Tree Diagrams and Venn Diagrams 8

GCSE C C C A Level P P P

Essential GCSE Maths 52.8

An unbiased 6-sided die is rolled twice.

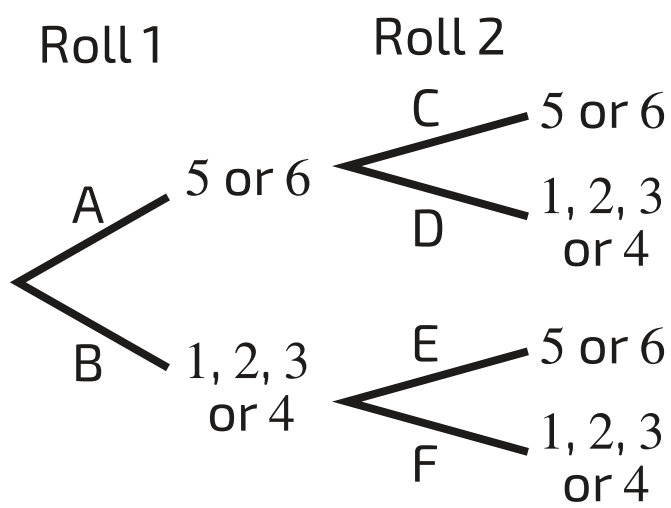

Figure 1: A tree diagram that needs the probabilities added.

Part A Complete the tree diagram

Find the probabilities A, B, C, D, E and F. Write your answers as exact fractions, cancelled down as far as possible. (Use `/` for the division line.)

A :

B :

C :

D :

E :

F :

Part B Rolling 5 or 6 twice in a row

What is the probability of rolling 5 or 6 twice in a row?

Part C Rolling a 5 or 6 exactly once

What is the probability of rolling a 5 or 6 exactly once?

Part D Rolling a 5 or 6 at least once

What is the probability of rolling a 5 or 6 at least once?

Tree Diagrams and Venn Diagrams 6

GCSE C C C A Level P P P

Essential GCSE Maths 52.6

A person has forty-eight socks. They have 32 identical white socks and 16 identical black socks. One morning the socks are all jumbled up in a drawer. The person pulls out two socks at random to wear, one after the other.

Draw a probability tree for the two socks the person picks.

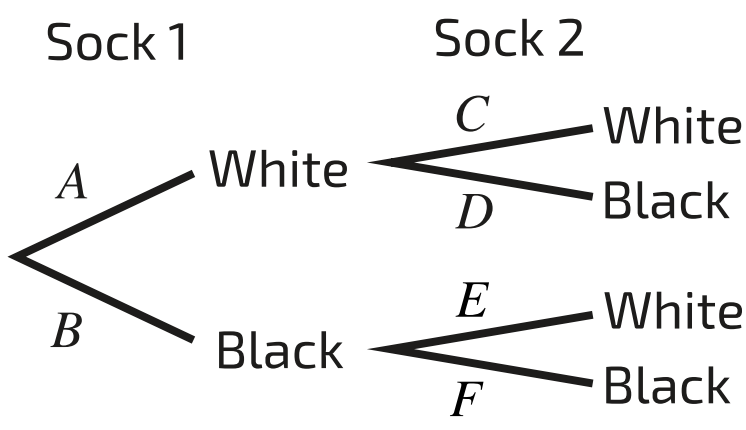

Figure 1: A tree diagram for picking white and black socks.

Part A Complete the tree diagram

Enter the probabilities A, B, C, D, E and F. Write your answers as exact fractions, cancelled down as far as possible. (Use `/` for the division line.)

A :

B :

C :

D :

E :

F :

Part B Calculate the probability of a pair of white socks

Calculate the probability $\mathrm{P}(\text{a pair of white socks})$.

Part C Calculate the probability of a matched pair of socks

Calculate the probability $\mathrm{P}(\text{a matched pair of socks})$.

Part D Calculate the probability of a mis-matched pair of socks

Calculate the probability $\mathrm{P}(\text{a mis-matched pair of socks})$.

Gameboard:

STEM SMART Single Maths 7 - Probability

Tree Diagrams and Venn Diagrams 4

GCSE C C C A Level P P P

Essential GCSE Maths 52.4

60 people in a town are surveyed about where they work and how they get to work. Of these people, a total of 40 drive to work, a total of 32 work in an office, and 26 drive to work and work in an office.

