

## Vectors: Diagrams and Proof 2ii

**Subject & topics:** Maths    **Stage & difficulty:** A Level P1

### Part A

#### Resultant vector

Find the resultant of the vectors  $4\underline{i} - \underline{j}$  and  $-2\underline{i} + 5\underline{j}$ .

$$\boxed{\quad} \underline{i} + \boxed{\quad} \underline{j}$$

### Part B

$\overrightarrow{MN}$

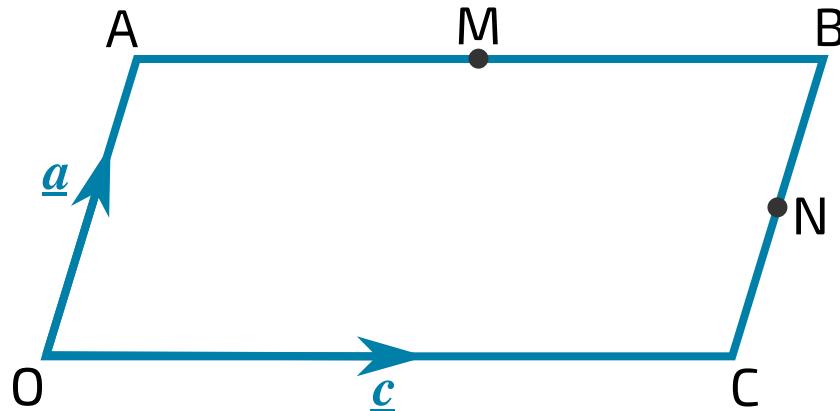


Figure 1: A parallelogram OABC.

In Figure 1, OABC is a parallelogram. M is the midpoint of AB. N is the midpoint of BC.  $\overrightarrow{OA} = \underline{a}$  and  $\overrightarrow{OC} = \underline{c}$ .

Find  $\overrightarrow{MN}$  in terms of  $\underline{a}$  and  $\underline{c}$ .

$$\boxed{\quad} \underline{a} + \boxed{\quad} \underline{c}$$





## Vectors: Diagrams and Proof 1ii

**Subject & topics:** Maths    **Stage & difficulty:** A Level P1

### Part A

$2s - t$

Given that  $\underline{s} = 3\underline{i} + 4\underline{j}$  and  $\underline{t} = 6\underline{i} - \underline{j}$ , find  $2\underline{s} - \underline{t}$ .

$$2\underline{s} - \underline{t} = \quad \underline{i} + \quad \underline{j}$$

### Part B

In terms of  $p$



**Figure 1:** Three points P, X and Q.

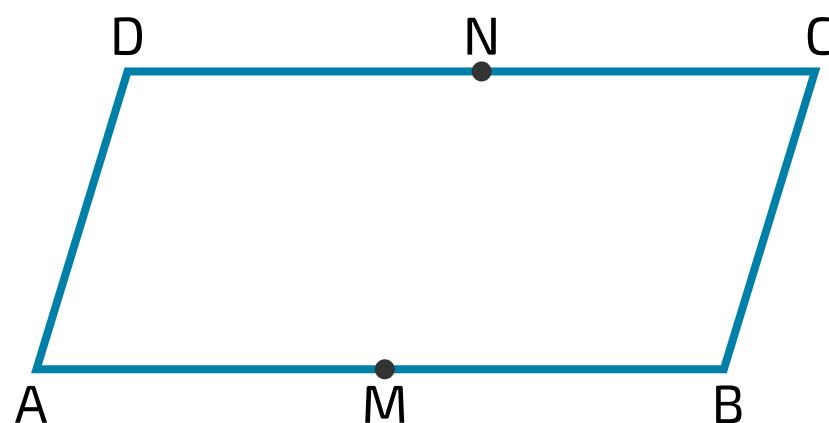
**Figure 1** shows three points P, X and Q such that  $\overrightarrow{XQ} = 3\overrightarrow{PX}$ .

Given that  $\overrightarrow{PX} = \underline{p}$ , find  $\overrightarrow{XQ}$  and  $\overrightarrow{QP}$  in terms of  $\underline{p}$ .

If a value is not an integer, enter the value as a decimal.

$$\overrightarrow{XQ} = \quad \underline{p}$$

$$\overrightarrow{QP} = \quad \underline{p}$$

**Part C****Proving AMCN is a parallelogram****Figure 2:** The parallelogram ABCD.

In **Figure 2** ABCD is a parallelogram. M and N are the mid-points of AB and DC.  $\overrightarrow{AB} = \underline{a}$  and  $\overrightarrow{AD} = \underline{b}$ . Use a vector method to prove that AMCN is also a parallelogram.

