



Physics. *You work it out.*

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Transformations of Graphs 3ii

A Level



The graph of $y = f(x)$ for $-2 \leq x \leq 2$ is shown in **Figure 1**.

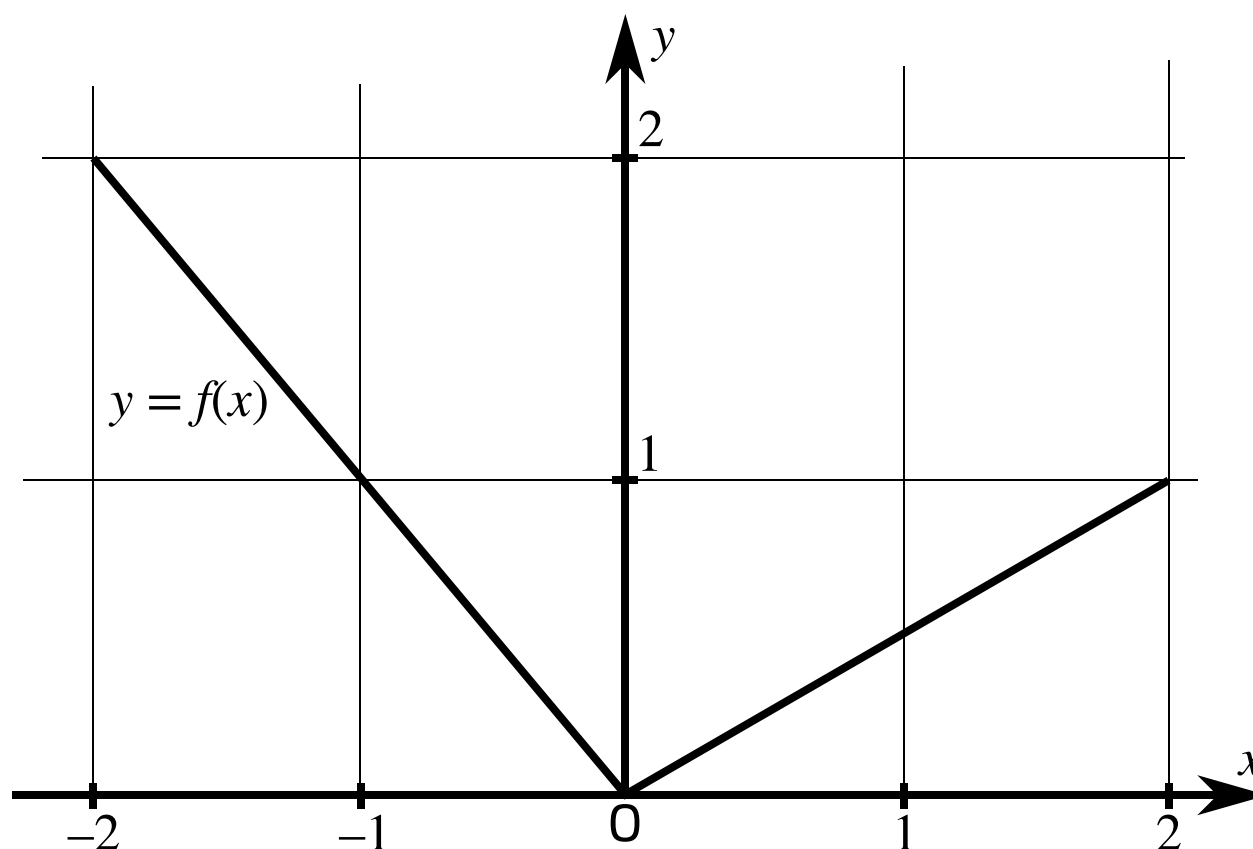


Figure 1: The graph of $y = f(x)$ for $-2 \leq x \leq 2$.

Part A Sketch $y = f(-x)$

Sketch the curve $y = f(-x)$ for $-2 \leq x \leq 2$.

What is the y -value of the curve $y = f(-x)$ when $x = 1$?

