Braking a Car

A Level

C C C

When a car of mass $1000\,\mathrm{kg}$ is travelling along a level road at a steady speed of $20\,\mathrm{m\,s^{-1}}$, its engine is working at $18\,\mathrm{kW}$.

Part A Resistive force

Find the magnitude of the resistive force due to friction, which may be taken to be constant.

Part B Braking force on the level

The engine is suddenly disconnected and the brakes applied, and the car comes to rest in $50\,\mathrm{m}$. Find the force, assumed constant, exerted by the brakes.

Part C Braking force on a slope

Find also the distance in which the car, travelling at $20\,\mathrm{m\,s^{-1}}$, would come to rest if the engine were disconnected and the same braking force applied on an upward incline of angle θ, where $\sin\theta = \frac{1}{20}$.

Part D Braking force downhill

By how much does this change if the car is travelling down the same hill at $20\,\mathrm{m\,s^{-1}}$?

Adapted with permission from UCLES, A Level Applied Mathematics, June 1961, Paper 1, Question 2.

Physics. You work it out.

Essential Pre-Uni Physics B9.2

A Level

Assume that extension is proportional to the tension.

An unstretched spring of natural length $30\,\text{cm}$ with spring constant $8.0\,\text{N}\,\text{cm}^{-1}$ is stretched by 20% of its original length. Work out how much elastic potential energy is stored in the spring. Give your answer to 2 significant figures.

Gameboard:
STEM SMART Double Maths 49 - Further Mechanics Revision

Restitution Between Spheres

Further A

Two small spheres A and B, with masses $0.30\,\mathrm{kg}$ and m respectively, lie at rest on a smooth horizontal surface. A is projected directly towards B with speed $6.0\,\mathrm{m\,s^{-1}}$ and hits B. The direction of motion of A is reversed in the collision. The speeds of A and B after the collision are $1.0\,\mathrm{m\,s^{-1}}$ and $3.0\,\mathrm{m\,s^{-1}}$ respectively. The coefficient of restitution between A and B is e.

Part A Mass

Find m.

Part B Coefficient of restitution

Find the value of e.

Part C Second collision?

B continues to move at $3.0\,\mathrm{m\,s^{-1}}$ and strikes a vertical wall at right angles and rebounds off the wall. The coefficient of restitution between B and the wall is f.

Find the range of values of f for which there will be a second collision between A and B. Fill in the gaps below.

[ ] [ ] f [ ] [ ]

Items:

-3 -2 -1 $-\frac{2}{3}$ $-\frac{1}{2}$ $-\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ 1 2 3 $<$ $>$ $\leq$ $\geq$ $=$

Part D Impulse from the wall

Find, in terms of f, the magnitude of the impulse that the wall exerts on B.

The following symbols may be useful: `f`

Part E Final speeds

Given that $f = \frac{3}{4}$,

Calculate the final speed of A.

Calculate the final speed of B.

Restitution: Sphere Collision 2

Further A

Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass $0.10\,\text{kg}$ and B has mass $0.40\,\text{kg}$. Immediately before the collision A is moving with speed $2.8\,\text{m s}^{-1}$ along the line of centres, and B is moving with speed $1.0\,\text{m s}^{-1}$ at an angle θ to the line of centres, where $\cos\theta = 0.80$, as shown in **Figure 1**. Immediately after the collision A is stationary.

