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STEM SMART Single Maths 25 - Defining Functions

Functions and Algebra 5ii

Subject & topics

Maths

Status

Not started

Stage & difficulty

A Level Challenge 1

The function f is defined by

$$f(x) = \frac{1}{\sqrt{x}} + 2, \quad x > 0.$$

The function g is defined for all real values of x by

$$g(x) = 10 - (x+3)^2.$$

Part A

Range of f

State the range of $f(x)$ as an inequality.

The following symbols may be useful: `<, <=, >, >=, f(x), x, y`

Part B

Inverse of f

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, x`

Part C

Range of g

State the range of $g(x)$ as an inequality.

The following symbols may be useful: `<, <=, >, >=, g(x), x, y`

Part D

Compound function of g

Find the value of $g(g(-1))$.

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STEM SMART Single Maths 25 - Defining Functions

Inverse Quadratic Function

Subject & topics

Maths | Functions | Graph Sketching

Status

Not started

Stage & difficulty

A Level Practice 2

Figure 1 shows the graph of $y = f(x)$, where

$$f(x) = 2 - x^2, \quad x \leq 0$$

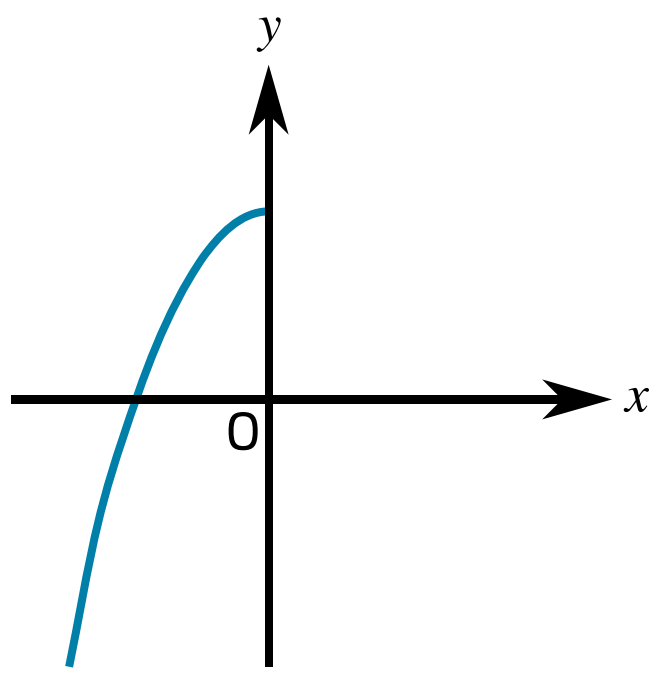


Figure 1: The graph of $y = f(x)$, for $x \leq 0$.

Part A

$f^2(-3)$

Evaluate $f^2(-3)$.

Part B

$f^{-1}(x)$

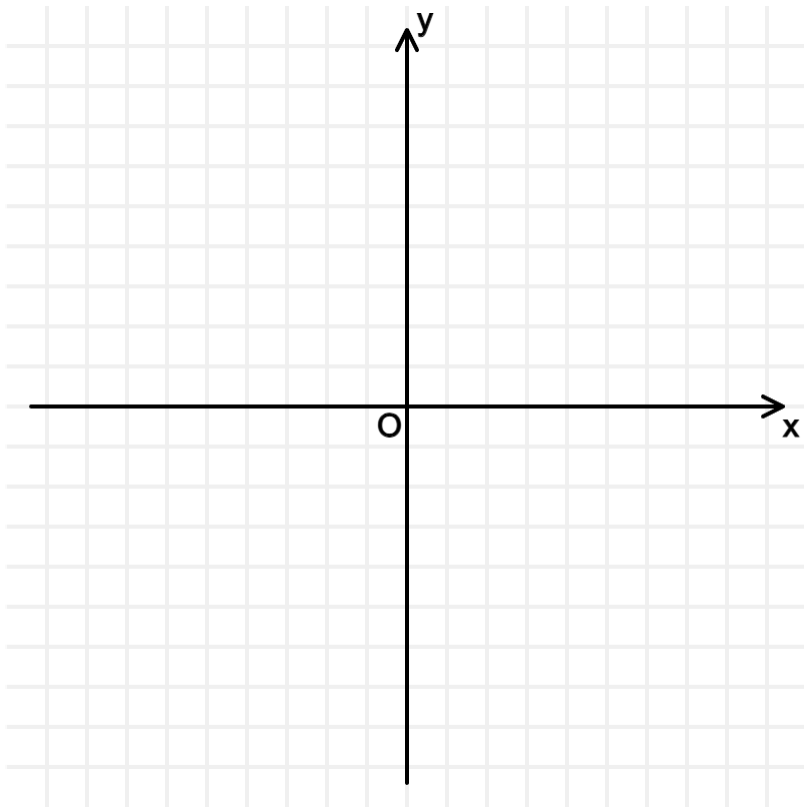
Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, x, y`

