Simplify Trig Expressions

A Level

Simplify the following trigonometric expressions.

Part A $\frac{1}{\cos^2 t - 1}$

Simplify the following expression to give a single trigonometric function:

$$\frac{1}{\cos^2 t - 1}$$

The following symbols may be useful: `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `t`, `tan()`

Part B $\frac{1-\sin^2 x}{\cos x}$

Simplify the following expression to give a single trigonometric function:

$$\frac{1 - \sin^2 x}{\cos x}$$

The following symbols may be useful: `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `tan()`, `x`

Part C $\sin\alpha\tan\alpha - \frac{1}{\cos\alpha}$

Simplify the following expression to give a single trigonometric function:

$$\sin\alpha\tan\alpha - \frac{1}{\cos\alpha}$$

The following symbols may be useful: `alpha, cos(), cosec(), cot(), sec(), sin(), tan()`

Part D $\tan w - \frac{\cos w}{1-\sin w}$

Simplify the following expression to give a single trigonometric function:

$$\tan w - \frac{\cos w}{1 - \sin w}$$

The following symbols may be useful: `cos(), cosec(), cot(), sec(), sin(), tan(), w`

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Functions: Reciprocal Trig 1ii

A Level

P P P

Part A $\sec \frac{1}{2}\alpha = 4$

Solve, for $0° < \alpha < 180°$, the equation $\sec \frac{1}{2}\alpha = 4$. Give your answer in degrees, to three significant figures.

Part B $\tan \beta = 7 \cot \beta$

Solve, for $0° < \beta < 180°$, the equation $\tan \beta = 7 \cot \beta$, and give the largest solution in degrees to three significant figures.

Give the smallest solution in degrees to three significant figures.

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Advanced Trig Identities 5ii

A Level

Solve, for $0° < \theta < 360°$, the equation $\sec^2\theta = 4\tan\theta - 2$.

Part A Smallest

Give the smallest solution to 3 significant figures.

Part B Second Smallest

Give the second smallest solution to 3 significant figures.

Part C Second Largest

Give the second largest solution to 3 significant figures.

Part D Largest

Give the largest solution to 3 significant figures.

Advanced Trig Identities 2ii

A Level

P P P

Part A $2\tan^2\theta - \frac{1}{\cos\theta}$

Express $2\tan^2\theta - \frac{1}{\cos\theta}$ in terms of $\sec\theta$.

The following symbols may be useful: `sec(), theta`

Part B **Solve**

Hence solve, for $0° < \theta < 360°$, the equation

$$2\tan^2\theta - \frac{1}{\cos\theta} = 4$$

Give the smallest solution to three significant figures.

Give the second smallest solution to four significant figures.

Give the second largest solution to four significant figures.

Give the largest solution to three significant figures.

