Vectors: Diagrams and Proof 1i

A Level

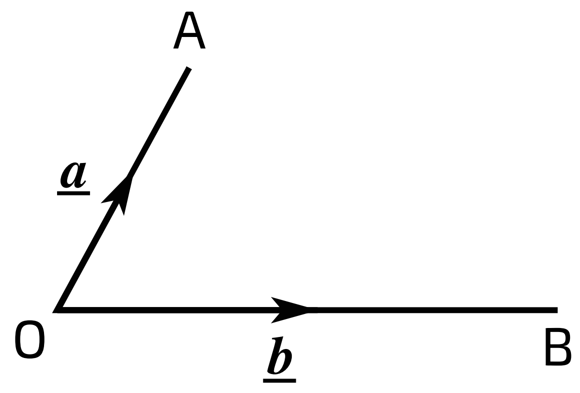

Figure 1: Three points O, A and B, and vectors joining them.

Figure 1 shows points O, A and B, with $\vec{OA} = \underline{\boldsymbol{a}}$ and $\vec{OB} = \underline{\boldsymbol{b}}$.

Part A Sketch

Make a sketch of the diagram and mark the points C and D such that $\vec{OC} = \underline{\boldsymbol{a}} + 2\underline{\boldsymbol{b}}$ and $\vec{OD} = 2\underline{\boldsymbol{a}} + \underline{\boldsymbol{b}}$.

Choose the correct sketch from the 3 options below.

