Matrices: nxm Rules 2i

Further A
P P P

The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$

Part A $\mathbf{AB}$

Find the matrix $\mathbf{AB}$

( )

Part B $\mathbf{BA} - 4\mathbf{C}$

Find the matrix given by $\mathbf{BA} - 4\mathbf{C}$.

( )

Adapted with permission from UCLES, A Level, June 2010, Paper 4725, Question 2.

2x2 Operations 2ii

Further A
P P P

The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} a & -1 \\ -3 & -2 \end{pmatrix}$.

Part A a

a satisfies the equation $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.

Find the value of a.

The following symbols may be useful: a

Part B Alternate value of a

Now take a to satisfy the equation $\mathbf{AB} = \begin{pmatrix} 7 & -4 \\ 9 & -7 \end{pmatrix}$.

Find the value of a.

The following symbols may be useful: a

Adapted with permission from UCLES, A Level, June 2007, Paper 4725, Question 1.

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STEM SMART Double Maths 9 - Matrices

2x2 Determinants and Inverses 1ii

Further A

P P P

The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -4 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}$. $\mathbf{I}$ denotes the 2×2 identity matrix.

Part A $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$

Find the matrix given by $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$.

$$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$$

Part B $\mathbf{A}^{-1}$

Find $\mathbf{A}^{-1}$.

$$\mathbf{A}^{-1} = \frac{1}{\square} \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$$

Items:

-5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 10 12 13 14 15

Part C $\left(\mathbf{AB}^{-1}\right)^{-1}$

Find $\left(\mathbf{AB}^{-1}\right)^{-1}$.

$$\left(\mathbf{AB}^{-1}\right)^{-1} = \frac{1}{\square}\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$$

Items:

-5 -4 -3 -2 -1 0 1 2 3 4 5 7 9 11 14 17 21 22

Adapted with permission from UCLES, A Level, Jan 2014, Paper 4725, Question 3.

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STEM SMART Double Maths 9 - Matrices

Matrices: 3x3 Determinants and Inverses 1i

Further A P P P

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$. The matrix $\mathbf{B}$ is such that $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$.

Part A $\det \mathbf{AB}$

Find $\det \mathbf{AB}$.

The following symbols may be useful: a

Part B $(\mathbf{AB})^{-1}$

Find $(\mathbf{AB})^{-1}$.

$$(\mathbf{AB})^{-1} = \frac{1}{\square} \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

Items:

-3 -2 -1 0 1 2 3 4 5 6 $a-2$ $a-3$ $a-6$ $2-a$ $3-a$ $6-a$ $3a-2$ $3a-3$ $3a-6$ $2-3a$ $3-3a$ $6-3a$

Part C $\mathbf{B}^{-1}$

Find $\mathbf{B}^{-1}$.

$$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 10.

Gameboard:

STEM SMART Double Maths 9 - Matrices

Physics. You work it out.

Home Gameboard Maths 3 Simultaneous Equations 3i

3 Simultaneous Equations 3i

Further A
P P P

The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

Part A a

Find the value of a in exact form, given that $\mathbf{B}$ is singular.

The following symbols may be useful: a

Part B $\mathbf{B}^{-1}$

Given that $\mathbf{B}$ is non-singular, find $\mathbf{B}^{-1}$.

$$\mathbf{B}^{-1} = \frac{1}{\boxed{}} \begin{pmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{pmatrix}$$

Items:

-4 -3 -2 -1 0 1 2 3 4 a $-a$ $2a$ $-2a$ $3a$ $-3a$ $a+2$ $a-2$ $a+4$ $a-4$ $a+6$ $a-6$ $2a+2$ $2a-2$ $3a+2$ $3a-2$

Part C Simultaneous equations

x, y and z satisfy the following simultaneous equations

$$\begin{aligned} -x + y + 3z &= 1 \\ 2x + y - z &= 4 \\ y + 2z &= -1 \end{aligned}$$

Use matrix methods to find x, y and z.

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Adapted with permission from UCLES, A Level, June 2005, Paper 4725, Question 7.

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STEM SMART Double Maths 9 - Matrices

Physics. You work it out.

Home Gameboard Maths Algebra Matrices Matrices - Intersecting Lines

Matrices - Intersecting Lines

Further A P P P University P P P

Two lines are described by

$$3x - 4y - 1 = 0$$
$$2x + py - 10 = 0.$$

where p is a constant. Use matrix notation to find the coordinates of the point of intersection of these two lines.

Part A Write in matrix form

Write these equations in matrix form $\mathbf{Ax} = \mathbf{b}$.

$$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

Part B Condition for no intersection

Use the matrix to find the value of p for which the lines do not intersect. Give your answer as an improper fraction.