Part C

Graph of $f^{-1}(x)$

Sketch the graph of $y = f^{-1}(x)$.



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Question deck:

STEM SMART Single Maths 25 - Defining Functions

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STEM SMART Single Maths 25 - Defining Functions

Function Types and Inverses

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

A Level Challenge 1

Figure 1 shows five different graphs, A, B, C, D and E, each for values of x such that $-a \leq x \leq a$ where a is a constant.

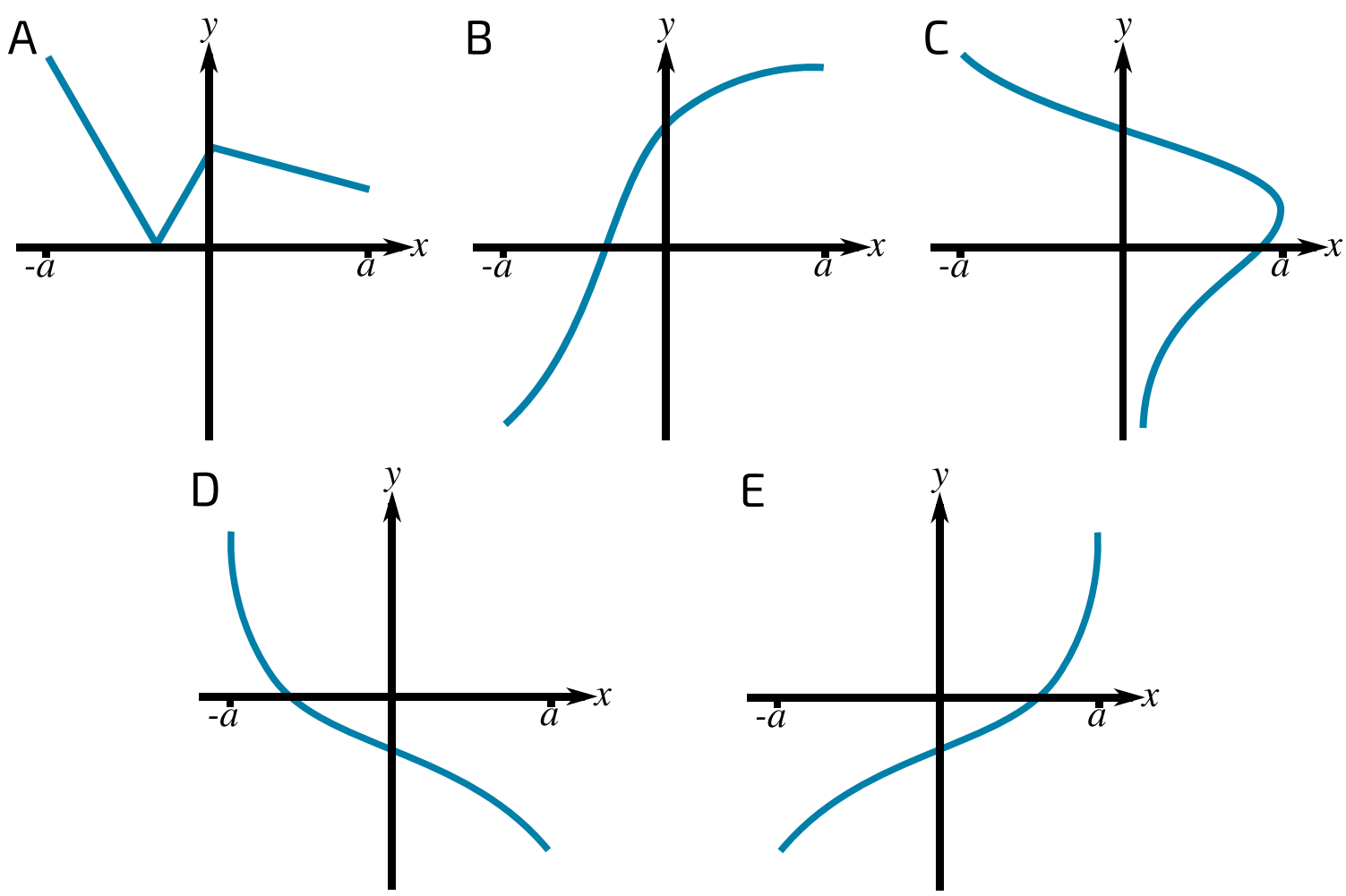


Figure 1: The set of five graphs, labelled A, B, C, D and E

Part A

Function

Which diagram does not show the graph of a function?

- A
- B
- C
- D
- E

Part B

One-to-one Function

Which diagram shows the graph of a function that is not one-to-one?

- A
- B
- C
