Special Relativity:

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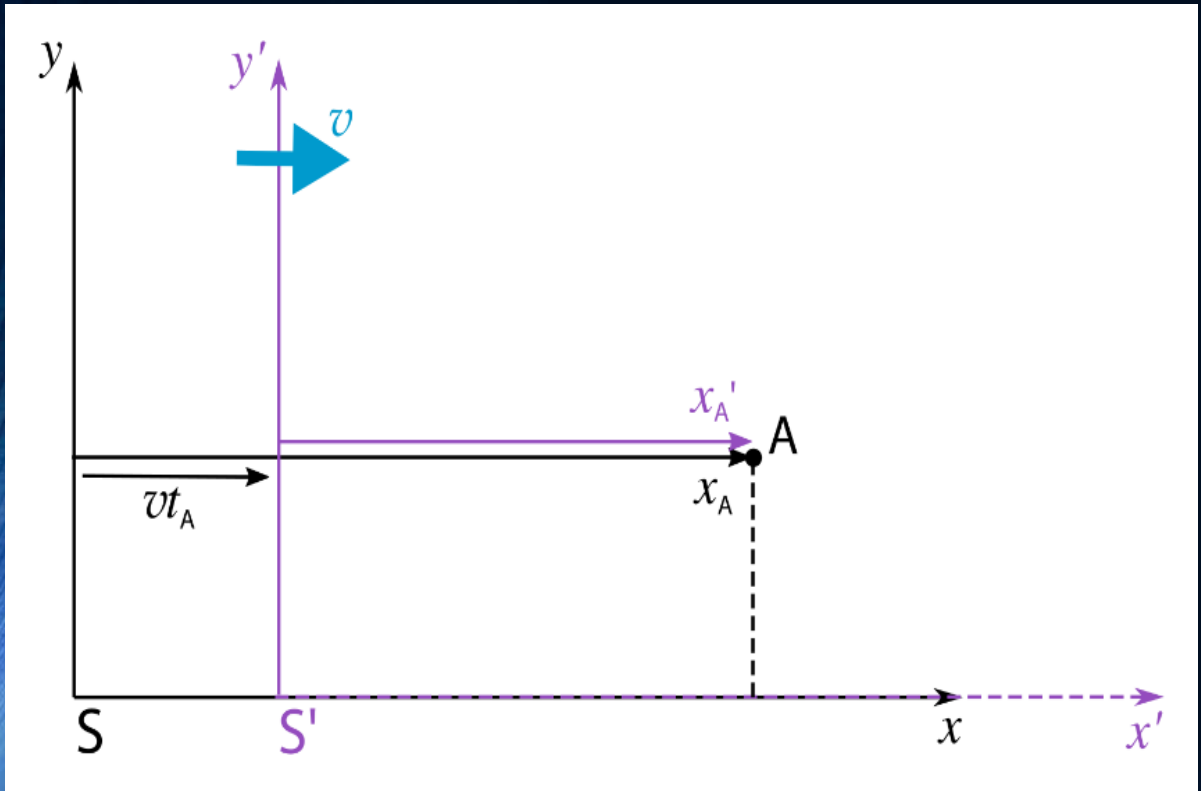
Frames of Reference:

Galilean Coordinate Transformations



- S’ moves with velocity v along the x-direction in frame S.
- If the origins of the two frames coincide at $t = t' = 0$, the transformation of an event in S to S’ is

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

- Implicit here the assumption that there is a universal ‘absolute’ time, running at the same rate in all inertial frames.

Inertial Frames of Reference

- A frame in which Newton's first law of motion is valid.
- What is Newton's first law?

  A body remains in a state of rest or uniform motion unless acted upon by a resultant force.

- You are in a car and cannot see outside but you can see a parcel in the boot of the car that is stationary according to you.

  What do you deduce about the motion of the car?

  What do you deduce (and why) if the parcel accelerates to the back of the car?

- A <u>non-inertial</u> frame is a frame which is accelerating.