Choose four items from the left and put them into order on the right to create a proof.

**Available items**

1. A parallelogram has two pairs of sides which are parallel and of equal length.

1. A parallelogram has two pairs of sides which are parallel. All sides of a parallelogram are the same length.

2.  $\overrightarrow{AM} = \overrightarrow{NC} = \frac{1}{2}\underline{a}$ . Therefore  $\overrightarrow{AM}$  is parallel to  $\overrightarrow{NC}$  and has the same length.

2.  $\overrightarrow{MB} = \overrightarrow{NC} = \frac{1}{2}\underline{a}$ . Therefore  $\overrightarrow{MB}$  is parallel to  $\overrightarrow{NC}$  and has the same length.

3.  $\overrightarrow{AD} = \overrightarrow{BC} = \underline{b}$ . Therefore  $\overrightarrow{AD}$  is parallel to  $\overrightarrow{BC}$  and has the same length.

3.  $\overrightarrow{AN} = \overrightarrow{MC} = \frac{1}{2}\underline{a} + \underline{b}$ . Therefore  $\overrightarrow{AN}$  is parallel to  $\overrightarrow{MC}$  and has the same length.

4. AMCN has two pairs of sides which are parallel and of equal length. Hence, AMCN is a parallelogram.

4. AMCN has four sides which are parallel and of equal length. Hence, AMCN is a parallelogram.

Adapted with permission from UCLES, A Level, 2000

Question deck:

[STEM SMART Single Maths 37 - Vectors: Geometrical Proofs &](#)

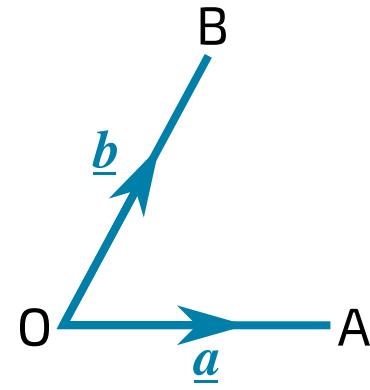
[Vectors in 3D](#)



## Vectors: Diagrams and Proof 2i

**Subject & topics:** Maths    **Stage & difficulty:** A Level P2

---



**Figure 1:** Points A and B and their position vectors with respect to the origin O.

In **Figure 1**, the points A and B have position vectors  $\underline{a}$  and  $\underline{b}$  with respect to the origin O.

**Part A**  
**Sketch**

Make a sketch of the diagram, and mark on the points C, D and E such that  $\overrightarrow{OC} = 2\underline{a}$ ,  $\overrightarrow{OD} = 2\underline{a} + \underline{b}$  and  $\overrightarrow{OE} = \frac{1}{3}\overrightarrow{OD}$ .

Which of the sketches below correctly shows this information?

