

Vectors: Diagrams and Proof 2ii

Subject & topics: Maths Stage & difficulty: A Level P1

Part A

Resultant vector

Find the resultant of the vectors $4\underline{i} - \underline{j}$ and $-2\underline{i} + 5\underline{j}$.

\underline{i} + \underline{j}

Part B

\overrightarrow{MN}

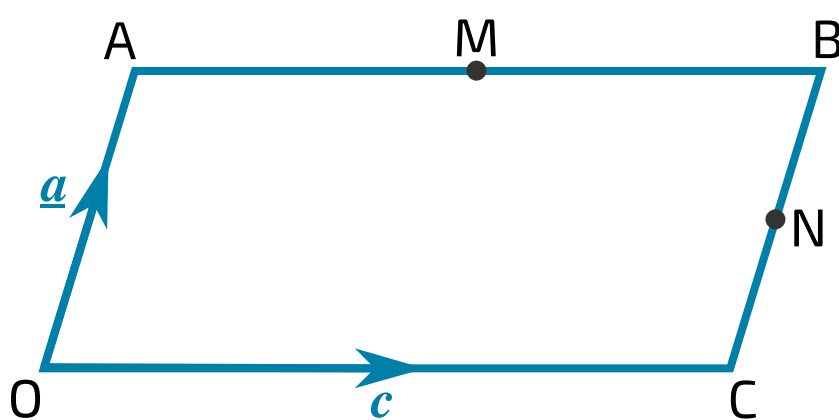


Figure 1: A parallelogram OABC.

In Figure 1, OABC is a parallelogram. M is the midpoint of AB. N is the midpoint of BC. $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OC} = \underline{c}$.

Find \overrightarrow{MN} in terms of \underline{a} and \underline{c} .

\underline{a} + \underline{c}



Vectors: Diagrams and Proof 1ii

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Part A
 $2\underline{s} - \underline{t}$

Given that $\underline{s} = 3\underline{i} + 4\underline{j}$ and $\underline{t} = 6\underline{i} - \underline{j}$, find $2\underline{s} - \underline{t}$.

$2\underline{s} - \underline{t} =$ $\underline{i} +$ \underline{j}

Part B
In terms of \underline{p}



Figure 1: Three points P, X and Q.

Figure 1 shows three points P, X and Q such that $\overrightarrow{XQ} = 3\overrightarrow{PX}$.

Given that $\overrightarrow{PX} = \underline{p}$, find \overrightarrow{XQ} and \overrightarrow{QP} in terms of \underline{p} .

If a value is not an integer, enter the value as a decimal.

$\overrightarrow{XQ} =$ \underline{p}
 $\overrightarrow{QP} =$ \underline{p}

Part C

Proving AMCN is a parallelogram

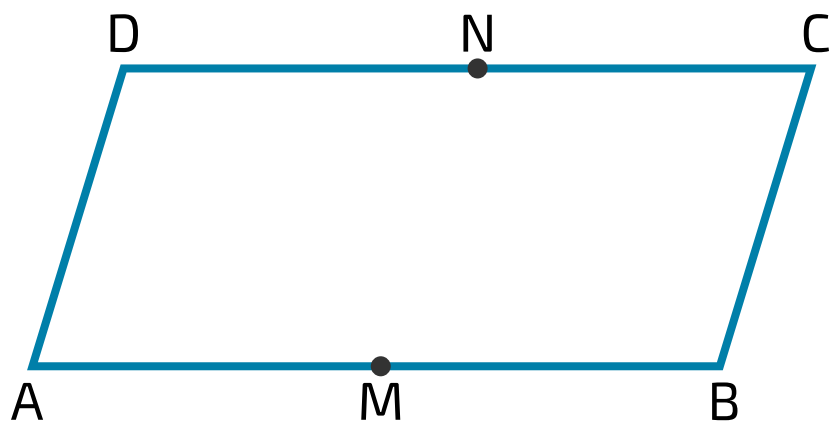


Figure 2: The parallelogram ABCD.

