Partial Fractions 1

A Level

Pre-Uni Maths for Sciences A5.1

The function $\frac{2x-1}{(3x-2)(x-1)}$ can be written as $\frac{A}{3x-2}+\frac{B}{x-1}$. Find A and B.

Part A Find A

Find the constant A.

Part B Find B

Find the constant B.

Created for isaacphysics.org Julia Riley

Partial Fractions 2

A Level

Pre-Uni Maths for Sciences A5.2

The function $\dfrac{w+2}{(w-1)(w+1)(2w+1)}$ can be written as $\dfrac{A}{(w-1)}+\dfrac{B}{(w+1)}+\dfrac{C}{(2w+1)}$. Using the substitution method find the constants A, B and C.

Part A Find A

Find the constant A.

The following symbols may be useful: A

Part B Find B

Find the constant B.

The following symbols may be useful: B

Part C Find C

Find the constant C.

The following symbols may be useful: C

Created for isaacphysics.org by Julia Riley

Gameboard:

STEM SMART Single Maths 26 - Partial fractions & Proof by Contradiction

Partial Fractions 3

A Level

P P P

The function $\frac{24t^2+31t+2}{(2t+1)^2(t+3)}$ can be written as $\frac{A}{(2t+1)^2}+\frac{B}{(2t+1)}+\frac{C}{(t+3)}$. Find the constants A, B and C.

Part A Find A

Find the constant A.

The following symbols may be useful: A

Part B Find B

Find the constant B.

The following symbols may be useful: B

Part C Find C

Find the constant C.

The following symbols may be useful: C

Created for isaacphysics.org by Jonathan Waugh

Gameboard:

STEM SMART Single Maths 26 - Partial fractions & Proof by Contradiction

Partial Fractions 4

A Level

P P P

The function $\frac{8a^2}{(x-a)(x+a)^2}$, where a is a constant, can be written as $\frac{A}{(x+a)^2} + \frac{B}{x+a} + \frac{C}{x-a}$. Find the constants A, B and C.

Part A Find A

Find the constant A.

The following symbols may be useful: `A, a`

Part B Find B

Find the constant B.

The following symbols may be useful: `B, a`

Part C Find C

Find the constant C.

The following symbols may be useful: `C, a`

Created for isaacphysics.org by Jonathan Waugh

Gameboard:

STEM SMART Single Maths 26 - Partial fractions & Proof by Contradiction

Improper Partial Fractions 1

A Level

Pre-Uni Maths for Sciences A5.3

Express $\frac{-6x^3+15x^2+x-11}{2x^2-5x-3}$ as partial fractions.

Created for isaacphysics.org by Matthew Rihan

Gameboard:

STEM SMART Single Maths 26 - Partial fractions & Proof by Contradiction

Proof and Odd Perfect Numbers

A Level

The **proper divisors** of a number are those factors which are not equal to the number itself. For the number 6,

- The divisors of 6 are 1, 2, 3 and 6.
- The proper divisors of 6 are 1, 2 and 3.

The number 6 is an example of a **perfect number**. A perfect number is a number for which the sum of its proper divisors is equal to the number itself. For the number 6,

$$1+2+3=6$$

In this question you will use proof by contradiction to show that an odd perfect number cannot be a square number.

Assumption:

We will assume that there is an odd perfect number, n, that is also a square number. Then $n=m^2$, where m is an integer.

Part A Reasoning: odd and even factors

- An even number multiplied by an even number is always an ______ number.
- An even number multiplied by an odd number is always an ______ number.
- An odd number multiplied by an odd number is always an ______ number.

Therefore, as n is an ______ number, the factors of n can only be ______ numbers.

Items:

odd **even**

Part B Reasoning: sum of proper divisors

As $n = m^2$, m is a factor of n.

Consider another factor of n. Call this factor p. As p is a factor of n, $q =$ ______ is also a factor of n. As $n = m^2$, $q = \frac{m^2}{p}$. Hence,

- If $p < m$, q ______ m.
- If $p > m$, q ______ m.

Therefore, with the exception of m, the factors of n occur in pairs. One factor in the pair is smaller than m, and the other factor is larger than m. Including m, the total number of factors of n is therefore an ______ number.

