



STEM SMART Single Maths 30 - Trigonometry: Reciprocal & Inverse Functions

Trigonometric Identities 2

A-level Maths Topic Summaries - Trigonometry

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level P2

Fill in the blanks to complete the summary notes on trigonometric identities involving the reciprocal trigonometric functions $\sec \theta$, $\operatorname{cosec} \theta$ and $\cot \theta$.

$$\boxed{} \equiv \frac{1}{\sin \theta}$$

$$\boxed{} \equiv \frac{1}{\cos \theta}$$

$$\boxed{} \equiv \frac{1}{\tan \theta}$$

$$\boxed{} \equiv \frac{\cos \theta}{\sin \theta}$$

$$1 + \tan^2 \theta \equiv \boxed{}$$

$$1 + \cot^2 \theta \equiv \boxed{}$$

Items:

 $\sec \theta$ $\operatorname{cosec} \theta$ $\cot \theta$ $\sec^2 \theta$ $\operatorname{cosec}^2 \theta$

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Reciprocal Trigonometric Functions

A-level Maths Topic Summaries - Trigonometry

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level P2

Fill in the blanks to complete the notes on reciprocal trigonometric functions.

Part A
 $\sin x$ and $\operatorname{cosec} x$

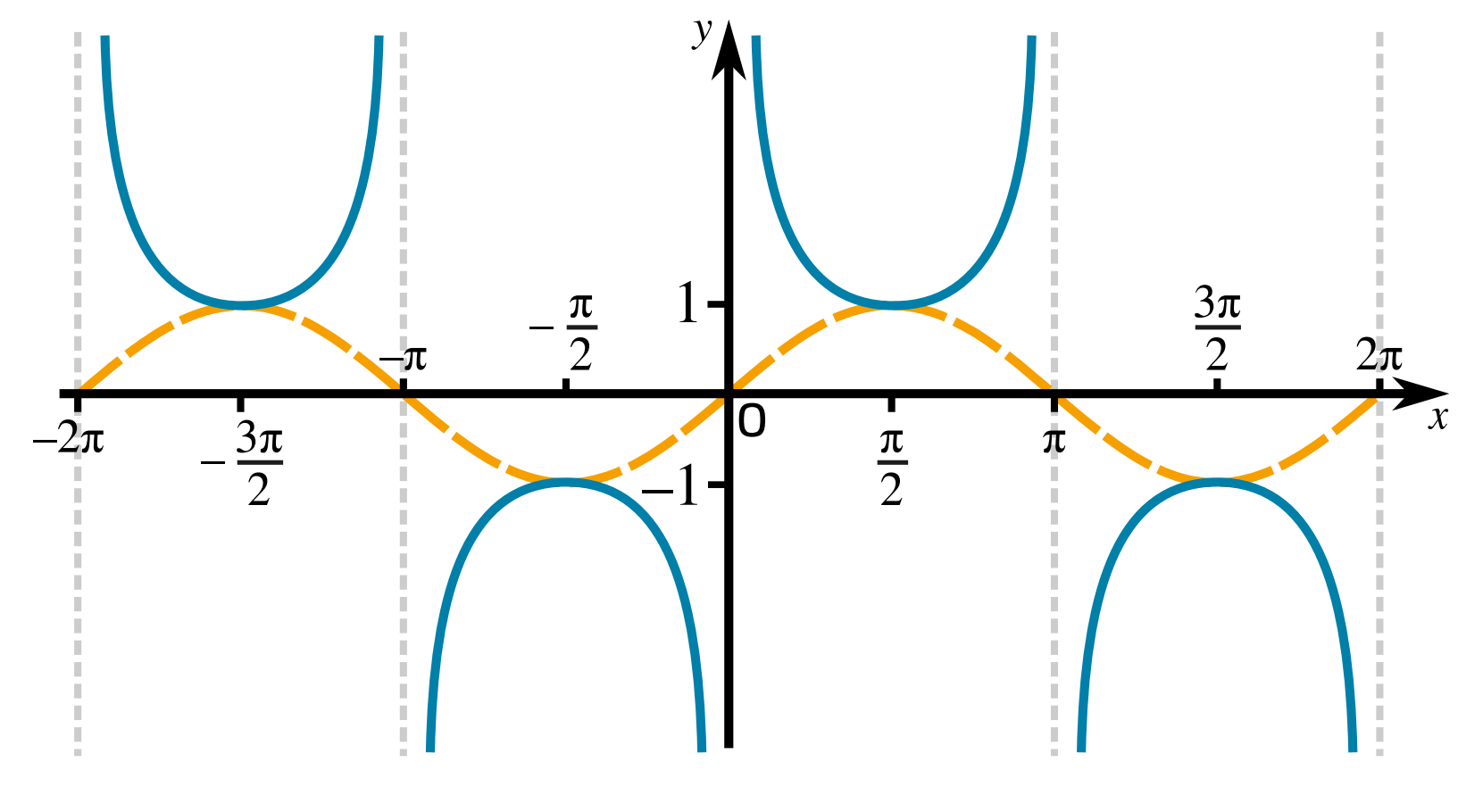


Figure 1: The graphs of $\operatorname{cosec} x$ (blue line) and $\sin x$ (yellow dashed line).

Key properties of $\operatorname{cosec} x$:

$\operatorname{cosec} x = \frac{1}{\text{[]}}$

- vertical asymptotes at $x = \text{[]}$, where n is any integer
- domain: []
- range: []

Items:

$\sin x$ $\cos x$ $\tan x$ $n\pi$ $2n\pi$ $(n + \frac{1}{2})\pi$ \mathbb{R} $x \neq n\pi$ $x \neq 2n\pi$ $x \neq (n + \frac{1}{2})\pi$ $x \leq -1$