The following symbols may be useful: y

Part B Sketch $y = f(-x) + 2$

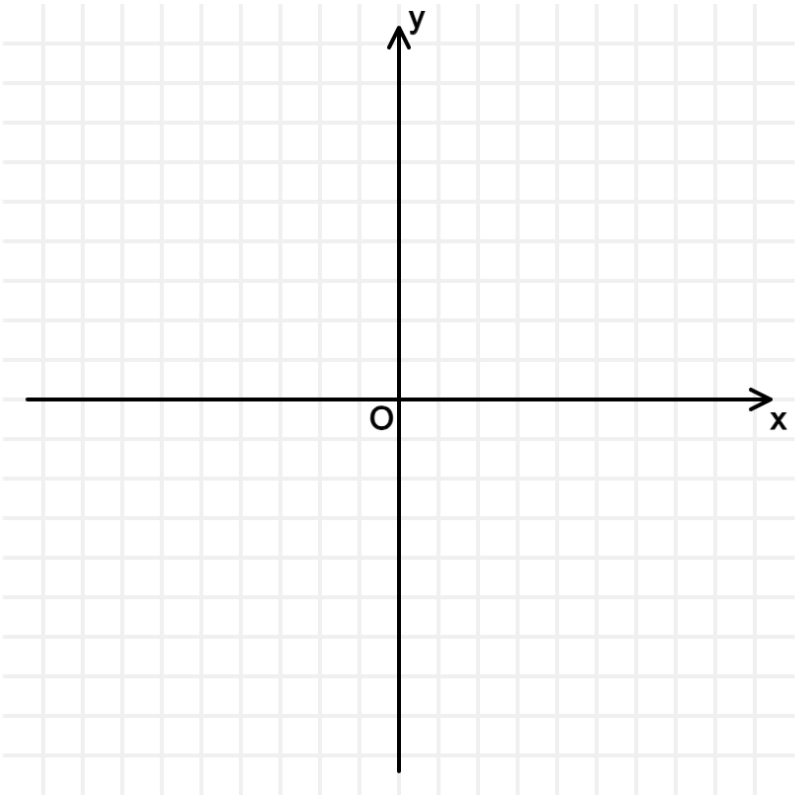
Sketch the curve $y = f(-x) + 2$ for $-2 \leq x \leq 2$.

What is the y -value of the curve $y = f(-x) + 2$ when $x = -2$?

The following symbols may be useful: y

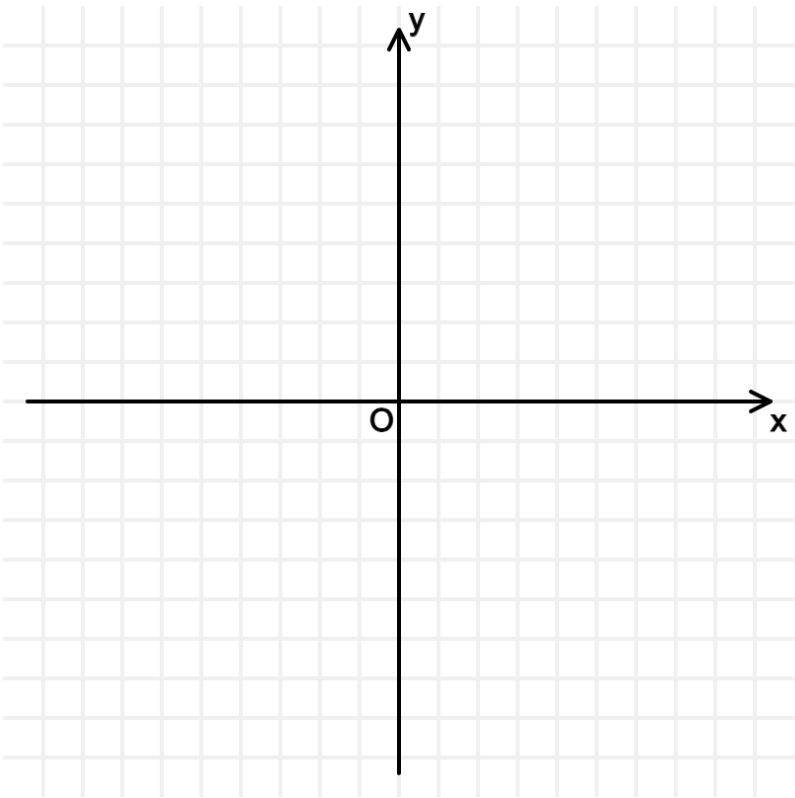
Part C Sketch $y = -\frac{1}{x^2}$

Sketch the curve $y = -\frac{1}{x^2}$.



Part D Sketch $y = 3 - \frac{1}{x^2}$

Sketch the curve $y = 3 - \frac{1}{x^2}$.



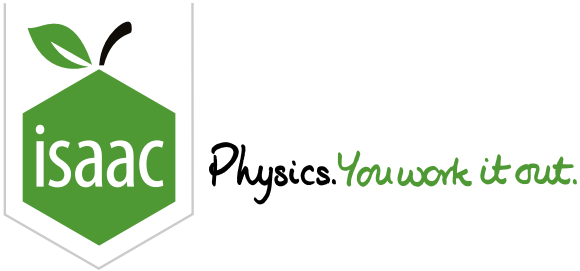
Part E State the equation

The curve $y = -\frac{1}{x^2}$ is stretched parallel to the y -axis by scale factor 2. State the equation of the transformed curve.

