Mechanics

A-level overview

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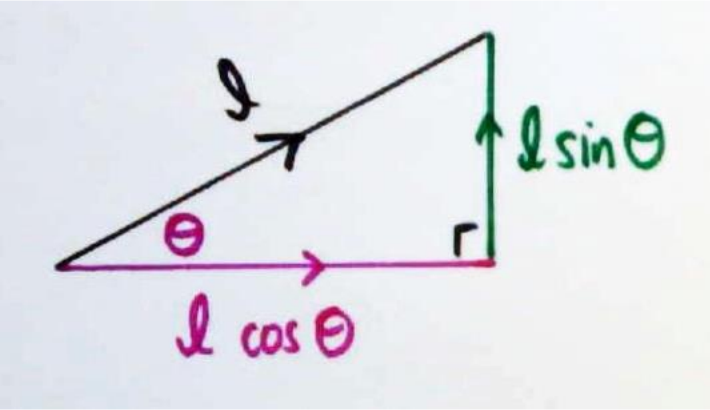
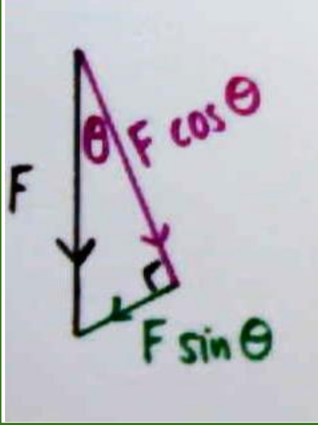
Vectors and Scalars

- **Vectors have magnitude (size) and direction**
  - **examples include velocity, displacement, force, acceleration**
- **Scalars have magnitude (size) alone**
  - **examples include mass, time, energy, speed, distance**
- **When analyzing motion problems, it helps to resolve [split] the vector into components (e.g. horizontal vs vertical)**

