



# Matrices: nxm Rules 2i

Further A

P

P

P

The matrices **A**, **B** and **C** are given by  $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$

Part A   **AB**

Find the matrix **AB**

()

Part B   **BA – 4C**

Find the matrix given by **BA – 4C**.

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# 2x2 Operations 2ii

Further A



The matrices **A** and **B** are given by  $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} a & -1 \\ -3 & -2 \end{pmatrix}$ .

**Part A**   *a*

*a* satisfies the equation  $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$ .

Find the value of *a*.

The following symbols may be useful: a

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**Part B**   Alternate value of *a*

Now take *a* to satisfy the equation  $\mathbf{AB} = \begin{pmatrix} 7 & -4 \\ 9 & -7 \end{pmatrix}$ .

Find the value of *a*.

The following symbols may be useful: a

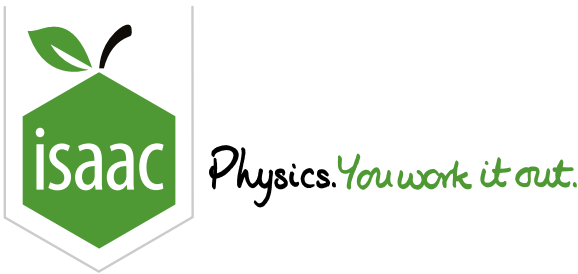
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# 2x2 Determinants and Inverses 1ii

Further A

P

P

P

The matrices **A** and **B** are given by  $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -4 & 5 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}$ . **I** denotes the  $2 \times 2$  identity matrix.

Part A    $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$

Find the matrix given by  $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$ .

(

)

Part B    $\mathbf{A}^{-1}$

Find  $\mathbf{A}^{-1}$ .

$\mathbf{A}^{-1} =$ 

1

Items:

-5

-4

-3

-2

-1

0

1

2

3

4

5

6

7

8

10

12

13

14

15

### Part C $(\mathbf{AB}^{-1})^{-1}$

Find  $(\mathbf{AB}^{-1})^{-1}$ .

$$(\mathbf{AB}^{-1})^{-1} = \frac{1}{\boxed{\phantom{000}}} \begin{pmatrix} \boxed{\phantom{000}} & \boxed{\phantom{000}} \\ \boxed{\phantom{000}} & \boxed{\phantom{000}} \end{pmatrix}$$

Items:

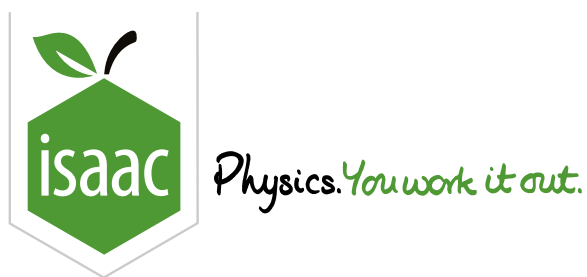
-5
-4
-3
-2
-1
0
1
2
3
4
5
7
9
11
14
17
21
22

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# Matrices: 3x3 Determinants and Inverses 1i

## Further A



The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$ . The matrix  $\mathbf{B}$  is such that  $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$ .

## Part A $\det \mathbf{AB}$

Find  $\det \mathbf{AB}$ .

The following symbols may be useful: a

## Part B $(\mathbf{AB})^{-1}$

Find  $(\mathbf{AB})^{-1}$ .

$$(\mathbf{AB})^{-1} = \frac{1}{\boxed{\phantom{000}}} \begin{pmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{pmatrix}$$

Items:

$\boxed{-3}$   $\boxed{-2}$   $\boxed{-1}$   $\boxed{0}$   $\boxed{1}$   $\boxed{2}$   $\boxed{3}$   $\boxed{4}$   $\boxed{5}$   $\boxed{6}$   $\boxed{a-2}$   $\boxed{a-3}$   $\boxed{a-6}$   $\boxed{2-a}$   $\boxed{3-a}$   $\boxed{6-a}$   $\boxed{3a-2}$   $\boxed{3a-3}$   
 $\boxed{3a-6}$   $\boxed{2-3a}$   $\boxed{3-3a}$   $\boxed{6-3a}$

Part C  $\mathbf{B}^{-1}$

Find  $\mathbf{B}^{-1}$ .

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## Further A



The matrix  $\mathbf{B}$  is given by  $\mathbf{B} = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$ .

## Part A *a*

Find the value of  $a$  in exact form, given that  $\mathbf{B}$  is singular.