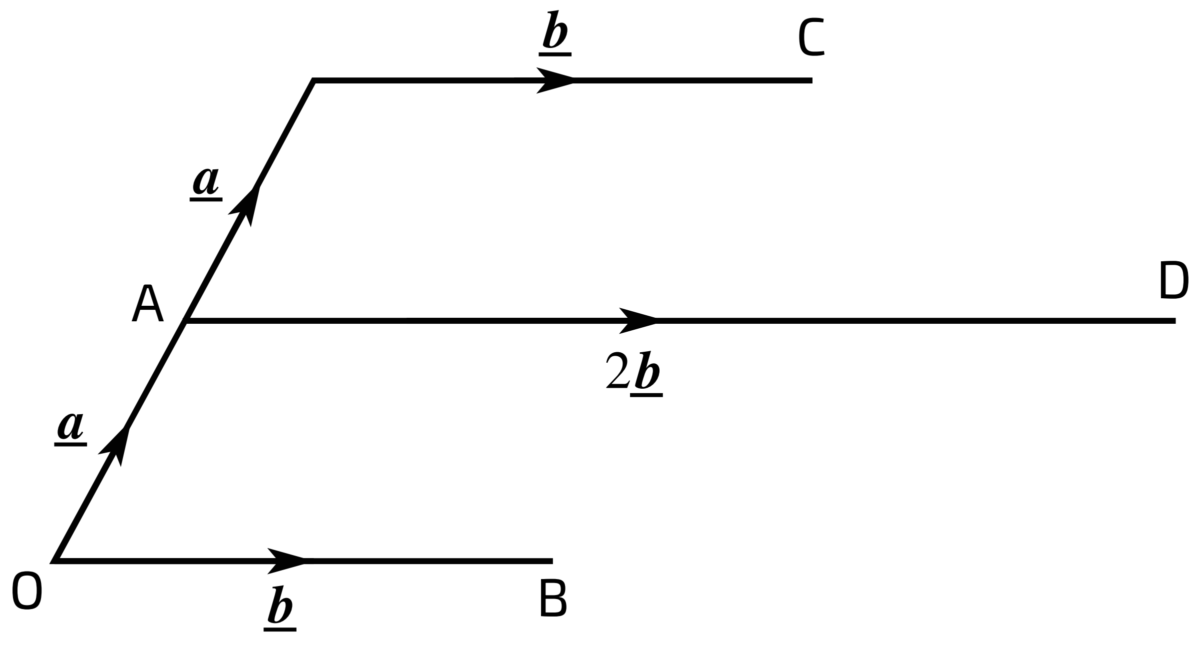

Figure 2: Option A

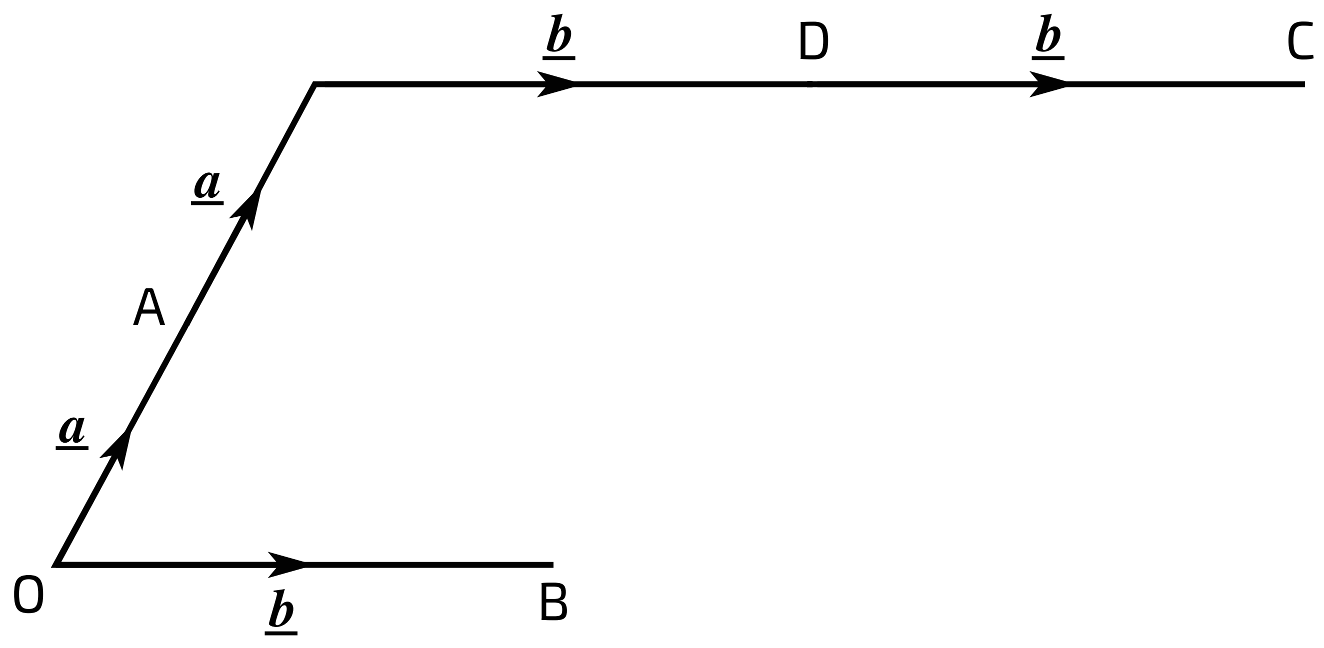

Figure 3: Option B

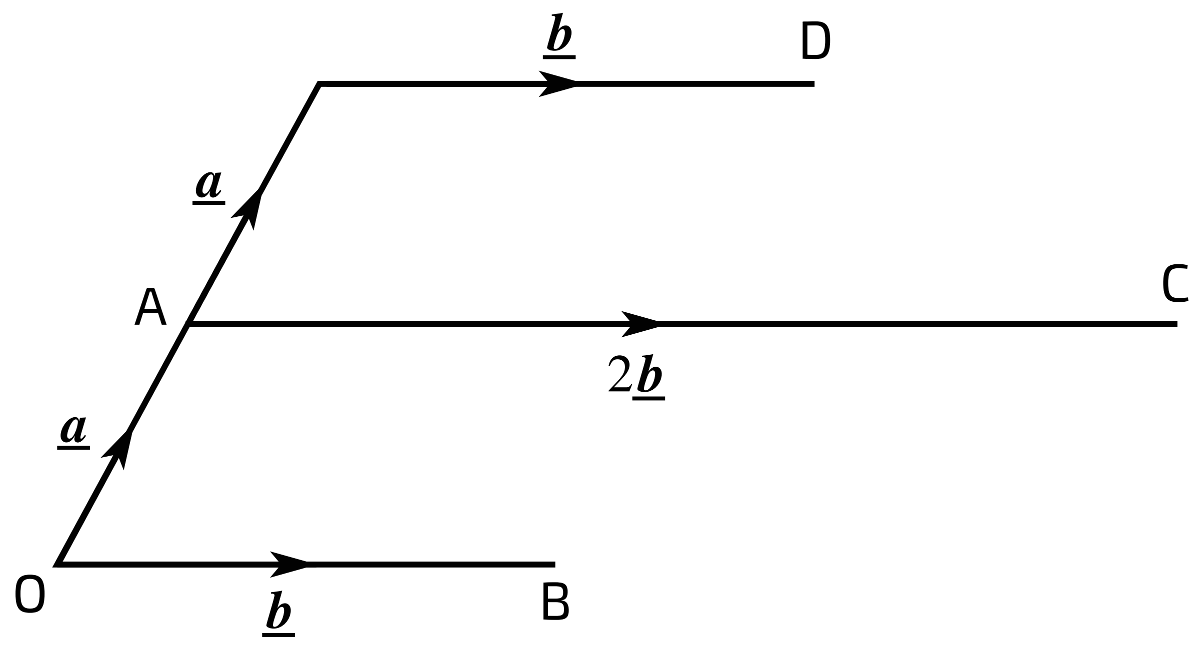

Figure 4: Option C

- Option A
- Option B

Option C

Part B Vector $\vec{DC}$

Express $\vec{DC}$ in terms of $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$, simplifying your answer.

