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Home Gameboard Maths Calculus Differentiation Differentiation from First Principles 2

Differentiation from First Principles 2

A Level
P P P

Differentiating a function $f(x)$ from first principles involves taking a limit. The derivative of $f(x)$ is given by the expression

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Part A Differentiate $4x^2 + 2x + 7$ from first principles

Differentiate $f(x) = 4x^2 + 2x + 7$ from first principles. Drag and drop options into the spaces below.

$f(x+h) = 4(x+h)^2 + 2(x+h) + 7$. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(4(x+h)^2 + 2(x+h) + 7) - (4x^2 + 2x + 7)}{h}.$$

Next, expanding the brackets in the numerator and simplifying gives

$$f'(x) = \lim_{h \to 0} \frac{\boxed{} - (4x^2 + 2x + 7)}{h}$$

$$f'(x) = \lim_{h \to 0} (\boxed{} + (\boxed{})h).$$

Finally, take the limit. As $h \to 0$, the terms containing h tend to 0. Therefore,

$$f'(x) = \boxed{}.$$

Items:

4 | $8x + 4$ | 7 | $4x^2 + 2x + 7 + 8hx + 2h + 4h^2$ | $4x^2 + 2x + 7 + 4hx + 2h + 4h^2$ | $8x + 2$ | $4x^2 + 4h^2$

Part B Differentiate $ax^2 + bx + c$ from first principles

Differentiate $f(x) = ax^2 + bx + c$, where a, b and c are constants, from first principles.

$f(x+h) = a(x+h)^2 + b(x+h) + c$. Substituting this into the expression for $f'(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(a(x+h)^2 + b(x+h) + c) - (ax^2 + bx + c)}{h}.$$

Next, expanding the brackets in the numerator and simplifying gives

$$f'(x) = \lim_{h \to 0} \frac{\boxed{} + (\boxed{})h + (\boxed{})h^2}{h}$$

$$f'(x) = \lim_{h \to 0} (\boxed{} + (\boxed{})h).$$

Finally, take the limit. As $h \to 0$, the terms containing h tend to 0. Therefore,

$$f'(x) = \boxed{}.$$

Items:

ab | $ax^2 + 2ahx + ah^2$ | a | $2a$ | 1 | $2ax + b$ | $b + ah$ | 0

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Home Gameboard Maths Differentiation (powers of x) 1i

Differentiation (powers of x) 1i

A Level
P P P

It is given that $f(x) = \frac{1}{x} - \sqrt{x} + 3$.

Part A Find $f'(x)$

Find $f'(x)$.

The following symbols may be useful: x

Part B Find $f''(x)$

Find $f''(4)$.

Used with permission from UCLES, A level, January 2010, Paper 4721, Question 9.

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Integration (powers of x) 2ii

A Level
P P P

Part A Find integral

Find $\int x(x^2 - 4)\mathrm{d}x$.

The following symbols may be useful: `c`, `x`

Part B Evaluate integral

Evaluate $\int_1^6 x(x^2 - 4)\mathrm{d}x$. Give the exact value of your answer as a decimal.

Part C Find integral

Find $\int \frac{6}{x^3}\mathrm{d}x$.

The following symbols may be useful: `c`, `x`

Used with permission from UCLES, A Level Maths, June 2007, OCR C2, Question 6

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Home Gameboard Maths Calculus

Calculus

A Level

C C C

Part A Integrating a factorised expression

Find $\int(x^2+9)(x-4)\mathrm{d}x$.

The following symbols may be useful: `c`, `x`

Part B Differentiation

A curve has the equation $y=\frac{1}{3}x^3-9x$.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The following symbols may be useful: `Derivative(y, x)`, `x`, `y`

Part C Stationary points

Find the coordinates of the stationary points of the curve $y = \frac{1}{3}x^3 - 9x$. Enter the x and y coordinates of the stationary point with the largest x coordinate.

Enter the x-coordinate of the stationary point with the largest (most positive) x:

The following symbols may be useful: x

Enter its corresponding y coordinate:

The following symbols may be useful: y

Part D Nature of stationary point

Determine the nature of the stationary point with the largest x-coordinate.