$-1 \leq x \leq 1$ $x \geq 1$ $x \leq -1 \text{ or } x \geq 1$

Part B

$\cos x$ and $\sec x$

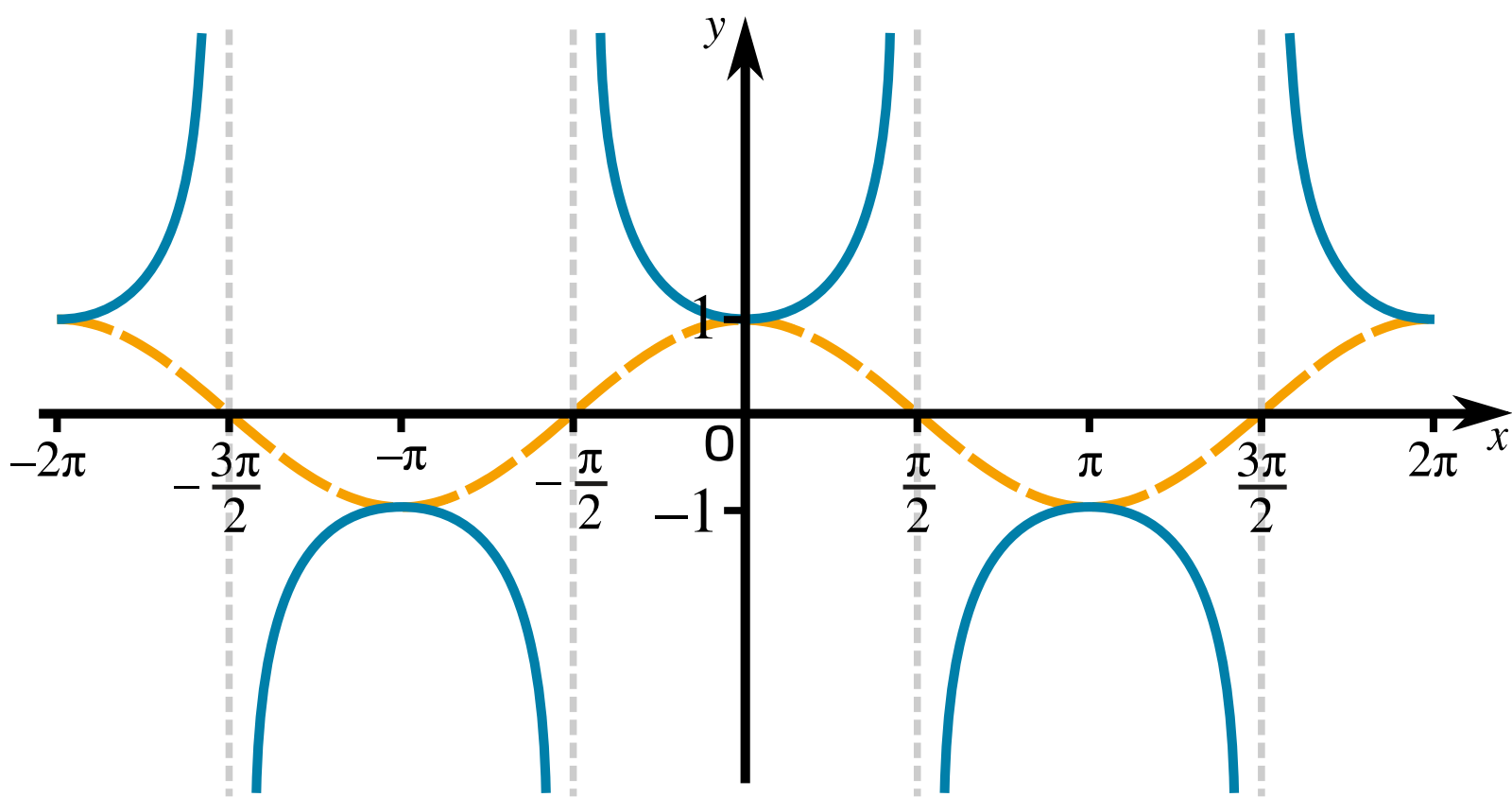


Figure 2: The graphs of $\sec x$ (blue line) and $\cos x$ (yellow dashed line).

Key properties of $\sec x$:

$\sec x = \frac{1}{\text{[]}}$

- vertical asymptotes at $x = \text{[]}$, where n is any integer
- domain: []
- range: []

Items:

$\sin x$

$\cos x$

$\tan x$

$n\pi$

$2n\pi$

$(n + \frac{1}{2})\pi$

\mathbb{R}

$x \neq n\pi$

$x \neq 2n\pi$

$x \neq (n + \frac{1}{2})\pi$

$x \leq -1$

$-1 \leq x \leq 1$

$x \geq 1$

$x \leq -1 \text{ or } x \geq 1$

Part C
 $\tan x$ and $\cot x$

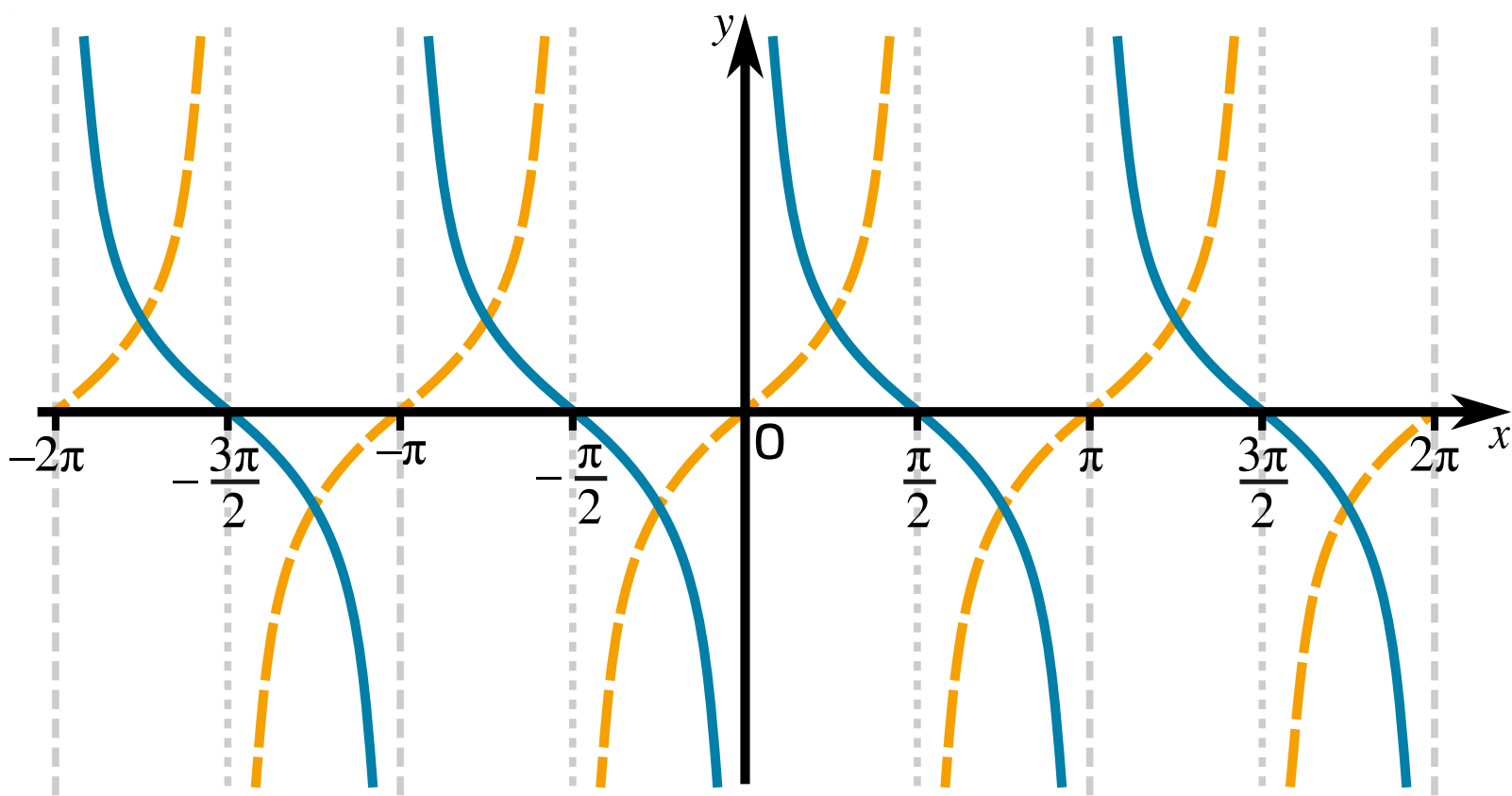


Figure 3: The graphs of $\cot x$ (blue line) and $\tan x$ (yellow dashed line).

Key properties of $\cot x$:

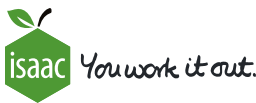
$\cot x = \frac{1}{\text{ }}$

- vertical asymptotes at $x = \text{ }$, where n is any integer
- domain:
- range:

Items:

$\sin x$ $\cos x$ $\tan x$ $n\pi$ $2n\pi$ $(n + \frac{1}{2})\pi$ \mathbb{R} $x \neq n\pi$ $x \neq 2n\pi$ $x \neq (n + \frac{1}{2})\pi$ $x \leq -1$

$-1 \leq x \leq 1$ $x \geq 1$ $x \leq -1 \text{ or } x \geq 1$



Inverse Trigonometric Functions

A-level Maths Topic Summaries - Trigonometry

Subject & topics: Maths | Geometry | Trigonometry Stage & difficulty: A Level P2

Fill in the blanks to complete the notes on inverse trigonometric functions.

Part A
 $\sin x$ and $\arcsin x$

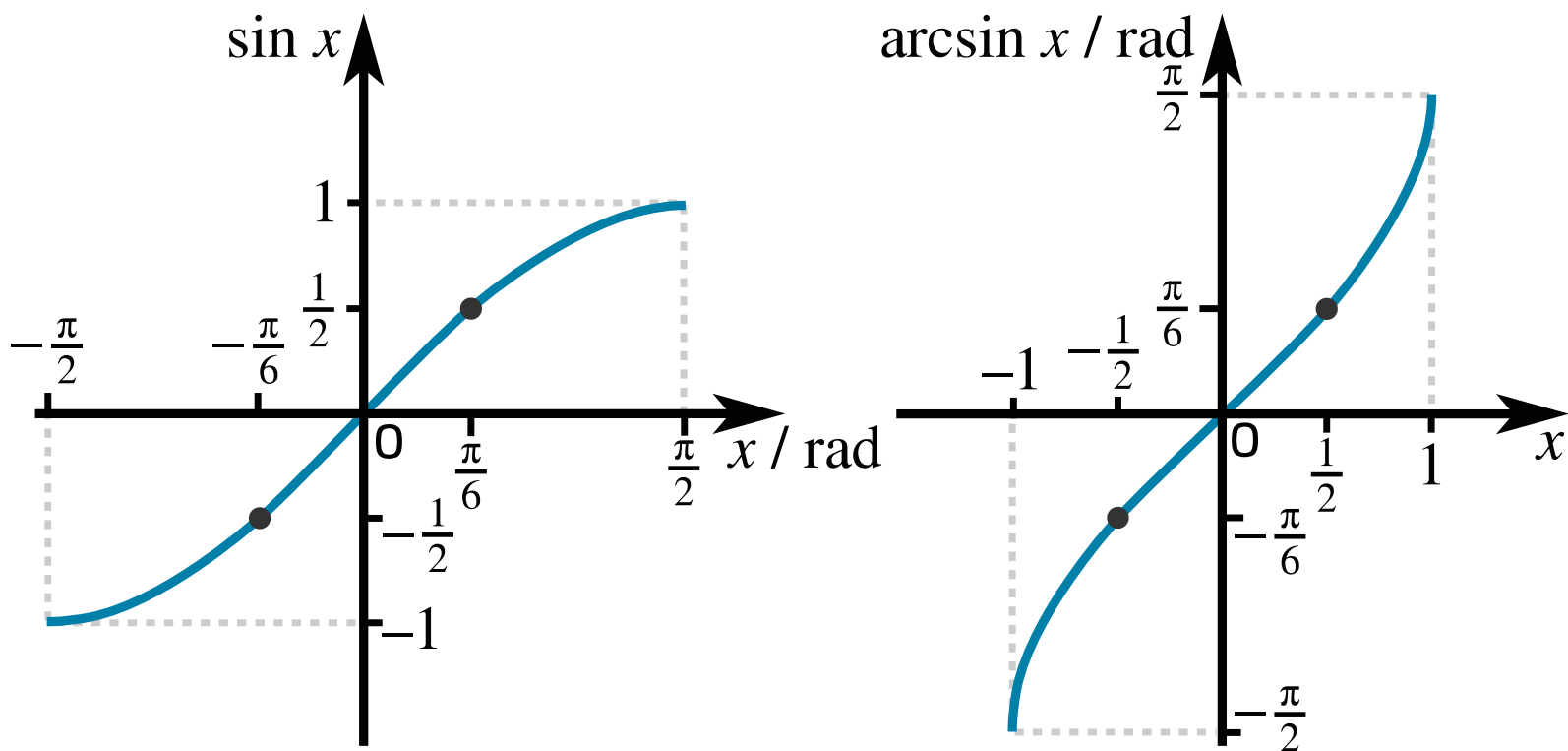


Figure 1: The graphs of $\sin x$ and $\arcsin x$.

