Functions and Algebra 3i

A Level

The functions f and g are defined for all real values of x by

$$f(x) = |2x + a| + 3a \quad \text{and} \quad g(x) = 5x - 4a,$$

where a is a positive constant.

Part A Range

Find the range of $f(x)$.

Fill in the inequality below.

$\square$ $\square$ $\square$

Items:

$f(x)$ $<$ $\leq$ $>$ $\geq$ $< f(x) <$ $\leq f(x) \leq$ $< f(x)$ **or** $f(x) <$ $\leq f(x)$ **or** $f(x) \leq$ $\frac{a}{3}$ $\frac{a}{2}$ a

$2a$ $3a$ $4a$ 0 $-\frac{a}{3}$ $-\frac{a}{2}$ $-a$ $-2a$

Part B Inverse function of $f(x)$

Fill in the blanks to explain why the function $f(x)$ has no inverse.

The function $f(x)$ is not $\square$. For example, $f(0) = 4a$ and $f(\square)$ also equals $4a$. Hence, $f(x)$ has no inverse.

Items:

many-to-many **many-to-one** $-2a$ $-a$ $2a$ a **one-to-one** **one-to-many**

Part C Inverse function of $g(x)$

Find an expression for $g^{-1}(x)$.

The following symbols may be useful: `a, x`

Part D Solve for x

Solve for x the equation $g(f(x)) = 31a$.

Give the value of x furthest from 0.

The following symbols may be useful: `a, x`

Give the value of x that is closest to 0.

The following symbols may be useful: `a, x`

Inverse Quadratic Function

A Level

Figure 1 shows the graph of $y = f(x)$, where

$$f(x) = 2 - x^2, \quad x \leq 0$$

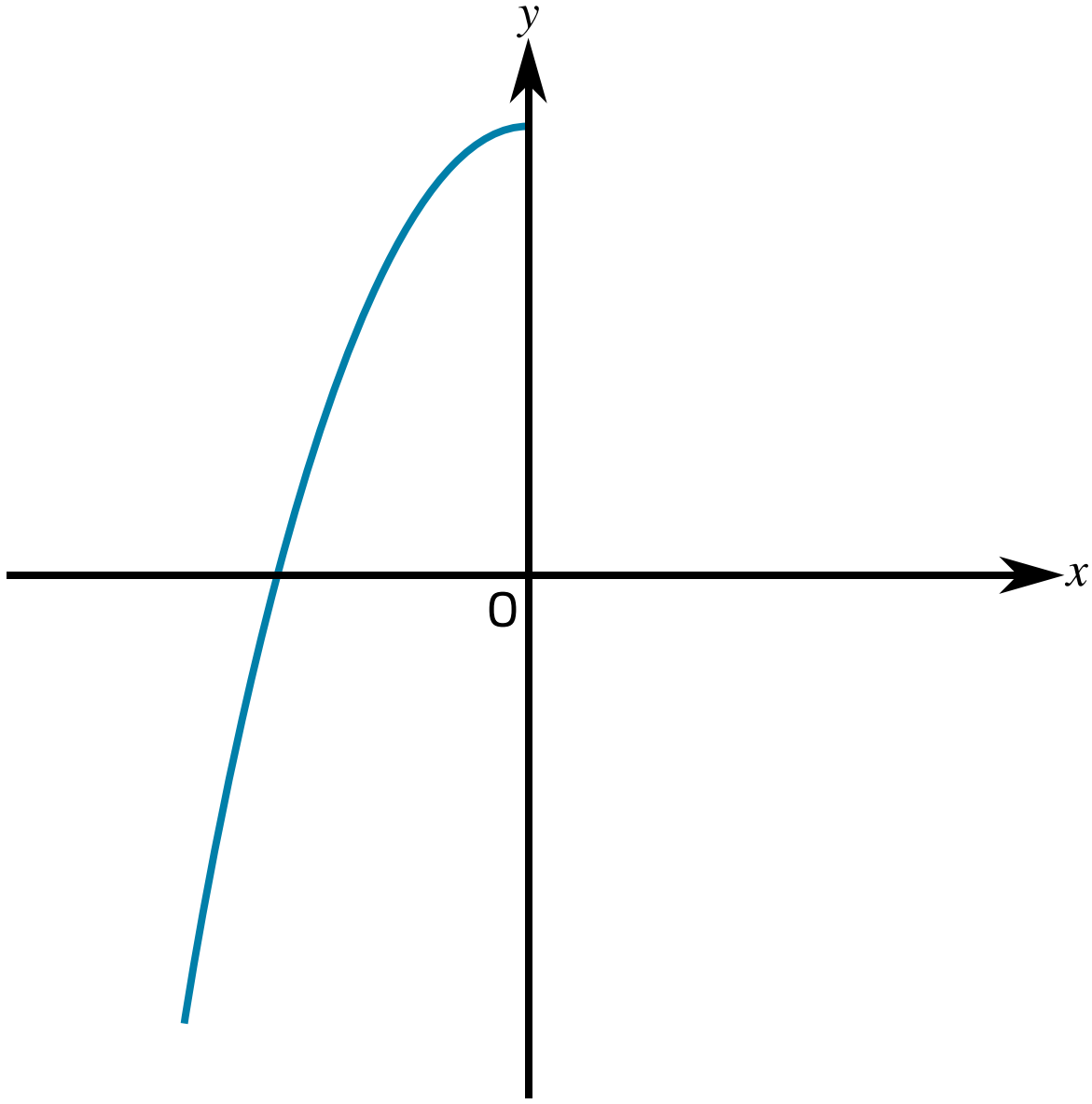


Figure 1: The graph of $y = f(x)$, for $x \leq 0$.

Part A $f^2(-3)$

Evaluate $f^2(-3)$.

