Advanced Trig Identities 4ii

A Level

P P P

The acute angle A is such that $\tan A = 2$.

Part A $\operatorname{cosec} A$

Find the exact value of $\operatorname{cosec} A$.

The following symbols may be useful: A

Part B $\tan B$

The angle B is such that $\tan(A+B) = 3$. Using an appropriate identity, find the exact value of $\tan B$.

The following symbols may be useful: B

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Probability 3.7

A Level

A system consists of 1000 particles which can be in either one of two states A and B; any particle has a probability of 0.9 of being in state A. Find the following.

Part A 900 particles in state A

Find the probability that there are exactly 900 particles in state A. Give your answer to 3 sf.

Part B 110 particles in state B

Find the probability that there are exactly 110 particles in state B. Give your answer to 3 sf.

Part C Between 880 and 920 particles in state A

Find the probability that there are more than 880 but less than 920 particles in state A. Give your answer to 3 sf.

Part D Less than 120 in state B

Find the probability that there are less than 120 particles in state B. Give your answer to 3 sf.

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Modulus 3i

A Level

P P P

Part A $|x - 250| < 10$

Solve the inequality $|x - 250| < 10$, and give the upper bound in the form $x < a$ or $x \leq a$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `x`

Solve the inequaltity $|x - 250| < 10$, and give the lower bound in the form $x > a$ or $x \geq a$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `x`

Part B $|1.02^n - 250| < 10$

Hence determine the range of the integers n which satisfy the inequality

$$|1.02^n - 250| < 10.$$

Give the upper bound in the form $n < a$ or $n \leq a$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `n`

Give the lower bound in the form $n > a$ or $n \geq a$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `n`

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Trapezium Rule 4i

A Level

P P P

Figure 1 shows the curve $y = 1.25^x$.