In order to define an inverse function, we need to restrict the domain of $\sin x$ to the interval .

The inverse of $\sin x$ is written as $\arcsin x$ or $\sin^{-1} x$. The domain and range of the inverse function are

- domain:
- range:

Items:

Part B

$\cos x$ and $\arccos x$

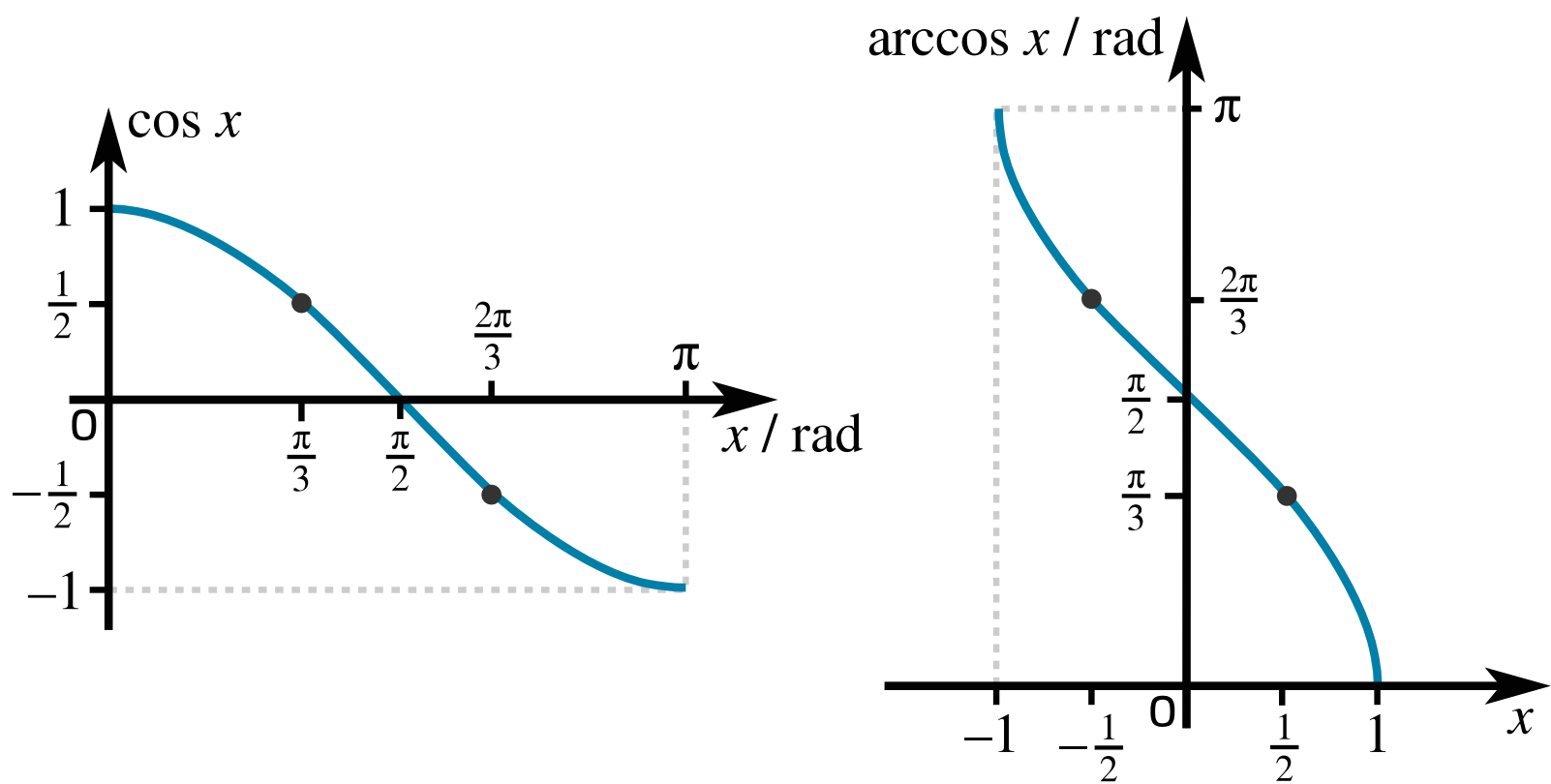


Figure 2: The graphs of $\cos x$ and $\arccos x$.

In order to define an inverse function, we need to restrict the domain of $\cos x$ to the interval .

The inverse of $\cos x$ is written as $\arccos x$ or $\cos^{-1} x$. The domain and range of the inverse function are

- domain:
- range:

Items:

$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$-1 \leq x \leq 1$

$0 \leq x \leq \pi$

\mathbb{R}

Part C
tan x and arctan x

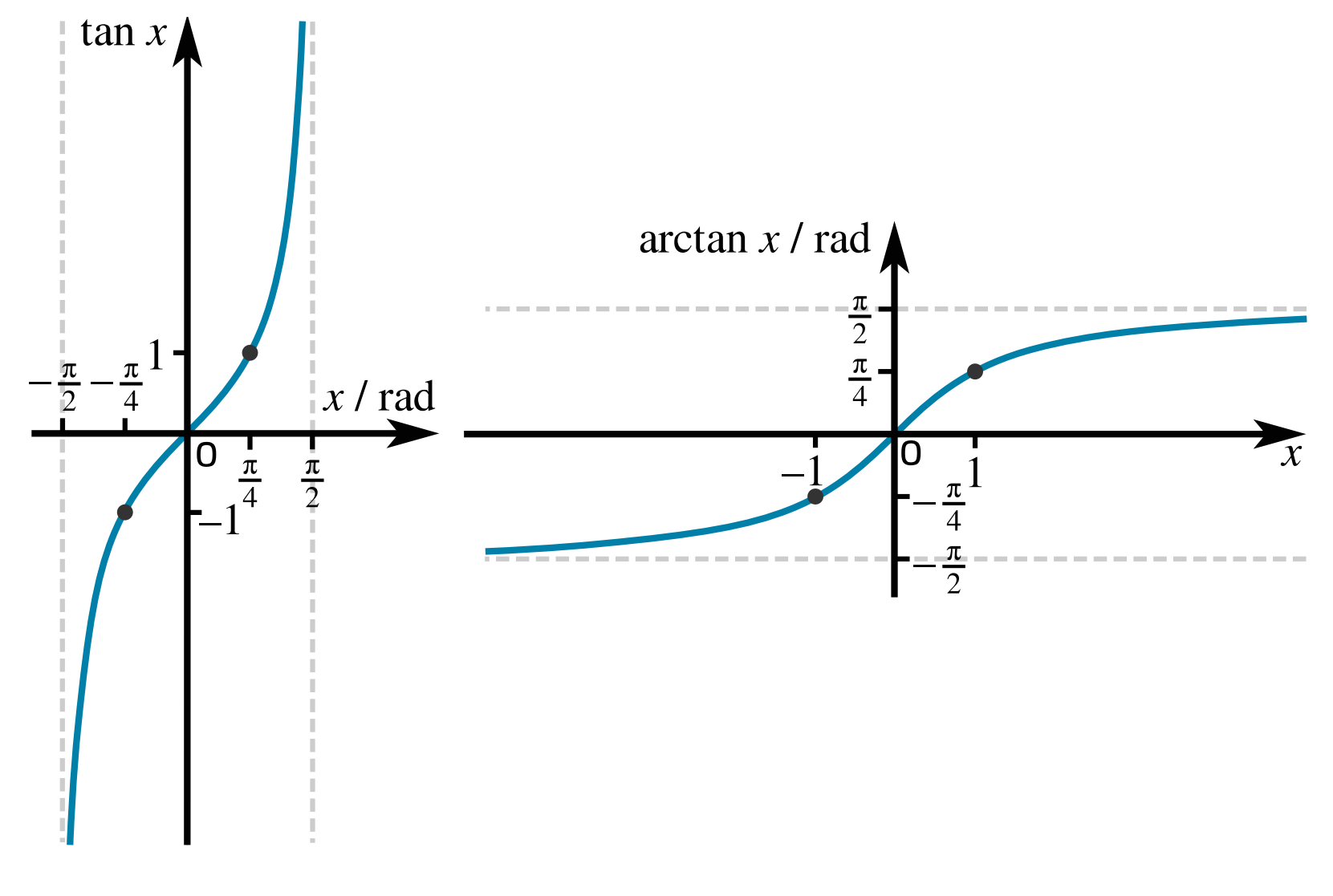


Figure 3: The graphs of $\tan x$ and $\arctan x$. Asymptotes are drawn as grey dashed lines.

In order to define an inverse function, we need to restrict the domain of $\tan x$ to the interval

The inverse of $\cos x$ is written as $\arctan x$ or $\tan^{-1} x$. The domain and range of the inverse function are

- domain:
- range:

The inverse function has horizontal asymptotes passing through on the vertical axis.

Items:

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

$-1 < x < 1$

$0 < x < 1$

\mathbb{R}

$\pm \frac{\pi}{2}$

± 1

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Simplify Trig Expressions

Pre-Uni Maths for Sciences H3.1

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level P1

Simplify the following trigonometric expressions.

Part A

$$\frac{1}{\cos^2 t - 1}$$

Simplify the following expression to give a single trigonometric function:

$$\frac{1}{\cos^2 t - 1}$$

The following symbols may be useful: $\cos()$, $\operatorname{cosec}()$, $\cot()$, $\sec()$, $\sin()$, t , $\tan()$

Part B

$$\frac{1 - \sin^2 x}{\cos x}$$

Simplify the following expression to give a single trigonometric function:

$$\frac{1 - \sin^2 x}{\cos x}$$

The following symbols may be useful: $\cos()$, $\operatorname{cosec}()$, $\cot()$, $\sec()$, $\sin()$, $\tan()$, x

Part C

$$\sin \alpha \tan \alpha - \frac{1}{\cos \alpha}$$

Simplify the following expression to give a single trigonometric function:

$$\sin \alpha \tan \alpha - \frac{1}{\cos \alpha}$$

The following symbols may be useful: α , $\cos()$, $\operatorname{cosec}()$, $\cot()$, $\sec()$, $\sin()$, $\tan()$

Part D

$$\tan w - \frac{\cos w}{1 - \sin w}$$

Simplify the following expression to give a single trigonometric function:

$$\tan w - \frac{\cos w}{1 - \sin w}$$

The following symbols may be useful: $\cos()$, $\operatorname{cosec}()$, $\cot()$, $\sec()$, $\sin()$, $\tan()$, w

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Question deck:

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Functions: Reciprocal Trig 1ii

Subject & topics: Maths**Stage & difficulty:** A Level P2**Part A**

$$\sec \frac{1}{2}\alpha = 4$$

Solve, for $0^\circ < \alpha < 180^\circ$, the equation $\sec \frac{1}{2}\alpha = 4$. Give your answer in degrees, to three significant figures.

Part B

$$\tan \beta = 7 \cot \beta$$

Solve, for $0^\circ < \beta < 180^\circ$, the equation $\tan \beta = 7 \cot \beta$. Give your answers to three significant figures.

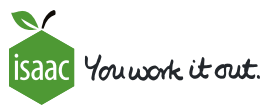
Enter your answers in order from lowest value of β to highest.

 (lowest value) (highest value)

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Advanced Trig Identities 5ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

Solve, for $0^\circ < \theta < 360^\circ$, the equation $\sec^2 \theta = 4 \tan \theta - 2$. Give inexact answers to 1 dp.

Enter your answers in order from lowest value of θ to highest.

(lowest value)

(highest value)

