

Derivatives of Standard Functions 2

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Complete the table of derivatives for these useful standard functions.

In these functions a and k are constants.

Function, $y(x)$	Derivative, $\frac{dy}{dx}$
ax^n	<input type="text"/>
$a \sin kx$	<input type="text"/>
$a \cos kx$	<input type="text"/>
$a \tan kx$	<input type="text"/>
$a e^{kx}$	<input type="text"/>
$a \ln kx$	<input type="text"/>

Items:

- $\frac{a}{x}$
- $\frac{ak}{x}$
- $ak \sec^2 kx$
- anx^{n-1}
- $ak \cos kx$
- ake^{kx}
- ae^{kx}
- $-ak \sin kx$



The Chain Rule

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Fill in the blanks below to complete the summary notes on the chain rule.

Part A Example calculation

The chain rule is used to differentiate a function of a function. If y is a function of u , and u is a function of x , then

$$\frac{dy}{dx} = \frac{dy}{\boxed{}} \frac{\boxed{}}{dx}$$

For example, we can write the function $y = (4x^2 + 7)^5$ as $y = u^5$, where $u = 4x^2 + 7$. Then

$$\frac{dy}{du} = \boxed{}$$

$$\frac{du}{dx} = \boxed{}$$

$$\therefore \frac{dy}{dx} = 5u^4 \times 8x$$

u is a variable we have introduced to help with the calculation. After carrying out the differentiation, we back substitute to write the answer in terms of x only.

$$\frac{dy}{dx} = \boxed{}$$

Items:

- 8x
- du
- dx
- dy
- 40x(4x + 7)⁴
- 5u⁴

Part B

The chain rule in practice

The chain rule is useful in practical situations. A common scenario is that the amount of liquid in a container is changing, either because it is being filled up or because it is leaking. The volume of liquid in the container V is a function of the height of the liquid in the container h , and h changes with time t . The chain rule gives a relationship between the rate of change of the volume and the rate of change of the height.

$$\frac{dV}{\boxed{}} = \frac{dV}{\boxed{}} \frac{dh}{\boxed{}}$$

Another useful relation is the relationship between $\frac{dy}{dx}$ and $\frac{dx}{dy}$:

$$\frac{dx}{dy} = \frac{1}{\boxed{}}$$

Items:

- dh
- dt
- dV
- dx
- dy
- $\frac{dy}{dx}$
- $\frac{dx}{dy}$



Powers Using Chain Rule 1

Pre-Uni Maths for Sciences J4.6

Subject & topics: Maths | Calculus | Differentiation Stage & difficulty: A Level P3

Part A

Differentiate $w = (4s + 3)^3$

Find $\frac{dw}{ds}$ if $w = (4s + 3)^3$.

The following symbols may be useful: s

Part B

First derivative of $z = (b - aw)^4$

Find $\frac{dz}{dw}$ when $z = (b - aw)^4$, where a and b are constants.

The following symbols may be useful: a, b, w

Part C

Second derivative of $z = (b - aw)^4$

Find $\frac{d^2z}{dw^2}$ when $z = (b - aw)^4$, where a and b are constants.

The following symbols may be useful: a, b, w

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STEM SMART Single Maths 32 - The Chain Rule



Differentiating Exponentials 1

Pre-Uni Maths for Sciences J4.4

Subject & topics: Maths | Calculus | Differentiation Stage & difficulty: A Level P3

Part A

Differentiate $\beta e^{-\alpha t}$

Differentiate $\beta e^{-\alpha t}$ with respect to t , where α and β are constants.

The following symbols may be useful: alpha, beta, e, t

Part B

Differentiate $Ce^{\beta m} + D$

Differentiate $Ce^{\beta m} + D$ with respect to m , where β , C and D are constants.

The following symbols may be useful: C, D, beta, e, m

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Differentiating Trig Functions 2

Pre-Uni Maths for Sciences J4.2

Subject & topics: Maths | Calculus | Differentiation Stage & difficulty: A Level P3

Part A

Differentiate $s = r \sin(\alpha\theta)$

Find $\frac{ds}{d\theta}$ if $s = r \sin(\alpha\theta)$, where r and α are constants.

The following symbols may be useful: alpha, cos(), r, sin(), tan(), theta

Part B

Differentiate $q = l \cos(\alpha - 2\beta\theta)$

Find $\frac{dq}{d\theta}$ if $q = l \cos(\alpha - 2\beta\theta)$, where l , α and β are constants.

