Home   Gameboard   Maths   Calculus   Integration   Integrating Powers 2

Integrating Powers 2

A Level Further A

P P P   P P P

Part A   Integrate ax^{-8}

Find the indefinite integral of ax^{-8}, where a is a constant.

The following symbols may be useful: `a, k, x`

Part B   Integrate $\frac{4}{x^2}$

Find $\displaystyle\int_{1}^{2} \frac{4}{x^2}\,\mathrm{d}x$.

The following symbols may be useful: `k, x`

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Indefinite Integrals 1

A Level Further A

Find the following indefinite integrals.

Part A   Integrate $(3x - 1)(x + 1)$

Find $\int (3x - 1)(x + 1)\,\mathrm{d}x.$

The following symbols may be useful: `k, x`

Part B   Integrate $\left(\sqrt{p} - \frac{1}{p}\right)^2$

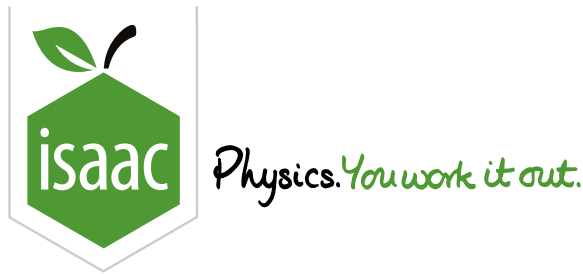Find $\displaystyle\int \left(\sqrt{p} - \frac{1}{p}\right)^2 \mathrm{d}p.$

The following symbols may be useful: `k, p`

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Indefinite Integrals 2

A Level   Further A

Find the following indefinite integrals.

Part A   **Integrate** $\frac{q^2+3}{q^{\frac{5}{2}}}$

Find $\displaystyle\int \frac{q^2 + 3}{q^{\frac{5}{2}}}\,\mathrm{d}q.$

The following symbols may be useful: `k, q`

Part B   **Integrate** $2z(z^2 - 1)(z^2 + 1)$

Find $\int 2z(z^2 - 1)(z^2 + 1)\mathrm{d}z.$

The following symbols may be useful: `k, z`

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Integrating Powers 4

A Level Further A

C C C C C C

Part A   **Integrate** $\frac{A}{r^7} - \frac{B}{r^{13}}$

Find $\int_a^\infty \left(\frac{A}{r^7} - \frac{B}{r^{13}} \right) dr.$

(The force between, for example, two atoms of an inert gas, a distance r apart is given by $\left(\frac{A}{r^7} - \frac{B}{r^{13}} \right)$, where A and B are (negative) constants; the first term is the attractive force between them (the van der Waals interaction, due to their fluctuating induced dipoles) and the second is the repulsive force due to the overlap of their electron shells. The integral describes the potential energy of such a system i.e. the work done bringing one atom from infinity to within a distance a of the other atom.)

Find $\int_a^\infty \left(\frac{A}{r^7} - \frac{B}{r^{13}} \right) dr.$

The following symbols may be useful: `A, B, a`

Part B   **Integrate** $\frac{C}{x^2} + D$

Find $\int_{x_1}^{x_2} \left(\frac{C}{x^2} + D \right) dx.$

(The function $\left(\frac{C}{x^2} + D \right)$, where C and D are constants, could describe the component of an electric field in the x-direction due to a combination of the field due to a point charge at the origin and a uniform field in the x-direction. The integral is then the potential difference between two points x_1 and x_2 on the x-axis.)

Find $\int_{x_1}^{x_2} \left(\frac{C}{x^2} + D \right) dx.$

The following symbols may be useful: `C, D, x_1, x_2`

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Functions from Differential Equations 2ii

A Level

P P P

The gradient of a curve is given by $\frac{dy}{dx} = 6x - 4$. The curve passes through the distinct points $(2, 5)$ and $(p, 5)$.

Part A   Equation of curve

Find the equation of the curve.

The following symbols may be useful: x, y

Part B   Find p

Find the value of p.

The following symbols may be useful: p

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Equation of Curve

A Level

P P P

A function $v(u)$ is such that $\dfrac{\mathrm{d}v}{\mathrm{d}u} = \dfrac{1}{3}u^{\frac{1}{3}}\left(1 - \dfrac{1}{u}\right)$ and $v(8) = -1$.

Find the equation of the function $v(u)$.

