Fantasy Football Analytics

Isaac T. Petersen

To our daughter, Maisie.

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Preface

This is a book in progress—it is incomplete. I will continue to add to and update it as I am able.

How to Contribute

This is an open-access textbook. My goal is to share data analysis strategies for free! Anyone is welcome to contribute to the project. If you would like to contribute, please consider one of the following:

- open an issue[1] or create a pull request[2] on the book's GitHub repository[3].
- buy me a coffee[4]—Support me in developing this (free!) resource for fantasy football analytics… Even a cup of coffee helps me stay awake!

The GitHub repository for the book is located here: https://github.com/isaactpetersen/Fantasy-Football-Analytics-Textbook. If you have data or analysis examples that are you willing to share and include in the book, feel free to contact me.

Open Access

This is an open-access book. This means that it is freely available for anyone to access.

[1]https://github.com/isaactpetersen/Fantasy-Football-Analytics-Textbook/issues
[2]https://github.com/isaactpetersen/Fantasy-Football-Analytics-Textbook/pulls
[3]https://github.com/isaactpetersen/Fantasy-Football-Analytics-Textbook
[4]https://www.buymeacoffee.com/isaactpetersen

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Citation

The APA-style citation for the book is:

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 url = {https://github.com/isaactpetersen/Fantasy-Football-Analytics-Textbook}
}
```

[5] https://creativecommons.org/licenses/by/4.0/

About the Author

I am an Assistant Professor in the Department of Psychological and Brain Sciences at the University of Iowa. I am a licensed psychologist with expertise in child clinical psychology. Why am I writing about fantasy football and data analysis? Because fantasy football involves the intersection of two things I love: sports and statistics.

Through my training, I have learned the value of statistics for answering important questions that I find interesting. In graduate training, I came to the realization that statistics are relevant not only for psychology and science, but also for domains that I enjoy as hobbies, including sports and fantasy sports. I have played in a longstanding fantasy football league for over 20 years (since my junior year of high school) with old friends from high school. I wanted to apply what I was learning about statistics to help others improve their performance in fantasy football and to help people—including those who might not otherwise be interested—to learn statistics. So I began blogging online about the value of applying statistics to improve decision making in fantasy football. Apparently, many people were interested in learning statistics when they could apply them to a domain that they find interesting like fantasy football. My blog eventually became FantasyFootballAnalytics.net[6], a website that uses advanced statistics to help people win their fantasy football leagues.

In terms of my `R` and statistics background, I have published many peer-reviewed publications[7] that employ advanced statistical methods, have published a book on psychological assessment[8] (Petersen, 2024b, 2024c) that includes applied examples in `R`, and have published the `petersenlab` `R` package[9] (Petersen, 2024a) on the Comprehensive R Archive Network (CRAN). I am also a co-author of the `ffanalytics` `R` package[10] that provides free utilities for downloading fantasy football projections and additional fantasy-relevant data, and for calculating projected points given your league settings.

6 http://fantasyfootballanalytics.net
7 https://developmental-psychopathology.lab.uiowa.edu/publications
8 https://www.routledge.com/9781032413068
9 https://cran.r-project.org/web/packages/petersenlab/index.html
10 https://github.com/FantasyFootballAnalytics/ffanalytics

Accessibility

I strive to follow principles of accessibility[11] (archived at https://perma.cc/8XJ9-Q6QJ) to make the book content accessible to people with visual impairments and physical disabilities. If there are additional ways I can make the content more accessible, please let me know.

Acknowledgments

I thank Dr. Benjamin Motz, who provided consultation and many helpful resources based on his fantasy football statistics class. I also thank key members of FantasyFootballAnalytics.net[12], including Val Pinskiy, Andrew Tungate, Dennis Andersen, and Adam Peterson, who helped develop and provide fantasy football-related resources and who helped sharpen my thinking about the topic. I also thank Professor Patrick Carroll, who taught me the value of statistics for answering important questions.

[11] https://bookdown.org/yihui/rmarkdown-cookbook/html-accessibility.html
[12] http://fantasyfootballanalytics.net

1

Introduction

1.1 About this Book

How can we use information to make predictions about uncertain events? This book is about empiricism (basing theories on observed data) and judgment, prediction, and decision making in the context of uncertainty. The book provides an introduction to modern analytical techniques used to make informed predictions, test theories, and draw conclusions from a given dataset.

This book was originally written for a undergraduate-level course entitled, "Fantasy Football: Predictive Analytics and Empiricism". The chapters provide an overview of topics that each could have its own class and textbook, such as causal inference[1], factor analysis[2], cluster analysis[3], principal component analysis[4], machine learning[5], cognitive biases[6], modern portfolio theory[7], data visualization[8], simulation[9], etc. The book gives readers an overview of the breadth of the approaches to prediction and empiricism. As a consequence, the book does not cover any one technique or approach in great depth.

1.2 What is Fantasy Football?

Fantasy football is an online game where participants assemble (i.e., "draft") imaginary teams composed of real-life National Football League (NFL) players. In this game, participants compete against their opponents (e.g.,

[1] causal-inference.qmd
[2] factor-analysis.qmd
[3] cluster-analysis.qmd
[4] pca.qmd
[5] machine-learning.qmd
[6] cognitive-bias.qmd
[7] modern-portfolio-theory.qmd
[8] data-visualization.qmd
[9] simulation.qmd

friends/coworkers/classmates), accumulating points based on players' actual statistical performances in games. The goal is to outscore one's opponent each week to win matches and ultimately claim victory in the league.

1.3 Why Focus on Fantasy Football?

I was fortunate to have an excellent instructor who taught me the value of learning statistics to answer interesting and important questions. That is, I do not find statistics intrinsically interesting; rather, I find them interesting because of what they allow me to do. Many students find statistics intimidating in part because of how it is typically taught—with examples like dice rolls and coin flips that are (seemingly irrelevant and) boring to students. My contention is that applied examples are a more effective lens to teach many concepts in psychology and data analysis. It can be more engaging and relatable to learn statistics in the applied context of sports, a domain that is more intuitive to many. Many people play fantasy sports. This book involves applying statistics to a particular domain (football). People actually want to learn statistical principles and methods when they can apply them to interesting questions (e.g., sports). In my opinion [and supported by evidence; Motz (2013)], this is a much more effective way of engaging people and teaching statistics than in the context of abstract coin flips and dice rolls. Fantasy football relies heavily on prediction—trying to predict which players will perform best and selecting them accordingly. In this way, fantasy football provides a plethora of decision making opportunities in the face of uncertainty, and a wealth of data for analyzing these decisions. However, unlike many other applied domains in psychology, fantasy football (1) allows a person to see the accuracy of their predictions on a timely basis and (2) provides a safe environment for friendly competition. Thus, it provides a unique domain to evaluate—and improve—the accuracy of various prediction models.

1.4 Educational Value

Skills in statistics, statistical programming, and data analysis are highly valuable. This book includes practical and conceptual tools that build a foundation for critical thinking. The book aims to help readers evaluate theory in the light of evidence (and vice versa) and to refine decision making in the context of uncertainty. Readers will learn about the ways that psychological science (and related disciplines) poses questions, formulates hypotheses, designs studies to test those questions, and interprets the findings, collectively with the aim to answer questions, improve decision making, and solve problems.

Of course, this is not a traditional psychology textbook. However, the book incorporates important psychological concepts, such as cognitive biases in judgment and prediction, etc. In the modern world of big data, research and society need people who know how to make sense of the information around us. Psychology is in a prime position to teach applied statistics to a wide variety of students, most of whom will not have careers as psychologists. Psychology can teach the importance of statistics given humans' cognitive biases. It can also teach about how these biases can influence how people interpret statistics. This book will teach readers the applications of statistics (prediction) and research methods (empiricism) to answer questions they find interesting, while applying scientific and psychological rigor.

1.5 Learning Objectives

This book aims to help readers accomplish the following learning objectives:

- Apply empirical inference and appreciate the value it provides over speculative supposition.
- Ask educated questions when confronted with decisions in the face of uncertainty.
- Understand human decision making, including common heuristics and cognitive biases and how to mitigate them analytically.
- Engage in critical thinking about causality, including devising plausible alternative explanations for observed effects.
- Understand causal inference including confounding, causal pathways, and counterfactuals.
- Think empirically about human behavior and performance.
- Describe the strengths and weaknesses of humans versus computers in prediction scenarios.
- Apply basic skills in statistical programming using `R` to manipulate and summarize datasets and to conduct data analysis.
- Critically evaluate the strengths and limitations of different statistical models and methodologies used in predicting uncertain events, enhancing their understanding of statistical inference and model selection.
- Use various analytical techniques for predicting the outcome of uncertain events, and for uncovering latent causes of patterns in observed data.
- Interpret findings from various statistical approaches and evaluate the accuracy of predictions.

- Engage in iterative problem-solving processes, refining analytical approaches based on feedback and outcomes, and adapting strategies accordingly.
- Communicate statistical findings and analyses in both written and oral formats, demonstrating proficiency in presenting complex information to diverse audiences.
- Make sense of big data.
- Use practical analytical skills that can be applied in future research and job settings.

1.6 Disclosures

I am the Owner of Fantasy Football Analytics, LLC, which operates https://fantasyfootballanalytics.net.

1.7 Disclaimer

"*This material probably won't win you fantasy football championships. You could take what we learn and apply it to fantasy football and you might become 5 percent more likely to win. Or… Consider the broader relevance of this. You could learn data analysis and figure out ways to apply it to other systems. And you could be making a six-figure salary within the next five years.*" – Benjamin Motz, Ph.D.

2

Intro to Football and Fantasy

This chapter provides a brief primer on (American) football and fantasy football. If you are already familiar with fantasy football, feel free to skip this chapter.

2.1 Football

Football is the most widely watched sport in the United States.[1]

2.1.1 The Objective

The goal in football is for a team to score more points than their opponent. A game lasts 60 minutes, and it is separated into four 15-minute quarters. The team with the most points when the time runs out wins.

2.1.2 The Roster

2.1.2.1 Overview

Each team has 11 players on the field at a time. The particular players who are on the field will depend on the situation, but usually includes one of the three subsets of players:

1. Offense
2. Defense
3. Special Teams

[1] https://news.gallup.com/poll/610046/football-retains-dominant-position-favorite-sport.aspx (archived at https://perma.cc/X2UG-RAAK); https://www.statista.com/statistics/1430289/most-watched-sports-leagues-usa/ (archived at https://perma.cc/JNU6-S96A)

An example formation is depicted in Figure 2.1.

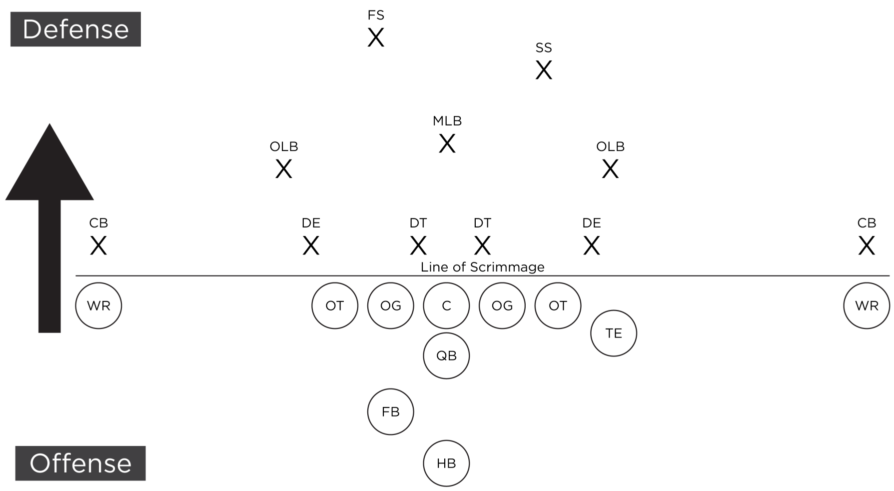

Figure 2.1 An Example Football Formation for the Offense and Defense. The solid line indicates the line of scrimmage. The arrow indicates the direction the offense tries to advance the ball.

2.1.2.2 Offense

The offense is on the field when the team has the ball.

Players on offense include:

- Quarterback (QB)
- Running Back (RB)
  - Halfback (HB) or Tailback (TB)
  - Fullback (FB)
- Wide Receiver (WR)
- Tight End (TE)
- Offensive Linemen (OL), part of the "Offensive Line"
  - Center (C)
  - Offensive Guard (OG)
  - Offensive Tackle (OT)

The quarterback is the most important player on the offense. They help lead the team down the field. The quarterback receives the ball from the Center at the beginning of the play, and they can either hand the ball off (typically to a Running Back or Fullback), pass the ball (typically to a Wide Receiver or

Tight End), or run the ball. Quarterbacks tend to have a strong arm for throwing the ball far and accurately. Some quarterbacks are fast and are considered "dual threats" to pass or run.

Running Backs take a hand-off from the Quarterback to execute a running play (i.e., a rush). They may also catch short passes from the Quarterback or help protect (i.e., block for) the Quarterback from the defensive players who are trying to tackle the Quarterback. Halfbacks and Tailbacks tend to be quick and agile. Fullbacks tend to be strong and powerful.

Wide Receivers catch passes from the Quarterback to execute a passing play. On running plays, they provide protection for the player running the ball (e.g., the Running Back) so the ball carrier can get as far as possible without being tackled. Wide receivers tend to be tall, fast, have good hands (can catch the ball well), and can jump high.

Tight Ends block for running and passing plays, and they catch passes from the Quarterback. Tight ends tend to be strong and have good hands.

Offensive Linemen block for running and passing plays. On passing plays, they provide protection for the Quarterback so the Quarterback has time to pass the ball without being tackled. On running plays, they provide protection for the player running the ball (e.g., the Running Back) so the ball carrier can get as far as possible without being tackled. Offensive Linemen tend to be large so they can provide adequate protection for the Quarterback and Running Back.

2.1.2.3 Defense

The defense is on the field when the team does not have the ball (i.e., when the opposing team has the ball).

Players on defense include:

- Defensive Linemen (DL), part of the "Defensive Line"
  - Defensive End (DE)
  - Defensive Tackle (DT)
- Linebacker (LB)
  - Middle (or Inside) Linebacker (MLB)
  - Outside Linebacker (OLB)
- Defensive Back (DB), part of the "Secondary"
  - Cornerback (CB)
  - Safety (S)
    * Free Safety (FS)
    * Strong Safety (SS)

The players on the defense attempt to tackle the offensive players for as short of gains as possible and attempt to prevent completed passes.

On passing plays, Defensive Linemen try to apply pressure to the Quarterback and try to tackle the Quarterback behind the line of scrimmage before the Quarterback can throw the ball (i.e., a sack). On rushing plays, Defensive Linemen try to tackle the ball carrier to prevent the ball carrier from advancing the ball (i.e., gaining yards). Defensive Linemen tend to be large yet quick so they can apply pressure to the Quarterback.

Linebackers are versatile in that, on a given play, they may attempt to a) "blitz" to sack the Quarterback, b) stop the Running Back, or c) prevent a completed pass. Linebackers tend to be strong yet agile.

Defensive Backs are specialist pass defenders. The main role of Cornerbacks is to cover the Wide Receivers. Safeties serve as the last line of defense for longer passes. Defensive Backs tend to be quick and agile.

2.1.2.4 Special Teams

The special teams involves specialist players who are on the field during all kicking plays including kickoffs, field goals, and punts.

Players on special teams include:

- Kicker (K)
- Punter (P)
- Holder
- Long Snapper
- Punt Returner
- Kick Returner
- and other players intended to block for or to tackle the ball carrier

On a field goal attempt, the Long Snapper snaps the ball to the Holder, who holds the ball for the Kicker. The Kicker attempts field goals and, during kickoffs, kicks the ball to the opposing team. During kickoffs, the Kick Returner catches the kicked ball and returns it for as many yards as possible. During a punt play, the Long Snapper snaps the ball to the Punter who kicks (i.e., punts) the ball to the opposing team. The Punt Returner catches the punted ball and returns it for as many yards as possible.

2.1.3 The Field

The football field is rectangular and is 120 yards long and 53 1/3 yards wide (109.73 m x 48.77 m).[2] At each end of the 120-yard field is a team's end zone.

[2] One yard is equal to three feet. A yard is just smaller than a meter (0.9144 meters).

Each end zone is 10 yards long (9.14 m). Thus, the distance from one end zone to the other end zone is 100 yards (91.44 m). Behind each end zone is a field goal post. A diagram of a football field is depicted in Figure 2.2.

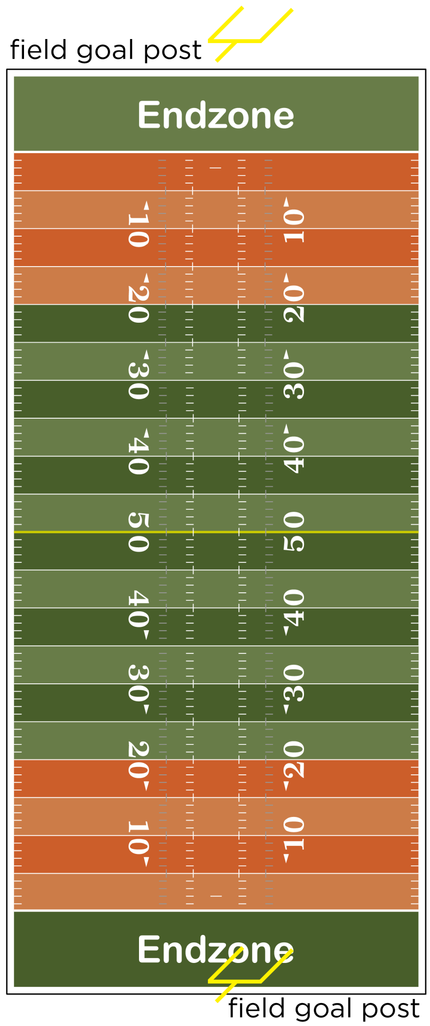

Figure 2.2 A Diagram of a Football Field. The yard markers depict the distance from the nearest end zone. The orange shaded area is called the "red zone", where chances of scoring points are highest. The original figure was modified to depict field goal posts. (Figure retrieved from https://commons.wikimedia.org/wiki/File:American_football_field.svg)

2.1.4 The Gameplay

At the beginning of the game, there is a coin flip to determine which teams receives the ball first and which team takes which side of the field. During the kickoff, the kicking team kicks the ball to the receiving team, who has the option to return the kick. The offense starts their possession at the 25 yard line—if there is no return (i.e., a touchback)—or wherever the kick returner is tackled or goes out of bounds.

The team with the ball (i.e., the offense) has four opportunities ("downs") to advance the ball (i.e., gain) 10 yards. A team can advance the ball either by running it or by throwing (i.e., passing) and catching it. At the end of a rushing play, the ball advances to wherever the ball carrier is tackled or goes out of bounds (i.e., wherever the player is "down"). At the end of a passing play, if the thrown ball is caught (i.e., a completed pass), the ball advances to wherever the ball carrier is tackled or goes out of bounds. If the thrown ball is not caught in bounds before the ball hits the ground (i.e., an incomplete pass), the ball does not advance. Wherever the ball is advanced to dictates where the next play begins. The yard position on the field where the next play takes place from is known as the "line of scrimmage". Neither team can cross the line the line of scrimmage until the next play begins. To begin the play, the ball is placed on the line of scrimmage and the Center gives (or "snaps") the ball to the Quarterback.

If the team advances the ball 10 or more yards within four downs, the team receives a "first down" and is awarded a new set of downs—four more downs to advance the ball 10 more yards. If the team advances the ball all the way to the other team's end zone, they score a touchdown. If the team fails to advance the ball 10 or more yards within four downs, the team loses the ball, and the other team takes possession at that spot on the field. There are risks of giving the other team the ball with a short distance to score. Thus, on fourth down, instead of trying to advance the ball for a first down, a team may choose to kick a field goal—to get points—or to punt.

A field goal involves a kicker kicking the ball with an intent to kick the ball through the field goal posts ("uprights"). To score points by making a field goal, the kicked ball must go between the uprights (extended vertically) and over the cross bar.

Punting involves a punter kicking the ball to the other team with an intent to give their opponent worse field position, thus making it harder for the other team to score. The punting team tries to pin the opponent as close as possible to the opponent's end zone (i.e., as far as possible from the own team's end zone), so they have a longer distance to go to score a touchdown.

There are multiple ways that ball possession can switch from the offense to the other team. After scoring a touchdown, field goal, or safety, there is a kickoff, in which the scoring team kicks the ball to the opponent. Another way that the ball switches possession to the other team is if the team commits a turnover. The defense can force a turnover by an interception, fumble recovery, or turnover on downs. A turnover due to an interception occurs when a defensive player catches the quarterback's pass. A turnover due to a fumble recovery occurs when an offensive player, who had possession of the ball, loses the ball before being down or scoring a touchdown and the ball is recovered by the opponent. A turnover on downs occurs when the team attempts on fourth down to achieve the remainder of the needed 10 yards to go but fails.

Other football-related situations include tackles for loss and sacks. A tackle for loss occurs when a ball carrier is tackled behind the line of scrimmage. A sack occurs when a Quarterback is tackled with the ball behind the line of scrimmage. A pass defended occurs when a defensive player knocks down the ball in the air so that the indended receiver cannot catch the ball.

2.1.5 The Scoring

The goal of the team with the ball (i.e., the offense) is to score points. It can do this by either advancing the ball into the other team's end zone (6 points) or by kicking a field goal (3 points). Advancing the ball in the other team's end zone is called a touchdown. After a touchdown, the offense chooses to attempt either a point-after-touchdown (PAT) or a two-point conversion. A PAT is a short kick attempt from the 15-yard line (i.e., 15 yards away from the end zone) that, if it goes through the goal posts ("uprights") and over the cross bar, is worth 1 point. A two-point conversion is a single-scoring opportunity from the 3-yard line (i.e., 3 yards away from the end zone). If the offense scores (i.e., advances the ball into the end zone) from the 3-yard line, the team is awarded 2 points.

A team can kick a field goal from any distance as long as the kick goes through the goal posts. The current record for the longest field goal is 66 yards (by Justin Tucker in 2021).

A safety occurs when the offense is tackled with the ball in their own end zone. When a safety occurs, the opposing team (i.e., defense) is awarded two points and the ball.

2.1.6 Glossary of Terms

- running play ("run") or rushing play (or "rush")—the attempt by an offensive player, typically the Running Back or Quarterback, to advance the ball "on the ground" by running it—not by passing it forward
- passing play (or "pass")—the attempt by an offensive player, typically the Quarterback, to advance the ball by throwing it forward to an offensive player

- passing attempt—the attempt to advance the ball by passing it (i.e., a thrown pass)
- rushing attempt—the attempt to advance the ball by running it
- passing completion—a thrown pass that is succesfully caught by an offensive player
- passing incompletion—a thrown pass that is not caught by an offensive player
- passing yards—the distance (in yards) the player advanced the ball by throwing it
- rushing yards—the distance (in yards) the player advanced the ball by running it
- receving yards—the distance (in yards) the player advanced the ball by catching thrown passes and then running with it further upfield
- kick/punt return yards—the distance (in yards) the player advanced the ball by returning kicks or punts
- turnover return yards—the distance (in yards) the player advanced the ball by returning turnovers
- reception—a pass that is caught by the offensive player
- touchdown—advancing the ball into the opponent's end zone either by a) throwing a completed pass that ends up in the end zone, b) running it into the end zone, c) catching it in the end zone, or d) catching it and then running it into the end zone
- passing touchdown—advancing the ball into the opponent's end zone either by throwing a completed pass that ends up in the end zone
- rushing touchdown—advancing the ball into the opponent's end zone either by running it into the end zone
- receiving touchdown—advancing the ball into the opponent's end zone either by catching it in the end zone or by catching it and then running it into the end zone
- kick/punt return touchdown—advancing the ball into the opponent's end zone when returning a kick or punt
- turnover return touchdown—advancing the ball into the opponent's end zone when returning a turnover (i.e., interception or fumble)
- two-point conversion—a single-scoring opportunity from the 3-yard line (i.e., 3 yards away from the end zone) that is an option given to a team that scores a touchdown; if the offense scores (i.e., advances the ball into the end zone) from the 3-yard line, the team is awarded 2 points
- block—when the defense/special teams blocks a kick or field goal by hitting the ball just after it is kicked to prevent the ball from going far
- kickoff—the kicking team kicks the ball to the receiving team, who has the option to return the kick
- field goal—a kicker kicks the ball with an intent to kick the ball through the field goal posts ("uprights"). To score points by making a field goal, the kicked ball must go between the uprights (extended vertically) and over the cross bar. If the field goal attempt is successful, the team gains 3 points.

- point after touchdown (PAT)—a short kick attempt from the 15-yard line (i.e., 15 yards away from the end zone) that, if it goes through the goal posts ("uprights") and over the cross bar, is worth 1 point
- extra point returned—if the defense/special teams returns the ball into the opponent's end zone during a point after touchdown (PAT) attempt, it is worth 2 points
- punt—a punter kicks the ball to the other team with an intent to give their opponent worse field position, thus making it harder for the other team to score
- fumble lost—when an offensive player, who had possession of the ball, loses the ball before being down or scoring a touchdown and the ball is recovered by the opponent
- fumble forced—when a defensive player knocks the ball out of the hands of an offensive player, who had possession of the ball
- fumble recovery—when a defensive player recovers a fumble by the opponent
- interception—when a defensive player catches a pass from an offensive player
- tackle—when a player brings down the ball carrier
- tackle solo—when a player is the main tackler (i.e., the primary player to bring down the ball carrier)
- tackle assist—when a player is one of two or more players who, together, bring down the ball carrier
- tackle for loss—when an offensive player is tackled with the ball behind the line of scrimmage
- sack—when a Quarterback is tackled with the ball behind the line of scrimmage
- pass defended—when a defensive player knocks down the ball in the air so that the indended receiver cannot catch the ball
- safety—when the offense is tackled with the ball in their own end zone

2.2 Fantasy Football

2.2.1 Overview of Fantasy Football

Fantasy football is one of the most widely played games in the history of games. It is estimated that around 62 million people play fantasy sports[3], of whom around 29 million play fantasy football.[4] As noted in the Introduction[5],

[3]https://thefsga.org/industry-demographics/ (archived at https://perma.cc/9PB8-ZDJJ)

[4]https://www.statista.com/topics/10895/fantasy-sports-in-the-us/ (archived at https://perma.cc/8YSN-UUNT)

[5]intro.qmd

fantasy football is an online game where participants assemble (i.e., “draft”) imaginary teams composed of real-life National Football League (NFL) players.[6] The participants are in charge of managing and making strategic decisions for their imaginary team to have the best possible team that will score the most points. Thus, the participants are called “managers”. Managers make decisions such as selecting which players to draft, selecting which players to play (i.e., “start”) on a weekly basis, identifying players to pick up from the remaining pool of available players (i.e., waiver wire), and making trades with other teams. Fantasy football relies heavily on prediction—trying to predict which players will perform best and selecting them accordingly.

2.2.2 The Fantasy League

A fantasy football “league” is composed of various imaginary (i.e., “fantasy”) teams—and their associated manager. In the fantasy league, the managers’ fantasy teams play against each other. A fantasy league is commonly composed of 8, 10, or 12 fantasy teams, but leagues can have more or fewer teams.

2.2.3 The Roster of a Fantasy Team

On a given roster, a manager has a “starting lineup” and a “bench”. Each week, the manager decides which players on their roster to put in the starting lineup, and which to keep on the bench. In many leagues, a starting lineup is composed of offensive players, a kicker, and defense/special teams:

Offensive players:

Table 2.1 Offensive Players in the Starting Lineup

Position	Typical Number of Players in Starting Lineup
Quarterback (QB)	1
Running Back (RB)	2
Wide Receiver (WR)	2
Tight End (TE)	1
Flex Position	1

A “flex position” is a flexible position that can involve a player from various positions: e.g., a Running Back, Wide Receiver, or Tight End.

Kickers:

[6]Fantasy leagues are also available for baseball[7], basketball[8], and many other sports.

- one Kicker (K)

Defense/Special Teams:

- one Team Defense (DST/D/DEF) or multiple Individual Defensive Players (IDP)

2.2.4 Scoring

2.2.4.1 Scoring Overview

In the game of fantasy football, managers accumulate points on a weekly basis based on players' actual statistical performances in NFL games. Managers receive points for only those players who are on their starting lineup (not players on their bench). A manager's goal is to outscore their opponent each week to win matches and ultimately claim victory in the league. Scoring settings can differ from league to league.

Below are common scoring settings for fantasy leagues.

2.2.4.2 Offensive Players

Table 2.2 Common Scoring Settings for Offensive Players

Statistical category	Points
Rushing or receiving TD	6
Returning a kick or punt for a TD	6
Returning or recovering a fumble for a TD	6
Passing TD	4
Passing INT	−2
Fumble lost	−2
Rushing, passing, or receiving 2-point conversion	2
Rushing or receiving yards	1 point per 10 yards
Passing yards	1 point per 25 yards

Note: "TD" = touchdown; "INT" = interception

Other common (but not necessarily standard) statistical categories include:

- receptions (called "point per reception" [PPR] leagues)

- return yards
- passing attempts
- rushing attempts

2.2.4.3 Kickers

Table 2.3 Common Scoring Settings for Kickers

Statistical category	Points
FG made: 50+ yards	5
FG made: 40–49 yards	4
FG made: 39 yards or less	3
Rushing, passing, or receiving 2-point conversion	2
Point after touchdown attempt made	1
Point after touchdown attempt missed	−1
Missed FG: 0–39 yards	−2
Missed FG: 40–49 yards	−1

Note: "FG" = field goal

2.2.4.4 Team Defense/Special Teams

Table 2.4 Common Scoring Settings for Team Defense/Special Teams

Statistical category	Points
Defensive or special teams TD	3
Interception	2
Fumble recovery	2
Blocked punt, PAT, or FG	2
Safety	2
Sack	1

Note: "TD" = touchdown; "PAT" = point after touchdown; "FG" = field goal

2.2.4.5 Individual Defensive Players

Table 2.5 Common Scoring Settings for Individual Defensive Players

Statistical category	Points
Tackle solo	1
Tackle assist	0.5
Tackle for loss	1
Sack	2
Interception	4
Fumble forced	2
Fumble recovery	2
TD	6
Safety	2
Pass defended	1
Blocked kick	2
Extra point returned	2

Note: "TD" = touchdown

Other common (but not necessarily standard) statistical categories include:

- turnover return yards

2.2.4.6 Common Scoring Abbreviations

- "TD" = touchdown
- "INT" = interception
- "yds" = yards
- "ATT" = attempts
- "2-pt conversion" = two-point conversion
- "FG" = field goal
- "PAT" = point after touchdown (i.e., extra point/point after attempt)

3

Getting Started with R for Data Analysis

The book uses R for statistical analyses (http://www.r-project.org). R is a free software environment; you can download it at no charge here: https://cran.r-project.org.

3.1 Initial Setup

To get started, follow the following steps:

1. Install R: https://cran.r-project.org
2. Install RStudio Desktop: https://posit.co/download/rstudio-desktop
3. After installing RStudio, open RStudio and run the following code in the console to install several key R packages:

```
install.packages(
  c("petersenlab","ffanalytics","nflreadr","nflfastR","nfl4th","nflplotR",
  "gsisdecoder","progressr","lubridate","tidyverse","psych"))
```

Note 1: If you are in Dr. Petersen's class

If you are in Dr. Petersen's class, also perform the following steps:

1. Set up a free account on GitHub.com[a].
2. Download GitHub Desktop: https://desktop.github.com

[a] https://github.com

3.2 Installing Packages

You can install R packages using the following syntax:

```
install.packages("INSERT_PACKAGE_NAME_HERE")
```

For instance, you can use the following code to install the `nflreadr` package:

```
install.packages("nflreadr")
```

3.3 Load Packages

```
library("ffanalytics")
library("nflreadr")
library("nflfastR")
library("nfl4th")
library("nflplotR")
library("progressr")
library("lubridate")
library("tidyverse")
```

3.4 Download Football Data

3.4.1 Players

```
nfl_players <- progressr::with_progress(
  nflreadr::load_players())
```

3.4.2 Teams

```
nfl_teams <- progressr::with_progress(
  nflreadr::load_teams(current = TRUE))
```

3.4.3 Player Info

3.4.4 Rosters

A Data Dictionary for rosters is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_rosters.html

```
nfl_rosters <- progressr::with_progress(
  nflreadr::load_rosters(seasons = TRUE))

nfl_rosters_weekly <- progressr::with_progress(
  nflreadr::load_rosters_weekly(seasons = TRUE))
```

3.4.5 Game Schedules

A Data Dictionary for game schedules data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_schedules.html

```
nfl_schedules <- progressr::with_progress(
  nflreadr::load_schedules(seasons = TRUE))
```

3.4.6 The Combine

A Data Dictionary for data from the combine is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_combine.html

```
nfl_combine <- progressr::with_progress(
  nflreadr::load_combine(seasons = TRUE))
```

3.4.7 Draft Picks

A Data Dictionary for draft picks data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_draft_picks.html

```
nfl_draftPicks <- progressr::with_progress(
  nflreadr::load_draft_picks(seasons = TRUE))
```

3.4.8 Depth Charts

A Data Dictionary for data from weekly depth charts is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_depth_charts.html

```
nfl_depthCharts <- progressr::with_progress(
  nflreadr::load_depth_charts(seasons = TRUE))
```

3.4.9 Play-By-Play Data

To download play-by-play data from prior weeks and seasons, we can use the `load_pbp()` function of the `nflreadr` package. We add a progress bar using the `with_progress()` function from the `progressr` package because it takes a while to run. A Data Dictionary for the play-by-play data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_pbp.html

> Note 2: Downloading play-by-play data
>
> Note: the following code takes a while to run.

```
nfl_pbp <- progressr::with_progress(
  nflreadr::load_pbp(seasons = TRUE))
```

3.4.10 4th Down Data

```
nfl_4thdown <- nfl4th::load_4th_pbp(seasons = 2014:2023)
```

3.4.11 Participation

A Data Dictionary for the participation data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_participation.html

```
nfl_participation <- progressr::with_progress(
  nflreadr::load_participation(
    seasons = TRUE,
    include_pbp = TRUE))
```

3.4.12 Historical Weekly Actual Player Statistics

We can download historical week-by-week actual player statistics using the `load_player_stats()` function from the `nflreadr` package. A Data Dictionary for statistics for offensive players is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_player_stats.html. A Data Dictionary for statistics for defensive players is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_player_stats_def.html.

```
nfl_actualStats_offense_weekly <- progressr::with_progress(
  nflreadr::load_player_stats(
    seasons = TRUE,
    stat_type = "offense"))

nfl_actualStats_defense_weekly <- progressr::with_progress(
  nflreadr::load_player_stats(
    seasons = TRUE,
    stat_type = "defense"))

nfl_actualStats_kicking_weekly <- progressr::with_progress(
  nflreadr::load_player_stats(
    seasons = TRUE,
    stat_type = "kicking"))
```

3.4.13 Injuries

A Data Dictionary for injury data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_injuries.html

```
nfl_injuries <- progressr::with_progress(
  nflreadr::load_injuries(seasons = TRUE))
```

3.4.14 Snap Counts

A Data Dictionary for snap counts data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_snap_counts.html

```
nfl_snapCounts <- progressr::with_progress(
  nflreadr::load_snap_counts(seasons = TRUE))
```

3.4.15 ESPN QBR

A Data Dictionary for ESPN QBR data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_espn_qbr.html

```
nfl_espnQBR_seasonal <- progressr::with_progress(
  nflreadr::load_espn_qbr(
    seasons = TRUE,
    summary_type = c("season")))

nfl_espnQBR_weekly <- progressr::with_progress(
  nflreadr::load_espn_qbr(
    seasons = TRUE,
    summary_type = c("weekly")))

nfl_espnQBR_weekly$game_week <- as.character(nfl_espnQBR_weekly$game_week)

nfl_espnQBR <- bind_rows(
  nfl_espnQBR_seasonal,
  nfl_espnQBR_weekly
)
```

3.4.16 NFL Next Gen Stats

A Data Dictionary for NFL Next Gen Stats data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_nextgen_stats.html

```
nfl_nextGenStats_pass_weekly <- progressr::with_progress(
  nflreadr::load_nextgen_stats(
    seasons = TRUE,
    stat_type = c("passing")))

nfl_nextGenStats_rush_weekly <- progressr::with_progress(
  nflreadr::load_nextgen_stats(
    seasons = TRUE,
    stat_type = c("rushing")))

nfl_nextGenStats_rec_weekly <- progressr::with_progress(
  nflreadr::load_nextgen_stats(
    seasons = TRUE,
    stat_type = c("receiving")))

nfl_nextGenStats_weekly <- bind_rows(
  nfl_nextGenStats_pass_weekly,
```

```
  nfl_nextGenStats_rush_weekly,
  nfl_nextGenStats_rec_weekly
)
```

3.4.17 Advanced Stats from PFR

A Data Dictionary for PFR passing data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_pfr_passing.html

```
nfl_advancedStatsPFR_pass_seasonal <- progressr::with_progress(
  nflreadr::load_pfr_advstats(
    seasons = TRUE,
    stat_type = c("pass"),
    summary_level = c("season")))

nfl_advancedStatsPFR_pass_weekly <- progressr::with_progress(
  nflreadr::load_pfr_advstats(
    seasons = TRUE,
    stat_type = c("pass"),
    summary_level = c("week")))

nfl_advancedStatsPFR_rush_seasonal <- progressr::with_progress(
  nflreadr::load_pfr_advstats(
    seasons = TRUE,
    stat_type = c("rush"),
    summary_level = c("season")))

nfl_advancedStatsPFR_rush_weekly <- progressr::with_progress(
  nflreadr::load_pfr_advstats(
    seasons = TRUE,
    stat_type = c("rush"),
    summary_level = c("week")))

nfl_advancedStatsPFR_rec_seasonal <- progressr::with_progress(
  nflreadr::load_pfr_advstats(
    seasons = TRUE,
    stat_type = c("rec"),
    summary_level = c("season")))

nfl_advancedStatsPFR_rec_weekly <- progressr::with_progress(
  nflreadr::load_pfr_advstats(
    seasons = TRUE,
    stat_type = c("rec"),
```

```
    summary_level = c("week")))

nfl_advancedStatsPFR_def_seasonal <- progressr::with_progress(
  nflreadr::load_pfr_advstats(
    seasons = TRUE,
    stat_type = c("def"),
    summary_level = c("season")))

nfl_advancedStatsPFR_def_weekly <- progressr::with_progress(
  nflreadr::load_pfr_advstats(
    seasons = TRUE,
    stat_type = c("def"),
    summary_level = c("week")))

nfl_advancedStatsPFR <- bind_rows(
  nfl_advancedStatsPFR_pass_seasonal,
  nfl_advancedStatsPFR_pass_weekly,
  nfl_advancedStatsPFR_rush_seasonal,
  nfl_advancedStatsPFR_rush_weekly,
  nfl_advancedStatsPFR_rec_seasonal,
  nfl_advancedStatsPFR_rec_weekly,
  nfl_advancedStatsPFR_def_seasonal,
  nfl_advancedStatsPFR_def_weekly,
)
```

3.4.18 Player Contracts

A Data Dictionary for player contracts data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_contracts.html

```
nfl_playerContracts <- progressr::with_progress(
  nflreadr::load_contracts())
```

3.4.19 FTN Charting Data

A Data Dictionary for FTN Charting data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_ftn_charting.html

```
nfl_ftnCharting <- progressr::with_progress(
  nflreadr::load_ftn_charting(seasons = TRUE))
```

3.4.20 Fantasy Player IDs

A Data Dictionary for fantasy player ID data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_ff_playerids.html

```
nfl_playerIDs <- progressr::with_progress(
  nflreadr::load_ff_playerids())
```

3.4.21 FantasyPros Rankings

A Data Dictionary for FantasyPros ranking data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_ff_rankings.html

```
#nfl_rankings <- progressr::with_progress( # currently throws error
#  nflreadr::load_ff_rankings(type = "all"))

nfl_rankings_draft <- progressr::with_progress(
  nflreadr::load_ff_rankings(type = "draft"))

nfl_rankings_weekly <- progressr::with_progress(
  nflreadr::load_ff_rankings(type = "week"))

nfl_rankings <- bind_rows(
  nfl_rankings_draft,
  nfl_rankings_weekly
)
```

3.4.22 Expected Fantasy Points

A Data Dictionary for expected fantasy points data is located at the following link: https://nflreadr.nflverse.com/articles/dictionary_ff_opportunity.html

```
nfl_expectedFantasyPoints_weekly <- progressr::with_progress(
  nflreadr::load_ff_opportunity(
    seasons = TRUE,
    stat_type = "weekly",
    model_version = "latest"
  ))

nfl_expectedFantasyPoints_pass <- progressr::with_progress(
  nflreadr::load_ff_opportunity(
    seasons = TRUE,
```

```
    stat_type = "pbp_pass",
    model_version = "latest"
  ))

nfl_expectedFantasyPoints_rush <- progressr::with_progress(
  nflreadr::load_ff_opportunity(
    seasons = TRUE,
    stat_type = "pbp_rush",
    model_version = "latest"
  ))

nfl_expectedFantasyPoints_weekly$season <- as.integer(nfl_expectedFantasyPoints_weekly$season)

nfl_expectedFantasyPoints_offense <- bind_rows(
  nfl_expectedFantasyPoints_pass,
  nfl_expectedFantasyPoints_rush
)
```

3.5 Data Dictionary

Data Dictionaries are metadata that describe the meaning of the variables in a datset. You can find Data Dictionaries for the various NFL datasets at the following link: https://nflreadr.nflverse.com/articles/index.html.

3.6 Create a Data Frame

Here is an example of creating a data frame:

```
players <- data.frame(
  ID = 1:12,
  name = c(
    "Ken Cussion",
    "Ben Sacked",
    "Chuck Downfield",
    "Ron Ingback",
    "Rhonda Ball",
    "Hugo Long",
```

```
    "Lionel Scrimmage",
    "Drew Blood",
    "Chase Emdown",
    "Justin Time",
    "Spike D'Ball",
    "Isac Ulooz"),
  position = c("QB","QB","QB","RB","RB","WR","WR","WR","WR","TE","TE","LB"),
  age = c(40, 30, 24, 20, 18, 23, 27, 32, 26, 23, NA, 37)
  )

fantasyPoints <- data.frame(
  ID = c(2, 7, 13, 14),
  fantasyPoints = c(250, 170, 65, 15)
)
```

3.7 Variable Names

To see the names of variables in a data frame, use the following syntax:

```
names(nfl_players)
```

```
 [1] "status"                   "display_name"
 [3] "first_name"               "last_name"
 [5] "esb_id"                   "gsis_id"
 [7] "suffix"                   "birth_date"
 [9] "college_name"             "position_group"
[11] "position"                 "jersey_number"
[13] "height"                   "weight"
[15] "years_of_experience"      "team_abbr"
[17] "team_seq"                 "current_team_id"
[19] "football_name"            "entry_year"
[21] "rookie_year"              "draft_club"
[23] "college_conference"       "status_description_abbr"
[25] "status_short_description" "gsis_it_id"
[27] "short_name"               "smart_id"
[29] "headshot"                 "draft_number"
[31] "uniform_number"           "draft_round"
[33] "season"
```

```
names(players)
```

```
[1] "ID"       "name"     "position" "age"
```

```
names(fantasyPoints)
```

```
[1] "ID"            "fantasyPoints"
```

3.8 Logical Operators

3.8.1 Is Equal To: ==

```
players$position == "RB"
```

```
 [1] FALSE FALSE FALSE  TRUE  TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```

3.8.2 Is Not Equal To: !=

```
players$position != "RB"
```

```
 [1]  TRUE  TRUE  TRUE FALSE FALSE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
```

3.8.3 Is Greater Than: >

```
players$age > 30
```

```
 [1]  TRUE FALSE FALSE FALSE FALSE FALSE FALSE  TRUE FALSE FALSE    NA  TRUE
```

3.8.4 Is Less Than: <

```
players$age < 30
```

```
 [1] FALSE FALSE  TRUE  TRUE  TRUE  TRUE  TRUE FALSE  TRUE  TRUE    NA FALSE
```

3.8.5 Is Greater Than or Equal To: `>=`

```
players$age >= 30
```

```
 [1]  TRUE  TRUE FALSE FALSE FALSE FALSE FALSE  TRUE FALSE FALSE    NA  TRUE
```

3.8.6 Is Less Than or Equal To: `<=`

```
players$age <= 30
```

```
 [1] FALSE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE FALSE  TRUE  TRUE    NA FALSE
```

3.8.7 Is In a Value of Another Vector: `%in%`

```
players$position %in% c("RB","WR")
```

```
 [1] FALSE FALSE FALSE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE FALSE FALSE FALSE
```

3.8.8 Is Not In a Value of Another Vector: `!(%in%)`

```
!(players$position %in% c("RB","WR"))
```

```
 [1]  TRUE  TRUE  TRUE FALSE FALSE FALSE FALSE FALSE FALSE  TRUE  TRUE  TRUE
```

3.8.9 Is Missing: `is.na()`

```r
is.na(players$age)
```

```
 [1] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE  TRUE FALSE
```

3.8.10 Is Not Missing: `!is.na()`

```r
!is.na(players$age)
```

```
 [1]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE FALSE  TRUE
```

3.8.11 And: `&`

```r
players$position == "WR" & players$age > 26
```

```
 [1] FALSE FALSE FALSE FALSE FALSE FALSE  TRUE  TRUE FALSE FALSE FALSE FALSE
```

3.8.12 Or: `|`

```r
players$position == "WR" | players$age > 23
```

```
 [1]  TRUE  TRUE  TRUE FALSE FALSE  TRUE  TRUE  TRUE  TRUE FALSE    NA  TRUE
```

3.9 Subset

To subset a data frame, use brackets to specify the subset of rows and columns to keep, where the value/vector before the comma specifies the rows to keep, and the value/vector after the comma specifies the columns to keep:

```r
dataframe[rowsToKeep, columnsToKeep]
```

You can subset by using any of the following:

- numeric indices of the rows/columns to keep (or drop)
- names of the rows/columns to keep (or drop)
- values of `TRUE` and `FALSE` corresponding to which rows/columns to keep

3.9.1 One Variable

To subset one variable, use the following syntax:

```
players$name
```

```
 [1] "Ken Cussion"     "Ben Sacked"      "Chuck Downfield"  "Ron Ingback"
 [5] "Rhonda Ball"     "Hugo Long"       "Lionel Scrimmage" "Drew Blood"
 [9] "Chase Emdown"    "Justin Time"     "Spike D'Ball"     "Isac Ulooz"
```

or:

```
players[,"name"]
```

```
 [1] "Ken Cussion"     "Ben Sacked"      "Chuck Downfield"  "Ron Ingback"
 [5] "Rhonda Ball"     "Hugo Long"       "Lionel Scrimmage" "Drew Blood"
 [9] "Chase Emdown"    "Justin Time"     "Spike D'Ball"     "Isac Ulooz"
```

3.9.2 Particular Rows of One Variable

To subset one variable, use the following syntax:

```
players$name[which(players$position == "RB")]
```

```
[1] "Ron Ingback" "Rhonda Ball"
```

or:

```
players[which(players$position == "RB"), "name"]
```

```
[1] "Ron Ingback" "Rhonda Ball"
```

3.9.3 Particular Columns (Variables)

To subset particular columns/variables, use the following syntax:

3.9.3.1 Base `R`

```
subsetVars <- c("name","age")

players[,c(2,4)]
```

```
              name age
1       Ken Cussion  40
2        Ben Sacked  30
3   Chuck Downfield  24
4       Ron Ingback  20
5       Rhonda Ball  18
6         Hugo Long  23
7  Lionel Scrimmage  27
8        Drew Blood  32
9      Chase Emdown  26
10      Justin Time  23
11     Spike D'Ball  NA
12       Isac Ulooz  37
```

```
players[,c("name","age")]
```

```
              name age
1       Ken Cussion  40
2        Ben Sacked  30
3   Chuck Downfield  24
4       Ron Ingback  20
5       Rhonda Ball  18
6         Hugo Long  23
7  Lionel Scrimmage  27
8        Drew Blood  32
9      Chase Emdown  26
10      Justin Time  23
11     Spike D'Ball  NA
12       Isac Ulooz  37
```

```
players[,subsetVars]
```

```
              name age
1       Ken Cussion  40
2        Ben Sacked  30
3   Chuck Downfield  24
4       Ron Ingback  20
5       Rhonda Ball  18
```

```
6         Hugo Long  23
7  Lionel Scrimmage  27
8        Drew Blood  32
9      Chase Emdown  26
10      Justin Time  23
11     Spike D'Ball  NA
12       Isac Ulooz  37
```

Or, to drop columns:

```
dropVars <- c("name","age")

players[,-c(2,4)]
```

```
   ID position
1   1       QB
2   2       QB
3   3       QB
4   4       RB
5   5       RB
6   6       WR
7   7       WR
8   8       WR
9   9       WR
10 10       TE
11 11       TE
12 12       LB
```

```
players[,!(names(players) %in% c("name","age"))]
```

```
   ID position
1   1       QB
2   2       QB
3   3       QB
4   4       RB
5   5       RB
6   6       WR
7   7       WR
8   8       WR
9   9       WR
10 10       TE
11 11       TE
12 12       LB
```

```
players[,!(names(players) %in% dropVars)]
```

```
   ID position
1   1       QB
2   2       QB
3   3       QB
4   4       RB
5   5       RB
6   6       WR
7   7       WR
8   8       WR
9   9       WR
10 10       TE
11 11       TE
12 12       LB
```

3.9.3.2 Tidyverse

```
players %>%
  select(name, age)
```

```
                name age
1        Ken Cussion  40
2         Ben Sacked  30
3    Chuck Downfield  24
4        Ron Ingback  20
5        Rhonda Ball  18
6          Hugo Long  23
7   Lionel Scrimmage  27
8         Drew Blood  32
9       Chase Emdown  26
10       Justin Time  23
11      Spike D'Ball  NA
12        Isac Ulooz  37
```

```
players %>%
  select(name:age)
```

```
                name position age
1        Ken Cussion       QB  40
2         Ben Sacked       QB  30
3    Chuck Downfield       QB  24
```

```
4       Ron Ingback       RB  20
5       Rhonda Ball       RB  18
6         Hugo Long       WR  23
7  Lionel Scrimmage       WR  27
8        Drew Blood       WR  32
9      Chase Emdown       WR  26
10      Justin Time       TE  23
11     Spike D'Ball       TE  NA
12       Isac Ulooz       LB  37
```

```
players %>%
  select(all_of(subsetVars))
```

```
               name age
1       Ken Cussion  40
2        Ben Sacked  30
3   Chuck Downfield  24
4       Ron Ingback  20
5       Rhonda Ball  18
6         Hugo Long  23
7  Lionel Scrimmage  27
8        Drew Blood  32
9      Chase Emdown  26
10      Justin Time  23
11     Spike D'Ball  NA
12       Isac Ulooz  37
```

Or, to drop columns:

```
players %>%
  select(-name, -age)
```

```
   ID position
1   1       QB
2   2       QB
3   3       QB
4   4       RB
5   5       RB
6   6       WR
7   7       WR
8   8       WR
9   9       WR
10 10       TE
11 11       TE
12 12       LB
```

```r
players %>%
  select(-c(name:age))
```

```
   ID
1   1
2   2
3   3
4   4
5   5
6   6
7   7
8   8
9   9
10 10
11 11
12 12
```

```r
players %>%
  select(-all_of(dropVars))
```

```
   ID position
1   1       QB
2   2       QB
3   3       QB
4   4       RB
5   5       RB
6   6       WR
7   7       WR
8   8       WR
9   9       WR
10 10       TE
11 11       TE
12 12       LB
```

3.9.4 Particular Rows

To subset particular rows, use the following syntax:

3.9.4.1 Base R

```r
subsetRows <- c(4,5)

players[c(4,5),]
```

```
  ID         name position age
4  4 Ron Ingback       RB  20
5  5 Rhonda Ball       RB  18
```

```r
players[subsetRows,]
```

```
  ID         name position age
4  4 Ron Ingback       RB  20
5  5 Rhonda Ball       RB  18
```

```r
players[which(players$position == "RB"),]
```

```
  ID         name position age
4  4 Ron Ingback       RB  20
5  5 Rhonda Ball       RB  18
```

3.9.4.2 Tidyverse

```r
players %>%
  filter(position == "WR")
```

```
  ID             name position age
1  6        Hugo Long       WR  23
2  7 Lionel Scrimmage       WR  27
3  8       Drew Blood       WR  32
4  9     Chase Emdown       WR  26
```

```r
players %>%
  filter(position == "WR", age <= 26)
```

```
  ID         name position age
1  6    Hugo Long       WR  23
2  9 Chase Emdown       WR  26
```

```
players %>%
  filter(position == "WR" | age >= 26)
```

```
  ID              name position age
1  1       Ken Cussion       QB  40
2  2        Ben Sacked       QB  30
3  6         Hugo Long       WR  23
4  7 Lionel Scrimmage       WR  27
5  8        Drew Blood       WR  32
6  9      Chase Emdown       WR  26
7 12        Isac Ulooz       LB  37
```

3.9.5 Particular Rows and Columns

To subset particular rows and columns, use the following syntax:

3.9.5.1 Base R

```
players[c(4,5), c(2,4)]
```

```
         name age
4 Ron Ingback  20
5 Rhonda Ball  18
```

```
players[subsetRows, subsetVars]
```

```
         name age
4 Ron Ingback  20
5 Rhonda Ball  18
```

```
players[which(players$position == "RB"), subsetVars]
```

```
         name age
4 Ron Ingback  20
5 Rhonda Ball  18
```

3.9.5.2 Tidyverse

```
players %>%
  filter(position == "RB") %>%
  select(all_of(subsetVars))
```

```
         name age
1 Ron Ingback  20
2 Rhonda Ball  18
```

3.10 View Data

3.10.1 All Data

To view data, use the following syntax:

```
View(players)
```

3.10.2 First 6 Rows/Elements

To view only the first six rows (if a data frame) or elements (if a vector), use the following syntax:

```
head(nfl_players)
```

```
# A tibble: 6 x 33
  status display_name    first_name last_name esb_id  gsis_id suffix birth_date
  <chr>  <chr>           <chr>      <chr>     <chr>   <chr>   <chr>  <chr>
1 RET    'Omar Ellison    'Omar      Ellison   ELL711~ 00-000~ <NA>   <NA>
2 ACT    A'Shawn Robinson A'Shawn    Robinson  ROB367~ 00-003~ <NA>   1995-03-21
3 ACT    A.J. Arcuri      A.J.       Arcuri    ARC716~ 00-003~ <NA>   <NA>
4 RES    A.J. Bouye       Arlandus   Bouye     BOU651~ 00-003~ <NA>   1991-08-16
5 ACT    A.J. Brown       Arthur     Brown     BRO413~ 00-003~ <NA>   1997-06-30
6 ACT    A.J. Cann        Aaron      Cann      CAN364~ 00-003~ <NA>   1991-10-03
# i 25 more variables: college_name <chr>, position_group <chr>,
#   position <chr>, jersey_number <int>, height <dbl>, weight <int>,
#   years_of_experience <chr>, team_abbr <chr>, team_seq <int>,
#   current_team_id <chr>, football_name <chr>, entry_year <int>,
#   rookie_year <int>, draft_club <chr>, college_conference <chr>,
#   status_description_abbr <chr>, status_short_description <chr>,
#   gsis_it_id <int>, short_name <chr>, smart_id <chr>, headshot <chr>, ...
```

```
head(nfl_players$display_name)
```

```
[1] "'Omar Ellison"    "A'Shawn Robinson" "A.J. Arcuri"     "A.J. Bouye"
[5] "A.J. Brown"       "A.J. Cann"
```

3.11 Data Characteristics

3.11.1 Data Structure

```
str(nfl_players)
```

```
nflvrs_d [20,039 x 33] (S3: nflverse_data/tbl_df/tbl/data.table/data.frame)
 $ status                  : chr [1:20039] "RET" "ACT" "ACT" "RES" ...
 $ display_name            : chr [1:20039] "'Omar Ellison" "A'Shawn Robinson" "A.J. Arcuri" "A.J. Bouye" ..
 $ first_name              : chr [1:20039] "'Omar" "A'Shawn" "A.J." "Arlandus" ...
 $ last_name               : chr [1:20039] "Ellison" "Robinson" "Arcuri" "Bouye" ...
 $ esb_id                  : chr [1:20039] "ELL711319" "ROB367960" "ARC716900" "BOU651714" ...
 $ gsis_id                 : chr [1:20039] "00-0004866" "00-0032889" "00-0037845" "00-0030228" ...
 $ suffix                  : chr [1:20039] NA NA NA NA ...
 $ birth_date              : chr [1:20039] NA "1995-03-21" NA "1991-08-16" ...
 $ college_name            : chr [1:20039] NA "Alabama" "Michigan State" "Central Florida" ...
 $ position_group          : chr [1:20039] "WR" "DL" "OL" "DB" ...
 $ position                : chr [1:20039] "WR" "DT" "T" "CB" ...
 $ jersey_number           : int [1:20039] 84 91 61 24 11 60 6 81 63 20 ...
 $ height                  : num [1:20039] 73 76 79 72 72 75 76 69 76 72 ...
 $ weight                  : int [1:20039] 200 330 320 191 226 325 220 190 280 183 ...
 $ years_of_experience     : chr [1:20039] "2" "8" "2" "8" ...
 $ team_abbr               : chr [1:20039] "LAC" "NYG" "LA" "CAR" ...
 $ team_seq                : int [1:20039] NA 1 NA 1 1 1 1 NA NA NA ...
 $ current_team_id         : chr [1:20039] "4400" "3410" "2510" "0750" ...
 $ football_name           : chr [1:20039] NA "A'Shawn" "A.J." "A.J." ...
 $ entry_year              : int [1:20039] NA 2016 2022 2013 2019 2015 2019 NA NA NA ...
 $ rookie_year             : int [1:20039] NA 2016 2022 2013 2019 2015 2019 NA NA NA ...
 $ draft_club              : chr [1:20039] NA "DET" "LA" NA ...
 $ college_conference      : chr [1:20039] NA "Southeastern Conference" "Big Ten Conference" "American A
 $ status_description_abbr : chr [1:20039] NA "A01" "A01" "R01" ...
 $ status_short_description: chr [1:20039] NA "Active" "Active" "R/Injured" ...
 $ gsis_it_id              : int [1:20039] NA 43335 54726 40688 47834 42410 48335 NA NA NA ...
 $ short_name              : chr [1:20039] NA "A.Robinson" "A.Arcuri" "A.Bouye" ...
```

```
 $ smart_id           : chr [1:20039] "3200454c-4c71-1319-728e-d49d3d236f8f" "3200524f-4236-7960-bf20
 $ headshot           : chr [1:20039] NA "https://static.www.nfl.com/image/private/f_auto,q_auto/leag
 $ draft_number         : int [1:20039] NA 46 261 NA 51 67 NA NA NA NA ...
 $ uniform_number          : chr [1:20039] NA "91" "61" "24" ...
 $ draft_round             : chr [1:20039] NA NA NA NA ...
 $ season               : int [1:20039] NA NA NA NA NA NA NA NA NA NA ...
 - attr(*, "nflverse_type")= chr "players"
 - attr(*, "nflverse_timestamp")= POSIXct[1:1], format: "2024-03-01 01:18:40"
```

3.11.2 Data Dimensions

Number of rows and columns:

```r
dim(nfl_players)
```

```
[1] 20039    33
```

3.11.3 Number of Elements

```r
length(nfl_players$display_name)
```

```
[1] 20039
```

3.11.4 Number of Missing Elements

```r
length(nfl_players$college_name[which(is.na(nfl_players$college_name))])
```

```
[1] 12127
```

3.11.5 Number of Non-Missing Elements

```r
length(nfl_players$college_name[which(!is.na(nfl_players$college_name))])
```

```
[1] 7912
```

```
length(na.omit(nfl_players$college_name))
```

```
[1] 7912
```

3.12 Create New Variables

To create a new variable, use the following syntax:

```
players$newVar <- NA
```

Here is an example of creating a new variable:

```
players$newVar <- 1:nrow(players)
```

3.13 Recode Variables

Here is an example of recoding a variable:

```
players$oldVar1 <- NA
players$oldVar1[which(players$position == "QB")] <- "quarterback"
players$oldVar1[which(players$position == "RB")] <- "running back"
players$oldVar1[which(players$position == "WR")] <- "wide receiver"
players$oldVar1[which(players$position == "TE")] <- "tight end"

players$oldVar2 <- NA
players$oldVar2[which(players$age < 30)] <- "young"
players$oldVar2[which(players$age >= 30)] <- "old"
```

Recode multiple variables:

```
players %>%
  mutate(across(c(
    oldVar1:oldVar2),
    ~ case_match(
      .,
      c("quarterback","old","running back") ~ 0,
      c("wide receiver","tight end","young") ~ 1)))
```

```
   ID             name position age oldVar1 oldVar2
1   1      Ken Cussion       QB  40       0       0
2   2       Ben Sacked       QB  30       0       0
3   3  Chuck Downfield       QB  24       0       1
4   4      Ron Ingback       RB  20       0       1
5   5      Rhonda Ball       RB  18       0       1
6   6        Hugo Long       WR  23       1       1
7   7 Lionel Scrimmage       WR  27       1       1
8   8       Drew Blood       WR  32       1       0
9   9     Chase Emdown       WR  26       1       1
10 10      Justin Time       TE  23       1       1
11 11     Spike D'Ball       TE  NA       1      NA
12 12       Isac Ulooz       LB  37      NA       0
```

3.14 Rename Variables

```
players <- players %>%
  rename(
    newVar1 = oldVar1,
    newVar2 = oldVar2)
```

Using a vector of variable names:

```
varNamesFrom <- c("oldVar1","oldVar2")
varNamesTo <- c("newVar1","newVar2")

players <- players %>%
  rename_with(~ varNamesTo, all_of(varNamesFrom))
```

