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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

Functions and Algebra 3i

Subject & topics

Maths

Status

Not started

Stage & difficulty

A Level Practice 2

The functions f and g are defined for all real values of x by

$$f(x) = \left|2x + a\right| + 3a \quad \text{and} \quad g(x) = 5x - 4a,$$

where a is a positive constant.

Part A

Range

Find the range of $f(x)$.

Fill in the inequality below.

$\square$ $\square$ $\square$

Items:

$f(x)$ $<$ $\leq$ $>$ $\geq$ $< f(x) <$ $\leq f(x) \leq$ $< f(x)$ **or** $f(x) <$ $\leq f(x)$ **or** $f(x) \leq$ $\frac{a}{3}$ $\frac{a}{2}$ a $2a$

$3a$ $4a$ 0 $-\frac{a}{3}$ $-\frac{a}{2}$ $-a$ $-2a$

Part B

Inverse function of $f(x)$

Fill in the blanks to explain why the function $f(x)$ has no inverse.

The function $f(x)$ is not [ ]. For example, $f(0) = 4a$ and $f($[ ]$)$ also equals $4a$. Hence, $f(x)$ has no inverse.

Items:

$-a$ | $2a$ | many-to-many | $-2a$ | one-to-many | a | one-to-one | many-to-one

Part C

Inverse function of $g(x)$

Find an expression for $g^{-1}(x)$.

The following symbols may be useful: a, x

Part D

Solve for x

Solve for x the equation $g\big(f(x)\big) = 31a$. Give your solutions in ascending order.

$x =$ [ ] a

$x =$ [ ] a

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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

Inverse Quadratic Function

Subject & topics

Maths | Functions | Graph Sketching

Status

Not started

Stage & difficulty

A Level Practice 2

Figure 1 shows the graph of $y = f(x)$, where

$$f(x) = 2 - x^2, \quad x \leq 0$$

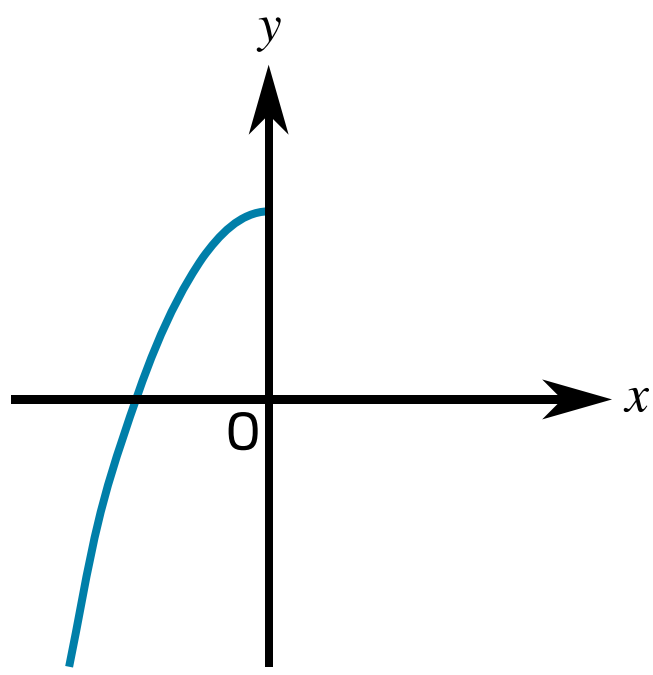


Figure 1: The graph of $y = f(x)$, for $x \leq 0$.

Part A

$f^2(-3)$

Evaluate $f^2(-3)$.

Need some help?

Part B

$f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

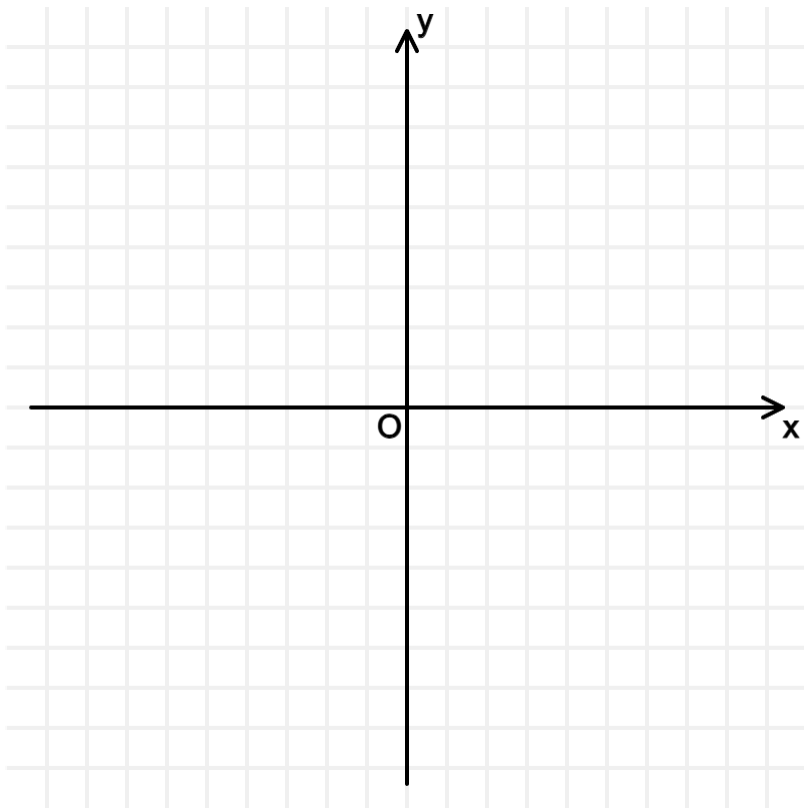
The following symbols may be useful: `f, x, y`

Need some help?

