Equilibrium Gerrymandering

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Gerrymandering

The legislative body (Congress) is made up of the Senate and the House of Representatives

- There are 435 member of the House of Representatives
- Each state elects some Representatives based on population
- Representative seats are allocated after every census

The state decides how to elect their representatives

- The state is divided into districts with each district electing one Representative
- The only restriction (almost) on the districts is that they are about the same size
- Some states have state congress draw the districts, others have a bipartisan committee
- Districts are redrawn after every census

State congress is also elected by districts.

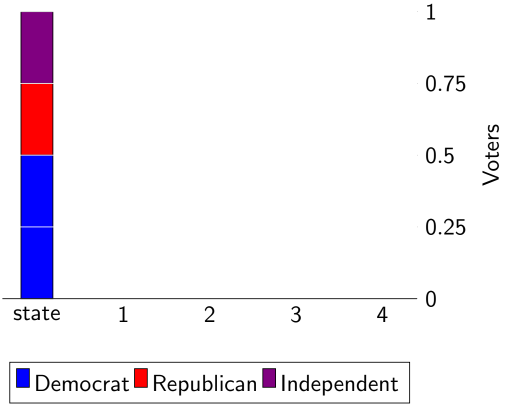
Today's Presentation

Simple State

State population

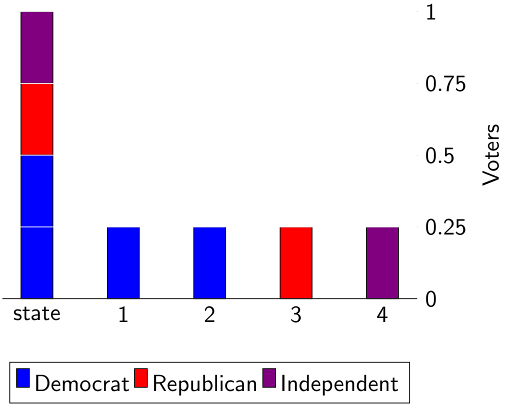
- 50% Democrat
- 25% Republican
- 25% Independent

The designer needs to divide up the population into districts.



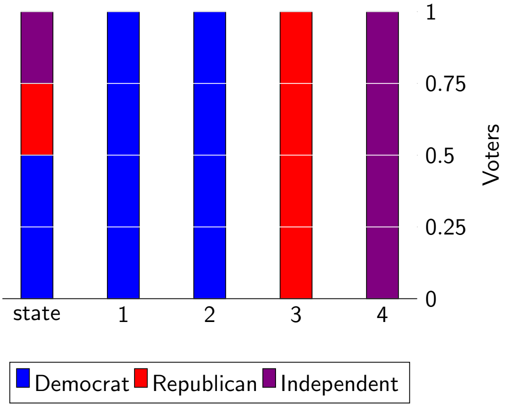
Proportional Districts

We could separate the political ideologies (with Democrats getting twice as many districts because there are twice as many of them)



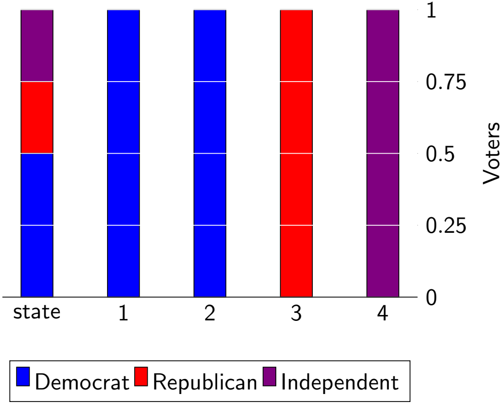
Proportional Districts

Rescale the bars to be proportions of the district.



