

# Functions and Algebra 3i

Subject & topics

Maths

Status

Not started

Stage & difficulty

A Level Practice 2

The functions  $f$  and  $g$  are defined for all real values of  $x$  by

$$f(x) = |2x + a| + 3a \quad \text{and} \quad g(x) = 5x - 4a,$$

where  $a$  is a positive constant.

Part A

Range

Find the range of  $f(x)$ .

Fill in the inequality below.

Items:

$f(x)$

$<$

$\leq$

$>$

$\geq$

$< f(x) <$

$\leq f(x) \leq$

$< f(x) \text{ or } f(x) <$

$\leq f(x) \text{ or } f(x) \leq$

$\frac{a}{3}$

$\frac{a}{2}$

$a$

$2a$

$3a$

$4a$

$0$

$-\frac{a}{3}$

$-\frac{a}{2}$

$-a$

$-2a$

Part B

Inverse function of  $f(x)$

Fill in the blanks to explain why the function  $f(x)$  has no inverse.

The function  $f(x)$  is not . For example,  $f(0) = 4a$  and  $f(\text{})$  also equals  $4a$ . Hence,  $f(x)$  has no inverse.

Items:

- 
- 
- 
- 
- 
- 
- 
- 

Part C

Inverse function of  $g(x)$

Find an expression for  $g^{-1}(x)$ .

The following symbols may be useful: a, x

Part D

Solve for  $x$

Solve for  $x$  the equation  $g(f(x)) = 31a$ . Give your solutions in ascending order.

- $x = \text{} a$
- $x = \text{} a$

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# Inverse Quadratic Function

Subject & topics

Maths | Functions | Graph Sketching

Status

Not started

Stage & difficulty

A Level Practice 2

Figure 1 shows the graph of  $y = f(x)$ , where

$$f(x) = 2 - x^2, \quad x \leq 0$$

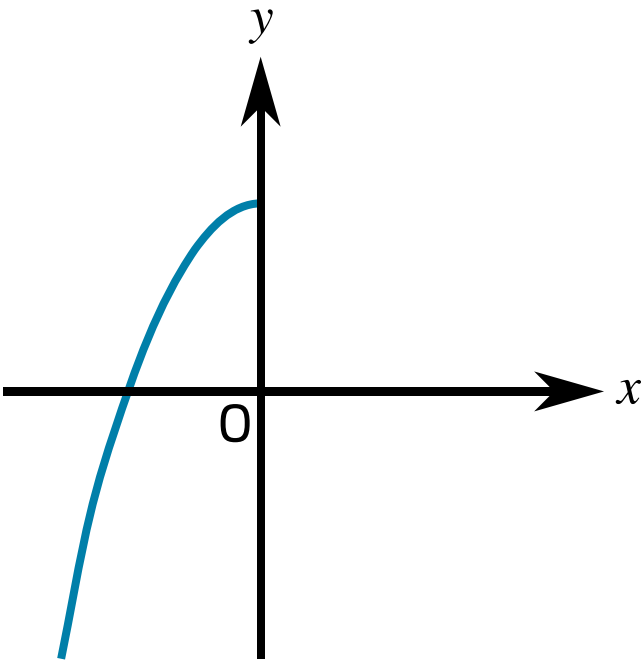


Figure 1: The graph of  $y = f(x)$ , for  $x \leq 0$ .

Part A

$f^2(-3)$

Evaluate  $f^2(-3)$ .

Need some help?

Part B

$f^{-1}(x)$

Find an expression for  $f^{-1}(x)$ .

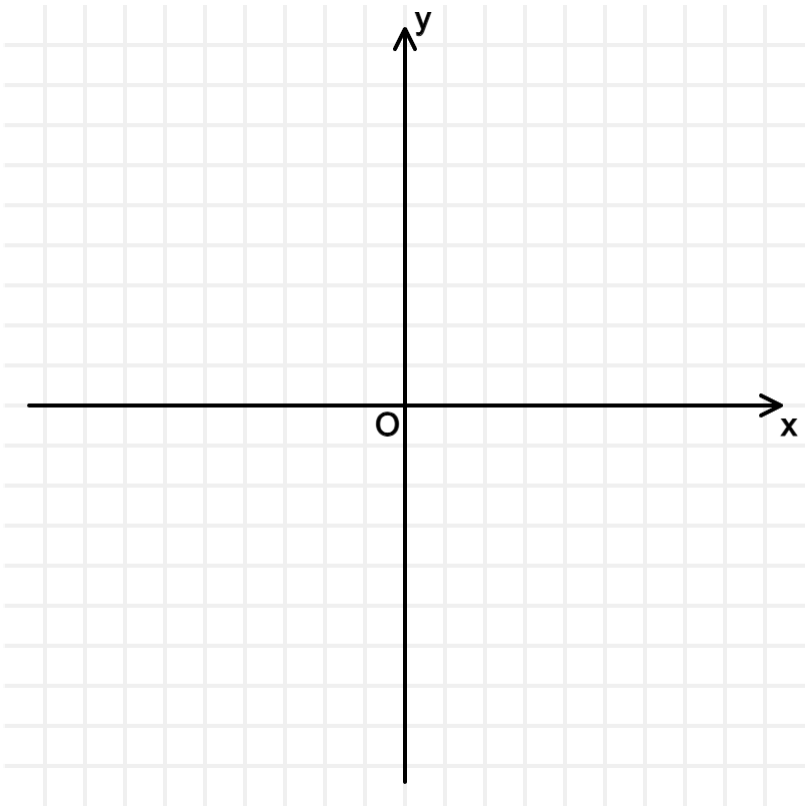
The following symbols may be useful: f, x, y

Need some help?

