Fluid Physics and Flight

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Summary of session

- The fundamental equations: continuity & energy
- Pressure at height
- Pressure and speed
- Wings
  - Lift
  - Induced Drag
  - Other drag

Density

- Density is the mass of each unit of volume of a material.
- Density$=\dfrac{\text{Mass}}{\text{Volume}}$ $\qquad \rho = \dfrac{m}{V}$
- Density is measured in kg/m^3 or g/cm^3.
- Air has a density of about $1.20 kg/m^3$
- Question: is moist air more or less dense than dry air?

Pressure

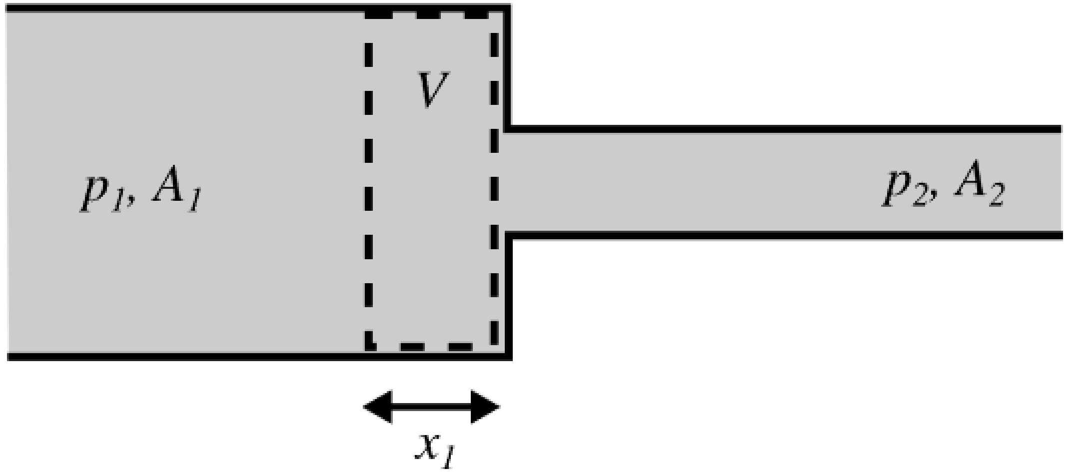
- Pressure measures how tightly a force is ‘focused’ on a point.
- $\text{Pressure} = \frac{\text{Force}}{\text{Area}}$ $p = \frac{F}{A}$ Unit: N/cm^2 or $N/m^2 = Pa$

Low Pressure = little damage	High Pressure = lots of damage
Same force, large surface area	Same force, small surface area
Flat end of drawing pin	Sharp end of drawing pin
Snow shoes, wide wheels on tractors	most tools, ice skates, nails