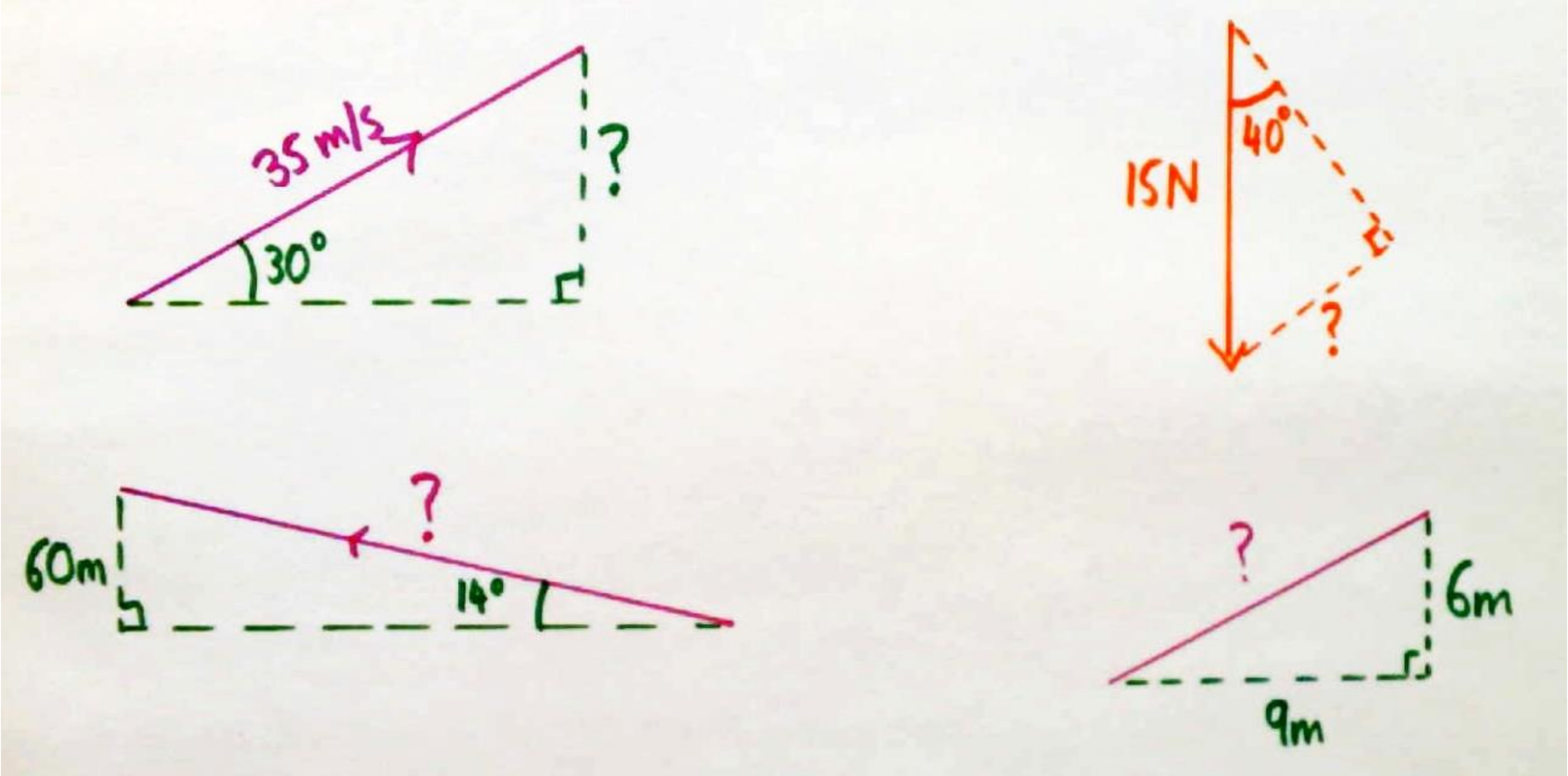




Resolving perpendicular and parallel to a surface

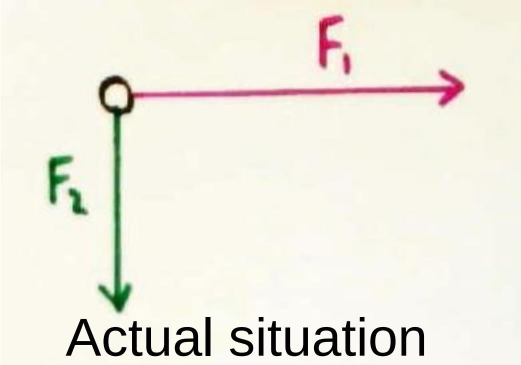
Component practice

- Work out the missing quantities

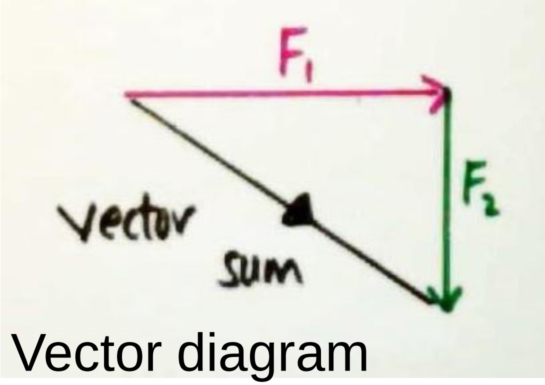


Adding vectors

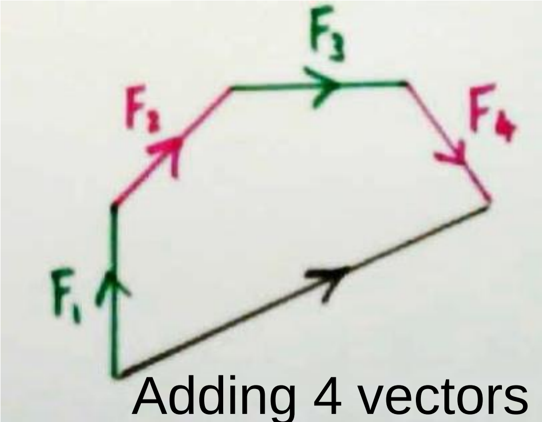
- **Vector addition is best shown by joining the vector arrows nose-to-tail.**



Actual situation

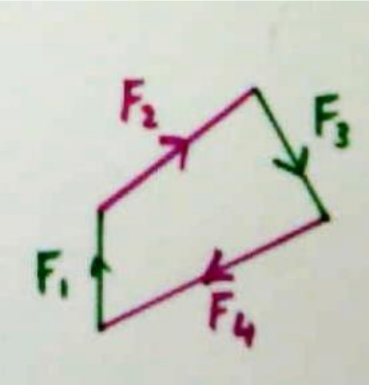


Vector diagram

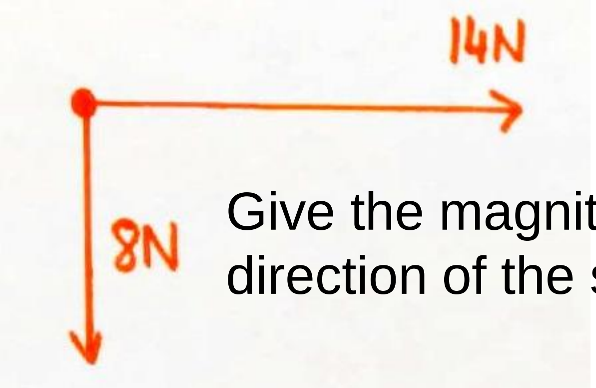


Adding 4 vectors

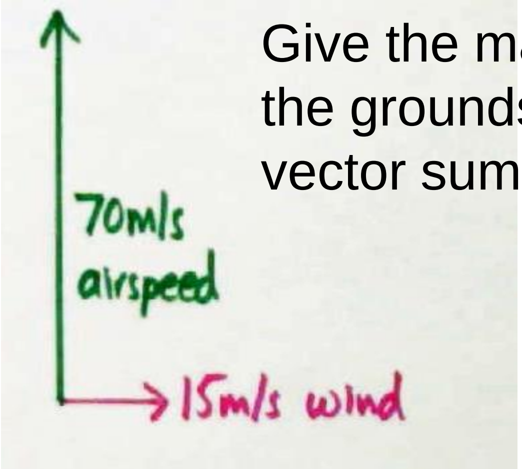
If arrows form a loop,
vector sum = 0



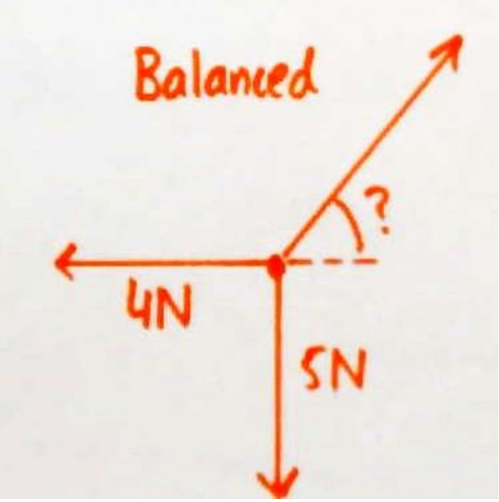
Vector addition practice



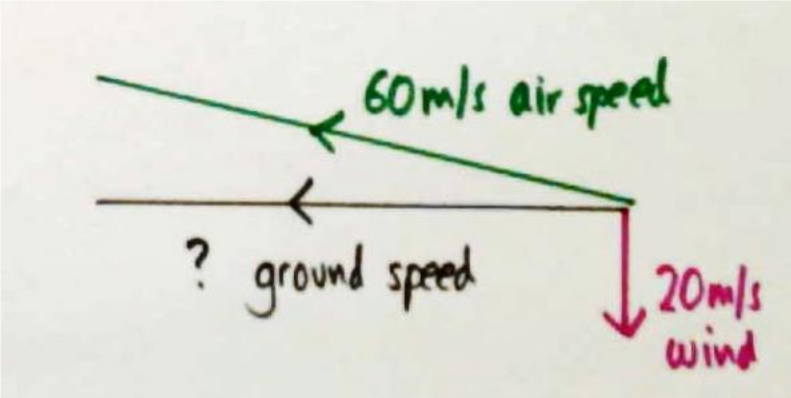
Give the magnitude and direction of the sum