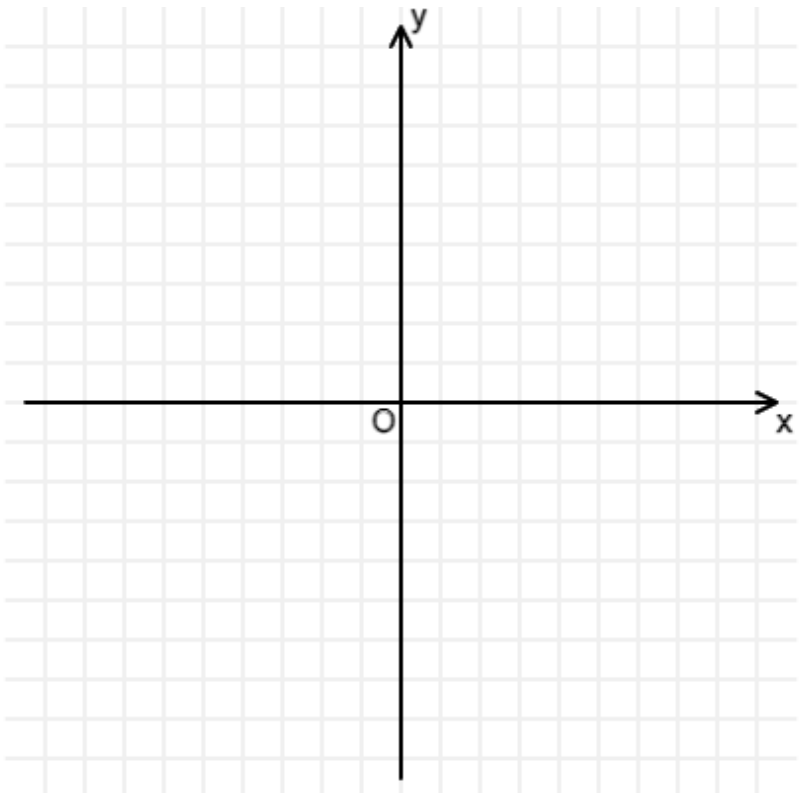
Part B $f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, x, y`

Part C **Graph of $f^{-1}(x)$**

Sketch the graph of $y = f^{-1}(x)$.



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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

Function Types and Inverses

A Level

C C C

Figure 1 shows five different graphs, A, B, C, D and E, each for values of x such that $-a \leq x \leq a$ where a is a constant.

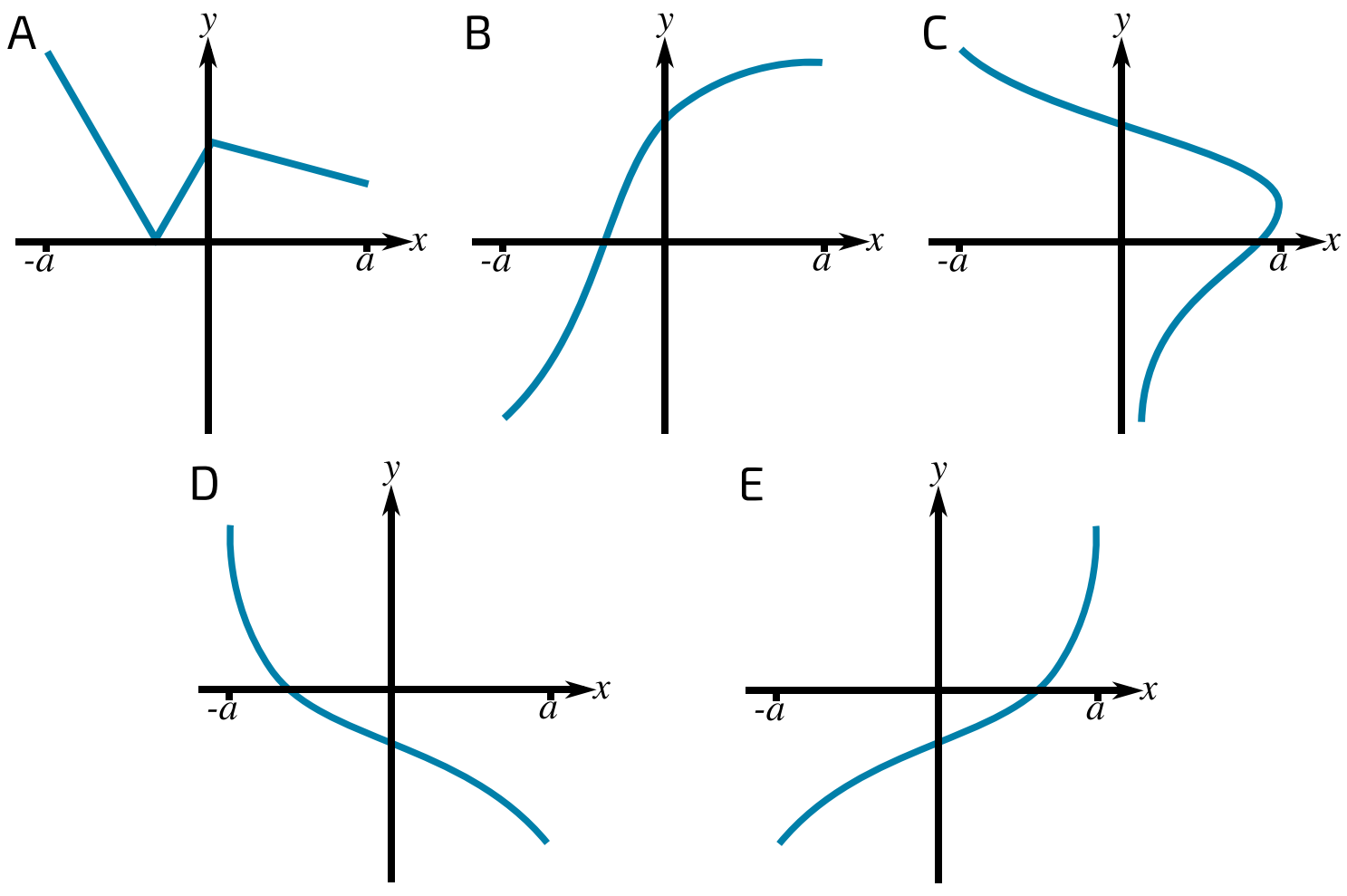


Figure 1: The set of five graphs, labelled A, B, C, D and E

Part A Function

Which diagram does not show the graph of a function?

