Powers Using Chain Rule 2

A Level

P P P

Pre-Uni Maths for Sciences J4.8

Part A First and second derivatives of $u = 2(3-2v)^{\frac{3}{2}}$

Find $\frac{\mathrm{d}u}{\mathrm{d}v}$ when $u = 2(3-2v)^{\frac{3}{2}}$.

The following symbols may be useful: v

Find $\frac{\mathrm{d}^2u}{\mathrm{d}v^2}$ when $u = 2(3-2v)^{\frac{3}{2}}$.

The following symbols may be useful: v

Part B Differentiate $p = \frac{1}{\sqrt{3a+8}}$

Find $\frac{dp}{\mathrm{d}a}$ if $p = \frac{1}{\sqrt{3a+8}}$.

The following symbols may be useful: a

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Chain Rule 2

A Level P P P

Pre-Uni Maths for Sciences J6.2

Part A Differentiate $E = B\sin^2(\omega t)$.

Find $\frac{\mathrm{d}E}{\mathrm{d}t}$ if $E = B\sin^2(\omega t)$, where B and ω are constants.

The following symbols may be useful: `B, E, cos(), omega, sin(), t, tan()`

Part B Differentiate $y = \mathrm{e}^{-\frac{x^2}{2\sigma^2}}$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \mathrm{e}^{-\frac{x^2}{2\sigma^2}}$, where σ is a constant.

The following symbols may be useful: `e, sigma, x`

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Gameboard:
STEM SMART Double Maths 11 - The Chain & Product Rules

Physics. You work it out.

Home Gameboard Maths Calculus Differentiation Differentiating Exponentials 3

Differentiating Exponentials 3

A Level

P P P

Pre-Uni Maths for Sciences J4.7

Part A Tangent to $y = \mathrm{e}^{2x} - \mathrm{e}^{-2x}$

Find the equation of the tangent to the curve $y = \mathrm{e}^{2x} - \mathrm{e}^{-2x}$ at the point $x = \frac{1}{2}$.

The following symbols may be useful: `e`, `x`, `y`

Part B Stationary point of $u = 2e^{3v} - 3v$

Find the coordinates and nature of the stationary point of the function $u = 2e^{3v} - 3v$.

Find the v coordinate of the stationary point.

The following symbols may be useful: v

Find the u coordinate of the stationary point.

The following symbols may be useful: u

Determine the nature of the stationary point.

- Minimum
- Maximum

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Physics. You work it out.

Home Gameboard Maths Calculus Differentiation Differentiating Natural Logs

Differentiating Natural Logs

A Level
P P P

Pre-Uni Maths for Sciences J4.10

Part A Differentiate $u = \ln(2v+3)$

Find $\dfrac{\mathrm{d}u}{\mathrm{d}v}$ if $u = \ln(2v+3)$.

The following symbols may be useful: v

Part B Stationary point of $p = 2\ln(2q) - 3q$

Find the coordinates and nature of the stationary point of the function $p = 2\ln(2q) - 3q$.

Give the q-coordinate of the stationary point.

The following symbols may be useful: q

Give the p-coordinate of the stationary point.

The following symbols may be useful: p

Determine the nature of the stationary point.

- Maximum
- Minimum

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Physics. You work it out.

Differentiating Trig Functions 3

A Level

P P P

Pre-Uni Maths for Sciences J4.3

Part A Velocity and acceleration if $x = A\cos(\omega t + \phi)$

The displacement x of an oscillating particle at time t is given by $x = A\cos(\omega t + \phi)$ where A, ω and ϕ are constants; find expressions for the velocity (the rate of change of displacement) and acceleration (the rate of change of velocity) of the particle.

Find an expression for the velocity (the rate of change of displacement) of the particle.

The following symbols may be useful: `A, a, cos(), omega, phi, sin(), t, tan(), v`

Find an expression for the acceleration (the rate of change of velocity) of the particle.

The following symbols may be useful: `A, a, omega, phi, t, v`

Part B Stationary points of the function $y = \cos(\omega t) + \sin(\omega t)$

Consider the function $y = \cos(\omega t) + \sin(\omega t)$, where ω is a positive constant.

Find the stationary points of the function in the range $0 < t < \frac{2\pi}{\omega}$. How many are there? The stationary point with the lowest value of t is at (t_1, y_1) and the stationary point with the second lowest value of t is at (t_2, y_2). Find the values of t and y at (t_1, y_1) and (t_2, y_2).

How many stationary points are there?

- 0
- 1
- 2
- 3
- 4

Find t_1, the t-coordinate of the stationary point with the lowest value of t in the range $0 < t < \frac{2\pi}{\omega}$.

The following symbols may be useful: `omega, pi, t_1, y_1`

Find t_2, the t-coordinate of the stationary point with the second lowest value of t in the range $0 < t < \frac{2\pi}{\omega}$.

The following symbols may be useful: `omega, pi, t_2, y_2`

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Gameboard:
STEM SMART Double Maths 11 - The Chain & Product Rules

Product Rule 2

A Level

P P P

Pre-Uni Maths for Sciences J6.4

Part A Differentiate $y = \alpha t^2 e^{\beta t}$

Find $\frac{\mathrm{d}y}{\mathrm{d}t}$ where $y = \alpha t^2 \mathrm{e}^{\beta t}$, given that α and β are constants.