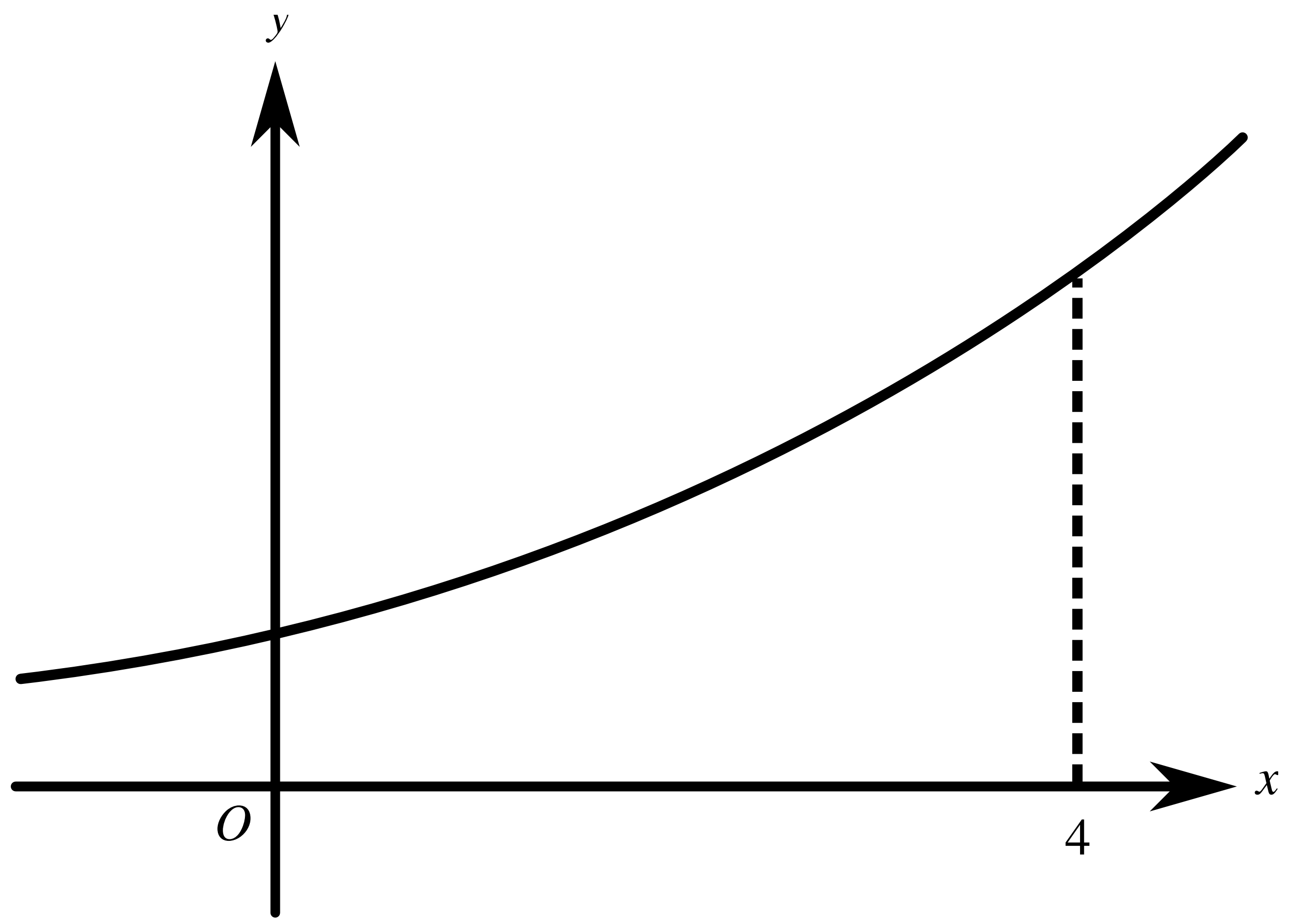

Figure 1: The curve $y = 1.25^x$.

Part A x-Coordinate

A point on the curve has y-coordinate 2 , calculate its x-coordinate, giving your answer to 3 significant figures.

Part B Derivative of y

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

The following symbols may be useful: `Derivative(y, x), e, ln(), log(), x`

Part C Trapezium Rule

Use the trapezium rule with 4 intervals to estimate the area of the region bounded by the curve, the axes and the line $x = 4$. Give your answer to three significant figures.

Part D Overestimate or Underestimate?

Is the estimate found in part C an overestimate or an underestimate?

- Overestimate
- Underestimate

Part E More Accurate Estimates

How could the trapezium rule could be used to find a more accurate estimate of the shaded region?

- Use rectangles instead of trapezia. Their shape will better fit this particular curve, and so give a more accurate approximation.
- Double the number of trapezia, keeping their width the same. Using more trapezia always results in a better approximation.
- Use the same number of trapezia, but reduce the width of the trapezia. Narrower trapezia are a better fit to the curve as they reduce the surplus area between the tops of the trapezia and the curve, and so will yield a better approximation to the area.
- Use a larger number of (narrower) trapezia over the same interval. This will reduce the surplus area between the tops of the trapezia and the curve, and so give a more accurate approximation.

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Data Analysis 5.4

A Level
C C C

The distribution of the velocities of gas molecules in one dimension is given by a normal distribution with mean zero and variance which is proportional to the temperature of the gas.

At room temperature, $T_1 = 300\,\mathrm{K}$, 20% of the molecules have speeds greater than $370\,\mathrm{m\,s^{-1}}$. At a higher temperature, T_2, 30% of the molecules have speeds greater than $370\,\mathrm{m\,s^{-1}}$. Answer the following questions and hence deduce the value of T_2.

Part A σ_1 at T_1

At room temperature, $T_1 = 300\,\mathrm{K}$, 20% of the molecules have speeds greater than $370\,\mathrm{m\,s^{-1}}$; find the value of the standard deviation in the velocities σ_1. Give your answer to 2 sf.

Part B σ_2 at T_2

At T_2, 30% of the molecules have speeds greater than $370\,\mathrm{m\,s^{-1}}$; find the value of the standard deviation in the velocities σ_2. Give your answer to 2 sf.

Part C The value of T_2

Use your answers to parts A and B and the information given above to deduce the temperature T_2. Give your answer to 2 sf.

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Parametrics & Implicit Differentiation

A Level
C C C

Part A Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

The parametric equations of a curve are

$$x = \frac{1}{\sqrt{2+t}} \text{ and } y = t^3 - 3t \text{ for } -2 < t \leq 0$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

The following symbols may be useful: `Derivative(y, x)`, `t`

Part B Stationary Point

The parametric equations of a curve are

$$x = \frac{1}{\sqrt{2+t}} \text{ and } y = t^3 - 3t \text{ for } -2 < t \leq 0$$

Give the x and y coordinates of the stationary point. Write your answer in the form (x,y) with no spaces.

What is the nature of the stationary point?

- Minimum
- Maximum
- Point of inflection

Part C Domain and Range

The parametric equations of a curve are

$$x = \frac{1}{\sqrt{2+t}} \text{ and } y = t^3 - 3t \text{ for } -2 < t \leq 0$$

State the domain of the resultant function. Write your answer in the form $x \geq a$, $x > a$, $x \leq a$, or $x < a$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `x`

State the upper bound of the range. Write your answer in the form $y \geq a$, $y > a$, $y \leq a$, or $y < a$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `y`

Give the lower bound of the range. Write your answer in the form $y \geq b$, $y > b$, $y \leq b$, or $y < b$.

