

STEM SMART Single Maths 35 - Integration by Substitution

## Integration by Substitution

A-level Maths Topic Summaries - Calculus

**Subject & topics:** Maths | Calculus | Integration     **Stage & difficulty:** A Level P3

Fill in the blanks to complete the notes on integration by substitution.

### Part A

#### Indefinite integrals

We will illustrate the process for the integral  $I = \int \frac{6x}{(2x-7)^{\frac{1}{2}}} dx$ .

1. Define a new variable  $u$ . Let  $u = 2x - 7$ . Then  $x =$  .

2. Rewrite the integrand in terms of  $u$ .

$$I = \int \text{} dx = \int (3u^{\frac{1}{2}} + 21u^{-\frac{1}{2}}) dx$$

3. Differentiate  $u = 2x - 7$  to get  $\frac{du}{dx} =$  , then make use of the chain rule to change the integral from an integral with respect to  $x$  into an integral with respect to  $u$ .

$$dx = \frac{dx}{du} du = \frac{1}{\frac{du}{dx}} du$$

$$\therefore I = \int \text{} du$$

4. Carry out the integration. This gives  $I =$    $+ c$ .

5. Finally, back-substitute for  $u$  to give the answer in terms of the original variable,  $x$ . This gives

$$I = \text{} + c.$$

Items:

2

$\frac{u+7}{2}$

$u^{\frac{3}{2}} + 21u^{\frac{1}{2}}$

$\frac{3}{2}u^{\frac{1}{2}} + \frac{21}{2}u^{-\frac{1}{2}}$

$\frac{3u+21}{u^{\frac{1}{2}}}$

$(2x-7)^{\frac{3}{2}} + 21(2x-7)^{\frac{1}{2}}$

Part B

Definite integrals

We will illustrate the process for the integral  $J = \int_4^5 \frac{6x}{(2x - 7)^{\frac{1}{2}}} \, dx$ . The first few steps are the same as for indefinite integrals.

- 1. Define a new variable  $u$ .
- 2. Rewrite the integrand in terms of  $u$ .
- 3. Use the chain rule to change the integral from an integral with respect to  $x$  into an integral with respect to  $u$ .

These steps give us

$\therefore J = \int_{x=4}^{x=5} \boxed{\phantom{000000}} \, du$

Next we consider the limits of the integral.

- 4. Use  $u = 2x - 7$  to change the limits of the integral from values of  $x$  to the equivalent values of  $u$ .

$J = \int_{u=\boxed{\phantom{00}}}^{u=\boxed{\phantom{00}}} \boxed{\phantom{000000}} \, du$

- 5. Carry out the integration.

$J = \left[ \boxed{\phantom{000000}} \right]_{\boxed{\phantom{00}}}^{\boxed{\phantom{00}}}$

Items:

1

2

3

$\frac{3}{2}u^{\frac{1}{2}} + \frac{21}{2}u^{-\frac{1}{2}}$

$u^{\frac{3}{2}} + 21u^{\frac{1}{2}}$



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## Integration by Substitution 1

Pre-Uni Maths for Sciences K3.1

Subject &amp; topics: Maths | Calculus | Integration

Stage &amp; difficulty: A Level P3

Part A

**Integrate**  $\sin(c\theta)$ 

Find  $\int \sin(c\theta) \, d\theta$ , where  $c$  is a constant.

The following symbols may be useful:  $C$ ,  $c$ ,  $k$ ,  $\theta$

Part B

**Integrate**  $e^{\alpha v}$ 

Find  $\int e^{\alpha v} \, dv$ , where  $\alpha$  is a constant.

The following symbols may be useful:  $\alpha$ ,  $e$ ,  $k$ ,  $v$

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## Integration by Substitution 2

Pre-Uni Maths for Sciences K3.2

Subject &amp; topics: Maths | Calculus | Integration

Stage &amp; difficulty: A Level P3

Part A

**Integrate**  $(bv + c)^2$ 

Find  $\int (bv + c)^2 dv$ , where  $b$  and  $c$  are constants.

The following symbols may be useful:  $b$ ,  $c$ ,  $k$ ,  $v$

Part B

**Integrate**  $a(y - b)^3$ 

Find  $\int_0^b a(y - b)^3 dy$ , where  $a$  and  $b$  are constants.

The following symbols may be useful:  $a$ ,  $b$

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## Integration: General 4ii

**Subject & topics:** Maths**Stage & difficulty:** A Level P3**Part A****Integrate  $(4 - 3x)^7$** 

Find  $\int (4 - 3x)^7 dx$ .

The following symbols may be useful:  $c$ ,  $x$

**Part B****Integrate  $(4 - 3x)^{-1}$** 

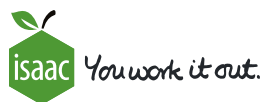
Find  $\int (4 - 3x)^{-1} dx$ .

The following symbols may be useful:  $c$ ,  $x$

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## Integration by Substitution 5ii

**Subject & topics:** Maths     **Stage & difficulty:** A Level P3

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Use the substitution  $u = 2x - 5$  to find the exact value of  $\int_{\frac{5}{2}}^3 (4x - 8)(2x - 5)^7 dx$ .

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## Integration by Substitution 5i

**Subject & topics:** Maths      **Stage & difficulty:** A Level P3

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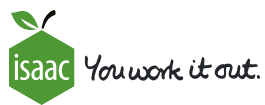
Use the substitution  $u = 2x + 1$  to evaluate  $\int_0^{\frac{1}{2}} \frac{4x - 1}{(2x + 1)^5} dx$ .

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## Integration by Substitution 4ii

**Subject & topics:** Maths    **Stage & difficulty:** A Level P3

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Use the substitution  $u = \sqrt{x+2}$  to find the exact value of  $\int_{-1}^7 \frac{x^2}{\sqrt{x+2}} dx$ .

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## Integration by Substitution 4i

**Subject & topics:** Maths    **Stage & difficulty:** A Level P3

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Use the substitution  $u = 1 + \sqrt{x}$  to find the exact value of  $\int_4^9 \frac{1}{1 + \sqrt{x}} dx$  in the form  $a + b \ln(c)$ , where  $a$ ,  $b$ , and  $c$  are positive constants to be found.

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## Integration by Substitution 3

Pre-Uni Maths for Sciences K3.3

Subject &amp; topics: Maths | Calculus | Integration

Stage &amp; difficulty: A Level P3

Part A

Integrate  $\frac{3}{(z+1)^2}$ Find  $\int_0^2 \frac{3}{(z+1)^2} dz$ .

Part B

Integrate  $\frac{e^{-\alpha x}}{(1+e^{-\alpha x})^4}$ Find  $\int \frac{e^{-\alpha x}}{(1+e^{-\alpha x})^4} dx$ , where  $\alpha$  is a constant.

The following symbols may be useful: C, alpha, c, e, k, x

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