Centre of Mass: Lamina 1

Further A

A uniform square lamina ABCD of side $6\ \mathrm{cm}$ has a semicircular piece, with AB as diameter, removed (see **Figure 1**).

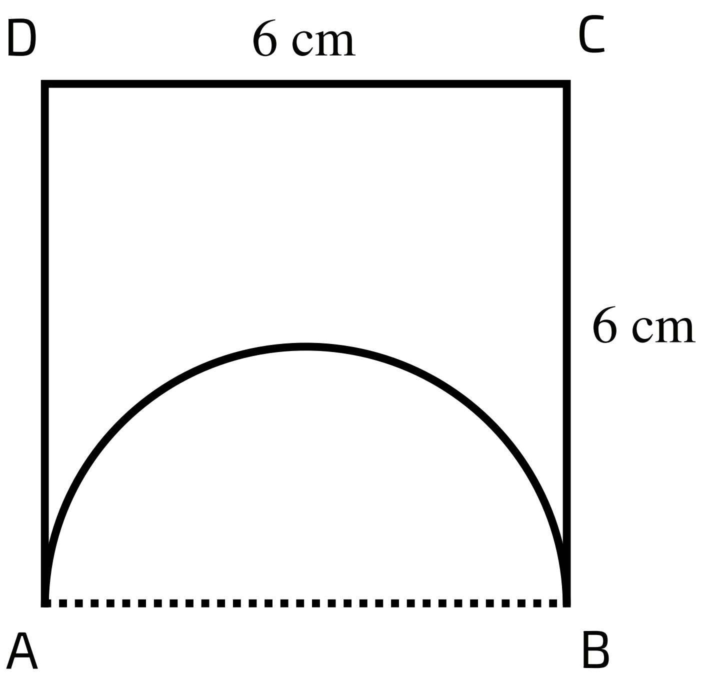

Figure 1: The lamina described in the question.

Part A Position of centre of mass

Find the distance of the centre of mass of the remaining shape from CD. Give your answer to 3 s.f.

Part B Angle to the vertical

The remaining shape is suspended from a fixed point by a string attached at C and hangs in equilibrium.

Find the acute angle between CD and the vertical. Give your answer to 3 s.f.

Used with permission from UCLES A Level, January 2013, Paper 4729, Question 4.

Toppling Block

A Level

This problem involves friction, which is not covered in some Physics A Levels. For more information please check with your teacher.

A rectangular block with a square base of side $10\,\mathrm{cm}$ rests on a rough horizontal surface. A slowly increasing horizontal force is applied to one vertical face. If this force is applied near the bottom of the face then as the force increases the block will slide before it topples. If the force is applied near the top then the block topples over before it starts to slide. When the force is applied $20\,\mathrm{cm}$ from the bottom, the block sometimes slides and sometimes topples.

Find the coefficient of friction between the block and the surface (to 2 significant figures).

Used with permission from UCLES, Higher School Certificate Physics, Paper 2.

Gameboard:

STEM SMART Double Maths 44 - Centres of Mass

Double Decker Bus

A Level

This problem involves centre of mass calculations and tipping points, which are not covered in some Physics A Levels. For more information please check with your teacher.

A double-decker bus of mass $M = 12000\,\mathrm{kg}$ has a width at its base of $w = 2.00\,\mathrm{m}$. Its centre of mass when empty is a height $y_0 = 0.900\,\mathrm{m}$ above the ground and equidistant between the two sides of the bus. Thirty-two passengers, each of mass $m = 75.0\,\mathrm{kg}$, are seated on each deck, symmetrically placed from side to side. The centre of mass of those on the lower deck is at a height $y_1 = 1.10\,\mathrm{m}$ above the ground and that of those on the upper deck is at a height $y_2 = 2.80\,\mathrm{m}$ above the ground.

Part A Angle of tilt

Find the maximum angle of tilt θ (the angle that the base of the bus makes with the horizontal) at which the bus would not topple.