3.15 Convert the Types of Variables

One variable:

```
players$factorVar <- factor(players$ID)
players$numericVar <- as.numeric(players$age)
```

```
players$integerVar <- as.integer(players$newVar1)
players$characterVar <- as.character(players$newVar2)
```

Multiple variables:

```
players %>%
  mutate(across(c(
    ID,
    age),
    as.numeric))
```

```
   ID            name position age      newVar1 newVar2 factorVar numericVar
1   1     Ken Cussion       QB  40  quarterback     old         1         40
2   2      Ben Sacked       QB  30  quarterback     old         2         30
3   3 Chuck Downfield       QB  24  quarterback   young         3         24
4   4     Ron Ingback       RB  20 running back   young         4         20
5   5     Rhonda Ball       RB  18 running back   young         5         18
6   6       Hugo Long       WR  23 wide receiver  young         6         23
7   7 Lionel Scrimmage      WR  27 wide receiver  young         7         27
8   8      Drew Blood       WR  32 wide receiver    old         8         32
9   9    Chase Emdown       WR  26 wide receiver  young         9         26
10 10     Justin Time       TE  23     tight end  young        10         23
11 11    Spike D'Ball       TE  NA     tight end   <NA>        11         NA
12 12      Isac Ulooz       LB  37          <NA>    old        12         37
   integerVar characterVar
1          NA          old
2          NA          old
3          NA        young
4          NA        young
5          NA        young
6          NA        young
7          NA        young
8          NA          old
9          NA        young
10         NA        young
11         NA         <NA>
12         NA          old
```

```
players %>%
  mutate(across(
    age:newVar1,
    as.character))
```

```
   ID          name position  age      newVar1 newVar2 factorVar numericVar
```

```
1   1      Ken Cussion      QB   40  quarterback     old         1         40
2   2       Ben Sacked      QB   30  quarterback     old         2         30
3   3  Chuck Downfield      QB   24  quarterback   young         3         24
4   4      Ron Ingback      RB   20 running back   young         4         20
5   5      Rhonda Ball      RB   18 running back   young         5         18
6   6        Hugo Long      WR   23 wide receiver  young         6         23
7   7 Lionel Scrimmage      WR   27 wide receiver  young         7         27
8   8       Drew Blood      WR   32 wide receiver    old         8         32
9   9     Chase Emdown      WR   26 wide receiver  young         9         26
10 10      Justin Time      TE   23    tight end   young        10         23
11 11     Spike D'Ball      TE <NA>    tight end    <NA>        11         NA
12 12       Isac Ulooz      LB   37         <NA>     old        12         37
   integerVar characterVar
1          NA          old
2          NA          old
3          NA        young
4          NA        young
5          NA        young
6          NA        young
7          NA        young
8          NA          old
9          NA        young
10         NA        young
11         NA         <NA>
12         NA          old
```

```
players %>%
  mutate(across(where(is.factor), as.character))
```

```
   ID             name position age       newVar1 newVar2 factorVar numericVar
1   1      Ken Cussion       QB  40   quarterback     old         1         40
2   2       Ben Sacked       QB  30   quarterback     old         2         30
3   3  Chuck Downfield       QB  24   quarterback   young         3         24
4   4      Ron Ingback       RB  20  running back   young         4         20
5   5      Rhonda Ball       RB  18  running back   young         5         18
6   6        Hugo Long       WR  23 wide receiver   young         6         23
7   7 Lionel Scrimmage       WR  27 wide receiver   young         7         27
8   8       Drew Blood       WR  32 wide receiver     old         8         32
9   9     Chase Emdown       WR  26 wide receiver   young         9         26
10 10      Justin Time       TE  23     tight end   young        10         23
11 11     Spike D'Ball       TE  NA     tight end    <NA>        11         NA
12 12       Isac Ulooz       LB  37          <NA>     old        12         37
   integerVar characterVar
1          NA          old
2          NA          old
```

```
3           NA        young
4           NA        young
5           NA        young
6           NA        young
7           NA        young
8           NA          old
9           NA        young
10          NA        young
11          NA         <NA>
12          NA          old
```

3.16 Merging/Joins

3.16.1 Overview

Merging (also called joining) merges two data objects using a shared set of variables called "keys." The keys are the variable(s) that uniquely identify each row (i.e., they account for the levels of nesting). In some data objects, the key might be the player's identification number (e.g., `player_id`). However, some data objects have multiple keys. For instance, in long form data objects, each participant may have multiple rows corresponding to multiple seasons. In this case, the keys may be `player_id` and `season`. If a participant has multiple rows corresponding to seasons and games/weeks, the keys are `player_id`, `season`, and `week`. In general, each row should have a value on each of the keys; there should be no missingness in the keys.

To merge two objects, the key(s) that will be used to match the records must be present in both objects. The keys are used to merge the variables in object 1 (`x`) with the variables in object 2 (`y`). Different merge types select different rows to merge.

Note: if the two objects include variables with the same name (apart from the keys), `R` will not know how you want each to appear in the merged object. So, it will add a suffix (e.g., `.x`, `.y`) to each common variable to indicate which object (i.e., object `x` or object `y`) the variable came from, where object `x` is the first object—i.e., the object to which object `y` (the second object) is merged. In general, apart from the keys, you should not include variables with the same name in two objects to be merged. To prevent this, either remove or rename the shared variable in one of the objects, or include the shared variable as a key. However, as described above, you should include it as a key ***only*** if it uniquely identifies each row in terms of levels of nesting.

3.16.2 Data Before Merging

Here are the data in the `players` object:

```
players
```

```
   ID           name position age      newVar1 newVar2 factorVar numericVar
1   1      Ken Cussion       QB  40   quarterback     old         1         40
2   2       Ben Sacked       QB  30   quarterback     old         2         30
3   3  Chuck Downfield       QB  24   quarterback   young         3         24
4   4      Ron Ingback       RB  20  running back   young         4         20
5   5      Rhonda Ball       RB  18  running back   young         5         18
6   6        Hugo Long       WR  23 wide receiver   young         6         23
7   7 Lionel Scrimmage       WR  27 wide receiver   young         7         27
8   8       Drew Blood       WR  32 wide receiver     old         8         32
9   9     Chase Emdown       WR  26 wide receiver   young         9         26
10 10      Justin Time       TE  23     tight end   young        10         23
11 11     Spike D'Ball       TE  NA     tight end    <NA>        11         NA
12 12       Isac Ulooz       LB  37          <NA>     old        12         37
   integerVar characterVar
1          NA          old
2          NA          old
3          NA        young
4          NA        young
5          NA        young
6          NA        young
7          NA        young
8          NA          old
9          NA        young
10         NA        young
11         NA         <NA>
12         NA          old
```

```
dim(players)
```

```
[1] 12 10
```

The data are structured in ID form. That is, every row in the dataset is uniquely identified by the variable, `ID`.

Here are the data in the `fantasyPoints` object:

```
fantasyPoints
```

```
  ID fantasyPoints
1  2           250
2  7           170
3 13            65
4 14            15
```

```
dim(fantasyPoints)
```

```
[1] 4 2
```

3.16.3 Types of Joins

3.16.3.1 Visual Overview of Join Types

Below is a visual that depicts various types of merges/joins. Object `x` is the circle labeled as `x`. Object `y` is the circle labeled as `y`. The area of overlap in the Venn diagram indicates the rows on the keys that are shared between the two objects (e.g., the same `player_id`, `season`, and `week`). The non-overlapping area indicates the rows on the keys that are unique to each object. The shaded blue area indicates which rows (on the keys) are kept in the merged object from each of the two objects, when using each of the merge types. For instance, a left outer join keeps the shared rows and the rows that are unique to object `x`, but it drops the rows that are unique to object `y`.

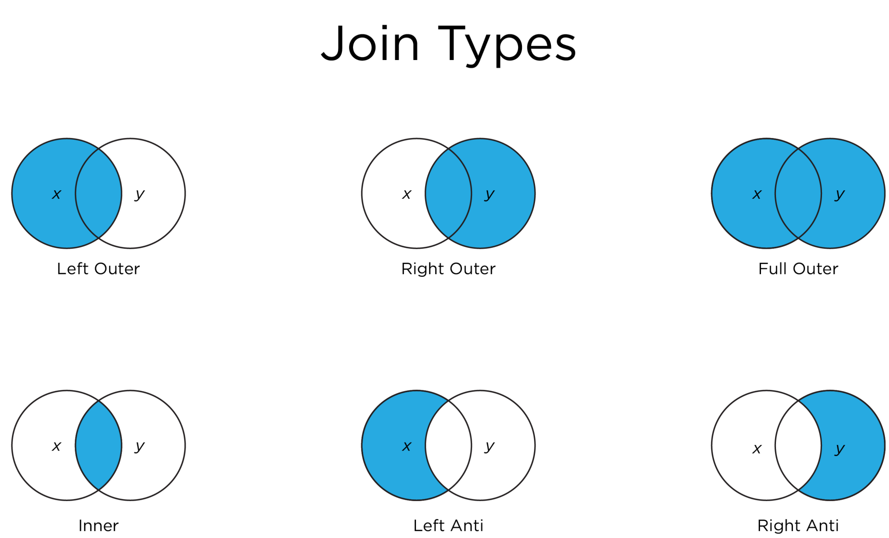

Figure 3.1 Types of merges/joins

3.16.3.2 Full Outer Join

A full outer join includes all rows in `x` **or** `y`. It returns columns from `x` and `y`. Here is how to merge two data frames using a full outer join (i.e., "full join"):

```r
fullJoinData <- full_join(
  players,
  fantasyPoints,
  by = "ID")

fullJoinData
```

```
   ID             name position age       newVar1 newVar2 factorVar numericVar
1   1      Ken Cussion       QB  40   quarterback     old         1         40
2   2       Ben Sacked       QB  30   quarterback     old         2         30
3   3  Chuck Downfield       QB  24   quarterback   young         3         24
4   4      Ron Ingback       RB  20  running back   young         4         20
5   5      Rhonda Ball       RB  18  running back   young         5         18
6   6        Hugo Long       WR  23 wide receiver   young         6         23
7   7 Lionel Scrimmage       WR  27 wide receiver   young         7         27
8   8       Drew Blood       WR  32 wide receiver     old         8         32
9   9     Chase Emdown       WR  26 wide receiver   young         9         26
10 10      Justin Time       TE  23     tight end   young        10         23
11 11     Spike D'Ball       TE  NA     tight end    <NA>        11         NA
12 12       Isac Ulooz       LB  37          <NA>     old        12         37
13 13             <NA>     <NA>  NA          <NA>    <NA>      <NA>         NA
14 14             <NA>     <NA>  NA          <NA>    <NA>      <NA>         NA
   integerVar characterVar fantasyPoints
1          NA          old            NA
2          NA          old           250
3          NA        young            NA
4          NA        young            NA
5          NA        young            NA
6          NA        young            NA
7          NA        young           170
8          NA          old            NA
9          NA        young            NA
10         NA        young            NA
11         NA         <NA>            NA
12         NA          old            NA
13         NA         <NA>            65
14         NA         <NA>            15
```

```
dim(fullJoinData)
```

```
[1] 14 11
```

3.16.3.3 Left Outer Join

A left outer join includes all rows in `x`. It returns columns from `x` and `y`. Here is how to merge two data frames using a left outer join ("left join"):

```
leftJoinData <- left_join(
  players,
  fantasyPoints,
  by = "ID")

leftJoinData
```

```
   ID            name position age      newVar1 newVar2 factorVar numericVar
1   1     Ken Cussion       QB  40   quarterback     old         1         40
2   2      Ben Sacked       QB  30   quarterback     old         2         30
3   3 Chuck Downfield       QB  24   quarterback   young         3         24
4   4      Ron Ingback       RB  20  running back   young         4         20
5   5     Rhonda Ball       RB  18  running back   young         5         18
6   6       Hugo Long       WR  23 wide receiver   young         6         23
7   7 Lionel Scrimmage      WR  27 wide receiver   young         7         27
8   8      Drew Blood       WR  32 wide receiver     old         8         32
9   9    Chase Emdown       WR  26 wide receiver   young         9         26
10 10     Justin Time       TE  23     tight end   young        10         23
11 11     Spike D'Ball      TE  NA     tight end    <NA>        11         NA
12 12      Isac Ulooz       LB  37          <NA>     old        12         37
   integerVar characterVar fantasyPoints
1          NA          old            NA
2          NA          old           250
3          NA        young            NA
4          NA        young            NA
5          NA        young            NA
6          NA        young            NA
7          NA        young           170
8          NA          old            NA
9          NA        young            NA
10         NA        young            NA
11         NA         <NA>            NA
12         NA          old            NA
```

```
dim(leftJoinData)
```

```
[1] 12 11
```

3.16.3.4 Right Outer Join

A right outer join includes all rows in `y`. It returns columns from `x` and `y`. Here is how to merge two data frames using a right outer join ("right join"):

```
rightJoinData <- right_join(
  players,
  fantasyPoints,
  by = "ID")

rightJoinData
```

```
  ID           name position age      newVar1 newVar2 factorVar numericVar
1  2      Ben Sacked       QB  30   quarterback     old         2         30
2  7 Lionel Scrimmage      WR  27 wide receiver   young         7         27
3 13            <NA>     <NA>  NA          <NA>    <NA>      <NA>         NA
4 14            <NA>     <NA>  NA          <NA>    <NA>      <NA>         NA
  integerVar characterVar fantasyPoints
1         NA          old           250
2         NA        young           170
3         NA         <NA>            65
4         NA         <NA>            15
```

```
dim(rightJoinData)
```

```
[1]  4 11
```

3.16.3.5 Inner Join

An inner join includes all rows that are in **both** `x` **and** `y`. An inner join will return one row of `x` for each matching row of `y`, and can duplicate values of records on either side (left or right) if `x` and `y` have more than one matching record. It returns columns from `x` and `y`. Here is how to merge two data frames using an inner join:

```
innerJoinData <- inner_join(
  players,
  fantasyPoints,
```

```
  by = "ID")

innerJoinData
```

```
  ID           name position age      newVar1 newVar2 factorVar numericVar
1  2     Ben Sacked       QB  30   quarterback     old         2         30
2  7 Lionel Scrimmage     WR  27 wide receiver   young         7         27
  integerVar characterVar fantasyPoints
1         NA          old           250
2         NA        young           170
```

```
dim(innerJoinData)
```

```
[1]  2 11
```

3.16.3.6 Semi Join

A semi join is a filter. A left semi join returns all rows from `x` **with** a match in `y`. That is, it filters out records from `x` that are not in `y`. Unlike an inner join, a left semi join will never duplicate rows of `x`, and it includes columns from only `x` (not from `y`). Here is how to merge two data frames using a left semi join:

```
semiJoinData <- semi_join(
  players,
  fantasyPoints,
  by = "ID")

semiJoinData
```

```
  ID           name position age      newVar1 newVar2 factorVar numericVar
1  2     Ben Sacked       QB  30   quarterback     old         2         30
2  7 Lionel Scrimmage     WR  27 wide receiver   young         7         27
  integerVar characterVar
1         NA          old
2         NA        young
```

```
dim(semiJoinData)
```

```
[1]  2 10
```

3.16.3.7 Anti Join

An anti join is a filter. A left anti join returns all rows from `x` **without** a match in `y`. That is, it filters out records from `x` that are in `y`. It returns columns from only `x` (not from `y`). Here is how to merge two data frames using a left anti join:

```
antiJoinData <- anti_join(
  players,
  fantasyPoints,
  by = "ID")

antiJoinData
```

```
   ID           name position age      newVar1 newVar2 factorVar numericVar
1   1     Ken Cussion       QB  40   quarterback     old         1         40
2   3 Chuck Downfield       QB  24   quarterback   young         3         24
3   4     Ron Ingback       RB  20  running back   young         4         20
4   5     Rhonda Ball       RB  18  running back   young         5         18
5   6       Hugo Long       WR  23 wide receiver   young         6         23
6   8      Drew Blood       WR  32 wide receiver     old         8         32
7   9    Chase Emdown       WR  26 wide receiver   young         9         26
8  10     Justin Time       TE  23     tight end   young        10         23
9  11    Spike D'Ball       TE  NA     tight end    <NA>        11         NA
10 12      Isac Ulooz       LB  37          <NA>     old        12         37
   integerVar characterVar
1          NA          old
2          NA        young
3          NA        young
4          NA        young
5          NA        young
6          NA          old
7          NA        young
8          NA        young
9          NA         <NA>
10         NA          old
```

```
dim(antiJoinData)
```

```
[1] 10 10
```

3.16.3.8 Cross Join

A cross join combines each row in `x` with each row in `y`.

```
crossJoinData <- cross_join(
  players,
  fantasyPoints)

crossJoinData
```

```
   ID.x             name position age       newVar1 newVar2 factorVar
1     1      Ken Cussion       QB  40    quarterback     old         1
2     1      Ken Cussion       QB  40    quarterback     old         1
3     1      Ken Cussion       QB  40    quarterback     old         1
4     1      Ken Cussion       QB  40    quarterback     old         1
5     2       Ben Sacked       QB  30    quarterback     old         2
6     2       Ben Sacked       QB  30    quarterback     old         2
7     2       Ben Sacked       QB  30    quarterback     old         2
8     2       Ben Sacked       QB  30    quarterback     old         2
9     3  Chuck Downfield       QB  24    quarterback   young         3
10    3  Chuck Downfield       QB  24    quarterback   young         3
11    3  Chuck Downfield       QB  24    quarterback   young         3
12    3  Chuck Downfield       QB  24    quarterback   young         3
13    4      Ron Ingback       RB  20   running back   young         4
14    4      Ron Ingback       RB  20   running back   young         4
15    4      Ron Ingback       RB  20   running back   young         4
16    4      Ron Ingback       RB  20   running back   young         4
17    5      Rhonda Ball       RB  18   running back   young         5
18    5      Rhonda Ball       RB  18   running back   young         5
19    5      Rhonda Ball       RB  18   running back   young         5
20    5      Rhonda Ball       RB  18   running back   young         5
21    6        Hugo Long       WR  23  wide receiver   young         6
22    6        Hugo Long       WR  23  wide receiver   young         6
23    6        Hugo Long       WR  23  wide receiver   young         6
24    6        Hugo Long       WR  23  wide receiver   young         6
25    7 Lionel Scrimmage       WR  27  wide receiver   young         7
26    7 Lionel Scrimmage       WR  27  wide receiver   young         7
27    7 Lionel Scrimmage       WR  27  wide receiver   young         7
28    7 Lionel Scrimmage       WR  27  wide receiver   young         7
29    8       Drew Blood       WR  32  wide receiver     old         8
30    8       Drew Blood       WR  32  wide receiver     old         8
31    8       Drew Blood       WR  32  wide receiver     old         8
32    8       Drew Blood       WR  32  wide receiver     old         8
33    9     Chase Emdown       WR  26  wide receiver   young         9
34    9     Chase Emdown       WR  26  wide receiver   young         9
35    9     Chase Emdown       WR  26  wide receiver   young         9
36    9     Chase Emdown       WR  26  wide receiver   young         9
37   10      Justin Time       TE  23      tight end   young        10
38   10      Justin Time       TE  23      tight end   young        10
```

```
39  10      Justin Time      TE  23     tight end   young        10
40  10      Justin Time      TE  23     tight end   young        10
41  11     Spike D'Ball      TE  NA     tight end    <NA>        11
42  11     Spike D'Ball      TE  NA     tight end    <NA>        11
43  11     Spike D'Ball      TE  NA     tight end    <NA>        11
44  11     Spike D'Ball      TE  NA     tight end    <NA>        11
45  12       Isac Ulooz      LB  37          <NA>     old        12
46  12       Isac Ulooz      LB  37          <NA>     old        12
47  12       Isac Ulooz      LB  37          <NA>     old        12
48  12       Isac Ulooz      LB  37          <NA>     old        12
   numericVar integerVar characterVar ID.y fantasyPoints
1          40         NA          old    2           250
2          40         NA          old    7           170
3          40         NA          old   13            65
4          40         NA          old   14            15
5          30         NA          old    2           250
6          30         NA          old    7           170
7          30         NA          old   13            65
8          30         NA          old   14            15
9          24         NA        young    2           250
10         24         NA        young    7           170
11         24         NA        young   13            65
12         24         NA        young   14            15
13         20         NA        young    2           250
14         20         NA        young    7           170
15         20         NA        young   13            65
16         20         NA        young   14            15
17         18         NA        young    2           250
18         18         NA        young    7           170
19         18         NA        young   13            65
20         18         NA        young   14            15
21         23         NA        young    2           250
22         23         NA        young    7           170
23         23         NA        young   13            65
24         23         NA        young   14            15
25         27         NA        young    2           250
26         27         NA        young    7           170
27         27         NA        young   13            65
28         27         NA        young   14            15
29         32         NA          old    2           250
30         32         NA          old    7           170
31         32         NA          old   13            65
32         32         NA          old   14            15
33         26         NA        young    2           250
34         26         NA        young    7           170
```

```
35          26          NA          young   13              65
36          26          NA          young   14              15
37          23          NA          young    2             250
38          23          NA          young    7             170
39          23          NA          young   13              65
40          23          NA          young   14              15
41          NA          NA           <NA>    2             250
42          NA          NA           <NA>    7             170
43          NA          NA           <NA>   13              65
44          NA          NA           <NA>   14              15
45          37          NA            old    2             250
46          37          NA            old    7             170
47          37          NA            old   13              65
48          37          NA            old   14              15
```

```
dim(crossJoinData)
```

```
[1] 48 12
```

3.17 Transform Data from Long to Wide

Depending on the analysis, it may be important to restructure the data to be in long or wide form. When the data are in wide form, each player has only one row. When the data are in long form, each player has multiple rows—e.g., a row for each game. The data structure is called wide or long form because a dataset in wide form has more columns and fewer rows (i.e., it appears wider and shorter), whereas a dataset in long form has more rows and fewer columns (i.e., it appears narrower and taller).

