Arithmetic Series 3i

A Level

A sequence u_1, u_2, u_3, ... is defined by

$$u_1 = 8 \quad \text{and} \quad u_{n+1} = u_n + 3.$$

Part A u_5

Find u_5.

The following symbols may be useful: `u_5`

Part B Terms in the sequence

The n^{th} term of the sequence can be written in the form $u_n = pn + q$. State the values of p and q.

$p =$ ☐

$q =$ ☐

Part C Type of sequence

What type of sequence is it?

- Arithmetic progression
- Geometric progression
- Periodic sequence

Part D Value of N

Find the value of N such that $\sum_{n=1}^{2N} u_n - \sum_{n=1}^{N} u_n = 1256$.

The following symbols may be useful: `N`

Geometric Series 5i

A Level

P P P

Part A Value of k

The first term of a geometric progression is 50 and the common ratio is 0.8. Use logarithms to find the smallest value of k such that the k^{th} term is less than 0.15.

The following symbols may be useful: k

Part B Common ratio and first term

In a different geometric progression, the second term is -3 and the sum to infinity is 4. Find the common ratio. Hence, find the first term.

Enter your answers below. If a value is not a whole number, enter the value as a decimal.

common ratio $=$ ☐

first term $=$ ☐

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Binomial: All Rational n 1i

A Level

Part A Partial Fractions

Given that $\dfrac{3x+4}{(1+x)(2+x)^2} \equiv \dfrac{A}{1+x} + \dfrac{B}{2+x} + \dfrac{C}{(2+x)^2}$, find A, B, and C.

If a value is not a whole number, enter the value as a decimal.

$A =$ ☐

$B =$ ☐

$C =$ ☐

Part B Expand

Hence or otherwise expand $\dfrac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2.

The following symbols may be useful: x

Part C Values of x

State the set of values of x for which the expansion in the above part is valid.

Construct your answer from the items below.

Items:

x $<$ $\leq$ $>$ $\geq$ $< x <$ $\leq x \leq$ $> x$ **or** $x >$ $\geq x$ **or** $x \geq$ -4 -2 $-\frac{3}{2}$ -1 $-\frac{1}{2}$ 0 $\frac{1}{2}$ 1 $\frac{3}{2}$ 2 4

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Circles 4i

A Level

A circle with centre C has the equation $x^2 + y^2 - 10x + 4y + 4 = 0$.

Part A Find the coordinates of C

By completing the square for x and y find the coordinates of C.

Enter the x and y coordinates below. If a value is not a whole number, enter the value as a decimal.

(____ , ____)

Part B Find radius

Find the radius of the circle.

Part C Find tangent

Find the equation of the tangent to the circle at the point P$(8, 2)$. Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers.

The following symbols may be useful: `x, y`

Part D Find area

The circle meets the y axis at Q and the tangent to the circle at P (as in part C) meets the y axis at R. Find the area of triangle PQR.

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Modulus 1i

A Level

Part A Transformations

Which two of the transformations described below are needed to transform the graph of $y = |x|$ to the graph of $y = |2(x + 3)|$?

- [ ] A translation by 3 units in the negative x direction.
- [ ] A stretch parallel to the y-axis with scale factor 2.
- [ ] A translation by 3 units in the positive x direction.
- [ ] A stretch parallel to the x-axis with scale factor 2.
- [ ] A translation by 6 units in the negative x direction.
- [ ] A stretch parallel to the y-axis with scale factor 3.

Part B Inequality

Solve the inequality $|x| > |2(x + 3)|$, and give the upper bound for the solution in the form $x < a$ or $x \leq a$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `x`

Give the lower bound for the solution in the form $x > a$ or $x \geq a$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `x`

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Revision

Functions: Reciprocal Trig 3i

A Level

It is given that θ is the acute angle such that $\sec\theta\sin\theta = 36\cot\theta$.

Part A Value of $\tan\theta$

Find $\tan\theta$.

The following symbols may be useful: `cos(), sin(), tan(), theta`

Part B Value of $\tan\left(\theta - \frac{\pi}{4}\right)$

Hence, using an appropriate formula, find the exact value of $\tan\left(\theta - \frac{\pi}{4}\right)$.

