Lab 1 Report

Introduction

The Saturn V rocket is a rocket being developed by NASA for use in the Apollo program. In order for the future of the Apollo missions and other NASA programs to be successful, the gravitational phenomena affecting the rocket, as well as its capabilities need to be well understood. This report provides a brief overview of the gravitational potential of the Earth and Moon, the gravitational forces on the Saturn V throughout its mission, as well as the altitude and flight time of the rocket itself. Gravitational potential and forces are calculated at various distances from the Earth, and depicted on plots. The height and flight time of the Saturn V have been theoretically calculated based on the physical properties of the rocket, and are compared to the experimentally determined results from recent tests. This analysis allows for a better understanding of the challenges the Saturn V will face, its current capabilities, and the areas in which further research is necessary.

The Gravitational Potential of the Earth Moon System

Gravitational potential (represented by the Φ symbol), is a measure of the potential energy per unit mass an object has due to the gravity of another body, such as the Earth or moon. Potential depends on the distance of the object to the body, which can be thought of as the amount of work needed to bring a mass from very far away, to that distance. Potential is given as:

$$\Phi = -\frac{GM}{r}$$

In this equation, M is the mass of the body, G is the gravitational constant with numerical value $6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$, and r is the distance from an object to the body. When considering the potential of a system with multiple bodies, the total potential is the sum of the potential from each individual body. Thus, for the Earth Moon system, the potential of an object a distance r_1 from the Earth and r_2 from the moon is given as:

$$\Phi = -G\left(\frac{M_{earth}}{r_1} + \frac{M_{moon}}{r_2}\right)$$

Figure 1 shows the potential of the Earth-Moon system over distances -1.5 to 1.5 times the distance between the Earth and moon. This chart was created in Python by calculating and plotting the potential from the above equation for a large number of positions near the Earth. Both the Earth and moon were approximated as point particles, with a potential of 0 for all positions inside of them. The color scale showing potential is logarithmic, to better show how potential decreases with distance. Using a logarithmic scale shows a less extreme distinction between the very high potential close to the Earth, and the lower potential at much greater distances. Note how the potential of the Earth is significantly greater than that of the moon, due to the Earth's much larger mass.

The Gravitational Force of the Earth-Moon System

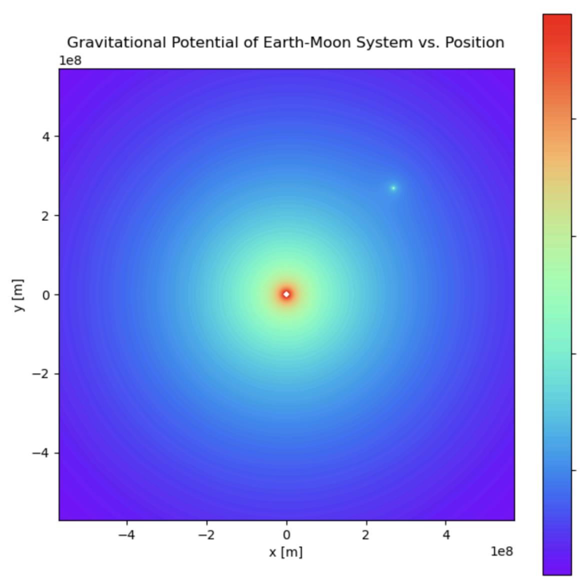
At any point in its flight, the Saturn V rocket will experience forces exerted by both the Earth, and the moon. The forces are vector quantities, meaning they have both a magnitude and direction. The gravitational force between two objects of masses m_1 and m_2, and separation of magnitude r is given as:

$$F = -\frac{Gm_1m_2}{r^2}\hat{r}$$

In this equation, $\hat{r}$ is a unit vector, which has magnitude 1 in the direction of the force. In a simple case like this, the force always goes directly from one object to another. During the Saturn V's flight, it will experience forces from both the Earth, and from the moon, with the total magnitude and direction being the sum of the two forces in each direction:

$$F = -Gm_{saturn\ V}\left(\frac{M_{earth}}{r_1^2}\hat{r_1} + \frac{M_{moon}}{r_2^2}\hat{r_2}\right)$$

In this equation r_1 and r_2 (and the corresponding unit vectors) are the displacements from the rocket to the Earth and moon respectively. Figure 2 shows a plot of the force on the Saturn V over the same range of positions. In this plot, the color of the line at any given position gives the magnitude of the force, and the corresponding arrow gives the direction. This plot was created in Python by evaluating the magnitude and direction of the force on the rocket over a large number of positions. The vertical and horizontal direction displacements between both the Earth and the moon were considered separately, and corresponding forces were calculated for each body. These components were then added together, to give a net force due to both objects. The magnitude and direction of this force was calculated from the two components. A logarithmic scale is also used to better show the force at greater distances. Note that net force drops off quickly with distance, and usually points towards the Earth except at positions near to the moon, where the net force starts to veer towards the moon.