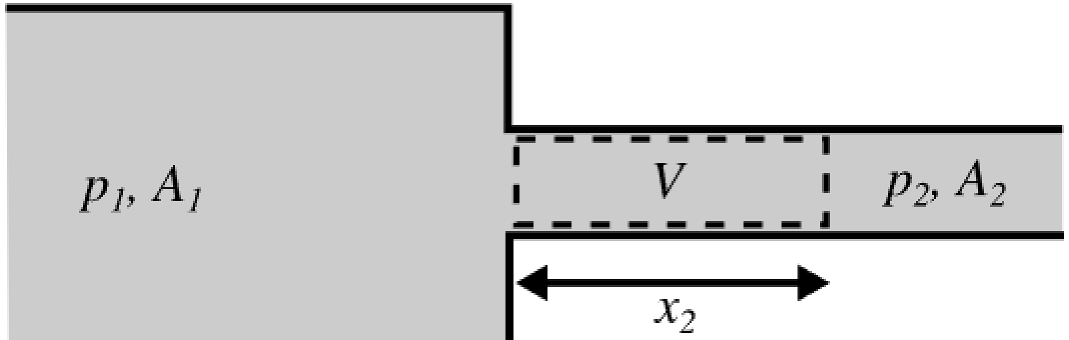
Continuity

- Incompressible fluid
- Volume flow per second

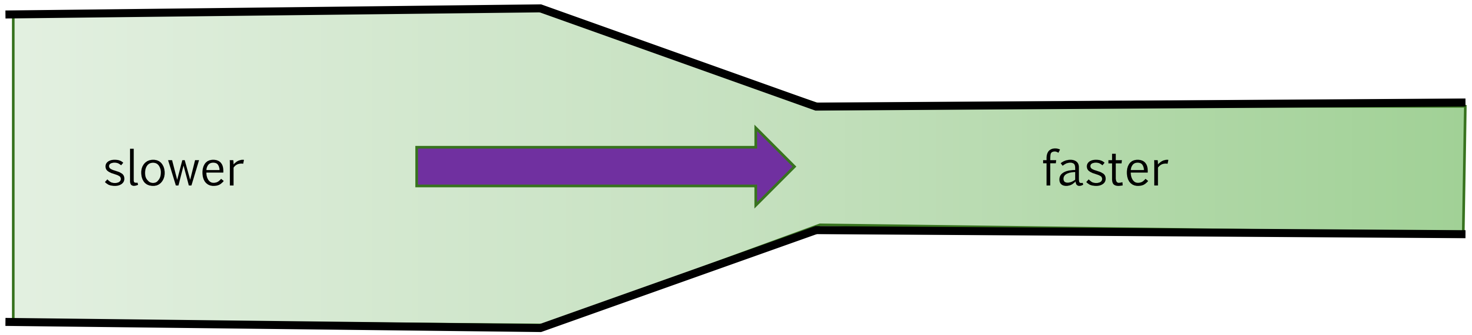
$$\frac{V}{t} = \frac{Ax}{t} = Av$$



- $A_1 v_1 = A_2 v_2$
- Fluid speeds up when pipe narrows



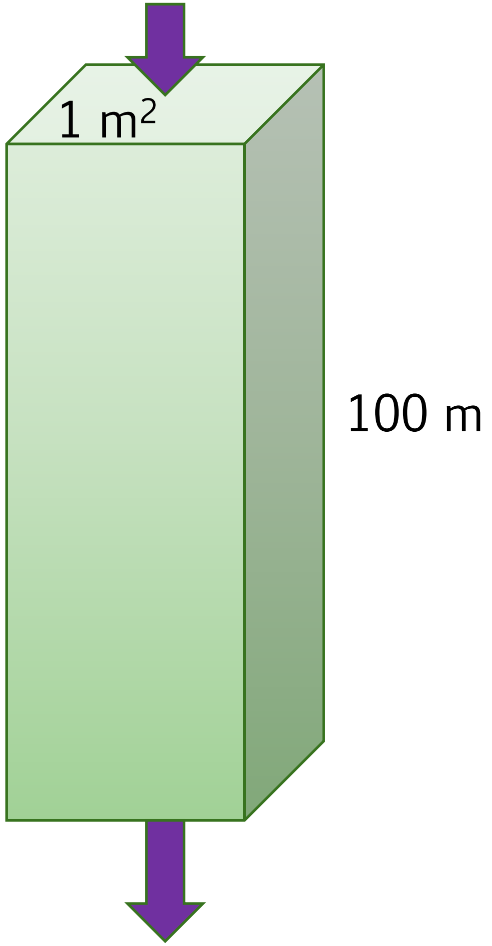
How does it speed up?



- Pressure must drop.
- Work done on gas =-$\Delta(Fx) = -\Delta(pAx) = -\Delta(pV) = -V\Delta p$
- Gain in potential energy =$mg\Delta h = \rho V g\Delta h$
- Gain in kinetic energy = $\Delta\left(\frac{1}{2}mv^2\right) = \Delta\left(\frac{1}{2}\rho V v^2\right) = \frac{1}{2}\rho V\ \Delta v^2$
- Conservation of energy: $-\Delta p = \rho g\Delta h + \frac{1}{2}\rho\Delta v^2$,
  so $p + \rho gh + \frac{1}{2}\rho v^2$=const.

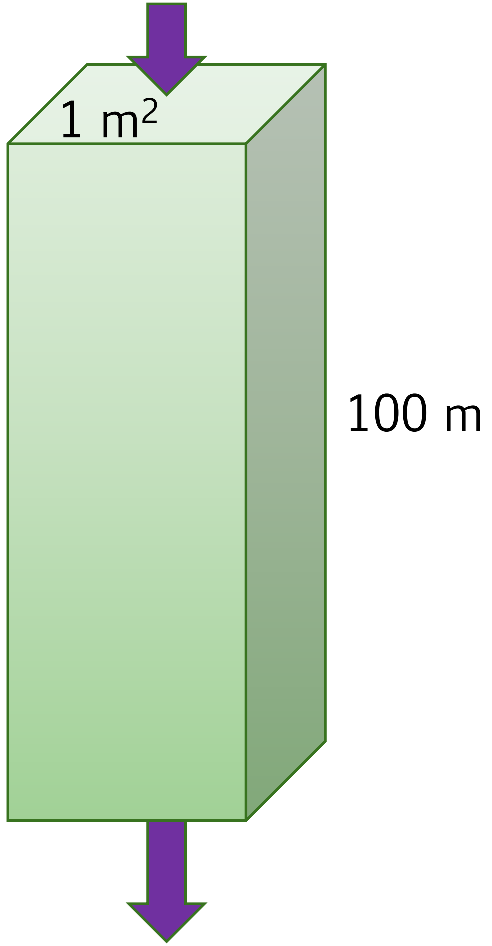
Pressure and Height 1

- As you climb, the air pressure drops
- We work out the drop for each 100m
  - $p + \rho g h + \frac{1}{2}\rho v^2$=const.
  - Assume stationary air, $p + \rho g h$ = constant
  - $\Delta p = -\rho g \Delta h$
  - For 100m, pressure drop is
- When pressure drops, density drops
  - $\rho = \frac{M}{V} = \frac{Nm}{V}$ and $pV = NkT$
  - $\rho = \frac{mp}{kT}$ so $\Delta p = -\frac{mgp}{kT}\Delta h$ and $\frac{dp}{dh} = -\frac{mg}{kT}p$



