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STEM SMART Single Maths 34 - Integrating Rational Functions

Integrals of Standard Functions 2

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** A Level P3

Complete the table of integrals for these standard functions.

In these functions a and k are constants.

Function, $y(x)$	**Integral,** $\int y(x)\mathrm{d}x$
$ax^n, n \neq -1$	
$a\sin kx$	
$a\cos kx$	
$a\sec^2 kx$	
$a\mathrm{e}^{kx}$	
$\dfrac{a}{x}$	

Items:

$-\frac{a}{x^2}+c$ $\frac{a}{k}\tan kx+c$ $\frac{a}{n+1}x^{n+1}+c$ $\frac{a}{k}\cos kx+c$ $-\frac{a}{k}\cos kx+c$ $\frac{a}{k}\mathrm{e}^{kx}+c$ $-\frac{a}{k}\sin kx+c$

$\frac{a}{k}\sin x+c$ $a\ln|x|+c$ $a\ln x+c$

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Integrating Rational Functions

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** A Level P3

Fill in the blanks to complete the notes on integrating rational functions.

In these functions a, p and q are constants.

A rational function is a function of the form $\frac{u(x)}{v(x)}$, where $u(x)$ and $v(x)$ are [ ]. To integrate a rational function, we start by splitting the function into [ ] and then use the integration rules below.

Function, $y(x)$	**Integral,** $\int y(x)\mathrm{d}x$
$\frac{a}{x}$	[ ]
$\frac{a}{px+q}$	[ ]
$\frac{a}{(bx+c)^2}$	[ ]
$a\frac{f'(x)}{f(x)}$	[ ]

Note that the last of these rules applies to [ ] function $f(x)$.

Items:

any | partial fractions | polynomials | $a\ln\lvert x\rvert + c$ | $\frac{a}{p}\ln\lvert px+q\rvert + c$ | $-\frac{a}{p(px+q)} + c$ | $a\ln\lvert f(x)\rvert + c$

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Partial Fractions 2ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

Part A
Partial Fractions

Express $\dfrac{x-1}{x(x+1)}$ in partial fractions.

The following symbols may be useful: x

Part B
Integral

Hence find the exact value of $\displaystyle\int_1^2 \frac{x-1}{x(x+1)}\mathrm{d}x$.

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Partial Fractions 1i

Subject & topics: Maths **Stage & difficulty:** A Level P2

Part A
Partial Fractions

Express $\frac{2+x^2}{(1+2x)(1-x)^2}$ in the form $\frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$.

The following symbols may be useful: x

Part B
Integration

Hence find $\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2}\,\mathrm{d}x$ in exact form.

The following symbols may be useful: `ln(), log()`

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Partial Fractions 4ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

Part A
Partial Fractions

Express $\frac{7-2x}{(x-2)^2}$ in the form $\frac{A}{x-2} + \frac{B}{(x-2)^2}$, where A, and B are constants.

The following symbols may be useful: x

Part B
Integral

Hence find the exact value of $\int_4^5 \frac{7-2x}{(x-2)^2}\,\mathrm{d}x$.

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STEM SMART Single Maths 34 - Integrating Rational Functions

Integration by Substitution 4

Pre-Uni Maths for Sciences K3.4

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** A Level P3

Part A

Integrate $\frac{1}{b(x+a)}$

Find $\int_0^a \frac{1}{b(x+a)}\,\mathrm{d}x$, where a and b are constants.

The following symbols may be useful: `a, b, k, x`

Part B

Integrate $\frac{x}{1+x^2}$

Find $\int_0^1 \frac{x}{1+x^2}\,\mathrm{d}x$.

The following symbols may be useful: `k, x`

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Integration by Substitution 5

Pre-Uni Maths for Sciences K3.5

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** A Level P3

Part A

Integrate $\frac{x^3}{a^5+ax^4}$

Find $\int_a^{2a} \frac{x^3}{a^5+ax^4}\,\mathrm{d}x$, where a is a constant.

The following symbols may be useful: a

Part B

Integrate $\tan\beta$

By writing $\tan\beta = \frac{\sin\beta}{\cos\beta}$, find $\int_0^{\frac{\pi}{4}} \tan\beta\,\mathrm{d}\beta$.

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Algebraic Division 1ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

Part A

Quotient and Remainder

Find the quotient when $3x^3 - x^2 + 10x - 3$ is divided by $x^2 + 3$.

The following symbols may be useful: x

Give the remainder.

The following symbols may be useful: x

Part B

Integral

Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3}\,\mathrm{d}x.$$

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STEM SMART Single Maths 34 - Integrating Rational Functions

Integration with Partial Fractions 4

Pre-Uni Maths for Sciences K5.4

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** Further A P2

Part A

Find A, B and C

Write the function $\dfrac{2z^2 - z - 3}{(z+2)(z^2 - 2z - 1)}$ in the form $\dfrac{A}{z+2} + \dfrac{B + Cz}{z^2 - 2z - 1}$.

Drag and drop the correct values in the expression below.

$$\frac{\square}{z+2} + \frac{\square + \square z}{z^2 - 2z - 1}$$

Items:

-5 -4 -3 -2 -1 0 1 2 3 4 5

Part B

Integrate

Hence find $\displaystyle\int_1^2 \frac{2z^2 - z - 3}{(z+2)(z^2 - 2z - 1)}\,\mathrm{d}z$.

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