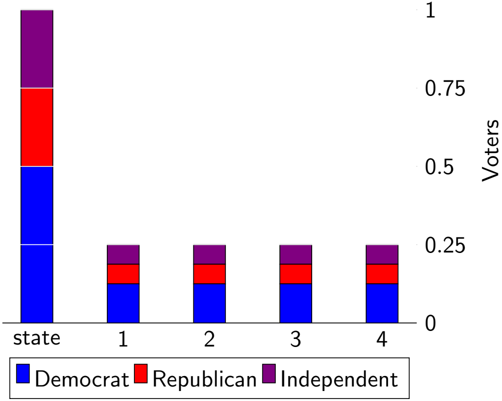
Proportional Districts

With porportional districts, the fraction of districts won by a party matches the fraction of voters for that party.



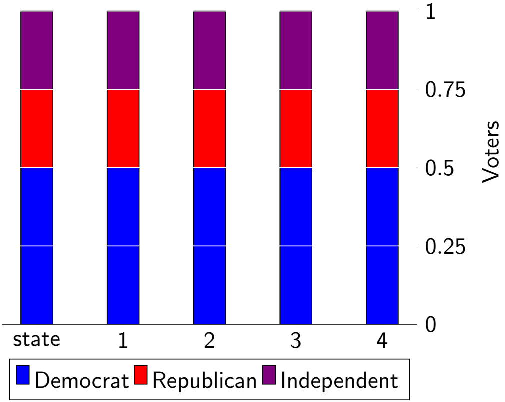
Identical Districts (Democrat Favoring)

You could make every district identical.



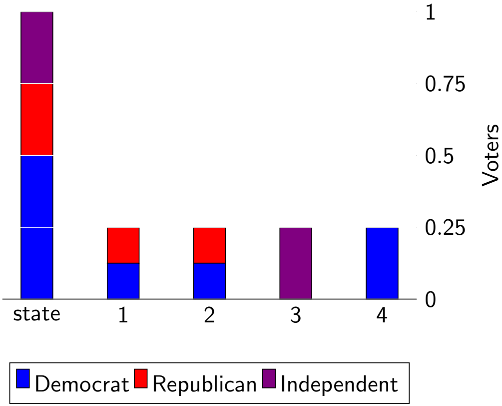
Identical Districts (Democrat Favoring)

Rescaling the districts again, we see this favors the Democrats.



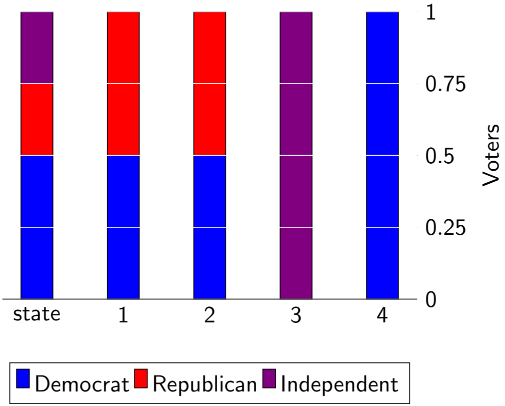
Packing and Cracking (Republican Favoring)

Here is the districting that most favors Republicans. It features "packing" and "cracking".



Packing and Cracking (Republican Favoring)

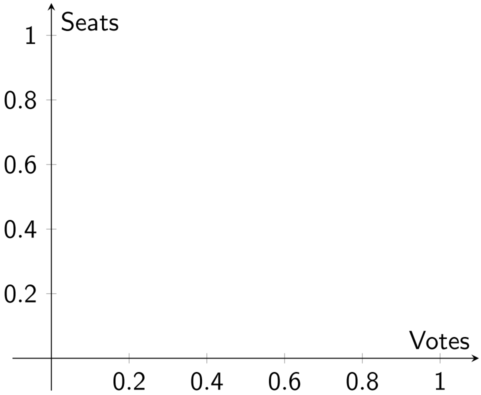
Rescaling the districts again, we see that the Democrats win 37.5% of districts on average.



Seat-vote Curves

The expected fraction of seats won isn't all that matters.
The fraction of seats won by a party graphed against the fraction of votes won is called the seat-vote curve.