The following symbols may be useful: `alpha, beta, e, t`

Part B Differentiate $\tan\theta$

Find the derivative w.r.t. θ of $\tan\theta$ by writing it as $\frac{\sin\theta}{\cos\theta}$.

The following symbols may be useful: `cos(), cosec(), cot(), sec(), sin(), tan(), theta`

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STEM SMART Double Maths 11 - The Chain & Product Rules

Differentiation: Products 4ii

A Level
P P P

Differentiate with respect to x, simplifying your answers where possible.

Part A $\sin x \tan x$

Differentiate $\sin x \tan x$.

The following symbols may be useful: x

Part B $x^2(x+1)^6$

Differentiate $x^2(x+1)^6$.

The following symbols may be useful: x

Gameboard:
STEM SMART Double Maths 11 - The Chain & Product Rules

Physics. You work it out.

Home Gameboard Maths Differentiation: Quotients 2ii

Differentiation: Quotients 2ii

A Level
P P P

Differentiate with respect to x, simplifying your answers where possible.

Part A $\frac{\ln x}{x}$

$y = \frac{\ln x}{x}$

The following symbols may be useful: `Derivative(y, x)`, `ln()`, `log()`, `x`, `y`

Part B $\frac{x^2}{\ln x}$

$y = \frac{x^2}{\ln x}$

The following symbols may be useful: `Derivative(y, x)`, `ln()`, `log()`, `x`, `y`

Part C **Stationary point of $y = \frac{x^2}{\ln x}$**

Determine the exact x-coordinate of the stationary point of the curve $y = \frac{x^2}{\ln x}$.

The following symbols may be useful: `e`, `ln()`, `x`

Gameboard:

STEM SMART Double Maths 11 - The Chain & Product Rules

Differentiation: Chain Rule 5ii

A Level

P P P

Leaking oil is forming a circular patch on the surface of the sea. The area of the patch is increasing at a rate of 250 square metres per hour. Find the rate at which the radius of the patch is increasing at the instant the area of the patch is 1900 square metres. Give your answer in metres per hour correct to two significant figures.

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Differentiation: Chain Rule 5i

A Level

P P P

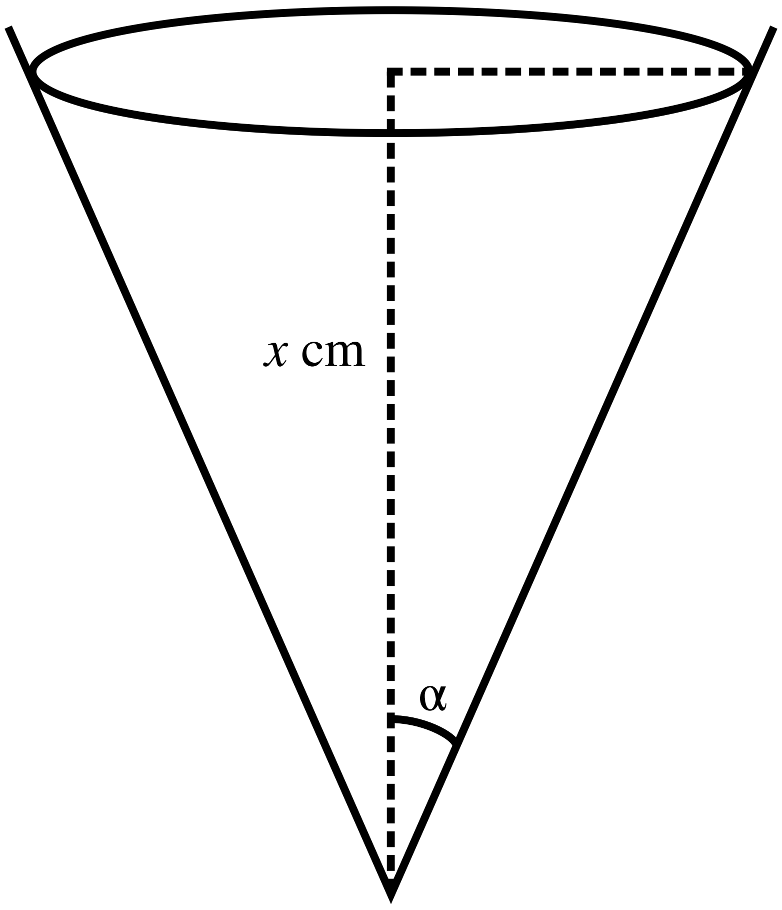

Figure 1: A diagram of a container in the form of a right circular cone.

Figure 1 shows a container in the form of a right circular cone. The angle between the axis and the slant height is α, where $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$. Initially the container is empty, and then liquid is added at the rate of $14\,\mathrm{cm}^3\,\mathrm{min}^{-1}$. The depth of liquid in the container at time t minutes is $x\,\mathrm{cm}$.

Part A Volume in Terms of Depth

Find an expression for the volume, $V\,\mathrm{cm}^3$, of liquid in the container when the depth is $x\,\mathrm{cm}$ in terms of x. [The volume of a cone is $\frac{1}{3}\pi r^2 h$.]

The following symbols may be useful: `V`, `pi`, `x`

Part B Rate of Change

Find the rate at which the depth of the liquid is increasing at the instant when the depth is $8\,\mathrm{cm}$. Give your answer in $\mathrm{cm\,min^{-1}}$ correct to two significant figures.