Part C

Graph of $f^{-1}(x)$

Sketch the graph of $y = f^{-1}(x)$.



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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

Function Types and Inverses

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

A Level Challenge 1

Figure 1 shows five different graphs, A, B, C, D and E, each for values of x such that $-a \leq x \leq a$ where a is a constant.

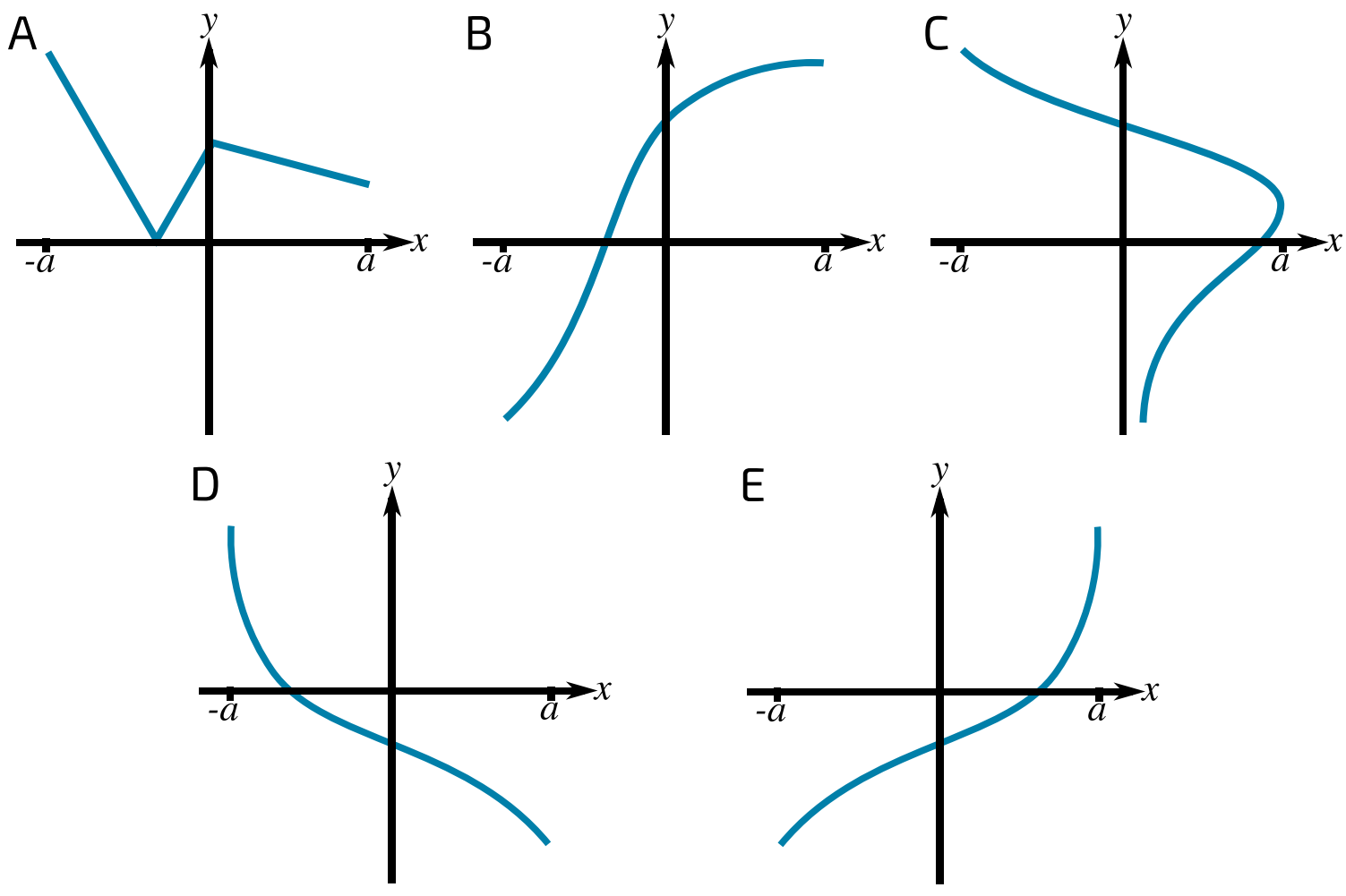


Figure 1: The set of five graphs, labelled A, B, C, D and E

Part A

Function

Which diagram does not show the graph of a function?

- A
- B
- C
- D
- E

Part B

One-to-one Function

Which diagram shows the graph of a function that is not one-to-one?

- A
- B
- C
- D
- E

Part C

Inverses

It is given that two of the diagrams illustrate functions that are inverses of each other. Identify one of these two diagrams.

