



Compound Angle Formulae

A-level Maths Topic Summaries - Trigonometry

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level P3

Fill in the blanks to complete the summary notes on compound angle formulae.

$\sin(A + B) \equiv \sin A \cos B$

$\sin(A - B) \equiv \sin A \cos B$

$\cos(A + B) \equiv \cos A \cos B$

$\cos(A - B) \equiv \cos A \cos B$

$\tan(A + B) \equiv \frac{\tan A}{1}$

$\tan(A - B) \equiv \frac{\tan A}{1}$

Items:

-
-
-
-
-
-
-
-



Double Angle Formulae

A-level Maths Topic Summaries - Trigonometry

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level P3

Fill in the blanks to complete the summary notes on double angle formulae.

There is one identity for $\sin(2A)$ and one for $\tan(2A)$. There are three identities for $\cos(2A)$ which are related by the identity $\sin^2 A + \cos^2 A \equiv 1$.

$\sin(2A) \equiv$

$\cos(2A) \equiv \cos^2 A$

$\cos(2A) \equiv 1$

$\cos(2A) \equiv$ $- 1$

$\tan(2A) \equiv \frac{\text{$

Items:

-
-
-
-
-
-
-
-



STEM SMART Single Maths 31 - Compound & Double-Angle Formulae

Combining Sin and Cos Terms - R Form

A-level Maths Topic Summaries - Trigonometry

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level P3
Fill in the blanks to complete the summary notes on R form.

The compound angle formulae can be used to rewrite the sum of two sine or cosine terms as a single sine or cosine function. We will illustrate this by writing $3 \sin x + 4 \cos x$ in the form $R \sin(x + \alpha)$.

$$R \sin(x + \alpha) \equiv 3 \sin x + 4 \cos x$$

Expanding $R \sin(x + \alpha)$ using the compound angle formula for $\sin(A + B)$,

$$R \sin(x + \alpha) \equiv R \sin x \cos \alpha + R \cos x \sin \alpha$$

Equating coefficients of $\sin x$ and $\cos x$ in the two expressions for $R \sin(x + \alpha)$ gives

$$3 = \boxed{} \quad (1)$$

$$4 = \boxed{} \quad (2)$$

We now solve (1) and (2) simultaneously to find R and α .

- To find R we square (1) and (2), add them together, and make use of the identity $\sin^2 \theta + \cos^2 \theta \equiv 1$. We end up with $\boxed{}$, and hence $R = \boxed{}$.
- To find α we divide (2) by (1) to get $\boxed{}$. We then use arctan to find $\alpha = \boxed{}$ (to 3 sf).

Therefore $3 \sin x + 4 \cos x$ can be written as $\boxed{} \sin(x + \boxed{})$.

Items:

$3^2 + 4^2 = R^2$
 3
 4
 5
 36.9°
 53.1°
 $R \cos \alpha$
 $R \sin \alpha$
 $\tan \alpha = \frac{4}{3}$

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Question deck:

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Formulae



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Trigonometry: Double Angles 1ii

Subject & topics: Maths **Stage & difficulty:** A Level C1**Part A****The form $a \sin^2 \theta + b \sin \theta + c = 0$**

Express the equation $(\operatorname{cosec} \theta)(3 \cos 2\theta + 7) + 11 = 0$ in the form $a \sin^2 \theta + b \sin \theta + c = 0$, where a , b , and c are constants and $a > 0$.

Enter the values of a , b and c :

 $a =$ $b =$ $c =$ **Part B****Solve**

Hence solve, for $-180^\circ < \theta < 180^\circ$, the equation $(\operatorname{cosec} \theta)(3 \cos 2\theta + 7) + 11 = 0$.

Give your answers in degrees, to three significant figures.

Enter your answers in order from lowest value of θ to highest.

 (lowest value) (highest value)

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Formulae



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Trigonometry: Double Angles 2ii

Subject & topics: Maths**Stage & difficulty:** A Level C1**Part A****sin Double Angle**

Write down the identity expressing $\sin 2\theta$ in terms of $\sin \theta$ and $\cos \theta$.

The following symbols may be useful: theta

Part B**sin 2α**

Given that $\sin \alpha = \frac{1}{4}$ and α is acute, find the exact value of $\sin 2\alpha$.

