



## Linear regression 3.1

**Subject & topics:** Maths | Statistics | Hypothesis Tests    **Stage & difficulty:** A Level P2

An experiment was carried out to find the acceleration of a ball rolling down a straight ramp. Assuming that the ball starts from rest and that the acceleration is constant, the relationship between the time  $t$  it takes to roll a distance  $s$  down the ramp is such that

$$s = \frac{1}{2}at^2$$

where  $a$  is the acceleration.

Two students A and B carry out the experiment one after the other using the same apparatus but making their own measurements of  $s$  and  $t$ . They decide to measure the time it takes for the ball to roll a fixed distance i.e. the distance is the independent variable and the time is the dependent variable.

### Part A

#### Rearrange the equation

Rearrange the expression

$$s = \frac{1}{2}at^2$$

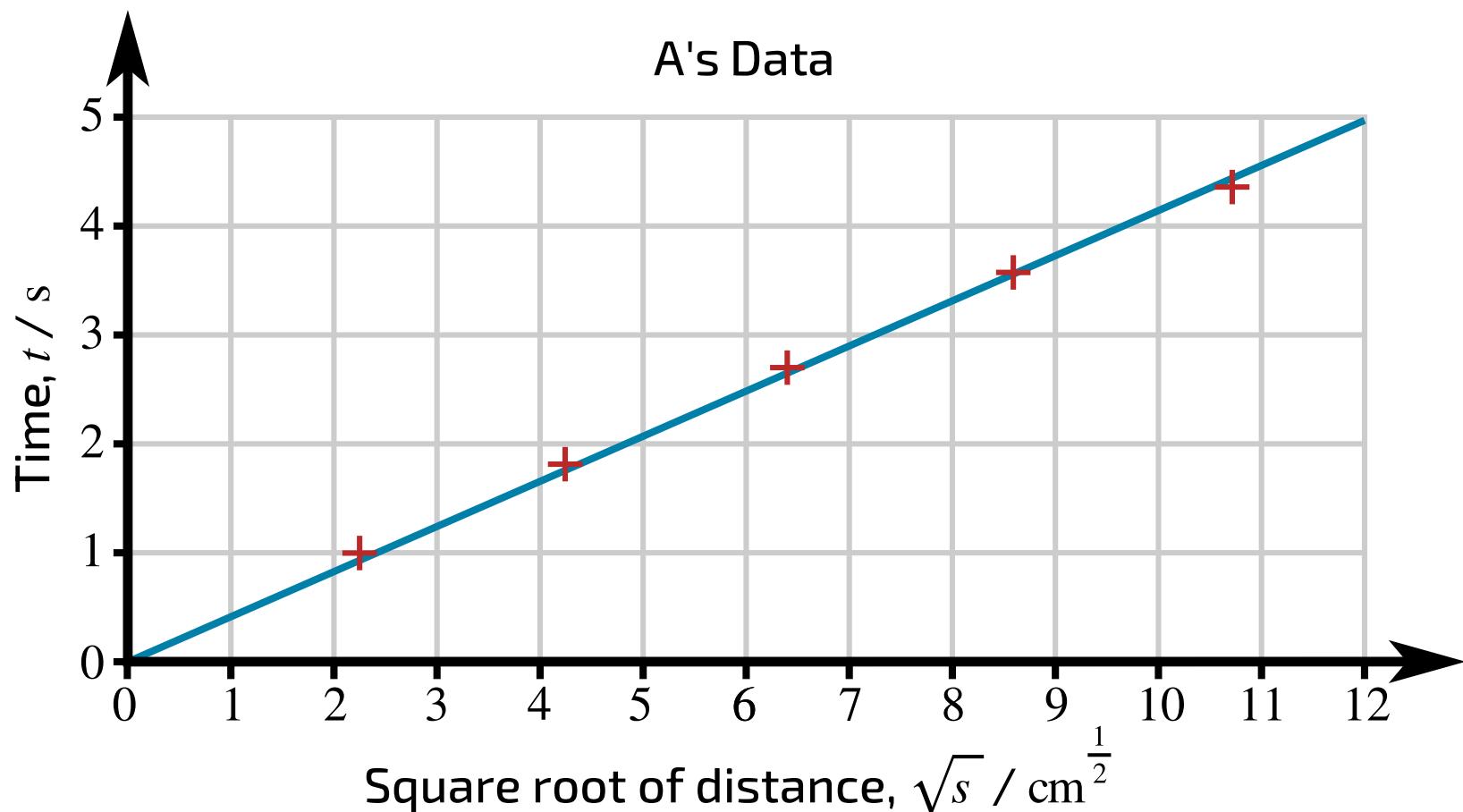
to obtain an equation for  $t$  (i.e. make  $t$  the subject of the equation).