Give the magnitude and direction of the groundspeed, which is the vector sum of wind and airspeed



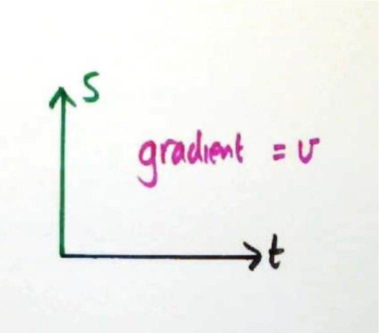
Give the magnitude and direction of the diagonal force needed to balance the other two forces

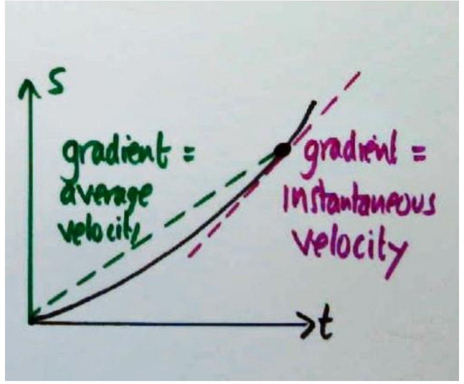


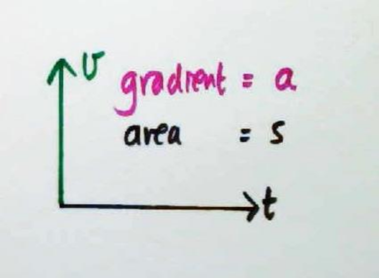
Give the direction of the airspeed and magnitude of the groundspeed

Describing motion

- Displacement s is the vector giving position
  - Gradient of s-t graph gives velocity
  - In two-dimensions, s can have x and y components
- Velocity is the rate of change of displacement
  - Starting velocity is written u, final velocity is v
  - Velocity is the gradient of an s-t graph
  - Displacement = average velocity x time, so $s = \frac{u+v}{2} t$
  - Displacement is the area under a v-t graph
- Acceleration is the rate of change of velocity
  - Acceleration is the gradient of a v-t graph
  - $a = \frac{v-u}{t}$







Constant acceleration situations

- Where force is steady (e.g. motion under gravity alone)
- We use the equations $s = \frac{u+v}{2}t$ and $a = \frac{v-u}{t}$ (so $v = u + at$)
  - Putting the equations together gives
  - $s = \frac{u+v}{2} \times \frac{v-u}{a} = \frac{v^2-u^2}{2a}$, and also $s = \frac{u+(u+at)}{2}t = ut + \frac{1}{2}at^2$
  - If the right equation is selected, we can solve a problem.
- Example: calculate s if a=6ms^{-2}, u=0ms^{-1}, v=50ms^{-1}
  - We want the equation with no t as t is not involved
  - $s = \frac{v^2-u^2}{2a} = \frac{50^2-0^2}{2\times 6} = 208\mathrm{m}$

Constant acceleration practice

1. How fast is a tennis ball falling 3.5s after it has been dropped from a height of 100m?

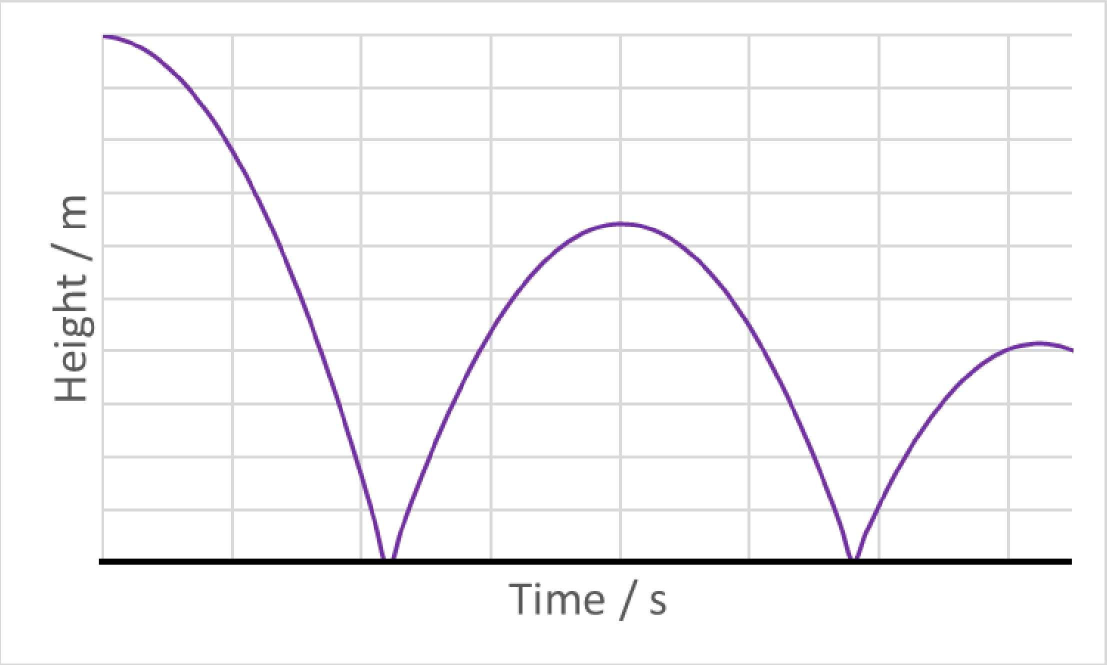
2. How far will the tennis ball have fallen?