- D
- E

Part C

Inverses

It is given that two of the diagrams illustrate functions that are inverses of each other. Identify one of these two diagrams.

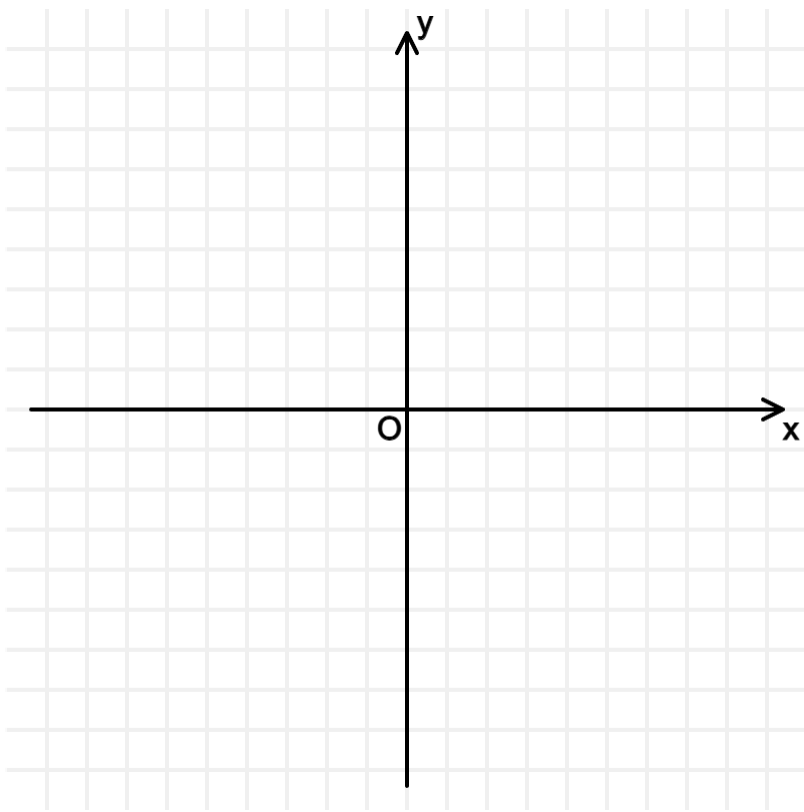
- A
- B
- C
- D
- E

Part D

Sketch

The graph in E has equation $y = f(x)$. Sketch the graph of $y = \left|f(x)\right|$.

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.



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STEM SMART Single Maths 25 - Defining Functions

Functions and Algebra 3i

Subject & topics

Maths

Status

Not started

Stage & difficulty

A Level Practice 2

The functions f and g are defined for all real values of x by

$$f(x) = \left|2x + a\right| + 3a \quad \text{and} \quad g(x) = 5x - 4a,$$

where a is a positive constant.

Part A

Range

Find the range of $f(x)$.

Fill in the inequality below.

