

Differentiation from First Principles 1



Pre-Uni Maths for Sciences J3.1 & J3.2

To differentiate a function $f(x)$ from first principles involves taking a limit. The derivative of $f(x)$ is given by the expression

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}.$$

Part A Differentiate x^3 from first principles

Differentiate x^3 from first principles. Drag and drop options into the spaces below.

In this question $f(x) = x^3$. Therefore, $f(x + h) =$. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\text{} - x^3}{h}.$$

Next, expand the brackets in the numerator and simplify:

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h}$$
$$f'(x) = \lim_{h \rightarrow 0} \frac{\text{}}{h} = \lim_{h \rightarrow 0} \text{}.$$

Finally, take the limit. As $h \rightarrow 0$, the term containing x^2 is unchanged (because it does not depend on h), but the terms containing xh and h^2 tend to 0. Therefore,

$$f'(x) = \text{}.$$

Items:

Part B Differentiate $2x^3 + 5$ from first principles

Differentiate $2x^3 + 5$ from first principles. Drag and drop options into the spaces below.

In this question $f(x) = 2x^3 + 5$. Therefore, $f(x + h) =$. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\text{} - (2x^3 + 5)}{h}.$$

Next, just as in part A, expand the brackets in the numerator. After simplification, this produces:

$$f'(x) = \lim_{h \rightarrow 0} \text{}.$$

Finally, take the limit. As $h \rightarrow 0$, the term containing x^2 is unchanged (because it does not depend on h), but the terms containing xh and h^2 tend to 0. Therefore,

$$f'(x) = \text{}.$$

Items:



Physics. *You work it out.*

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Differentiating Powers 1

Pre-Uni Maths for Sciences J1.1

A Level



Part A Differentiate $y = x^4$

Find $\frac{dy}{dx}$ if $y = x^4$.

The following symbols may be useful: x

Part B Gradient of $x = t^2$

Find the gradient of the curve $x = t^2$ at the points $t = 0$, $t = 3$ and $t = -3$.

Find the gradient at $t = 0$.

Find the gradient at $t = 3$.

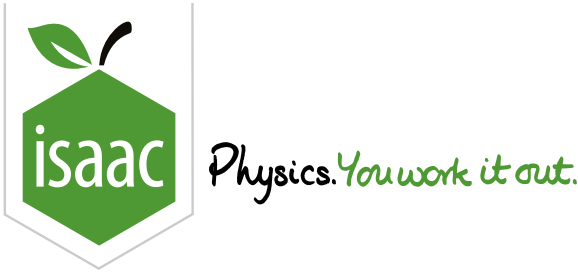
Find the gradient at $t = -3$.

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Differentiating Powers 3

Pre-Uni Maths for Sciences J1.4

A Level

P

P

P

Part A

Gradient of curve $t = 4s^{-\frac{3}{4}}$

Find the gradient of the curve $t = 4s^{-\frac{3}{4}}$ at the point $s = 16$.

Part B

First derivative of $x = bt^{\frac{3}{2}}$

Find $\frac{dx}{dt}$ if $x = bt^{\frac{3}{2}}$, where b is a constant.

The following symbols may be useful: b , t

Part C

Second derivative of $x = bt^{\frac{3}{2}}$

Find $\frac{d^2x}{dt^2}$ if $x = bt^{\frac{3}{2}}$, where b is a constant.

The following symbols may be useful: b , t

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Physics. *You work it out.*

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Differentiation (powers of x) 3ii

A Level



Find $\frac{dy}{dx}$ in each of the following cases.

Part A Algebraic fraction

$$y = \frac{(3x)^2 \times x^4}{x}.$$

The following symbols may be useful: x

Part B Cube root

$$y = \sqrt[3]{x}.$$

The following symbols may be useful: x

Part C Reciprocal

$$y = \frac{1}{2x^3}.$$

The following symbols may be useful: x

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Physics. *You work it out.*

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Differentiation (powers of x) 1ii

A Level
P P P

It is given that $y = 6x^3 + \frac{4}{\sqrt{x}} + 5x$.

Part A Find derivative

Find $\frac{dy}{dx}$.

The following symbols may be useful: x

Part B Find second derivative

Find $\frac{d^2y}{dx^2}$.

