



Functions and Algebra 5ii

A Level

The function f is defined by

$$f(x) = \frac{1}{\sqrt{x}} + 2, \quad x > 0.$$

The function g is defined for all real values of x by

$$g(x) = 10 - (x + 3)^2.$$

Part A Range of f

State the range of $f(x)$ as an inequality.

The following symbols may be useful: $<$, \leq , $>$, \geq , $f(x)$, x , y

Part B Inverse of f

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: f , x

Part C Range of g

State the range of $g(x)$ as an inequality.

The following symbols may be useful: $<$, \leq , $>$, \geq , $g(x)$, x , y

Part D Compound function of g

Find the value of $g(g(-1))$.

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Inverse Quadratic Function

A Level



Figure 1 shows the graph of $y = f(x)$, where

$$f(x) = 2 - x^2, \quad x \leq 0$$

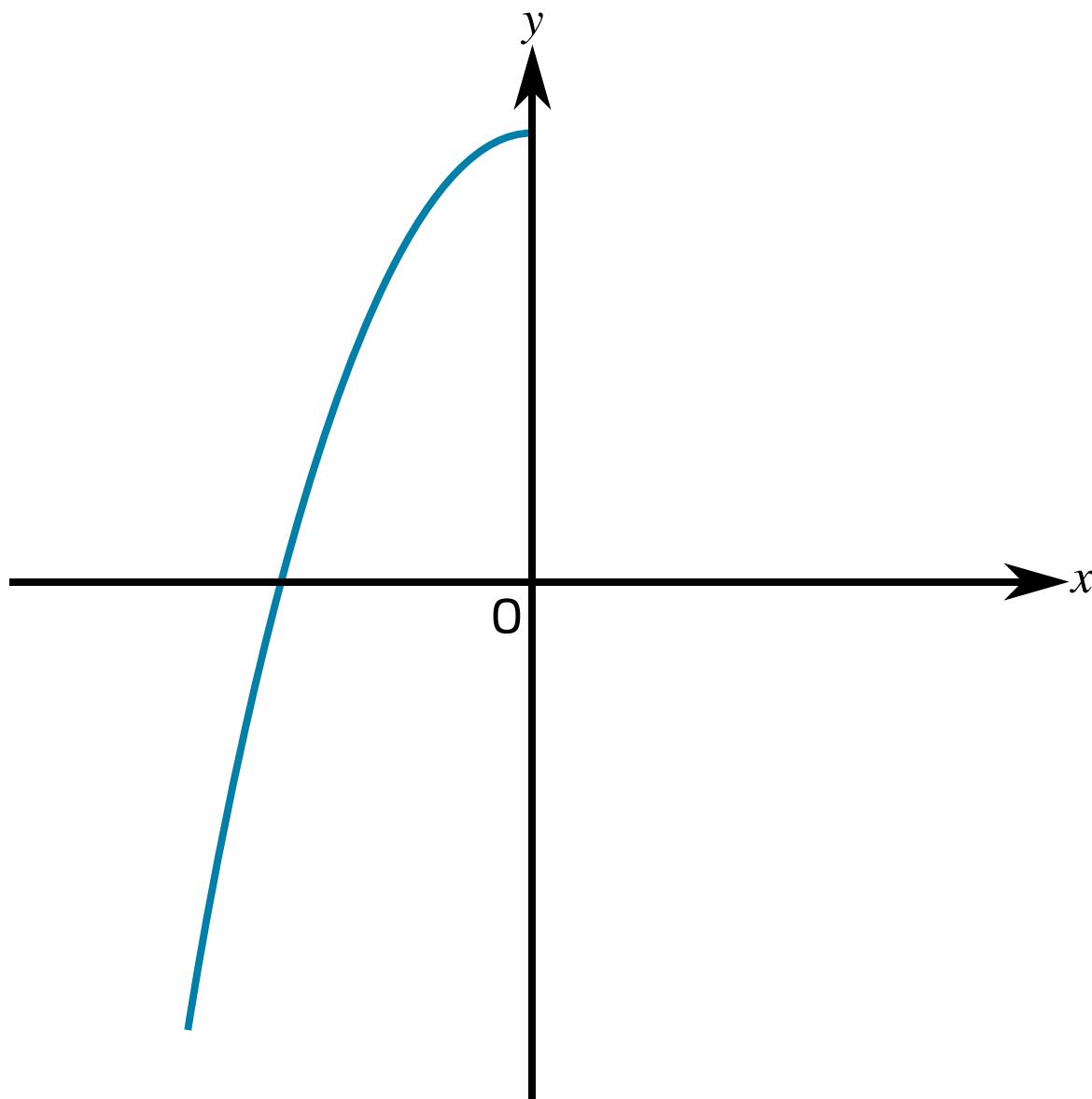


Figure 1: The graph of $y = f(x)$, for $x \leq 0$.