Part B Angle of tilt to the left

If all the passengers on the right-hand side of the bus get off, leaving those on the left-hand side, the centre of mass of the remaining passengers is a horizontal distance $x_0 = 0.400\,\mathrm{m}$ from the left-hand side of the bus.

Find the maximum angle α (the angle the base of the bus makes with the horizontal) by which the bus can now tilt to the left without toppling.

Part C Angle of tilt to the right

What is the angle β (the angle the base of the bus makes to the horizontal) by which the bus must now tilt to the right in order to topple?

Centre of Mass: Arc on a Pivot

Further A

A uniform semicircular arc ACB is freely pivoted at A. The arc has mass $0.3\,\mathrm{kg}$ and is held in equilibrium by a force of magnitude P applied at B. The line of action of this force lies in the same plane as the arc, and is perpendicular to AB. The diameter AB has length $4\,\mathrm{cm}$ and makes an angle θ with the downward vertical (see **Figure 1**).

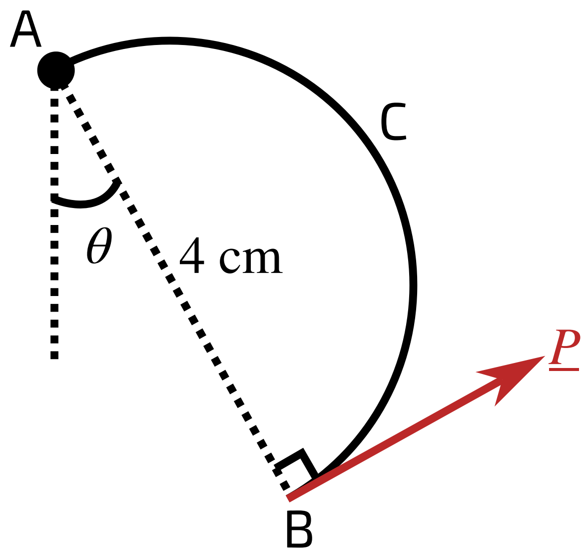

Figure 1: The semicircular arc ACB.

Part A Force at A

Given that $\theta = 0°$, find the magnitude of the force acting on the arc at A. Give your answer to 3 s.f.

Part B Perpendicular force

Given instead that $\theta = 30°$, find the value of P. Give your answer to 3 s.f.

Used with permission from UCLES A Level, June 2013, Paper 4729, Question 3.

Gameboard:

STEM SMART Double Maths 44 - Centres of Mass

Centre of Mass: Prism Topple

Further A

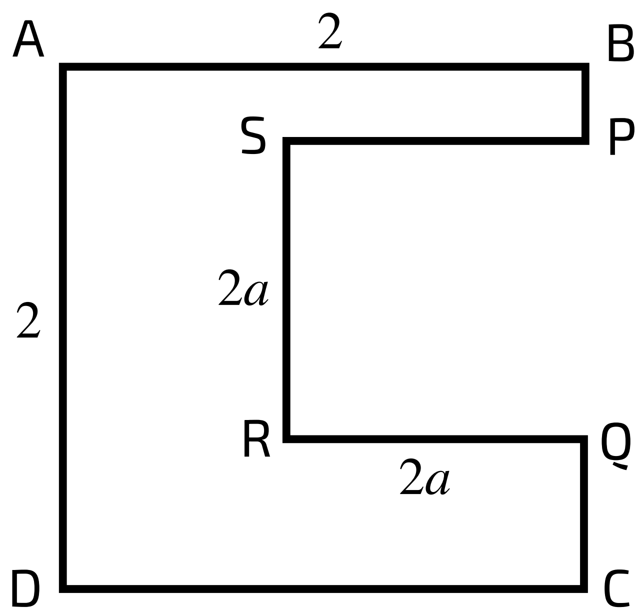

Figure 1: Cross-section of the prism described in the question.

Figure 1 shows the cross-section through the centre of mass of a uniform solid prism. The cross-section is a square ABCD of side 2 with a square PQRS of side $2a$ removed.

It is given that the centre of mass of the prism lies on the line through R and S.

