Binomial: All Rational n 3ii

A Level

Part A Expansion

Expand $(a+x)^{-2}$ in ascending powers of x up to and including the term x^2.

The following symbols may be useful: `a, x`

Part B Value of a

When $(1-x)(a+x)^{-2}$ is expanded, the coefficient of x^2 is 0. Find the value of a.

The following symbols may be useful: `a, x`

Used with permission from UCLES A-level Maths papers, 2003-2017.

Binomial: All Rational n 1i

A Level

Part A Partial Fractions

Given that $\frac{3x+4}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$, find A, B, and C.

Find A.

The following symbols may be useful: A

Find B.

The following symbols may be useful: B

Find C.

The following symbols may be useful: C

Part B Expand

Hence or otherwise expand $\frac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2.

The following symbols may be useful: x

Part C Values of x

State the set of values of x for which the expansion in the above part is valid.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $x < a$
- $x \leq a$
- $x > a$
- $x \geq a$
- $a < x < b$
- $a \leq x \leq b$
- $x < a$ or $x > b$
- $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

Gameboard:

STEM SMART Single Maths 44 - Algebra & Series Revision

Physics. You work it out.

Arithmetic Series 3i

A Level

A sequence u_1, u_2, u_3, ... is defined by

$$u_1 = 8 \quad \text{and} \quad u_{n+1} = u_n + 3.$$

Part A u_5

Find u_5.

The following symbols may be useful: `u_5`

Part B Terms in the Sequence

The n^{th} term of the sequence can be written in the form $u_n = pn + q$. State the values of p and q.

Give the value of p.

The following symbols may be useful: `p`

Give the value of q.

The following symbols may be useful: `q`

Part C Type of Sequence

What type of sequence is it?

- Arithmetic progression
- Geometric progression
- Periodic sequence

Part D Value of N

Find the value of N such that $\sum_{n=1}^{2N} u_n - \sum_{n=1}^{N} u_n = 1256$.

The following symbols may be useful: `N`

Gameboard:

STEM SMART Single Maths 44 - Algebra & Series Revision

Geometric Series 5i

A Level

Part A Value of k

The first term of a geometric progression is 50 and the common ratio is 0.8. Use logarithms to find the smallest value of k such that the k^{th} term is less than 0.15.

The following symbols may be useful: `k`

Part B First Term and Common Ratio

In a different geometric progression, the second term is -3 and the sum to infinity is 4.

Find the common ratio.

The following symbols may be useful: `r`

Hence find the first term.

Gameboard:

STEM SMART Single Maths 44 - Algebra & Series Revision

Polynomials, Factors and Roots 5i

A Level

The cubic polynomial $2x^3 - x^2 + kx + 18$ is denoted by $f(x)$. It is given that $(x + 3)$ is a factor of $f(x)$.

Part A Value of k

Find the value of k.

The following symbols may be useful: k

Part B Factorise

Factorise $f(x)$ completely.

The following symbols may be useful: x

Part C Roots of $f(x) = 0$

Give the highest (most positive) root of the equation $f(x) = 0$.

The following symbols may be useful: x

Gameboard:

STEM SMART Single Maths 44 - Algebra & Series Revision

Inequalities 1i

A Level

The length of a rectangular children's playground is $10\,\mathrm{m}$ more than its width. The width of the playground is x metres.

Part A Linear inequalities

The perimeter of the playground is greater than $64\,\mathrm{m}$. Write down a linear inequality in x in the form $ax + b > c$. (You do not have to simplify the inequality or solve for x.)

The following symbols may be useful: `<`, `>`, `x`

Part B Quadratic inequalities

The playground is less than $299\,\mathrm{m}^2$. Write down an inequality of the form $(x - a)(x + b) < 0$, where a and b are positive integers.

