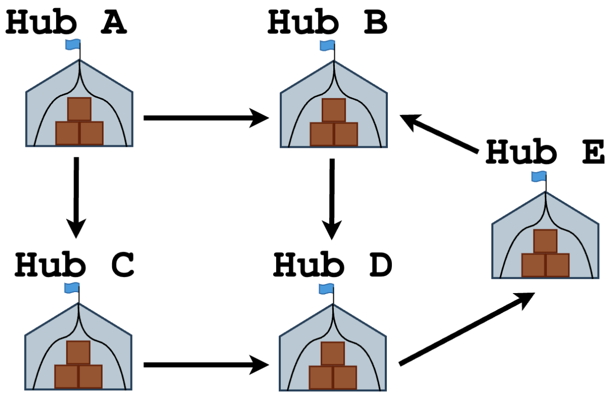
Week 26 Extension Question: Drones and Distribution Hubs

A delivery company operates drones between several distribution hubs in a city. The hubs are connected via **one-way** air corridors, and the company needs to ensure that a drone starting from a specific hub can deliver to all hubs without needing to land or recharge.

The map of the air corridors is as follows:

- Hub A → Hub B
- Hub A → Hub C
- Hub B → Hub D
- Hub C → Hub D
- Hub D → Hub E
- Hub E → Hub B

Each hub is a **vertex**, and each air corridor is a **directed edge** from one hub to another. (You may find it helpful to draw this graph)



<u>Questions</u>

(a) From which single hub(s) can a drone be launched such that it can eventually reach all other hubs, following only the allowed one-way corridors?

Answer: A

(b) Which edge is not necessary for the hub you chose in (a) to be able to reach every other node?

Answer: B -> D

(c) A **strongly connected** graph is one where every node is reachable from each node. Add a single air-corridor to make this delivery network strongly connected.

Answer: Add an edge from any other node to A

(d) Now consider a much larger network with thousands of hubs and even more edges. Some computer scientists are discussing pure reachability without any consideration for efficiency. Is there any way to abstract loops in this setup?

Suggested answer:

Any node in a loop can be reached from any other node in that same cycle. So we can abstract the entire loop up to a single component such that any edges to nodes in the loop now go to the component, and any edge that stems from a node in the loop now stems from the component. The loop is a <u>strongly connected component</u>: any node is reachable from any other node in the component. This allows us to reduce the number of nodes in a graph when testing for reachability.