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Home Gameboard Maths Polynomials, Factors and Roots 4i

# Polynomials, Factors and Roots 4i

A Level
P P P

The polynomial $f(x)$ is given by $f(x) = 2x^3 + 9x^2 + 11x - 8$.

## Part A Factors

Using the factor theorem decide whether $(2x-1)$ is a factor of $f(x)$ or not.

- $(2x-1)$ is not a factor of $f(x)$
- $(2x-1)$ is a factor of $f(x)$

## Part B Find quadratic factor

Express $f(x)$ as a product of a linear factor and a quadratic factor.

The following symbols may be useful: x

## Part C Real roots

State the number of real roots to the equation $f(x) = 0$.

[Home](#) [Gameboard](#) Maths Algebraic Division 5ii

# Algebraic Division 5ii

A Level

P P P

## Part A Quotient and Remainder 1

Find the quotient and remainder when $3x^4 - x^3 - 3x^2 - 14x - 8$ is divided by $x^2 + x + 2$.

Give the quotient.

The following symbols may be useful: x

Give the remainder.

The following symbols may be useful: x

## Part B Quotient and Remainder 2

Find the quotient and remainder when $4x^3 + 8x^2 - 5x + 12$ is divided by $2x^2 + 1$.

Give the quotient.

The following symbols may be useful: x

Give the remainder.

The following symbols may be useful: x

# Algebraic Division 5i

A Level

P P P

## Part A Quotient and Remainder

Find the quotient and remainder when $x^4+1$ is divided by $x^2+1$.

State the quotient.

The following symbols may be useful: x

State the remainder.

## Part B Find $f(x)$

When the polynomial $f(x)$ is divided by $x^2+1$, the quotient is $x^2+4x+2$ and the remainder is $x-1$. Find $f(x)$, simplifying your answer.

The following symbols may be useful: x



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Home Gameboard Maths Algebraic Division 3ii

# Algebraic Division 3ii

A Level
P P P

The cubic polynomial $ax^3 - 4x^2 - 7ax + 12$ is denoted by $f(x)$.

## Part A Value of $a$

Given that $(x - 3)$ is a factor of $f(x)$, find the value of the constant $a$.

The following symbols may be useful: a

## Part B Remainder

Using this value of $a$, find the remainder when $f(x)$ is divided by $(x + 2)$.



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# Proof and Hollow Pyramids

A Level
P P P

A hollow pyramid shape can be made by stacking identical spheres.

## Part A Square-based pyramids

The diagram below shows the first three pyramids in a sequence of square-based hollow pyramids.

**Figure 1:** The first three square-based hollow pyramids, with sides made up of 4, 5 and 6 identical spheres. These are the pyramids for $k = 1$, $k = 2$ and $k = 3$.

Let the number of spheres that make up the $k$th pyramid in the sequence be $S_k$. From the list below, choose the correct expression for $S_k$.

- $8k + 21$
- $4k + 5$
- $8k + 13$
- $16k - 11$

## Part B Triangle-based pyramids

The diagram below shows the first three pyramids in a sequence of triangle-based hollow pyramids.

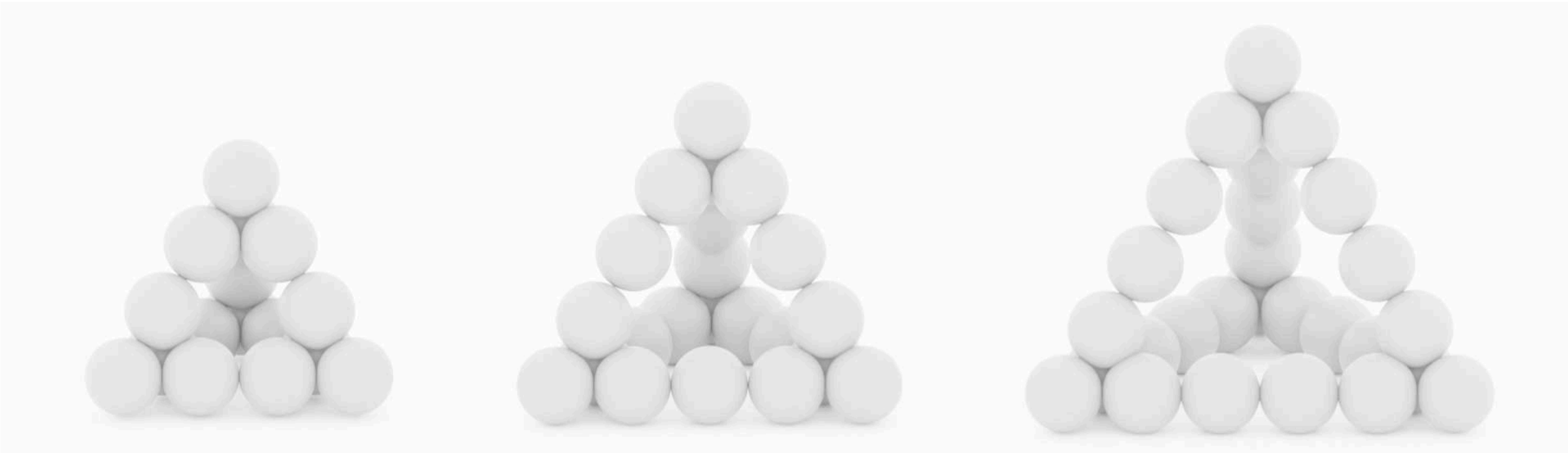

**Figure 2:** The first three triangle-based hollow pyramids, with sides made up of 4, 5 and 6 identical spheres. These are the pyramids for $n = 1$, $n = 2$ and $n = 3$.

