SPC/BPhO China Summer School: Question Sheet

Rotational Mechanics & Special Relativity

How to solve physics problems

Here is some general advice which you should apply to every physics problem you attempt. It is important that you develop these techniques as they will make the problems easier to digest and understand, and provide you with a framework in which to think and apply the physics that you know.

1. **Always draw a diagram**

   A diagram can help to clarify your thoughts and begin to understand what the question is asking. It is **vital** that you use the diagram to define the symbols that you need (see 3 below). Make it big enough (this means at least 1/3 of a page!) and be tidy – use pencil as well as ink and use a ruler where necessary.

2. **Think about the physics**

   Ask yourself what physics might be applicable to the question, and **write it down in words**. Try to **understand** the problem qualitatively before writing down any equations. **Do not just write down equations!**

3. **Stay in symbols until the end**

   At school you may have been taught to do your calculations numerically, often without writing anything down but just putting the numbers into your calculator – **we advise that you don't do this now!** You will give yourself a huge advantage if you delay substituting in your numerical values until the very last line as it enables you to find your mistakes and save effort (often things cancel out so you don't need to work so hard).

4. **Check the dimensions**

   Think about the dimensions (or units) of each quantity as you write it down. You will find this a discipline which helps enormously to avoid mistakes and will help your understanding. Make sure that the **dimensions** of your final algebraic equation match on each side before you make a numerical substitution. When making a numerical substitution, write down the **units** of your answer. E.g. $2.95\ \mathrm{J\ kg^{-1}}$.

5. **Does the answer make sense?**

   You will probably have an idea of what looks about right, and what is clearly wrong e.g. the radius of the Earth as $10\ \mathrm{m}$ should ring some alarm bells. Many mistakes are simple arithmetic errors involving powers of tem. If in doubt, check your substitutions. When giving the final numerical answer to a problem use a sensible number of significant figures.

 Dr Lisa Jardine-Wright

Session 1

Mechanics Problems:

1. **IP Question (L1): Human Tower**
2. **IP Question (L2):** Hanging a Picture
3. **IP Question (L3): Two Cubes**
4. **IP Question (L4):** A Hailstorm
5. **IP Question (L2):** Hammer & Nail
6. **IP Question (L3): Pop-up Toy**
7. **IP Question (L3):** Energy of a Bullet
8. **IP Question (L4):** Ballistic Pendulum

Session 2

Kinematics Problems:

9. **IP Question (L1):** Hansel & Gretel
10. **IP Question (L3): Stop the Train**
11. **IP Question (L3): Jet in the Fog**
12. **IP Question (L4):** Swimming to a Boat

Rotational Equilibrium Problems

13. **IP Question (L3): Shelf and Brackets**
14. **IP Question (L4):** Weight of a Lorry
15. **IP Question (L4): A Hinged Rod**
16. **IP Question (L5):** Three Cylinders
17. **IP Question (L6):** Two Spheres

Session 3

Circular Motion Questions:

18. **IP Question (L4):** Cornering on a Smooth Surface
19. **IP Question (L4): Car on a Roundabout**
20. **IP Question (L5):** Circular Pendulum

Frames of Reference (& Collisions) Problems

21. The driver of a train A travelling at $120\ \mathrm{km\,h^{-1}}$ sees another train B $2\ \mathrm{km}$ ahead of them which is travelling in the same direction along the same track at a constant speed of $60\ \mathrm{km\,h^{-1}}$. (Speeds given are measured relative to the ground).
    a. Deduce the speed of train B in the frame of reference train A.
    b. If both drivers continue at constant speed, how long would it take for train A to collide with train B?
    c. If the driver of train A instantly applied their brakes, what is the minimum deceleration needed to avoid a collision in the frame of train B?

22. A car of mass $1\ \mathrm{tonne}\ (1000\ \mathrm{kg})$ is going at $10\ \mathrm{m\,s^{-1}}$ and is approaching a lorry of mass $5\ \mathrm{tonnes}$ moving at $10\ \mathrm{m\,s^{-1}}$ in the opposite direction.
    a. Find the momentum of the two vehicles in the frame of reference fixed with respect to the road.
    b. Find the momentum of the two vehicles in the frame in which the car is stationary.

Special Relativity Problems

23. A positive kaon (K^+) has a lifetime of $0.1237\ \mu s$ in its own rest frame. It has a speed $0.990c$ relative to a laboratory frame of reference. How far can it travel in the laboratory frame during its lifetime according to
    a. classical physics and
    b. special relativity?

