# A Locally Conformal Finite-Difference Time-Domain (FDTD) Algorithm for Modeling Three-Dimensional Perfectly Conducting Objects

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***Abstract*— A novel conformal finite-difference time-domain (CFDTD) technique for locally distorted contours that accurately model curved metallic objects is presented in this paper. This approach is easy to implement and is numerically stable. Several examples are presented to demonstrate that the new method yields results that are far more accurate than those generated by the conventional staircasing approach. Example geometries include cylindrical and spherical cavities, and a circular microstrip patch antenna. Accuracy of the scheme is demonstrated by comparing the results derived from analytical and Method of Moments (MoM) techniques.**

***Index Terms*— Finite-difference time-domain (FDTD), locally conformal grid, Maxwell solver.**

## I. Introduction

The staircasing approach to analyzing objects with curved metallic surfaces using the Yee algorithm [1] not only introduces errors [2] due to inaccurate approximation of the geometry, but can also generate spurious solutions. Several techniques have been proposed in the literature for overcoming these difficulties. These include the globally curvilinear grid technique [3], which requires a special structured type of mesh that may be difficult to generate when modeling an arbitrary shaped object, and the contour path finite-difference time-domain (CPFDTD) scheme [4] that deforms the grid only locally to accommodate the curvature of the surface. Although simple and efficient, CPFDTD frequently leads to instabilities because of noncausal and nonreciprocal *nearest neighbor* approximation [5], [6]. Various modifications in the CPFDTD algorithm have been proposed in the literature to obviate this instability problem [7], [8]; however, the bookkeeping, mesh generation, and programming are considerably more complex in these schemes than they are in the conventional FDTD algorithm.

In a recent communication, the present authors have reported a simple yet accurate technique for the FDTD analysis of curved two-dimensional (2-D) perfectly conducting bodies [9] using a locally conformal grid. In this paper, we extend this technique to three-dimensional cases and illustrate its application by investigating a number of test geometries.

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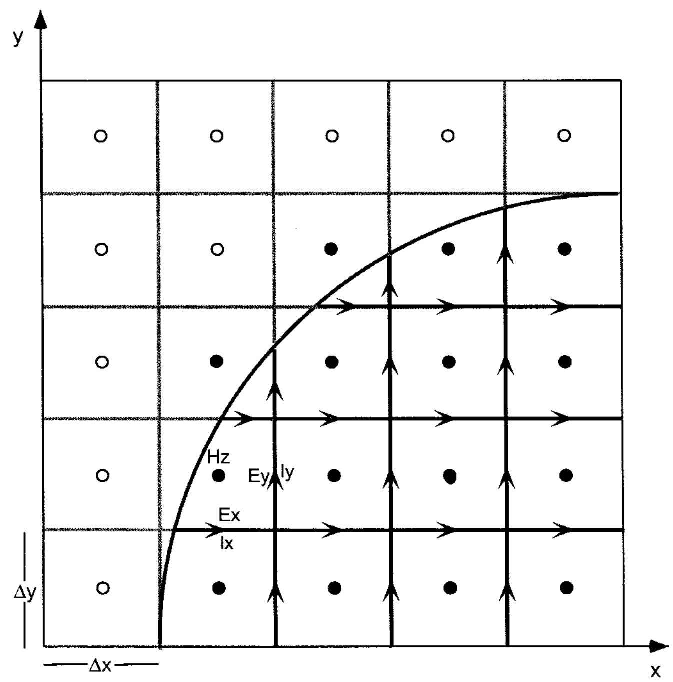


Fig. 1. Cross-sectional view of the quarter of the spherical cavity.

## II. Conformal Finite-Difference Time-Domain (CFDTD) Algorithm

Consider the cross section of a spherical cavity resonator shown in Fig. 1, where, for convenience, we display only one of its quadrants. The mesh for this structure is depicted by the solid dark lines in the figure, which also shows an overlay of a uniform Cartesian mesh. The undistorted cells are treated in the usual way in the CFDTD algorithm, for both the $E$- and $H$-fields. For the distorted cells, no special treatment is needed for the electric field, which is assumed to be constant along the edge of a cell that resides within the cavity, and zero if it is located on the metallic surface or within the conductor. However, the treatment of the $H$-field is somewhat different. It is assumed to be located at the center of the corresponding *undistorted* cell (the Cartesian cell obtained by removing the partial filling) for the purpose of numerical calculations, irrespective of whether the location of the center is inside or outside the computational domain. It is also assumed to be constant over the region of the cell that is inside the cavity. Thus, the updating procedure of the $H$-field is changed little from the conventional FDTD scheme.