The following symbols may be useful: `pi, theta`

Part C Value of $\tan(2\theta)$

Using an appropriate formula, find the exact value of $\tan(2\theta)$.

The following symbols may be useful: `theta`

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Revision

Trigonometry: Double Angles 2i

A Level

Part A Proof

Simplify $\sin(2\theta)\big(\tan\theta + \cot\theta\big)$ as far as possible.

The following symbols may be useful: `theta`

Part B Exact Value

Hence find the exact value of $\tan\frac{\pi}{12} + \tan\frac{\pi}{8} + \cot\frac{\pi}{12} + \cot\frac{\pi}{8}$.

Part C Solve

Using your answer to part A, solve the equation $\sin(4\theta)\big(\tan\theta + \cot\theta\big) = 1$ for $0 < \theta < \frac{\pi}{2}$, to three significant figures, giving your answer in **radians**.

Part D $\left(1-\cos\left(2\theta\right)\right)^2\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)^3$

Using you answer to part A, express $\left(1-\cos\left(2\theta\right)\right)^2\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)^3$ in terms of $\sin\theta$.

The following symbols may be useful: `theta`

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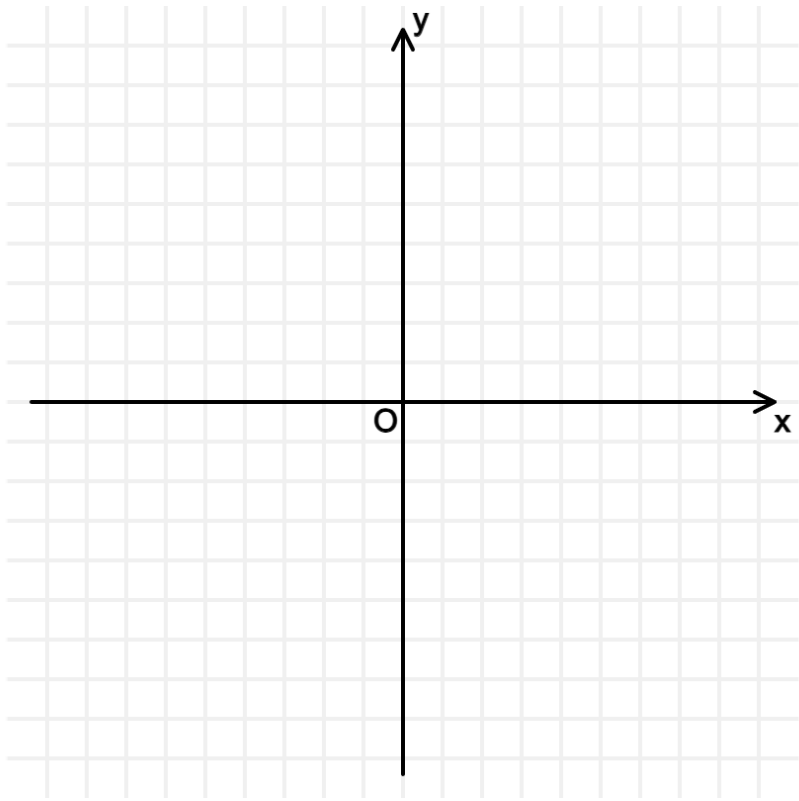
Revision