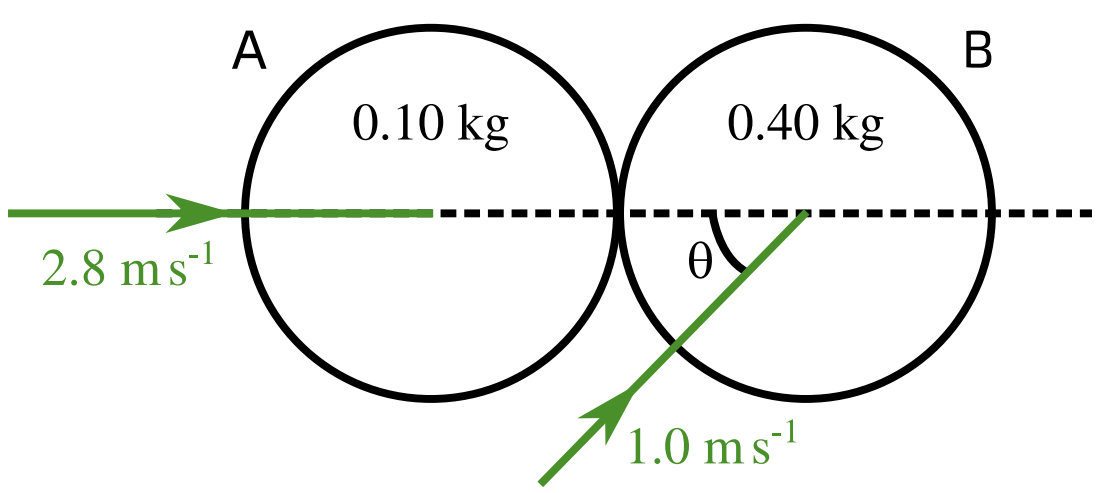

Figure 1: The spheres A and B as they collide.

Part A Coefficient of restitution

Find the value of the coefficient of restitution between A and B.

Part B Change in direction

Find the angle turned through by the direction of motion of B as a result of the collision.

Physics. You work it out.

Two Spinning Balls

A Level

C C C

A ball of mass m_1 is suspended from a fixed point O by a light string of length $l = 5.0\,\mathrm{m}$ and a second ball of mass m_2 is suspended from the first by a light string of the same length.

When the system is rotating steadily with angular velocity ω about the vertical through O , the balls describe horizontal circles of radii $3.0\,\mathrm{m}$ and $7.0\,\mathrm{m}$ with their centres $4.0\,\mathrm{m}$ and $7.0\,\mathrm{m}$ respectively below O .

Part A Angular speed

Find the value of ω

Part B Ratio of masses

What is the ratio of the masses of the balls? Express your answer as a decimal greater than one.

Gameboard:

STEM SMART Double Maths 49 - Further Mechanics

Revision

Centre of Mass: Lamina 2

Further A

Figure 1 shows a uniform lamina BCD in the shape of a quarter circle of radius $6.0\,\text{cm}$.

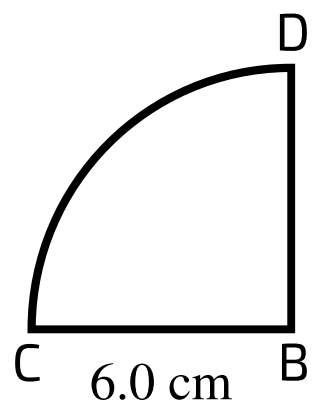

Figure 1: Lamina BCD.

Part A Centre of mass of BCD

Find the distance of the centre of mass of the lamina from B.

Part B Distance from BD

A uniform rectangular lamina ABDE is such that AB is $12\,\mathrm{cm}$ and AE is $6.0\,\mathrm{cm}$. A single plane object ABCDE is formed by attaching the rectangular lamina ABDE to the lamina BCD along BD, as shown in **Figure 2**. The mass of ABDE is $3.0\,\mathrm{kg}$ and the mass of BCD is $2.0\,\mathrm{kg}$.