The following symbols may be useful: `u, v`

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Integration and Area 3i

A Level

P P P

Figure 1 shows the graph of $y = 1 - 3x^{-\frac{1}{2}}$.
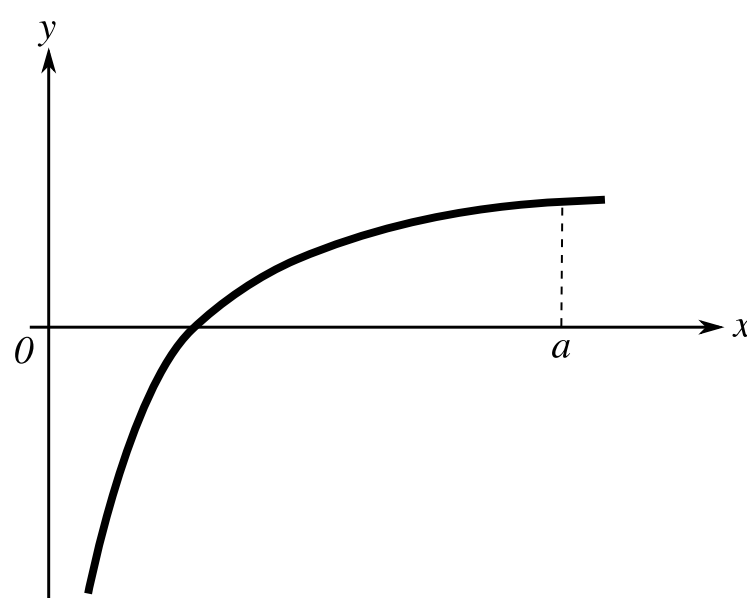

Figure 1: Graph of $y = 1 - 3x^{-\frac{1}{2}}$.

Part A   Find intersection with x-axis

Find the x-coordinate of the intersection of that curve with the x-axis.

The following symbols may be useful: x

Part B   Find a

The region enclosed by the curve, the x-axis and the line $x = a$ (where $a > 9$) has an area equal to 4 square units, find the value of a.

The following symbols may be useful: a

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Home　　Gameboard　　Maths　　Calculus　　Integration　　Area Under a Curve 1

Area Under a Curve 1

A Level

P　P　P

A graph of the functions $y = (x - 2)(x + 1)$ and $y = x + 1$ is shown in **Figure 1**. Find the areas of the shaded regions labelled A and B. A is the region between P and Q enclosed by the curve $y = (x - 2)(x + 1)$ and the x-axis; B is the region between Q and R below the curve $y = (x - 2)(x + 1)$ and above the x-axis.
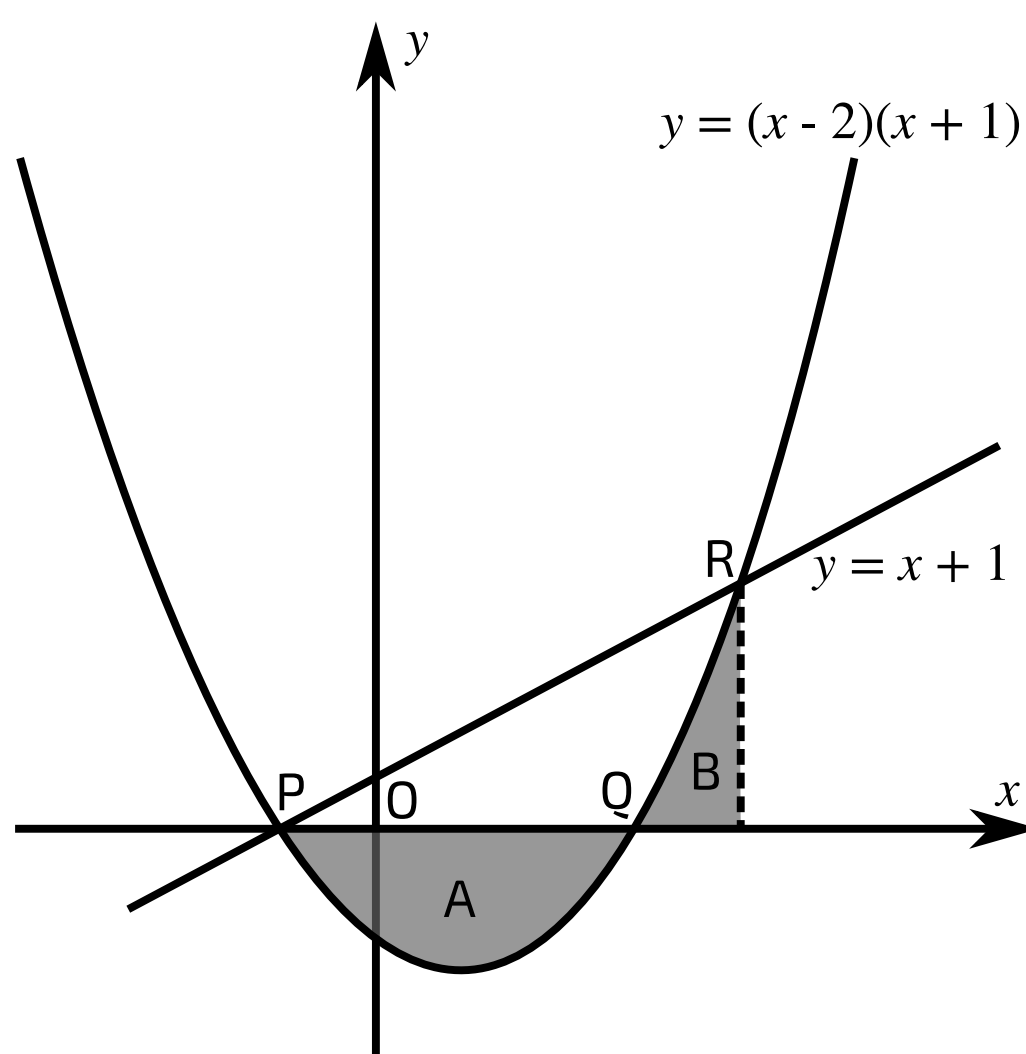

