TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

Students should note that the real paper is 20 questions to be done in 75 minutes

PAPER 2   **D513/02**

2022   <mark>30 minutes</mark>

Additional materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

This paper is the second of two papers.

This paper <mark>contains 8 questions</mark>. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt <mark>all 8</mark> questions. Each question is worth one mark.

You can use the question paper for rough working or notes, but **no extra paper** is allowed.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

PV4

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5   A straight line L passes through $(1,2)$.

Let P be the statement

> **if** the y-intercept of L is negative, **then** the x-intercept of L is positive.

Which of the following statements **must** be true?

    I   P

    II   the converse of P

    III the contrapositive of P

A   none of them

B   I only

C   II only

D   III only

E   I and II only

F   I and III only

G   II and III only

H   I, II and III

6 A list consists of n integers.

Consider the following statements:

P: n is odd.

Q: The median of the list is one of the numbers in the list.

Which one of the following is true?

A P is **necessary and sufficient** for Q.

B P is **necessary** but **not sufficient** for Q.

C P is **sufficient** but **not necessary** for Q.

D P is **not necessary** and **not sufficient** for Q.

8 A selection, S, of n terms is taken from the arithmetic sequence 1, 4, 7, 10, … , 70.

Consider the following statement:

$(*)$ There are two distinct terms in S whose sum is 74.

What is the smallest value of n for which $(*)$ is **necessarily** true?

A 12

B 13

C 14

D 21

E 22

F 23

10   Which of the following statements is/are true?

   I   **For all** real numbers x and **for all** positive integers n, $x < n$

   II   **For all** real numbers x, **there exists** a positive integer n such that $x < n$

   III   **There exists** a real number x such that **for all** positive integers n, $x < n$

A   none of them

B   I only

C   II only

D   III only

E   I and II only

F   I and III only

G   II and III only

H   I, II and III

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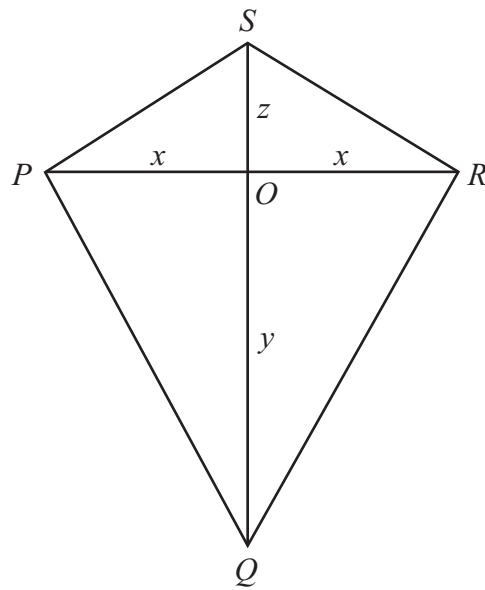
The diagram shows a kite *PQRS* whose diagonals meet at *O*.

$OP = x$
$OQ = y$
$OR = x$
$OS = z$

Which of the following is **necessary and sufficient** for angle *SPQ* to be a right angle?

A $x = y = z$

B $2x = y + z$

C $x^2 = yz$

D $y = z$

E $y^2 = x^2 + z^2$

14   Consider the two inequalities:

$$|x+5| < |x+11|$$

$$|x+11| < |x+1|$$

Which one of the following is correct?

A   There is no real number for which both inequalities are true.

B   There is exactly one real number for which both inequalities are true.

C   The real numbers for which both inequalities are true form an interval of length 1.

D   The real numbers for which both inequalities are true form an interval of length 2.

E   The real numbers for which both inequalities are true form an interval of length 3.

F   The real numbers for which both inequalities are true form an interval of length 4.

G   The real numbers for which both inequalities are true form an interval of length 5.

17 A student answered the following question:

> a and b are non-zero real numbers.
>
> Prove that the equation $x^3 + ax^2 + b = 0$ has three distinct real roots if
> $$27b\left(b + \frac{4a^3}{27}\right) < 0$$

Here is the student's solution:

I   We differentiate $y = x^3 + ax^2 + b$ to get $\dfrac{dy}{dx} = 3x^2 + 2ax = x(3x + 2a)$

Solving $\dfrac{dy}{dx} = 0$ shows that the stationary points are at $(0, b)$ and $\left(-\dfrac{2a}{3}, b + \dfrac{4a^3}{27}\right)$

II If $27b\left(b + \dfrac{4a^3}{27}\right) < 0$, then b and $b + \dfrac{4a^3}{27}$ must have opposite signs, and so one of the stationary points is above the x-axis and one is below.