Frames of Reference

Platform view

Frames of Reference

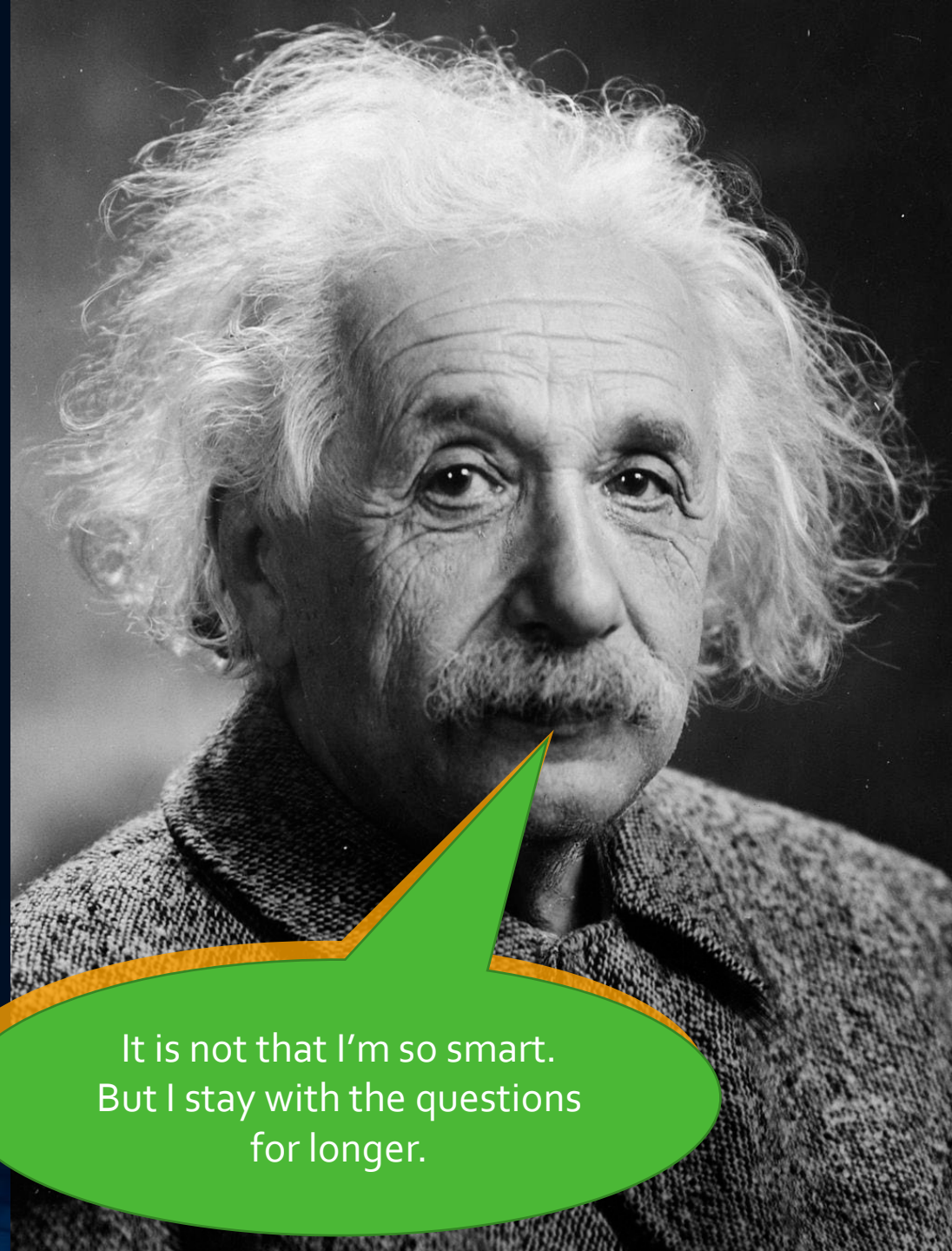
Platform view

Enter Einstein

Galileo's relativity is not the whole story!

- it is actually an approximation to the real truth that works for us in everyday life.

Proposed that there is a single relativity principle for both dynamics and EM but it is NOT Galilean.



Special Theory of Relativity: Historical Context

The "Aether"

- Mid to late 19th century the wave theory of light was generally accepted and naturally explained by

  Reflection, refraction, interference, diffraction and polarisation.