$\square$ $\square$ $\square$

Items:

$f(x)$ $<$ $\leq$ $>$ $\geq$ $< f(x) <$ $\leq f(x) \leq$ $< f(x)$ **or** $f(x) <$ $\leq f(x)$ **or** $f(x) \leq$ $\frac{a}{3}$ $\frac{a}{2}$ a $2a$

$3a$ $4a$ 0 $-\frac{a}{3}$ $-\frac{a}{2}$ $-a$ $-2a$

Part B

Inverse function of $f(x)$

Fill in the blanks to explain why the function $f(x)$ has no inverse.

The function $f(x)$ is not [ ]. For example, $f(0) = 4a$ and $f($[ ]$)$ also equals $4a$. Hence, $f(x)$ has no inverse.

Items:

$-2a$ | one-to-one | many-to-many | many-to-one | $-a$ | a | one-to-many | $2a$

Part C

Inverse function of $g(x)$

Find an expression for $g^{-1}(x)$.

The following symbols may be useful: `a, x`

Part D

Solve for x

Solve for x the equation $g\big(f(x)\big) = 31a$. Give your solutions in ascending order.

$x =$ [ ] a

$x =$ [ ] a

Question deck:

STEM SMART Single Maths 25 - Defining Functions

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STEM SMART Single Maths 25 - Defining Functions

Modulus 3ii

Subject & topics

Maths

Status

Not started

Stage & difficulty

A Level Practice 1

Solve the inequality $\left|2x-5\right| > \left|x+1\right|$.

Construct your answer from the items below.

$\square$ $\square$ $\square$

Items:

x $<$ $\leq$ $>$ $\geq$ $< x <$ $\leq x \leq$ $> x$ **or** $x >$ $\geq x$ **or** $x \geq$ -6 -4 -2 $-\frac{3}{2}$ $-\frac{4}{3}$

-1 $-\frac{1}{2}$ 0 $\frac{1}{2}$ 1 $\frac{4}{3}$ $\frac{3}{2}$ 2 4 6

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STEM SMART Single Maths 25 - Defining Functions

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STEM SMART Single Maths 25 - Defining Functions

Modulus Functions 2

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

A Level Practice 2
Further A Practice 1

Part A

Divergence of $y = \left|\frac{1}{x}\right|$

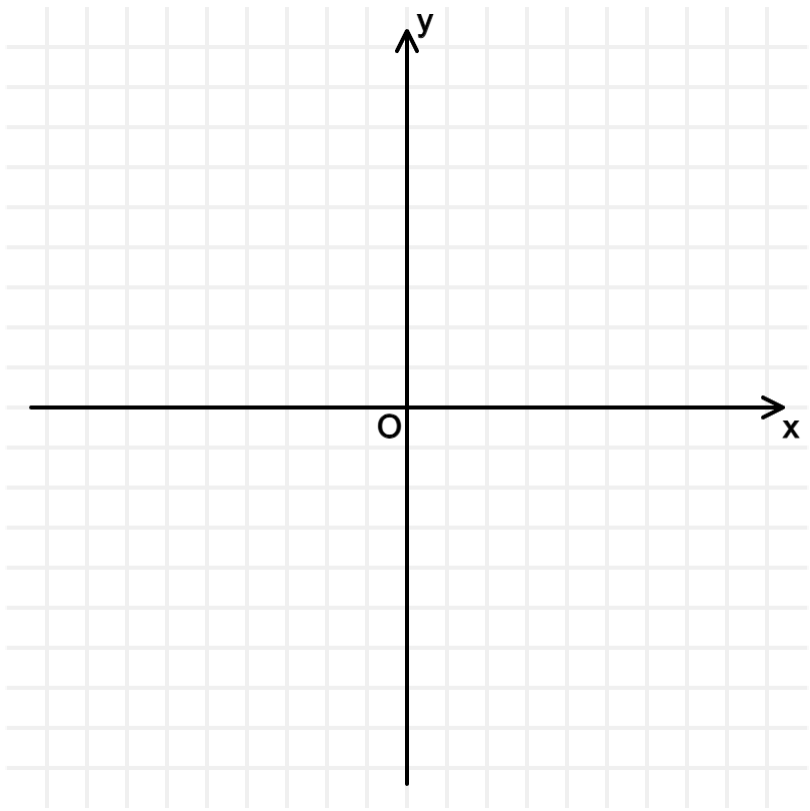
Does the function $y = \left|\frac{1}{x}\right|$ diverge anywhere? Where?