The following symbols may be useful: alpha

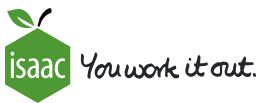
Part C**Solve**

Solve in degrees, for $0^\circ < \beta < 90^\circ$, the equation $5 \sin 2\beta \sec \beta = 3$, giving your answer in degrees to three significant figures.

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Formulae



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Trigonometry: Combined Angles 5i

Subject & topics: Maths **Stage & difficulty:** A Level P2

The value of $\tan 10^\circ$ is denoted by p . Find, in terms of p , the value of:

Part A
 $\tan 55^\circ$

$\tan 55^\circ$

The following symbols may be useful: p

Part B
 $\tan 5^\circ$

$\tan 5^\circ$

The following symbols may be useful: p

Part C
 $\tan \theta$

$\tan \theta$, where θ satisfies the equation $3 \sin (\theta + 10^\circ) = 7 \cos (\theta - 10^\circ)$.

The following symbols may be useful: p, theta

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STEM SMART Single Maths 31 - Compound & Double-Angle
Formulae



Functions: Reciprocal Trig 2i

Subject & topics: Maths **Stage & difficulty:** A Level P2

It is given that A and B are angles such that

$$\sec^2 A - \tan A = 13 \quad \text{and} \quad \sin B \sec^2 B = 27 \cos B \operatorname{cosec}^2 B.$$

Part A

Largest value of $\tan(A - B)$

Find the largest possible exact value of $\tan(A - B)$.

The following symbols may be useful: A, B

Part B

Smallest value of $\tan(A - B)$

Give the smallest possible value of $\tan(A - B)$.

The following symbols may be useful: A, B

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STEM SMART Single Maths 31 - Compound & Double-Angle Formulae

Trigonometry and R-Form 1

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level C1

Part A

$5 \cos x + 12 \sin x$

Express $5 \cos x + 12 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.

State the value of R :

$R =$

Give the value of α in degrees, to three significant figures:

$\alpha =$

Part B

Transformations

Give details of a pair of transformations which transform the curve $y = \cos x$ to the curve $y = 5 \cos x + 12 \sin x$.

Available items

- Translation in the positive x direction by α .
- Stretch parallel to the y -axis by a factor of R .
- Stretch parallel to the y -axis by a factor of $\frac{1}{R}$.
- Stretch parallel to the x -axis by a factor of R .
- Translation in the positive y direction by α .
- Stretch parallel to the x -axis by a factor of $\frac{1}{R}$.
- Translation in the negative y direction by α .
- Translation in the negative x direction by α .

Part C

Solve

Solve, for $0^\circ < x < 360^\circ$, the equation $5 \cos x + 12 \sin x = 2$, giving your answers in degrees correct to the nearest 0.1° .

Enter your answers in order from lowest value of θ to highest.

(lowest value)

(highest value)

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Formulae



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Trigonometry: Combined Angles 3i

Subject & topics: Maths **Stage & difficulty:** A Level P2

In **Figure 1**, ABCD represents a rectangular table with sides 3.5 m and 1.5 m. It has been turned so it wedges in a passage of width 2.5 m.