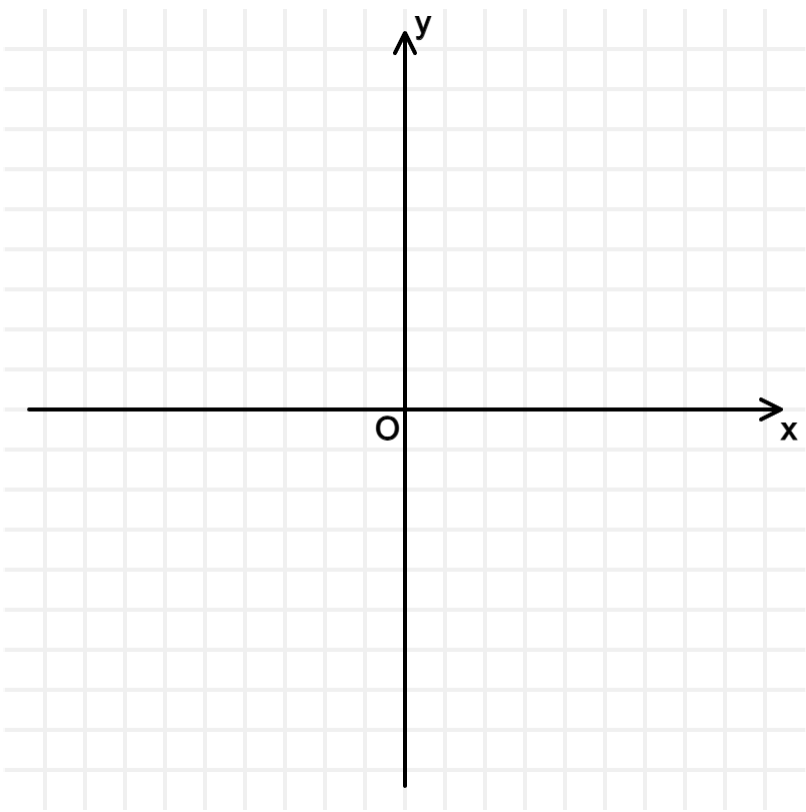
- A
- B
- C
- D
- E

Part D

Sketch

The graph in E has equation $y = f(x)$. Sketch the graph of $y = \left|f(x)\right|$.

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.



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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

Combined Transformations

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

A Level Practice 2

The function f is defined by $f(x) = \sqrt{mx + 7} - 4$, where $x \geqslant -\frac{7}{m}$ and m is a positive constant. **Figure 1** shows the curve $y = f(x)$.

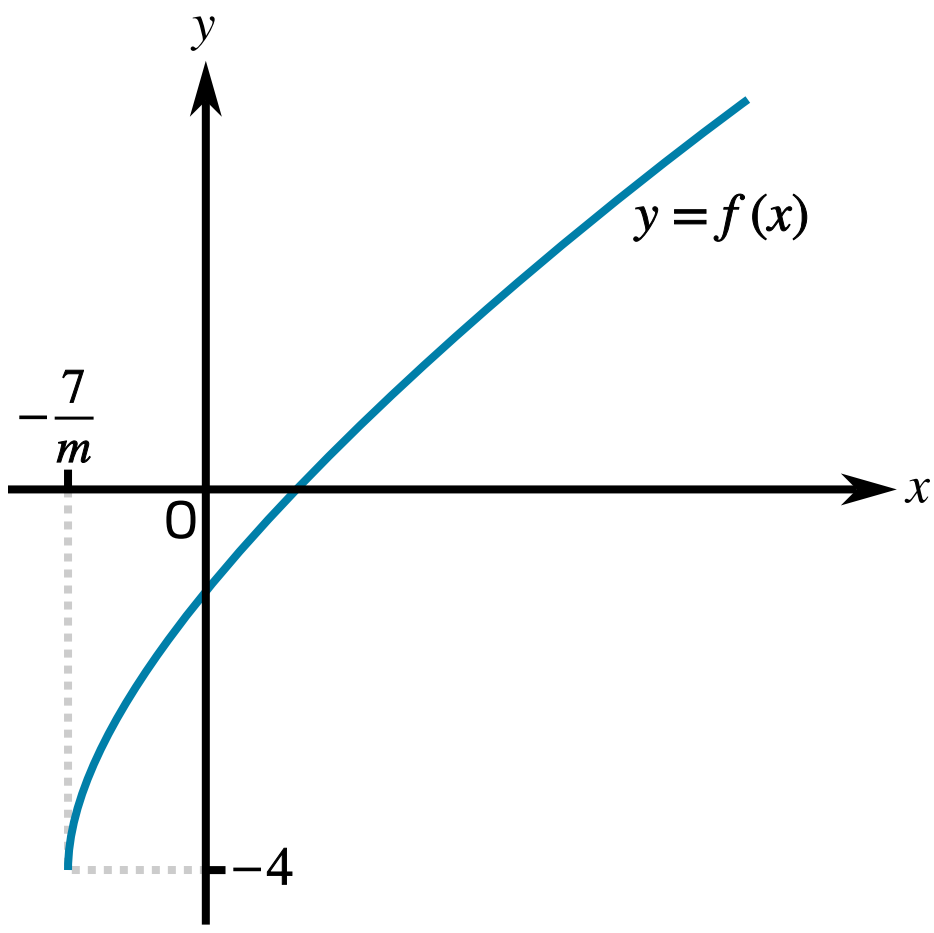

Figure 1: The curve $y = f(x)$

Part A

Translation of the curve $y = \sqrt{x}$

A sequence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations.

Available items

- Stretch the curve in the y direction by a factor of $\frac{1}{m}$.
- Translate the curve 7 units in the negative y direction.
- Stretch the curve in the x direction by a factor of m.
- Stretch the curve in the x direction by a factor of $\frac{1}{m}$.
- Translate the curve 4 units in the negative y direction.
- Translate the curve 4 units in the negative x direction.
- Translate the curve 7 units in the positive x direction.
- Translate the curve 7 units in the negative x direction.
- Translate the curve 4 units in the positive y direction.

Part B

$f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, m, x`

Part C

Values of m

It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Thus it can be deduced that neither curve meets the line $y = x$. Hence determine the possible values of m.

Construct your answer from the items below.

[ ] [ ] [ ]

Items:

m $<$ $\leq$ $>$ $\geq$ $< m <$ $\leq m \leq$ $> m$ **or** $m >$ $\geq m$ **or** $m \geq$ -28 -14 -8 -7

-4 -2 -1 0 1 2 4 7 8 14 28

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Curve Sketching and Combined Transformations 3i

Subject & topics

Maths

Status

Not started

Stage & difficulty

A Level Practice 2

The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x)$$

where k is a positive constant. **Figure 1** shows the curve with equation $y = f(x)$.