The following symbols may be useful: x

Part B

Graph of $y = \left|\frac{1}{x}\right|$

Sketch the graph of $y = \left|\frac{1}{x}\right|$.



Part C

Divergence of $y = \left|\frac{1}{x^2-4}\right|$

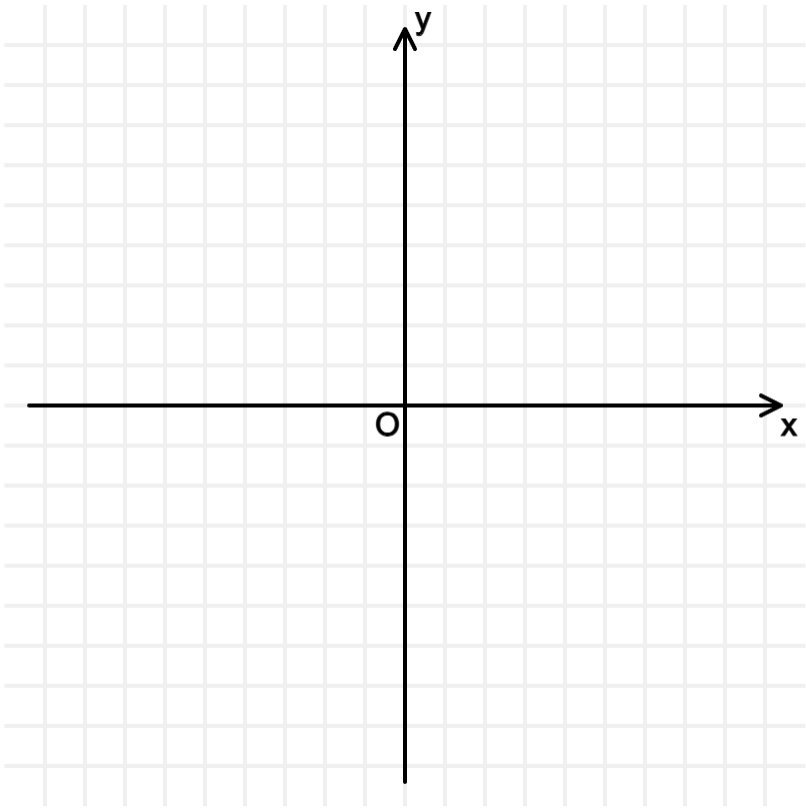
Does the function $y = \left|\frac{1}{x^2 - 4}\right|$ diverge anywhere? Where?

The following symbols may be useful: `x, ±`

Part D

Graph of $y = \left|\frac{1}{x^2-4}\right|$

Sketch the graph of $y = \left|\dfrac{1}{x^2 - 4}\right|$.



Part E

Solve equation graphically

Solve the equation $|x| = \left|\dfrac{1}{x}\right|$ graphically and give the solution as a single expression.

The following symbols may be useful: `x, ±`

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Question deck:

STEM SMART Single Maths 25 - Defining Functions

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STEM SMART Single Maths 25 - Defining Functions

Curve Sketching and Combined Transformations 3i

Subject & topics

Maths

Status

Not started

Stage & difficulty

A Level Practice 2

The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x)$$

where k is a positive constant. **Figure 1** shows the curve with equation $y = f(x)$.