The following symbols may be useful: x , y

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Transformations of Graphs 1i

A Level

P

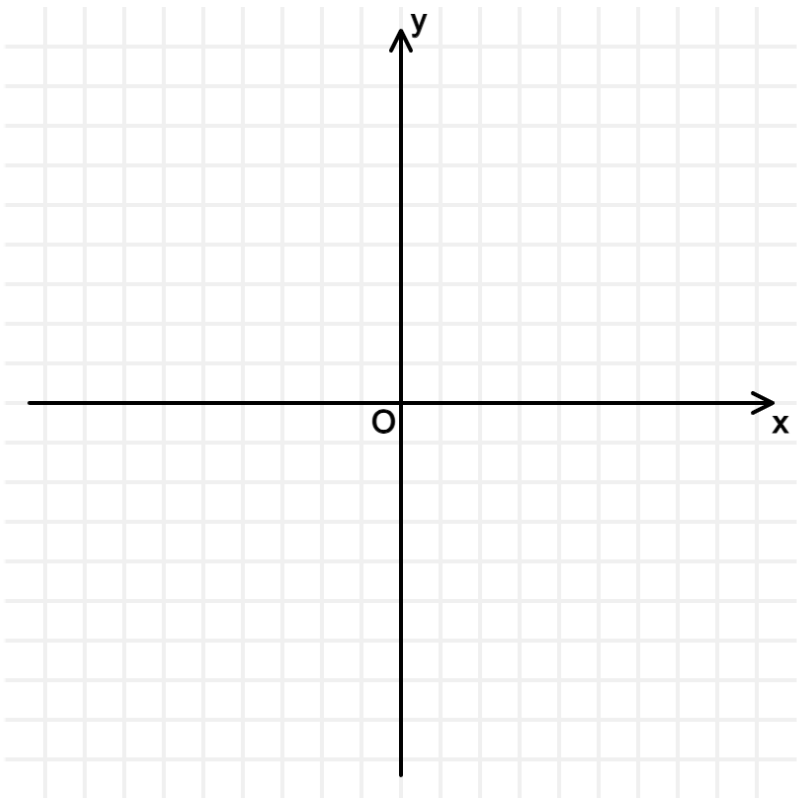
P

P

Part A

Sketch y

Find the roots of the curve $y = x^2(3 - x)$ and sketch it.



Part B

Translate y

The curve $y = x^2(3 - x)$ is translated by two units in the positive direction parallel to the x axis.

State the equation of the curve after this transformation.

The following symbols may be useful: x , y

Part C Find transformation of y

Which of these describes the transformation of the curve $y = x^2(3 - x)$ to $y = \frac{1}{2}x^2(3 - x)$?

- ☐ A stretch of scale factor 2 parallel to the y -axis.
- ☐ A stretch of scale factor 2 parallel to the x -axis.
- ☐ A stretch of scale factor $\frac{1}{2}$ parallel to the y -axis.
- ☐ A stretch of scale factor $\frac{1}{2}$ parallel to the x -axis.
-

Part D Vertical translation of $f(x)$

The curve $y = f(x)$ passes through the point P with coordinates (2, 5).

State the coordinates of the point corresponding to P on the curve $y = f(x) + 2$.

(,)

Part E Lateral stretching of $f(x)$

The curve $y = f(x)$ passes through the point P with coordinates (2, 5).

State the coordinates of the point corresponding to P on the curve $y = f(2x)$.

(,)

Part F Find transformation of $f(x)$

Which of the following describes the single transformation that maps the curve $y = f(x)$ onto $y = f(x + 4)$?

- ☐ A translation of 4 units parallel to the x -axis.
 - ☐ A translation of -4 units parallel to the x -axis.
 - ☐ A translation of -4 units parallel to the y -axis.
 - ☐ A translation of 4 units parallel to the y -axis.
-

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Gameboard:

**STEM SMART Double Maths 17 - Transformations and
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Reflection and Symmetry

A Level



Pre-Uni Maths for Sciences E2.10

The following questions ask you to deduce the symmetry properties of a number of functions. There are three choices:

- even - a function for which $f(x) = f(-x)$ which is also described as being symmetric about the vertical axis,
- odd - a function for which $f(x) = -f(-x)$ which is also described as being antisymmetric about the vertical axis (or symmetric about zero),
- neither even nor odd.

Where relevant you may assume that a and b are non-zero constants.

Part A Even functions

Decide which of the following functions are even.

- ☐ $\frac{a}{x^2} + b$
- ☐ $(x - a)(x + b) \ (a \neq b)$
- ☐ $ax^2 + bx^4$
- ☐ $x^2(a + bx)$
- ☐ $(x - a)(x + a)$
- ☐ $a \cos x$
- ☐ ax^2
- ☐ $a(x + b)^2$
- ☐ $\frac{a}{x^2} + bx^2$
- ☐ $ax^2 + b$
- ☐ $a \sin x$
-

Part B Odd functions

Decide which of the following functions are odd.

☐ ax

☐ $x^2(a + bx)$

☐ $\frac{a}{x} + \frac{b}{x^3}$

☐ $x^{\frac{1}{3}}$

☐ $\frac{a}{x}$

☐ $\frac{a}{x} + b$

☐ $\frac{a}{x} + bx^3$

☐ $a \sin x$

☐ $(x + a)^{\frac{1}{3}}$

☐ $x(a + bx^2)$

☐ $a \tan x$

Part C Neither odd nor even functions

Decide which of the following functions are neither odd nor even.

☐ $x(ax^2 + b)$

☐ $\cos x + \sin x$

☐ $(x - a)(x + a)^2$

☐ $a\left(\frac{1}{x^2} - \frac{1}{b^2}\right)$

☐ $a(b - x)^{\frac{1}{2}}$

☐ $a \tan(x + 45^\circ)$

☐ $(x - a)(x + a)$

☐ $ax - b$

☐ $x^2(ax + b)$

☐ $\frac{a}{(x - b)^2}$

☐ $ax^{\frac{1}{2}}$

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Circles 1ii

A Level



The circle with equation $x^2 + y^2 - 6x - k = 0$ has radius 4.

