



STEM SMART Double Maths 2 - Quadratic &amp; Cubic Equations

## Completing the Square 3

Essential GCSE Maths 29.3

Subject &amp; topics: Maths | Algebra | Quadratics

Stage &amp; difficulty: GCSE P3, A Level P1

Express the following in completed square form.

Give your answers in the form  $b(x + a)^2 + c$ , use improper (top heavy) fractions rather than mixed fractions in your answers.

Part A

$$2x^2 - 8x + 2$$

$$2x^2 - 8x + 2$$

The following symbols may be useful: x

Part B

$$3x^2 - 18x - 7$$

$$3x^2 - 18x - 7$$

The following symbols may be useful: x



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## Graphs of Quadratic Functions 4

Essential GCSE Maths 27.4

Subject &amp; topics: Maths | Functions | Graph Sketching

Stage &amp; difficulty: GCSE P2, A Level P1

Without drawing graphs, find for each function:

(i) the  $y$ -intercept    (ii) where the graph crosses the  $x$ -axis.

**Part A**

$$y = x^2 + x - 2$$

$$y = x^2 + x - 2$$

(i) the  $y$ -intercept

 $y =$  

(ii) the  $x$ -intercepts

lower value:  $x =$  higher value:  $x =$

**Part B**

$$y = x^2 + 6x + 5$$

$$y = x^2 + 6x + 5$$

(i) the  $y$ -intercept

$$y = \text{[ ]}$$

(ii) the  $x$ -intercepts

lower value:  $x = \text{[ ]}$

higher value:  $x = \text{[ ]}$

**Part C**

$$y = x^2 - 8x + 15$$

$$y = x^2 - 8x + 15$$

(i) the  $y$ -intercept

$$y = \text{[ ]}$$

(ii) the  $x$ -intercepts

lower value:  $x = \text{[ ]}$

higher value:  $x = \text{[ ]}$

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## Graphs of Quadratic Functions 11

Essential GCSE Maths 27.11

**Subject & topics:** Maths | Functions | Graph Sketching**Stage & difficulty:** GCSE C3, A Level C1

The formula  $s = ut + \frac{1}{2}at^2$  is used to calculate the height  $s$  of projectiles (such as balls) as a function of time.

Plot or sketch a graph of  $s$  against  $t$  for  $0 \leq t \leq 7$ , given that  $u = 29.43 \text{ m/s}$  and  $a = -9.81 \text{ m/s}^2$ .

### Part A

#### What is the maximum height?

What is the maximum height reached? Give your answer to 3 sf.

### Part B

#### How long to return to its starting height?

How long does a projectile modelled by this graph take to return to its starting height? You may assume the projectile was launched at  $t = 0$ . Give your answer to 3 sf.

### Part C

#### What is the relative position of the projectile?

At  $t = 7 \text{ s}$ , what is the height of the projectile relative to its starting position? Give your answer to 3 sf.

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# Quadratics: Graphs and Discriminants 2ii

**Subject & topics:** Maths      **Stage & difficulty:** A Level P1

The quadratic equation  $x^2 + kx + k = 0$  has no real roots for  $x$ .

Part A

Find discriminant

Write down the discriminant of  $x^2 + kx + k$  in terms of  $k$ .

The following symbols may be useful: k

Part B

Possible values of  $k$

Hence find the set of values  $k$  can take.

Construct your answer from the items below.

Items:

<

>

$k$

$< k <$

$\leq k \leq$

$> k$  or  $k >$

$\geq k$  or  $k \geq$

$\leq$

$\geq$

-4

-3

-2

-1

0

1

2

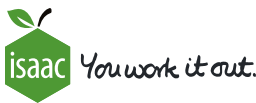
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# Quadratic Inequalities

Pre-Uni Maths for Sciences A2.5

**Subject & topics:** Maths | Algebra | Manipulation      **Stage & difficulty:** GCSE C3, A Level P1

Solve the following quadratic inequalities.

Part A

$3x^2 - 2x - 8 \leq 0$

Solve the inequality  $3x^2 - 2x - 8 \leq 0$ .

Construct your answer from the items below.

Items:

- <
- >
- $x$
- $< x <$
- $\leq x \leq$
- $< x \text{ or } x <$
- $\leq x \text{ or } x \leq$
- $\leq$
- $\geq$
- $-\frac{4}{3}$
- $\frac{4}{3}$
- $-2$
- $2$



Part B

$-2x^2 + 5 < 7x + 11$

Solve the inequality  $-2x^2 + 5 < 7x + 11$ .

Construct your answer from the items below.

