Rational Confidence and Unknown Unknowns

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Confidence-Knowledge Graph

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- Charles Darwin noticed it and remarked, "Ignorance more frequently begets confidence than does knowledge."
- Mark Twain made a similar observation, "When I was a boy of 14, my father was so ignorant I could hardly stand to have the old man around. But when I got to be 21, I was astonished at how much the old man had learned in seven years."
- In *As You Like It*, William Shakespeare put it clearly, "the fool doth think he is wise, but the wise man knows himself to be a fool."

Questions

- What confidence-knowledge graphs are rationalizable?

Are you above average?

Many studies with similar results

- Lake Wobegon Effect
- 88% of drivers think they are above average
- Svenson 1981
- Buunk and Van Yperen

The opposite effect also exists

Questions

We now have two big questions

- What confidence-knowledge graphs are rationalizable?
- How many rational people can think they're above average?

Unknown unknowns

You're reading papers in a new field you want to study.

- You don't know how many paper there are on the subject
- You may not find every paper that has been written
- You may not have time to carefully study every paper you find

"We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns, the ones we don't know we don't know."

-United States Secretary of Defense Donald Rumsfeld

Set up

You will stop reading papers for one of three reasons

- You stop if you've read all the papers
  - Suppose there are 0, 1, 2, or 3 papers
  - Each is equally likely
- You stop if you don't find any more papers
  - After reading each paper (0, 1, or 2), if there is another paper you find it with probability 2/3
- You stop if you run out of budget
  - You have a budget of 2 papers

Confidence-knowledge graph

Conditional on the number of papers you've read, what is the probability that you've read everything available?

Consider first someone who's read zero papers

- It's possible that there aren't any papers (ex ante 25%)
- It's possible that there are papers they didn't find ($\frac{1}{3}$ conditional on existence)

$$prob = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{3}{4}\frac{1}{3}} = \frac{1}{2} = 50\%$$

Confidence-knowledge graph

We can compute the confidence level of someone who has read one paper

$$prob = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{2}{3}\frac{1}{3}} = \frac{3}{5} = 60\%$$

Confidence-knowledge graph

We can compute the confidence level of someone who has read two papers

- Say don't have the budget to read a third paper, so they don't search for one
    - There are equally likely to be two or three papers, so confidence is 50%
- Say they do still search for a third even though they won't read it
    - If they find a third paper, confidence is 0%
    - If they don't find a third, confidence is 75%
    - On average, the confidence level will be 50%

Confidence-knowledge graph

Result

- Not the 45 degree line
- Can slope downward
- Hump-shaped
- Charlie Munger's book

Comparisons

Are you above average?

- Mass of independent learners
- Each person guesses whether they are (strictly) above average or not
- Payoff of 1 if guess is correct 0 if incorrect
- What fraction of the population thinks they are above average?
- It depends on the realization of the number of papers

Comparisons

Suppose there aren't any papers on the subject

- Everyone has read zero papers
- 100% of the population guesses (correctly) they are not above average

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Suppose there is one paper on the subject

- People who have read one paper think they're above average (correctly)
    - There is a 60% chance there is only one paper
    - Two-thirds of the population will have read one paper

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- Everyone has read zero papers
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Suppose there is one paper on the subject

- People who have read one paper think they're above average (correctly)
  - There is a 60% chance there is only one paper
  - Two-thirds of the population will have read one paper

Suppose there are two or three papers on the subject

- People who have read two papers know they're above average
- 67% of the population thinks they are above average while only 44% actually are

Conclusions

Confidence-knowledge graphs

- Can be downward sloping or hump-shaped
- Can we bound all possible graphs?