The points $A(3, a)$ and $B(-1, 0)$ lie on the circumference of the circle, with $a > 0$.

Part A Centre

By completing the square for x and y find the coordinates of the centre of the circle.

(,)

Part B Value of k

Find the value of k .

The following symbols may be useful: k

Part C Length AB

Calculate the length of AB, giving your answer in simplified surd form.

Part D Equation

Find the equation of the line AB. Give your answer in the form $y = mx + c$.

The following symbols may be useful: x , y

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Circles 3ii

A Level

A circle has centre $(3, 1)$ and radius 5, and a line has equation $y = 2x$.

Part A Circle equation

Write down the equation of the circle.

The following symbols may be useful: x , y

Part B Intersection points

Find the coordinates of the points of intersection of the line and the circle.

(,)

Part C Point on the line

Find the coordinates of the point on the line which is closest to the centre of the circle.

(,)

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Circles 2i

A Level
P P P

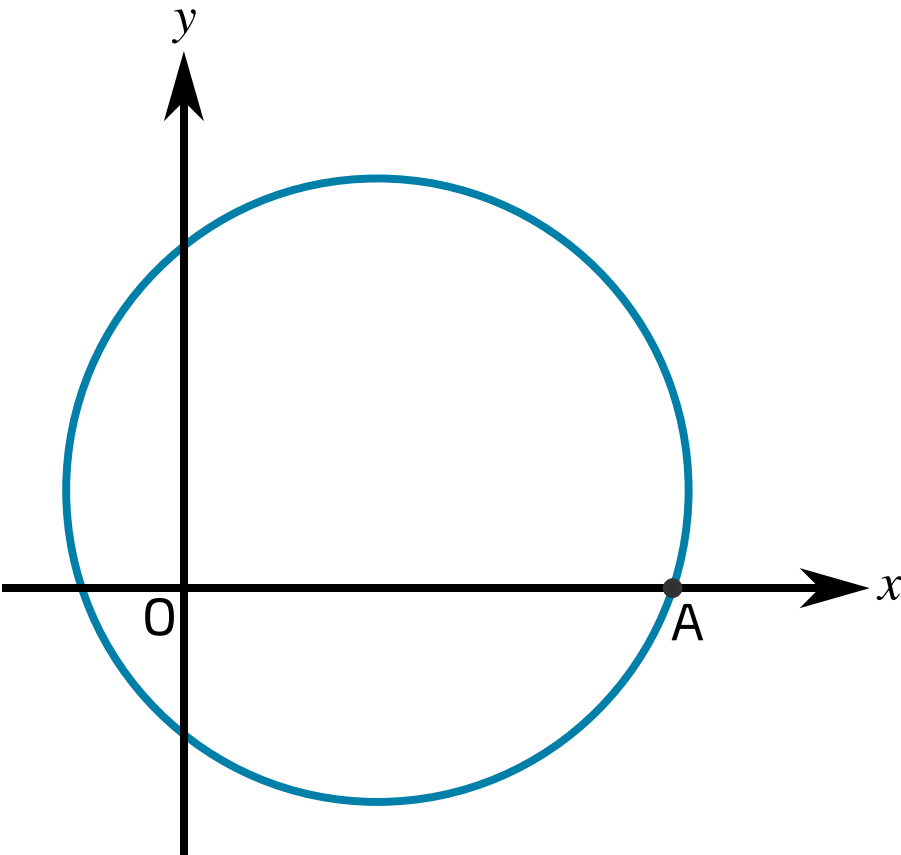


Figure 1: The circle with equation $x^2 + y^2 - 8x - 6y - 20 = 0$.

Figure 1 shows the circle with equation $x^2 + y^2 - 8x - 6y - 20 = 0$. The circle crosses the positive x axis at point A.

Part A Find centre

By completing the square for x and y find the coordinates of the centre of the circle.

(,)

Part B Find radius

Find the radius of the circle.

Part C Tangent to the circle at A

Find the equation of the tangent to the circle at A. Give your answer in the form $y = mx + c$.

The following symbols may be useful: x , y

Part D Another tangent to the circle

A second tangent to the circle is parallel to the tangent at A. Find the equation of this second tangent in the form $y = mx + c$.

The following symbols may be useful: x , y

Part E Find a radius

Another circle has its centre at the origin O and radius r . This circle lies wholly inside the first circle and $r > 0$. Find the upper bound for r . Give your answer as an inequality.

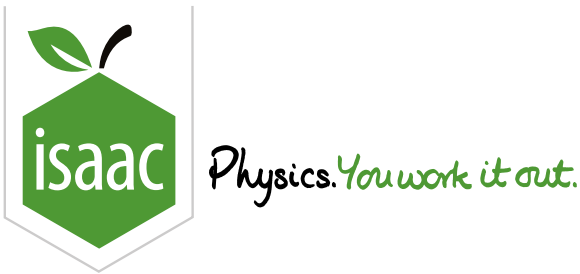
The following symbols may be useful: $<$, \leq , $>$, \geq , r

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Transformations and Area 2i

Further A



Part A Enlargement

The matrix **A** represents an enlargement, centre $(0, 0)$, with scale factor $\sqrt{2}$.