In this state, the Democrats will get between 50 and 75 percent of the vote.



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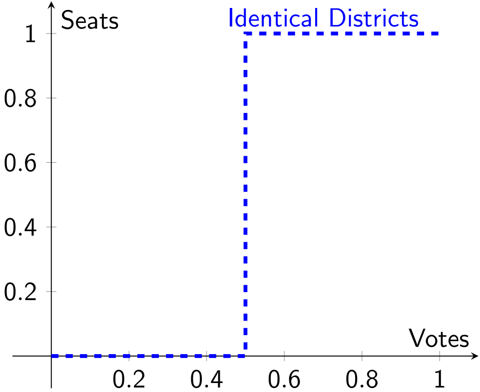
In the identical districts, the Democrats always win all the seats.



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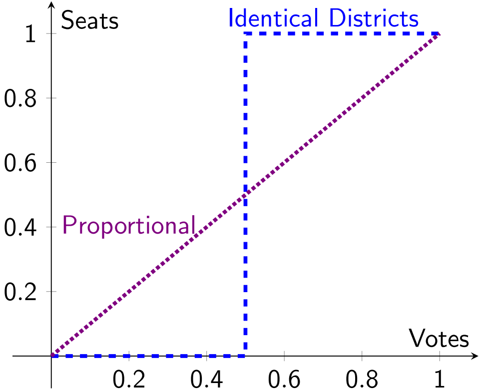
In the proportional districts, the seat-vote curve is equal to the 45 degree line.



Seat-vote Curves

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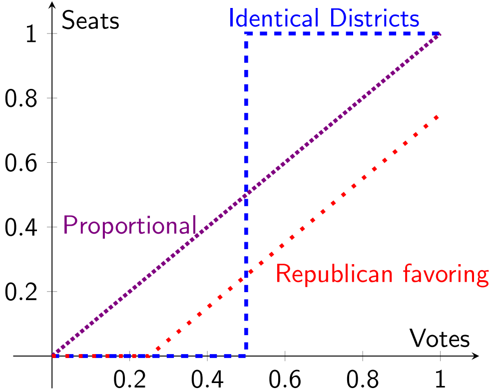
In this state, the Democrats will get between 50 and 75 percent of the vote.

In the Republican favoring districts, the seat-vote curve still has a slope of one but is shifted down.



Roadmap

Maximal Districting

- Find the districting scheme to maximize seats for a party
- Splitting distributions

Equilibrium Districting

- Find the districting scheme to maximize welfare of state's voters
- Seat-vote curves are important
- In equilibrium seat-vote curves are steep and slope is negatively related to state size

Empirical Evidence

- Estimate seat-vote curves for each state
- The curves are very steep
- Slope is approximated well by state size

Setup

The variable of interest is the policy.

- The policy is a number from 0 to 1 (0 being Democrat and 1 Republican)
- State i gets to elect n_i fraction of the Representatives
- S_i is the fraction of seats won Democrats
- The policy chosen is equal to the average Representative, $1 - \sum_{i=1}^{M} n_i S_i$

Setup

There are M states each with a unit mass of voters.

- State i has π_{Di}, π_{Ri}, and π_{Ii} fraction of Democrats, Republicans, and Independents
- A Democrat's preferred policy is 0, a Republican's is 1, and Independents' preferred policy is distributed between 0 and 1
- A voter gets a payoff of $-\left(\hat{\theta}-\theta\right)^2$ if $\hat{\theta}$ is the chosen policy and θ is their preferred policy
- Voters are not strategic
    - Democrats vote Democrat
    - Republicans vote Republican
    - The fraction of Independents that vote Democrat is drawn from a uniform distribution

Setup

The discrict designer's problem.

