STEM SMART Single Maths 29 - Radians & Small Angle Approximations

Radians

A-level Maths Topic Summaries - Trigonometry

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level P2

Fill in the blanks to complete the notes on radians below.

Part A

Radians and degrees

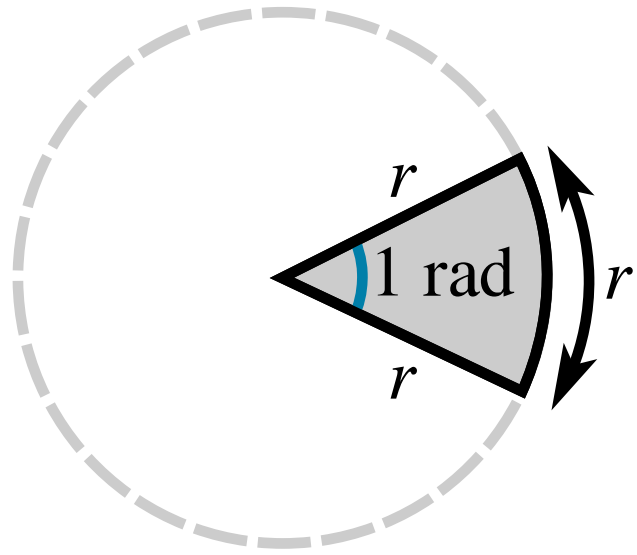

Figure 1: Illustrating the definition of the radian.

Radians are an alternative unit for measuring [ ]. The diagram above illustrates the definition of the radian. 1 radian is the angle at the centre of a circle of radius r that is subtended by a circular arc of length [ ].

The circumference of a circle is $2\pi r$. Hence, there are [ ] radians in one complete circle.

$$360^\circ = [\ ]\ \text{rad}$$

To convert between degrees and radians, we can use the formulae

$$\theta\ \text{rad} = [\ ] \times \theta^\circ$$

$$\theta^\circ = [\ ] \times \theta\ \text{rad}$$

Items:

angles r $\frac{180}{\pi}$ 2π $\frac{\pi}{180}$

Part B

Table of common values

Complete the table of common values

θ°	θ rad	θ°	θ rad
0		120	
30		150	
45		180	
60		270	
90		360	

Items:

0 π 2π $\frac{\pi}{2}$ $\frac{3\pi}{2}$ $\frac{\pi}{3}$ $\frac{2\pi}{3}$ $\frac{\pi}{4}$ $\frac{\pi}{6}$ $\frac{5\pi}{6}$

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Arcs, Sectors and Segments

A-level Maths Topic Summaries - Radians in Geometry

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level P2

Fill in the blanks to complete the notes on arc length, sector area and segments below.

Part A

Arc length and sector area

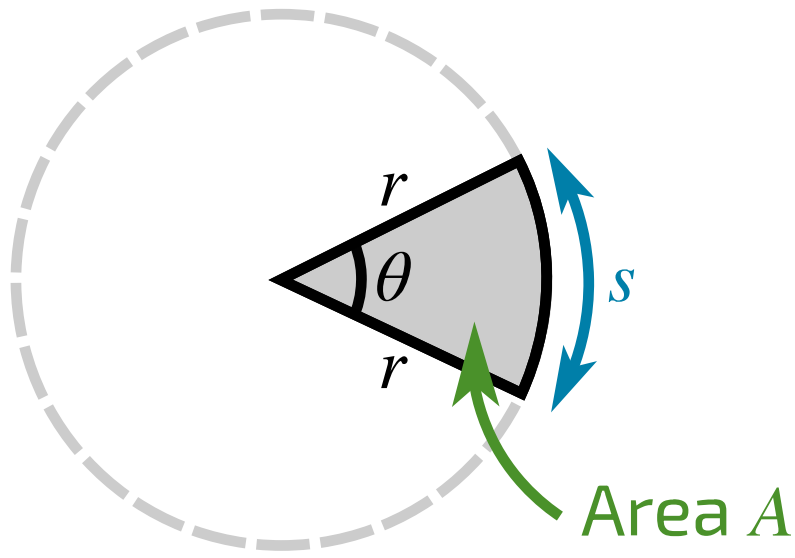

Figure 1: Arc length s and sector area A.

When angles are measured in [ ], expressions for calculating arc length and sector area are particularly simple.

