Ratings and Reputation: Extended Abstract

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Abstract

This paper studies an information designer with reputation concerns. Each period a firm seeks to raise debt to finance a project of uncertain quality. The firm may higher a credit rating agency. The credit rating agency is able to perform an investigation an obtain a metric of product quality. The credit rating designs a rating system contingent on their observed metric to maximize profit. Investors observe the credit rating before making investment decisions. The correlation between the rating agency's metric and project quality is uncertain and beliefs about this correlation play the role of the rating agency's reputation. Investors attempt to learn about project quality to make investment decisions, but also learn the metric's correlation so they know how much to trust future ratings. The rating agency faces a trade-off in designing the rating protocol. A rating protocol that is more informative about project quality is also more revealing about the metric's correlation. In the best pooling equilibrium, reputation concerns have a different affect on different agencies. The rating agencies with a low reputation issue more revealing ratings than they would in a static game to try to build their reputation. The rating agencies with a high reputation reveal even less information than they would in a static game to try to protect their reputation. In some cases, there can be separating equilibria. These equilibria all have the same form.

> The high quality (high correlation) rating agency must give very revealing ratings until they are able to correctly predict quality a number of times. Then they can conceal information by playing the static optimal for a rating agency known to be the high type for the remainder of periods.

Sender (she) has access to some random variable Y and can design any informative experiments to reveal information about Y. It is as a standard Bayesian Persuasion setup. Receiver (he) needs to take an action and doesn't care about Y per se. What is important to receiver is some other variable Z that is potentially correlated with Y, but the correlation is uncertain. Each period, Y and Z are drawn, a message is sent based on Y, receiver takes and action, and then payoffs are realized (based on Z). Y and Z are drawn from some joint distribution $H_\theta(y,z)$ independently each period, where θ is the unknown parameter of the distribution capturing the correlation.

Say that Z takes one of two values, $z \in \{0,1\}$. Also, θ takes one of two values, $\theta \in \{H, L\}$. μ will denote the probability that $z = 1$, and x will denote the probability that $\theta = H$. To keep the interpretation of θ, assume that the marginal distributions do not change when θ changes

$$H_H(z) = H_L(z) \ \ \forall z; \qquad\qquad H_H(y) = H_L(y) \ \ \forall y \tag{1}$$

but the conditional distributions do change.

$$\mu_H(y) = H_H(z|y) \neq H_L(z|y) = \mu_L(y) = \mu_0 \tag{2}$$

For simplicity, I've also added the assumption here that if $\theta = L$, then Y is completely uninformative of Z, but everything works without this. Call $F(\mu)$ the distribution of $\mu_H(y)$. It is the distribution of posteriors that would be realized if the high type fully revealed Y.

For any given message, if μ_H is the belief that would have been induced if θ was known to be high, then the belief actually induced is

$$\mu = x\mu_H + (1-x)\mu_0. \tag{3}$$

So, we can go back and forth between problems of picking μ or picking μ_H.

Call $w(\mu)$ the flow payoff Sender gets when Receiver has beliefs μ and takes their optimal action. The objective for Sender becomes

$$V(x) = \max_{\Delta(\mu)\in\Delta[0,1]} \quad \mathbb{E}_\mu \left[w(\mu) + \delta \mathbb{E}_z \left[V(x') \right] \right] \tag{4}$$

subject to two constraints. The first constraint is the normal martingale condition on beliefs.

$$\mathbb{E}[\mu] = \mu_0 \tag{5}$$

The second condition is that Sender cannot reveal more information than she has. This means that the distribution of posteriors they induce needs to be less well informed than the distribution caused by fully revealing Y. This is a second order stochastic dominance condition.

$$\Delta(\mu) \preceq_{SOSD} F\left(\frac{\mu}{x} - \frac{1-x}{x}\mu_0\right)\frac{1}{x} \tag{6}$$

If for the high θ type Y perfectly reveals Z, then this simply becomes a condition that the support of $\Delta(\mu)$ lives in the interval $[(1-x)\mu_0, x + (1-x)\mu_0]$. Generally, it is a strictly stronger restriction than that.

Proposition 1. *If $w(\mu)$ is convex, the optimal policy is to reveal all information.*

This isn't quite as trivial as it may seem initially. This is because the chosen posterior μ affects the evolution of the reputation x.