Pressure and Height 2

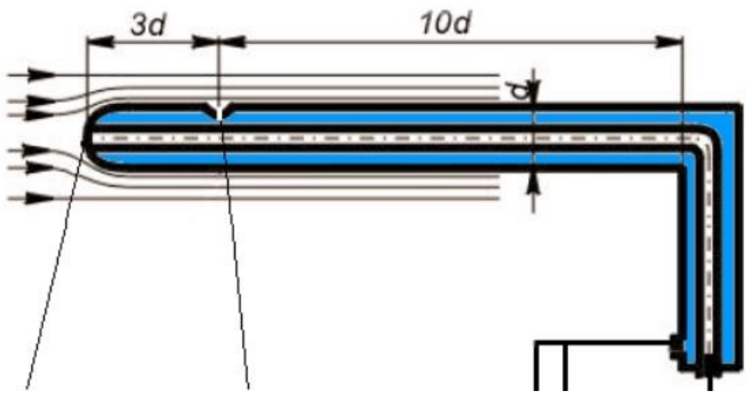
- When pressure drops, density drops
  - $\frac{dp}{dh} = -\frac{mg}{kT}p$ so $p = p_0 \exp\left(-\frac{mgh}{kT}\right)$
  - this is at constant temperature *T*.
- Suppose $T = T_0 - rh$
  - $\frac{dp}{dh} = -\frac{mgp}{k(T_0 - rh)}$ and $\int \frac{dp}{p} = \int \frac{mg\,dh}{kr(h - T_0/r)}$
  - $p = p_0 \left(1 - \frac{hr}{T_0}\right)^{mg/kr}$
  - International standard atmosphere $r = 6.5$K/km
- Extra challenge (DALR)
  - Calculate Lapse Rate $r = \frac{dT}{dh} = \frac{dT}{dp} \times \frac{dp}{dh}$
  - $pV = NkT$, molecular heat capacity $c_V = \frac{5}{2}k$ (diatomic)
  - Adiabatic (no heat flow): $Nc_V\ \delta T = -p\ \delta V = -\delta(pV) + V\ \delta p$
  - $\frac{dT}{dh} = -\frac{2mg}{7k}$, this is a drop of for each km ascended



Pressure and Speed 1

- As you slow down, the pressure rises
- Pressure increase on stopping air
  - $p + \rho g h + \frac{1}{2}\rho v^2$=const.
  - Horizontal air, $p + \frac{1}{2}\rho v^2 = \text{constant}$
  - $\Delta p = -\frac{1}{2}\rho \Delta v^2$
  - If you bring the air to rest $\Delta p = \frac{1}{2}\rho v^2$
  - This is termed the dynamic pressure q
  - For 1.2kg/m^3 air at 50m/s, the pressure drop is...

  - Dynamic pressure is measured in pitot tubes for giving pilots information about the air speed



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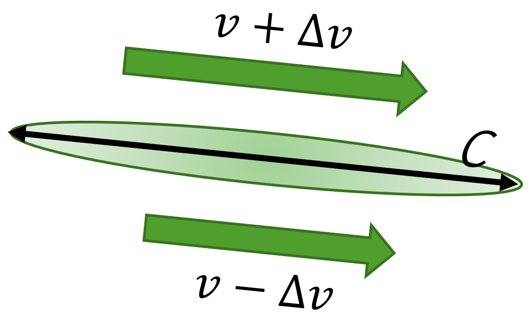
Pressure and Speed 2

- As you speed up, the pressure drops
- We work out the drop for each 1m/s
  - $p + \rho g h + \frac{1}{2}\rho v^2$=const.
  - Horizontal air, $p + \frac{1}{2}\rho v^2$ = constant
  - $\Delta p = -\frac{1}{2}\rho \Delta v^2 = -\rho v\ \Delta v$
- For air with circulation speed Δv
  - pressure on top decreases $\Delta p = -\rho v\ \Delta v$
  - pressure underneath increases $\Delta p = +\rho v\ \Delta v$
  - pressure difference = $2\rho v\ \Delta v$
  - force $L = 2\rho v\ \Delta v\ A = 2\rho v\ \Delta v\ Cb$
  - force per wingspan length $\frac{L}{b} = \rho v \times 2C\ \Delta v$
  - $\frac{L}{b} = \rho v \times$ circulation. This is the Kutta-Joukowski Law