- A
- B
- C
- D
- E

Part B One-to-one Function

Which diagram shows the graph of a function that is not one-to-one?

- A
- B
- C
- D
- E

Part C Inverses

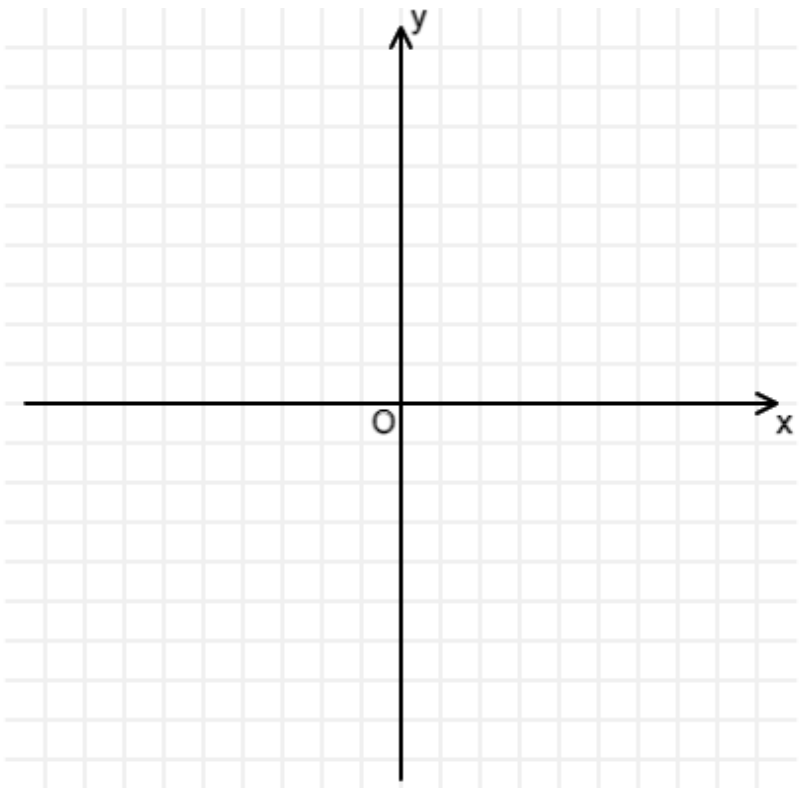
It is given that two of the diagrams illustrate functions that are inverses of each other. Identify one of these two diagrams.

- A
- B
- C
- D
- E

Part D Sketch

The graph in E has equation $y = f(x)$. Sketch the graph of $y = \left|f(x)\right|$.

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.



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Combined Transformations

A Level

The function f is defined by $f(x) = \sqrt{mx + 7} - 4$, where $x \geqslant -\frac{7}{m}$ and m is a positive constant. **Figure 1** shows the curve $y = f(x)$.

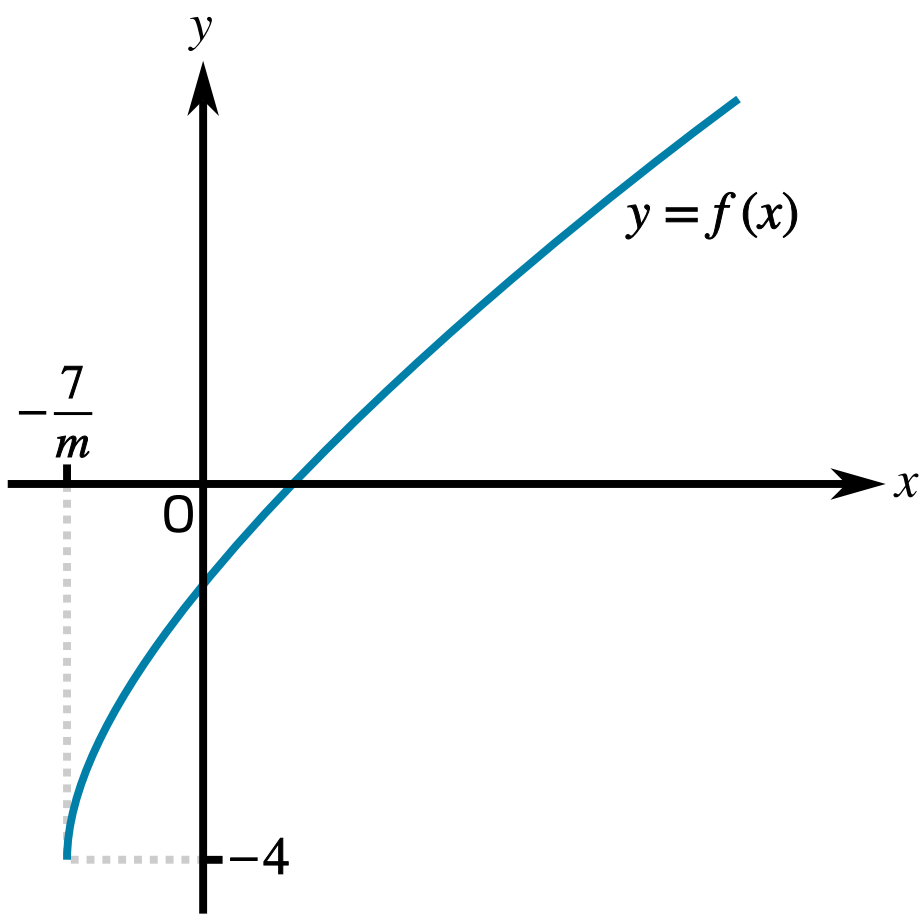

Figure 1: The curve $y = f(x)$

Part A Translation of the curve $y = \sqrt{x}$

A sequence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations.

