of m

It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Thus it can be deduced that neither curve meets the line $y = x$. Hence determine the set of possible values of m, and give the upper bound in the form $m < a$ or $m \le a$.

The following symbols may be useful: `<, <=, >, >=, m`

Give the lower bound in the form $m > a$ or $m \ge a$.

The following symbols may be useful: `<, <=, >, >=, m`