STEM SMART Single Maths 39 - Projectiles & Parametric Equations

Projectiles and Trajectories

A-level Maths Topic Summaries - Vectors

Subject & topics: Maths **Stage & difficulty:** A Level P3

Fill in the boxes to complete the notes on projectiles and trajectories.

Part A
Projectiles

A **projectile** is an object that has been set into motion, and for which the only force acting on the object is gravity. An example is a cricket ball that has been thrown into the air, if we neglect the effect of air resistance.

Figure 1 shows the path of a projectile launched from the origin with a speed u at an angle θ above the horizontal. The velocity at a later time t is $\underline{v}$.

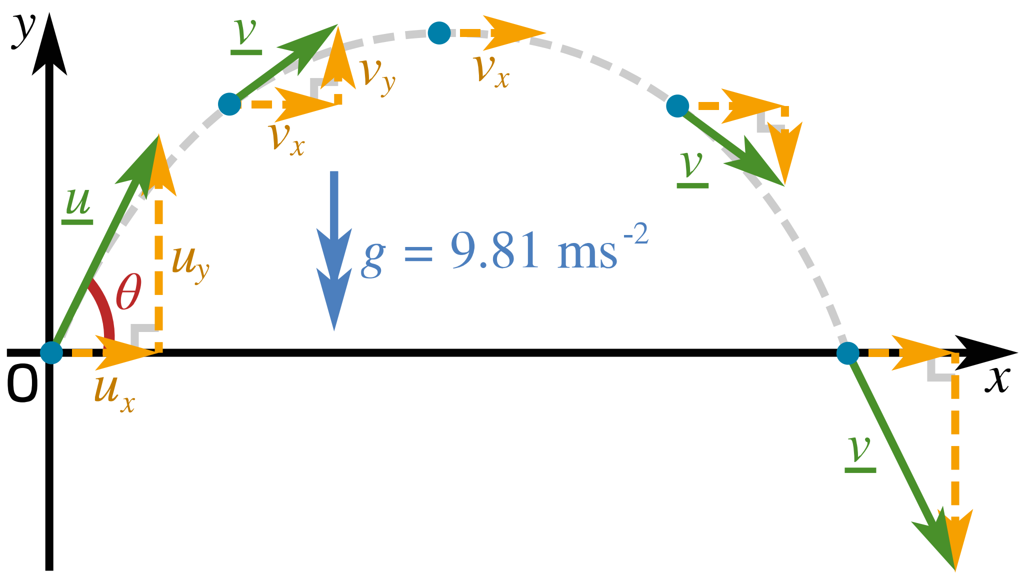

Figure 1: An example of projectile motion.

The projectile is launched with a speed u at an angle θ above the horizontal. Hence, the x-component of the initial velocity is [ ] and the y-component of the initial velocity is [ ].

Horizontal motion:

The only force acting on a projectile comes from gravity. Gravity acts [ ]. There is no force acting on the object [ ]. Therefore, the horizontal component of the acceleration is [ ]. Hence, the horizontal component of the velocity does not change.

$$a_x = 0, \quad v_x = [\ ], \quad x = [\ ]\,t$$

Vertical motion:

We use the equations for motion with constant acceleration (suvat equations) to describe the motion in the vertical direction.

$$a_x = [\ ], \quad v_y = [\ ] + [\ ]\,t,$$
$$y = [\ ]\,t + \tfrac{1}{2}[\ ]\,t^2$$

Items:

$u\cos\theta$ | $u\sin\theta$ | 0 | downwards | horizontally | $(-g)$

Part B

Trajectories

The path followed by a projectile is its **trajectory**.

From part A, we know that if a projectile is launched from the origin with a speed u at an angle θ above the horizontal, the x and y coordinates of the projectile at time t are

$x = \boxed{}\, t$

$y = \boxed{}\, t + \frac{1}{2} \boxed{}\, t^2$

These equations are linked by the time, t. We can rearrange the equation for x into the form $t = \boxed{}$, then substitute for t in the equation for y to get an equation relating y and x:

$$y = \boxed{} - \boxed{}\, x^2$$

This equation is the trajectory of the projectile. Its shape is a $\boxed{}$.

