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STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Integration by Parts

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Integration   **Stage & difficulty:** A Level P3

Fill in the blanks to complete the notes on integration by substitution.

Integration by parts uses the product rule (in reverse) to write an integral that is hard to carry out in terms of an integral that is more straightforward. The formula for integration by parts is

$$\int u \boxed{} \, \mathrm{d}x = \boxed{} - \int \boxed{} \, \mathrm{d}x$$

where $u(x)$ and $v(x)$ are functions of x.

As an example, let us integrate $\int x \cos x \, \mathrm{d}x$. The integrand is a product of two terms. We choose $u = x$ and $\dfrac{\mathrm{d}v}{\mathrm{d}x} = \boxed{}$. Therefore, $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \boxed{}$ and $v = \boxed{}$ (by convention we leave out the constant of integration until the end of the calculation). Using the formula gives

$$\int x \cos x \, \mathrm{d}x = \boxed{} - \int \boxed{} \, \mathrm{d}x$$

The integral on the left hand side is difficult, but the integral on the right hand side can be done easily. In some calculations, you may need to carry out integration by parts more than once.

Items:

$\boxed{\dfrac{\mathrm{d}v}{\mathrm{d}x}}$ $\boxed{u\,v}$ $\boxed{\dfrac{\mathrm{d}u}{\mathrm{d}x}\,v}$ $\boxed{\cos x}$ $\boxed{1}$ $\boxed{\sin x}$ $\boxed{x \sin x}$ $\boxed{1 \times \sin x}$

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Integration - Trig Manipulations 1ii

Subject & topics: Maths   **Stage & difficulty:** A Level P2
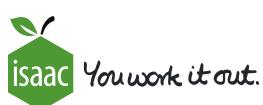
Use integration to find the exact value of $\displaystyle\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} \left(9 - 6\cos^2 4x\right)\,\mathrm{d}x$.

The following symbols may be useful: `pi`

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Integration - Trig Manipulations 3ii

Subject & topics: Maths   **Stage & difficulty:** A Level P2

Find $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1 - 2\sin^2 x}{1 + 2\sin x \cos x}\,\mathrm{d}x$, giving your answer in the form $a\ln b$.

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Integration by Substitution 2i

Subject & topics: Maths   **Stage & difficulty:** A Level P3
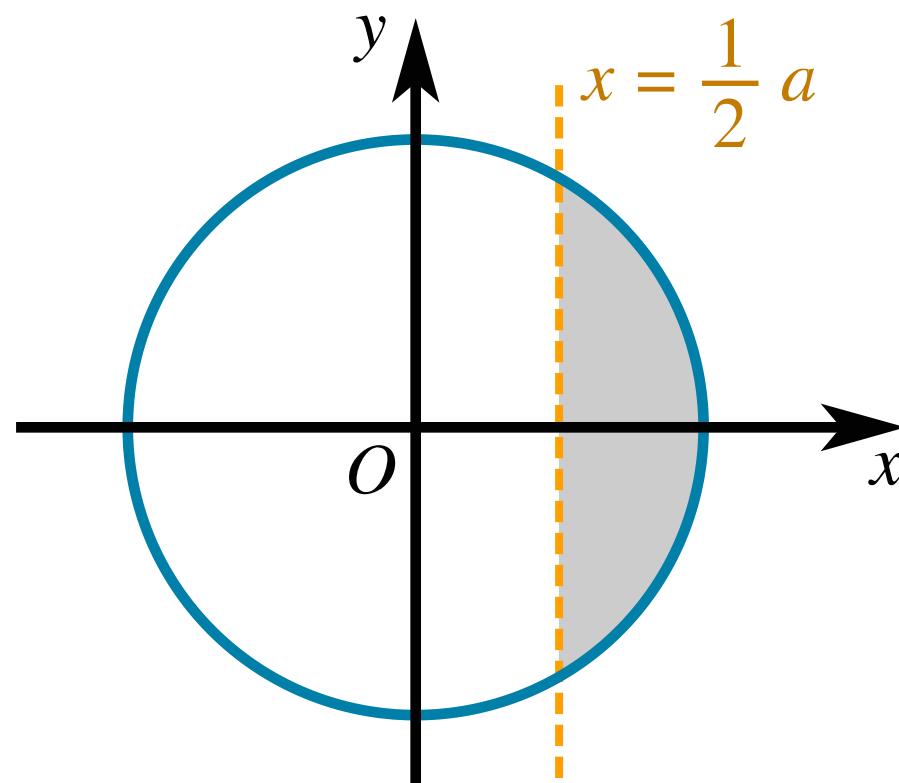

