

Home Gameboard Maths Geometry Vectors Crossing Paths

Crossing Paths



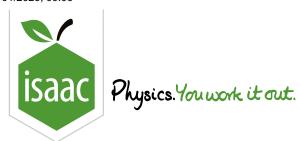
A person is walking northwards with velocity $\underline{\boldsymbol{v}}=1.0\underline{\boldsymbol{j}}\,\mathrm{m\,s^{-1}}$ from a point $\underline{\boldsymbol{r}}=500\underline{\boldsymbol{i}}\,\mathrm{m}$. At the same time a second person starts walking from a point $\underline{\boldsymbol{s}}=-500\underline{\boldsymbol{i}}+500\underline{\boldsymbol{j}}\,\mathrm{m}$ with velocity $\underline{\boldsymbol{u}}=1.0\underline{\boldsymbol{i}}+u_y\underline{\boldsymbol{j}}\,\mathrm{m\,s^{-1}}$.

Part A Time Taken

Find the time T in seconds that passes between the walkers setting off, and their paths crossing.

Part B Required Speed

Find the speed u_y required for the two people to meet.



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Maths Geometry

Vectors

Vector Equations of Motion 2

Vector Equations of Motion 2



In one dimension, motion with constant acceleration is modelled with the suvat equations. There are analogous vector equations for motion with constant acceleration in two or three dimensions:

$$oldsymbol{\underline{v}} = oldsymbol{\underline{u}} + oldsymbol{\underline{a}}t \hspace{1cm} oldsymbol{\underline{s}} = rac{1}{2}(oldsymbol{\underline{u}} + oldsymbol{\underline{v}})t \hspace{1cm} oldsymbol{\underline{s}} = oldsymbol{\underline{u}}t + rac{1}{2}oldsymbol{\underline{a}}t^2$$

In these equations the acceleration is \underline{a} and the time over which the acceleration takes place is t. The initial velocity is \underline{u} , the final velocity is \underline{v} , and \underline{s} is the change in displacement during the period of acceleration.

(The equivalent expression to $v^2=u^2+2as$ involves the scalar product and is not needed in this question.)

Part A Find an expression for velocity

A particle moves in the x-y plane with the constant acceleration $\begin{pmatrix} 4 \\ 2 \end{pmatrix} \, \mathrm{m} \, \mathrm{s}^{-2}$. Find an expression for the velocity of the particle after $t \, \mathrm{s}$ given that it has velocity $\begin{pmatrix} -19 \\ -10 \end{pmatrix} \, \mathrm{m} \, \mathrm{s}^{-1}$ initially.

Give an expression for the *x* component of the velocity.

The following symbols may be useful: t

Give an expression for the *y* component of the velocity.

The following symbols may be useful: t

Part B Find a position

A horse is running in a large, flat, rectangular field. The field is modelled using the x-y plane, with the origin at one corner. When the horse is at the position $\underline{p} = 50\underline{i} + 70\underline{j}$ relative to the origin it is moving with a velocity of $\begin{pmatrix} 12 \\ -6 \end{pmatrix}$ m s⁻¹. The horse slows down. Its acceleration is $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$ m s⁻². What is the position of the horse relative to the origin after 3 seconds?

Give your answer as coordinates, (x, y).



Part C Find a value of t

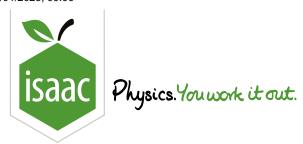
At time $t=0\,\mathrm{s}$ a particle is at the origin, moving with a velocity $\underline{\boldsymbol{u}}_1$. The particle accelerates for $4\,\mathrm{s}$ with acceleration $\begin{pmatrix} 10\\16 \end{pmatrix} \mathrm{m\,s^{-2}}$. When $t=4\,\mathrm{s}$, the particle has a displacement of $\begin{pmatrix} 240\\384 \end{pmatrix} \mathrm{m}$ from the origin.

Suppose instead that the particle starts at the origin at time $t=0\,\mathrm{s}$ moving with the velocity $-\underline{\boldsymbol{u}}_1$. If the particle accelerates with acceleration $\begin{pmatrix} 10\\16 \end{pmatrix} \mathrm{m\,s^{-2}}$ it will still arrive at the point $\begin{pmatrix} 240\\384 \end{pmatrix} \mathrm{m}$. At what time t does this occur?

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STEM SMART Single Maths 38 - Vector Equations of Motion



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Maths

Vectors & Calculus 2i

Vectors & Calculus 2i



A projectile has velocity $inom{A}{5-gt}$ $\mathrm{m\,s^{-1}}$, where A is a constant.

Part A Displacement

The particle is at $\binom{5}{10}$ when t=0.

Find an expression for the x-component of the particle's displacement, in metres, as a function of t.

The following symbols may be useful: A, g, t

Find an expression for the y-component of the particle's displacement, in metres, as a function of t.

The following symbols may be useful: A, $\,$ g, $\,$ t

Part B Force

Find an expression for the force on the particle, given that it has mass $m \lg$. Give your answer in the form $a\underline{i} + bj$ where \underline{i} and j are unit vectors in the x and y directions respectively.

