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Home Gameboard Maths Vectors: Diagrams and Proof 2ii

Vectors: Diagrams and Proof 2ii

A Level
P P P

Part A Resultant vector

Find the resultant of the vectors $4\underline{\boldsymbol{i}} - \underline{\boldsymbol{j}}$ and $-2\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}}$.

[ ] $\underline{\boldsymbol{i}} +$ [ ] $\underline{\boldsymbol{j}}$

Part B $\overrightarrow{\mathrm{MN}}$

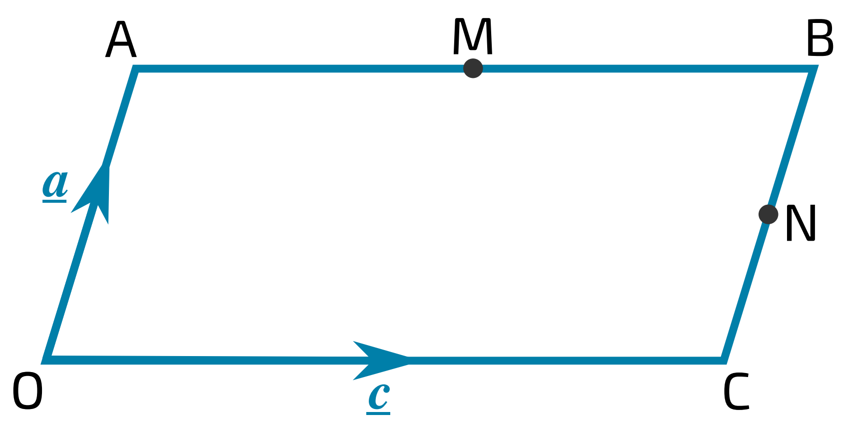

Figure 1: A parallelogram OABC.

In **Figure 1**, OABC is a parallelogram. M is the midpoint of AB. N is the midpoint of BC. $\overrightarrow{\mathrm{OA}} = \underline{\boldsymbol{a}}$ and $\overrightarrow{\mathrm{OC}} = \underline{\boldsymbol{c}}$.

Find $\overrightarrow{\mathrm{MN}}$ in terms of $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{c}}$.

The following symbols may be useful: `a, c`

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Vectors: Diagrams and Proof 1ii

A Level
P P P

Part A $2\underline{\boldsymbol{s}} - \underline{\boldsymbol{t}}$

Given that $\underline{\boldsymbol{s}} = 3\underline{\boldsymbol{i}} + 4\underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{t}} = 6\underline{\boldsymbol{i}} - \underline{\boldsymbol{j}}$, find $2\underline{\boldsymbol{s}} - \underline{\boldsymbol{t}}$.

$$2\underline{\boldsymbol{s}} - \underline{\boldsymbol{t}} = \boxed{}\ \underline{\boldsymbol{i}} + \boxed{}\ \underline{\boldsymbol{j}}$$

Part B In terms of $\underline{\boldsymbol{p}}$

Figure 1: Three points P, X and Q.

Figure 1 shows three points P, X and Q such that $\overrightarrow{\mathrm{XQ}} = 3\overrightarrow{\mathrm{PX}}$.

Given that $\overrightarrow{\mathrm{PX}} = \underline{\boldsymbol{p}}$, find $\overrightarrow{\mathrm{XQ}}$ and $\overrightarrow{\mathrm{QP}}$ in terms of $\underline{\boldsymbol{p}}$.

If a value is not an integer, enter the value as a decimal.

$$\overrightarrow{\mathrm{XQ}} = \boxed{}\ \underline{\boldsymbol{p}}$$

$$\overrightarrow{\mathrm{QP}} = \boxed{}\ \underline{\boldsymbol{p}}$$

Part C Proving AMCN is a parallelogram

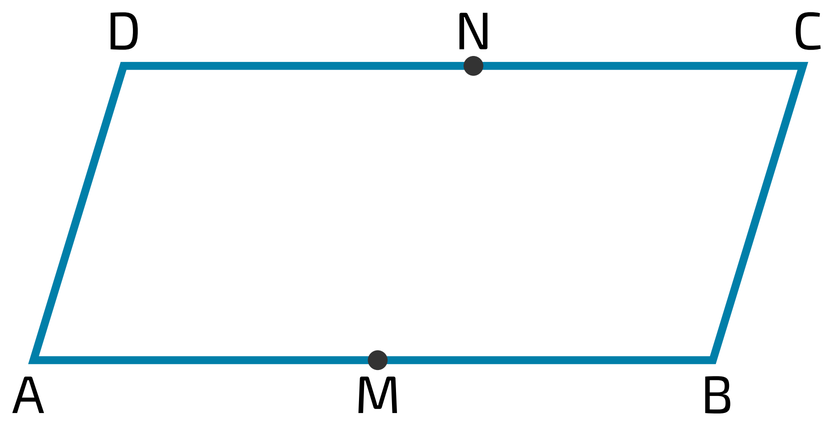

Figure 2: The parallellogram ABCD.

