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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Expand and Simplify Binomials

Pre-Uni Maths for Sciences C1.1

Subject & topics: Maths | Algebra | Series   **Stage & difficulty:** A Level P1

Part A

$(x+1)^4$

Expand and simplify $(x+1)^4$.

The following symbols may be useful: x

Part B

$(z+2a)^3$

Expand and simplify $(z+2a)^3$.

The following symbols may be useful: a, z

Part C

$(a-b)^5$

Expand and simplify $(a-b)^5$.

The following symbols may be useful: a, b

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Find Coefficients 2

Pre-Uni Maths for Sciences C1.3

Subject & topics: Maths | Algebra | Series   **Stage & difficulty:** A Level P1

Without expanding the binomials, find:

Part A

Coefficient of $x^4 y^6$

The coefficient of $x^4 y^6$ in the expansion of $(x^2 + 3y^2)^5$.

Part B

Coefficient of x^{20}

The coefficient of x^{20} in the expansion of $(x^2 + 3x)^{12}$.

Part C

The coefficient of ab^7

The coefficient of ab^7 in the expansion of $(a + \frac{1}{4}b)^8$.

Part D
Constant term

The constant term in the expansion of $\left(\dfrac{x^2}{2} - \dfrac{8}{x} \right)^9$.

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Group and Expand

Pre-Uni Maths for Sciences C1.5

Subject & topics: Maths | Algebra | Series   **Stage & difficulty:** A Level P1

Expand $(1 - 2x + 3x^2)^7$ in ascending powers of x as far as x^3.

The following symbols may be useful: x

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Binomial: All Rational n 2i

Subject & topics: Maths   **Stage & difficulty:** A Level P2

Part A

Expansion

Expand $(1 - 4x)^{\frac{1}{4}}$ in ascending powers of x, up to and including the term in x^3.

The following symbols may be useful: x

Part B

Values of a and b

The term of lowest degree in the expansion of

$$\left(1 + ax\right)\left(1 + bx^2\right)^7 - \left(1 - 4x\right)^{\frac{1}{4}}$$

in ascending powers of x is the term in x^3. Find the values of the constants a and b.

$a = $ ☐

$b = $ ☐

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Binomial: All Rational n 1i

Subject & topics: Maths   **Stage & difficulty:** A Level P2

Part A
Partial Fractions

Given that $\dfrac{3x + 4}{(1 + x)(2 + x)^2} \equiv \dfrac{A}{1 + x} + \dfrac{B}{2 + x} + \dfrac{C}{(2 + x)^2}$, find A, B, and C.

If a value is not a whole number, enter the value as a decimal.

$A = \boxed{}$

$B = \boxed{}$

$C = \boxed{}$

Part B
Expand

Hence or otherwise expand $\dfrac{3x + 4}{(1 + x)(2 + x)^2}$ in ascending powers of x, up to and including the term in x^2.

The following symbols may be useful: x

Part C
Values of x

State the set of values of x for which the expansion in the above part is valid.

Construct your answer from the items below.

Items:

x $<$ $\leq$ $>$ $\geq$ $< x <$ $\leq x \leq$ $> x$ **or** $x >$ $\geq x$ **or** $x \geq$ -4 -2 $-\dfrac{3}{2}$ -1 $-\dfrac{1}{2}$ 0

$\dfrac{1}{2}$ 1 $\dfrac{3}{2}$ 2 4

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Maclaurin Series - Binomial

Subject & topics: Maths | Algebra | Series   **Stage & difficulty:** A Level P2, Further A P1

Part A
Expand $(1 + r)^{\frac{1}{3}}$

Expand $(1 + r)^{\frac{1}{3}}$ up to the term in r^3.

The following symbols may be useful: r

Part B
Find $1.1^{\frac{1}{3}}$

Hence, using your expansion, find $(1.1)^{\frac{1}{3}}$ to 3 decimal places.

Part C
Find $9^{\frac{1}{3}}$

Now, using your expansion again, find $9^{\frac{1}{3}}$ to 2 decimal places.

Part D

Electric field on the axis of a charged sheet

The electric field E on the axis of a uniformly charged circular sheet at a distance z from the centre of the sheet is given by

$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + a^2}} \right]$$

where σ is the charge per unit area on the sheet and a is the radius of the sheet. Show that in the limit when $z \gg a$ the field on the axis is such that $E \approx \frac{A}{z^2}$ and find A.

The following symbols may be useful: `A, a, epsilon_0, sigma, z`

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Maclaurin Series - Cos & Sin 1

Pre-Uni Maths for Sciences C3.6

Subject & topics: Maths | Algebra | Series   **Stage & difficulty:** Further A C2, University C1

Part A
Find the cosine of the angle 0.2 rad

Find, using a Maclaurin expansion, the cosine of the angle $0.2\,\mathrm{rad}$, correct to 3 decimal places.

Part B
Find the sine of the angle 0.08 rad

Find, using a Maclaurin expansion, the sine of the angle 0.08 rad, correct to 2 significant figures.

Part C
Potential energy of mass on pendulum

A pendulum consists of a point mass m suspended on a light string of length l. When the string makes an angle of ϕ to the vertical its potential energy relative to the point where $\phi = 0$ is given by $mgl(1 - \cos\phi)$. Show that for $\phi \ll 1$ the potential energy is given approximately by $A_0\phi^2$ and find an expression for A_0.

The following symbols may be useful: `g, l, m`

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

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Maclaurin Series - ln

Pre-Uni Maths for Sciences C3.2

Subject & topics: Maths | Algebra | Series   **Stage & difficulty:** Further A P2, University P1

Part A

Expand $\ln(1+z)$ and hence $\ln(2+4y)$

(i) Write down the Maclaurin expansion of $\ln(1+z)$ up to the term in z^3.

The following symbols may be useful: z

(ii) By re-writing $\ln(2+4y)$ in the form $A + \ln(1+z)$, where A is a constant, find the Maclaurin expansion of $\ln(2+4y)$ up to the term in y^3.

The following symbols may be useful: y, z

Part B

Expand $\ln\left(\frac{1+q}{1-q}\right)$

Find the first 4 non-zero terms in the Maclaurin expansion of $\ln\left(\frac{1+q}{1-q}\right)$.

The following symbols may be useful: q

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Maclaurin Series - Exponentials 3

Pre-Uni Maths for Sciences C3.5

Subject & topics: Maths | Algebra | Series   **Stage & difficulty:** Further A C2, University C1

Part A
Expand $A\mathrm{e}^{-\alpha t}$

Expand $A\mathrm{e}^{-\alpha t}$ up to the term in t^2.

The following symbols may be useful: `A, alpha, p, t`

Part B
Expand $\mathrm{e}^p - \mathrm{e}^{-p}$

Find the first two non-zero terms in the Maclaurin expansion of $\mathrm{e}^p - \mathrm{e}^{-p}$.

The following symbols may be useful: `A, alpha, p, t`

Part C
Energy decay in oscillations

A lightly damped oscillatory system has a period T. The total energy of the system at time t is given by $E(t)$. One period later its energy $E(t + T) = E(t)\mathrm{e}^{-\gamma T}$.

(i) Find an expression for the fractional change in energy in one cycle.

The following symbols may be useful: `T, e, gamma`

(ii) On the assumption that $\gamma T \ll 1$ find an approximate expression for the fractional change in energy in one cycle.

The following symbols may be useful: `T, e, gamma`

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