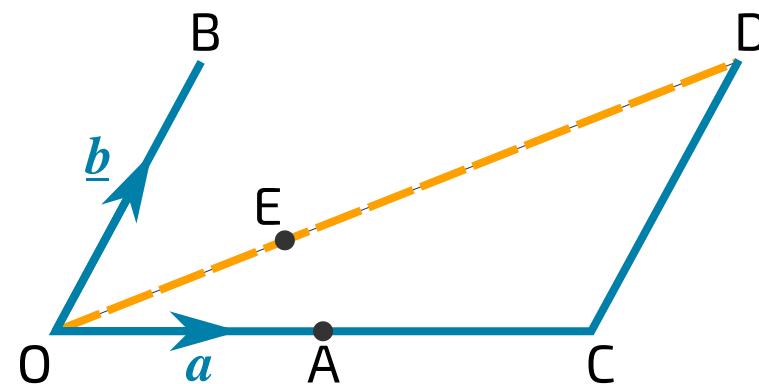


Figure 2: Option A

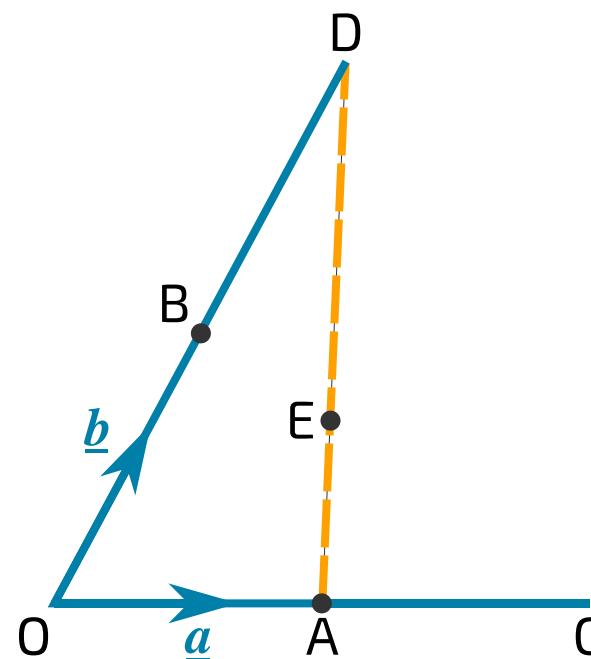


Figure 3: Option B

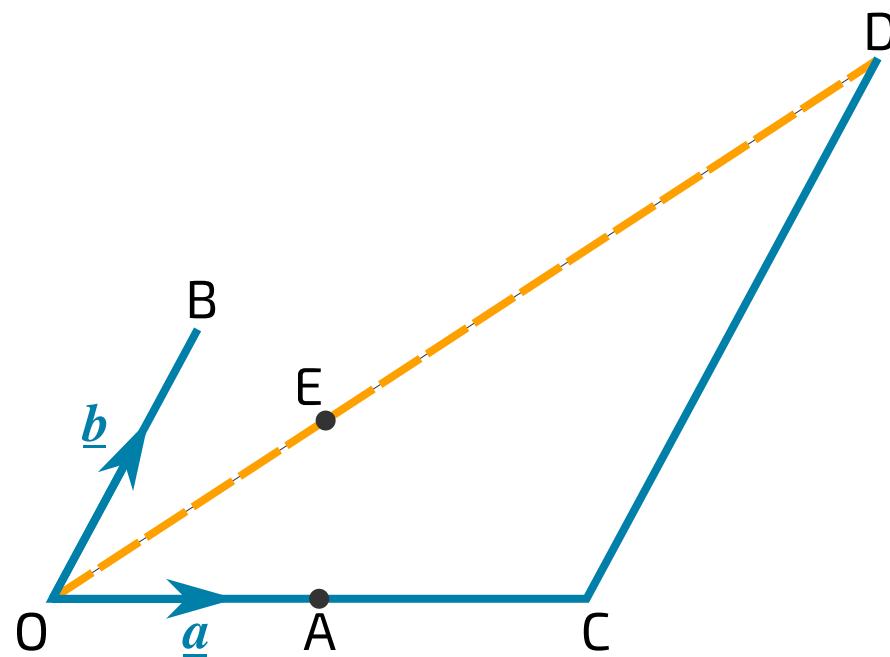


Figure 4: Option C

- Option A
- Option B
- Option C

**Part B****Position vector of E**

Give the position vector of E with respect to A.

$$\boxed{\quad} \underline{a} + \boxed{\quad} \underline{b}$$

**Part C****Proof**

Hence prove that E lies on the line joining A and B.

Choose three items from the choices on the left and put them into order on the right to create a proof.

**Available items**

1. Point E lies on the line joining A and B if the vector  $\overrightarrow{OE}$  is a scalar multiple of the vector  $\overrightarrow{OB}$ , i.e.  $\overrightarrow{OE} = k\overrightarrow{OB}$ .

1. Point E lies on the line joining A and B if the vector  $\overrightarrow{AE}$  is a scalar multiple of the vector  $\overrightarrow{AB}$ , i.e.  $\overrightarrow{AE} = k\overrightarrow{AB}$ .

2.  $\overrightarrow{AE} = \frac{1}{3}(\underline{b} - \underline{a})$  and  $\overrightarrow{AB} = \underline{b} - \underline{a}$ . Therefore,  $\overrightarrow{AE} = \frac{1}{3}\overrightarrow{AB}$ .

2.  $\overrightarrow{AE} = \underline{b} - \underline{a}$  and  $\overrightarrow{AB} = \frac{1}{3}(\underline{b} - \underline{a})$ . Therefore,  $\overrightarrow{AE} = 3\overrightarrow{AB}$ .

3.  $\overrightarrow{OE}$  is a scalar multiple of  $\overrightarrow{OD}$ . Hence, E lies on the line joining O and D.

3.  $\overrightarrow{AE}$  is a scalar multiple of  $\overrightarrow{AB}$ . Hence, E lies on the line joining A and B.

Question deck:

**STEM SMART Single Maths 37 - Vectors: Geometrical Proofs &**

**Vectors in 3D**



## 3D Vectors 2ii

**Subject & topics:** Maths    **Stage & difficulty:** A Level P3

The points A and B have position vectors  $\underline{a}$  and  $\underline{b}$  relative to an origin O, where  $\underline{a} = 4\underline{i} + 3\underline{j} - 2\underline{k}$  and  $\underline{b} = -7\underline{i} + 5\underline{j} + 4\underline{k}$ .

