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Complex Numbers: Equations to Quartics 3i

Further A



Part A $z^4 = 16$

The roots of the equation $z^4 = 16$ can be written in the form $x + iy$.

Give the solution with positive x .

The following symbols may be useful: i

Give the solution with negative x .

The following symbols may be useful: i

Give the solution with positive y .

The following symbols may be useful: i

Give the solution with negative y .

The following symbols may be useful: i

Part B Another quartic

The previous part can be used to solve the equation $w^4 = 16(1 - w)^4$. The solutions can be written in the form $x + iy$.

Of the two roots that are purely real - that is, $\text{Im}(w) = 0$ - give the larger root.

Of the two roots that are purely real - that is, $\text{Im}(w) = 0$ - give the smaller root.

The other two roots are complex, and can be written in the form $x + iy$.

Give the complex root with positive y .

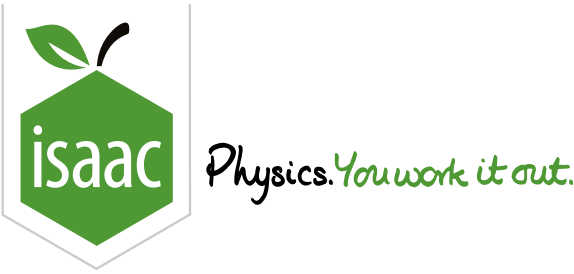
The following symbols may be useful: i

Give the complex root with negative y .

The following symbols may be useful: i

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Argand Diagrams: Solving Inequalities 3i