The following symbols may be useful: alpha, beta, cos(), l, sin(), tan(), theta

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Differentiating Natural Logs

Pre-Uni Maths for Sciences J4.10

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Part A

Differentiate $u = \ln (2v + 3)$

Find $\frac{du}{dv}$ if $u = \ln (2v + 3)$.

The following symbols may be useful: v

Part B

Stationary point of $p = 2 \ln (2q) - 3q$

Find the coordinates and nature of the stationary point of the function $p = 2 \ln (2q) - 3q$.

Give the q -coordinate of the stationary point.

The following symbols may be useful: q

Give the p -coordinate of the stationary point.

The following symbols may be useful: p

Determine the nature of the stationary point.

- ☐ Minimum
- ☐ Maximum

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Differentiation: Chain Rule 1ii

Subject & topics: Maths **Stage & difficulty:** A Level P2

The volume, $V \text{ m}^3$, of liquid in a container is given by

$$V = (3h^2 + 4)^{\frac{3}{2}} - 8$$

where $h \text{ m}$ is the depth of the liquid.

Part A**Rate of Change (a)**

Find the value of $\frac{dV}{dh}$ when $h = 0.6$, giving your answer to four significant figures.

Part B**Rate of Change (b)**

Liquid is leaking from the container. It is observed that, when the depth of the liquid is 0.6 m , the depth is decreasing at a rate of 0.015 m per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is 0.6 m . Answer to four significant figures.

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Differentiating Exponentials 3

Pre-Uni Maths for Sciences J4.7

Subject & topics: Maths | Calculus | Differentiation **Stage & difficulty:** A Level P3

Part A

Tangent to $y = e^{2x} - e^{-2x}$

Find the equation of the tangent to the curve $y = e^{2x} - e^{-2x}$ at the point $x = \frac{1}{2}$.

The following symbols may be useful: e, x, y

Part B

Stationary point of $u = 2e^{3v} - 3v$

Find the coordinates and nature of the stationary point of the function $u = 2e^{3v} - 3v$.

Find the v coordinate of the stationary point.

The following symbols may be useful: v

Find the u coordinate of the stationary point.

The following symbols may be useful: u

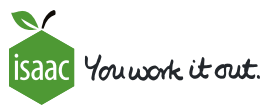
Determine the nature of the stationary point.

☐ Minimum

☐ Maximum

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Chain Rule 2

Pre-Uni Maths for Sciences J6.2

Subject & topics: Maths | Calculus | Differentiation Stage & difficulty: A Level P3

Part A

Differentiate $E = B \sin^2(\omega t)$.

Find $\frac{dE}{dt}$ if $E = B \sin^2(\omega t)$, where B and ω are constants.

The following symbols may be useful: B, E, cos(), omega, sin(), t, tan()

Part B

Differentiate $y = e^{-\frac{x^2}{2\sigma^2}}$

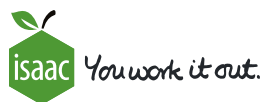
Find $\frac{dy}{dx}$ if $y = e^{-\frac{x^2}{2\sigma^2}}$, where σ is a constant.

The following symbols may be useful: e, sigma, x

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Further Derivatives of Exponentials and Logarithms

Subject & topics: Maths | Calculus | Differentiation**Stage & difficulty:** A Level P3, Further A P1

This question uses the chain rule to find the derivatives of several functions involving exponentials and logarithms.

Part A

Rewriting a^x

Use the rules for exponentials and logarithms to write $y = a^x$, where a is a positive constant, in the form $y = e^{bx}$, where b is a constant. Enter an expression for b in terms of a .

The following symbols may be useful: a , $\ln()$, $\log()$

Part B

Differentiating a^x

Using your answer to part A, use the chain rule to find an expression for $\frac{dy}{dx}$ for the function $y = a^x$. Give your answer in the form $f(a)a^x$, where $f(a)$ is a function of a to be determined.

The following symbols may be useful: a , $\ln()$, $\log()$

Part C**Differentiating $\log_a(x)$**

Use the chain rule to find an expression for $\frac{dy}{dx}$ for the function $y = \log_a(x)$.

The following symbols may be useful: a , $\ln()$, $\log()$

Part D**Differentiate e^{e^x}**

Use the chain rule to find an expression for $\frac{dy}{dx}$ for the function $y = e^{e^x}$.

The following symbols may be useful: e , x

Part E**Differentiate $\ln(\ln(x))$**

Use the chain rule to find an expression for $\frac{dy}{dx}$ for the function $y = \ln(\ln x)$.

The following symbols may be useful: $\ln()$, $\log()$, x

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