Sketching Inverse Trigonometric Functions 2

A Level

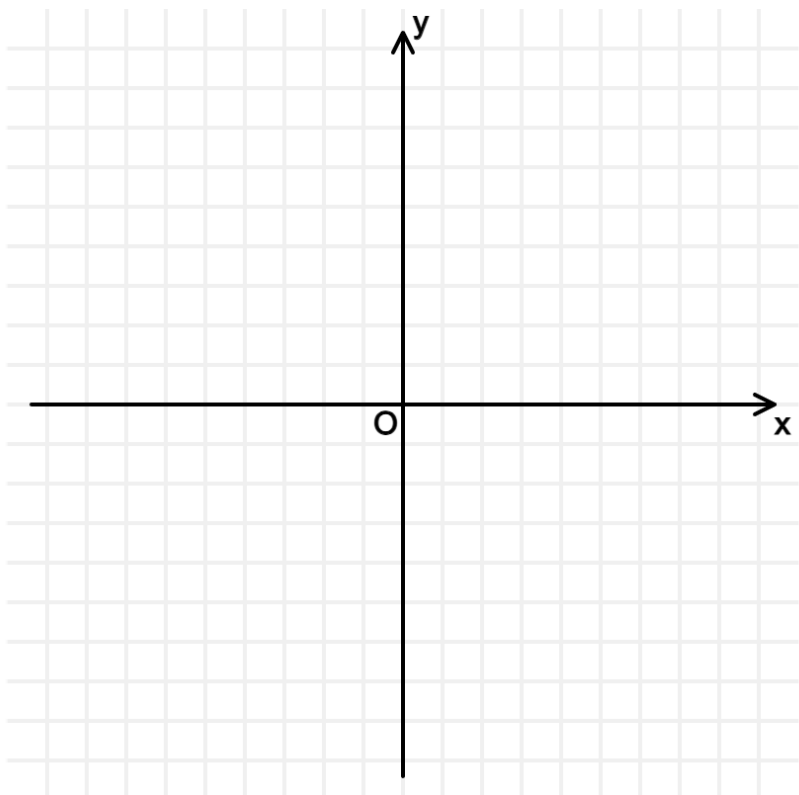
Part A Sketch $\arccos(x-1)-1$

Sketch the graph of $y = \arccos(x-1) - 1$.



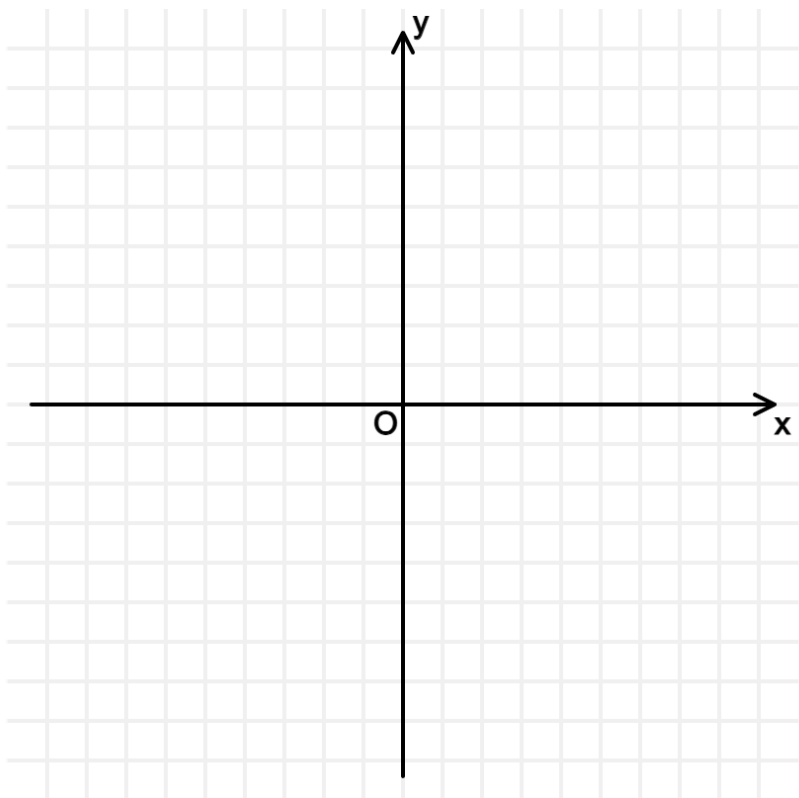
Part B Sketch $\frac{\pi}{2} - 2\arctan x$

Sketch the graph of $y = \frac{\pi}{2} - 2\arctan x$.



Part C Sketch $\arcsin(2x-1) - \frac{\pi}{2}$

Sketch the graph of $y = \arcsin(2x-1) - \frac{\pi}{2}$.



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