

Gameboard

Maths Calculus

Differential Equations

Integrating Factors 1

Integrating Factors 1

Pre-Uni Maths for Science L2.1



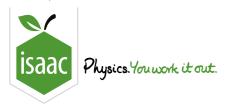
Find the general solution of the differential equation

$$xrac{\mathrm{d}y}{\mathrm{d}x}+(a+x)y=\mathrm{e}^{-x}.$$

Find the general solution for y as a function of x.

The following symbols may be useful: , a, c, e, k, x, y

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Maths

Calculus Differential Equations

RC Circuit (Integrating Factors)

RC Circuit (Integrating Factors)

Pre-Uni Maths for Science L2.7



A circuit consists of a capacitor C, a resistor R and a switch in series with a battery of emf V_0 . The switch is initially open and the capacitor is uncharged. At t=0 the switch is closed. The equation for the charge q on the capacitor as a function of time t after the switch is closed is

$$R\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{q}{C} = V_0.$$

Find how the charge on the capacitor varies with time t given that q=0 at t=0.

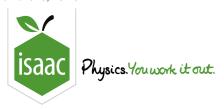
Find the equation for the charge q on the capacitor as a function of time t.

The following symbols may be useful: C, R, V_0, e, q, t

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STEM SMART Double Maths 41 - 1st & 2nd Order Differential Equations



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Maths

Calculus Differential Equations Undamped Pendulum (2nd Order)

Undamped Pendulum (2nd Order)

Further A University





Pre-Uni Maths for Science L2.6

The equation describing the small-angle oscillations of a simple pendulum is

$$rac{\mathrm{d}^2 heta}{\mathrm{d}t^2} = -rac{g}{l} heta$$

where θ is its angular displacement from the vertical at time t, l is the length of the pendulum and g is the acceleration due to gravity. Find an expression for θ as a function of t given that $\theta=\alpha$ and $\frac{\mathrm{d}\theta}{\mathrm{d}t}=\beta$ at t=0.

Find the equation for θ as a function of t given that $\theta = \alpha$ and $\frac{d\theta}{dt} = \beta$ at t = 0.

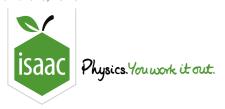
The following symbols may be useful: alpha, beta, g, 1, t, theta

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STEM SMART Double Maths 41 - 1st & 2nd Order

Differential Equations



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Maths

Calculus Differential Equations Mass on Spring (2nd Order)

Mass on Spring (2nd Order)

Further A University



Pre-Uni Maths for Science L2.8

A mass m on a spring is subjected to a damping force. The equation describing its displacement x from its equilibrium position as a function of time t is

$$mrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx - brac{\mathrm{d}x}{\mathrm{d}t},$$

where -kx is the force from the spring and $-b \frac{\mathrm{d}x}{\mathrm{d}t}$ is the force due to damping. The damping coefficient b is related to the spring constant k by $k=rac{4b^2}{25m}.$ Find an expression for the subsequent motion of the mass given that x=0 and $rac{\mathrm{d}x}{\mathrm{d}t}=V$ at t=0.

Find the equation describing the subsequent motion of the mass given that x=0 and $\frac{\mathrm{d}x}{\mathrm{d}t}=V$ at t=0. Give your answer in terms of the constants b, m, and V.

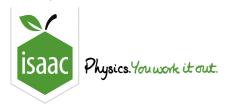
The following symbols may be useful: V, b, e, m, t, x

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Differential Equations



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Maths Calculus

Differential Equations

Damped Pendulum (2nd Order)

Damped Pendulum (2nd Order)

Pre-Uni Maths for Science L2.2



The equation describing the displacement x of the bob of a damped pendulum from its equilibrium position is given by

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega_0^2 x - 2\gamma \frac{\mathrm{d}x}{\mathrm{d}t}$$

where ω_0 is the angular frequency of undamped oscillations of the pendulum and γ is related to the damping. Assuming $\omega_0 > \gamma$ find an equation for x at time t given that x = X and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ at t = 0. (You will find it helpful to define a new constant ω_1 such that $\omega_1^2 = \omega_0^2 - \gamma^2$.)