Show these data on a Venn diagram.

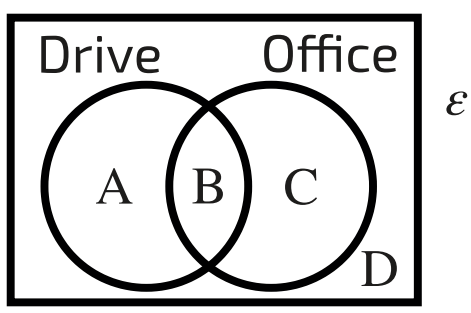

Figure 1: An unpopulated Venn diagram for the survey.

Part A Complete the Venn diagram

Enter the values A, B, C and D.

$A =$ ☐

$B =$ ☐

$C =$ ☐

$D =$ ☐

Part B Calculate the probability a person works in an office

If one person from the survey is chosen at random, calculate the probability that they work in an office.

Part C Calculate the probability...

If one person from the survey is chosen at random, calculate the probability that they drive to work but don't work in an office.

Part D Calculate the probability a person neither drives nor works in an office

If one person from the survey is chosen at random, calculate the probability that they neither drive to work nor work in an office.

Gameboard:

STEM SMART Single Maths 7 - Probability

Data Analysis 3.9

A Level

A radioactive source emits alpha-particles randomly. 100 measurements were made of the number n of alpha-particles emitted in a fixed time T and the results are displayed in unknown reference \ref{maths_3.9_fig1}. From the chart estimate the mean, variance and standard deviation of the number of particles emitted in time T. Find the probabilities that $n < 3$, that $2 < n \leq 5$ and that $n \geq 7$.

(The distribution shown is approximately what is called a Poisson distribution which is used extensively in statistical analyses.)

