Differentiation and Gradients: Beyond Polynomials 4ii

A Level

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in each of the following cases.

Part A Derivative 1

$$y = x^3 e^{2x}$$

The following symbols may be useful: `Derivative(y, x), e, ln(), log(), x, y`

Part B Derivative 2

$$y = \ln\left(3 + 2x^2\right)$$

The following symbols may be useful: `Derivative(y, x), e, ln(), log(), x, y`

Part C Derivative 3

$$y = \frac{x}{2x+1}$$

The following symbols may be useful: `Derivative(y, x), e, ln(), log(), x, y`

Differentiation and Gradients: Beyond Polynomials 3i

A Level

It is given that $y = 5^{x-1}$.

Part A Logarithm rules

Use the laws of logarithms to rearrange $y = 5^{x-1}$ to give an expression for x in the form $x = a + \frac{\ln y}{\ln b}$, where a and b are positive integers.

The following symbols may be useful: `ln(), log(), x, y`

Part B Derivative

Find an expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

The following symbols may be useful: `Derivative(x,y), e, ln(), log(), y`

Part C Gradient

Hence find the exact value of the gradient of the curve $y = 5^{x-1}$ at the point $(3, 25)$.

The following symbols may be useful: `, ln(), log()`

Gameboard:

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Physics. You work it out.

Differentiation: Products 3ii

A Level

Find the exact value of the x-coordinate of the stationary point of the curve $y = x \ln x$.

The following symbols may be useful: `e, x`

Gameboard:

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Differentiation: Quotients 3ii

A Level

Part A Differentiate

A curve has equation $y = \frac{2x+1}{3x-1}$. Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

The following symbols may be useful: `Derivative(y, x)`, `ln()`, `log()`, `x`, `y`

Part B Tangent

Hence find the equation of the tangent to this curve at the point $(1, \frac{3}{2})$, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

The following symbols may be useful: `x`, `y`

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Differentiation: Chain Rule 4i

A Level

Earth is being added to a pile so that, when the height of the pile is h metres, its volume is V cubic metres, where

$$V = (h^6 + 16)^{\frac{1}{2}} - 4$$

.

Part A Rate of Change (a)

Find the value of $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 2$, to three significant figures.

Part B Rate of Change (b)

The volume of the pile is increasing at a constant rate of 8 cubic metres per hour. Find the rate in metres per hour, at which the height of the pile is increasing at the instant when $h = 2$. Give your answer correct to 2 significant figures.

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Differentiation: Implicit 4i

A Level

Find the equation of the normal to the curve $x^3 + 2x^2y = y^3 + 15$ at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

The following symbols may be useful: `x, y`

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Differentiation: Synoptic Problems 5i

A Level

Part A Gradient

Find the gradient of the curve $x^2 + xy + y^2 = 3$ at the point $(-1, -1)$.

The following symbols may be useful: `Derivative(y, x)`

Part B Coordinates

A curve C has parametric equations

$$x = 2t^2 - 1, \quad y = t^3 + t.$$

Find the coordinates of the point on C at which the tangent is parallel to the y-axis.

Give the x-value.

Give the y-value.

Part C Rates of Change

A curve C has parametric equations

$$x = 2t^2 - 1, \quad y = t^3 + t.$$

Find the values of t for which x and y have the same rate of change with respect to t.

Give the larger value of t.

The following symbols may be useful: `t`

Give the smaller value of t.

The following symbols may be useful: `t`

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Differentiation: Synoptic Problems 1i

A Level

P P P

Figure 1 shows the curve $y = (\ln x)^2$.

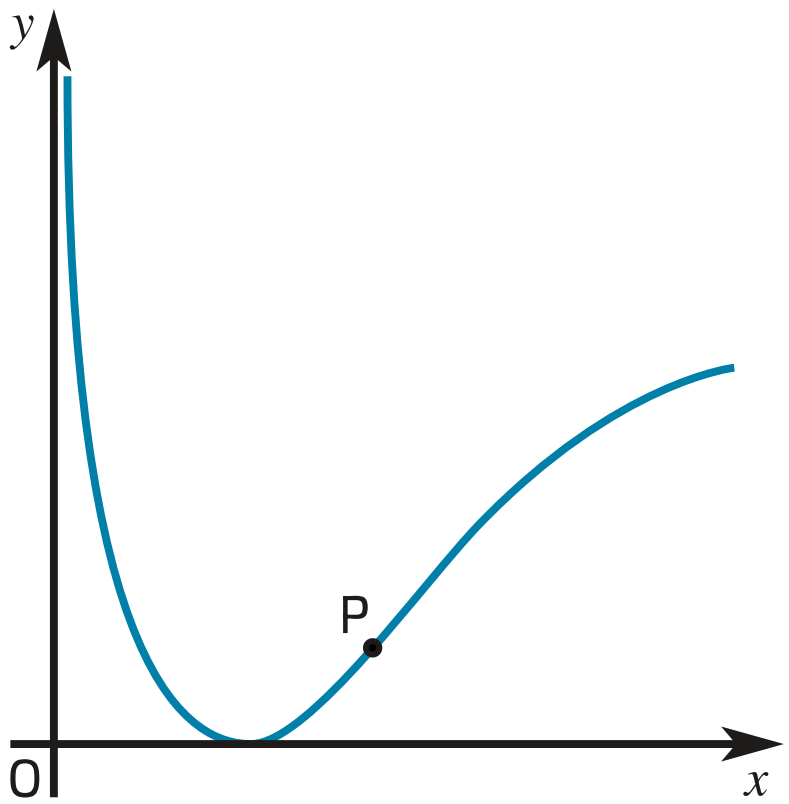

Figure 1: The curve $y = (\ln x)^2$.

Part A First derivative

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y,x), ln(), log(), x`

Part B Second derivative

Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

The following symbols may be useful: `Derivative (y , x , x), ln(), log(), x`

Part C The point P

The point P on the curve is the point at which the gradient takes its maximum value.

What is the nature of point P?

- A maximum
- A minimum
- A horizontal point of inflection
- An inclined point of inflection
- An asymptote
- None of the above

Part D The point Q

The tangent to the curve at P meets the y-axis at the point Q, which has coordinates $(0, q)$. What is the value of q?

Adapted with permission from UCLES A-level Maths papers, 2003-2017.

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Parametric Equations 4i

A Level

A curve has parametric equations

$$x = 2\sin t, \qquad y = \cos 2t + 2\sin t$$

for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

Part A Derivative

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of t.

The following symbols may be useful: `Derivative(y, x), cos(), cosec(), cot(), sec(), sin(), t, tan(), x, y`

Part B Coordinates

Find the x-coordinate of the stationary point.

The following symbols may be useful: `x`

Find the y-coordinate of the stationary point.

The following symbols may be useful: `y`

Part C Equation

Find the cartesian equation of the curve.

The following symbols may be useful: `x, y`

Part D Range

Find the range of values x can take.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $x < a$
- $x \leq a$
- $x > a$
- $x \geq a$
- $a < x < b$
- $a \leq x \leq b$
- $x < a$ or $x > b$
- $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: `n`

Part E Sketch

Hence sketch the curve.