In Figure 2 ABCD is a parallelogram. M and N are the mid-points of AB and DC. $\overrightarrow{AB} = \underline{a}$ and $\overrightarrow{AD} = \underline{b}$. Use a vector method to prove that AMCN is also a parallelogram.

Choose four items from the left and put them into order on the right to create a proof.

Available items

1. A parallelogram has two pairs of sides which are parallel and of equal length.
1. A parallelogram has two pairs of sides which are parallel. All sides of a parallelogram are the same length.
2. $\overrightarrow{AM} = \overrightarrow{NC} = \frac{1}{2}\underline{a}$. Therefore \overrightarrow{AM} is parallel to \overrightarrow{NC} and has the same length.
2. $\overrightarrow{MB} = \overrightarrow{NC} = \frac{1}{2}\underline{a}$. Therefore \overrightarrow{MB} is parallel to \overrightarrow{NC} and has the same length.
3. $\overrightarrow{AD} = \overrightarrow{BC} = \underline{b}$. Therefore \overrightarrow{AD} is parallel to \overrightarrow{BC} and has the same length.
3. $\overrightarrow{AN} = \overrightarrow{MC} = \frac{1}{2}\underline{a} + \underline{b}$. Therefore \overrightarrow{AN} is parallel to \overrightarrow{MC} and has the same length.
4. AMCN has two pairs of sides which are parallel and of equal length. Hence, AMCN is a parallelogram.
4. AMCN has four sides which are parallel and of equal length. Hence, AMCN is a parallelogram.

Adapted with permission from UCLES, A Level, 2000



Vectors: Diagrams and Proof 2i

Subject & topics: Maths Stage & difficulty: A Level P2

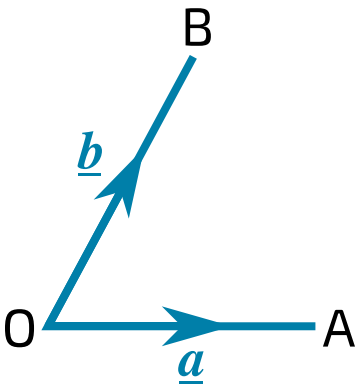


Figure 1: Points A and B and their position vectors with respect to the origin O.

In **Figure 1**, the points A and B have position vectors a and b with respect to the origin O.

Part A
Sketch

Make a sketch of the diagram, and mark on the points C, D and E such that $\overrightarrow{OC} = 2\underline{a}$, $\overrightarrow{OD} = 2\underline{a} + \underline{b}$ and $\overrightarrow{OE} = \frac{1}{3}\overrightarrow{OD}$.

Which of the sketches below correctly shows this information?