Complete the matrix **A** using the items below.

$$\mathbf{A} = \begin{pmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{pmatrix}$$

Items:

- −2

$-\sqrt{2}$

−1

$-\frac{\sqrt{2}}{2}$

0

$\frac{\sqrt{2}}{2}$

1

$\sqrt{2}$

2
-

Part B Matrix B

The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$.

Which of the following transformations is represented by **B**?

- ☐ Stretch, scale factor $\frac{\sqrt{2}}{2}$ parallel to the y axis.
- ☐ Rotation, about the origin, 45° clockwise.
- ☐ Reflection in the line $y = \frac{x}{\sqrt{2}}$
- ☐ Enlargement, centre $(0, 0)$, scale factor $\frac{1}{\sqrt{2}}$.

Part C Successive transformations

C is given by $\mathbf{C} = \mathbf{AB}$. Find **C**.

$\mathbf{C} = \begin{pmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{pmatrix}$

Items:

- −2

−√2

−1

− $\frac{\sqrt{2}}{2}$

− $\frac{1}{2}$

0

$\frac{1}{2}$

$\frac{\sqrt{2}}{2}$

1

√2

2

Part D Transformed area

Find the area of the image of the unit square under the transformation represented by **C**.

Part E Unit square

Which of the figures below shows the unit square and its image under the transformation represented by **C**?

