Hypothesis Testing: Meeples

A Level P P P

A manufacturer of meeples (a type of playing piece used in many board games) finds that, on average, 10% of the meeples they produce are defective. In order to improve the production of their meeples, they purchase new equipment to reduce the number of defects. In order to test the new equipment, a member of their quality assurance team collects a random sample of 80 meeples produced using the new equipment and finds that 3 of them have defects. Test at the 5% significance level whether the new equipment has reduced the number of defects.

Let the random variable X be the number of meeples with defects. Then $X \sim \mathrm{B}(\_\_\_\_, p)$.

The null and alternative hypotheses are:

$$\mathrm{H}_0 : p \;\_\_\_\_\;\_\_\_\_ \qquad \mathrm{H}_1 : p \;\_\_\_\_\;\_\_\_\_$$

The test statistic, $\mathrm{P}(X \leq \_\_\_\_) = \_\_\_\_$.

Comparing this to the significance level, we find that $\mathrm{P}(X \leq \_\_\_\_)$ ____ ____.

Therefore we ____ H_0 at the 5% level. There ____ evidence to suggest that the new equipment has reduced the number of meeples with defects.

Items:

$>$ $<$ $=$ $\neq$ **reject** **do not reject** **is** **is insufficient** 0.1 0.05 0.025 3 5 8 10

70 72 80 88 90 0.0107 0.0246 0.0353 0.0527 0.0880

Created for isaacphysics.org by Matthew Rihan

Hypothesis Testing 3.2

A Level P P P Further A P P P

An experimenter uses a piece of test equipment to make measurements of samples she has manufactured. There is a 10% probability that the equipment will fail when she makes a measurement. She makes some changes to the equipment which she hopes will increase its reliability. Answer the following questions about how she will test whether her changes have increased the reliability.

Part A Carrying out 30 measurements

The experimenter would like to test the hypothesis, at the 2% level of significance, that her changes have increased the reliability of her equipment. She decides to make 30 measurements and there are no failures.

Assuming that the failure rate is unchanged find the probability that there are no failures. Give your answer to 2 s.f.

Deduce what the experimenter can conclude about the changes she has made.

- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; therefore the changes have not improved the reliability of the test equipment.
- Given the original failure rate she would have expected about 3 failures, so there is sufficient evidence to support her hypothesis
- The changes have not improved the reliability of the test equipment.
- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; there is insufficient evidence to indicate that the changes have improved the reliability of the test equipment.
- There is sufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.

Part B Carrying out more measurements

The experimenter realises that she needs more evidence to test, at the 2% level of significance, whether or not her changes have increased the reliability of her equipment. She decides to make more measurements. Assuming that she still gets no failures, how many measurements will she have to make in total to provide support for her hypothesis at the 2% level of significance?

Part C Carrying out 60 measurements

The experimenter makes 60 measurements in all and gets one failure.

Assuming that the failure rate is unchanged find the probability that she will get one failure or no failures. Give your answer to 2 s.f.

Hence deduce what the experimenter can conclude about the changes she has made.

- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is insufficient evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- There is sufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- There is sufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is insufficient evidence to support the hypothesis that the changes have improved the reliability of the test equipment
- There is insufficient evidence at the 2% level to reject the null hypothesis that the changes have had no effect; at the 2% level there is evidence to support the hypothesis that the changes have improved the reliability of the test equipment.
- The number of failures is less than the 6 expected so the null hypothesis can be rejected; the reliability of the test equipment has therefore improved.

Created for isaacphysics.org by Julia Riley

Gameboard:
STEM SMART Double Maths 20 - Hypothesis Tests, Combinatorics & Probability

Physics. You work it out.

Home Gameboard Maths Statistics Hypothesis Tests Hypothesis Testing 3.3

Hypothesis Testing 3.3

A Level P P P Further A P P P

According to one theory the probability that a particular result will be obtained in an experiment is 0.6 whereas according to another theory the probability is 0.7. The experiment is carried out 100 times and the particular result is obtained 76 times.

Part A Testing $p = 0.6$

Consider the first theory which suggests that $p = 0.6$; assume the null hypothesis is that $p = 0.6$ and the alternative hypothesis is that $p > 0.6$. Find the critical region for the test (in this case if X is the number of times the particular result is obtained the critical region has one part with $X \geq X_h$). If the theory is to be tested at the 2% level of confidence, deduce X_h.

