Differentiating Sums and Differences 3

A Level

C C C

Pre-Uni Maths for Sciences J1.9

Part A Velocity if $s = ut + bt^2$

A particle is moving in one dimension. Its displacement s at time t is given by $s = ut + bt^2$, where u and b are constants. The velocity v of the particle at time t is given by the rate of change of displacement with time, i.e. $v = \dfrac{\mathrm{d}s}{\mathrm{d}t}$.

Find an expression for the velocity.

The following symbols may be useful: `b, t, u, v`

Part B Acceleration if $s = ut + bt^2$

A particle is moving in one dimension. Its displacement s at time t is given by $s = ut + bt^2$, where u and b are constants. The acceleration a of the particle at time t is given by the rate of change of velocity with time.

Find an expression for the acceleration.

The following symbols may be useful: `a, b, t, u`

Part C Velocity if $x = \alpha t + \beta t^3$

The displacement of a body at time t is given by $x = \alpha t + \beta t^3$ where $\alpha = 4\,\mathrm{m\,s^{-1}}$ and $\beta = 5\,\mathrm{m\,s^{-3}}$. Use the fact that the velocity is the rate of change of displacement to find the velocity of the body at $t = 2\,\mathrm{s}$.

Find the velocity of the body at $t = 2\,\mathrm{s}$.

Part D Acceleration if $x = \alpha t + \beta t^3$

The displacement of a body at time t is given by $x = \alpha t + \beta t^3$ where $\alpha = 4\,\mathrm{m\,s^{-1}}$ and $\beta = 5\,\mathrm{m\,s^{-3}}$. Use the fact that the acceleration is the rate of change of velocity to find the acceleration of the body at $t = 2\,\mathrm{s}$.

Find the acceleration of the body at $t = 2\,\mathrm{s}$.

Created for isaacphysics.org by Julia Riley

Acceleration f(t) 2ii

A Level

A particle P travels in a straight line. The velocity of P at time t seconds after it passes through a fixed point A is given by $(0.6t^2 + 3)\,\mathrm{m\,s^{-1}}$.

Part A Velocity at A

Find the velocity of P when it passes through A. Give your answer to 1 significant figure.

Part B Displacement at $t = 1.5\,\mathrm{s}$

Find the displacement of P from A when $t = 1.5\,\mathrm{s}$. Give your answer to 3 significant figures.

Part C Velocity at $a = 6\,\mathrm{m\,s^{-2}}$

Find the velocity of P when it has an acceleration of $6\,\mathrm{m\,s^{-2}}$. Give your answer to 2 significant figures.

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Gameboard:

STEM SMART Single Maths 22 - Variable Acceleration

Acceleration f(t) 1i

A Level

P P P

A particle P moves in a straight line. At time $t\,\mathrm{s}$ after passing through a point O of the line the displacement of P from O is $x\,\mathrm{m}$ where $x = 0.06t^3 - 0.45t^2 - 0.24t$.

Part A Velocity of P

Find the velocity of P when $t = 0\,\mathrm{s}$.

Part B Acceleration of P

Find the acceleration of P when $t = 0\,\mathrm{s}$.

Part C Minimum velocity of P

Find the speed of P when it is at its minimum velocity. Give your answer to 3 significant figures.

Part D Positive value of t

Find the positive value of t when the direction of motion of P changes. Give your answer to 3 significant figures.

Gameboard:

STEM SMART Single Maths 22 - Variable Acceleration

Acceleration f(t) 3i

A Level

A particle starts from rest at the point A and travels in a straight line. The displacement $s\,\mathrm{m}$ of the particle from A at time $t\,\mathrm{s}$ after leaving A is given by

$$s = 0.001t^4 - 0.04t^3 + 0.6t^2, \quad \text{for } 0 \leq t \leq 10$$

Part A Velocity $t = 10$

Find the velocity of the particle when $t = 10$.

Part B Velocity $t = 20$

The acceleration of the particle for $t \geq 10$ is $(0.8 - 0.08t)\,\mathrm{m\,s^{-2}}$.

Calculate the velocity of the particle when $t = 20$.

Part C Displacement $t = 20$

Find the displacement from A of the particle when $t = 20$. Give your answer to 3 significant figures.

Gameboard:

STEM SMART Single Maths 22 - Variable Acceleration

Acceleration f(t) 4i

A Level

A car is travelling along a straight horizontal road with velocity $32.5\,\mathrm{m\,s^{-1}}$. The driver applies the brakes and the car decelerates at $(8 - 0.6t)\,\mathrm{m\,s^{-2}}$, where $t\,\mathrm{s}$ is the time which has elapsed since the brakes were first applied.

Part A Velocity

Find an expression for the velocity of the car when it is decelerating.

The following symbols may be useful: `t`

Part B Time taken

Find the time taken to bring the car to rest.

Part C Distance travelled

Find the total distance travelled by the car whilst it is decelerating.