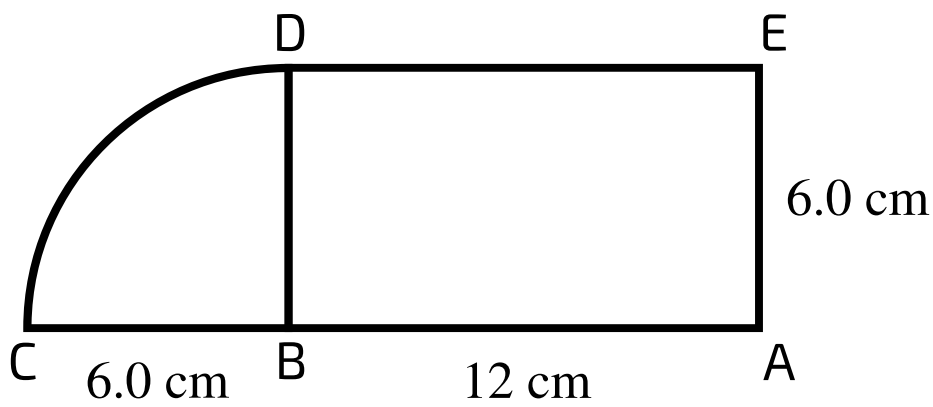

Figure 2: Object ABCDE, formed by joining laminas BCD and ABDE.

Find the distance of the centre of mass of the object ABCDE from BD.

Part C Distance from AC

Find the distance of the centre of mass of the object ABCDE from AC.

Part D Angle to vertical

The object ABCDE is freely suspended at C and rests in equilibrium.

Calculate the angle that AC makes with the vertical.

Adapted with permission from UCLES A Level, June 2009, Paper 4729, Question 5.

Gameboard:
STEM SMART Double Maths 49 - Further Mechanics Revision

Centre of Mass by Integration 2

Further A

The region bounded by the y-axis and the curves $y = \sin(2x)$ and $y = \sqrt{2}\cos x$ for $0 \leq x \leq \frac{\pi}{4}$ is occupied by a uniform lamina.

Find the exact value of the x-coordinate of the centre of mass of the lamina.

The following symbols may be useful: `pi, x`

Used with permission from UCLES, A Level, June 2014, Paper 4731/01, Question 3

Gameboard:
STEM SMART Double Maths 49 - Further Mechanics Revision

Differential Equations: Resistive Forces 2i

Further A

A particle of mass $0.2\,\mathrm{kg}$ travels in a straight line on a smooth horizontal surface.

At time t seconds it is $x\,\mathrm{m}$ from a fixed point O and is moving away from O with velocity $v\,\mathrm{m\,s^{-1}}$.

A force of magnitude $\frac{1}{2}\left(12-\frac{1}{4}v\right)^{\frac{1}{2}}\,\mathrm{N}$ acts on the particle in the direction of motion.

At time $t=0$ the particle is at O and has velocity $12\,\mathrm{m\,s^{-1}}$.

Part A Maximum velocity

State the maximum possible velocity of the particle.

Part B Expression for v

Find an expression for v in terms of t, valid while the particle is accelerating.

The following symbols may be useful: `t`

Part C Distance travelled

Hence find the distance travelled by the particle as its velocity increases from $12\,\mathrm{m\,s^{-1}}$ to $32\,\mathrm{m\,s^{-1}}$.

Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, June 2017, Paper 4730, Question 3.

Differential Equations: General Applications 2ii

Further A

A particle P of mass $0.2\,\text{kg}$ is suspended from a fixed point O by a light elastic string of natural length $0.7\,\text{m}$ and modulus of elasticity $3.5\,\text{N}$. P is at the equilibrium position when it is projected vertically downwards with speed $1.6\,\text{m}\,\text{s}^{-1}$. At time $t\,\text{s}$ after being set in motion P is $x\,\text{m}$ below the equilibrium position and has velocity $v\,\text{m}\,\text{s}^{-1}$.

The tension, T, in the string is expressed as

$$T = \frac{3.5(0.392 + x)}{0.7}\,\text{N}$$

The equilibrium position of P is $1.092\,\text{m}$ below O, and the strength of gravity is $9.8\,\text{N}\,\text{kg}^{-1}$.

Part A SHM

Prove that P moves with simple harmonic motion, and calculate the amplitude.

Part B Find x

Calculate x when $t = 0.4$. Give your answer to 3 significant figures.

Part C Find v

Calculate the velocity of P when $t = 0.4$. Give your answer to 3 significant figures.