Modeling Languages for Optimization

DS 775

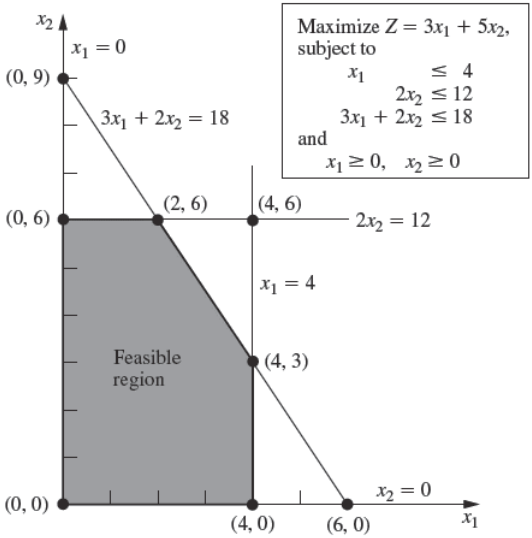
Topics

- Optimization Programming Language (OPL)
  - overview of IDE
  - programming basics
- Numerical Methods for Linear Programming

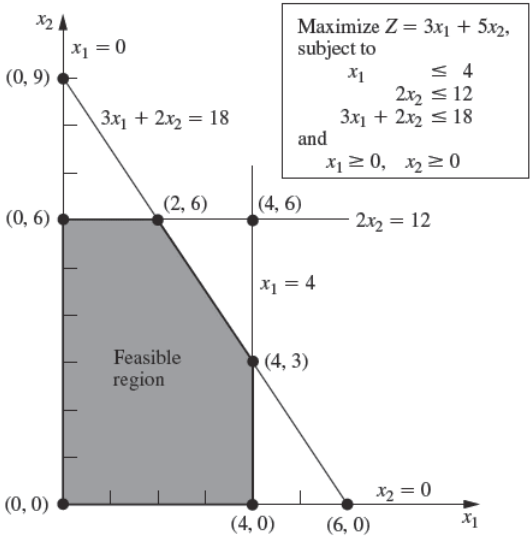
Two Numerical Methods

- Simplex Method (Dantzig 1947)
  - widely considered one of top 10 numerical algorithms from the 20th century (link below)
- Interior Point Method (Karmakar 1984)

Wyndor Problem - Simplex Method



Wyndor Problem - Interior Point Method



Simplex - Pros and Cons

- Pros
  - n variables, usually works with $O(n)$ operations
  - exploits geometry to visit vertices
  - good for small problems
- Cons
  - worst case $O(2^n)$ operations
  - gets expensive for large problems

Interior Point - Pros and Cons

- Pros
  - modern methods require $O(n^3 L)$ operations
  - better for large, sparse problems
- Cons
  - harder to understand
  - for small problems Simplex is usually faster

Optimization Programming Language (OPL)

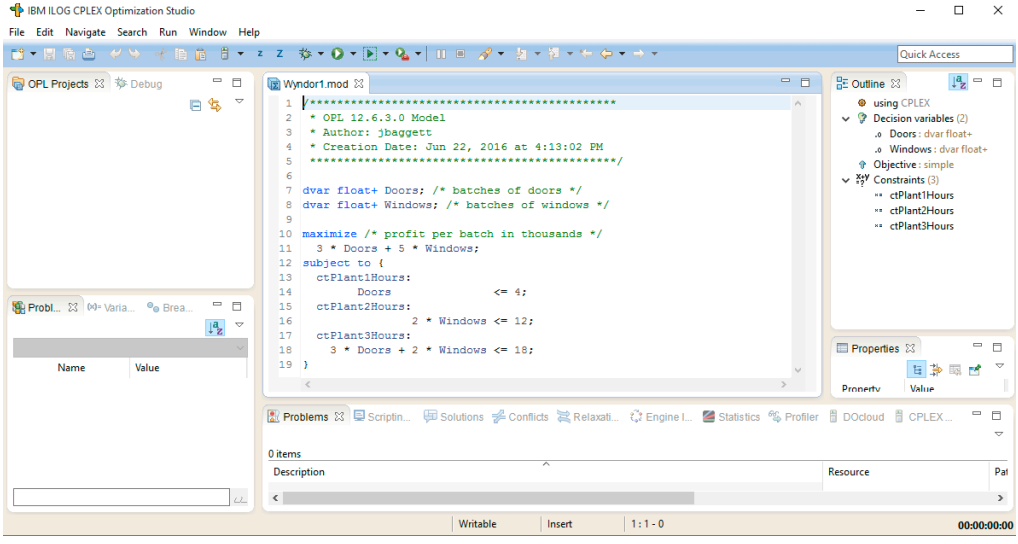
- easier model specification
- separate model from data
- relies on IBM CPLEX solvers for numerics
- interfaces to R, Python, C, and others
- similar to AMPL, LINGO, and others
- also can use with constraint programming

A simple OPL model

```
dvar float+ Doors; /* batches of doors */
dvar float+ Windows; /* batches of windows */

maximize /* profit per batch in thousands */
  3 * Doors + 5 * Windows;
subject to {
  ctPlant1Hours:
        Doors                <= 4;
  ctPlant2Hours:
               2 * Windows <= 12;
  ctPlant3Hours:
    3 * Doors + 2 * Windows <= 18;
}
```

Introduction to Optimization Studio



Separating the model and data

Indexing using names

Infeasible constraints

Model and Data - Separate Files

A Transportation Problem

- Shipment of products pairs of cities
  - transport has per product cost
  - transport cannot exceed a given limit
  - transport amount subject to supply and demand constraints
  - Assumes total supply $=$ total demand
  - Minimize total cost subject
- Usually constraint equations are *sparse*
  - each equation involves only a few of the many decision variables

A Transportation Problem - a picture

Install Example into Workspace

First Model

Sparse Model with Tuples