Physics. You work it out.

Probabilities: Ducks

GCSE C C C A Level P P P

Some ducks are swimming on a pond. The heads, wings, and tails of these ducks have a variety of colour combinations. There are 11 ducks in total, and their colours are shown below.

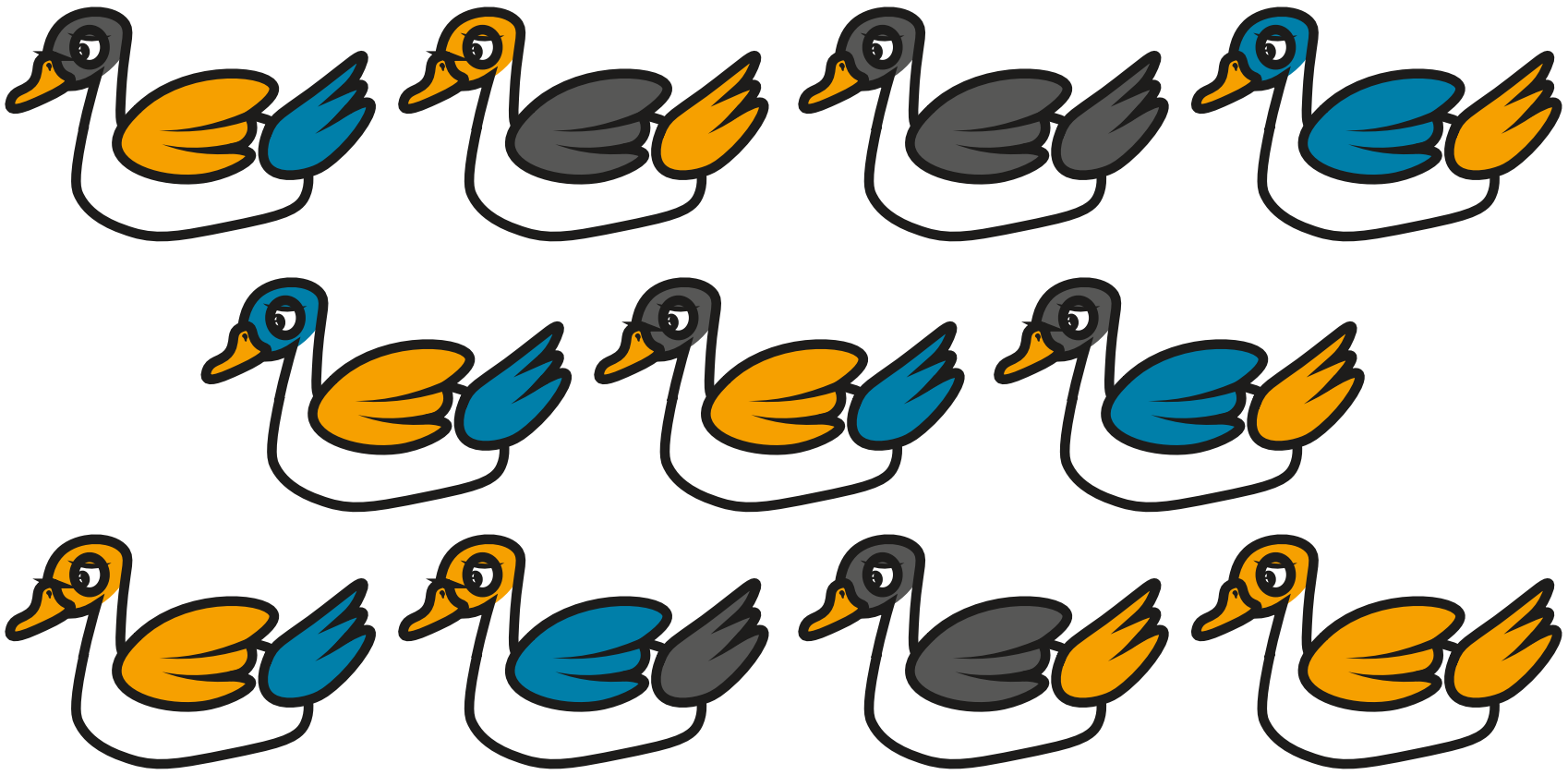

Figure 1: The colour combinations of the 11 ducks.

Suppose that a conservationist chose one of the ducks at random for an examination. What are the following probabilities?

Give your answers as fractions, simplified as far as possible.

Part A A yellow head

What is the probability that the duck has a yellow head?

Part B Head, wing and tail the same colour

What is the probability that the head, wing and tail of the duck are the same colour?

Part C Conditional probability 1

If the duck has a yellow head, what is the probability that it has a grey wing?

Part D Conditional probability 2

What is the probability that the duck has a grey head, given that it has a yellow wing?

Part E More complicated condition 1

What is the probability that the duck has a yellow tail, given that the head and wing of the duck are the same colour?

Part F More complicated condition 2

The chosen duck satisfies at least one of these conditions:

- The head and tail of the duck are the same colour.
- The head and wing of the duck are both blue.
- The duck has a yellow head.