Figure 1: A graph of the functions $y = (x - 2)(x + 1)$ and $y = x + 1$. A is the region between P and Q enclosed by the curve $y = (x - 2)(x + 1)$ and the x-axis; B is the region between Q and R below the curve $y = (x - 2)(x + 1)$ and above the x-axis.

Part A　　**Region A**

Find the area of the region A. Give your answer in the form of an improper fraction.

Part B   Region B

Find the area of the region B. Give your answer in the form of an improper fraction.

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Part B   Region B

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Area Under a Curve 2

A Level   Further A

A graph of the functions $y = x^2 + 3$ and $y = 4x$ is shown in **Figure 1**. Find the area of the shaded region labelled A, the region between the line $y = 4x$ and the curve $y = x^2 + 3$.
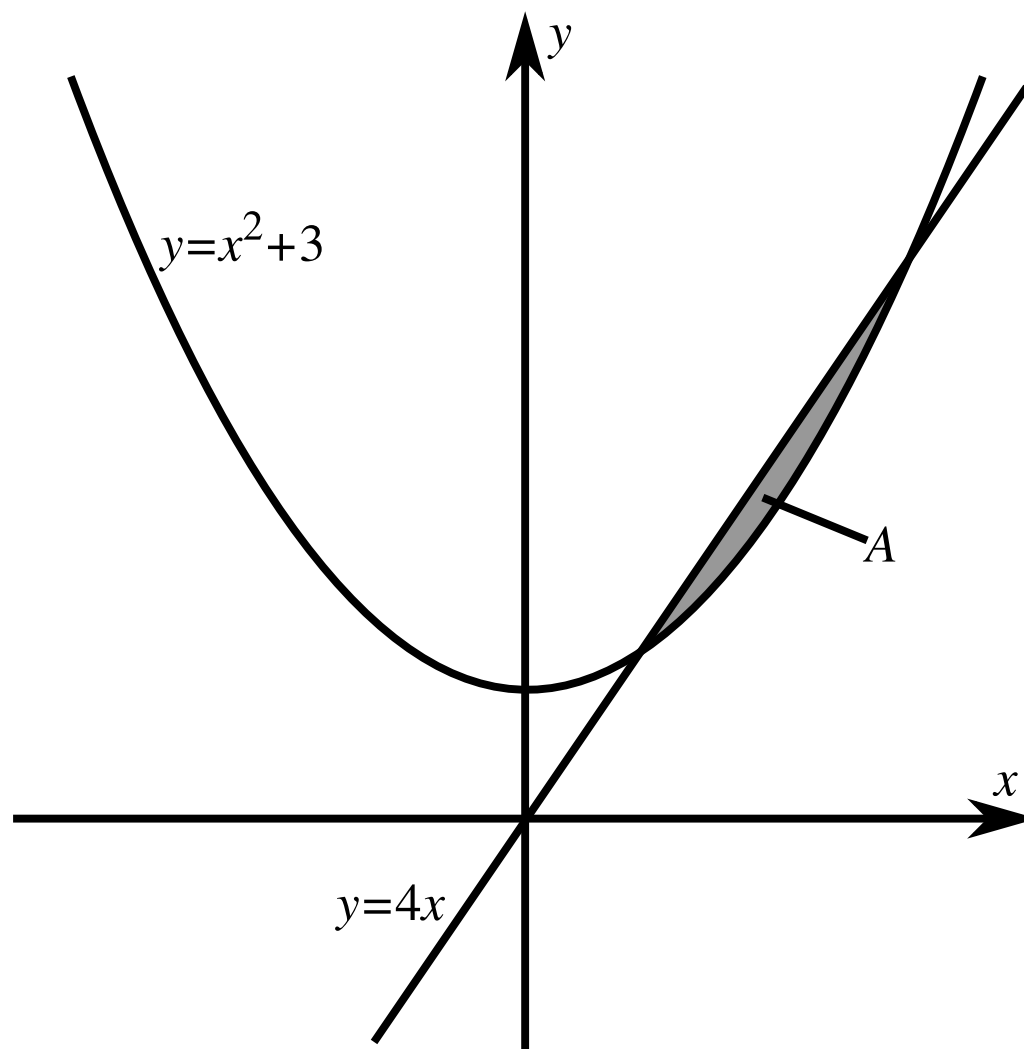

Figure 1: A graph of the functions $y = x^2 + 3$ and $y = 4x$. The shaded area A is the region between the line $y = 4x$ and the curve $y = x^2 + 3$.