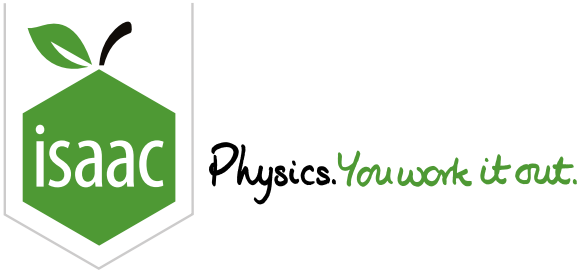
The following symbols may be useful: x

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Differentiating Sums and Differences 2

Pre-Uni Maths for Sciences J1.5

A Level

P

P

P

Part A Differentiate $ax^3 + \frac{b}{x} + c$

Differentiate $ax^3 + \frac{b}{x} + c$ with respect to x (a , b and c are constants).

The following symbols may be useful: a , b , c , x

Part B Differentiate $(2m + 3)(m - 1)$

Differentiate $(2m + 3)(m - 1)$ with respect to m .

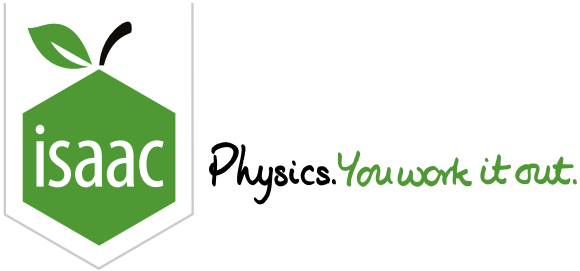
The following symbols may be useful: m

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Gradient Function: Tangents and Normals 1ii

A Level
P P P

Part A

Equation of tangent

Find the equation of the tangent to the curve $y = 7 + 6x - x^2$ at the point P where $x = 5$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

The following symbols may be useful: x , y

Part B

Mid-point coordinate

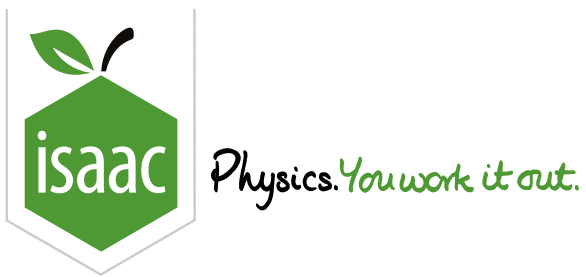
This tangent meets the x -axis at Q . Find the x -coordinate of the mid-point of PQ .

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Gradient Function: Tangents and Normals 1i



A curve has equation $y = x^2 + x$.

Part A Gradient

Find the gradient of the curve at the point where $x = 2$.

Part B Normal

Find the equation of the normal to the curve at the point for which $x = 2$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

The following symbols may be useful: x , y

Part C Find k

Find the smallest value of k for which the line $y = kx - 4$ is a tangent to the curve.

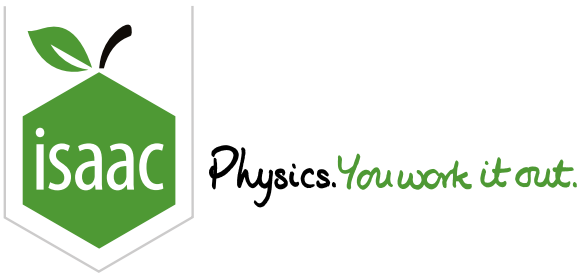
The following symbols may be useful: k

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Differentiating Powers 4

Pre-Uni Maths for Sciences J1.6

A Level

P

P

P

Part A

Derivative of $v = Bu^{-3}$

Find $\frac{dv}{du}$ if $v = Bu^{-3}$, where B is a constant.

The following symbols may be useful: B , u

Part B

Force if potential $V = \frac{q^2}{4\pi\epsilon_0 r}$

The electrostatic potential energy V of two equal charges q a distance r apart is given by $V = \frac{q^2}{4\pi\epsilon_0 r}$, where ϵ_0 and q are constants. The force between the two charges is given by $-\frac{dV}{dr}$; find an expression for this force.

The following symbols may be useful: ϵ_0 , π , q , r

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