Arc length

Arc length, $s =$ [ ]

Sector area

Sector area, $A =$ [ ]

Sector perimeter

The perimeter of a segment is equal to the arc length plus twice the radius.

Sector perimeter $=$ [ ]

Items:

degrees **radians** $r\theta$ $r\theta + 2r$ $\frac{1}{2}r^2\theta$

Part B

Segments

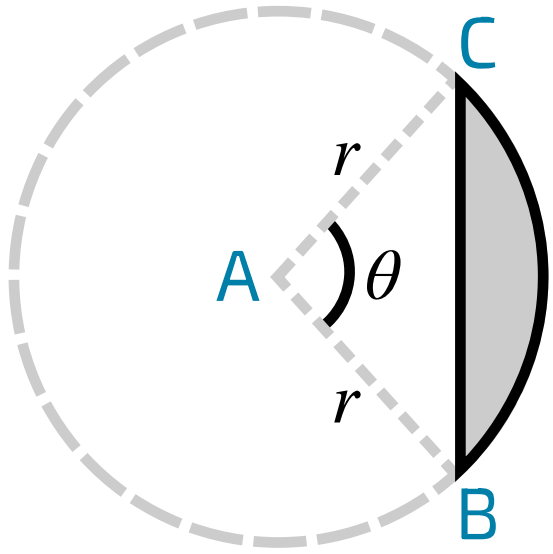

Figure 2: A segment of a circle.

Calculations involving segments of a circle, such as that in **Figure 2**, are common.

Segment area

The area of triangle ABC can be calculated using the formula $\mathrm{Area} = \frac{1}{2}ab\sin\theta$. The area of the segment can be found by subtracting the area of triangle ABC from the area of the sector.

Area of triangle $\mathrm{ABC} =$ ☐

$\therefore$ Segment area $=$ ☐

Segment perimeter

The length of the chord BC can be found using the cosine rule, $c^2 = a^2 + b^2 - 2bc\cos A$. The perimeter of the segment can be found by adding the lengths of the arc BC and the cord BC.

Length of chord $\mathrm{BC} =$ ☐

$\therefore$ Segment perimeter $=$ ☐

Items:

$\boxed{\frac{1}{2}r^2\sin\theta}$ $\boxed{\sqrt{2r^2 - 2r^2\cos\theta}}$ $\boxed{\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta}$ $\boxed{r\theta + \sqrt{2r^2 - 2r^2\cos\theta}}$

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Small Angle Approximations

A-level Maths Topic Summaries - Trigonometry

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level P2

Fill in the blanks to complete the notes on small angle approximations below.

When θ is small, we can approximate $\sin\theta$, $\cos\theta$ and $\tan\theta$ using the following polynomial expressions.

$$\sin\theta \approx \square$$

$$\cos\theta \approx 1 - \square$$

$$\tan\theta \approx \square$$

These expressions are only valid when θ is measured in $\square$.

Items:

θ $\frac{\theta^2}{2}$ **degrees** **radians**

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Radians-problems involving area 5ii

Subject & topics: Maths **Stage & difficulty:** A Level P1

Figure 1 shows a sector OAB of a circle, centre O and radius $8\,\mathrm{cm}$. The angle AOB is $46\,^\circ$.

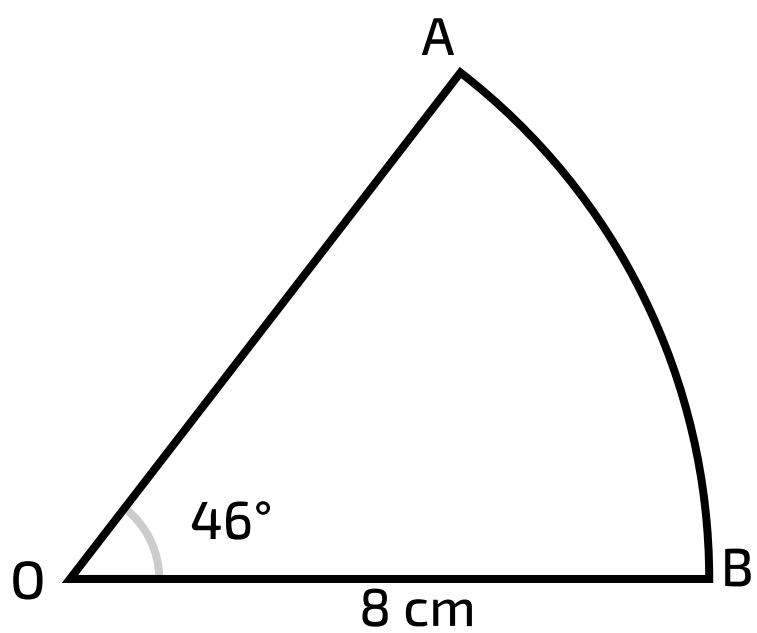

Figure 1: Sector AOB.