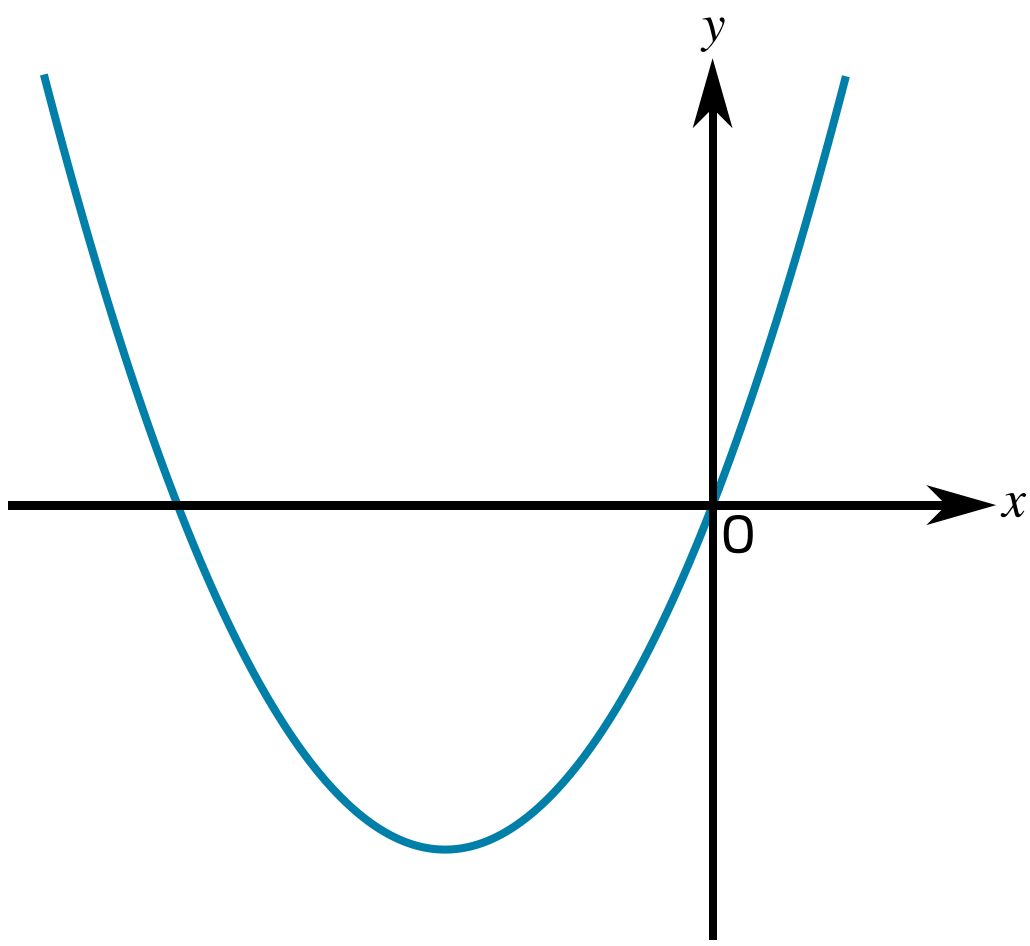

Figure 1: The graph of $y = f(x)$

Part A

Transformations

Give a sequence of transformations that will transform the curve $y = x^2$ to the curve $y = f(x)$.

Available items

- Translate the curve 2 units in the negative y direction.
- Translate the curve 2 units in the positive x direction.
- Translate the curve 2 units in the positive y direction.
- Translate the curve 2 units in the negative x direction.
- Stretch the curve in the y direction by a factor of $\frac{1}{k}$.
- Stretch the curve in the x direction by a factor of k.
- Translate the curve 4 units in the positive x direction.
- Stretch the curve in the x direction by a factor of $\frac{1}{k}$.
- Translate the curve 4 units in the negative y direction.
- Translate the curve 4 units in the positive y direction.
- Translate the curve 4 units in the negative x direction.
- Stretch the curve in the y direction by a factor of k.

Part B

Range

Find the range of $f(x)$ as a single inequality in terms of k.

The following symbols may be useful: `<, <=, >, >=, f(x), k, x, y`

Part C

Find k

It is given that there are three distinct values of x which satisfy the equation $\left|f(x)\right| = 20$.

Find the value of k.

Part D

Solve $\left|f(x)\right| = 20$

Using the value of k from part C, find the three distinct values of x which satisfy the equation $\left|f(x)\right| = 20$. Give any irrational values to 3 sf.

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Question deck:

STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

Modulus Functions 2

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

A Level Practice 2
Further A Practice 1

Part A

Divergence of $y = |\frac{1}{x}|$

Does the function $y = \left|\frac{1}{x}\right|$ diverge anywhere? Where?

