Crossing Paths

A Level P P P Further A P P P

A person is walking northwards with velocity $\underline{\boldsymbol{v}} = 1.0\underline{\boldsymbol{j}}\,\mathrm{m\,s^{-1}}$ from a point $\underline{\boldsymbol{r}} = 500\underline{\boldsymbol{i}}\,\mathrm{m}$. At the same time a second person starts walking from a point $\underline{\boldsymbol{s}} = -500\underline{\boldsymbol{i}} + 500\underline{\boldsymbol{j}}\,\mathrm{m}$ with velocity $\underline{\boldsymbol{u}} = 1.0\underline{\boldsymbol{i}} + u_y\underline{\boldsymbol{j}}\,\mathrm{m\,s^{-1}}$.

Part A Time Taken

Find the time T in seconds that passes between the walkers setting off, and their paths crossing.

Part B Required Speed

Find the speed u_y required for the two people to meet.

Vector Equations of Motion 2

A Level

In one dimension, motion with constant acceleration is modelled with the $suvat$ equations. There are analogous vector equations for motion with constant acceleration in two or three dimensions:

$$\underline{\boldsymbol{v}} = \underline{\boldsymbol{u}} + \underline{\boldsymbol{a}}t \qquad \underline{\boldsymbol{s}} = \frac{1}{2}(\underline{\boldsymbol{u}} + \underline{\boldsymbol{v}})t \qquad \underline{\boldsymbol{s}} = \underline{\boldsymbol{u}}t + \frac{1}{2}\underline{\boldsymbol{a}}t^2$$

In these equations the acceleration is $\underline{\boldsymbol{a}}$ and the time over which the acceleration takes place is t. The initial velocity is $\underline{\boldsymbol{u}}$, the final velocity is $\underline{\boldsymbol{v}}$, and $\underline{\boldsymbol{s}}$ is the change in displacement during the period of acceleration.

(The equivalent expression to $v^2 = u^2 + 2as$ involves the scalar product and is not needed in this question.)

Part A Find an expression for velocity

A particle moves in the x-y plane with the constant acceleration $\begin{pmatrix} 4 \\ 2 \end{pmatrix} \mathrm{m\,s^{-2}}$. Find an expression for the velocity of the particle after $t\,\mathrm{s}$ given that it has velocity $\begin{pmatrix} -19 \\ -10 \end{pmatrix} \mathrm{m\,s^{-1}}$ initially.

Give an expression for the x component of the velocity.

The following symbols may be useful: `t`

Give an expression for the y component of the velocity.

The following symbols may be useful: `t`

Part B Find a position

A horse is running in a large, flat, rectangular field. The field is modelled using the x-y plane, with the origin at one corner. When the horse is at the position $\underline{\boldsymbol{p}} = 50\underline{\boldsymbol{i}} + 70\underline{\boldsymbol{j}}$ relative to the origin it is moving with a velocity of $\begin{pmatrix} 12 \\ -6 \end{pmatrix} \mathrm{m\,s^{-1}}$. The horse slows down. Its acceleration is $\begin{pmatrix} -4 \\ 2 \end{pmatrix} \mathrm{m\,s^{-2}}$. What is the position of the horse relative to the origin after 3 seconds?

Give your answer as coordinates, (x, y).

(,)

Part C Find a value of t

At time $t = 0\,\mathrm{s}$ a particle is at the origin, moving with a velocity $\underline{\boldsymbol{u}}_1$. The particle accelerates for $4\,\mathrm{s}$ with acceleration $\begin{pmatrix} 10 \\ 16 \end{pmatrix} \mathrm{m\,s^{-2}}$. When $t = 4\,\mathrm{s}$, the particle has a displacement of $\begin{pmatrix} 240 \\ 384 \end{pmatrix} \mathrm{m}$ from the origin.

Suppose instead that the particle starts at the origin at time $t = 0\,\mathrm{s}$ moving with the velocity $-\underline{\boldsymbol{u}}_1$. If the particle accelerates with acceleration $\begin{pmatrix} 10 \\ 16 \end{pmatrix} \mathrm{m\,s^{-2}}$ it will still arrive at the point $\begin{pmatrix} 240 \\ 384 \end{pmatrix} \mathrm{m}$. At what time t does this occur?

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Gameboard:
STEM SMART Single Maths 38 - Vector Equations of Motion

Vectors & Calculus 2i

A Level

A projectile has velocity $\begin{pmatrix} A \\ 5 - gt \end{pmatrix} \text{m s}^{-1}$, where A is a constant.

Part A Displacement

The particle is at $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$ when $t = 0$.

Find an expression for the x-component of the particle's displacement, in metres, as a function of t.

The following symbols may be useful: `A, g, t`

Find an expression for the y-component of the particle's displacement, in metres, as a function of t.