Functions: Inverse Trig 3ii

A Level

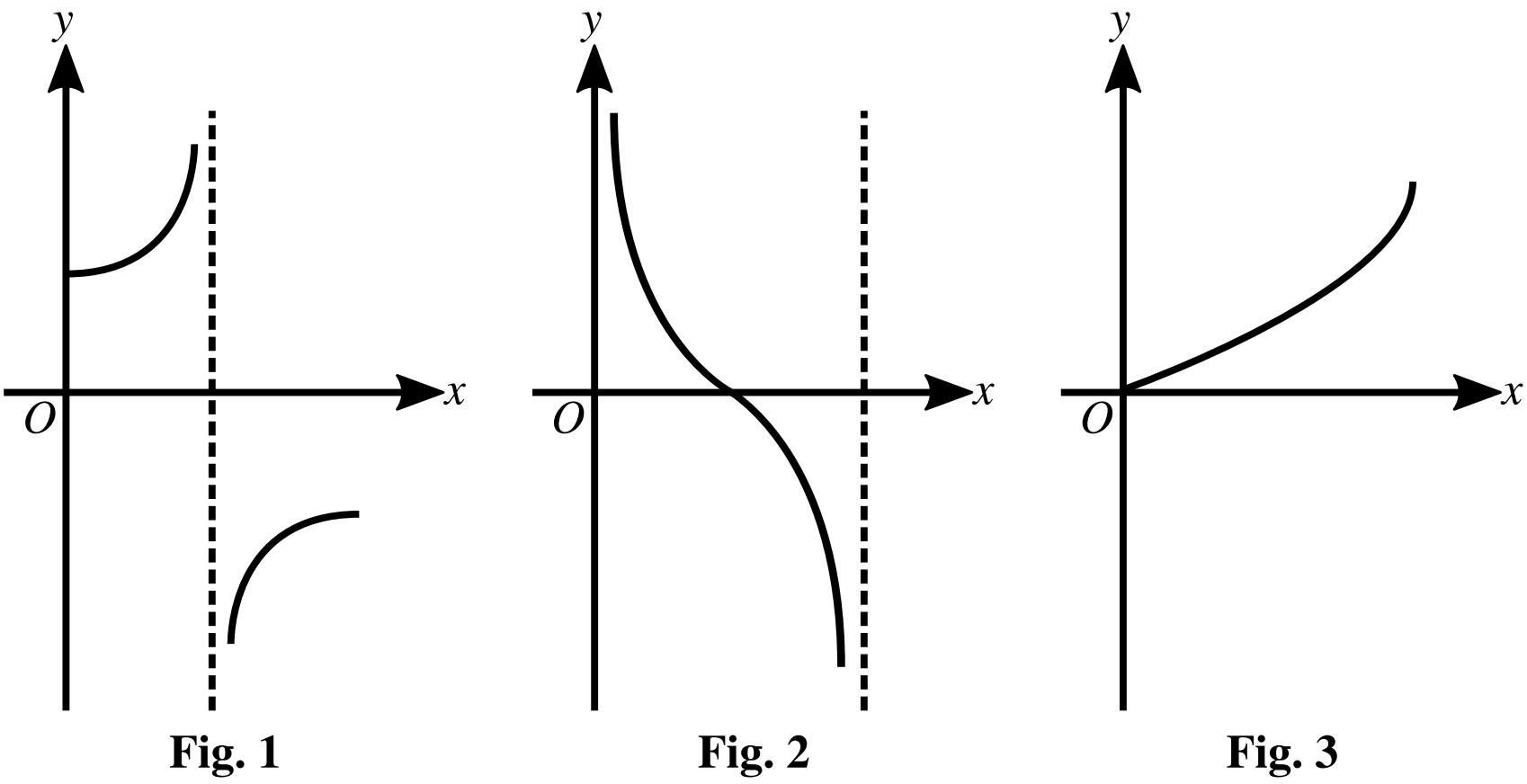

Figure 1: A diagram showing sections of curves

Each diagram in **Figure 1** shows part of a curve, the equation of which is one of the following:

$$y = \sin^{-1} x, \quad y = \cos^{-1} x, \quad y = \tan^{-1} x, \quad y = \sec x, \quad y = \operatorname{cosec} x, \quad y = \cot x$$

Part A **Fig. 1**

Which equation corresponds to **Fig. 1**?

- $y = \operatorname{cosec} x$
- $y = \tan^{-1} x$
- $y = \sec x$
- $y = \cot x$
- $y = \sin^{-1} x$
- $y = \cos^{-1} x$

Part B **Fig. 2**

Which equation corresponds to **Fig. 2**?

- $y = \sin^{-1} x$
- $y = \cos^{-1} x$
- $y = \operatorname{cosec} x$
- $y = \cot x$
- $y = \sec x$
- $y = \tan^{-1} x$

Part C **Fig. 3**

Which equation corresponds to **Fig. 3**?

- $y = \sin^{-1} x$
- $y = \cot x$
- $y = \sec x$
- $y = \operatorname{cosec} x$
- $y = \tan^{-1} x$
- $y = \cos^{-1} x$