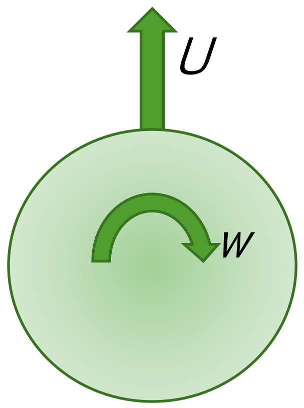
Circulation $\Gamma = \oint v\ dL$

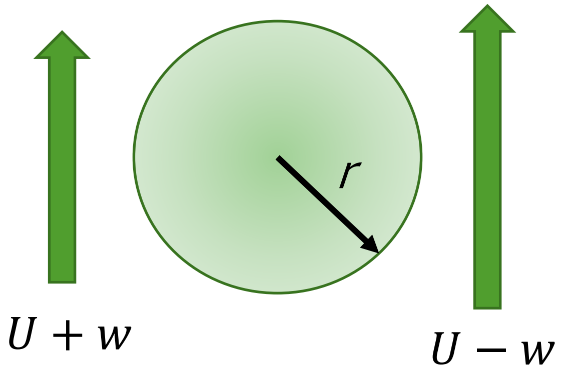
$= (v + \Delta v)C\ -\ (v - \Delta v)C$

$= 2C\ \Delta v$

Pressure and Speed 3

- The Magnus effect – a spinning cylinder
  - circulation = $2\pi r\, w$
  - force = $\rho L U \times 2\pi r w$
  - actual forces are different
    - fast motion changes streamlines
    - onset of turbulence must be taken into account
  - this effect is used in marine propulsion
  - and also in many ball sports!





Norsepower rotor

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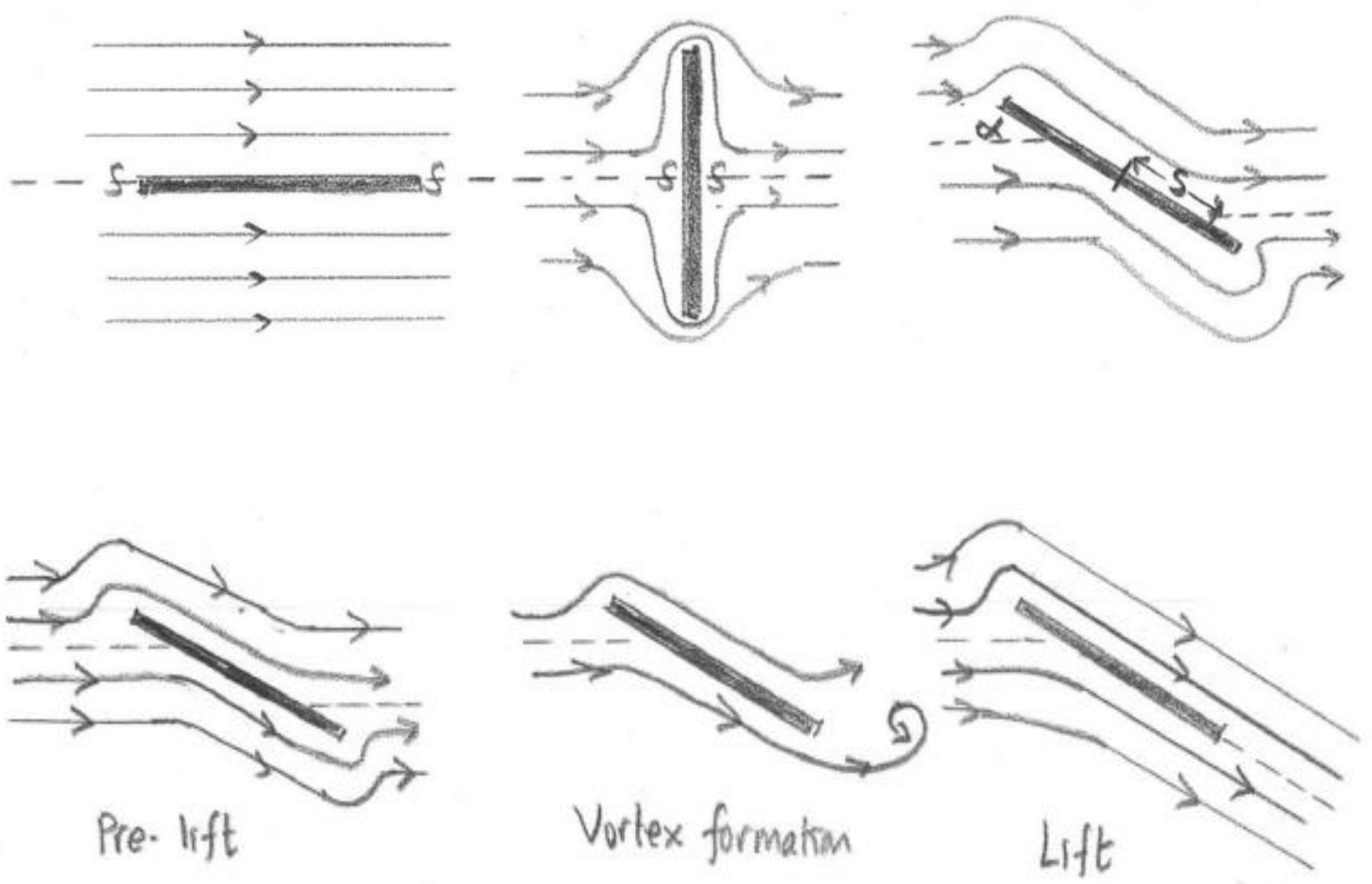
Basketball Magnus

YouTube: How Ridiculous: What happens when a spinning basketball is thrown off a dam!