The following symbols may be useful:  $a$ ,  $s$ ,  $t$

**Part B**  
**A's data**

A graph of time against the square root of distance using A's data is shown in **Figure 1**; the time  $t$  is in s and the square root of the distance  $\sqrt{s}$  is in  $\text{cm}^{\frac{1}{2}}$ . The uncertainties in the values are smaller than the sizes of the crosses. The line of best fit, assuming that it goes through the point  $(0, 0)$ , is shown; it has the following parameters,

$$t = 0.4137\sqrt{s} \quad r = 0.9998 \quad r^2 = 0.9996$$

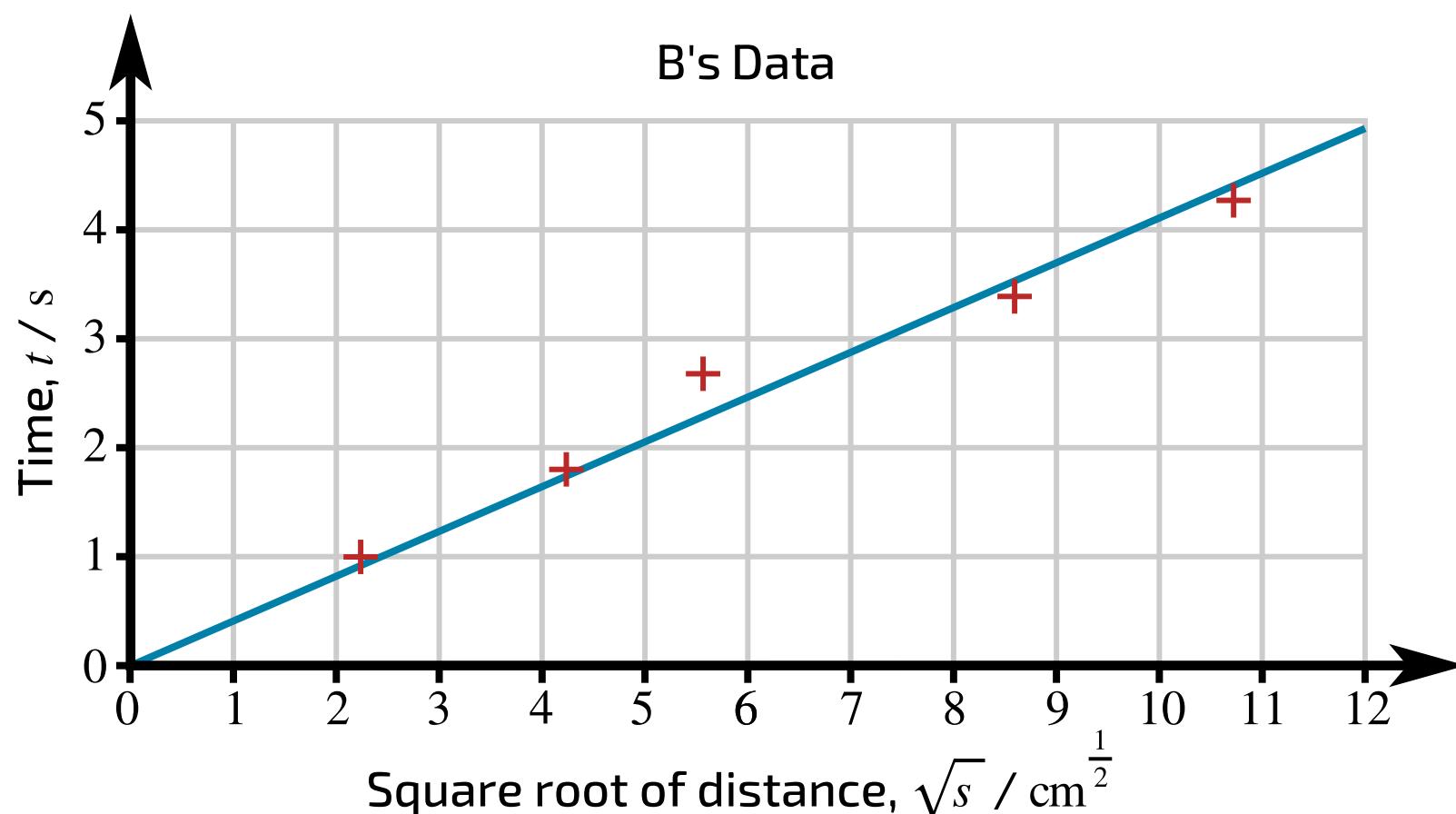


**Figure 1:** A graph of  $t$  against  $\sqrt{s}$  using A's data, showing the line of best fit.

Deduce the value of the acceleration down the ramp.

