Double Angles 2

Pre-Uni Maths for Science H4.9

A Level

P P P

Part A $\tan(2\theta)$

Using the formula for $\tan(A \pm B)$, derive the double angle formula for $\tan(2\theta)$.

The following symbols may be useful: `cos(), sin(), tan(), theta`

Part B $\tan(4\theta)$

Now prove that $\tan 4\theta = \dfrac{k}{1 - 6\tan^2\theta + \tan^4\theta}$ and give an expression for k in its simplest form in terms of $\tan\theta$.

The following symbols may be useful: `k, theta`

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Trigonometry: Double Angles 1ii

A Level

C C C

Part A   **The form** $a\sin^2\theta + b\sin\theta + c = 0$

Express the equation $(\operatorname{cosec}\theta)(3\cos 2\theta + 7) + 11 = 0$ in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b, and c are constants and $a > 0$.

Give the value of a.

The following symbols may be useful: a

Give the value of b.

The following symbols may be useful: b

Give the value of c.

The following symbols may be useful: c

Hence solve, for $-180° < \theta < 180°$, the equation $(\operatorname{cosec} \theta)(3 \cos 2\theta + 7) + 11 = 0$. Give your answers in degrees, to three significant figures.

Give the highest (most positive) solution.

Give the lowest (most negative) solution.

Gameboard:
STEM SMART Single Maths 31 - Compound and Double-Angle Formulae

Trigonometry: Double Angles 2ii

Part A $\sin$ **Double Angle**

Write down the identity expressing $\sin 2\theta$ in terms of $\sin \theta$ and $\cos \theta$.

The following symbols may be useful: `theta`

Part B $\sin 2\alpha$

Given that $\sin \alpha = \frac{1}{4}$ and α is acute, find the exact value of $\sin 2\alpha$.

The following symbols may be useful: `alpha`

Part C   **Solve**

Solve in degrees, for $0° < \beta < 90°$, the equation $5 \sin 2\beta \sec \beta = 3$, giving your answer in degrees to three significant figures.

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Gameboard:
STEM SMART Single Maths 31 - Compound and Double-Angle Formulae

Trigonometry: Combined Angles 5i

The value of $\tan 10°$ is denoted by p. Find, in terms of p, the value of:

Part A $\tan 55°$

$\tan 55°$

The following symbols may be useful: p

Part B $\tan 5°$

$\tan 5°$

The following symbols may be useful: p

Part C $\tan \theta$

$\tan \theta$, where θ satisfies the equation $3 \sin \left(\theta + 10° \right) = 7 \cos \left(\theta - 10° \right)$.

The following symbols may be useful: p, theta

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Gameboard:

STEM SMART Single Maths 31 - Compound and Double-Angle Formulae

Functions: Reciprocal Trig 2i

It is given that A and B are angles such that

$$\sec^2 A - \tan A = 13 \quad \text{and} \quad \sin B \sec^2 B = 27 \cos B \csc^2 B.$$

Part A   Largest value of $\tan(A - B)$

Find the largest possible exact value of $\tan(A - B)$.

The following symbols may be useful: A, B

Part B   Smallest value of $\tan(A - B)$

Give the smallest possible value of $\tan(A - B)$.

The following symbols may be useful: A, B

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Gameboard:

STEM SMART Single Maths 31 - Compound and Double-Angle Formulae

Trigonometry and R-Form 1

Part A $5\cos x + 12\sin x$

Express $5\cos x + 12\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

State the value of R.

The following symbols may be useful: R

Give the value of α in degrees, to three significant figures.

Give details of a pair of transformations which transform the curve $y = \cos x$ to the curve $y = 5\cos x + 12\sin x$.

Available items

Translation in the positive x direction by α.

Translation in the negative x direction by α.

Stretch parallel to the y-axis by a factor of R.

Translation in the negative y direction by α.

Stretch parallel to the x-axis by a factor of R.

Stretch parallel to the y-axis by a factor of $\frac{1}{R}$.

Translation in the positive y direction by α.

Stretch parallel to the x-axis by a factor of $\frac{1}{R}$.

Part C   Solve

Solve, for $0° < x < 360°$, the equation $5\cos x + 12\sin x = 2$, giving your answers correct to the nearest $0.1°$.

Give the smallest solution, in degrees, to four significant figures.