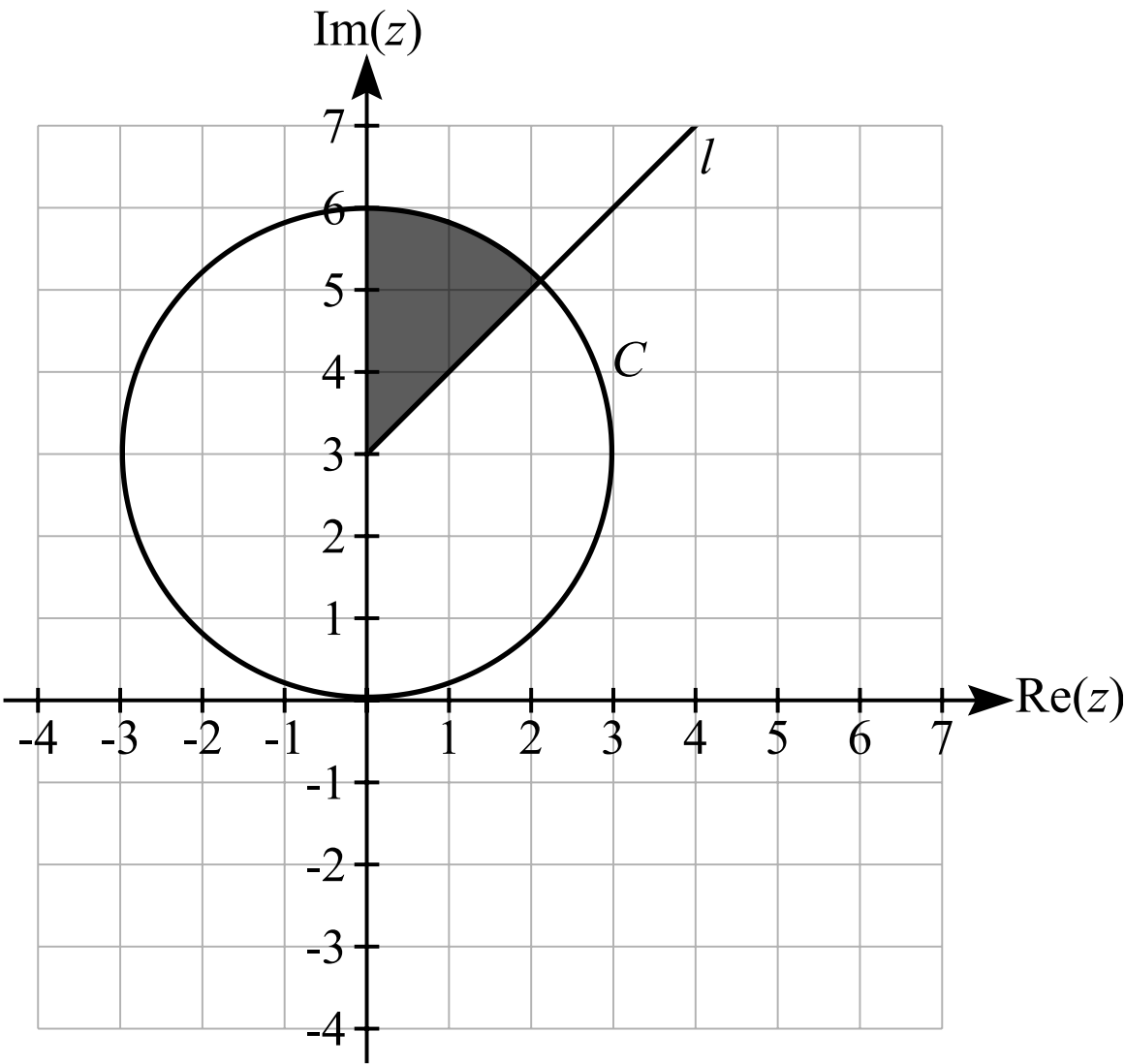


Figure 1: C and l are shown on a single Argand diagram.

The Argand diagram above shows a half-line l and a circle C . The circle has centre $3i$ and passes through the origin.

Part A **Equation of C**

Give the equation of C in the form

$$|z - z_1| = a$$

where z_1 is complex and a is real.

The following symbols may be useful: i , z

Part B **Equation of l**

The equation of l can be written in the form

$$\arg(z - z_1) = a\pi$$

where z_1 is complex and a is real.

Give z_1 in the form $x + iy$.

The following symbols may be useful: i , π , z_2

Give the value of a in exact form.

The following symbols may be useful: a

Part C Inequalities

The shaded region is defined by two inequalities. [The shaded region includes the boundaries.]

They are in the form

$$|z - z_1| \text{ inequality } a$$

and

$$b\pi \leq \arg(z - z_2)$$

$$\arg(z - z_2) \leq c\pi$$

where inequality indicates either $<$, $>$, \geq or \leq .

Give the full expression for the first inequality.

The following symbols may be useful: $<$, \leq , $>$, \geq , i , z

Give the value of b in the second inequality.

The following symbols may be useful: b

Give the value of c in the third inequality.

The following symbols may be useful: c

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Complex Numbers: De Moivre 4ii

Further A



Part A $\cos^6 \theta$

By expressing $\cos \theta$ in terms of $e^{i\theta}$, write $\cos^6 \theta$ in terms of $\cos(n\theta)$.

Give your answer in the form

$$\cos^6 \theta = f(\cos 6\theta, \cos 4\theta, \cos 2\theta)$$

The following symbols may be useful: $\cos()$, θ

Part B Solutions to an equation

Hence solve, for $0 \leq \theta \leq \pi$,

$$\cos 6\theta + 6 \cos 4\theta + 2 \cos 2\theta = 3.$$

Give your solutions in increasing order, in radians, to 3 significant figures.

What is θ_1 ?

What is θ_2 ?

What is θ_3 ?

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Algebra and Roots: Cubics with Substitution 3i

Further A



The cubic equation $x^3 + 2x^2 + 3x + 4 = 0$ has roots α , β and γ .

Part A Substitution

Use the substitution $x = \frac{1}{u+1}$ to find a cubic equation in u with the form $au^3 + bu^2 + cu + d = 0$ where a , b , c and d are integers.

