



Physics. You work it out.

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Area Between Two Curves 3ii

A Level



Figure 1 shows part of the curve $y = \frac{1}{x}$.

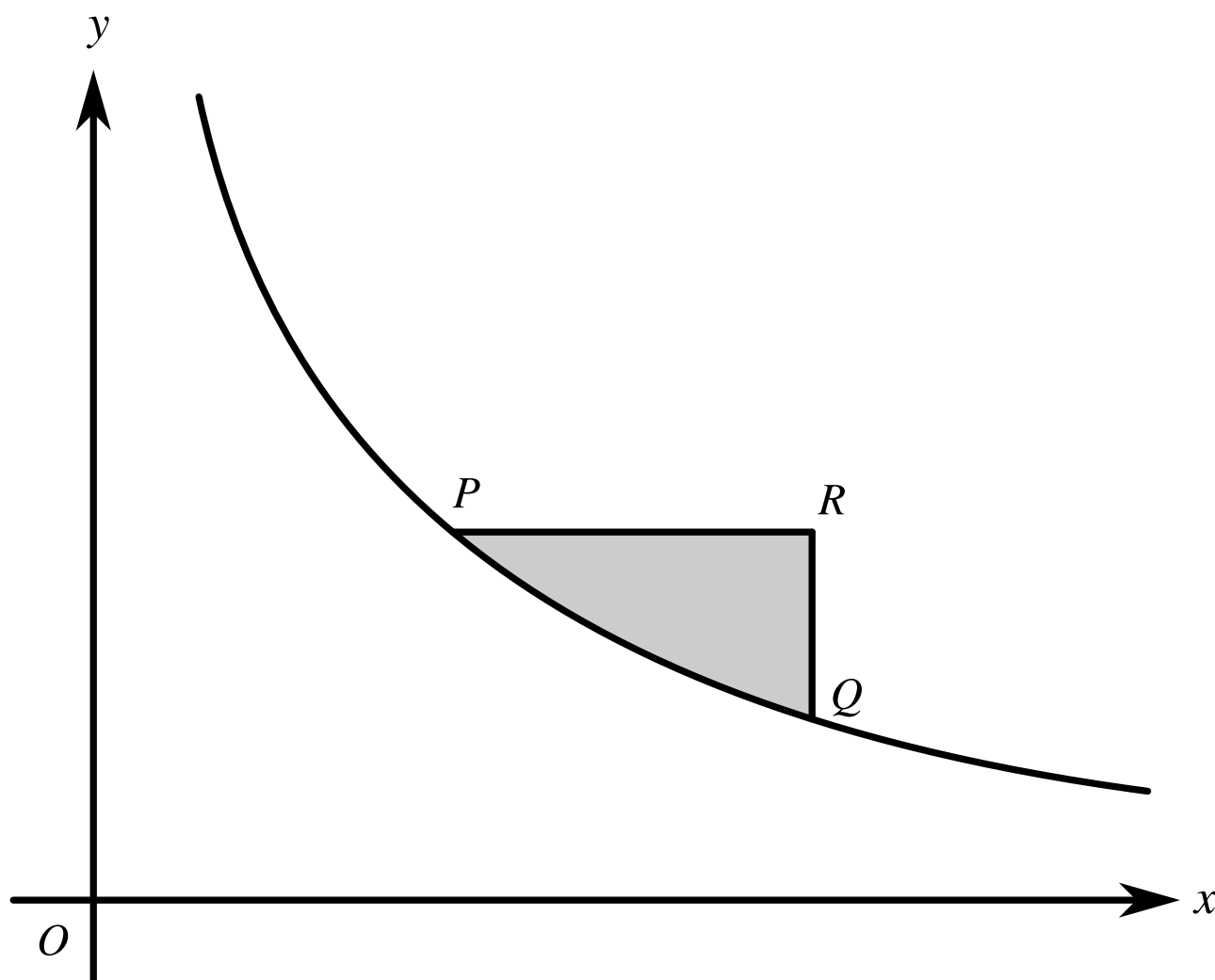


Figure 1: Part of the curve $y = \frac{1}{x}$.

