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Poisson Distribution: Crystals

Further A

P P P

In a rock, small crystal formations occur at a constant average rate of 3.2 per cubic metre.

Part A Assumptions

State a further assumption needed to model the number of crystal formations in a fixed volume of rock by a Poisson distribution.

- Crystals must occur independently of one another
- The rock must be at least a cubic metre in size
- The rock must contain 3 or more crystals

Part B Exactly five crystal formations

In the remainder of the question, you should assume that a Poisson model is appropriate.

Calculate the probability that in one cubic metre of rock there are exactly 5 crystal formations. Give your answer to 3 s.f.

Part C At least three crystal formations

Calculate the probability that in 0.74 cubic metres of rock there are at least 3 crystal formations. Give your answer to 3 s.f.

Part D At least 36 crystal formations

Calculate the probability that in 10 cubic metres of rock there are at least 36 crystal formations. Give your answer to 3 s.f.

Adapted with permission from UCLES, A level, June 2012, OCR , Question 4

Poisson Distribution - Quasars

Further A University

Currently an average of 24 quasars have been found per square degree of sky. On the assumption that their distribution is random and independent so that it follows a Poisson distribution find the following.

Part A Probability of less than 15 quasars

Find the probability that there are fewer than 15 quasars in a randomly chosen area of 1 square degree. Give your answer to 3 s.f.

Part B Probability between 24 and 30 quasars

Find the probability that there are no fewer than 24 and no more than 30 quasars in a randomly chosen area of 1 square degree. Give your answer to 3 s.f.

Part C Probability of no quasars

A square with an area of 1 square degree is divided into 16 smaller squares with equal areas. One of these smaller squares is randomly selected. Find the probability that there are no quasars in this smaller area. Give your answer to 3 s.f.

Part D Probability of at least 2 quasars

Again considering the 1 degree square divided into 16 smaller squares of equal area, find the probability that at least 8 of the smaller squares contain at least 2 quasars. Give your answer to 3 s.f.

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Poisson Distribution - Radioactivity

Further A University

A radioactive source produces gamma rays which travel to a detector. An average of 17 gamma rays arrive at the detector every 10 seconds.

The detector counts and displays the number of gamma rays arriving during a time interval set by the user. In this time interval the detector can detect a maximum of 5 gamma rays before saturating; thus, if the number of gamma rays arriving at the detector in this time interval is > 5, the counter will read 5. (For example, supposing 8 gamma rays arrive at the detector in the given time interval the counter will read 5.)

Part A Probability that the detector saturates in $2\,\mathrm{s}$

The detector is set to count gamma rays for 2 seconds. Calculate the probability that the reading on the counter is not the same as the number of gamma rays that arrive at the detector. Give your answer to 3 sf.

Part B The most probable number of gamma rays in $2\,\mathrm{s}$

The detector is set to count gamma rays for 2 seconds. Find the most probable number of gamma rays arriving at the detector.

Part C Expected number of gamma rays in $2\,\mathrm{s}$

The detector is set to count gamma rays for 2 seconds; find the expected number of gamma rays it will detect giving your answer to 3 sf.

Find the expected value of the percentage error in the measurement of the number of gamma rays arriving at the detector. (This is the percentage difference between the expected value of the reading on the detector that you have just derived, and the mean number of gamma rays arriving at the detector.) Give your answer to 2 sf.

$\%$

Part D Probability that the detector saturates in $1\,\mathrm{s}$

The detector is adjusted to count gamma rays for 1 second; it still saturates when the number of counts exceeds 5. Calculate the probability that the reading on the counter is not the same as the number of gamma rays that arrive at the detector. Give your answer to 3 sf.

Part E Expected number of gamma rays in $1\,\mathrm{s}$

The detector is set to count gamma rays for 1 second; find the expected number of gamma rays it will detect giving your answer to 3 sf.

Find the expected value of the percentage error in the measurement of the number of gamma rays arriving at the detector. (This is the percentage difference between the expected value of the reading on the detector that you have just derived, and the mean number of gamma rays arriving at the detector.) Give your answer to 2 sf.

$\%$

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Poisson Distribution: Random Variables

Further A

P P P

This question looks at the properties of a random variable which follows a Poisson distribution, and also the sum of two independent random variables which each have Poisson distributions.