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Functions: Inverse Trig 3ii

Subject & topics: Maths

Stage & difficulty: A Level P2

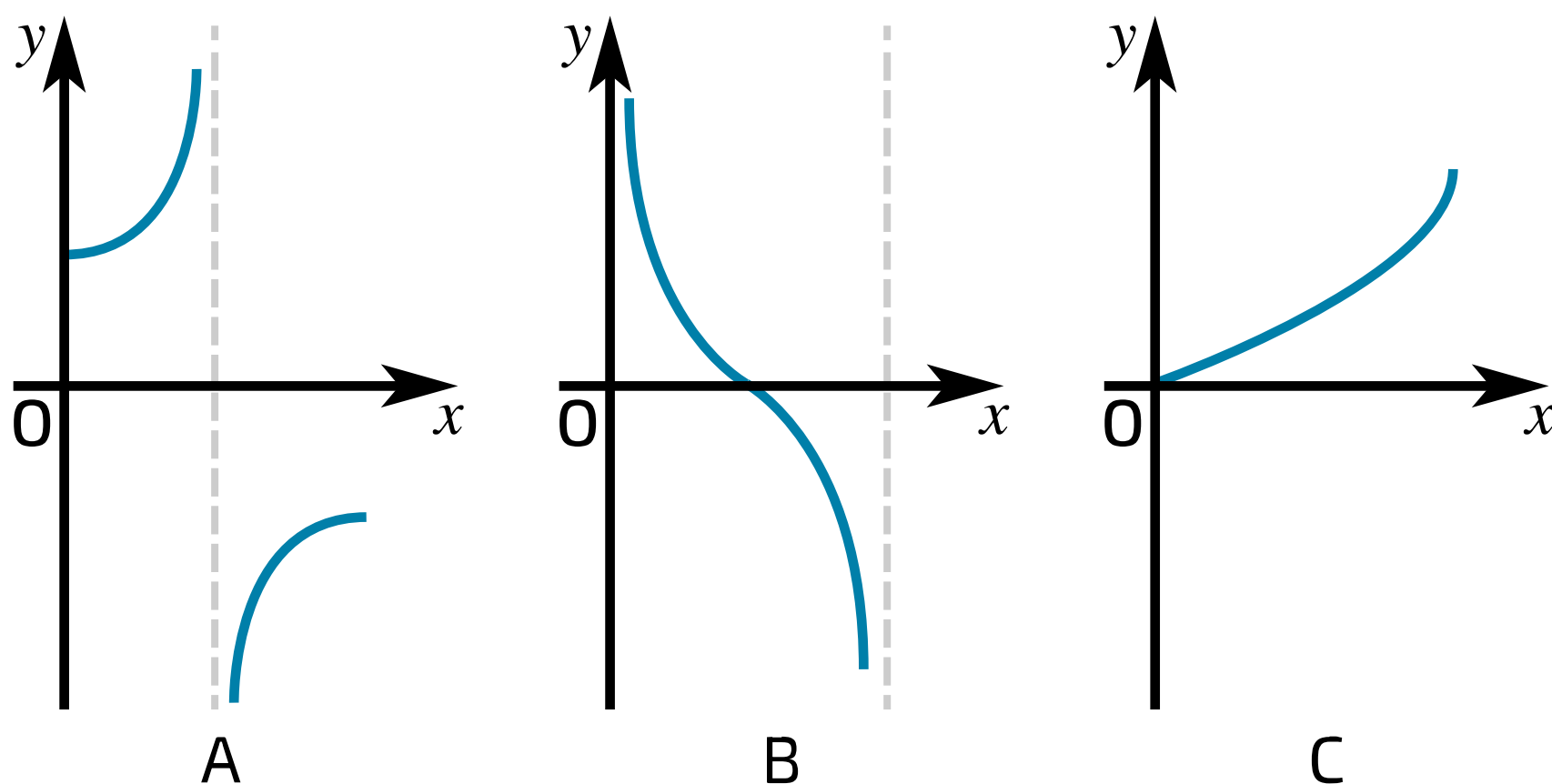


Figure 1: Graphs A, B and C.

Each diagram in **Figure 1** shows part of a curve, the equation of which is one of the following:

$$y = \arcsin x, \quad y = \arccos x, \quad y = \arctan x, \quad y = \sec x, \quad y = \operatorname{cosec} x, \quad y = \cot x$$

Part A

Graph A

Which equation corresponds to graph A?

- ☐ $y = \cot x$
- ☐ $y = \arctan x$
- ☐ $y = \arcsin x$
- ☐ $y = \operatorname{cosec} x$
- ☐ $y = \arccos x$
- ☐ $y = \sec x$

Part B

Graph B

Which equation corresponds to graph B?

- ☐ $y = \arcsin x$
- ☐ $y = \sec x$
- ☐ $y = \operatorname{cosec} x$
- ☐ $y = \arctan x$
- ☐ $y = \cot x$
- ☐ $y = \arccos x$

Part C

Graph C

Which equation corresponds to graph C?

- ☐ $y = \sec x$
- ☐ $y = \arccos x$
- ☐ $y = \arcsin x$
- ☐ $y = \cot x$
- ☐ $y = \arctan x$
- ☐ $y = \operatorname{cosec} x$

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Sketching Reciprocal Trigonometric Functions

Subject & topics: Maths | Functions | Graph Sketching **Stage & difficulty:** A Level P2

Part A

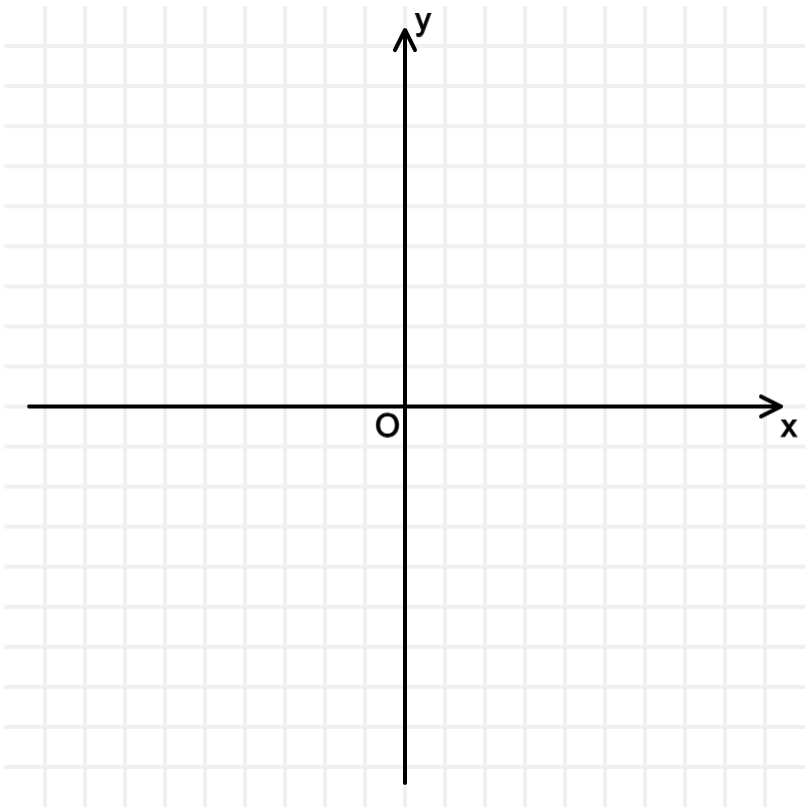
Sketch $2 \sec x + 2$

Sketch the graph of $y = 2 \sec x + 2$ in the interval $0^\circ \leq x \leq 360^\circ$.