The following symbols may be useful: u

Part B $\left(\frac{1}{\alpha} - 1\right) \left(\frac{1}{\beta} - 1\right) \left(\frac{1}{\gamma} - 1\right)$

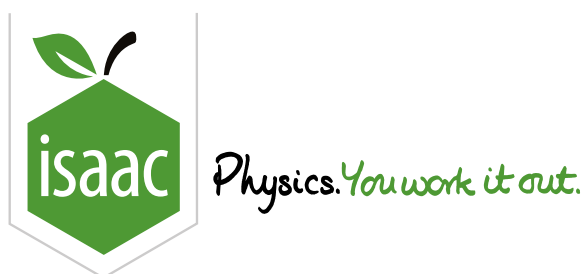
Hence, find the value of $\left(\frac{1}{\alpha} - 1\right) \left(\frac{1}{\beta} - 1\right) \left(\frac{1}{\gamma} - 1\right)$ as a single fraction.

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Matrices - Linear Equations 1

Further A



Use matrix notation to solve the following set of three equations:

$$x + ky - z = 3$$

$$3x + ky = 1$$

$$-x + 4y + z = 3.$$

where k is a constant.

Part A Matrix form

Write these equations in matrix form $\mathbf{Ax} = \mathbf{b}$.

If the matrix \mathbf{A} is written in the form

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

give the values of the following elements of the matrix \mathbf{A} .

Give the value of a_{12} .

The following symbols may be useful: k

Give the value of a_{21} .

Give the value of a_{23} .

Give the value of a_{32} .

Part B No unique solution

Find an expression for the condition that there is no unique solution to this set of equations.

The following symbols may be useful: k

Part C The inverse matrix

Find \mathbf{A}^{-1} , the inverse of \mathbf{A} .

If the matrix \mathbf{A}^{-1} is written in the form

$$\mathbf{A}^{-1} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

give expressions for the following elements of \mathbf{A}^{-1} .

Give an expression for α_{13} .

The following symbols may be useful: k

Give an expression for α_{21} .

The following symbols may be useful: k

Give an expression for α_{31} .

Give an expression for α_{32} .

The following symbols may be useful: k

Part D Solution to the set of equations

Using \mathbf{A}^{-1} , find the solutions for x , y and z in terms of k .

Find an expression for x .

The following symbols may be useful: k

Find an expression for y .

The following symbols may be useful: k

Find an expression for z .

The following symbols may be useful: k

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Matrices - Transformations 2

Further A



A, **B** and **C** are 3×3 matrices such that $\mathbf{C} = \mathbf{B}\mathbf{A}$ and

$$\mathbf{B} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and

$$\mathbf{C} = \begin{pmatrix} p & 0 & q \\ 0 & r & 0 \\ s & 0 & t \end{pmatrix}$$

Part A Matrix \mathbf{B}^{-1}

Find \mathbf{B}^{-1} . Give expressions for the elements of \mathbf{B}^{-1} on the leading diagonal i.e. β_{11} , β_{22} and β_{33} .

Give an expression for β_{11} .

The following symbols may be useful: k

Give an expression for β_{22} .

The following symbols may be useful: k

Give an expression for β_{33} .

Part B **Matrix \mathbf{A}**

Use \mathbf{B}^{-1} to deduce the form of the matrix \mathbf{A} .

Give your answer by writing the elements in each row in brackets in the form (a_{m1}, a_{m2}, a_{m3}) where $m = 1, 2$ or 3 . Thus, if $a_{21} = 1$, $a_{22} = 2$ and $a_{23} = 0$, type: (1,2,0) with no spaces.

Give the elements in the top row ($m = 1$) of the matrix, writing them in the form indicated above.

Give the elements in the second row ($m = 2$) of the matrix, writing them in the form indicated above.

Give the elements in the bottom row ($m = 3$) of the matrix, writing them in the form indicated above.

Part C Transformation produced by **A**

If the matrix **A** represents rotation anticlockwise about the *y*-axis through an angle *D* deduce the expressions for *p*, *q*, *r*, *s* and $\frac{r}{k}$.

Deduce the expression for *p*.

The following symbols may be useful: D, cos(), k, sin(), tan()

Deduce an expression for *q*.

The following symbols may be useful: D, cos(), k, sin(), tan()

Deduce an expression for $\frac{r}{k}$.

The following symbols may be useful: cos(), k, r, sin(), tan()

Deduce an expression for *s*.

The following symbols may be useful: D, cos(), k, sin(), tan()

Deduce an expression for *t*.

