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# Partial Fractions 2ii

A Level



## Part A Partial Fractions

Express  $\frac{x-1}{x(x+1)}$  in partial fractions.

The following symbols may be useful:  $x$

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## Part B Integral

Hence find the exact value of  $\int_1^2 \frac{x-1}{x(x+1)} dx$ .

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# Partial Fractions 1i

A Level



## Part A Partial Fractions

Express  $\frac{2+x^2}{(1+2x)(1-x)^2}$  in the form  $\frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$ .

The following symbols may be useful:  $x$

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## Part B Integration

Hence find  $\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2} dx$  in exact form.

The following symbols may be useful:  $\int$ ,  $\ln()$ ,  $\log()$

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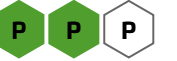


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# Partial Fractions 4ii

A Level



## Part A   Partial Fractions

Express  $\frac{7-2x}{(x-2)^2}$  in the form  $\frac{A}{x-2} + \frac{B}{(x-2)^2}$ , where  $A$ , and  $B$  are constants.

The following symbols may be useful: x

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## Part B   Integral

Hence find the exact value of  $\int_4^5 \frac{7-2x}{(x-2)^2} dx$ .

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# Integration by Substitution 4

Pre-Uni Maths for Science K3.5

A Level



## Part A Integrate $\frac{1}{b(x+a)}$

Find  $\int_0^a \frac{1}{b(x+a)} dx$ , where  $a$  and  $b$  are constants.

The following symbols may be useful:  $a$ ,  $b$ ,  $k$ ,  $x$

## Part B Integrate $\frac{x}{1+x^2}$

Find  $\int_0^1 \frac{x}{1+x^2} dx$ .

The following symbols may be useful:  $k$ ,  $x$

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# Integration by Substitution 5

Pre-Uni Maths for Science K3.6

A Level



**Part A** Integrate  $\frac{x^3}{a^5 + ax^4}$

Find  $\int_a^{2a} \frac{x^3}{a^5 + ax^4} dx$ .

The following symbols may be useful: a

**Part B** Integrate  $\tan \beta$

By writing  $\tan \beta = \frac{\sin \beta}{\cos \beta}$ , find  $\int_0^{\frac{\pi}{4}} \tan \beta d\beta$ .

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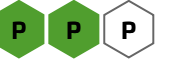


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# Algebraic Division 1ii

A Level



## Part A Quotient and Remainder

Find the quotient when  $3x^3 - x^2 + 10x - 3$  is divided by  $x^2 + 3$ .

The following symbols may be useful: x

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Give the remainder.

The following symbols may be useful: x

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## Part B Integral

Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3} dx.$$

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# Integration by Substitution 1i

A Level



Let  $I = \int \frac{1}{x(1 + \sqrt{x})^2} dx$ .

## Part A Substitution

Using the substitution  $u = \sqrt{x}$ , transform  $I$  into the integral  $\int f(u) du$ . Give the function  $f(u)$  in terms of  $u$ .

The following symbols may be useful:  $f$ ,  $u$

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## Part B Partial Fractions

Express  $\frac{2}{u(1+u)^2}$  in the form  $\frac{A}{u} + \frac{B}{1+u} + \frac{C}{(1+u)^2}$ .

The following symbols may be useful:  $u$

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## Part C Integrate

Hence find  $I$ .

The following symbols may be useful:  $I$ ,  $c$ ,  $x$

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# Integration by Substitution 1ii

A Level



## Part A Partial Fractions

Given that  $\frac{2t}{(t+1)^2}$  can be expressed in the form  $\frac{A}{t+1} + \frac{B}{(t+1)^2}$ , find the values of the constants  $A$  and  $B$ .

State the value of  $A$ .

The following symbols may be useful: A

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State the value of  $B$ .

The following symbols may be useful: B

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## Part B Show

Show that the substitution  $t = \sqrt{2x - 1}$  transforms  $\int \frac{1}{x + \sqrt{2x - 1}} dx$  to  $\int \frac{2t}{(t + 1)^2} dt$ .

**More practice questions?**

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## Part C    Exact Value

Hence find the exact value of  $\int_1^5 \frac{1}{x + \sqrt{2x - 1}} dx$ .

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# Integration With Partial Fractions 2

A Level Further A  
P P P P P P

Pre-Uni Maths for Science A4.9

Write the function  $\frac{2z^2 - z - 3}{(z + 2)(z^2 - 2z - 1)}$  in the form  $\frac{A}{z + 2} + \frac{B + Cz}{z^2 - 2z - 1}$ . Hence find

$$\int_1^2 \frac{2z^2 - z - 3}{(z + 2)(z^2 - 2z - 1)} dz.$$

## Part A Find A

Find the constant  $A$

## Part B Find B

Find the constant  $B$ .

## Part C Find C

Find the constant  $C$ .

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## Part D Integrate

Hence find  $\int_1^2 \frac{2z^2 - z - 3}{(z + 2)(z^2 - 2z - 1)} dz$ .

The following symbols may be useful:  $\cos()$ ,  $\operatorname{cosec}()$ ,  $\operatorname{cosech}()$ ,  $\cosh()$ ,  $\cot()$ ,  $\coth()$ ,  $\ln()$ ,  $\log()$ ,  $\sec()$ ,  $\operatorname{sech}()$ ,  $\sin()$ ,  $\sinh()$ ,  $\tan()$ ,  $\tanh()$ ,  $z$

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