The following symbols may be useful: a

## Part B $\mathbf{B}^{-1}$

Given that  $\mathbf{B}$  is non-singular, find  $\mathbf{B}^{-1}$ .

$$\mathbf{B}^{-1} = \frac{1}{\boxed{\phantom{000}}} \begin{pmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{pmatrix}$$

Items:

$-4$   $-3$   $-2$   $-1$   $0$   $1$   $2$   $3$   $4$   $a$   $-a$   $2a$   $-2a$   $3a$   $-3a$   $a+2$   $a-2$   $a+4$   $a-4$   
 $a+6$   $a-6$   $2a+2$   $2a-2$   $3a+2$   $3a-2$

Part C    Simultaneous equations

$x$ ,  $y$  and  $z$  satisfy the following simultaneous equations

$$-x + y + 3z = 1$$
$$2x + y - z = 4$$
$$y + 2z = -1$$

Use matrix methods to find  $x$ ,  $y$  and  $z$ .

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# Matrices - Intersecting Lines

## Further A University



Two lines are described by

$$3x - 4y - 1 = 0$$

$$2x + py - 10 = 0.$$

where  $p$  is a constant. Use matrix notation to find the coordinates of the point of intersection of these two lines.

### Part A Write in matrix form

Write these equations in matrix form  $\mathbf{Ax} = \mathbf{b}$ .

$$\begin{pmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{pmatrix}$$

### Part B Condition for no intersection

Use the matrix to find the value of  $p$  for which the lines do not intersect. Give your answer as an improper fraction.

The following symbols may be useful:  $p$

Part C    The inverse matrix

Find  $\mathbf{A}^{-1}$ , the inverse of  $\mathbf{A}$ .

$\mathbf{A}^{-1} = \frac{1}{\phantom{0000}} \begin{pmatrix} \phantom{00} & \phantom{00} \\ \phantom{00} & \phantom{00} \end{pmatrix}$

Items:

- −4

−3

−2

−1

0

1

2

3

4

$p$

$−p$

$p + 4$

$p − 4$

$p + 8$

$p − 8$

$3p + 4$

$3p − 4$

$3p + 8$

$3p − 8$

Part D    Components of point of intersection

Using  $\mathbf{A}^{-1}$  find the the point of intersection in terms of  $p$ .

$\left( \begin{pmatrix} \phantom{00} \\ \phantom{00} \end{pmatrix}, \begin{pmatrix} \phantom{00} \\ \phantom{00} \end{pmatrix} \right)$

Items:

- 18

20

28

30

38

40

$p + 4$

$p + 8$

$p + 40$

$p + 80$

$3p + 4$

$3p + 8$

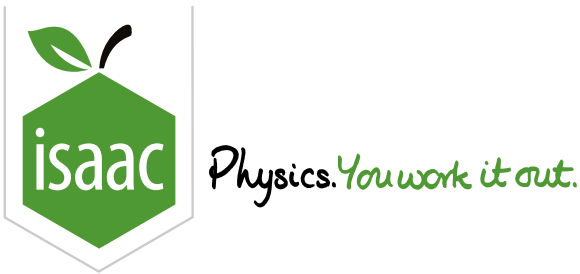
$3p + 40$

$3p + 80$

Part E    A value for  $p$

If the  $y$ -component of the point of intersection is equal to 2, find the value of  $p$ .

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# Matrices - Linear Equations 2

Further A University

P

P

P

P

P

P

Use matrix notation to solve the following set of three equations for  $x$ ,  $y$  and  $z$ :

$$x + cy = c$$
$$x - y + 2z = -c$$
$$2x - 2y - z = 2.$$

Part A

Write in matrix form

Write these equations in matrix form  $\mathbf{R}\mathbf{x} = \mathbf{p}$ .

$x$

$y$

$z$

=

Part B

Determinant of the matrix

Find the determinant of  $\mathbf{R}$  in terms of  $c$ .

The following symbols may be useful:  $c$

### Part C Condition for no unique solution

Deduce the value of  $c$  for which there is no unique solution.

## Part D The inverse matrix

Find the inverse matrix  $\mathbf{R}^{-1}$ .

$$\mathbf{R}^{-1} = \frac{1}{\boxed{\phantom{000}}} \begin{pmatrix} \boxed{\phantom{000}} & \boxed{\phantom{000}} & \boxed{\phantom{000}} \\ \boxed{\phantom{000}} & \boxed{\phantom{000}} & \boxed{\phantom{000}} \\ \boxed{\phantom{000}} & \boxed{\phantom{000}} & \boxed{\phantom{000}} \end{pmatrix}$$

Items:

[illegible]

### Part E Solution to the set of equations if $c = 1$

Using  $\mathbf{R}^{-1}$ , find the solutions for  $x$ ,  $y$  and  $z$  if  $c = 1$ .