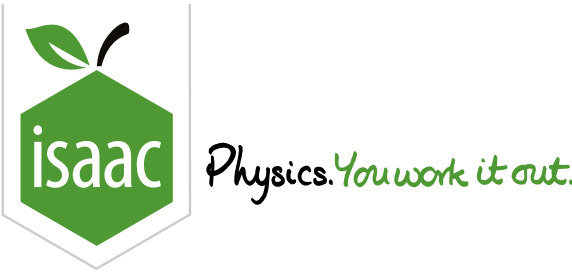
The point P has coordinates $(a, \frac{1}{a})$ and the point Q has coordinates $(2a, \frac{1}{2a})$, where a is a positive constant. The point R is such that PR is parallel to the x -axis and QR is parallel to the y -axis. The region shaded in the diagram is bounded by the curve and by the lines PR and QR .

Find the area of the shaded region.

The following symbols may be useful: a , e , $\ln()$, $\log()$

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Integration by Substitution 1

Pre-Uni Maths for Sciences 5.4.1

A Level
P P P

Part A Integrate $\sin(c\theta)$

Find $\int \sin(c\theta) \, d\theta$, where c is a constant.

The following symbols may be useful: C , c , k , θ

Part B Integrate $e^{\alpha v}$

Find $\int e^{\alpha v} \, dv$, where α is a constant.

The following symbols may be useful: α , e , k , v

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Area Between Two Curves 1ii

A Level



Figure 1 shows the curve $y = e^{3x} - 6e^{2x} + 32$.

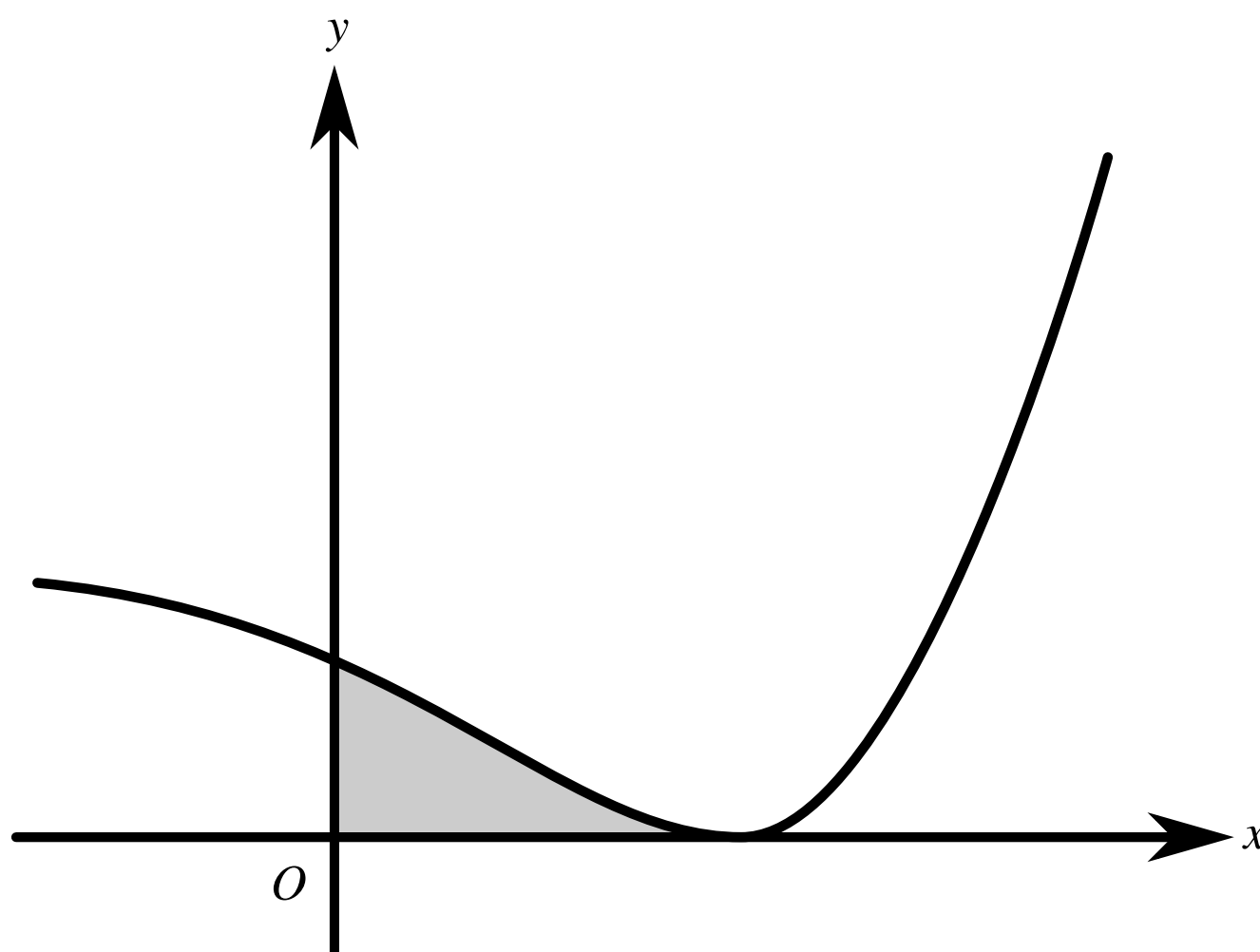


Figure 1: The curve $y = e^{3x} - 6e^{2x} + 32$.

