



STEM SMART Double Maths 30 - Inverse Trigonometric Calculus & Polar Coordinates

Calculus: Improper Integration 1i

Subject & topics: Maths **Stage & difficulty:** Further A P2

Evaluate $\int_0^{\infty} 2xe^{-x} dx$.

Note that $xe^{-x} \rightarrow 0$ as $x \rightarrow \infty$.

Adapted with permission from UCLES, A Level, Sample Paper 2017, Paper Y541, Question 5.



Calculus: Improper Integration 2i

Subject & topics: Maths **Stage & difficulty:** Further A P2

A large cuboid block is covered with a tarpaulin.

The tarpaulin is placed symmetrically over the block.

It is in contact with the block where the block is horizontal, and the shape of the tarpaulin where it hangs over the ends of the block can be modelled by the function:

$$f(x) = \frac{1}{x^2}$$

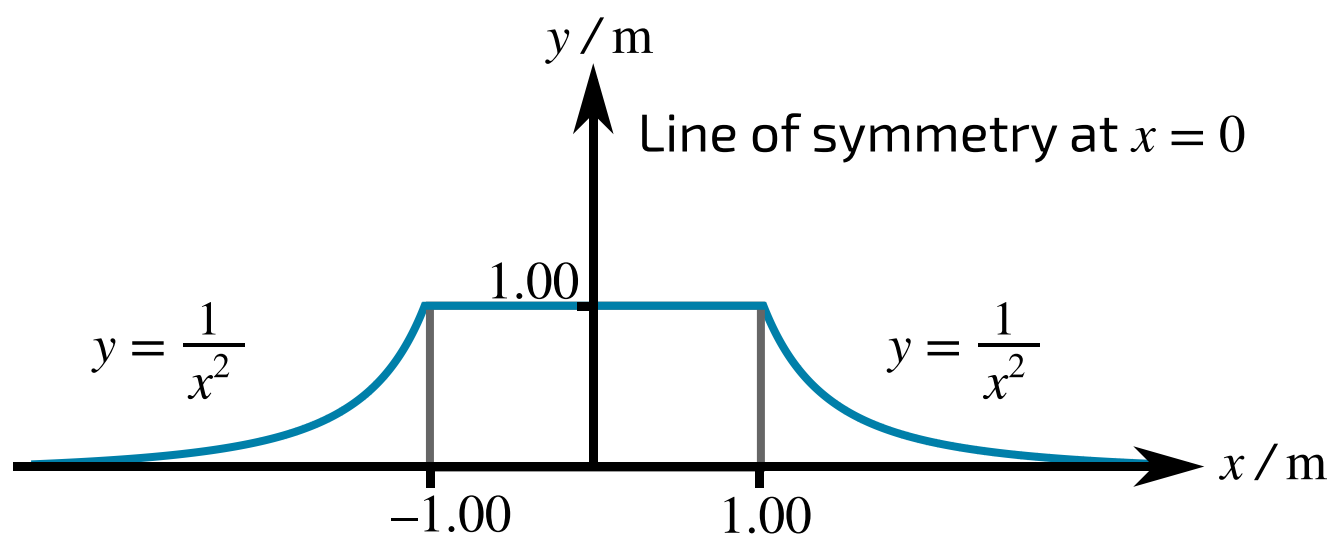


Figure 1: Diagram of the tarpaulin.

If the block is 2.00 m in length and 1.00 m in height, find the area of the cross-section of this system, which is shown in **Figure 1**.

Give your answer to 3 significant figures.

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Mean Values 1

Pre-Uni Maths for Sciences K6.1, K6.2, K6.3 & K6.4

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** Further A P2

Find the mean values of the following.

Part A

$\frac{x}{\sqrt{4-x^2}}$ **between 0 and 2**

Find the mean value of

$$\frac{x}{\sqrt{4-x^2}}$$

between $x = 0$ and $x = 2$.

Part B

$\frac{\sin(2\theta)}{(1-\cos^2\theta)^3}$ **between $\frac{\pi}{6}$ and $\frac{\pi}{2}$**

Find the mean value of

$$\frac{\sin(2\theta)}{(1-\cos^2\theta)^3}$$

between $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{2}$, giving your answer in exact form.

The following symbols may be useful: pi

Part C

$\frac{1}{1+e^{-2t}}$ between 0 and 3

Find the mean value of

$$\frac{1}{1 + e^{-2t}}$$

between $t = 0$ and $t = 3$, giving your answer in exact form.

The following symbols may be useful: e, ln(), log()

Part D

$4b \tan \phi$ between 0 and b

The mean value of the function $4b \tan \phi$ between $\phi = 0$ and $\phi = b$ is equal to $2 \ln 2$, where $b < \frac{\pi}{2}$. Deduce the value of b .