Items:

<

>

$x$

$< x <$

$\leq x \leq$

$< x \text{ or } x <$

$\leq x \text{ or } x \leq$

$\leq$

$\geq$

$-\frac{3}{2}$

$\frac{3}{2}$

$-2$

$2$

Created for isaacphysics.org by Julia Riley

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## Powerful Quadratics

**Subject & topics:** Maths | Algebra | Quadratics**Stage & difficulty:** GCSE C3, A Level C1**Part A****Find  $x$** 

Find all the solutions of

$$(x^2 - 9x + 19)^{x^2 - 3x + 2} = 1$$

**Part B****Find  $y$** 

Find all the solutions of

$$(y^2 - 13y + 41)^{y^2 + 6y + 8} = 1$$

Created for isaacscience.org by Matthew Rihan

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## Quadratic Equations 5

Pre-Uni Maths for Sciences B1.10

Subject &amp; topics: Maths | Algebra | Quadratics

Stage &amp; difficulty: GCSE C3, A Level C1

Show that the solution to the equation  $mp^2 + bp + k = 0$  can be written as  $p = -\gamma \pm \sqrt{\gamma^2 - \omega^2}$ .

### Part A

#### Find an expression for $\gamma$

Hence find an expression for  $\gamma$  in terms of one or more of the constants  $m$ ,  $b$  and  $k$  in the original equation.

The following symbols may be useful:  $b$ ,  $\gamma$ ,  $k$ ,  $m$ ,  $\omega$

### Part B

#### Find an expression for $\omega$

Also give an expression for  $\omega$  in terms of one or more of the constants  $m$ ,  $b$  and  $k$ .

The following symbols may be useful:  $b$ ,  $\gamma$ ,  $k$ ,  $m$ ,  $\omega$

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## Linear-Quadratic 6

Pre-Uni Maths for Science B2.10

**Subject & topics:** Maths | Algebra | Simultaneous Equations      **Stage & difficulty:** GCSE C3, A Level P1

A particle of mass  $M$ , travelling at speed  $u$ , collides head-on and elastically with a stationary particle of mass  $m$ . After the collision the particles of mass  $M$  and of mass  $m$  travel at (non-zero) speeds  $v$  and  $w$  respectively.

By applying the laws of conservation of momentum and kinetic energy we can write down two simultaneous equations for the collision:  $Mu = Mv + mw$  and  $\frac{1}{2}Mu^2 = \frac{1}{2}Mv^2 + \frac{1}{2}mw^2$

### Part A

$w$  (speed of particle of mass  $m$  after collision)

Find an expression for  $w$ , the speed of the particle of mass  $m$  after the collision, in terms of  $u$ ,  $M$  and  $m$ .

The following symbols may be useful:  $M$ ,  $m$ ,  $u$ ,  $v$ ,  $w$

### Part B

$v$  (speed of particle of mass  $M$  after collision)

Find the corresponding expression for  $v$ , the speed of the particle of mass  $M$  after the collision, in terms of  $u$ ,  $M$  and  $m$ .

The following symbols may be useful:  $M$ ,  $m$ ,  $u$ ,  $v$ ,  $w$

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## Algebra and Roots: Cubics 2ii

**Subject & topics:** Maths    **Stage & difficulty:** Further A P1

The cubic equation  $x^3 - 6x^2 + kx + 10 = 0$  has roots  $p - q$ ,  $p + q$  and  $p$ .

**Part A** $p$ 

Find  $p$  by considering the sum of roots.

The following symbols may be useful:  $p$

**Part B** $q$ 

Find  $q$  by considering the product of roots. Take  $q$  to be positive.

The following symbols may be useful:  $q$

Part C  
 $k$

Find  $k$ .

The following symbols may be useful:  $k$

Adapted with permission from UCLES, A Level, OCR FP1 Specimen paper, Paper 4725, Question 2.

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## Algebra and Roots: Cubics 1i

**Subject & topics:** Maths    **Stage & difficulty:** Further A P2

The cubic equation  $3x^3 - 9x^2 + 6x + 2 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

**Part A**

$$\alpha + \beta + \gamma$$

Give the value of  $\alpha + \beta + \gamma$ .

**Part B**

$$\alpha\beta + \alpha\gamma + \beta\gamma$$

Give the value of  $\alpha\beta + \alpha\gamma + \beta\gamma$ .

**Part C**

$$\alpha\beta\gamma$$

Give the value of  $\alpha\beta\gamma$ .

Part D

Transformed cubic

The cubic equation  $x^3 + ax^2 + bx + c = 0$  has roots  $\alpha^2$ ,  $\beta^2$  and  $\gamma^2$ .

Find  $a$ .

The following symbols may be useful: a

Find  $b$ .

The following symbols may be useful: b

Find  $c$ .

The following symbols may be useful: c

Adapted with permission from UCLES, A Level, June 2012, Paper 4725, Question 10.