- Minimum
- Maximum
- Neither/Inconclusive

Part E Tangent to the curve

Given that $24x + 3y + 2 = 0$ is the equation of the tangent to the curve $y = \frac{1}{3}x^3 - 9x$ at the point (p, q), find the values of p and q.

(i) Enter value of p:

The following symbols may be useful: p

(ii) Enter value of q:

The following symbols may be useful: q

Part F Normal to the curve

Find the equation of the normal to the curve $y = \frac{1}{3}x^3 - 9x$ at the point (p, q) you found in Part E.

Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers

The following symbols may be useful: x, y

Modified by Sally Waugh with permission from UCLES, A Level, June 2005, Paper 4721, Question 10.

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Home Gameboard Maths Calculus Integration Area Under a Curve 2

Area Under a Curve 2

A Level C C C Further A P P P

A graph of the functions $y = x^2 + 3$ and $y = 4x$ is shown in **Figure 1**. Find the area of the shaded region labelled A, the region between the line $y = 4x$ and the curve $y = x^2 + 3$.

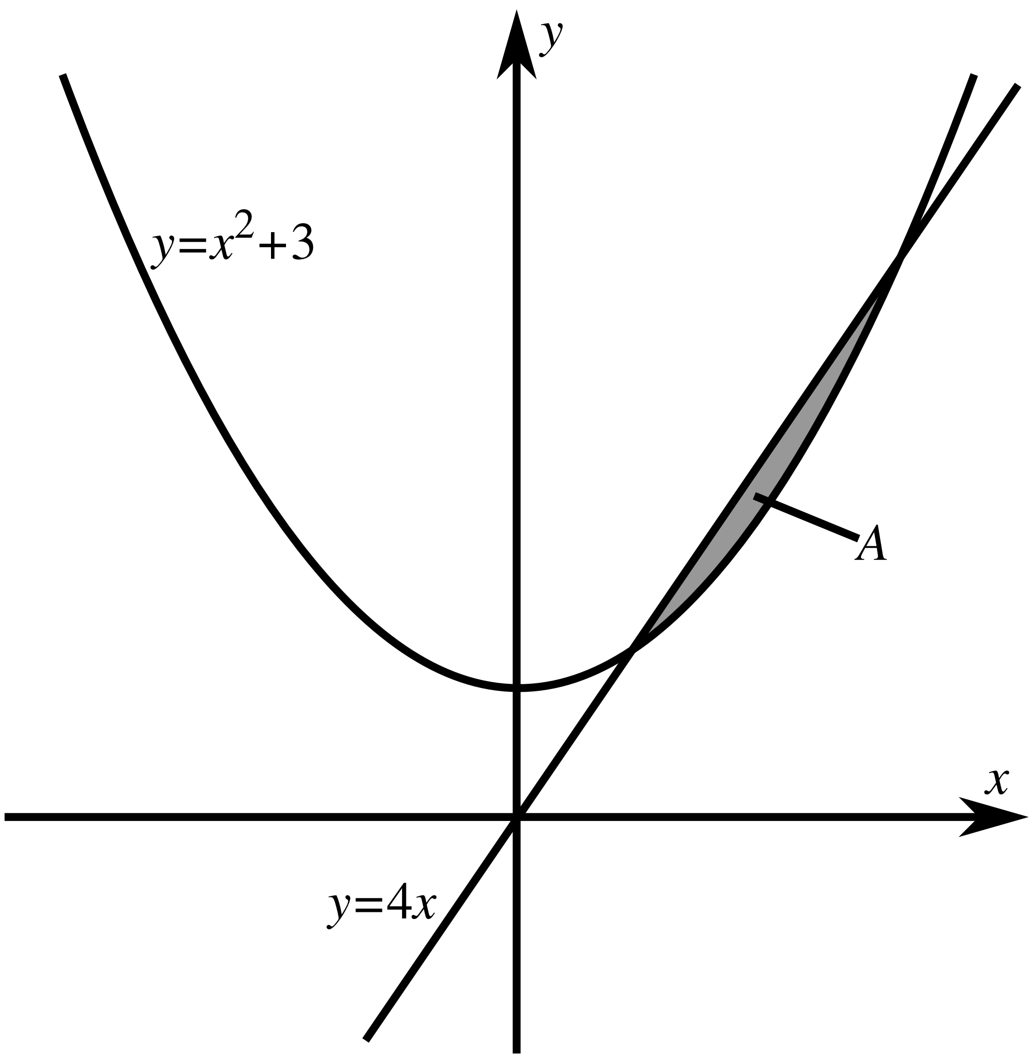

Figure 1: A graph of the functions $y = x^2 + 3$ and $y = 4x$. The shaded area A is the region between the line $y = 4x$ and the curve $y = x^2 + 3$.

Find the area of the region A. Give your answer in the form of an improper fraction.

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Functions from Differential Equations 2i

A Level

P P P

The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 + a$, where a is a constant. The curve passes through the points $(-1, 2)$ and $(2, 17)$. Find the equation of the curve.

The following symbols may be useful: `x`, `y`

Used with permission from UCLES, A level, June 2009, Paper 4722, Question 6.

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Area of Isosceles Triangle

A Level C C C Further A P P P

