Further Mechanics

A-level overview

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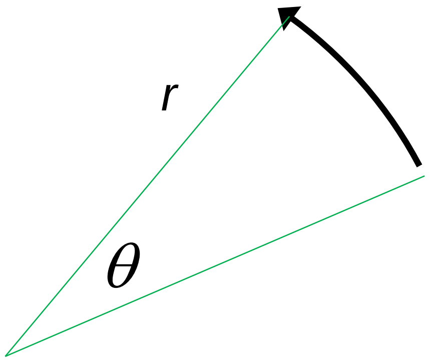
https://isaacphysics.org/pages/remote_learning

Circular Motion

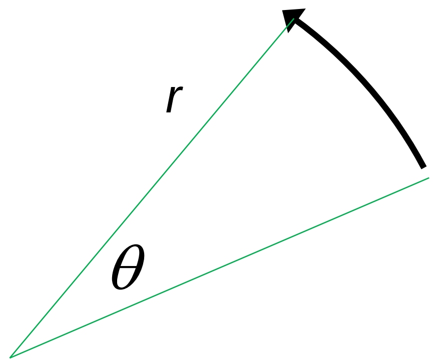
Measuring angles



Distance travelled

$$= \frac{\theta}{360°} \times 2\pi r$$

Measuring angles



Distance travelled

$$s = r\theta$$

Speed

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\frac{\theta}{t} = r\omega$$

Angular velocity ω (rad s^{-1})

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Angle practice

1. Convert 3° into radians:

2. Calculate the angular velocity if
   a) time period $T = 4.5$s
   b) frequency $f = 50$Hz
   c) speed $v = 15$ms^{-1} and radius $r = 12$m

3. How far will an object travel in 4.0s on a 2m radius circle if
   a) $\omega = 3.5$ rad s^{-1}
   b) $f = 1.45$ Hz
   c) $T = 60$ s

Centripetal acceleration

An object travelling on a circular path

- is continually changing direction,
- therefore is changing velocity (or momentum)
- therefore is accelerating, so
- a resultant force must be acting on it.

The acceleration is towards the centre of the circle

centripetal acceleration: $a = \frac{v^2}{r} = \omega^2 r = v\omega$

in terms of time period:

resultant force needed:

Centripetal practice

1. Calculate the force needed to make a 1200kg car go round a 9.5m radius roundabout at $13ms^{-1}$.

2. Calculate the acceleration of a 300g mass whirled round in a horizontal circle of 2.5m radius once every 1.25s.

3. Calculate the speed of the rim of a space station of radius 40m if it is to simulate Earth's gravity ($a = 9.8\ ms^{-2}$)

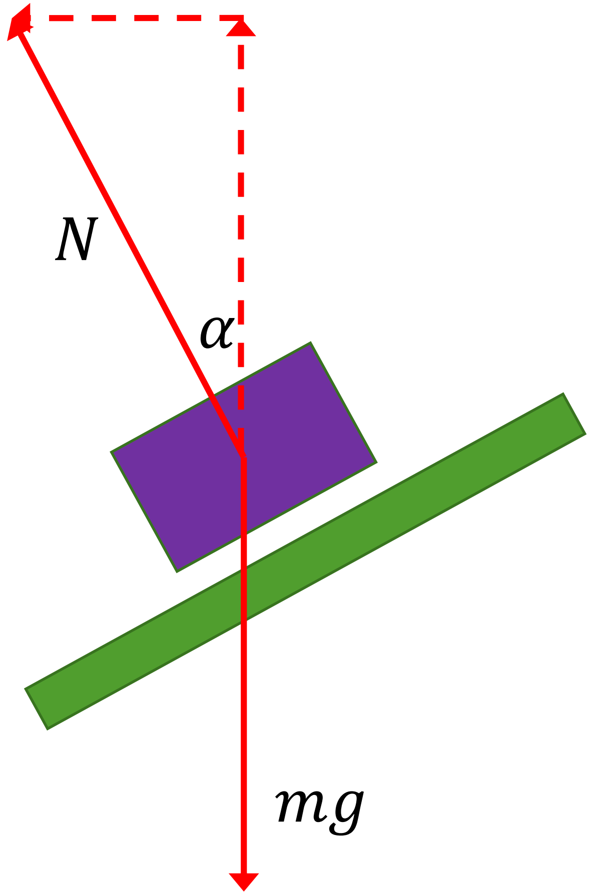
Orbits

Force on a satellite $F = \frac{GMm}{r^2}$ $G = 6.67 \times 10^{-11}\mathrm{Nm}^2\mathrm{kg}^{-2}$

