STEM SMART Single Maths 26 - Partial fractions & Proof by Contradiction

Partial Fractions

A-level Maths Topic Summaries - Algebra

Subject & topics: Maths | Algebra | Manipulation **Stage & difficulty:** A Level P3

Fill in the blanks to complete the notes on partial fractions below.

Just as we can add algebraic fractions together, we can also reverse the process and split them apart. For example,

$$\frac{5x-1}{(x+1)(x-2)} \equiv \frac{2}{x+1} + \frac{3}{\boxed{}}$$

In the expression on the left, the denominator is factorised into brackets of the form $(ax+b)$. These brackets are linear [ ]. The terms on the right hand side are called [ ]. There is one term for each of the [ ]. The numerators of the terms on the right hand side are [ ] and may be integers or fractions.

Sometimes a linear factor is [ ]. For example,

$$\frac{18}{(x+1)^2(x-2)} \equiv \boxed{} + \frac{3}{x-2}$$

On the left hand side the factor $(x+1)$ is squared. On the right hand side, we need to include a term involving [ ] as well as $(x+1)$.

Items:

constants | **factors** | **partial fractions** | **repeated** | $(x+1)^2$ | $x-2$ | $(x-2)^2$ | $\frac{6}{x+1}$ | $-\frac{6}{(x+1)^2}$ | $-\frac{6}{(x+1)^2}+\frac{6}{x+1}$

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Partial Fractions 1

Pre-Uni Maths for Sciences A5.1

Subject & topics: Maths | Algebra | Manipulation **Stage & difficulty:** A Level P3

The function $\dfrac{2x-1}{(3x-2)(x-1)}$ can be written as $\dfrac{A}{3x-2}+\dfrac{B}{x-1}$. Find A and B.

Part A

Find A

Find the constant A.

Part B

Find B

Find the constant B.

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Partial Fractions 2

Pre-Uni Maths for Sciences A5.2

Subject & topics: Maths | Algebra | Manipulation **Stage & difficulty:** A Level P3

The function $\dfrac{w+2}{(w-1)(w+1)(2w+1)}$ can be written as $\dfrac{A}{(w-1)}+\dfrac{B}{(w+1)}+\dfrac{C}{(2w+1)}$. Using the substitution method find the constants A, B and C.

Part A

Find A

Find the constant A.

The following symbols may be useful: A

Part B

Find B

Find the constant B.

The following symbols may be useful: B

Part C

Find C

Find the constant C.

The following symbols may be useful: C

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Partial Fractions 3

Subject & topics: Maths | Algebra | Manipulation **Stage & difficulty:** A Level P3

The function $\frac{24t^2+31t+2}{(2t+1)^2(t+3)}$ can be written as $\frac{A}{(2t+1)^2}+\frac{B}{(2t+1)}+\frac{C}{(t+3)}$. Find the constants A, B and C.

Part A

Find A

Find the constant A.

The following symbols may be useful: A

Part B

Find B

Find the constant B.

The following symbols may be useful: B

Part C

Find C

Find the constant C.

The following symbols may be useful: C

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Improper Partial Fractions 1

Pre-Uni Maths for Sciences A5.3

Subject & topics: Maths | Algebra | Manipulation **Stage & difficulty:** A Level P3

Express $\dfrac{-6x^3 + 15x^2 + x - 11}{2x^2 - 5x - 3}$ as partial fractions.

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isaac You work it out.

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Proof Applied to Surface Areas

Subject & topics: Maths | Number | Arithmetic **Stage & difficulty:** A Level P3

Consider a sphere with a radius r cm, where r is a rational number. Using proof by contradiction, show that the side length of a cube with the same surface area cannot also be a rational number of cm.

Once you have worked out a proof yourself, consider the model proof below. Drag and drop answers into the blank spaces to complete the proof. You may use the same answer more than once.

Assumption:
Consider a sphere of radius r cm, where r is a rational number. Let the side length of a cube with the same surface area as the sphere be a cm. Assume that a is a rational number, in which case $a = \frac{b}{c}$, where b and c are integers with no common factor.

Reasoning:
The surface area of the sphere is [ ]. Because r is a rational number, $r = \frac{p}{q}$, where p and q are integers with no common factor. Hence, the surface area of the sphere may be written as [ ].

The surface area of the cube is [ ]. Using $a = \frac{b}{c}$, the surface area may be written as [ ].

The surface area of the sphere and the cube are equal. Hence, $4\pi\left(\frac{p}{q}\right)^2 = 6\left(\frac{b}{c}\right)^2$. Re-arranging this equation to give an expression for [ ] produces [ ].