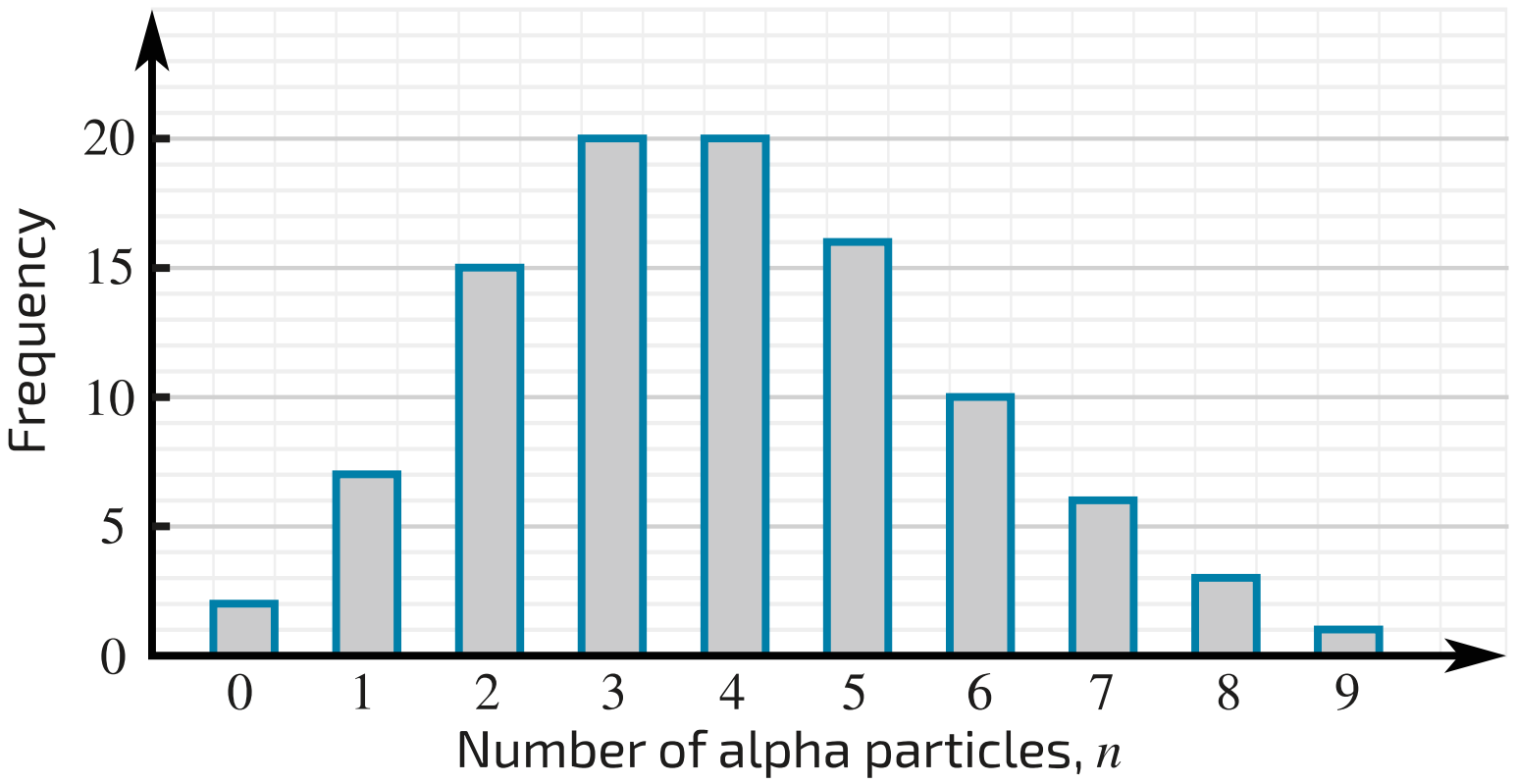

Figure 1: A bar chart showing the frequency with which a given number n of alpha-particles were emitted in a fixed time T.

Part A The mean

Find the mean number of particles emitted in time T. Give your answer to 2 sf.

Part B The variance

Find the variance in the number of particles emitted in time T. Give your answer to 2 sf.

Part C The standard deviation

Find the standard deviation in the number of particles emitted in time T. Give your answer to 2 sf.

Part D Probability $n < 3$

Find the probability that $n < 3$. Give your answer to 2 sf.

Part E Probability $2 < n \leq 5$

Find the probability that $2 < n \leq 5$. Give your answer to 2 sf.

Part F Probability that $n \geq 7$

Find the probability that $n \geq 7$. Give your answer to 2 sf.

Created for isaacphysics.org by Julia Riley

Gameboard:

STEM SMART Single Maths 7 - Probability

Probability 3.8

A Level

A bag contains four red and five yellow objects. Three of the objects are removed and not replaced. Deduce the probability distribution and the cumulative probability distribution for X, the number of red objects removed, giving your answers in their simplest exact form.

Part A The probability distribution

The probability distribution $f(x)$ can be defined as

$$f(x) = \mathrm{P}(X = x)$$

where X is the number of red objects removed. Find $f(x)$ for $x = 0,\ 1,\ 2,\ 3$.

Write your answers as exact fractions, cancelled down as far as possible. (Use / for the division line.)

$f(0) = \mathrm{P}(X = 0) =$ [ ]

$f(1) = \mathrm{P}(X = 1) =$ [ ]

$f(2) = \mathrm{P}(X = 2) =$ [ ]

$f(3) = \mathrm{P}(X = 3) =$ [ ]

Part B The cumulative probability distribution

The cumulative probability distribution $F(x)$ can be defined as

$$F(x) = \mathrm{P}(X \leq x)$$

where X is the number of red objects removed. Find $F(x)$ for $x = 0,\ 1,\ 2,\ 3$.

Write your answers as exact fractions, cancelled down as far as possible. (Use `/` for the division line.)

$F(0) = \mathrm{P}(X \leq 0) =$ ☐

$F(1) = \mathrm{P}(X \leq 1) =$ ☐

$F(2) = \mathrm{P}(X \leq 2) =$ ☐

$F(3) = \mathrm{P}(X \leq 3) =$ ☐

Created by Julia Riley for isaacphysics.org

Gameboard:

STEM SMART Single Maths 7 - Probability

Probability 3.10

A Level

Two devices produce random outputs of 0, 1 or $2\,\text{V}$ with probabilities shown in the table. The two devices are connected in series so that they produce an output which is the sum of their individual outputs.

Output / V	0	1	2
Device 1 - Probability	0.25	0.25	a
Device 2 - Probability	b	0.2	0.4

Answer the following.

Part A Values of a and b

Deduce the values of a and b. Give your answers as decimals.

$a =$ ☐

$b =$ ☐

Part B Probability distribution for the sum

Find the probability distribution for X, the sum of the two outputs.