Writing $F = ma$ using $a = \frac{v^2}{r}$:

Writing $F = ma$ using $a = r\omega^2 = \frac{4\pi^2 r}{T^2}$:

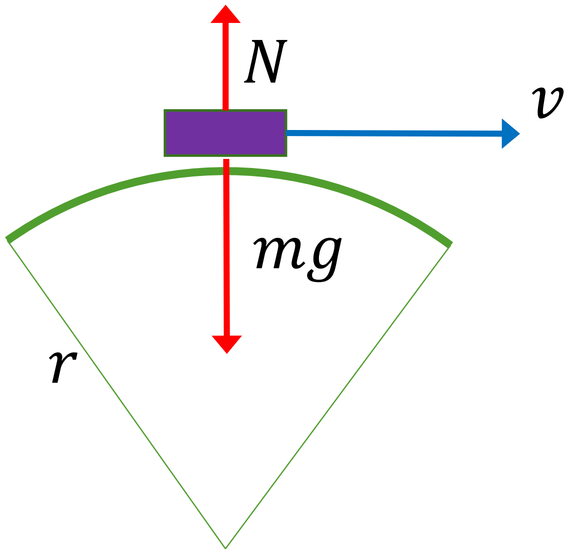
Banked turns



Vertically: $N\cos\alpha = mg$

Horizontally: $N\sin\alpha = ma = \frac{mv^2}{r}$

Vertical circles



Hump-back bridge

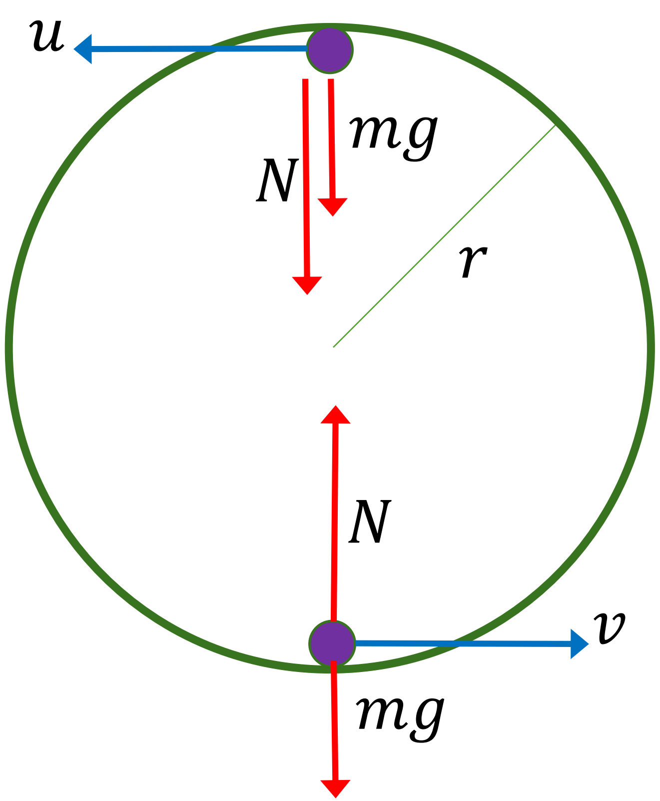
$$mg - N = \frac{mv^2}{r}$$

car flies off when...

Loop-the-loop

at top of loop

at base of loop



Advanced circle practice

1. A car leaves the ground as it goes over a hump back bridge at $17ms^{-1}$. What is the radius of the bridge?

2. What angle of bank would an aeroplane need if flying at $70ms^{-1}$ if the pilot wished to turn through 180° in 60s?

3. Calculate the normal reaction on a 75kg rider on a rollercoaster as it goes through the lowest point at a radius of 8.5m at a speed of $16ms^{-1}$.

Oscillations

Simple harmonic motion

Where there is a force

proportional to displacement from equilibrium, and

directed towards equilibrium point $F = -kx,\ a = -\omega^2 x,\ \omega^2 = \dfrac{k}{m}$

Solution $x = A\cos(\omega t)$ $\omega = \dfrac{2\pi}{T} = 2\pi f$

$v = -A\omega\sin(\omega t)$ $v_{max} = A\omega$ $v^2 = \omega^2(A^2 - x^2)$

$a = -A\omega^2\cos(\omega t)$ $a_{max} = A\omega^2$ $a = -\omega^2 x$

For pendulum of length *L*, mass *m*, angle θ to vertical:

$$F \approx -mg\sin\theta = -mg\frac{x}{L} \quad a = \frac{F}{m} = -\frac{g}{L}x \quad \omega^2 = \frac{g}{L}$$