Part A $f^2(-3)$

Evaluate $f^2(-3)$.

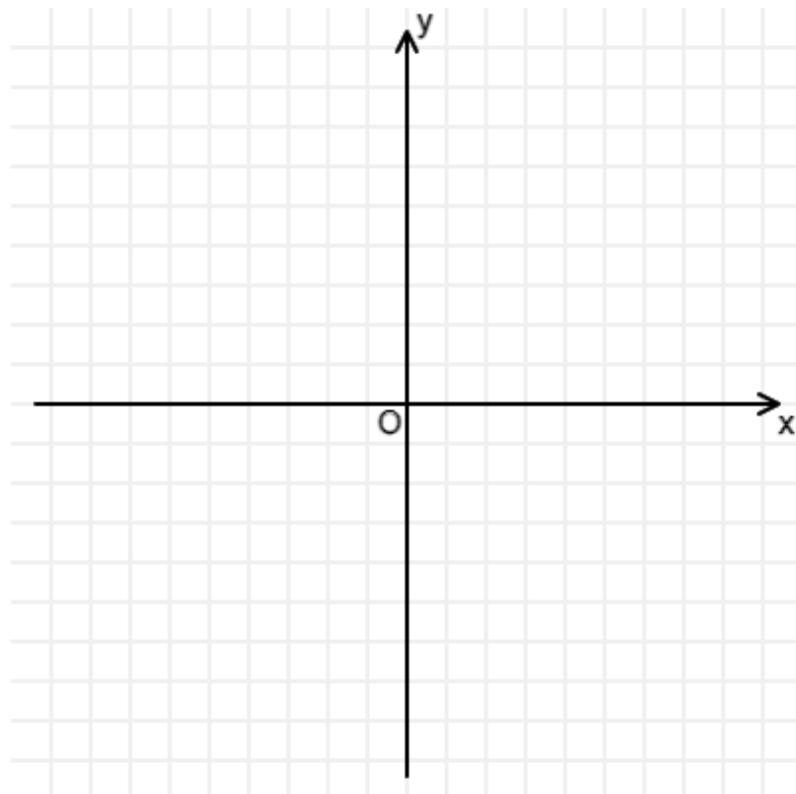
Part B $f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: f , x , y

Part C Graph of $f^{-1}(x)$

Sketch the graph of $y = f^{-1}(x)$.



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Function Types and Inverses

A Level

c	c	c
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Figure 1 shows five different graphs, A, B, C, D and E, each for values of x such that $-a \leq x \leq a$ where a is a constant.

