Essential GCSE Maths 54.1

GCSE C C C A Level P P P

A hospital has a total of 9760 employees. They are divided into 4 categories:

Doctors	Nurses	Scientists	Administrators
1240	5020	1080	?

Part A Number of administrators

How many administrators work for the hospital?

Part B Number of employees surveyed

A stratified sample is required for a survey of employees' opinions. 5% of the employees will be surveyed.

How many employees will be included in the survey?

Part C Numbers of people from each category

A stratified sample is required for a survey of employees' opinions. 5% of the employees will be surveyed.

Work out how many people must be randomly selected from each of the four employee categories to give a stratified sample.

How many doctors?

How many nurses?

How many scientists?

How many administrators?

Woodland Survey

A Level

A conservationist wishes to collect data on the number of trees in a woodland nature reserve that have been parasitised by mistletoe. The reserve consists of birch, aspen and oak trees.

Part A Conducting a census

Why would it be impractical to conduct a census to collect data from every tree in the woodland?

- The act of collecting data from a tree would destroy it.
- The population would need to be classified into distinct strata.
- It would be very time-consuming and expensive.
- Collecting this data might spread the mistletoe
- A sampling frame would be needed.

Part B Type of sample

The conservationist wishes to determine if any particular species of tree is more likely to be parasitised by mistletoe. To do so, they decide to survey the first 10 birches, 10 aspen and 10 oak trees that they come across in the reserve.

What type of sample are they collecting?

Part C Disadvantage of sampling

What is a disadvantage of the type of sampling described in Part B?

- This non-random sampling could introduce bias.
- A sampling frame is needed in order to select the sample.
- It is very time-consuming and expensive.

Part D Species data type

What type of data is the species of a tree?

Part E Number of trees data type

What type of data is the number of trees that have been parasitised by mistletoe?

- Discrete quantitative data.
- Continuous quantitative data.
- Discrete qualitative data.
- Continuous qualitative data.

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Researcher Survey

A Level

A research institute wishes to collect data from its employees on the methods by which they travel to work. The institute has an alphabetised list of its 450 employees.

Part A Systematic sample

Describe how they could take a systematic sample of size 30.

Assign numbers from 1 to [ ] to the employees on the alphabetised list. Calculate $450 \div$ [ ] $=$ [ ]. Generate a random number between 1 and [ ]. Select the employee corresponding to that number and then select every [ ] th employee on the list after that.

Items:

15 30 450

Part B Alternative method

One researcher suggests that it would be much easier to collect a sample by speaking to the first 30 employees in the canteen.

What type of sampling would this be?

Give one disadvantage of this type of sampling.

- This method would be time-consuming to carry out.
- Increasing the size of the sample can be very expensive.
- This method requires the use of a sampling frame.
- This method is likely to introduce bias towards employees who use the canteen.
- The sample is unlikely to be representative of the different groups among the employees.

Part C Stratified sample

The research institute also wishes to gather data from its research staff as to which new pieces of lab equipment will be required over the next year. The research staff within the institute consist of 40 geneticists, 25 ecologists and 55 epidemiologists. It is thought that the different types of staff will have different requirements for lab equipment.

Describe how the institute could collect a stratified sample of size 20 from its research staff.

There are a total of 120 research staff. The sample is $\square \div 120 = \square$ of the research staff.

The institute should survey $40 \times \square \approx \square$ geneticists, $25 \times \square \approx \square$ ecologists and $55 \times \square \approx \square$ epidemiologists. The staff should be selected randomly from each group, by generating random numbers from 1 to $\square$ for geneticists, 1 to $\square$ for ecologists and 1 to $\square$ for epidemiologists and selecting the corresponding members of staff, discarding and replacing any duplicate numbers within each group.

Items:

9 $\frac{5}{24}$ 20 5 55 6 25 10 4 $\frac{1}{6}$ $\frac{1}{3}$ 7 40 $\frac{11}{24}$

Probability 5.1

A Level P P P Further A P P P

Find the following probabilities.

