isaac *You work it out.*

Question

Integrating Factors 2

Pre-Uni Maths for Sciences L2.2

Subject & topics: Maths | Calculus | Differential Equations   **Stage & difficulty:** Further A P3, University P1

Find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + (a + x)y = \mathrm{e}^{-x}.$$

Find the general solution for y as a function of x.

The following symbols may be useful: a, c, e, k, x, y

Created for isaacphysics.org by Julia Riley

isaac *You work it out.*

Question

Differential Equations: First Order - Integrating Factor 2i

Subject & topics: Maths   **Stage & difficulty:** Further A P2

Part A
General solution

Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{x} = \sin 2x,$$

giving your answer in the form $y = f(x) + \frac{c}{x}$ where c is a constant of integration.

The following symbols may be useful: `c, cos(), sin(), x, y`

Part B
Particular solution

In a particular case, it is given that $y = \frac{2}{\pi}$ when $x = \frac{1}{4}\pi$.

Find the value of c in this case.

Part C
Approximation

Write down a function to which y approximates when x is large and positive.

The following symbols may be useful: `cos()`, `sin()`, `tan()`, `x`

Adapted with permission from UCLES, A Level, January 2008 , Paper 4727, Question 5.

Question deck:

STEM SMART Double Maths 40 - 1st & 2nd Order Differential Equations

Part C
Approximation

isaac *You work it out.*

Question

2nd Order Homogeneous 1

Subject & topics: Maths | Calculus | Differential Equations   **Stage & difficulty:** Further A P3

The variables x and y satisfy the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} - 15y = 0$$

Find the solution of the equation given that when $x = 0$, $y = 5$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -1$.

The following symbols may be useful: `e, x, y`

Created for isaacphysics.org by Matthew Rihan

Question deck:

STEM SMART Double Maths 40 - 1st & 2nd Order Differential
Equations

isaac *You work it out.*

Question

2nd Order Homogeneous 2

Subject & topics: Maths | Calculus | Differential Equations　　　**Stage & difficulty:** Further A P3

The variables z and t satisfy the differential equation

$$4\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + 12\frac{\mathrm{d}z}{\mathrm{d}t} + 9z = 0$$

Find the solution of the equation given that when $t = 0$, $z = 4$ and $\frac{\mathrm{d}z}{\mathrm{d}t} = 1$.

The following symbols may be useful: `e, t, z`

Created for isaacphysics.org by Matthew Rihan

Question deck:

STEM SMART Double Maths 40 - 1st & 2nd Order Differential Equations

isaac *You work it out.*

Question

Mass on Spring (2nd Order)

Pre-Uni Maths for Sciences L3.6

Subject & topics: Maths | Calculus | Differential Equations **Stage & difficulty:** Further A P3, University P1

A mass m on a spring is subjected to a damping force. The equation describing its displacement x from its equilibrium position as a function of time t is

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -kx - b\frac{\mathrm{d}x}{\mathrm{d}t},$$

where $-kx$ is the force from the spring and $-b\frac{\mathrm{d}x}{\mathrm{d}t}$ is the force due to damping. The damping coefficient b is related to the spring constant k by $k = \frac{4b^2}{25m}$. Find an expression for the subsequent motion of the mass given that $x = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = V$ at $t = 0$.

> Find the equation describing the subsequent motion of the mass given that $x = 0$ and $\frac{\mathrm{d}x}{dt} = V$ at $t = 0$. Give your answer in terms of the constants b, m, and V.
>
> The following symbols may be useful: `V, b, e, m, t, x`

Created for isaacphysics.org by Julia Riley

Question deck:

STEM SMART Double Maths 40 - 1st & 2nd Order Differential Equations

isaac *You work it out.*

Question

Damped Pendulum (2nd Order)

Pre-Uni Maths for Sciences L3.3

Subject & topics: Maths | Calculus | Differential Equations   **Stage & difficulty:** Further A P3, University P1

The equation describing the displacement x of the bob of a damped pendulum from its equilibrium position is given by

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega_0^2 x - 2\gamma \frac{\mathrm{d}x}{\mathrm{d}t}$$

where ω_0 is the angular frequency of undamped oscillations of the pendulum and γ is related to the damping. Assuming $\omega_0 > \gamma$ find an equation for x at time t given that $x = X$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ at $t = 0$. (You will find it helpful to define a new constant ω_1 such that $\omega_1^2 = \omega_0^2 - \gamma^2$.)