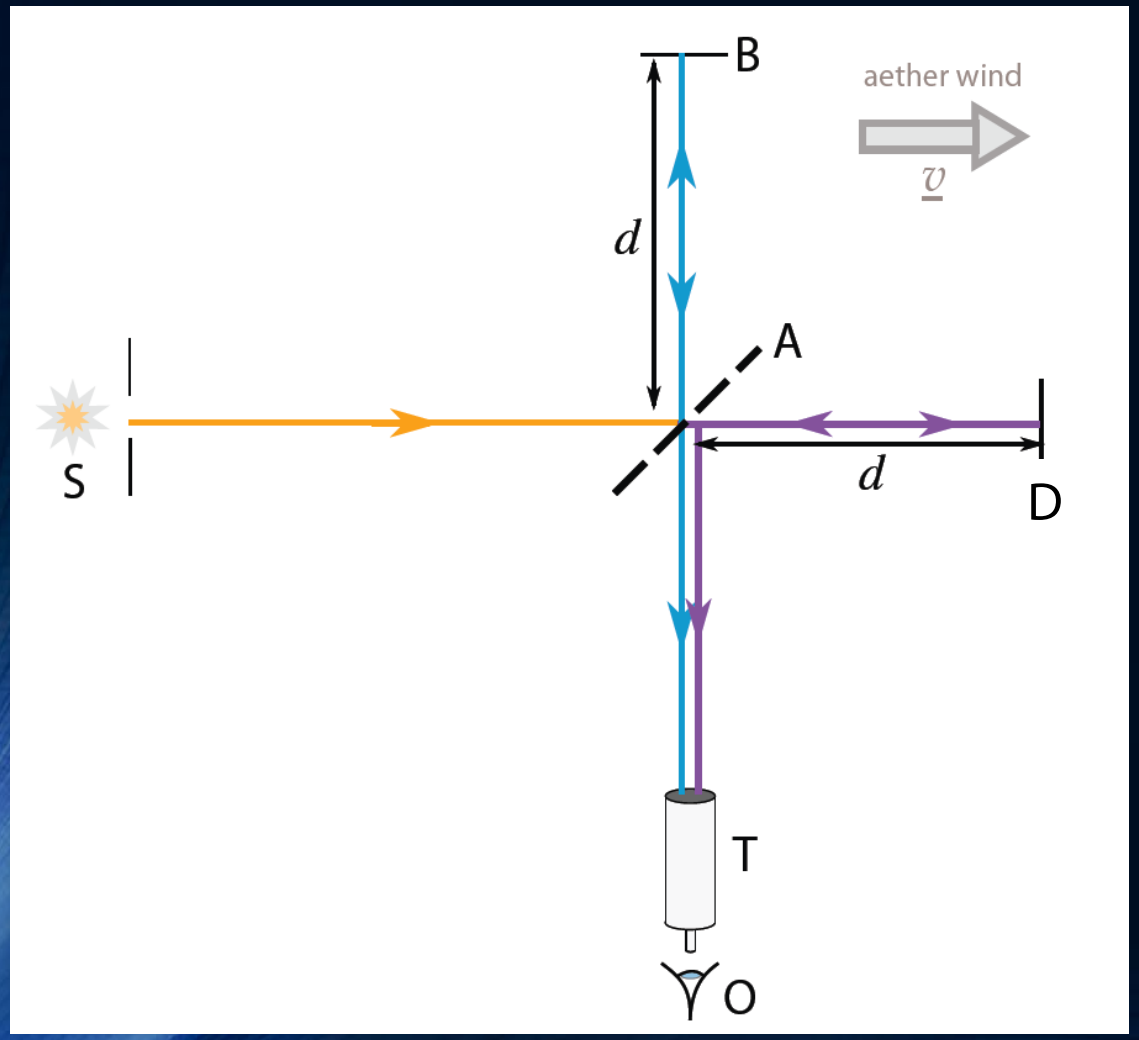
- It was thought that, like all other known waves, light must travel in a medium – called the luminiferous aether.

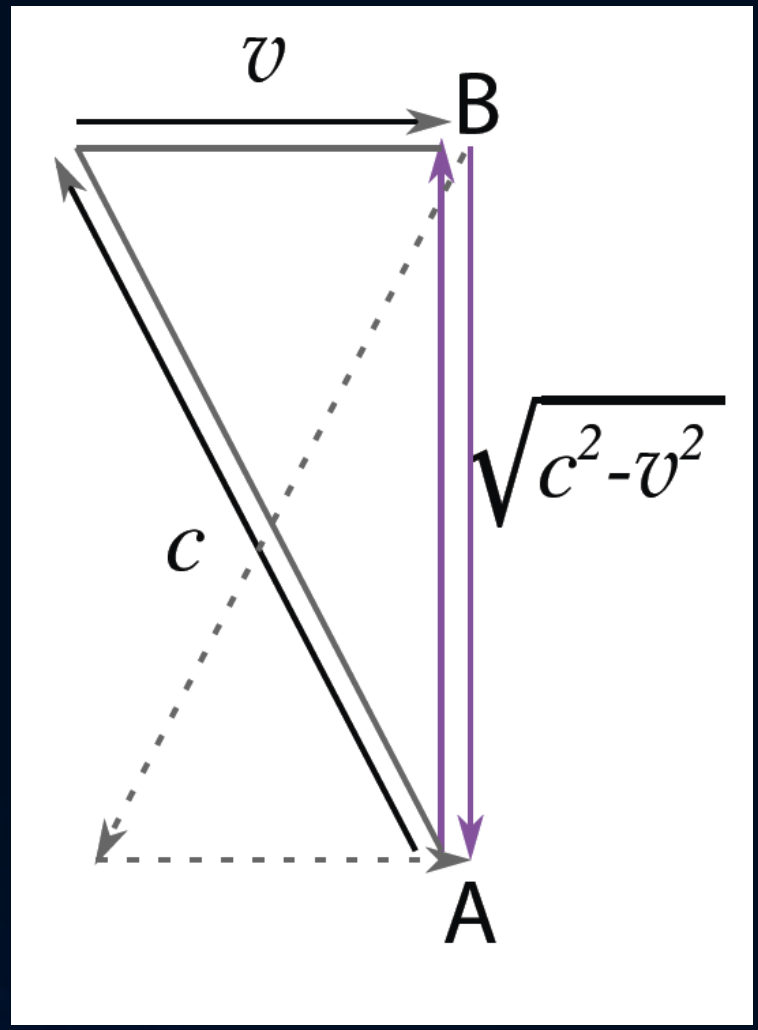
- Maxwell's equations predicted electromagnetic waves – light – travelling with a speed $c = \sqrt{\frac{1}{\epsilon_0 \mu_0}} \approx 3 \times 10^8 \text{ m s}^{-1}$