The following symbols may be useful: `<`, `>`, `x`

Part C Solving inequalities

By solving the inequalities from previous parts, determine the set of possible values of x.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $x < a$
- $x \leq a$
- $x > a$
- $x \geq a$
- $a < x < b$
- $a \leq x \leq b$
- $x < a$ or $x > b$
- $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

Used with permission from UCLES, A level, January 2010, Paper 4722, Question 11

Gameboard:

STEM SMART Single Maths 44 - Algebra & Series Revision

Modulus 1i

A Level

Part A Transformations

Which two of the transformations described below are needed to transform the graph of $y = |x|$ to the graph of $y = |2(x + 3)|$?

- [ ] A stretch parallel to the x-axis with scale factor 2.
- [ ] A stretch parallel to the y-axis with scale factor 2.
- [ ] A translation by 3 units in the negative x direction.
- [ ] A translation by 3 units in the positive x direction.
- [ ] A translation by 6 units in the negative x direction.
- [ ] A stretch parallel to the y-axis with scale factor 3.

Part B Inequality

Solve the inequality $|x| > |2(x + 3)|$, and give the upper bound for the solution in the form $x < a$ or $x \leq a$.

The following symbols may be useful: `<, <=, >, >=, x`

Give the lower bound for the solution in the form $x > a$ or $x \geq a$.

The following symbols may be useful: `<, <=, >, >=, x`

Gameboard:

STEM SMART Single Maths 44 - Algebra & Series Revision

Curve Sketching and Combined Transformations 3ii

A Level

Part A Transforming the curve $y = \ln x$

The curve $y = \ln x$ is transformed by:

1. A reflection in the x-axis
2. A stretch with scale factor 3 parallel to the y-axis
3. A translation in the positive y-direction by $\ln 4$

Find the equation of the resulting curve, giving your answer in the form $y = \ln(g(x))$.

The following symbols may be useful: `x, y`

Part B Graph Transformation

Figure 1 shows the curve with equation $y = f(x)$. It is given that $f(-7) = 0$ and that there are stationary points at $(-2, -6)$, and $(0, 0)$.

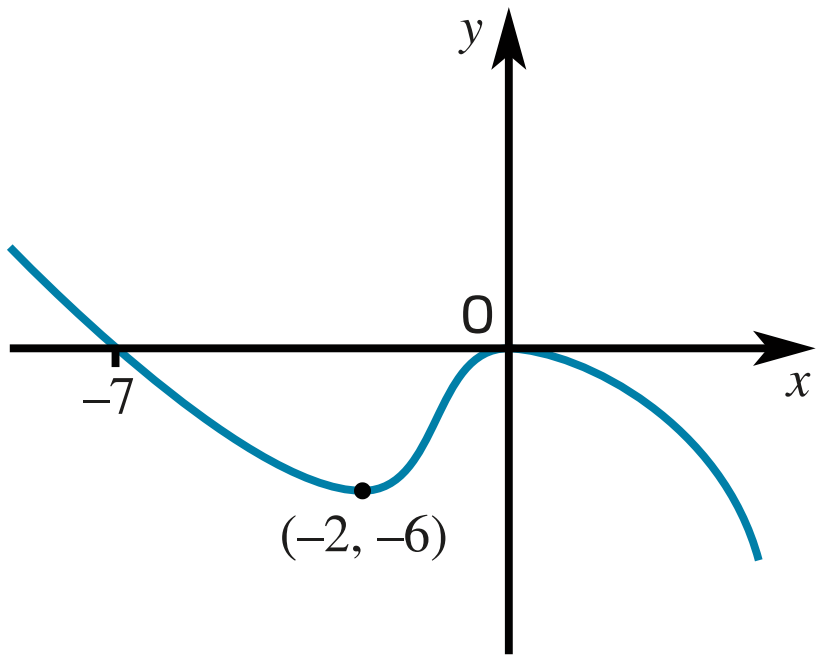

Figure 1: The curve with equation $y = f(x)$.

Sketch the curve with equation $y = -4f(x + 3)$.

Which three transformations from the list below together describe the transformation from $y = f(x)$ to $y = -4f(x + 3)$?

- [ ] A translation by 3 units in the negative x direction.
- [ ] A reflection in the y-axis.
- [ ] A reflection in the x-axis.
- [ ] A translation by 3 units in the positive x direction.
- [ ] A stretch parallel to the x-axis with scale factor 4.
- [ ] A stretch parallel to the y-axis with scale factor 4.