The following symbols may be useful: D, cos(), k, sin(), tan()

Part D Reflection in the $z = 0$ plane

If \mathbf{C} represents reflection in the $z = 0$ plane deduce the values of r and D .

Deduce the value of r .

Deduce the value of the angle D . Give your answer in radians and assume $0 \leq D < 2\pi$.

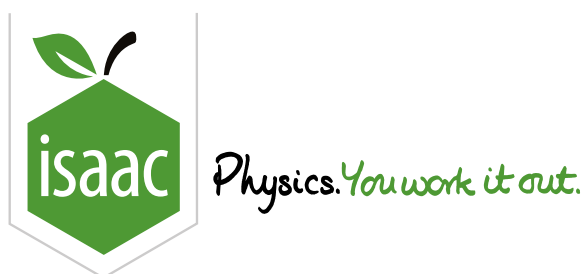
The following symbols may be useful: π

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Polar Coordinates: General 3i

Further A



The equation of a curve, in polar coordinates, is

$$r = \sqrt{3} + \tan \theta, \quad \text{for } -\frac{1}{3}\pi \leq \theta \leq \frac{1}{4}\pi.$$

Part A Tangent at the pole

Find the equation of the tangent at the pole in the form $\theta = \alpha$.

The following symbols may be useful: π , θ

Part B Greatest value of r

State the greatest value of r .

Part C Corresponding value of θ

State the value of θ at which r takes its greatest value.

The following symbols may be useful: π

Part D Sketch the curve

Sketch the curve.

Which curve in **Figure 1** most resembles your sketch?

