Expand and Simplify Binomials

Further A

Pre-Uni Maths for Sciences 3.3.1

Part A $(x+1)^4$

Expand and simplify $(x+1)^4$.

The following symbols may be useful: x

Part B $(z+2a)^3$

Expand and simplify $(z+2a)^3$.

The following symbols may be useful: a, z

Part C $(a-b)^5$

Expand and simplify $(a-b)^5$.

The following symbols may be useful: a, b

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Find Coefficients 2

Further A

P P P

Pre-Uni Maths for Sciences 3.3.3

Without expanding the binomials, find:

Part A Coefficient of x^4y^6

The coefficient of x^4y^6 in the expansion of $(x^2 + 3y^2)^5$.

Part B Coefficient of x^{20}

The coefficient of x^{20} in the expansion of $(x^2 + 3x)^{12}$.

Part C The coefficient of ab^7

The coefficient of ab^7 in the expansion of $(a + \frac{1}{4}b)^8$.

Part D Constant term

The constant term in the expansion of $\left(\frac{x^2}{2} - \frac{8}{x}\right)^9$.

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Group and Expand

Further A

P P P

Pre-Uni Maths for Sciences 3.3.8

Expand $(1 - 2x + 3x^2)^7$ in ascending powers of x as far as x^3.

The following symbols may be useful: x

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Binomial: All Rational n 2i

A Level

Part A Expansion

Expand $(1-4x)^{\frac{1}{4}}$ in ascending powers of x, up to and including the term in x^3.

The following symbols may be useful: x

Part B Values of a and b

The term of lowest degree in the expansion of

$$(1+ax)(1+bx^2)^7-(1-4x)^{\frac{1}{4}}$$

in ascending powers of x is the term in x^3. Find the values of the constants a and b.

What is the value of a?

The following symbols may be useful: a

What is the value of b?

The following symbols may be useful: b

Gameboard:

STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Binomial: All Rational n 1i

A Level

Part A Partial Fractions

Given that $\frac{3x+4}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$, find A, B, and C.

Find A.

The following symbols may be useful: A

Find B.

The following symbols may be useful: B

Find C.

The following symbols may be useful: C

Part B Expand

Hence or otherwise expand $\frac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2.

The following symbols may be useful: x

Part C Values of x

State the set of values of x for which the expansion in the above part is valid.

Construct your answer from the items below.

Items:

x | $<$ | $\leq$ | $>$ | $\geq$ | $< x <$ | $\leq x \leq$ | $> x$ **or** $x >$ | $\geq x$ **or** $x \geq$ | -4 | -2 | $-\frac{3}{2}$ | -1

$-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | 2 | 4

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Maclaurin Series - Binomial

Further A

Part A Expand $(1+r)^{\frac{1}{3}}$ and find $1.1^{\frac{1}{3}}$ and $9^{\frac{1}{3}}$

Expand $(1+r)^{\frac{1}{3}}$ up to the term in r^3.

The following symbols may be useful: `r`

Hence, using your expansion, find $(1.1)^{\frac{1}{3}}$ to 3 decimal places.

Now, using your expansion again, find $9^{\frac{1}{3}}$ to 2 decimal places.

Part B Electric field on the axis of a charged sheet

The electric field E on the axis of a uniformly charged circular sheet at a distance z from the centre of the sheet is given by

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + a^2}}\right]$$

where σ is the charge per unit area on the sheet and a is the radius of the sheet. Show that in the limit when $z \gg a$ the field on the axis is such that $E \approx \frac{A}{z^2}$ and find A.

The following symbols may be useful: `A, a, epsilon_0, sigma, z`

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Maclaurin Series - Cos & Sin 1

Further A

C C C

Pre-Uni Maths for Sciences 6.3.6

Part A Find the cosine of the angle 0.2 rad

Find, using a Maclaurin expansion, the cosine of the angle $0.2\,\mathrm{rad}$, correct to 3 decimal places.

Part B Find the sine of the angle 0.08 rad

Find, using a Maclaurin expansion, the sine of the angle 0.08 rad, correct to 2 significant figures.

Part C Potential energy of mass on pendulum

A pendulum consists of a point mass m suspended on a light string of length l. When the string makes an angle of ϕ to the vertical its potential energy relative to the point where $\phi = 0$ is given by $mgl(1 - \cos\phi)$. Show that for $\phi \ll 1$ the potential energy is given approximately by $A_0\phi^2$ and find an expression for A_0.

The following symbols may be useful: `g, l, m`

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STEM SMART Double Maths 27 - Binomial & Maclaurin Expansions

Maclaurin Series - ln

Pre-Uni Maths for Sciences 6.3.2

Further A

Part A Expand $\ln(1+z)$ and hence $\ln(2+4y)$

(i) Write down the Maclaurin expansion of $\ln(1+z)$ up to the term in z^3.

The following symbols may be useful: `z`

(ii) By re-writing $\ln(2+4y)$ in the form $A+\ln(1+z)$, where A is a constant, find the Maclaurin expansion of $\ln(2+4y)$ up to the term in y^3.

The following symbols may be useful: `y, z`

Part B Expand $\ln(\frac{1+q}{1-q})$

Find the first 4 non-zero terms in the Maclaurin expansion of $\ln\left(\frac{1+q}{1-q}\right)$.

The following symbols may be useful: `q`

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Maclaurin Series - Exponentials 2

Further A

Pre-Uni Maths for Sciences 6.3.4

Part A Expand $Ae^{-\alpha t}$

Expand $Ae^{-\alpha t}$ up to the term in t^2.

The following symbols may be useful: `A, alpha, p, t`

Part B Expand $e^{p} - e^{-p}$

Find the first two non-zero terms in the Maclaurin expansion of $e^{p} - e^{-p}$.

The following symbols may be useful: `A, alpha, p, t`

Part C Energy decay in oscillations

A lightly damped oscillatory system has a period T. The total energy of the system at time t is given by $E(t)$. One period later its energy $E(t+T) = E(t)\mathrm{e}^{-\gamma T}$.

(i) Find an expression for the fractional change in energy in one cycle.

The following symbols may be useful: `T, e, gamma`

(ii) On the assumption that $\gamma T \ll 1$ find an approximate expression for the fractional change in energy in one cycle.

The following symbols may be useful: `T, e, gamma`

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