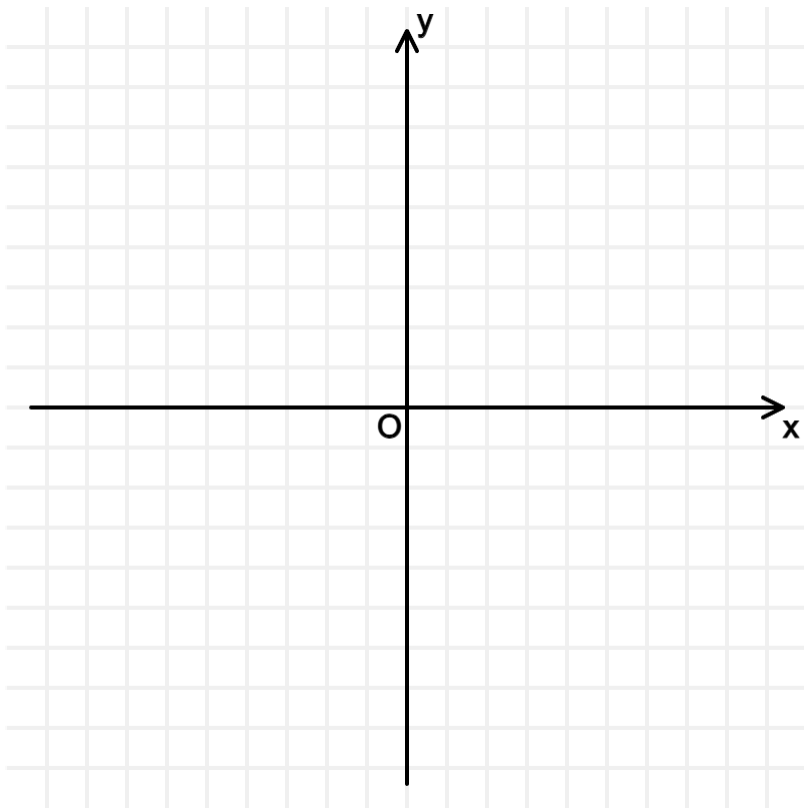
The following symbols may be useful: x

Need some help?

Part B

Graph of $y = \left|\frac{1}{x}\right|$

Sketch the graph of $y = \left|\frac{1}{x}\right|$.



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Part C

Divergence of $y = \left|\frac{1}{x^2-4}\right|$

Does the function $y = \left|\frac{1}{x^2 - 4}\right|$ diverge anywhere? Where?

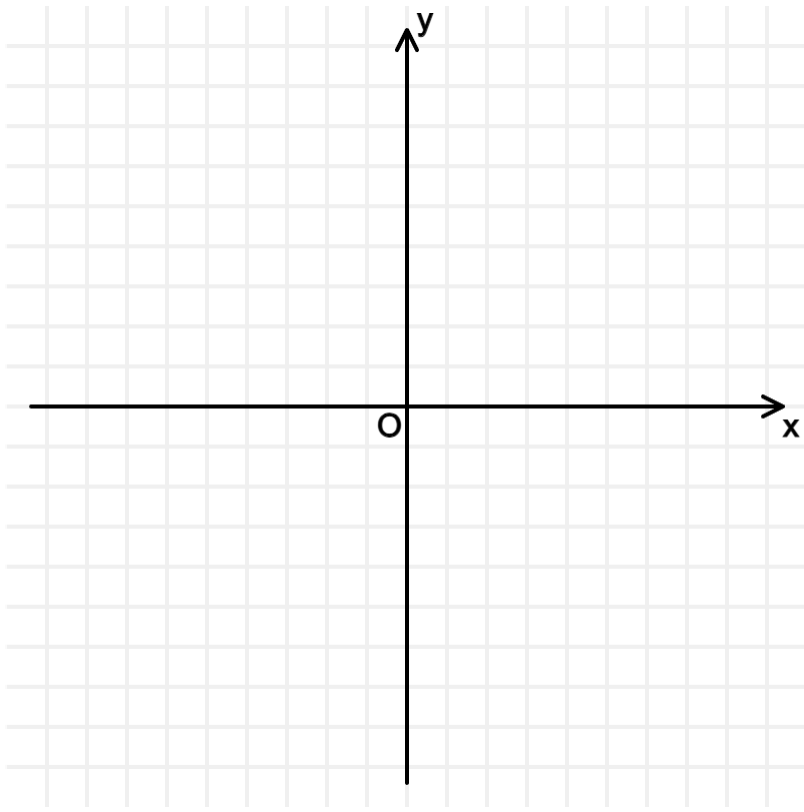
The following symbols may be useful: `x, ±`

Need some help?

Part D

Graph of $y = \left|\frac{1}{x^2-4}\right|$

Sketch the graph of $y = \left|\dfrac{1}{x^2-4}\right|$.



Need some help?

Part E

Solve equation graphically

Solve the equation $|x| = \left|\dfrac{1}{x}\right|$ graphically and give the solution as a single expression.

The following symbols may be useful: `x, ±`

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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

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Sketching a Cubic Modulus Function

Pre-Uni Maths for Sciences E4.7

Subject & topics

Maths | Functions | Graph Sketching

Status

Not started

Stage & difficulty

A Level Challenge 2

The function f is defined by $f(x) = 2x^3 - x^2 - 4x - 4$.

Part A
Quotient

Find the quotient when $f(x)$ is divided by $x - 2$.

The following symbols may be useful: x

Need some help?

Part B

First stationary point of $f(x)$

Find the co-ordinates and nature of the stationary point of $f(x)$ with the larger x-coordinate.

The stationary point (,) is a .

Items:

0 -16 -3 -2 2 1 **minimum point** -7 -4 3 -55 29 -132 **maximum point**

point of inflection 92 4 -1

Need some help?

Part C

Second stationary point of $f(x)$

Find the co-ordinates and nature of the stationary point of $f(x)$ with the smaller x-coordinate.

The stationary point (,) is a .

