# Functions from Differential Equations 2ii

A Level

The gradient of a curve is given by $\frac{dy}{dx} = 6x - 4$. The curve passes through the distinct points $(2, 5)$ and $(p, 5)$.

## Part A Equation of curve

Find the equation of the curve.

The following symbols may be useful: `x, y`

## Part B Find $p$

Find the value of $p$.

The following symbols may be useful: `p`

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# Functions from Differential Equations 1ii

A Level

The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 12\sqrt{x}$. The curve passes through the point $(4,\ 50)$. Find the equation of the curve.

The following symbols may be useful: `c, x, y`

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# Equation of Curve

A Level

A function $v(u)$ is such that $\frac{\mathrm{d}v}{\mathrm{d}u} = \frac{1}{3}u^{\frac{1}{3}}\left(1 - \frac{1}{u}\right)$ and $v(8) = -1$.

Find the equation of the function $v(u)$.

The following symbols may be useful: `u, v`

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# Functions from Differential Equations 1i

A Level

A curve has an equation which satisfies $\frac{dy}{dx} = kx(2x - 1)$ for all values of $x$.The point $P(2, 7)$ lies on the curve and the gradient of the curve at $P$ is $9$.

## Part A Find $k$

Find the value of the constant $k$. Give your answer as an improper fraction.

The following symbols may be useful: `k`

## Part B Equation of curve

Find the equation of the curve.

The following symbols may be useful: `x, y`

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# Integrating to Find An Area 1

A Level

This question is about the use of integration to find the area of one or more regions between a curve and the $x$-axis.

## Part A Calculating the area under $y = x^2 + 1$

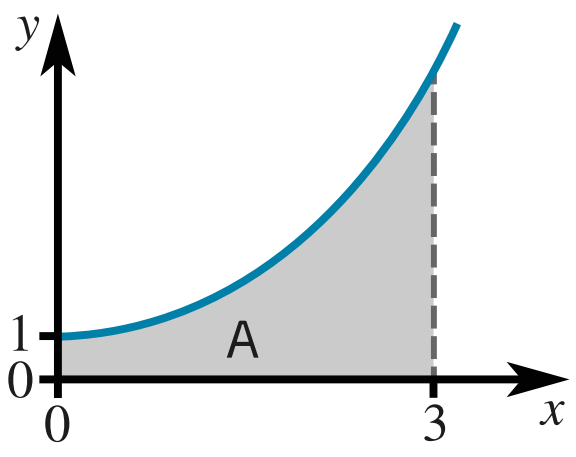


**Figure 1:** The graph of $y = x^2 + 1$

**Figure 1** shows the curve $y = x^2 + 1$. The region between the curve and the $x$-axis, bounded by the lines $x = 0$ and $x = 3$, is labelled $A$. Calculate the area of $A$.

## Part B Calculating the area above $y = x^2 - 1$

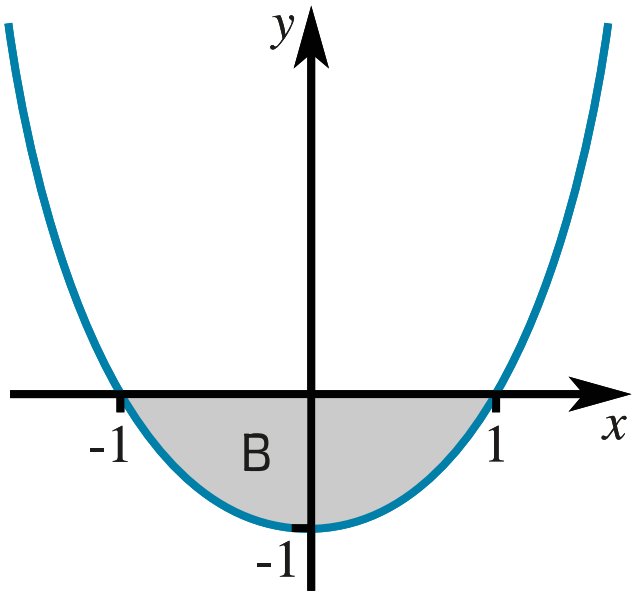


**Figure 2:** The graph of $y = x^2 - 1$

**Figure 2** shows the curve $y = x^2 - 1$. The region between the curve and the $x$-axis, bounded by the lines $x = -1$ and $x = 1$, is labelled $B$. Calculate the area of $B$.

