Physics. You work it out.

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Confidence Intervals - Scales

Further A

P P P

A set of bathroom scales is known to operate with an error which is normally distributed. One morning a man weighs himself 4 times. The 4 values for his mass, in kg, which can be considered to be a random sample are as follows:

$$62.6,\ 62.8,\ 62.1,\ 62.5$$

Part A Confidence interval

Find a 95% confidence interval for his mass. Give the lower and upper confidence limits to 3 sf.

(kg , kg)

Part B Width of a confidence interval

Based on these results, a $y\%$ confidence interval has a width of 0.4822. Find y.

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t-tests: Chemicals

Further A

P P P

The senior technician of a university science department is considering using a new supplier of chemicals for student laboratory experiments. These chemicals do not need to be of a very high grade of purity, but must be sufficiently pure that the experiments can be conducted efficiently and safely.

The new supplier states that the average level of impurity in the chemicals it supplies is no more than 4%. The senior technician carefully measures the impurity in a random sample of eight containers of these chemicals and finds the percentages to be as follows.

$$3.9 \quad 4.5 \quad 4.7 \quad 4.3 \quad 4.9 \quad 3.4 \quad 4.5 \quad 5.0$$

Part A Null and alternative hypotheses

State the appropriate null and alternative hypotheses for the usual t-test for examining whether the supplier is meeting the stated standard.

$$\mathrm{H}_0 : \square \ \square \ 4 \qquad \mathrm{H}_1 : \square \ \square \ 4$$

Items:

$=$ $<$ s $\overline{x}$ σ $\neq$ μ $>$

Part B $\mathrm{N}(0,1)$ distribution

Explain why the corresponding test based on the $\mathrm{N}(0,1)$ distribution cannot be used. Select all that apply.

- [ ] The true value of μ is unknown.
- [ ] The sample is large.
- [ ] The true value of σ^2 is unknown.
- [ ] The sample is small.

Part C Necessary condition

What condition is necessary for the correct use of the t-test?

- The underlying distribution is normal.
- The underlying distribution is uniform.
- The underlying distribution is unknown.

Part D t-test

Carry out the t-test, using a 5% significance level. Fill in the gaps below.

The test statistic, $t =$ [ ]. The critical value at the 5% level is $t_{\text{crit}} =$ [ ].

Comparing these, we find that t [ ].

Therefore we [ ] H_0 at the 5% level. There [ ] evidence to suggest that the average level of impurity in the chemicals is [ ] the supplier stated.

Items:

lower than | **not as** | 1.895 | **reject** | **higher than** | 2.306 | $> t_{\text{crit}}$ | 1.990 | $< t_{\text{crit}}$ | $\neq t_{\text{crit}}$ | 2.365 | 1.860 | **is** | 2.127 | **is insufficient** | 2.274 | 0.606 | **do not reject**

Part E Confidence interval

Provide a two-sided 95% confidence interval for the true mean percentage of impurity in the supplier's chemicals. Give the lower and upper bounds to 3 sf.

([ ] $\%$, [ ] $\%$)

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STEM SMART Double Maths 44 - t-tests

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t-tests: Geography Fieldwork

Further A P P P

A group of students were tested in Geography before and after a fieldwork course. The marks of 10 randomly selected students are shown in the table.

Student	A	B	C	D	E	F	G	H	I	J
Mark before fieldwork	19	84	84	99	59	19	29	49	54	69
Mark after fieldwork	23	98	83	88	68	33	28	53	58	88

Part A t-test

Use a suitable t-test, at the 5% level of significance, to test whether the students' performance has improved. Fill in the gaps below.

The null and alternative hypotheses are:

$$\mathrm{H}_0 : \mu_2 \boxed{} \mu_1 \qquad \mathrm{H}_1 : \mu_2 \boxed{} \mu_1$$

The test statistic, $t =$ [ ]. The critical value is $t_{\text{crit}} =$ [ ].

Comparing these, we find that t [ ] t_{crit}.

Therefore we [ ] H_0 at the 5% level. There [ ] evidence to suggest that the fieldwork has improved the students' performance.

Items:

2.262 | 2.075 | 2.228 | 1.812 | **reject** | $=$ | 1.969 | **do not reject** | **is** | 1.829 | $<$ | $>$ | 1.833 | **is insufficient**

Part B Assumption

Which of these is a necessary assumption in applying the test?

- The differences are uniformly distributed.
- The differences have a positive mean.
- The differences are normally distributed.

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t-tests: Runners

Further A P P P

A national athletics coach suspects that, on average, 200-metre runners' indoor times exceed their outdoor times by more than $0.1\,\mathrm{s}$. In order to test this, the coach randomly selects eight 200-metre runners and records their indoor and outdoor times. The results, in seconds, are shown in the table.

Runner	A	B	C	D	E	F	G	H
Indoor time	21.5	21.8	20.9	21.2	21.4	21.4	21.2	21.0
Outdoor time	21.1	21.7	20.7	20.9	21.3	21.0	21.1	20.8

Stating suitable hypotheses, test the coach's suspicion at the 2.5% level of significance.