Give the largest solution, in degrees, to four significant figures.
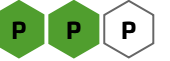

Gameboard:
STEM SMART Single Maths 31 - Compound and Double-Angle Formulae

Trigonometry: Combined Angles 3i

A Level
P P P

In **Figure 1**, $ABCD$ represents a rectangular table with sides $3.5\,\text{m}$ and $1.5\,\text{m}$. It has been turned so it wedges in a passage of width $2.5\,\text{m}$.
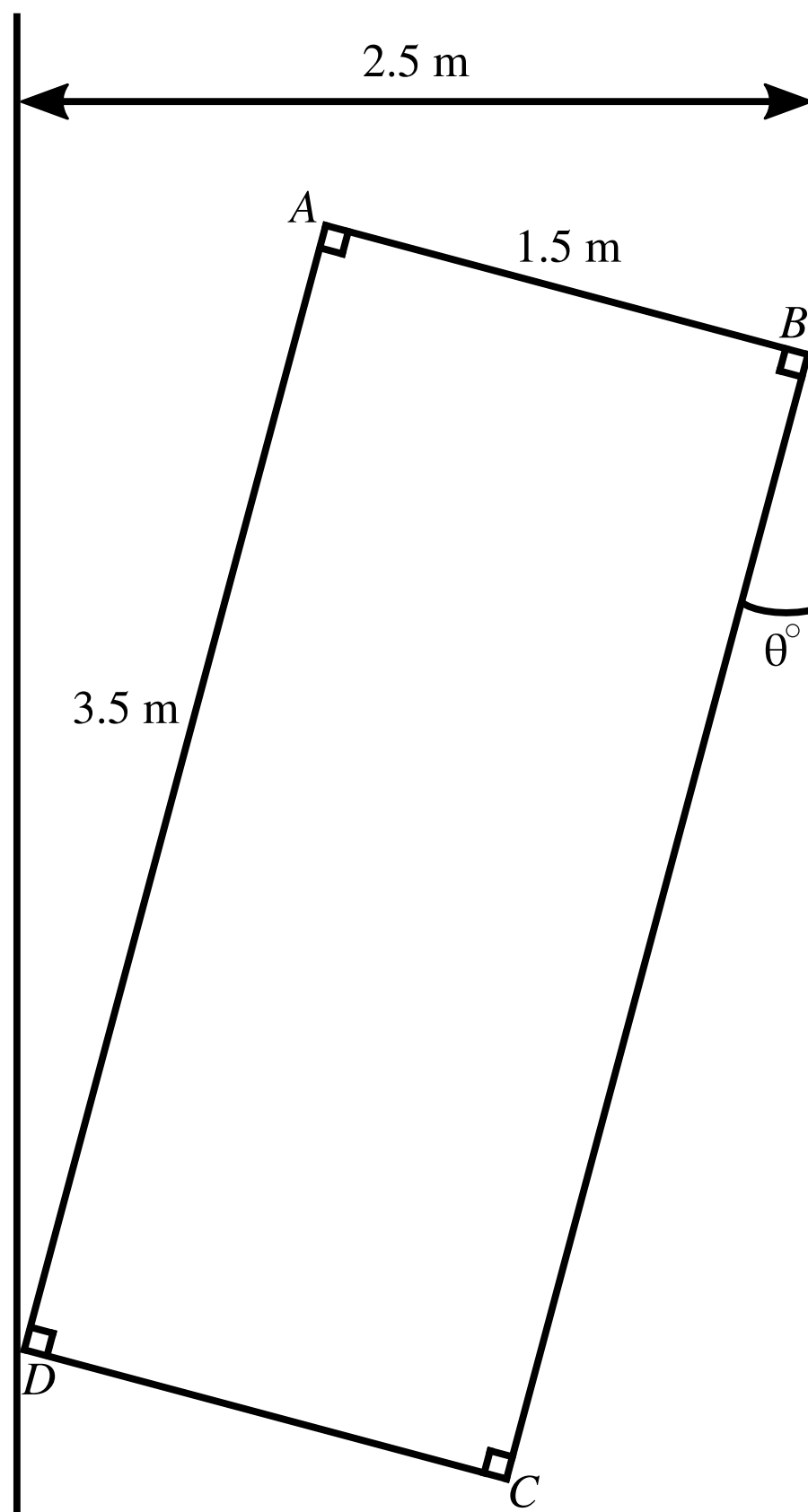
Figure 1: The rectangular table $ABCD$.

Part A   Value of $7 \sin \theta° + 3 \cos \theta°$

Given that θ is the acute angle between the longer side and the passage, as shown in the diagram, find the exact value of $7 \sin \theta + 3 \cos \theta$.

The following symbols may be useful: `cos()`, `sin()`, `tan()`, `theta`

Part B   The form $R \sin (\theta° + \alpha°)$

Express $7 \sin \theta + 3 \cos \theta$ in the form $R \sin (\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

Give the exact value of R.

The following symbols may be useful: `R`

Give the value of α to 3 significant figures.

Part C   Find θ

Find θ, to 3 significant figures.

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Gameboard:
STEM SMART Single Maths 31 - Compound and Double-Angle Formulae

Addition of Angles 7

Pre-Uni Maths for Science H4.10

Two waves

$$\psi_1 = A\cos\left(2\pi ft - \left(\frac{2\pi}{\lambda}\right)x + \phi\right)$$

and

$$\psi_2 = A\cos\left(2\pi ft - \left(\frac{2\pi}{\lambda}\right)x - \phi\right)$$

interfere, such that the resultant wave is given by $\psi = \psi_1 + \psi_2$. Express ψ as the product of two terms.

Express ψ as the product of two terms.

The following symbols may be useful: `A, f, lambda, phi, pi, t, x`

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