The following symbols may be useful: `a, b`

Part C Proof

Prove that $ABCD$ is a parallelogram.

Sides $\vec{AD}$ and ______ are both equal to ______. Therefore they are parallel and of equal length.

Sides $\vec{AB}$ and ______ are both equal to ______. Therefore they are parallel and of equal length.

The quadrilateral $ABCD$ has two pairs of parallel sides of equal length. Therefore $ABCD$ is a parallelogram.

Items:

$\mathbf{a}+\mathbf{b}$ | $\vec{CD}$ | $\vec{DC}$ | $\mathbf{b}-\mathbf{a}$ | $\vec{CB}$ | $\mathbf{a}-\mathbf{b}$ | $-\mathbf{a}-\mathbf{b}$ | $\vec{BC}$

Adapted with permission from UCLES, A Level, Jan 2003

3D Vectors 2i

A Level

$ABCD$ is a quadrilateral. You are given four pieces of information:

- Relative to a fixed origin O, the position vector of A is $2\underline{\boldsymbol{i}} + 5\underline{\boldsymbol{j}} + 8\underline{\boldsymbol{k}}$.
- Relative to a fixed origin O, the position vector of B is $5\underline{\boldsymbol{i}} + 9\underline{\boldsymbol{j}} + 8\underline{\boldsymbol{k}}$.
- The vector $\vec{BC} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$.
- The vector $\vec{BD} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$.

Part A Finding AB

Find the vector $\vec{AB}$. Give your answer in the form `(x,y,z)` with the commas and without the spaces.

Part B Finding CD

Find the vector $\vec{CD}$. Give your answer in the form `(x,y,z)` with the commas and without the spaces.

Part C Finding AD

Find the vector $\vec{AD}$. Give your answer in the form `(x,y,z)` with the commas and without the spaces.

Part D Type of quadrilateral

The shape $ABCD$ lies in a plane. What type of quadrilateral is $ABCD$?

- An irregular quadrilateral
- A kite
- A square
- A rhombus
- A trapezium
- A rectangle
- A parallelogram

Created for isaacphysics.org by Jonathan Waugh

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STEM SMART Double Maths 15 - Trigonometry, Vectors & Kinematics Revision

Constant Acceleration 2i

A Level

A stone is released from rest on a bridge and falls vertically into a lake. The stone has velocity $14\,\mathrm{m\,s^{-1}}$ when it enters the lake.

Part A Distance fallen

Calculate the distance the stone falls before it enters the lake.

Part B Time on entering the lake

Calculate the time after its release when it enters the lake. Give your answer to 3 significant figures.

Part C Acceleration

The lake is $15\,\mathrm{m}$ deep and the stone has velocity $20\,\mathrm{m\,s^{-1}}$ immediately before it reaches the bed of the lake.

Given that there is no sudden change in the velocity of the stone when it enters the lake, find the acceleration of the stone while it is falling through the lake. Give your answer to 2 significant figures.

Used with permission from UCLES, A Level, June 2016, OCR M1, Question 1

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Physics. You work it out.

Home Gameboard Maths Kinematics Graphs

Kinematics Graphs

A Level

A man drives a car on a horizontal straight road. At $t = 0$ where the time t is in seconds, the car runs out of petrol. At this instant the car is running at $12\,\mathrm{m\,s^{-1}}$. The car decelerates uniformly, coming to rest when $t = 8$. The man then walks back along the road at $0.7\,\mathrm{m\,s^{-1}}$ until he reaches a petrol station a distance of $420\,\mathrm{m}$ from his car. After his arrival at the petrol station it takes him $250\,\mathrm{s}$ to obtain a can of petrol. He is then given a lift back to his car on a motorcycle. The motorcycle starts from rest and accelerates uniformly until its speed is $20\,\mathrm{m\,s^{-1}}$; it then decelerates uniformly, coming to rest at the stationary car at time $t = T$.

Part A (t, v) graph

Sketch the shape of the (t, v) graph for the man for $0 \leq t \leq T$. [Your sketch need not be drawn to scale yet numerical values should be shown].

To see an example sketch, answer the following question: What is the largest positive value of v on the graph?

Part B Finding deceleration

Find the deceleration of the car for $0 < t < 8$.

Part C Finding T

Find the value of T.

Used with permission from UCLES, A Level Maths, January 2006, OCR M1, Question 5

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Projectiles: General 3ii

A Level

A golfer hits a ball from a point O on horizontal ground with a velocity of $55\,\mathrm{m\,s^{-1}}$ at an angle of 20° above the horizontal. The ball first hits the ground at a point A and the time of flight is t seconds. Assume that there is no air resistance.