Find an equation for x at time t given that x=X and $\dfrac{\mathrm{d}x}{\mathrm{d}t}=0$ at t=0.

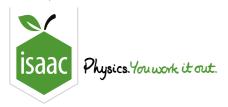
The following symbols may be useful: X, e, gamma, omega_1, t, x

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Differential Equations



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Gameboard Maths

Calculus

Differential Equations

Inhomogeneous Equation (2nd Order)

Inhomogeneous Equation (2nd Order)



Pre-Uni Maths for Science L2.4

Find the solution of the equation

$$rac{\mathrm{d}^2 p}{\mathrm{d}q^2} - 4rac{\mathrm{d}p}{\mathrm{d}q} + 3p = 3q - 1$$

given that p=2 and $rac{\mathrm{d}p}{\mathrm{d}q}=-1$ when q=0.

Find the solution of the equation.

The following symbols may be useful: e, p, q

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Differential Equations



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Maths

Calculus Differential Equations

Forced Oscillator (2nd Order)

Forced Oscillator (2nd Order)

Further A University



Pre-Uni Maths for Science L2.5

The equation of motion of a forced oscillator is given by

$$rac{\mathrm{d}^2z}{\mathrm{d}t^2}+\omega_0^2z=Z_0\sin(\omega_1t)$$

Given that $\omega_0
eq \omega_1$ find the solution for z given that z=0 and $\frac{\mathrm{d}z}{\mathrm{d}t}=0$ at t=0.

Find the solution for z given that z=0 and $\frac{\mathrm{d}z}{\mathrm{d}t}=0$ at t=0.

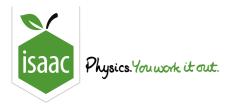
The following symbols may be useful: Z_0, omega_0, omega_1, t, z

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Differential Equations



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Maths

Differential Equations: General Applications 2i

Differential Equations: General Applications 2i



During an industrial process substance X is converted into substance Z. Some of the substance X goes through an intermediate phase, and is converted into substance Y, before being converted into substance Z. The situation is modelled by

$$rac{\mathrm{d}y}{\mathrm{d}t} = 0.3x - 0.2y \quad ext{ and } \quad rac{\mathrm{d}z}{\mathrm{d}t} = 0.2y + 0.1x$$

where x, y and z are the amounts in kg of X, Y and Z at time t hours after the process starts.

Initially there is $10 \,\mathrm{kg}$ of substance X and nothing of substances Y and Z. The amount of substance X decreases exponentially. The initial rate of decrease is $4 \,\mathrm{kg} \,\mathrm{hour}^{-1}$.

Part A Expression for x

Find an expression for x.

The following symbols may be useful: e, t

Part B	dx	dy	$\perp dz$
raitb	$\frac{dt}{dt}$	<u>d</u> -	- dt

Show that $rac{\mathrm{d}x}{\mathrm{d}t}+rac{\mathrm{d}y}{\mathrm{d}t}+rac{\mathrm{d}z}{\mathrm{d}t}=k$ where k is a constant.

State the value of k.

Comment on this result in the context of the industrial process.

The total amount of all three substances is constant throughout the process
The total amount of all three substances is constant throughout the process

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		The total	amount of	all three	substances	decreases	throughout	the process
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Part C Expression for y

Find an expression for y in terms of t.

The following symbols may be useful: e, $\,\mathrm{t}$

Part D Maximum amount of Y

Determine the maximum amount of substance \boldsymbol{Y} present during the process.

Part E $\hspace{1.5cm}$ Time to produce $9\,\mathrm{kg}$ of substance Z

How long does it take to produce $9 \, \mathrm{kg}$ of substance Z? Give your answer to $3 \, \mathrm{significant}$ figures.

Adapted with permission from UCLES, A Level, Sample Paper 2017, Paper Y541, Question 11.