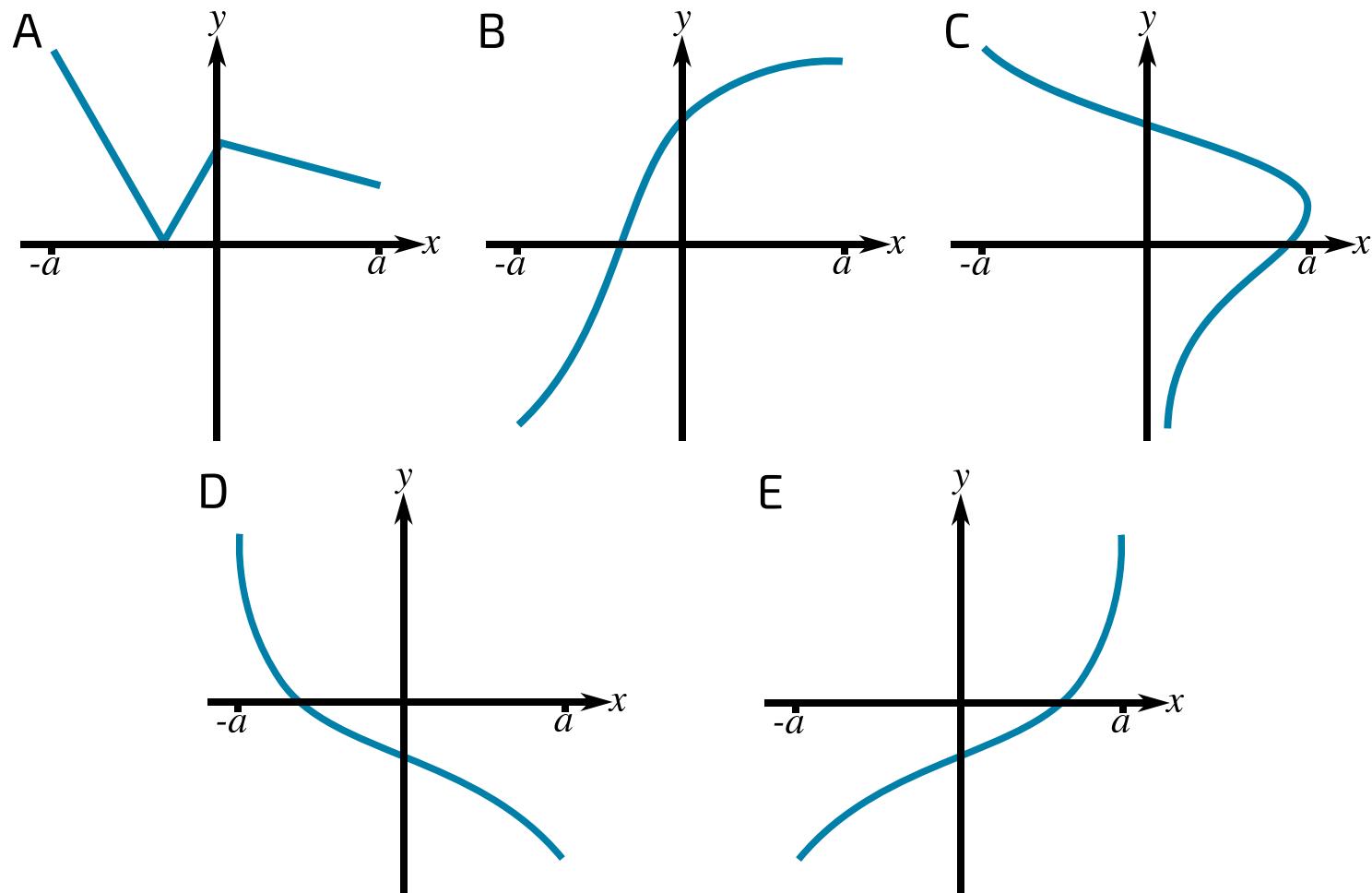


Figure 1: The set of five graphs, labelled A, B, C, D and E

Part A Function

Which diagram does not show the graph of a function?

- A
 - B
 - C
 - D
 - E
-

Part B One-to-one Function

Which diagram shows the graph of a function that is not one-to-one?

- A
 - B
 - C
 - D
 - E
-

Part C Inverses

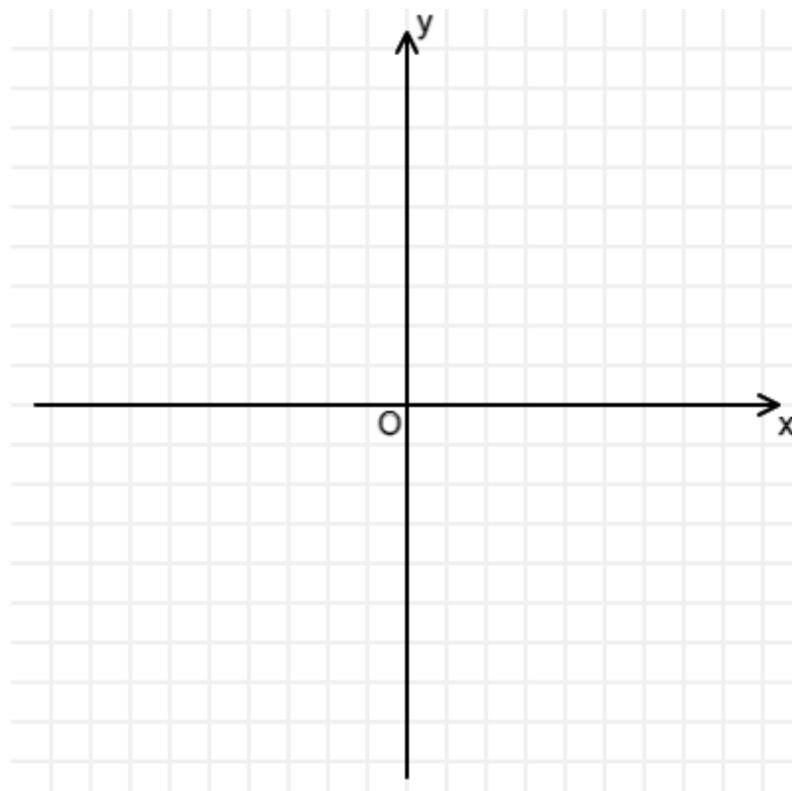
It is given that two of the diagrams illustrate functions that are inverses of each other. Identify one of these two diagrams.