Items:

$u\cos\theta$ | $u\sin\theta$ | $\tan\theta$ | $\frac{x}{u\cos\theta}$ | $(-g)$ | $\frac{g}{2u^2\cos^2\theta}$ | parabola

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STEM SMART Single Maths 39 - Projectiles & Parametric Equations

Parametric Equations

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Fill in the boxes to complete the notes on parametric equations.

Part A

Parametric equations

A [ ] equation is an equation that is written in terms of the variables x and y of the Cartesian coordinate system. For example, $x^2 + y^2 = 25$ is a Cartesian equation for a circle.

Another way to write the equations of lines and curves is to use **parametric equations**. In parametric equations, x and y are defined separately in terms of a third variable called a [ ] (often t or θ). For example, a parametric form for a circle is

$$x = 5\cos\theta, \quad y = 5\sin\theta, \quad 0 \leq \theta < 2\pi$$

Each value of the parameter corresponds to a particular point on the curve. Changing the value of the parameter [ ] you along the curve. The parameterisation of a curve is [ ]. The same curve can be written in a parametric form in many different ways.

Items:

Cartesian | moves | not unique | parameter

Part B

Derivatives and integrals

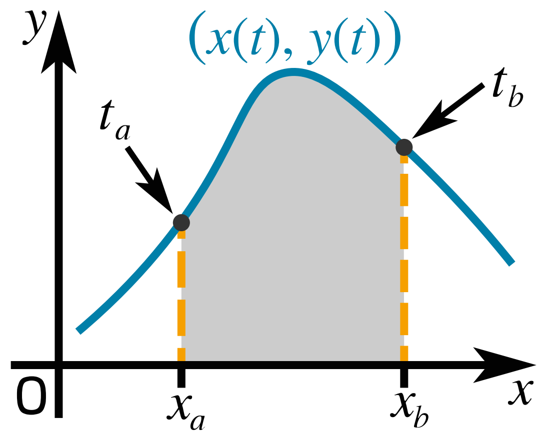

Figure 1: Illustrating the area under a curve for an equation in parametric form.

To find the gradient of a curve given in parametric form we make use of the chain rule. If the parameter is t, the gradient is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\boxed{}}{\boxed{}}$$

To find the area under a curve, we also make use of the chain rule. We turn the integral in terms of x into an integral in terms of t. (This is effectively integration by substitution.)

$$\int_{x_a}^{x_b} y(x)\,\mathrm{d}x = \int_{\boxed{}}^{\boxed{}} y(t)\boxed{}\,\mathrm{d}t$$

Items:

t_a t_b $\frac{\mathrm{d}x}{\mathrm{d}t}$ $\frac{\mathrm{d}y}{\mathrm{d}t}$

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Projectiles: Trajectories 4ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

A particle P is projected with speed $40\,\mathrm{m\,s^{-1}}$ at an angle of 35° above the horizontal from a point O.

Part A

Magnitude of velocity

For the instant $3\,\mathrm{s}$ after projection, calculate the magnitude of the velocity of P. Give your answer to 3 significant figures.

Part B

Direction of velocity

For the instant $3\,\mathrm{s}$ after projection, calculate the direction of the velocity of P. Give your answer as an angle, in degrees, below the horizontal to 3 significant figures.

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Projectiles: Trajectories 1i

Subject & topics: Maths **Stage & difficulty:** A Level P2

A stone is projected horizontally with speed $7\,\mathrm{m\,s^{-1}}$ from a point O on the edge of a vertical cliff. The horizontal and upward vertical displacements of the stone from O at any subsequent time, t seconds, are $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively. Assume that there is no air resistance.

Part A

y in terms of x

In this question, use the value $g = 9.8\,\mathrm{m\,s^{-2}}$ for the acceleration under gravity.

By first expressing x and y in terms of t, find an expression for y in terms of x.

The following symbols may be useful: x, y

Part B

Distance between cliff and stone

The stone hits the sea at a point which is $20\,\mathrm{m}$ below the level of O.

Find the distance between the foot of the cliff and the point where the stone hits the sea. Give your answer to 3 significant figures.

Part C

Speed and direction of motion

Find the speed of the stone immediately before it hits the sea. Give your answer to 2 significant figures.