Part A

The substitution $x = a \sin \theta$

By using the substitution $x = a \sin \theta$, find the exact value of

$$\int_{\frac{1}{2}a}^{a} \sqrt{(a^2 - x^2)}\, dx$$

The following symbols may be useful: `a, pi`

Part B
Area of a segment

The diagram shows the circle $x^2 + y^2 = a^2$ and the line $x = \frac{1}{2}a$. Find the area of the shaded region, giving your answer in an exact form.

The following symbols may be useful: `a, pi`

Question deck:

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Equations

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Integration by Parts 4ii

Subject & topics: Maths **Stage & difficulty:** A Level P3

Find $\displaystyle\int_0^1 16x\mathrm{e}^{4x}\,\mathrm{d}x$, in an exact form.

The following symbols may be useful: e

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Integration by Parts 5

Pre-Uni Maths for Sciences K4.5

Subject & topics: Maths | Calculus | Integration   **Stage & difficulty:** A Level P3

Find, by integrating by parts twice, $\displaystyle\int_{0}^{\pi/3} \mathrm{e}^{-x} \sin x \, \mathrm{d}x$.

The following symbols may be useful: `e, pi`

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Integration by Parts 2ii

Subject & topics: Maths   **Stage & difficulty:** A Level P3

Find the exact value of $\int_1^8 \dfrac{1}{\sqrt[3]{x}} \ln x \, \mathrm{d}x$, giving your answer in the form $A \ln 2 + B$, where A and B are constants to be found.

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STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Integration of Differential Equations 1ii

Subject & topics: Maths   **Stage & difficulty:** A Level P3

The gradient of a curve at the point (x, y), where $x > -2$, is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3y^2(x+2)}$$

The points $(1, 2)$ and $(q, 1.5)$ lie on the curve. Find the value of q, giving your answer correct to 3 significant figures.

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STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Integration of Differential Equations 4i

Subject & topics: Maths   **Stage & difficulty:** A Level P3

Part A
Derivative

If $y = \operatorname{cosec} x$ then find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), ln(), log(), sec(), sin(), tan(), x, y`

Part B
Solve

Solve the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\sin x \tan x \cot t$$

given that $x = \frac{\pi}{6}$ when $t = \frac{\pi}{2}$.

The following symbols may be useful: `arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), ln(), log(), sec(), sin(), t, tan(), x`

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STEM SMART Single Maths 36 - Integration by Parts & Differential Equations

Constructing Differential Equations 1i

Subject & topics: Maths   **Stage & difficulty:** A Level P2

A cylindrical container has a height of $200\,\mathrm{cm}$. The container was initially full of a chemical but there is a leak from a hole in the base (**Figure 1**). When the leak is noticed, the container is half-full and the level of the chemical is dropping at a rate of $1\,\mathrm{cm\,min}^{-1}$.

It is required to find for how many minutes the container has been leaking. To model the situation it is assumed that, when the depth of the chemical remaining is $x\,\mathrm{cm}$, the rate at which the level is dropping is proportional to $\sqrt{x}$.