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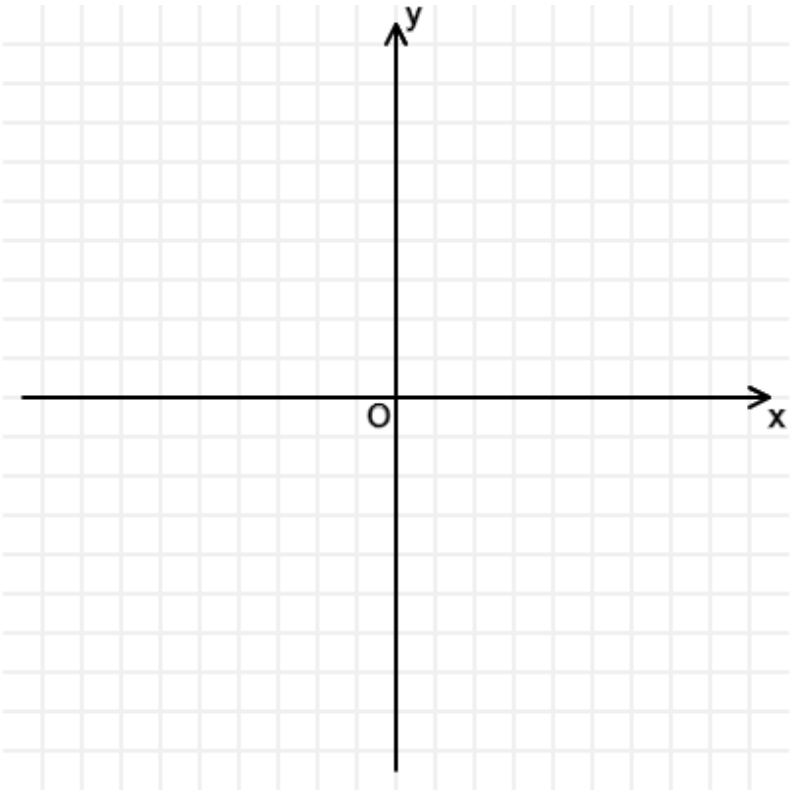
Sketching Reciprocal Trigonometric Functions

A Level

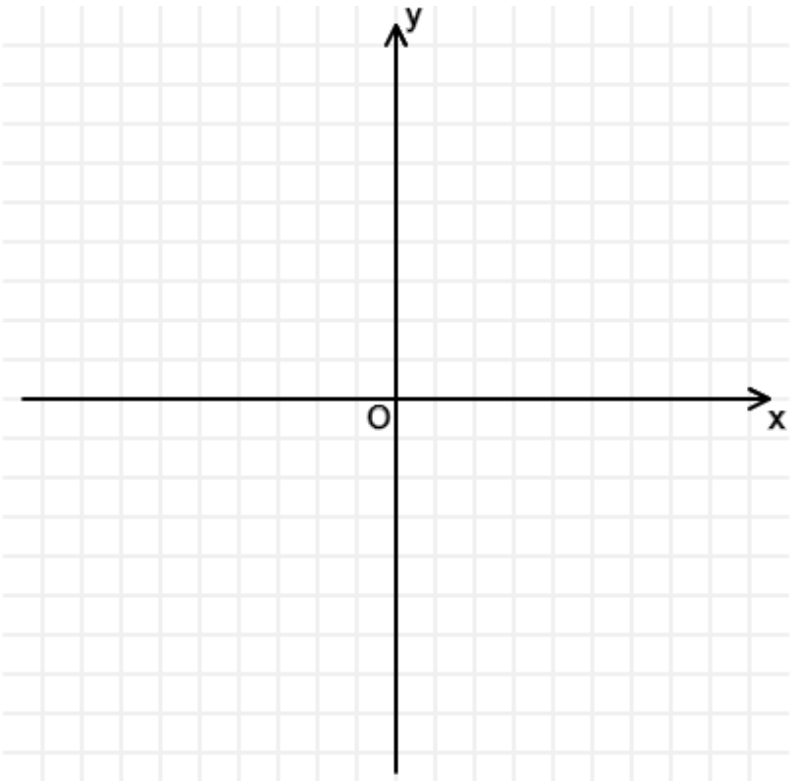
Part A Sketch $2\sec x + 2$

Sketch the graph of $y = 2\sec x + 2$ in the interval $0° \leq x \leq 360°$.