3. How much time will pass before the ball hits the ground?

4. How much time would it take if it were thrown downwards at $12ms^{-1}$ rather than dropped?

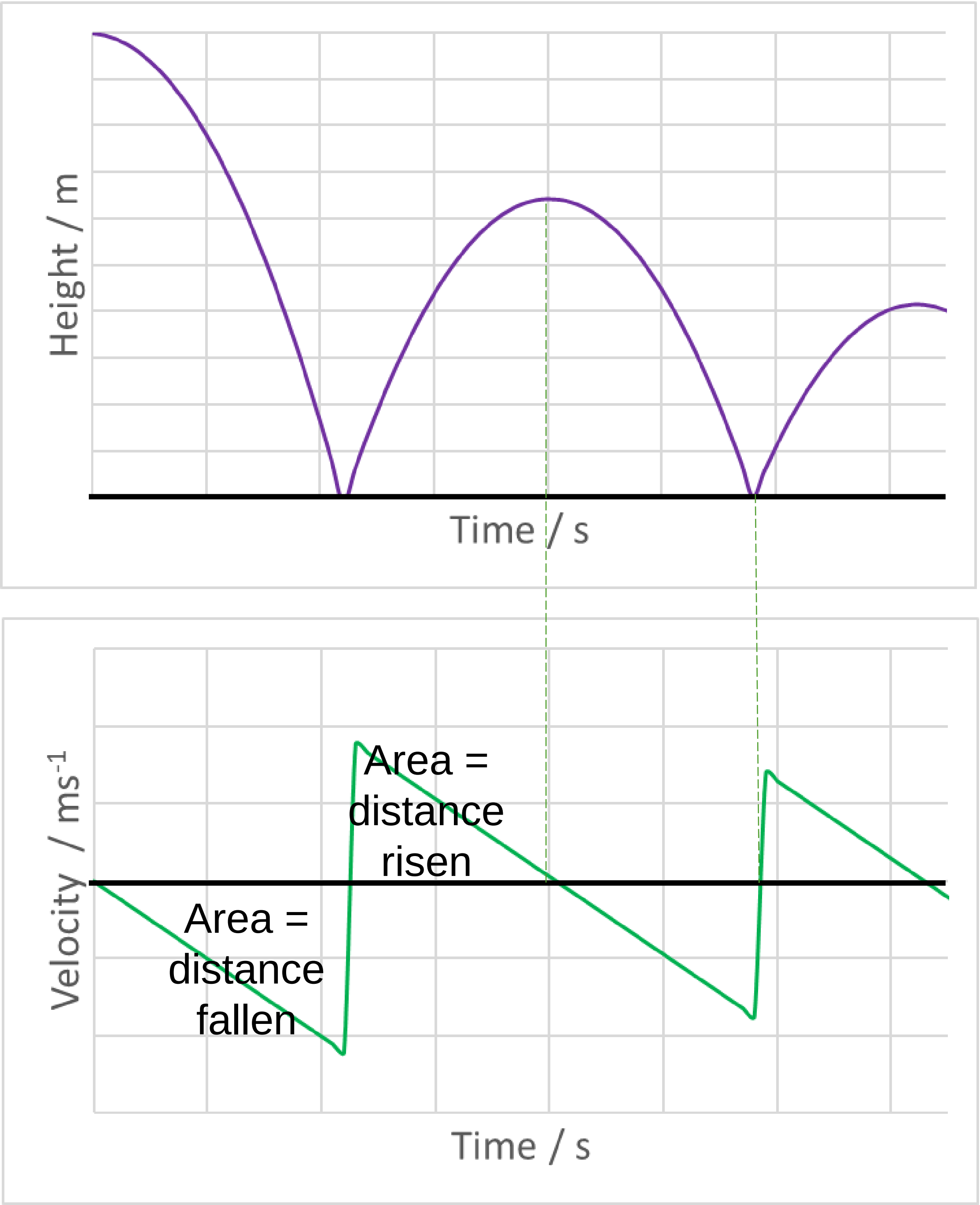
A bouncing ball

- Draw the velocity-time graph for a ball dropped onto a hard surface.
- The displacement-time graph is given here.



A bouncing ball

- When ball is falling, velocity is negative.
- At the bounce point, the velocity is quickly changed (by the force) to a high positive value
- While the ball is not in contact with the ground, the v-t graph is a straight line with gradient $-9.8 ms^{-2}$: this is its acceleration.



Motion in 2d - trajectories

› Vertical (y) and horizontal (x) motion are separate

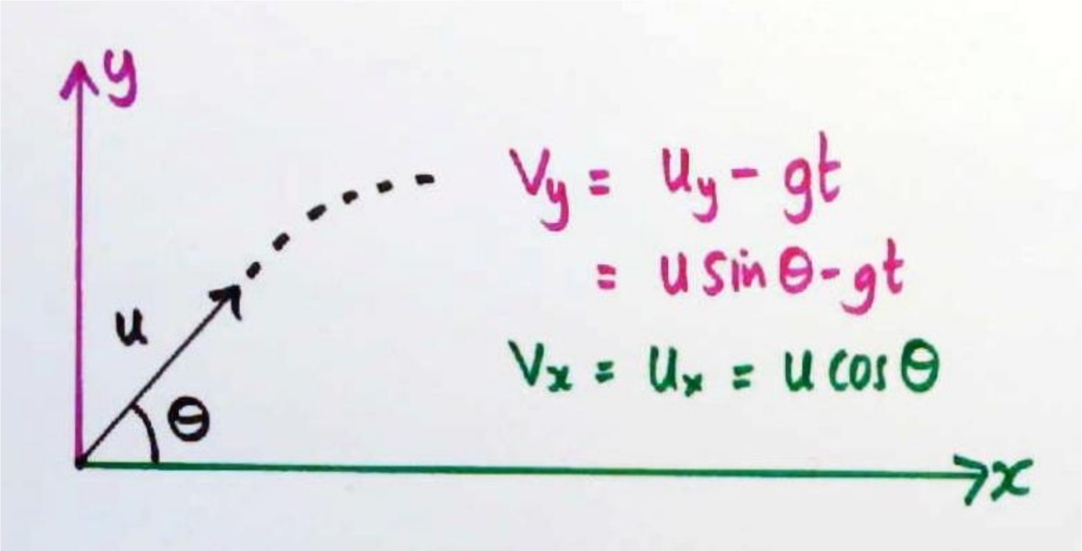
- $s_y = u_y t + \frac{1}{2} a_y t^2$ while $s_x = u_x t$ as usually $a_x = 0$
- "A ball is thrown horizontally at $4.5 ms^{-1}$ from a height of 2.5m. How high is it after it has been in the air for 0.3s?"

