Momentum and Kinetic Energy 3.1

A Level

Quantities:

p momentum $(\mathrm{kg\,m\,s^{-1}})$
m mass (kg)
v speed $(\mathrm{m\,s^{-1}})$
V accelerating voltage (V)

E kinetic energy (J)
λ wavelength (m)
q charge (C)
h Planck constant $(\mathrm{J\,s})$

Equations:

$$p = mv \qquad E = \frac{1}{2}mv^2 \qquad E = qV \qquad \lambda = \frac{h}{p}$$

Use the equations above to derive expressions without v for:

Part A Kinetic energy

the kinetic energy, E, in terms of p and m.

The following symbols may be useful: `E, V, h, lambda, m, p, q, v`

Part B Momentum

the momentum, p, in terms of E and m.

The following symbols may be useful: `E, V, h, lambda, m, p, q, v`

Part C Momentum of an accelerated particle

the momentum of an accelerated particle in terms of V, m and q.

The following symbols may be useful: `E, V, h, lambda, m, p, q, v`

Part D Wavelength of an accelerated particle

the wavelength of an accelerated particle in terms of V and q.

The following symbols may be useful: `E, V, h, lambda, m, p, q, v`

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Home   Gameboard   Physics   Electricity   Components   Photon Flux for an LED 7.1

Photon Flux for an LED 7.1

A Level

Quantities:

Φ_q photon flux (s^{-1})
E photon energy (J)
λ wavelength of light (m)
I electric current (A)
t duration (s)

V potential difference (V)
e electron charge (magnitude) (C)
P LED power (W)
n number of electrons or photons
h Planck's constant $(\mathrm{J\ s})$

Equations:

$$E = eV \qquad E = \frac{hc}{\lambda} \qquad \Phi_\mathrm{q} = \frac{n}{t} \qquad ne = It \qquad P = IV$$

Use the equations above to derive expressions for:

Part A The current

the current I in terms of Φ_q and e.

The following symbols may be useful: `I, P, Phi_q, V, c, e, h, lambda`

Part B The potential difference across an LED

the potential difference across a conducting LED V in terms of h, c, e and λ.

The following symbols may be useful: `I, P, Phi_q, V, c, e, h, lambda`

Part C The power of the LED

the power of the LED P in terms of h, c, λ, and Φ_q.

The following symbols may be useful: `I, P, Phi_q, V, c, e, h, lambda`

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Standing Waves on a String 15.1

A Level

Quantities:

f frequency (Hz)
n harmonic (no unit)
λ wavelength (m)
T tension in string (N)

ℓ length of vibrating string (m)
μ linear mass density $(\mathrm{kg\,m^{-1}})$
c speed of progressive wave $(\mathrm{m\,s^{-1}})$
M mass of vibrating string (kg)

Equations:

$$c^2 = \frac{T}{\mu} \qquad \mu = \frac{M}{\ell} \qquad \lambda = \frac{2\ell}{n} \qquad c = f\lambda$$

Use the equations above to derive expressions for:

Part A The fundamental frequency in terms of λ, μ and T

the fundamental frequency f_1 in terms of λ, μ and T.

The following symbols may be useful: `T, f_1, lambda, mu`

Part B The fundamental frequency in terms of ℓ, μ and T.

the fundamental frequency f_1 in terms of ℓ, μ and T.

The following symbols may be useful: `T, f_1, l, mu`

Part C The frequency of the n^{th} harmonic f_n in terms of ℓ, n, μ and T

the frequency of the n^{th} harmonic f_n in terms of ℓ, n, μ and T.