**Part C****B's data**

A graph of time against the square root of distance using B's data is shown in [Figure 2](#); the time  $t$  is in s and the square root of the distance  $\sqrt{s}$  is in  $\text{cm}^{\frac{1}{2}}$ . The uncertainties in the values are smaller than the sizes of the crosses. The line of best fit, assuming that it goes through the point  $(0, 0)$  and using all five points, is shown. It is clear from looking at the graph that although the  $r$  value is very high, there is an obvious outlier; on discussing the results with Student A, Student B concludes that they have either measured or recorded the distance incorrectly in this case. The line is fitted again omitting the outlier.



[Figure 2](#): A graph of  $t$  against  $\sqrt{s}$  using B's data, showing the line of best fit.

The best fit lines have the following parameters:

$$t = 0.4109\sqrt{s} \quad r = 0.9976 \quad r^2 = 0.9951$$

$$t = 0.4005\sqrt{s} \quad r = 0.9997 \quad r^2 = 0.9993.$$

Decide which of the two you think is the best fit line when the outlier is omitted and give its  $r$  value.

**Part D****Acceleration using B's data**

Find the value of the acceleration down the ramp using the more appropriate set of B's data.

**Part E****Time to roll 150 cm**

Use A's data to estimate the time it will take the ball to roll 1.5 m down the ramp giving your answer to 3 sf.

**Part F****Difference in time estimates**

The best fit line to the data can be used to estimate the time it will take the ball to roll any given distance down the ramp. Find the percentage difference between the estimates obtained by A and B, giving your answer to 1 sf.



## Linear regression 3.2

**Subject & topics:** Maths | Statistics | Hypothesis Tests    **Stage & difficulty:** A Level C1

An experiment is carried out to measure the resistance  $R$  of a semiconductor as a function of absolute temperature  $T$ . Theory suggests that above a certain temperature

$$R = R_0 e^{\frac{b}{T}}$$

where  $R_0$  and  $b$  are constants.

### Part A

#### Rearrange the equation

By taking the natural logarithms of both sides of the equation show that it can be written

$$\ln R = a + f(T)$$

where  $a$  is a constant and  $f(T)$  is a function of  $T$ . Find expressions for  $a$  and  $f(T)$ .

Find an expression for  $a$ .

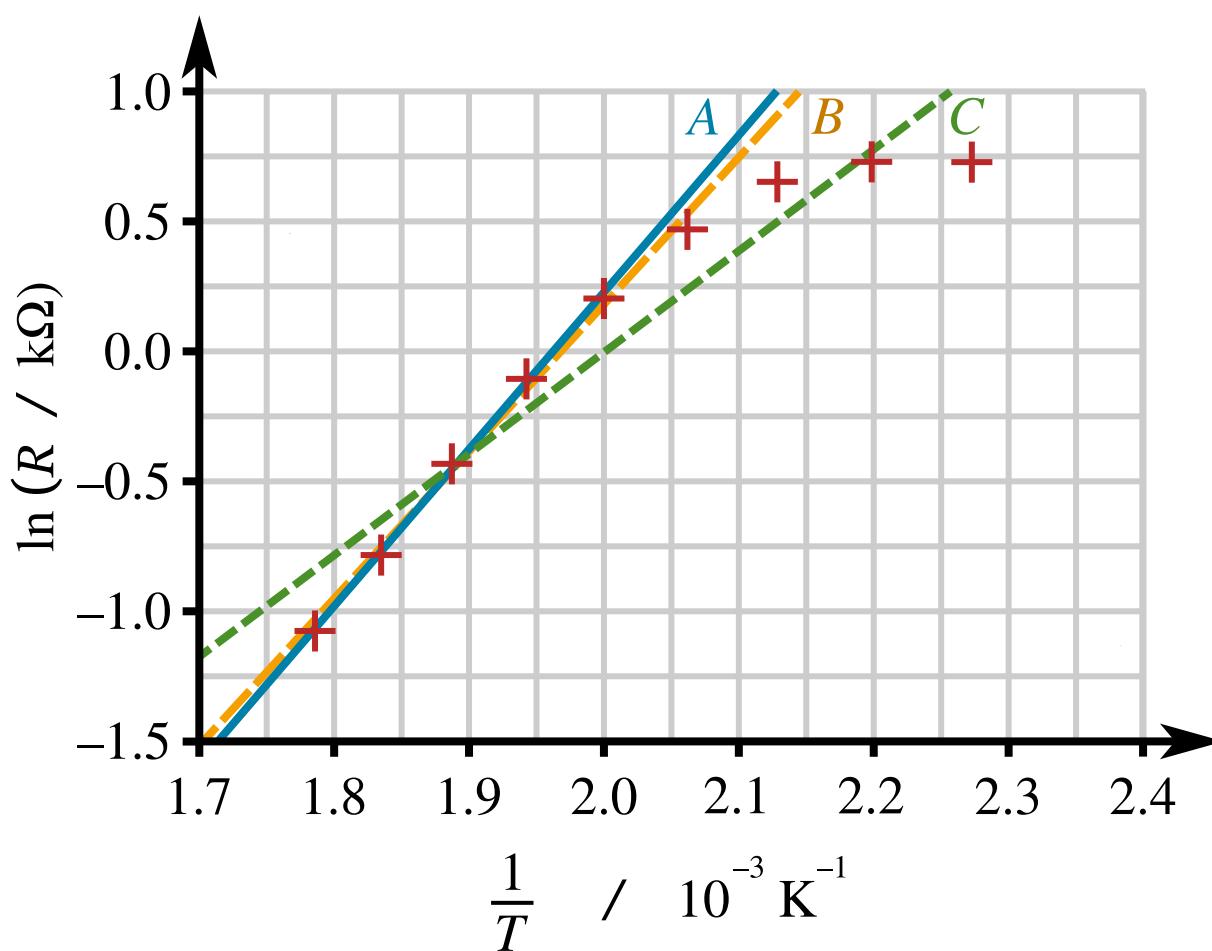
The following symbols may be useful:  $R_0$ ,  $T$ ,  $a$ ,  $b$ ,  $e$ ,  $f(T)$