Part A Value of a

Find the exact value of a.

The following symbols may be useful: a

Part B Position of centre of mass

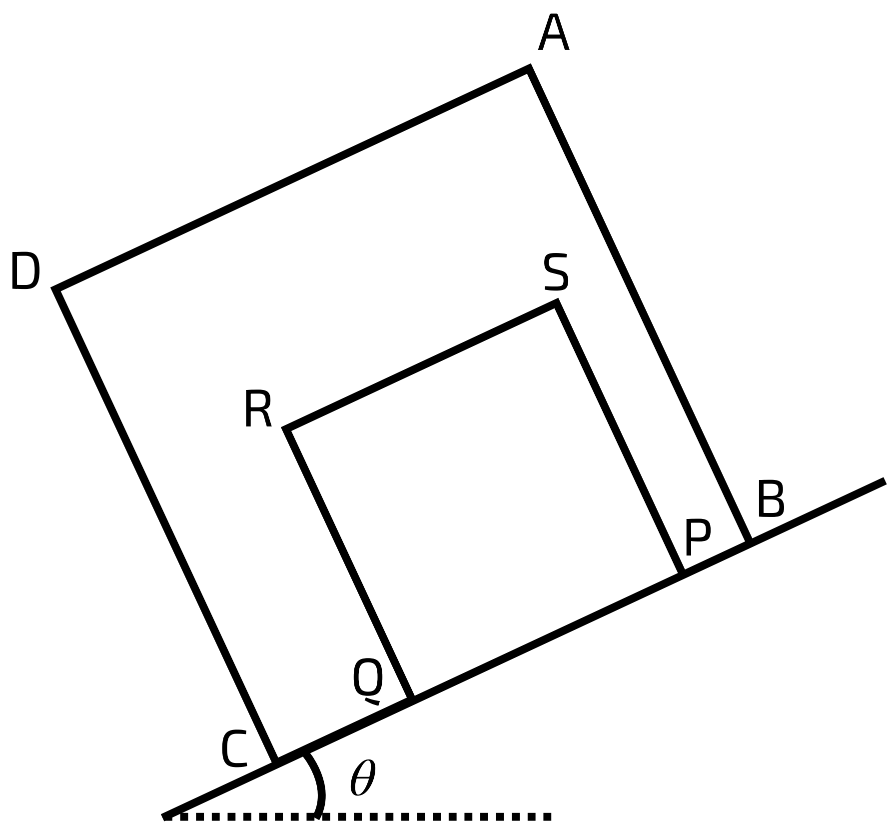

Figure 2: The prism on an inclined plane.

The prism is now placed on an inclined plane which makes an angle θ with the horizontal. BC lies along a line of greatest slope with B higher than C, as shown in **Figure 2**. The plane is slowly tilted until the prism topples, without slipping, when $\theta = 30^\circ$.

Find the exact distance of the centre of mass of the prism from CD.

Adapted with permission from OCR A Level, June 2018, Paper 4729, Question 7.

Gameboard:

STEM SMART Double Maths 44 - Centres of Mass

Space Monster Attack

Further A

C C C

This problem involves centre of mass calculation by integration, which is not covered in some Physics A Levels. For more information please check with your teacher.

Part A C.O.M. of triangle

A lamina in the shape of an isosceles triangle, of height b and base $2a$, is resting on its shortest side.

By considering the triangle as a collection of infinitesimally small rectangles, find the distance D of the centre of mass from the base of the triangle.

The following symbols may be useful: `a, b, tan(), theta`

Part B C.O.M. of ship

The spaceship shown in the figure consists of four sections, a square bridge, two sets of rectangular living quarters and a triangular engine bay. The engine bay is twice as dense as the other parts of the ship.

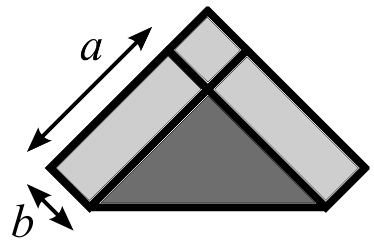

Figure 2: Diagram of spaceship.