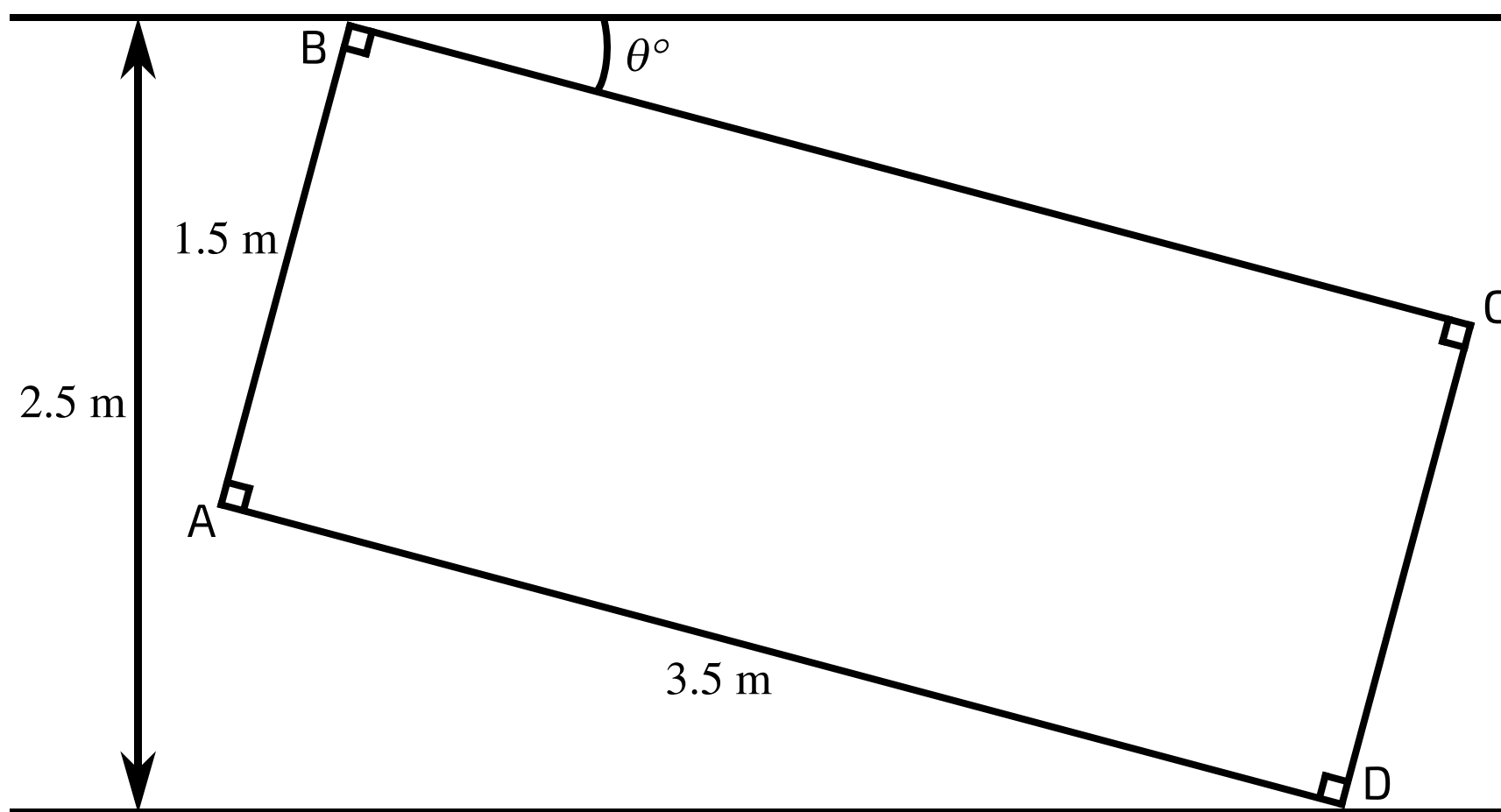


Figure 1: The rectangular table ABCD.

Part A

Value of $7 \sin \theta^\circ + 3 \cos \theta^\circ$

Given that θ is the acute angle between the longer side and the passage, as shown in the diagram, find the exact value of $7 \sin \theta + 3 \cos \theta$.

The following symbols may be useful: $\cos()$, $\sin()$, $\tan()$, theta

Part B

The form $R \sin (\theta^{\circ} + \alpha^{\circ})$

Express $7 \sin \theta + 3 \cos \theta$ in the form $R \sin (\theta + \alpha)$, where $R > 0$ and $0^{\circ} < \alpha < 90^{\circ}$.

Give the exact value of R .

The following symbols may be useful: R

Give the value of α to 3 significant figures.

Part C

Find θ

Find θ , to 3 significant figures.

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Wave Interference

Pre-Uni Maths for Sciences H4.10

Subject & topics: Maths | Geometry | Trigonometry **Stage & difficulty:** A Level C3

Two waves

$$\psi_1 = A \cos \left(2\pi f t - \left(\frac{2\pi}{\lambda} \right) x + \phi \right)$$

and

$$\psi_2 = A \cos \left(2\pi f t - \left(\frac{2\pi}{\lambda} \right) x - \phi \right)$$

interfere, such that the resultant wave is given by $\psi = \psi_1 + \psi_2$. Express ψ as the product of two trigonometric functions.

Express ψ as the product of two trigonometric functions.

The following symbols may be useful: A, f, lambda, phi, pi, t, x

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