Projectiles: Trajectories 4ii

A Level

A particle P is projected with speed $40\,\mathrm{m\,s^{-1}}$ at an angle of 35° above the horizontal from a point O.

Part A Magnitude of velocity

For the instant $3\,\mathrm{s}$ after projection, calculate the magnitude of the velocity of P. Give your answer to 3 significant figures.

Part B Direction of velocity

For the instant $3\,\mathrm{s}$ after projection, calculate the direction of the velocity of P. Give your answer as an angle, in degrees, below the horizontal to 3 significant figures.

Projectiles: Trajectories 1i

A Level

A stone is projected horizontally with speed $7\,\mathrm{m\,s^{-1}}$ from a point O on the edge of a vertical cliff. The horizontal and upward vertical displacements of the stone from O at any subsequent time, t seconds, are $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively. Assume that there is no air resistance.

Part A x in terms of y

In this question, use the value $g = 9.8\,\mathrm{m\,s^{-2}}$ for the acceleration under gravity.

By first expressing x and y in terms of t, find an expression for y in terms of x.

The following symbols may be useful: `x`, `y`

Part B Distance between cliff and stone

The stone hits the sea at a point which is $20\,\mathrm{m}$ below the level of O.

Find the distance between the foot of the cliff and the point where the stone hits the sea. Give your answer to 3 significant figures.

Part C Speed and direction of motion

Find the speed of the stone immediately before it hits the sea. Give your answer to 2 significant figures.

Find the direction of motion of the stone immediately before it hits the sea. Give your answer as an angle below the horizontal to 3 significant figures.

Used with permission from UCLES, A Level, January 2006, OCR M1, Question 6

Gameboard:
STEM SMART Single Maths 39 - Projectiles & Parametric Equations

Projectiles: Trajectories 4i

A Level

A particle is projected with speed $7\,\mathrm{m\,s^{-1}}$ at an angle of elevation of 30° from a point O and moves freely under gravity. The horizontal and vertically upwards displacements of the particle from O at any subsequent time $t\,\mathrm{s}$ are $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively.

Part A x & y in terms of t

In this question, use the value $g = 9.8\,\mathrm{m\,s^{-2}}$ for the acceleration under gravity.

Express x in terms of t. When entering your answer, use fractions and surds rather than decimals.

The following symbols may be useful: `cos(), sin(), t, tan(), x`

Express y in terms of t. When entering your answer, use fractions rather than decimals.

The following symbols may be useful: `cos(), sin(), t, tan(), y`

Part B y in terms of x

Hence find the equation, y in terms of x, for the trajectory of the particle.

The following symbols may be useful: `x, y`

Part C Values of x

Calculate the smaller of two values of x when $y = 0.6$. Give your answer as an exact surd.

Calculate the larger of two values of x when $y = 0.6$. Give your answer as an exact surd.

Part D Direction of motion

Find the direction of motion of the particle when $y = 0.6$ and the particle is rising. Give your answer as an angle from the horizontal and to 3 significant figures.

Adapted with permission from UCLES, A Level, OCR M2, June 2011, Question 5

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Parametric Equations 3ii

A Level

Figure 1 shows the curve with parametric equations

$$x = a\sin\theta,\ \ y = a\theta\cos\theta,$$

where a is a positive constant and $-\pi \leq \theta \leq \pi$. The curve meets the positive y-axis at A and the positive x-axis at B.