The following symbols may be useful: `<`, `<=`, `>`, `>=`, `y`

Part D Sketch

The parametric equations of a curve are

$$x = \frac{1}{\sqrt{2+t}} \text{ and } y = t^3 - 3t \text{ for } -2 < t \leq 0$$

Sketch the graph of this function.

Easier question?

Part E $\frac{\mathrm{d}y}{\mathrm{d}x}$

Figure 2 shows the curve with equation $x^2 + y^3 - 8x - 12y = 4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q.

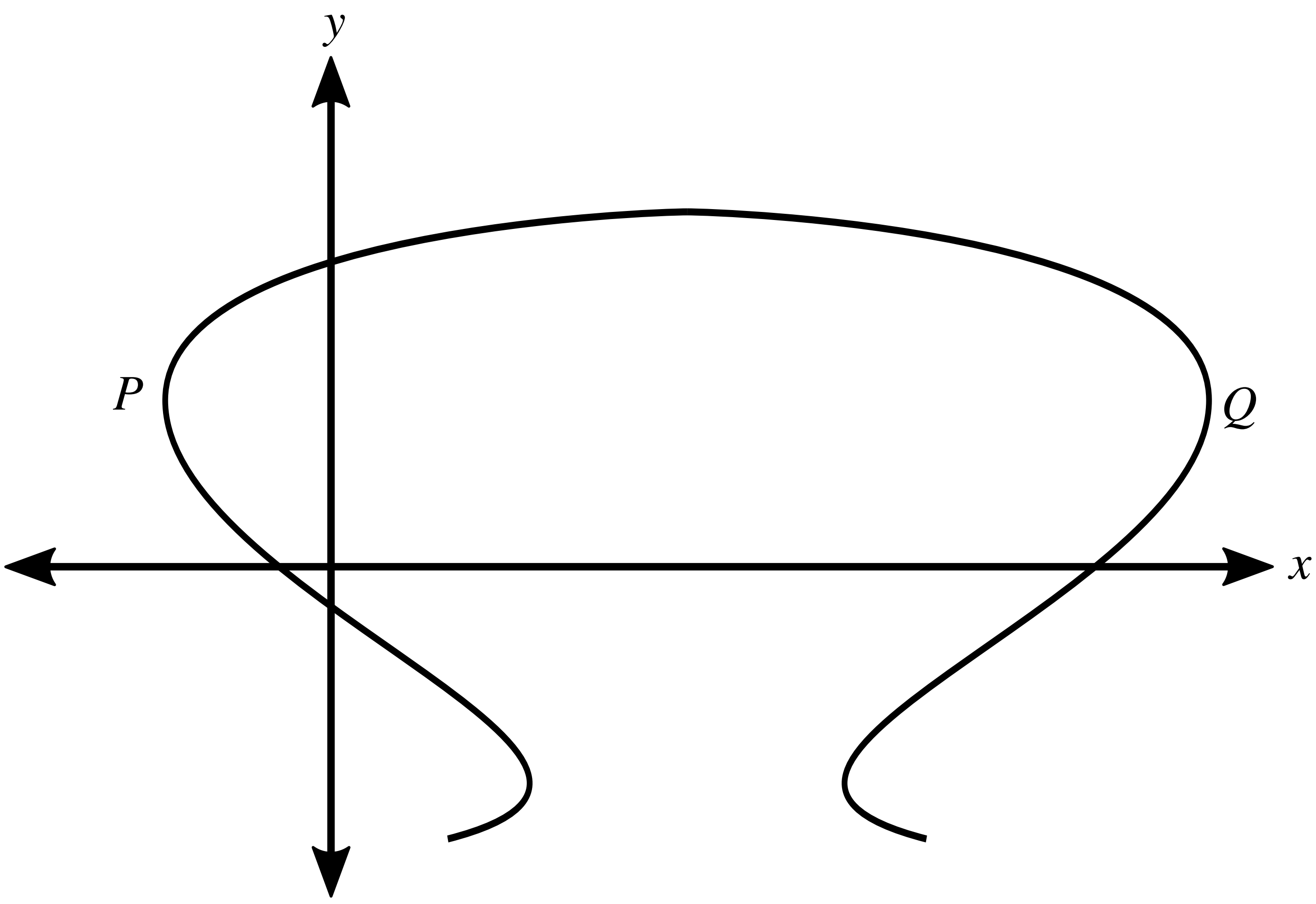

Figure 2: A diagram of the curve $x^2 + y^3 - 8x - 12y = 4$.