## Part C Integrating $5x(x^2 - 1)$

Calculate the value of the integral $\int_{-1}^{1} 5x(x^2 - 1)\mathrm{d}x$.

## Part D The region enclosed between $y = 5x(x^2 - 1)$ and the $x$-axis

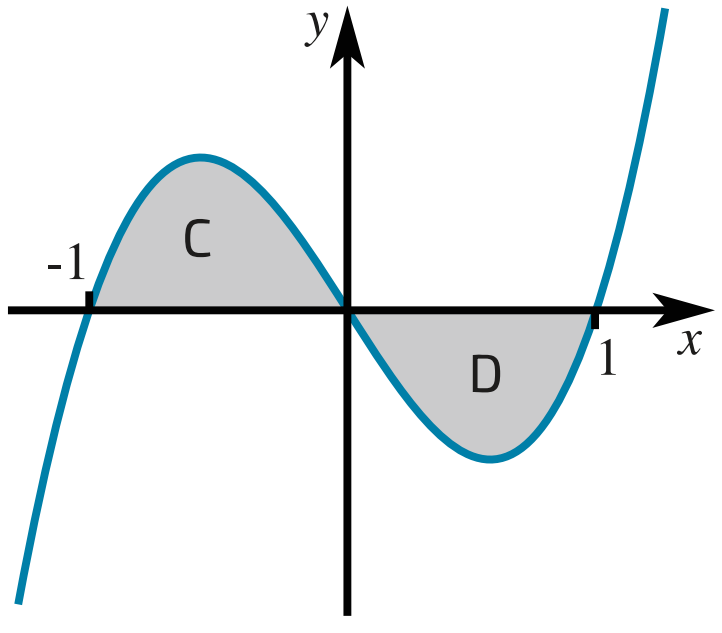


**Figure 3:** The graph of $y = 5x(x^2 - 1)$

Use **Figure 3** to explain why $\int_{-1}^{1} 5x(x^2 - 1)\mathrm{d}x$ does not give the total area of the regions enclosed by the curve $y = 5x(x^2 - 1)$ and the $x$-axis between $x = -1$ and $x = +1$. Drag and drop options into the spaces provided to complete your answer.

The curve $y = 5x(x^2 - 1)$ intercepts the $x$-axis at $x = -1$, $x = 0$ and $x = 1$. For $-1 < x < 0$ the curve is [ ] the $x$-axis. Hence, the value of $\int_{-1}^{0} 5x(x^2 - 1)\mathrm{d}x$ is [ ] and equal to the area of region [ ]. However, for $0 < x < 1$ the curve is [ ] the $x$-axis. Hence, the value of the integral $\int_{0}^{1} 5x(x^2 - 1)\mathrm{d}x$ is [ ]. The area of the region labelled [ ] is given by $-\int_{0}^{1} 5x(x^2 - 1)\mathrm{d}x$.

Thee total area of the shaded regions is $\int_{-1}^{0} 5x(x^2 - 1)\mathrm{d}x - \int_{0}^{1} 5x(x^2 - 1)\mathrm{d}x$. This is not the same as $\int_{-1}^{1} 5x(x^2 - 1)\mathrm{d}x$, which has a value of $0$ as the curve has [ ] symmetry and the contributions from the parts above and below the $x$-axis cancel out exactly.

Items:

**above** | $C$ | **reflective** | $D$ | **positive** | **negative** | **rotational** | **below**

## Part E Calculating an area for $y = 5x(x^2 - 1)$

Calculate the area of the region enclosed between the curve $y = 5x(x^2 - 1)$ and the $x$-axis for $x$ between $-1$ and $1$.