- A grid can help
  - use one side to work out the time
  - then use the other side to answer the problem

Vertical motion	Horizontal motion
$a_y = -10\ ms^{-2}$	$a_x = 0$
$u_y = 0\ ms^{-1}$	$u_x = v_x = 4.5\ ms^{-1}$
$v_y =$	
$s_y =$?	$s_x =$
$t = 0.3\ s$	

Motion in 2d – diagonal starts

- Use vectors to break the initial motion into vertical and horizontal components.
- Then work separately with the horizontal and vertical quantities.



Motion in 2d - example

A ball is thrown at $14ms^{-1}$ at an angle of 50° to the ground.

What is its maximum height?

How far from the start will it be when it lands?

<table>
<tr><th>Vertical motion</th><th>Horizontal motion</th></tr>
<tr><td>$a_y = -9.8\ ms^{-2}$</td><td>$a_x = 0$</td></tr>
<tr><td>$u_y =$</td><td rowspan="2">$u_x = v_x =$</td></tr>
<tr><td>$v_y =$</td></tr>
<tr><td>$s_y =$</td><td>$s_x =$</td></tr>
<tr><td colspan="2">$t =$</td></tr>
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Motion in 2d practice

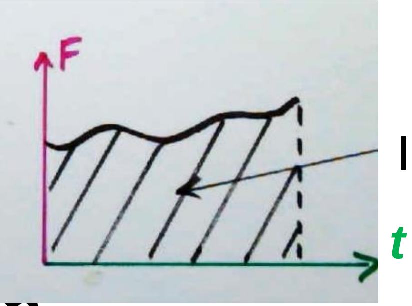
1. A lump of putty is fired at a wall horizontally at 4.0m/s. If the wall is 2.5m away, how far below the aiming point does the putty hit the wall?

2. How far uprange of a village should an aid package be dropped from an aircraft flying at 90ms^{-1} at a height of 2500m?

Momentum

› **Momentum = mass x velocity**
  - **Momentum is a vector quantity, measured in $kgms^{-1}$ or Ns**
  - **Internal forces within a system do not change the total momentum**

- **Impulse = Force x time**
  - **If the force varies,**
    - **impulse = average force x time**
    - **impulse is area under F-t graph**



- **Impulse = change of momentum**

Newton's Laws of motion

- **First Law – if no unbalanced force acts on an object, its momentum will not change.**
  - this means that its velocity will not change. It will either remain stationary, or moving at a steady speed in the same direction.
- **Second Law – the rate of change of momentum is proportional to the force applied**
  - momentum change = impulse (in SI units) $mv - mu = Ft$
  - for steady mass systems, we have $F = ma$
- **Third Law – where two objects interact, the forces which act on the two objects are equal and opposite**
  - the momentum gain of the one object is equal to the loss of the other

Momentum practice

1. Calculate the final speed of a 720kg car accelerated by an 850N force from rest for 2.5s.

2. Calculate the force needed to keep a conveyor belt going at $1.5 ms^{-1}$ if 32kg of initially stationary sand lands on it every second.

3. The speed of a 30g projectile is measured by firing it into a 5.00kg bag of sand. When the projectile embeds itself into the bag, the bag starts moving at $0.89 ms^{-1}$. How fast was the projectile travelling?

Drag

- Drag is a force between an object and its surrounding fluid (liquid or gas) which resists its motion.
- Depending on the size and speed of the object, and the density and viscosity (thickness) of the fluid, the drag will either be proportional to the speed or to speed^2.
- In the case of objects falling through air,
  - Drag is proportional to speed^2.
  - If D = 2.8N when v = 21ms^{-1}, what will D be when v=35ms^{-1}?

    $D = kv^2$ so $k = \frac{D}{v^2} = \frac{2.8}{21^2} = 0.00635$ Ns^{2}m^{-2}

    New drag = $kv^2 = 0.00635 \times 35^2 = 7.8$ N

Terminal velocity

- When an object falls, two forces act – drag and weight
- Initially, $v = 0$ and acceleration = g
- As object speeds up,
  - drag increases, and
  - acceleration reduces
- When D=mg, it has a steady speed called terminal velocity

Acceleration

$$= \frac{\text{resultant force}}{\text{mass}}$$

$$= \frac{mg - kv^2}{m} = g - \frac{kv^2}{m}$$

If speed is constant $a = 0$

so $g = \frac{kv^2}{m}$ $v = \sqrt{\frac{mg}{k}}$

Drag practice

In these questions, assume that drag is proportional to speed^2.