Items:

$-\frac{1}{2}$ -1 $-\frac{5}{2}$ $-\frac{2}{3}$ -7 -3 $-\frac{145}{27}$ $\frac{3}{2}$ -7 -6 $-\frac{77}{27}$ $\frac{1}{3}$ $-\frac{3}{2}$ $-\frac{64}{27}$ $-\frac{176}{27}$ **point of inflection**

$\frac{1}{2}$ $\frac{2}{3}$ **maximum point** $-\frac{1}{3}$ **minimum point** $-\frac{11}{2}$ 1

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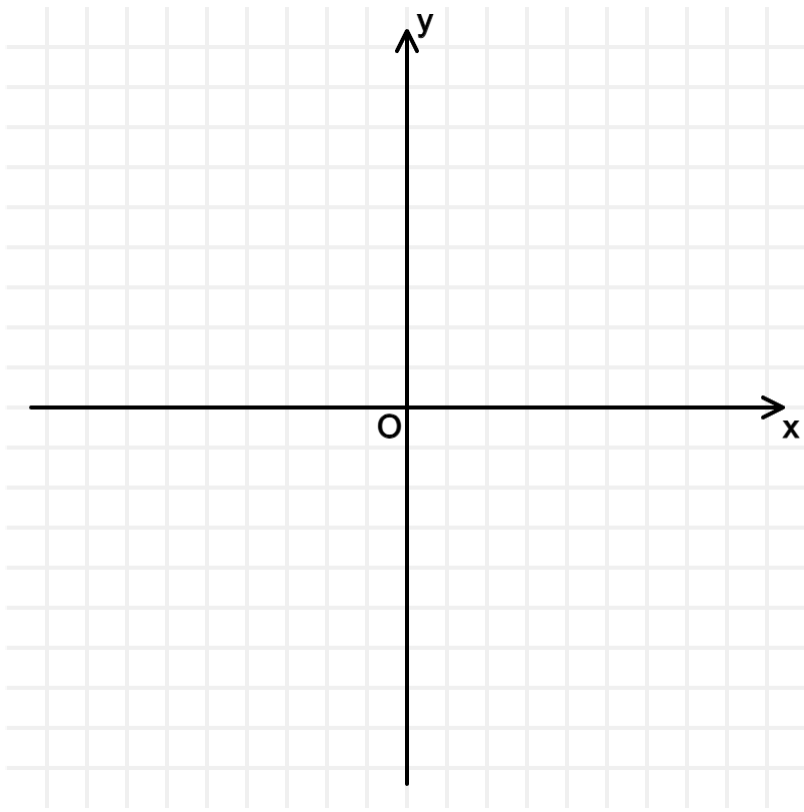
Part D

Sketch of $y = f(x)$

Sketch the graph of $y = f(x)$.



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Part E

Sketch of $y = |f(x)|$

Sketch the graph of $y = |f(x)|$, then pick the graph that corresponds to $y = |f(x)|$ from the options below.

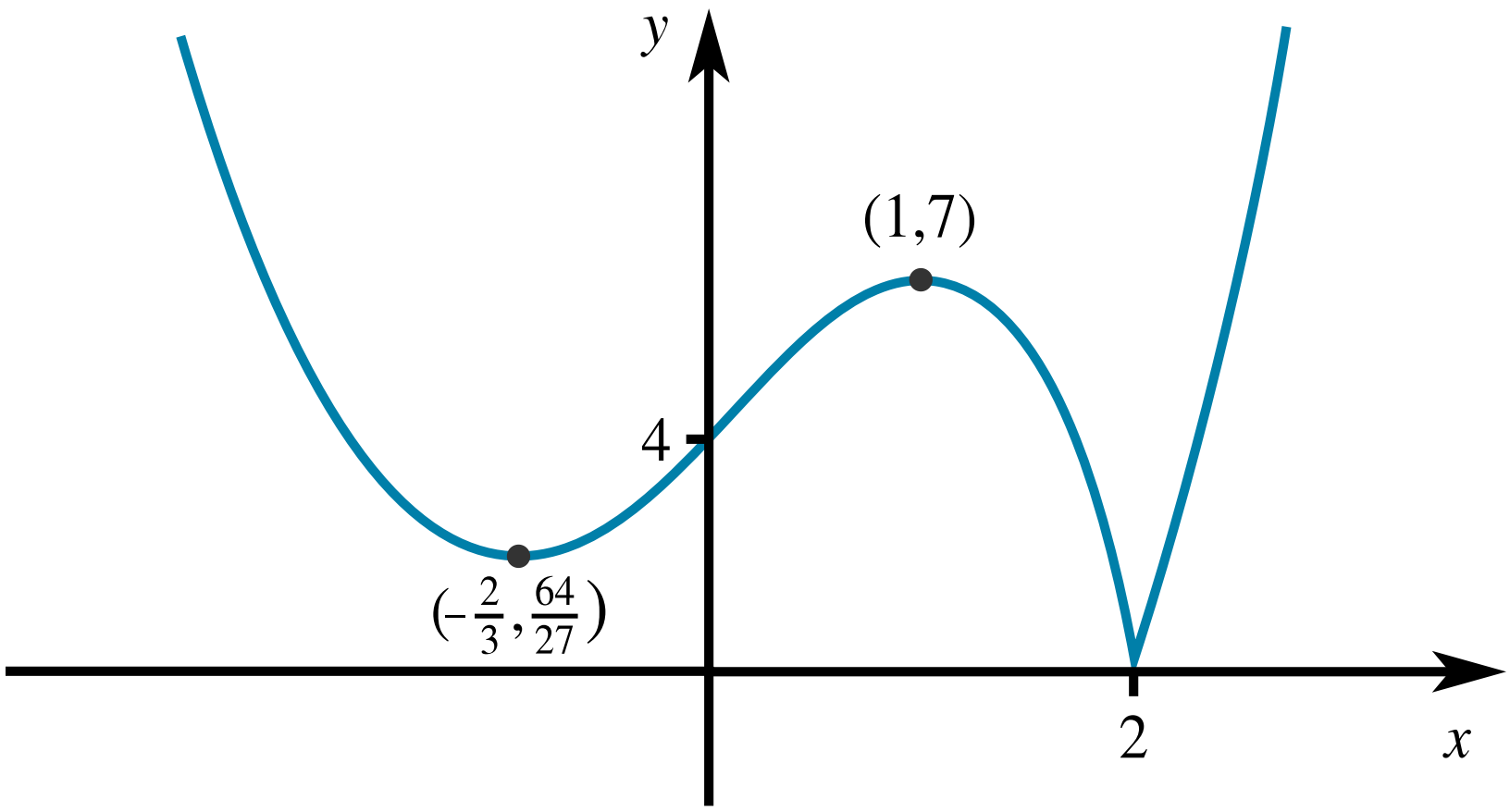


Figure 1: Option (i)

