STEM SMART Single Maths 38 - Vector Equations of Motion

Constant Acceleration - More Than One Dimension

A-level Maths Topic Summaries - Vectors

Subject & topics: Maths **Stage & difficulty:** A Level P3

Fill in the boxes to complete the notes on motion in more than one dimension with constant acceleration.

When we are working in more than one dimension, if the acceleration of an object is **a constant vector**, we can use the vector form of the equations for motion with constant acceleration (the vector suvat equations). These equations look very similar to the equations for motion in one dimension, except that the acceleration $\underline{\boldsymbol{a}}$, change in displacement $\underline{\boldsymbol{s}}$, initial velocity $\underline{\boldsymbol{u}}$, and final velocity $\underline{\boldsymbol{v}}$ are now vectors.

$$\underline{\boldsymbol{v}} = \underline{\boldsymbol{u}} + \underline{\boldsymbol{a}}t$$

$$\underline{\boldsymbol{s}} = \frac{\boxed{} + \underline{\boldsymbol{v}}}{2}t$$

$$\underline{\boldsymbol{s}} = \underline{\boldsymbol{u}}\,t + \tfrac{1}{2}\boxed{}\,t^2$$

$$\underline{\boldsymbol{s}} = \boxed{}\,t - \tfrac{1}{2}\underline{\boldsymbol{a}}t^2$$

We can split each of these vector equations into separate equations for each component, linked together by the time t. For example, in two dimensions we can write $\underline{\boldsymbol{v}} = \underline{\boldsymbol{u}} + \underline{\boldsymbol{a}}t$ as the pair of equations

$$v_x = \boxed{} + \boxed{}\,t$$

$$v_y = \boxed{} + \boxed{}\,t$$

Note that the vector form of the equation $v^2 = u^2 + 2as$ is missing from the list above. It is more complicated than the other equations as it requires the scalar product.

Items:

$\boxed{a\_x}$ $\boxed{a\_y}$ $\boxed{t}$ $\boxed{u\_x}$ $\boxed{u\_y}$ $\boxed{\underline{\boldsymbol{a}}}$ $\boxed{\underline{\boldsymbol{s}}}$ $\boxed{\underline{\boldsymbol{u}}}$ $\boxed{\underline{\boldsymbol{v}}}$

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STEM SMART Single Maths 38 - Vector Equations of Motion

Variable Acceleration - More Than One Dimension

A-level Maths Topic Summaries - Vectors

Subject & topics: Maths | Geometry | Vectors **Stage & difficulty:** A Level P3

Fill in the boxes to complete the notes on motion in more than one dimension with variable acceleration.

Part A

The equations of motion

When we are working in more than one dimension, if the acceleration of an object is **a constant**, we can use the vector form of the equations for motion with constant acceleration (the vector suvat equations).

If the acceleration of an object **varies with time**, we have to use calculus. The relationships between displacement, velocity and acceleration look very similar to the relationships for motion with variable acceleration in one dimension.

Let us write the displacement from the origin as $\underline{\boldsymbol{x}}(t)$, the velocity as $\underline{\boldsymbol{v}}(t)$ and the acceleration as $\underline{\boldsymbol{a}}(t)$, where t is time. We have the following relationships.

$$\underline{\boldsymbol{v}}(t) = \frac{\mathrm{d}\boxed{}}{\mathrm{d}t}$$

$$\underline{\boldsymbol{a}}(t) = \frac{\mathrm{d}\boxed{}}{\mathrm{d}t}$$

$$\underline{\boldsymbol{a}}(t) = \frac{\mathrm{d}^2\boxed{}}{\mathrm{d}t^2}$$

$$\underline{\boldsymbol{x}}(t) = \int \boxed{}\, \mathrm{d}t$$

$$\underline{\boldsymbol{v}}(t) = \int \boxed{}\, \mathrm{d}t$$

Items:

$\boxed{\underline{\boldsymbol{x}}(t)}$ $\boxed{\underline{\boldsymbol{v}}(t)}$ $\boxed{\underline{\boldsymbol{a}}(t)}$