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We now offer a possible justification for the fact that we always locate the $H$-field at the center of the cell, regardless of whether it is partially filled or not. We begin with the observation that, conventionally, the updating of the magnetic field in a regular cell is carried out by summing up the contributions of the electric fields tangential to the boundaries of the cell. Next, we consider the case where the cell is partially filled with a lossy material. It may be verified that the updating of the $H$-field as well as its location remains unchanged with the introduction of this modification, and this is the basis of our argument for following the procedure for time-stepping of the $H$-field that we have presented above. However, it is also obvious that the electric field in the region interior to the conducting medium becomes negligible in the limit of a perfectly conducting filling, and, hence, we need only retain the contributions of the $E$-field on the portions of the cell contour that are outside of the conducting region.

The above procedure enable us to employ the regular FDTD equations to update the magnetic field by using the electric field values along the distorted contour that are appropriately weighted with the lengths of the contours. The updating equation for the $H$-field along the $z$-direction reads

$$\begin{aligned} H_z^{n+1/2}(i,j,k) &= H_z^{n-1/2}(i,j,k) + \frac{\Delta t}{\mu * \mathrm{Area}(i,j,k)} \\ &\quad \times \{E_x^n(i,j,k) * l_x(i,j,k) - E_x^n(i,j-1,k) \\ &\quad * l_x(i,j-1,k) - E_y^n(i,j,k) * l_y(i,j,k) \\ &\quad + E_y^n(i-1,j,k) * l_y(i-1,j,k)\}. \qquad (1) \end{aligned}$$

where $l_x$ and $l_y$ are the cell lengths along the $x$ and $y$ directions, respectively.

Once the $H$-fields have been computed, the $E$-fields are updated in the conventional manner by using the adjacent $H$-field values. For example, the updated $E$-field along the $x$-direction is derived from

$$\begin{aligned} &E_x^{n+1}(i,j,k) \\ &\quad = E_x^n(i,j,k) \\ &\qquad + \frac{\Delta t}{\varepsilon \times \Delta y}\{H_z^{n+1/2}(i,j+1,k) - H_z^{n+1/2}(i,j,k)\} \\ &\qquad - \frac{\Delta t}{\varepsilon \times \Delta z}\{H_y^{n+1/2}(i+1,j,k) - H_y^{n+1/2}(i,j,k)\} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2) \end{aligned}$$

where, $\Delta y$ and $\Delta z$ are the step size along $y$ and $z$ directions.

Since all the field values are updated without *borrowing* from any of the adjacent cells, as is done in CPFDTD, the associated stability problems are no longer present in this scheme. However, the stability of this algorithm is still governed by the nature of the mesh and the choice of the time step. Numerical experiments have shown that a time step of 50 to 70% of the Courant limit associated with the *undistorted* cell is adequate to ensure the stability of the algorithm, provided that the following conditions are met.

1) The area of the distorted cell (partially filled) is greater than 1.5% (for 50% of Courant limit) and 2.5% (for 70% of limit) of the area of the undistorted cell area.

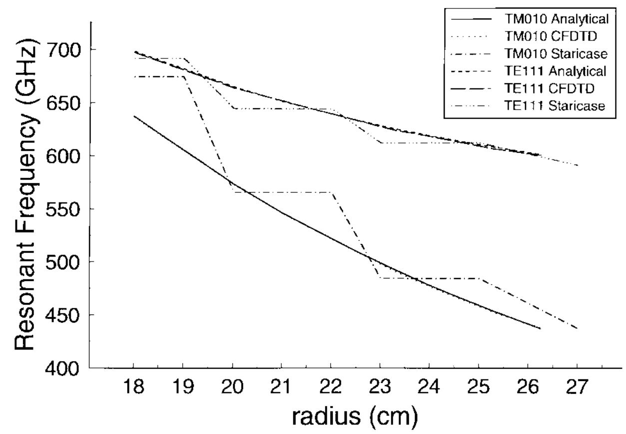


Fig. 2. Resonant frequencies of cylindrical cavities calculated using various methods.

2) The ratio between the maximum length of the side of a cell and its area is less than 15 (for 50% of Courant limit) and 10 (for 70% of limit).

Finally, if desired, the CPFDTD scheme can be used to supplement the CFDTD algorithm to deal with the cells in which the above conditions are violated.