Find the direction of motion of the stone immediately before it hits the sea. Give your answer as an angle below the horizontal to 3 significant figures.

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Projectiles: Trajectories 4i

Subject & topics: Maths **Stage & difficulty:** A Level P2

A particle is projected with speed $7\,\mathrm{m\,s^{-1}}$ at an angle of elevation of 30° from a point O and moves freely under gravity. The horizontal and vertically upwards displacements of the particle from O at any subsequent time $t\,\mathrm{s}$ are $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively.

Part A
x & y in terms of t

In this question, use the value $g = 9.8\,\mathrm{m\,s^{-2}}$ for the acceleration under gravity.

Express x in terms of t. When entering your answer, use fractions and surds rather than decimals.

The following symbols may be useful: `cos(), sin(), t, tan(), x`

Express y in terms of t. When entering your answer, use fractions rather than decimals.

The following symbols may be useful: `cos(), sin(), t, tan(), y`

Part B
y in terms of x

Hence find the equation, y in terms of x, for the trajectory of the particle.

The following symbols may be useful: `x, y`

Part C

Values of x

Calculate the smaller of two values of x when $y = 0.6$. Give your answer as an exact surd.

Calculate the larger of two values of x when $y = 0.6$. Give your answer as an exact surd.

Part D

Direction of motion

Find the direction of motion of the particle when $y = 0.6$ and the particle is rising. Give your answer as an angle from the horizontal and to 3 significant figures.

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Parametric Equations 3ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

Figure 1 shows the curve with parametric equations

$$x = a\sin\theta,\ y = a\theta\cos\theta,$$

where a is a positive constant and $-\pi \le \theta \le \pi$. The curve meets the positive y-axis at A and the positive x-axis at B.

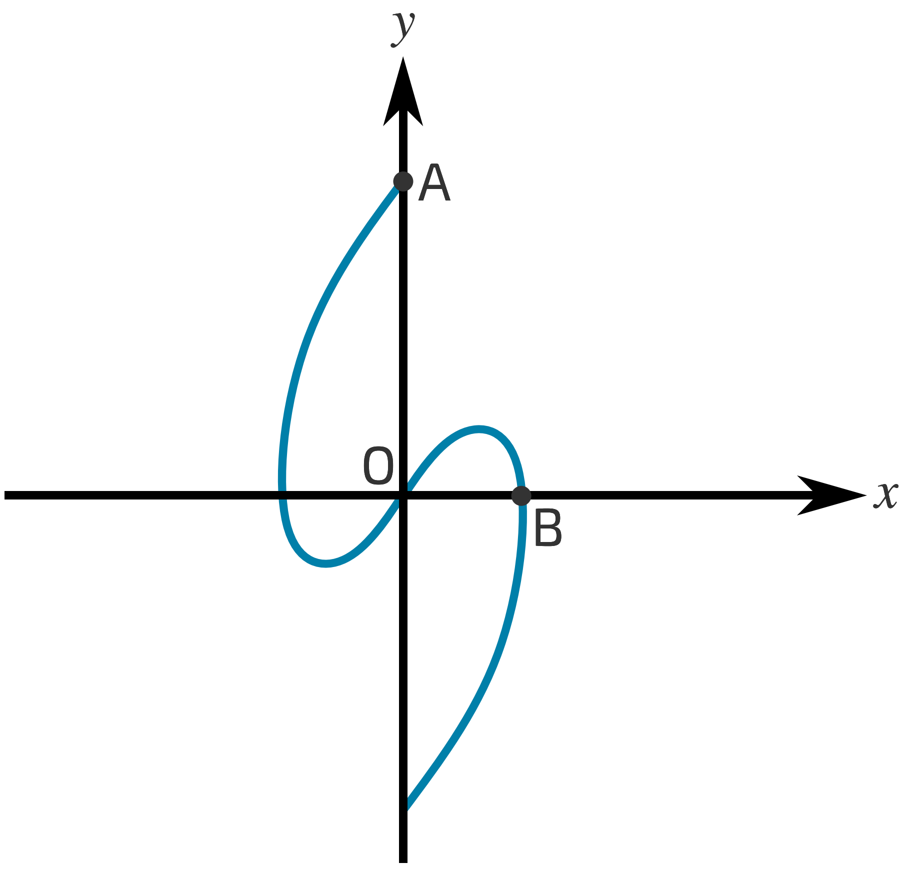

Figure 1: The graph defined by $x = a\sin\theta, y = a\theta\cos\theta$ for $-\pi \le \theta \le \pi$.

