Physics. You work it out.

Home Maths Statistics Probability Poisson Distribution: Crystals

Poisson Distribution: Crystals

Further A

P P P

In a rock, small crystal formations occur at a constant average rate of 3.2 per cubic metre.

Part A Assumptions

State a further assumption needed to model the number of crystal formations in a fixed volume of rock by a Poisson distribution.

- Crystals must occur independently of one another
- The rock must be at least a cubic metre in size
- The rock must contain 3 or more crystals

Part B Exactly five crystal formations

In the remainder of the question, you should assume that a Poisson model is appropriate.

Calculate the probability that in one cubic metre of rock there are exactly 5 crystal formations. Give your answer to 3 sf.

Part C At least three crystal formations

Calculate the probability that in 0.74 cubic metres of rock there are at least 3 crystal formations. Give your answer to 3 sf.

Part D At least 36 crystal formations

Calculate the probability that in 10 cubic metres of rock there are at least 36 crystal formations. Give your answer to 3 sf.

Adapted with permission from UCLES, A level, June 2012, OCR , Question 4

Physics. You work it out.

Home Maths Statistics Probability Poisson distribution - quasars

Poisson distribution - quasars

Further A University

Currently an average of 24 quasars have been found per square degree of sky. On the assumption that their distribution is random and independent so that it follows a Poisson distribution find the following.

Part A Probability of less than 15 quasars

Find the probability that there are fewer than 15 quasars in a randomly chosen area of 1 square degree. Give your answer to 3 sf.

Part B Probability between 24 and 30 quasars

Find the probability that there are no fewer than 24 and no more than 30 quasars in a randomly chosen area of 1 square degree. Give your answer to 3 sf.

Part C Probability of no quasars

A square with an area of 1 square degree is divided into 16 smaller squares with equal areas. One of these smaller squares is randomly selected. Find the probability that there are no quasars in this smaller area. Give your answer to 3 sf.

Part D Probability of at least 2 quasars

Again considering the 1 degree square divided into 16 smaller squares of equal area, find the probability that at least 8 of the smaller squares contain at least 2 quasars. Give your answer to 3 sf.

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Physics. You work it out.

Home Maths Statistics Probability Poisson distribution - radioactivity

Poisson distribution - radioactivity

Further A University

A radioactive source produces gamma rays which travel to a detector. An average of 17 gamma rays arrive at the detector every 10 seconds.

The detector counts and displays the number of gamma rays arriving during a time interval set by the user. In this time interval the detector can detect a maximum of 5 gamma rays before saturating; thus, if the number of gamma rays arriving at the detector in this time interval is > 5, the counter will read 5. (For example, supposing 8 gamma rays arrive at the detector in the given time interval the counter will read 5.)

Part A Probability that the detector saturates in $2\,\mathrm{s}$

The detector is set to count gamma rays for 2 seconds. Calculate the probability that the reading on the counter is not the same as the number of gamma rays that arrive at the detector. Give your answer to 3 sf.

Part B The most probable number of gamma rays in $2\,\mathrm{s}$

The detector is set to count gamma rays for 2 seconds. Find the most probable number of gamma rays arriving at the detector.

Part C Expected number of gamma rays in $2\,\mathrm{s}$

The detector is set to count gamma rays for 2 seconds; find the expected number of gamma rays it will detect giving your answer to 3 sf.

Find the expectation value of the percentage error in the measurement of the number of gamma rays arriving at the detector. (This is the percentage difference between the expectation value of the reading on the detector that you have just derived, and the mean number of gamma rays arriving at the detector.) Give your answer to 2 sf.

Part D Probability that the detector saturates in $1\,\mathrm{s}$

The detector is adjusted to count gamma rays for 1 second; it still saturates when the number of counts exceeds 5. Calculate the probability that the reading on the counter is not the same as the number of gamma rays that arrive at the detector. Give your answer to 3 sf.

Part E Expected number of gamma rays in $1\,\mathrm{s}$

The detector is set to count gamma rays for 1 second; find the expected number of gamma rays it will detect giving your answer to 3 sf.

Find the expectation value of the percentage error in the measurement of the number of gamma rays arriving at the detector. (This is the percentage difference between the expectation value of the reading on the detector that you have just derived, and the mean number of gamma rays arriving at the detector.) Give your answer to 2 sf.

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Poisson Distribution: Random Variables

Further A

P P P

This question looks at the properties of a random variable which follows a Poisson distribution, and also the sum of two independent random variables which each have Poisson distributions.