What is the probability that the tail of the duck is not grey?

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Probability 5.1

A Level P P P Further A P P P

Find the following probabilities.

Part A $\mathrm{P}(\mathrm{Y})$ and $\mathrm{P}(\mathrm{Y}|\mathrm{X})$

It is given that $\mathrm{P}(\mathrm{X}) = 0.3$, $\mathrm{P}(\mathrm{X} \cup \mathrm{Y}) = 0.6$ and $\mathrm{P}(\mathrm{X} \cap \mathrm{Y}) = 0.2$.

Find $\mathrm{P}(\mathrm{Y})$.

Find $\mathrm{P}(\mathrm{Y}|\mathrm{X})$, giving your answer as an exact fraction.

Part B $\mathrm{P}(\mathrm{C} \cap \mathrm{D})$ and $\mathrm{P}(\mathrm{C}|\mathrm{D}')$

It is given that $\mathrm{P}(\mathrm{C}) = 0.6$, $\mathrm{P}(\mathrm{D}) = 0.5$ and $\mathrm{P}((\mathrm{C} \cup \mathrm{D})') = 0.3$.

Find $\mathrm{P}(\mathrm{C} \cap \mathrm{D})$.

Find $\mathrm{P}(\mathrm{C}|\mathrm{D}')$.

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Gameboard:

STEM SMART Single Maths 23 - Conditional Probability

Probabilities: Employment

A Level

Data about employment of people in their thirties and forties in a small rural area are shown in the following table.

	Unemployed	Employed
Thirties	206	412
Forties	358	305

A person from this area in these age groups is chosen at random. Let T be the event that the person is in their thirties and let E be the event that the person is employed.

Part A $\mathrm{P}(\mathrm{T})$

Find $\mathrm{P}(\mathrm{T})$.

Part B $\mathrm{P}(\mathrm{T}\ \mathrm{and}\ \mathrm{E})$

Find $\mathrm{P}(\mathrm{T}\ \mathrm{and}\ \mathrm{E})$.

Part C Independent events?

Are T and E independent events? Fill in the blanks below to complete the argument.

If T and E are independent, P(E|T) = [ ], i.e. the probability of being unemployed is [ ] irrespective of age.
Using the values in the table, P(E|T) = [ ] and [ ]. Therefore T and E [ ] independent events.

Items:

are P(T) P(E) $\frac{2}{3}$ $\frac{412}{1281}$ $\frac{412}{717}$ $P(E) = \frac{206}{427}$ **the same** $P(E) = \frac{239}{427}$ **are not** **random**

Part D Unemployed and in their thirties

Given that the person chosen is unemployed, find the probability that the person is in their forties.

Adapted with permission from UCLES, A Level, CIE, January 2005, Paper 6 Probability & Statistics 1 (S1), Question 7

Gameboard:

STEM SMART Single Maths 23 - Conditional Probability

Probability 5.2

A Level P P P Further A P P P

Consider the situation in which $\mathrm{P}(\mathrm{X}) = 0.3$, $\mathrm{P}(\mathrm{X} \cup \mathrm{Y}) = 0.7$ and $\mathrm{P}(\mathrm{Y}) = k$. Find the value of k in the following situations.

Part A X and Y mutually exclusive

Find the value of k if X and Y are mutually exclusive.

Part B X and Y independent

Find the value of k if X and Y are independent. Give your answer as an exact fraction.

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Probability 5.3

A Level P P P Further A P P P

Part A Substandard samples

A laboratory has two devices A and B which produce samples for an experiment. Device A has produced 100 samples of which 5% are substandard. Device B has produced 25 of which 4% are substandard. An experimenter has found a substandard sample. Assuming that samples are chosen at random, what is the probability that it was produced by device B?

Part B Equipment failure

In hot weather the antiquated air-conditioning system in Professor A's laboratory may break down. On any given hot day, there is a 5% chance that the air-conditioning system breaks down. If the air-conditioning breaks down, the probability this will lead to the Professor's equipment failing by the end of the day is 0.3. If the air-conditioning does not break down, the probability that the equipment fails by the end of the day is only 0.05.

One hot day the Professor checks their lab first thing in the morning and the air-conditioning and equipment are both working. When the Professor gets ready to leave at the end of the day, they notice that their equipment has failed. What is the probability that the failure was not due to a breakdown of the air-conditioning system?

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Probability 5.4

A Level C C C Further A C C C

The probability of a randomly selected person in a population having a particular genetic trait is 0.00001. A test for this trait successfully detects it, if present, 99.9% of the time, and only returns a false positive 0.1% of the time.

Part A Probability after one test

A person tests positive for the trait. Find the probability that they actually have the genetic trait. Give your answer to 3 significant figures.

Part B Probability after two tests

In order to improve accuracy, individuals are instructed to take the test twice, regardless of the result of the first test.

What is the probability that an individual receives a positive result from both tests? Give your answer to 3 significant figures.

Find the probability, given that they have tested positive twice, that they actually have the genetic trait. Give your answer to 3 significant figures.

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