Part C

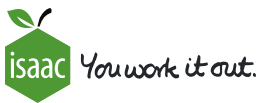
Graph of  $f^{-1}(x)$

Sketch the graph of  $y = f^{-1}(x)$ .



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## Function Types and Inverses

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

A Level Challenge 1

Figure 1 shows five different graphs, A, B, C, D and E, each for values of  $x$  such that  $-a \leq x \leq a$  where  $a$  is a constant.

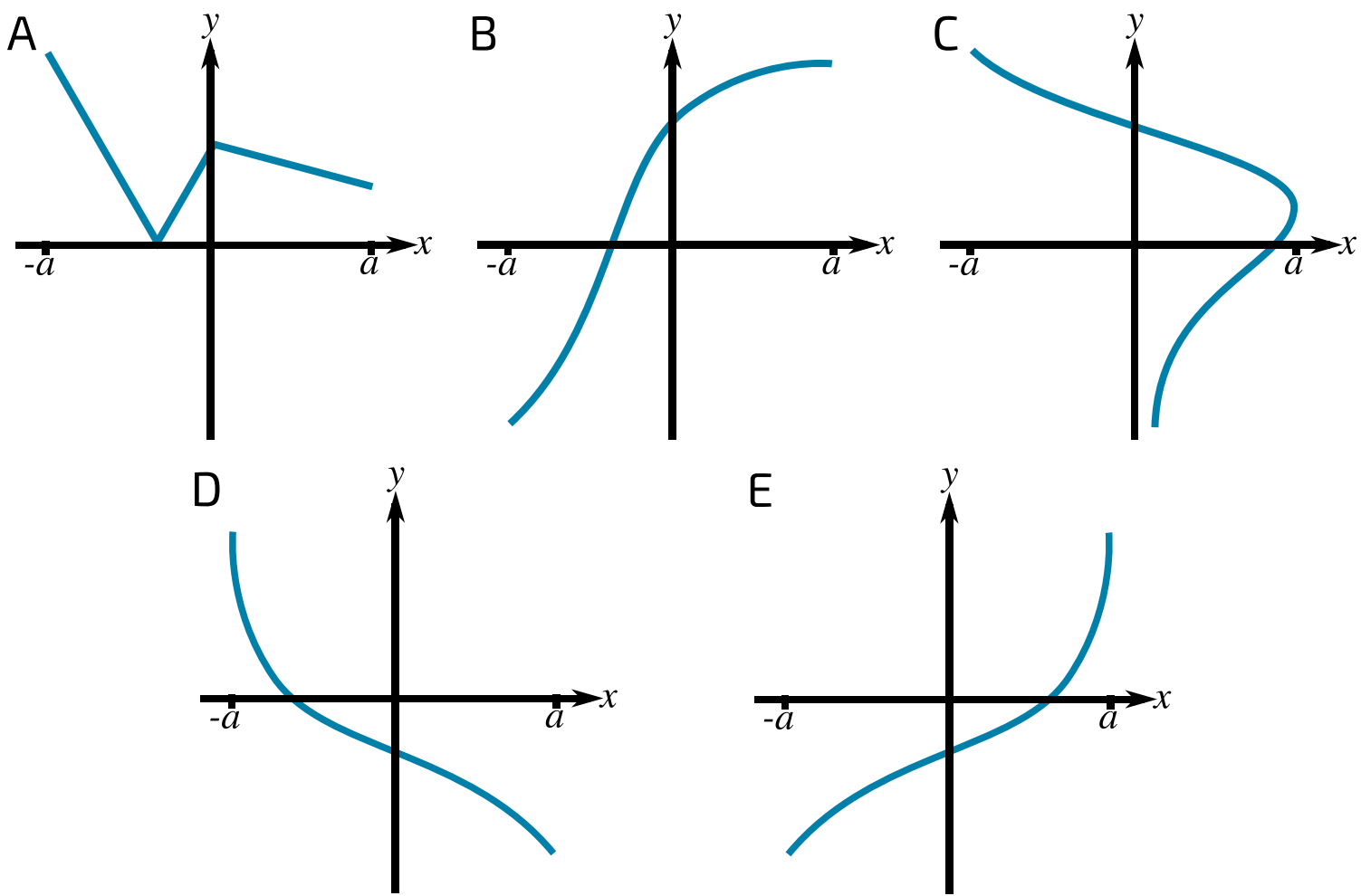


Figure 1: The set of five graphs, labelled A, B, C, D and E

Part A

Function

Which diagram does not show the graph of a function?

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E

Part B

One-to-one Function

Which diagram shows the graph of a function that is not one-to-one?

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E

Part C

Inverses

It is given that two of the diagrams illustrate functions that are inverses of each other. Identify one of these two diagrams.

☐

A

☐

B

☐

C

☐

D

☐

E

Part D

Sketch

The graph in E has equation  $y = f(x)$ . Sketch the graph of  $y = |f(x)|$ .

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.

