Integration - Trig Manipulations 1ii

A Level

P P P

Use integration to find the exact value of $\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} (9 - 6\cos^2 4x)\,\mathrm{d}x$.

The following symbols may be useful: `pi`

Integration - Trig Manipulations 3ii

A Level

Find $\int_0^{\frac{\pi}{4}} \frac{1-2\sin^2 x}{1+2\sin x\cos x}\mathrm{d}x$, giving your answer in the form $a\ln b$.

Gameboard:

STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Integration - Trig Manipulations 3i

A Level

Part A Simplify

Simplify as far as possible $\frac{1}{1-\tan x} - \frac{1}{1+\tan x}$.

The following symbols may be useful: x

Part B Integrate

Hence evaluate $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} (\frac{1}{1-\tan x} - \frac{1}{1+\tan x})\mathrm{d}x$, giving your answer in the form $a\ln(b)$.

The following symbols may be useful: pi

Gameboard:

STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Integration by Substitution 2i

A Level

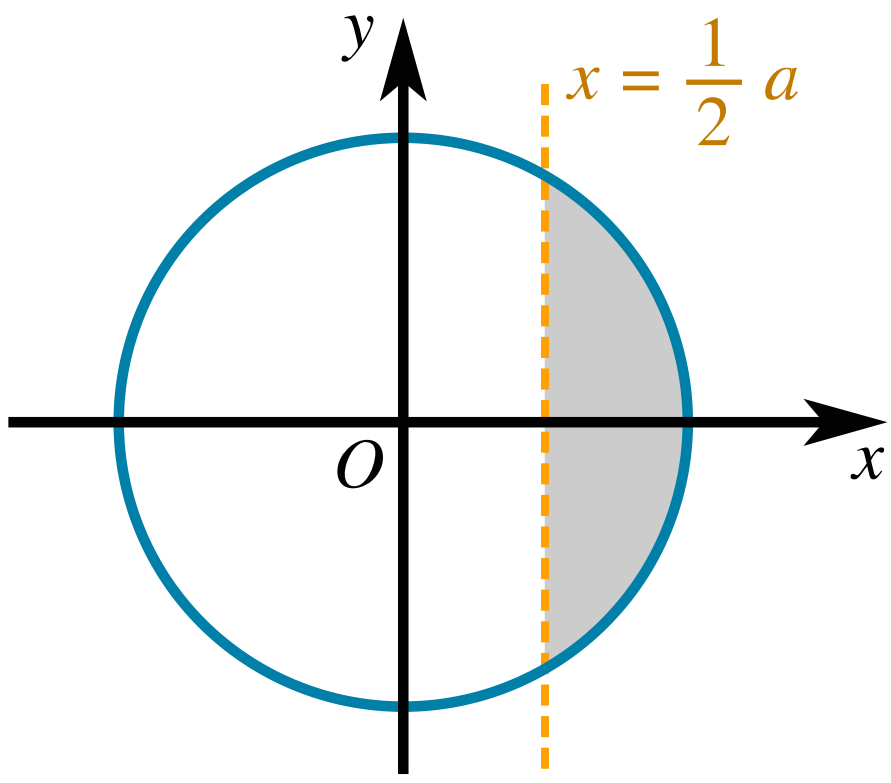
Part A The substitution $x = a\sin\theta$

By using the substitution $x = a\sin\theta$, find the exact value of

$$\int_{\frac{1}{2}a}^{a} \sqrt{(a^2 - x^2)}\,dx$$

The following symbols may be useful: `a, pi`

Part B Area of a segment



The diagram shows the circle $x^2 + y^2 = a^2$ and the line $x = \frac{1}{2}a$. Find the area of the shaded region, giving your answer in an exact form.

The following symbols may be useful: `a, pi`

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Gameboard:

STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Integration by Parts 3ii

A Level

Evaluate $\int_0^{\frac{\pi}{2}} x\cos x\,\mathrm{d}x$, giving your answer in an exact form.

The following symbols may be useful: `pi`

Gameboard:
STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Integration by Parts 5

A Level

P P P

Pre-Uni Maths for Sciences K4.5

Find, by integrating by parts twice, $\displaystyle\int_0^{\pi/3} \mathrm{e}^{-x} \sin x \, \mathrm{d}x$.

The following symbols may be useful: `e, pi`

Created for isaacphysics.org by Julia Riley

Gameboard:

STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Integration by Parts 2ii

A Level

Find the exact value of $\int_1^8 \frac{1}{\sqrt[3]{x}} \ln(x) \mathrm{d}x$, giving your answer in the form $A\ln(2) + B$, where A and B are constants to be found.

Gameboard:

STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Integration of Differential Equations 1ii

A Level

The gradient of a curve at the point (x, y), where $x > -2$, is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3y^2(x+2)}$$

The points $(1, 2)$ and $(q, 1.5)$ lie on the curve. Find the value of q, giving your answer correct to 3 significant figures.

Gameboard:
STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Integration of Differential Equations 4i

A Level

Part A Derivative

If $y = \operatorname{cosec} x$ then find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x)`, `arccos()`, `arccosec()`, `arccot()`, `arcsec()`, `arcsin()`, `arctan()`, `cos()`, `cosec()`, `cot()`, `ln()`, `log()`, `sec()`, `sin()`, `tan()`, `x`, `y`

Part B Solve

Solve the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\sin x \tan x \cot t$$

given that $x = \frac{\pi}{6}$ when $t = \frac{\pi}{2}$.