Here are the data in the `nfl_actualStats_offense_weekly` object. The data are structured in "player-season-week form". That is, every row in the dataset is uniquely identified by the variables, `player_id`, `season`, and `week`. This is an example of long form, because each player has multiple rows.

Original data:

```
dataLong <- nfl_actualStats_offense_weekly %>%
  select(player_id, player_display_name, season, week, fantasy_points)

dim(dataLong)
```

```
[1] 129739      5
```

```
names(dataLong)
```

```
[1] "player_id"           "player_display_name" "season"
[4] "week"                "fantasy_points"
```

Below, we widen the data by two variables (`season` and `week`), using `tidyverse`, so that the data are now in "player form" (where each row is uniquely identified by the `player_id` variable):

```
dataWide <- dataLong %>%
  pivot_wider(
    names_from = c(season, week),
    names_glue = "{.value}_{season}_week{week}",
    values_from = fantasy_points)

dim(dataWide)
```

```
[1] 4021  530
```

```
names(dataWide)
```

```
  [1] "player_id"                   "player_display_name"
  [3] "fantasy_points_1999_week1"   "fantasy_points_1999_week2"
  [5] "fantasy_points_1999_week4"   "fantasy_points_1999_week7"
  [7] "fantasy_points_1999_week8"   "fantasy_points_1999_week9"
  [9] "fantasy_points_1999_week10" "fantasy_points_1999_week11"
 [11] "fantasy_points_1999_week12" "fantasy_points_1999_week13"
 [13] "fantasy_points_1999_week14" "fantasy_points_1999_week15"
 [15] "fantasy_points_1999_week16" "fantasy_points_1999_week5"
 [17] "fantasy_points_1999_week6"   "fantasy_points_1999_week17"
 [19] "fantasy_points_1999_week18" "fantasy_points_1999_week3"
 [21] "fantasy_points_1999_week19" "fantasy_points_1999_week20"
 [23] "fantasy_points_1999_week21" "fantasy_points_2000_week1"
 [25] "fantasy_points_2000_week12" "fantasy_points_2000_week14"
 [27] "fantasy_points_2000_week15" "fantasy_points_2000_week6"
 [29] "fantasy_points_2000_week10" "fantasy_points_2000_week4"
 [31] "fantasy_points_2000_week5"   "fantasy_points_2000_week7"
 [33] "fantasy_points_2000_week8"   "fantasy_points_2000_week9"
 [35] "fantasy_points_2000_week11" "fantasy_points_2000_week13"
 [37] "fantasy_points_2000_week2"   "fantasy_points_2000_week16"
 [39] "fantasy_points_2000_week17" "fantasy_points_2000_week3"
 [41] "fantasy_points_2000_week18" "fantasy_points_2000_week19"
 [43] "fantasy_points_2000_week21" "fantasy_points_2000_week20"
```

```
 [45] "fantasy_points_2001_week15" "fantasy_points_2001_week17"
 [47] "fantasy_points_2001_week1"  "fantasy_points_2001_week3"
 [49] "fantasy_points_2001_week4"  "fantasy_points_2001_week5"
 [51] "fantasy_points_2001_week6"  "fantasy_points_2001_week9"
 [53] "fantasy_points_2001_week11" "fantasy_points_2001_week12"
 [55] "fantasy_points_2001_week13" "fantasy_points_2001_week14"
 [57] "fantasy_points_2001_week16" "fantasy_points_2001_week2"
 [59] "fantasy_points_2001_week7"  "fantasy_points_2001_week8"
 [61] "fantasy_points_2001_week10" "fantasy_points_2001_week19"
 [63] "fantasy_points_2001_week18" "fantasy_points_2001_week20"
 [65] "fantasy_points_2001_week21" "fantasy_points_2002_week3"
 [67] "fantasy_points_2002_week1"  "fantasy_points_2002_week2"
 [69] "fantasy_points_2002_week4"  "fantasy_points_2002_week6"
 [71] "fantasy_points_2002_week7"  "fantasy_points_2002_week8"
 [73] "fantasy_points_2002_week9"  "fantasy_points_2002_week5"
 [75] "fantasy_points_2002_week10" "fantasy_points_2002_week11"
 [77] "fantasy_points_2002_week12" "fantasy_points_2002_week13"
 [79] "fantasy_points_2002_week14" "fantasy_points_2002_week15"
 [81] "fantasy_points_2002_week16" "fantasy_points_2002_week17"
 [83] "fantasy_points_2002_week19" "fantasy_points_2002_week20"
 [85] "fantasy_points_2002_week21" "fantasy_points_2002_week18"
 [87] "fantasy_points_2003_week2"  "fantasy_points_2003_week4"
 [89] "fantasy_points_2003_week5"  "fantasy_points_2003_week7"
 [91] "fantasy_points_2003_week8"  "fantasy_points_2003_week9"
 [93] "fantasy_points_2003_week10" "fantasy_points_2003_week11"
 [95] "fantasy_points_2003_week13" "fantasy_points_2003_week14"
 [97] "fantasy_points_2003_week15" "fantasy_points_2003_week17"
 [99] "fantasy_points_2003_week1"  "fantasy_points_2003_week3"
[101] "fantasy_points_2003_week6"  "fantasy_points_2003_week12"
[103] "fantasy_points_2003_week16" "fantasy_points_2003_week18"
[105] "fantasy_points_2003_week19" "fantasy_points_2003_week20"
[107] "fantasy_points_2003_week21" "fantasy_points_2004_week2"
[109] "fantasy_points_2004_week5"  "fantasy_points_2004_week6"
[111] "fantasy_points_2004_week10" "fantasy_points_2004_week16"
[113] "fantasy_points_2004_week17" "fantasy_points_2004_week1"
[115] "fantasy_points_2004_week3"  "fantasy_points_2004_week7"
[117] "fantasy_points_2004_week8"  "fantasy_points_2004_week9"
[119] "fantasy_points_2004_week11" "fantasy_points_2004_week12"
[121] "fantasy_points_2004_week13" "fantasy_points_2004_week14"
[123] "fantasy_points_2004_week15" "fantasy_points_2004_week4"
[125] "fantasy_points_2004_week19" "fantasy_points_2004_week20"
[127] "fantasy_points_2004_week21" "fantasy_points_2004_week18"
[129] "fantasy_points_2005_week1"  "fantasy_points_2005_week2"
[131] "fantasy_points_2005_week3"  "fantasy_points_2005_week4"
[133] "fantasy_points_2005_week6"  "fantasy_points_2005_week7"
```

```
[135] "fantasy_points_2005_week8"  "fantasy_points_2005_week10"
[137] "fantasy_points_2005_week11" "fantasy_points_2005_week13"
[139] "fantasy_points_2005_week14" "fantasy_points_2005_week15"
[141] "fantasy_points_2005_week16" "fantasy_points_2005_week17"
[143] "fantasy_points_2005_week5"  "fantasy_points_2005_week9"
[145] "fantasy_points_2005_week12" "fantasy_points_2005_week18"
[147] "fantasy_points_2005_week19" "fantasy_points_2005_week20"
[149] "fantasy_points_2005_week21" "fantasy_points_2006_week4"
[151] "fantasy_points_2006_week1"  "fantasy_points_2006_week2"
[153] "fantasy_points_2006_week3"  "fantasy_points_2006_week10"
[155] "fantasy_points_2006_week11" "fantasy_points_2006_week12"
[157] "fantasy_points_2006_week13" "fantasy_points_2006_week14"
[159] "fantasy_points_2006_week5"  "fantasy_points_2006_week6"
[161] "fantasy_points_2006_week7"  "fantasy_points_2006_week15"
[163] "fantasy_points_2006_week16" "fantasy_points_2006_week17"
[165] "fantasy_points_2006_week8"  "fantasy_points_2006_week9"
[167] "fantasy_points_2006_week18" "fantasy_points_2006_week19"
[169] "fantasy_points_2006_week20" "fantasy_points_2006_week21"
[171] "fantasy_points_2007_week1"  "fantasy_points_2007_week2"
[173] "fantasy_points_2007_week3"  "fantasy_points_2007_week5"
[175] "fantasy_points_2007_week9"  "fantasy_points_2007_week16"
[177] "fantasy_points_2007_week17" "fantasy_points_2007_week14"
[179] "fantasy_points_2007_week4"  "fantasy_points_2007_week6"
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[189] "fantasy_points_2007_week21" "fantasy_points_2007_week18"
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[195] "fantasy_points_2008_week3"  "fantasy_points_2008_week4"
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[215] "fantasy_points_2009_week2"  "fantasy_points_2009_week3"
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[221] "fantasy_points_2009_week14" "fantasy_points_2009_week15"
[223] "fantasy_points_2009_week16" "fantasy_points_2009_week17"
```

```
[225] "fantasy_points_2009_week1"  "fantasy_points_2009_week4"
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[229] "fantasy_points_2009_week10" "fantasy_points_2009_week18"
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[245] "fantasy_points_2010_week15" "fantasy_points_2010_week16"
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[279] "fantasy_points_2012_week3"  "fantasy_points_2012_week4"
[281] "fantasy_points_2012_week5"  "fantasy_points_2012_week7"
[283] "fantasy_points_2012_week8"  "fantasy_points_2012_week9"
[285] "fantasy_points_2012_week11" "fantasy_points_2012_week16"
[287] "fantasy_points_2012_week1"  "fantasy_points_2012_week6"
[289] "fantasy_points_2012_week10" "fantasy_points_2012_week14"
[291] "fantasy_points_2012_week15" "fantasy_points_2012_week17"
[293] "fantasy_points_2012_week19" "fantasy_points_2012_week20"
[295] "fantasy_points_2012_week21" "fantasy_points_2012_week18"
[297] "fantasy_points_2013_week1"  "fantasy_points_2013_week2"
[299] "fantasy_points_2013_week3"  "fantasy_points_2013_week4"
[301] "fantasy_points_2013_week5"  "fantasy_points_2013_week7"
[303] "fantasy_points_2013_week8"  "fantasy_points_2013_week9"
[305] "fantasy_points_2013_week10" "fantasy_points_2013_week11"
[307] "fantasy_points_2013_week12" "fantasy_points_2013_week13"
[309] "fantasy_points_2013_week14" "fantasy_points_2013_week15"
[311] "fantasy_points_2013_week16" "fantasy_points_2013_week17"
[313] "fantasy_points_2013_week6"  "fantasy_points_2013_week19"
```

```
[315] "fantasy_points_2013_week20" "fantasy_points_2013_week21"
[317] "fantasy_points_2013_week18" "fantasy_points_2014_week3"
[319] "fantasy_points_2014_week4"  "fantasy_points_2014_week16"
[321] "fantasy_points_2014_week17" "fantasy_points_2014_week1"
[323] "fantasy_points_2014_week2"  "fantasy_points_2014_week5"
[325] "fantasy_points_2014_week6"  "fantasy_points_2014_week7"
[327] "fantasy_points_2014_week8"  "fantasy_points_2014_week9"
[329] "fantasy_points_2014_week10" "fantasy_points_2014_week11"
[331] "fantasy_points_2014_week12" "fantasy_points_2014_week13"
[333] "fantasy_points_2014_week14" "fantasy_points_2014_week15"
[335] "fantasy_points_2014_week19" "fantasy_points_2014_week20"
[337] "fantasy_points_2014_week21" "fantasy_points_2014_week18"
[339] "fantasy_points_2015_week4"  "fantasy_points_2015_week5"
[341] "fantasy_points_2015_week11" "fantasy_points_2015_week12"
[343] "fantasy_points_2015_week13" "fantasy_points_2015_week14"
[345] "fantasy_points_2015_week15" "fantasy_points_2015_week16"
[347] "fantasy_points_2015_week1"  "fantasy_points_2015_week2"
[349] "fantasy_points_2015_week3"  "fantasy_points_2015_week6"
[351] "fantasy_points_2015_week8"  "fantasy_points_2015_week9"
[353] "fantasy_points_2015_week10" "fantasy_points_2015_week17"
[355] "fantasy_points_2015_week19" "fantasy_points_2015_week20"
[357] "fantasy_points_2015_week21" "fantasy_points_2015_week7"
[359] "fantasy_points_2015_week18" "fantasy_points_2016_week5"
[361] "fantasy_points_2016_week6"  "fantasy_points_2016_week7"
[363] "fantasy_points_2016_week8"  "fantasy_points_2016_week10"
[365] "fantasy_points_2016_week11" "fantasy_points_2016_week12"
[367] "fantasy_points_2016_week13" "fantasy_points_2016_week14"
[369] "fantasy_points_2016_week15" "fantasy_points_2016_week16"
[371] "fantasy_points_2016_week17" "fantasy_points_2016_week19"
[373] "fantasy_points_2016_week20" "fantasy_points_2016_week21"
[375] "fantasy_points_2016_week1"  "fantasy_points_2016_week2"
[377] "fantasy_points_2016_week3"  "fantasy_points_2016_week4"
[379] "fantasy_points_2016_week9"  "fantasy_points_2016_week18"
[381] "fantasy_points_2017_week1"  "fantasy_points_2017_week2"
[383] "fantasy_points_2017_week3"  "fantasy_points_2017_week4"
[385] "fantasy_points_2017_week5"  "fantasy_points_2017_week6"
[387] "fantasy_points_2017_week7"  "fantasy_points_2017_week8"
[389] "fantasy_points_2017_week10" "fantasy_points_2017_week11"
[391] "fantasy_points_2017_week12" "fantasy_points_2017_week13"
[393] "fantasy_points_2017_week14" "fantasy_points_2017_week15"
[395] "fantasy_points_2017_week16" "fantasy_points_2017_week17"
[397] "fantasy_points_2017_week19" "fantasy_points_2017_week20"
[399] "fantasy_points_2017_week21" "fantasy_points_2017_week9"
[401] "fantasy_points_2017_week18" "fantasy_points_2018_week1"
[403] "fantasy_points_2018_week2"  "fantasy_points_2018_week3"
```

```
[405] "fantasy_points_2018_week4"  "fantasy_points_2018_week5"
[407] "fantasy_points_2018_week6"  "fantasy_points_2018_week7"
[409] "fantasy_points_2018_week8"  "fantasy_points_2018_week9"
[411] "fantasy_points_2018_week10" "fantasy_points_2018_week12"
[413] "fantasy_points_2018_week13" "fantasy_points_2018_week14"
[415] "fantasy_points_2018_week15" "fantasy_points_2018_week16"
[417] "fantasy_points_2018_week17" "fantasy_points_2018_week19"
[419] "fantasy_points_2018_week20" "fantasy_points_2018_week21"
[421] "fantasy_points_2018_week11" "fantasy_points_2018_week18"
[423] "fantasy_points_2019_week1"  "fantasy_points_2019_week2"
[425] "fantasy_points_2019_week3"  "fantasy_points_2019_week4"
[427] "fantasy_points_2019_week5"  "fantasy_points_2019_week6"
[429] "fantasy_points_2019_week7"  "fantasy_points_2019_week8"
[431] "fantasy_points_2019_week9"  "fantasy_points_2019_week11"
[433] "fantasy_points_2019_week12" "fantasy_points_2019_week13"
[435] "fantasy_points_2019_week14" "fantasy_points_2019_week15"
[437] "fantasy_points_2019_week16" "fantasy_points_2019_week17"
[439] "fantasy_points_2019_week18" "fantasy_points_2019_week10"
[441] "fantasy_points_2019_week19" "fantasy_points_2019_week20"
[443] "fantasy_points_2019_week21" "fantasy_points_2020_week1"
[445] "fantasy_points_2020_week2"  "fantasy_points_2020_week3"
[447] "fantasy_points_2020_week4"  "fantasy_points_2020_week5"
[449] "fantasy_points_2020_week6"  "fantasy_points_2020_week7"
[451] "fantasy_points_2020_week8"  "fantasy_points_2020_week9"
[453] "fantasy_points_2020_week10" "fantasy_points_2020_week11"
[455] "fantasy_points_2020_week12" "fantasy_points_2020_week14"
[457] "fantasy_points_2020_week15" "fantasy_points_2020_week16"
[459] "fantasy_points_2020_week17" "fantasy_points_2020_week18"
[461] "fantasy_points_2020_week19" "fantasy_points_2020_week20"
[463] "fantasy_points_2020_week21" "fantasy_points_2020_week13"
[465] "fantasy_points_2021_week1"  "fantasy_points_2021_week2"
[467] "fantasy_points_2021_week3"  "fantasy_points_2021_week4"
[469] "fantasy_points_2021_week5"  "fantasy_points_2021_week6"
[471] "fantasy_points_2021_week7"  "fantasy_points_2021_week8"
[473] "fantasy_points_2021_week10" "fantasy_points_2021_week11"
[475] "fantasy_points_2021_week12" "fantasy_points_2021_week13"
[477] "fantasy_points_2021_week14" "fantasy_points_2021_week15"
[479] "fantasy_points_2021_week16" "fantasy_points_2021_week17"
[481] "fantasy_points_2021_week18" "fantasy_points_2021_week19"
[483] "fantasy_points_2021_week20" "fantasy_points_2021_week9"
[485] "fantasy_points_2021_week21" "fantasy_points_2021_week22"
[487] "fantasy_points_2022_week1"  "fantasy_points_2022_week2"
[489] "fantasy_points_2022_week3"  "fantasy_points_2022_week4"
[491] "fantasy_points_2022_week5"  "fantasy_points_2022_week6"
[493] "fantasy_points_2022_week7"  "fantasy_points_2022_week8"
```

```
[495] "fantasy_points_2022_week9"  "fantasy_points_2022_week10"
[497] "fantasy_points_2022_week12" "fantasy_points_2022_week13"
[499] "fantasy_points_2022_week14" "fantasy_points_2022_week15"
[501] "fantasy_points_2022_week16" "fantasy_points_2022_week17"
[503] "fantasy_points_2022_week18" "fantasy_points_2022_week19"
[505] "fantasy_points_2022_week11" "fantasy_points_2022_week20"
[507] "fantasy_points_2022_week21" "fantasy_points_2022_week22"
[509] "fantasy_points_2023_week1"  "fantasy_points_2023_week4"
[511] "fantasy_points_2023_week7"  "fantasy_points_2023_week11"
[513] "fantasy_points_2023_week14" "fantasy_points_2023_week16"
[515] "fantasy_points_2023_week13" "fantasy_points_2023_week15"
[517] "fantasy_points_2023_week17" "fantasy_points_2023_week19"
[519] "fantasy_points_2023_week2"  "fantasy_points_2023_week3"
[521] "fantasy_points_2023_week5"  "fantasy_points_2023_week6"
[523] "fantasy_points_2023_week8"  "fantasy_points_2023_week12"
[525] "fantasy_points_2023_week18" "fantasy_points_2023_week10"
[527] "fantasy_points_2023_week21" "fantasy_points_2023_week22"
[529] "fantasy_points_2023_week9"  "fantasy_points_2023_week20"
```

3.18 Transform Data from Wide to Long

Conversely, we can also restructure data from wide to long.