Find the area of the region A. Give your answer in the form of an improper fraction.

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Area Under a Curve 3

A Level Further A

A graph of the functions $y = \dfrac{1}{2\sqrt{x}}$ and $y = 2x\sqrt{x}$ for $x \geq 0$ is shown in **Figure 1**. Find the area of the shaded region OPQR.
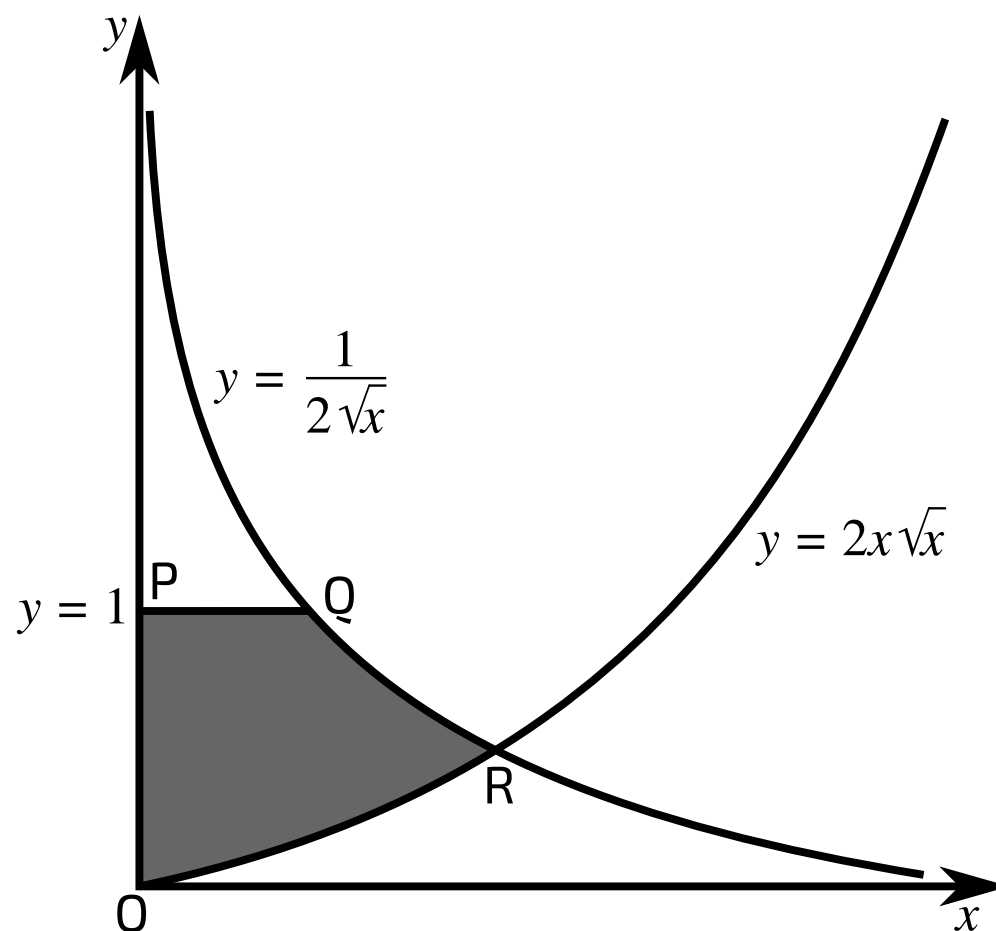

Figure 1: A graph of the functions $y = \dfrac{1}{2\sqrt{x}}$ and $y = 2x\sqrt{x}$ for $x \geq 0$. The shaded region OPQR is bounded by the line $x = 0$, the line $y = 1$, the curve $y = \dfrac{1}{2\sqrt{x}}$ and the curve $y = 2x\sqrt{x}$.

Part A   The x coordinate of Q

Deduce the x coordinate of the point Q.

Part B   The x coordinate of R

Find the x coordinate of the point R.

Part C   The area of OPQR

Find the area of the shaded region OPQR, giving your answer in an exact form.

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Part B   The x coordinate of R

Find the x coordinate of the point R.