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# Integration and Area 1ii

A Level

**Figure 1** shows part of the curve $y = x^2 - 3x$ and the line $x = 5$.

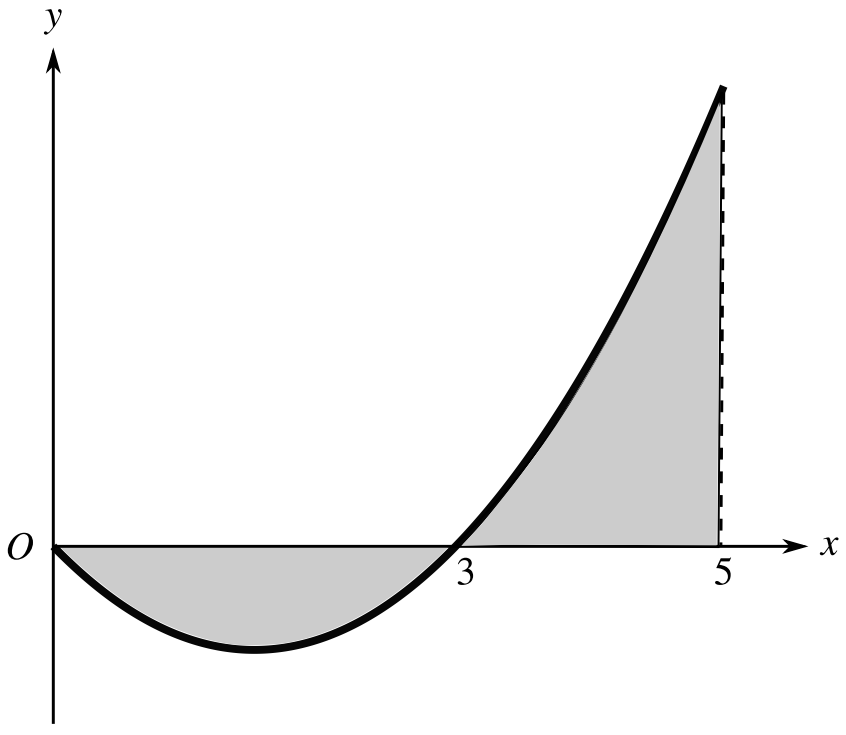


**Figure 1:** A graph of $y = x^2 - 3x$ and the line $x = 5$

## Part A Area of shaded regions

Which of the following expressions gives the total area of the regions shaded in **Figure 1**? Explain your choice.

- $\int_0^5 (x^2 - 3x)\,\mathrm{d}x$
- $\int_3^5 (x^2 - 3x)\,\mathrm{d}x - \int_0^3 (x^2 - 3x)\,\mathrm{d}x$
- $\int_0^3 (x^2 - 3x)\,\mathrm{d}x + \int_3^5 (x^2 - 3x)\,\mathrm{d}x$

## Part B Find area

Use integration to find the total area of the shaded regions. Give your answer to $3$ significant figures.

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# Area Under a Curve 1

A Level

P P P

A graph of the functions $y = (x - 2)(x + 1)$ and $y = x + 1$ is shown in **Figure 1**. Find the areas of the shaded regions labelled A and B. A is the region between P and Q enclosed by the curve $y = (x - 2)(x + 1)$ and the $x$-axis; B is the region between Q and R below the curve $y = (x - 2)(x + 1)$ and above the $x$-axis.