III If the cubic has three distinct real roots, then one of the stationary points is above the x-axis and one is below.

IV Hence if $27b\left(b + \dfrac{4a^3}{27}\right) < 0$, then the equation has three distinct real roots.

Which one of the following options best describes the student's solution?

A It is a completely correct solution.

B The student has instead proved the converse of the statement in the question.

C The solution is wrong, because the student should have stated step II after step III.

D The solution is wrong, because the student should have shown the converse of the result in step II.

E The solution is wrong, because the student should have shown the converse of the result in step III.

18 P, Q, R and S show the graphs of

$$y = (\cos x)^{\cos x}, \quad y = (\sin x)^{\sin x}, \quad y = (\cos x)^{\sin x} \text{ and } y = (\sin x)^{\cos x}$$

for $0 < x < \dfrac{\pi}{2}$ in some order.
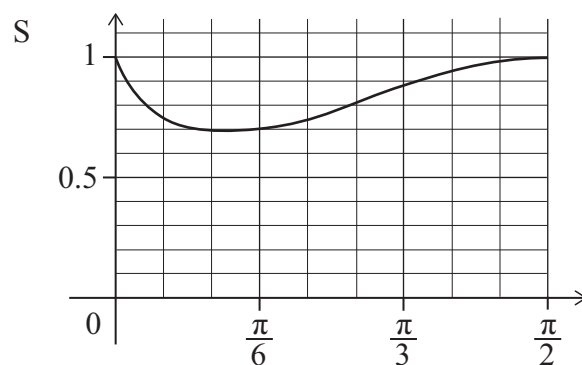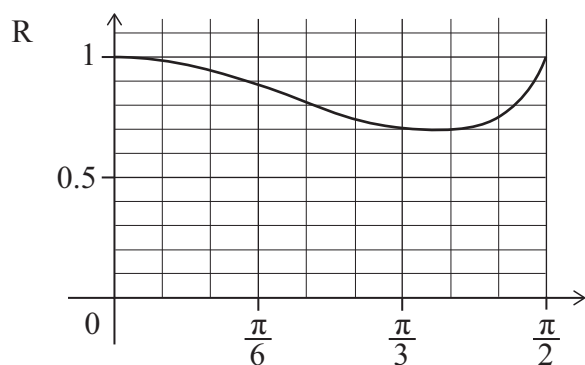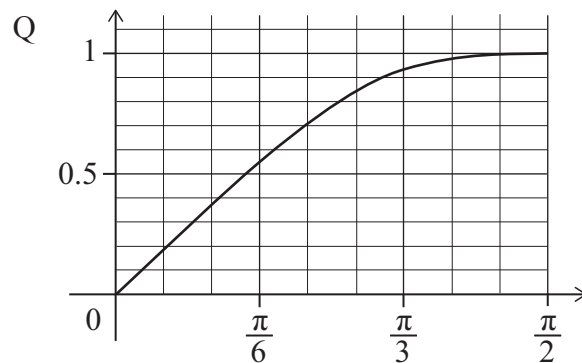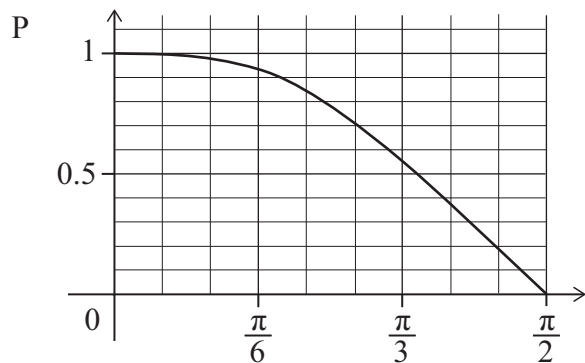
Which row in the following table correctly identifies the graphs?

	$y = (\cos x)^{\cos x}$	$y = (\sin x)^{\sin x}$	$y = (\cos x)^{\sin x}$	$y = (\sin x)^{\cos x}$
A	P	Q	R	S
B	P	Q	S	R
C	Q	P	R	S
D	Q	P	S	R
E	R	S	P	Q
F	R	S	Q	P
G	S	R	P	Q
H	S	R	Q	P

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