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Question deck:

**STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities**





# Combined Transformations

Subject & topics

Maths | Functions | General Functions

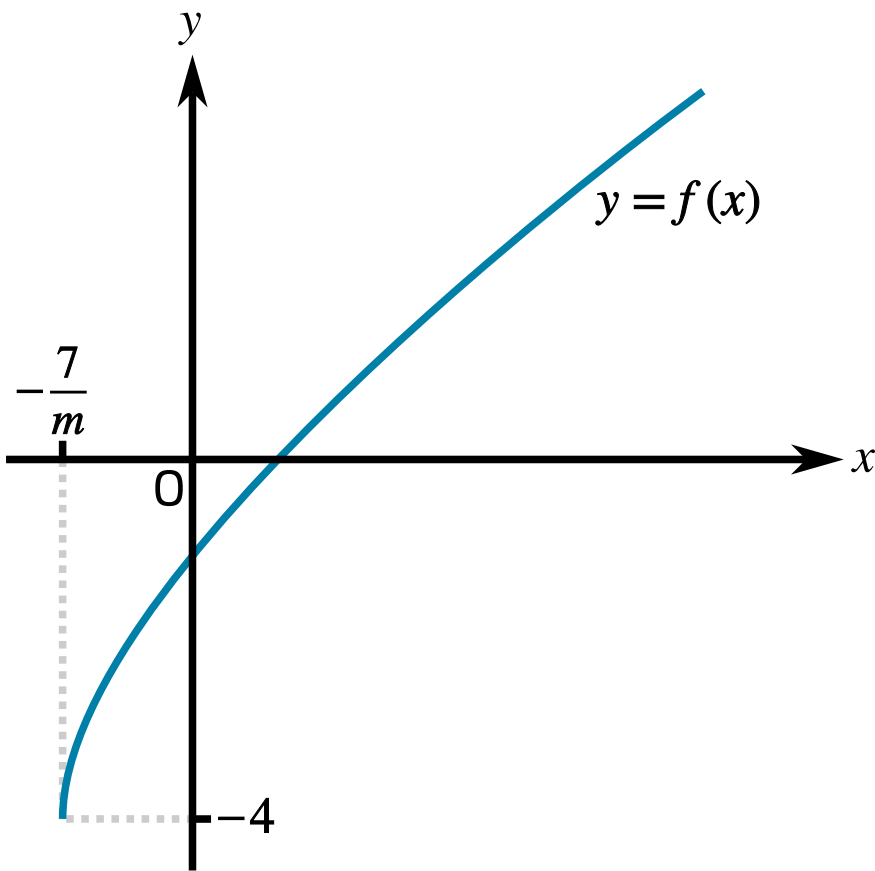
Status

Not started

Stage & difficulty

A Level Practice 2

The function  $f$  is defined by  $f(x) = \sqrt{mx + 7} - 4$ , where  $x \geqslant -\frac{7}{m}$  and  $m$  is a positive constant. **Figure 1** shows the curve  $y = f(x)$ .



**Figure 1:** The curve  $y = f(x)$

Part A

Translation of the curve  $y = \sqrt{x}$

ence of transformations maps the curve  $y = \sqrt{x}$  to the curve  $y = f(x)$ . Give details of these transformations.

ible items

nslate the curve 4 units in the negative  $y$  direction.

atch the curve in the  $x$  direction by a factor of  $\frac{1}{m}$ .

nslate the curve 7 units in the negative  $x$  direction.

nslate the curve 4 units in the positive  $y$  direction.

nslate the curve 4 units in the negative  $x$  direction.

atch the curve in the  $x$  direction by a factor of  $m$ .

atch the curve in the  $y$  direction by a factor of  $\frac{1}{m}$ .

nslate the curve 7 units in the negative  $y$  direction.

nslate the curve 7 units in the positive  $x$  direction.

Need some help?

Part B

$f^{-1}(x)$

Find an expression for  $f^{-1}(x)$ .

The following symbols may be useful: f, m, x

Need some help?

Part C

Values of  $m$

It is given that the curves  $y = f(x)$  and  $y = f^{-1}(x)$  do not meet. Thus it can be deduced that neither curve meets the line  $y = x$ . Hence determine the possible values of  $m$ .

Construct your answer from the items below.

Items:

$m$

$<$

$\leq$

$>$

$\geq$

$< m <$

$\leq m \leq$

$> m \text{ or } m >$

$\geq m \text{ or } m \geq$

$-28$

$-14$

$-8$

$-7$

$-4$

$-2$

$-1$

$0$

$1$

$2$

$4$

$7$

$8$

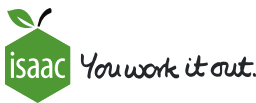
$14$

$28$

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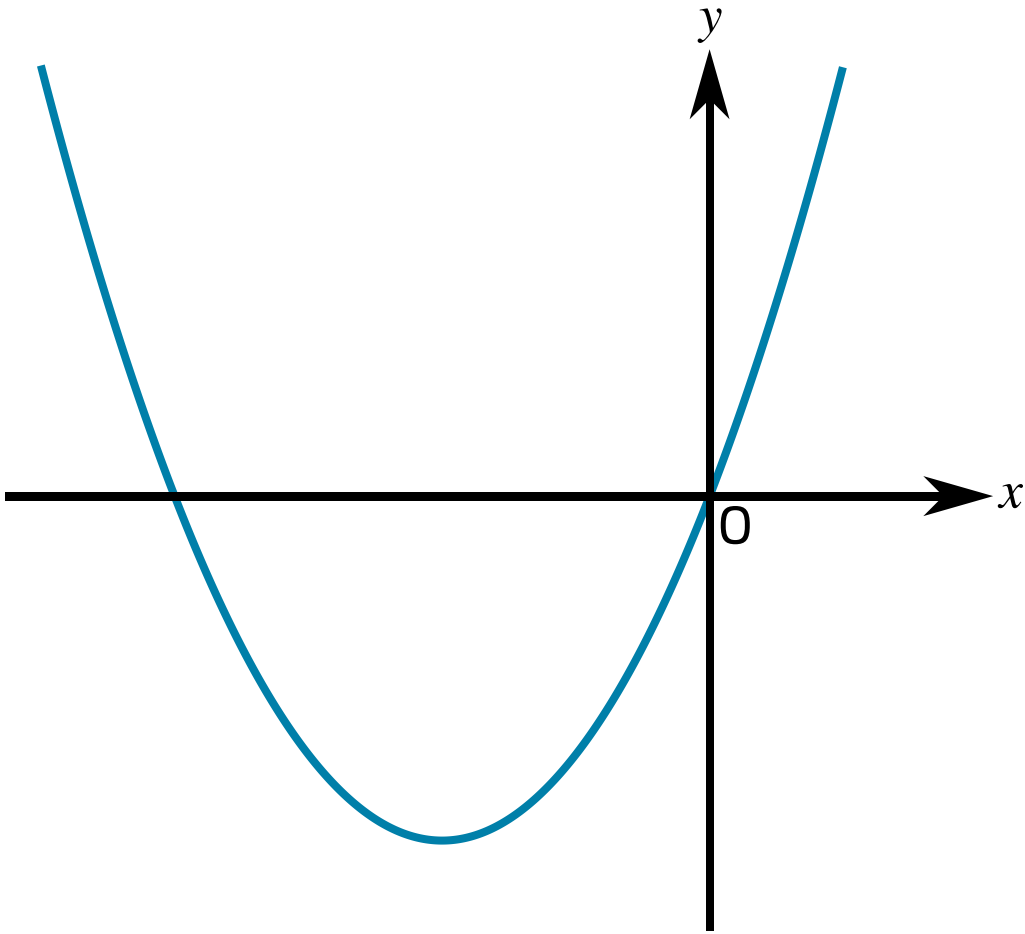
## Curve Sketching and Combined Transformations 3i

Subject & topics		
Maths		
Status	Stage & difficulty	<div><div></div><div></div><div></div></div>
Not started	A Level Practice 2	

The function  $f$  is defined for all real values of  $x$  by

$$f(x) = k(x^2 + 4x)$$

where  $k$  is a positive constant. **Figure 1** shows the curve with equation  $y = f(x)$ .



**Figure 1:** The graph of  $y = f(x)$

Part A

Transformations

Give a sequence of transformations that will transform the curve  $y = x^2$  to the curve  $y = f(x)$ .

Available items

- Translate the curve 2 units in the negative  $y$  direction.
- Translate the curve 2 units in the positive  $x$  direction.
- Translate the curve 2 units in the positive  $y$  direction.
- Translate the curve 2 units in the negative  $x$  direction.
- Stretch the curve in the  $y$  direction by a factor of  $\frac{1}{k}$ .
- Stretch the curve in the  $x$  direction by a factor of  $k$ .
- Translate the curve 4 units in the positive  $x$  direction.
- Stretch the curve in the  $x$  direction by a factor of  $\frac{1}{k}$ .
- Translate the curve 4 units in the negative  $y$  direction.
- Translate the curve 4 units in the positive  $y$  direction.
- Translate the curve 4 units in the negative  $x$  direction.
- Stretch the curve in the  $y$  direction by a factor of  $k$ .

Part B

Range

Find the range of  $f(x)$  as a single inequality in terms of  $k$ .

The following symbols may be useful:  $<$ ,  $<=$ ,  $>$ ,  $>=$ ,  $f(x)$ ,  $k$ ,  $x$ ,  $y$

Part C

Find  $k$

It is given that there are three distinct values of  $x$  which satisfy the equation  $|f(x)| = 20$ .

Find the value of  $k$ .

Part D

Solve  $|f(x)| = 20$

Using the value of  $k$  from part C, find the three distinct values of  $x$  which satisfy the equation  $|f(x)| = 20$ . Give any irrational values to 3 sf.

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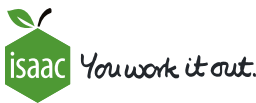
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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

## Modulus Functions 2

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

A Level Practice 2  
Further A Practice 1

Part A

Divergence of  $y = \left|\frac{1}{x}\right|$

Does the function  $y = \left|\frac{1}{x}\right|$  diverge anywhere? Where?

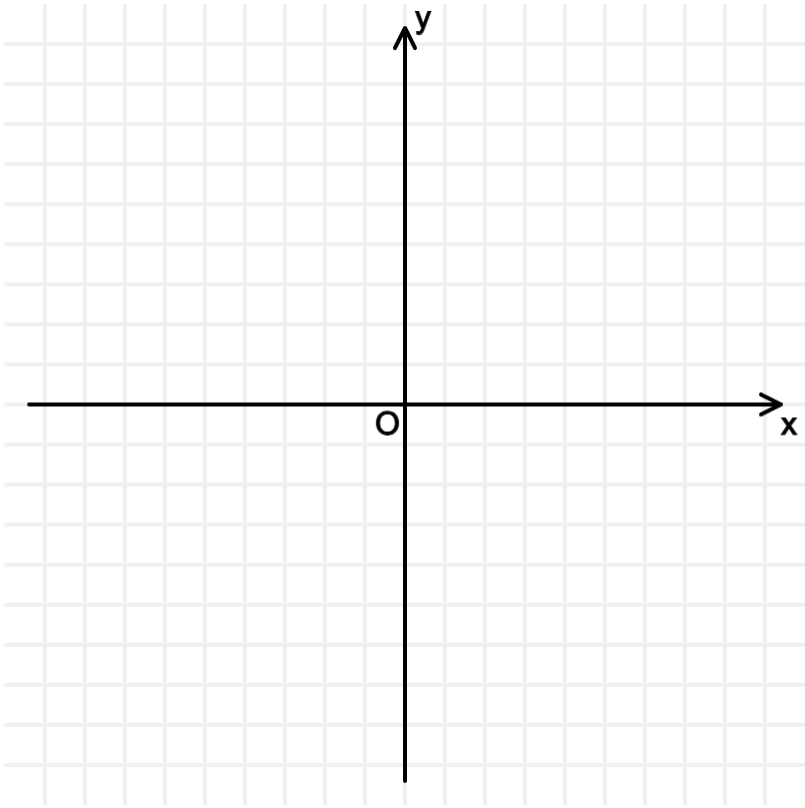
The following symbols may be useful: x

Need some help?