Aether Experiments

- Two experiments to deduce the properties of the aether were in direct contradiction to each other
  1. “stellar aberration” → evidence for a stationary aether – absolute frame.
  2. “Michelson-Morley experiment”

Michaelson-Morley





Aether Experiments

- Two experiments to deduce the properties of the aether were in direct contradiction to each other
  1. "stellar aberration" → evidence for a stationary aether – absolute frame.
  2. "Michelson-Morley experiment" → the aether was at rest w.r.t. the Earth i.e. dragged along by the Earth – no absolute frame, no aether?

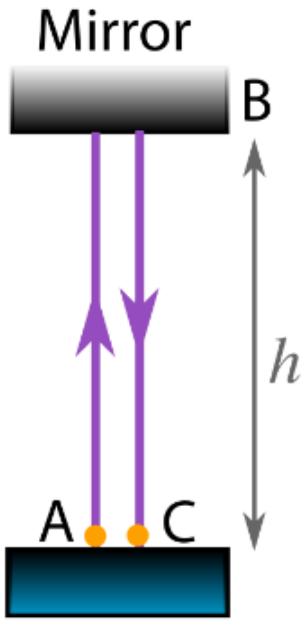
- Contradictions:
  a) Newton's Laws do not need absolute space – they apply in all inertial frames.
  b) Maxwell's equations are not Galilean invariant because the speed of light is the same in all frames.

Einstein's Solution:

There is a single relativity principle for both dynamics and EM but it is NOT Galilean.

- The Postulates:
  1. The laws of Physics are the same in all inertial frames;
  2. The speed of light in a vacuum is the same in all inertial frames.

The Light Clock



Train frame, S'
with the light clock

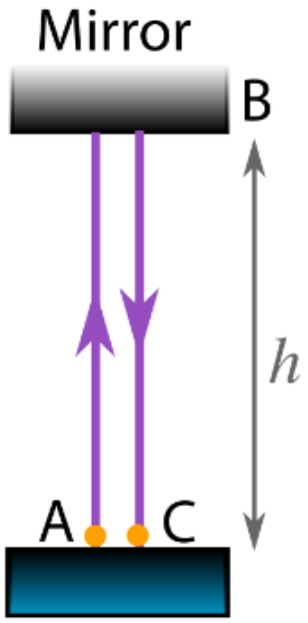
Time for A to B to C

$$\Delta t'_{AC} = \frac{2h}{c}$$

Physics

$$Speed = \frac{distance}{time}$$

The Light Clock



Train frame, S'
with the light clock

Time for A to B to C

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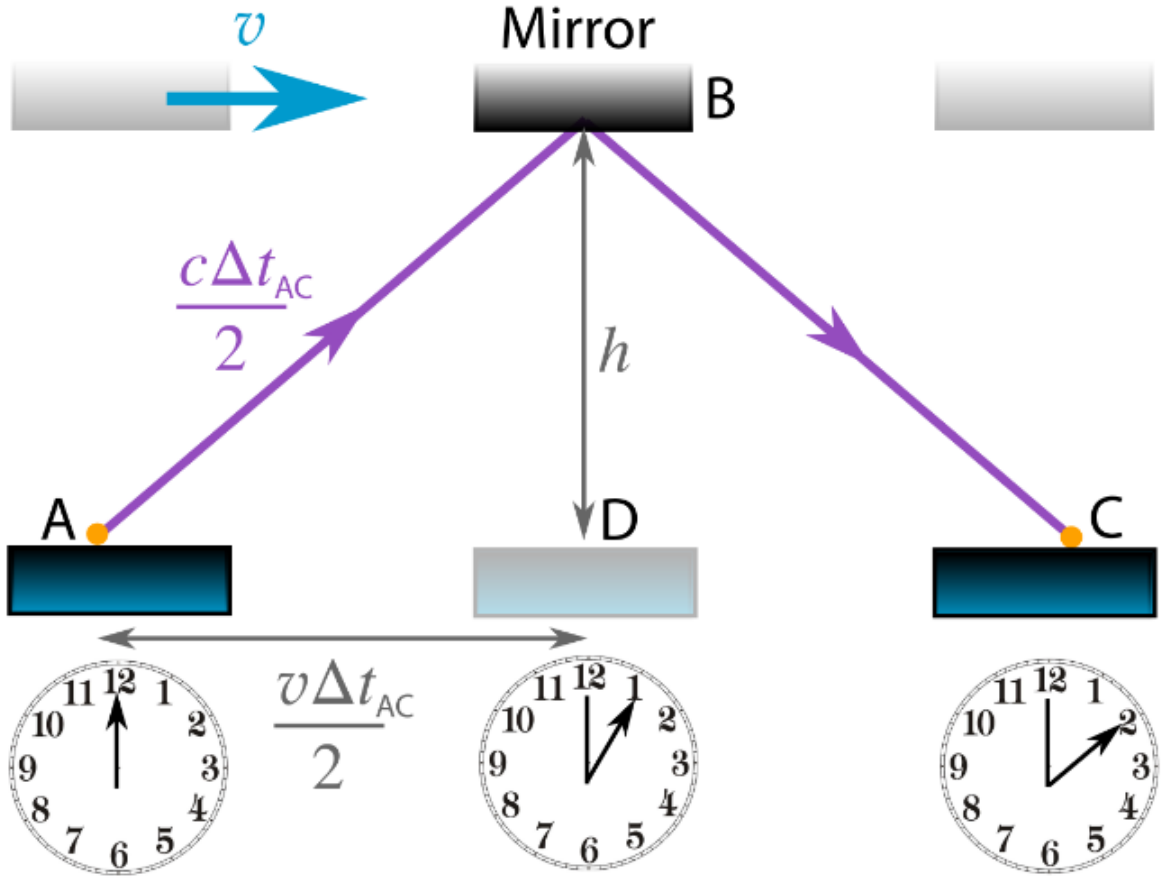