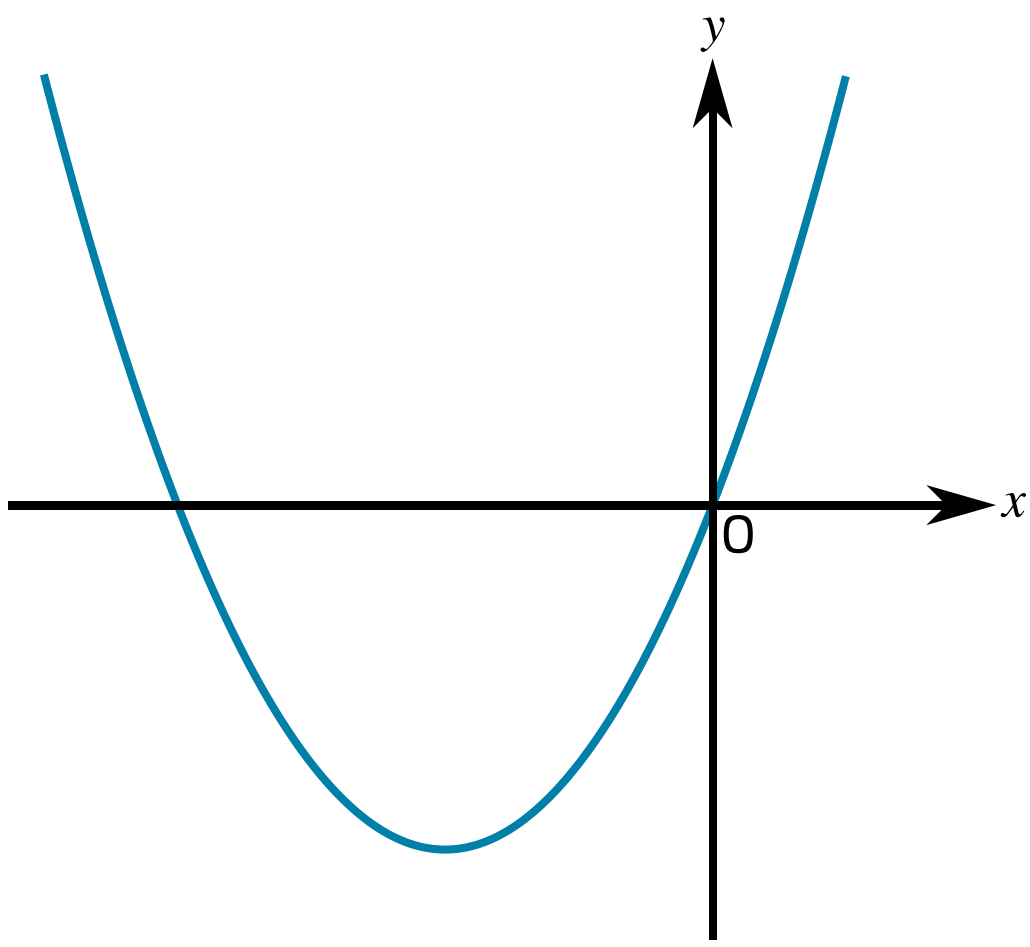

Figure 1: The graph of $y = f(x)$

Part A

Transformations

Give a sequence of transformations that will transform the curve $y = x^2$ to the curve $y = f(x)$.

Available items

- Translate the curve 2 units in the negative y direction.
- Stretch the curve in the x direction by a factor of k.
- Translate the curve 4 units in the positive y direction.
- Translate the curve 2 units in the positive y direction.
- Translate the curve 4 units in the negative y direction.
- Translate the curve 2 units in the positive x direction.
- Stretch the curve in the x direction by a factor of $\frac{1}{k}$.
- Translate the curve 4 units in the negative x direction.
- Translate the curve 2 units in the negative x direction.
- Stretch the curve in the y direction by a factor of $\frac{1}{k}$.
- Translate the curve 4 units in the positive x direction.
- Stretch the curve in the y direction by a factor of k.

Part B

Range

Find the range of $f(x)$ as a single inequality in terms of k.

The following symbols may be useful: `<, <=, >, >=, f(x), k, x, y`

Part C

Find k

It is given that there are three distinct values of x which satisfy the equation $\left|f(x)\right| = 20$.

Find the value of k.

Part D

Solve $\left|f(x)\right| = 20$

Using the value of k from part C, find the three distinct values of x which satisfy the equation $\left|f(x)\right| = 20$. Give any irrational values to 3 sf.

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( )
( )

Question deck:

STEM SMART Single Maths 25 - Defining Functions

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STEM SMART Single Maths 24 - Defining Functions

Combined Transformations

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

A Level Practice 2

The function f is defined by $f(x) = \sqrt{mx + 7} - 4$, where $x \geqslant -\frac{7}{m}$ and m is a positive constant. **Figure 1** shows the curve $y = f(x)$.