Part B Testing $p = 0.7$

Now consider the second theory which suggests that $p = 0.7$; assume the null hypothesis is that $p = 0.7$ and the alternative hypothesis is that $p \neq 0.7$. Find the critical region for the test (in this case if X is the number of times the particular result is obtained the critical region has two parts $X \leq X_l$ and $X \geq X_h$). If the theory is to be tested at the 2% level of confidence, deduce X_l and X_h.

Find X_l.

Find X_h

Part C Conclusions from the tests

What can be concluded at the 2% level from the tests?

- There is insufficient evidence to reject the hypothesis that $p = 0.6$; there is insufficient evidence to reject the hypothesis that $p = 0.7$.
- The hypotheses that $p > 0.6$ and that $p = 0.7$ are correct.
- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; the hypothesis that $p = 0.7$ is correct.
- There is insufficient evidence to reject the hypothesis that $p = 0.6$; there is sufficient evidence to reject the hypothesis that $p = 0.7$ and to support the theory that $p \neq 0.7$.
- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; there is insufficient evidence to reject the hypothesis that $p = 0.7$.
- There is sufficient evidence to reject the hypothesis that $p = 0.6$ and to support the theory that $p > 0.6$; there is sufficient evidence to reject the hypothesis that $p = 0.7$ and to support the theory that $p \neq 0.7$.

Created for isaacphysics.org by Julia Riley

Gameboard:
STEM SMART Double Maths 20 - Hypothesis Tests, Combinatorics & Probability

Physics. You work it out.

Home Gameboard Maths Statistics Probability Permutations and Combinations 1

Permutations and Combinations 1

A Level

P P P

This question is about the number of possible orders when rearranging the letters of the word NEVER.

Part A Distinct permutations

The five letters of the word NEVER are arranged in a random order in a straight line.

How many different orders of the letters are possible?

Part B Orders with adjacent Es

The five letters of the word NEVER are arranged in random order in a straight line.

In how many of the possible orders are the two E's next to each other?

Part C Orders with one E in the first two letters

The five letters of the word NEVER are arranged in random order in a straight line.

Find the probability that the first two letters in the order include exactly one letter E.

Physics. You work it out.

Home Gameboard Maths Statistics Probability Permutations and Combinations 3

Permutations and Combinations 3

A Level
P P P

This question is about the number of possible orders of 7 students sitting on a bench.

Part A Number of permutations

A group of 7 students sit in a random order on a bench. Find the number of orders in which they can sit.

Part B Permutations where two students must be adjacent

A group of 7 students sit in a random order on a bench. The 7 students include Tom and Jerry. Find the probability that Tom and Jerry sit next to each other.

Part C Permutations where no boys are adjacent

A group of 7 students sit in random order on a bench. The students consist of 3 girls and 4 boys. Find the probability that no two boys sit next to each other.

Part D Permutations where all girls are adjacent

A group of 7 students sit in a random order on a bench. The students consist of 3 girls and 4 boys. Find the probability that all three girls sit next to each other.

Used with permission from UCLES, A level, June 2011, Paper 4732, Question 6

Gameboard:

STEM SMART Double Maths 20 - Hypothesis Tests, Combinatorics & Probability

Physics. You work it out.

Permutations and Combinations 4

A Level

This question is about students taking numbered cards out of a bag to make 4-digit numbers, and finding the probabilities of the results they might get.

Part A How many 4-digit numbers can be made?

A bag contains 9 discs numbered $1, 2, 3, 4, 5, 6, 7, 8, 9$. Andrea chooses 4 discs at random, without replacement, and places them in a row.

How many different 4-digit numbers can be made?

Part B How many odd 4-digit numbers?

A bag contains 9 discs numbered $1, 2, 3, 4, 5, 6, 7, 8, 9$. Andrea chooses 4 discs at random, without replacement, and places them in a row.

How many different **odd** 4-digit numbers can be made?

Part C Numbers with 3 odd digits

A bag contains 9 discs numbered $1, 2, 3, 4, 5, 6, 7, 8, 9$. Martin chooses 4 discs at random, without replacement.

