Differentiation from First Principles 2

A Level

P P P

Pre-Uni Maths for Sciences J3.3 & J3.4

Differentiating a function $f(x)$ from first principles involves taking a limit. The derivative of $f(x)$ is given by the expression

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Part A Differentiate $4x^2 + 2x + 7$ from first principles

Differentiate $f(x) = 4x^2 + 2x + 7$ from first principles. Drag and drop options into the spaces below.

$f(x+h) = 4(x+h)^2 + 2(x+h) + 7$. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(4(x+h)^2 + 2(x+h) + 7) - (4x^2 + 2x + 7)}{h}.$$

Next, expanding the brackets in the numerator and simplifying gives

$$f'(x) = \lim_{h \to 0} \frac{\boxed{} - (4x^2 + 2x + 7)}{h}$$

$$f'(x) = \lim_{h \to 0} (\boxed{} + (\boxed{})h).$$

Finally, take the limit. As $h \to 0$, the terms containing h tend to 0. Therefore,

$$f'(x) = \boxed{}.$$

Items:

4 | $4x^2 + 4h^2$ | $4x^2 + 2x + 7 + 4hx + 2h + 4h^2$ | $8x + 4$ | $8x + 2$ | $4x^2 + 2x + 7 + 8hx + 2h + 4h^2$ | 7

Part B Differentiate $ax^2 + bx + c$ from first principles

Differentiate $f(x) = ax^2 + bx + c$, where a, b and c are constants, from first principles.

$f(x+h) = a(x+h)^2 + b(x+h) + c$. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(a(x+h)^2 + b(x+h) + c) - (ax^2 + bx + c)}{h}.$$

Next, expanding the brackets in the numerator and simplifying gives

$$f'(x) = \lim_{h \to 0} \frac{\boxed{} + (\boxed{})h + (\boxed{})h^2}{h}$$

$$f'(x) = \lim_{h \to 0} (\boxed{} + (\boxed{})h).$$

Finally, take the limit. As $h \to 0$, the terms containing h tend to 0. Therefore,

$$f'(x) = \boxed{}.$$

Items:

$ax^2 + 2ahx + ah^2$ | $2ax + b$ | ab | a | 1 | $2a$ | $b + ah$ | 0

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Differentiation (powers of x) 1i

A Level
P P P

It is given that $f(x) = \frac{1}{x} - \sqrt{x} + 3$.

Part A Find $f'(x)$

Find $f'(x)$.

The following symbols may be useful: x

Part B Find $f''(4)$

Find $f''(4)$.

Used with permission from UCLES, A level, January 2010, Paper 4721, Question 9.

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STEM SMART Single Maths 17 - Calculus Revision

Integration (powers of x) 2ii

A Level

P P P

Part A Find integral

Find $\int x(x^2-4)\mathrm{d}x$.

The following symbols may be useful: `C, c, k, x`

Part B Evaluate integral

Evaluate $\int_1^6 x(x^2-4)\mathrm{d}x$. Give the exact value of your answer as a decimal.

Part C Find integral

Find $\int \frac{6}{x^3}\mathrm{d}x$.

The following symbols may be useful: `C, c, k, x`

Used with permission from UCLES, A Level Maths, June 2007, OCR C2, Question 6

Gameboard:
STEM SMART Single Maths 17 - Calculus Revision

Area Under a Curve 2

A Level

P P P

Pre-Uni Maths for Sciences K2.8

A graph of the functions $y = x^2 + 3$ and $y = 4x$ is shown in **Figure 1**. Find the area of the shaded region labelled A, the region between the line $y = 4x$ and the curve $y = x^2 + 3$.

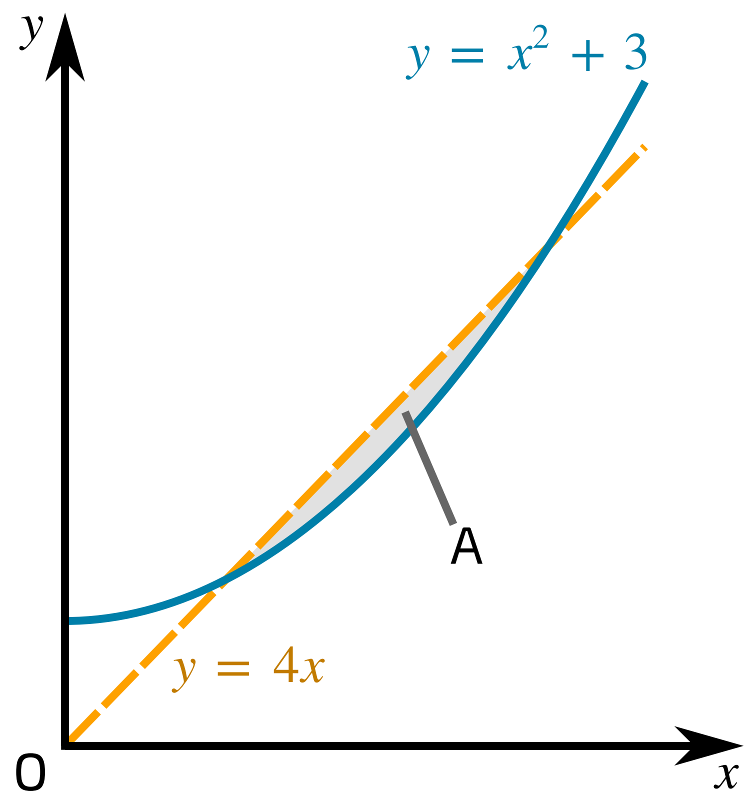

Figure 1: A graph of the functions $y = x^2 + 3$ and $y = 4x$. The shaded region A is the region between the line $y = 4x$ and the curve $y = x^2 + 3$.