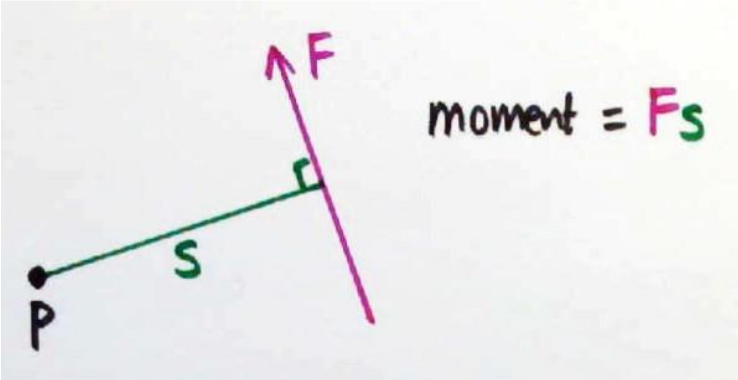
1. If an 80kg person skydives and has a terminal velocity of 72m/s, calculate their acceleration when falling at 45m/s.

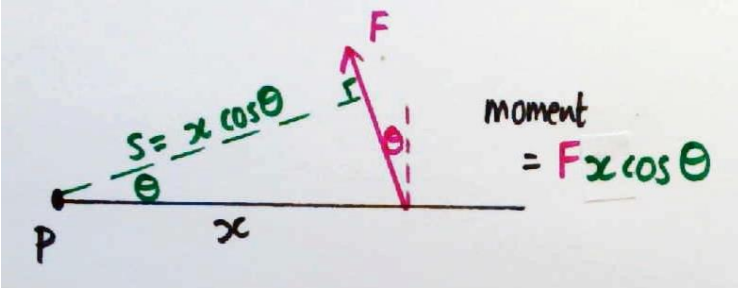
2. When falling at terminal velocity, their parachute opens, and the drag increases by a factor of 85. What will the acceleration be the moment after the parachute has opened?

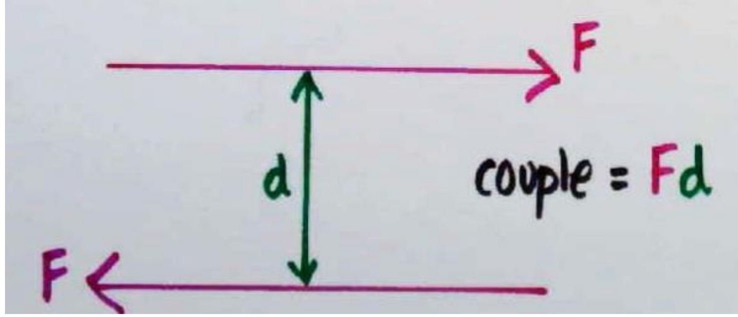
3. Calculate the terminal velocity with the parachute open.

Moments

- The moment of a force about a point = Force x perpendicular distance
- Let + mean ↺ and – mean ↻
- Principle of moments: Equilibrium if sum of moments = 0
- You choose where to put the point
- A couple has the same moment about any point in the plane
- Draw weight from centre of mass

