isaac You work it out.

STEM SMART Single Maths 32 - The Chain Rule

Derivatives of Standard Functions 2

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Complete the table of derivatives for these useful standard functions.

In these functions a and k are constants.

Function, $y(x)$	**Derivative,** $\frac{\mathrm{d}y}{\mathrm{d}x}$
ax^n	
$a\sin kx$	
$a\cos kx$	
$a\tan kx$	
$a\mathrm{e}^{kx}$	
$a\ln kx$	

Items:

$\frac{a}{x}$ $\frac{ak}{x}$ $ak\sec^2 kx$ anx^{n-1} $ak\cos kx$ $ak\mathrm{e}^{kx}$ $a\mathrm{e}^{kx}$ $-ak\sin kx$

Created for isaacphysics.org by Jonathan Waugh

STEM SMART Single Maths 32 - The Chain Rule

The Chain Rule

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Fill in the blanks below to complete the summary notes on the chain rule.

Part A

Example calculation

The chain rule is used to differentiate a function of a function. If y is a function of u, and u is a function of x, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\square}\frac{\square}{\mathrm{d}x}$$

For example, we can write the function $y = (4x^2+7)^5$ as $y = u^5$, where $u = 4x^2+7$. Then

$$\frac{\mathrm{d}y}{\mathrm{d}u} = \square$$

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \square$$

$$\therefore \quad \frac{\mathrm{d}y}{\mathrm{d}x} = 5u^4 \times 8x$$

u is a variable we have introduced to help with the calculation. After carrying out the differentiation, we back substitute to write the answer in terms of x only.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \square$$

Items:

$8x$ $\mathrm{d}u$ $\mathrm{d}x$ $\mathrm{d}y$ $40x(4x+7)^4$ $5u^4$

Part B

The chain rule in practice

The chain rule is useful in practical situations. A common scenario is that the amount of liquid in a container is changing, either because it is being filled up or because it is leaking. The volume of liquid in the container V is a function of the height of the liquid in the container h, and h changes with time t. The chain rule gives a relationship between the rate of change of the volume and the rate of change of the height.

$$\frac{\mathrm{d}V}{\square} = \frac{\mathrm{d}V}{\square}\frac{\mathrm{d}h}{\square}$$

Another useful relation is the relationship between $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}x}{\mathrm{d}y}$:

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{\square}$$

Items:

$\boxed{\mathrm{d}h}$ $\boxed{\mathrm{d}t}$ $\boxed{\mathrm{d}V}$ $\boxed{\mathrm{d}x}$ $\boxed{\mathrm{d}y}$ $\boxed{\frac{\mathrm{d}y}{\mathrm{d}x}}$ $\boxed{\frac{\mathrm{d}x}{\mathrm{d}y}}$

Created for isaacphysics.org by Jonathan Waugh

Question deck:

STEM SMART Single Maths 32 - The Chain Rule

STEM SMART Single Maths 32 - The Chain Rule

Powers Using Chain Rule 1

Pre-Uni Maths for Sciences J4.6

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Part A

Differentiate $w = (4s + 3)^3$

Find $\dfrac{\mathrm{d}w}{\mathrm{d}s}$ if $w = (4s + 3)^3$.

The following symbols may be useful: `s`

Part B

First derivative of $z = (b - aw)^4$

Find $\dfrac{\mathrm{d}z}{\mathrm{d}w}$ when $z = (b - aw)^4$, where a and b are constants.

The following symbols may be useful: `a, b, w`

Part C

Second derivative of $z = (b - aw)^4$

Find $\dfrac{\mathrm{d}^2 z}{\mathrm{d}w^2}$ when $z = (b - aw)^4$, where a and b are constants.

The following symbols may be useful: `a, b, w`

Created for isaacphysics.org by Julia Riley

Question deck:

STEM SMART Single Maths 32 - The Chain Rule

STEM SMART Single Maths 32 - The Chain Rule

Differentiating Exponentials 1

Pre-Uni Maths for Sciences J4.4

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Part A

Differentiate $\beta \mathrm{e}^{-\alpha t}$

Differentiate $\beta \mathrm{e}^{-\alpha t}$ with respect to t, where α and β are constants.

The following symbols may be useful: `alpha, beta, e, t`

Part B

Differentiate $C\mathrm{e}^{\beta m} + D$

Differentiate $C\mathrm{e}^{\beta m} + D$ with respect to m, where β, C and D are constants.

The following symbols may be useful: `C, D, beta, e, m`

Created for isaacphysics.org by Julia Riley

Question deck:

STEM SMART Single Maths 32 - The Chain Rule

STEM SMART Single Maths 32 - The Chain Rule

Differentiating Trig Functions 2

Pre-Uni Maths for Sciences J4.2

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Part A

Differentiate $s = r\sin(\alpha\theta)$

Find $\frac{\mathrm{d}s}{\mathrm{d}\theta}$ if $s = r\sin(\alpha\theta)$, where r and α are constants.

The following symbols may be useful: `alpha`, `cos()`, `r`, `sin()`, `tan()`, `theta`

Part B

Differentiate $q = l\cos(\alpha - 2\beta\theta)$

Find $\frac{\mathrm{d}q}{\mathrm{d}\theta}$ if $q = l\cos(\alpha - 2\beta\theta)$, where l, α and β are constants.