Find the area of the region A. Give your answer in the form of an improper fraction.

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STEM SMART Single Maths 17 - Calculus Revision

Functions from Differential Equations 2i

A Level

P P P

The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + a$, where a is a constant. The curve passes through the points $(-1, 2)$ and $(2, 17)$. Find the equation of the curve.

The following symbols may be useful: `x`, `y`

Used with permission from UCLES, A level, June 2009, Paper 4722, Question 6.

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Physics. You work it out.

Home Gameboard Maths Calculus

Calculus

A Level
C C C

Part A Integrating a factorised expression

Find $\int (x^2+9)(x-4)\mathrm{d}x$.

The following symbols may be useful: `C, c, k, x`

Part B Differentiation

A curve has the equation $y=\frac{1}{3}x^3-9x$.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), x, y`

Part C Stationary points

Find the coordinates of the stationary points of the curve $y=\frac{1}{3}x^3-9x$. Enter the x and y coordinates of the stationary point with the largest x coordinate.

(____ , ____)

Part D Nature of stationary point

Determine the nature of the stationary point with the largest x-coordinate.

- Minimum
- Maximum
- Neither/Inconclusive

Part E Tangent to the curve

Given that $24x + 3y + 2 = 0$ is the equation of the tangent to the curve $y = \frac{1}{3}x^3 - 9x$ at the point (p, q), find the values of p and q.

(i) Enter value of p:

The following symbols may be useful: p

(ii) Enter value of q:

The following symbols may be useful: q

Part F Normal to the curve

Find the equation of the normal to the curve $y = \frac{1}{3}x^3 - 9x$ at the point (p, q) you found in Part E.

Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers

The following symbols may be useful: x, y

Modified by Sally Waugh with permission from UCLES, A Level, June 2005, Paper 4721, Question 10.

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Minimisation: Surface Area

A Level

Figure 1 shows a solid shape, which is made out of a cuboid and two half-cylinders.

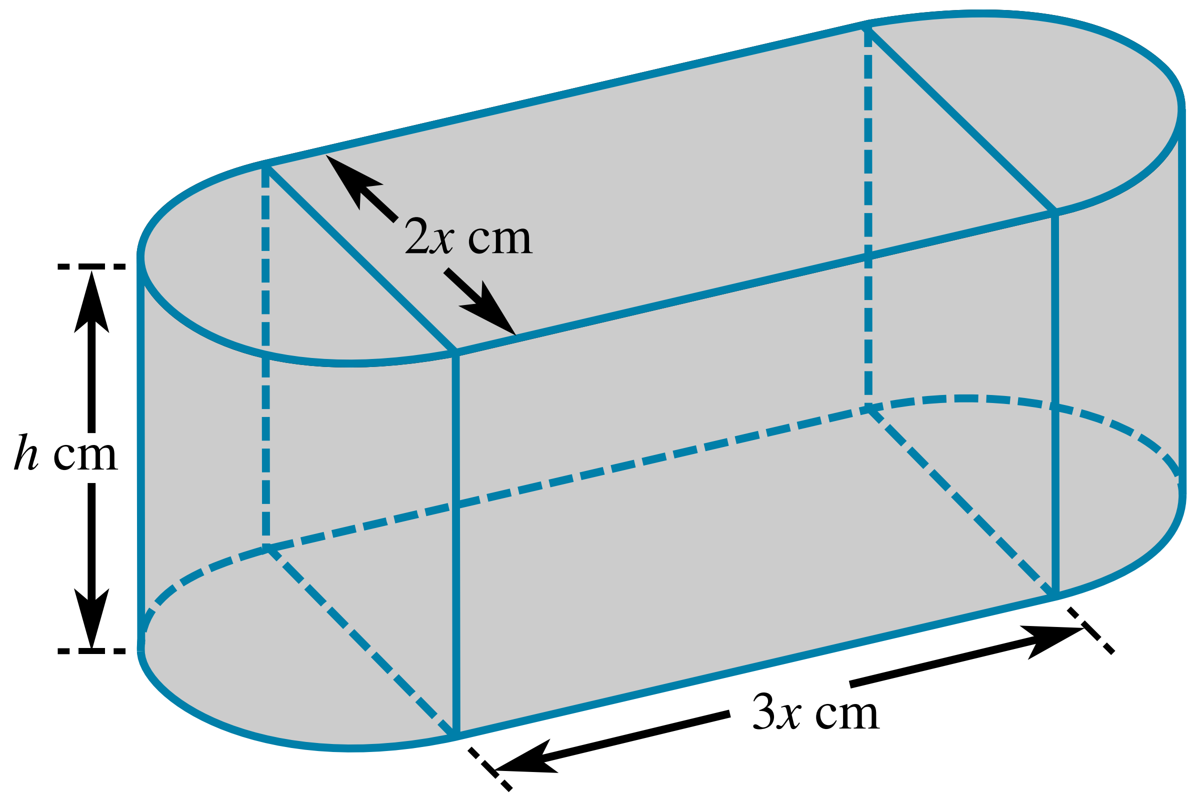

Figure 1: A solid shape made out of a cuboid and two half-cylinders.

Part A Expression for surface area

Find an expression for the surface area of the shape S in terms of π, x and h.

The following symbols may be useful: `S`, `h`, `pi`, `x`

Part B Expression for volume

Find an expression for the volume of the shape V in terms of π, x and h.

The following symbols may be useful: `V, h, pi, x`

Part C Minimum surface area

If the volume of the shape is $32\,000\,\mathrm{cm}^3$, find the value of x for which the surface area is a minimum. Give your answer to 3 significant figures.

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