Part A

Points O, A and B

What is the value of θ corresponding to the origin?

$\theta =$ ☐

What are the coordinates of A?

$(0,$ ☐ $)$

What are the coordinates of B?

$($ ☐ $, 0)$

Items:

$-\pi$ | $-\frac{\pi}{2}$ | 0 | $\frac{\pi}{2}$ | π | $-2a$ | $-\pi a$ | $-a$ | $-\frac{\pi}{2}a$ | a | $\frac{\pi}{2}a$ | $2a$ | πa

Part B

Gradient

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x)`, `arccos()`, `arccosec()`, `arccot()`, `arcsec()`, `arcsin()`, `arctan()`, `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `tan()`, `theta`, `x`, `y`

Part C

Tangent equation

Find the equation for the tangent to the curve at the origin.

The following symbols may be useful: `x`, `y`

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Parametric Equations 1ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

A curve is defined by the parametric equations

$$x = \sin^2\theta, y = 4\sin\theta - \sin^3\theta$$

where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

Part A

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x)`, `arccos()`, `arccosec()`, `arccot()`, `arcsec()`, `arcsin()`, `arctan()`, `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `tan()`, `theta`, `x`, `y`

Part B

Point on the curve

Find the coordinates of the point on the curve at which the gradient is 2.

Give your answers as exact fractions.

x-coordinate:

y-coordinate:

Part C

Stationary points

Drag and drop answers into the boxes below to complete the argument showing that the curve has no stationary points.

If the curve has stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x}$ $\boxed{}$ at those points. Hence, using the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ found in part A,

$$\boxed{} - 3\sin^2\theta = 0$$
$$\Rightarrow \sin\theta = \pm\sqrt{\frac{\boxed{}}{3}}$$

However, $\sin\theta$ obeys the inequality $\boxed{} \leq \sin\theta \leq \boxed{}$ so there is no value of θ that satisfies $\sin\theta = \pm\sqrt{\frac{\boxed{}}{3}}$. Therefore, there are no stationary points.

Items:

$\boxed{-2}$ $\boxed{-1}$ $\boxed{<0}$ $\boxed{=0}$ $\boxed{0}$ $\boxed{>0}$ $\boxed{1}$ $\boxed{2}$ $\boxed{4}$ $\boxed{5}$ $\boxed{6}$ $\boxed{7}$

Part D

Cartesian equation

Find a cartesian equation of the curve, giving your answer in the form $y^2 = f(x)$.

The following symbols may be useful: `x`, `y`

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Parametric Integration 1

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** A Level P3

The curve C has parametric equations

$$x = 2t^2 - 3 \qquad y = t(4 - t^2)$$

The curve crosses the x-axis at the points A and B and the region R is enclosed by the loop of the curve, as shown in **Figure 1**.