Part A $\mathrm{E}(X)$ for a Poisson distribution

A random variable X is modelled by the Poisson distribution $X \sim \mathrm{Po}(5.9)$. Write down the value of the expectation of X, $\mathrm{E}(X)$.

Part B $\mathrm{Var}(X)$ for a Poisson distribution

A random variable X is modelled by the Poisson distribution $X \sim \mathrm{Po}(\lambda)$. The ratio $\dfrac{\mathrm{P}(X=6)}{\mathrm{P}(X=4)} = 0.3$. Find the exact value of the variance of X, $\mathrm{Var}(X)$.

Part C Finding a

Consider the two random variables $X \sim \mathrm{Po}(\lambda)$ and $Y \sim \mathrm{Po}(2\lambda)$. Find an expression for the value a such that $\mathrm{P}(X=a) = \mathrm{P}(Y=a)$. Give your answer in the form $\dfrac{f(\lambda)}{\ln(2)}$, where $f(\lambda)$ is a function of λ.

The following symbols may be useful: `lambda`, `ln()`, `log()`

Part D Summing Poisson variables

The independent random variables X and Y have the distributions $X \sim \mathrm{Po}(\lambda)$ and $Y \sim \mathrm{Po}(2\lambda)$. A third random variable $T = X + Y$.

If $\mathrm{P}(T = 5) = \frac{1}{2}\mathrm{P}(X = 5)$, find an expression for λ. Give your answer in the form $\frac{1}{a}\ln b$, where a and b are integers.

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Physics. You work it out.

Home Gameboard Maths Statistics Random Variables Geometric Distribution: Darts

Geometric Distribution: Darts

Further A

P P P

A darts player aims repeatedly for the bull's-eye. He counts it a success if his dart hits the bull's-eye. It may be assumed that, on each throw, the probability of success is $\frac{1}{4}$, independently of all other throws.

Part A First success on fourth throw

Find the probability that, in a series of throws, the first success occurs on the fourth throw. Give your answer as a fraction.

Part B First success between fourth and eighth throws

Find the probability that, in a series of throws, the first success occurs between the fourth and eighth throws (inclusive). Give your answer to 3 s.f.

Part C Second success on third throw

Find the probability that, in a series of throws, the second success occurs on the third throw. Give your answer as a fraction.

Adapted with permission from OCR June 2000, Mathematics Paper 4, Question 14.

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Geometric Distribution

Further A

P P P

Part A Random variable X

The random variable X has a geometric distribution with parameter $\frac{1}{3}$. Find P$(X = 3)$.

Part B Random variable Y

The random variable Y has a geometric distribution with mean 4. Find $\mathrm{Var}(Y)$.

Adapted with permission from OCR November 1999, Mathematics Paper 4, Question 6.

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Geometric Distribution - Pulsar

Further A P P P

A pulsar produces regular pulses with a period $T = 1.33\,\mathrm{s}$; the brightness of the pulses varies randomly from pulse to pulse. The pulses arrive at a receiver which can only detect them when the brightness exceeds a certain level. In one set of measurements lasting for $5054\,\mathrm{s}$, 570 pulses are bright enough to be detected.

It is assumed that Pulse n arrives at nT, i.e. Pulse 1 arrives at T, Pulse 2 at $2T$ and so on. N is the number of pulses that have arrived up until and including the first one bright enough to be detected; you may assume it follows a geometric distribution $N \sim \mathrm{Geo}(p)$.

Part A Probability of detecting a pulse

Deduce the probability that a pulse will be detected.

Part B First pulse detected is Pulse 5

Find the probability that the first pulse bright enough to be detected by the receiver is Pulse 5. Give your answer to 3 s.f.

Part C First pulse detected is after Pulse 5

Find the probability that the first pulse bright enough to be detected by the receiver arrives after Pulse 5. Give your answer to 3 s.f.

Part D First pulse detected is before Pulse 5

Find the probability that the first pulse bright enough to be detected by the receiver arrives before Pulse 5. Give your answer to 3 s.f.

Part E Probability less than 5%

Find the smallest value of n for which the probability that Pulse n is the first detected pulse is less than 5%, i.e. P$(N = n) < 5\%$.

Part F Probability less than 20%

Find the smallest value of n for which the probability that the first detected pulse is after Pulse n is less than 20%, i.e. P$(N > n) < 20\%$.