Find an expression for $T_n$, the number of spheres that make up the $n$th pyramid in this sequence.

The following symbols may be useful: `T_n, n`

## Part C Is rearrangement possible?

Prove that it is not possible to rearrange the spheres making up any square-based pyramid to produce a triangle-based pyramid (of any size) without either having spheres left over or needing extra spheres.

Once you have worked out a proof yourself, consider the model proof below. Drag and drop answers into the blank spaces to complete the proof. You may use the same answer more than once.

We will use proof by deduction.

**Reasoning:**

The number of spheres making up the $k$th hollow square-based pyramid is given by $8k + 13$. For any positive ______ value of $k$, $8k$ is ______. Hence, $8k + 13$ is always ______.

The number of spheres making up the $n$th hollow triangle-based pyramid is given by ______. For any positive ______ value of $n$, $6n$ is ______. Hence, ______ is always even.

Therefore, the number of spheres required to make a hollow square-based pyramid ______ the same as the number of spheres required to make a hollow triangle-based pyramid.

**Conclusion:**

Hence, it is not possible to rearrange the spheres making up any square-based pyramid to produce a triangle-based pyramid (of any size) without either having spheres left over or needing extra spheres.

Items:

integer | odd | $6n + 10$ | can never be | fractional | even | rational | $10n + 6$ | is always

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# Induction: Sequences 1i

Further A

P P P

The sequence $u_1, u_2, u_3 \ldots$ is defined by $u_1 = 2$ and $u_{n+1} = \frac{u_n}{1+u_n}$ for $n \geq 1$.

## Part A $u_2$, $u_3$, and $u_4$

Find $u_2$.

The following symbols may be useful: u_2

Find $u_3$.

The following symbols may be useful: u_3

Find $u_4$.

The following symbols may be useful: u_4

## Part B $u_n$ in terms of $n$

Hence, suggest an expression for $u_n$ in terms of $n$ and use induction to prove your suggestion is correct.

The following symbols may be useful: n, u_n

Home Gameboard Maths Induction: Divisibility 1i

# Induction: Divisibility 1i

Further A

P P P

The sequence $u_1, u_2, u_3 \ldots$ is defined by $u_n = 5^n + 2^{n-1}$.

## Part A $u_1$, $u_2$ and $u_3$

Find $u_1$.

The following symbols may be useful: u_1

Find $u_2$.

The following symbols may be useful: u_2

Find $u_3$.

The following symbols may be useful: u_3

## Part B Divisibility

Hence, suggest a positive integer, other than 1, which divides exactly into every term of the sequence and prove it with induction by considering $u_{n+1} + u_n$.

# Induction: Matrices 2i

Further A

P P P

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$.

$\mathbf{M}^n$ can be expressed in the form

$$\mathbf{M}^n = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

## Part A $\mathbf{M}^4$

Give an expression for $\alpha + \beta + \gamma + \delta$ when $n = 4$.

## Part B $\mathbf{M}^n$

Hence, suggest a suitable form for $\mathbf{M}^n$ in terms of $n$ and prove it with induction. Give an expression for $\alpha + \beta + \gamma + \delta$.

The following symbols may be useful: n

Adapted with permission from UCLES, A Level, Jan 2012, Paper 4725, Question 7.

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Home Gameboard Maths Number Arithmetic Divisibility by Exhaustion

# Divisibility by Exhaustion

A Level Further A

A sequence $u_n$ is defined by $u_n = n^7 - n$, where $n \in \mathbb{N}$. The first four terms of this sequence are

$$0, 126, 2184, 16380, \ldots$$

What is the largest integer that will divide every term of this sequence?

## Part A Factorise $u_n$

Factorise $u_n$ completely.

The following symbols may be useful: n

## Part B Divisibility by $2$

Using your expression from part A, prove that every term in the sequence is divisible by $2$.

Once you have worked out a proof yourself, consider the model proof below. Drag and drop answers into the blank spaces to complete the proof.

We know that the factors of $u_n$ from part A.

When $n$ is even, it is divisible by $2$ and we can see that [ ] is a factor of $u_n$, so $u_n$ is divisible by $2$.