**Part A**

**Length  $\overrightarrow{AB}$**

Find the length of  $\overrightarrow{AB}$ . Give your answer as an exact surd.

**Part B**

**Unit vector**

Find the unit vector in the direction of  $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$ . Give your answer in terms of the unit vectors  $\underline{i}$ ,  $\underline{j}$  and  $\underline{k}$ .

The following symbols may be useful:  $i$ ,  $j$ ,  $k$

Adapted with permission from UCLES, A Level, January 2007 & June 2011, OCR C4

Question deck:

[STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D](#)



## STEM SMART Single Maths 37 - Vectors: Geometrical Proofs &amp; Vectors in 3D

**3D Vectors 1ii****Subject & topics:** Maths    **Stage & difficulty:** A Level P3

ABCD is a parallelogram. The position vectors of A, B and C are given respectively by

$$\underline{a} = 2\underline{i} + \underline{j} + 3\underline{k} \quad \underline{b} = 3\underline{i} - 2\underline{j} \quad \underline{c} = \underline{i} - \underline{j} - 2\underline{k}$$

**Part A****Position of D**

Find the position vector of D. Give your answer in terms of the unit vectors  $\underline{i}$ ,  $\underline{j}$  and  $\underline{k}$ .

The following symbols may be useful:  $i$ ,  $j$ ,  $k$

**Part B****Unit vector**

Find the unit vector in the direction  $\overrightarrow{OD}$  where O is the fixed origin. Give your answer in terms of the unit vectors  $\underline{i}$ ,  $\underline{j}$  and  $\underline{k}$ .

The following symbols may be useful:  $i$ ,  $j$ ,  $k$

Adapted with permission from UCLES, A Level, June 2005, OCR C4, Question 5

Question deck:

[STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D](#)



## 3D Vectors 2i

**Subject & topics:** Maths    **Stage & difficulty:** A Level P2

ABCD is a quadrilateral. You are given four pieces of information:

- Relative to a fixed origin O, the position vector of A is  $2\underline{i} + 5\underline{j} + 8\underline{k}$ .
- Relative to a fixed origin O, the position vector of B is  $5\underline{i} + 9\underline{j} + 8\underline{k}$ .
- The vector  $\overrightarrow{BC} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$ .
- The vector  $\overrightarrow{BD} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$ .

### Part A

#### Finding $\overrightarrow{AB}$

Find the vector  $\overrightarrow{AB}$ . Give your answer in  $\underline{i}, \underline{j}, \underline{k}$  form.

$$\overrightarrow{AB} = \boxed{\phantom{0}} \underline{i} + \boxed{\phantom{0}} \underline{j} + \boxed{\phantom{0}} \underline{k}$$

### Part B

#### Finding $\overrightarrow{CD}$

Find the vector  $\overrightarrow{CD}$ . Give your answer in  $\underline{i}, \underline{j}, \underline{k}$  form.

$$\overrightarrow{CD} = \boxed{\phantom{0}} \underline{i} + \boxed{\phantom{0}} \underline{j} + \boxed{\phantom{0}} \underline{k}$$

**Part C****Finding  $\overrightarrow{AD}$** 

Find the vector  $\overrightarrow{AD}$ . Give your answer in  $\underline{i}, \underline{j}, \underline{k}$  form.

$$\overrightarrow{AD} = \boxed{\phantom{0}} \underline{i} + \boxed{\phantom{0}} \underline{j} + \boxed{\phantom{0}} \underline{k}$$

**Part D****Type of quadrilateral**

The shape ABCD lies in a plane. What type of quadrilateral is ABCD?

- A rhombus
- A parallelogram
- An irregular quadrilateral
- A kite
- A trapezium
- A square
- A rectangle

Created for isaacphysics.org by Jonathan Waugh

Question deck:

[STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D](#)



## STEM SMART Single Maths 37 - Vectors: Geometrical Proofs &amp; Vectors in 3D

## Vectors in 3D

**Subject & topics:** Maths | Geometry | Vectors    **Stage & difficulty:** A Level P3

Given that  $\underline{a} = 6\underline{i} + (p - 10)\underline{j} + (3p - 5)\underline{k}$ , and that  $|\underline{a}| = 11$ , find the possible values of  $p$ .