Application - wings

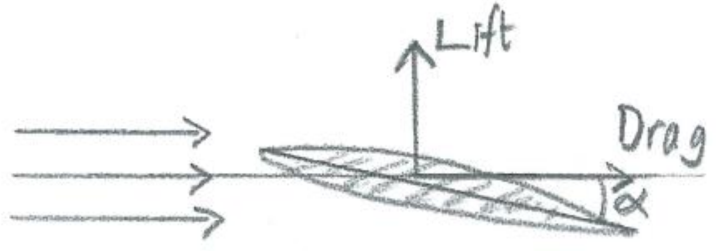


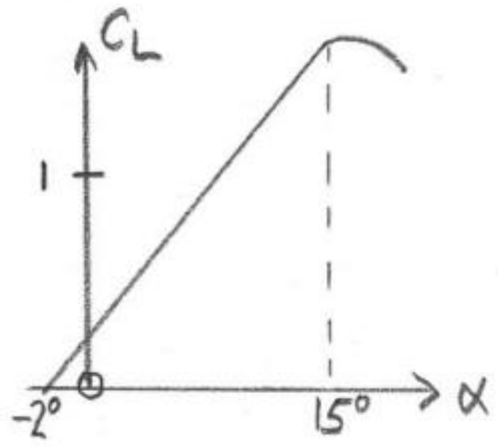
The lift equation

$$L = \frac{\rho S v^2}{2} \times C_L$$

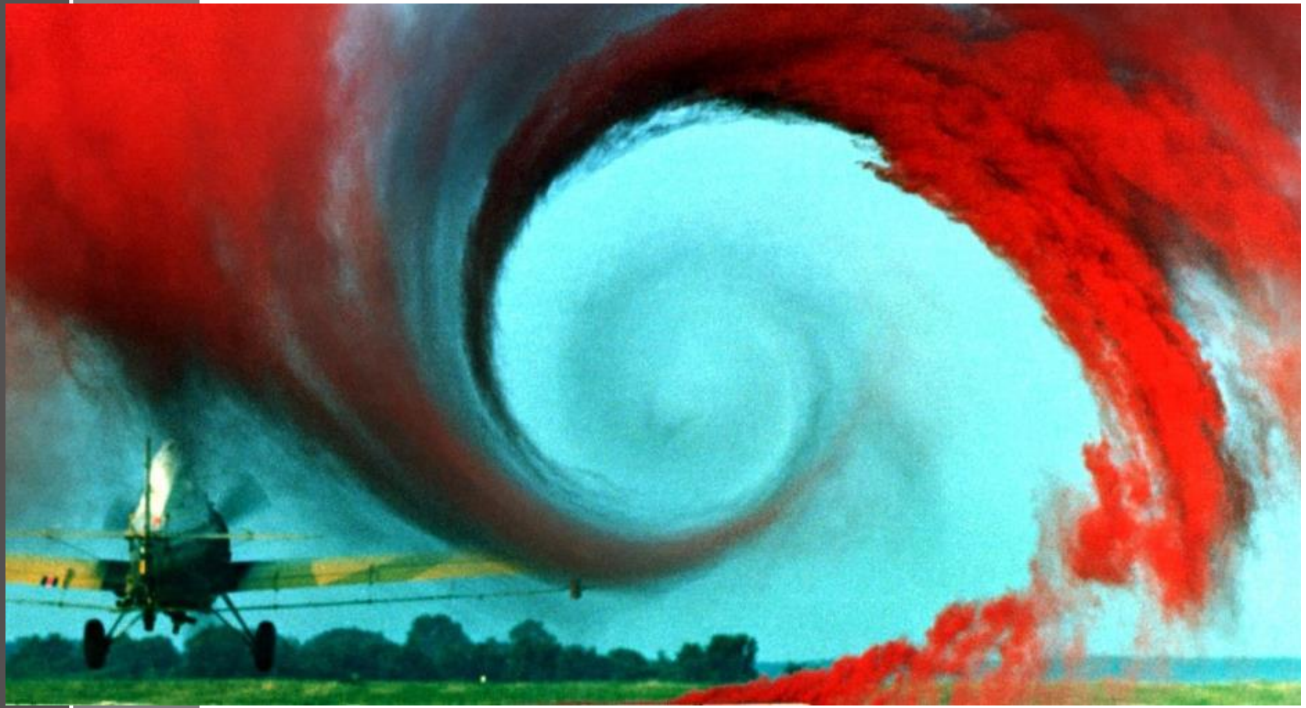
$$L = \frac{1}{2}\rho S v^2 = \Gamma v b \rho$$