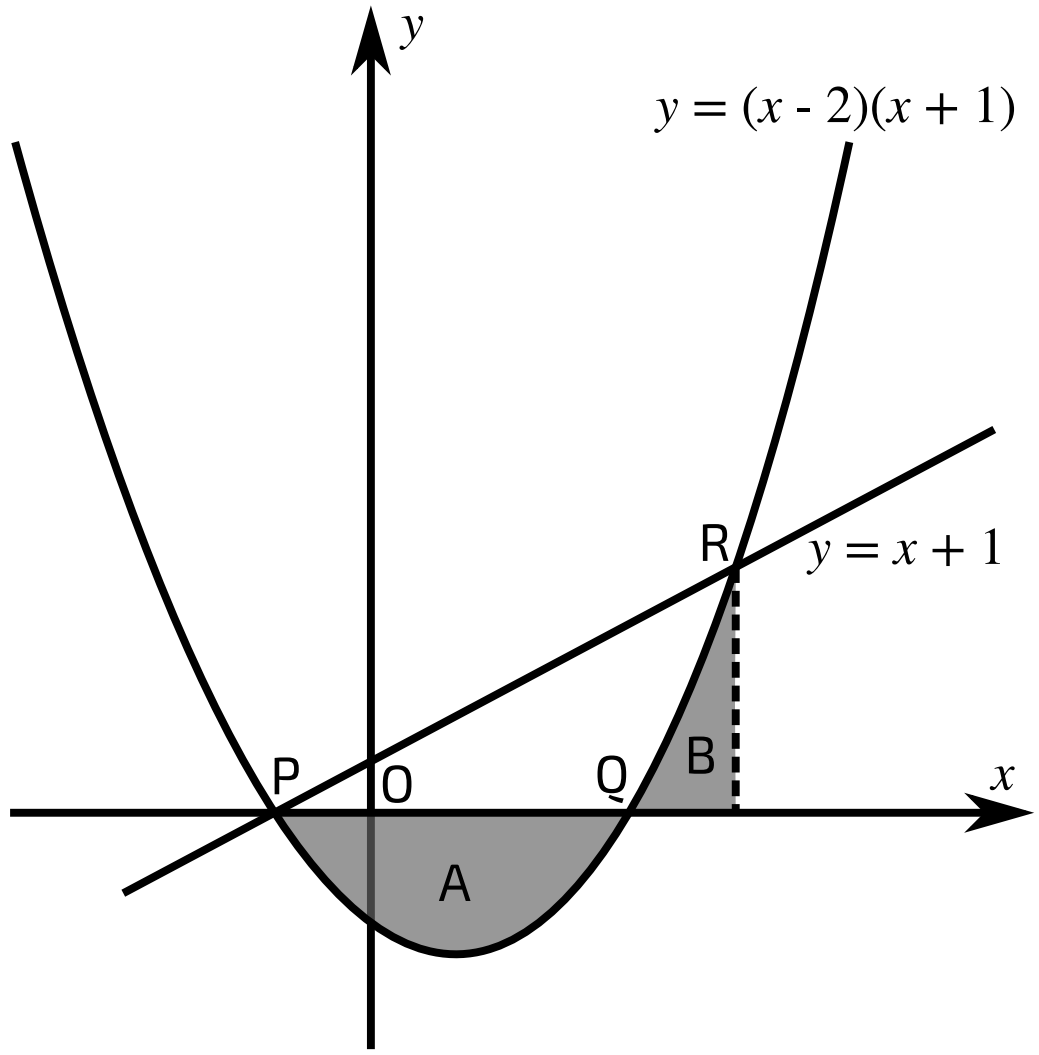


**Figure 1:** A graph of the functions $y = (x - 2)(x + 1)$ and $y = x + 1$. A is the region between P and Q enclosed by the curve $y = (x - 2)(x + 1)$ and the $x$-axis; B is the region between Q and R below the curve $y = (x - 2)(x + 1)$ and above the $x$-axis.

## Part A Region A

Find the area of the region A. Give your answer in the form of an improper fraction.

## Part B Region B

Find the area of the region B. Give your answer in the form of an improper fraction.

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# Integration and Area 3i

A Level

**Figure 1** shows the graph of $y = 1 - 3x^{-\frac{1}{2}}$.

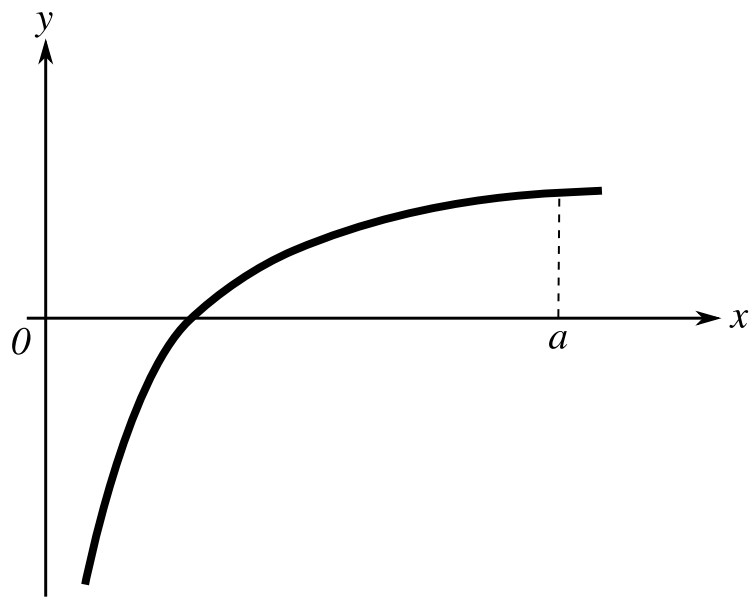


**Figure 1:** Graph of $y = 1 - 3x^{-\frac{1}{2}}$.

## Part A Find intersection with $x$-axis

Find the $x$-coordinate of the intersection of that curve with the $x$-axis.