Available items

- Translate the curve 4 units in the positive y direction.
- Translate the curve 4 units in the negative y direction.
- Stretch the curve in the x direction by a factor of $\frac{1}{m}$.
- Translate the curve 7 units in the positive x direction.
- Stretch the curve in the y direction by a factor of $\frac{1}{m}$.
- Stretch the curve in the x direction by a factor of m.
- Translate the curve 7 units in the negative y direction.
- Translate the curve 4 units in the negative x direction.
- Translate the curve 7 units in the negative x direction.

Part B $f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: `f, m, x`

Part C Values of m

It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Thus it can be deduced that neither curve meets the line $y = x$. Hence determine the set of possible values of m, and give the upper bound in the form $m < a$ or $m \leq a$.

The following symbols may be useful: `<, <=, >, >=, m`

Give the lower bound in the form $m > a$ or $m \geq a$.

The following symbols may be useful: `<, <=, >, >=, m`

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Curve Sketching and Combined Transformations 3i

A Level

The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x)$$

where k is a positive constant. **Figure 1** shows the curve with equation $y = f(x)$.

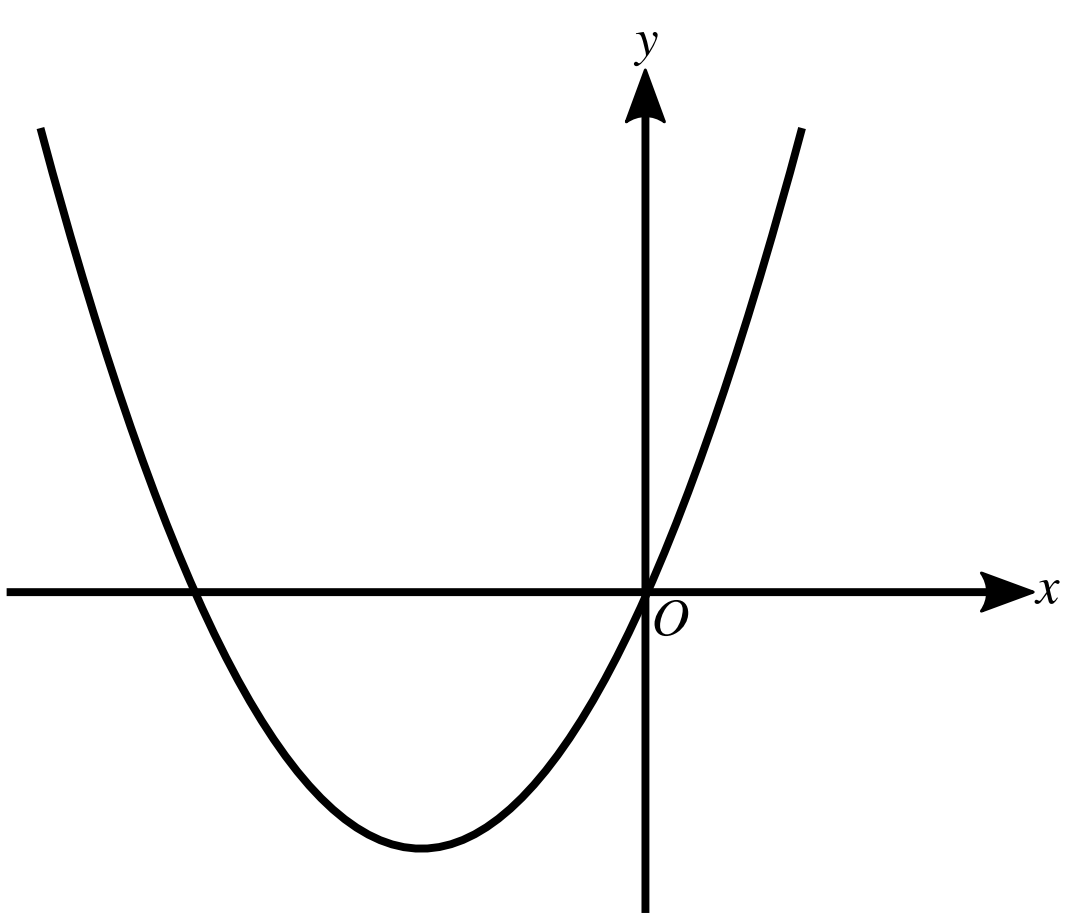

Figure 1: The graph of $y = f(x)$

Part A Transformations

The curve $y = x^2$ can be transformed to the curve $y = f(x)$ by the following sequence of transformations

$$\text{a translation parallel to the } x\text{-axis,}$$

$$\text{a translation parallel to the } y\text{-axis,}$$

$$\text{a stretch.}$$

Give details, in terms of k where appropriate, of these transformations.

Give the number of units that the curve is translated in the x direction, assuming right to be positive.

The following symbols may be useful: k

Give the number of units that the curve is translated in the y direction, assuming up to be positive.

The following symbols may be useful: k

Give the stretch factor of the transformation.

The following symbols may be useful: k

Part B Range

Find the range of $f(x)$ as a single inequality in terms of k.

The following symbols may be useful: `<, <=, >, >=, f(x), k, x, y`

Part C $\left|f(x)\right| = 20$

It is given that there are three distinct values of x which satisfy the equation $\left|f(x)\right| = 20$. Find the value of k and determine exactly the three values of x which satisfy the equation in this case.

State the value of k.

The following symbols may be useful: k

Give the rational value of x which satisfies this equation.

The following symbols may be useful: x

Give one of the irrational solutions for x in its simplest exact form.