Part A x -coordinate

Give the exact x -coordinate of the minimum point and verify that the y -coordinate of the minimum point is 0.

The following symbols may be useful: x

Part B Area of shaded region

Find the exact area of the shaded region enclosed by the curve and the coordinate axes.

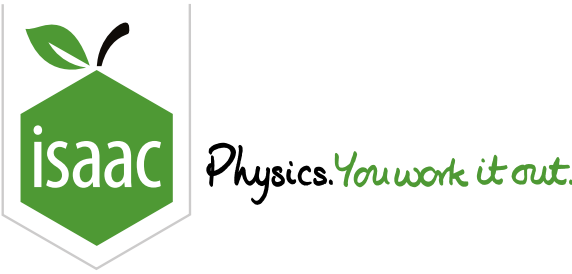
The following symbols may be useful: $\ln()$, $\log()$

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Integration by Substitution 3

Pre-Uni Maths for Sciences 5.4.3

A Level
P P P

Part A Integrate $\frac{3}{(z+1)^2}$

Find $\int_0^2 \frac{3}{(z+1)^2} dz$.

Part B Integrate $\frac{e^{-\alpha x}}{(1+e^{-\alpha x})^4}$

Find $\int \frac{e^{-\alpha x}}{(1+e^{-\alpha x})^4} dx$, where α is a constant.

The following symbols may be useful: C, alpha, c, e, k, x

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Integration by Substitution 4

Pre-Uni Maths for Sciences 5.4.4

A Level
P P P

Part A Integrate $\frac{1}{b(x+a)}$

Find $\int_0^a \frac{1}{b(x+a)} dx$, where a and b are constants.

The following symbols may be useful: a , b , k , x

Part B Integrate $\frac{x}{1+x^2}$

Find $\int_0^1 \frac{x}{1+x^2} dx$.

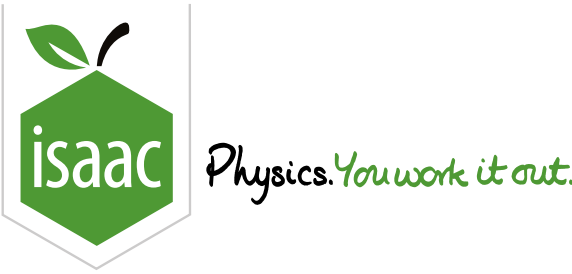
The following symbols may be useful: k , x

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Integration by Substitution 2ii

A Level

P

P

P

Use the substitution $u = 1 + \ln(x) + x$ to find $\int \frac{3(x + 1)(1 - \ln(x) - x)}{x(1 + \ln(x) + x)}dx$.

The following symbols may be useful: c , I , c , k , x

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Integration by Parts 1

Pre-Uni Maths for Sciences 5.4.8

A Level
P P P

Part A Integrate $(s + 1)e^{-\frac{s}{\alpha}}$

Find $\int_0^\alpha (s + 1)e^{-\frac{s}{\alpha}} \mathrm{d}s$, where α is a constant.

The following symbols may be useful: alpha, e

Part B Integrate $x \sin x$

Find $\int_0^{\frac{\pi}{2}} x \sin x \mathrm{d}x$.

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Integration by Parts 2

Pre-Uni Maths for Sciences 5.4.9

A Level
P P P

Part A Integrate $z^2 \cos z$

Find $\int z^2 \cos z \, dz$.

The following symbols may be useful: c , c , k , z

Part B Integrate $3t^2 \ln t$

Find $\int_1^2 3t^2 \ln t \, dt$.