First rewrite the updated reputation x' in terms of the chosen posterior μ.

$$x' = \frac{x\mu_H}{x\mu_H + (1-x)\mu_0} = \frac{\mu - (1-x)\mu_0}{\mu} \tag{7}$$

Now the objective becomes

$$V(x) = \max_{\Delta(\mu)\in\Delta[0,1]} \mathbb{E}\left[w(\mu) + \delta\mu V\left(\frac{\mu-(1-x)\mu_0}{\mu}\right) + \delta(1-\mu)V\left(\frac{x-\mu+(1-x)\mu_0}{1-\mu}\right)\right] \tag{8}$$

subject to Bayes plausibility and the second-order stochastic dominance condition.

Then a contraction mapping arguement proves the theorem. Suppose $V(\cdot)$ is a weakly convex function of x'. A simple calculation shows that (8) is a strictly convex function of μ. Essentially by the definition of second-order stochastic dominance, this will be maximized by the most revealing signal. Then show that the resulting value function is a convex function of the prior reputation x. This is also a fairly quick calculation. Simply put, the fow payoff incentives perfectly align with the reputational incentives.

Next I go to a specific case for $w(\mu)$. Take $w(\mu)$ to be the simple step function.

$$w(\mu) = \begin{cases} 0 & \text{if } \mu < \bar{\mu} \\ 1 & \text{if } \mu \geq \bar{\mu} \end{cases} \tag{9}$$

This is the go to example seen in Kamenica and Gentzkow and other papers.

Consider the optimal strategy if there is only one period (no reputational concerns). As usual, Sender chooses to pool beliefs on $\bar{\mu}$ and reveal beliefs below some threshold. This can only be done if the reputation is sufficiently high.

$$x \geq \frac{\bar{\mu} - \mu_0}{1 - \mu_0} \tag{10}$$

If the reputation is below that cutoff, any strategy gives a payoff of zero. It can be shown that the payoff above this cutoff is always a concave function of the reputation x.

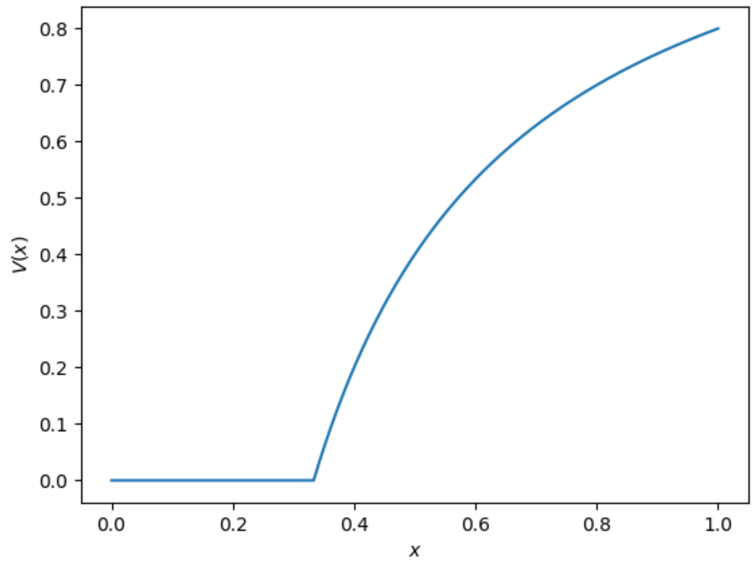

Figure 1: The value of the one period game as a function of the Sender's reputation. It is flat for the first region, then possibly jumps up, then is concave the rest of the way.

This kink makes the function convex for low reputation and concave for high reputations. Consider the second to last period. Revealing more information about Y also reveals more information aout the correlation θ. This implies that for low reputations, motivational concerns will cause Sender to be more revealing than in the static game. For high reputations, motivational concerns will cause Sender to be less revealing than in the static game.

This was done with Sender not knowing their own value θ. If Sender does know the value, the best pooling equilibrium can be obtained through similar means. There can also generally be seperating equilibria. These always take the same form. The low type does the static optimal for a Sender known to be low type. The high type sends a highly informative signal until their signal has been correct a sufficient number of times, then they play the static optimal for a Sender known to be the high type. There is no seperating equilibrium if the low type has no information (Y is uninformative of Z).

My main application for this is a credit rating agency. They observe many metrics of firm health and give a rating. Investors choose how much of the bond to buy. Later, investors observe if the furm defaults on the bond and updates their belief about the quality of the credit rating agency's metrics of firm health.

This could also be a model of a university giving grades to students. The university wants its students to get good jobs, but also wants to build a strong reputation. This model explains why everyone at Harvard gets A's and the grades are not very informative, but at Slippery Rock State the grades distinguish between students a lot.