Original data:

```
dataWide <- nfl_actualStats_offense_weekly %>%
  select(player_id, player_display_name, season, week, recent_team, opponent_team)

dim(dataWide)
```

```
[1] 129739      6
```

```
names(dataWide)
```

```
[1] "player_id"           "player_display_name" "season"
[4] "week"                "recent_team"         "opponent_team"
```

Data in long form, transformed from wide form using `tidyverse`:

```
dataLong <- dataWide %>%
  pivot_longer(
    cols = c(recent_team, opponent_team),
    names_to = "role",
    values_to = "team")

dim(dataLong)
```

```
[1] 259478      6
```

```
names(dataLong)
```

```
[1] "player_id"           "player_display_name" "season"
[4] "week"                "role"                "team"
```

3.19 Loops

If you want to perform the same computation multiple times, it can be faster to do it in a loop compared to writing out the same computation many times. For instance, here is a loop that runs from 1 to 12 (the number of players in the `players` object), incrementing by 1 after each iteration. The loop prints each element of a vector (i.e., the player's name) and the loop index (`i`) that indicates where the loop is in terms of its iterations:

```
for(i in 1:length(players$ID)){
  print(paste("The loop is at index:", i, sep = " "))
  print(paste("My favorite player is:", players$name[i], sep = " "))
}
```

```
[1] "The loop is at index: 1"
[1] "My favorite player is: Ken Cussion"
[1] "The loop is at index: 2"
[1] "My favorite player is: Ben Sacked"
[1] "The loop is at index: 3"
[1] "My favorite player is: Chuck Downfield"
[1] "The loop is at index: 4"
[1] "My favorite player is: Ron Ingback"
[1] "The loop is at index: 5"
[1] "My favorite player is: Rhonda Ball"
```

```
[1] "The loop is at index: 6"
[1] "My favorite player is: Hugo Long"
[1] "The loop is at index: 7"
[1] "My favorite player is: Lionel Scrimmage"
[1] "The loop is at index: 8"
[1] "My favorite player is: Drew Blood"
[1] "The loop is at index: 9"
[1] "My favorite player is: Chase Emdown"
[1] "The loop is at index: 10"
[1] "My favorite player is: Justin Time"
[1] "The loop is at index: 11"
[1] "My favorite player is: Spike D'Ball"
[1] "The loop is at index: 12"
[1] "My favorite player is: Isac Ulooz"
```

3.20 Calculations

3.20.1 Historical Actual Player Statistics

In addition to week-by-week actual player statistics, we can also compute historical actual player statistics as a function of different timeframes, including season-by-season and career statistics.

3.20.1.1 Career Statistics

First, we can compute the players' career statistics using the `calculate_player_stats()`, `calculate_player_stats_def()`, and `calculate_player_stats_kicking()` functions from the `nflfastR` package for offensive players, defensive players, and kickers, respectively.

> **Note 3: Calculating players' career statistics**
>
> Note: the following code takes a while to run.

```
nfl_actualStats_offense_career <- nflfastR::calculate_player_stats(
  nfl_pbp,
  weekly = FALSE)

nfl_actualStats_defense_career <- nflfastR::calculate_player_stats_def(
  nfl_pbp,
```

```
  weekly = FALSE)

nfl_actualStats_kicking_career <- nflfastR::calculate_player_stats_kicking(
  nfl_pbp,
  weekly = FALSE)
```

3.20.1.2 Season-by-Season Statistics

Second, we can compute the players' season-by-season statistics.

```
seasons <- unique(nfl_pbp$season)

nfl_pbp_seasonalList <- list()
nfl_actualStats_offense_seasonalList <- list()
nfl_actualStats_defense_seasonalList <- list()
nfl_actualStats_kicking_seasonalList <- list()
```

> **Note 4: Calculating players' season-by-season statistics**
>
> Note: the following code takes a while to run.

```
pb <- txtProgressBar(
  min = 0,
  max = length(seasons),
  style = 3)

for(i in 1:length(seasons)){
  # Subset play-by-play data by season
  nfl_pbp_seasonalList[[i]] <- nfl_pbp %>%
    filter(season == seasons[i])

  # Compute actual statistics by season
  nfl_actualStats_offense_seasonalList[[i]] <-
    nflfastR::calculate_player_stats(
      nfl_pbp_seasonalList[[i]],
      weekly = FALSE)

  nfl_actualStats_defense_seasonalList[[i]] <-
    nflfastR::calculate_player_stats_def(
      nfl_pbp_seasonalList[[i]],
      weekly = FALSE)

  nfl_actualStats_kicking_seasonalList[[i]] <-
```

```
    nflfastR::calculate_player_stats_kicking(
      nfl_pbp_seasonalList[[i]],
      weekly = FALSE)

  nfl_actualStats_offense_seasonalList[[i]]$season <- seasons[i]
  nfl_actualStats_defense_seasonalList[[i]]$season <- seasons[i]
  nfl_actualStats_kicking_seasonalList[[i]]$season <- seasons[i]

  print(
    paste("Completed computing projections for season: ", seasons[i], sep = ""))

  # Update the progress bar
  setTxtProgressBar(pb, i)
}

# Close the progress bar
close(pb)

nfl_actualStats_offense_seasonal <- nfl_actualStats_offense_seasonalList %>%
  bind_rows()
nfl_actualStats_defense_seasonal <- nfl_actualStats_defense_seasonalList %>%
  bind_rows()
nfl_actualStats_kicking_seasonal <- nfl_actualStats_kicking_seasonalList %>%
  bind_rows()
```

3.20.1.3 Week-by-Week Statistics

We already load players' week-by-week statistics above. Nevertheless, we could compute players' weekly statistics from the play-by-play data using the following syntax:

```
nfl_actualStats_offense_weekly <- nflfastR::calculate_player_stats(
  nfl_pbp,
  weekly = TRUE)

nfl_actualStats_defense_weekly <- nflfastR::calculate_player_stats_def(
  nfl_pbp,
  weekly = TRUE)

nfl_actualStats_kicking_weekly <- nflfastR::calculate_player_stats_kicking(
  nfl_pbp,
  weekly = TRUE)
```

3.20.2 Historical Actual Fantasy Points

Specify scoring settings:

3.20.2.1 Weekly

3.20.2.2 Seasonal

3.20.2.3 Career

3.20.3 Player Age

```
# Reshape from wide to long format
nfl_actualStats_offense_weekly_long <- nfl_actualStats_offense_weekly %>%
  pivot_longer(
    cols = c(recent_team, opponent_team),
    names_to = "role",
    values_to = "team")

# Perform separate inner join operations for the home_team and away_team
nfl_actualStats_offense_weekly_home <- inner_join(
  nfl_actualStats_offense_weekly_long,
  nfl_schedules,
  by = c("season","week","team" = "home_team")) %>%
  mutate(home_away = "home_team")

nfl_actualStats_offense_weekly_away <- inner_join(
  nfl_actualStats_offense_weekly_long,
  nfl_schedules,
  by = c("season","week","team" = "away_team")) %>%
  mutate(home_away = "away_team")

nfl_actualStats_defense_weekly_home <- inner_join(
  nfl_actualStats_defense_weekly,
  nfl_schedules,
  by = c("season","week","team" = "home_team")) %>%
  mutate(home_away = "home_team")

nfl_actualStats_defense_weekly_away <- inner_join(
  nfl_actualStats_defense_weekly,
  nfl_schedules,
  by = c("season","week","team" = "away_team")) %>%
  mutate(home_away = "away_team")
```

```
nfl_actualStats_kicking_weekly_home <- inner_join(
  nfl_actualStats_kicking_weekly,
  nfl_schedules,
  by = c("season","week","team" = "home_team")) %>%
  mutate(home_away = "home_team")

nfl_actualStats_kicking_weekly_away <- inner_join(
  nfl_actualStats_kicking_weekly,
  nfl_schedules,
  by = c("season","week","team" = "away_team")) %>%
  mutate(home_away = "away_team")

# Combine the results of the join operations
nfl_actualStats_offense_weekly_schedules_long <- bind_rows(
  nfl_actualStats_offense_weekly_home,
  nfl_actualStats_offense_weekly_away)

nfl_actualStats_defense_weekly_schedules_long <- bind_rows(
  nfl_actualStats_defense_weekly_home,
  nfl_actualStats_defense_weekly_away)

nfl_actualStats_kicking_weekly_schedules_long <- bind_rows(
  nfl_actualStats_kicking_weekly_home,
  nfl_actualStats_kicking_weekly_away)

# Reshape from long to wide
player_game_gameday_offense <- nfl_actualStats_offense_weekly_schedules_long %>%
  distinct(player_id, season, week, game_id, home_away, team, gameday) %>% #, .keep_all = TRUE
  pivot_wider(
    names_from = home_away,
    values_from = team)

player_game_gameday_defense <- nfl_actualStats_defense_weekly_schedules_long %>%
  distinct(player_id, season, week, game_id, home_away, team, gameday) %>% #, .keep_all = TRUE
  pivot_wider(
    names_from = home_away,
    values_from = team)

player_game_gameday_kicking <- nfl_actualStats_kicking_weekly_schedules_long %>%
  distinct(player_id, season, week, game_id, home_away, team, gameday) %>% #, .keep_all = TRUE
  pivot_wider(
    names_from = home_away,
    values_from = team)
```

```
# Merge player birthdate and the game date
player_game_birthdate_gameday_offense <- left_join(
  player_game_gameday_offense,
  unique(nfl_players[,c("gsis_id","birth_date")]),
  by = c("player_id" = "gsis_id")
)

player_game_birthdate_gameday_defense <- left_join(
  player_game_gameday_defense,
  unique(nfl_players[,c("gsis_id","birth_date")]),
  by = c("player_id" = "gsis_id")
)

player_game_birthdate_gameday_kicking <- left_join(
  player_game_gameday_kicking,
  unique(nfl_players[,c("gsis_id","birth_date")]),
  by = c("player_id" = "gsis_id")
)

player_game_birthdate_gameday_offense$birth_date <- ymd(player_game_birthdate_gameday_offense$birth
player_game_birthdate_gameday_offense$gameday <- ymd(player_game_birthdate_gameday_offense$gameday

player_game_birthdate_gameday_defense$birth_date <- ymd(player_game_birthdate_gameday_defense$birth
player_game_birthdate_gameday_defense$gameday <- ymd(player_game_birthdate_gameday_defense$gameday

player_game_birthdate_gameday_kicking$birth_date <- ymd(player_game_birthdate_gameday_kicking$birth
player_game_birthdate_gameday_kicking$gameday <- ymd(player_game_birthdate_gameday_kicking$gameday

# Calculate player's age for a given week as the difference between their birthdate and the game d
player_game_birthdate_gameday_offense$age <- interval(
  start = player_game_birthdate_gameday_offense$birth_date,
  end = player_game_birthdate_gameday_offense$gameday
) %>%
  time_length(unit = "years")

player_game_birthdate_gameday_defense$age <- interval(
  start = player_game_birthdate_gameday_defense$birth_date,
  end = player_game_birthdate_gameday_defense$gameday
) %>%
  time_length(unit = "years")

player_game_birthdate_gameday_kicking$age <- interval(
  start = player_game_birthdate_gameday_kicking$birth_date,
  end = player_game_birthdate_gameday_kicking$gameday
```

```
) %>%
  time_length(unit = "years")

# Merge with player info
player_age_offense <-  left_join(
  player_game_birthdate_gameday_offense,
  nfl_players %>% select(-birth_date, -season),
  by = c("player_id" = "gsis_id"))

player_age_defense <-  left_join(
  player_game_birthdate_gameday_defense,
  nfl_players %>% select(-birth_date, -season),
  by = c("player_id" = "gsis_id"))

player_age_kicking <-  left_join(
  player_game_birthdate_gameday_kicking,
  nfl_players %>% select(-birth_date, -season),
  by = c("player_id" = "gsis_id"))

# Add game_id to weekly stats to facilitate merging
nfl_actualStats_game_offense_weekly <- nfl_actualStats_offense_weekly %>%
  left_join(
    player_age_offense[,c("season","week","player_id","game_id")],
    by = c("season","week","player_id"))

nfl_actualStats_game_defense_weekly <- nfl_actualStats_defense_weekly %>%
  left_join(
    player_age_offense[,c("season","week","player_id","game_id")],
    by = c("season","week","player_id"))

nfl_actualStats_game_kicking_weekly <- nfl_actualStats_kicking_weekly %>%
  left_join(
    player_age_offense[,c("season","week","player_id","game_id")],
    by = c("season","week","player_id"))

# Merge with player weekly stats
player_age_stats_offense <- left_join(
  player_age_offense %>% select(-position, -position_group),
  nfl_actualStats_game_offense_weekly,
  by = c(c("season","week","player_id","game_id")))

player_age_stats_defense <- left_join(
  player_age_defense %>% select(-position, -position_group),
  nfl_actualStats_game_defense_weekly,
```

```
  by = c(c("season","week","player_id","game_id")))

player_age_stats_kicking <- left_join(
  player_age_kicking %>% select(-position, -position_group),
  nfl_actualStats_game_kicking_weekly,
  by = c(c("season","week","player_id","game_id")))

player_age_stats_offense$years_of_experience <- as.integer(player_age_stats_offense$years_of_exper
player_age_stats_defense$years_of_experience <- as.integer(player_age_stats_defense$years_of_exper
player_age_stats_kicking$years_of_experience <- as.integer(player_age_stats_kicking$years_of_exper

# Merge player info with seasonal stats
player_seasonal_offense <- left_join(
  nfl_actualStats_offense_seasonal,
  nfl_players %>% select(-position, -position_group, -season),
  by = c("player_id" = "gsis_id")
)

player_seasonal_defense <- left_join(
  nfl_actualStats_defense_seasonal,
  nfl_players %>% select(-position, -position_group, -season),
  by = c("player_id" = "gsis_id")
)

player_seasonal_kicking <- left_join(
  nfl_actualStats_kicking_seasonal,
  nfl_players %>% select(-position, -position_group, -season),
  by = c("player_id" = "gsis_id")
)

# Calculate age
season_startdate <- nfl_schedules %>%
  group_by(season) %>%
  summarise(startdate = min(gameday, na.rm = TRUE))

player_seasonal_offense <- player_seasonal_offense %>%
  left_join(
    season_startdate,
    by = "season"
  )

player_seasonal_defense <- player_seasonal_defense %>%
  left_join(
    season_startdate,
```

```
    by = "season"
  )

player_seasonal_kicking <- player_seasonal_kicking %>%
  left_join(
    season_startdate,
    by = "season"
  )

player_seasonal_offense$age <- interval(
  start = player_seasonal_offense$birth_date,
  end = player_seasonal_offense$startdate
) %>%
  time_length(unit = "years")

player_seasonal_defense$age <- interval(
  start = player_seasonal_defense$birth_date,
  end = player_seasonal_defense$startdate
) %>%
  time_length(unit = "years")

player_seasonal_kicking$age <- interval(
  start = player_seasonal_kicking$birth_date,
  end = player_seasonal_kicking$startdate
) %>%
  time_length(unit = "years")
```

3.21 Plotting

3.21.1 Rushing Yards per Carry By Player Age

```
# Prepare Data
rushing_attempts <- nfl_pbp %>%
  dplyr::filter(
    season_type == "REG") %>%
    filter(
      rush == 1,
      rush_attempt == 1,
      qb_scramble == 0,
      qb_dropback == 0,
```

```
      !is.na(rushing_yards))

rb_yardsPerCarry <- rushing_attempts %>%
  group_by(rusher_id, season) %>%
  summarise(
    ypc = mean(rushing_yards, na.rm = TRUE),
    rush_attempts = n(),
    .groups = "drop") %>%
  ungroup() %>%
  left_join(
    nfl_players %>% select(-season),
    by = c("rusher_id" = "gsis_id")
  ) %>%
  filter(
    position_group == "RB",
    rush_attempts >= 50) %>%
  left_join(
    season_startdate,
    by = "season"
  )

rb_yardsPerCarry$age <- interval(
  start = rb_yardsPerCarry$birth_date,
  end = rb_yardsPerCarry$startdate
) %>%
  time_length(unit = "years")

# Create Plot
ggplot2::ggplot(
  data = rb_yardsPerCarry,
  ggplot2::aes(
    x = age,
    y = ypc)) +
  ggplot2::geom_point() +
  ggplot2::geom_smooth() +
  ggplot2::labs(
    x = "Rushing Back Age (years)",
    y = "Rushing Yards per Carry/season",
    title = "2023 NFL Rushing Yards Per Carry per Season by Player Age",
    subtitle = "(minimum 50 rushing attempts)"
  ) +
  ggplot2::theme_classic()
```

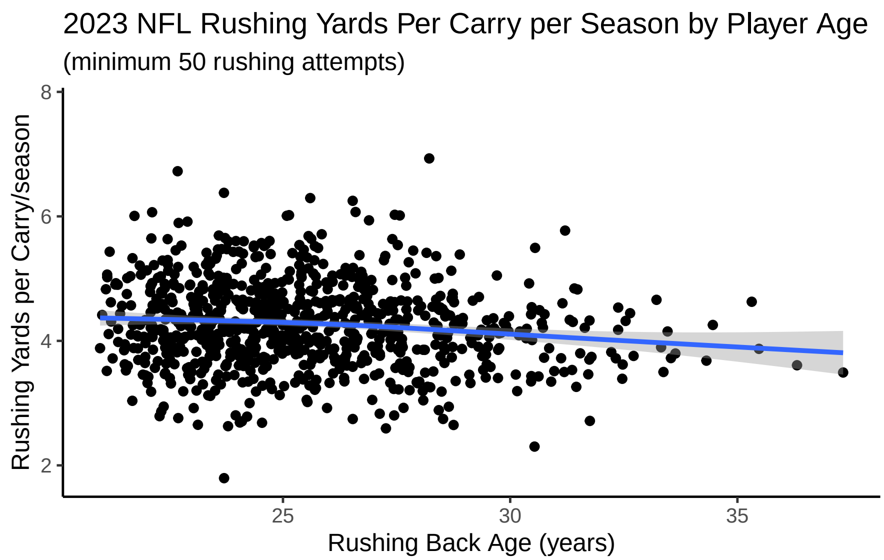


Figure 3.2 2023 NFL Rushing Yards Per Carry per Season by Player Age

```
# Subset Data
rb_seasonal <- player_seasonal_offense %>%
  filter(position_group == "RB")

# Create Plot
ggplot2::ggplot(
  data = rb_seasonal,
  ggplot2::aes(
    x = age,
    y = rushing_epa)) +
  ggplot2::geom_point() +
  ggplot2::geom_smooth() +
  ggplot2::labs(
    x = "Rushing Back Age (years)",
    y = "Rushing EPA/season",
    title = "2023 NFL Rushing Expected Points Added (EPA) per Season by Player Age"
  ) +
  ggplot2::theme_classic()
```

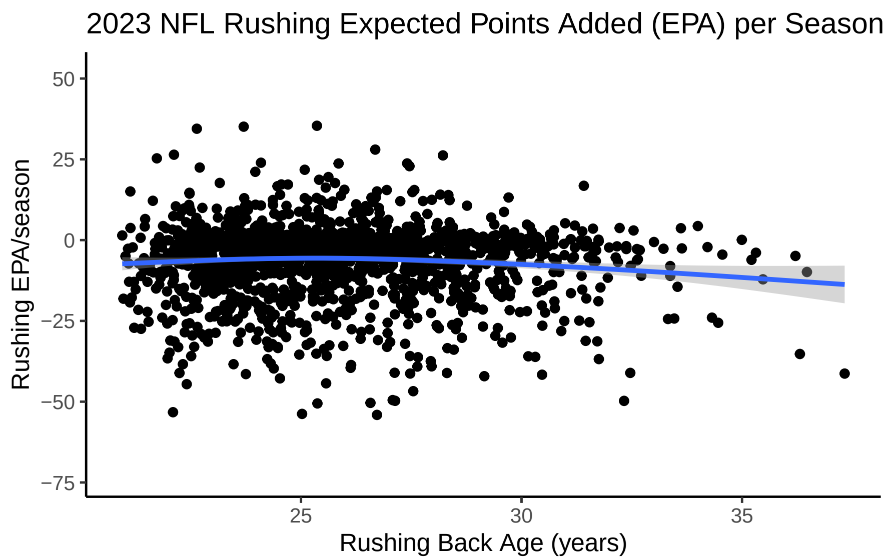


Figure 3.3 2023 NFL Rushing Expected Points Added (EPA) per Season by Player Age

3.21.2 Defensive and Offensive EPA per Play

Expected points added (EPA) per play by the team with possession.

```
pbp_regularSeason <- nfl_pbp %>%
  dplyr::filter(
    season == 2023,
    season_type == "REG") %>%
  dplyr::filter(!is.na(posteam) & (rush == 1 | pass == 1))

epa_offense <- pbp_regularSeason %>%
  dplyr::group_by(team = posteam) %>%
  dplyr::summarise(off_epa = mean(epa, na.rm = TRUE))

epa_defense <- pbp_regularSeason %>%
  dplyr::group_by(team = defteam) %>%
  dplyr::summarise(def_epa = mean(epa, na.rm = TRUE))

epa_combined <- epa_offense %>%
  dplyr::inner_join(epa_defense, by = "team")

ggplot2::ggplot(
```

```
  data = epa_combined,
  ggplot2::aes(
    x = off_epa,
    y = def_epa)) +
  nflplotR::geom_mean_lines(
    ggplot2::aes(
      x0 = off_epa ,
      y0 = def_epa)) +
  nflplotR::geom_nfl_logos(
    ggplot2::aes(
      team_abbr = team),
      width = 0.065,
      alpha = 0.7) +
  ggplot2::labs(
    x = "Offense EPA/play",
    y = "Defense EPA/play",
    title = "2023 NFL Offensive and Defensive EPA per Play"
  ) +
  ggplot2::theme_classic() +
  ggplot2::scale_y_reverse()
```

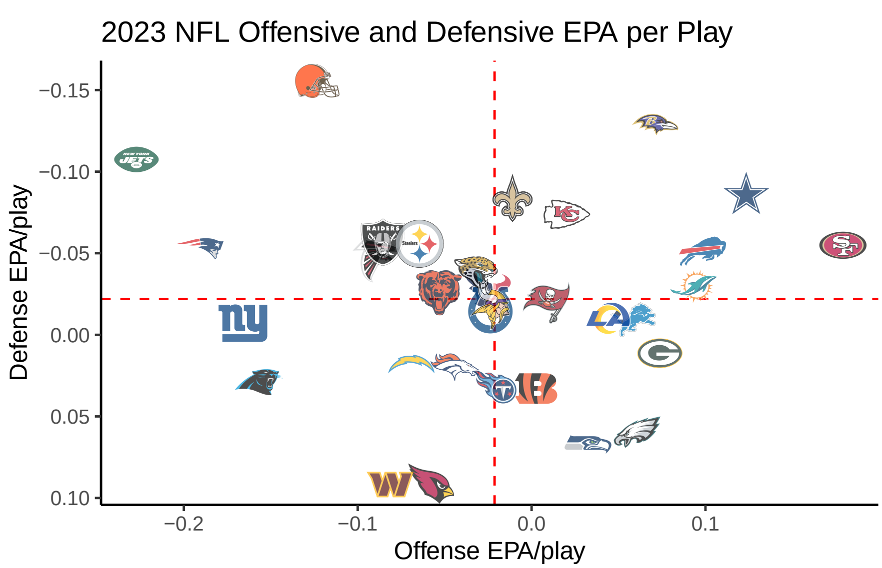


Figure 3.4 2023 NFL Offensive and Defensive EPA per Play

4

Player Evaluation

4.1 Getting Started

4.1.1 Load Packages

```
library("tidyverse")
```

4.2 Overview

Evaluating players for fantasy football could be thought of as similar to the process of evaluating companies when picking stocks to buy. You want to evaluate and compare various assets so that you get the assets with the best value.