- Take state distribution, $x = [\pi_D, \pi_R, \pi_I]$, as given
- Choose a district distribution, $x_k \in \Delta^2$
- Choose the fraction of districts to have the distribution x_k, μ_k

The districts must add up to the state population.

$$\sum_{k=1}^{K} \mu_k x_k = \mathbb{E}[x_k]$$
$$= x$$

Setup

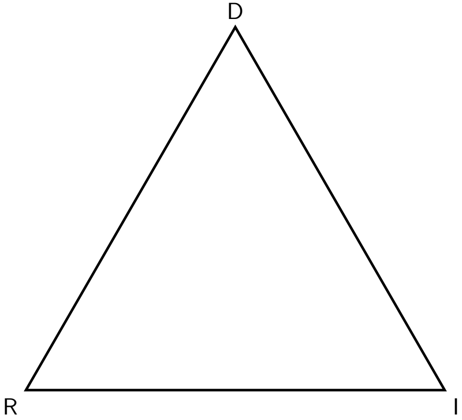
Call $\chi_k(x_k)$ the probability of winning a district with distribution x_k. As long as π_{Dk} and π_{Rk} are less than $\frac{1}{2}$, this is equal to

$$\chi_k(x_k) = \frac{\frac{1}{2} - \pi_{Dk}}{1 - \pi_{Rk} - \pi_{Dk}}$$

The districter's problem is

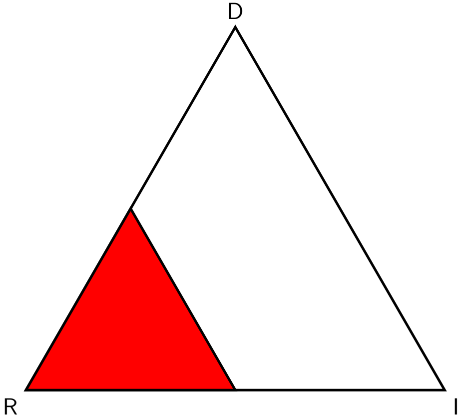
$$\max_{x_k \subset \Delta^2, \mu \in \Delta^{K-1}} \sum_{k=1}^{K} \mu_k \chi_k(x_k)$$
$$\text{s.t.} \sum_{k=1}^{K} \mu_k x_k = x.$$

Triangle



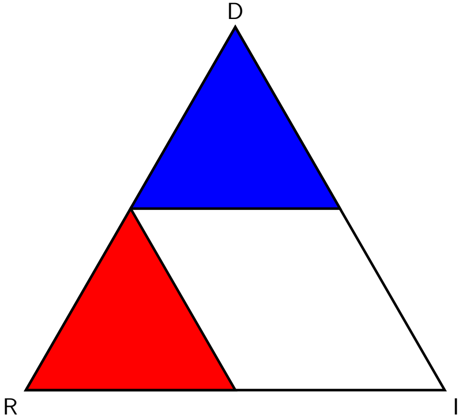
The distribution of voters lies in a simplex.

Triangle



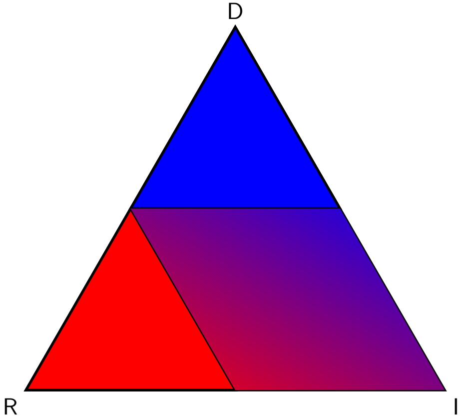
If more than 50 percent is Republican, the Republican candidate will win.

Triangle



If more than 50 percent is Democrat, the Democrat candidate will win.

