

Gameboard

Maths

Complex Numbers: Manipulations 3i

Complex Numbers: Manipulations 3i



The complex number 2+i is denoted by z, and the complex conjugate of z is denoted by z^* .

Part A z^2

Express z^2 in the form x+iy, where x and y are exact real numbers.

The following symbols may be useful: i

Part B $4z-z^2$

Express $4z - z^2$ in the form x + iy, where x and y are exact real numbers.

The following symbols may be useful: i

Part C zz^*

Express zz^* in the form x+iy, where x and y are exact real numbers.

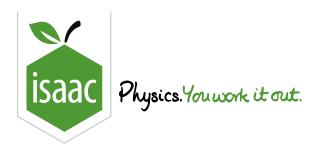
The following symbols may be useful: i

Part D $\frac{z+1}{z-1}$

Express $\frac{z+1}{z-1}$ in the form x+iy, where x and y are exact real numbers.

The following symbols may be useful: i

Adapted with permission from UCLES, A Level, OCR FP1 Specimen paper, Paper 4725, Question 3.



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Maths

Complex Numbers: Manipulations 1i

Complex Numbers: Manipulations 1i



The complex number z has modulus $2\sqrt{3}$ and argument $-\frac{\pi}{3}$.

Part A 2

Find z in the form z=x+iy, where x and y are are exact real numbers.

The following symbols may be useful: i, z

Part B $\frac{1}{(z^*-5i)^2}$

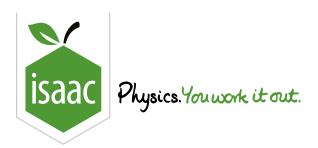
Find $\frac{1}{(z^*-5i)^2}$ in the form x+iy, where x and y are exact real numbers.

The following symbols may be useful: i

Adapted with permission from UCLES, A Level, June 2016, Paper 4725, Question 2.

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STEM SMART Double Maths 31 - Complex Numbers & Argand Diagrams



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Maths

Complex Numbers: x+iy and Euler 3i

Complex Numbers: x+iy and Euler 3i



The complex number z satisfies the equation

$$z + 2iz^* = 12 + 9i$$

Part A 2

Find z in the form z = x + iy.

The following symbols may be useful: i, z

Part B Modulus-Argument

z can also be expressed in the form

$$z = r(\cos\theta + i\sin\theta)$$

Find r.

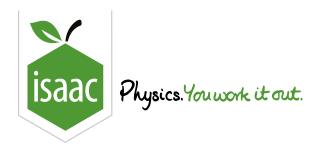
The following symbols may be useful: r

Find θ to 3 significant figures in radians.

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Maths

Complex Numbers: Equations to Quartics 1ii

Complex Numbers: Equations to Quartics 1ii



Part A Square roots

The square roots of the complex number 5+12i can be expressed in the form x+iy.

Give the square root with positive x and positive y.

The following symbols may be useful: ${\tt i}$

Give the square root with negative x and negative y.

The following symbols may be useful: i

Part B $(3-2i)^2$

Find $(3-2i)^2$ in the form x+iy where x and y are exact.

The following symbols may be useful: i

Part C Roots of quartic

				4	مناهر مربيم ممالا
The answers to the	previous	parts can b	oe used 1	to solve	the quartic

$$z^4 - 10z^2 + 169 = 0$$

The roots to the quartic can be expressed in the form x + iy.

Give the root with positive x and positive y.

The following symbols may be useful: i

Give the root with positive x and negative y.

The following symbols may be useful: i

Give the root with negative x and positive y.

The following symbols may be useful: i

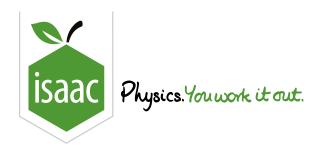
Give the root with negative x and negative y.

The following symbols may be useful: i

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 9.

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Maths

Applying Complex Numbers 2ii

Applying Complex Numbers 2ii



One root of the cubic equation $x^3 + bx^2 + cx - 15 = 0$, where b and c are real, is the complex number 2 + i.

Part A Complex root

Find the other complex root in the form x+iy.

The following symbols may be useful: i

Part B Real root

Find the real root.

Part C

Find b.

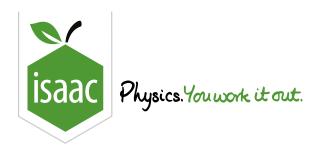
The following symbols may be useful: b

Part	c			
	Find c .			
	The following symbols may be useful: c			
				_

Adapted with permission from Sally Waugh.