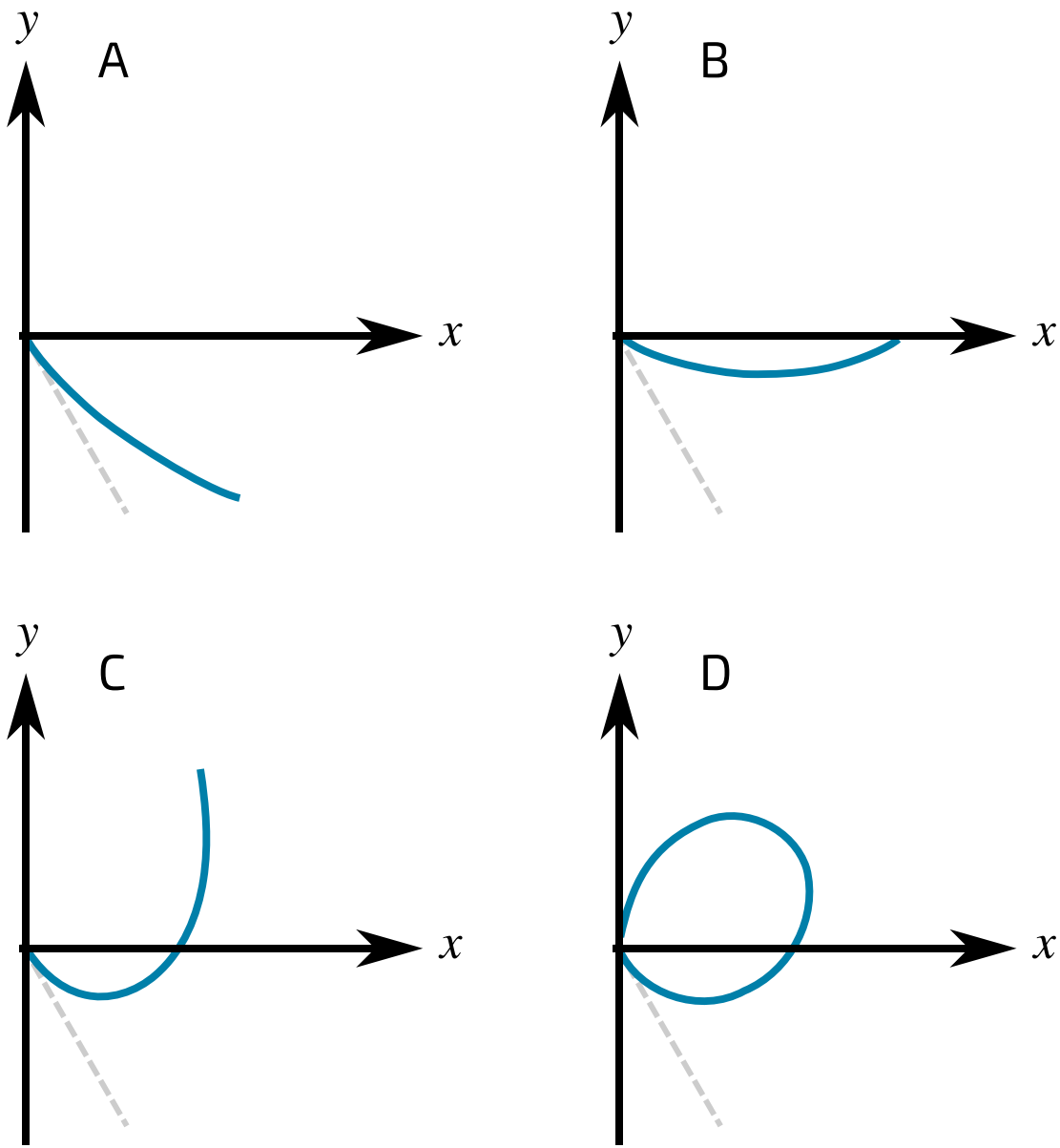


Figure 1: Four curves.

- ☐ Curve A
- ☐ Curve B
- ☐ Curve C
- ☐ Curve D

Part E Area of region

Given that

$$\int \tan x \, dx = \ln |\sec x| + C,$$

find the exact area of the region enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{4}\pi$.

The following symbols may be useful: $\ln()$, $\log()$, π

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Vectors: Lines and Planes 3i

Further A



The plane Π passes through the points $(1, 2, 1)$, $(2, 3, 6)$ and $(4, -1, 2)$.

Part A Cartesian equation of Π

Find a cartesian equation of the plane Π .

Give your answer in the form $ax + by + cz = 19$.

The following symbols may be useful: x , y , z

Part B Intersection of l and Π

The line l has equation $r = \begin{pmatrix} -1 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$.

Find the value of λ at the point of intersection of Π and l .

Part C Angle between l and Π

Find the acute angle between Π and l .

Give your answer in degrees to 3 significant figures.



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Vectors: Geometry 2i

Further A



(In this question, the notation $\triangle ABC$ denotes the area of the triangle ABC .)

The vector product of two vectors \underline{p} and \underline{q} is given by $\underline{p} \times \underline{q} = |\underline{p}| |\underline{q}| \sin \theta \underline{\hat{n}}$ where θ is the angle between \underline{p} and \underline{q} , with $0 \leq \theta \leq \pi$, and $\underline{\hat{n}}$ is a unit vector perpendicular to both \underline{p} and \underline{q} in the right-handed sense.

The points P , Q and R have position vectors $p\underline{i}$, $q\underline{j}$ and $r\underline{k}$ respectively, relative to the origin O , where p , q and r are positive. The points O , P , Q and R are joined to form a tetrahedron.

Part A Sketch tetrahedron

Draw a sketch of the tetrahedron.

Which of these sketches is correct?