SHM formula practice

The 600g mass on a 1.50m pendulum is moved 5.0cm to the side then released at t=0

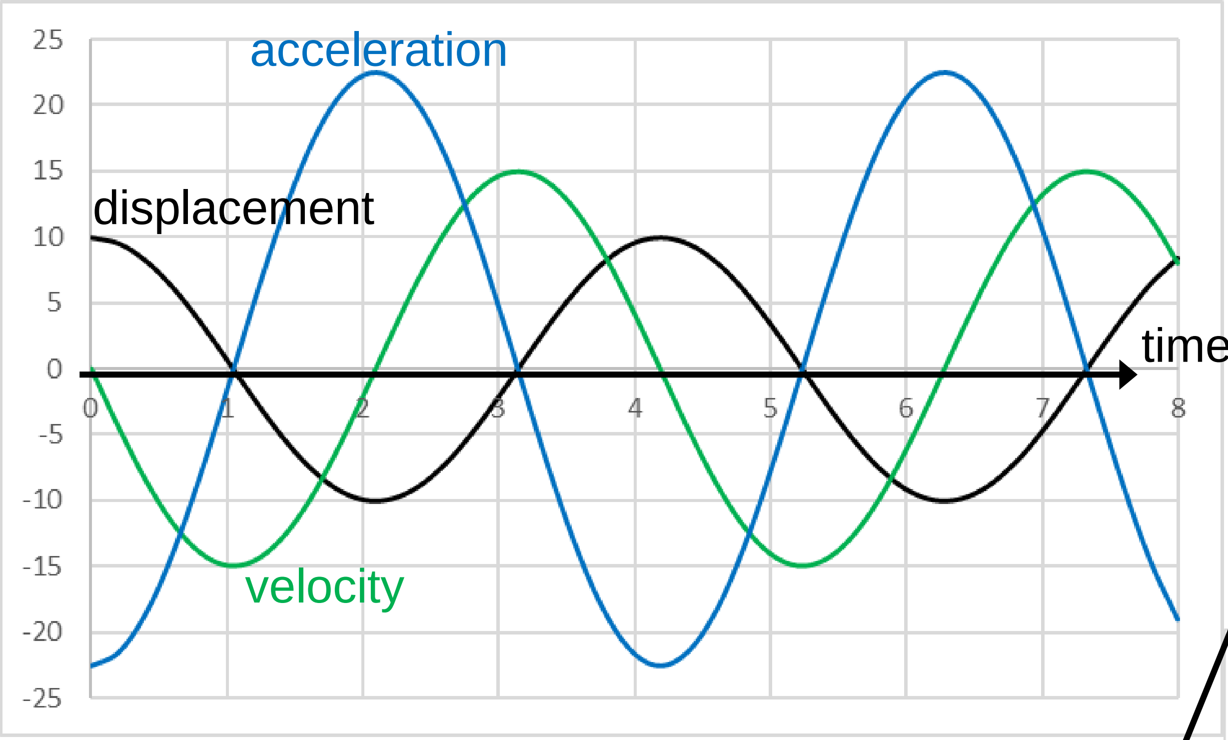
1. **Calculate the angular frequency ω.**

2. **Calculate the maximum speed of the bob.**

3. **Calculate the acceleration when t=0.32s.**

4. **Calculate the speed when x=2.5cm**

SHM graphs



<u>Time graphs</u>

Here, ω=1.5 rad s^{-1}.