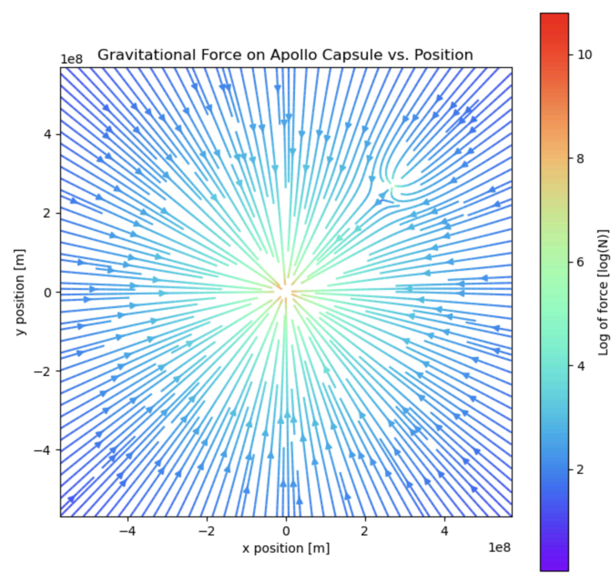


Figure 1 (left): Gravitational potential of the Earth-Moon System.
Figure 2 (right): Gravitational force on Saturn V Rocket.

Projected Performance of the Saturn V Stage 1

The Saturn V rocket is planned to have an initial, "wet", mass of $m_0 = 2.8 \times 10^6\ kg$ and a final, "dry" mass of $m_f = 7.5 \times 10^5\ kg$. The rocket is also expected to consume fuel at a constant rate of $\dot{m} = 1.3 \times 10^4\ kg\ s^{-1}$ throughout its flight.This information allows for calculation of the flight time T, given by:

$$T = \frac{m_0 - m_f}{\dot{m}}$$

Computing this value with the known data in Python gives a flight time of $T = 157.7\ s$. The change in velocity of the rocket over a time interval can be found using the equation:

$$\Delta V = v_e \ln\left(\frac{m_0}{m_0 - \dot{m}t}\right) - gt$$

In this equation v_e is the fuel velocity ($v_e = 2.4 \times 10^3\ m\ s^{-1}$ for the Saturn V). The final height of the rocket can be found by integrating this equation over the time interval in which the rocket is in the air:

$$h = \int_0^T \Delta V dt$$

This integral was computed in Python using the quad function in the scipy Python package. This function allows for numerical integration of functions with a high level of accuracy. This calculation gave a resulting final height of $h = 74.094\ km$. The Saturn V stage 1 is thus expected to have a total burn time $157.7\ s$ and achieve a final height of $74.094\ km$.

Discussion and Future Work

There are some approximations made in this work that leave room for additional research and analysis. Both the Earth and the moon were approximated as point particles for all potential and force calculations, which can limit their accuracy in some cases. Additional research should be done to create a more complex model of the Earth and moon's gravitational effects.

Additionally, the most recent experimental data from the prototypes gives a burn time of $160\ s$ and a height of $70\ km$. While these are close to the values calculated in this report, there are discrepancies. This likely due to the fact that fuel was approximated as being consumed at a constant rate, and the fact that no outside conditions were taken into account (such as drag or weather). The lower flight time and greater altitude reflected in this report may be due to a greater consumption of fuel at certain points in the rocket's flight, which lead to a higher altitude reached slightly faster. Conditions like wind and the shape of the rocket may also have effects on flight behavior that are not accounted for in this analysis. While the calculations in this report do closely reflect the experimentally determined data, further analysis could give more detailed and accurate descriptions of the rocket's flight.

The data in this report provides an overview of the gravitational effects of the Earth and moon on the Saturn V, as well as predictions for the rocket's performance. Further research is needed to describe these phenomena with a higher degree of accuracy.