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Differentiation from First Principles 1



Pre-Uni Maths for Sciences J3.1 & J3.2

To differentiate a function $f(x)$ from first principles involves taking a limit. The derivative of $f(x)$ is given by the expression

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

Part A Differentiate x^3 from first principles

Differentiate x^3 from first principles. Drag and drop options into the spaces below.

In this question $f(x) = x^3$. Therefore, $f(x+h) =$. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\text{} - x^3}{h}.$$

Next, expand the brackets in the numerator and simplify:

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\text{}}{h} = \lim_{h \rightarrow 0} \text{}.$$

Finally, take the limit. As $h \rightarrow 0$, the term containing x^2 is unchanged (because it does not depend on h), but the terms containing xh and h^2 tend to 0. Therefore,

$$f'(x) = \text{}.$$

Items:

Part B

Differentiate $2x^3 + 5$ from first principles

Differentiate $2x^3 + 5$ from first principles. Drag and drop options into the spaces below.

In this question $f(x) = 2x^3 + 5$. Therefore, $f(x + h) =$. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{ \text{ } - (2x^3 + 5) }{h}.$$

Next, just as in part A, expand the brackets in the numerator. After simplification, this produces:

$$f'(x) = \lim_{h \rightarrow 0} \text{ }.$$

Finally, take the limit. As $h \rightarrow 0$, the term containing x^2 is unchanged (because it does not depend on h), but the terms containing xh and h^2 tend to 0. Therefore,

$$f'(x) = \text{ }.$$

Items:

- $2x^3h^3 + 5$

$6x^2$

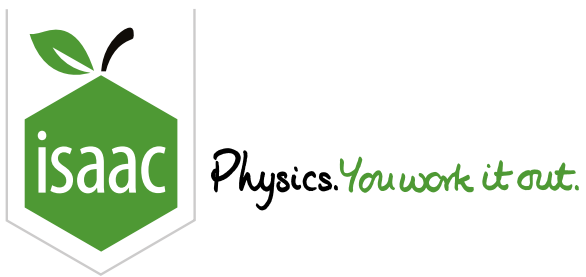
$2(x + h)^3 + 5$

$2x^3 + 5h$

$6x^2 + 5$

$6x^2 + 6xh + 2h^2 + 5$

$6x^2 + 6xh + 2h^2$



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Differentiation from First Principles 3

Pre-Uni Maths for Sciences J3.5



Differentiating from first principles involves taking a limit. The derivative of y with respect to x is given by

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x}.$$

In this expression δy is the small change in y produced by δx , a small change in x .

The value of $\frac{dy}{dx}$ at a point on a curve is the gradient of the tangent to the curve at that point.

Part A Expand $(x + a)^4$

Expand $(x + a)^4$ and simplify as far as possible.

The following symbols may be useful: a , x

Part B Differentiate $y = 9x^4 - 8x$ from first principles

Differentiate $y = 9x^4 - 8x$ from first principles. Drag and drop options into the spaces below.

Consider the coordinates (x, y) of a point on the curve $y = 9x^4 - 8x$. When x increases by δx to $x + \delta x$, y changes to $y + \delta y =$. Substituting this into the expression for the derivative,

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{(y + \delta y) - y}{\delta x} = \frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{(\text{ }) - (9x^4 - 8x)}{\delta x}.$$

Using the answer to part A gives

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{ \text{ } - (9x^4 - 8x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} (\text{ } + \text{ } \delta x + 36x(\delta x)^2 + 9(\delta x)^3).$$

Finally, take the limit. As $\delta x \rightarrow 0$, the terms containing δx tend to 0. Therefore,

$$\frac{dy}{dx} = \text{ }.$$

Items:

$9(x + \delta x)^4 - 8(x + \delta x)$

$x^4 - 8x + 4x^3(\delta x) - 8(\delta x) + 6x^2(\delta x)^2 + 4x(\delta x)^3 + (\delta x)^4$

$9x^4 + 36x^3(\delta x)$

$54x^2$

$36x^3 - 8$

$9x^4 - 8x + 36x^3(\delta x) - 8\delta x + 54x^2(\delta x)^2 + 36x(\delta x)^3 + 9(\delta x)^4$