Part B

Differentiating and integrating vectors

When we **differentiate a vector**, we differentiate each component separately. For example,

$$\underline{\boldsymbol{a}}(t) = \frac{\mathrm{d}\underline{\boldsymbol{v}}(t)}{\mathrm{d}t} = \begin{pmatrix} \dfrac{\mathrm{d}v_x(t)}{\mathrm{d}t} \\ \dfrac{\mathrm{d}v_y(t)}{\mathrm{d}t} \end{pmatrix}$$

Let us calculate the acceleration if the velocity is given by $\underline{\boldsymbol{v}}(t) = \begin{matrix} 12t^2 \\ 6t - 7 \end{matrix}$.

$$\underline{\boldsymbol{a}}(t) = \frac{\mathrm{d}\underline{\boldsymbol{v}}(t)}{\mathrm{d}t} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

When we **integrate a vector**, we integrate each component separately. For example,

$$\underline{\boldsymbol{x}}(t) = \int \underline{\boldsymbol{v}}(t)\,\mathrm{d}t = \begin{pmatrix} \int v_x(t)\,\mathrm{d}t \\ \int v_y(t)\,\mathrm{d}t \end{pmatrix}$$

Let us calculate the displacement if the velocity is given by $\underline{\boldsymbol{v}}(t) = \begin{matrix} 12t^2 \\ 6t - 7 \end{matrix}$.

$$\underline{\boldsymbol{x}}(t) = \quad \underline{\boldsymbol{v}}(t)\,\mathrm{d}t = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

There is one constant of integration for each component, c_x and c_y.

Items:

$\boxed{6}$ $\boxed{24t}$ $\boxed{3t^2 - 7t + c\_y}$ $\boxed{4t^3 + c\_x}$

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Crossing Paths

Subject & topics: Maths | Geometry | Vectors **Stage & difficulty:** A Level P2, Further A P1

A person is walking northwards with velocity $\underline{\boldsymbol{v}} = 1.0\underline{\boldsymbol{j}}\,\mathrm{m\,s^{-1}}$ from a point $\underline{\boldsymbol{r}} = 500\underline{\boldsymbol{i}}\,\mathrm{m}$. At the same time a second person starts walking from a point $\underline{\boldsymbol{s}} = -500\underline{\boldsymbol{i}} + 500\underline{\boldsymbol{j}}\,\mathrm{m}$ with velocity $\underline{\boldsymbol{u}} = 1.0\underline{\boldsymbol{i}} + u_y\underline{\boldsymbol{j}}\,\mathrm{m\,s^{-1}}$.

Part A
Time Taken

Find the time T in seconds that passes between the walkers setting off, and their paths crossing.

Part B
Required Speed

Find the speed u_y required for the two people to meet.

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Vector Equations of Motion 2

Subject & topics: Maths | Geometry | Vectors **Stage & difficulty:** A Level P3

In one dimension, motion with constant acceleration is modelled with the $suvat$ equations. There are analogous vector equations for motion with constant acceleration in two or three dimensions:

$$\underline{\boldsymbol{v}} = \underline{\boldsymbol{u}} + \underline{\boldsymbol{a}}t \qquad \underline{\boldsymbol{s}} = \frac{1}{2}(\underline{\boldsymbol{u}} + \underline{\boldsymbol{v}})t \qquad \underline{\boldsymbol{s}} = \underline{\boldsymbol{u}}t + \frac{1}{2}\underline{\boldsymbol{a}}t^2$$

In these equations the acceleration is $\underline{\boldsymbol{a}}$ and the time over which the acceleration takes place is t. The initial velocity is $\underline{\boldsymbol{u}}$, the final velocity is $\underline{\boldsymbol{v}}$, and $\underline{\boldsymbol{s}}$ is the change in displacement during the period of acceleration.

(The equivalent expression to $v^2 = u^2 + 2as$ involves the scalar product and is not needed in this question.)