The following symbols may be useful: `T, f_n, l, mu, n`

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Home Gameboard Physics Mechanics Circular Motion Banked Tracks for Turning 17.1

Banked Tracks for Turning 17.1

A Level

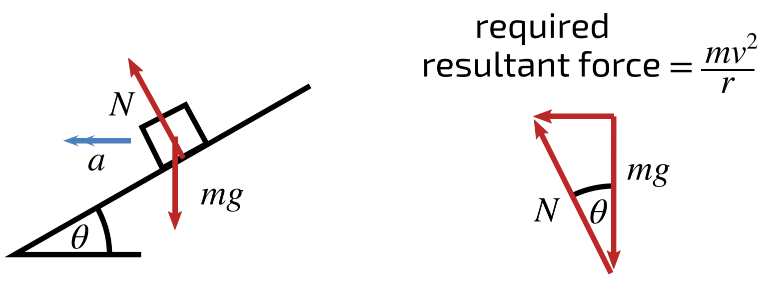

Figure 1: A diagram of a car traveling along a sloping track with the forces on the car resolved. The car is travelling perpendicular to the page.

Quantities:

m mass (kg)
r radius of path (m)
ω angular velocity ($\mathrm{rad\,s^{-1}}$)
N Normal contact force on car from track (N)

a acceleration inwards ($\mathrm{m\,s^{-2}}$)
v speed ($\mathrm{m\,s^{-1}}$)
θ angle of track ($^\circ$)
t_{p} period of orbit (s)

Equations:

$$F = ma \qquad a_{\text{centripetal}} = \frac{v^2}{r} \qquad v = r\omega \qquad t_{\mathrm{p}} = \frac{2\pi r}{v}$$

A car of weight mg travels at constant speed v around a smooth, banked track of radius r and slope θ above the horizontal, and remains at a constant height up the slope. Use diagrams, such as the one above, to write down expressions for:

Part A N in terms of m, g and θ

N in terms of m, g and θ.

The following symbols may be useful: `N, cos(), g, m, sin(), tan(), theta`

Part B v in terms of g, r and θ

v in terms of g, r and θ.

The following symbols may be useful: `N, cos(), g, r, sin(), tan(), theta`

Part C t_{p} in terms of r, g and θ

t_{p} in terms of r, g and θ.

The following symbols may be useful: `cos(), g, pi, r, sin(), t_p, tan(), theta`

Part D N in terms of m, g, r and v

N in terms of m, g, r and v.

The following symbols may be useful: `N, g, m, r, v`

Part E a in terms of v and ω

a in terms of v and ω.

The following symbols may be useful: `a, omega, v`

Part F ω in terms of g, r and θ

ω in terms of g, r and θ.

The following symbols may be useful: `cos(), g, omega, r, sin(), tan(), theta`

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Physics. You work it out.

Orbits 26.1

A Level

Quantities:

G Newton's gravitational constant $\left(\mathrm{N\,m^2\,kg^{-2}}\right)$
g gravitational field strength $\left(\mathrm{N\,kg^{-1}}\right)$
E electric field strength $\left(\mathrm{N\,C^{-1}}\right)$
B magnetic flux density (T)
a centripetal acceleration $\left(\mathrm{m\,s^{-2}}\right)$
ϵ_0 permittivity of free space $\left(\mathrm{F\,m^{-1}}\right)$
q, Q charge (C)
r radius of orbit (m)

F centripetal force (N)
T orbital period (s)
m, M mass (kg)
v velocity $\left(\mathrm{m\,s^{-1}}\right)$

Equations:

$$g = \frac{GM}{r^2} \qquad E = \frac{Q}{4\pi\epsilon_0 r^2} \qquad a = \frac{v^2}{r} \qquad F = ma \qquad v = \frac{2\pi r}{T}$$

$$F = mg \qquad F = qE \qquad F = qvB \qquad r^3 \propto T^2$$

A moon of mass m in a circular orbit around a planet of mass M.

Part A v using G, M, r

Use the equations above to obtain v in terms of G, M and r.

The following symbols may be useful: `G, M, r, v`

Part B Kepler's Third Law

Use the equations above to derive Kepler's Third Law, $r^3 \propto T^2$.

What is the constant of proportionality r^3/T^2 in terms of G and M?