$9x^4 + 9(\delta x)^4 - 8x - 8(\delta x)$

$9x^3 - 8$

Part C Gradient of tangent

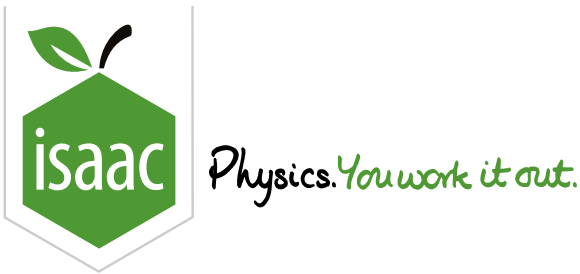
Find the gradient of the tangent to the curve at the point $(1, 1)$.

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Differentiation (powers of x) 3ii

A Level

P

P

P

Find $\frac{dy}{dx}$ in each of the following cases.

Part A Algebraic fraction

$$y = \frac{(3x)^2 \times x^4}{x}.$$

The following symbols may be useful: x

Part B Cube root

$$y = \sqrt[3]{x}.$$

The following symbols may be useful: x

Part C Reciprocal

$$y = \frac{1}{2x^3}.$$

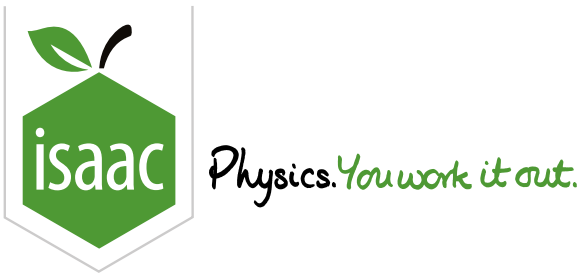
The following symbols may be useful: x

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Differentiating Powers 4

Pre-Uni Maths for Sciences J1.6

A Level

P

P

P

Part A

Derivative of $v = Bu^{-3}$

Find $\frac{dv}{du}$ if $v = Bu^{-3}$, where B is a constant.

The following symbols may be useful: B , u

Part B

Force if potential $V = \frac{q^2}{4\pi\epsilon_0 r}$

The electrostatic potential energy V of two equal charges q a distance r apart is given by $V = \frac{q^2}{4\pi\epsilon_0 r}$, where ϵ_0 and q are constants. The force between the two charges is given by $-\frac{dV}{dr}$; find an expression for this force.

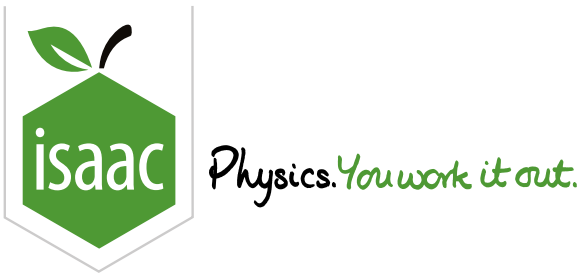
The following symbols may be useful: ϵ_0 , π , q , r

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Gradient Function: Tangents and Normals 1i

A Level
P P P

A curve has equation $y = x^2 + x$.

Part A Gradient

Find the gradient of the curve at the point where $x = 2$.

Part B Normal

Find the equation of the normal to the curve at the point for which $x = 2$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

The following symbols may be useful: x , y

Part C Find k

Find the smallest value of k for which the line $y = kx - 4$ is a tangent to the curve.

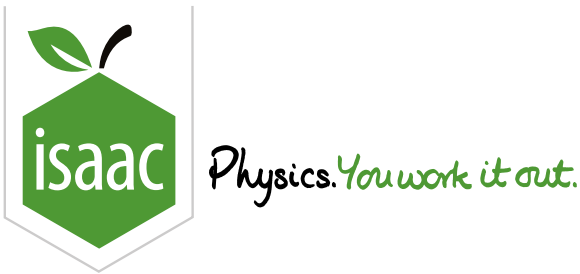
The following symbols may be useful: k

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Stationary Points 2ii



Part A Find coordinate

Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 + 4$.