Part A Value of t

Calculate the value of t to 3 significant figures.

Part B Distance OA

Calculate the distance OA.

Part C Speed of motion

Calculate the speed of motion of the ball $2.6\,\mathrm{s}$ after the golfer hits the ball. Give your answer to 3 significant figures.

Part D Direction of motion

Calculate the direction of motion of the ball $2.60\,\mathrm{s}$ after the golfer hits the ball. Give your answer, measured below the horizontal, to 3 significant figures.

Applying Trigonometry

A Level

A rower attempts to row across a river from a place A where the banks of the river are straight and parallel. They wish to reach a point B which is directly opposite to A on the other bank (i.e. on a line perpendicular to the bank at point A).

They start to row towards B, keeping the boat aligned in a direction parallel to $\underline{\boldsymbol{j}}$, but discover that there is a current flowing in a direction $\underline{\boldsymbol{i}}$ parallel to the banks, such that their resultant travel is along a vector $\underline{\boldsymbol{v}} = \underline{\boldsymbol{i}} + 4\underline{\boldsymbol{j}}$.

Part A Speed of the rower

Find the magnitude of vector $\underline{\boldsymbol{v}}$.

The following symbols may be useful: v

Part B Angle between $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{i}}$

Find the angle between vectors $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{i}}$. Give your answer to no more than 3 sig figs.

Part C Direction of travel

If they are to arrive at B, but can adjust their rowing speed to cross to the other bank in the same time that it would take if there were no current, in what direction should they actually row? Give your answer as a vector in terms of the unit vectors i and j.

The following symbols may be useful: `i, j, v_row`

What is the angle between $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{j}}$? Give your answer to no more than 3 sig figs.

Part D A tower in the distance

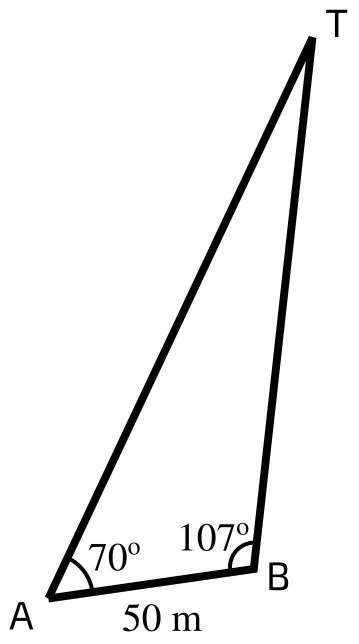

Figure 1: The tower, and points A and B.

Some walkers see a tower, T, in the distance and want to know how far away it is. They take a bearing from point A and walk for $50\,\mathrm{m}$ in a straight line before taking another bearing from point B. They find that the angle TAB is 70° and angle TBA is 107° (see **Figure 1**).

Find the distance of the tower from A. Give your answer to three significant figures.

Part E Distance from C

They continue walking in the same direction for another $100\,\mathrm{m}$ to a point C, so that AC is $150\,\mathrm{m}$. What is the distance of the tower from C? Give your answer to three significant figures.

Part F Shortest distance from A to C

Find the shortest distance of the walkers from the tower as they walk from A to C. Give your answer to three significant figures.

Part G Area swept out

D is the point on AC such that TD is the shortest distance of the walkers from the tower.

Find the area of ground represented by the triangle ATD. Give your answer in km^2 and to 3 significant figures.

Adapted with permission from SAW 2017 and UCLES, A Level, January 2009, Paper 4722, Question 5.

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Trigonometry: Double Angles 4i

A Level

Part A Proof

By first expanding $\cos(2\theta + \theta)$, rewrite $\cos 3\theta$ in terms of $\cos\theta$.

The following symbols may be useful: `cos()`, `theta`

Part B $\cos 6\theta$

Hence write $\cos 6\theta$ in terms of $\cos\theta$.

The following symbols may be useful: `cos()`, `theta`

Part C The equation $1 + \cos 6\theta = 18\cos^2\theta$

Give the smallest positive solution to the equation

$$1 + \cos 6\theta = 18\cos^2\theta$$

in degrees.

Used with permission from UCLES A-level Maths papers, 2003-2017.

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Trigonometry: Combined Angles 4ii

A Level
C C C

Part A Combined Angles

Express $8\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.

Give the value of R.

The following symbols may be useful: R

Give the value of α to three significant figures.

Part B Solve

Hence solve, for $0^\circ < \theta < 360^\circ$, the equation $8\sin\theta - 6\cos\theta = 9$, giving your answers in degrees to three significant figures.

Give the smallest solution.

Give the largest solution.

Part C Maximum Value

Hence find the greatest possible value of

$$32\sin x - 24\cos x - (16\sin y - 12\cos y)$$

as the angles x and y vary.