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STEM SMART Single Maths 33 - The Product Rule & Implicit Differentiation

The Product and Quotient Rules

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Differentiation   **Stage & difficulty:** A Level P3

Fill in the blanks below to complete the summary notes on the product and quotient rules.

The function $y = u(x)v(x)$ is a ⬚ of the functions of $u(x)$ and $v(x)$. To differentiate a function like this we use the ⬚ rule:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \boxed{}\, v + u\, \boxed{}$$

The function $y = \dfrac{u(x)}{v(x)}$ is a ⬚ of the functions of $u(x)$ and $v(x)$. To differentiate a function like this we use the ⬚ rule:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\boxed{}\, v - u\, \boxed{}}{v^2}$$

Items:

| product | quotient | $\dfrac{\mathrm{d}v}{\mathrm{d}x}$ | $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ |

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STEM SMART Single Maths 33 - The Product Rule & Implicit Differentiation

Implicit Differentiation

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Differentiation   **Stage & difficulty:** A Level P3

Fill in the blanks below to complete the summary notes on implicit differentiation.

Equations of the form $y = f(x)$, such as $y = 2x + 1$ or $y = 3\tan x$, give y as an ⬚ function of x.
Equations which are not in the form $y = f(x)$, such as $e^y + x^3y^2 = 2x$, give y as an ⬚ function of x.

To find the gradient of an implicit function, we need to know how to differentiate functions of y with respect to x.

- e^y is a function of y. To differentiate a function of y with respect to x we use the ⬚ rule.

$$\frac{d(e^y)}{dx} = \frac{d(e^y)}{\boxed{}} \frac{\boxed{}}{dx}$$

$$= \boxed{}\frac{dy}{dx}$$

- x^3y^2 is a function of both x and y. To differentiate terms of this form with respect to x we first use the ⬚ rule. Then, to differentiate the function of y, we use the chain rule.

$$\frac{d(x^3y^2)}{dx} = \frac{d(x^3)}{dx}y^2 + x^3\frac{d(y^2)}{dx}$$

$$= \frac{d(x^3)}{dx}y^2 + x^3\frac{d(y^2)}{\boxed{}}\frac{dy}{dx}$$

$$= \boxed{} + \boxed{}\frac{dy}{dx}$$

Items:

chain   explicit   implicit   product dx dy $3x^2y^2$ e^y $2x^3y$

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STEM SMART Single Maths 33 - The Product Rule & Implicit Differentiation

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Differentiation: Products 4ii

Subject & topics: Maths   **Stage & difficulty:** A Level P2

Differentiate with respect to x, simplifying your answers where possible.

Part A

$\sin x \tan x$

Differentiate $\sin x \tan x$.

The following symbols may be useful: x

Part B

$x^2 (x + 1)^6$

Differentiate $x^2 (x + 1)^6$.

The following symbols may be useful: x

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Differentiation

 You work it out.

STEM SMART Single Maths 33 - The Product Rule & Implicit Differentiation

Differentiation: Products 2ii

Subject & topics: Maths   **Stage & difficulty:** A Level P2

Given that $y = 4x^2 \ln x$, answer the following.

Part A
First Derivative

Find an expression for $\frac{dy}{dx}$.

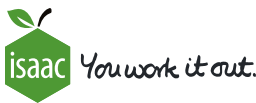The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part B
Second Derivative

Find the value of $\frac{d^2y}{dx^2}$, when $x = e^2$.

The following symbols may be useful: `Derivative(y , x , x), ln(), log(), x, y`

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Differentiation: Quotients 2ii

Subject & topics: Maths   **Stage & difficulty:** A Level P2

Differentiate with respect to x, simplifying your answers where possible.

Part A

$\dfrac{\ln x}{x}$

$y = \dfrac{\ln x}{x}$

The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part B

$\dfrac{x^2}{\ln x}$

$y = \dfrac{x^2}{\ln x}$

The following symbols may be useful: `Derivative(y, x), ln(), log(), x, y`

Part C
Stationary point of $y = \frac{x^2}{\ln x}$

Determine the exact x-coordinate of the stationary point of the curve $y = \frac{x^2}{\ln x}$.

The following symbols may be useful: `e, ln(), x`

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Question deck:

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Part C
Stationary point of $y = \frac{x^2}{\ln x}$

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Differentiation: Quotients 3i

Subject & topics: Maths   **Stage & difficulty:** A Level P2

A curve has equation $y = \frac{x^2+4}{x+2}$.

Part A
Derivative

Find an expression for $\frac{dy}{dx}$ in terms of x.

The following symbols may be useful: `Derivative(y, x), x, y`

Part B
Normal

Find the equation of the normal to the curve at the point $\left(1, \frac{5}{3}\right)$, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

The following symbols may be useful: `x, y`

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Implicit Differentiation 1

Pre-Uni Maths for Sciences J6.7

Subject & topics: Maths | Calculus | Differentiation   **Stage & difficulty:** A Level P3

Part A

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $x^2 + y^2 = r^2$.

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ if $x^2 + y^2 = r^2$, giving your answer as a simple function of x and y.

The following symbols may be useful: `x, y`

Part B

Find gradient of tangent to $x^2 - xy + y^2 = 7$

Consider the curve $x^2 - xy + y^2 = 7$.

Find as a function of x and y the gradient of the tangent to the curve $x^2 - xy + y^2 = 7$.

The following symbols may be useful: `x, y`

Hence evaluate the slope at the point $(-1, 2)$.