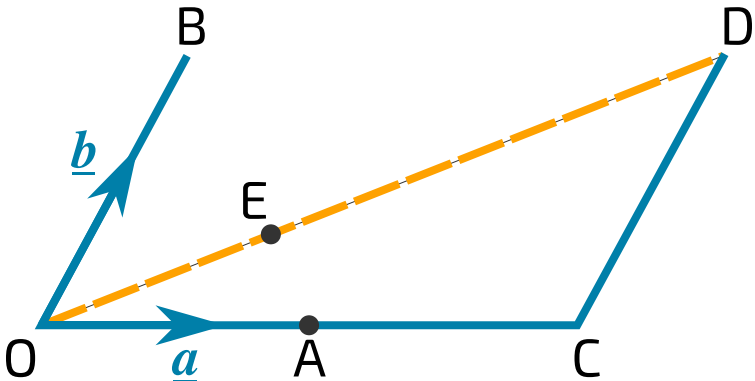


Figure 2: Option A

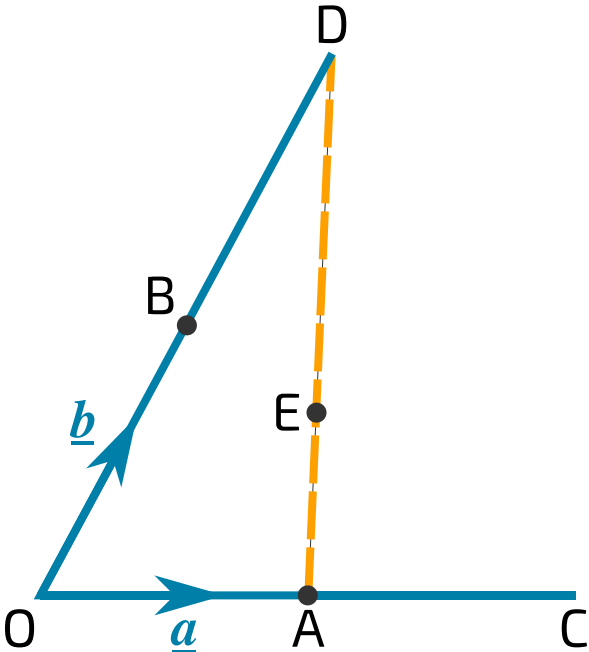


Figure 3: Option B

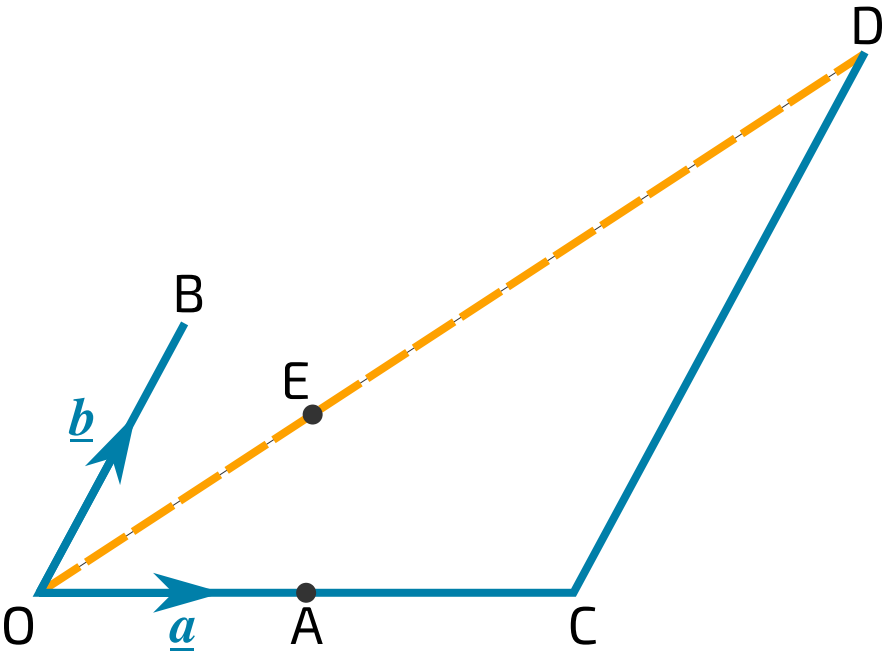


Figure 4: Option C

- ☐ Option A
- ☐ Option B
- ☐ Option C

Part B
Position vector of E

Give the position vector of E with respect to A.

a + **b**

Part C
Proof

Hence prove that E lies on the line joining A and B.

Choose three items from the choices on the left and put them into order on the right to create a proof.

Available items

1. Point *E* lies on the line joining A and B if the vector \overrightarrow{OE} is a scalar multiple of the vector \overrightarrow{OB} , i.e. $\overrightarrow{OE} = k\overrightarrow{OB}$.
1. Point E lies on the line joining A and B if the vector \overrightarrow{AE} is a scalar multiple of the vector \overrightarrow{AB} , i.e. $\overrightarrow{AE} = k\overrightarrow{AB}$.
2. $\overrightarrow{AE} = \frac{1}{3}(\underline{b} - \underline{a})$ and $\overrightarrow{AB} = \underline{b} - \underline{a}$. Therefore, $\overrightarrow{AE} = \frac{1}{3}\overrightarrow{AB}$.
2. $\overrightarrow{AE} = \underline{b} - \underline{a}$ and $\overrightarrow{AB} = \frac{1}{3}(\underline{b} - \underline{a})$. Therefore, $\overrightarrow{AE} = 3\overrightarrow{AB}$.
3. \overrightarrow{OE} is a scalar multiple of \overrightarrow{OD} . Hence, E lies on the line joining O and D.
3. \overrightarrow{AE} is a scalar multiple of \overrightarrow{AB} . Hence, E lies on the line joining A and B.