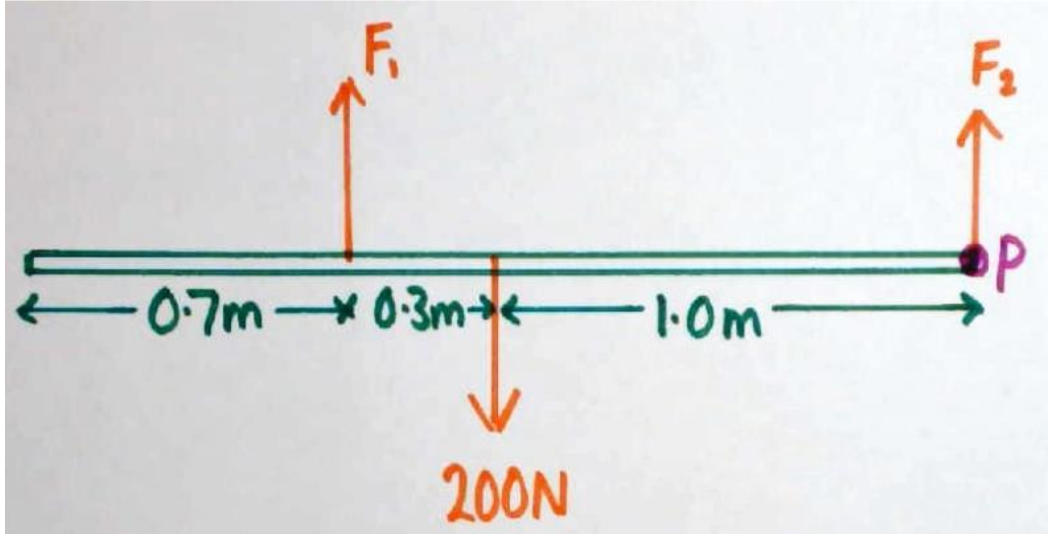






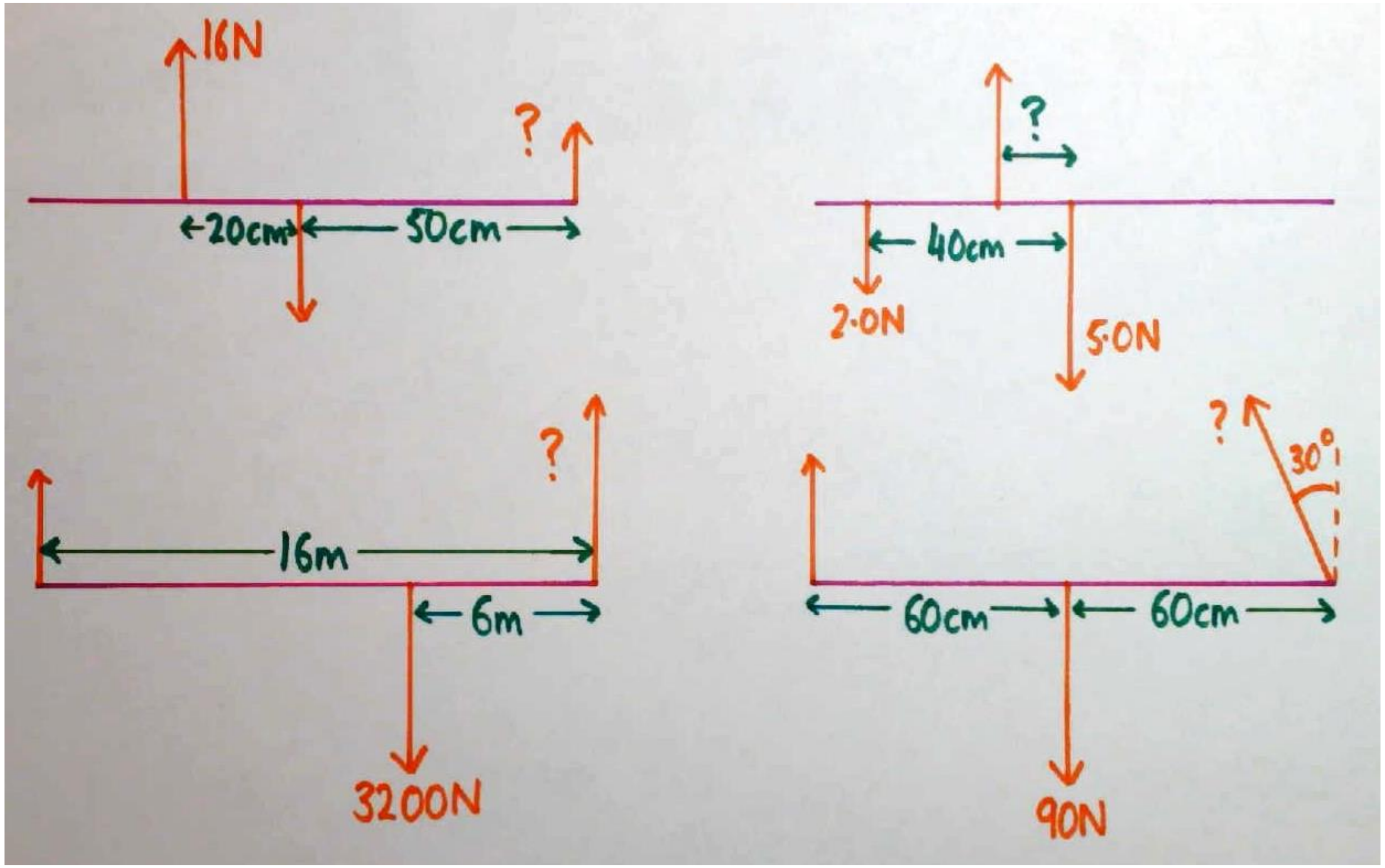
Moments – two force problems

- Aim: to calculate the force F_1.
- Take moments about P
  - Moment of $F_2 = 0$
  - Moment of $F_1 = -1.3\,F_1$
  - Moment of $W = 200x1 = 200$
- For equilibrium
  - $200 - 1.3F_1 = 0$
  - $F_1 = 154N$
- For bonus point
  - $F_2 + F_1 = 200N$
  - $F_2 = 200N - F_1 = 200N - 154N = 46N$

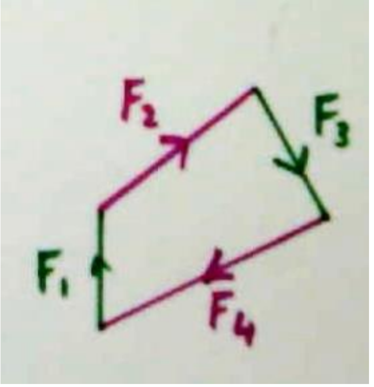


Moments practice

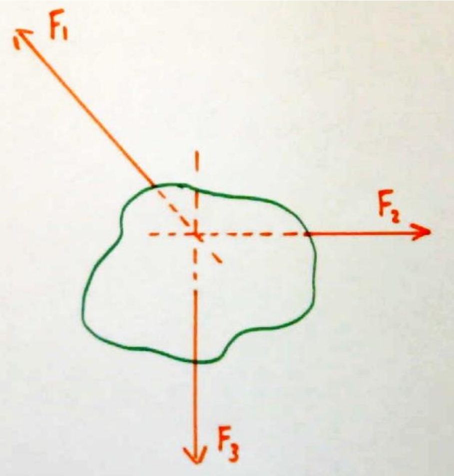
- **Work out the force or distance labelled '?'.**



Equilibrium



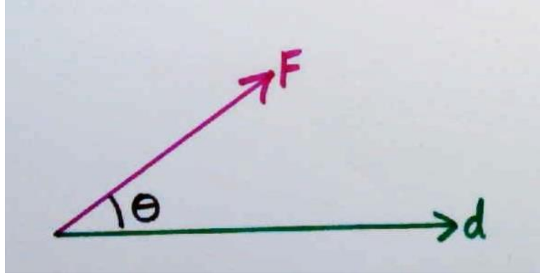
- An object will be in equilibrium if
- 1. There is no resultant force on it
  - This means that the x-components of all forces sum to zero (as do the y-components and z-components)
  - Alternatively, if the forces are drawn as nose-to-tail arrows, they form a closed shape
- 2. The total moment about a point is zero (you can choose any point)
  - If all of the force lines pass through a point on the object, this will automatically be true.