## III. Numerical Results

To demonstrate both the accuracy and stability of the proposed approach, we have calculated the resonant frequencies of the dominant TE and TM modes of cylindrical cavities and the two lowest order resonant frequencies of spherical cavities. The usefulness of the approach has been further demonstrated by computing the resonant frequency of a circular microstrip patch antenna. All of these results were obtained by using 8192 time steps. Additional runs were carried out with up to 16 000 time steps and no instability was ever observed.

### A. Cylindrical Cavity

The proposed algorithm was employed to calculate the resonant frequencies of the dominant TE and TM modes for circularly cylindrical cavities, with heights of 30 cm and radii ranging from 18 to 26 cm. In each case, the spatial discretization was chosen to be 3 cm. The results are shown in Fig. 2 and are compared with those obtained by using the staircase approximation, as well as with the analytical results. In the above figure, the legend CFDTD refers to the present conformal FDTD method. The worst case and the average error due to staircasing is 8.45% and 3.76%, respectively, while the corresponding figure for the present scheme is only 0.3% and 0.14%.

### B. Spherical Cavity

To demonstrate the full three-dimensional capability of the algorithm, two lowest order resonant frequencies of the spherical cavity have been computed. Calculations were carried out for spheres with radii ranging from 14 to 24 cm by using a spatial discretization of 4 cm in size. It is evident that this choice of the discretization leads to a relatively coarse

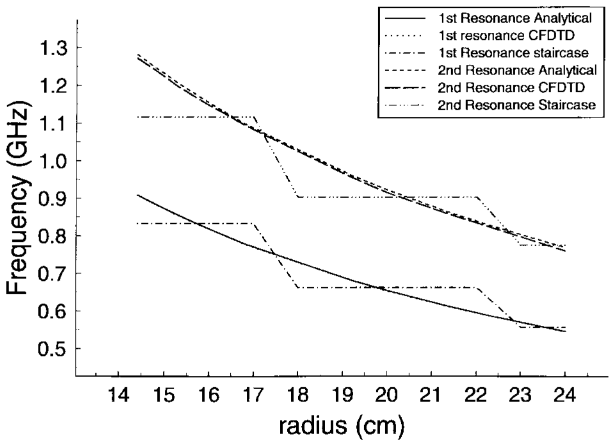


Fig. 3. Resonant frequencies of spherical cavities calculated using various methods.

mesh and, hence, provides a stringent test for the algorithm. Fig. 3 shows the comparison between the resonant frequencies predicted by the present technique, the analytical results, and those derived by using the conventional staircasing approach. It can be seen that accurate results are obtained with the CFDTD method even with the relatively course mesh that has been employed. The worst and average staircasing errors are 12.96% and 4.8%, respectively, while the same for the present scheme are only 1.14% and 0.69%, respectively.

### *C. Circular Microstrip Patch*

The last example considered was a probe-fed circular microstrip patch antenna with the following parameters: dielectric constant 4.0, thickness 4.0 mm, and radius 10 mm. The cell size along the thickness of the substrate was 0.8 mm while it was 2.5 mm along the other two directions. The fundamental resonant frequency of the patch obtained by using the present method, the staircased FDTD and the MoM were, 3.75, 3.68, and 3.80 GHz, respectively. The CFDTD result is 1.31% lower than the one predicted by MoM, whereas the one computed by using the staircased FDTD algorithm is 3.16% lower. The difference between the CFDTD and MoM results for the resonant frequency can be attributed to the fact that the radius of the feed probe modeled with the MoM is finite, whereas it is assumed to be infinitesimal in the FDTD simulation.

## IV. Conclusion

In this paper, we have shown that a simple accurate stable and locally conformal FDTD algorithm, previously developed for the 2-D case, can be successfully extended to three-dimensional geometries. The scheme has been found to be numerically stable, because the problem associated with borrowing from adjacent cell, as for instance in CPFDTD, is not present here. Furthermore, the algorithm generates no spurious solutions and yields results that are considerably more accurate than those obtained by using the staircasing approach. There are several attractive features of this algorithm: first, the bookkeeping involved is relatively simple compared to those required in other explicit nonorthogonal FDTD schemes. Second, an automatic mesh generation procedure can be utilized in conjunction with this algorithm because the specifications for the mesh are not very demanding. Finally, the allowable time step for stable calculations can be much higher ($\sim$ order of magnitude) than dictated by the smallest-size distorted cell, and this can result in a considerable saving of the computation time.

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