Find the probability that the 4 digits include at least 3 odd digits.

Part D Digits that add up to 28

A bag contains 9 discs numbered 1, 2, 3, 4, 5, 6, 7, 8, 9. Martin chooses 4 discs at random, without replacement.

Find the probability that the 4 digits add up to 28.

Used with permission from UCLES, A level, January 2012, Paper 4732, Question 9

Gameboard:
STEM SMART Double Maths 20 - Hypothesis Tests, Combinatorics & Probability

Physics. You work it out.

Home Gameboard Maths Statistics Probability Probabilities: Employment

Probabilities: Employment

A Level

Data about employment of people in their thirties and forties in a small rural area are shown in the following table.

	Unemployed	Employed
Thirties	206	412
Forties	358	305

A person from this area in these age groups is chosen at random. Let T be the event that the person is in their thirties and let E be the event that the person is employed.

Part A $\mathsf{P}(\mathsf{T})$

Find $\mathsf{P}(\mathsf{T})$.

Part B $\mathsf{P}(\mathsf{T} \text{ and } \mathsf{E})$

Find $\mathsf{P}(\mathsf{T} \text{ and } \mathsf{E})$.

Part C Independent events?

Are T and E independent events? Fill in the blanks below to complete the argument.

If T and E are independent, $\mathrm{P}(\mathrm{E}|\mathrm{T})$ = ____, i.e. the probability of being unemployed is ____ irrespective of age.
Using the values in the table, $\mathrm{P}(\mathrm{E}|\mathrm{T})$ = ____ and ____. Therefore T and E ____ independent events.

Items:

$\frac{2}{3}$ | **are not** | $\mathrm{P}(\mathrm{T})$ | $\mathrm{P}(\mathrm{E}) = \frac{239}{427}$ | **are** | **the same** | $\frac{412}{717}$ | $\frac{412}{1281}$ | **random** | $\mathrm{P}(\mathrm{E})$ | $\mathrm{P}(\mathrm{E}) = \frac{206}{427}$

Part D Unemployed and in their thirties

Given that the person chosen is unemployed, find the probability that the person is in their forties.

Adapted with permission from UCLES, A Level, CIE, January 2005, Paper 6 Probability & Statistics 1 (S1), Question 7

Gameboard:
STEM SMART Double Maths 20 - Hypothesis Tests, Combinatorics & Probability

Physics. You work it out.

Home Gameboard Maths Statistics Probability Probability 5.3

Probability 5.3

A Level Further A

Part A Substandard samples

A laboratory has two devices A and B which produce samples for an experiment. Device A has produced 100 samples of which 5% are substandard. Device B has produced 25 of which 4% are substandard. An experimenter has found a substandard sample. Assuming that samples are chosen at random, what is the probability that it was produced by device B?

Part B Equipment failure

In hot weather the antiquated air-conditioning system in Professor A's laboratory may break down. On any given hot day, there is a 5% chance that the air-conditioning system breaks down. If the air-conditioning breaks down, the probability this will lead to the Professor's equipment failing by the end of the day is 0.3. If the air-conditioning does not break down, the probability that the equipment fails by the end of the day is only 0.05.

One hot day the Professor checks their lab first thing in the morning and the air-conditioning and equipment are both working. When the Professor gets ready to leave at the end of the day, they notice that their equipment has failed. What is the probability that the failure was not due to a breakdown of the air-conditioning system?

Created by for isaacphysics.org by Julia Riley

Gameboard:
STEM SMART Double Maths 20 - Hypothesis Tests, Combinatorics & Probability

Probability 5.4

The probability of a randomly selected person in a population having a particular genetic trait is 0.00001. A test for this trait successfully detects it, if present, 99.9% of the time, and only returns a false positive 0.1% of the time.

Part A Probability after one test

A person tests positive for the trait. Find the probability that they actually have the genetic trait. Give your answer to 3 significant figures.

Part B Probability after two tests

In order to improve accuracy, individuals are instructed to take the test twice, regardless of the result of the first test.

What is the probability that an individual receives a positive result from both tests? Give your answer to 3 significant figures.

Find the probability, given that they have tested positive twice, that they actually have the genetic trait. Give your answer to 3 significant figures.

Created by for isaacphysics.org by Julia Riley