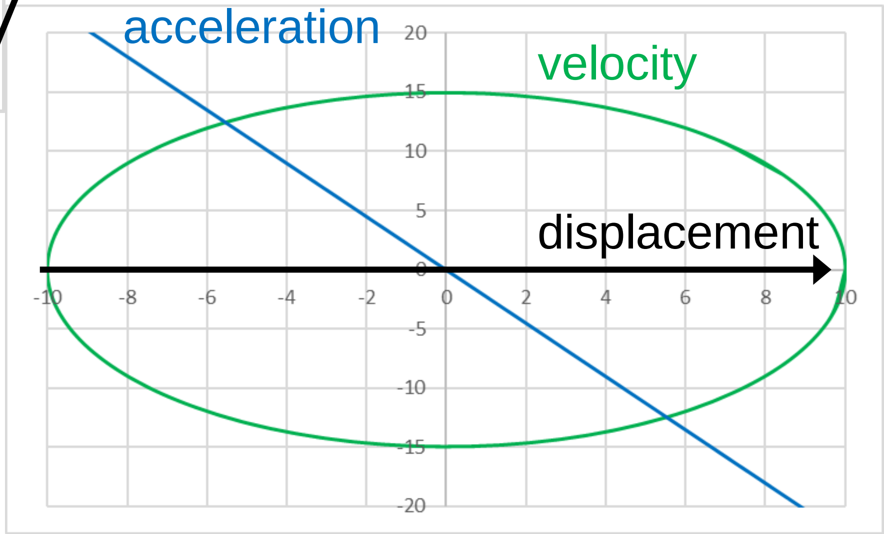
A=x_{max}=10, v_{max}=15, a_{max}=22.5

v peaks ¼ cycle before x

a peaks ¼ cycle before v

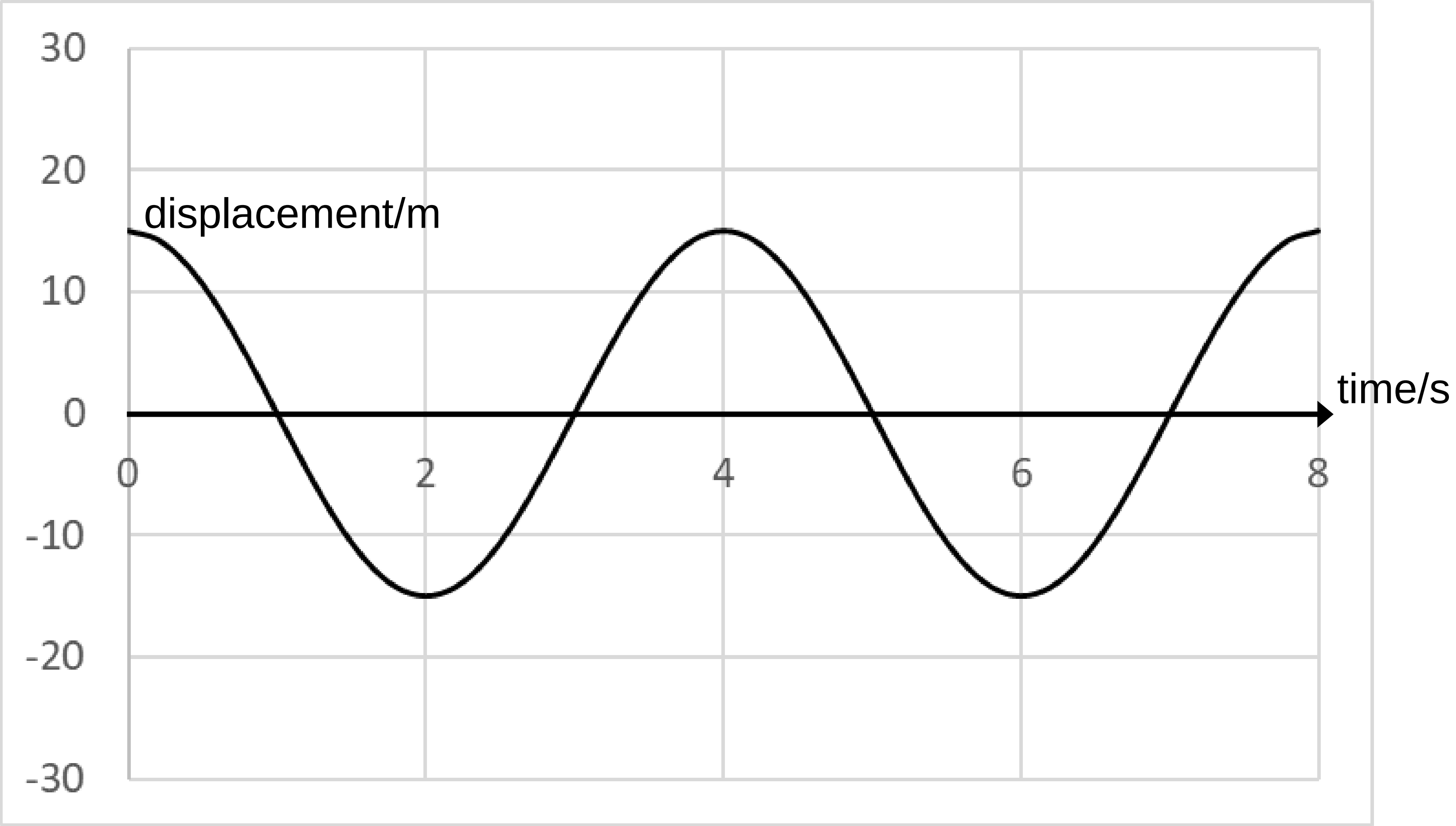
When plotted against displacement

- velocity graph makes ellipse
- acceleration graph straight with negative gradient



SHM graph practice

Sketch and label the velocity-time graph for this oscillation.