24. Two events occur in the same place in the laboratory frame and are separated by a time interval of $2\ \mathrm{s}$. The time interval between these two events when measured from the rocket frame is $4\ \mathrm{s}$.
    a. Deduce the Lorentz factor γ and hence calculate the speed of the rocket relative to the laboratory frame.
    b. Calculate the distance between the two events in the rocket frame.

25. A metre stick which is $2\ \mathrm{cm}$ wide is aligned in the north-south direction. How fast and which direction relative to an observer is the metre stick moving if its length appears the same as its width?

Relativistic Kinematics *is the application of the Special Theory of Relativity to space and time. Please read the following general advice on how to tackle kinematical relativity problems before proceeding.*

Some or all of the following 'rules' can be applied to solve any kinematical problem in Special Relativity. If you apply the rules carefully, without first muddling yourself with too much potentially confusing thought about what contracts and what dilates etc., you will get the right answer.

(i) **Identify the events**. Label them A, B, C etc. Thus event A may be the flash of a light, B the spaceship exploding, C the arrival of a message at the Earth etc.

(ii) **Draw diagrams showing the events in the relevant frames of reference.** Thus you might show events A and B as seen both in the Earth frame and in the rocket frame.

(iii) **Write down the *intervals* between the events in all frames.** Set these equal to known quantities where you can and put a question mark where you can't. Thus you might write

$$\Delta x'_{\mathrm{AB}} = l_0;\ \Delta x_{\mathrm{AB}} = ?\,;\ \Delta t'_{\mathrm{AB}} = \frac{l_0}{c};\ \Delta t_{\mathrm{AB}} = ?$$

(iv) **Apply the Lorentz transformation to the intervals to find the unknown values**. If S' is the frame moving at speed v parallel to, and in the direction of, the positive x axis of frame S, then the Lorentz transformation of the interval, $(\Delta x, \Delta y, \Delta z, \Delta t)$, between two events as observed in the S frame, and the interval $(\Delta x', \Delta y', \Delta z', \Delta t')$ between the s*ame* two events as observed in the S' frame is:

$$\Delta t' = \gamma\left(\Delta t - {v\Delta x}/{c^2}\right); \qquad \Delta t = \gamma\left(\Delta t' + {v\Delta x'}/{c^2}\right)$$

$$\Delta x' = \gamma(\Delta x - v\Delta t); \qquad \Delta x = \gamma(\Delta x' + v\Delta t') \qquad \text{Where } \gamma = \sqrt{\frac{1}{(1-v^2/c^2)}}$$

$$\Delta y' = \Delta y; \qquad \Delta y = \Delta y'$$

$$\Delta z' = \Delta z; \qquad \Delta z = \Delta z'$$

26. A spaceship sets off from Earth for a distant destination, travelling in a straight line at a uniform speed of $3c/5$. Ten years later, as measured on the Earth, a second spaceship sets off in the same direction with a speed of $4c/5$. The captains of the two vessels are twins.

    a. For how long, in the Earth's frame, does each of the spaceships travel before the second spaceship catches up with the first?

    Consider three events: (A) the slower spaceship leaves the Earth; (B) the faster spaceship leaves the Earth; and (C) the faster spaceship catches up with the slower spaceship.

    b. If the event A has coordinates $x = 0,\ t = 0$ in the Earth's frame, what are the coordinates of the other two events, also as observed in the Earth's frame?

    c. By transforming these events to frames moving with the slower and faster spaceships respectively, determine which of the twins is older, and by how much, when the faster spaceship catches up with the slower spaceship.

27. As a spaceship passes the Earth with a speed of $0.8c$, observers on this spaceship and on the Earth agree that the time is 12:00 in both places. Thirty minutes later, as measured on the spaceship's clock, the spaceship passes an interplanetary navigation station fixed relative to the Earth. The clock on the interplanetary navigation station reads Earth time.

a. What is the time on the navigation station clock as the spaceship passes?
b. How far from the Earth, as measured in the Earth's frame, is the navigation station?
c. As the spaceship passes the navigation station, it reports back to Earth by radio. When, according to a clock on the Earth, is the signal received?
d. There is an immediate reply from Earth. When, according to the spaceship's clock, is the reply received at the spaceship?