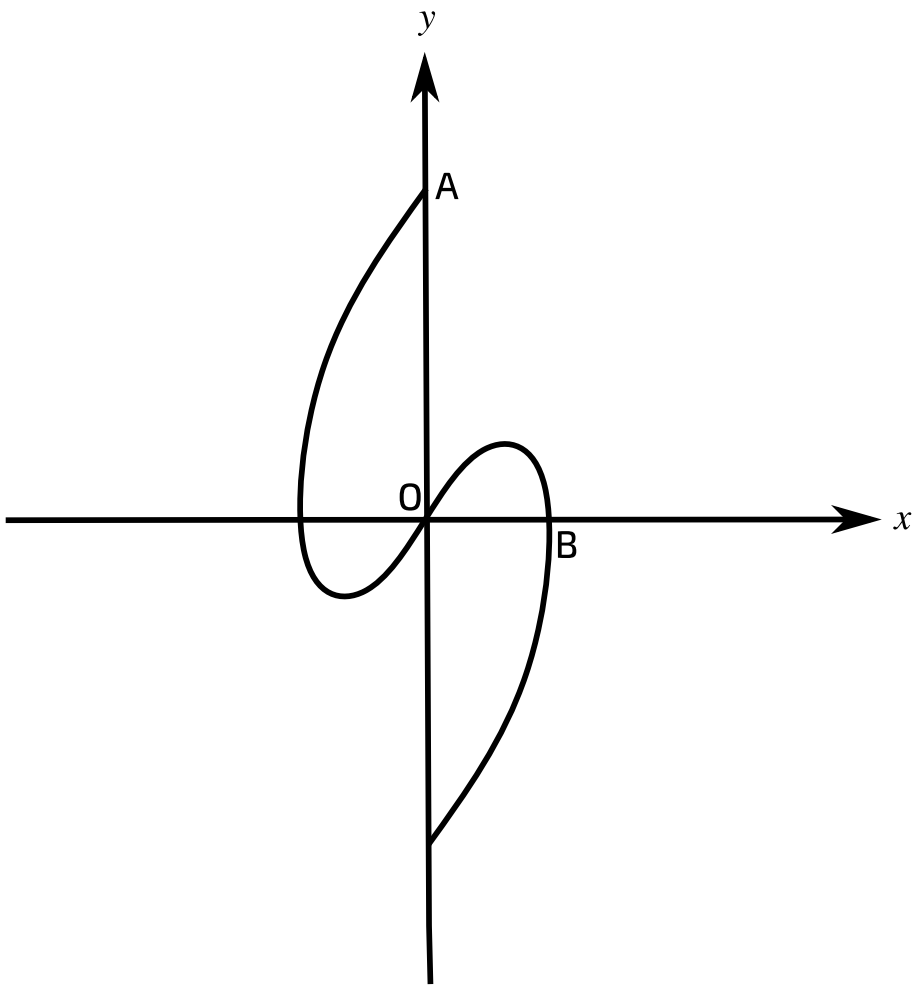

Figure 1: The graph defined by $x = a\sin\theta, y = a\theta\cos\theta$ for $-\pi \leq \theta \leq \pi$.

Part A Points O, A and B

Write down the value of θ corresponding to the origin.

The following symbols may be useful: `arccos(), arcsin(), arctan(), cos(), cosec(), cot(), pi, sec(), sin(), tan(), theta`

State the y coordinate of A.

The following symbols may be useful: `a, arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), pi, sec(), sin(), tan(), y`

State the x coordinate of B.

The following symbols may be useful: `a, arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), pi, sec(), sin(), tan(), x`

Part B Gradient

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `, Derivative(y, x), arccos(), arccosec(), arccot(), arcsec(), arcsin(), arctan(), cos(), cosec(), cot(), dx, dy, sec(), sin(), tan(), theta, x, y`

Part C Tangent equation

Find the equation for the tangent to the curve at the origin.

The following symbols may be useful: `x, y`

Gameboard:

STEM SMART Single Maths 39 - Projectiles & Parametric Equations

Parametric Equations 1ii

A Level

A curve is defined by the parametric equations

$$x = \sin^2\theta, y = 4\sin\theta - \sin^3\theta$$

where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

Part A Differential $\frac{\mathrm{d}y}{\mathrm{d}x}$

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x)`, `arccos()`, `arccosec()`, `arccot()`, `arcsec()`, `arcsin()`, `arctan()`, `cos()`, `cosec()`, `cot()`, `dx`, `dy`, `sec()`, `sin()`, `tan()`, `theta`, `x`, `y`

Part B Point on the curve

Find the coordinates of the point on the curve at which the gradient is 2.

Find the x coordinate.

Find the y coordinate.

Part C Stationary points

Drag and drop answers into the boxes below to complete the argument showing that the curve has no stationary points.

If the curve has stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x}$ [ ] at those points. Hence, using the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ found in part A,

$$[\ ] - 3\sin^2\theta = 0$$

$$\Rightarrow \sin\theta = \pm\sqrt{\frac{[\ ]}{3}}$$

However, $\sin\theta$ obeys the inequality [ ] $\leq \sin\theta \leq$ [ ] so there is no value of θ that satisfies $\sin\theta = \pm\sqrt{\frac{[\ ]}{3}}$. Therefore, there are no stationary points.

Items:

-2 -1 < 0 $= 0$ 0 > 0 1 2 4 5 6 7

Part D Cartesian equation

Find a cartesian equation of the curve, giving your answer in the form $y^2 = f(x)$.