In **Figure 2** ABCD is a parallelogram. M and N are the mid-points of AB and DC. $\overrightarrow{AB} = \underline{\boldsymbol{a}}$ and $\overrightarrow{AD} = \underline{\boldsymbol{b}}$. Use a vector method to prove that AMCN is also a parallelogram.

Choose four items from the left and put them into order on the right to create a proof.

Available items

1. A parallelogram has two pairs of sides which are parallel and of equal length.

1. A parallelogram has two pairs of sides which are parallel. All sides of a parallelogram are the same length.

2. $\overrightarrow{AM} = \overrightarrow{NC} = \frac{1}{2}\underline{\boldsymbol{a}}$. Therefore $\overrightarrow{AM}$ is parallel to $\overrightarrow{NC}$ and has the same length.

2. $\overrightarrow{MB} = \overrightarrow{NC} = \frac{1}{2}\underline{\boldsymbol{a}}$. Therefore $\overrightarrow{MB}$ is parallel to $\overrightarrow{NC}$ and has the same length.

3. $\overrightarrow{AD} = \overrightarrow{BC} = \underline{\boldsymbol{b}}$. Therefore $\overrightarrow{AD}$ is parallel to $\overrightarrow{BC}$ and has the same length.

3. $\overrightarrow{AN} = \overrightarrow{MC} = \frac{1}{2}\underline{\boldsymbol{a}} + \underline{\boldsymbol{b}}$. Therefore $\overrightarrow{AN}$ is parallel to $\overrightarrow{MC}$ and has the same length.

4. AMCN has two pairs of sides which are parallel and of equal length. Hence, AMCN is a parallelogram.

4. AMCN has four sides which are parallel and of equal length. Hence, AMCN is a parallelogram.

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Vectors: Diagrams and Proof 2i

A Level

P P P

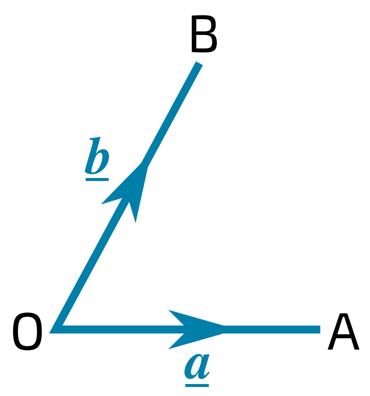

Figure 1: Points A and B and their position vectors with respect to the origin O.

In **Figure 1**, the points A and B have position vectors $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ with respect to the origin O.

Part A Sketch

Make a sketch of the diagram, and mark on the points C, D and E such that $\overrightarrow{OC} = 2\underline{\boldsymbol{a}}$, $\overrightarrow{OD} = 2\underline{\boldsymbol{a}} + \underline{\boldsymbol{b}}$ and $\overrightarrow{OE} = \frac{1}{3}\overrightarrow{OD}$.

Which of the sketches below correctly shows this information?