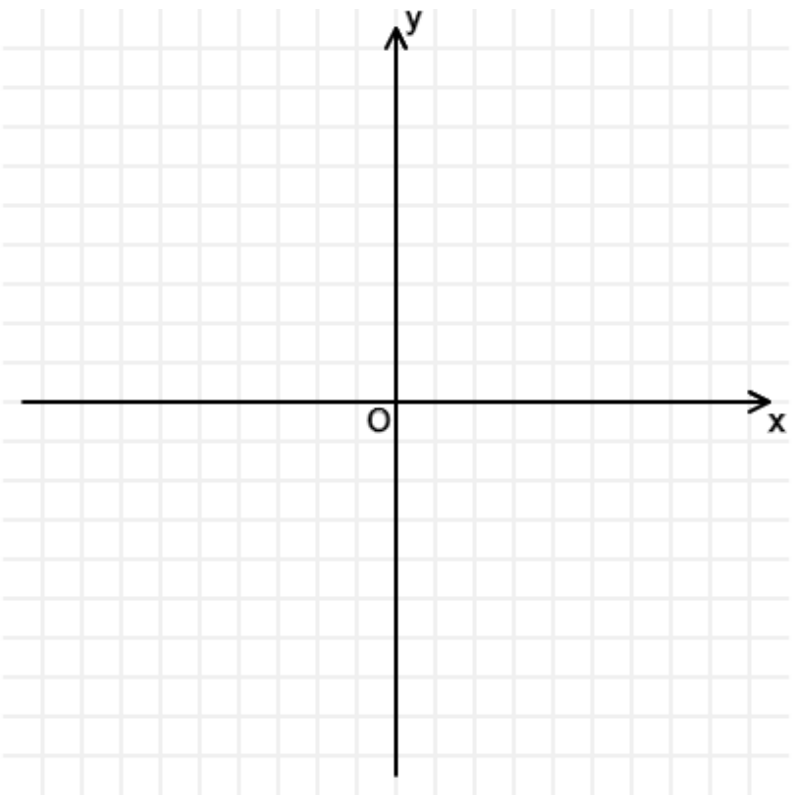
Part B **Sketch** $\cot\left(x+\frac{\pi}{4}\right)-1$

Sketch the graph of $y=\cot\left(x+\frac{\pi}{4}\right)-1$ in the interval $0\leq x\leq 2\pi$.



Part C **Sketch** $-\operatorname{cosec}\frac{x}{2}-3$

Sketch the graph of $y=-\operatorname{cosec}\frac{x}{2}-3$ in the interval $-2\pi\leq x\leq 2\pi$.



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Functions: Inverse Trig 1ii

A Level

Figure 1 shows the graph of $y = -\arcsin(x-1)$.