Platform frame, S
Standing watching the light clock pass by on the train

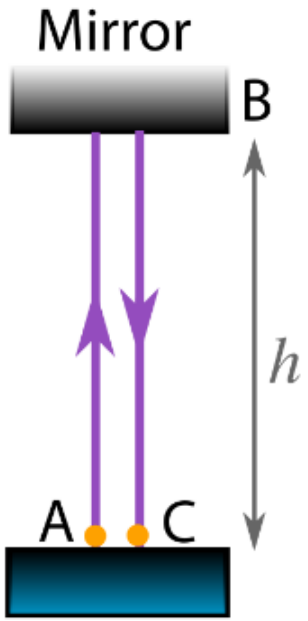
Physics

$$Speed = \frac{distance}{time}$$

The Light Clock

Distance pulse travels $c\Delta t_{AC}$

$$(c\Delta t_{AC})^2 = (v\Delta t_{AC})^2 + 4h^2$$



Train frame, S'
with the light clock

Time for A to B to C

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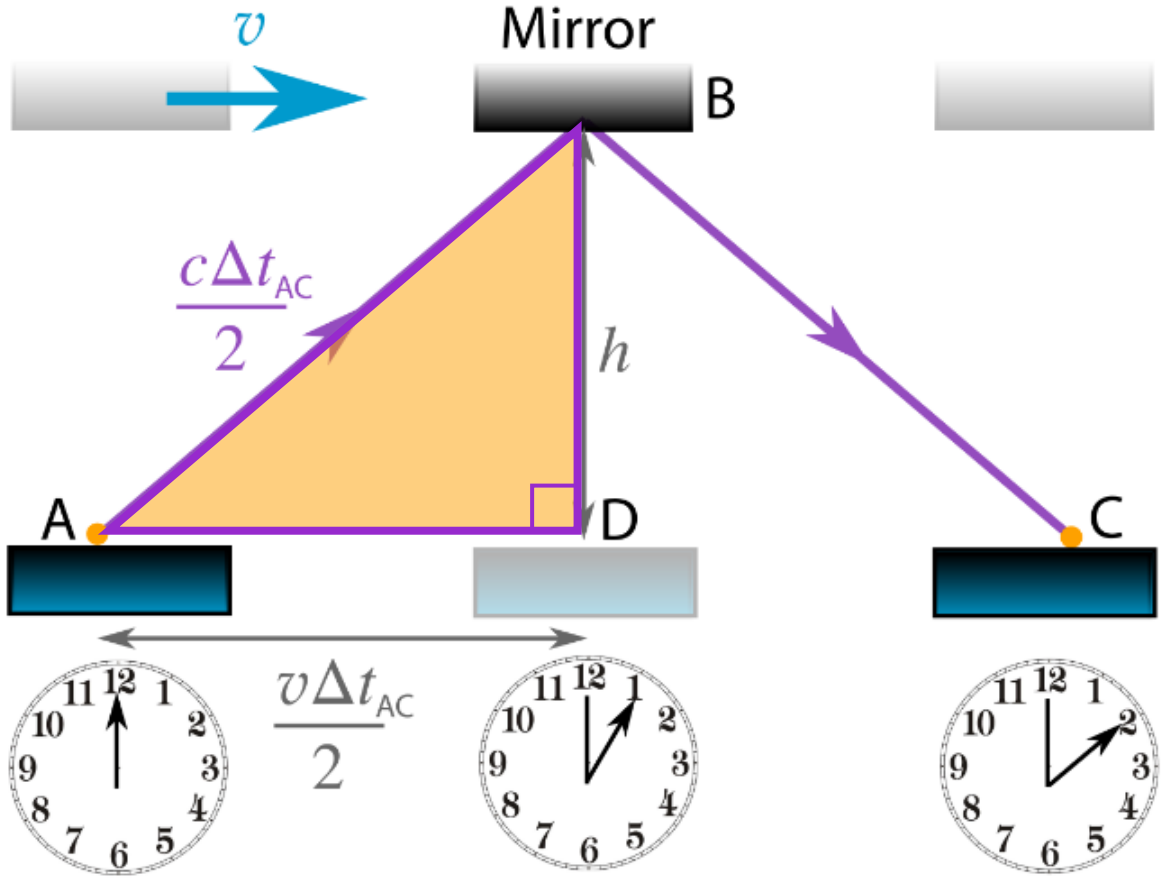