Find  $f(T)$ .

The following symbols may be useful:  $R_0$ ,  $T$ ,  $a$ ,  $b$ ,  $e$ ,  $f(T)$

**Part B****Lines of best fit**

In **Figure 1**,  $\ln R$ , where  $R$  is in  $k\Omega$ , is plotted as a function of  $\frac{1}{T}$ , where  $\frac{1}{T}$  is in  $10^{-3} \text{ K}^{-1}$ . Thus if  $R = 1000 \Omega$ , then  $\ln R = \ln(1 \text{ k}\Omega) = 0$  and, if  $T = 500 \text{ K}$ , then  $\frac{1}{T} = 0.002 \text{ K}^{-1} = 2 \times 10^{-3} \text{ K}^{-1}$ .



**Figure 1:** A plot of  $\ln R$  against  $\frac{1}{T}$ ; three lines of best fit, *A* (blue, solid), *B* (yellow, long-dashed) and *C* (green, short-dashed) fitted to different ranges of the data, are shown.

Three lines of best fit *A*, *B* and *C* are fitted to different ranges of the data as shown in **Figure 1**.

The parameters of the three fitted lines are:

$$\begin{array}{lllll} \text{I : } & a = -11.197 & b = 5.686 & r = 0.997 & r^2 = 0.994 \\ \text{II : } & a = -11.857 & b = 6.042 & r = 0.999 & r^2 = 0.998 \\ \text{III : } & a = -7.816 & b = 3.906 & r = 0.955 & r^2 = 0.913 \end{array}$$

where  $a$  and  $b$  are as defined in Part A and the initial equation, and  $b$  has units of  $10^3 \text{ K}$ .

Match the lines to the parameters.

I corresponds to line .

II corresponds to line .

III corresponds to line .

Items:

- A**    **B**    **C**

**Part C****Deductions from the graphs**

Theory suggests that above a certain temperature

$$R = R_0 e^{\frac{b}{T}}$$

where  $R_0$  and  $b$  are constants.

Using the information from the graphs in part B, suggest, to 1 sf, the temperature above which the theory is valid.

**Part D****Estimate the energy gap**

According to the theory the energy gap between the insulating and the conducting energy bands in a semiconductor is  $E_g = 2kb$ , where  $k$  is the Boltzmann constant ( $k = 1.4 \times 10^{-23} \text{ J K}^{-1}$ ). Select the line of best fit from Part B which best fits the theory and deduce the value of  $b$ ; hence estimate  $E_g$ .

**Part E****Resistance when  $T = 520 \text{ K}$** 

Use the equation of the line of best fit from Part B to deduce the resistance at 520 K.

**Part F****Resistance when  $T = 450 \text{ K}$** 

By looking at the graph in Part B, deduce the resistance when  $T = 450 \text{ K}$  giving your answer to 1 significant figure.

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Question deck:

[STEM SMART Double Maths 33 - Regression, Correlation & Hypothesis Tests](#)



## Correlation Hypothesis Testing 1

**Subject & topics:** Maths | Statistics | Hypothesis Tests    **Stage & difficulty:** A Level P3

In each part, carry out a hypothesis test for the requested type of correlation at the stated significance level.