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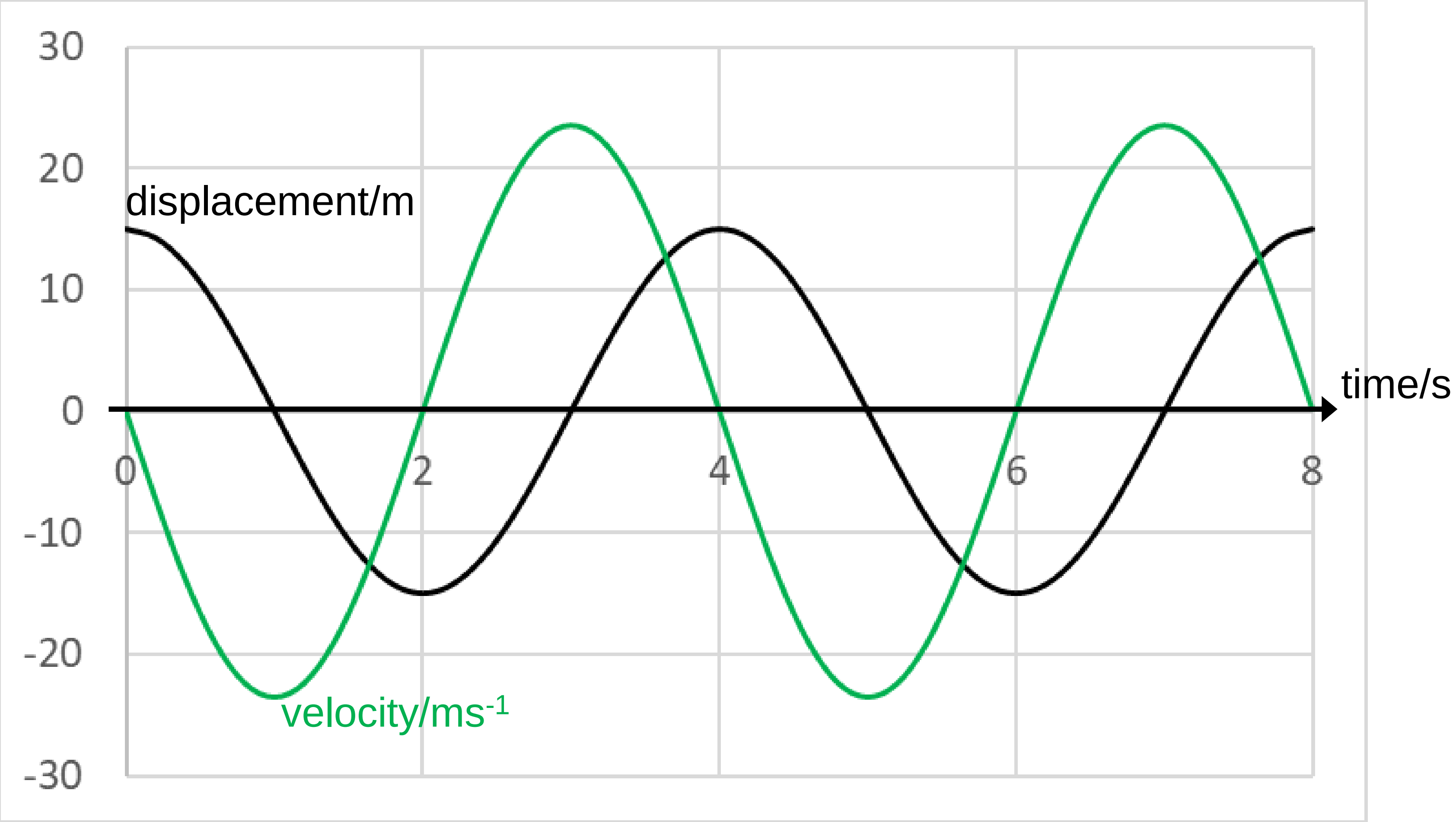


SHM energy

Kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2 \sin^2(\omega t)$

Potential energy $P = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t)$

and as $\omega^2 = \frac{k}{m}$ **it follows that** $k = m\omega^2$ **and so**

Total energy

$$E = K + P = \frac{1}{2}kA^2\{\sin^2(\omega t) + \cos^2(\omega t)\} = \frac{1}{2}kA^2$$