Find the distance D from the centre of mass of the ship to the ship's base in terms of b given $a = 3b$.

The following symbols may be useful: `a, b, tan(), theta`

Part C C.O.M. of fleet

These spaceships are designed so they can dock together in a fleet of four, as shown in **Figure 3**.

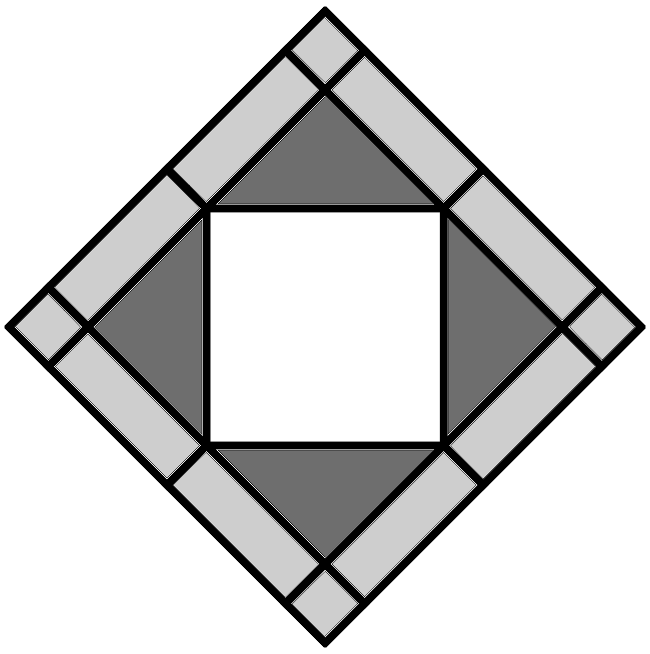

Figure 3: Four spaceships docked together.

Unfortunately whilst docked the fleet floats close to an asteroid on which lurks a ravenous Bugblatter Beast of Traal. The beast swallows one of the ships whole leaving the formation looking like **Figure 4**.

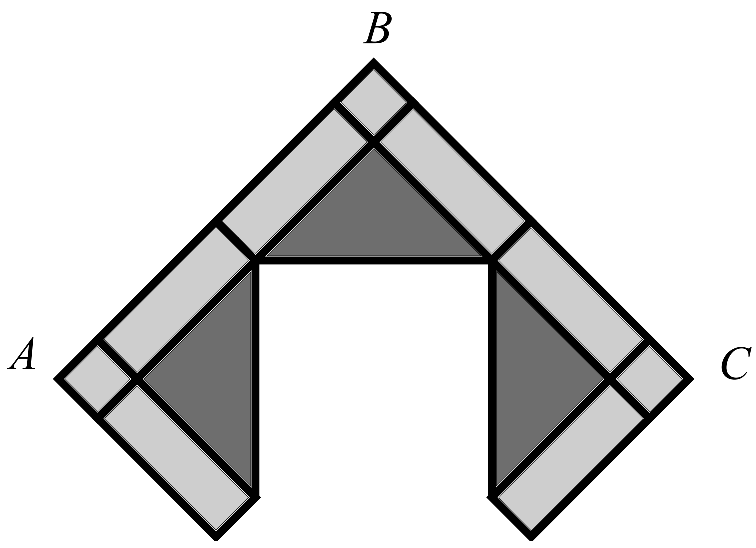

Figure 4: Three remaining spaceships in formation.

Find the distance D from the centre of the square made by four ships to the centre of mass of the remaining fleet.

The following symbols may be useful: `a, b, tan(), theta`

Part D Beast attack

In an attempt to stop the other ships escaping the Bugblatter Beast disables the main engines of the remaining ships using its electrified tentacles. Unbeknownst to it though, each of the ships has a tiny docking engine at the front tip of the bridge. The beast grabs the bridges of ships A and C on their very corners and pulls them in equal and opposite vertical directions, trying to rip the ships apart with its two monstrous heads.