### Part A

#### Positive correlation

A sample of size  $n = 17$  has a correlation coefficient of  $r = 0.601$ . Test at the 1% significance level whether the population from which the sample was taken has positive correlation, then fill in the blanks below.

The null hypothesis is that the population has no correlation. The alternative hypothesis is that the population exhibits positive correlation.

$$H_0 : \rho = 0$$

For a one-tailed test, the critical value of the correlation coefficient for a sample of size 17 is  at the 1% significance level.

The correlation coefficient for the sample is 0.601. This is  than the critical value. Hence, we  the null hypothesis. There is evidence that the population exhibits positive correlation.

Items:

- 0.5577
- 0.5742
- 0.6055
- $H_1 : \rho < 0$
- $H_1 : \rho \neq 0$
- $H_1 : \rho > 0$
- reject
- do not reject
- smaller
- equal to
- larger

**Part B****Negative correlation**

A researcher believes that average speeds for athletes running the 400 m at a particular track are lower if there has been a larger amount of rainfall earlier in the day. The researcher times one particular athlete at the same time every day on ten different autumn days. They record the depth of rainfall (in mm) before each run, and work out the athlete's average speed. The researcher calculates that the correlation coefficient is  $-0.713$ . Test at the 5% significance level whether an athlete's average speed and the amount of rainfall are indeed negatively correlated at this track.

**Available items**

1. The null hypothesis is that there is no correlation. The alternative hypothesis is there is negative correlation.

1. The null hypothesis is that there is negative correlation. The alternative hypothesis is that there is no correlation.

2.  $H_0 : \rho = 0$        $H_1 : \rho > 0$

2.  $H_0 : \rho = 0$        $H_1 : \rho < 0$

3. For a one-tailed test, the critical value of the correlation coefficient for a sample of size 5 is 0.8054 at the 10% significance level.

3. For a one-tailed test, the critical value of the correlation coefficient for a sample of size 10 is 0.5494 at the 5% significance level.

4. The correlation coefficient for the sample is  $-0.713$ . This is negative, and has a magnitude greater than the critical value ( $| -0.713 | > 0.5494$ ).

4. The correlation coefficient for the sample is  $-0.713$ , and  $-0.713 < 0.5494$ .

5. Hence, we do not reject the null hypothesis. There is not significant evidence for negative correlation between an athlete's average speed and the amount of rainfall.

5. Hence, we reject the null hypothesis. There is evidence that an athlete's average speed and the amount of rainfall have negative correlation.

**Part C****Any (linear) correlation**

An author wonders whether the amount of time their cat sits next to them is correlated with the number of words they write during the day. Over fifty-three days, the author records the number of words they write and for how long the cat sits nearby, and finds  $r = 0.3300$ . Test the data at the 1% significance level.

Choose from the options below to construct a complete hypothesis test.

This question is looking for correlation in either direction. A two-tailed test is needed.

$$H_0 : \rho = 0$$

$$H_1 : \rho \neq 0$$

This question is looking for positive correlation. A one-tailed test is needed.

$$H_0 : \rho = 0$$

$$H_1 : \rho > 0$$

- For a one-tailed test, the critical value of the correlation coefficient for a sample of size 53 is 0.3188 at the 1% significance level.
- For a two-tailed test, the critical value of the correlation coefficient for a sample of size 53 is 0.3509 at the 1% significance level.
- The correlation coefficient for the sample is 0.3300, and  $0.3300 < 0.3509$ .
- The correlation coefficient for the sample is 0.3300, and  $0.3300 > 0.3188$ .
- Hence, we do not reject the null hypothesis. There is not significant evidence that the number of words the author writes is correlated with the amount of time their cat sits near them.
- Hence, we reject the null hypothesis. There is evidence that the number of words the author writes is correlated with the amount of time their cat sits near them.

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## STEM SMART Double Maths 33 - Regression, Correlation &amp; Hypothesis Tests

## Correlation Hypothesis Testing 2

**Subject & topics:** Maths | Statistics | Hypothesis Tests    **Stage & difficulty:** A Level P3

A town planner believes that on summer weekday afternoons the amount of traffic into the centre of their town is higher when the temperature is higher. They want to test this hypothesis at the 1% significance level.

Every weekday (Monday to Friday) for six weeks they monitor the traffic on the main roads into town, and record the mean afternoon temperature, and they find that the correlation coefficient is  $-0.4517$ .

### Part A

#### Initial conclusion

Without doing any calculations, which of these statements can the town planner make? Choose all that apply.

