# Straight Lines: Coordinates and Lengths 2i

A Level

The points $A$, $B$, and $C$ have coordinates $(5, 1)$, $(p, 7)$, and $(8, 2)$ respectively.

## Part A Possible values of $p$

Given that the distance between the points $A$ and $B$ is twice the distance between points $A$ and $C$, calculate the possible values of $p$. Enter the smallest possible value of $p$.

The following symbols may be useful: p

## Part B Midpoint of $AB$

Given also that the line passing through $A$ and $B$ has equation $y = 3x - 14$, find the coordinates of the midpoint of $AB$. Enter the $x$ and $y$ coordinates below.

Enter the $x$ coordinate:

The following symbols may be useful: x

Enter the $y$ coordinate:

The following symbols may be useful: y

Used with permission from UCLES, A Level, January 2006, Paper 4721, Question 9.

# Straight Lines: Coordinates and Lengths 1ii

A Level

## Part A Find coordinate

The line segment joining the points $(-2, 7)$ and $(-4, p)$ has gradient $4$. Find the value of $p$.

The following symbols may be useful: `p`

## Part B Find coordinates and midpoint

The line segment joining the points $(-2, 7)$ and $(6, q)$ has midpoint $(m, 5)$. Find $m$ and $q$. Enter the values of $m$ and $q$ below.

Enter the value of $m$:

The following symbols may be useful: `m`

Enter the value of $q$:

The following symbols may be useful: `q`

## Part C Find coordinate from length

The line segment joining the points $(-2, 7)$ and $(d, 3)$ has length $2\sqrt{13}$. Find the two possible values of of $d$. Enter the greatest possible value of $d$.

The following symbols may be useful: d

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Used with permission from UCLES, A Level, January 2013, Paper 4721, Question 6.

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# Straight lines: gradients and normals 2i

A Level

$A$ is the point $(2, 7)$ and $B$ is the point $(-1, -2)$.

## Part A Equation of line

Find the equation of the line through $A$ parallel to the line $y = 4x - 5$, giving your answer in the form $y = mx + c$.

The following symbols may be useful: `x, y`

## Part B Length of $AB$

Calculate the length of $AB$, giving your answer in simplified surd form.

## Part C Find equation of line

Find the equation of the line which passes through the midpoint of $AB$, and which is perpendicular to $AB$. Give your answer in the form $ax + by + c = 0$, where $a$, $b$, and $c$ are integers.

The following symbols may be useful: `x, y`



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# Straight lines: gradients and normals 3ii

A Level

The points $A(1, 3)$, $B(7, 1)$, and $C(-3, -9)$ are joined to form a triangle.

## Part A Show right angle

Show that this triangle is right angled, and determine whether the right angle is located at $A$, $B$, or $C$.

- $A$
- $C$
- $B$

## Part B Triangle in circle

The points $A$, $B$ and $C$ lie on the circumference of a circle.

Find the $x$ coordinate of the centre of the circle.

The following symbols may be useful: x

Find the $y$ coordinate of the centre of the circle.

The following symbols may be useful: y

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# Logarithmic Plots 1

A Level

The logarithms to base $10$ of two variables, $x$ and $y$, are plotted against each other below.