Give your answers as decimals (use 3 sf if rounding is necessary).

x	$\mathrm{P}(X = x)$
0	
1	
2	
3	
4	

Created by Julia Riley for isaacphysics.org

Gameboard:

STEM SMART Single Maths 7 - Probability

Probability 3.3

A Level

C C C

A system consists of three magnets side by side; they can be oriented with their north poles pointing upwards (U) or downwards (D). The first one on the left is always U. If one magnet points upwards (U) then the probability that the one next to it points upwards (i.e. the arrangement UU) is $\frac{1}{4}$ and the probability that it points downwards (i.e. UD) is $\frac{3}{4}$; similarly if one magnet points downwards (D) then the probability that the one next to it points downwards (i.e. DD) is $\frac{1}{4}$ and the probability that it points upwards (DU) is $\frac{3}{4}$. Thus the probability of the arrangement being UUD is $\frac{1}{4} \times \frac{3}{4}$. Construct a tree diagram to illustrate the probabilities of getting the various different arrangements. Hence find the following probabilities.

Part A 3 magnets pointing upwards

Find the probability that all three magnets are pointing upwards. Give your answer in exact form.

Part B 2 magnets pointing upwards

Find the probability that only two magnets are pointing upwards. Give your answer in its simplest exact form.

Part C 1 magnet pointing upwards

Find the probability that only one magnet is pointing upwards. Give your answer in exact form.

Part D No magnets pointing upwards

What is the probability that no magnets are pointing upwards?

Created by Julia Riley for isaacphysics.org

Gameboard:

STEM SMART Single Maths 7 - Probability

Probability 3.6

A Level

C C C

In some magnetic materials each atom or molecule is itself a permanent magnet. Consider a very simple one-dimensional model of such a material which consists of 20 such mini-magnets arranged in a line; each mini-magnet has two possible orientations which are equally likely and which we will call 'up' and 'down'. Find the probabilities of obtaining the following arrangements.

Part A All pointing up

Find the probability that all 20 mini-magnets are pointing up. Give your answer to 3 sf.

Part B Exactly 10 pointing up

Find the probability that exactly 10 mini-magnets are pointing down. Give your answer to 3 sf.

Part C 10 or more pointing up

Find the probability that 10 or more are pointing up. Give your answer to 3 sf.

Part D Alternately up and down

What is the probability that in the line the mini-magnets are alternately up (U) and then down (D) or down (D) and then up (U) i.e. UDUD... or DUDU...? Give your answer to 3 sf.

Created by Julia Riley for isaacphysics.org

Gameboard:
STEM SMART Single Maths 7 - Probability

Probability 3.7

A Level

A system consists of 1000 particles which can be in either one of two states A and B; any particle has a probability of 0.9 of being in state A. Find the following.

Part A 900 particles in state A

Find the probability that there are exactly 900 particles in state A. Give your answer to $3\ \mathrm{sf}$.

Part B 110 particles in state B

Find the probability that there are exactly 110 particles in state B. Give your answer to $3\ \mathrm{sf}$.

Part C Between 880 and 920 particles in state A

Find the probability that there are more than 880 but less than 920 particles in state A. Give your answer to $3\ \mathrm{sf}$.

Part D Less than 120 in state B

Find the probability that there are less than 120 particles in state B. Give your answer to $3\ \mathrm{sf}$.

Created by Julia Riley for isaacphysics.org

Gameboard:

STEM SMART Single Maths 7 - Probability

Jury Duty

GCSE C C C

A Level C C C

You are serving on a jury in the Crown Court. The defendant has been accused of a serious crime, however the **only** evidence is that their DNA is a perfect match to the perpetrator's DNA found at the crime scene.

The expert in genetic analysis tells you that the chance of a false positive (i.e. an innocent person matching that DNA) is 1 in $3\,000\,000$. The prosecution lawyer says in their summing up speech that this means that as the defendant matches the DNA, the chance that they are innocent is less than $0.000\,04\%$, This means that there is a $99.999\,96\%$ chance that the person is guilty, and as this is *beyond reasonable doubt* you and your jury colleagues should decide that the person is guilty.

Back in the jury room, the other jurors know that you have mathematical knowledge and ask you for your view on the matter.

If you think it relevant, you may also assume that the crime was definitely committed by a British person, and that the population of Britain is $67\,000\,000$.

Based on the information given, what is your best estimate of the probability that the defendant is guilty? Give your answer to two significant figures.

Created for isaacphysics.org by Anton Machacek.