- There is no evidence that the amount of traffic and afternoon temperature are correlated.
- The correlation coefficient is negative. There may be a negative correlation between the amount of traffic and afternoon temperature.
- The correlation coefficient of their sample is negative, so there is a negative correlation between the amount of traffic on summer afternoons and temperature.
- The correlation coefficient is negative, so there is definitely no positive correlation between the amount of traffic and temperature.
- The correlation coefficient is negative, so there is no evidence that the amount of traffic is positively correlated with temperature.

**Part B****Choosing a hypothesis test**

Using the given data, which of the following hypothesis tests would it be most useful for the town planner to carry out?

- A hypothesis test to see if the amount of traffic and afternoon temperature are negatively correlated at the 1% significance level.
- A hypothesis test to see if the amount of traffic and afternoon temperature are negatively correlated at the 20% significance level.
- A hypothesis test to see if the amount of traffic and afternoon temperature are negatively correlated at the 50% significance level.

**Part C****Null and alternative hypotheses**

The town planner carries out the most useful test listed in part B.

Drag and drop symbols into the spaces below to state the null and alternative hypotheses for this test, where  $\rho$  represents the population correlation coefficient and  $r$  represents the sample correlation coefficient.

$H_0:$

$H_1:$

Items:

ρ r < = > 0 1

**Part D****Carrying out the test**

Carry out the hypothesis test, and make a conclusion. Then fill in the blanks below.

The critical value of the correlation coefficient is .

Comparing the town planner's value to the critical value gives .

Therefore,  the null hypothesis. There  significant evidence that there is a  correlation between the amount of traffic in summer and afternoon temperature.

Items:

- 0.4226    0.8822    0.9343     $| -0.4517 | > 0.4226$      $| -0.4517 | < 0.9343$      $| -0.4517 | < 0.8822$     reject  
 do not reject    is    may be    is not    negative    positive    no

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## STEM SMART Double Maths 33 - Regression, Correlation &amp; Hypothesis Tests

## Spearman's Rank Test 1

**Subject & topics:** Maths | Statistics | Hypothesis Tests    **Stage & difficulty:** Further A P2

In an examination consisting of 4 separate subjects, a group of 10 students obtained the following marks. The overall mark was out of 375, and the Physics and Chemistry marks were each out of 100.

Student	A	B	C	D	E	F	G	H	I	J
Overall	274	255	246	245	229	228	219	213	205	176
Physics	76	77	67	65	58	60	52	63	47	45
Chemistry	82	—	65	67	—	64	68	54	51	38

Find Spearman's rank correlation coefficients between each pair of data to test whether there is evidence of a positive correlation between them.

**Part A****Overall and Physics marks**

Find Spearman's rank correlation coefficient  $r_s$  for the relationship between the overall mark and the Physics mark, giving your answer to 3 sf.

**Part B****Overall and Physics significance**

Test at the 1% significance level whether there is a positive association between the overall mark and that obtained in Physics.

Find the appropriate critical value for Spearman's rank correlation coefficient, giving your answer to 3 sf.

What do you conclude about whether there is a positive association at the 1% significance level between the overall mark and that obtained in Physics?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Spearman's rank correlation coefficient is [ ] the calculated value; the null hypothesis that there is [ ] association between the two values can therefore be [ ] at the 1% level and there is [ ] evidence for a positive association at this level.

Items:

- no
- equal to
- significant
- rejected
- negative
- less than
- accepted
- positive
- greater than

**Part C****Overall and Chemistry marks**

Find Spearman's rank correlation coefficient  $r_s$  for the relationship between the overall mark and the Chemistry mark, giving your answer to 3 sf.

**Part D****Overall and Chemistry significance**

Test at the 1% significance level whether there is a positive association between the overall mark and that obtained in Chemistry.

Find the appropriate critical value for Spearman's rank correlation coefficient, giving your answer to 3 sf.

What do you conclude about whether there is a positive association at the 1% significance level between the overall mark and that obtained in Chemistry?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Spearman's rank correlation coefficient is  the calculated value; the null hypothesis that there is  association between the two values cannot therefore be  at the 1% level, and there is no  evidence for a positive association at this level.

Items:

- 

**Part E****Physics and Chemistry marks**

Find Spearman's rank correlation coefficient  $r_s$  for the relationship between the Physics mark and the Chemistry mark, giving your answer to 3 sf.

**Part F****Physics and Chemistry significance**

Test at the 1% significance level whether there is a positive association between the Physics mark and that obtained in Chemistry.

Find the appropriate critical value for Spearman's rank correlation coefficient, giving your answer to 3 sf.

What do you conclude about whether there is a positive association at the 1% significance level between the mark obtained in Physics and that obtained in Chemistry?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Spearman's rank correlation coefficient is  the calculated value; the null hypothesis that there is  association between the two values cannot therefore be  at the 1% level, and there is no  evidence for a positive association at this level.