Question deck:

STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D



STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D

3D Vectors 2ii

Subject & topics: Maths **Stage & difficulty:** A Level P3

The points A and B have position vectors \underline{a} and \underline{b} relative to an origin O, where $\underline{a} = 4\underline{i} + 3\underline{j} - 2\underline{k}$ and $\underline{b} = -7\underline{i} + 5\underline{j} + 4\underline{k}$.

Part A

Length \overrightarrow{AB}

Find the length of \overrightarrow{AB} . Give your answer as an exact surd.

Part B

Unit vector

Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. Give your answer in terms of the unit vectors \underline{i} , \underline{j} and \underline{k} .

The following symbols may be useful: \underline{i} , \underline{j} , \underline{k}

Adapted with permission from UCLES, A Level, January 2007 & June 2011, OCR C4

Question deck:

STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D



STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D

3D Vectors 1ii

Subject & topics: Maths **Stage & difficulty:** A Level P3

ABCD is a parallelogram. The position vectors of A, B and C are given respectively by

$$\underline{a} = 2\underline{i} + \underline{j} + 3\underline{k} \qquad \underline{b} = 3\underline{i} - 2\underline{j} \qquad \underline{c} = \underline{i} - \underline{j} - 2\underline{k}$$

Part A

Position of D

Find the position vector of D. Give your answer in terms of the unit vectors \underline{i} , \underline{j} and \underline{k} .

The following symbols may be useful: \underline{i} , \underline{j} , \underline{k}

Part B

Unit vector

Find the unit vector in the direction \overrightarrow{OD} where O is the fixed origin. Give your answer in terms of the unit vectors \underline{i} , \underline{j} and \underline{k} .

The following symbols may be useful: \underline{i} , \underline{j} , \underline{k}

Adapted with permission from UCLES, A Level, June 2005, OCR C4, Question 5

Question deck:

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STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D

3D Vectors 2i

Subject & topics: Maths **Stage & difficulty:** A Level P2

ABCD is a quadrilateral. You are given four pieces of information:

- Relative to a fixed origin O , the position vector of A is $2\underline{i} + 5\underline{j} + 8\underline{k}$.
- Relative to a fixed origin O , the position vector of B is $5\underline{i} + 9\underline{j} + 8\underline{k}$.
- The vector $\overrightarrow{BC} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$.
- The vector $\overrightarrow{BD} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$.

Part A

Finding \overrightarrow{AB}

Find the vector \overrightarrow{AB} . Give your answer in $\underline{i}, \underline{j}, \underline{k}$ form.

$$\overrightarrow{AB} = \text{ } \underline{i} + \text{ } \underline{j} + \text{ } \underline{k}$$

Part B

Finding \overrightarrow{CD}

Find the vector \overrightarrow{CD} . Give your answer in $\underline{i}, \underline{j}, \underline{k}$ form.

$$\overrightarrow{CD} = \text{ } \underline{i} + \text{ } \underline{j} + \text{ } \underline{k}$$

Part C

Finding \overrightarrow{AD}

Find the vector \overrightarrow{AD} . Give your answer in $\underline{i}, \underline{j}, \underline{k}$ form.

$\overrightarrow{AD} =$ $\underline{i} +$ $\underline{j} +$ \underline{k}

Part D

Type of quadrilateral

The shape ABCD lies in a plane. What type of quadrilateral is ABCD?

- ☐ A rhombus
- ☐ A parallelogram
- ☐ An irregular quadrilateral
- ☐ A kite
- ☐ A trapezium
- ☐ A square
- ☐ A rectangle

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Question deck:
STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D



Vectors in 3D

Subject & topics: Maths | Geometry | Vectors **Stage & difficulty:** A Level P3

Given that $\underline{a} = 6\underline{i} + (p - 10)\underline{j} + (3p - 5)\underline{k}$, and that $|\underline{a}| = 11$, find the possible values of p .