Gameboard:
STEM SMART Single Maths 22 - Variable Acceleration

Calculus and Vectors 1ii

A Level

A particle P of mass $0.2\,\mathrm{kg}$ moves on a smooth horizontal plane. Initially it is projected with velocity $0.8\,\mathrm{m\,s^{-1}}$ from a fixed point O towards another fixed point A. At time $t\,\mathrm{s}$ after projection, P is $x\,\mathrm{m}$ from O and is moving with velocity $v\,\mathrm{m\,s^{-1}}$, with the direction OA being positive. A force of $(1.5t - 1)\,\mathrm{N}$ acts on P in the direction parallel to OA.

Part A Expression for v

Find an expression for v in terms of t.

(Use fractions rather than decimals when entering your answer.)

The following symbols may be useful: `t, v`

Part B Time when $v = 0.8\,\mathrm{m\,s^{-1}}$

Find the time (in seconds) when the velocity of P is next $0.8\,\mathrm{m\,s^{-1}}$.

Give your answer to 3 sf.

Part C Times through O

Find the first time when P subsequently passes through O.

Find the second time when P subsequently passes through O.

Part D Distance in third second

Find the distance P travels in the third second of its motion.

Gameboard:

STEM SMART Single Maths 22 - Variable Acceleration

General Kinematics 1ii

A Level

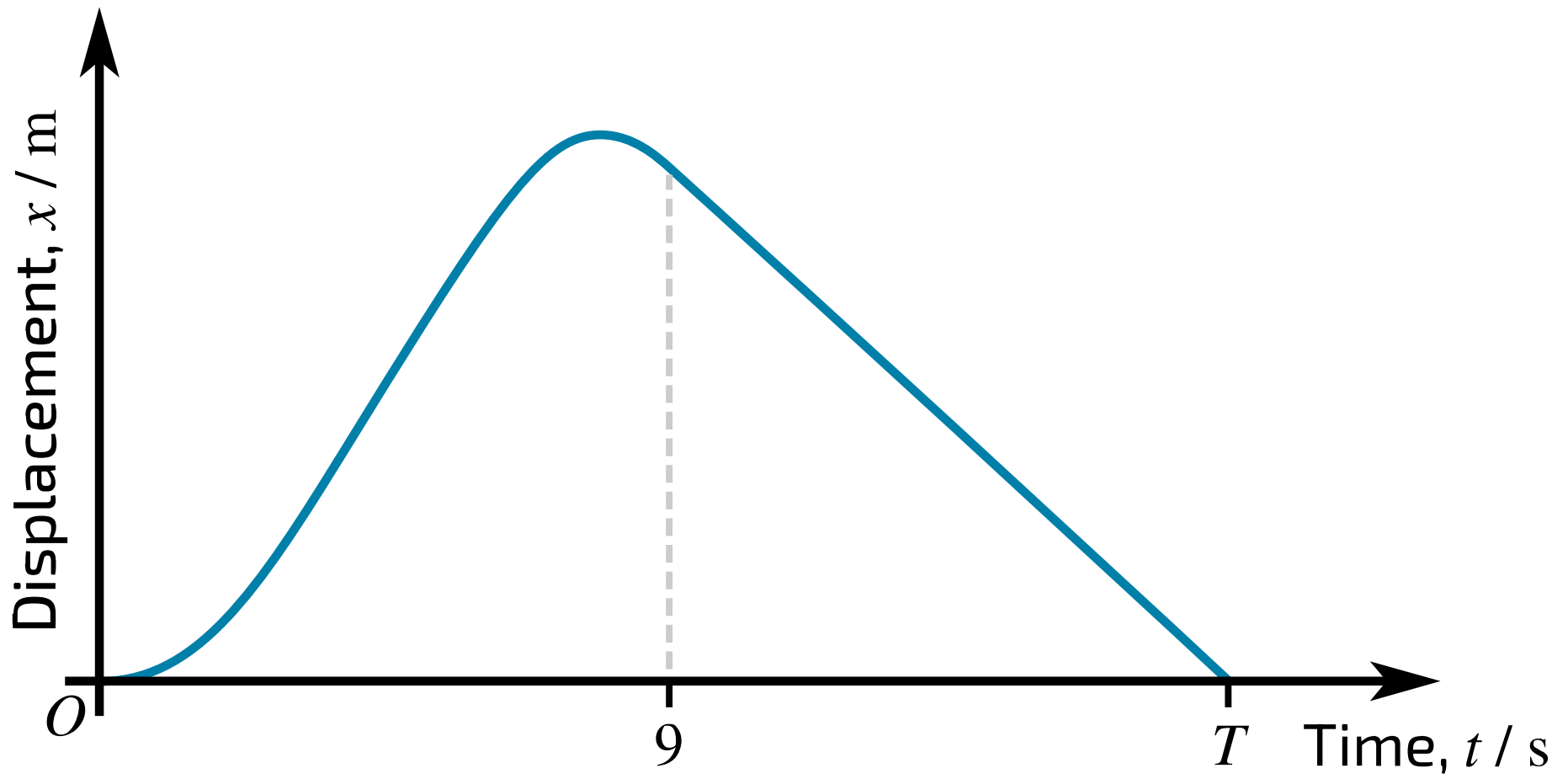

Figure 1: Distance-time graph showing the motion of the particle between A and B.