Part B

Graph of  $y = \left|\frac{1}{x}\right|$

Sketch the graph of  $y = \left|\frac{1}{x}\right|$ .



Need some help?

Part C

Divergence of  $y = \left|\frac{1}{x^2-4}\right|$

Does the function  $y = \left|\frac{1}{x^2-4}\right|$  diverge anywhere? Where?

The following symbols may be useful:  $x$ ,  $\pm$

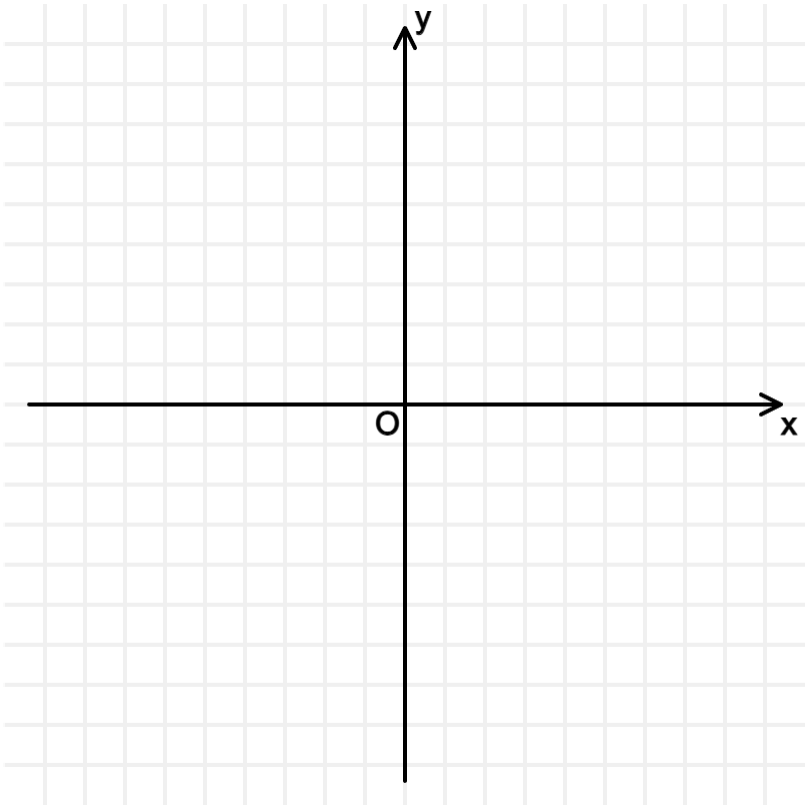
Need some help?



Part D

Graph of  $y = \left| \frac{1}{x^2 - 4} \right|$

Sketch the graph of  $y = \left| \frac{1}{x^2 - 4} \right|$ .



Need some help?

Part E

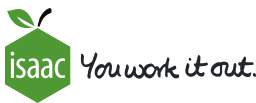
Solve equation graphically

Solve the equation  $|x| = \left| \frac{1}{x} \right|$  graphically and give the solution as a single expression.

The following symbols may be useful:  $x$ ,  $\pm$

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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

# Sketching a Cubic Modulus Function

Pre-Uni Maths for Sciences E4.7

Subject & topics

Maths | Functions | Graph Sketching

Status

Not started

Stage & difficulty

A Level Challenge 2

The function  $f$  is defined by  $f(x) = 2x^3 - x^2 - 4x - 4$ .

Part A

Quotient

Find the quotient when  $f(x)$  is divided by  $x - 2$ .

The following symbols may be useful: x

Need some help?

Part B

First stationary point of  $f(x)$

Find the co-ordinates and nature of the stationary point of  $f(x)$  with the larger  $x$ -coordinate.

The stationary point (  ,  ) is a .

Items:

- 
- 

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Part C

Second stationary point of  $f(x)$

Find the co-ordinates and nature of the stationary point of  $f(x)$  with the smaller  $x$ -coordinate.

The stationary point (  ,  ) is a .

Items:

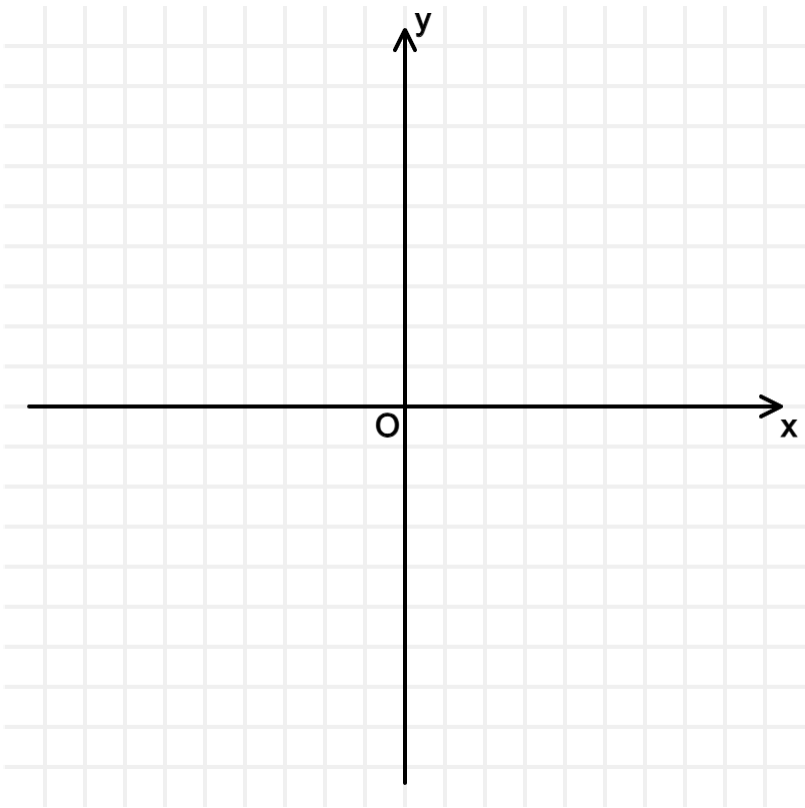
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Part D