The ships' docking engines can point in any direction, and the captain knows that if she can ensure the fleet does not rotate then the beast will get confused and forget about its quarry.

Assuming the docking engines of A and C can produce exactly enough force to save the crew if pointed in the right direction. What is the correct angle, θ to point the engines? Give the answer as the fractional value of $\tan(\theta)$.

The following symbols may be useful: `a`, `b`, `tan()`, `theta`

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Gameboard:

STEM SMART Double Maths 44 - Centres of Mass

Centre of Mass: Cones on Strings

Further A

A **solid uniform cone** has height $8\,\mathrm{cm}$, base radius $5\,\mathrm{cm}$ and mass $4\,\mathrm{kg}$.

A **uniform conical shell** has height $10\,\mathrm{cm}$, base radius $5\,\mathrm{cm}$ and mass $0.4\,\mathrm{kg}$.

The two shapes are joined together so that the circumferences of their circular bases coincide.

Part A Position of centre of mass

Find the distance of the centre of mass of the shape from the common circular base. Give your answer to 3 s.f.

Part B String tensions

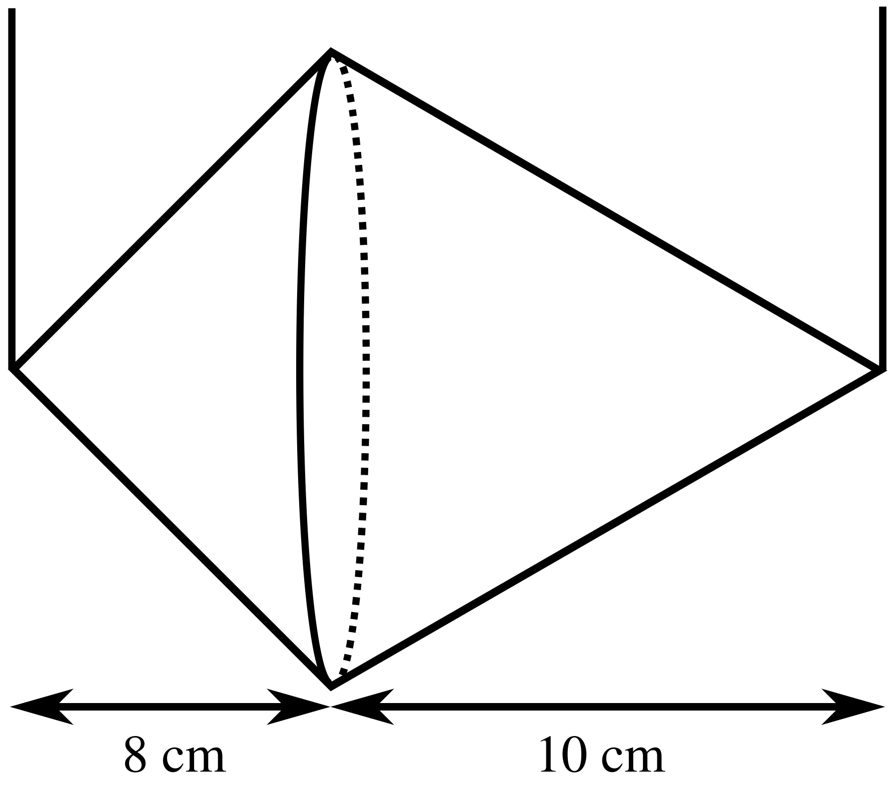

Figure 1: The situation described in the question.

The object is suspended with a string attached to the vertex of the cone and another string attached to the vertex of the conical shell. The object is in equilibrium with the strings vertical and the axis of symmetry of the object horizontal, as shown in **Figure 1**.

Find the tension in the string attached to the **conical shell**. Give your answer to 3 s.f.

Find the tension in the string attached to the **solid cone**. Give your answer to 3 s.f.

Used with permission from OCR A Level, June 2013, Paper 4729, Question 4.

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Centre of Mass by Integration

Further A

The region bounded by the x-axis, the lines $x = 1$ and $x = 2$, and the curve $y = kx^2$, where k is a positive constant, is occupied by a uniform lamina.