Part A

Convert angle to radians

Express $46\,^\circ$ in radians, correct to 3 significant figures.

Part B

Arc length

Find the length of the arc AB.

Part C

Area of sector

Find the area of the sector OAB.

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Radians-problems involving area 2ii

Subject & topics: Maths **Stage & difficulty:** A Level P1

Figure 1 shows two congruent triangles, BCD and BAE, where ABC is a straight line. In triangle BCD, BD $= 8\,\text{cm}$, CD $= 11\,\text{cm}$ and angle CBD $= 65\,^\circ$. The points E and D are joined by an arc of a circle with centre B and radius $8\,\text{cm}$.

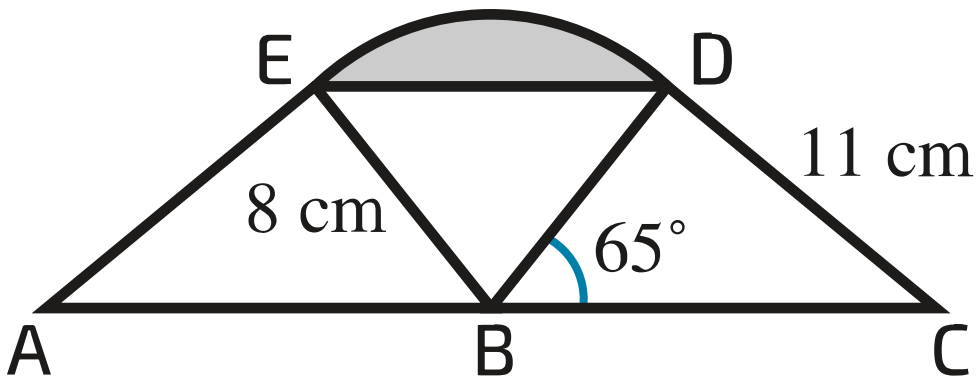

Figure 1: Diagram of the triangles.

Part A

Angle BCD

Find angle BCD. Give your answer in radians, correct to 3 significant figures.

Part B

Angle EBD

Find the angle EBD, giving your answer in radians correct to 3 significant figures.

Part C

Area of shaded segment

Hence find the area (in cm^2) of the shaded segment bounded by the chord ED and the arc ED, giving your answer correct to 3 significant figures.

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Radians-problems involving area 1ii

Subject & topics: Maths **Stage & difficulty:** A Level P1

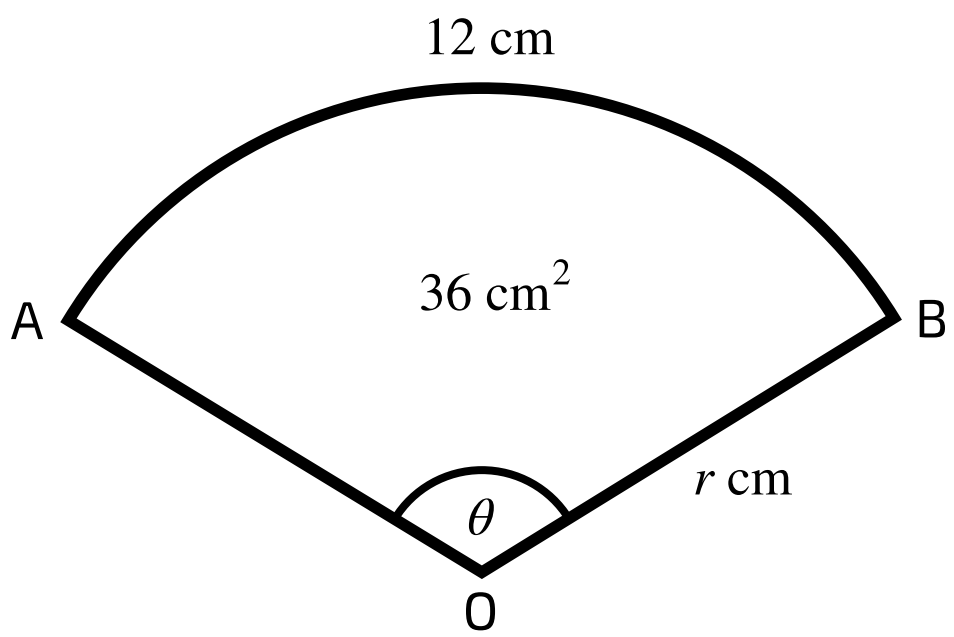

Figure 1: The sector OAB.