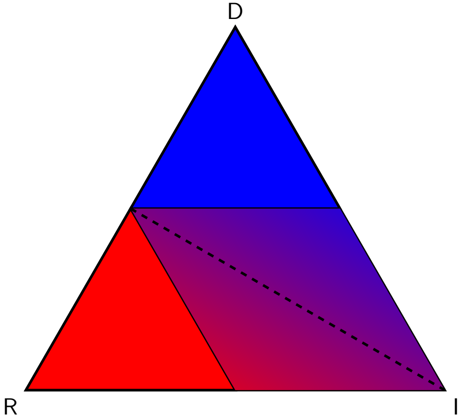
Triangle



The probability of the Republican winning in the middle region is

$$\chi = \frac{\frac{1}{2} - \pi_D}{1 - \pi_R - \pi_D}$$

Triangle



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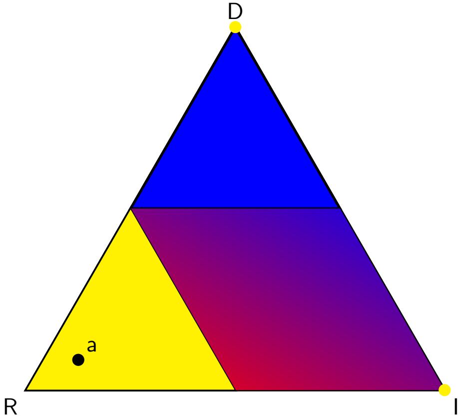
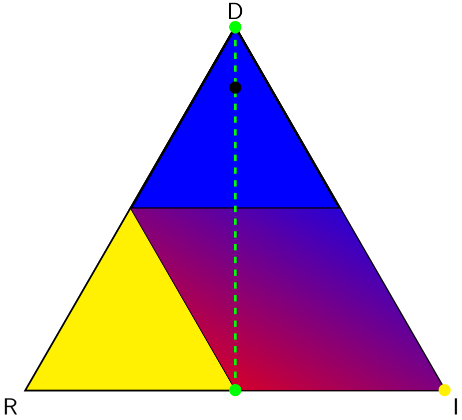


The yellow districts are undominated.

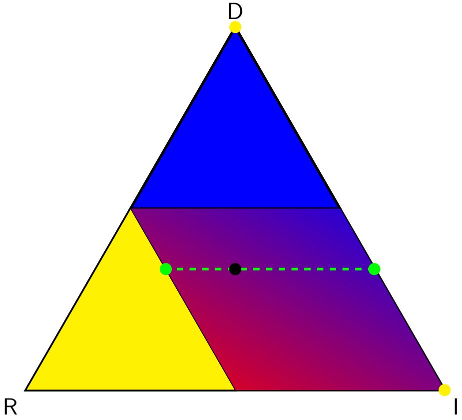
Triangle



All other districts are dominated.

Districts in the blue have value of zero.

Triangle

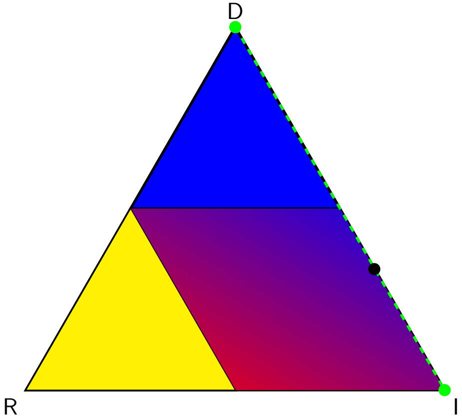


Holding π_D fixed,

$$\chi = \frac{\frac{1}{2} - \pi_D}{1 - \pi_R - \pi_D}$$

is a convex function of π_R.

