# Integration by Substitution 1

**Pre-Uni Maths for Science K3.2**

A Level

P P P

**Part A**      **Integrate** $\sin(c\theta)$

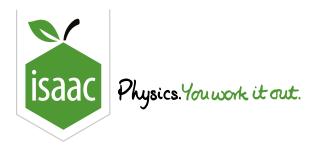Find $\int \sin(c\theta)\,\mathrm{d}\theta$, where $c$ is a constant.

The following symbols may be useful: `C, c, k, theta`

**Part B**      **Integrate** $e^{\alpha v}$

Find $\int e^{\alpha v}\,\mathrm{d}v$, where $\alpha$ is a constant.

The following symbols may be useful: `alpha, e, k, v`

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# Integration by Substitution 2

**Pre-Uni Maths for Science K3.3**

### Part A    Integrate $(bv + c)^2$

Find $\displaystyle\int (bv + c)^2 \mathrm{d}v$, where $b$ and $c$ are constants.

The following symbols may be useful: `b, c, k, v`
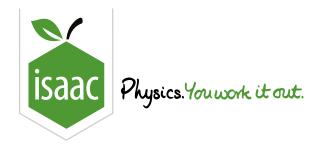
### Part B    Integrate $a(y - b)^3$

Find $\displaystyle\int_0^b a(y - b)^3 \mathrm{d}y$, where $a$ and $b$ are constants.

The following symbols may be useful: `a, b`

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Gameboard:

**STEM SMART Single Maths 34 - Integration by Substitution**

# Integration: General 4ii

**Part A**    **Integrate** $(4 - 3x)^7$

Find $\displaystyle\int (4 - 3x)^7 \mathrm{d}x.$

The following symbols may be useful: c, x

**Part B**    **Integrate** $(4 - 3x)^{-1}$

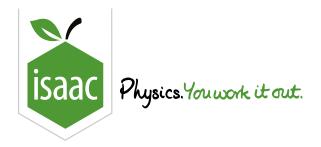Find $\displaystyle\int (4 - 3x)^{-1} \mathrm{d}x.$

The following symbols may be useful: c, x

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Gameboard:

**STEM SMART Single Maths 34 - Integration by Substitution**

# Integration by Substitution 5ii

Use the substitution $u = 2x - 5$ to find the exact value of $\displaystyle\int_{\frac{5}{2}}^{3} (4x - 8)(2x - 5)^7 \mathrm{d}x$.

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Gameboard:

**STEM SMART Single Maths 34 - Integration by Substitution**

# Integration by Substitution 5i

Use the substitution $u = 2x + 1$ to evaluate $\displaystyle\int_0^{\frac{1}{2}} \frac{4x - 1}{(2x + 1)^5} \,\mathrm{d}x$.

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Gameboard:

**STEM SMART Single Maths 34 - Integration by Substitution**

# Integration by Substitution 4ii

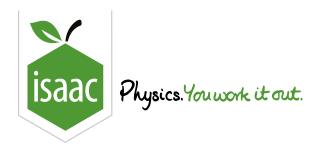Use the substitution $u = \sqrt{x+2}$ to find the exact value of $\displaystyle\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}\,\mathrm{d}x$.

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Gameboard:

**STEM SMART Single Maths 34 - Integration by Substitution**
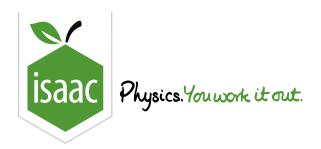
# Integration by Substitution 4i

Use the substitution $u = 1 + \sqrt{x}$ to find the exact value of $\displaystyle\int_4^9 \frac{1}{1 + \sqrt{x}}\,\mathrm{d}x$ in the form $a + b\ln(c)$, where $a$, $b$, and $c$ are positive constants to be found.

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Gameboard:

**STEM SMART Single Maths 34 - Integration by Substitution**

# Integration by Substitution 3

**Pre-Uni Maths for Science K3.4**

**Part A**    **Integrate** $\frac{3}{(z+1)^2}$

Find $\displaystyle\int_0^2 \frac{3}{(z+1)^2}\,\mathrm{d}z.$

**Part B**    **Integrate** $\frac{e^{-\alpha x}}{(1+e^{-\alpha x})^4}$
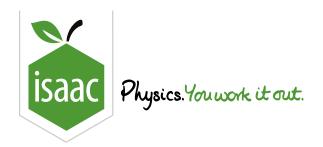
Find $\displaystyle\int \frac{\mathrm{e}^{-\alpha x}}{(1+\mathrm{e}^{-\alpha x})^4}\,\mathrm{d}x,$ where $\alpha$ is a constant.

The following symbols may be useful: `C, alpha, c, e, k, x`

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Gameboard:

**STEM SMART Single Maths 34 - Integration by Substitution**

# Integration by Substitution 2i

In this question, $I$ denotes the definite integral $\int_2^5 \dfrac{5-x}{2+\sqrt{x-1}}\,\mathrm{d}x$. The value of $I$ can be found using two different methods.

## Part A     Substitution

Show that the substitution $u = \sqrt{x-1}$ transforms $I$ to $\int_1^2 \left(4u - 2u^2\right)\mathrm{d}u$ and hence find the exact value of $I$.

The following symbols may be useful: I

## Part B     Rationalisation

Simplify as far as possible $\left(2 + \sqrt{x-1}\right)\left(2 - \sqrt{x-1}\right)$.

The following symbols may be useful: x

By first multiplying the numerator and denominator of $\frac{5-x}{2+\sqrt{x-1}}$ by $2 - \sqrt{x-1}$, find the exact value of $I$.

The following symbols may be useful: I

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