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STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Binomial Expansion - Rational n

A-level Maths Topic Summaries - Series

Subject & topics: Maths | Algebra | Series **Stage & difficulty:** A Level P3

Fill in the blanks below to complete the notes on binomial expansion for rational powers of n.

Brackets of the form $(1+x)^n$, where n is any [ ] number, can be expanded using the expression

$$(1+x)^n = 1 + [\ ]\ x + \frac{n(n-1)}{2!}\ [\ ] + \frac{n(n-1)(n-2)}{3!}\ [\ ] + \ldots$$

The right hand side is a sum of positive integer powers of x, plus a constant. Sums of this form are called [ ].

The right hand side has an infinite number of terms. The right hand side converges to the left hand side as long as [ ].

If $|x|$ is small, we can approximate $(1+x)^n$ using the [ ] terms of the series. The [ ] terms we use, the better our approximation will be.

Items:

first few | more | power series | rational | n | x^2 | x^3 | $|x|<1$ | $|x|=1$ | $|x|>1$

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STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Binomial: All Rational n 2ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

Part A

Expansion

Expand $(1+4x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients.

The following symbols may be useful: x

Part B

Values of x

State the set of values of x for which the expansion is valid. You may use the symbols $<$ $>$ $=$ x and $|x|$ which can be entered as `abs(x)`.

The following symbols may be useful: `<, >, abs(), x`

Part C

Value of k

In the expansion of

$$(1+kx)(1+4x)^{\frac{1}{2}},$$

the coefficient of x is 7. Find the value of the constant k and hence the coefficient of x^2.

$k =$

coefficient of $x^2 =$

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STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Binomial: All Rational n 4ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

Part A

Expansion

Expand $(2+x)^{-2}$ in ascending powers of x up to and including the term in x^3.

The following symbols may be useful: x

Part B

Set of Values

State the set of values of x for which the expression is valid. You may use the symbols $<$ $>$ $=$ x and $|x|$ which can be entered as `abs(x)`.

The following symbols may be useful: `<`, `>`, `abs()`, x

Part C

Coefficient

Hence find the coefficient of x^3 in the expansion of $\frac{1+x^2}{(2+x)^2}$.

The following symbols may be useful: x

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STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Binomial: All Rational n 1i

Subject & topics: Maths **Stage & difficulty:** A Level P2

Part A

Partial Fractions

Given that $\frac{3x+4}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$, find A, B, and C.

If a value is not a whole number, enter the value as a decimal.

$A =$ ____

$B =$ ____

$C =$ ____

Part B

Expand

Hence or otherwise expand $\frac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2.

The following symbols may be useful: x

Part C

Values of x

State the set of values of x for which the expansion in the above part is valid.

Construct your answer from the items below.

Items:

x $<$ $\leq$ $>$ $\geq$ $< x <$ $\leq x \leq$ $> x$ **or** $x >$ $\geq x$ **or** $x \geq$ -4 -2 $-\frac{3}{2}$ -1 $-\frac{1}{2}$ 0 $\frac{1}{2}$ 1 $\frac{3}{2}$ 2 4

Question deck:

STEM SMART Single Maths 28 - Binomial Theorem - Rational n

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STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Binomial: All Rational n 3i

Subject & topics: Maths **Stage & difficulty:** A Level P2

Find the coefficient of x^2 in the expansion in ascending powers of x of

$$\sqrt{\frac{1+ax}{4-x}}$$

giving your answer in terms of a.

The following symbols may be useful: a

Question deck:

STEM SMART Single Maths 28 - Binomial Theorem - Rational n

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STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Binomial Expansion 5

Pre-Uni Maths for Sciences C2.7

Subject & topics: Maths | Algebra | Series **Stage & difficulty:** A Level P3

Rewrite each of the following expressions in the form $a(1+b)^n$, where a is an integer and b is a fraction such that $|b|<1$. Hence, using the appropriate binomial expansion, find the value of each of them correct to 4 dp.

Part A

$\sqrt{36.1}$

Rewrite $\sqrt{36.1}$ in the form $a(1+b)^n$, where a is an integer, b is a fraction such that $|b|<1$, and $n=\frac{1}{2}$.

$\sqrt{36.1} =$ ☐ $(1+$ ☐ $)^{\frac{1}{2}}$

Hence, using the appropriate binomial expansion, find the value of $\sqrt{36.1}$ correct to 4 dp.

$\sqrt{36.1} =$ ☐

Part B

$\sqrt[3]{1.09}$

Rewrite $\sqrt[3]{1.09}$ in the form $a(1+b)^n$, where a is an integer, b is a fraction such that $|b|<1$ and $n=\frac{1}{3}$.

$\sqrt[3]{1.09} =$ ☐ $(1+$ ☐ $)^{\frac{1}{3}}$

Hence, using the appropriate binomial expansion, find the value of $\sqrt[3]{1.09}$ correct to 4 dp.

$\sqrt[3]{1.09} =$ ☐

Part C

$\frac{1}{\sqrt{1.04}}$

Rewrite $\frac{1}{\sqrt{1.04}}$ in the form $a(1+b)^n$, where a is an integer, b is a fraction such that $|b|<1$ and $n=-\frac{1}{2}$.

$\frac{1}{\sqrt{1.04}} =$ ☐ $(1+$ ☐ $)^{-\frac{1}{2}}$

Hence, using the appropriate binomial expansion, find the value of $\frac{1}{\sqrt{1.04}}$ correct to 4 dp.

$\frac{1}{\sqrt{1.04}} =$ ☐

Part D

$\sqrt[3]{125.4}$

Rewrite $\sqrt[3]{125.4}$ in the form $a(1+b)^n$, where a is an integer, b is a fraction such that $|b|<1$ and $n=\frac{1}{3}$.

$\sqrt[3]{125.4} =$ ☐ $(1+$ ☐ $)^{\frac{1}{3}}$

Hence, using the appropriate binomial expansion, find the value of $\sqrt[3]{125.4}$ correct to 4 dp.

$\sqrt[3]{125.4} =$ ☐

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Question deck:

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STEM SMART Single Maths 28 - Binomial Theorem - Rational n

Maclaurin Series - Potential due to Dipole

Pre-Uni Maths for Sciences C3.9

Subject & topics: Maths | Algebra | Series **Stage & difficulty:** Further A C3, University C1

Part A

Potential on the axis

An electric dipole consists of two charges $+q$ and $-q$ separated in the z direction by a very small distance a. The electric potential $V(z)$ a distance z away from the centre of the dipole in a direction along the line joining the two charges is given by

$$V(z) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{z-\frac{a}{2}} - \frac{1}{z+\frac{a}{2}}\right).$$

Find an approximate expression for $V(z)$ (assume that $z \gg a$ and obtain the first non-zero term in the Maclaurin (or binomial) expansion of $V(z)$).

The following symbols may be useful: `a, epsilon_0, pi, q, z`

Part B

Potential in any direction

An electric dipole consists of two charges $+q$ and $-q$ separated by a distance a. The electric potential V a distance r from the centre of the dipole in a direction making an angle θ to the line joining the two charges is given approximately by

$$V \approx \frac{q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{r^2 - ar\cos\theta}} - \frac{1}{\sqrt{r^2 + ar\cos\theta}}\right).$$

Assuming that $r \gg a$ show that $V \approx \frac{A\cos\theta}{r^2}$ and find an expression for A.

The following symbols may be useful: `a, epsilon_0, pi, q, r`

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