Let μ_i and μ_o be the population means for indoor and outdoor times, respectively. The null and alternative hypotheses are:

$$\mathrm{H}_0 : \mu_i - \mu_o = \square \qquad \mathrm{H}_1 : \mu_i - \mu_o > \square$$

The test statistic, $t =$ ______. The critical value is $t_{\text{crit}} =$ ______.

Comparing these, we find that t ______ t_{crit}.

Therefore we ______ H_0 at the 2.5% level. There ______ evidence to suggest that 200-metre runners' indoor times exceed their outdoor times by more than $0.1\,\mathrm{s}$.

Items:

$=$ | $>$ | is | 5.308 | 0.8284 | 0 | 1.895 | $\neq$ | 1.860 | $<$ | 0.2 | 2.365 | 2.306 | reject | do not reject | 0.225 | 4.965 | 0.1 | is insufficient | 2.758 | 2.949

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t-tests: Stopping Distance

Further A P P P

A racing driver claims that they can bring their car to rest from a speed of $150\,\mathrm{km\,h^{-1}}$ in a shorter distance with radial-ply tyres than with cross-ply tyres. As evidence for this claim, they list the stopping distances on a race track in twelve random trials, six with radial-ply tyres and six with cross-ply tyres.

	Stopping distances (m)					
Radial-ply tyres, r	123	164	139	161	118	135
Cross-ply tyres, c	156	137	134	173	155	151

Part A Assumptions

State two assumptions that are required in order to test the driver's claim with a suitable t-test.

- [ ] The stopping distances for radial-ply tyres and cross-ply tyres follow a Poisson distribution.
- [ ] The stopping distances for radial-ply tyres and cross-ply tyres have equal variances.
- [ ] The stopping distances for radial-ply tyres and cross-ply tyres follow a normal distribution.
- [ ] The stopping distances for radial-ply tyres and cross-ply tyres have equal means.
- [ ] The stopping distances for radial-ply tyres and cross-ply tyres follow a uniform distribution.

Part B Hypothesis test

Test at the 5% significance level whether the racing driver's claim is justified.

The null and alternative hypotheses are:

$$\mathrm{H}_0 : \mu_r \; \square \; \mu_c \qquad \mathrm{H}_1 : \mu_r \; \square \; \mu_c$$

Calculating the difference as $\overline{r} - \overline{c}$, the test statistic, $t =$ ☐. The critical value is $t_{\text{crit}} =$ ☐.

Comparing these, we find that t ☐ t_{crit}.

Therefore we ☐ H_0 at the 5% level. There ☐ evidence to suggest that stopping distances are shorter with radial-ply tyres than with other types of tyre.

Items:

$>$ | **is insufficient** | -0.984 | -1.133 | -1.782 | **reject** | **is** | $=$ | $\neq$ | -1.242 | **do not reject** | $<$ | -1.812 | -2.228 | -2.179

Part C Confidence interval

Find a 95% confidence interval for the difference, $\mu_r - \mu_c$, between the mean stopping distances of radial-ply tyres and cross-ply tyres. Give the lower and upper bounds to 3 sf.

(☐ m , ☐ m)

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t-tests: Carton Packing

Further A

P P P

Two machines A and B both pack cartons in a factory. The mean packing times are compared by timing the packing of 10 randomly chosen cartons from machine A and 8 randomly chosen cartons from machine B. The times taken for each machine to pack these cartons, $a\,\mathrm{seconds}$ and $b\,\mathrm{seconds}$ respectively, are summarised below.

$$\sum a = 221.4 \qquad \sum a^2 = 4920.9 \qquad \sum b = 199.2 \qquad \sum b^2 = 4980.3$$

The packing times have independent normal distributions with equal variance.

Part A Hypothesis test

Carry out a test, at the 1% significance level, of whether the population mean packing times differ for the two machines.

The null and alternative hypotheses are:

$$\mathrm{H}_0 : \mu_a \ \boxed{}\ \mu_b \qquad \mathrm{H}_1 : \mu_a \ \boxed{}\ \mu_b$$

Calculating the difference as $\bar{b} - \bar{a}$, the test statistic, $t =$ [ ]. The critical value is $t_{\text{crit}} =$ [ ].

Comparing these, we find that t [ ] t_{crit}.

Therefore we [ ] H_0 at the 1% level. There [ ] evidence to suggest that there is a difference between the mean packing times of the machines.

Items:

2.878 | **is** | **is insufficient** | $=$ | $>$ | 2.921 | $<$ | 3.941 | 3.711 | 2.552 | 2.583 | 3.645 | **do not reject** | **reject** | $\neq$ | 3.876

Part B Evidence for larger difference

Find the largest possible value of the constant c for which there is evidence at the 1% significance level that $\mu_b - \mu_a > c$.

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