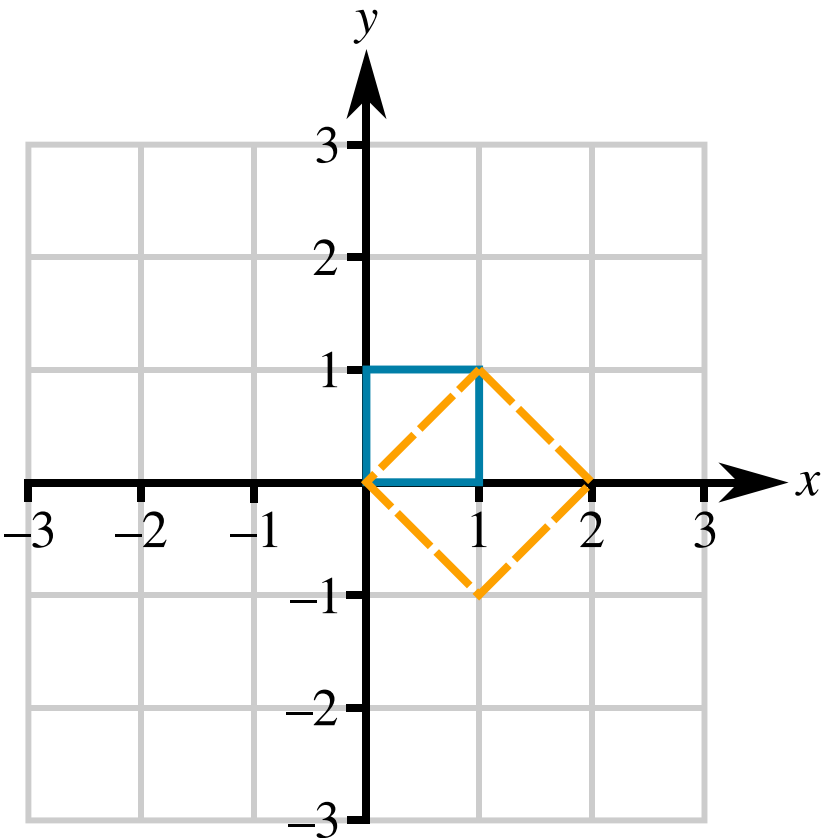


Figure 1: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

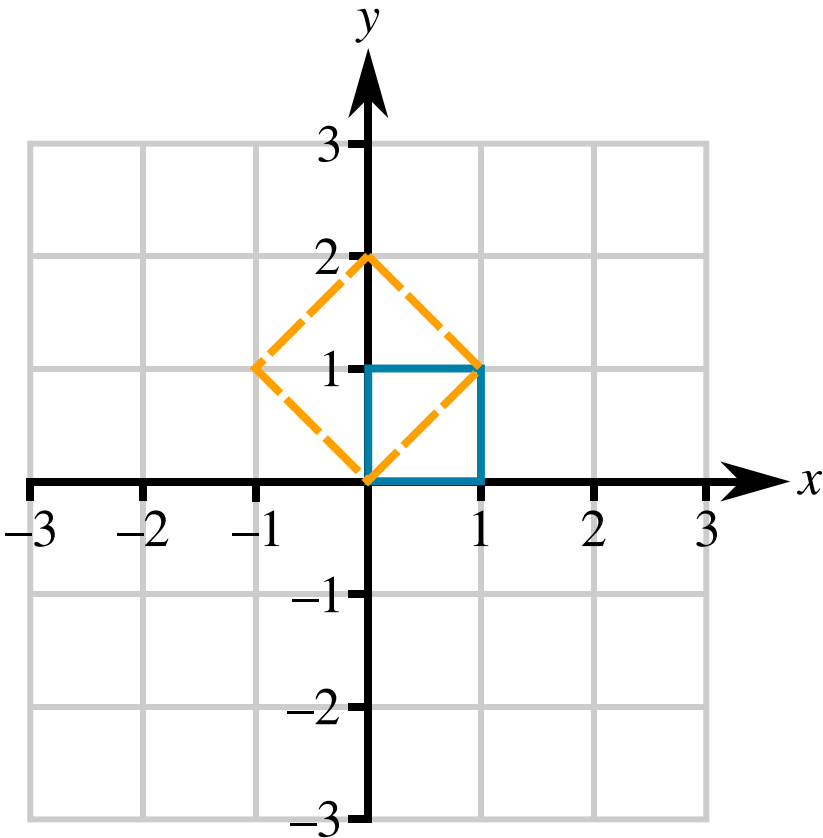


Figure 2: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

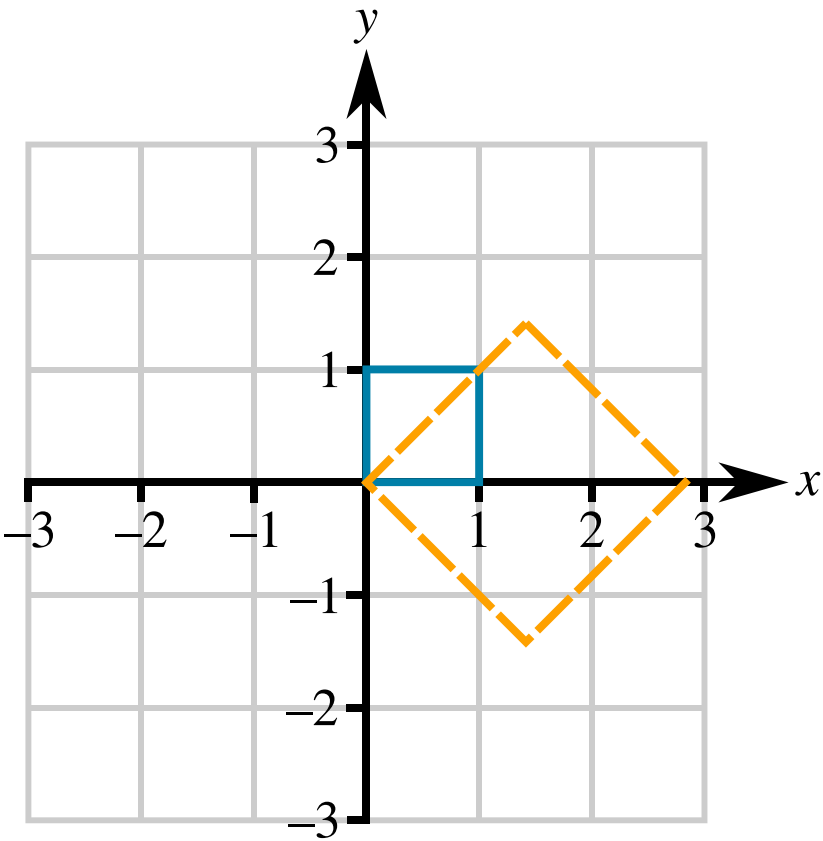


Figure 3: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

