Stationary Points 1

A Level Further A

Part A Number of stationary points of $y = 2x^3 - 24x - 5$

Find the position and nature of the stationary points of the function $y = 2x^3 - 24x - 5$.

How many stationary points are there?

- 0
- 3
- 1
- 2
- 4

Part B First stationary point of $y = 2x^3 - 24x - 5$

Find the position and nature of the stationary points of the function $y = 2x^3 - 24x - 5$.

Find x_1, the x coordinate of the stationary point with the lowest value of x.

The following symbols may be useful: `x_1, y_1`

Find y_1, the y coordinate of the stationary point (x_1, y_1).

The following symbols may be useful: `x_1, y_1`

What is the nature of this stationary point?

- Minimum
- Maximum

Part C Second stationary point of $y = 2x^3 - 24x - 5$

Find the position and nature of the stationary points of the function $y = 2x^3 - 24x - 5$.

Find x_2, the x coordinate of the stationary point with the second lowest value of x.

The following symbols may be useful: `x_2, y_2`

Find y_2, the y coordinate of the stationary point (x_2, y_2).

The following symbols may be useful: `x_2, y_2`

What is the nature of this stationary point?

- Minimum
- Maximum

Part D Number of stationary points of $y = 2x^3 - 5x^2 + 4x + 6$

Find the position and nature of the stationary points of the function $y = 2x^3 - 5x^2 + 4x + 6$.

How many stationary points are there?

- 2
- 3
- 0
- 4
- 1

Part E First stationary point of $y = 2x^3 - 5x^2 + 4x + 6$

Find the position and nature of the stationary points of the function $y = 2x^3 - 5x^2 + 4x + 6$.

Find x_1, the x coordinate of the stationary point with the lowest value of x.

The following symbols may be useful: `x_1, y_1`

Find y_1, the y coordinate of the stationary point (x_1, y_1). (Give your answer in the form of an improper fraction.)

The following symbols may be useful: `x_1, y_1`

What is the nature of this stationary point?

- Minimum
- Maximum

Part F Second stationary point of $y = 2x^3 - 5x^2 + 4x + 6$

Find the position and nature of the stationary points of the function $y = 2x^3 - 5x^2 + 4x + 6$.

Find x_2, the x coordinate of the stationary point with the second lowest value of x.

The following symbols may be useful: `x_2, y_2`

Find y_2, the y coordinate of the stationary point (x_2, y_2).

The following symbols may be useful: `x_2, y_2`

What is the nature of this stationary point?

- ○ Maximum
- ○ Minimum

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Maxima and Minima: Problems 2ii

A Level

A curve has equation $y = 3x^3 - 7x + \frac{2}{x}$

Part A Verify stationary point

Verify the curve has a stationary point when $x = 1$.

More practice questions?

Part B Nature of stationary point

Determine the nature of this stationary point.

- Maximum
- Neither/inconclusive
- Minimum

Part C Tangent to curve

The tangent to the curve at this stationary point meets the y-axis at the point Q. Find the y-coordinate of Q.

Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Stationary Points 1ii

A Level

The curve $y = x^3 - kx^2 + x - 3$ has two stationary points.

Part A Differentiate

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `k, x`

Part B Find k

Given that there is a stationary point when $x = 1$, find the value of k.

The following symbols may be useful: `k`

Part C Differentiate twice

Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

The following symbols may be useful: x

Hence determine whether the stationary point is a minimum or a maximum.

- Maximum
- Minimum

Part D Find coordinate

Find the x-coordinate of the other stationary point.

The following symbols may be useful: x

Used with permission from UCLES, A Level, June 2009, Paper 4721, Question 10.

Gameboard:
STEM SMART Single Maths 11 - Using Differentiation

Physics. You work it out.

Stationary Points 2ii

A Level

Part A Find coordinate

Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 + 4$. Enter the x and y coordinates of the stationary point with the greatest x coordinate.

Enter the x-coordinate:

The following symbols may be useful: x

Enter the y-coordinate:

The following symbols may be useful: y

Part B Stationary point

Determine whether the stationary point whose coordinates you entered is a maximum point or a minimum point.

- Inconclusive
- Maximum
- Minimum

Part C Range of x

For what range of values of x does $x^3 - 3x^2 + 4$ decrease as x increases?

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $x < a$
- $x \leq a$
- $x > a$
- $x \geq a$
- $a < x < b$
- $a \leq x \leq b$
- $x < a$ or $x > b$
- $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: n

Used with permission from UCLES, A level, January 2006, Paper 4721, Question 6

Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Maxima and Minima: Problems 1ii

A Level

Figure 1: The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is x metres, and the area of the enclosure is $A\,\mathrm{m}^2$.

Part A Express as equation

Show that A can be expressed in the form $px - qx^2$, and find this expression.

The following symbols may be useful: x

Part B Use differentiation

Use differentiation to find the maximum value of A.

The following symbols may be useful: A

Used with permission from UCLES, A Level, June 2007, Paper 4721, Question 5.

Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Maxima and Minima: Problems 1i

A Level

A cuboid has an volume of exactly $8\,\mathrm{m}^3$. The base of the cuboid is a square with side length x metres. The surface area of the cuboid is $A\,\mathrm{m}^2$.

Part A Find expression for A

Show that A can be expressed in the form $ax^2 + \frac{b}{x}$, where a and b are constants, and find this expression.

The following symbols may be useful: x

Part B Find $\frac{\mathrm{d}A}{\mathrm{d}x}$

Find $\frac{\mathrm{d}A}{\mathrm{d}x}$.

The following symbols may be useful: x

Part C Find minimum

Find the value of x which gives the smallest surface area of the cuboid.

The following symbols may be useful: x

Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Differentiating Powers 6

A Level Further A

A quadratic function has the form $y = a + bx + cx^2$ where a, b and c are constants. It has a stationary point at $(2, 2)$ and, at $x = 1$, the tangent to the curve has a gradient of -2. Find the values of a, b and c. (In practice, with the information given, you will need to find b and c before you can find a.)

Part A The value of b

Find the value of b.

The following symbols may be useful: b

Part B The value of c

Find the value of c.

The following symbols may be useful: c

Part C The value of a

Find the value of a.

The following symbols may be useful: a

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Gameboard:

STEM SMART Single Maths 11 - Using Differentiation

Stationary Points 3

A Level Further A

C C C P P P

Part A Find the maximum height of a projectile

A particle is fired upwards into the air with a initial speed w and moves subsequently under the influence of gravity with an acceleration g downwards, such that its height h at time t is given by $h = wt - \frac{1}{2}gt^2$, where w and g are constants. Find an expression for its maximum height above its initial position.

The following symbols may be useful: `g, h, w`

Part B Examine the potential energy of two molecules

The potential energy of two molecules separated by a distance r is given by

$$U = U_0\left(\left(\frac{a}{r}\right)^{12} - 2\left(\frac{a}{r}\right)^{6}\right)$$

where U_0 and a are positive constants. The equilibrium separation of the two molecules occurs when the potential energy is a minimum; find expressions for the equilibrium separation and the value of the potential energy at this separation.

(a) Find an expression for the equilibrium separation of the molecules.

The following symbols may be useful: `U, U_0, a, r`

(b) Find an expression for the potential energy when the molecules are at their equilibrium separation.

The following symbols may be useful: `U, U_0, a, r`

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