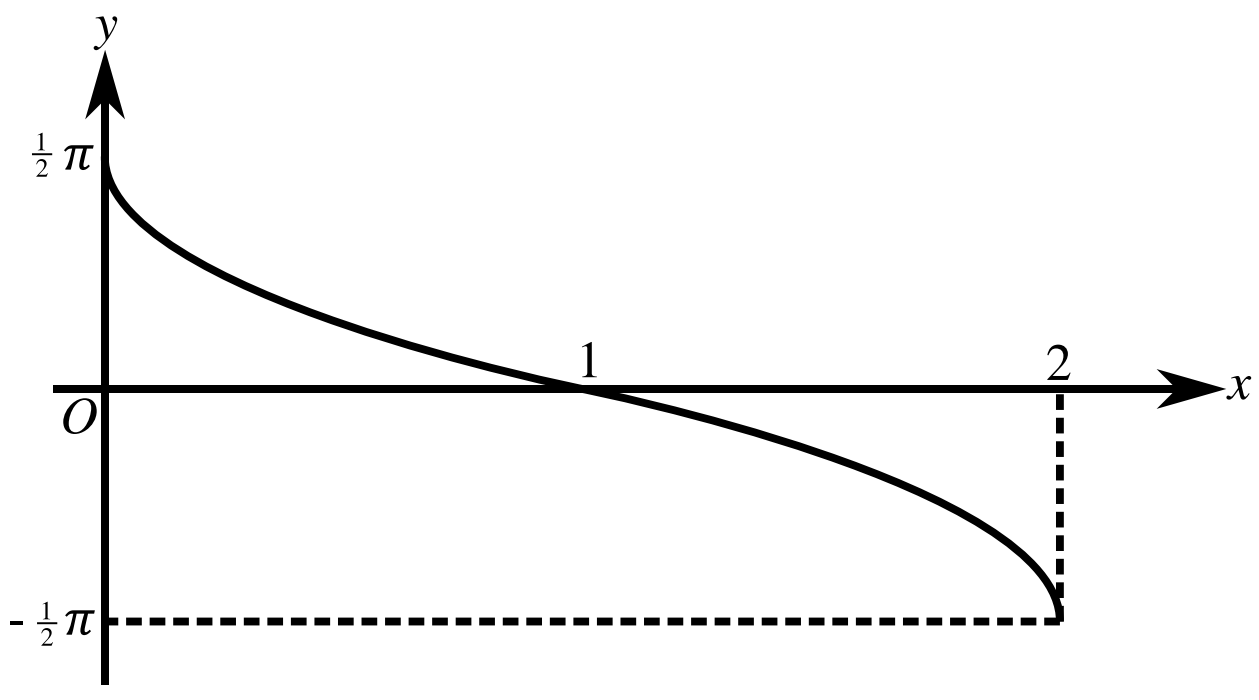

Figure 1: A graph of the function $y = -\arcsin(x-1)$

Part A Transformations

Give details of the pair of geometrical transformations which transform the graph of $y = -\arcsin(x-1)$ to the graph of $y = \arcsin x$.

- Reflect in y-axis, translate by 1 in $-y$-direction
- Reflect in $y = -x$, translate by 1 in $+x$-direction
- Reflect in x-axis, translate by 1 in $-x$-direction
- Reflect in y-axis, translate by 1 in $-x$-direction
- Reflect in x-axis, translate by 1 in $+x$-direction
- Reflect in line $y = x$, translate by 1 in $+y$-direction
- Reflect in y-axis, translate by 1 in $+y$-direction

Part B Sketch

Sketch the graph of $y = \left| -\arcsin(x-1) \right|$.

To see an example sketch, answer the following question: For what value of y does the curve meet the y-axis?

The following symbols may be useful: `pi`

Part C Solutions

Find the exact solutions of the equation $\left| -\arcsin(x-1) \right| = \frac{\pi}{3}$.

Give the largest exact solution.

The following symbols may be useful: `x`

Give the smallest exact solution.

The following symbols may be useful: `x`

Gameboard:

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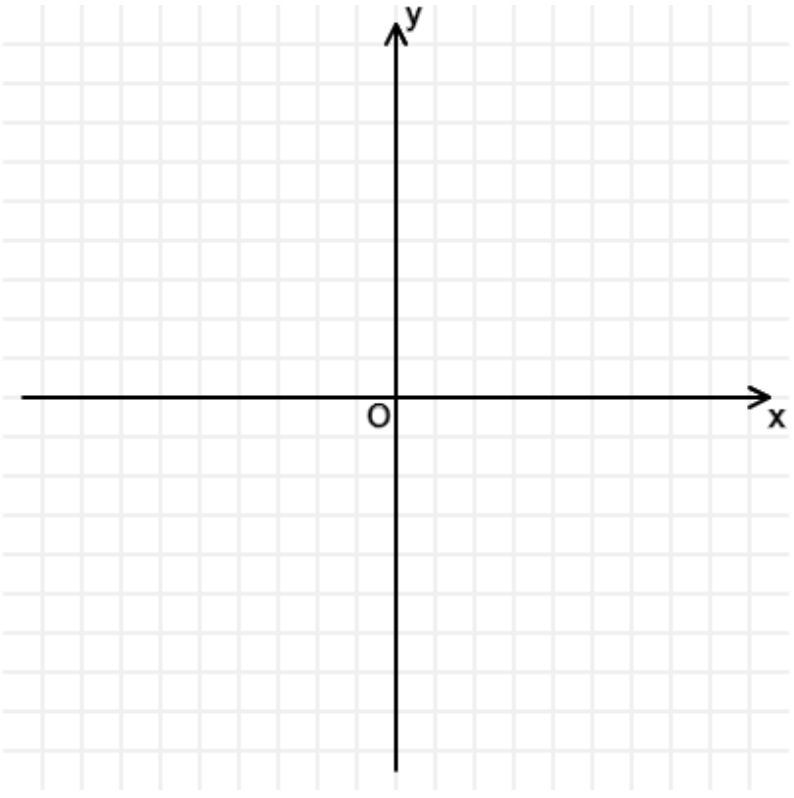
Sketching Inverse Trigonometric Functions