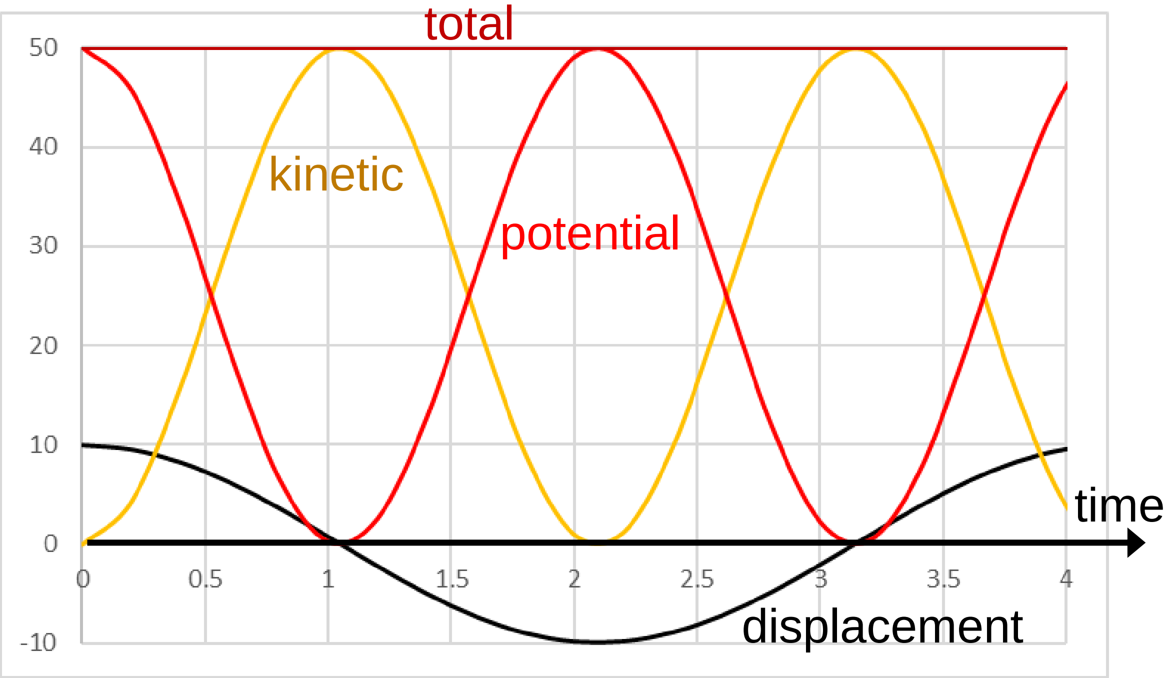
Energy practice

1. Calculate the total energy in a 3.0cm amplitude oscillation of a spring with constant 200Nm^{-1}.

2. Calculate the potential energy of the oscillation in q1 2.64s after it was released if its mass is 500kg.

3. Calculate the kinetic energy of the oscillation in q1 when x = 1.5cm.

Energy graphs

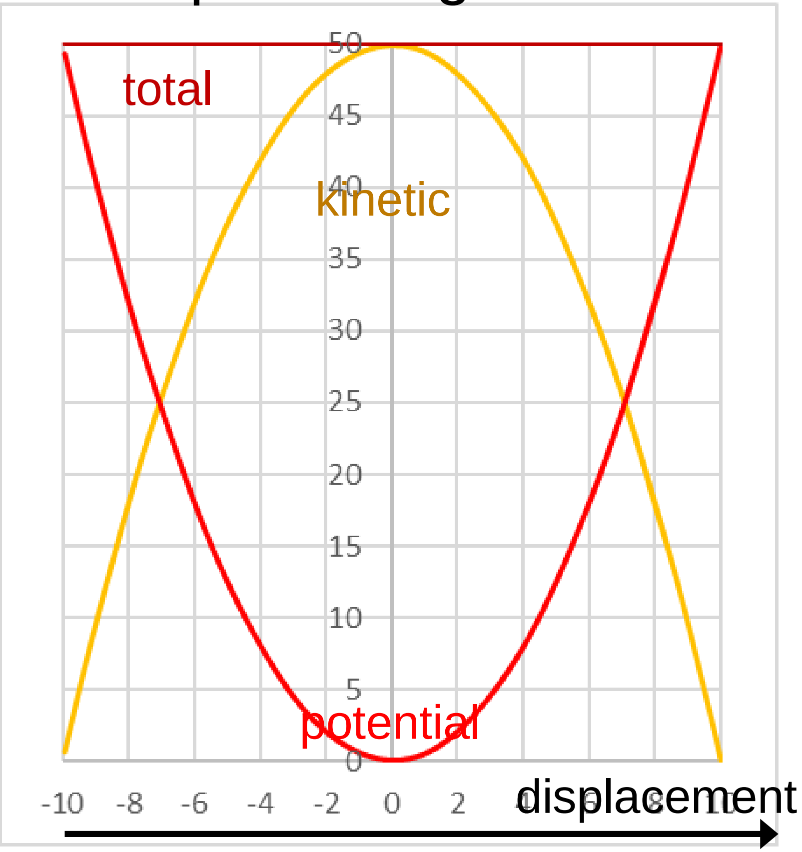


Energy curves have twice the frequency of displacement or velocity.