- A
 - B
 - C
 - D
 - E
-

Part D Sketch

The graph in E has equation $y = f(x)$. Sketch the graph of $y = |f(x)|$.

To prevent any sharp changes in your curve from being smoothed out, sketch your curve as two sections.



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Functions and Algebra 3i

A Level



The functions f and g are defined for all real values of x by

$$f(x) = |2x + a| + 3a \quad \text{and} \quad g(x) = 5x - 4a,$$

where a is a positive constant.

Part A Range

Find the range of $f(x)$.

Fill in the inequality below.

Items:

- | | | | | | | | | | | | |
|--------|------|--------|-----|----------------|----------------|------------------|-----------------------------|-----------------------------------|---------------|---------------|-----|
| $f(x)$ | $<$ | \leq | $>$ | \geq | $< f(x) <$ | $\leq f(x) \leq$ | $< f(x) \text{ or } f(x) <$ | $\leq f(x) \text{ or } f(x) \leq$ | $\frac{a}{3}$ | $\frac{a}{2}$ | a |
| $2a$ | $3a$ | $4a$ | 0 | $-\frac{a}{3}$ | $-\frac{a}{2}$ | $-a$ | $-2a$ | | | | |

Part B Inverse function of $f(x)$

Fill in the blanks to explain why the function $f(x)$ has no inverse.

The function $f(x)$ is not . For example, $f(0) = 4a$ and $f(\text{ })$ also equals $4a$. Hence, $f(x)$ has no inverse.

Items:

many-to-many

one-to-many

many-to-one

$-2a$

$-a$

one-to-one

a

$2a$

Part C Inverse function of $g(x)$

Find an expression for $g^{-1}(x)$.

The following symbols may be useful: a , x

Part D Solve for x

Solve for x the equation $g(f(x)) = 31a$.

Give the value of x furthest from 0.

The following symbols may be useful: a , x

Give the value of x that is closest to 0.

The following symbols may be useful: a , x



Modulus 3ii

A Level



Solve the inequality $|2x - 5| > |x + 1|$.

Construct your answer from the items below.

Items:

x	$<$	\leq	$>$	\geq	$< x <$	$\leq x \leq$	$> x$ or $x >$	$\geq x$ or $x \geq$	-6	-4	-2	$-\frac{3}{2}$
$-\frac{4}{3}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{4}{3}$	$\frac{3}{2}$	2	4	6		