The following symbols may be useful: b, pi

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Mean Values 2

Pre-Uni Maths for Sciences K6.5, K6.6 & K6.7

Subject & topics: Maths | Calculus | Integration Stage & difficulty: Further A P2

Find the mean values of the following.

Part A

$\sin\left(\frac{\pi x}{a}\right)$ between 0 and a

Find the mean value of $\sin\left(\frac{\pi x}{a}\right)$ between $x = 0$ and $x = a$.

The following symbols may be useful: a, pi

Part B

$\sin\left(\frac{\pi x}{a}\right)$ between $-a$ and a

Find the mean value of $\sin\left(\frac{\pi x}{a}\right)$ between $x = -a$ and $x = a$.

The following symbols may be useful: a, pi

Part C **$\sin^2\left(\frac{\pi x}{a}\right)$ between 0 and a**

Find the mean value of $\sin^2\left(\frac{\pi x}{a}\right)$ between $x = 0$ and $x = a$.

The following symbols may be useful: a, pi

Part D **$\sin^2\left(\frac{\pi x}{a}\right)$ between $-a$ and a**

Find the mean value of $\sin^2\left(\frac{\pi x}{a}\right)$ between $x = -a$ and $x = a$.

The following symbols may be useful: a, pi

Part E **$x \sin^2\left(\frac{\pi x}{2a}\right)$ between 0 and $2a$**

Find the mean value of $x \sin^2\left(\frac{\pi x}{2a}\right)$ between $x = 0$ and $x = 2a$.

The following symbols may be useful: a, pi

Part F

$x \sin^2\left(\frac{\pi x}{2a}\right)$ between $-a$ and a

Find the mean value of $x \sin^2\left(\frac{\pi x}{2a}\right)$ between $x = -a$ and $x = a$.

The following symbols may be useful: a, pi

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Calculus: Inverse Trigonometry 2i

Subject & topics: Maths Stage & difficulty: Further A P2

Part A

Derivative of $\arcsin x$

Find the derivative of $\arcsin x$

The following symbols may be useful: x

Part B

Implicit differentiation

Given that

$$\arcsin 2x + \arcsin y = \frac{1}{2} \pi$$

find the exact value of $\frac{dy}{dx}$ when $x = \frac{1}{4}$.

Adapted with permission from UCLES, A Level, January 2009, Paper 4726, Question 3.

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STEM SMART Double Maths 30 - Inverse Trigonometric Calculus & Polar Coordinates

Integration Using Inverse Trig 1

Subject & topics: Maths | Calculus | Integration**Stage & difficulty:** A Level C1, Further A P2

Find the following integrals.

Part A

$$\frac{1}{\sqrt{1-x^2}}$$

Find the indefinite integral

$$\int \frac{dx}{\sqrt{1-x^2}}$$

using a suitable trigonometric substitution.

The following symbols may be useful: C, arccos(), arcsin(), arctan(), c, k, x

Part B

$$\frac{5}{\sqrt{9-x^2}}$$

Find the integral

$$\int_{\frac{3}{\sqrt{2}}}^{\frac{3\sqrt{3}}{2}} \frac{5 dx}{\sqrt{9-x^2}}$$

giving your answer in exact form.

The following symbols may be useful: pi

Part C

$$\frac{2}{\sqrt{1-2x^2}}$$

Find the integral

$$\int_0^{\frac{1}{2}} \frac{2 \, dx}{\sqrt{1-2x^2}}$$

giving your answer in exact form.

The following symbols may be useful: pi

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STEM SMART Double Maths 30 - Inverse Trigonometric Calculus & Polar Coordinates

Integration Using Inverse Trig 2

Subject & topics: Maths | Calculus | Integration**Stage & difficulty:** A Level C1, Further A P2

Find the following integrals.

Part A

$$\frac{1}{1+x^2}$$

Find the indefinite integral

$$\int \frac{dx}{1+x^2}$$

using a suitable trigonometric substitution.

The following symbols may be useful: C, arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), c, k, x

Part B

$$\frac{4}{4x^2+9}$$

Find the integral

$$\int_0^{\infty} \frac{4 dx}{4x^2+9}$$

giving your answer in exact form.

The following symbols may be useful: pi

Part C

$$\frac{3}{x\sqrt{4x^2-1}}$$

Find the integral

$$\int_1^{\infty} \frac{3 \, dx}{x\sqrt{4x^2-1}}$$

giving your answer in exact form.

The following symbols may be useful: pi

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Polar Coordinates: General 1ii

Subject & topics: Maths **Stage & difficulty:** Further A P2

A curve C has the cartesian equation $x^3 + y^3 = axy$, where $x \geq 0, y \geq 0$ and $a > 0$.

Part A

Polar equation of C

Express the polar equation of C in the form $r = f(\theta)$.