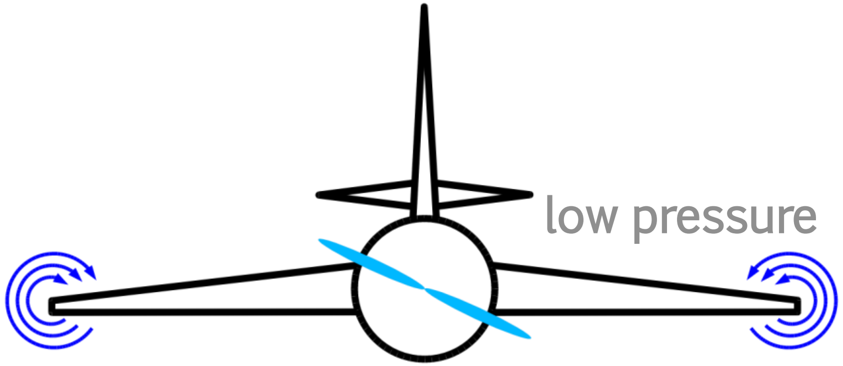
$$\Gamma = \frac{S v C_L}{2b}$$





3d Aerodynamics – Tip vortices





Flow round wing tip creates tip vortices

Air behind aircraft is moving downwards

Air behind, and to side is moving upwards

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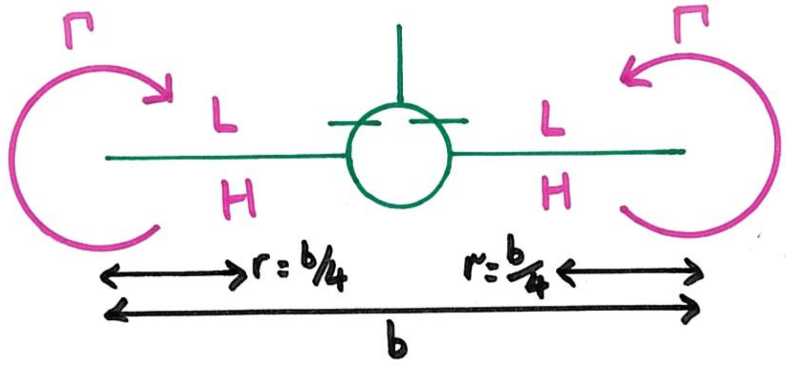
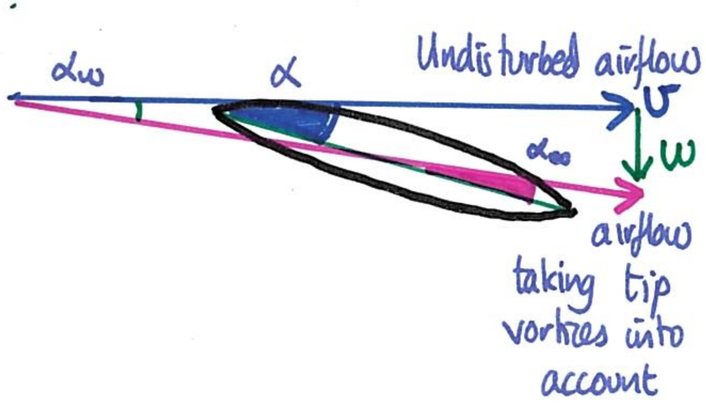
Wingtip vortices





For wingtip vortex touching mid-wing point

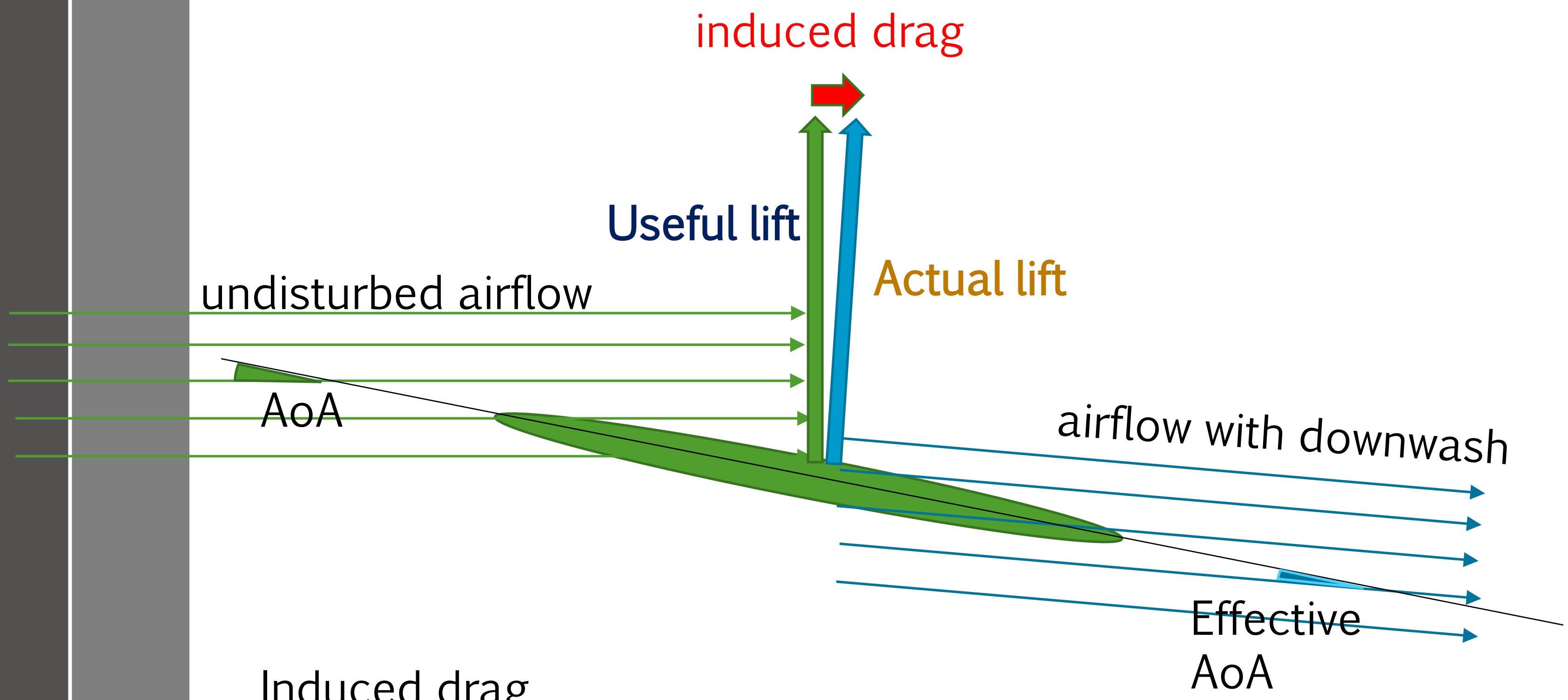
$$\Gamma = 2\,\pi r w = \tfrac{1}{2}\pi b w$$