The following symbols may be useful: x

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Modulus Functions 2

A Level Further A

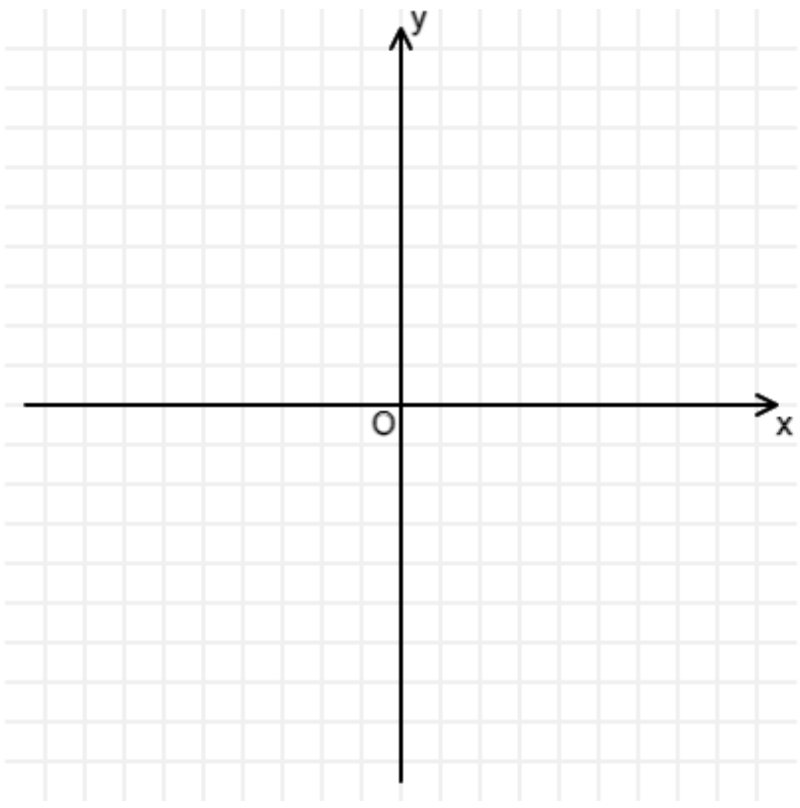
Part A Divergence of $y = |\frac{1}{x}|$

Does the function $y = \left|\frac{1}{x}\right|$ diverge anywhere? Where?

The following symbols may be useful: x

Part B Graph of $y = |\frac{1}{x}|$

Sketch the graph of $y = \left|\frac{1}{x}\right|$.



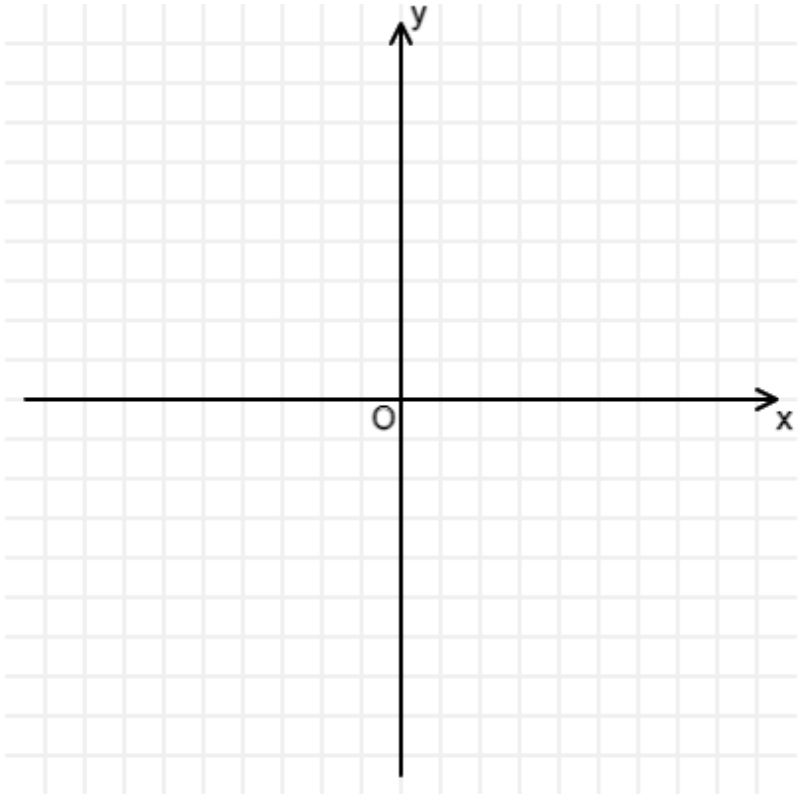
Part C Divergence of $y = \left|\frac{1}{x^2-4}\right|$

Does the function $y = \left|\dfrac{1}{x^2-4}\right|$ diverge anywhere? Where?

The following symbols may be useful: `x, ±`

Part D Graph of $y = \left|\frac{1}{x^2-4}\right|$

Sketch the graph of $y = \left|\dfrac{1}{x^2-4}\right|$.



Part E Solve equation graphically

Solve the equation $|x| = \left|\dfrac{1}{x}\right|$ graphically and give the solution as a single expression.

The following symbols may be useful: `x, ±`

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Sketching a Cubic Modulus Function

A Level

C C C

The function f is defined by $f(x) = 2x^3 - x^2 - 4x - 4$.

Part A Quotient

Find the quotient when $f(x)$ is divided by $x - 2$.

The following symbols may be useful: x

Part B First stationary point of $f(x)$

Find the co-ordinates and nature of the stationary point of $f(x)$ with the larger x-coordinate.

The stationary point (,) is a .

Items:

-7 -2 -1 -3 **minimum point** -132 3 -4 -55 29 1 0 **maximum point**

point of inflection 2 92 -16 4

Part C Second stationary point of $f(x)$

Find the co-ordinates and nature of the stationary point of $f(x)$ with the smaller x-coordinate.