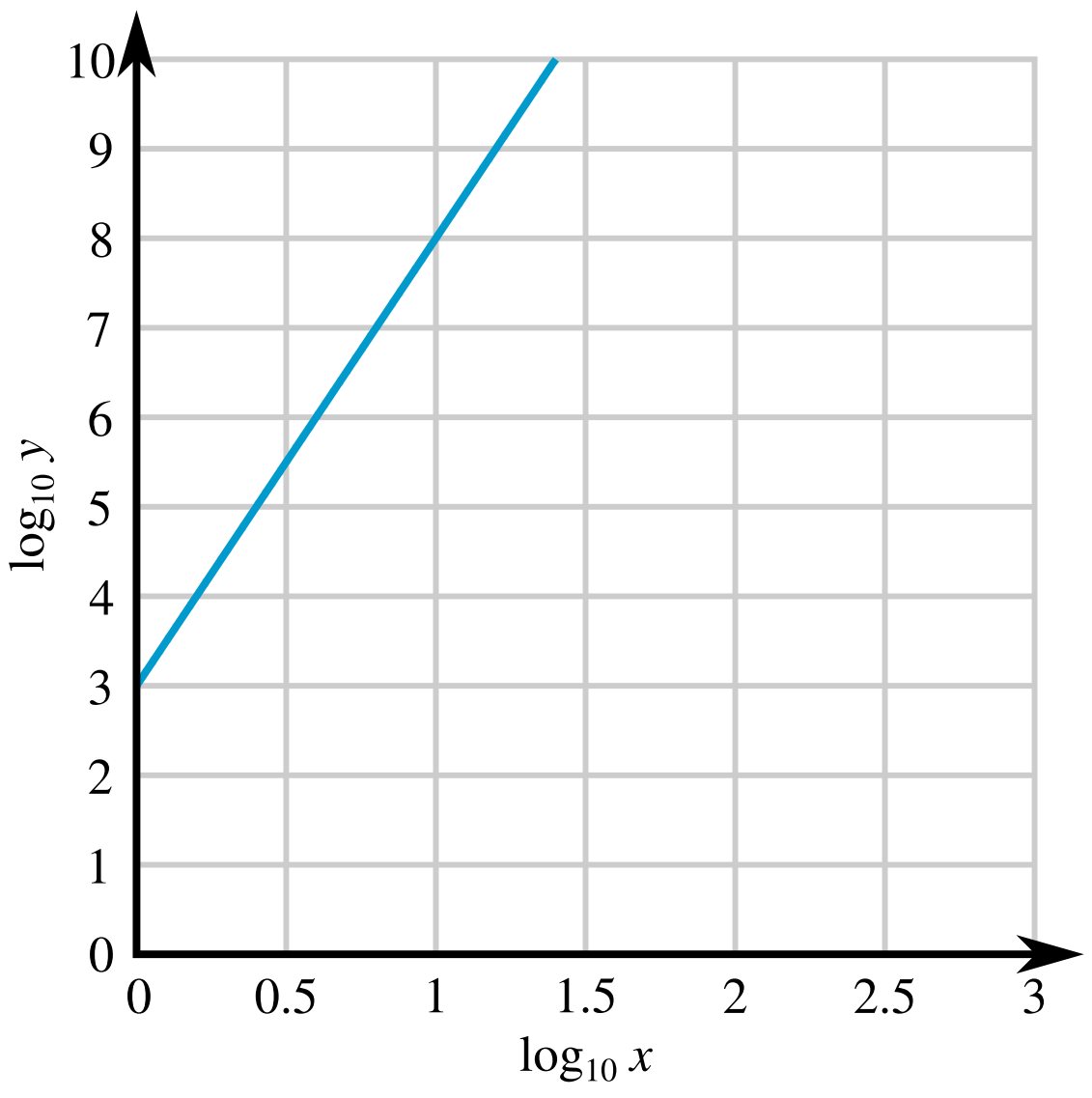


**Figure 1:** A plot of $\log_{10} y$ against $\log_{10} x$.

Use this plot to determine the relationship between $x$ and $y$. Give your answer in the form $y = ax^b$, where $a$ and $b$ are constants.

The following symbols may be useful: `x, y`

Adapted for Isaac Physics from NST IA Biology preparation work

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# Logarithmic Plots 3

A Level

By plotting a graph of $\ln F$ against $\ln r$, a student finds that the relationship between the gravitational force, $F$, on a pair of objects with fixed masses is given by

$$F = \frac{10^8}{r^2}$$

where $r$ is the separation between them.

## Part A Find the gradient

What was the gradient of the graph?

## Part B Find the intercept

What was the intercept of the graph? Give your answer to 2 significant figures.