The isosceles triangle shown in **Figure 1** has a base of length $2b$ and perpendicular height h. The length p of the perimeter of the triangle is fixed. Find an expression in terms of p for the value of b which will maximise the area A of the triangle. Find an expression for this maximum area.

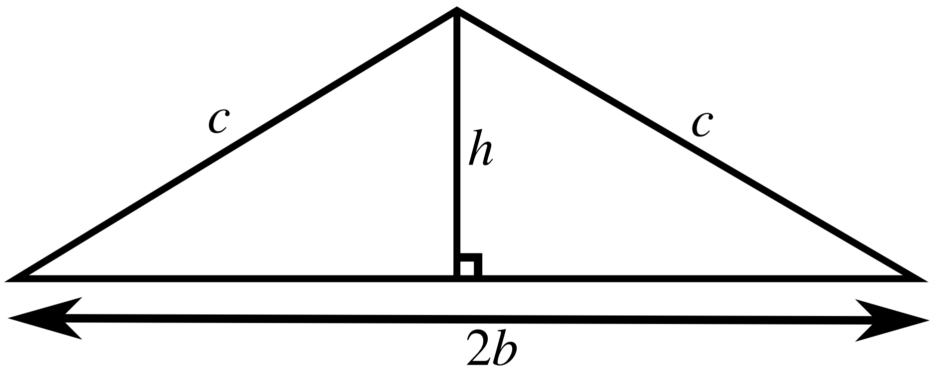

Figure 1: An isosceles triangle with a base of length $2b$, perpendicular height h and sides of length c.

Part A Area A and perimeter p

Write down the equation for the area A of the triangle in terms of b and h.

The following symbols may be useful: `A, b, h`

Find the equation for the perimeter p of the triangle in terms of b and h.

- $p = 2b + 2\sqrt{4b^2 + h^2}$
- $p = 2b + \sqrt{b^2 + h^2}$
- $p = b + 2\sqrt{b^2 + h^2}$
- $p = b + \sqrt{b^2 + h^2}$
- $p = 2b + \sqrt{4b^2 + h^2}$
- $p = 2\left(b + \sqrt{b^2 + h^2}\right)$

Using the above, obtain an equation for A in terms of p and b.

The following symbols may be useful: `A, b, p`

Part B Expressions for b and h

Using the equation for A you found in Part A, find an **expression** in terms of p for the value of b which will maximise the area A of the triangle. (Since p is fixed you may treat it as a constant.)

Hint: you may not know how to differentiate the expression for A, but note that since A is positive it will be a maximum when A^2 is a maximum.

The following symbols may be useful: p

Find, in terms of p, the expression for h corresponding to this value of b.

The following symbols may be useful: p

Part C The maximum area

Using your result from Part B, find an expression for the maximum area in terms of p.

The following symbols may be useful: p

Part D Check that the area is a maximum

Find, at the value of b deduced above, an expression in terms of p for the second derivative of A^2 with respect to b; convince yourself that the value of the second derivative indicates that the value of A^2, and hence of A, is a maximum.

The following symbols may be useful: p

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