Part A Find the x-coordinate

Find the exact x-coordinate of the centre of mass of the lamina.

The following symbols may be useful: x

Part B Find k

Given that the x- and y- coordinates of the centre of mass of the lamina are equal, find the exact value of k.

The following symbols may be useful: k

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Gameboard:

STEM SMART Double Maths 44 - Centres of Mass

Solid of Revolution Centre of Mass

Further A

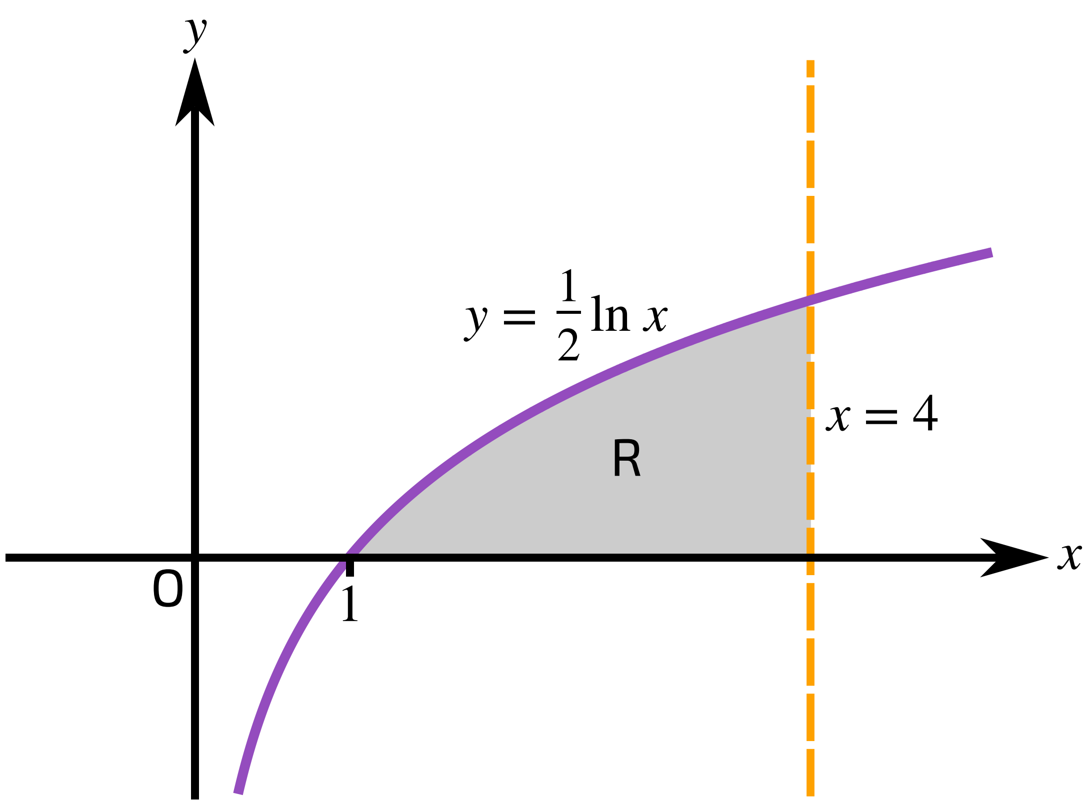

Figure 1: The region R.

Figure 1 shows the curve with equation $y = \frac{1}{2}\ln x$. The region R, shaded in **Figure 1**, is bounded by the curve, the x-axis and the line $x = 4$. A uniform solid of revolution is formed by rotating R completely about the y-axis to form a solid of volume V.

Part A Volume of revolution

Find the exact volume V of the solid of revolution. Give your answer in the form $V = \frac{\pi}{4}\left(a\ln 2 + b\right)$, where a and b are integers.

The following symbols may be useful: `V`, `a`, `b`, `ln()`, `pi`

Part B Centre of mass

Find the exact y-coordinate of the centre of mass of the solid.

The following symbols may be useful: `ln()`, `y`

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