Binomial: All Rational n 2ii

A Level

Part A Expansion

Expand $(1+4x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients.

The following symbols may be useful: `x`

Part B Values of x

State the set of values of x for which the expansion is valid. You may use the symbols $< > = \ x$ and $|x|$ which can be entered as `abs(x)`.

The following symbols may be useful: `<, >, abs(), x`

Part C Value of k

In the expansion of

$$(1+kx)(1+4x)^{\frac{1}{2}},$$

the coefficient of x is 7. Find the value of the constant k and hence the coefficient of x^2.

State the value of k.

The following symbols may be useful: k

State the coefficient of x^2.

Binomial: All Rational n 4ii

A Level

Part A Expansion

Expand $(2+x)^{-2}$ in ascending powers of x up to and including the term in x^3.

The following symbols may be useful: x

Part B Set of Values

State the set of values of x for which the expression is valid. You may use the symbols $< > = \ x$ and $|x|$ which can be entered as `abs(x)`.

The following symbols may be useful: `<, >, abs(), x`

Part C Coefficient

Hence find the coefficient of x^3 in the expansion of $\frac{1+x^2}{(2+x)^2}$.

The following symbols may be useful: x

Gameboard:

STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Binomial: All Rational n 1i

A Level

Part A Partial Fractions

Given that $\frac{3x+4}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$, find A, B, and C.

Find A.

The following symbols may be useful: A

Find B.

The following symbols may be useful: B

Find C.

The following symbols may be useful: C

Part B **Expand**

Hence or otherwise expand $\frac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2.

The following symbols may be useful: x

Part C Values of x

State the set of values of x for which the expansion in the above part is valid.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $x < a$
- $x \leq a$
- $x > a$
- $x \geq a$
- $a < x < b$
- $a \leq x \leq b$
- $x < a$ or $x > b$
- $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

Gameboard:

STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Binomial: All Rational n 3i

A Level

P P P

Find the coefficient of x^2 in the expansion in ascending powers of x of

$$\sqrt{\frac{1+ax}{4-x}}$$

giving your answer in terms of a.

The following symbols may be useful: a

Gameboard:

STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Binomial Expansion 1

A Level P P P Further A P P P

Rewrite each of the following expressions in the form $a(1+b)^n$, where $|b|<1$. Hence, using the appropriate binomial expansion, find the value of each of them correct to 4 dp.

Part A $\sqrt{36.1}$

Rewrite $\sqrt{36.1}$ in the form $a(1+b)^n$, where $|b|<1$ and $n=\frac{1}{2}$. Hence, using the appropriate binomial expansion, find the value correct to 4 dp.

Give the value of a.

Give the value of b as a simplified fraction.

Using the binomial theorem, evaluate $\sqrt{36.1}$ correct to 4 dp.

Part B $\sqrt[3]{1.09}$

Rewrite $\sqrt[3]{1.09}$ in the form $a(1+b)^n$, where $|b|<1$ and $n=\frac{1}{3}$. Hence, using the appropriate binomial expansion, find the value correct to 4 dp.

Give the value of a.

Give the value of b as a simplified fraction.

Using the binomial theorem, evaluate $\sqrt[3]{1.09}$ correct to 4 dp.

Part C $1/\sqrt{1.04}$

Rewrite $\dfrac{1}{\sqrt{1.04}}$ in the form $a(1+b)^n$, where $|b|<1$ and $n=-\frac{1}{2}$. Hence, using the appropriate binomial expansion, find the value correct to 4 dp.

Give the value of a.

Give the value of b as a simplified fraction.

Using the binomial theorem, evaluate $\dfrac{1}{\sqrt{1.04}}$ correct to 4 dp.

Part D $\sqrt[3]{125.4}$

Rewrite $\sqrt[3]{125.4}$ in the form $a(1+b)^n$, where $|b|<1$ and $n=\frac{1}{3}$. Hence, using the appropriate binomial expansion, find the value correct to 4 dp.

Give the value of a.

Give the value of b as a simplified fraction.

Using the binomial theorem, evaluate $\sqrt[3]{125.4}$ correct to 4 dp.

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Gameboard:

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Binomial: All Rational n 5i

A Level

Part A Expansion

Expand $(1-3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3.

The following symbols may be useful: x

Part B Coefficient of x^3

Hence find the coefficient of x^3 in the expansion of $(1-3(x+x^3))^{-\frac{1}{3}}$

Gameboard:

STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Maclaurin Series - Potential due to Dipole

Further A University

Part A Potential on the axis

An electric dipole consists of two charges $+q$ and $-q$ separated in the z direction by a very small distance a. The electric potential $V(z)$ a distance z away from the centre of the dipole in a direction along the line joining the two charges is given by

$$V(z) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{z-\frac{a}{2}} - \frac{1}{z+\frac{a}{2}}\right).$$

Find an approximate expression for $V(z)$ (assume that $z \gg a$ and obtain the first non-zero term in the Maclaurin (or binomial) expansion of $V(z)$).

The following symbols may be useful: `a, epsilon_0, pi, q, z`

Part B Potential in any direction

An electric dipole consists of two charges $+q$ and $-q$ separated by a distance a. The electric potential V a distance r from the centre of the dipole in a direction making an angle θ to the line joining the two charges is given approximately by

$$V \approx \frac{q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{r^2 - ar\cos\theta}} - \frac{1}{\sqrt{r^2 + ar\cos\theta}}\right).$$

Assuming that $r \gg a$ show that $V \approx \frac{A\cos\theta}{r^2}$ and find an expression for A.

The following symbols may be useful: `a, epsilon_0, pi, q, r`

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