Comparisons

- Can be above or below 50%
- Not a mean vs. median trick
- Below when realization is low, above when realization is high

Model

You start learning at time zero until you stop for one of three reasons

- $t^I \sim \mathcal{I}(t)$ is the amount of information available
- $t^S \sim \mathcal{S}(t)$ is the amount of information you would conditionally find
- $t^B \sim \mathcal{B}(t)$ is your budget for information

You stop learning at the minimum of those three times.
$\hat{t} = \min\{t^I, t^S, t^B\}$

Definitions

Definition

Given some distributions, $\mathcal{I}(t)$, $\mathcal{S}(t)$, and $\mathcal{B}(t)$, the **confidence-knowledge graph**, $f : \mathbb{R}_+ \to [0,1]$, is the average posterior probability of having all information conditional on stopping at time t.

So, it's just the probability that $\hat{t} = t^I$ on average.

Definition

A function $g : \mathbb{R}_+ \to [0,1]$ is rationalizable if there exists some distributions $\mathcal{I}(t)$, $\mathcal{S}(t)$, and $\mathcal{B}(t)$ such that $g(t)$ is the confidence-knowledge graph of those distributions.

Result

What confidence-knowledge graphs are rationalizable?

Theorem

Every function $g : \mathbb{R}_+ \to [0,1)$ *is rationalizable.*

We don't even need the budget for this result.

Intuition

Consider the following distributions

$$\mathcal{I}(t) = 1 - e^{-2t}, \qquad \mathcal{S}(t) = 1 - e^{-t}, \qquad \mathcal{B}(t) = 0$$

At the point where you stopped, it's twice as likely that you were stopped by $\mathcal{I}(t)$ than by $\mathcal{S}(t)$.

- Any flat confidence-knowledge graph can be obtained by scaling the arrival rate for $\mathcal{I}(t)$ up or down.

Intuition

Generally, the average confidence level is just the ration of hazard rates.

$$p = \frac{\sigma^{\mathcal{I}}}{\sigma^{\mathcal{I}} + \sigma^{\mathcal{S}} + \sigma^{\mathcal{B}}}$$

So, the confidence-knowledge graph will be increasing (or decreasing) whenever the hazard rate for $\mathcal{I}(t)$ is increasing (or decreasing) relative to the hazard rate of $\mathcal{S}(t)$.

Proof

Since hazard rates are practically unrestrained, any function is rationalizable.

Let $g(t)$ be some function mapping into $[0, 1)$.

- Remove the budget because it isn't needed
- Set $\mathcal{S}(t) = 1 - e^{-t}$
- You will obtain the $g(t)$ for the confidence-knowledge graph if the hazard rate of $\mathcal{I}(t) = \frac{g(t)}{1-g(t)} = r(t)$.

The differential equation $\frac{\mathcal{I}'(t)}{1-\mathcal{I}(t)} = r(t)$ has a solution.

$$\mathcal{I}(t) = e^{\int_0^t -r(\xi)\ d\xi} \int_0^x e^{-\int_0^\zeta -r(\xi)\ d\xi} r(\zeta)\ d\zeta$$

Comparisons

How many people can think they're above average?

- Consider the example from earlier.

$$\mathcal{I}(t) = 1 - e^{-2t}, \qquad \mathcal{S}(t) = 1 - e^{-t}, \qquad \mathcal{B}(t) = 0$$

Everyone can believe they are above average.

- Likelihood of being above average, not above average of averages

Result

Theorem

For any value $p \in [0,1]$, there exist distributions of $\mathcal{I}(t)$, $\mathcal{S}(t)$, and $\mathcal{B}(t)$ such that p fraction of the population believes they are above average.

There is an easy way and a hard way to show this.

Implications

What does this mean for studies of overconfidence?

- Any confidence-knowledge graph is rationalizable
- Any comparison to the average is rationalizable
- Studies where you see an objective outcome aren't immune
- What if the agents communicate?

Literature

There are a million studies documenting overconfidence

- Kruger and Dunning (1999), Buunk and Van Yperen (1991), Svenson (1981), Malmendier and Tate (2005)

There are several answers to the 88% are above average fact

- Benoît and Dubra (2011), Zábojník (2004), Brocas and Carillo (2007), Köszegi (2006), Moore and Healy (2008)

- Modica and Rustichini (1999)

What now?

- Some application
- Endogenize search