Part B

Sketch $\cot \left(x + \frac{\pi}{4}\right) - 1$

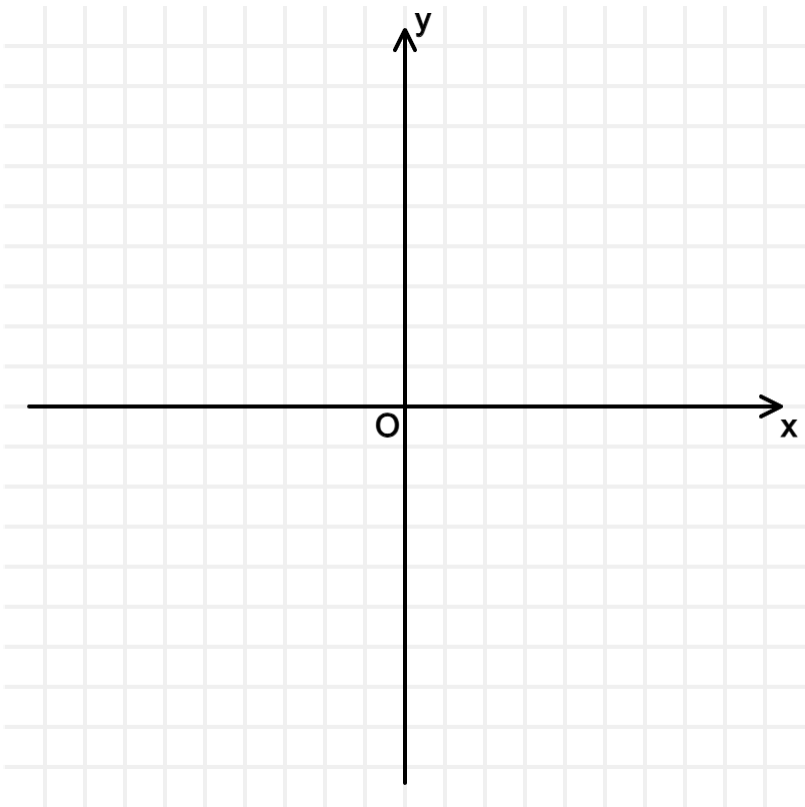
Sketch the graph of $y = \cot \left(x + \frac{\pi}{4}\right) - 1$ in the interval $0 \leq x \leq 2\pi$.



Part C

Sketch $-\operatorname{cosec} \frac{x}{2} - 3$

Sketch the graph of $y = -\operatorname{cosec} \frac{x}{2} - 3$ in the interval $-2\pi \leq x \leq 2\pi$.



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Functions: Inverse Trig 1ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

Figure 1 shows the graph of $y = -\arcsin(x - 1)$.

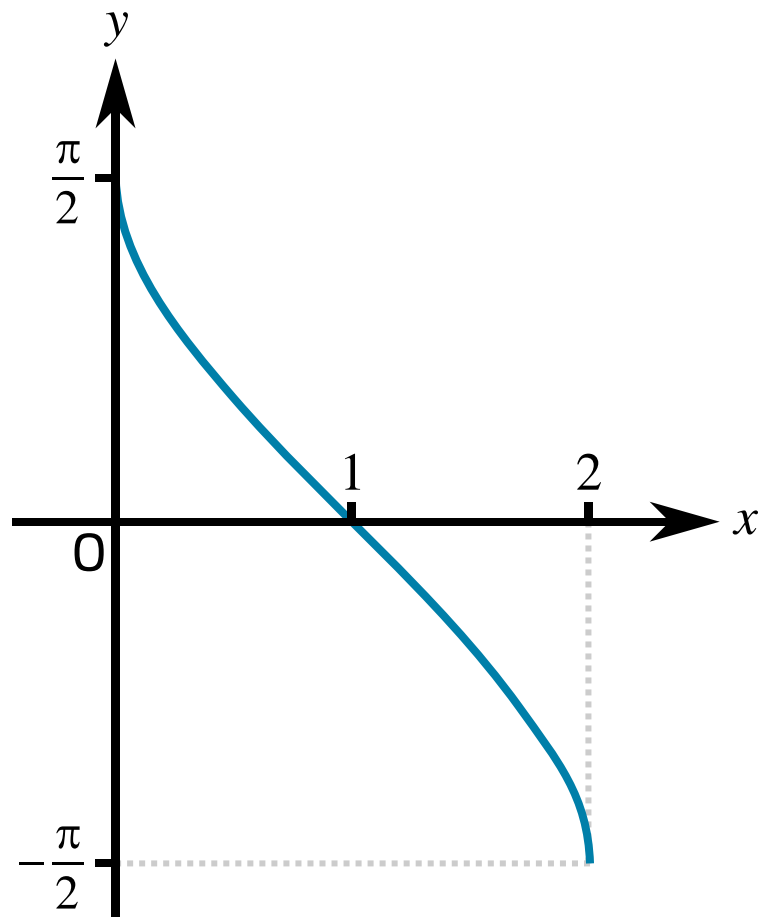


Figure 1: A graph of the function $y = -\arcsin(x - 1)$

Part A

Transformations

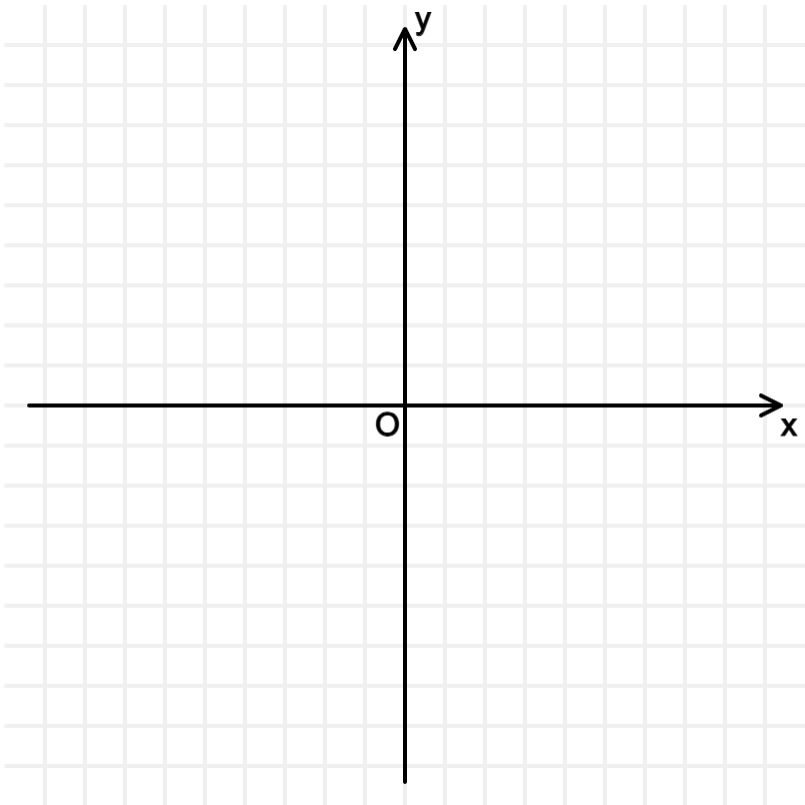
Give details of the pair of geometrical transformations which transform the graph of $y = -\arcsin(x - 1)$ to the graph of $y = \arcsin x$.