Find an equation for x at time t given that $x = X$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ at $t = 0$.

The following symbols may be useful: `X, e, gamma, omega_1, t, x`

Created for isaacphysics.org by Julia Riley

Question deck:

STEM SMART Double Maths 40 - 1st & 2nd Order Differential Equations

isaac *You work it out.*

Question

Second Order Differential Equation 2

Pre-Uni Maths for Sciences L3.2

Subject & topics: Maths | Calculus | Differential Equations   **Stage & difficulty:** Further A P3, University P1

Find the solution of the equation

$$\frac{\mathrm{d}^2 p}{\mathrm{d}q^2} - 4\frac{\mathrm{d}p}{\mathrm{d}q} + 3p = 3q - 1$$

given that $p = 2$ and $\frac{\mathrm{d}p}{\mathrm{d}q} = -1$ when $q = 0$.
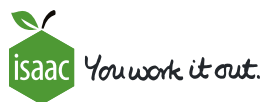
> Find the solution of the equation.
>
> The following symbols may be useful: `e, p, q`

Created for isaacphysics.org by Julia Riley

Question deck:

STEM SMART Double Maths 40 - 1st & 2nd Order Differential
Equations

isaac *You work it out.*

Question

Forced Oscillator (2nd Order)

Pre-Uni Maths for Sciences L3.4

Subject & topics: Maths | Calculus | Differential Equations   **Stage & difficulty:** Further A P3, University P1

The equation of motion of a forced oscillator is given by

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + \omega_0^2 \, z = Z_0 \, \sin(\omega_1 \, t)$$

Given that $\omega_0 \neq \omega_1$ find the solution for z given that $z = 0$ and $\frac{\mathrm{d}z}{\mathrm{d}t} = 0$ at $t = 0$.

> Find the solution for z given that $z = 0$ and $\frac{\mathrm{d}z}{\mathrm{d}t} = 0$ at $t = 0$.
>
> The following symbols may be useful: `Z_0, omega_0, omega_1, t, z`

Created for isaacphysics.org by Julia Riley

Question deck:

STEM SMART Double Maths 40 - 1st & 2nd Order Differential Equations

isaac *You work it out.*

Question

Differential Equations: General Applications 2i

Subject & topics: Maths   **Stage & difficulty:** Further A P2

During an industrial process substance X is converted into substance Z. Some of the substance X goes through an intermediate phase, and is converted into substance Y, before being converted into substance Z. The situation is modelled by

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 0.3x - 0.2y \quad \text{and} \quad \frac{\mathrm{d}z}{\mathrm{d}t} = 0.2y + 0.1x$$

where x, y and z are the amounts in kg of X, Y and Z at time t hours after the process starts.

Initially there is $10\,\mathrm{kg}$ of substance X and nothing of substances Y and Z. The amount of substance X decreases exponentially. The initial rate of decrease is $4\,\mathrm{kg\,hour^{-1}}$.

Part A

Expression for x

Find an expression for x.

The following symbols may be useful: `e, t`

Part B

$$\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\mathrm{d}z}{\mathrm{d}t}$$

Show that $\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\mathrm{d}z}{\mathrm{d}t} = k$ where k is a constant.

State the value of k.

Comment on this result in the context of the industrial process.

- ○ The total amount of all three substances is zero throughout the process.

- ○ The total amount of all three substances decreases throughout the process.

- ○ The total amount of all three substances increases throughout the process.

- ○ The total amount of all three substances is constant throughout the process.

Part C

Expression for y

Find an expression for y in terms of t.

The following symbols may be useful: `e, t`

Part D

Maximum amount of Y

Determine the maximum amount of substance Y present during the process.

Part E
Time to produce $9\,\mathrm{kg}$ of substance Z

How long does it take to produce $9\,\mathrm{kg}$ of substance Z? Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, Sample Paper 2017, Paper Y541, Question 11.

Part E
Time to produce $9\,\mathrm{kg}$ of substance Z