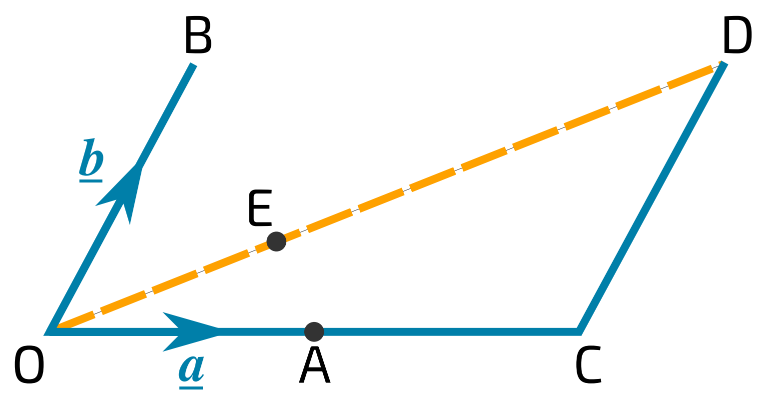

Figure 2: Option A

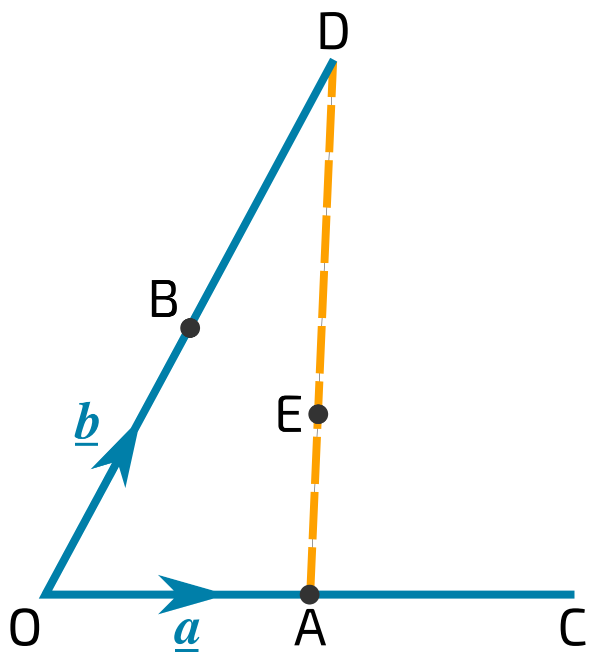

Figure 3: Option B

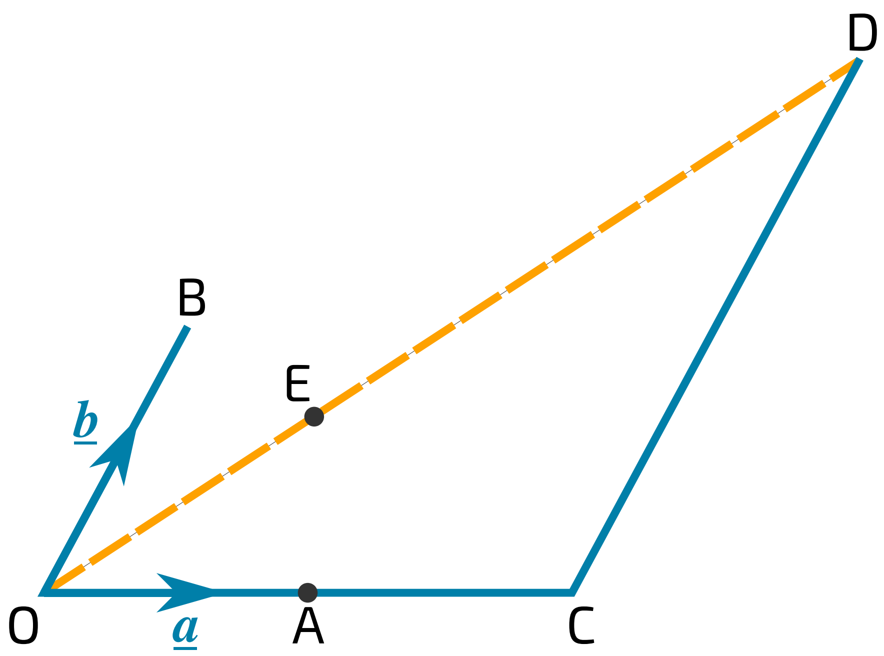

Figure 4: Option C

- Option A

Option B

Option C

Part B Position vector of E

Give the position vector of E with respect to A.

The following symbols may be useful: `a, b`

Part C Proof

Hence prove that E lies on the line joining A and B.

Choose three items from the choices on the left and put them into order on the right to create a proof.

Available items

1. Point E lies on the line joining A and B if the vector $\overrightarrow{\mathrm{OE}}$ is a scalar multiple of the vector $\overrightarrow{\mathrm{OB}}$, i.e. $\overrightarrow{\mathrm{OE}} = k\overrightarrow{\mathrm{OB}}$.

1. Point E lies on the line joining A and B if the vector $\overrightarrow{\mathrm{AE}}$ is a scalar multiple of the vector $\overrightarrow{\mathrm{AB}}$, i.e. $\overrightarrow{\mathrm{AE}} = k\overrightarrow{\mathrm{AB}}$.