Enter the x - and y -coordinates of the stationary point with the greatest x -coordinate. If a value is not a whole number, enter the value as a decimal.

(,)

Part B Stationary point

Determine whether the stationary point whose coordinates you entered is a maximum point or a minimum point.

- ☐ Maximum
- ☐ Inconclusive
- ☐ Minimum

Part C Range of x

For what range of values of x does $x^3 - 3x^2 + 4$ decrease as x increases?

Construct your answer from the items below.

Items:

<

>

x

$< x <$

$\leq x \leq$

$< x \text{ or } x <$

$\leq x \text{ or } x \leq$

\leq

\geq

-2

-1

0

1

2

3

4

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Stationary Points 3

Pre-Uni Maths for Sciences J2.5

A Level



Part A Maximum height of a projectile

A particle is fired upwards into the air with a initial speed w and moves subsequently under the influence of gravity with an acceleration g downwards, such that its height h at time t is given by $h = wt - \frac{1}{2}gt^2$, where w and g are constants. Find an expression for its maximum height above its initial position.

The following symbols may be useful: g , h , w

Part B Potential energy of two molecules

The potential energy of two molecules separated by a distance r is given by

$$U = U_0 \left(\left(\frac{a}{r} \right)^{12} - 2 \left(\frac{a}{r} \right)^6 \right)$$

where U_0 and a are positive constants. The equilibrium separation of the two molecules occurs when the potential energy is a minimum.

Find an expression for the equilibrium separation of the molecules.

The following symbols may be useful: U , U_0 , a , r

Find an expression for the potential energy when the molecules are at their equilibrium separation.

The following symbols may be useful: U , U_0 , a , r

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Maxima and Minima: Problems 1ii

A Level
P P P

Figure 1 shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is x metres, and the area of the enclosure is $A \text{ m}^2$.



Figure 1: The rectangular enclosure.

Part A Express as equation

Show that A can be expressed in the form $px - qx^2$, and find this expression.

The following symbols may be useful: A , x

Part B Use differentiation

Use differentiation to find the maximum value of the area of the enclosure, $A \text{ m}^2$.

Enter your value of A :

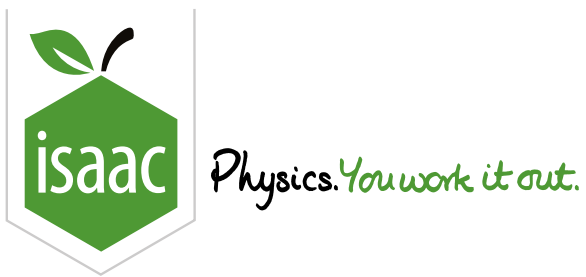
The following symbols may be useful: A

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Minimising the Area

Pre-Uni Maths for Sciences J2.2

A Level



A rectangular cuboid has a base with sides of length a and b and a height c . Its volume V and height c are fixed.

Part A Volume V and surface area A

Write down the equation for the volume V of the rectangular cuboid in terms of a , b and c .

The following symbols may be useful: v , a , b , c

Write down the equation for the surface area A of the rectangular cuboid in terms of a , b and c .

The following symbols may be useful: A , a , b , c

From your equation for V deduce an expression for b in terms of V , a and c . Hence, by substitution, obtain an equation for A in terms of V , a and c .

The following symbols may be useful: A , v , a , c

Part B Expressions for a and b

Differentiate with respect to a the expression for A you found in Part A (since V and c are fixed you may treat them as constants). Hence find in terms of V and c an expression for the value of a for which the surface area A is minimised.

The following symbols may be useful: v , c

Find, in terms of V and c , the expression for b corresponding to this value of a .

The following symbols may be useful: v , c

Part C The minimum area

Find an expression for the minimum surface area in terms of V and c .

The following symbols may be useful: v , c

Part D Check that the area is a minimum

Find, at the value of a deduced in Part B, an expression in terms of V and c for the second derivative of A with respect to a ; convince yourself that the value of the second derivative indicates that the value of A is a minimum at this point.

The following symbols may be useful: v , c