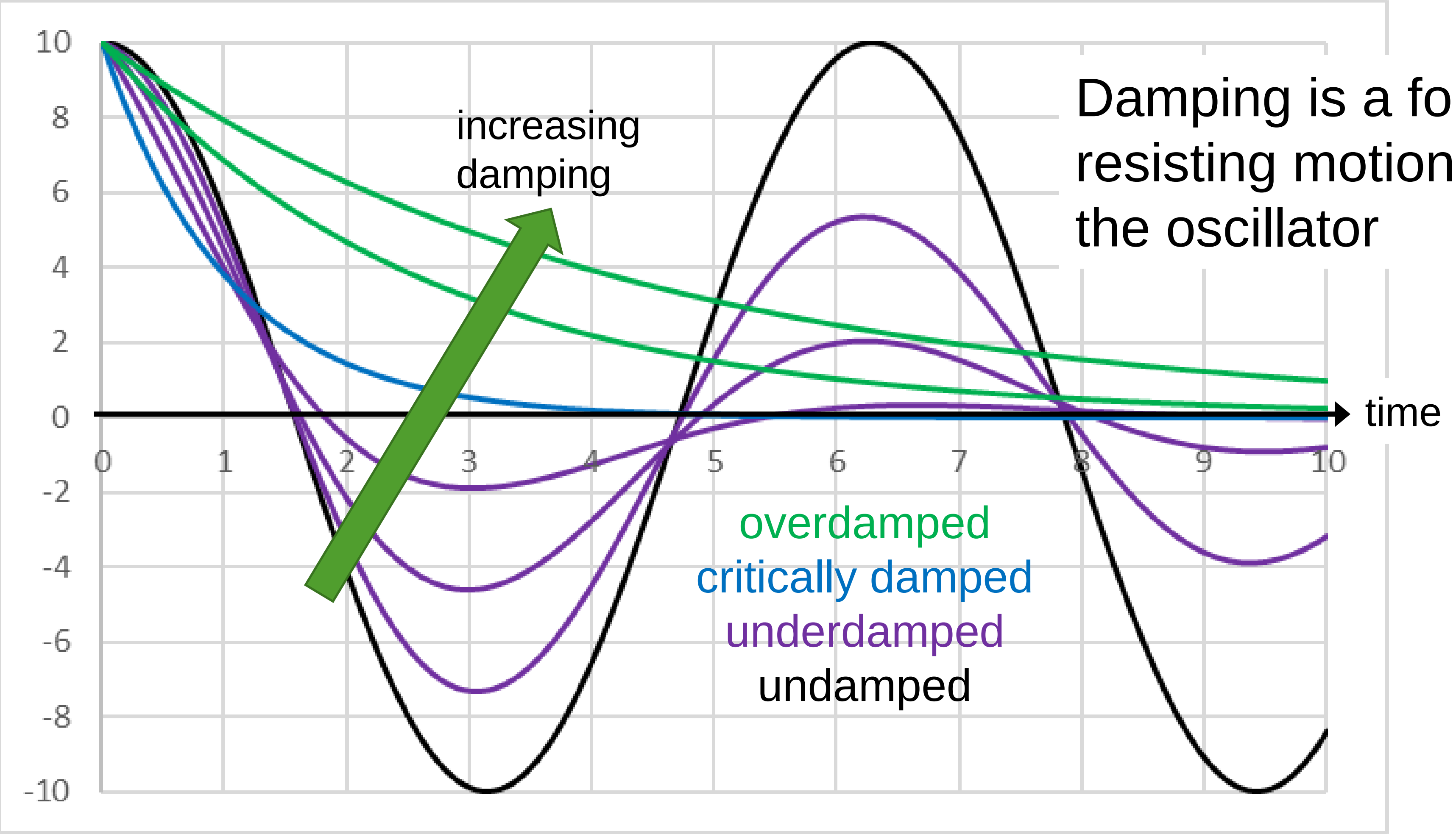
Potential peaks when $x = A$ and $v = 0$

