Differentiation: Products 3ii

A Level

Find the exact value of the x-coordinate of the stationary point of the curve $y = x \ln x$.

The following symbols may be useful: `e, x`

Differentiation: Quotients 3ii

A Level

Part A Differentiate

A curve has equation $y = \frac{2x+1}{3x-1}$. Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part B Tangent

Hence find the equation of the tangent to this curve at the point $(1, \frac{3}{2})$, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

The following symbols may be useful: `x, y`

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Integration by Substitution 1ii

A Level

Use the substitution $x = \frac{1}{3}\sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{6}} \frac{1}{(1-9x^2)^{\frac{3}{2}}}\,\mathrm{d}x$$

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Area Between Two Curves 1ii

A Level

P P P

Figure 1 shows the curve $y = \mathrm{e}^{3x} - 6\mathrm{e}^{2x} + 32$.

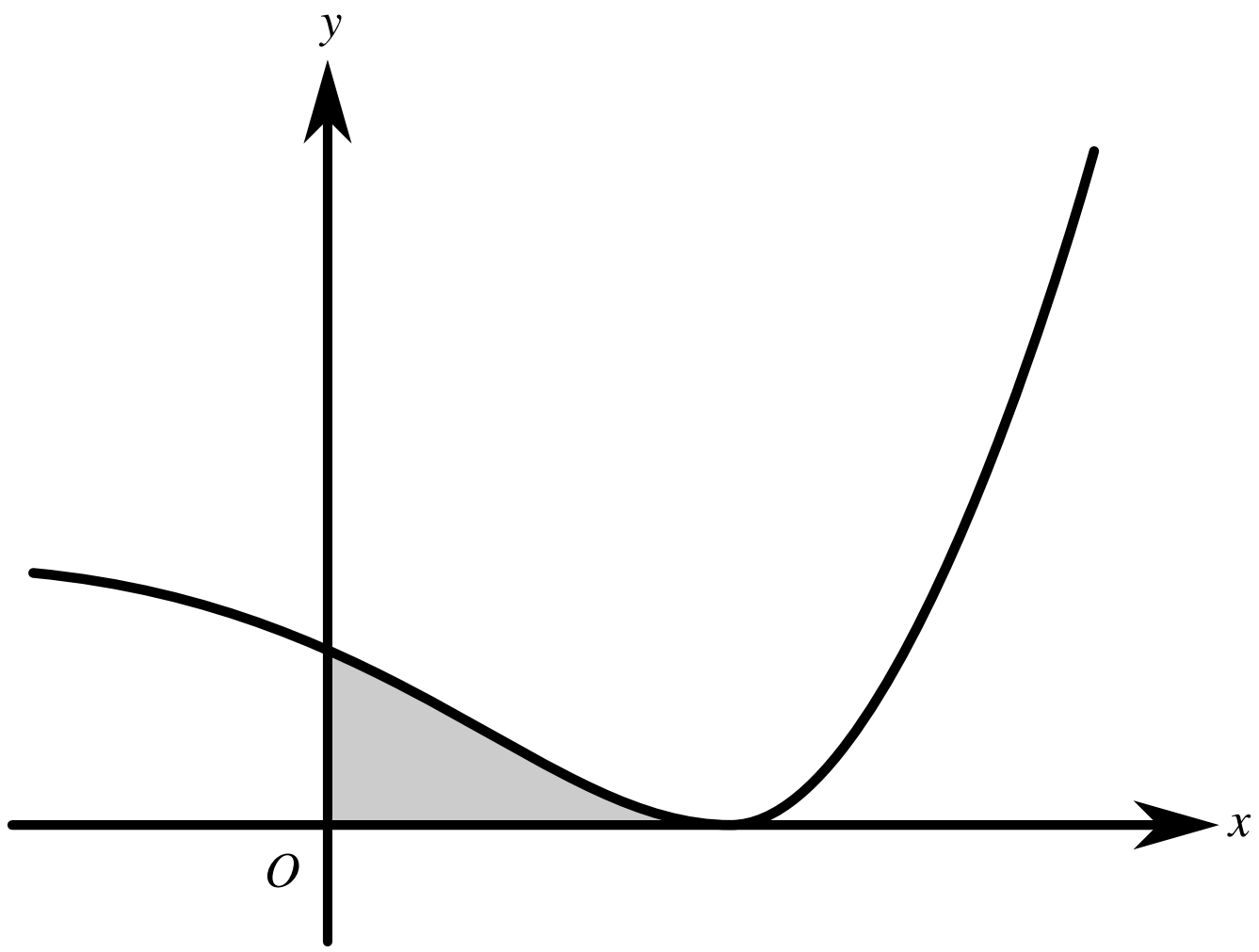

Figure 1: The curve $y = \mathrm{e}^{3x} - 6\mathrm{e}^{2x} + 32$.

Part A x-coordinate

Give the exact x-coordinate of the minimum point and verify that the y-coordinate of the minimum point is 0.

The following symbols may be useful: x

Part B Area of shaded region

Find the exact area of the shaded region enclosed by the curve and the coordinate axes.

The following symbols may be useful: `ln(), log()`

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Differentiation: Chain Rule 4i

A Level

Earth is being added to a pile so that, when the height of the pile is h metres, its volume is V cubic metres, where

$$V = (h^6 + 16)^{\frac{1}{2}} - 4.$$

Part A Rate of change of volume

Find the value of $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 2$, to three significant figures.

Part B Rate of change of height

The volume of the pile is increasing at a constant rate of 8 cubic metres per hour. Find the rate in metres per hour, at which the height of the pile is increasing at the instant when $h = 2$. Give your answer correct to 2 significant figures.

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Modelling - Advanced 2ii

A Level

P P P

At time t seconds, the radius of a spherical balloon is $r\,\mathrm{cm}$. The balloon is being inflated so that the rate of increase of its radius is inversely proportional to the square root of its radius. When $t = 5$, $r = 9$ and, at this instant, the radius is increasing at $1.08\,\mathrm{cm\,s^{-1}}$.

Part A Differential equation

Write down a differential equation to model this situation. Your answer should include a constant k, whose value you do not need to determine yet.

The following symbols may be useful: `Derivative(r, t), k, r, t`

Part B Solution

Solve the differential equation to express r in terms of t. Your answer should include some numerical constants, which should be converted to exact fractions.

The following symbols may be useful: `r, t`

Part C Initial condition

How much air (in cm^3) is in the balloon initially? Write your answer as an exact expression.

The following symbols may be useful: `pi`

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Differentiation: Implicit 4i

A Level

Find the equation of the normal to the curve $x^3 + 2x^2y = y^3 + 15$ at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

The following symbols may be useful: `x, y`

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Parametric Equations 4ii

A Level

A curve has parametric equations $x = \frac{1}{t} - 1$ and $y = 2t + \frac{1}{t^2}$

Part A Gradient function

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, simplifying your answer.

The following symbols may be useful: `Derivative(y, x), t, x, y`

Part B Stationary point

Find the x-coordinate of the stationary point.

The following symbols may be useful: x

Find the y-coordinate of the stationary point.

The following symbols may be useful: y

By considering the gradient of the curve on either side of this point, determine its nature.

- Point of inflection
- Minimum
- Maximum

Part C Cartesian equation

Find the cartesian equation of the curve.

The following symbols may be useful: x, y

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Integration by Parts 3i

A Level

Find $\int_0^{\pi} \left(x^2 + 5x + 7\right) \sin x \, \mathrm{d}x$.

The following symbols may be useful: `pi`