The following symbols may be useful: `A, g, t`

Part B Force

Find an expression for the force on the particle, given that it has mass $m\,\text{kg}$. Give your answer in the form $a\underline{\boldsymbol{i}} + b\underline{\boldsymbol{j}}$ where $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$ are unit vectors in the x and y directions respectively.

The following symbols may be useful: `A, g, i, j, m`

Part C Value of A

The projectile hits a target at the coordinates $\begin{pmatrix} 20 \\ 0 \end{pmatrix}$.

What is the value of A? Give your answer to 2 significant figures. In your calculation, use the approximation $g \approx 10\,\mathrm{m\,s^{-2}}$ and assume that the target is hit at $t > 0$.

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Gameboard:
STEM SMART Single Maths 38 - Vector Equations of Motion

Vectors & Calculus 1i

A Level

A planet moves through space. The force on the planet is given by

$$\underline{\boldsymbol{F}} = \begin{pmatrix} -mAB^2\cos Bt \\ -mAB^2\sin Bt \end{pmatrix}$$

where A and B are numerical constants and m is the mass of the planet.

Part A Velocity

Given that the velocity of the planet when $t = 0$ is $\begin{pmatrix} 0 \\ AB \end{pmatrix}$.

Find an expression for the x-component of the velocity of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Find an expression for the y-component of the velocity of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part B Displacement

Given that the displacement of the planet when $t = 0$ is $\begin{pmatrix} A \\ 0 \end{pmatrix}$.

Find an expression for the x-component of the displacement of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Find an expression for the y-component of the displacement of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part C Modulus

Find an expression for the modulus of the displacement. Simplify your answer as far as possible.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part D Shape of path

What is the shape of the path that the planet follows?

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Gameboard:
STEM SMART Single Maths 38 - Vector Equations of Motion

Particles Moving on a Surface

A Level

C C C

A particle Q of mass $0.2\,\mathrm{kg}$ is projected horizontally with velocity $4\,\mathrm{m\,s^{-1}}$ from a fixed point A on a smooth horizontal surface. At time $t\,\mathrm{s}$ after projection Q is $x\,\mathrm{m}$ from A and is moving away from A with velocity $v\,\mathrm{m\,s^{-1}}$. There is a force of $3\cos 2t\,\mathrm{N}$ acting on Q in the positive x-direction.

Part A Expression for velocity

Find an expression for the velocity of Q at time t.

The following symbols may be useful: `cos(), sin(), t, tan(), v`

Part B Maximum and minimum

State the minimum and maximum values of the velocity of Q as t varies.

minimum value of Q: ☐

maximum value of Q: ☐

Part C Average velocity

Find the average velocity of Q between the times $t = \pi$ and $t = \frac{3}{2}\pi$. Give your answer to 3 significant figures.

Part D Particle's velocity

A particle P moves in a plane. Its displacement from the starting point, $\underline{\boldsymbol{R}}$, varies with time, t, as follows:

$$\underline{\boldsymbol{R}} = \begin{pmatrix} 2t^2 \sin \pi t - 1 \\ 1 + t^3 \end{pmatrix}$$

Where displacement is measured in metres and time is measured in seconds.

What is the x-component of the particle's velocity?

The following symbols may be useful: `cos(), pi, sin(), t, tan()`

What is the y-component of the particle's velocity?

The following symbols may be useful: `cos(), pi, sin(), t, tan()`

Part E Speed of particle

Find the speed of the particle when $t = 2$. Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, June 2016, OCR M3, Question 2

Gameboard:

STEM SMART Single Maths 38 - Vector Equations of Motion

Vector Equations of Motion 1

A Level

This question looks at three different uses of calculus in vector problems.

Part A Integrating to find particle displacement

A particle moves in the x-y plane with velocity $\underline{\boldsymbol{v}} = \begin{pmatrix} 2t\mathrm{e}^{-2t^2} \\ 3t\mathrm{e}^{-4t^2} \end{pmatrix}$. Find an expression for the displacement of the particle at time t, given that the particle is at the origin when $t = 0$.

Enter an expression for the x-component of the displacement.

The following symbols may be useful: `e, t`

Enter an expression for the y-component of the displacement.

The following symbols may be useful: `t`

Part B Finding a maximum speed

At a time $t\,\mathrm{s}$ a particle moves in the x-y plane with velocity $\underline{\boldsymbol{v}} = \begin{pmatrix} 2t\mathrm{e}^{-2t^2} \\ 3 \end{pmatrix} \mathrm{m\,s^{-1}}$. What is the maximum speed of the particle? Give your answer as an expression in terms of e.

The following symbols may be useful: `e`

Part C Distance of closest approach to the origin

The displacement of a particle is given by the expression $\underline{\boldsymbol{s}} = \begin{pmatrix} \mathrm{e}^{3t} \\ \mathrm{e}^{6t} - 5 \end{pmatrix}$. Find the shortest distance between the particle and the origin during the particle's motion. Give your answer in the form $\frac{\sqrt{a}}{2}$.

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