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# Stationary Points 1

Pre-Uni Maths for Sciences J2.1

A Level  
P P P

Find the coordinates, nature and number of the stationary points of the following functions.

**Part A**  $y = 2x^3 - 24x - 5$

$$y = 2x^3 - 24x - 5$$

Give the stationary points in order of increasing  $x$ -coordinate.

- There is a  at (, )
- There is a  at (, )

**Part B**  $y = 2x^3 - 5x^2 + 4x + 6$

$$y = 2x^3 - 5x^2 + 4x + 6$$

Give the stationary points in order of increasing  $x$ -coordinate. Enter fractions as improper fractions in their simplest form (e.g. 4/3).

- There is a  at (, )
- There is a  at (, )

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# Maxima and Minima: Problems 2ii



A curve has equation  $y = 3x^3 - 7x + \frac{2}{x}$

## Part A Verify stationary point

Fill in the blanks to verify that the curve  $y = 3x^3 - 7x + \frac{2}{x}$  has a stationary point when  $x = 1$ .

At a stationary point the value of  $\frac{dy}{dx}$  is .

For the curve, the first derivative is

$$\frac{dy}{dx} = \text{} x^2 + \text{} + \text{} x^{-2}$$

Hence, when  $x = 1$ ,

$$\frac{dy}{dx} = \text{}$$

Therefore, the curve has a stationary point when  $x = 1$ .

## Part B Nature of stationary point

Fill in the blanks to determine the nature of the stationary point when  $x = 1$ .

The second derivative of  $y = 3x^3 - 7x + \frac{2}{x}$  is:

$$\frac{d^2y}{dx^2} = \boxed{\phantom{000}} x + \boxed{\phantom{000}} x^{-3}$$

Hence, when  $x = 1$ ,

$$\frac{d^2y}{dx^2} = \boxed{\phantom{000}}$$

Therefore, the stationary point when  $x = 1$  is (maximum / minimum / point of inflection):

## Part C Tangent to curve

The tangent to the curve at this stationary point meets the  $y$ -axis at the point Q. Find the  $y$ -coordinate of Q.

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# Stationary Points 1ii

A Level  
P P P

The curve  $y = x^3 - kx^2 + x - 3$  has two stationary points.

**Part A   Differentiate**

Find  $\frac{dy}{dx}$ .

The following symbols may be useful:  $k$ ,  $x$

---

**Part B   Find  $k$**

Given that there is a stationary point when  $x = 1$ , find the value of  $k$ .

The following symbols may be useful:  $k$

---

Part C    Differentiate twice

Find  $\frac{d^2y}{dx^2}$ .

The following symbols may be useful:  $x$

---

Hence determine whether the stationary point is a minimum or a maximum.

☐ Minimum

☐ Maximum

---

Part D    Find coordinate

Find the  $x$ -coordinate of the other stationary point.

The following symbols may be useful:  $x$

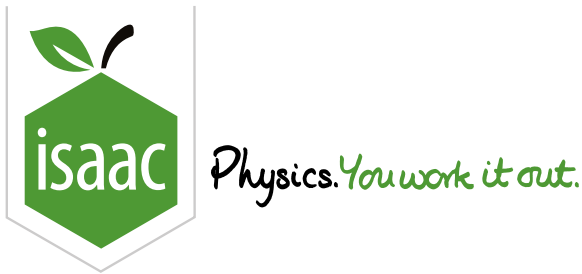
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# Stationary Points 2ii



**Part A**   Find coordinate

Find the coordinates of the stationary points on the curve  $y = x^3 - 3x^2 + 4$ .

Enter the  $x$  and  $y$  coordinates of the stationary point with the greatest  $x$  coordinate. If a value is not a whole number, enter the value as a decimal.

(  ,  )

**Part B**   Stationary point

Determine whether the stationary point whose coordinates you entered is a maximum point or a minimum point.

- ☐ Maximum
- ☐ Minimum
- ☐ Inconclusive

Part C    Range of  $x$

For what range of values of  $x$  does  $x^3 - 3x^2 + 4$  decrease as  $x$  increases?

Construct your answer from the items below.

Items:

<

>

$x$

$< x <$

$\leq x \leq$

$< x \text{ or } x <$

$\leq x \text{ or } x \leq$

$\leq$

$\geq$

$-2$

$-1$

$0$

$1$

$2$

$3$

$4$

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# Maxima and Minima: Problems 1ii

A Level  
P P P

**Figure 1** shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is  $x$  metres, and the area of the enclosure is  $A \text{ m}^2$ .