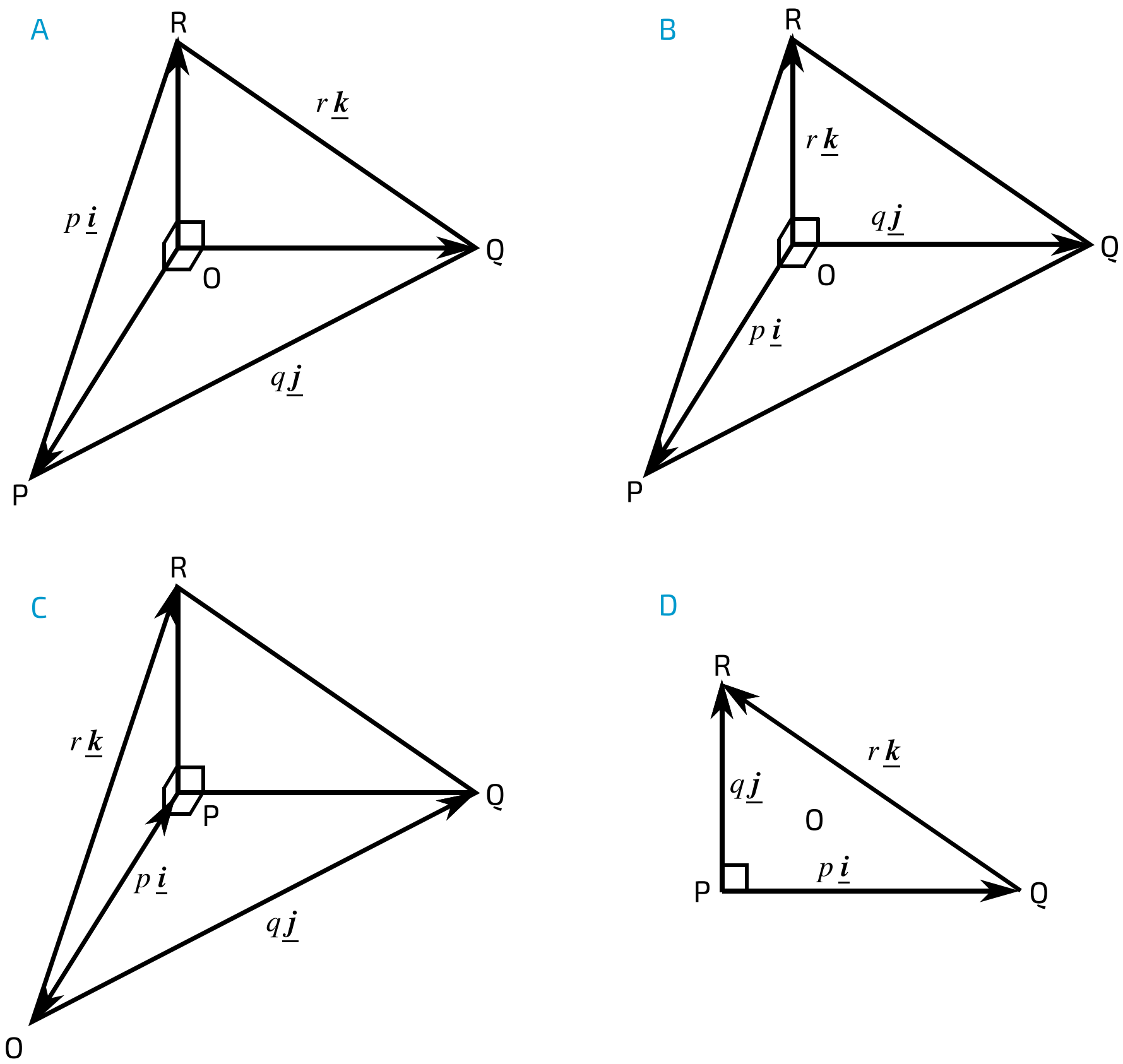


Figure 1: Four sketches.

- ☐ Sketch A
- ☐ Sketch B
- ☐ Sketch C
- ☐ Sketch D

Part B Triangle areas

Write down the values of $\triangle OPQ$, $\triangle OQR$ and $\triangle ORP$.

What is $\triangle OPQ$?

The following symbols may be useful: p , q , r

What is $\triangle OQR$?

The following symbols may be useful: p , q , r

What is $\triangle ORP$?

Part C Vector product and area

Use the definition of the vector product to show that $k|\overrightarrow{RP} \times \overrightarrow{RQ}|$ is equal to the area of one of the tetrahedron's faces, where k is a constant to be found.

Which area is $k|\overrightarrow{RP} \times \overrightarrow{RQ}|$ equal to?

☐ $\triangle ORP$

☐ $\triangle OQR$

☐ $\triangle OPQ$

☐ $\triangle PQR$

What is the value of k ?

Part D Relationship between areas

Show that we can find an equation of the form

$$(\triangle OPQ)^2 + (\triangle OQR)^2 + (\triangle ORP)^2 = \alpha(\triangle PQR)^2$$

where α is a constant to be found.

What is α ?

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