The following symbols may be useful: A, g, i, j, m

${\bf Part \ C} \qquad {\bf Value \ of} \ A$

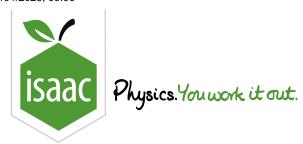
The projectile hits a target at the coordinates $\binom{20}{0}$.

What is the value of A? Give your answer to 2 significant figures. In your calculation, use the approximation $g \approx 10\,\mathrm{m\,s^{-2}}$ and assume that the target is hit at t>0.

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Maths

Vectors & Calculus 1i

Vectors & Calculus 1i



A planet moves through space. The force on the planet is given by

$$oldsymbol{\underline{F}} = egin{pmatrix} -mAB^2\cos Bt \ -mAB^2\sin Bt \end{pmatrix}$$

where A and B are numerical constants and m is the mass of the planet.

Part A Velocity

Given that the velocity of the planet when t=0 is $\begin{pmatrix} 0 \\ AB \end{pmatrix}$.

Find an expression for the x-component of the velocity of the planet as a function of time.

The following symbols may be useful: A, B, cos(), sin(), t, tan()

Find an expression for the *y*-component of the velocity of the planet as a function of time.

The following symbols may be useful: A, B, cos(), sin(), t, tan()

Part B Displacement

Given that the displacement of the planet when t=0 is $\begin{pmatrix} A \\ 0 \end{pmatrix}$.

Find an expression for the x-component of the displacement of the planet as a function of time.

The following symbols may be useful: A, B, cos(), sin(), t, tan()

Find an expression for the *y*-component of the displacement of the planet as a function of time.

The following symbols may be useful: A, B, cos(), sin(), t, tan()

Part C Modulus

Find an expression for the modulus of the displacement. Simplify your answer as far as possible.

The following symbols may be useful: A, B, cos(), sin(), t, tan()

Part D Shape of path

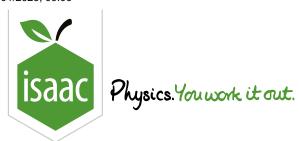
What is the shape of the path that the planet follows?

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<u>STEM SMART Single Maths 38 - Vector Equations of</u>

Motion



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Maths

Particles Moving on a Surface

Particles Moving on a Surface



A particle Q of mass $0.2\,\mathrm{kg}$ is projected horizontally with velocity $4\,\mathrm{m\,s^{-1}}$ from a fixed point A on a smooth horizontal surface. At time $t\,\mathrm{s}$ after projection Q is $x\,\mathrm{m}$ from A and is moving away from A with velocity $v\,\mathrm{m\,s^{-1}}$. There is a force of $3\cos 2t\,\mathrm{N}$ acting on Q in the positive x-direction.

Part C Average velocity

Find the average velocity of Q between the times $t=\pi$ and $t=\frac{3}{2}\pi$. Give your answer to 3 significant figures.

Part D Particle's velocity

A particle P moves in a plane. Its displacement from the starting point, \underline{R} , varies with time, t, as follows:

$$oldsymbol{R} = egin{pmatrix} 2t^2 \sin \pi t - 1 \ 1 + t^3 \end{pmatrix}$$

Where displacement is measured in metres and time is measured in seconds.

What is the x-component of the particle's velocity?

The following symbols may be useful: cos(), pi, sin(), t, tan()

What is the *y*-component of the particle's velocity?

The following symbols may be useful: cos(), pi, sin(), t, tan()

Part E Speed of particle

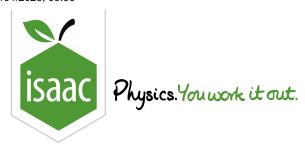
Find the speed of the particle when t=2. Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, June 2016, OCR M3, Question 2

Gameboard:

<u>STEM SMART Single Maths 38 - Vector Equations of</u>

Motion



<u>Gameboard</u>

Maths

Geometry

Vectors

Vector Equations of Motion 1

Vector Equations of Motion 1



This question looks at three different uses of calculus in vector problems.

Part A Integrating to find particle displacement

A particle moves in the x-y plane with velocity $\underline{v}=\begin{pmatrix} 2t{
m e}^{-2t^2} \\ 3t{
m e}^{-4t^2} \end{pmatrix}$. Find an expression for the displacement of

the particle at time t, given that the particle is at the origin when t=0.

Enter an expression for the x-component of the displacement.

The following symbols may be useful: e, t

Enter an expression for the *y*-component of the displacement.

The following symbols may be useful: t

Part B Finding a maximum speed

At a time t s a particle moves in the x-y plane with velocity $\underline{v} = \begin{pmatrix} 2t \mathrm{e}^{-2t^2} \\ 3 \end{pmatrix} \mathrm{m} \, \mathrm{s}^{-1}$. What is the maximum speed of the particle? Give your answer as an expression in terms of e .

The following symbols may be useful: e

Part C Distance of closest approach to the origin

The displacement of a particle is given by the expression $\underline{s} = \begin{pmatrix} \mathrm{e}^{3t} \\ \mathrm{e}^{6t} - 5 \end{pmatrix}$. Find the shortest distance between the particle and the origin during the particle's motion. Give your answer in the form $\frac{\sqrt{a}}{2}$.

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