The following symbols may be useful: `arccos()`, `arccosec()`, `arccot()`, `arcsec()`, `arcsin()`, `arctan()`, `cos()`, `cosec()`, `cot()`, `ln()`, `log()`, `sec()`, `sin()`, `t`, `tan()`, `x`

Gameboard:
STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Constructing Differential Equations 1i

A Level

A cylindrical container has a height of $200\,\mathrm{cm}$. The container was initially full of a chemical but there is a leak from a hole in the base (**Figure 1**). When the leak is noticed, the container is half-full and the level of the chemical is dropping at a rate of $1\,\mathrm{cm\,min^{-1}}$.

It is required to find for how many minutes the container has been leaking. To model the situation it is assumed that, when the depth of the chemical remaining is $x\,\mathrm{cm}$, the rate at which the level is dropping is proportional to $\sqrt{x}$.

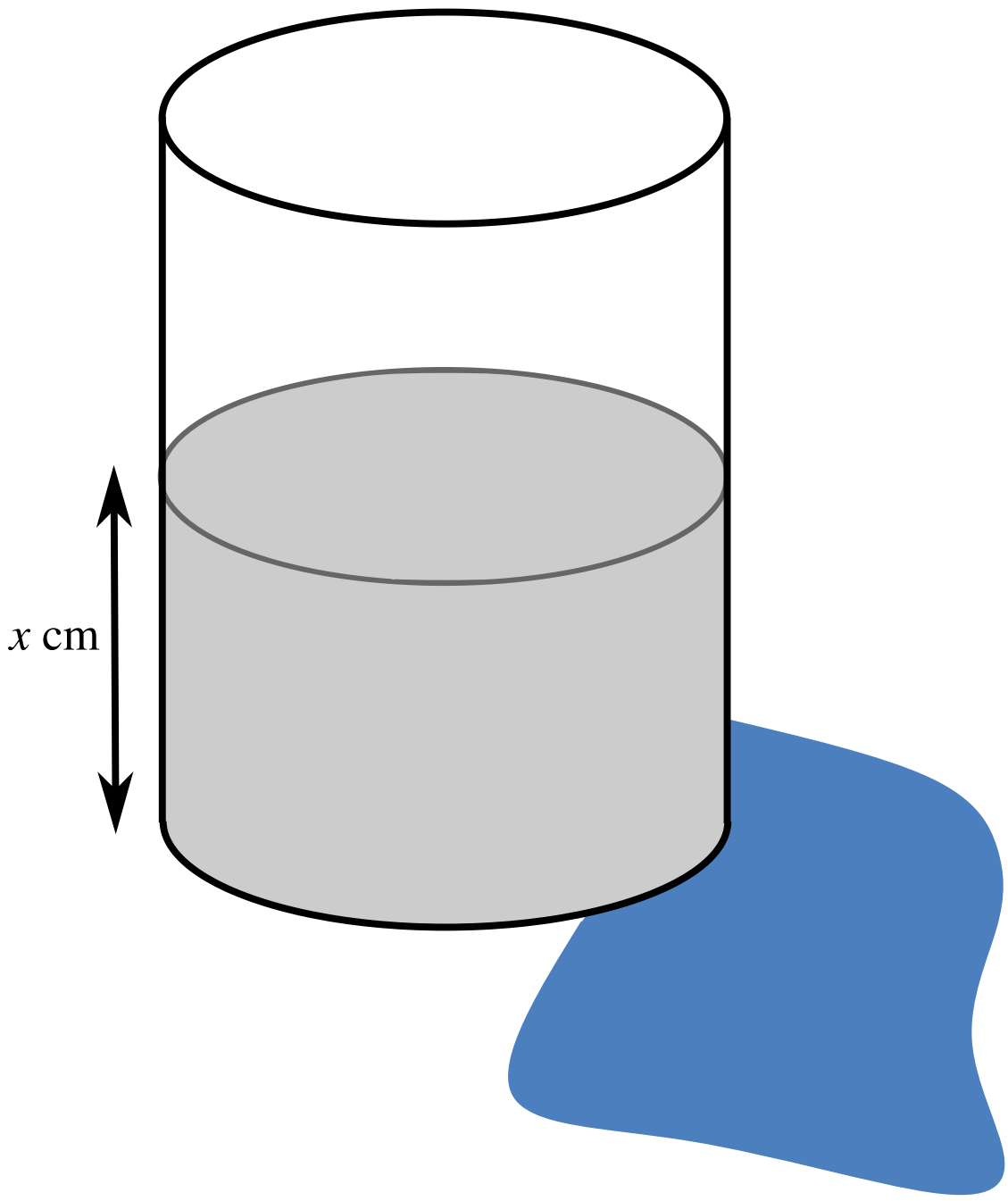

Figure 1: Cylindrical container that is leaking from its base.

Part A Differential equation

State an appropriate differential equation for the rate of change of height of chemical in the tank.

The following symbols may be useful: `Derivative(x, t), k, t, x`

Part B Solve

Solve this differential equation, giving x in terms of t, the time in minutes since the leak began.

The following symbols may be useful: `t, x`

Part C Time

Calculate the length of time that the container has been leaking for. Give your answer to 3 significant figures.