The following symbols may be useful: x

## Part B Find $a$

The region enclosed by the curve, the $x$-axis and the line $x = a$ (where $a > 9$) has an area equal to $4$ square units, find the value of $a$.

The following symbols may be useful: a

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# Area Under a Curve 3

A Level Further A

C C C P P P

A graph of the functions $y = \dfrac{1}{2\sqrt{x}}$ and $y = 2x\sqrt{x}$ for $x \geq 0$ is shown in **Figure 1**. Find the area of the shaded region OPQR.

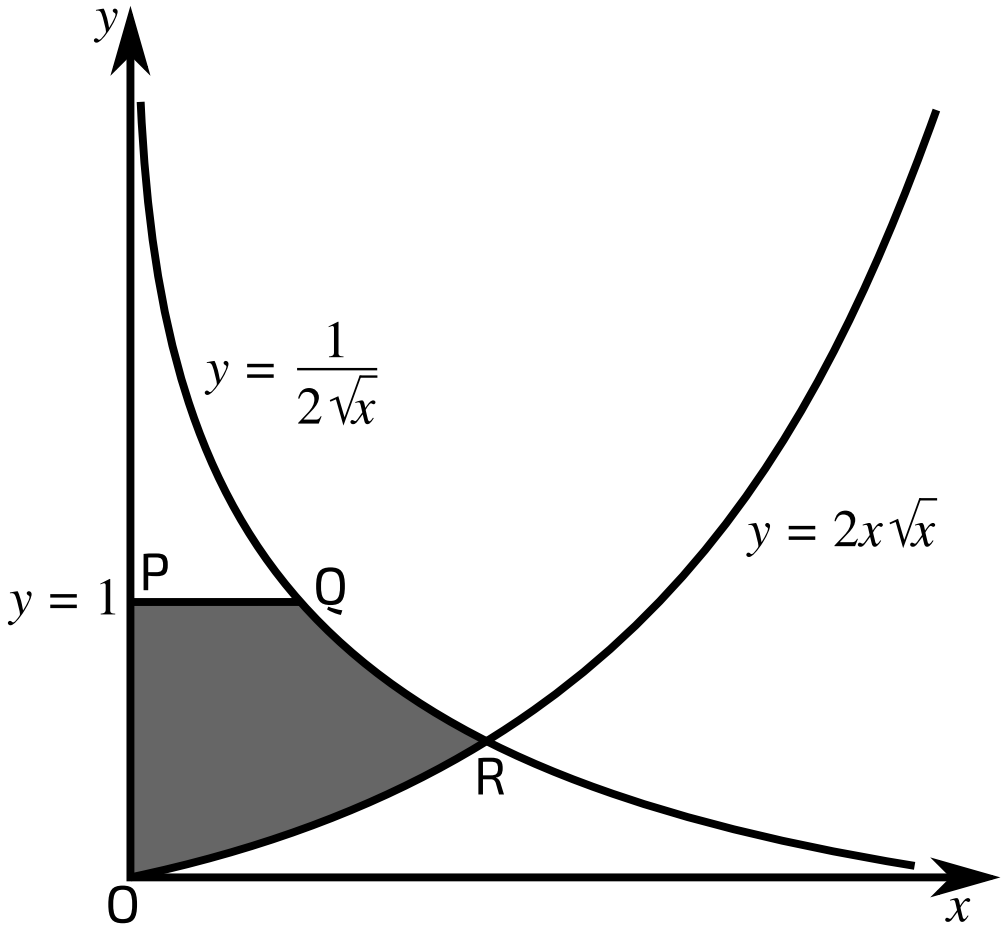


**Figure 1:** A graph of the functions $y = \dfrac{1}{2\sqrt{x}}$ and $y = 2x\sqrt{x}$ for $x \geq 0$. The shaded region OPQR is bounded by the line $x = 0$, the line $y = 1$, the curve $y = \dfrac{1}{2\sqrt{x}}$ and the curve $y = 2x\sqrt{x}$.

## Part A The $x$ coordinate of Q

Deduce the $x$ coordinate of the point Q.

## Part B The $x$ coordinate of R

Find the $x$ coordinate of the point R.

## Part C The area of OPQR

Find the area of the shaded region OPQR, giving your answer in an exact form.

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