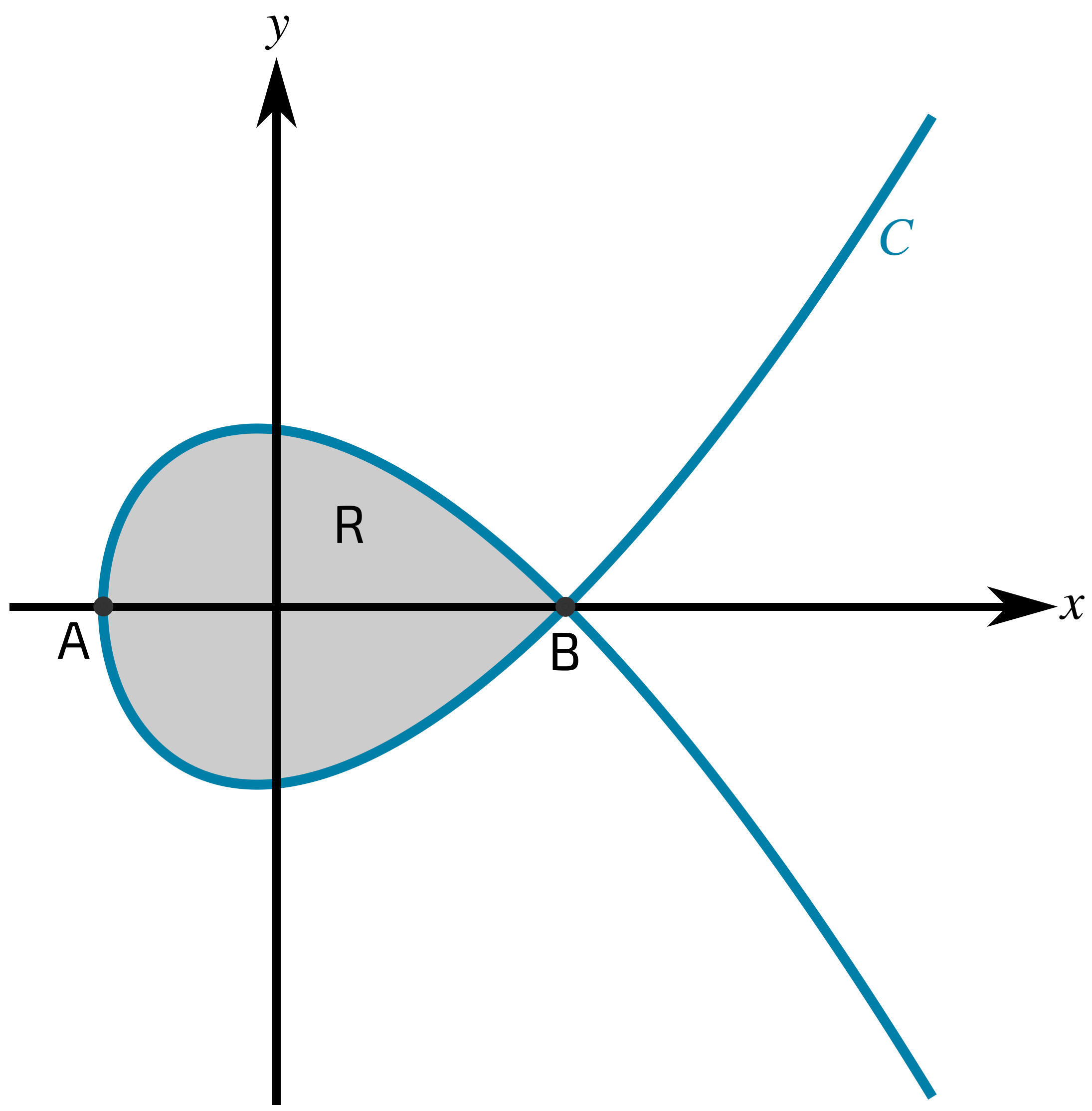

Figure 1: A graph of the curve C.

Part A

Point A

Find the x-coordinate of the point A.

Part B

Point B

Find the x-coordinate of the point B.

Part C

Area of R

The region R is enclosed by the loop of the curve, as shown in **Figure 1**. Find the exact value of the area of R.

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Parametric Equations 4i

Subject & topics: Maths **Stage & difficulty:** A Level P2

A curve has parametric equations

$$x = 2\sin t, \qquad y = \cos 2t + 2\sin t$$

for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

Part A

Derivative

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of t.

The following symbols may be useful: `Derivative(y, x), cos(), cosec(), cot(), sec(), sin(), t, tan(), x, y`

Part B

Coordinates

Find the (x, y) coordinates of the stationary point.

If a value is not a whole number, enter the value as a decimal.

(,)

Part C

Equation

Find the Cartesian equation of the curve.

The following symbols may be useful: x, y

Part D

Range

Find the range of values that x can take.

Construct your answer from the items below.

Items:

$<$ $>$ x $< x <$ $\leq x \leq$ $> x$ or $x >$ $\geq x$ or $x \geq$ $\leq$ $\geq$ -2 -1 0 1 2 3 4

Part E

Sketch

Hence sketch the curve.