Part C Stationary points

The curve $y = -4f(x + 3)$ has two stationary points. Find the coordinates of the stationary point with the largest y-value.

Give the x-value of the stationary point with the largest y-value.

The following symbols may be useful: x

Give the y-value of the stationary point with the largest y-value.

The following symbols may be useful: y

Used with permission from UCLES A-level Maths papers, 2003-2017.

Gameboard:

STEM SMART Single Maths 44 - Algebra & Series Revision

Functions and Algebra 3ii

A Level

C C C

The functions f, g, and h are defined for all real values of x by

$$f(x) = |x|, \quad g(x) = 3x + 5, \quad \text{and} \quad h(x) = g\big(g(x)\big).$$

Part A Solve for x

Solve the equation $g(x + 2) = f(-12)$.

The following symbols may be useful: `x`

Part B $h^{-1}(x)$

Find $h^{-1}(x)$ in its simplest form.

The following symbols may be useful: `h, x`

Part C $x + f(x)$

Determine the values of x for which

$$x + f(x) = 0.$$

You may use symbols $|x|$ written as abs(), $=$, $<$, $>$, $<=$, $>=$.

The following symbols may be useful: `<, <=, >, >=, abs(), x`

Used with permission from UCLES A-level Maths papers, 2003-2017.

Gameboard:

STEM SMART Single Maths 44 - Algebra & Series Revision

Functions: Graphs and Inverse Functions 3i

A Level

Figure 1 shows the curve $y = f(x)$, where f is the function defined for all real values of x by

$$f(x) = 3 + 4\mathrm{e}^{-x}.$$

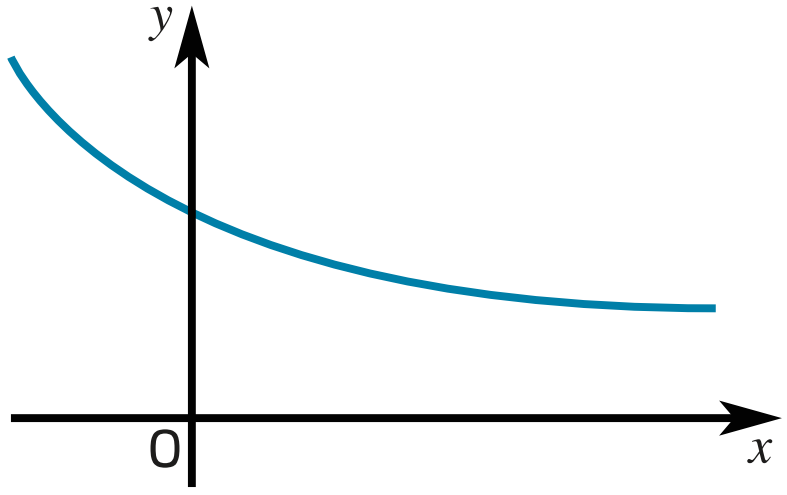

Figure 1: The curve $y = f(x)$.

Part A Range

State the range of $f(x)$ as a single inequality.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `f`, `x`, `y`

Part B $f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, x`

State the domain of $f^{-1}(x)$.

The following symbols may be useful: `f, x`

Part C **Intersection with $y = x$**

The straight line $y = x$ meets the curve $y = f(x)$ at the point P. By using an iterative process based on the equation $x = f(x)$, with a starting value of 3, find the coordinates of the point P. Give each coordinate correct to 3 decimal places.

What is the x-coordinate?

What is the y-coordinate?

Part D **Relation of P to the curves**

How is the point P related to the curves $y = f(x)$ and $y = f^{-1}(x)$?

- The point P is where $y = f(x)$ and $y = f^{-1}(x)$ intersect.
- The point P is the maximum of $y = f(x)$ and the minimum of $y = f^{-1}(x)$.
- The point P is a point of inflection for both $y = f(x)$ and $y = f^{-1}(x)$.

Used with permission from UCLES A-level Maths papers, 2003-2017.