Part A $P(Y)$ and $P(Y|X)$

It is given that $P(X) = 0.3$, $P(X \cup Y) = 0.6$ and $P(X \cap Y) = 0.2$.

Find $P(Y)$.

Find $P(Y|X)$, giving your answer as an exact fraction.

Part B $P(C \cap D)$ and $P(C|D')$

It is given that $P(C) = 0.6$, $P(D) = 0.5$ and $P((C \cup D)') = 0.3$.

Find $P(C \cap D)$.

Find $P(C|D')$.

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Probability 5.2

A Level P P P Further A P P P

Consider the situation in which $\mathrm{P}(\mathrm{X}) = 0.3$, $\mathrm{P}(\mathrm{X} \cup \mathrm{Y}) = 0.7$ and $\mathrm{P}(\mathrm{Y}) = k$. Find the value of k in the following situations.

Part A X and Y mutually exclusive

Find the value of k if X and Y are mutually exclusive.

Part B X and Y independent

Find the value of k if X and Y are independent. Give your answer as an exact fraction.

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Probabilities: Employment

A Level

Data about employment of people in their thirties and forties in a small rural area are shown in the following table.

	Unemployed	Employed
Thirties	206	412
Forties	358	305

A person from this area in these age groups is chosen at random. Let T be the event that the person is in their thirties and let E be the event that the person is employed.

Part A $P(T)$

Find $P(T)$.

Part B $P(T \text{ and } E)$

Find $P(T \text{ and } E)$.

Part C Independent events?

Are T and E independent events? Fill in the blanks below to complete the argument.

If T and E are independent, P(E|T) = [ ], i.e. the probability of being unemployed is [ ] irrespective of age.
Using the values in the table, P(E|T) = [ ] and [ ]. Therefore T and E [ ] independent events.

Items:

are not	$\frac{2}{3}$	**are**	**the same**	$\frac{412}{717}$	$P(E) = \frac{206}{427}$	$P(E) = \frac{239}{427}$	$\frac{412}{1281}$	P(E)	P(T)	**random**

Part D Unemployed and in their thirties

Given that the person chosen is unemployed, find the probability that the person is in their forties.

Adapted with permission from UCLES, A Level, CIE, January 2005, Paper 6 Probability & Statistics 1 (S1), Question 7

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Probability 5.3

A Level P P P Further A P P P

Part A Substandard samples

A laboratory has two devices A and B which produce samples for an experiment. Device A has produced 100 samples of which 5% are substandard. Device B has produced 25 of which 4% are substandard. An experimenter has found a substandard sample. Assuming that samples are chosen at random, what is the probability that it was produced by device B?

Part B Equipment failure

In hot weather the antiquated air-conditioning system in Professor A's laboratory may break down. On any given hot day, there is a 5% chance that the air-conditioning system breaks down. If the air-conditioning breaks down, the probability this will lead to the Professor's equipment failing by the end of the day is 0.3. If the air-conditioning does not break down, the probability that the equipment fails by the end of the day is only 0.05.

One hot day the Professor checks their lab first thing in the morning and the air-conditioning and equipment are both working. When the Professor gets ready to leave at the end of the day, they notice that their equipment has failed. What is the probability that the failure was not due to a breakdown of the air-conditioning system?

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Probability 5.4

A Level C C C Further A C C C

The probability of a randomly selected person in a population having a particular genetic trait is 0.00001. A test for this trait successfully detects it, if present, 99.9% of the time, and only returns a false positive 0.1% of the time.

Part A Probability after one test

A person tests positive for the trait. Find the probability that they actually have the genetic trait. Give your answer to 3 significant figures.

Part B Probability after two tests

In order to improve accuracy, individuals are instructed to take the test twice, regardless of the result of the first test.

What is the probability that an individual receives a positive result from both tests? Give your answer to 3 significant figures.

Find the probability, given that they have tested positive twice, that they actually have the genetic trait. Give your answer to 3 significant figures.

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