As b, c, p and q are all integers, this expression implies [ ] must be [ ] number. However, π is not [ ] number.

Conclusion:
The assumption that a is rational has resulted in a contradiction. Hence, the assumption cannot be true. Therefore, the side length of a cube with the same surface area as a sphere of radius r cm, where r is a rational number, cannot be a rational number of cm.

Items:

a real | a^3 | a rational | an irrational | $\pi = \frac{3b^2p^2}{2c^2q^2}$ | $\frac{3b^2q^2}{2c^2p^2}$ | $4\pi r^2$ | $6a^2$ | $\pi = \frac{3b^2q^2}{2c^2p^2}$ | $4\pi\left(\frac{p}{q}\right)^2$

$\boxed{6\left(\frac{b}{c}\right)^2}$ $\boxed{\pi}$

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Proof and Odd Perfect Numbers

Subject & topics: Maths | Number | Arithmetic **Stage & difficulty:** A Level P3

The **proper divisors** of a number are those factors which are not equal to the number itself. For the number 6,

- The divisors of 6 are 1, 2, 3 and 6.
- The proper divisors of 6 are 1, 2 and 3.

The number 6 is an example of a **perfect number**. A perfect number is a number for which the sum of its proper divisors is equal to the number itself. For the number 6,

$$1 + 2 + 3 = 6$$

In this question you will use proof by contradiction to show that an odd perfect number cannot be a square number.

Assumption:

We will assume that there is an odd perfect number, n, that is also a square number. Then $n = m^2$, where m is an integer.

Part A

Reasoning: odd and even factors

- An even number multiplied by an even number is always an [ ] number.
- An even number multiplied by an odd number is always an [ ] number.
- An odd number multiplied by an odd number is always an [ ] number.

Therefore, as n is an [ ] number, the factors of n can only be [ ] numbers.

Items:

odd even

Part B

Reasoning: sum of proper divisors

As $n = m^2$, m is a factor of n.

Consider another factor of n. Call this factor p. As p is a factor of n, $q =$ ______ is also a factor of n. As $n = m^2$, $q = \frac{m^2}{p}$. Hence,

- If $p < m$, q ______ m.
- If $p > m$, q ______ m.

Therefore, with the exception of m, the factors of n occur in pairs. One factor in the pair is smaller than m, and the other factor is larger than m. Including m, the total number of factors of n is therefore an ______ number.

For any value of n, one of the factor pairs is 1 and n. The number of proper divisors (factors other than n itself) is therefore an ______ number. As we have shown in part A that all of the factors of n are ______ numbers, the sum of the proper divisors of n is therefore an ______ number.

Items:

odd $\frac{n}{p}$ **even** $\frac{p}{n}$ $<$ $>$

Part C

Conclusion

Our starting assumption was that n is an odd perfect number and also a square number.

The definition of a perfect number means that the sum of the proper divisors of n is equal to ______. The sum of the proper divisors must therefore be an ______ number.

However, in part B we have shown that if n is an odd number which is also a square number, the sum of the proper divisors has to be an ______ number.

Therefore, we have arrived at a contradiction. We conclude that there are no odd perfect numbers that are also square numbers.

Items:

n n^2 **even** **odd** $2n$

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Partial Fractions Applied to Other Functions

Pre-Uni Maths for Sciences A5.5, A5.6 & A5.7

Subject & topics: Maths | Algebra | Manipulation **Stage & difficulty:** A Level C3

Express the following functions in partial fraction form.

Part A

A trigonometric function

Express the function $\dfrac{\cos y}{(\cos y+1)(2\cos y+1)}$ in the form $\dfrac{A}{\cos y+1}+\dfrac{B}{2\cos y+1}$, where A and B are constants.

The following symbols may be useful: `cos(), sin(), tan(), y`

Part B

An exponential function

Express the function $\dfrac{\mathrm{e}^{2x}+5}{(\mathrm{e}^x-1)(\mathrm{e}^x-2)(\mathrm{e}^x-3)}$ in the form $\dfrac{A}{\mathrm{e}^x-1}+\dfrac{B}{\mathrm{e}^x-2}+\dfrac{C}{\mathrm{e}^x-3}$, where A, B and C are constants.

The following symbols may be useful: `e, x`

Part C

A logarithmic function

Express the function $\dfrac{5\ln z + 20}{(\ln z)^2 + \ln(z^2) + 1}$ in the form $\dfrac{A}{(\ln z + 1)^2} + \dfrac{B}{\ln z + 1}$, where A and B are constants.

The following symbols may be useful: `ln()`, `log()`, `z`

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