The following symbols may be useful: `alpha`, `beta`, `cos()`, `l`, `sin()`, `tan()`, `theta`

Created for isaacphysics.org by Julia Riley

Question deck:

STEM SMART Single Maths 32 - The Chain Rule

STEM SMART Single Maths 32 - The Chain Rule

Differentiating Natural Logs

Pre-Uni Maths for Sciences J4.10

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Part A

Differentiate $u = \ln(2v + 3)$

Find $\frac{\mathrm{d}u}{\mathrm{d}v}$ if $u = \ln(2v + 3)$.

The following symbols may be useful: v

Part B

Stationary point of $p = 2\ln(2q) - 3q$

Find the coordinates and nature of the stationary point of the function $p = 2\ln(2q) - 3q$.

Give the q-coordinate of the stationary point.

The following symbols may be useful: q

Give the p-coordinate of the stationary point.

The following symbols may be useful: p

Determine the nature of the stationary point.

- Minimum
- Maximum

Created for isaacphysics.org by Julia Riley

Question deck:

STEM SMART Single Maths 32 - The Chain Rule

STEM SMART Single Maths 32 - The Chain Rule

Differentiation: Chain Rule 1ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

The volume, $V\,\mathrm{m}^3$, of liquid in a container is given by

$$V = \left(3h^2 + 4\right)^{\frac{3}{2}} - 8$$

where $h\,\mathrm{m}$ is the depth of the liquid.

Part A

Rate of Change (a)

Find the value of $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 0.6$, giving your answer to four significant figures.

Part B

Rate of Change (b)

Liquid is leaking from the container. It is observed that, when the depth of the liquid is $0.6\,\mathrm{m}$, the depth is decreasing at a rate of $0.015\,\mathrm{m}$ per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is $0.6\,\mathrm{m}$. Answer to four significant figures.

Question deck:

STEM SMART Single Maths 32 - The Chain Rule

STEM SMART Single Maths 32 - The Chain Rule

Differentiating Exponentials 3

Pre-Uni Maths for Sciences J4.7

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Part A

Tangent to $y = \mathrm{e}^{2x} - \mathrm{e}^{-2x}$

Find the equation of the tangent to the curve $y = \mathrm{e}^{2x} - \mathrm{e}^{-2x}$ at the point $x = \frac{1}{2}$.

The following symbols may be useful: `e`, `x`, `y`

Part B

Stationary point of $u = 2\mathrm{e}^{3v} - 3v$

Find the coordinates and nature of the stationary point of the function $u = 2\mathrm{e}^{3v} - 3v$.

Find the v coordinate of the stationary point.

The following symbols may be useful: v

Find the u coordinate of the stationary point.

The following symbols may be useful: u

Determine the nature of the stationary point.

- Minimum
- Maximum

Created for isaacphysics.org by Julia Riley

Question deck:

STEM SMART Single Maths 32 - The Chain Rule

STEM SMART Single Maths 32 - The Chain Rule

Chain Rule 2

Pre-Uni Maths for Sciences J6.2

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Part A

Differentiate $E = B\sin^2(\omega t)$.

Find $\frac{\mathrm{d}E}{\mathrm{d}t}$ if $E = B\sin^2(\omega t)$, where B and ω are constants.

The following symbols may be useful: `B, E, cos(), omega, sin(), t, tan()`

Part B

Differentiate $y = \mathrm{e}^{-\frac{x^2}{2\sigma^2}}$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \mathrm{e}^{-\frac{x^2}{2\sigma^2}}$, where σ is a constant.

The following symbols may be useful: `e, sigma, x`

Created for isaacphysics.org by Julia Riley

Question deck:

STEM SMART Single Maths 32 - The Chain Rule

STEM SMART Single Maths 32 - The Chain Rule

Further Derivatives of Exponentials and Logarithms

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3, Further A P1

This question uses the chain rule to find the derivatives of several functions involving exponentials and logarithms.

Part A

Rewriting a^x

Use the rules for exponentials and logarithms to write $y = a^x$, where a is a positive constant, in the form $y = e^{bx}$, where b is a constant. Enter an expression for b in terms of a.

The following symbols may be useful: `a`, `ln()`, `log()`

Part B

Differentiating a^x

Using your answer to part A, use the chain rule to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the function $y = a^x$. Give your answer in the form $f(a)a^x$, where $f(a)$ is a function of a to be determined.

The following symbols may be useful: `a`, `ln()`, `log()`

Part C

Differentiating $\log_a(x)$

Use the chain rule to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the function $y = \log_a(x)$.

The following symbols may be useful: `a`, `ln()`, `log()`

Part D

Differentiate $\mathrm{e}^{\mathrm{e}^x}$

Use the chain rule to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the function $y = \mathrm{e}^{\mathrm{e}^x}$.

The following symbols may be useful: `e`, `x`

Part E

Differentiate $\ln(\ln(x))$

Use the chain rule to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the function $y = \ln(\ln x)$.

The following symbols may be useful: `ln()`, `log()`, `x`

Created for isaacphysics.org by Jonathan Waugh