Items:

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## STEM SMART Double Maths 33 - Regression, Correlation &amp; Hypothesis Tests

## Using Pearson's PMCC 1

**Subject & topics:** Maths | Statistics | Hypothesis Tests    **Stage & difficulty:** Further A P2

Find Pearson's product moment correlation coefficient for the following sets of statistics.

### Part A

#### Data set 1

Find the correlation coefficient for the following set of statistics, giving your answer to 3 sf.

$$\Sigma x = 275, \Sigma x^2 = 10781, \Sigma y = 251, \Sigma y^2 = 8407, \Sigma xy = 7842, n = 10$$

### Part B

#### Data set 2

Find the correlation coefficient for the following set of statistics, giving your answer to 3 sf.

$$\Sigma x = 907, \Sigma x^2 = 105944, \Sigma y = 289, \Sigma y^2 = 10916, \Sigma xy = 14929, n = 12$$

### Part C

#### Data set 3

Find the correlation coefficient for the following set of statistics, giving your answer to 3 sf.

$$\Sigma(x - \bar{x})^2 = 1592, \Sigma(y - \bar{y})^2 = 2473, \Sigma(x - \bar{x})(y - \bar{y}) = -1280, n = 10$$

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## STEM SMART Double Maths 33 - Regression, Correlation &amp; Hypothesis Tests

## Using Pearson's PMCC 2

**Subject & topics:** Maths | Statistics | Hypothesis Tests

**Stage & difficulty:** Further A P2, University P1

An electrical device has been designed and is being tested. The summary statistics for the relationship between the input and output voltages ( $V_i$  and  $V_o$  respectively) from the device are as follows.

$$\Sigma V_i = 302, \Sigma V_i^2 = 15532, \Sigma V_o = 228, \Sigma V_o^2 = 10980, \Sigma V_i V_o = 12296, n = 6$$

### Part A

#### The correlation coefficient

Find Pearson's product moment correlation coefficient for the set of statistics, giving your answer to 3 sf.

**Part B****Significance of the correlation between input and output**

Test at the 1% significance level whether there is a correlation between the input and output voltages.

Find the appropriate critical value for Pearson's product moment correlation coefficient, giving your answer to 3 sf.

What do you conclude about whether there is a correlation at the 1% significance level between the input and output voltages?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Pearson's product moment correlation coefficient is  the calculated value; the null hypothesis that there is  correlation between the two values can therefore be  at the 1% level and there is  evidence for a correlation at this level.

Items:

**Part C****The regression line**

Calculate the equation of the regression line  $V_o = a + bV_i$  relating the output voltage ( $V_o$ ) to the input voltage ( $V_i$ ).

Give the values of  $a$  and  $b$  to 3 sf.

$$V_o = \boxed{\phantom{00}} + \boxed{\phantom{00}} V_i$$

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## Correlation - Attenuation of Gamma Rays

**Subject & topics:** Maths | Statistics | Hypothesis Tests    **Stage & difficulty:** A Level C2, Further A P2

An experiment to investigate the attenuation of gamma rays by lead was carried out by measuring the count rate as a function of the number of lead sheets placed in front of a gamma ray source. The following results were obtained.

Number of lead sheets	0	1	2	3	4	5	6
Gamma ray count rate	13106	4301	1469	474	163	63	17

### Part A

#### Rank correlation coefficient

Write down Spearman's rank correlation coefficient for these data.

### Part B

#### Linear product moment correlation coefficient

Find the Pearson product moment correlation coefficient between the two sets of data, giving your answer to 4 sf.

**Part C****Significance of the linear product moment correlation coefficient**

Test at the 1% significance level whether there is a negative correlation between the count rate and the number of lead sheets.

Find the appropriate critical value for Pearson's product moment correlation coefficient, giving your answer to 4 sf.

What do you conclude about whether there is a negative correlation at the 1% significance level between the count rate and the number of lead sheets?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Pearson's product moment correlation coefficient is [ ] the modulus of the calculated value; the [ ] hypothesis cannot be rejected so there is [ ] evidence for a correlation at the 1% level.

Items:

[no significant] [sufficient] [accepted] [alternative] [less than] [null] [equal to] [rejected] [greater than]

**Part D****Logarithmic product moment correlation coefficient**

Consider now the relationship between the natural log of the counts and the number of lead sheets.

Find the Pearson product moment correlation coefficient between the natural log of the count rate and the number of lead sheets, giving your answer to 4 sf.

**Part E****Significance of the logarithmic product moment correlation coefficient**

Test at the 1% significance level whether there is a negative correlation between the natural log of the count rate and the number of lead sheets.