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Modulus Functions 2

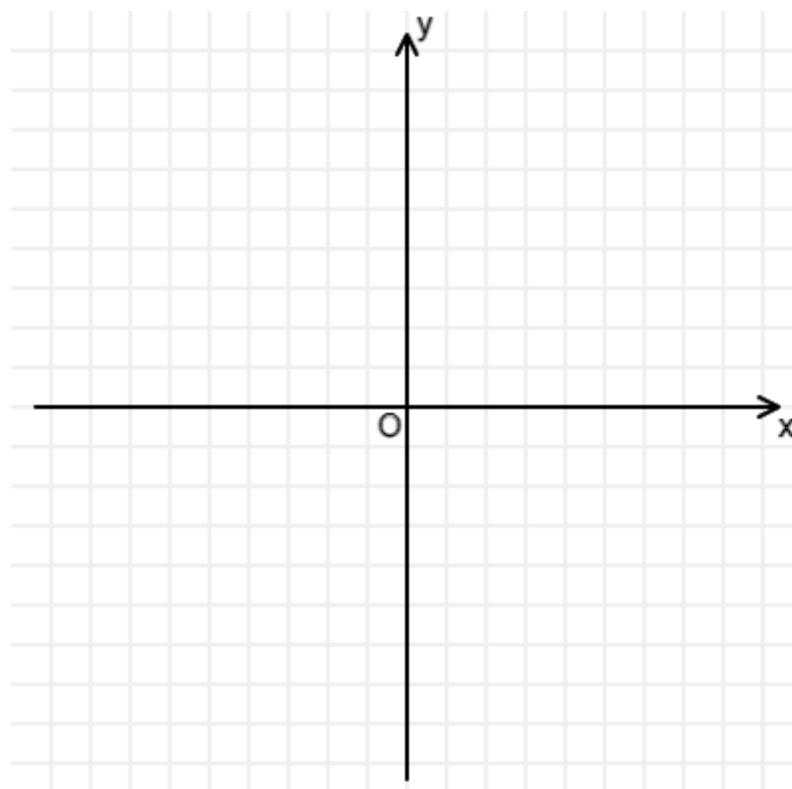
Part A Divergence of $y = |\frac{1}{x}|$

Does the function $y = \left| \frac{1}{x} \right|$ diverge anywhere? Where?

The following symbols may be useful: ∞

Part B Graph of $y = |\frac{1}{x}|$

Sketch the graph of $y = \left| \frac{1}{x} \right|$.



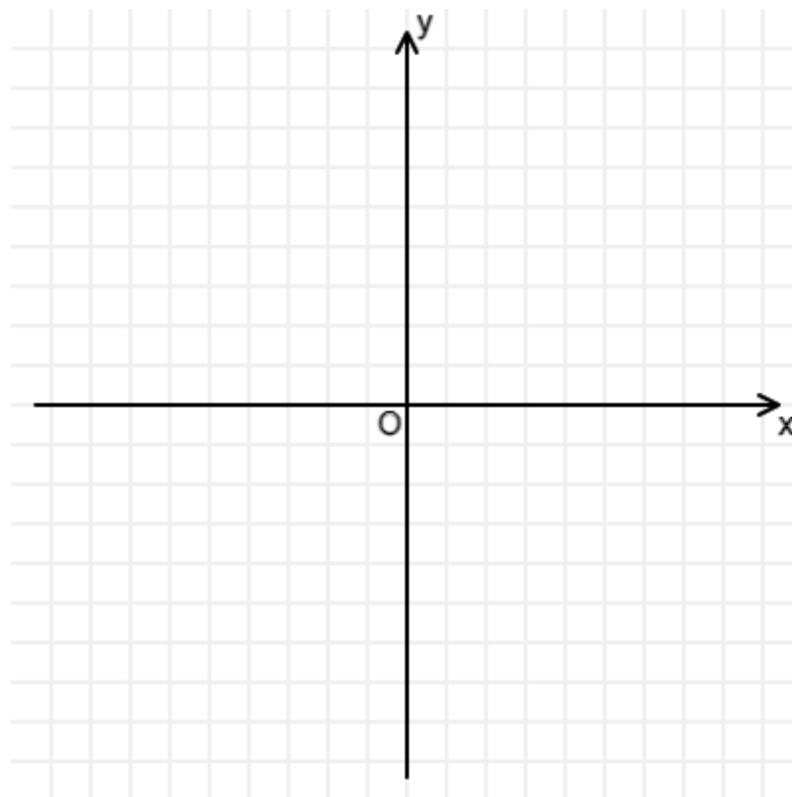
Part C Divergence of $y = \left| \frac{1}{x^2 - 4} \right|$

Does the function $y = \left| \frac{1}{x^2 - 4} \right|$ diverge anywhere? Where?

The following symbols may be useful: x, \pm

Part D Graph of $y = \left| \frac{1}{x^2 - 4} \right|$

Sketch the graph of $y = \left| \frac{1}{x^2 - 4} \right|$.



Part E Solve equation graphically

Solve the equation $|x| = \left| \frac{1}{x} \right|$ graphically and give the solution as a single expression.