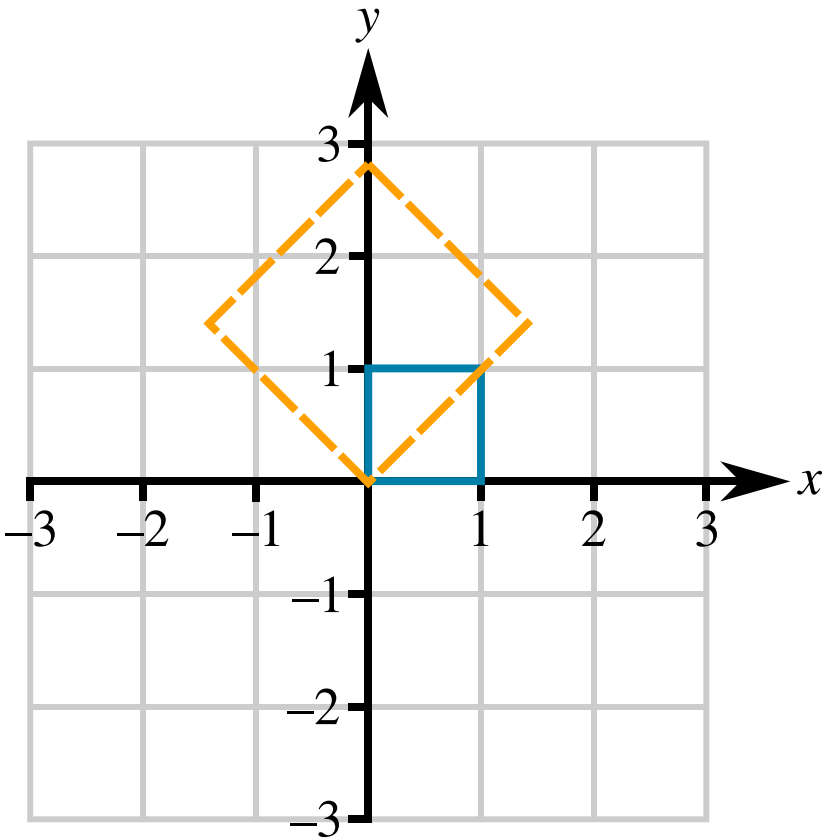


Figure 4: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

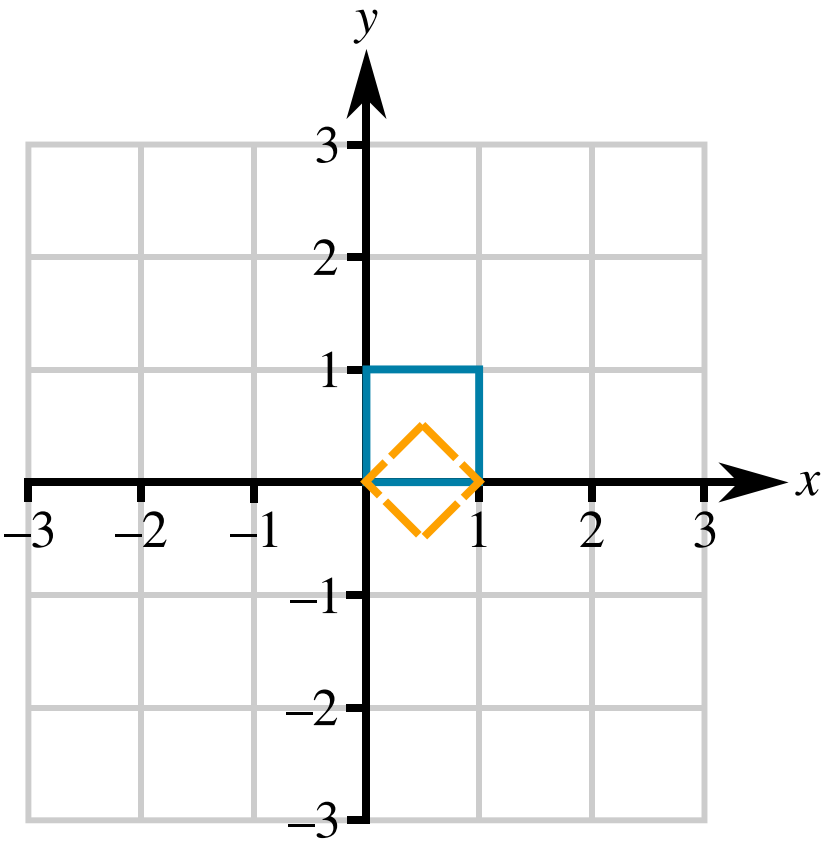


Figure 5: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

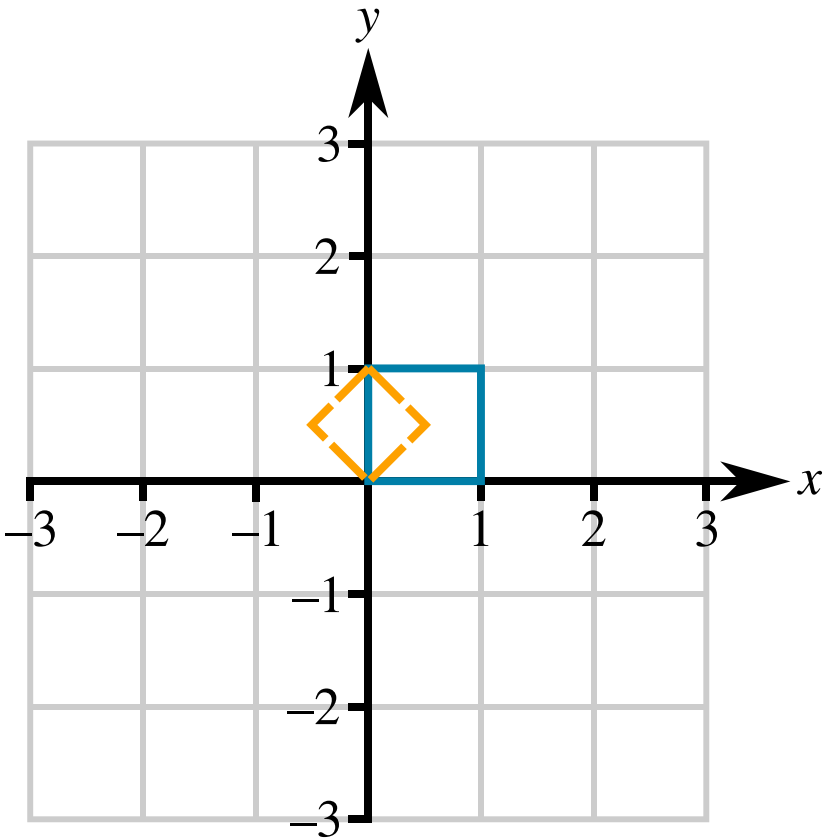


Figure 6: The unit square, shown with a solid blue line, and its image, shown with a dashed yellow line.