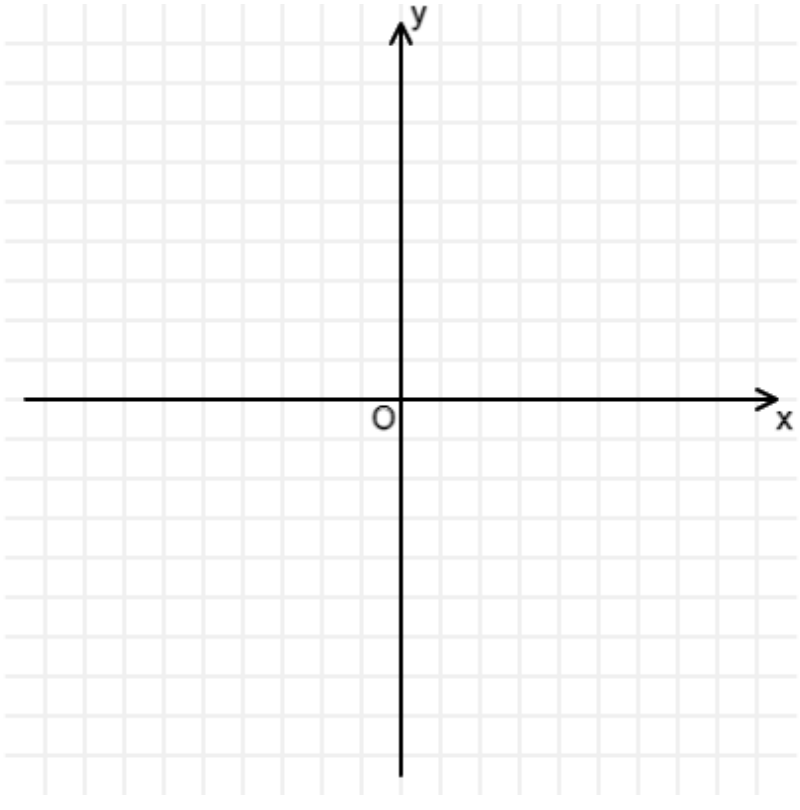
The stationary point ([ ] , [ ]) is a [ ] .

Items:

$-\frac{145}{27}$	$-\frac{5}{2}$	**point of inflection**	-3	$-\frac{11}{2}$	$-\frac{1}{2}$	$\frac{2}{3}$	-6	$-\frac{176}{27}$	**minimum point**	$\frac{1}{2}$	$-\frac{77}{27}$	
$-\frac{1}{3}$	-7	**maximum point**	1	-7	$-\frac{3}{2}$	-1	$-\frac{64}{27}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{2}$		

Part D Sketch of $y = f(x)$

Sketch the graph of $y = f(x)$.



Part E Sketch of $y = |f(x)|$

Sketch the graph of $y = |f(x)|$, then pick the graph that corresponds to $y = |f(x)|$ from the options below.