The following symbols may be useful: `G, M, pi`

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Capacitors and Resistors 33.1

A Level

Quantities:

R resistance (Ω)
C capacitance (F)
t time (s)
Q charge on capacitor (C)
I current in circuit (A)

V_{C} voltage across capacitor (V)
V_{R} voltage across resistor (V)
V_0 initial or max voltage (V)
Q_0 initial or max charge (C)
I_0 initial current (A)

Equations:

$$Q = CV_{\mathrm{C}} \qquad V_{\mathrm{R}} = IR \qquad I = I_0\, e^{-t/RC}$$

When discharging:

$$V_{\mathrm{C}} = V_{\mathrm{R}} \qquad V_{\mathrm{C}} = V_0\, e^{-t/RC}$$

When charging:

$$V_{\mathrm{C}} + V_{\mathrm{R}} = V_0 \qquad V_{\mathrm{C}} = V_0\,(1 - e^{-t/RC})$$

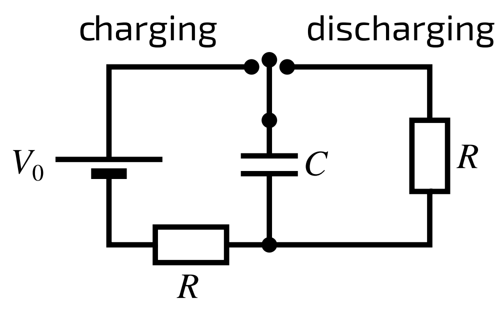

Figure 1: A circuit for charging and discharging a capacitor.

Use the equations above to write down expressions for:

Part A Q discharging

the charge Q versus time, when discharging.

The following symbols may be useful: `C, C, Q, Q_0, R, V_0, e, t`

Part B Q charging

the charge Q versus time, when charging.

The following symbols may be useful: `C, C, Q, Q_0, R, V_0, e, t`

Part C Q_0 using V_0, C

the initial charge Q_0 in terms of V_0 and C.

The following symbols may be useful: `C, Q_0, V_0`

Part D V_R discharging

the voltage V_R across the resistor versus time, when discharging.

The following symbols may be useful: `C, R, V_0, V_R, e, t`

Part E V_R charging

the voltage V_R across the resistor versus time, when charging.

The following symbols may be useful: `C, R, V_0, V_R, e, t`

Part F Q using I

Q in terms of I when discharging.

The following symbols may be useful: `C, I, Q, R`

Part G Q using $\mathrm{d}Q/\mathrm{d}t$

Q in terms of $\mathrm{d}Q/\mathrm{d}t = -I$ when discharging.

The following symbols may be useful: `C, Derivative(Q, t), Q, R`

Part H I_0 using V_0, R

I_0 in terms of V_0 and R when discharging.

The following symbols may be useful: `I_0, R, V_0`

Part I I_0 using Q_0, R, C

I_0 in terms of Q_0, R and C when discharging.

The following symbols may be useful: `C, I_0, Q_0, R`

Part J Discharge time if $I = I_0$

the time to completely discharge if the current were constant at I_0.

The following symbols may be useful: `C`, `R`

Part K Fraction of Q_0 at $t = RC$

the fraction of Q_0 still on the capacitor after a time RC. (Give your answer in terms of e, the base of natural exponents).

The following symbols may be useful: `C`, `R`, `e`

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Deriving Kinetic Theory 31

A Level

This question is designed to take you through a derivation using kinetic theory. The end result is an expression for the pressure of an ideal gas. It is the only derivation that you are required to reproduce in most A Level syllabuses. Attempt all parts of the question, in order. You might find that this question page will take longer to complete than most questions on Isaac Physics.

We create a mathematical model using Newton's Laws for the particles in a gas. When we have done this, we find it predicts many aspects of bulk gas behaviour correctly. To do this, we assume that the gas is an **ideal gas**.

Example context: explaining how the volume, pressure and temperature of a gas change by considering the collisions of the particles in the gas with each other and the walls of the container. This allows you to predict the thermodynamic behaviour of a gas without having to do an experiment.