Sketch of  $y = f(x)$

Sketch the graph of  $y = f(x)$ .



Need some help?

Part E

Sketch of  $y = |f(x)|$

Sketch the graph of  $y = |f(x)|$ , then pick the graph that corresponds to  $y = |f(x)|$  from the options below.

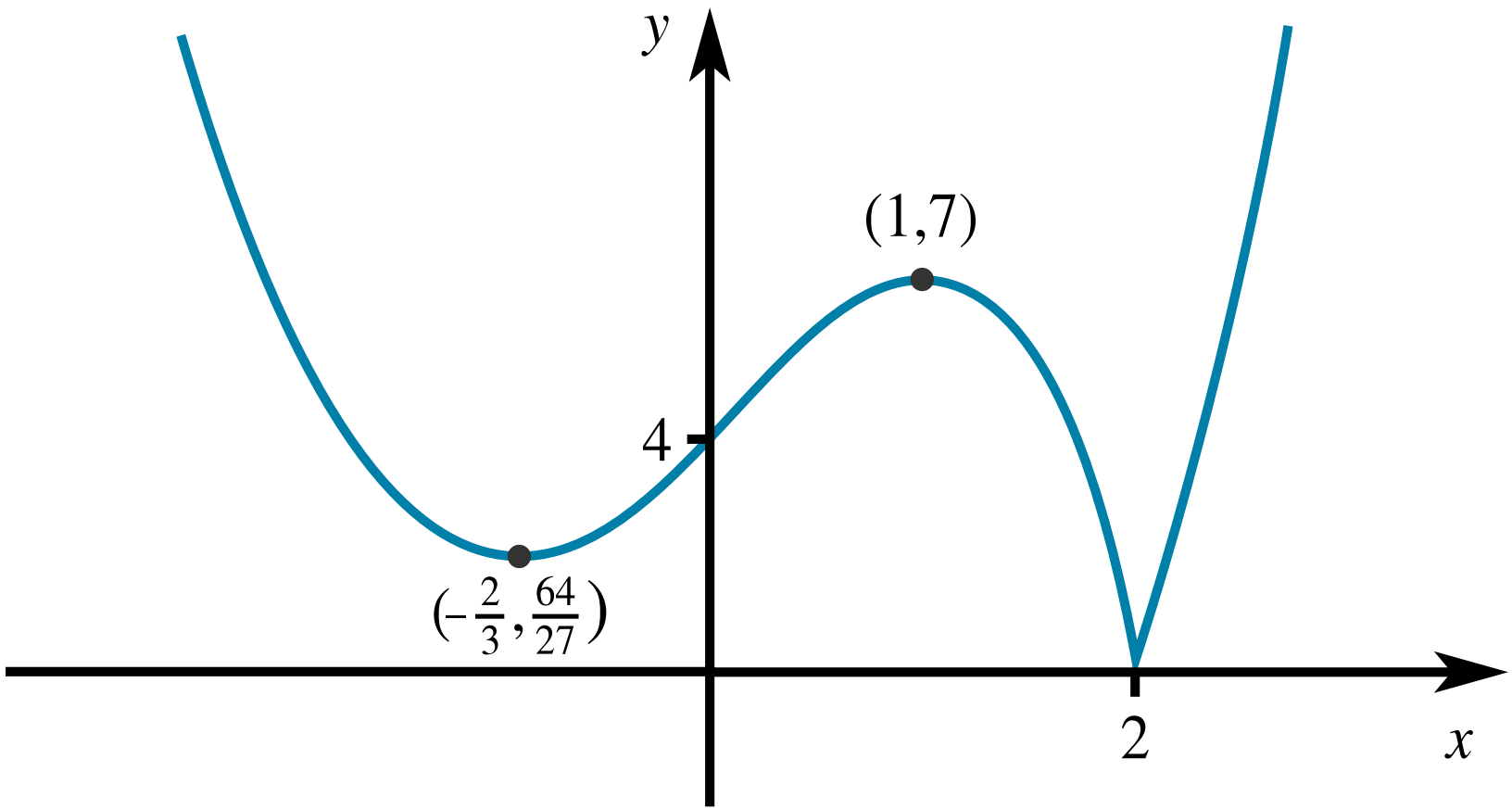


Figure 1: Option (i)