- ☐ Reflect in line $y = x$, translate by 1 in $+y$ -direction
- ☐ Reflect in $y = -x$, translate by 1 in $+x$ -direction
- ☐ Reflect in y -axis, translate by 1 in $+y$ -direction
- ☐ Reflect in y -axis, translate by 1 in $-y$ -direction
- ☐ Reflect in y -axis, translate by 1 in $-x$ -direction
- ☐ Reflect in x -axis, translate by 1 in $+x$ -direction
- ☐ Reflect in x -axis, translate by 1 in $-x$ -direction

Part B

Sketch

Sketch the graph of $y = \left| -\arcsin (x - 1) \right|$. To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.



Part C

Solutions

Find the solutions of the equation $\left| -\arcsin (x - 1) \right| = \frac{\pi}{3}$. Give your answers to 3 sf.

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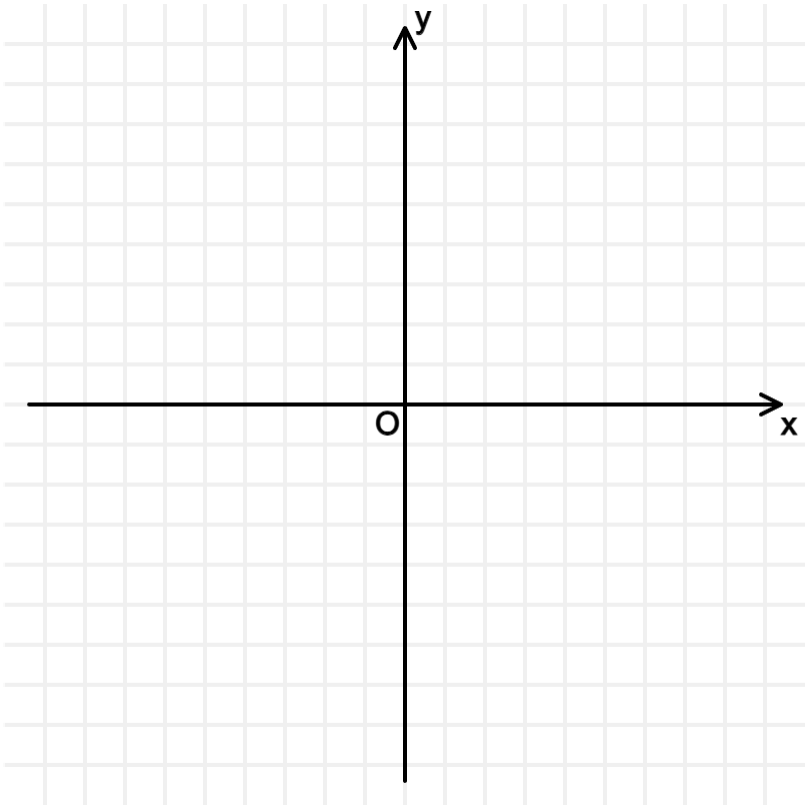


Sketching Inverse Trigonometric Functions

Subject & topics: Maths | Functions | Graph Sketching **Stage & difficulty:** A Level P2

Part A
Sketch $\arcsin \frac{x}{3} + \frac{\pi}{2}$

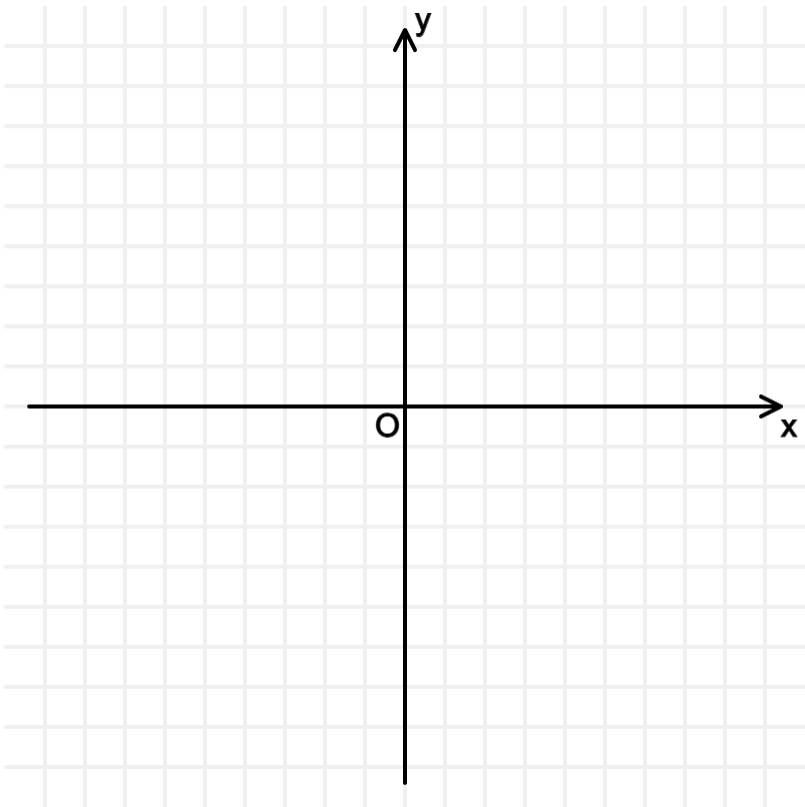
Sketch the graph of $y = \arcsin \frac{x}{3} + \frac{\pi}{2}$.



Part B

Sketch $2 \arctan (x - 1)$

Sketch the graph of $y = 2 \arctan (x - 1)$.



Part C

Sketch $\arccos\left(-\frac{x}{2} + \frac{1}{2}\right) - \frac{\pi}{3}$

Sketch the graph of $y = \arccos\left(-\frac{x}{2} + \frac{1}{2}\right) - \frac{\pi}{3}$.