The following symbols may be useful: a, cos(), r, sin(), tan(), theta

Part B

Range of θ

Given that $0 \leq \theta \leq \beta$.

Find β .

The following symbols may be useful: pi

Part C

Line of symmetry

The line $\theta = \alpha$ is a line of symmetry of C .

Find and simplify an expression for $r = f\left(\frac{1}{2}\pi - \theta\right)$.

The following symbols may be useful: a, cos(), r, sin(), tan(), theta

Hence find the value of α .

The following symbols may be useful: pi

Part D

r at line of symmetry

Find the value of r when $\theta = \frac{1}{4}\pi$.

The following symbols may be useful: a

Part E
Sketch of C

Sketch the curve C .

Which curve in **Figure 1** is closest to your sketch?

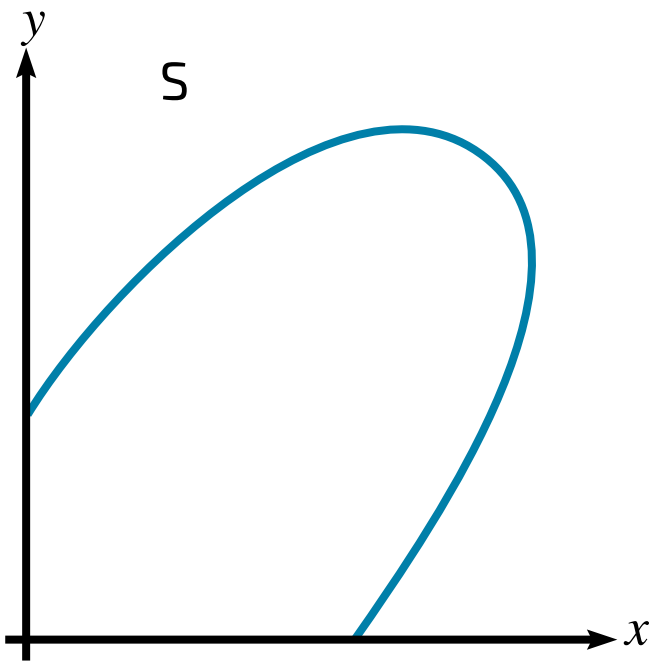
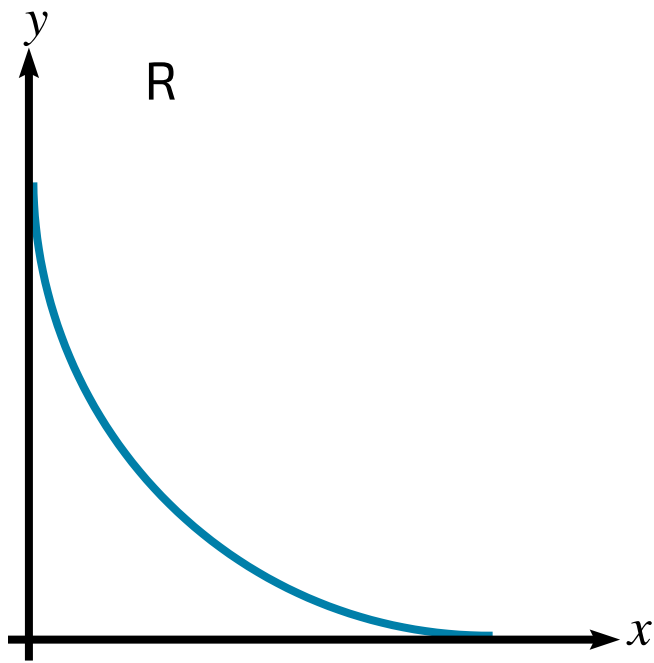
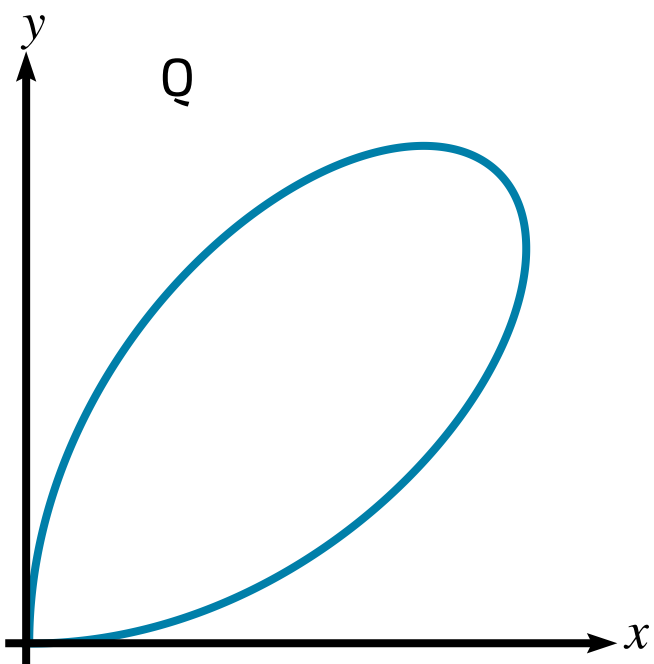
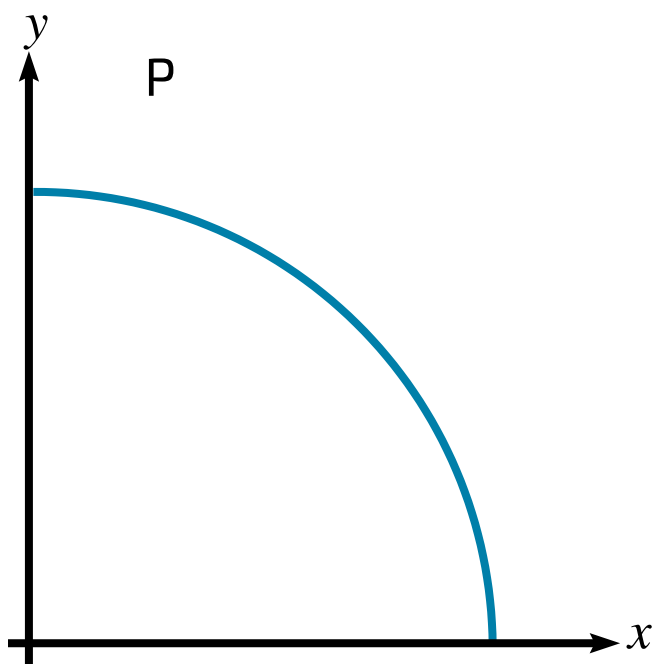