2. $\overrightarrow{\mathrm{AE}} = \frac{1}{3}(\underline{\boldsymbol{b}} - \underline{\boldsymbol{a}})$ and $\overrightarrow{\mathrm{AB}} = \underline{\boldsymbol{b}} - \underline{\boldsymbol{a}}$. Therefore, $\overrightarrow{\mathrm{AE}} = \frac{1}{3}\overrightarrow{\mathrm{AB}}$.

2. $\overrightarrow{\mathrm{AE}} = \underline{\boldsymbol{b}} - \underline{\boldsymbol{a}}$ and $\overrightarrow{\mathrm{AB}} = \frac{1}{3}(\underline{\boldsymbol{b}} - \underline{\boldsymbol{a}})$. Therefore, $\overrightarrow{\mathrm{AE}} = 3\overrightarrow{\mathrm{AB}}$.

3. $\overrightarrow{\mathrm{OE}}$ is a scalar multiple of $\overrightarrow{\mathrm{OD}}$. Hence, E lies on the line joining O and D.

3. $\overrightarrow{\mathrm{AE}}$ is a scalar multiple of $\overrightarrow{\mathrm{AB}}$. Hence, E lies on the line joining A and B.

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Home Gameboard Maths 3D Vectors 2ii

3D Vectors 2ii

A Level
P P P

The points A and B have position vectors $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ relative to an origin O, where $\underline{\boldsymbol{a}} = 4\underline{\boldsymbol{i}} + 3\underline{\boldsymbol{j}} - 2\underline{\boldsymbol{k}}$ and $\underline{\boldsymbol{b}} = -7\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}} + 4\underline{\boldsymbol{k}}$.

Part A Length $\overrightarrow{AB}$

Find the length of $\overrightarrow{AB}$. Give your answer as an exact surd.

Part B Unit vector

Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. Give your answer in terms of the unit vectors $\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{k}}$.

The following symbols may be useful: `i, j, k`

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3D Vectors 1ii

A Level

P P P

ABCD is a parallelogram. The position vectors of A, B and C are given respectively by

$$\underline{\boldsymbol{a}} = 2\underline{\boldsymbol{i}} + \underline{\boldsymbol{j}} + 3\underline{\boldsymbol{k}} \qquad \underline{\boldsymbol{b}} = 3\underline{\boldsymbol{i}} - 2\underline{\boldsymbol{j}} \qquad \underline{\boldsymbol{c}} = \underline{\boldsymbol{i}} - \underline{\boldsymbol{j}} - 2\underline{\boldsymbol{k}}$$

Part A Position of D

Find the position vector of D. Give your answer in terms of the unit vectors $\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{k}}$.

The following symbols may be useful: `i`, `j`, `k`

Part B Unit vector

Find the unit vector in the direction $\overrightarrow{\mathrm{OD}}$ where O is the fixed origin. Give your answer in terms of the unit vectors $\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{k}}$.

The following symbols may be useful: `i`, `j`, `k`

Adapted with permission from UCLES, A Level, June 2005, OCR C4, Question 5

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Physics. You work it out.

3D Vectors 2i

A Level
P P P

ABCD is a quadrilateral. You are given four pieces of information:

- Relative to a fixed origin O, the position vector of A is $2\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}} + 8\underline{\boldsymbol{k}}$.
- Relative to a fixed origin O, the position vector of B is $5\underline{\boldsymbol{i}} + 9\underline{\boldsymbol{j}} + 8\underline{\boldsymbol{k}}$.
- The vector $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$.
- The vector $\overrightarrow{\mathrm{BD}} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$.

Part A Finding $\overrightarrow{\mathrm{AB}}$

Find the vector $\overrightarrow{\mathrm{AB}}$. Give your answer in $\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$, $\underline{\boldsymbol{k}}$ form.

$$\overrightarrow{\mathrm{AB}} = \square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}}$$

Part B Finding $\overrightarrow{\mathrm{CD}}$

Find the vector $\overrightarrow{\mathrm{CD}}$. Give your answer in $\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$, $\underline{\boldsymbol{k}}$ form.

$$\overrightarrow{\mathrm{CD}} = \square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}}$$

Part C Finding $\overrightarrow{AD}$

Find the vector $\overrightarrow{AD}$. Give your answer in $\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$, $\underline{\boldsymbol{k}}$ form.

$$\overrightarrow{AD} = \square\ \underline{\boldsymbol{i}} + \square\ \underline{\boldsymbol{j}} + \square\ \underline{\boldsymbol{k}}$$

Part D Type of quadrilateral

The shape ABCD lies in a plane. What type of quadrilateral is ABCD?

