

EDA

Q1 1) Show that $m(a + bX) = a + b m(X)$

$$\begin{aligned}
 m(a + bX) &= \frac{1}{N} \sum_{t=1}^N (a + bX) \\
 &= \frac{1}{N} \left[\sum_{t=1}^N a + \sum_{t=1}^N bX \right] \\
 &= \frac{1}{N} \sum_{t=1}^N a + \frac{1}{N} \sum_{t=1}^N bX \\
 &= \frac{1}{N} \sum_{t=1}^N a + b \frac{1}{N} \sum_{t=1}^N X \\
 &= [m(a) + b m(x)]
 \end{aligned}$$

- ① definition of sample mean
 ② distributive prop. of summation notation

2) Show that $\text{cov}(X, a+bY) = b \text{cov}(X, Y)$

$$\text{cov}(X, a+bY) = \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(y_i - m(Y))$$

① Def'n of covariance

$$= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(a+bY - m(a+bY)) \quad ②$$

② Sub. Y for $a+bY$

$$= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(a + b(m(Y))) \quad ③$$

③ Sub. $m(a+bm(Y))$ using part 1 of proof

$$= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(b(Y - m(Y))) \quad ④$$

④ a's cancel and factor out b

$$= b \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(Y - m(Y)) \quad ⑤$$

⑤ take out constant b.

$$b \text{cov}(X, Y)$$

⑥ Def'n of covariance

3) Show that $\text{cov}(a+bX, a+bX) = b^2 \text{cov}(X, X)$

$$\text{cov}(a+bX, a+bX) = \frac{1}{N} \sum_{i=1}^N (a+bX_i - m(a+bX))(a+bX_i - m(a+bX))$$

① from def'n of covariance

$$= \frac{1}{N} \sum_{i=1}^N ((a+bX_i) - (a+bm(X)))^2 \quad \text{② from part 1 } m(a+bX) = a+bm(X)$$

$$= \frac{1}{N} \sum_{i=1}^N (bX_i - bm(X))^2 = \frac{1}{N} \sum_{i=1}^N (b(X_i - m(X)))^2$$

$$= \frac{1}{N} \sum_{i=1}^N b^2 (X_i - m(X))^2 = b^2 \underbrace{\frac{1}{N} \sum_{i=1}^N (X_i - m(X))^2}_{\text{this is } \text{cov}(X, X)}$$

$$= \boxed{b^2 \text{cov}(X, X)}$$

4) a) Is a non-decreasing transformation of the median the median of the transformed variable? Quantile? IQR?

Example: $X = \{-2, 0, 1, 3, 5\}$ Post Trans.: $\{-18, 2, 7, 17, 27\}$

<u>Pre-Transformation</u>	<u>Post-Transformation</u>	$2+5X$
Median: 1	Median: 7	
25 th Quartile: 0	25 th Quartile: 2	$2+5X \checkmark$
75 th Quartile: 3	75 th Quartile: 17	
IQR: 3	IQR: 15	$2+5(3) = 17$
Range: 7	Range: 45	

Same order, but IQR and range don't scale by transformation (but scaled by the constn (5) here)

5) For above it works, but if took one number out of set $\{-2, 0, 1, 3, 5\}$ $g(m(x)) = 2.25$
 $\{-18, 2, 7, 27\}$ $m(g(x)) = 9.5$