Figure 1: Four curves.

- ☐ Curve P
- ☐ Curve Q
- ☐ Curve R
- ☐ Curve S

Adapted with permission from UCLES, A Level, June 11, Paper 4726, Question 4.

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STEM SMART Double Maths 30 - Inverse Trigonometric Calculus & Polar Coordinates



Polar Coordinates: General 2i

Subject & topics: Maths **Stage & difficulty:** Further A P2

A curve has polar equation $r = \cos \theta \sin 2\theta$, for $0 \leq \theta \leq \frac{1}{2}\pi$.

Part A

Maximum value of r

Find the maximum value of r .

Part B

Cartesian equation of the curve

Find a cartesian equation of the curve.
Give your answer in the form $(x^2 + y^2)^2 = f(x, y)$
The following symbols may be useful: x , y

Adapted with permission from UCLES, A Level, June 2012, Paper 4726, Question 2.

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STEM SMART Double Maths 30 - Inverse Trigonometric Calculus & Polar Coordinates

Polar Coordinates: Area 2i

Subject & topics: Maths **Stage & difficulty:** Further A P2

The equation of a curve, in polar coordinates, is

$$r = 2 \cos 2\theta \quad (-\pi < \theta \leq \pi).$$

Part A

Tangents at the poles

Find the values of θ which give the directions of the tangents at the pole.

Give your answers in terms of π , in order from lowest to highest (most negative to most positive).

$\theta_1 =$ π (lowest value)

$\theta_2 =$ π

$\theta_3 =$ π

$\theta_4 =$ π (highest value)

Part B**Area enclosed by one loop**

A loop of the curve is shown in the diagram.

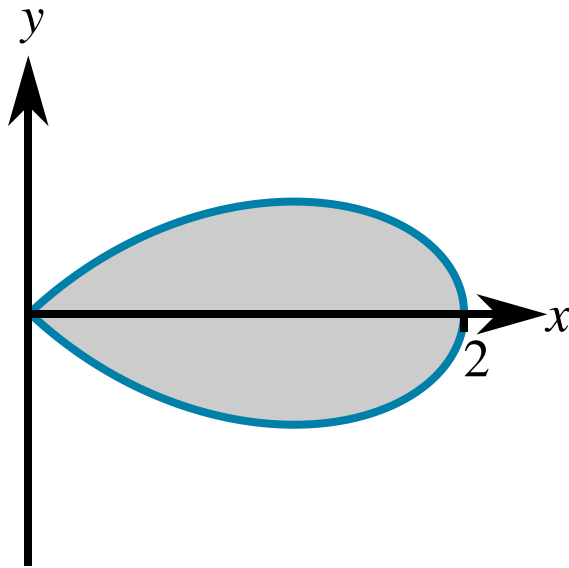


Figure 1: One loop of $r = 2 \cos 2\theta$.

Find the exact value of the area of the region enclosed by the loop.

The following symbols may be useful: π

Adapted with permission from UCLES, A Level, Specimen Paper, Paper 4726, Question 4.