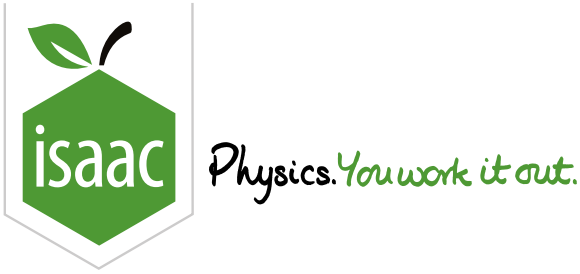
$$\left( \boxed{\phantom{000}}, \boxed{\phantom{000}}, \boxed{\phantom{000}} \right)$$

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Matrices - Linear Equations 3

Further A University



A system consists of three masses  $m_1$ ,  $m_2$  and  $m_3$  in a line; they each have the same mass  $m$ . The mass  $m_2$  is in the centre and connected by springs of spring constant  $k$  to  $m_1$  on the left and  $m_3$  on the right. The masses are all performing simple harmonic motion at the same angular frequency  $\omega$  such that their equations of motion are

$$\begin{aligned} -kx_1 + kx_2 &= -m\omega^2x_1 \\ kx_1 - 2kx_2 + kx_3 &= -m\omega^2x_2 \\ kx_2 - kx_3 &= -m\omega^2x_3. \end{aligned}$$

where  $x_1$ ,  $x_2$  and  $x_3$  are the displacements of  $m_1$ ,  $m_2$  and  $m_3$  respectively.

These equations can be written in matrix form

$$\begin{aligned} \mathbf{Ax} &= -m\omega^2\mathbf{x} \\ &= -m\omega^2\mathbf{Ix} \\ \Rightarrow (\mathbf{A} + m\omega^2\mathbf{I})\mathbf{x} &= 0 \end{aligned}$$

A matrix equation of this sort only has solutions if  $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$ . Use this to find the possible values of  $\omega^2$ . For each value of  $\omega$  find the relationship between  $x_1$ ,  $x_2$  and  $x_3$ .

Part A

The matrix **A**

Find the matrix **A**.

$$\mathbf{A} = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

Items:

- 2

-1

0

1

2

-3k

-2k

-k

k

2k

3k

Part B    The matrix  $\mathbf{A} + m\omega^2\mathbf{I}$

Find the matrix  $\mathbf{A} + m\omega^2\mathbf{I}$ .

$$\mathbf{A} + m\omega^2\mathbf{I} = \begin{pmatrix} \boxed{\phantom{0}} & \boxed{\phantom{0}} & \boxed{\phantom{0}} \\ \boxed{\phantom{0}} & \boxed{\phantom{0}} & \boxed{\phantom{0}} \\ \boxed{\phantom{0}} & \boxed{\phantom{0}} & \boxed{\phantom{0}} \end{pmatrix}$$

Items:

- 1

0

1

-2k

-k

k

2k

mω<sup>2</sup> - 2k

mω<sup>2</sup> - k

mω<sup>2</sup> + k

mω<sup>2</sup> + 2k

Part C    The possible values of  $\omega^2$

Using the fact that non-zero solutions to the equation  $(\mathbf{A} + m\omega^2\mathbf{I})\mathbf{x} = 0$  require that  $|\mathbf{A} + m\omega^2\mathbf{I}| = 0$ , deduce the three values of  $\omega^2$ . The three values,  $\omega_1^2$ ,  $\omega_2^2$  and  $\omega_3^2$ , are such that  $\omega_1^2 < \omega_2^2 < \omega_3^2$ .

$$\omega_1^2 = \boxed{\phantom{0}}, \omega_2^2 = \boxed{\phantom{0}} \text{ and } \omega_3^2 = \boxed{\phantom{0}}$$

Part D    The relationship between  $x_1$ ,  $x_2$  and  $x_3$

Since the determinant of the matrix is zero there are no unique solutions to the set of three equations; however, for each value of  $\omega^2$ ,  $x_1$ ,  $x_2$  and  $x_3$  have a fixed relationship to each other. On the assumption that  $x_1 = 1$ , find  $x_2$  and  $x_3$  for each of the three frequencies deduced in Part B.

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