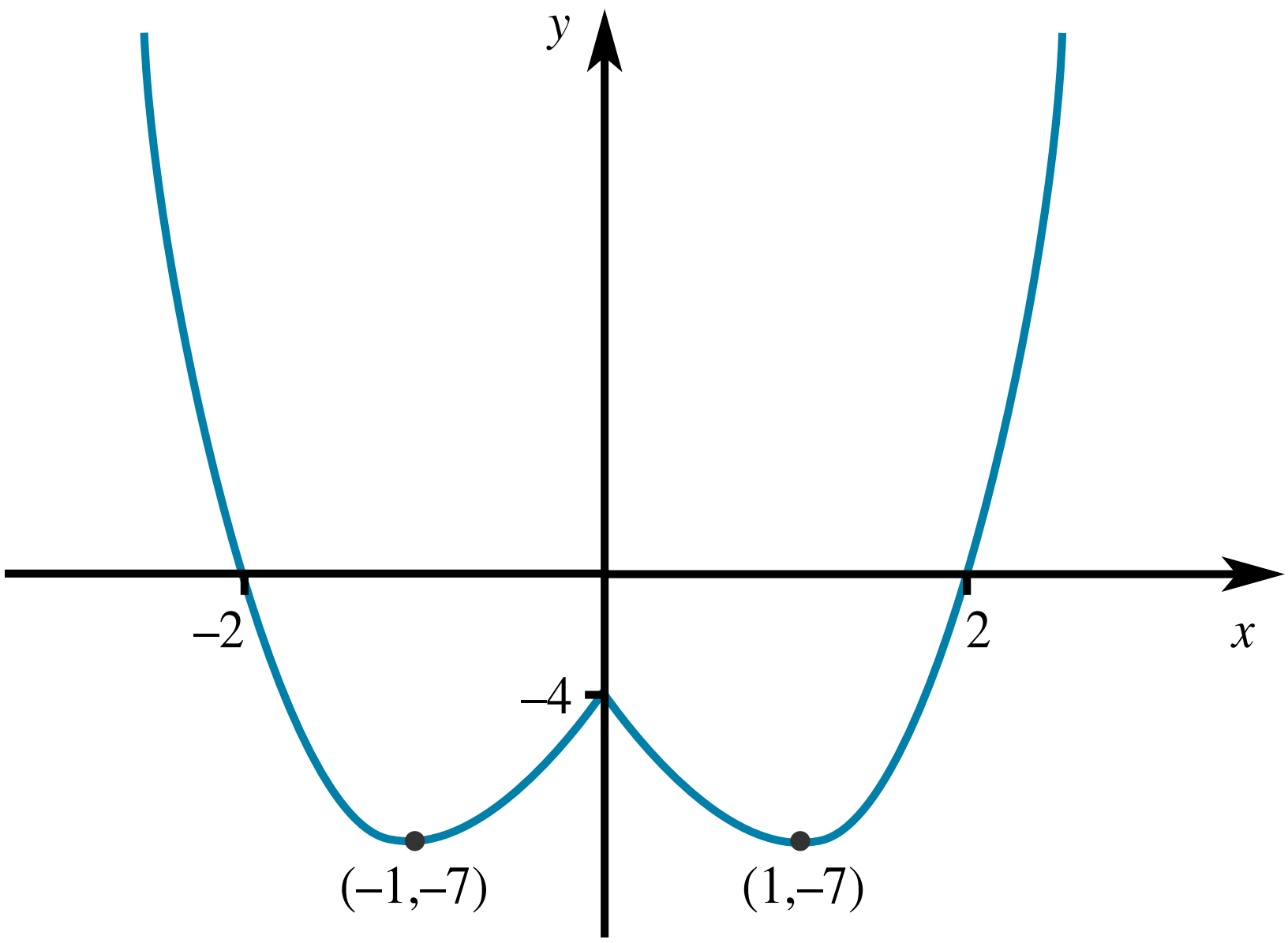


Figure 2: Option (ii)