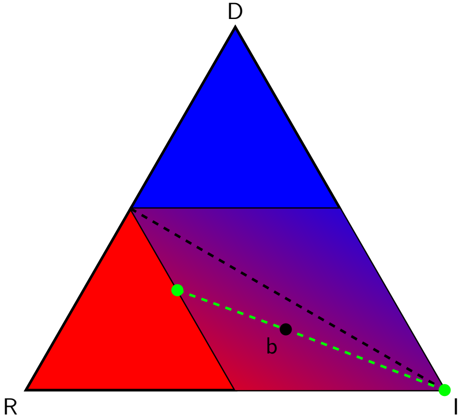
Triangle



When $\pi_R = 0$,

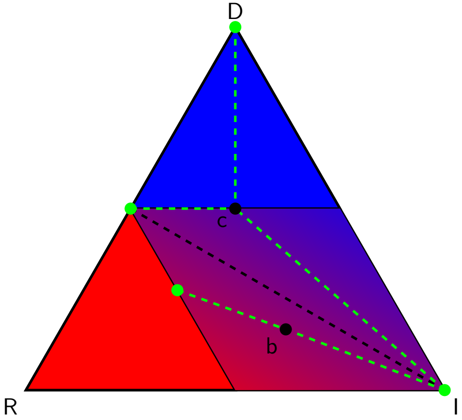
$$\chi = \frac{\frac{1}{2} - \pi_D}{1 - \pi_D}$$
$$\leq \frac{\frac{1}{2} - \pi_D + \frac{1}{2}\pi_D^2}{1 - \pi_D}$$
$$= \frac{1}{2}\frac{(1 - \pi_D)^2}{1 - \pi_D}$$
$$= \frac{1 - \pi_D}{2}$$

Triangle



The optimal districting is to split the state up into the undominated districts.

Triangle



The optimal districting is to split the state up into the undominated districts.

Optimal Districting

If $\pi_D > \pi_R$ you'll have some districts that are garunteed to lose.

- Fill $\pi_D - \pi_R$ districts with only Democrats (Packing)

You can garuntee wins in $2\pi_R$ districts.

- Fill the district exactly halfway with Republicans (any more is wasteful)
- Put remaining Democrats in districts garunteed to win (Cracking)

All remaining Independents are in their own districts.

- 50 percent of these districts are won in expectation

Maximal Value

The expected fraction of seats the Republicans can win with the optimal districting is

$$v_R = \begin{cases} 1 & \text{if } \pi_R \geq \frac{1}{2} \\ 2\pi_R + \frac{1}{2}\left(\pi_I - (\pi_R - \pi_D)\right) & \text{if } \pi_D < \pi_R < \frac{1}{2} \\ 2\pi_R + \frac{1}{2}\pi_I & \text{if } \pi_R \leq \pi_D. \end{cases} \quad (1)$$

This can be rewritten simply as

$$v_R = \min\left\{1\ ,\ 2\pi_R + \frac{1}{2}\left(\pi_I - \max\{0,\ \pi_R - \pi_D\}\right)\right\}. \quad (2)$$

Bipartisan Districts

In many states, a bipartisan committee or judicial group chooses the districts.

Consider the the districting to maximize the welfare of the state citizens. Remember the voter payoff equals

$$U(\hat{\theta}, \theta) = -\left(\hat{\theta} - \theta\right)^2$$

where $\hat{\theta}$ is the policy and θ is the preferred policy of the voter.

First we'll worry about the optimal fraction of seats the designer would like to gives the Democrats. Later we can think about if it is implementable by a districting scheme.

Seat-Vote Curves

Seat-vote curves are now necessary.

- The optimal policy depends on each voter's preferred policy.
- Since the independent voters move around each election, π_D, π_R, and π_I aren't enough information
- The best policy will depend on the vote share as well, v.
- Call $F_v(\theta)$ the distribution of preferred policies conditional on observing vote share v

The seat-vote curve is the fraction of seats allocated to Democrats as a function of the fraction of votes won by Democrats. The designer is choosing a seat-vote curve.

One State

First think about a single state in isolation.
Their problem is now

$$\max_{S(v)\in[0,1]} \int_0^1 -(1-S-\theta)^2 \, dF_v(\theta),$$

and has a simple solution,

$$\begin{aligned} 1 - S^*(v) &= \int_0^1 \theta dF_v(\theta) \\ &= \mathbb{E}\left[\theta \mid v\right]. \end{aligned}$$

One State

Most theory about seat-vote curve up until now comes from this equation.

$$1 - S^*(v) = \mathbb{E}\left[\theta \mid v\right]$$

One State

Most theory about seat-vote curve up until now comes from this equation.