There are various domains of criteria we can consider when evaluating a football player's fantasy prospects. Potential domains to consider include:

- athletic profile
- historical performance
- health
- age and career stage
- situational factors
- matchups
- cognitive and motivational factors
- fantasy value

The discussion that follows is based on my and others' *impressions* of some of the characteristics that may be valuable to consider when evaluating players. However, the extent to which any factor is actually relevant for predicting future performance is an empirical question and should be evaluated empirically.

4.3 Athletic Profile

Factors related to a player's athletic profile include factors such as:

- body shape
  - height
  - weight
  - hand size
  - wing span (arm length)
- body function
  - agility
  - strength
  - speed
  - acceleration/explosiveness
  - jumping ability

In terms of body shape, we might consider a player's height, weight, hand size, and wing span (arm length). Height allows players to see over opponents and to reach balls higher in the air. Thus, greater height is particularly valuable for Quarterbacks and Wide Receivers. Heavier players are tougher to budge and to tackle. Greater weight is particularly valuable for Linemen, Fullbacks, and Tight Ends, but it can also be valuable—to a deree—for Quarterbacks, Running Backs, and Wide Receivers. Hand size and wing span is particularly valuable for people catching the ball; thus, a larger hand size and longer wing span are particularly valuable for Wide Receivers and Tight Ends.

In terms of body function, we can consider a player's agility, strength, speed, acceleration/explosiveness, and jumping ability. For Wide Receivers, speed, explosiveness, and jumping ability are particularly valuable. For Running Backs, agility, strength, speed, and explosiveness are particularly valuable.

Many aspects of a player's athletic profile are available from the National Football League (NFL) Combine, which is especially relevant for evaluating rookies. We demonstrate how to import data from the NFL Combine in Section 3.4.6. There are also calculators that integrate information about body shape and information from the NFL Combine to determine a player's relative athletic score (RAS) for their position: https://ras.football/ras-calculator/

4.4 Historical Performance

4.4.1 Overview

> "The best predictor of future behavior is past behavior." – Unknown

> "Past performance does not guarantee future results." – A common disclaimer about investments.

Factors relating to historical performance to consider could include:

- performance in college
  - draft position
- performance in the NFL
- efficiency
- consistency

It is important to consider a player's past performance. However, the extent to which historical performance may predict future performance may depend on many factors such as (a) the similarity of the prior situation to the current situation, (b) how long ago the prior situation was, and (c) the extent to which the player (or situation) has changed in the interim. For rookies, the player does not have prior seasons of performance in the NFL to draw upon. Thus, when evaluating rookies, it can be helpful to consider their performance in college or in their prior leagues. However, there are large differences between the situation in college and the situation in the NFL, so prior success in college may not portend future success in the NFL. An indicator that intends to be prognostic of future performance, and that accounts for past performance, is a player's draft position—that is, how early (or late) was a player selected in the NFL Draft. The earlier a player was selected in the NFL Draft, the greater likelihood that the player will perform well.

For players who have played in the NFL, past performance becomes more relevant because, presumably, the prior situation is more similar (than was their situation in college) to their current situation. Nevertheless, lots of things change from game to game and season to season: injuries, coaches, coaching strategies, teammates, etc. So just because a player performed well or poorly in a given game or season does not necessarily mean that they will perform similarly in subsequent games/seasons. Nevertheless, historical performance is one of the best indicators we have.

We demonstrate how to import historical player statistics in Section 3.4.12. We demonstrate how to calculate historical player statistics in Section 3.20.1. We demonstrate how to calculate historical fantasy points in Section 3.20.2.

4.4.2 Efficiency

In addition to how many fantasy points a player scores in terms of historical performance, we also care about efficiency and consistency. How efficient were they given the number of opportunities they had? If they were relatively more efficient, they will likely score more points than many of their peers when given more opportunities. If they were relativelly inefficient, their capacity to score fantasy points may be more dependent on touches/opportunities. Efficiency might be operationalized by indicators such as yards per passing attempt, yards per rushing attempt, yards per target, yards per reception, etc.

4.4.3 Consistency

In terms of consistency, how consistent was the player they from game to game and from season to season? For instance, we could examine the standard deviations of players' fantasy points across games in a given season. However, the standard deviation tends to be upwardly biased as the mean increases. So, we can account for the player's mean fantasy points per game by dividing their game-to-game standard deviation of fantasy points (σ) by their mean fantasy points across games (μ). This is known as the coefficient of variation (CV), which is provided in Equation 4.1.

$$CV = \frac{\sigma}{\mu} \tag{4.1}$$

Players with a lower standard deviation and a lower coefficient of variation (of fantasy points across games) are more consistent. In the example below, Player 2 might be preferable to Player 1 because Player 2 is more consistent; Player 1 is more "boom-or-bust." Despite showing a similar mean of fantasy points across weeks, Player 2 shows a smaller week-to-week standard deviation and coefficient of variation.

```
set.seed(1)

playerScoresByWeek <- data.frame(
  player1_scores = rnorm(17, mean = 20, sd = 7),
  player2_scores = rnorm(17, mean = 20, sd = 4),
  player3_scores = rnorm(17, mean = 10, sd = 4),
  player4_scores = rnorm(17, mean = 10, sd = 1)
)

consistencyData <- data.frame(t(playerScoresByWeek))

weekNames <- paste("week", 1:17, sep = "")

names(consistencyData) <- weekNames
row.names(consistencyData) <- NULL

consistencyData$mean <- rowMeans(consistencyData[,weekNames])
consistencyData$sd <- apply(consistencyData, 1, sd)
consistencyData$cv <- consistencyData$sd / consistencyData$mean

consistencyData$player <- c(1, 2, 3, 4)

consistencyData <- consistencyData %>%
  select(player, mean, sd, cv, week1:week17)

round(consistencyData, 2)
```

```
  player  mean   sd   cv week1 week2 week3 week4 week5 week6 week7 week8 week9
1      1 20.60 6.47 0.31 15.61 21.29 14.15 31.17 22.31 14.26 23.41 25.17 24.03
2      2 20.61 3.35 0.16 23.78 23.28 22.38 23.68 23.13 20.30 12.04 22.48 19.78
3      3 10.32 2.65 0.26  4.49  8.34  8.42  9.76 14.40 13.05  9.34  8.99 12.79
4      4 10.19 1.11 0.11  9.39 10.34  8.87 11.43 11.98  9.63  8.96 10.57  9.86
  week10 week11 week12 week13 week14 week15 week16 week17
1  17.86  30.58  22.73  15.65   4.50  27.87  19.69  19.89
2  19.38  14.12  18.09  21.67  25.43  19.59  21.55  19.78
3  12.23   7.24   7.17  11.46  13.07   9.55  13.52  11.59
4  12.40   9.96  10.69  10.03   9.26  10.19   8.20  11.47
```

4.5 Health

Health-related factors to consider include:

- current injury status
- injury history

It is also important to consider a player's past and current health status. In terms of a player's current health status, it is important to consider whether they are injured or are playing at less than 100% of their typical health. In terms of a player's prior health status, one can consider their injury history, including the frequency and severity of injuries and their prognosis.

We demonstrate how to import injury reports in Section 3.4.13.

4.6 Age and Career Stage

Age and career stage-related factors include:

- age
- experience
- touches

A player's age is relevant because of important age-related changes in a player's speed, ability to recover from injury, etc. A player's experience is relevant because players develop knowledge and skills with greater experience. A player's prior touches/usage is also relevant, because it speaks to how many hits a player may have taken. For players who take more hits, it may be more likely that their bodies "break down" sooner.

4.7 Situational Factors

Situational factors one could consider include:

- team quality

- role on team
- teammates
- opportunity and usage
  - snap count
  - touches/targets
  - red zone usage

Football is a team sport. A player is embedded within a broader team context; it is important to consider the strength of their team context insofar as it may support— or detract from—a player's performance. For instance, for a Quarterback, it is important to consider how strong the pass blocking is from the Offensive Line. Will they have enough time to throw the ball, or will they be constantly under pressure to be sacked? It is also important to consider the strength of the pass catchers—the Wide Receivers and Tight Ends. For a Running Back, it is important to consider how strong the run blocking is from the Offensive Line. For a Wide Receiver, it is important to consider how strong the pass blocking is, and how strong the Quarterback is.

It is also important to consider a player's role on the team. Is the player a starter or a backup? Related to this, it is important to consider the strength of one's teammates. For a given Running Back, if a teammate is better at running the ball, this may take away from how much the player sees the field. For a given Wide Receiver, if a teammate is better at catching the ball, this may take some targets away from the player. However, the team's top defensive back is often matched up against the team's top Wide Receiver. So, if the team's top Wide Receiver is matched up against a particularly strong Defensive Back, the second- and third-best Wide Receivers may more targets than usual.

It is also important to consider a player's opportunity and usage, which are influenced by many factors, including the skill of the player, the skill of their teammates, the role of the player on the team, the coaching style, the strategy of the opposing team, game scripts, etc. In terms of the player's opportunity and usage, how many snaps do they get? How many touches and/or targets do they receive? Being on the field for more snaps and receiving more touches and/or targets means that the player has more opportunities to score fantasy points. Are they targeted in the red zone? Red zone targets are more likely to lead to touchdown scoring opportunities, which are particularly valuable in fantasy football.

4.8 Matchups

Matchup-related factors to consider include:

- strength of schedule
- weekly matchup

Another aspect to consider is how challenging their matchup(s) and strength of schedule is. For a Quarterback, it is valuable to consider how strong the oppenent's passing defense is. For a Running Back, how strong is the running defense? For a Wide Receiver, how strong is the passing defense and the Defensive Back that is likely to be assigned to guard them?

4.9 Cognitive and Motivational Factors

Other factors to consider include cognitive and motivational factors. Some coaches refer to these as the "X Factor" or "the intangibles." However, just as any other construct in psychology, we can devise ways to operationalize them. Insofar as they are observable, they are measurable.

Cognitive and motivational factors one could consider include:

- reaction time
- knowledge and intelligence
- work ethic and mental toughness
- incentives
    - contract performance incentives
    - whether they are in a contract year

A player's knowledge, intelligence, and reaction time can help them gain an upper-hand even when they may not be the fastest or strongest. A player's work ethic and mental toughness may help them be resilient and persevere in the face of challenges. Contact-related incentives may lead a player to put forth greater effort. For instance, a contract may have a performance incentive that provides a player greater compensation if they achieve a particular performance milestone (e.g., receiving yards). Another potential incentive is if a player is in what is called their "contract year" (i.e., the last year of their current contract). If a player is in the last year of their current contract, they have an incentive to perform well so they can get re-signed to a new contract.

4.10 Fantasy Value

4.10.1 Sources From Which to Evaluate Fantasy Value

There are several sources that one can draw upon to evaluate a player's fantasy value:

- expert or aggregated rankings
- layperson rankings
  - players' Average Draft Position (ADP) in other league snake drafts
  - players' Average Auction Value (AAV) in other league auction drafts
- expert or aggregated projections

4.10.1.1 Expert Fantasy Rankings

Fantasy rankings (by so-called "experts") are provided by many sources. To reduce some of the bias due to a given source, some services aggregate projections across sources, consistent with a "wisdom of the crowd" approach. FantasyPros[1] aggregates fantasy rankings across sources. Fantasy Football Analytics[2] creates fantasy rankings from projections that are aggregated across sources (see the webapp here: https://apps.fantasyfootballanalytics.net).

4.10.1.2 Layperson Fantasy Rankings: ADP and AAV

Average Draft Position (ADP) and Average Auction Value (AAV), are based on league drafts, mostly composed of everyday people. ADP is based on snake drafts, whereas AAV is based on auction drafts. Thus, ADP and AAV are consistent with a "wisdom of the crowd" approach, and I refer to them as forms of rankings by laypeople. ADP data are provided by FantasyPros[3]. AAV data are also provided by FantasyPros[4].

4.10.1.3 Projections

Projections are provided by various sources. Projections (and rankings, for that matter) are a bit of a black box. It is often unclear how they were derived

[1] https://www.fantasypros.com/nfl/rankings/consensus-cheatsheets.php
[2] https://fantasyfootballanalytics.net
[3] https://www.fantasypros.com/nfl/adp/overall.php
[4] https://www.fantasypros.com/nfl/auction-values/calculator.php

by a particular source. That is, it is unclear how much of the projection was based on statistical analysis versus conjecture.

To reduce some of the bias due to a given source, some services aggregate projections across sources, consistent with a "wisdom of the crowd" approach. Projections that are aggregated across sources are provided by Fantasy Football Analytics[5] (see the webapp here: https://apps.fantasyfootballanalytics.net) and by FantasyPros[6].

4.10.1.4 Benefits of Using Projections Rather than Rankings

It is important to keep in mind that rankings, ADP, and AAV are specific to roster and scoring settings of a particular league. For instance, in point-per-reception (PPR) leagues, players who catch lots of passes (Wide Receivers, Tight Ends, and some Running Backs) are valued more highly. As another example, Quarterbacks are valued more highly in 2-Quarterback leagues. Thus, if using rankings, ADP, or AAV, it is important to find ones from leagues that mirror—as closely as possible—your league settings.

Projected statistics (e.g., projected passing touchdowns) are agnostic to league settings and can thus be used to generate league-specific fantasy projections and rankings. Thus, projected statisitics may be more useful than rankings because they can be used to generate rankings for your particular league settings. For instance, if you know how many touchdowns, yards, and interceptions a Quarterback is a projected to throw (in addition to any other relevant categories for the player, e.g., rushing yards and touchdowns), you can calculate how many fantasy points the Quarterback is expected to gain in your league (or in any league). Thus, you can calculate ranking from projections, but you cannot reverse engineer projections from rankings.

4.10.2 Indices to Evaluate Fantasy Value

Based on the sources above (rankings, ADP, AAV, and projections), we can derive multiple indices to evaluate fantasy value. There are many potential indices that can be worthwhile to consider, including a player's:

- dropoff
- value over replacement player (VORP)
- uncertainty

[5] https://fantasyfootballanalytics.net
[6] https://www.fantasypros.com/nfl/auction-values/calculator.php

4.10.2.1 Dropoff

A player's *dropoff* is the difference between (a) the player's projected points and (b) the projected points of the next-best player at that position.

4.10.2.2 Value Over Replacement Player

Because players from some positions (e.g., Quarterbacks) tend to score more points than players from other positions (e.g., Wide Receivers), it would be inadvisable to compare players across different positions based on projected points. In order to more fairly compare players across positions, we can consider a player's value over a typical replacement player at that position (shortened to "value over replacement player"). A player's *value over a replacement player* (VORP) is the difference between (a) a player's projected fantasy points and (b) the fantasy points that you would be expected to get from a typical bench player at that position. Thus, VORP provides an index of how much added value a player provides.

4.10.2.3 Uncertainty

A player's *uncertainty* is how much variability there is in projections or rankings for a given player across sources. For instance, consider a scenario where three experts provide ratings about two players, Player A and Player B. Player A is projected to score 300, 310, and 290 points by experts 1, 2, and 3, respectively. Player B is projected to score 400, 300, and 200 points by experts 1, 2, and 3, respectively. In this case, both players are (on average) projected to score the same number of points (300).

```
exampleData <- data.frame(
  player = c(rep("A", 3), rep("B", 3)),
  expert = c(1:3, 1:3),
  projectedPoints = c(300, 310, 290, 400, 300, 200)
)

playerA_mean <- mean(exampleData$projectedPoints[which(exampleData$player == "A")])
playerB_mean <- mean(exampleData$projectedPoints[which(exampleData$player == "B")])

playerA_sd <- sd(exampleData$projectedPoints[which(exampleData$player == "A")])
playerB_sd <- sd(exampleData$projectedPoints[which(exampleData$player == "B")])

playerA_cv <- playerA_mean / playerA_sd
playerB_cv <- playerB_mean / playerB_sd
```

```
playerA_mean
```

```
[1] 300
```

```
playerB_mean
```

```
[1] 300
```

However, the players differ considerably in their uncertainty (i.e., the source-to-source variability in their projections), as operationalized with the standard deviation and coefficient variation of projected points across sources for a given player.

```
playerA_sd
```

```
[1] 10
```

```
playerB_sd
```

```
[1] 100
```

```
playerA_cv
```

```
[1] 30
```

```
playerB_cv
```

```
[1] 3
```

Here is a depiction of a density plot of projected points for a player with a low, medium, and high uncertainty:

```
playerA <- rnorm(1000000, mean = 150, sd = 5)
playerB <- rnorm(1000000, mean = 150, sd = 15)
playerC <- rnorm(1000000, mean = 150, sd = 30)

mydata <- data.frame(playerA, playerB, playerC)

mydata_long <- mydata %>%
  pivot_longer(
```

```
    cols = everything(),
    names_to = "player",
    values_to = "points"
  ) %>%
  mutate(
    name = case_match(
      player,
      "playerA" ~ "Player A",
      "playerB" ~ "Player B",
      "playerC" ~ "Player C",
    )
  )

ggplot2::ggplot(
  data = mydata_long,
  ggplot2::aes(
    x = points,
    fill = name
  )
) +
  ggplot2::geom_density(alpha = .3) +
  ggplot2::labs(
    x = "Players' Projected Points",
    title = "Density Plot of Projected Points for Three Players"
  ) +
  ggplot2::theme_classic() +
  ggplot2::theme(legend.title = element_blank())
```

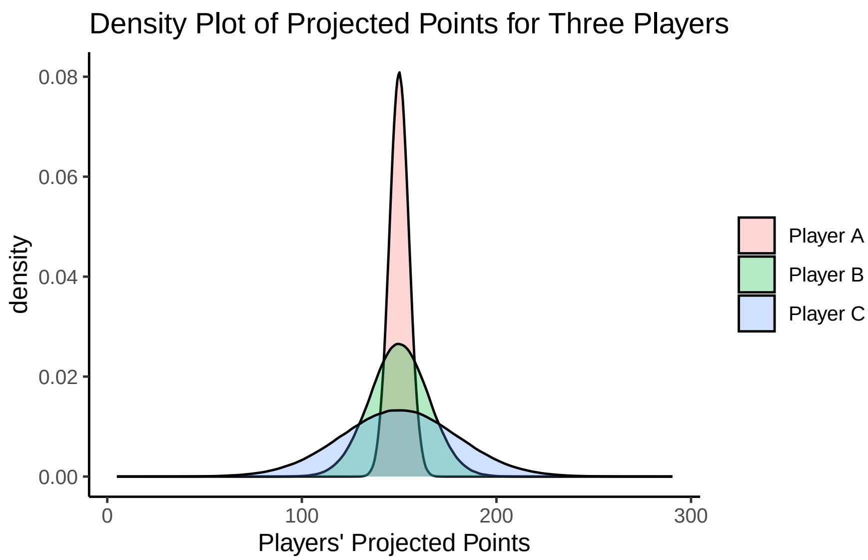


Figure 4.1 Density Plot of Projected Points for Three Players

Uncertainty is not necessarily a bad characteristic of a player's projected points. It just means we have less confidence about how the player may be expected to perform. Thus, players with greater uncertainty are risky and tend to have a higher upside (or ceiling) and a lower downside (or floor).