The following symbols may be useful: x, \pm



Curve Sketching and Combined Transformations 3i

The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x)$$

where k is a positive constant. **Figure 1** shows the curve with equation $y = f(x)$.

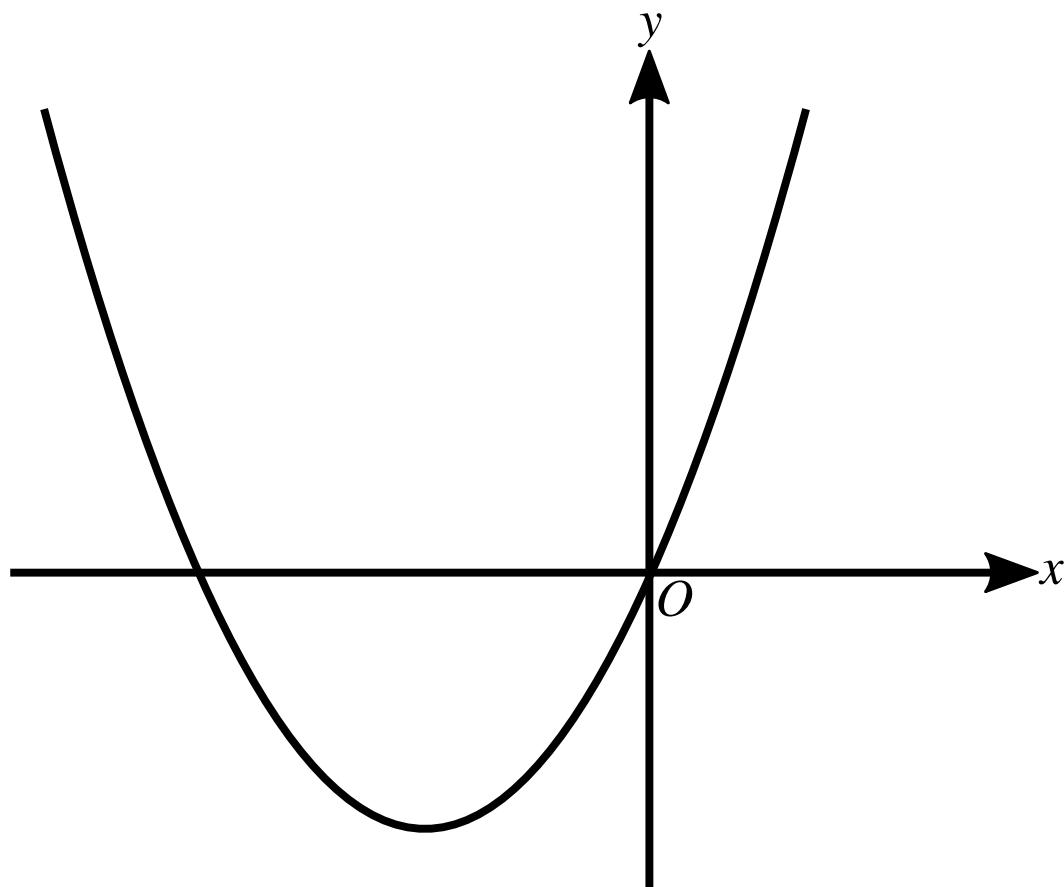


Figure 1: The graph of $y = f(x)$

Part A Transformations

The curve $y = x^2$ can be transformed to the curve $y = f(x)$ by the following sequence of transformations

a translation parallel to the x -axis,

a translation parallel to the y -axis,

a stretch.

Give details, in terms of k where appropriate, of these transformations.

Give the number of units that the curve is translated in the x direction, assuming right to be positive.

The following symbols may be useful: k

Give the number of units that the curve is translated in the y direction, assuming up to be positive.

The following symbols may be useful: k

Give the stretch factor of the transformation.

The following symbols may be useful: k

Part B Range

Find the range of $f(x)$ as a single inequality in terms of k .

The following symbols may be useful: $<$, \leq , $>$, \geq , $f(x)$, k , x , y

Part C $|f(x)| = 20$

It is given that there are three distinct values of x which satisfy the equation $|f(x)| = 20$. Find the value of k and determine exactly the three values of x which satisfy the equation in this case.

State the value of k .

The following symbols may be useful: k

Give the rational value of x which satisfies this equation.