**Figure 1:** The rectangular enclosure.

## Part A Express as equation

Show that  $A$  can be expressed in the form  $px - qx^2$ , and find this expression.

The following symbols may be useful:  $A$ ,  $x$

## Part B Use differentiation

Use differentiation to find the maximum value of the area of the enclosure,  $A \text{ m}^2$ .

Enter your value of  $A$ :

The following symbols may be useful:  $A$

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# Maxima and Minima: Problems 1i

A Level



A cuboid has a volume of exactly  $8 \text{ m}^3$ . The base of the cuboid is a square with side length  $x$  metres. The surface area of the cuboid is  $A \text{ m}^2$ .

## Part A Find expression for $A$

Show that  $A$  can be expressed in the form  $ax^2 + \frac{b}{x}$ , where  $a$  and  $b$  are constants, and find this expression.

The following symbols may be useful:  $x$

---

## Part B Find $\frac{dA}{dx}$

Find  $\frac{dA}{dx}$ .

The following symbols may be useful:  $x$

---

## Part C Find minimum

Find the value of  $x$  which gives the smallest surface area of the cuboid.

The following symbols may be useful:  $x$

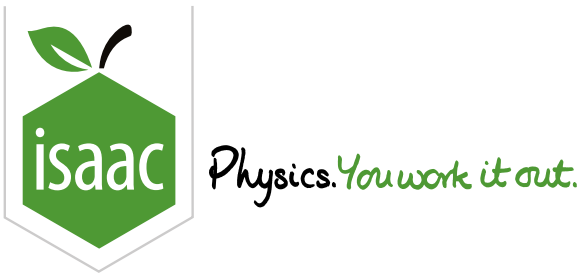
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# Differentiating Powers 7



## Pre-Uni Maths for Sciences J2.3

A quadratic function has the form  $y = a + bx + cx^2$  where  $a$ ,  $b$  and  $c$  are constants. It has a stationary point at  $(2, 2)$  and, at  $x = 1$ , the tangent to the curve has a gradient of  $-2$ . Find the values of  $a$ ,  $b$  and  $c$ . (In practice, with the information given, you will need to find  $b$  and  $c$  before you can find  $a$ .)

Drag and drop the correct values into the equation below.

$y = \square + \square x + \square x^2$

Items:

- 6

-5

-4

-3

-2

-1

0

1

2

3

4

5

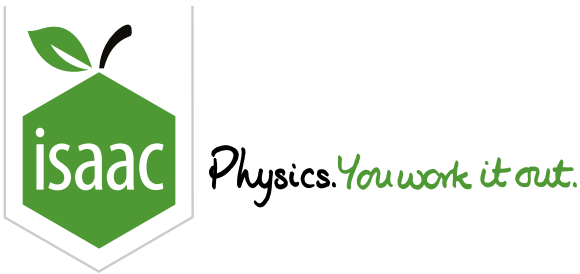
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# Stationary Points 3

Pre-Uni Maths for Sciences J2.5

A Level

C

C

C

Part A

Maximum height of a projectile

A particle is fired upwards into the air with a initial speed  $w$  and moves subsequently under the influence of gravity with an acceleration  $g$  downwards, such that its height  $h$  at time  $t$  is given by  $h = wt - \frac{1}{2}gt^2$ , where  $w$  and  $g$  are constants. Find an expression for its maximum height above its initial position.

The following symbols may be useful:  $g$ ,  $h$ ,  $w$

## Part B Potential energy of two molecules

The potential energy of two molecules separated by a distance  $r$  is given by

$$U = U_0 \left( \left( \frac{a}{r} \right)^{12} - 2 \left( \frac{a}{r} \right)^6 \right)$$

where  $U_0$  and  $a$  are positive constants. The equilibrium separation of the two molecules occurs when the potential energy is a minimum.

Find an expression for the equilibrium separation of the molecules.

The following symbols may be useful:  $U$ ,  $U_0$ ,  $a$ ,  $r$

---

Find an expression for the potential energy when the molecules are at their equilibrium separation.

The following symbols may be useful:  $U$ ,  $U_0$ ,  $a$ ,  $r$

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