Part A

Smaller value of p

Enter the smaller value of p :

Part B

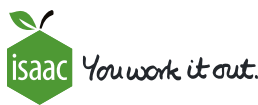
Larger value of p

Enter the larger value of p :

Adapted for Isaac Physics from NST IA Biology preparation work

Question deck:

STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D



STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D

Angles Between a 3D Vector and the Axes

Subject & topics: Maths | Geometry | Vectors **Stage & difficulty:** A Level P3

Find the angles between the vector $\underline{i} + 2\underline{j} + 3\underline{k}$ and the x -, y - and z -coordinate axes.

Part A

Angle with x -axis

What is the angle in degrees between the vector and the x -axis? Give your answer to 3 sf.

Part B

Angle with y -axis

What is the angle in degrees between the vector and the y -axis? Give your answer to 3 sf.

Part C

Angle with z -axis

What is the angle in degrees between the vector and the z -axis? Give your answer to 3 sf.

Adapted for Isaac Physics from NST IA Biology preparation work

Question deck:

STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D



Manipulating Vectors in 3D

Pre-Uni Maths for Sciences I1.10

Subject & topics: Maths | Geometry | Vectors Stage & difficulty: A Level C2

A vector $\underline{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$ has a length of 4.00 units.

Part A
Case 1

If \underline{u} lies in the (x, y) -plane, makes an angle of 30° with the x -direction and u_y is positive, find u_x .
Give your answer to 3 sf.

Part B
Case 2

If $u_x = u_y = 2$ and u_z is negative, find u_z .
Give your answer to 3 sf.

Part C
Case 3

If $u_z = 1$, $u_y = 2u_x$ and u_y is positive, find u_y .

Give your answer to 3 sf.

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Question deck:
STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D



3D Vectors 3ii

Subject & topics: Maths Stage & difficulty: A Level P3

Two points A and B have position vectors $3\underline{i} - \underline{j} + 2\underline{k}$ and $2\underline{j} + 3\underline{k}$ respectively.

Part A

Vector \overrightarrow{AB}

Find the vector \overrightarrow{AB} .

$$\overrightarrow{AB} = \text{ } \underline{i} + \text{ } \underline{j} + \text{ } \underline{k}$$

Hence find the length of \overrightarrow{AB} . Give your answer as an exact surd.

$$|\overrightarrow{AB}| = \sqrt{\text{ }}$$

Part B
Intersection

Show that the line through A and B does not intersect the line through the origin parallel to the vector \underline{i} .

One way to prove this is to use proof by contradiction. Fill in the blanks to complete the proof below.

Opening statement:

The line through the origin parallel to the vector \underline{i} is the x -axis. On the x -axis, $y = z =$. If the line through A and B intersects the x -axis, then there is a value of λ such that

$$\overrightarrow{OA} + \lambda \overrightarrow{AB} = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

where μ is the value of x where the line intersects the x -axis.

Calculations:

Putting in expressions for \overrightarrow{OA} and \overrightarrow{AB} ,

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \lambda \text{ } = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

This gives three equations, one for each of the x , y and z components:

$$3 + \text{ } \lambda = \mu, \quad -1 + \text{ } \lambda = 0 \quad \text{and} \quad 2 + \text{ } \lambda = 0$$

The second of these equations re-arranges to $\lambda =$, but the third equation rearranges to $\lambda =$. Hence, these equations are inconsistent and we have reached a contradiction.

Conclusion:

There is no point on the line through A and B for which y and z are both zero, so this line does not intersect the x -axis, and hence this line does not intersect the line through the origin parallel to the vector \underline{i} .

Items:

-3

-2

-1

0

1

2

3

$\frac{1}{3}$

$\frac{2}{3}$

$\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$

Used with permission from UCLES, A Level, January 2004, OCR P3, Question 5