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Proportional seat-vote curve

- All voters are either an extreme Democrat or extreme Republican, $supp(F(\theta)) \subset \{0, 1\}$
- The optimal policy is then equal to the fraction of voters that vote Republican
- $S^*(v) = v$
- The slope of the seat-vote curve is equal to one

One State

Most theory about seat-vote curve up until now comes from this equation.

$$1 - S^*(v) = \mathbb{E}\left[\theta \mid v\right]$$

Coate-Knight

- Most think the seat-vote curve should be flatter than proportional
- One additional vote is a more mild change in ideology
- Coate and Knight 2007, assumes independent voters are uniformly distributed on an interval with width 2τ
- This equation becomes their optimum

$$S^*(v) = \frac{1}{2} + (\pi_D - \pi_R)\left(\frac{1}{2} - \tau\right) + 2\tau\left(v - \frac{1}{2}\right)$$

Many States

In my model, there are many states and the national policy comes from the Representatives from every state.

$$\max_{S_i(v_i)} \mathbb{E}\left[\int_0^1 \left(1 - \sum_{j=1}^{M} n_j S_j(v_j) - \theta\right)^2 dF_{v_j}(\theta) \mid v_i\right]$$

$$1 - S_i^*(v_i) = \frac{1}{n_i}\left(\mathbb{E}\left[\theta \mid v_i\right] - \left(1 - \sum_{j \neq i}^{M} n_j \mathbb{E}\left[S_j(v_j)\right]\right)\right)$$

Still a simple solution

- The slope is now much higher.
- The slope is proportional to the state's size, $\frac{1}{n_i}$
- You account for who you expect every other state to elect

Explanation

The slope is steep.

- A 1 percent change nationally is a 50 percent change locally
- Goal is to move the average, so everyone bids more extreme
- Electoral college is winner-take-all

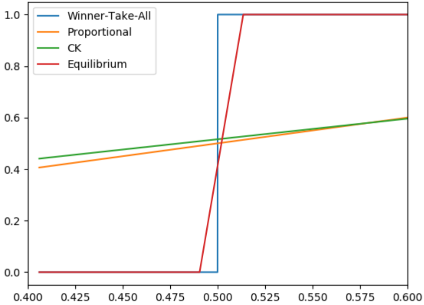

Figure: Seat-vote curves for Minnesota

Explanation

The slope larger for small states.

- A small state needs to flip all their Representatives to have much impact nationally
- Inversely proportional

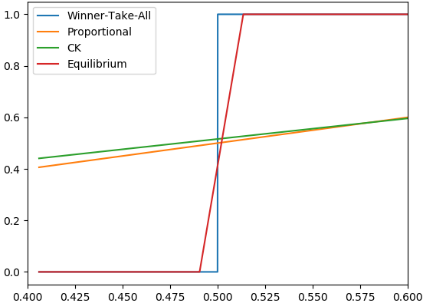

Figure: Seat-vote curves for Minnesota

Explanation

The optimal curve accounts for other states' actions.

- A state that leans Democrat should still elect Republicans if the rest of the country leans Democrat even more.

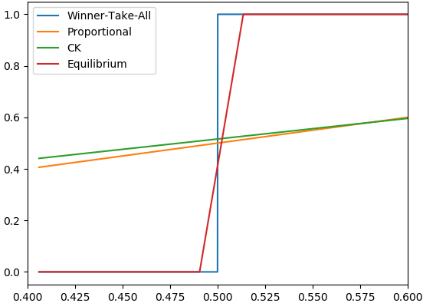

Figure: Seat-vote curves for Minnesota

Estimating Seat-Vote Curves

Data from Cooperative Congressional Election Study

- Individual survey
- 50,000+ data points
- Asked political identification
- Congressional disctrict

The number of representatives allocated to each state is also needed.