A particle travels along a straight line from a point A to a point B and then returns to A along the same straight line. During the first $9\,\mathrm{s}$ of the motion the displacement $x\,\mathrm{m}$ of the particle from A at time $t\,\mathrm{s}$ is given by $x = t^2 - \frac{1}{12}t^3$. The particle then travels at a constant speed of $2\frac{1}{4}\,\mathrm{m\,s^{-1}}$ until it reaches A at time $t = T$.

Part A Velocity expression

Find an expression for the velocity of the particle during the first $9\,\mathrm{s}$ of its motion.

The following symbols may be useful: `t`

Part B Time and distance

Find the time it takes the particle to reach B.

$\square$ s

Find the distance AB in metres. Give you answer as an exact fraction, simplified as far as possible.

$\square$ m

Part C Time taken

Find the value of T.

$\square$ s

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Gameboard:

STEM SMART Single Maths 22 - Variable Acceleration

Kinematics & Calculus

A Level

C C C

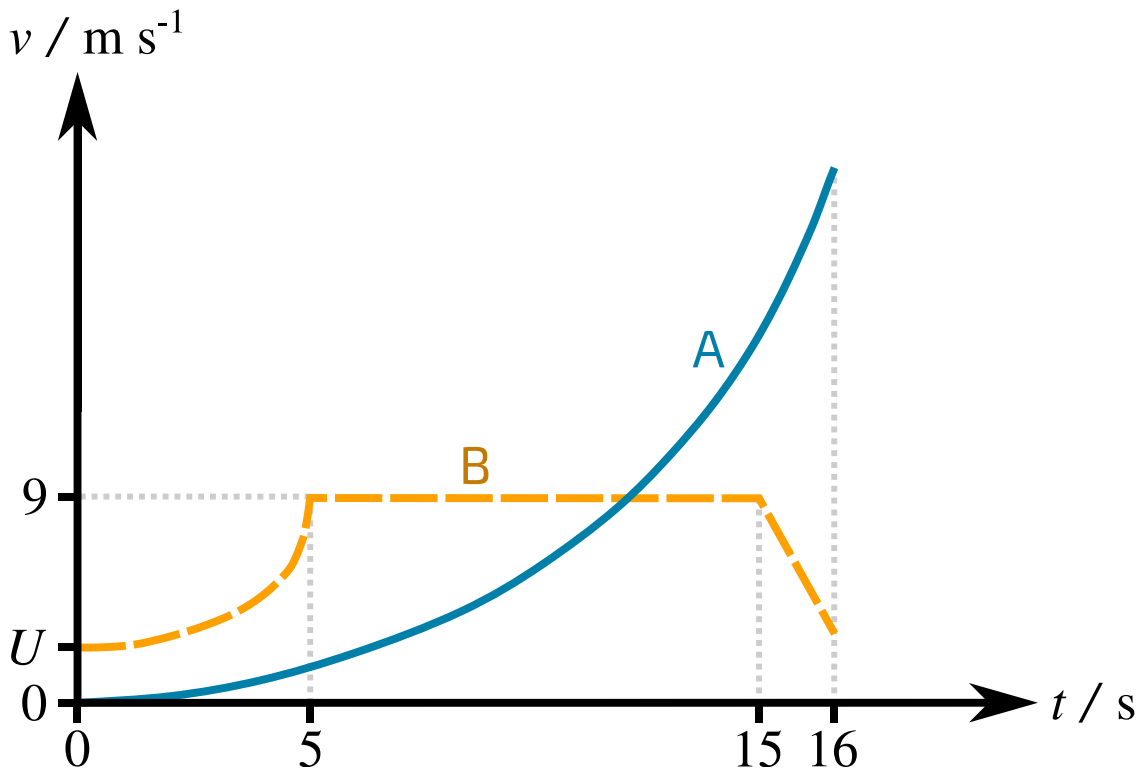

Figure 1: Velocity-time graph of the motion of two particles A and B along the same straight line.

The diagram shows the (t, v) graphs for two particles A and B which move on the same straight line. The units of v and t are $\mathrm{m\,s^{-1}}$ and s respectively. Both particles are at the point S on the line when $t = 0$. The particle A is initially at rest, and moves with acceleration $0.18t\,\mathrm{m\,s^{-2}}$ until the two particles collide when $t = 16\,\mathrm{s}$. The initial velocity of B is $U\,\mathrm{m\,s^{-1}}$ and B has variable acceleration for the first five seconds of its motion. For the next ten seconds of its motion B has a constant velocity of $9\,\mathrm{m\,s^{-1}}$; finally B moves with constant deceleration for one second before it collides with A.

Part A t for same velocity

Calculate the value of t at which the two particles have the same velocity.

Part B Calculate U

For $0 \leq t \leq 5$ the distance of B from S is $(Ut + 0.08t^3)\,\mathrm{m}$.

Calculate U.

Part C Distance from S

Calculate how far B is from S when $t = 5\,\mathrm{s}$.

Part D v_B when $t = 16\,\mathrm{s}$

Calculate the velocity of B when $t = 16\,\mathrm{s}$.

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