

# Braking a Car

A Level



When a car of mass  $1000\text{ kg}$  is travelling along a level road at a steady speed of  $20\text{ m s}^{-1}$ , its engine is working at  $18\text{ kW}$ .

## Part A     Resistive force

Find the magnitude of the resistive force due to friction, which may be taken to be constant.

## Part B     Braking force on the level

The engine is suddenly disconnected and the brakes applied, and the car comes to rest in  $50\text{ m}$ . Find the force, assumed constant, exerted by the brakes.

## Part C     Braking force on a slope

Find also the distance in which the car, travelling at  $20\text{ m s}^{-1}$ , would come to rest if the engine were disconnected and the same braking force applied on an upward incline of angle  $\theta$ , where  $\sin \theta = \frac{1}{20}$ .

## Part D     Braking force downhill

By how much does this change if the car is travelling down the same hill at  $20\text{ m s}^{-1}$ ?

# Essential Pre-Uni Physics B9.2

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Assume that extension is proportional to the tension.

A spring of natural length 30 cm with spring constant  $8.0 \text{ N cm}^{-1}$  stretches by 20 % of its natural length. Work out how much elastic potential energy is stored in the spring. Give your answer to 2 significant figures.

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Gameboard:

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# Restitution Between Spheres

Two small spheres A and B, with masses  $0.30\text{ kg}$  and  $m$  respectively, lie at rest on a smooth horizontal surface. A is projected directly towards B with speed  $6.0\text{ m s}^{-1}$  and hits B. The direction of motion of A is reversed in the collision. The speeds of A and B after the collision are  $1.0\text{ m s}^{-1}$  and  $3.0\text{ m s}^{-1}$  respectively. The coefficient of restitution between A and B is  $e$ .

## Part A    Mass

Find  $m$ .

## Part B    Coefficient of restitution

Find the value of  $e$ .

## Part C    Second collision?

B continues to move at  $3.0\text{ m s}^{-1}$  and strikes a vertical wall at right angles and rebounds off the wall. The coefficient of restitution between B and the wall is  $f$ .

Find the range of values of  $f$  for which there will be a second collision between A and B. Fill in the gaps below.

$f$

Items:

- 3

-2

-1

$-\frac{2}{3}$

$-\frac{1}{2}$

$-\frac{1}{3}$

0

$\frac{1}{3}$

$\frac{1}{2}$

$\frac{2}{3}$

1

2

3

<

>

$\leq$

$\geq$

=

**Part D     Impulse from the wall**

Find, in terms of  $f$ , the magnitude of the impulse that the wall exerts on B.

The following symbols may be useful:  $\mathfrak{f}$

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**Part E     Final speeds**

Given that  $f = \frac{3}{4}$ ,

Calculate the final speed of A.

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Calculate the final speed of B.

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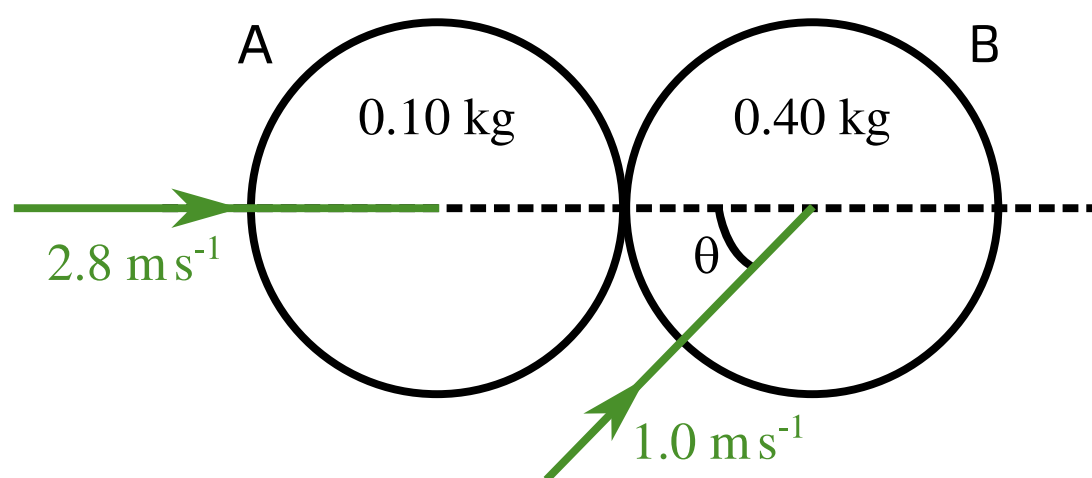
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# Restitution: Sphere Collision 2

Further A



Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass  $0.10 \text{ kg}$  and B has mass  $0.40 \text{ kg}$ . Immediately before the collision A is moving with speed  $2.8 \text{ m s}^{-1}$  along the line of centres, and B is moving with speed  $1.0 \text{ m s}^{-1}$  at an angle  $\theta$  to the line of centres, where  $\cos \theta = 0.80$ , as shown in **Figure 1**. Immediately after the collision A is stationary.



**Figure 1:** The spheres A and B as they collide.

## Part A Coefficient of restitution

Find the value of the coefficient of restitution between A and B.

## Part B Change in direction

Find the angle turned through by the direction of motion of B as a result of the collision.

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# Two Spinning Balls

A Level



A ball of mass  $m_1$  is suspended from a fixed point  $O$  by a light string of length  $l = 5.0 \text{ m}$  and a second ball of mass  $m_2$  is suspended from the first by a light string of the same length.

When the system is rotating steadily with angular velocity  $\omega$  about the vertical through  $O$ , the balls describe horizontal circles of radii  $3.0 \text{ m}$  and  $7.0 \text{ m}$  with their centres  $4.0 \text{ m}$  and  $7.0 \text{ m}$  respectively below  $O$ .