Platform frame, S

$$(c^2 - v^2)\Delta t_{AC}^2 = 4h^2$$

Physics

$$Speed = \frac{distance}{time}$$

The Light Clock

Distance pulse travels $c\Delta t_{AC}$

$$(c\Delta t_{AC})^2 = (v\Delta t_{AC})^2 + 4\mathrm{h}^2$$

Time for A to B to C

$$\Delta \mathrm{t}'_{\mathrm{AC}} = \frac{2h}{c}$$

$$(c^2 - v^2)\Delta t_{AC}^2 = 4\mathrm{h}^2$$

$$(c^2 - v^2)\Delta t_{AC}^2 = c^2 {\Delta t'_{AC}}^2$$

So what?

What if $v = \frac{4}{5}\mathrm{c}$?

$$\left(1 - \frac{v^2}{c^2}\right)\Delta t_{AC}^2 = {\Delta t'_{AC}}^2$$

$$\sqrt{\left(1 - \frac{v^2}{c^2}\right)}\,\Delta t_{AC} = \Delta t'_{\mathrm{AC}}$$

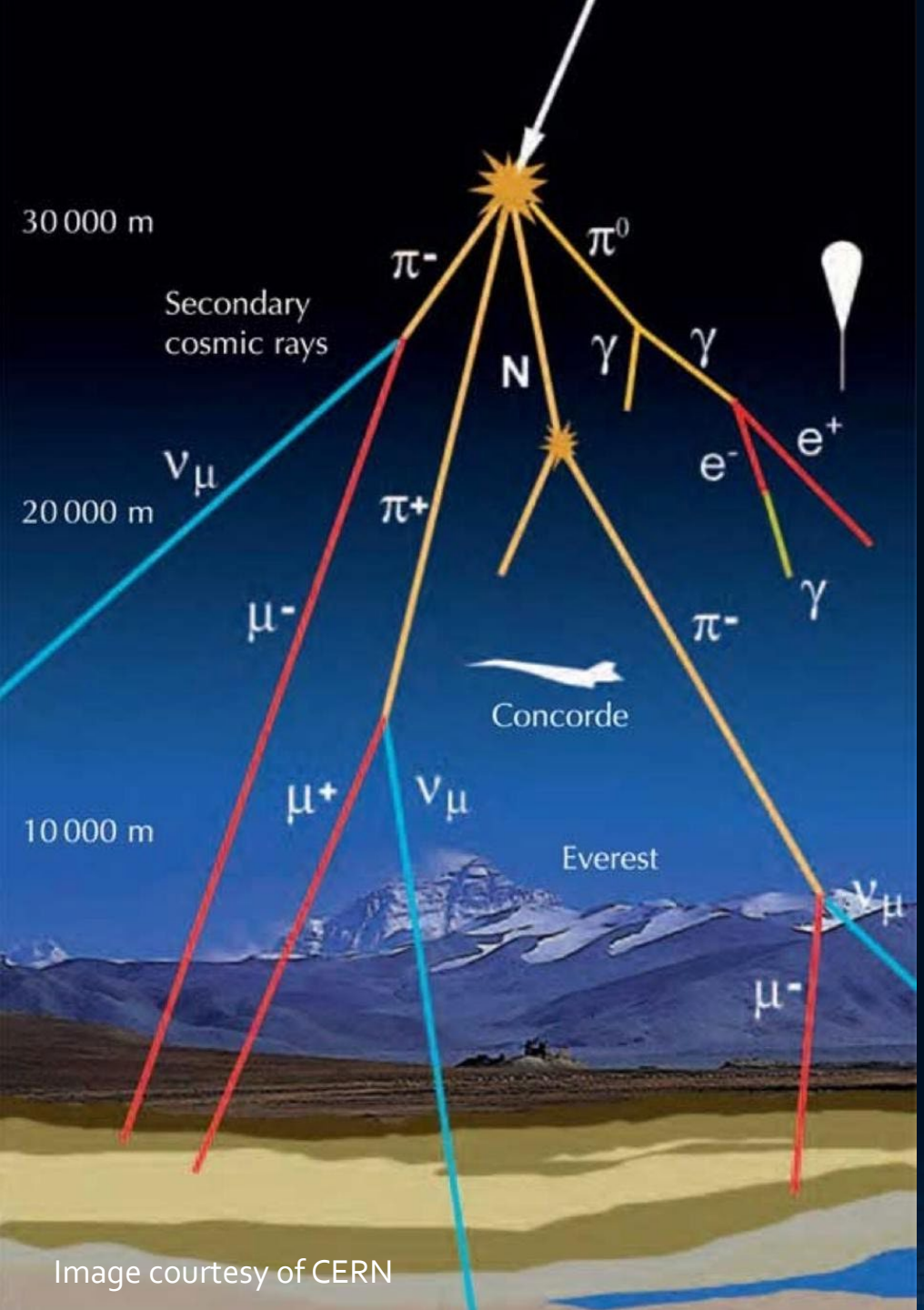


Image courtesy of CERN

Does this physics matter?