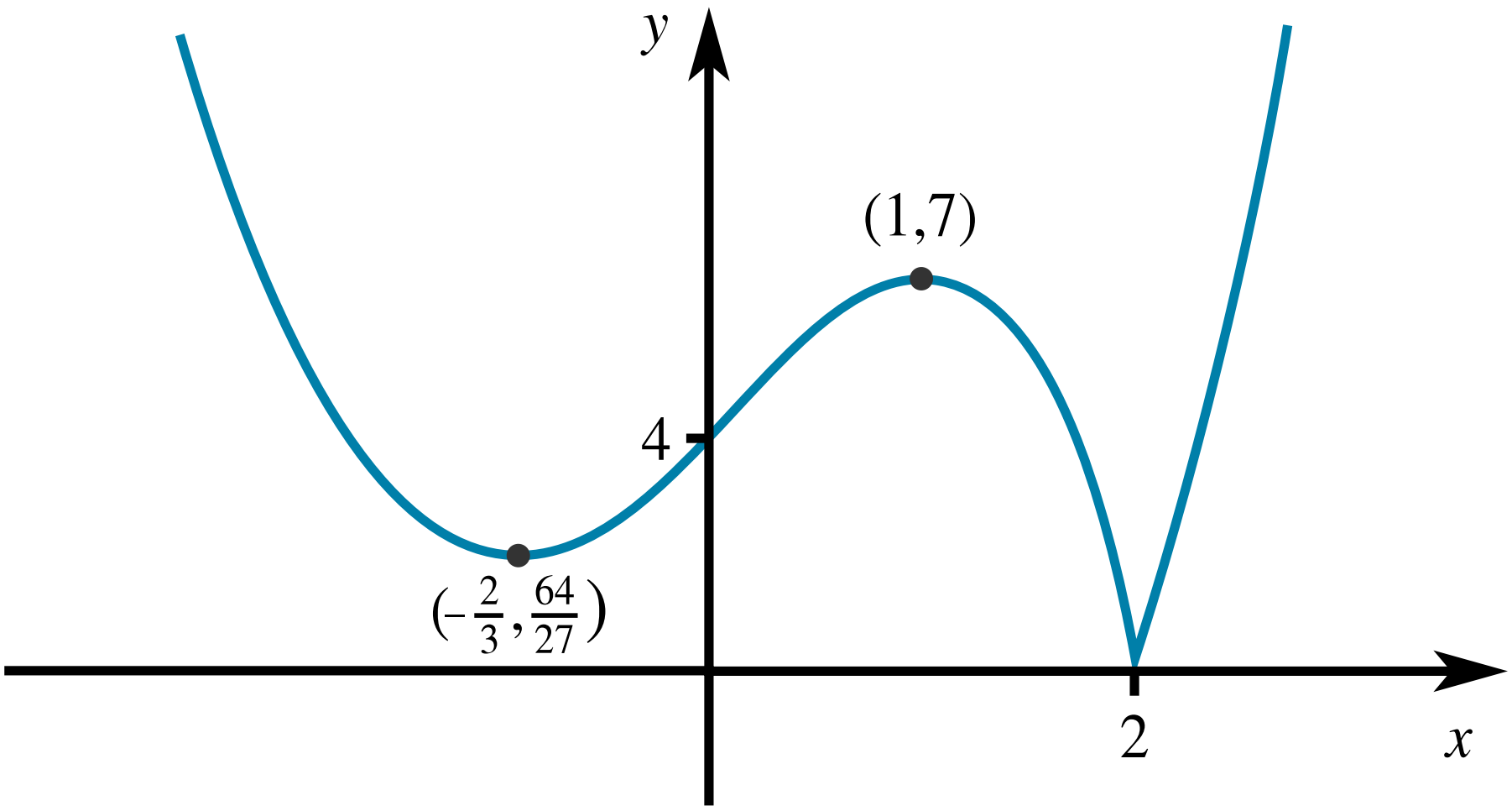


Figure 1: Option (i)

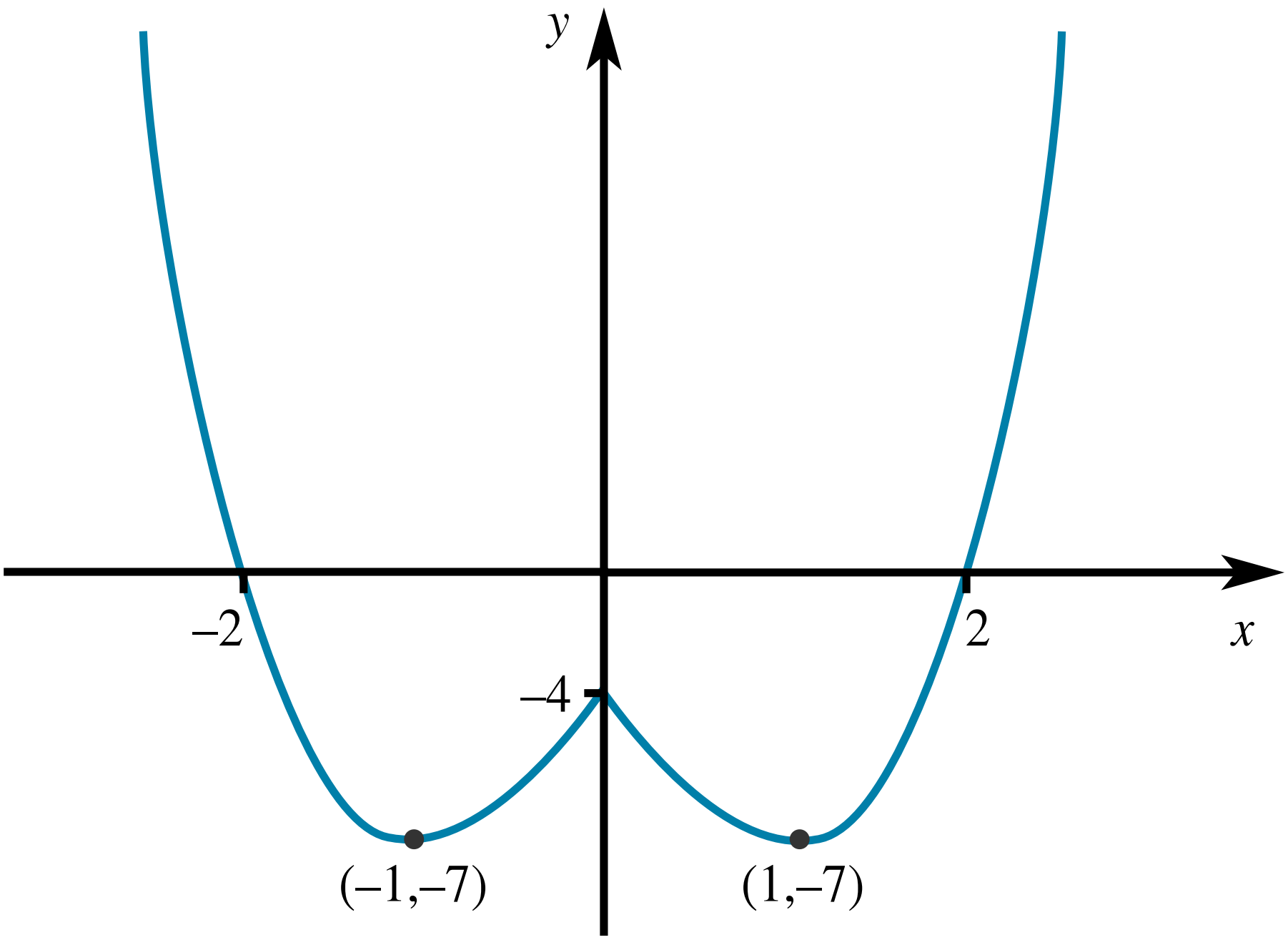


Figure 2: Option (ii)