$x\,\mathrm{cm}$
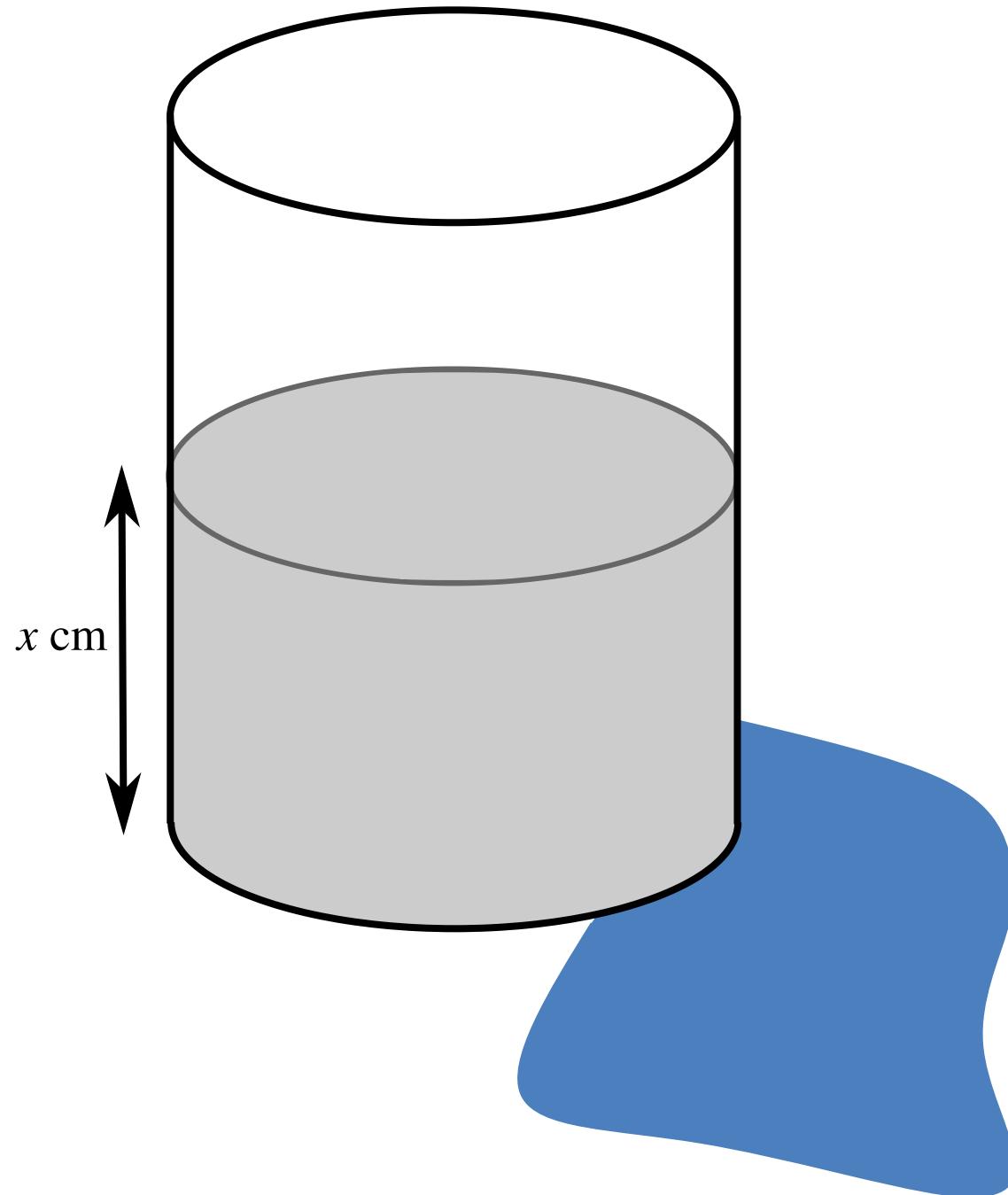
Figure 1: Cylindrical container that is leaking from its base.

Part A
Differential equation

State an appropriate differential equation for the rate of change of height of chemical in the tank.

The following symbols may be useful: `Derivative(x, t), k, t, x`

Part B
Solve

Solve this differential equation, giving x in terms of t, the time in minutes since the leak began.

The following symbols may be useful: `t, x`

Part C
Time

Calculate the length of time that the container has been leaking for. Give your answer to 3 significant figures.

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