- A parallelogram
- A trapezium
- A rhombus
- A rectangle
- An irregular quadrilateral
- A kite
- A square

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Vectors in 3D

A Level
P P P

Given that $\underline{\boldsymbol{a}} = 6\underline{\boldsymbol{i}} + (p-10)\underline{\boldsymbol{j}} + (3p-5)\underline{\boldsymbol{k}}$, and that $|\underline{\boldsymbol{a}}| = 11$, find the possible values of p.

Part A Smaller value of p

Enter the smaller value of p:

Part B Larger value of p

Enter the larger value of p:

Adapted for Isaac Physics from NST IA Biology preparation work

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Angles Between a 3D Vector and the Axes

A Level
P P P

Find the angles between the vector $\underline{\boldsymbol{i}} + 2\underline{\boldsymbol{j}} + 3\underline{\boldsymbol{k}}$ and the x, y and z coordinate axes.

Part A Angle with x axis

What is the angle in degrees between the vector and the x-axis? Give your answer to 3 sf.

Part B Angle with y axis

What is the angle in degrees between the vector and the y-axis? Give your answer to 3 sf.

Part C Angle with z axis

What is the angle in degrees between the vector and the z-axis? Give your answer to 3 sf.

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Physics. You work it out.

Home Gameboard Maths Geometry Vectors Manipulating Vectors in 3D

Manipulating Vectors in 3D

A Level
C C C

Pre-Uni Maths for Sciences I1.10

A vector $\underline{\boldsymbol{u}} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$ has a length of 4.00 units.

Part A Case 1

If $\underline{\boldsymbol{u}}$ lies in the (x, y)-plane, makes an angle of 30° with the x-direction and u_y is positive, find u_x.

Give your answer to 3 sf.

Part B Case 2

If $u_x = u_y = 2$ and u_z is negative, find u_z.

Give your answer to 3 sf.

Part C Case 3

If $u_z = 1$, $u_y = 2u_x$ and u_y is positive, find u_y.

Give your answer to 3 sf.

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3D Vectors 3ii

A Level

P P P

Two points A and B have position vectors $3\underline{\boldsymbol{i}} - \underline{\boldsymbol{j}} + 2\underline{\boldsymbol{k}}$ and $2\underline{\boldsymbol{j}} + 3\underline{\boldsymbol{k}}$ respectively.

Part A Vector $\overrightarrow{\mathrm{AB}}$

Find the vector $\overrightarrow{\mathrm{AB}}$.

$$\overrightarrow{\mathrm{AB}} = \boxed{}\ \underline{\boldsymbol{i}} + \boxed{}\ \underline{\boldsymbol{j}} + \boxed{}\ \underline{\boldsymbol{k}}$$

Hence find the length of $\overrightarrow{\mathrm{AB}}$. Give your answer as an exact surd.

$$|\overrightarrow{\mathrm{AB}}| = \sqrt{\boxed{}}$$

Part B Intersection

Show that the line through A and B does not intersect the line through the origin parallel to the vector $\underline{\boldsymbol{i}}$.

One way to prove this is to use proof by contradiction. Fill in the blanks to complete the proof below.

Opening statement:

The line through the origin parallel to the vector $\underline{\boldsymbol{i}}$ is the x-axis. On the x-axis, $y = z =$ ______. If the line through A and B intersects the x-axis, then there is a value of λ such that

$$\overrightarrow{\mathrm{OA}} + \lambda\overrightarrow{\mathrm{AB}} = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

where μ is the value of x where the line intersects the x-axis.

Calculations:

Putting in expressions for $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{AB}}$,

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \lambda \boxed{} = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

This gives three equations, one for each of the x, y and z components:

$$3 + \boxed{}\,\lambda = \mu, \quad -1 + \boxed{}\,\lambda = 0 \quad \text{and} \quad 2 + \boxed{}\,\lambda = 0$$

The second of these equations re-arranges to $\lambda =$ ______, but the third equation rearranges to $\lambda =$ ______. Hence, these equations are inconsistent and we have reached a contradiction.

Conclusion:

There is no point on the line through A and B for which y and z are both zero, so this line does not intersect the x-axis, and hence this line does not intersect the line through the origin parallel to the vector $\underline{\boldsymbol{i}}$.

Items:

-3 -2 -1 0 1 2 3 $\frac{1}{3}$ $\frac{2}{3}$ $\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$

Used with permission from UCLES, A Level, January 2004, OCR P3, Question 5