## Part A   Angular speed

Find the value of  $\omega$

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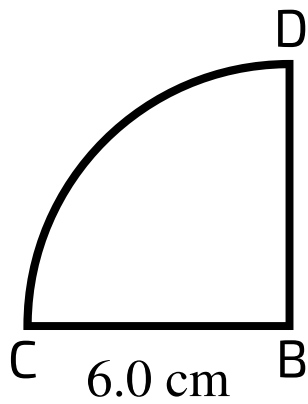
## Part B   Ratio of masses

What is the ratio of the masses of the balls? Express your answer as a decimal greater than one.

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# Centre of Mass: Lamina 2

**Figure 1** shows a uniform lamina BCD in the shape of a quarter circle of radius 6.0 cm.



**Figure 1:** Lamina BCD.

## Part A    Centre of mass of BCD

Find the distance of the centre of mass of the lamina from B.

Part B Distance from BD

A uniform rectangular lamina ABDE is such that AB is 12 cm and AE is 6.0 cm. A single plane object ABCDE is formed by attaching the rectangular lamina ABDE to the lamina BCD along BD, as shown in **Figure 2**. The mass of ABDE is 3.0 kg and the mass of BCD is 2.0 kg.

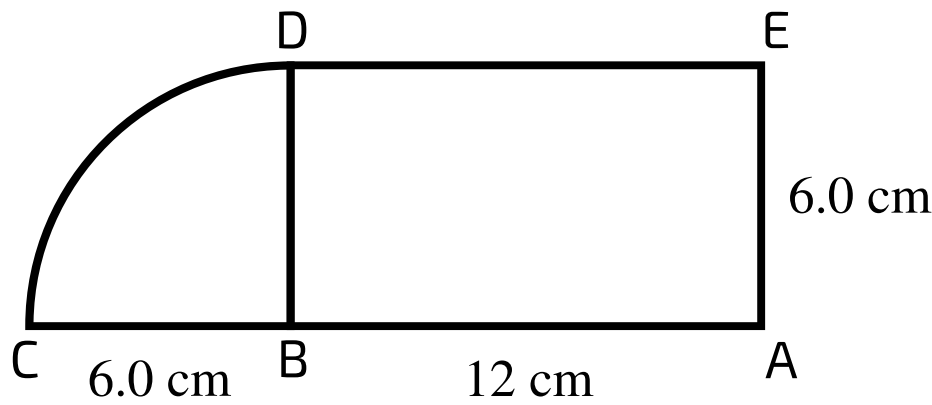


Figure 2: Object ABCDE, formed by joining laminas BCD and ABDE.

Find the distance of the centre of mass of the object ABCDE from BD.

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Part C Distance from AC

Find the distance of the centre of mass of the object ABCDE from AC.

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Part D Angle to vertical

The object ABCDE is freely suspended at C and rests in equilibrium.  
Calculate the angle that AC makes with the vertical.

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Adapted with permission from UCLES A Level, June 2009, Paper 4729, Question 5.



# Centre of Mass by Integration 2

Further A



The region bounded by the  $y$ -axis and the curves  $y = \sin(2x)$  and  $y = \sqrt{2} \cos x$  for  $0 \leq x \leq \frac{\pi}{4}$  is occupied by a uniform lamina.

Find the exact value of the  $x$ -coordinate of the centre of mass of the lamina.

The following symbols may be useful:  $\pi$ ,  $x$

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# Differential Equations: Resistive Forces 2i

Further A



A particle of mass  $0.2 \text{ kg}$  travels in a straight line on a smooth horizontal surface.

At time  $t$  seconds it is  $x \text{ m}$  from a fixed point  $O$  and is moving away from  $O$  with velocity  $v \text{ m s}^{-1}$ .

A force of magnitude  $\frac{1}{2} \left(12 - \frac{1}{4}v\right)^{\frac{1}{2}} \text{ N}$  acts on the particle in the direction of motion.

At time  $t = 0$  the particle is at  $O$  and has velocity  $12 \text{ m s}^{-1}$ .

## Part A   Maximum velocity

State the maximum possible velocity of the particle.

## Part B   Expression for $v$

Find an expression for  $v$  in terms of  $t$ , valid while the particle is accelerating.

The following symbols may be useful:  $t$

## Part C   Distance travelled

Hence find the distance travelled by the particle as its velocity increases from  $12 \text{ m s}^{-1}$  to  $32 \text{ m s}^{-1}$ .

Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, June 2017, Paper 4730, Question 3.

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# Differential Equations: General Applications 2ii

Further A



A particle  $P$  of mass  $0.2 \text{ kg}$  is suspended from a fixed point  $O$  by a light elastic string of natural length  $0.7 \text{ m}$  and modulus of elasticity  $3.5 \text{ N}$ .  $P$  is at the equilibrium position when it is projected vertically downwards with speed  $1.6 \text{ m s}^{-1}$ . At time  $t \text{ s}$  after being set in motion  $P$  is  $x \text{ m}$  below the equilibrium position and has velocity  $v \text{ m s}^{-1}$ .

The tension,  $T$ , in the string is expressed as

$$T = \frac{3.5(0.392 + x)}{0.7} \text{ N}$$

The equilibrium position of  $P$  is  $1.092 \text{ m}$  below  $O$ , and the strength of gravity is  $9.8 \text{ N kg}^{-1}$ .

## Part A SHM

Prove that  $P$  moves with simple harmonic motion, and calculate the amplitude.

## Part B Find $x$

Calculate  $x$  when  $t = 0.4$ . Give your answer to 3 significant figures.

## Part C Find $v$

Calculate the velocity of  $P$  when  $t = 0.4$ . Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, January 2007 , Paper 4730, Question 4.