A sector OAB of a circle of radius r cm has angle θ radians. The length of the arc of the sector is 12 cm and the area of the sector is $36\,\text{cm}^2$ (see **Figure 1**).

Part A

First equation

By considering the length of the arc of the sector, write down an equation involving r and θ, where one side of the equation is a numerical constant.

The following symbols may be useful: `r, theta`

Part B

Second equation

By considering the area of the sector, write down another equation involving r and θ, where one side of the equation is a numerical constant.

The following symbols may be useful: `r, theta`

Part C

Values of r and θ

Hence show that $r = 6\,\text{cm}$ and find the value of θ.

Part D

Area of segment

Find the area of the segment bounded by the arc AB and the chord AB. Give your answer to 3 sf.

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STEM SMART Single Maths 29 - Radians & Small Angle Approximations

Radians and Trig Functions 2i

Subject & topics: Maths **Stage & difficulty:** A Level P2

Figure 1 shows part of the curve $y = \cos 2x$, where x is in radians. The point A is the minimum point of this part of the curve.

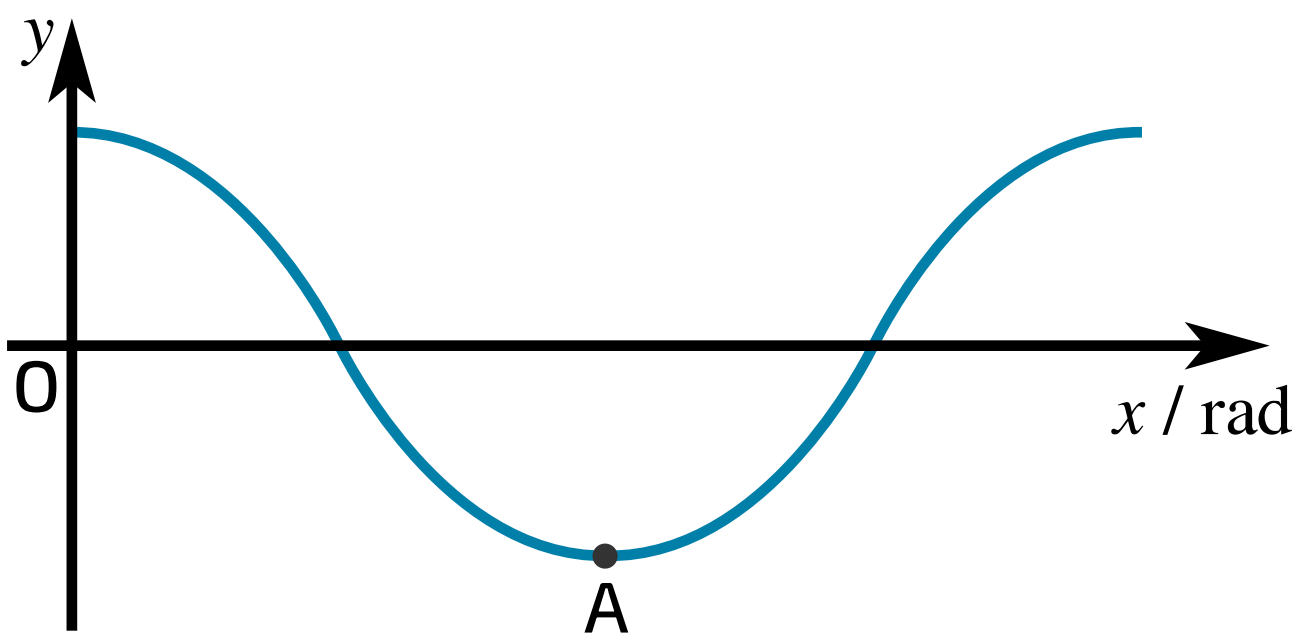

Figure 1: The graph of $y = \cos 2x$.