$$w = \frac{2\Gamma}{\pi b}$$

$$\alpha_w = \frac{w}{v} = \frac{2\Gamma}{\pi b v} = \frac{2}{\pi b v} \times \frac{S v C_L}{2b} = \frac{S C_L}{\pi b^2} = \frac{C_L}{\pi \mathcal{R}}$$

where $\mathcal{R} = b^2/S$ is the aspect ratio of the wing.

3d Aerodynamics – vortex and AoA



Induced drag

$$D_I = L\alpha_w = \frac{LC_L}{\pi\mathcal{R}} = \frac{L}{\pi\mathcal{R}} \times \frac{2L}{\rho S v^2} = \frac{2L^2}{\pi\rho b^2 v^2}$$

- Wingtip vortices create downwash
- This reduces the AoA and hence the lift
- The higher the aspect ratio, the less tip effects are important, and the less the AoA is reduced
- The lift now has a backward component – induced drag

Drag

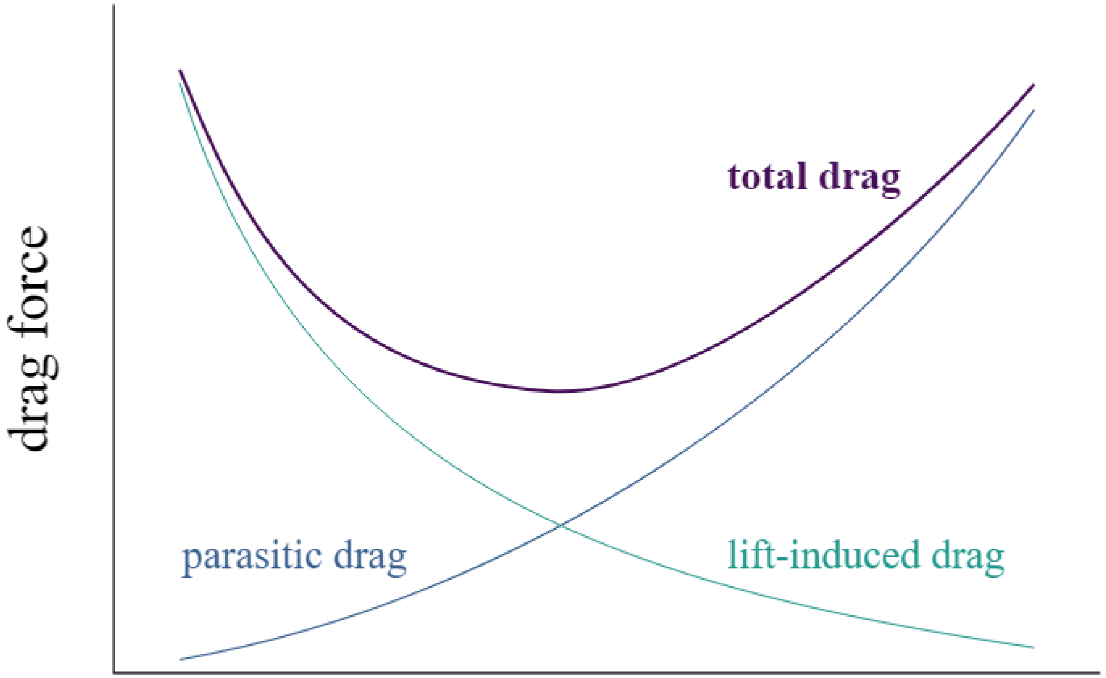
- Inertia drag (get out of the way), called parasitic drag

$$D_P = \frac{\rho S v^2}{2} \times C_D$$

- Viscous drag (sticky fluid)

$$D_V = \frac{\mu S v}{d}$$

3d Aerodynamics – Drag & Speed



$$D_P = \frac{C_D \rho S v^2}{2} \qquad D_I = \frac{2L^2}{\pi \rho b^2 v^2}$$

- As you fly faster
  - you get more parasite drag, and
  - you need less C_L
    - so you use a smaller AoA
    - you get less downwash
    - you have less induced drag
- Total drag = induced + parasitic
  - has a minimum value at one particular speed
  - this is the speed at which the lowest thrust is needed

