Week 28 Extension Question: Hash Tables

A hash table of size 10 stores student IDs.
The hash function is

$$H(x) = (sum\ of\ digits)\ mod\ 10.$$

When multiple student IDs hash to the same slot, **linear probing** takes place and the ID is stored in the next available slot.

The IDs in the table are: 2145, 3509, 1050, 6948, 6600, and one unknown number Y.

When Y is inserted, it collides with another number and is placed in another slot after three linear probes of +1 each.

Find all possible values for Y in terms of their digit-sum value, and give an example.

Answer:

Step 1: Hash the given IDs

Hash function: **H(x) = (sum of digits) mod 10**

- 2145 → 2+1+4+5 = 12 → **H=2**
- 3509 → 3+5+0+9 = 17 → **H=7**
- 1050 → 1+0+5+0 = 6 → **H=6**
- 6948 → 6+9+4+8 = 27 → **H=7** (collides with 3509)
- 6600 → 6+6+0+0 = 12 → **H=2** (collides with 2145)

Step 2: Insert with linear probing (+1)

- Put **2145** at slot **2**.
- Put **3509** at slot **7**.
- Put **1050** at slot **6**.
- **6948** hashes to 7 (occupied), probe to **8** → empty → put at **8**.
- **6600** hashes to 2 (occupied), probe to **3** → empty → put at **3**.

Current table (before Y):

Slot	0	1	2	3	4	5	6	7	8	9
Val			2145	6600			1050	3509	6948	

Step 3: Use the "three probes" clue for Y

"Placed after **three** linear probes of +1" means:

- Start at **H(Y)** (occupied) → probe to **H(Y)+1** (occupied) → probe to **H(Y)+2** (occupied) → insert at **H(Y)+3** (empty).

Looking at the table, the only run of **three consecutive occupied slots** is **6, 7, 8**, and the next slot **9** is empty.
 Therefore **H(Y) must be 6**.

So **the sum of Y's digits ≡ 6 (mod 10)**.
 Examples of valid Y (digit-sum = 6, 16, 26, …): **3300 (3+3+0+0=6)**, **1203 (1+2+0+3=6)**, **6000 (6+0+0+0=6)**, **9999 (9+9+9+9=36 → 36 mod 10 = 6)**.

Step 4: Final table (after inserting Y)

Y hashes to 6 → probe 7 (busy) → probe 8 (busy) → insert at **9**.

Slot	0	1	2	3	4	5	6	7	8	9
Val			2145	6600			1050	3509	6948	Y

Answer (concise): All possible Y have a digit-sum congruent to **6 mod 10** (e.g., **3300**).