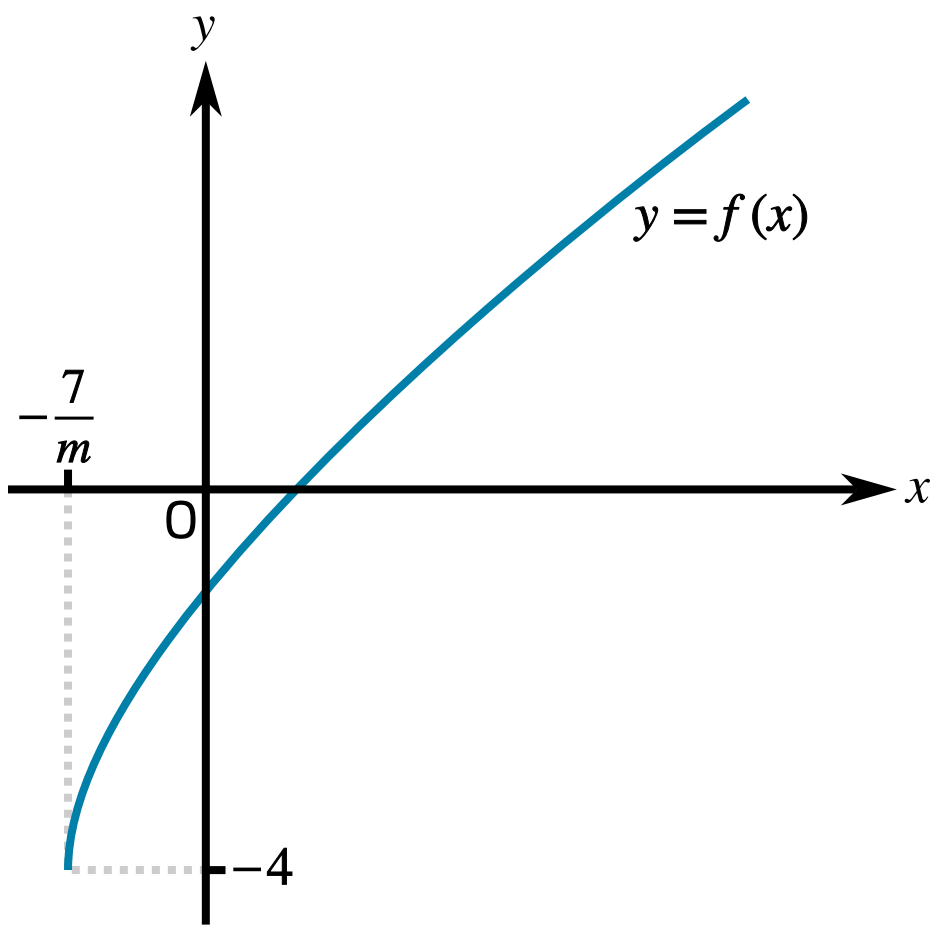

Figure 1: The curve $y = f(x)$

Part A

Translation of the curve $y = \sqrt{x}$

A sequence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations.

Available items

- Stretch the curve in the y direction by a factor of $\frac{1}{m}$.
- Translate the curve 7 units in the negative y direction.
- Stretch the curve in the x direction by a factor of m.
- Stretch the curve in the x direction by a factor of $\frac{1}{m}$.
- Translate the curve 4 units in the negative y direction.
- Translate the curve 4 units in the negative x direction.
- Translate the curve 7 units in the positive x direction.
- Translate the curve 7 units in the negative x direction.
- Translate the curve 4 units in the positive y direction.

Part B

$f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, m, x`

Part C

Values of m

It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Thus it can be deduced that neither curve meets the line $y = x$. Hence determine the possible values of m.

Construct your answer from the items below.

[ ] [ ] [ ]

Items:

m $<$ $\leq$ $>$ $\geq$ $< m <$ $\leq m \leq$ $> m$ **or** $m >$ $\geq m$ **or** $m \geq$ -28 -14 -8 -7

-4 -2 -1 0 1 2 4 7 8 14 28