- Measuring muons on Earth presented a dilemma.
- Their half-life is just 2.2 μs.
- How far can they travel in that time?
- The speed of the muon means that we need to take SR into account.

$$\sqrt{1-\frac{v^2}{c^2}}\,\Delta t = \Delta t'$$

Earth Muon

Staying Younger for Longer?

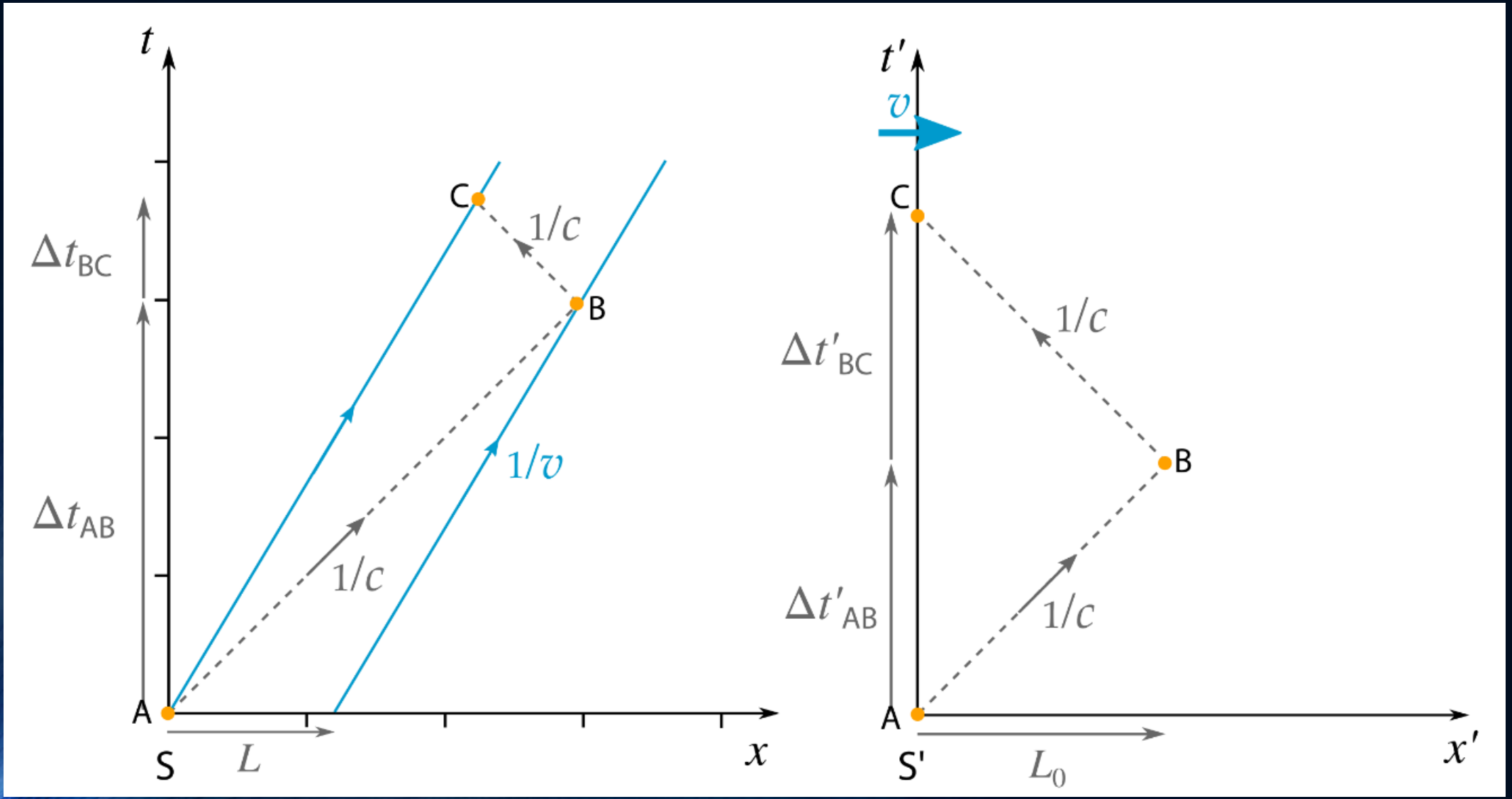
Not to scale!