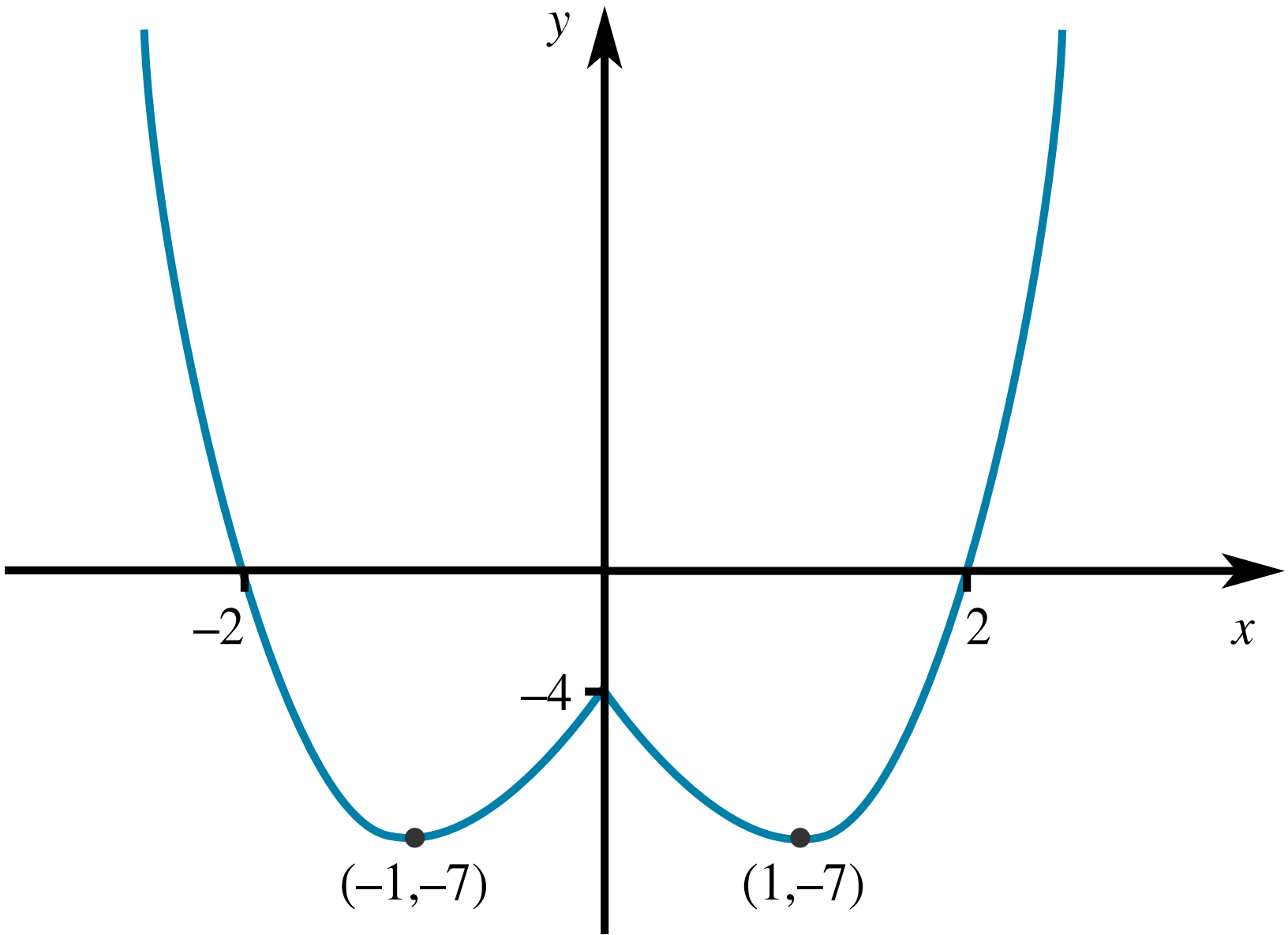


Figure 2: Option (ii)