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Question deck:

STEM SMART Single Maths 33 - The Product Rule & Implicit Differentiation

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STEM SMART Single Maths 33 - The Product Rule & Implicit Differentiation
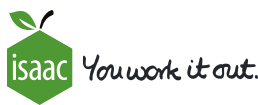
Differentiation: Products 4i

Subject & topics: Maths   **Stage & difficulty:** A Level P2

The equation of a curve has the form $y = e^{x^2}\left(ax^2 + b\right)$, where a and b are non-zero constants.

Part A

First Derivative

Find an expression for $\frac{dy}{dx}$.

The following symbols may be useful: `Derivative(y, x)`, `a`, `b`, `e`, `ln()`, `log()`, `x`, `y`

Part B

Second Derivative

Find an expression for $\frac{d^2y}{dx^2}$.

The following symbols may be useful: `Derivative(y, x, x)`, `a`, `b`, `e`, `ln()`, `log()`, `x`, `y`
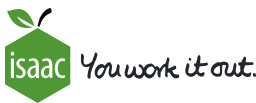
Part C
a in terms of b

It is given that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ can be expressed in the form $\mathrm{e}^{x^2}(cx^4 + d)$, where c and d are non-zero constants. Find an expression for a in terms of b.

The following symbols may be useful: `a, b`

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Question deck:

STEM SMART Single Maths 33 - The Product Rule & Implicit Differentiation

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STEM SMART Single Maths 33 - The Product Rule & Implicit Differentiation

Differentiation: Quotients 2i

Subject & topics: Maths   **Stage & difficulty:** A Level P2

Part A
Derivative

Given that $y = \frac{4\ln(x)-3}{4\ln(x)+3}$, find an expression for $\frac{dy}{dx}$.

The following symbols may be useful: `Derivative(y, x)`, `ln()`, `log()`, `x`, `y`

Part B
Gradient

Give the exact value of the gradient of the curve $y = \frac{4\ln(x)-3}{4\ln(x)+3}$ at the point where it crosses the x-axis.

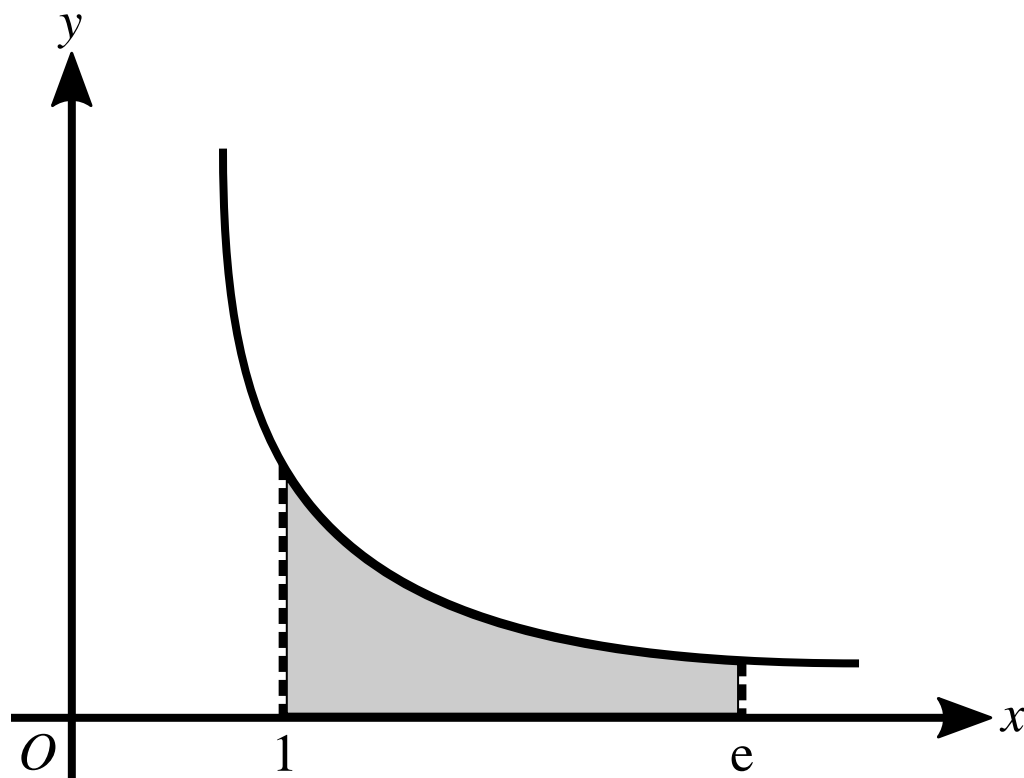The following symbols may be useful: `Derivative(y , x)`, `e`

Part C
Area

Figure 1 shows part of the curve with equation

$$y = \frac{2}{x^{\frac{1}{2}}\left(4\ln(x) + 3\right)}.$$

The region shaded in the diagram is bounded by the curve and the lines $x = 1$, $x = e$, and $y = 0$. Find the exact value of the integral I where

$$I = \int_{1}^{e} \pi y^2 \mathrm{d}x.$$

Figure 1: A diagram showing part of the curve with equation $y = \frac{2}{x^{\frac{1}{2}}(4\ln(x)+3)}$.

Give the exact value of I.

The following symbols may be useful: `I, pi`

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STEM SMART Single Maths 33 - The Product Rule & Implicit Differentiation

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STEM SMART Single Maths 33 - The Product Rule & Implicit Differentiation

Calculus: Inverse Trigonometry 2i

Subject & topics: Maths   **Stage & difficulty:** Further A P2

Part A
Derivative of $\arcsin x$

Find the derivative of $\arcsin x$

The following symbols may be useful: x

Part B
Implicit differentiation

Given that

$$\arcsin 2x + \arcsin y = \frac{1}{2}\pi$$

find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \frac{1}{4}$.

Adapted with permission from UCLES, A Level, January 2009, Paper 4726, Question 3.