A Level

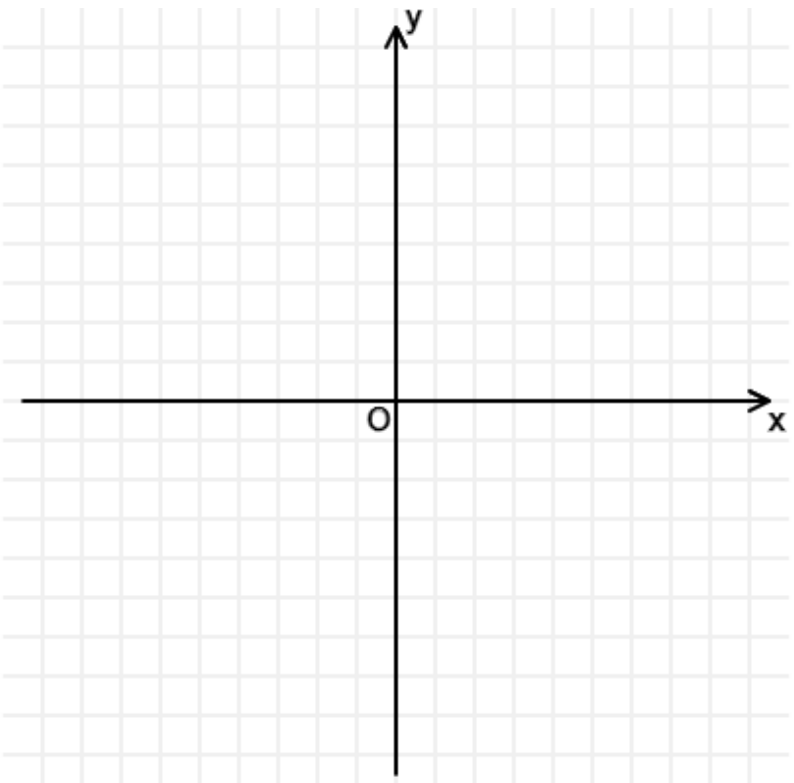
Part A Sketch $\arcsin \frac{x}{3} + \frac{\pi}{2}$

Sketch the graph of $y = \arcsin \frac{x}{3} + \frac{\pi}{2}$.



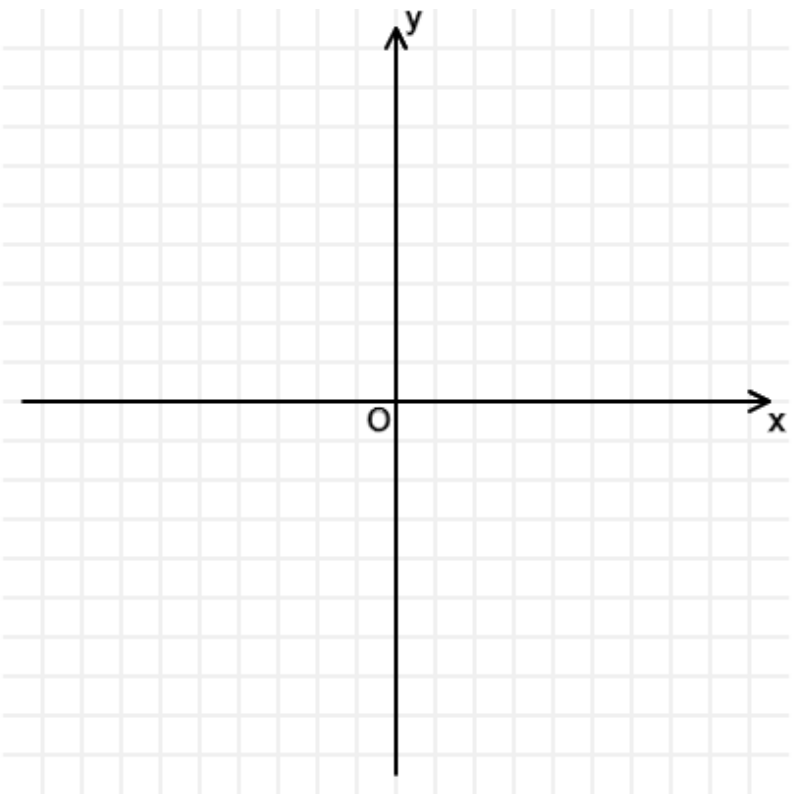
Part B **Sketch** $2\arctan(x-1)$

Sketch the graph of $y = 2\arctan(x-1)$.



Part C **Sketch** $\arccos\left(-\frac{x}{2}+\frac{1}{2}\right)-\frac{\pi}{3}$

Sketch the graph of $y = \arccos\left(-\frac{x}{2}+\frac{1}{2}\right)-\frac{\pi}{3}$.



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