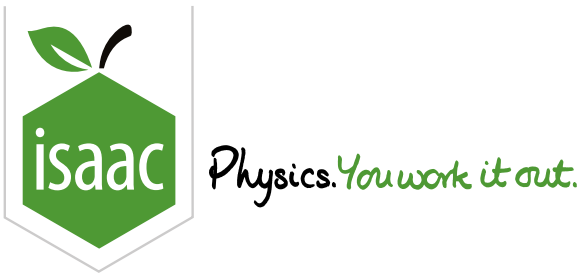
- ☐ **Figure 1**
- ☐ **Figure 2**
- ☐ **Figure 3**
- ☐ **Figure 4**
- ☐ **Figure 5**
- ☐ **Figure 6**

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Transformations - Successive 3i

Further A

The diagram in **Figure 1** shows the unit square OABC, and its image OAB'C' after a transformation.

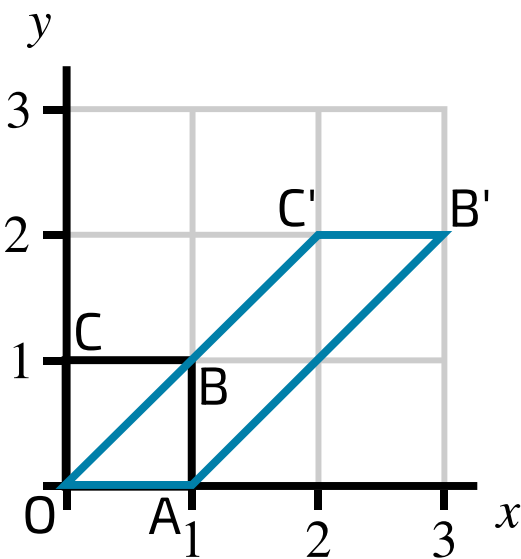


Figure 1: The unit square is shown in black, and the image after transformation is shown in blue.

Part A Matrix **X**

Find the matrix, **X**, for this transformation.

$\mathbf{X} = \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$

Items:

- 0

1

2

3

Part B Transformations P & Q

The transformation represented by **X** is equivalent to a transformation *P* followed by a transformation *Q*, which can be represented by the matrices **P** and **Q**.

Fill in the gaps below to describe a pair of possible transformations *P* and *Q*, and find the matrices **P** and **Q** that represent them.

- P* is a , and is represented by $\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$.
- Q* is a , and is represented by $\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$.

Now instead find the matrix that represents transformation *Q* followed by transformation *P*.

- Q* followed by *P* is represented by $\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$.

Items:

0

1

2

3

4

stretch

shear

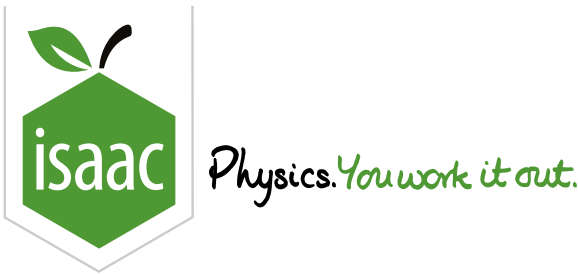
rotation

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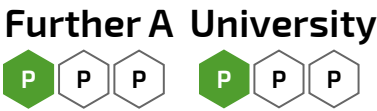
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Matrices - Transformations 1



P and **Q** are 3×3 matrices which carry out a reflection in the plane $y = 0$ and a rotation about the x -axis, respectively. The matrix **R** = **QP**.

Part A **Reflection in the $y = 0$ plane**

The 3×3 matrix **P** carries out a reflection in the plane $y = 0$.

Complete the matrix **P** using the items below.

P = $\begin{pmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{pmatrix}$

Items:

- −3

−2

−1

0

1

2

3

Part B Rotation about the x -axis

The 3×3 matrix \mathbf{Q} carries out an anticlockwise rotation about the x -axis through an angle A .

Complete the matrix \mathbf{Q} using the items below.

$$\mathbf{Q} = \begin{pmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{pmatrix}$$

Items:

- 1

0

1

-sin A

sin A

-cos A

cos A

-tan A

tan A
-

Part C Reflection followed by rotation

Find the 3×3 matrix $\mathbf{R} = \mathbf{QP}$ which carries out a reflection in the plane $y = 0$ followed by a rotation anticlockwise about the x -axis through an angle A .

$$\mathbf{R} = \begin{pmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{pmatrix}$$

Items:

- 1

0

1

-sin A

sin A

-cos A

cos A

-tan A

tan A
-

Part D **Reflection in the $z = 0$ plane**

If the value of A is such that the 3×3 matrix \mathbf{R} matrix represents a reflection in the plane $z = 0$, find the angle A . Assume $0 \leq A < 2\pi$ and give your answer in radians.

The following symbols may be useful: A , π

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