The following symbols may be useful: x

Give one of the irrational solutions for x in its simplest exact form.

The following symbols may be useful: x

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Combined Transformations

A Level



The function f is defined by $f(x) = \sqrt{mx + 7} - 4$, where $x \geq -\frac{7}{m}$ and m is a positive constant. **Figure 1** shows the curve $y = f(x)$.

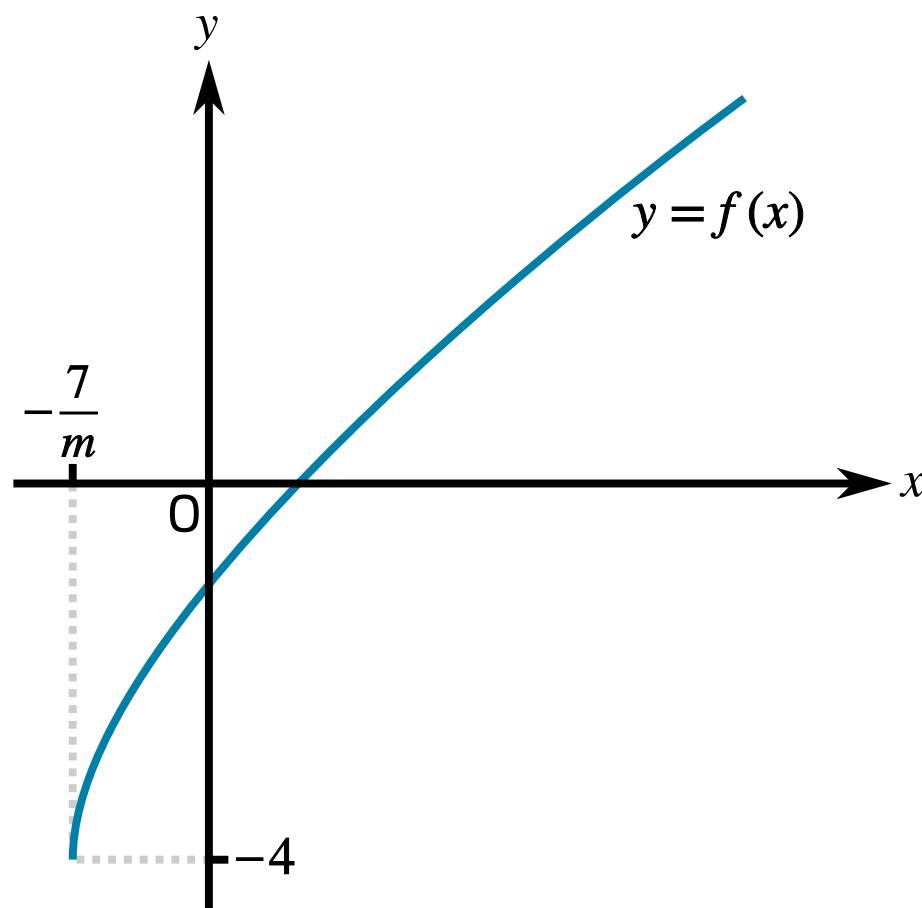


Figure 1: The curve $y = f(x)$

Part A Translation of the curve $y = \sqrt{x}$

A sequence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations.

Available items

Stretch the curve in the x direction by a factor of $\frac{1}{m}$.

Translate the curve 4 units in the negative x direction.

Translate the curve 7 units in the negative x direction.

Translate the curve 7 units in the positive x direction.

Translate the curve 4 units in the negative y direction.

Stretch the curve in the x direction by a factor of m .

Translate the curve 4 units in the positive y direction.

Translate the curve 7 units in the negative y direction.

Stretch the curve in the y direction by a factor of $\frac{1}{m}$.

Part B $f^{-1}(x)$

Find an expression for $f^{-1}(x)$.

The following symbols may be useful: f , m , x

Part C Values of m

It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Thus it can be deduced that neither curve meets the line $y = x$. Hence determine the possible values of m .

Construct your answer from the items below.

Items:

m	$<$	\leq	$>$	\geq	$< m <$	$\leq m \leq$	$> m$ or $m >$	$\geq m$ or $m \geq$	-28	-14	-8
-7	-4	-2	-1	0	1	2	4	7	8	14	28

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