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Maths

Complex Numbers: Equations to Quartics 1i

Complex Numbers: Equations to Quartics 1i



One root of the quadratic equation $z^2 + az + b = 0$, where a and b are real, is 16 - 30i.

Part A Other root

Give the other root in the form x+iy.

The following symbols may be useful: i

Part B a and b

Find the value of a

The following symbols may be useful: a

Find the value of b.

The following symbols may be useful: b

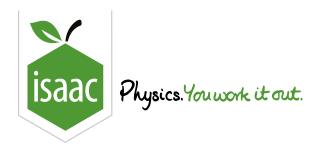
Part	C Quartic
	The quartic equation $z^4+az^2+b=0$ has roots in the form $x+iy$.
	Give the root with positive x and positive y .
	The following symbols may be useful: i
	Give the root with positive x and negative y .
	The following symbols may be useful: i
	Give the root with negative x and positive y .
	The following symbols may be useful: i
	Give the root with negative x and negative y .

Adapted with permission from UCLES, A Level, June 2011, Paper 4725, Question 9.

The following symbols may be useful: i

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Maths

Argand Diagrams and Simple Loci 2i

Argand Diagrams and Simple Loci 2i



The complex number a is denoted by $1+i\sqrt{3}$.

Part A

Find the value of |a|.

Find $\arg a$ in exact form.

The following symbols may be useful: pi

Part B Loci

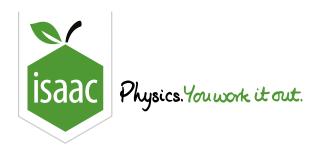
Sketch the loci given by |z-a|=|a| and ${
m arg}\,(z-a)={1\over 2}\pi$ on a single Argand diagram.

Easier question?

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Maths

Argand Diagrams: Using Loci 2i

Argand Diagrams: Using Loci 2i



The loci C_1 and C_2 are given by

$$|z| = |z - 4i|$$

and

$$rg z = rac{\pi}{6}$$

respectively.

Part A Loci of C_1 and C_2

Sketch the loci of \mathcal{C}_1 and \mathcal{C}_2 on a single Argand diagram.

Easier question?

Part B Intersection

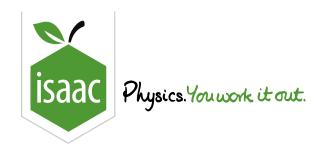
Hence find, in the form x+iy, the complex number represented by the point of intersection of C_1 and C_2 . Give your answer in exact form.

The following symbols may be useful: i

Adapted with permission from UCLES, A Level, June 2008, Paper 4725, Question 6.

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Maths

Argand Diagrams: Solving Inequalities 1ii

Argand Diagrams: Solving Inequalities 1ii



The loci L_1 and L_2 are given by

$$|z|=2$$

and

$$arg(z-3-i)=\pi$$

respectively.

Part A Equation of L_1

By writing z in the form x+iy, express the equation for L_1 in Cartesian form, simplifying your answer as far as possible.

The following symbols may be useful: x, y

Part B Loci

Sketch L_1 and L_2 on a single Argand diagram.

More practice questions?

Part C Inequalities

Indicate, by shading, the region of the Argand diagram for which

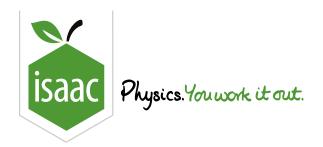
$$|z| \leq 2 ext{ and } 0 \leq rg(z-3-i) \leq \pi$$

More practice questions?

Adapted with permission from UCLES, A Level, June 2018, Paper 4725, Question 4.

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Maths

Argand Diagrams: Solving Inequalities 4ii

Argand Diagrams: Solving Inequalities 4ii



The loci L_1 and L_2 are given by

$$|z - 3 + 4i| = 5$$

and

$$|z| = |z - 6|$$

respectively.

Part A Equation of L_1

Give the equation of L_1 in the form $(x-a)^2+(y-b)^2=c^2$.

The following symbols may be useful: x, y

Part B Inequalities

Indicate, by shading, the region of the Argand diagram for which

$$|z-3+4i| \le 5 \text{ and } |z| \ge |z-6|$$

More practice questions?

Adapted with permission from UCLES, A Level, June 2018, Paper 4725, Question 4.