Part A

Period

State the period of $y = \cos 2x$.

The following symbols may be useful: `pi, t`

Part B

Coordinates of A

What are the coordinates of A?

In your answer give the x coordinate as a fraction of π.

(☐ π, ☐)

Part C

The inequality $\cos 2x \leqslant \frac{1}{2}$

Solve the inequality $\cos 2x \leqslant \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$, giving your answer as a range of angles x.

Construct your answer from the items below.

☐☐☐

Items:

$<$ $>$ x $< x <$ $\leq x \leq$ $< x$ **or** $x <$ $\leq x$ **or** $x \leq$ $\leq$ $\geq$ 0 $\frac{\pi}{6}$ $\frac{\pi}{3}$ $\frac{\pi}{2}$ $\frac{2\pi}{3}$ $\frac{5\pi}{6}$ π

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Radians and Trig Functions 2ii

Subject & topics: Maths **Stage & difficulty:** A Level P1

This question is about solving the equation $2\cos x = \tan 2x$ for $0 \leq x \leq \pi$.

Part A

The equation $2\cos x = \tan 2x$

Write down the exact values of $\cos\frac{\pi}{6}$ and $\tan\frac{\pi}{3}$ (where the angles are in radians).

- $\cos\frac{\pi}{6} =$ [ ]
- $\tan\frac{\pi}{3} =$ [ ]

To verify that $x = \frac{\pi}{6}$ is a solution of the equation $2\cos x = \tan 2x$, consider the two sides of the equation separately:

- When $x = \frac{\pi}{6}$, $2\cos x =$ [ ].
- When $x = \frac{\pi}{6}$, $\tan 2x =$ [ ].

The left hand side and right hand side are equal when $x = \frac{\pi}{6}$. Hence, $x = \frac{\pi}{6}$ is a solution of the equation $2\cos x = \tan 2x$.

Items:

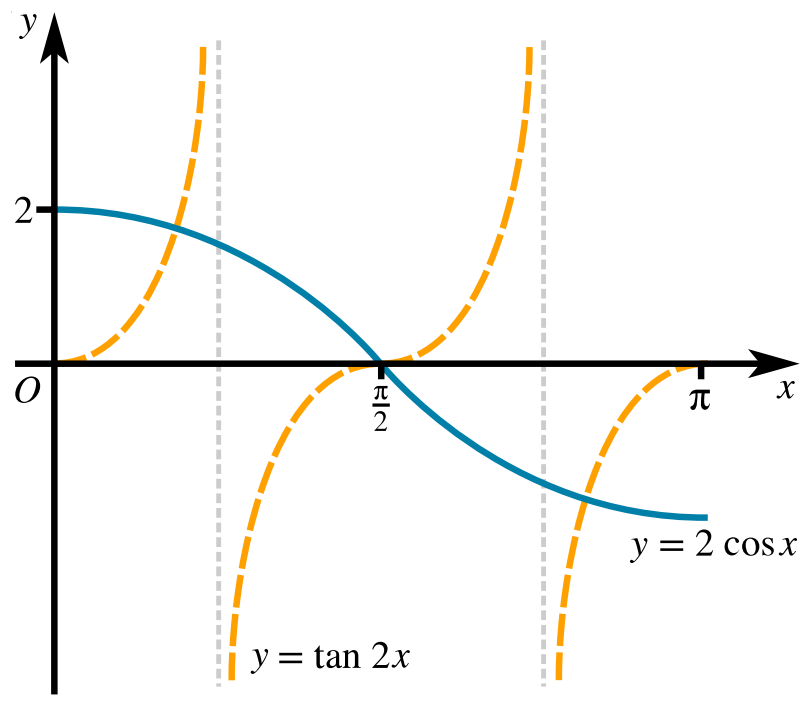
2 $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{3}}$ $\frac{\sqrt{3}}{2}$ $\sqrt{3}$ $\frac{1}{2}$ 3 1