Work done & energy transfer

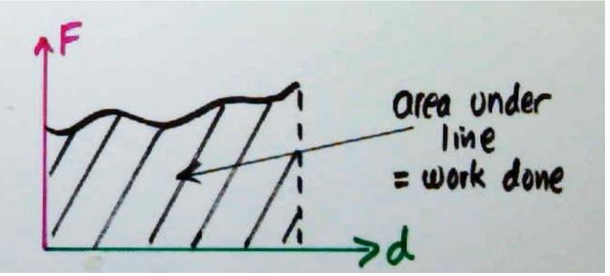
Energy transfer to an object

= work done on it
= Force on object x distance it moves in direction of force
= Distance x component of force in direction of distance
= $F d \cos\theta$ where θ is the angle between F and d



If force is not constant

work done is area under the F-d graph

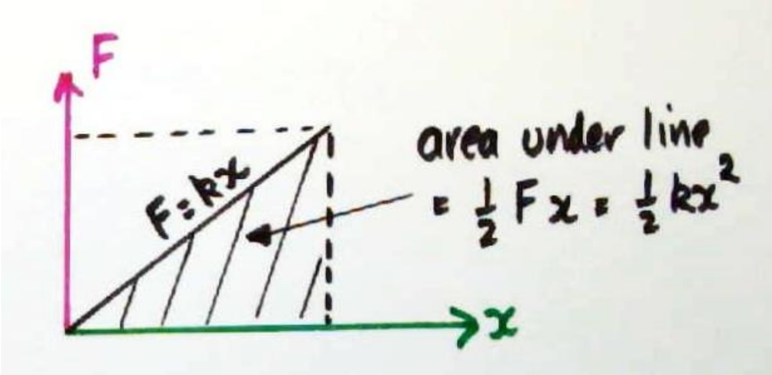


Gravitational Potential Energy

$F = mg$, $d\cos\theta = h$, so $E = mgh$

Elastic Potential Energy

As in diagram, $E = \frac{1}{2}Fx = \frac{1}{2}kx^2$



Kinetic energy & power

- Work done in accelerating an object from *u* to *v*
  - $E = Fs = mas$ but $2as = v^2 - u^2$ so $E = \frac{mv^2}{2} - \frac{mu^2}{2}$
  - Kinetic energy = energy to accelerate from *u*=0 to *v*= $\frac{mv^2}{2}$
- Power = Work done / Time taken
  - $P = \frac{E}{t} = \frac{Fs\cos\theta}{t} = F\frac{s}{t}cos\theta = Fv\cos\theta$
  - Power = velocity x component of force in direction of motion

Energy practice

1. Calculate the kinetic energy gained by a 12 000kg bus when it accelerates from $12ms^{-1}$ to $18ms^{-1}$.

2. A 200kg pumpkin is winched up until it gains 18kJ of gravitational potential energy.
   a) How high was it winched?
   b) When it is dropped, how fast will it be just before it hits the ground?

3. What is the top speed of a car with an engine power of 150kW if it faces resistance forces of 3300N?

4. What is the maximum extension of a $20Ncm^{-1}$ spring if 200J of work is done on it?

Solving 2-d motion problem

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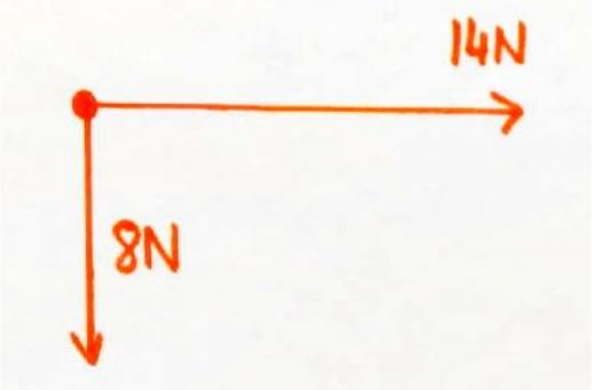
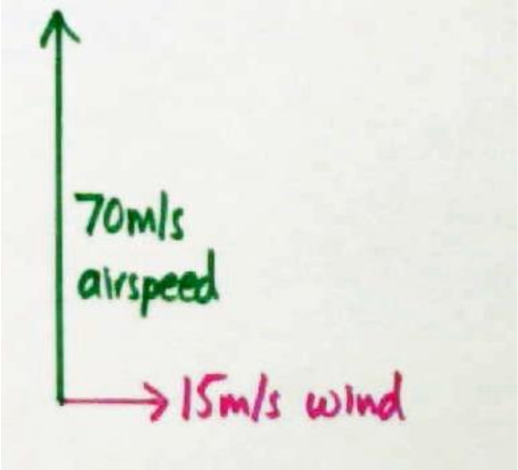
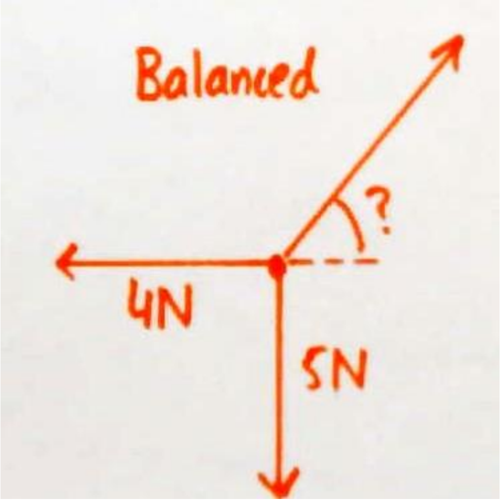
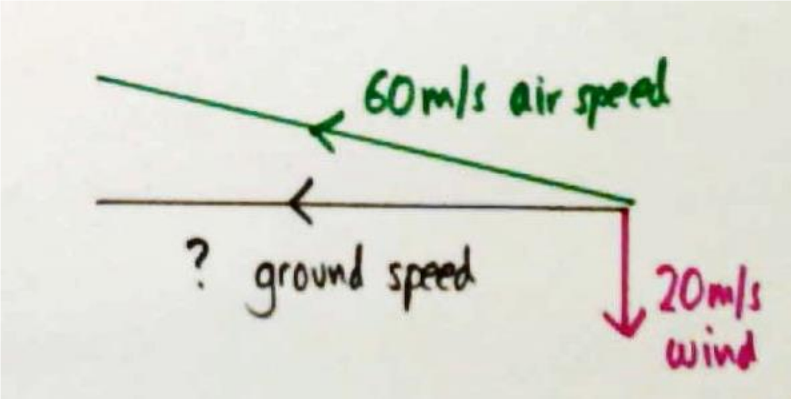
Vector addition practice









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