When $n$ is odd, we can write $n =$ [ ] in terms of $k$, where $k \in \mathbb{Z}$. Then the factor [ ] $=$ [ ] in terms of $k$, so the factor [ ] is divisible by $2$, and hence $u_n$ is divisible by $2$.

Therefore, $u_n$ is divisible by $2$ for any value of $n$. So every term in the sequence is divisible by $2$.

Items:

$n^2+n+1$ | $2k$ | $n+1$ | $2k+1$ | $n-1$ | $n^2-n+1$ | $n$

## Part C Divisibility by $3$

Using your expression from part A, prove that every term in the sequence is divisible by $3$.

Once you have worked out a proof yourself, consider the model proof below. Drag and drop answers into the blank spaces to complete the proof.

We know that the factors of $u_n$ from part A.

When $n$ is a multiple of $3$, it is divisible by $3$ and we can see that ___ is a factor of $u_n$, so $u_n$ is divisible by $3$.

When $n = 3k + 1$, where $k \in \mathbb{Z}$, then the factor ___ $=$ ___ in terms of $k$, so the factor ___ is divisible by $3$, and hence $u_n$ is divisible by $3$.

When $n = 3k + 2$, where $k \in \mathbb{Z}$, then the factor ___ $=$ ___ in terms of $k$, so the factor ___ is divisible by $3$, and hence $u_n$ is divisible by $3$.

Therefore, $u_n$ is divisible by $3$ for any value of $n$. So every term in the sequence is divisible by $3$.

Items:

$3k+3$ | $3k-3$ | $n+1$ | $n^2+n+1$ | $3k+2$ | $3k$ | $n$ | $n^2-n+1$ | $n-1$ | $3k+1$

## Part D Divisibility by $7$

Using your expression from part A, prove that every term in the sequence is divisible by $7$.

Once you have worked out a proof yourself, consider the model proof below. Drag and drop answers into the blank spaces to complete the proof. You may use the same answer more than once.

We know that the factors of $u_n$ from part A.

When $n$ is a multiple of $7$, it is divisible by $7$ and we can see that [ ] is a factor of $u_n$, so $u_n$ is divisible by $7$.

When $n = 7k+1$, where $k \in \mathbb{Z}$, then the factor [ ] $=$ [ ] in terms of $k$, so the factor [ ] is divisible by $7$, and hence $u_n$ is divisible by $7$.

When $n = 7k+2$, where $k \in \mathbb{Z}$, then the factor [ ] $=$ [ ] in terms of $k$, so the factor [ ] is divisible by $7$, and hence $u_n$ is divisible by $7$.

When $n = 7k+3$, where $k \in \mathbb{Z}$, then the factor [ ] $=$ [ ] in terms of $k$, so the factor [ ] is divisible by $7$, and hence $u_n$ is divisible by $7$.

When $n = 7k+4$, where $k \in \mathbb{Z}$, then the factor [ ] $=$ [ ] in terms of $k$, so the factor [ ] is divisible by $7$, and hence $u_n$ is divisible by $7$.

When $n = 7k+5$, where $k \in \mathbb{Z}$, then the factor [ ] $=$ [ ] in terms of $k$, so the factor [ ] is divisible by $7$, and hence $u_n$ is divisible by $7$.

When $n = 7k+6$, where $k \in \mathbb{Z}$, then the factor [ ] $=$ [ ] in terms of $k$, so the factor [ ] is divisible by $7$, and hence $u_n$ is divisible by $7$.

Therefore, $u_n$ is divisible by $7$ for any value of $n$. So every term in the sequence is divisible by $7$.

Items:

$n+1$ | $7k$ | $49k^2+35k+7$ | $n^2-n+1$ | $7k+7$ | $n-1$ | $49k^2+63k+21$ | $n^2+n+1$ | $n$

## Part E Largest Divisor

Prove that $42$ is the largest integer that will divide every term of $u_n$.

Once you have worked out a proof yourself, consider the model proof below. Drag and drop answers into the blank spaces to complete the proof. You may use the same answer more than once.

We know from earlier that $u_n$ is divisible by $2$, $3$ and $7$. So we know that $2 \times 3 \times 7 =$ ______ will divide $u_n$. Are there any larger integers that can do so?

Let's consider the first non-zero term, $126$. We find that $126 \div 42 =$ ______. This shows that the prime factorisation of $126$ is ______. Hence, the only larger factors of $126$ are (in order of increasing size) ______ and ______. Will these divide any other terms of $u_n$?

Looking at the next term, we find that $2184 \div$ ______ $= \frac{104}{3}$, so ______ does not divide $2184$. Considering our other factor, we find that $2184 \div$ ______ $= \frac{52}{3}$, so ______ does not divide $2184$ either.

Therefore, $42$ is the largest integer that will divide every term of $u_n$.

Items:

$42$ | $45$ | $7$ | $18$ | $5$ | $2$ | $2 \times 3^2 \times 7$ | $2^2 \times 3 \times 7$ | $126$ | $2 \times 3^2 \times 5$ | $3$ | $63$

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