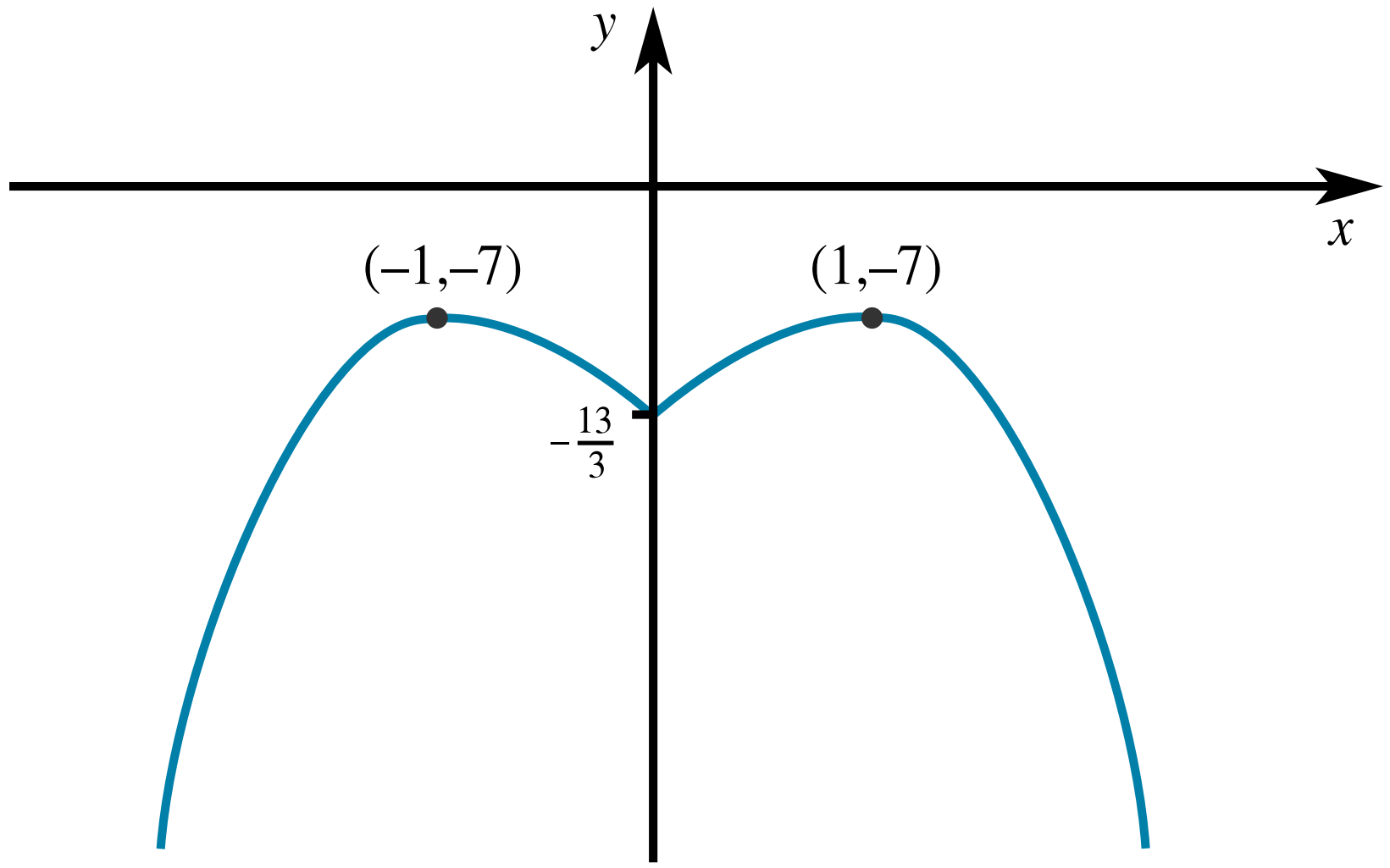


Figure 3: Option (iii)

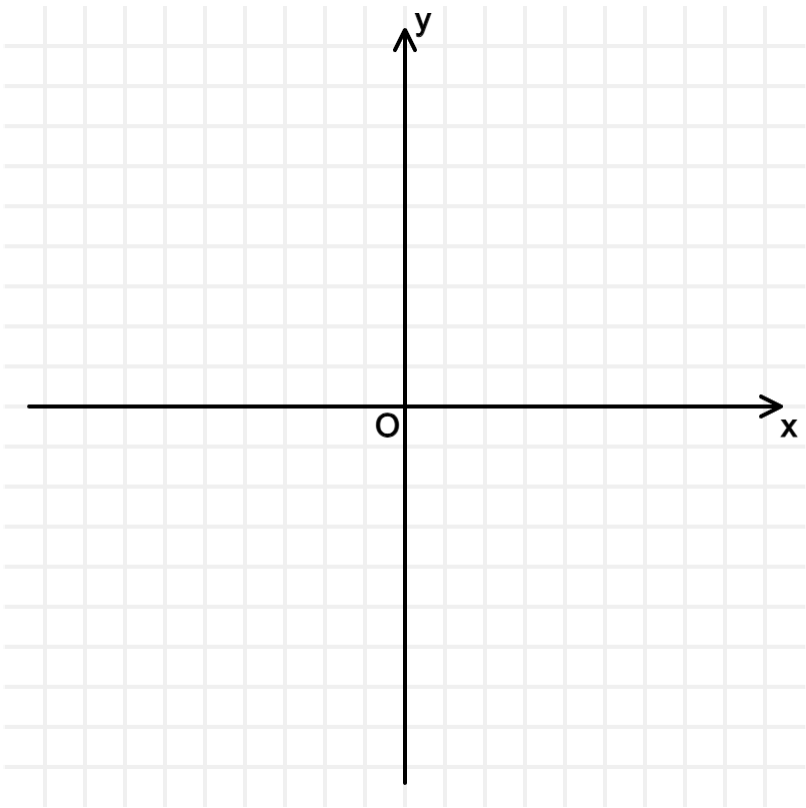
- Option (i)
- Option (ii)
- Option (iii)

Part F

Sketch of $y = f(|x|)$

Sketch the graph of $y = f(|x|)$.

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.



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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

Rational Inequality

Pre-Uni Maths for Sciences A2.10

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

Further A Practice 2

Solve the inequality

$$\frac{x+4}{x+2} \leq \frac{x+2}{x-1}$$

giving your answer using set notation.

The solution is $\{x : \square\square x \square\square\} \cup \{x : x \square\square\}$.

Items:

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9

10 $<$ $>$ $\leq$ $\geq$ $=$

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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

Rational Modulus Inequality

Pre-Uni Maths for Sciences E4.10

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

Further A Practice 2

Solve the inequality

$$\frac{-x^2-5x+24}{|x|+3} > 2$$

giving your answer using set notation.

The solution is $\{x :$ ☐ ☐ x ☐ ☐ $\}$.

Items:

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9

10 $<$ $>$ $\leq$ $\geq$ $=$

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