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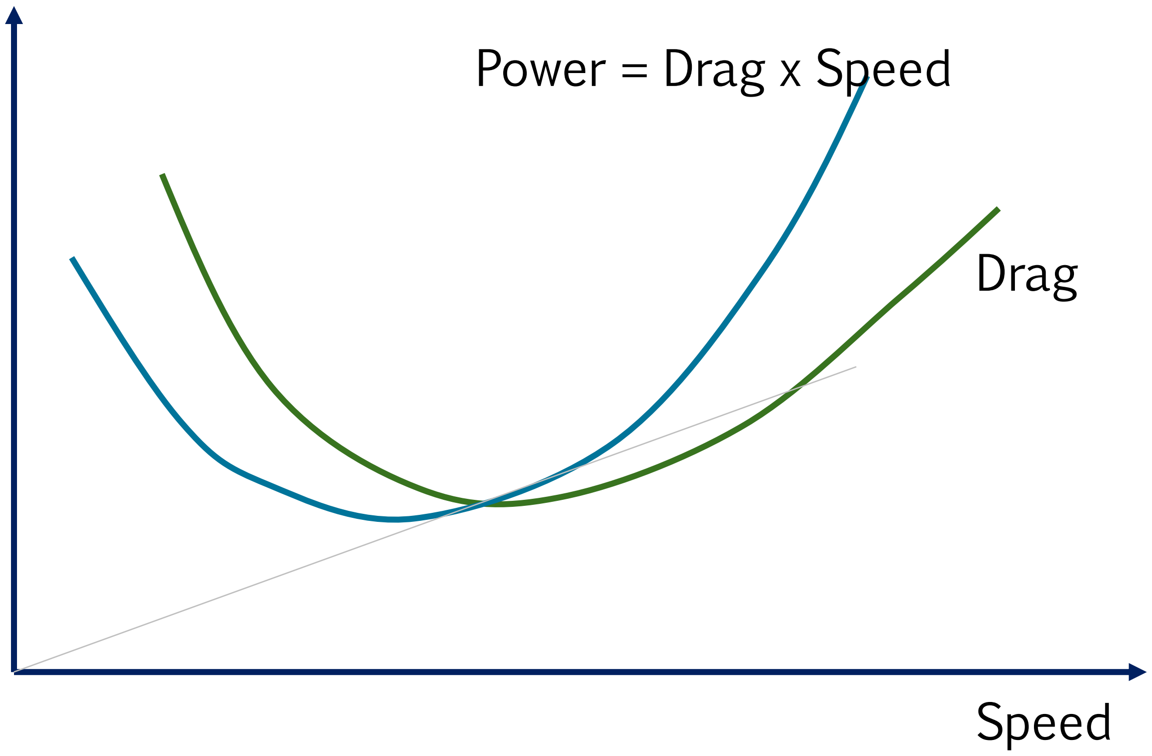
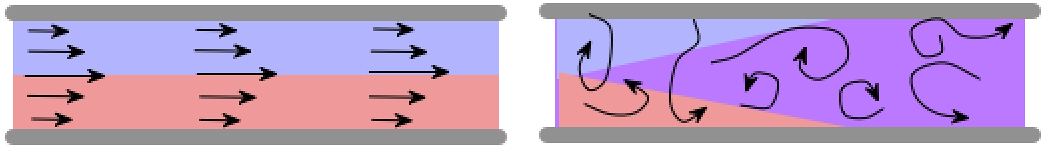
3d Aerodynamics – Power & Speed



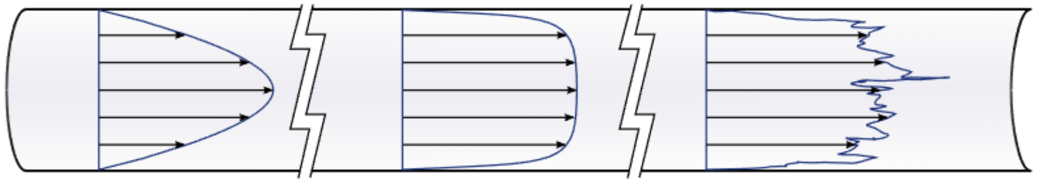
- Power = Drag x Speed
- Speed for least power is less than speed for least drag
- Speed for greatest endurance (time in the air) is less than speed for greatest range

Reynolds

$$\frac{D_P}{D_V} \propto \mathrm{Re} = \frac{\rho v d}{\mu}$$



Laminar Turbulent



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Applications

- Meteorology and Oceanography
- Industry (fluid delivery in pipes)
- Aerodynamics
- Mechanical engineering (flow through engines)
- Structural engineering (e.g. wind flow around buildings)
- Food and wearable technology
- Traffic flow

Thank you for listening.

Time for questions

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