Quantities:

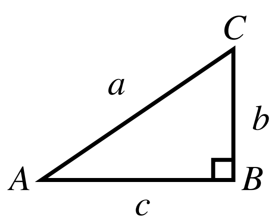
Δ "A change in"
m mass of a particle (kg)
F force (N)
p momentum of a particle $(\mathrm{kg\,m\,s^{-1}})$
t time taken (s)
c speed of a particle $(\mathrm{m\,s^{-1}})$
u, v, w components of velocity in the x, y, z directions $(\mathrm{m\,s^{-1}})$

s distance travelled (m)
P pressure of a gas $(\mathrm{N\,m^{-2}}$ or $\mathrm{Pa})$
A area of face $(\mathrm{m^2})$
V volume of gas $(\mathrm{m^3})$
N number of molecules
$\overline{c^2}$ mean-square speed $(\mathrm{m^2\,s^{-2}})$

Equations:

$$F = \frac{\Delta mv}{\Delta t} \qquad P = \frac{F}{A} \qquad v = \frac{s}{t}$$

$$p = m \times \text{velocity} \qquad \Delta p = p_{\text{after}} - p_{\text{before}} \qquad a^2 = b^2 + c^2 \text{ (Pythagoras)}$$

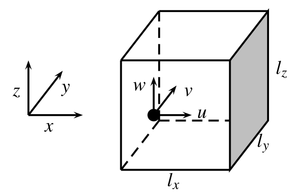


Assumptions about ideal gases:

- The volume of a particle is so small compared to the volume of the gas, we can ignore it.
- There are no attractive forces between particles, only collision forces.
- Particle movement is continuous and random.
- Particle collisions are perfectly elastic, so there is no loss of kinetic energy.
- Collision time is very short in comparison with the time between impacts.
- There are enough molecules for statistics to be applied.

Part A Volume of a box

The particle is in a box of dimensions l_x, l_y, l_z. The box represents the volume of the gas. The shaded faces represent the collisions.

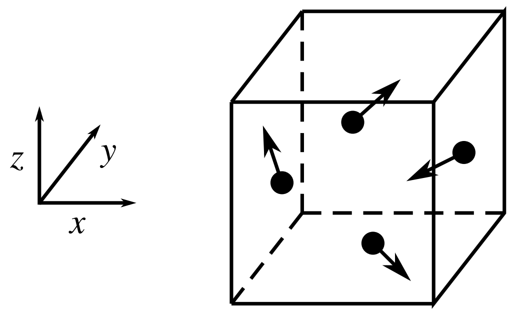


Write down the formula for the volume of the box V in terms of l_x, l_y and l_z.

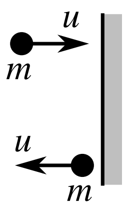
The following symbols may be useful: `V, l_x, l_y, l_z`

Part B **Change in momentum**

We can think of the gas as a group of N particles moving around randomly, hitting the sides of the container. As the motion is random, we expect the average speeds in different directions to be the same.



Let's consider one particle moving in the positive x direction. The particle collides with the container wall.



Write an expression for the change in momentum of the particle, Δp, in terms of m and u. Pay attention to which direction is positive.

The following symbols may be useful: `Deltap, m, p, u`

Part C **Average force F_{particle}**

Write an expression for the average force F_{particle} (written in the editor as F_{p}) on the particle (from the wall), to cause the change in momentum of the particle. The time between collisions with the wall is Δt.

The following symbols may be useful: `Deltat, F_p, m, u`

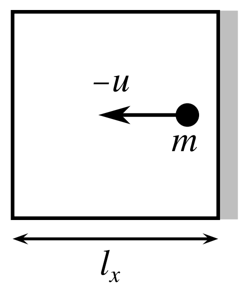
Part D **Average force F_{wall}**

Use Newton's Third Law of Motion to write down an expression for the average force, F_{wall} (written in the editor as F_{w}), of the particle on the wall over time Δt. Pay attention to the sign.

The following symbols may be useful: `Deltat, F_w, m, u`

Part E **Time Δt**

Between collisions, the particle will travel to the other side of the container and back again.