Part B

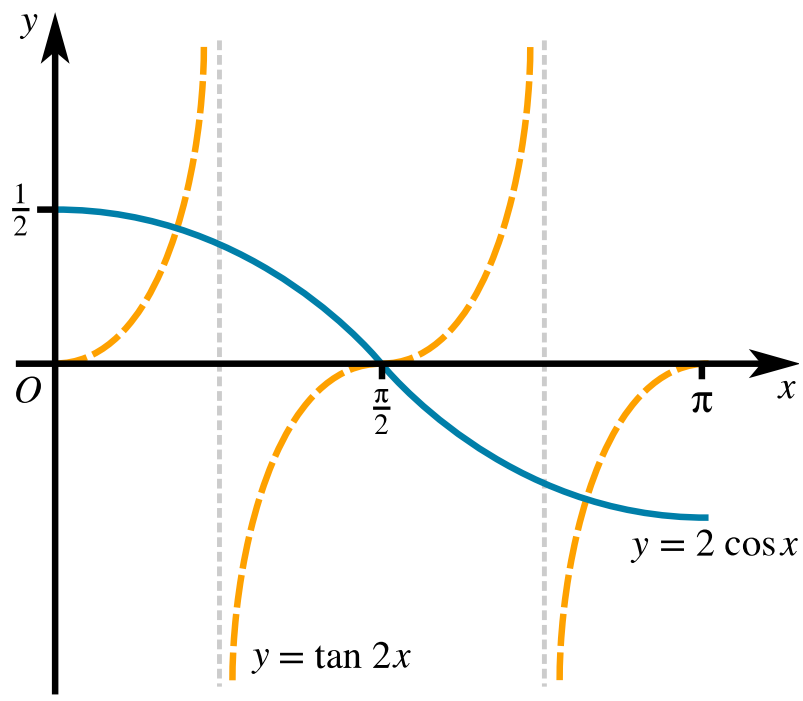
Sketch

Sketch, on a single diagram, the graphs of $y = 2\cos x$ and $y = \tan 2x$, for x (radians) such that $0 \leqslant x \leqslant \pi$.

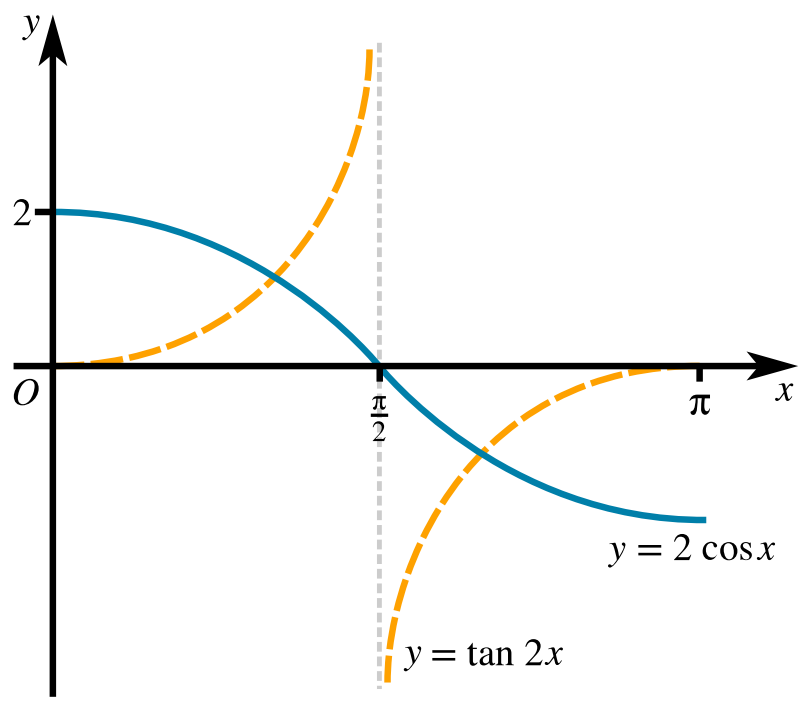
Choose the correct graph from the three options below.



Option A



Option B



Option C

Figure 1: Options A, B and C.

- A
- B
- C

Part C

Other solutions

Hence state, as a fraction of π, the two other values of x between 0 and π satisfying the equation $2\cos x = \tan 2x$.

smaller root $=$ π

larger root $=$ π

Adapted with permission from UCLES A-level Maths papers, 2003-2017.

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Radians and Trig Functions 1i

Subject & topics: Maths **Stage & difficulty:** A Level P2

A curve has equation $y = \sin(ax)$, where a is a positive constant and x is in radians.

Part A
Period

State the period of $y = \sin(ax)$, giving your answer in an exact form in terms of a.