Part G First and second detection

Find the probability that the first pulse bright enough to be detected is the third one and that the second pulse bright enough to be detected is the tenth. Give your answer to 3 s.f.

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Physics. You work it out.

Home Gameboard Maths Statistics Hypothesis Tests Quality of Tests: Accidents

Quality of Tests: Accidents

Further A
P P P

The number of accidents per month at a certain road junction has a Poisson distribution with mean 4.8. A new road sign is introduced warning drivers of the danger ahead, and in a subsequent month 2 accidents occurred.

Part A Hypothesis test

A hypothesis test at the 10% level is used to determine whether there were fewer accidents after the new road sign was introduced.

Find the critical region for this test and carry out the test. Fill in the gaps below.

Let the random variable X be the number of accidents per month at the road junction. Then $X \sim \mathrm{Po}(\lambda)$.

The null and alternative hypotheses are:

$$\mathrm{H}_0 : \lambda \; \square \; \square \qquad \mathrm{H}_1 : \lambda \; \square \; \square$$

The critical region is $\square \leq X \leq \square$.

$\square$ is $\square$ the critical region.

Therefore we $\square$ H_0. There $\square$ evidence to suggest that the road sign has decreased accidents.

Items:

$>$ $<$ $=$ reject do not reject within not in is is insufficient 0 1 2 3 4 5 2.8 4.8

Part B Type I error

Find the probability of a Type I error. Give your answer to 3 s.f.

Gameboard:

STEM SMART Double Maths 42 - The Poisson & Geometric Distributions & Tests

Physics. You work it out.

Home Gameboard Maths Statistics Hypothesis Tests Quality of Tests: Plant Research

Quality of Tests: Plant Research

Further A

P P P

In a research laboratory where plants are studied, the probability of a certain type of plant surviving was 0.35. The laboratory manager changed the growing conditions and wished to test whether the probability of a plant surviving had increased.

Part A Plant sample

The plants were grown in rows, and when the manager requested a random sample of 8 plants to be taken, the technician took all 8 plants from the front row. Explain what was wrong with the technician's sample.

- The sample was not random.
- The sample was too large.
- The sample should have been taken from the back row instead.

Part B Hypothesis test

A suitable sample of 8 plants was taken and 4 of these 8 plants survived.

Using a 5% significance level, find the critical region and carry out the test. Fill in the gaps below.

Let the random variable X be the number of plants that survived. Then $X \sim \mathrm{B}($ ____ $, p)$.

The null and alternative hypotheses are:

$\mathrm{H}_0 : p$ ____ ____ $\mathrm{H}_1 : p$ ____ ____

The critical region is ____ $\leq X \leq$ ____.

____ is ____ the critical region.

Therefore we ____ H_0. There ____ evidence to suggest that the probability of a plant surviving has improved.

Items:

$>$ $<$ $=$ $\neq$ **reject** **do not reject** **is** **is insufficient** **within** **not in** 0 1 2 3 4 5 6 7 8 0.45 0.35 0.65

Part C Type II error meaning

State the meaning of a Type II error in the context of the test in Part B.

- Saying that there is no improvement, when there is.
- Saying that there is improvement, when there isn't.
- Saying that there is improvement, when there is.
- Saying that there is no improvement, when there isn't.

Part D Type II error probability

Find the probability of a Type II error for the test in Part B if the probability of a plant surviving is now 0.4. Give your answer to 3 s.f.

Part E Power

Hence calculate the power of the test when the probability of a plant surviving is 0.4. Give your answer to 2 s.f.

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Quality of Tests: Cash Withdrawal

Further A P P P

Over a long period of time it is found that the time spent at cash withdrawal points follows a normal distribution with mean 2.1 minutes and standard deviation 0.9 minutes.

A new system is tried out, to speed up the procedure. The null hypothesis is that the mean time spent is the same under the new system as previously. It is decided to reject the null hypothesis and accept that the new system is quicker if the mean withdrawal time from a random sample of 20 cash withdrawals is less than 1.7 minutes.

Assume that, for the new system, the standard deviation is still 0.9 minutes, and the time spent still follows a normal distribution.

Part A Size

Calculate the size of the test. Give your answer to 3 s.f.

Part B Power

If the mean withdrawal time under the new system is actually 1.5 minutes, calculate the power of the test. Give your answer to 3 s.f.

Used with permission from UCLES, A Level, June 2003, Paper 9709/07/M/J/03, Question 5