Find the appropriate critical value for Pearson's product moment correlation coefficient, giving your answer to 4 sf.

What do you conclude about whether there is a negative correlation at the 1% significance level between the natural log of the count rate and the number of lead sheets?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Pearson's product moment correlation coefficient is [ ] the modulus of the calculated value; the [ ] hypothesis can be rejected at the 1% level and there is [ ] evidence for a negative correlation at this level.

Items:

**Part F****The equation of the regression line**

Calculate the equation of the regression line

$$\ln N = a + bm$$

where  $N$  is the gamma ray count and  $m$  is the number of lead sheets. Give the values of  $a$  and  $b$  to 4 sf.

$$\ln N = [ ] + [ ] m$$

**Part G****Attenuation constant**

Theory predicts that the relationship expected between the number of counts  $N$  and the number of lead sheets  $m$  is given by

$$N = N_0 e^{-\alpha m}$$

where  $\alpha$  is the attenuation constant and  $N_0$  is a constant.

Find the values of  $\alpha$  and  $N_0$ , giving your answers to 4 sf.

$$\alpha = \boxed{\phantom{0000}}$$

$$N_0 = \boxed{\phantom{0000}}$$

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## STEM SMART Double Maths 33 - Regression, Correlation &amp; Hypothesis Tests

## Correlation - a Torsional Pendulum

**Subject & topics:** Maths | Statistics | Hypothesis Tests

**Stage & difficulty:** Further A P2, University P1

An experiment was carried out to investigate the amplitude of the driven oscillations of a torsional pendulum as a function of the frequency of the driving force. The initial set of results (I) and an additional follow-up set of results (II) are shown below. (The amplitude is given in arbitrary units.)

### Initial results I

<b>Frequency (Hz)</b>	0.28	0.36	0.43	0.49	0.51	0.56
<b>Amplitude</b>	1.0	1.4	1.9	3.1	6.2	5.2

### Follow-up results II

<b>Frequency (Hz)</b>	0.52	0.53	0.54	0.64	0.87
<b>Amplitude</b>	15.2	12.5	7.9	1.6	0.5

#### Part A

#### Initial results I - Spearman's rank correlation coefficient

Find Spearman's rank correlation coefficient for the initial set of results I, giving your answer to 3 sf.

**Part B****Initial results I - significance of the rank correlation coefficient**

Test at the 5% significance level whether there is a correlation between the driving frequency and the amplitude of the oscillations for the initial results I, using Spearman's rank correlation coefficient.

Find the appropriate critical value for Spearman's rank correlation coefficient, giving your answer to 3 sf.

What do you conclude about whether there is a correlation at the 5% significance level between the driving frequency and the amplitude of the oscillations?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Spearman's rank correlation coefficient is  the calculated value; there is therefore  evidence to  the null hypothesis at the 5% level and  evidence for a correlation at this level.

Items:

- 
- 
- 
- 
- 
- 
- 
- 
- 
- 
- 

**Part C****Initial results I - Pearson's product moment correlation coefficient**

Find Pearson's product moment correlation coefficient for the initial set of results I, giving your answer to 3 sf.

**Part D****Initial results I - significance of the product moment correlation coefficient**

Test at the 5% significance level whether there is a correlation between the driving frequency and the amplitude of the oscillations for the initial results I, using Pearson's product moment correlation coefficient.

Find the appropriate critical value for Pearson's product moment correlation coefficient, giving your answer to 3 sf.

What do you conclude about whether there is a correlation at the 5% significance level between the driving frequency and the amplitude of the oscillations?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Pearson's product moment correlation coefficient is  the calculated value; there is therefore sufficient evidence to  the null hypothesis at the 5% level and  evidence for a correlation at this level.

Items:

less than    reject    positive    accept    equal to    significant    greater than    no    negative

**Part E****Results I and II - Pearson's product moment correlation coefficient**

Now consider the initial and follow-up results together (11 pairs of values in all). Find Pearson's product moment correlation coefficient for this data set consisting of the 11 pairs of values, giving your answer to 3 sf.

**Part F****Results I and II - significance of the product moment correlation coefficient**

Test at the 5% significance level whether there is a correlation between the driving frequency and the amplitude of the whole data set, using Pearson's product moment correlation coefficient.

Find the appropriate critical value for Pearson's product moment correlation coefficient, giving your answer to 3 sf.

What do you conclude now about whether there is a correlation at the 5% significance level between the driving frequency and the amplitude of the oscillations?

Choose the correct words to fill in the following sentence.

The appropriate critical value for Pearson's product moment correlation coefficient is  the calculated value; there is therefore  evidence for a correlation between the driving frequency and the amplitude of the oscillations.

Items:

no significant    greater than    less than    equal to    sufficient