The following symbols may be useful: `a, pi, t`

Part B
$\sin(ax) = k$

The two smallest positive solutions of $\sin(ax) = k$, where k is a positive constant, are $x = \frac{1}{5}\pi$ and $x = \frac{2}{5}\pi$.

Find the exact values of a and k.

$a =$ ☐

$k =$ ☐ $\sqrt{}$ ☐

Part C

$\sin(ax) = \sqrt{3}\cos(ax)$

Given instead that $\sin(ax) = \sqrt{3}\cos(ax)$, find the two smallest positive solutions for x, giving your answers in an exact form.

Enter your answers in order from lowest value of x to highest.

$x =$ ☐ $\frac{\pi}{a}$ (lowest value)

$x =$ ☐ $\frac{\pi}{a}$ (highest value)

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Small Angle Approximations 1i

Subject & topics: Maths **Stage & difficulty:** A Level P2

The small angle approximation is used when measuring distances in astronomy.

The two stars Alpha Centauri A and Alpha Centauri B are in a binary pair (they orbit one another). The distance between them is an average of 11 Astronomical Units, and they are an average of 4.4 light years from Earth.

$1\,\mathrm{AU} = 1\,\mathrm{Astronomical\ Unit} = 149\,597\,870\,700\,\mathrm{m}$
$1\,\mathrm{ly} = 1\,\mathrm{Light\ Year} = 9.4607 \times 10^{15}\,\mathrm{m}$

Assume that a telescope is pointing straight at Alpha Centauri A with the geometry shown in **Figure 1**.

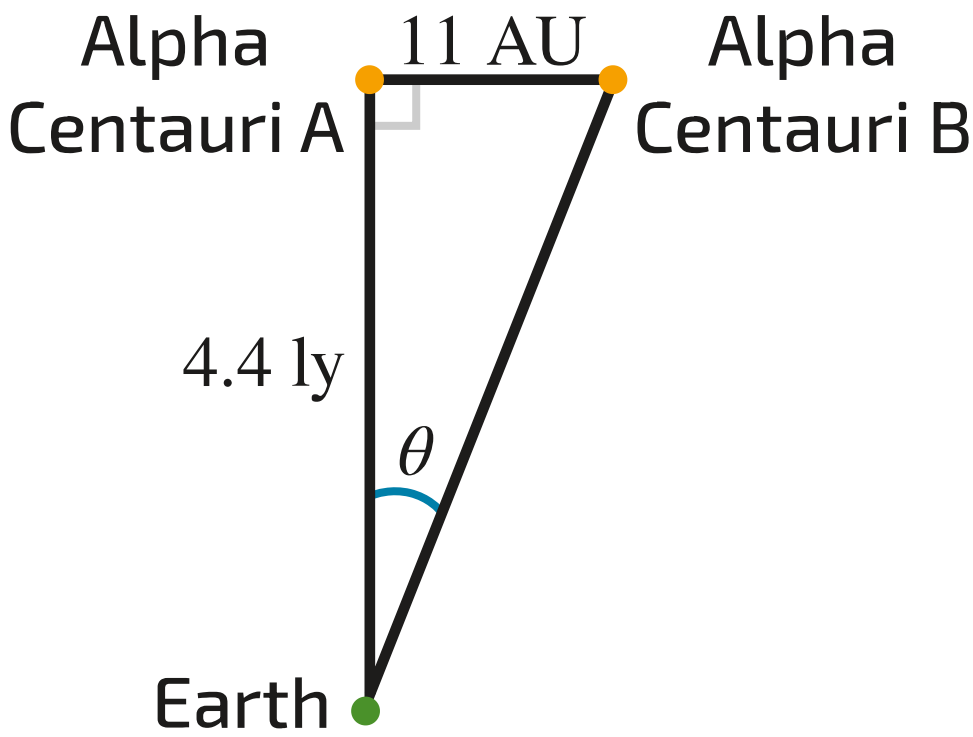

Figure 1: A telescope pointing straight at Alpha Centauri A

Use the small angle approximation to estimate θ, the angular separation between the stars as seen by the telescope. Give your answer to 2 significant figures.

Part A

Radians

Give the answer in radians.

Part B

Degrees

Give the answer in degrees.

Part C

Arc Seconds

Give the answer in Arc Seconds. (Where 1 arc second is one $\left(\frac{1}{3600}\right)^{\text{th}}$ of a degree.)