Part A $\mathrm{E}(X)$ for a Poisson distribution

A random variable X is modelled by the Poisson distribution $X \sim \mathrm{Po}(5.9)$. Write down the value of the expectation of X, $\mathrm{E}(X)$.

Part B $\mathrm{Var}(X)$ for a Poisson distribution

A random variable X is modelled by the Poisson distribution $X \sim \mathrm{Po}(\lambda)$. The ratio $\frac{\mathrm{P}(X=6)}{\mathrm{P}(X=4)} = 0.3$. Find the exact value of the variance of X, $\mathrm{Var}(X)$.

Part C Finding a

Consider the two random variables $X \sim \mathrm{Po}(\lambda)$ and $Y \sim \mathrm{Po}(2\lambda)$. Find an expression for the value a such that $\mathrm{P}(X=a) = \mathrm{P}(Y=a)$. Give your answer in the form $\frac{f(\lambda)}{\ln(2)}$, where $f(\lambda)$ is a function of λ.

The following symbols may be useful: `lambda`, `ln()`, `log()`

Part D Summing Poisson variables

The independent random variables X and Y have the distributions $X \sim \mathrm{Po}(\lambda)$ and $Y \sim \mathrm{Po}(2\lambda)$. A third random variable $T = X + Y$.

If $\mathrm{P}(T = 5) = \frac{1}{2}\mathrm{P}(X = 5)$, find an expression for λ. Give your answer in the form $\frac{1}{a}\ln b$, where a and b are integers.

Created for isaacphysics.org by Jonathan Waugh

Physics. You work it out.

Home Maths Statistics Random Variables Geometric Distribution: Darts

Geometric Distribution: Darts

Further A

P P P

A darts player aims repeatedly for the bull's-eye. He counts it a success if his dart hits the bull's-eye. It may be assumed that, on each throw, the probability of success is $\frac{1}{4}$, independently of all other throws.

Part A First success on fourth throw

Find the probability that, in a series of throws, the first success occurs on the fourth throw. Give your answer as a fraction.

Part B First success between fourth and eighth throws

Find the probability that, in a series of throws, the first success occurs between the fourth and eighth throws (inclusive). Give your answer to 3 sf.

Part C Second success on third throw

Find the probability that, in a series of throws, the second success occurs on the third throw. Give your answer as a fraction.

Adapted with permission from OCR June 2000, Mathematics Paper 4, Question 14.

Home Maths Statistics Random Variables Geometric Distribution

Geometric Distribution

Further A

Part A Random variable X

The random variable X has a geometric distribution with parameter $\frac{1}{3}$. Find $\mathrm{P}(X = 3)$.

Part B Random variable Y

The random variable Y has a geometric distribution with mean 4. Find $\mathrm{Var}(Y)$.

Adapted with permission from OCR November 1999, Mathematics Paper 4, Question 6.

Physics. You work it out.

Geometric distribution - pulsar

Further A

C C C

A pulsar produces regular pulses with a period $T = 1.33\,\mathrm{s}$; the brightness of the pulses varies randomly from pulse to pulse. The pulses arrive at a receiver which can only detect them when the brightness exceeds a certain level. In one set of measurements lasting for $5054\,\mathrm{s}$, 570 pulses are bright enough to be detected.

It is assumed that Pulse n arrives at nT, i.e. Pulse 1 arrives at T, Pulse 2 at $2T$ and so on. N is the number of pulses that have arrived up until and including the first one bright enough to be detected; you may assume it follows a geometric distribution $N \sim \mathrm{Geo}(p)$.

Part A Probability of detecting a pulse

Deduce the probability that a pulse will be detected.

Part B First pulse detected is Pulse 5

Find the probability that the first pulse bright enough to be detected by the receiver is Pulse 5. Give your answer to 3 sf.

Part C First pulse detected is after Pulse 5

Find the probability that the first pulse bright enough to be detected by the receiver arrives after Pulse 5. Give your answer to 3 sf.

Part D First pulse detected is before Pulse 5

Find the probability that the first pulse bright enough to be detected by the receiver arrives before Pulse 5. Give your answer to 3 sf.

Part E Probability less than 5%

Find the smallest value of n for which the probability that Pulse n is the first detected pulse is less than 5%, i.e. $\mathrm{P}(N = n) < 5\%$.

Part F Probability less than 20%

Find the smallest value of n for which the probability that the first detected pulse is after Pulse n is less than 20%, i.e. $\mathrm{P}(N > n) < 20\%$.

Part G First and second detection

Find the probability that the first pulse bright enough to be detected is the third one and that the second pulse bright enough to be detected is the tenth. Give your answer to 3 sf.

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