Find an expression for Δt in terms of u and l_x.

The following symbols may be useful: `Deltat, l_x, u`

Part F **New expression for F_{wall}**

Now substitute your expression for Δt from Part E into your equation in Part D and simplify it. This will give you a new expression for the force of the particle on the wall, F_{wall} (written in the editor as F_w), in terms of m, u, and l_x.

The following symbols may be useful: `F_w, l_x, m, u`

Part G **Average pressure P_1**

The average pressure exerted by the particle on the wall may be written as F_{wall}/A, where A is the area of the wall. Use your answer to Part F to find an expression for the average pressure P_1 due to this one molecule:

(a) in terms of u, m, l_x, l_y, and l_z;

The following symbols may be useful: `P_1, l_x, l_y, l_z, m, u`

(b) in terms of u, m, and the volume, V, of the container. (Use your answer from Part A).

The following symbols may be useful: `P_1, V, m, u`

Part H **P_1 if v_1, w_1 not zero**

We now have an expression for the pressure P_1 on the container due to the collision of one particle. From here on we refer to this particle as 'particle 1' and label its speed as c_1 and its velocity components as u_1, v_1 and w_1. There are actually N particles in the gas. They each have the same mass m, but will have different velocities. For example, 'particle 2' has velocity components u_2, v_2 and w_2, has speed c_2, and will cause a pressure P_2.

Up until now, we have assumed that our particle was only moving in the x direction. Does the expression for P_1 derived in question Part G change if v_1 and w_1 are not necessarily zero?

- Yes
- No

Part I **Pressure P_2**

By looking at your reasoning for particle 1, write down an expression for the pressure P_2 on the same wall in terms of m, u_2, v_2, w_2 and V.

The following symbols may be useful: `P_2, V, m, u_2, v_2, w_2`

Part J **Total pressure P**

The total pressure on this wall will be the sum of the pressures due to all of the individual particles: $P = P_1 + P_2 + \ldots$.

Use your expression from Part G(b) to write the equation for total pressure P in terms of m, V, u_1, u_2 and the other x components of velocity. Assume that all the particles have the same mass, m.

Which of these equations is correct?

- $P = mV\left(u_1^2 + u_2^2 + \ldots\right)$
- $P = m\left(u_1^2 + u_2^2 + \ldots\right)$
- $P = u_1^2 + u_2^2 + \ldots$
- $P = u_1^2 + u_2^2 + \ldots$
- $P = m\left(u_1 + u_2 + \ldots\right)$
- $P = \frac{m}{V}\left(u_1 + u_2 + \ldots\right)$
- $P = \frac{m}{V}\left(u_1^2 + u_2^2 + \ldots\right)$
- $P = mV\left(u_1 + u_2 + \ldots\right)$

Part K **Average squared x component of velocity $\overline{u^2}$**

Find an expression for the average squared x component of velocity $\overline{u^2}$ if there are N molecules whose squared velocity components are u_1^2, u_2^2 and so on.

Which of these equations is correct?

- $\overline{u^2} = \frac{u_1 + u_2 + u_3 + \ldots + u_N}{N}$
- $\overline{u^2} = \frac{u_1^2 + u_2^2 + u_3^2 + \ldots + u_N^2}{N}$
- $\overline{u^2} = u_1 + u_2 + u_3 + \ldots + u_N$
- $\overline{u^2} = (u_1 + u_2 + u_3 + \ldots + u_N)^2$
- $\overline{u^2} = \frac{(u_1 + u_2 + u_3 + \ldots + u_N)^2}{N}$
- $\overline{u^2} = (\frac{u_1 + u_2 + u_3 + \ldots + u_N}{N})^2$
- $\overline{u^2} = u_1^2 + u_2^2 + u_3^2 + \ldots + u_N^2$

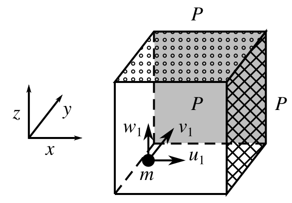
Part L **Re-write pressure P using m, V, N, $\overline{u^2}$**

Use your answer to Part K to re-write the pressure from Part J in terms of m, V, N and $\overline{u^2}$ (written as u^2 in the editor).