Kinetic peaks when $x = 0$ and $v = v_{max}$

Energy makes parabolae when plotted against x

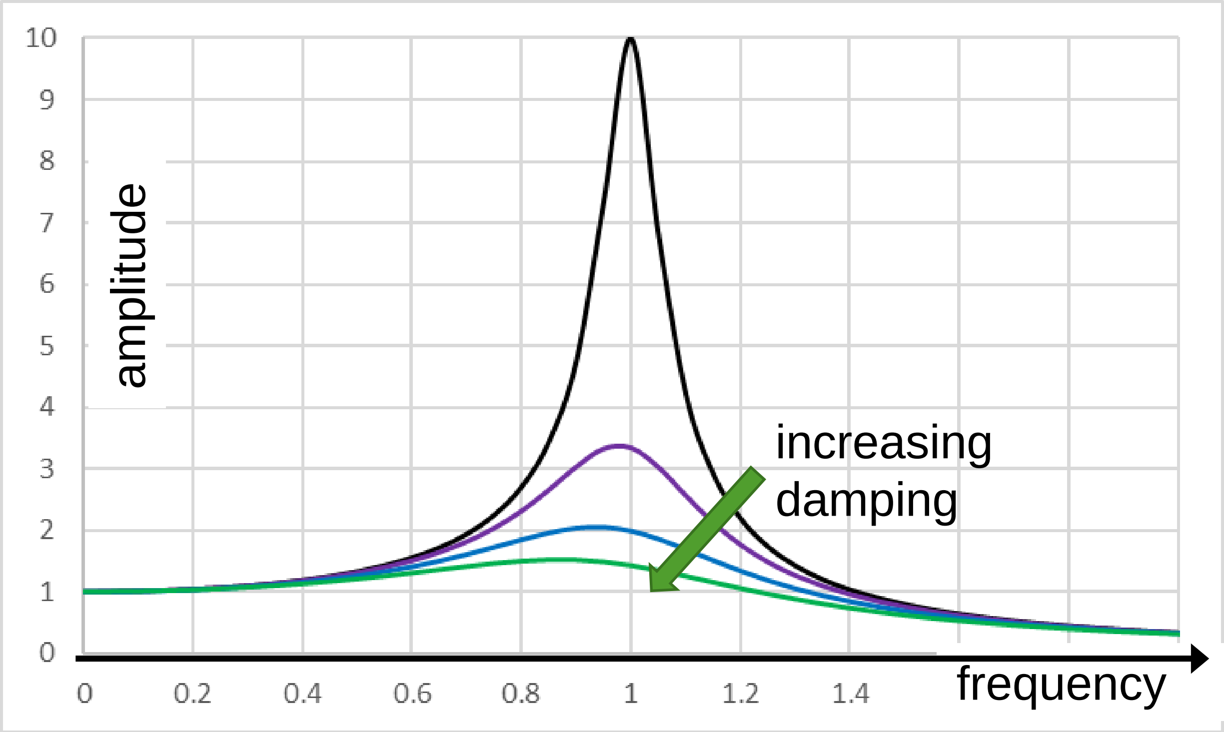


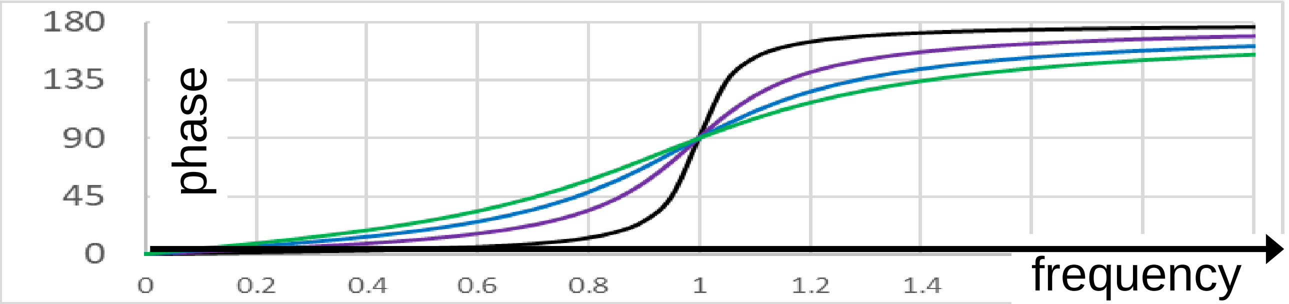
Damping



Damping is a force resisting motion applied to the oscillator

Resonance





The build up of large amplitude oscillations when a system is forced at its natural frequency.

Natural frequency is frequency of oscillations after system is displaced from equilibrium and released.

Links

A Level Topic Revision

https://isaacphysics.org/pages/a_level_topic_index#a_level_revision

Consolidation Programme

https://isaacphysics.org/pages/summer_programmes_2021