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x), x, y`

Part F Implicit Differentiation

The diagram shows the curve with equation $x^2 + y^3 - 8x - 12y = 4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q.

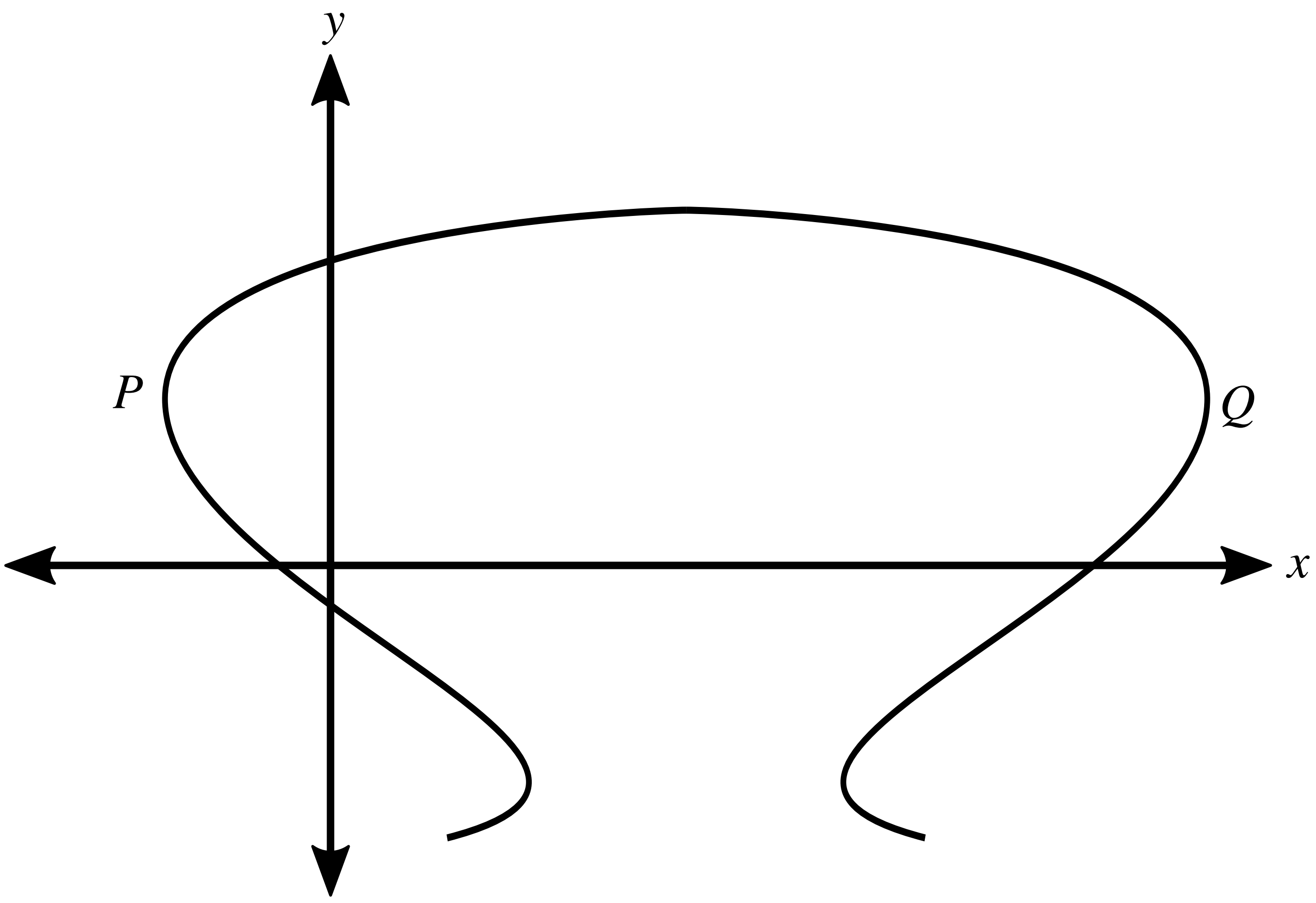

Figure 3: A diagram of the curve $x^2 + y^3 - 8x - 12y = 4$.

Give the x and y coordinates for point P. Write your answer in the form (x,y) without spaces.

Give the x and y coordinates for point Q. Write your answer in the form (x,y) without spaces.