Partial Fractions 2ii

A Level

P P P

Part A Partial Fractions

Express $\dfrac{x-1}{x(x+1)}$ in partial fractions.

The following symbols may be useful: x

Part B Integral

Hence find the exact value of $\displaystyle\int_1^2 \frac{x-1}{x(x+1)}\mathrm{d}x$.

Partial Fractions 1i

A Level

P P P

Part A Partial Fractions

Express $\frac{2+x^2}{(1+2x)(1-x)^2}$ in the form $\frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$.

The following symbols may be useful: x

Part B Integration

Hence find $\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2}\mathrm{d}x$ in exact form.

The following symbols may be useful: , `ln()`, `log()`

Gameboard:

STEM SMART Single Maths 35 - Integrating Rational Functions

Partial Fractions 4ii

A Level

Part A Partial Fractions

Express $\frac{7-2x}{(x-2)^2}$ in the form $\frac{A}{x-2} + \frac{B}{(x-2)^2}$, where A, and B are constants.

The following symbols may be useful: x

Part B Integral

Hence find the exact value of $\int_4^5 \frac{7-2x}{(x-2)^2}\mathrm{d}x$.

Gameboard:

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Integration by Substitution 4

A Level

P P P

Pre-Uni Maths for Science K3.5

Part A Integrate $\frac{1}{b(x+a)}$

Find $\int_0^a \frac{1}{b(x+a)}\mathrm{d}x$, where a and b are constants.

The following symbols may be useful: `a, b, k, x`

Part B Integrate $\frac{x}{1+x^2}$

Find $\int_0^1 \frac{x}{1+x^2}\mathrm{d}x$.

The following symbols may be useful: `k, x`

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Integration by Substitution 5

A Level

P P P

Pre-Uni Maths for Science K3.6

Part A Integrate $\frac{x^3}{a^5+ax^4}$

Find $\int_a^{2a} \frac{x^3}{a^5+ax^4}\mathrm{d}x$.

The following symbols may be useful: a

Part B Integrate $\tan\beta$

By writing $\tan\beta = \frac{\sin\beta}{\cos\beta}$, find $\int_0^{\frac{\pi}{4}} \tan\beta\,\mathrm{d}\beta$.

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Algebraic Division 1ii

A Level

Part A Quotient and Remainder

Find the quotient when $3x^3 - x^2 + 10x - 3$ is divided by $x^2 + 3$.

The following symbols may be useful: x

Give the remainder.

The following symbols may be useful: x

Part B Integral

Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3} \mathrm{d}x.$$

Gameboard:

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Integration by Substitution 1i

A Level

Let $I = \int \frac{1}{x\left(1+\sqrt{x}\right)^2}\mathrm{d}x$.

Part A Substitution

Using the substitution $u = \sqrt{x}$, transform I into the integral $\int f(u)\mathrm{d}u$. Give the function $f(u)$ in terms of u.

The following symbols may be useful: `f, u`

Part B Partial Fractions

Express $\frac{2}{u(1+u)^2}$ in the form $\frac{A}{u} + \frac{B}{1+u} + \frac{C}{(1+u)^2}$.

The following symbols may be useful: `u`

Part C Integrate

Hence find I.

The following symbols may be useful: `I, c, x`

Integration by Substitution 1ii

A Level

Part A Partial Fractions

Given that $\frac{2t}{(t+1)^2}$ can be expressed in the form $\frac{A}{t+1} + \frac{B}{(t+1)^2}$, find the values of the constants A and B.

State the value of A.

The following symbols may be useful: A

State the value of B.

The following symbols may be useful: B

Part B Show

Show that the substitution $t = \sqrt{2x-1}$ transforms $\int \frac{1}{x+\sqrt{2x-1}}\mathrm{d}x$ to $\int \frac{2t}{(t+1)^2}\mathrm{d}t$.

More practice questions?

Part C Exact Value

Hence find the exact value of $\displaystyle\int_1^5 \frac{1}{x+\sqrt{2x-1}}\mathrm{d}x$.

Gameboard:

STEM SMART Single Maths 35 - Integrating Rational Functions

Integration With Partial Fractions 2

A Level Further A

Pre-Uni Maths for Science A4.9

Write the function $\dfrac{2z^2 - z - 3}{(z+2)(z^2 - 2z - 1)}$ in the form $\dfrac{A}{z+2} + \dfrac{B + Cz}{z^2 - 2z - 1}$. Hence find $\displaystyle\int_1^2 \frac{2z^2 - z - 3}{(z+2)(z^2 - 2z - 1)}\,\mathrm{d}z$.

Part A Find A

Find the constant A

Part B Find B

Find the constant B.

Part C Find C

Find the constant C.

Part D Integrate

Hence find $\displaystyle\int_1^2 \frac{2z^2 - z - 3}{(z+2)(z^2 - 2z - 1)}\,\mathrm{d}z$.

The following symbols may be useful: `cos()`, `cosec()`, `cosech()`, `cosh()`, `cot()`, `coth()`, `ln()`, `log()`, `sec()`, `sech()`, `sin()`, `sinh()`, `tan()`, `tanh()`, `z`

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