The following symbols may be useful: p

Part C The inverse matrix

Find $\mathbf{A}^{-1}$, the inverse of $\mathbf{A}$.

$$\mathbf{A}^{-1} = \frac{1}{\square}\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$$

Items:

-4 -3 -2 -1 0 1 2 3 4 p $-p$ $p+4$ $p-4$ $p+8$ $p-8$ $3p+4$ $3p-4$ $3p+8$ $3p-8$

Part D Components of point of intersection

Using $\mathbf{A}^{-1}$ find the the point of intersection in terms of p.

$$\left(\frac{\square}{\square}, \frac{\square}{\square}\right)$$

Items:

18 20 28 30 38 40 $p+4$ $p+8$ $p+40$ $p+80$ $3p+4$ $3p+8$ $3p+40$ $3p+80$

Part E A value for p

If the y-component of the point of intersection is equal to 2, find the value of p.

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Matrices - Linear Equations 2

Further A P P P University P P P

Use matrix notation to solve the following set of three equations for x, y and z:

$$\begin{aligned} x + cy &= c \\ x - y + 2z &= -c \\ 2x - 2y - z &= 2. \end{aligned}$$

Part A Write in matrix form

Write these equations in matrix form $\mathbf{Rx} = \mathbf{p}$.

$$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

Part B Determinant of the matrix

Find the determinant of $\mathbf{R}$ in terms of c.

The following symbols may be useful: c

Part C Condition for no unique solution

Deduce the value of c for which there is no unique solution.

Part D The inverse matrix

Find the inverse matrix $\mathbf{R}^{-1}$.

$$\mathbf{R}^{-1} = \frac{1}{\square}\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

Items:

-3 -2 -1 0 1 2 3 4 5 c $-c$ $2c$ $-2c$ $5c$ $-5c$ $c+1$ $c-1$ $-c-1$ $2c+2$ $2c-2$ $-2c-2$ $5c+5$ $5c-5$ $-5c-5$

Part E Solution to the set of equations if $c = 1$

Using $\mathbf{R}^{-1}$, find the solutions for x, y and z if $c = 1$.

($\square$, $\square$, $\square$)

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Matrices - Linear Equations 3

Further A University

A system consists of three masses m_1, m_2 and m_3 in a line; they each have the same mass m. The mass m_2 is in the centre and connected by springs of spring constant k to m_1 on the left and m_3 on the right. The masses are all performing simple harmonic motion at the same angular frequency ω such that their equations of motion are

$$\begin{aligned} -kx_1 + kx_2 &= -m\omega^2 x_1 \\ kx_1 - 2kx_2 + kx_3 &= -m\omega^2 x_2 \\ kx_2 - kx_3 &= -m\omega^2 x_3 \,. \end{aligned}$$

where x_1, x_2 and x_3 are the displacements of m_1, m_2 and m_3 respectively.

These equations can be written in matrix form

$$\begin{aligned} \mathbf{Ax} &= -m\omega^2 \mathbf{x} \\ &= -m\omega^2 \mathbf{Ix} \\ \Rightarrow (\mathbf{A} + m\omega^2 \mathbf{I})\mathbf{x} &= 0 \end{aligned}$$

A matrix equation of this sort only has solutions if $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$. Use this to find the possible values of ω^2. For each value of ω find the relationship between x_1, x_2 and x_3.

Part A The matrix $\mathbf{A}$

Find the matrix $\mathbf{A}$.

$$\mathbf{A} = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

Items:

-2 -1 0 1 2 $-3k$ $-2k$ $-k$ k $2k$ $3k$

Part B The matrix $\mathbf{A} + m\omega^2\mathbf{I}$

Find the matrix $\mathbf{A} + m\omega^2\mathbf{I}$.

$$\mathbf{A} + m\omega^2\mathbf{I} = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

Items:

-1 0 1 $-2k$ $-k$ k $2k$ $m\omega^2 - 2k$ $m\omega^2 - k$ $m\omega^2 + k$ $m\omega^2 + 2k$

Part C The possible values of ω^2

Using the fact that non-zero solutions to the equation $(\mathbf{A} + m\omega^2\mathbf{I})\mathbf{x} = 0$ require that $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$, deduce the three values of ω^2. The three values, ω_1^2, ω_2^2 and ω_3^2, are such that $\omega_1^2 < \omega_2^2 < \omega_3^2$.

$\omega_1^2 =$ ☐, $\omega_2^2 =$ ☐ and $\omega_3^2 =$ ☐

Part D The relationship between x_1, x_2 and x_3

Since the determinant of the matrix is zero there are no unique solutions to the set of three equations; however, for each value of ω^2, x_1, x_2 and x_3 have a fixed relationship to each other. On the assumption that $x_1 = 1$, find x_2 and x_3 for each of the three frequencies deduced in Part B.

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