The following symbols may be useful: `N, P, V, m, u^2`

Part M $\overline{v^2}$ and $\overline{w^2}$

We now have an expression for the pressure of the particles on the right hand wall. As the particles are moving randomly, they exert the same pressure on the other walls as well.



We now take into account the fact that the molecules are not just moving in the x direction.

The y components of each molecule's velocity are written v_1, v_2, v_3 and so on. Use your answer to Part K to write expressions (when there are N particles) for:

the average squared y velocity component $\overline{v^2}$; (Which of these equations is correct?)

- $\overline{v^2} = \frac{v_1 + v_2 + v_3 + ... + v_N}{N}$
- $\overline{v^2} = v_1^2 + v_2^2 + v_3^2 + ... + v_N^2$
- $\overline{v^2} = v_1 + v_2 + v_3 + ... + v_N$
- $\overline{v^2} = (\frac{v_1 + v_2 + v_3 + ... + v_N}{N})^2$
- $\overline{v^2} = \frac{v_1^2 + v_2^2 + v_3^2 + ... + v_N^2}{N}$
- $\overline{v^2} = \frac{(v_1 + v_2 + v_3 + ... + v_N)^2}{N}$
- $\overline{v^2} = (v_1 + v_2 + v_3 + ... + v_N)^2$

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the average squared z velocity component $\overline{w^2}$; (Which of these equations is correct?)

- $\overline{w^2} = w_1^2 + w_2^2 + w_3^2 + ... + w_N^2$
- $\overline{w^2} = \frac{w_1 + w_2 + w_3 + ... + w_N}{N}$
- $\overline{w^2} = (w_1 + w_2 + w_3 + ... + w_N)^2$
- $\overline{w^2} = \frac{w_1^2 + w_2^2 + w_3^2 + ... + w_N^2}{N}$
- $\overline{w^2} = \frac{(w_1 + w_2 + w_3 + ... + w_N)^2}{N}$
- $\overline{w^2} = (\frac{w_1 + w_2 + w_3 + ... + w_N}{N})^2$
- $\overline{w^2} = w_1 + w_2 + w_3 + ... + w_N$

Part N Equations linking P, $\overline{v^2}$ and $\overline{w^2}$

In Part L, you wrote an equation linking P and $\overline{u^2}$.

By thinking of collisions with the back wall causing an equal pressure, write a similar equation linking P and $\overline{v^2}$ (written in the editor as v^2).

The following symbols may be useful: `N, P, V, m, v^2`

Then, by thinking of collisions with the top wall, write a similar equation linking P and $\overline{w^2}$ (written in the editor as w^2).

The following symbols may be useful: `N, P, V, m, w^2`

Part O Equation linking $\overline{c^2}$, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$

In the Example in the notes page, we saw that $c^2 = u^2 + v^2 + w^2$, where c^2 is the square speed of one molecule. Applied to particle 1 this means that $c_1^2 = u_1^2 + v_1^2 + w_1^2$.

Use this information to write an equation relating $\overline{c^2}$ (the mean square speed), to $\overline{u^2}$, $\overline{v^2}$ and $\overline{w^2}$ (the mean square velocity components). (In the editor, all of the mean square quantities are written without the lines above them.)

The following symbols may be useful: `c^2, u^2, v^2, w^2`

Part P Pressure P in terms of $\overline{c^2}$

Use your answers to Parts N and O to write an equation for the pressure P in terms of the mean square speed $\overline{c^2}$ (written in the editor as c^2).

The following symbols may be useful: `N, P, V, c^2, m`

Exercise

Copy the diagram of the box and see how far you can progress with the proof without looking at all the steps. Remember:

1. 1 particle
2. N particles
3. 3 dimensions.

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