Part A

Find an expression for velocity

A particle moves in the x-y plane with the constant acceleration $\begin{pmatrix} 4 \\ 2 \end{pmatrix} \mathrm{m\,s^{-2}}$. Find an expression for the velocity of the particle after $t\,\mathrm{s}$ given that it has velocity $\begin{pmatrix} -19 \\ -10 \end{pmatrix} \mathrm{m\,s^{-1}}$ initially.

Give an expression for the x component of the velocity.

The following symbols may be useful: `t`

Give an expression for the y component of the velocity.

The following symbols may be useful: `t`

Part B

Find a position

A horse is running in a large, flat, rectangular field. The field is modelled using the x-y plane, with the origin at one corner. When the horse is at the position $\underline{\boldsymbol{p}} = 50\underline{\boldsymbol{i}} + 70\underline{\boldsymbol{j}}$ relative to the origin it is moving with a velocity of $\begin{pmatrix} 12 \\ -6 \end{pmatrix} \mathrm{m\,s^{-1}}$. The horse slows down. Its acceleration is $\begin{pmatrix} -4 \\ 2 \end{pmatrix} \mathrm{m\,s^{-2}}$. What is the position of the horse relative to the origin after 3 seconds?

Give your answer as coordinates, (x, y).

(,)

Part C

Find a value of t

At time $t = 0\,\mathrm{s}$ a particle is at the origin, moving with a velocity $\underline{\boldsymbol{u}}_1$. The particle accelerates for $4\,\mathrm{s}$ with acceleration $\begin{pmatrix} 10 \\ 16 \end{pmatrix} \mathrm{m\,s^{-2}}$. When $t = 4\,\mathrm{s}$, the particle has a displacement of $\begin{pmatrix} 240 \\ 384 \end{pmatrix} \mathrm{m}$ from the origin.

Suppose instead that the particle starts at the origin at time $t = 0\,\mathrm{s}$ moving with the velocity $-\underline{\boldsymbol{u}}_1$. If the particle accelerates with acceleration $\begin{pmatrix} 10 \\ 16 \end{pmatrix} \mathrm{m\,s^{-2}}$ it will still arrive at the point $\begin{pmatrix} 240 \\ 384 \end{pmatrix} \mathrm{m}$. At what time t does this occur?

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Vectors & Calculus 2i

Subject & topics: Maths **Stage & difficulty:** A Level P3

A projectile has velocity $\begin{pmatrix} A \\ 5 - gt \end{pmatrix} \mathrm{m\,s^{-1}}$, where A is a constant.

Part A

Displacement

The particle is at $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$ when $t = 0$.

Find an expression for the x-component of the particle's displacement, in metres, as a function of t.

The following symbols may be useful: `A, g, t`

Find an expression for the y-component of the particle's displacement, in metres, as a function of t.

The following symbols may be useful: `A, g, t`

Part B

Force

Find an expression for the force on the particle, given that it has mass $m\,\mathrm{kg}$. Give your answer in the form $a\underline{\boldsymbol{i}} + b\underline{\boldsymbol{j}}$ where $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$ are unit vectors in the x and y directions respectively.

The following symbols may be useful: `A, g, i, j, m`

Part C

Value of A

The projectile hits a target at the coordinates $\begin{pmatrix} 20 \\ 0 \end{pmatrix}$.

What is the value of A? Give your answer to 2 significant figures. In your calculation, use the approximation $g \approx 10\,\mathrm{m\,s^{-2}}$ and assume that the target is hit at $t > 0$.

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Vectors & Calculus 1i

Subject & topics: Maths **Stage & difficulty:** A Level P3

A planet moves through space. The force on the planet is given by

$$\underline{\boldsymbol{F}} = \begin{pmatrix} -mAB^2\cos Bt \\ -mAB^2\sin Bt \end{pmatrix}$$

where A and B are numerical constants and m is the mass of the planet.

Part A

Velocity

Given that the velocity of the planet when $t = 0$ is $\begin{pmatrix} 0 \\ AB \end{pmatrix}$.