Estimating Seat-Vote Curves

Aggregating political identification

- Within each state to get π_{Di}, π_{Ri}, and π_{Ii}
- Within each district

Using the fraction of Democrats, Republicans, and Independents in each district, the seat-vote curve can be computed

- Draw the fraction of Independents to vote Democrat in a district from a uniform distribution
- Record the election winner and the Democrat vote share
- Add up across every district in the state to get a point on the seat-vote curve

Repeat 10,000 times and draw a smoothed average of fraction of seats won as a function of the fraction of votes won.

Estimates

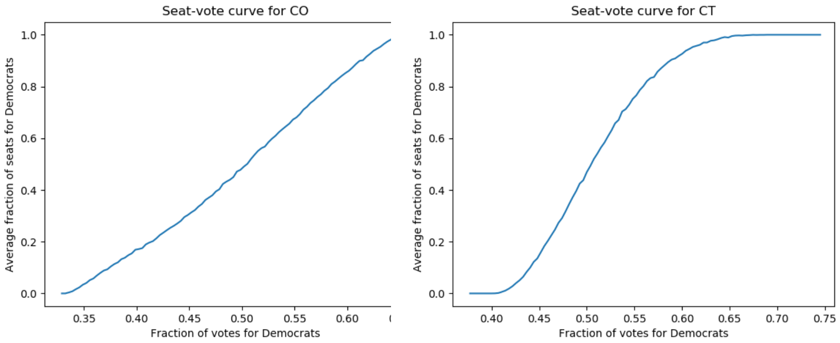

Figure: The estimated seat-vote curves for Colorado and Connecticut. Connecticut's curve is much steeper in the middle than Colorado's. Connecticut has the more responsive curve.

Regression

VARIABLES	(1) Model1	(2) Model2	(3) Model3	(4) Model3
Representatives	-0.375***	-0.335***		
	(0.0613)	(0.0649)		
Reps^2	0.00613***	0.00560***		
	(0.00136)	(0.00137)		
Dem_control		-3.805	-3.201**	-3.573**
		(2.319)	(1.316)	(1.378)
n_inverse			5.912***	5.817***
			(0.459)	(0.471)
Party_control				2.017
				(2.172)
Constant	7.777***	7.648***	3.849***	3.702***
	(0.406)	(0.407)	(0.209)	(0.262)
Observations	50	50	50	50
R-squared	0.492	0.520	0.826	0.829

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Fit

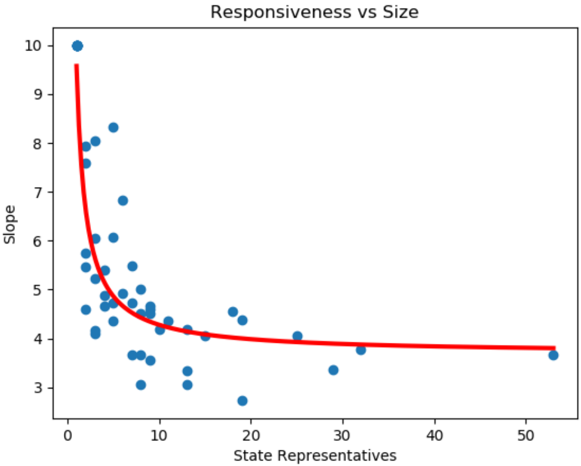

Figure: The responsiveness is approximately equal to $a\frac{1}{n_i} + \epsilon_i$.

Extension

National welfare computation

- National seat-vote curve should have low responsiveness
- Can be implemented by each state doing a low responsiveness
- Prisoner's Dilemma

Median congress member choosing policy.

- Only two real outcomes: Democrat majority or Republican majority
- Optimal for each state is a winner-take-all election
- Cutoff may not be exactly 50 percent
- Other states' strategies don't matter

Conclusion

Thank you.