Adapted for Isaac Physics from NST IA Biology preparation work

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# 3 Simultaneous Equations 3i

Further A

The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

## Part A $a$

Find the value of $a$ in exact form, given that $\mathbf{B}$ is singular.

The following symbols may be useful: a

## Part B $\mathbf{B}^{-1}$

$\mathbf{B}^{-1}$ can be written in the form $\mathbf{B}^{-1} = \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \zeta \\ \eta & \theta & \iota \end{pmatrix}$. You are given that $\mathbf{B}$ is non-singular.

Give an expression for $\alpha - \beta + \gamma - \delta + \epsilon - \zeta + \eta - \theta + \iota$ in terms of $a$.

The following symbols may be useful: a

## Part C Simultaneous equations

$x$, $y$ and $z$ satisfy the following simultaneous equations

$$-x + y + 3z = 1$$

$$2x + y - z = 4$$

$$y + 2z = -1$$

Use matrix methods to solve this question only.

Find $x$ in exact form.

The following symbols may be useful: x

Find $y$ in exact form.

The following symbols may be useful: y

Find $z$ in exact form.

The following symbols may be useful: z

Adapted with permission from UCLES, A Level, June 2005, Paper 4725, Question 7.

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# Matrices - Intersecting Lines

Further AUniversity

Two lines are described by

$$3x - 4y - 1 = 0$$
$$2x + py - 10 = 0.$$

where $p$ is a constant. Use matrix notation to find the coordinates of the point of intersection of these two lines.

## Part A Write in matrix form

Write these equations in matrix form $\mathbf{Ax} = \mathbf{b}$.

If the matrix $\mathbf{A}$ is written in the form

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

give the values of these matrix elements.

Give the value of $a_{11}$.

Give the value of $a_{12}$.

Give the value of $a_{21}$.

Give the value of $a_{22}$.

The following symbols may be useful: p

## Part B Condition for no intersection

Use the matrix to find the value of $p$ for which the lines do not intersect. Give your answer as an improper fraction.

The following symbols may be useful: p

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## Part C The inverse matrix

Find $\mathbf{A}^{-1}$, the inverse of $\mathbf{A}$.

If the matrix $\mathbf{A}^{-1}$ is written in the form

$$\mathbf{A}^{-1} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$$

give the values of these matrix elements

Give an expression for $\alpha_{11}$.

The following symbols may be useful: p

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Give an expression for $\alpha_{12}$.

The following symbols may be useful: p

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Give an expression for $\alpha_{21}$.

The following symbols may be useful: p

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Give an expression for $\alpha_{22}$.

The following symbols may be useful: p

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## Part D Components of point of intersection

Using $\mathbf{A}^{-1}$ obtain expressions for the $x$ and $y$ components for the point of intersection.

Give an expression for the $x$-component of the point of intersection.

The following symbols may be useful: p

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Give an expression for the $y$-component of the point of intersection.

The following symbols may be useful: p

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## Part E A value for $p$

If the $y$-component of the point of intersection is equal to 2, find the value of $p$.

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# Matrices - Linear Equations 2

Further AUniversity

Use matrix notation to solve the following set of three equations for $x$, $y$ and $z$:

$$x + cy = c$$
$$x - y + 3z = -c$$
$$2x - 2y - z = 2.$$

## Part A Determinant of the matrix

Write these equations in matrix form $\mathbf{Rx} = \mathbf{p}$. Hence deduce the determinant of $\mathbf{R}$ and find the value of $c$ for which there is no unique solution.

Find the determinant of $\mathbf{R}$.

The following symbols may be useful: c

Deduce the value of $c$ for which there is no unique solution.

## Part B The inverse matrix

Find the inverse matrix $\mathbf{R}^{-1}$.

If the matrix $\mathbf{R}^{-1}$ is written in the form

$$\mathbf{R}^{-1} = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{pmatrix}$$

give expressions for the elements of $\mathbf{R}^{-1}$ on the leading diagonal i.e. $\rho_{11}$, $\rho_{22}$ and $\rho_{33}$.