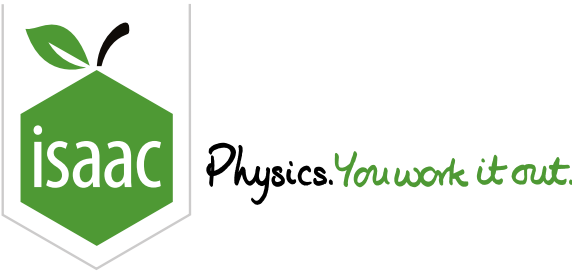
The following symbols may be useful: $\ln()$, t

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Integration by Parts 4

Pre-Uni Maths for Sciences 6.5.2

A Level
P P P

Find $\int \frac{\ln(\sin x)}{\cos^2 x} dx$.

The following symbols may be useful: C , c , k , x

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Integrating 1/x

A Level



The usual rule for integrating polynomials $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ breaks down for x^{-1} . In this question we explore the properties of this integral. This will give us insight into common $(\ln x)$ logarithms and also the exponential function (e^x) .

Part A Logarithms

The fundamental property of a logarithm is that $\log ab = \log a + \log b$ regardless of your choice of base. Here we will define

$$L(y) = \int_1^y \frac{1}{x} dx$$

and show that our function $L(y)$ does indeed have the property of a logarithm.

To work out $L(ab)$ we will break the integral into two sections

$$L(ab) = \int_1^{ab} \frac{1}{x} dx = \int_1^a \frac{1}{x} dx + \int_a^{ab} \frac{1}{x} dx$$

so

$$L(ab) = L(a) + \int_a^{ab} \frac{1}{x} dx.$$

Which substitution will be most suitable to express $\int_a^{ab} \frac{1}{x} dx$ in terms of our function L ?

- ☐ $z = \frac{x}{b}$
- ☐ $z = ax$
- ☐ $z = bx$
- ☐ $z = \frac{x}{a}$

Once the appropriate substitution has been made, we find that $\int_a^{ab} \frac{1}{x} dx$ is equal to

- ☐ $\int_1^b \frac{a}{z} dz = a L(b)$
- ☐ $\int_1^{ab} \frac{1}{z} dz = L(ab)$
- ☐ $\int_1^a \frac{1}{z} dz = L(a)$
- ☐ $\int_1^b \frac{1}{z} dz = L(b)$

You have shown that $L(ab) = L(a) + L(b)$ and therefore that our function $L(z) = \int_1^z \frac{1}{z} dz$ is some kind of logarithm.

Part B Logarithm base

As in the previous section, we write $L(a) = \int_1^a \frac{1}{x} dx$. By definition, this means that $\frac{dL}{dx} = \frac{1}{x}$.

We already know that $L(x)$ has the properties of a logarithm. This means that if $y = L(x)$, then $x = g^y$ for some unknown constant g , which will be the base of the logarithms.

Remembering that $\frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1}$, we can combine the information above to show that $\frac{d g^y}{dy}$ is equal to

- ☐ $\frac{1}{y}$
- ☐ g^y
- ☐ y
- ☐ $\frac{1}{g^y}$

One of the defining features of the exponential function e^x is that $\frac{d e^x}{dx} = e^x$. The number e is also the base of the natural logarithms $\ln(x)$.

It follows that $g^x = e^x$ and that accordingly $L(x) = \log_e x = \ln x$.

We therefore know (at least for positive x) that $\int \frac{1}{x} dx = \ln x + C$.

Part C Expansion first term

In this section, we investigate the exponential function and how it might be evaluated.

We make an assumption that the function $y = e^x$ can be written as polynomial with fixed co-efficients a_n :

$$e^x = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots + a_n x^n + \dots$$

What must be the value of a_0 ?

Part D Exponential expansion co-efficient

In this section, we continue to investigate the exponential function and how it might be evaluated.

We assume that the function $y = e^x$ can be written as polynomial with fixed co-efficients a_n :

$$e^x = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots + a_n x^n + \dots$$

One property of the exponential function e^x is that $\frac{d e^x}{d x} = e^x$.

Using this information, write an expression for $\frac{a_n}{a_{n-1}}$.

The following symbols may be useful: n

Part E Exponential expansion first terms

In this section, we continue to investigate the exponential function and how it might be evaluated.

We assume that the function $y = e^x$ can be written as polynomial with fixed co-efficients a_n :

$$e^x = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots + a_n x^n + \dots$$

Use the answers to the previous questions to write the expansion of e^x up to and including the x^4 term. Do **not** use factorial notation in your answer (write 24 rather than 4!).

The following symbols may be useful: e, x

Part F Value of e

Use your expansion up to and including the x^4 term from the last question to calculate the value of e to three significant figures.