The following symbols may be useful: `x, y`

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Parametric Equations 2i

A Level

A curve has parametric equations

$$x = \frac{1}{t+1}, y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

Part A Value of t

Show that the value of t at one of these points is -2 and find the value of t at the other point.

The following symbols may be useful: `t`

Part B Normal

Find the equation of the normal to the curve at the point for which $t = -2$, giving your answer in the form $y = f(x)$.

The following symbols may be useful: `x, y`

Part C Value of t

Find the value of t at the point where this normal meets the curve again.

The following symbols may be useful: `t`

Part D Cartesian Equation

Find a cartesian equation of the curve, giving your answer in the form $y = f(x)$.

The following symbols may be useful: `x, y`

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Physics. You work it out.

Parametric Integration 1

A Level

The curve C has parametric equations

$$x = 2t^2 - 3 \qquad y = t(4 - t^2)$$

The curve crosses the x-axis at the points A and B and the region R is enclosed by the loop of the curve, as shown in **Figure 1**.

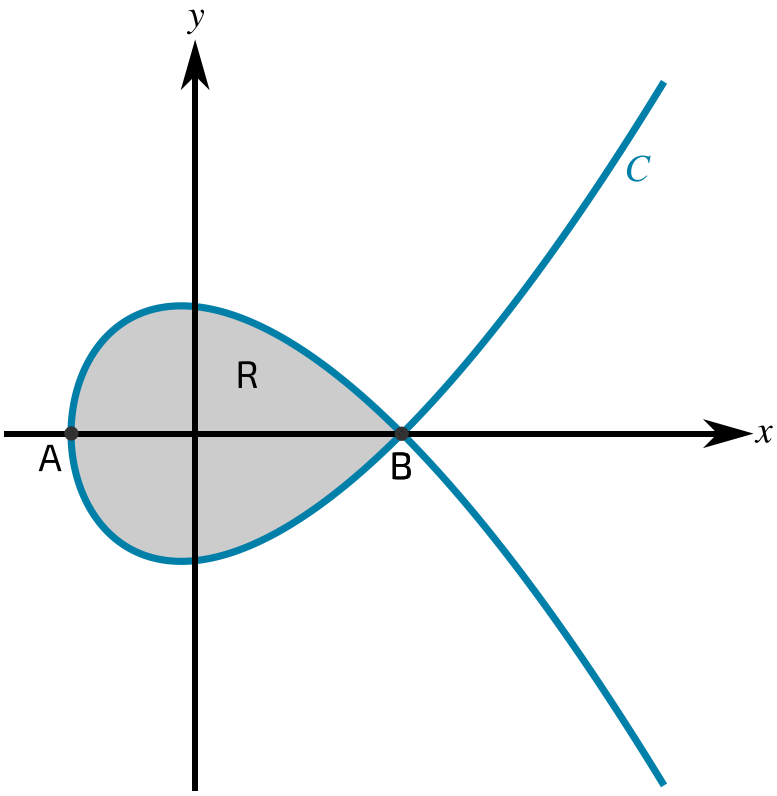

Figure 1: A graph of the curve C.

Part A Point A

Find the x-coordinate of the point A.

Part B Point B

Find the x-coordinate of the point B.

Part C Area of R

The region R is enclosed by the loop of the curve, as shown in **Figure 1**. Find the exact value of the area of R.

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Parametric Equations 4i

A Level

A curve has parametric equations

$$x = 2\sin t, \qquad y = \cos 2t + 2\sin t$$

for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

Part A Derivative

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of t.

The following symbols may be useful: `Derivative(y, x)`, `cos()`, `cosec()`, `cot()`, `sec()`, `sin()`, `t`, `tan()`, `x`, `y`

Part B Coordinates

Find the x-coordinate of the stationary point.

The following symbols may be useful: `x`

Find the y-coordinate of the stationary point.

The following symbols may be useful: `y`

Part C Equation

Find the cartesian equation of the curve.

The following symbols may be useful: `x, y`

Part D Range

Find the range of values x can take.

What form does your answer take? Choose from the list below, where a and b are constants and $a < b$, and then find a and/or b.

- $x < a$
- $x \leq a$
- $x > a$
- $x \geq a$
- $a < x < b$
- $a \leq x \leq b$
- $x < a$ or $x > b$
- $x \leq a$ or $x \geq b$

Write down the value of a.

Write down the value of b (or if your chosen form has no b, write "n").

The following symbols may be useful: `n`

Part E Sketch

Hence sketch the curve.