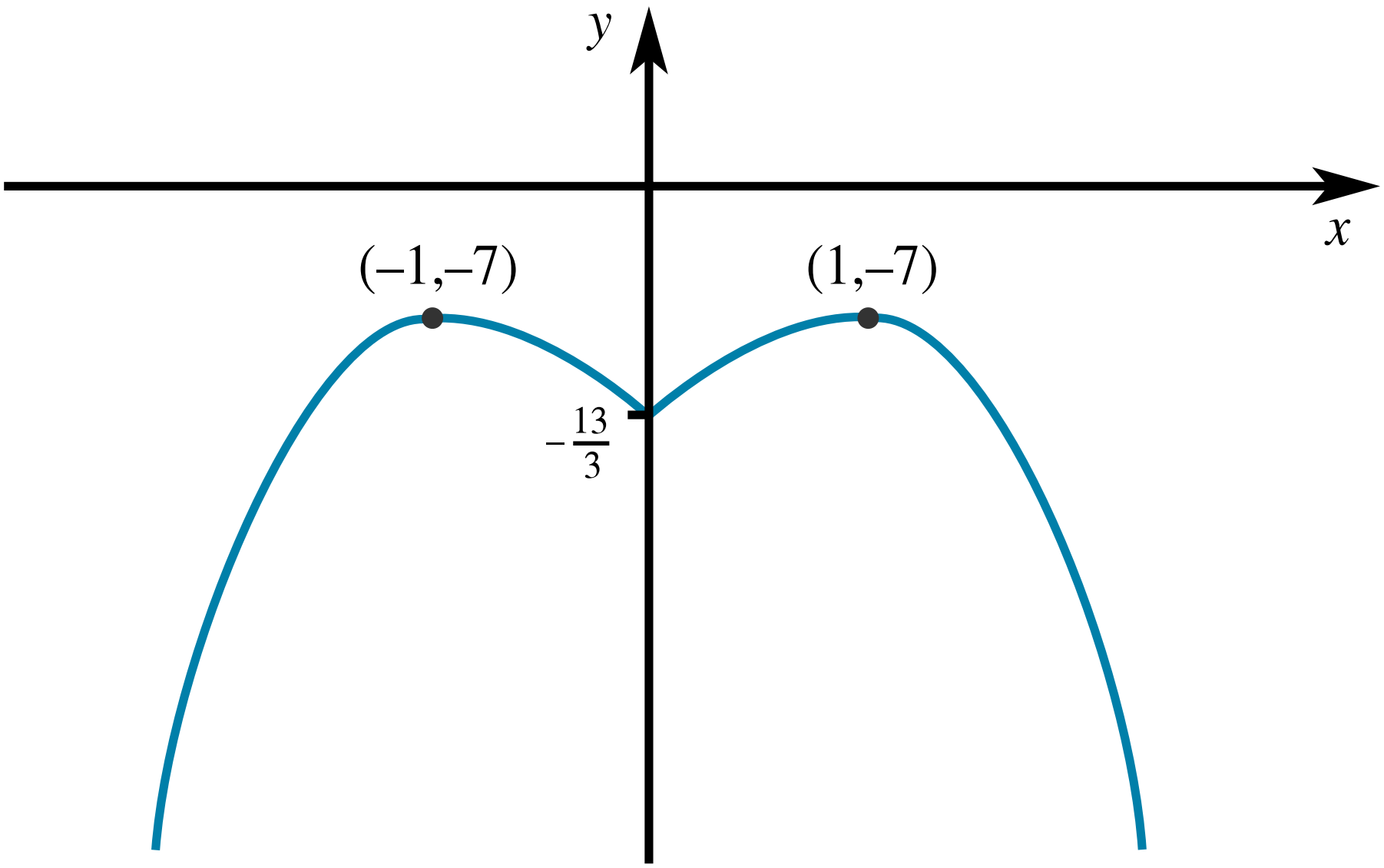


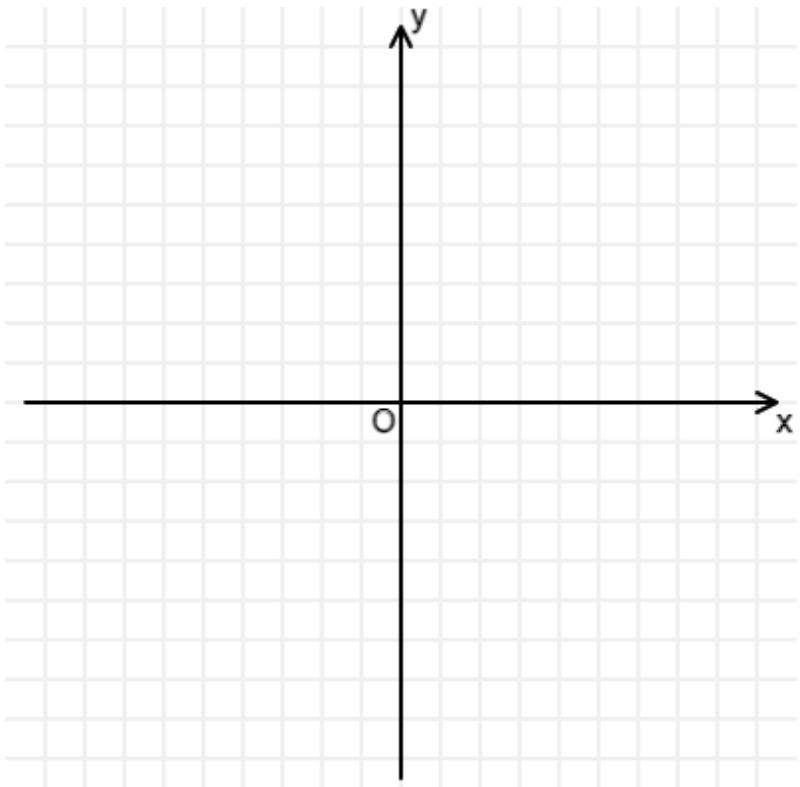
Figure 3: Option (iii)

- Option (i)
- Option (ii)
- Option (iii)

Part F Sketch of $y = f(|x|)$

Sketch the graph of $y = f(|x|)$.

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.



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Rational Inequality

Further A

Solve the inequality

$$\frac{x+4}{x+2} \leq \frac{x+2}{x-1}$$

giving your answer using set notation.

The solution is $\{x :$ [ ] [ ] x [ ] [ ] $\} \cup \{x : x$ [ ] [ ] $\}$.

Items:

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

$<$ $>$ $\leq$ $\geq$ $=$

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Rational Modulus Inequality

Further A

Solve the inequality

$$\frac{-x^2 - 5x + 24}{|x| + 3} > 2$$

giving your answer using set notation.

The solution is $\{x :$ [ ] [ ] x [ ] [ ] $\}$.

Items:

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

$<$ $>$ $\leq$ $\geq$ $=$

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