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STEM SMART Single Maths 35 - Integration by Substitution

Integration by Substitution

A-level Maths Topic Summaries - Calculus

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** A Level P3

Fill in the blanks to complete the notes on integration by substitution.

Part A

Indefinite integrals

We will illustrate the process for the integral $I = \int \frac{6x}{(2x-7)^{\frac{1}{2}}}\,\mathrm{d}x$.

1. Define a new variable u. Let $u = 2x - 7$. Then $x =$ [ ].

2. Rewrite the integrand in terms of u.

$$I = \int [\quad]\,\mathrm{d}x = \int \left(3u^{\frac{1}{2}} + 21u^{-\frac{1}{2}}\right)\mathrm{d}x$$

3. Differentiate $u = 2x - 7$ to get $\frac{\mathrm{d}u}{\mathrm{d}x} =$ [ ], then make use of the chain rule to change the integral from an integral with respect to x into an integral with respect to u.

$$\mathrm{d}x = \frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u = \frac{1}{\frac{\mathrm{d}u}{\mathrm{d}x}}\,\mathrm{d}u$$

$$\therefore I = \int [\quad]\,\mathrm{d}u$$

4. Carry out the integration. This gives $I =$ [ ] $+ c$.

5. Finally, back-substitute for u to give the answer in terms of the original variable, x. This gives $I =$ [ ] $+ c$.

Items:

2 | $\frac{u+7}{2}$ | $u^{\frac{3}{2}} + 21u^{\frac{1}{2}}$ | $\frac{3}{2}u^{\frac{1}{2}} + \frac{21}{2}u^{-\frac{1}{2}}$ | $\frac{3u+21}{u^{\frac{1}{2}}}$ | $(2x-7)^{\frac{3}{2}} + 21(2x-7)^{\frac{1}{2}}$

Part B

Definite integrals

We will illustrate the process for the integral $J = \int_4^5 \frac{6x}{(2x-7)^{\frac{1}{2}}}\,\mathrm{d}x$. The first few steps are the same as for indefinite integrals.

1. Define a new variable u.
2. Rewrite the integrand in terms of u.
3. Use the chain rule to change the integral from an integral with respect to x into an integral with respect to u.

These steps give us

$$\therefore J = \int_{x=4}^{x=5} \boxed{}\,\mathrm{d}u$$

Next we consider the limits of the integral.

4. Use $u = 2x - 7$ to change the limits of the integral from values of x to the equivalent values of u.

$$J = \int_{u=\boxed{}}^{u=\boxed{}} \boxed{}\,\mathrm{d}u$$

5. Carry out the integration.

$$J = \left[\boxed{}\right]_{\boxed{}}^{\boxed{}}$$

Items:

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{\frac{3}{2}u^{\frac{1}{2}} + \frac{21}{2}u^{-\frac{1}{2}}}$ $\boxed{u^{\frac{3}{2}} + 21u^{\frac{1}{2}}}$

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STEM SMART Single Maths 35 - Integration by Substitution

Integration by Substitution 1

Pre-Uni Maths for Sciences K3.1

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** A Level P3

Part A

Integrate $\sin(c\theta)$

Find $\int \sin(c\theta)\,\mathrm{d}\theta$, where c is a constant.

The following symbols may be useful: `C, c, k, theta`

Part B

Integrate $\mathrm{e}^{\alpha v}$

Find $\int \mathrm{e}^{\alpha v}\,\mathrm{d}v$, where α is a constant.

The following symbols may be useful: `alpha, e, k, v`

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Question deck:

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STEM SMART Single Maths 35 - Integration by Substitution

Integration by Substitution 2

Pre-Uni Maths for Sciences K3.2

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** A Level P3

Part A

Integrate $(bv + c)^2$

Find $\int (bv + c)^2 \, \mathrm{d}v$, where b and c are constants.

The following symbols may be useful: `b, c, k, v`

Part B

Integrate $a(y - b)^3$

Find $\int_0^b a(y - b)^3 \, \mathrm{d}y$, where a and b are constants.

The following symbols may be useful: `a, b`

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Question deck:

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STEM SMART Single Maths 35 - Integration by Substitution

Integration: General 4ii

Subject & topics: Maths **Stage & difficulty:** A Level P3

Part A

Integrate $(4-3x)^7$

Find $\int (4-3x)^7 \mathrm{d}x$.

The following symbols may be useful: `c, x`

Part B

Integrate $(4-3x)^{-1}$

Find $\int (4-3x)^{-1} \mathrm{d}x$.

The following symbols may be useful: `c, x`

Question deck:

STEM SMART Single Maths 35 - Integration by Substitution

isaac You work it out.

STEM SMART Single Maths 35 - Integration by Substitution

Integration by Substitution 5ii

Subject & topics: Maths **Stage & difficulty:** A Level P3

Use the substitution $u = 2x - 5$ to find the exact value of $\int_{\frac{5}{2}}^{3} (4x - 8)(2x - 5)^7 \mathrm{d}x$.

Question deck:

STEM SMART Single Maths 35 - Integration by Substitution

isaac You work it out.

STEM SMART Single Maths 35 - Integration by Substitution

Integration by Substitution 5i

Subject & topics: Maths **Stage & difficulty:** A Level P3

Use the substitution $u = 2x + 1$ to evaluate $\displaystyle\int_0^{\frac{1}{2}} \frac{4x - 1}{(2x + 1)^5} \mathrm{d}x$.

Question deck:

STEM SMART Single Maths 35 - Integration by Substitution

STEM SMART Single Maths 35 - Integration by Substitution

Integration by Substitution 4ii

Subject & topics: Maths **Stage & difficulty:** A Level P3

Use the substitution $u = \sqrt{x+2}$ to find the exact value of $\displaystyle\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}\mathrm{d}x$.

Question deck:

STEM SMART Single Maths 35 - Integration by Substitution

isaac You work it out.

STEM SMART Single Maths 35 - Integration by Substitution

Integration by Substitution 4i

Subject & topics: Maths **Stage & difficulty:** A Level P3

Use the substitution $u = 1 + \sqrt{x}$ to find the exact value of $\int_4^9 \frac{1}{1+\sqrt{x}}\mathrm{d}x$ in the form $a + b\ln(c)$, where a, b, and c are positive constants to be found.

Question deck:

STEM SMART Single Maths 35 - Integration by Substitution

isaac You work it out.

STEM SMART Single Maths 35 - Integration by Substitution

Integration by Substitution 3

Pre-Uni Maths for Sciences K3.3

Subject & topics: Maths | Calculus | Integration **Stage & difficulty:** A Level P3

Part A

Integrate $\frac{3}{(z+1)^2}$

Find $\displaystyle\int_0^2 \frac{3}{(z+1)^2}\,\mathrm{d}z$.

Part B

Integrate $\frac{e^{-\alpha x}}{(1+e^{-\alpha x})^4}$

Find $\displaystyle\int \frac{\mathrm{e}^{-\alpha x}}{(1+\mathrm{e}^{-\alpha x})^4}\,\mathrm{d}x$, where α is a constant.

The following symbols may be useful: `C, alpha, c, e, k, x`

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