Find an expression for the x-component of the velocity of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Find an expression for the y-component of the velocity of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part B

Displacement

Given that the displacement of the planet when $t = 0$ is $\begin{pmatrix} A \\ 0 \end{pmatrix}$.

Find an expression for the x-component of the displacement of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Find an expression for the y-component of the displacement of the planet as a function of time.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part C

Modulus

Find an expression for the modulus of the displacement. Simplify your answer as far as possible.

The following symbols may be useful: `A, B, cos(), sin(), t, tan()`

Part D

Shape of path

What is the shape of the path that the planet follows?

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Particles Moving on a Surface

Subject & topics: Maths **Stage & difficulty:** A Level C2

A particle Q of mass $0.2\,\mathrm{kg}$ is projected horizontally with velocity $4\,\mathrm{m\,s^{-1}}$ from a fixed point A on a smooth horizontal surface. At time $t\,\mathrm{s}$ after projection Q is $x\,\mathrm{m}$ from A and is moving away from A with velocity $v\,\mathrm{m\,s^{-1}}$. There is a force of $3\cos 2t\,\mathrm{N}$ acting on Q in the positive x-direction.

Part A

Expression for velocity

Find an expression for the velocity of Q at time t.

The following symbols may be useful: `cos()`, `sin()`, `t`, `tan()`, `v`

Part B

Maximum and minimum

State the minimum and maximum values of the velocity of Q as t varies.

minimum value of Q:

maximum value of Q:

Part C

Average velocity

Find the average velocity of Q between the times $t = \pi$ and $t = \frac{3}{2}\pi$. Give your answer to 3 significant figures.

Part D

Particle's velocity

A particle P moves in a plane. Its displacement from the starting point, $\underline{\boldsymbol{R}}$, varies with time, t, as follows:

$$\underline{\boldsymbol{R}} = \begin{pmatrix} 2t^2 \sin \pi t - 1 \\ 1 + t^3 \end{pmatrix}$$

where displacement is measured in metres and time is measured in seconds.

What is the x-component of the particle's velocity?

The following symbols may be useful: `cos(), pi, sin(), t, tan()`

What is the y-component of the particle's velocity?

The following symbols may be useful: `cos(), pi, sin(), t, tan()`

Part E

Speed of particle

Find the speed of the particle when $t = 2$. Give your answer to 3 significant figures.

Adapted with permission from UCLES, A Level, June 2016, OCR M3, Question 2

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STEM SMART Single Maths 38 - Vector Equations of Motion

Vector Equations of Motion 1

Subject & topics: Maths | Geometry | Vectors **Stage & difficulty:** A Level P3

This question looks at three different uses of calculus in vector problems.

Part A

Integrating to find particle displacement

A particle moves in the x-y plane with velocity $\underline{\boldsymbol{v}} = \begin{pmatrix} 2t\mathrm{e}^{-2t^2} \\ 3t\mathrm{e}^{-4t^2} \end{pmatrix}$. Find an expression for the displacement of the particle at time t, given that the particle is at the origin when $t = 0$.

Enter an expression for the x-component of the displacement.

The following symbols may be useful: `e, t`

Enter an expression for the y-component of the displacement.

The following symbols may be useful: `t`

Part B

Finding a maximum speed

At a time t s a particle moves in the x-y plane with velocity $\underline{\boldsymbol{v}} = \begin{pmatrix} 2t\mathrm{e}^{-2t^2} \\ 3 \end{pmatrix} \mathrm{m\,s^{-1}}$. What is the maximum speed of the particle? Give your answer as an expression in terms of e.

The following symbols may be useful: e

Part C

Distance of closest approach to the origin

The displacement of a particle is given by the expression $\underline{\boldsymbol{s}} = \begin{pmatrix} \mathrm{e}^{3t} \\ \mathrm{e}^{6t} - 5 \end{pmatrix}$. Find the shortest distance between the particle and the origin during the particle's motion. Give your answer in the form $\dfrac{\sqrt{a}}{2}$.

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