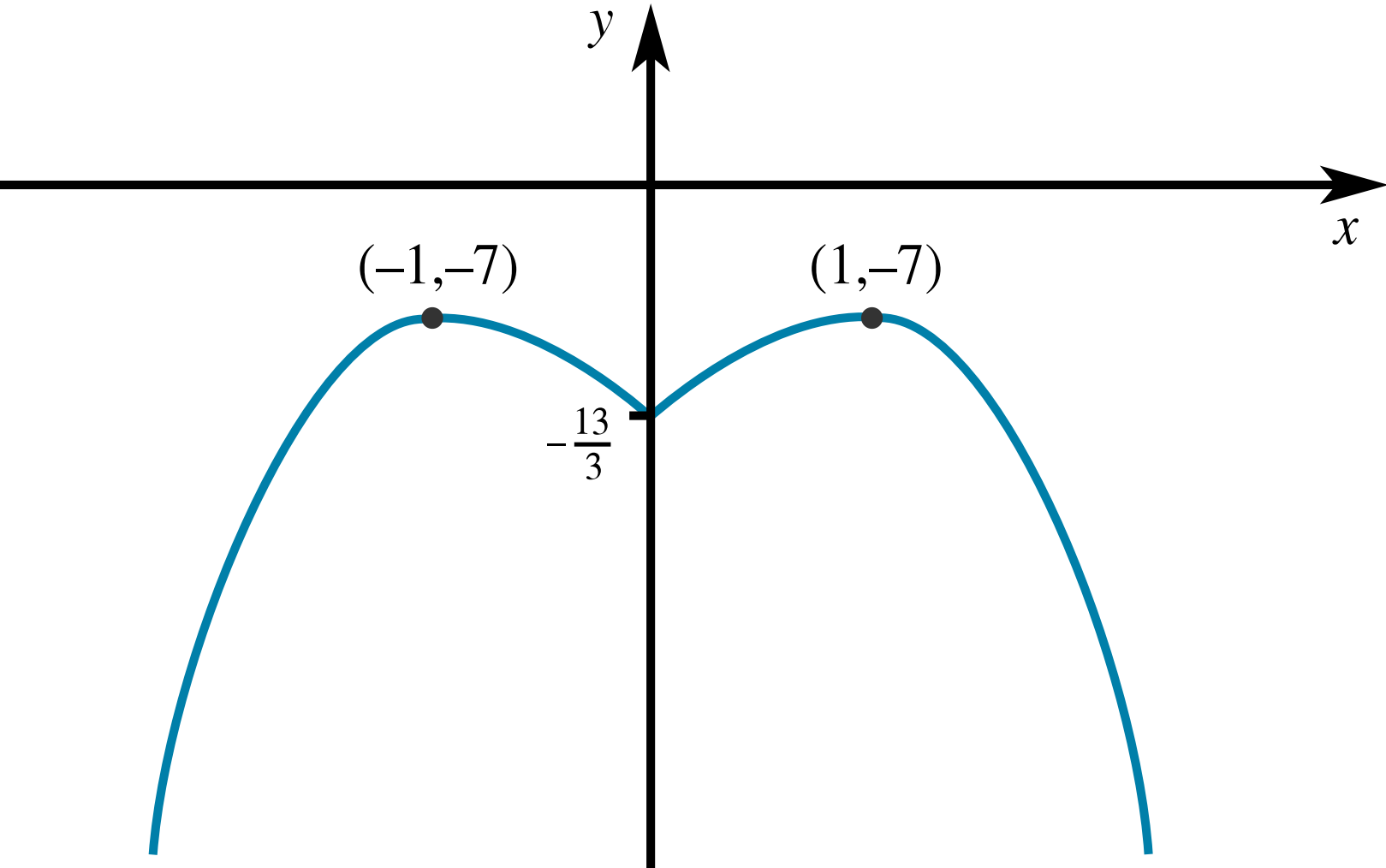


Figure 3: Option (iii)

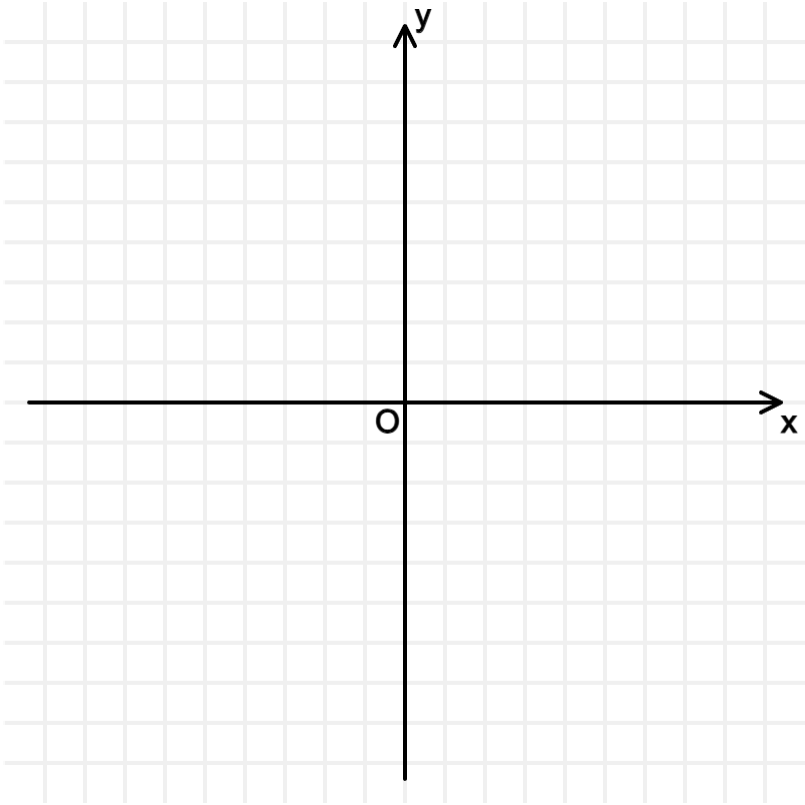
- ☐ Option (i)
- ☐ Option (ii)
- ☐ Option (iii)

Part F

Sketch of  $y = f(|x|)$

Sketch the graph of  $y = f(|x|)$ .

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.

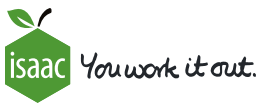


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Question deck:

**STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities**



# Rational Inequality

Pre-Uni Maths for Sciences A2.10

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

Further A Practice 2

Solve the inequality

$$\frac{x + 4}{x + 2} \leq \frac{x + 2}{x - 1}$$

giving your answer using set notation.

The solution is  $\{x : \text{ } \text{ } x \text{ } \text{ }\} \cup \{x : x \text{ } \text{ }\}$ .

Items:

-10

-9

-8

-7

-6

-5

-4

-3

-2

-1

0

1

2

3

4

5

6

7

8

9

10

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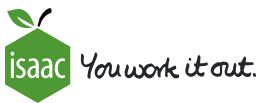
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STEM SMART Double Maths 23 - Defining Functions & Rational Inequalities

# Rational Modulus Inequality

Pre-Uni Maths for Sciences E4.10

Subject & topics

Maths | Functions | General Functions

Status

Not started

Stage & difficulty

Further A Practice 2

Solve the inequality

$$\frac{-x^2 - 5x + 24}{|x| + 3} > 2$$

giving your answer using set notation.

The solution is  $\{x :$  $x$  $\}.$

Items:

-10

-9

-8

-7

-6

-5

-4

-3

-2

-1

0

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