**Part A**

**Smaller value of  $p$**

Enter the smaller value of  $p$ :

**Part B**

**Larger value of  $p$**

Enter the larger value of  $p$ :

Adapted for Isaac Physics from NST IA Biology preparation work

Question deck:

[STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D](#)



STEM SMART Single Maths 37 - Vectors: Geometrical Proofs &amp; Vectors in 3D

## Angles Between a 3D Vector and the Axes

**Subject & topics:** Maths | Geometry | Vectors**Stage & difficulty:** A Level P3

Find the angles between the vector  $\underline{i} + 2\underline{j} + 3\underline{k}$  and the  $x$ -,  $y$ - and  $z$ -coordinate axes.

**Part A****Angle with  $x$ -axis**

What is the angle in degrees between the vector and the  $x$ -axis? Give your answer to 3 sf.

**Part B****Angle with  $y$ -axis**

What is the angle in degrees between the vector and the  $y$ -axis? Give your answer to 3 sf.

**Part C****Angle with  $z$ -axis**

What is the angle in degrees between the vector and the  $z$ -axis? Give your answer to 3 sf.

Adapted for Isaac Physics from NST IA Biology preparation work

Question deck:

**STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D**



## STEM SMART Single Maths 37 - Vectors: Geometrical Proofs &amp; Vectors in 3D

## Manipulating Vectors in 3D

Pre-Uni Maths for Sciences I1.10

**Subject & topics:** Maths | Geometry | Vectors    **Stage & difficulty:** A Level C2

A vector  $\underline{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$  has a length of 4.00 units.

### Part A

#### Case 1

If  $\underline{u}$  lies in the  $(x, y)$ -plane, makes an angle of  $30^\circ$  with the  $x$ -direction and  $u_y$  is positive, find  $u_x$ .

Give your answer to 3 sf.

### Part B

#### Case 2

If  $u_x = u_y = 2$  and  $u_z$  is negative, find  $u_z$ .

Give your answer to 3 sf.

**Part C**  
**Case 3**

If  $u_z = 1$ ,  $u_y = 2u_x$  and  $u_y$  is positive, find  $u_y$ .

Give your answer to 3 sf.

Created for isaacphysics.org by Julia Riley.

Question deck:

**STEM SMART Single Maths 37 - Vectors: Geometrical Proofs & Vectors in 3D**



## STEM SMART Single Maths 37 - Vectors: Geometrical Proofs &amp; Vectors in 3D

## 3D Vectors 3ii

**Subject & topics:** Maths    **Stage & difficulty:** A Level P3

Two points A and B have position vectors  $3\underline{i} - \underline{j} + 2\underline{k}$  and  $2\underline{j} + 3\underline{k}$  respectively.

**Part A****Vector  $\overrightarrow{AB}$** 

Find the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \boxed{\phantom{0}} \underline{i} + \boxed{\phantom{0}} \underline{j} + \boxed{\phantom{0}} \underline{k}$$

Hence find the length of  $\overrightarrow{AB}$ . Give your answer as an exact surd.

$$|\overrightarrow{AB}| = \sqrt{\boxed{\phantom{000}}}$$

## Part B

### Intersection

Show that the line through A and B does not intersect the line through the origin parallel to the vector  $\underline{i}$ .

One way to prove this is to use proof by contradiction. Fill in the blanks to complete the proof below.

**Opening statement:**

The line through the origin parallel to the vector  $\underline{i}$  is the  $x$ -axis. On the  $x$ -axis,  $y = z = \boxed{\phantom{0}}$ . If the line through A and B intersects the  $x$ -axis, then there is a value of  $\lambda$  such that

$$\overrightarrow{OA} + \lambda \overrightarrow{AB} = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

where  $\mu$  is the value of  $x$  where the line intersects the  $x$ -axis.

**Calculations:**

Putting in expressions for  $\overrightarrow{OA}$  and  $\overrightarrow{AB}$ ,

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \lambda \boxed{\phantom{0}} = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

This gives three equations, one for each of the  $x$ ,  $y$  and  $z$  components:

$$3 + \boxed{\phantom{0}} \lambda = \mu, \quad -1 + \boxed{\phantom{0}} \lambda = 0 \quad \text{and} \quad 2 + \boxed{\phantom{0}} \lambda = 0$$

The second of these equations re-arranges to  $\lambda = \boxed{\phantom{0}}$ , but the third equation rearranges to  $\lambda = \boxed{\phantom{0}}$ . Hence, these equations are inconsistent and we have reached a contradiction.

**Conclusion:**

There is no point on the line through A and B for which  $y$  and  $z$  are both zero, so this line does not intersect the  $x$ -axis, and hence this line does not intersect the line through the origin parallel to the vector  $\underline{i}$ .

Items:

- |                          |                          |                          |                          |                          |                          |                          |                          |                          |                          |
|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| <input type="checkbox"/> |
|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|