For any value of n, one of the factor pairs is 1 and n. The number of proper divisors (factors other than n itself) is therefore an ______ number. As we have shown in part A that all of the factors of n are ______ numbers, the sum of the proper divisors of n is therefore an ______ number.

Items:

$\frac{n}{p}$ $\frac{p}{n}$ **odd** $>$ **even** $<$

Part C Conclusion

Our starting assumption was that n is an odd perfect number and also a square number.

The definition of a perfect number means that the sum of the proper divisors of n is equal to ______. The sum of the proper divisors must therefore be an ______ number.

However, in part B we have shown that if n is an odd number which is also a square number, the sum of the proper divisors has to be an ______ number.

Therefore, we have arrived at a contradiction. We conclude that there are no odd perfect numbers that are also square numbers.

Items:

$2n$ n **even** **odd** n^2

Created for isaacphysics.org by Jonathan Waugh

Gameboard:

STEM SMART Single Maths 26 - Partial fractions & Proof by Contradiction

Proof Applied to Surface Areas

A Level

P P P

Consider a sphere with a radius r cm, where r is a rational number. Using proof by contradiction, show that the side length of a cube with the same surface area cannot also be a rational number of cm.

Once you have worked out a proof yourself, consider the model proof below. Drag and drop answers into the blank spaces to complete the proof. You may use the same answer more than once.

Assumption:
Consider a sphere of radius r cm, where r is a rational number. Let the side length of a cube with the same surface area as the sphere be a cm. Assume that a is a rational number, in which case $a = \frac{b}{c}$, where b and c are integers with no common factor.

Reasoning:
The surface area of the sphere is _____. Because r is a rational number, $r = \frac{p}{q}$, where p and q are integers with no common factor. Hence, the surface area of the sphere may be written as _____.

The surface area of the cube is _____. Using $a = \frac{b}{c}$, the surface area may be written as _____.

The surface area of the sphere and the cube are equal. Hence, $4\pi\left(\frac{p}{q}\right)^2 = 6\left(\frac{b}{c}\right)^2$. Re-arranging this equation to give an expression for _____ produces _____.

As b, c, p and q are all integers, _____ must be _____ number. However, π is not _____ number.

Conclusion:
The assumption that a is rational has resulted in a contradiction. Hence, the assumption cannot be true. Therefore, the side length of a cube with the same surface area as a sphere of radius r cm, where r is a rational number, cannot be a rational number of cm.

Items:

$4\pi r^2$ | $\frac{3b^2q^2}{2c^2p^2}$ | an irrational | a^3 | a rational | $\pi = \frac{3b^2p^2}{2c^2q^2}$ | $4\pi\left(\frac{p}{q}\right)^2$ | π | $\pi = \frac{3b^2q^2}{2c^2p^2}$ | a real | $6\left(\frac{b}{c}\right)^2$ | $6a^2$

Created for isaacphysics.org by J.Waugh

Gameboard:

STEM SMART Single Maths 26 - Partial fractions & Proof by Contradiction

Partial Fractions Applied to Other Functions

A Level

Pre-Uni Maths for Sciences A5.5, A5.6 & A5.7

Express the following functions in partial fraction form.

Part A A trigonometric function

Express the function $\dfrac{\cos y}{(\cos y+1)(2\cos y+1)}$ in the form $\dfrac{A}{\cos y+1}+\dfrac{B}{2\cos y+1}$, where A and B are constants.

The following symbols may be useful: `cos()`, `sin()`, `tan()`, `y`

Part B An exponential function

Express the function $\dfrac{\mathrm{e}^{2x}+5}{(\mathrm{e}^x-1)(\mathrm{e}^x-2)(\mathrm{e}^x-3)}$ in the form $\dfrac{A}{\mathrm{e}^x-1}+\dfrac{B}{\mathrm{e}^x-2}+\dfrac{C}{\mathrm{e}^x-3}$, where A, B and C are constants.

The following symbols may be useful: `e`, `x`

Part C A logarithmic function

Express the function $\dfrac{5\ln z + 20}{(\ln z)^2 + \ln(z^2) + 1}$ in the form $\dfrac{A}{(\ln z + 1)^2} + \dfrac{B}{\ln z + 1}$, where A and B are constants.

The following symbols may be useful: `ln()`, `log()`, `z`

Created for isaacphysics.org by Jonathan Waugh