Give an expression for $\rho_{11}$

The following symbols may be useful: c

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Give an expression for $\rho_{22}$

The following symbols may be useful: c

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Give an expression for $\rho_{33}$.

The following symbols may be useful: c

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## Part C Solution to the set of equations if $c = 1$

Using $\mathbf{R}^{-1}$, find the solutions for $x$, $y$ and $z$ if $c = 1$.

Find the value of $x$.

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Find the value of $y$.

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Find the value of $z$.

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# Matrices - Linear Equations 3

Further AUniversity

C C C P P P

A system consists of three masses $m_1$, $m_2$ and $m_3$ in a line; they each have the same mass $m$. The mass $m_2$ is in the centre and connected by springs of spring constant $k$ to $m_1$ on the left and $m_3$ on the right. The masses are all performing simple harmonic motion at the same angular frequency $\omega$ such that their equations of motion are

$$\begin{aligned} -kx_1 + kx_2 &= -m\omega^2 x_1 \\ kx_1 - 2kx_2 + kx_3 &= -m\omega^2 x_2 \\ kx_2 - kx_3 &= -m\omega^2 x_3. \end{aligned}$$

where $x_1$, $x_2$ and $x_3$ are the displacements of $m_1$, $m_2$ and $m_3$ respectively.

These equations can be written in matrix form

$$\begin{aligned} \mathbf{A}\mathbf{x} &= -m\omega^2\mathbf{x} \\ &= -m\omega^2\mathbf{I}\mathbf{x} \\ \Rightarrow (\mathbf{A} + m\omega^2\mathbf{I})\mathbf{x} &= 0 \end{aligned}$$

A matrix equation of this sort only has solutions if $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$. Use this to find the possible values of $\omega^2$. For each value of $\omega$ find the relationship between $x_1$, $x_2$ and $x_3$.

## Part A The matrix $\mathbf{A}$

If the matrix $\mathbf{A}$ is written in the form

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

deduce the expressions for the following elements of $\mathbf{A}$.

Give the expression for $a_{11}$.

The following symbols may be useful: `k, m`

Give the expression for $a_{21}$.

The following symbols may be useful: `k, m`

Give the expression for $a_{22}$.

The following symbols may be useful: `k, m`

Give the expression for $a_{31}$.

The following symbols may be useful: `k, m`

## Part B The possible values of $\omega^2$

Write down the matrix $\mathbf{A} + m\omega^2\mathbf{I}$. Using the fact that solutions to the equation $\mathbf{A} + m\omega^2\mathbf{I} = 0$, require that $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$ deduce the three values of $\omega^2$. The three values, $\omega_1^2$, $\omega_2^2$ and $\omega_3^2$, are such that $\omega_1^2 < \omega_2^2 < \omega_3^2$.

Give an expression for the $11$ component (i.e. the component in row $1$, column $1$) of $\mathbf{A} + m\omega^2\mathbf{I}$.

The following symbols may be useful: `k, m, omega`

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Find an expression for $\omega_1^2$.

The following symbols may be useful: `k, m`

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Find an expression for $\omega_2^2$.

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Find an expression for $\omega_3^2$.

The following symbols may be useful: `k, m`

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## Part C The relationship between $x_1$, $x_2$ and $x_3$

Since the determinant of the matrix is zero there are no unique solutions to the set of three equations; however, for each value of $\omega^2$, $x_1$, $x_2$ and $x_3$ have a fixed relationship to each other. On the assumption that $x_1 = 1$, find $x_2$ and $x_3$ for each of the three frequencies deduced in Part B. Give your answers using the format $1,a,b$ with no spaces, where $x_1 = 1$, $x_2 = a$ and $x_3 = b$.

Given that $x_1 = 1$, find $x_2$ and $x_3$ for $\omega_1^2$.

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Given that $x_1 = 1$, find $x_2$ and $x_3$ for $\omega_2^2$.

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Given that $x_1 = 1$, find $x_2$ and $x_3$ for $\omega_3^2$.

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