$$\sqrt{\left(1-\frac{v^2}{c^2}\right)}\,\Delta t = \Delta t'$$

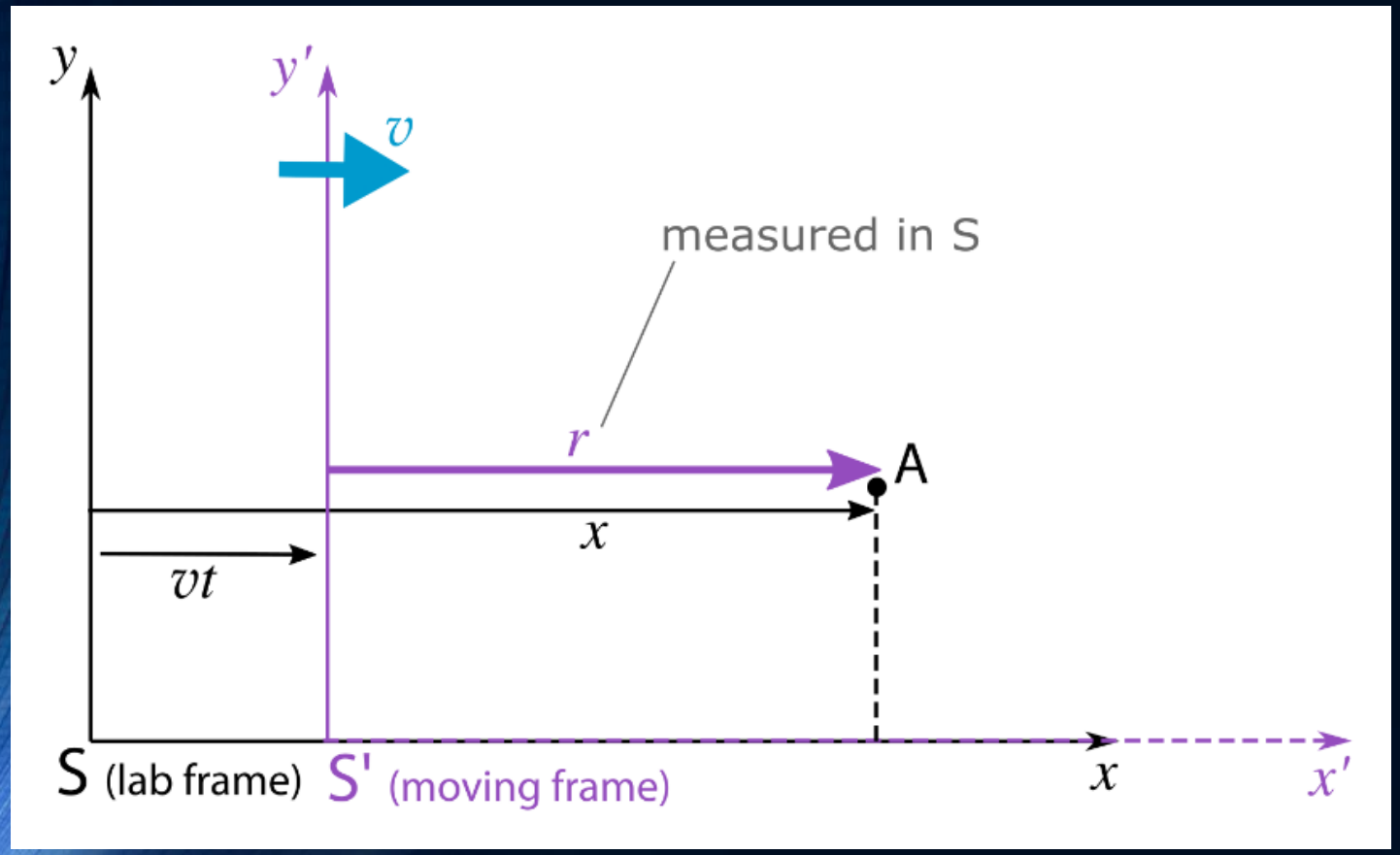
- One twin, Bob, stays on Earth.
- Alice, takes her rocket into space travelling at 4/5 of the speed of light.
- After 5 years have passed for Bob, how many have passed for Alice?

Questions 23 & 24.

Length Contraction



Lorentz Transformations



$$r = x - vt$$

- Ruler of length r_0 in frame S' has length r in S such that

$$r_0 = \gamma r$$
$$x' = \gamma r$$
$$x' = \gamma(x - vt)$$

- And

$$x = \gamma(x' + vt')$$

- To find times:

$$x = \gamma[\gamma(x - vt) + vt']$$

$$t' = \gamma(t - vx/c^2)$$

Lorentz Transformations

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

$$\Delta y' = \Delta y$$

$$\Delta z' = \Delta z$$

$$\Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right)$$

$$\Delta x = \gamma(\Delta x' + v\Delta t')$$

$$\Delta y = \Delta y'$$

$$\Delta z = \Delta z'$$

With

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$