4.11 Putting it Altogether

After performing an evaluation of the relevant domain(s) for a given player, then one must integrate the evaluation information across domains to make a judgment about a player's overall value. When thinking about a player's value, it can be worth thinking of a player's upside and a player's downside. Player that are more consistent may show higher downside but a lower upside. Younger, less experienced players may show a higher upside but a lower downside.

The extent to which you prioritize a higher upside versus a higher downside may depend on many factors. For instance, when drafting players, you may prioritize drafting players with the highest downside (i.e., the safest players), whereas you may draft sleepers (i.e., players with higher upside) for your bench. When choosing which players to start in a given week, if you are predicted to beat a team handily, it may make sense to start the players with the highest downside. By contrast, if you are predicted to lose to a team by a good margin, it may make sense to start the players with the highest upside.

5

The Fantasy Draft

5.1 Getting Started

5.1.1 Load Packages

5.2 Types of Fantasy Drafts

There are several types of drafts in fantasy football. The most common types of drafts are snake drafts and auction drafts.

5.2.1 Snake Draft

In a snake draft, the participants (i.e., managers) are assigned a draft order. In the first round, the managers draft in that order. In the second round, the managers draft in reverse order. It continues to "snake" in this way, round after round, so that the person who has the first pick in a given round has the last pick in the next round, and whoever has the last pick in a given round has the first pick in the next round.

5.2.2 Auction Draft

In an auction draft, the managers are assigned a nomination order and there is a salary cap (e.g., $200). The first manager chooses which player to nominate. Then, the managers bid on that player like in an auction. In order to bid, the manager must raise the price by at least $1. If two managers want to obtain the same player, they may continue to raise the amount until one manager backs out and is no longer to bid by raising the price. The highest bidder wins (i.e., drafts) that player. Then, the second manager nominates a player, and the managers bid on that player. This process repeats until all teams have drafted their allotment of players.

5.2.3 Comparison

Snake drafts are more common than auction drafts. Snake drafts tend to be quicker than auction drafts. However, auction drafts are more fair than snake drafts. In an auction draft, unlike a snake draft, all players are available to all teams. For instance, in a snake draft, the first 9 players drafted are unavailable to the 10th pick of the first round. So, if you have the 10th pick and want the top-ranked player, this player would not be available to you in the snake draft. However, in the auction draft, every player is available to every manager, so long as the manager is able and willing to bid enough.

5.3 Draft Strategy

5.3.1 Overview

There is no one "right" draft strategy. Sometimes it works best to "zig" when everyone else is "zagging". For instance, if you notice that everyone else is drafting Wide Receivers, this may mean that other managers are over-valuing Wide Receivers, and this could be a nice opportunity to draft a Running Back for good value.

In general, you will first want to generate the rankings you will use to select which players to prioritize. You may generate your rankings based one or more of the following:

- your evaluation of players[1]
- expert or aggregated rankings
- layperson rankings
  - players' Average Draft Position (ADP) in other league drafts (for snake drafts)
  - players' Average Auction Value (AAV) in other league drafts (for auction drafts[2])
- expert or aggregated projections
- indices derived from rankings and projections

Section 4.10.1 describes where to obtain aggregated rankings, aggregated projections, ADP, and AAV data.

[1] player-evaluation.qmd
[2] sec-draftStrategyAuction

An important concept in the draft is "dropoff", which is described in Section 4.10.2.1. *Dropoff* at a given position, is the difference—in terms of projected fantasy points—between (a) the best available player remaining at that position and (b) the second-best available player remaining at that position. If there is a bigger dropoff at a given position, there may be greater value in drafting the top player from that position. For instance, consider the following scenario: "Quarterback A" is projected to score 325 points, and "Quarterback B" is projected to score 320 points. "Tight End A" is projected to score 230 points, and "Tight End B" is projected to score 150 points. In this example, there is a much greater dropoff for Tight Ends than there is for Quarterbacks. Thus, even though "Quarterback A" is projected to score more points than "Tight End A", "Tight End A" may be more valuable because there is still a good Quarterback available if someone else drafts "Quarterback A".

Another important concept is a player's value over a typical replacement player at that position (shortened to "value over replacement player"; VORP), which is described in Section 4.10.2.2.

Another important concept is a player's uncertainty, which is described in Section 4.10.2.3.

In both snake and auction draft formats, your goal is to draft the team whose weekly starting lineup scores the most points and thus the collection of players with the greatest VORP. For your starting lineup, it may make sense—especially with your earliest selections—when comparing two players with equivalent VORP, to prioritize players with higher consistency and lower uncertainty, because they may be considered "safer" with a higher floor. However, when drafting players for your bench, it make make more sense to prioritize high-risk, high reward players with greater uncertainty, because they may have a higher ceiling. Players with a higher ceiling have a potential to be "sleepers"—players who are valued low (i.e., with a high ADP or low AAV) and who outperform their valuation. Note that, although players with greater uncertainty are high-risk, high-reward players, selecting this kind of a player for your bench (i.e., in a late round or for a small cost) is a *lower* risk selection, because you have less to lose with later/lower-cost picks. That is, even though the *player* is higher risk, selecting a higher risk player for your bench is a lower risk *decision*.

The Spurs in the National Basketball Association (NBA) were well-reputed for excelling in this draft strategy[3] (archived at https://perma.cc/X7NW-WZC6). They frequently used their second-round picks to draft high-risk, high-reward players. Sometimes, the secound round pick was a bust, but they have little to lose with a failed second round pick. Other times, their second round picks—including Willie Anderson, DeJuan Blair, Goran Dragic, Luis Scola, and Manu Ginóbili—greatly outperformed expectations. Thanks, in

[3] https://harvardsportsanalysis.org/2013/11/beating-the-nba-draft-does-any-team-outperform-expectations/

part, to this draft strategy, the team showed strong extended success for nearly three decades from 1989 through the late-2010s.

However, the draft strategies to achieve the "optimal lineup" differ between snake versus auction drafts.

5.3.2 Snake Draft

In general, your goal is to draft the team whose weekly starting lineup has the greatest VORP. Consequently, you are often looking to pick the player with the highest VORP at a given selection, while keeping in mind (a) the dropoff of players at other positions and (b) which players may be available at subsequent picks so that you do not sacrifice too much later value with a given selection. For instance, if a particular Quarterback has a slightly higher VORP than a particular Running Back, but the Quarterback is likely to be available at the manager's next pick but the Running Back is likely to be unavailable at their next pick, it might make more sense to draft the Running Back.

5.3.3 Auction Draft

According to an analysis[4] by the Harvard Sports Analysis Collective (archived at https://perma.cc/P7RX-92UU), the majority of the manager's salary cap should be spent on the starting lineup, and you should spend less on bench players. This is known as the "stars and scrubs" draft strategy. Based on the analysis, the author recommended applying a 10% premium to the top players and a 10% discount to the lower-tiered players. The idea behind the approach is that a player on your bench does not contribute to the team's points and, thus, most players drafted to your bench do not contribute much to the team's points throughout the season. That said, bench players can be important in the case of a starter's injury or under-performance. So, it is recommended to draft starters with lower uncertainty who are safer. In contrast to your starting lineup, you may look to draft players on your bench who have greater uncertainty for their high reward potential in a low-risk selection given the lower price.

An alternative to the "stars and scrubs" approach is to wait to draft more "high-value" players after other managers have over-paid for players.

[4] https://harvardsportsanalysis.wordpress.com/wp-content/uploads/2012/04/fantasyfootballdraftanalysis1.pdf

6

Research Methods

6.1 Getting Started

6.1.1 Load Packages

6.2 Sample vs Population

In research, it is important to distinguish between the sample and the target population. The target *population* is who you want your study's findings to generalize to. For instance, if we want our findings to lead to inferences we can draw regarding all current NFL players, then NFL players are our target population. However, despite our best efforts to recruit all NFL players into our study, we may not succeed in doing that. The participants (i.e., people or players) who we successfully recruit to be in our study represent our *sample*.

It is rare for the sample to include all people who are in the target population. It can be costly to recruit large samples, and many potential participants may decline to participate for a variety of reasons (insufficient time, lack of interest in the study, distrust of scientists, etc.). Thus, our goals are (a) to recruit as many people from the population as possible and (b) for the sample to be as *representative* of the population as possible.

For increasing the representativeness of the sample (with respect to the population), we might conduct a *random sample*, in which each person in the population (i.e., each NFL player) has equal likelihood of being selected. For instance, we might randomly select 250 players to recruit to the study. True random samples, though strong in aspiration, are difficult and costly to achieve. In reality, many researchers conduct convenience sampling. A convenience sample is recruited because it is convenient (i.e., less costly and time-consuming).

For instance, many studies examine college students—in part, because they are easy to recruit. If our target population is NFL players but we are unable to recruit NFL players into our study, we could easily recruit a large sample of college students. Although the convenience sample may afford a very large sample, the college student sample may not be representative of the target population (NFL players). Thus, the findings in our study may not *generalize* to NFL players—that is, what we learn in college students may not apply in the same way among NFL players. For instance, if we learn that consumption of sports drinks (compared to drinking only water) improves running speed among college students, that may not be the case among NFL players.

6.3 Research Designs

There are three broad types of research designs:

- experiment
- correlational/observational study
- case study

6.3.1 Experiment

In an *experiment*, there are one or more things (i.e., variables) that we manipulate to see how the manipulation influences the process of interest. The variable that we manipulate is the *independent variable*. By contast, the *dependent variable* is the variable that we evaluate to determine whether it was influenced by the manipulation (i.e., by the independent variable).Besides the independent and dependent variables, the researcher attempts to hold everything else constant through processes including standardization and random assignment. *Standardization* involves using the same procedures to assess each participant, so that scores can be fairly compared across participants (and groups). Random assignment involves randomly assigning participants to conditions of the independent variable, so the people in each condition are comparable and do not differ systematically.

For instance, we may be interested to evaluate whether players perform better (e.g., run faster) if they drink a sports drink compared when they drink only water. Our hypothesis might be that players will be expected to perform better when they drink a sports drink (compared to when they drink only water). To this this research question and hypothesis, we might conduct an experiment by randomly assigning some players during practice to receive a sports drink and some players to receive only water. In this case, our independent variable is whether the player receives a sports drink. Our dependent variable might be their 40-yard dash time during practice.

6.3.2 Correlational/Observational Study

In a correlational (aka observational) study, we do not manipulate a variable to see how the manipulation influences another variable. Instead, we examine how two variables, a predictor and an outcome variable, are associated. The hypothesized cause is called the predictor variable. The hypothesized effect is called the outcome variable. In this way, the predictor variable is similar to the independent variable, and the outcome variable is similar to the dependent variable. However, unlike the independent and dependent variables in an experiment, the predictor and outcome variables in a correlational study are not manipulated.

For instance, to use a correlational study to test the possibility that players who drink sports drinks perform better than players who drink only water, we could examine whether the players who drink sports drinks during a game score more fantasy points than players who drink only water during the game. In this case, our predictor variable is whether the players drinks sports drinks during a game. Our outcome variable is the number of fantasy points the player scored.

6.3.2.1 Correlation Does Not Imply Causation

As the maxim goes, "correlation does not imply causation"—just because two variables are associated does not necessarily mean that they are causally related.

Just because `X` is associated with `Y` does not mean that `X` causes `Y`. Consider that you find an association between variables `X` and `Y`.

There are several reasons why you might observe an association between `X` and `Y`:

- `X` causes `Y`
- `Y` causes `X`
- a third variable (i.e., confound), `Z`, influences both `X` and `Y`
- the association between `X` and `Y` is spurious

For instance, one possibility is that the association we observed reflects our hypothesis that `X` causes `Y`, as depicted in Figure 6.1. That is, consumption of more sports drink may improve players' performance.

However, a second possibility is that the association reflects the opposite direction of effect, where `Y` actually causes `X`, as depicted in Figure 6.2. For instance, greater performance may lead players to drink more sports drink (rather than the reverse).

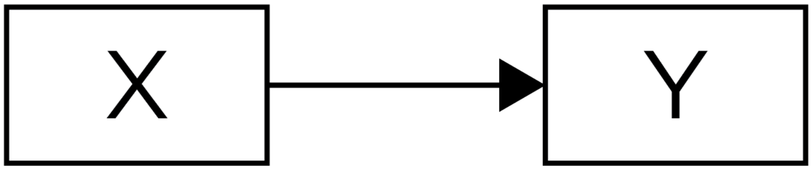


Figure 6.1 Hypothesized Causal Effect Based on an Observed Association Between X and Y, Such That X Causes Y.

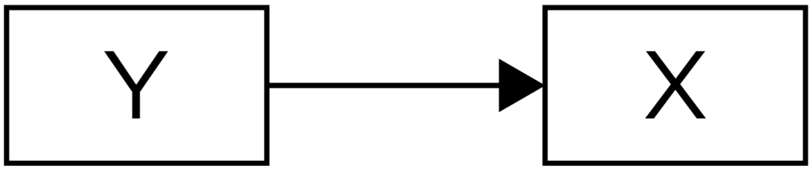


Figure 6.2 Reverse (Opposite) Direction of Effect From the Hypothesized Effect, Where Y Causes X.

A third possibility is that the association could reflect the influence of a third variable. If a third variable is a common cause of each and accounts for their association, it is a *confound*. An observed association between X and Y could reflect a confound—i.e., a cause (Z) that influences both X and Y, which explains why X and Y are correlated even though they are not causally related. A third variable confound that is a common cause of both X and Y is depicted in Figure 6.3. For instance, it may not be that sport drink consumption per se influences player performance; rather, it may be that players who are more intelligent or have more financial resources tend to drink more sports drinks and also tend to perform better. In this case, intelligence or financial resources may be a confound that influences both sports drink consumption and player performance, but sports drink consumptions—though correlated with player performance—does not influence player performance.

For another example, consider that ice cream sales are associated with shark attacks. It is unlikely that more people eating ice creams leads to shark attacks. There is a likely a third variable—heat waves—that is a confound because it influences both ice cream sales and shark attacks and explains their association.

Lastly, the association might be spurious. It might just reflect random variation (i.e., chance), and that when tested on an independent sample, what appeared as an association in the original dataset may not hold when testing the association in a new dataset.

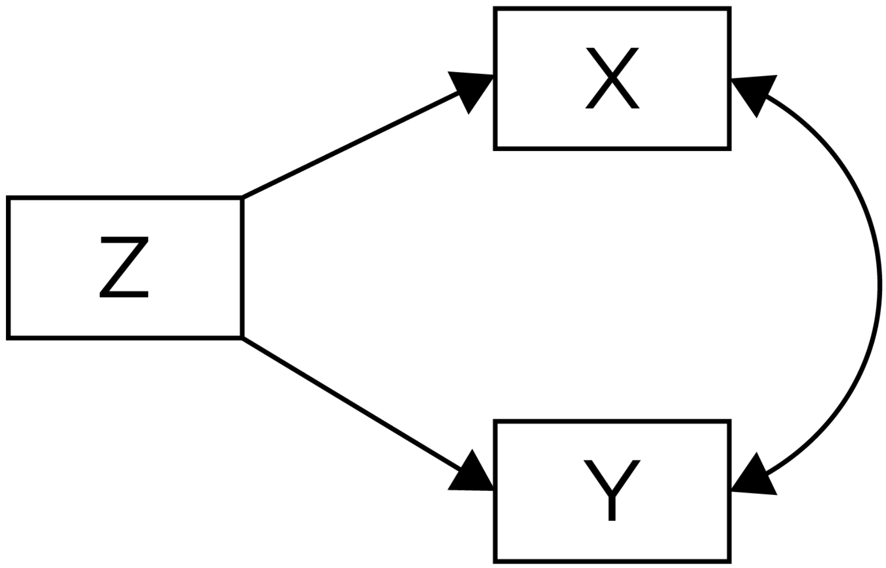


Figure 6.3 Confounded Association Between `X` and `Y` due to a Common Cause, `Z`.

6.3.3 Case Study

In a case study, we assess a small sample of individuals (commonly only one person or a few people), often with rich qualitative information. Themes may be coded from the qualitative information, which may help inform inferences about whether some process may have played a role in influencing the outcome of interest. The inferences are then drawn in a subjective, qualitative way. Testimonials and anecdotes are examples are case studies.

For instance, to use a case study to evalute the possibilty that players who drink sports drinks perform better than players who drink only water, we could conduct an in-depth interview with a player. In the interview, we might ask the player how they performed in games with versus without a sports drink and have them discuss whether they believe the sports drink improved their performance (and if so, how). Then, based on the player's responses, we might code the responses to extract themes and to make a qualitative judgement of whether or not the player likely performed better during games in which they had a sports drink.

6.3.4 Other Features of the Research Design

6.3.4.1 Number of Timepoints

In addition to whether the research design is an experiment, correlational/observational study, or a case study, a research design can also have one or multiple timepoints. The differing number of timepoints allow studies to be characterized as one of the following:

- cross-sectional
- longitudinal

6.3.4.1.1 Cross-Sectional

A *cross-sectional study* is a study with one timepoint.

For instance, in a cross-sectional study evaluating whether having a sports drink improves player performance, we might assess players' drinking behavior and performance during only game 1.

Cross-sectional studies are more common than longitudinal studies because cross-sectional studies are less costly and time-consuming. They can provide a helpful starting point to test findings more rigorously in subsequent longitudinal studies.

6.3.4.1.2 Longitudinal Design

A *longitudinal study* is a study with more than one timepoint. When the same measures are assessed at each of multiple timepoints, we refer to this as a "repeated measures" design.

In a longitudinal study evaluating whether having a sports drink improves player performance, we might assess players' drinking behavior and performance during each game of the season, and possibly across multiple seasons.

Longitudinal studies are less common than cross-sectional studies because longitudinal studies are more costly and time-consuming. Nevertheless, longitudinal studies can allow us test our hypotheses more rigorously, because they can allow us to test whether changes in the predictor/indepdnent variable leads to changes in the outcome/dependent variable. Thus, compared to cross-sectional studies, longitudinal studies can provide greater confidence in causal inferences.

6.3.4.2 Within- or Between-Subject

A research design can also be within-subject, between-subject, or both. A study can involve both within-subject and between-subject comparisons if one predictor/independent variable is within-subject and another predictor/independent variable is between-subject.

6.3.4.2.1 Within-Subject Design

A *within-subject design* is one in which each participant (i.e., person or player) receives multiple levels of the independent variable (or predictor).

For instance, in an experiment evaluating whether having a sports drink improves player performance, we might assign players to drink the sports drink in the first half of the game and to drink only water in the second half of the game. Or we could assign some of the players to drink sports drink in the first half and water in the second half, and assign the other players to drink water in the first half and sports drink in the second half.

In a correlational study evaluating whether having a sports drink improves player performance, we might evaluate how within-person changes in sports drink consumption are associated with within-person changes in performance. That is, we could evaluate, when a given player has a sports drink (or more sports drinks), do they perform better than when the same individual has only water (or fewer sports drinks)?

Within-subject designs tend to have greater statistical power than between-subject designs. However, within-subject designs often have *carryover effects*. For instance, consider the study in which we assign players to drink only water in the first and third quarters and to drink sports drink in the second and fourth quarters (an A-B-A-B design). Drinking sports drink in the second quarter could increase how much hydration a player has throughout the rest of the game, which could lead to altered performance in the third and fourth quarters that is not due to what they drink in third and fourth quarters.

6.3.4.2.2 Between-Subject Design

A *between-subject design* is one in which each participant (i.e., person or player) receives only one level of the independent variable.

For instance, in an experiment evaluating whether having a sports drink improves player performance, we might assign some players to drink the sports drink but the other players to drink only water.

In a correlational study evaluating whether having a sports drink improves player performance, we might evaluate whether people who drink sports drinks tend to perform better than players who drink only water. Or, we could evaluate whether players who drink more sports drinks perform better than players who drink fewer sports drinks (i.e., whether the number of sports drinks consumed during a game is correlated with player performance).

6.4 Research Design Validity

Research design validity involves the accuracy of inferences from a study. There are three types of research design validity:

- internal validity
- external validity
- conclusion validity

6.4.1 Internal Validity

Internal validity is the extent to which we can be confident that the associations identified in the study are causal.

6.4.2 External Validity

External validity is the extent to which we can be confident that findings from the study play out similarly in the real world—that is, the findings generalize to the target population.

6.4.3 Tradeoffs Between Internal and External Validity

There is a tradeoff between internal and external validity—a single research design cannot have both high internal and high external validity. Each study and design has weaknesses. Some research designs are better suited for making causal inferences, whereas other designs tend to be better suited for making inferences that generalize to the real world. The research design that is best suited to making causal inferences is an experiment because it is the design in which the researcher has the greatest control over the variables. Thus, experiments tend to have higher internal validity than other research designs. However, by manipulating one variable and holding everything else constant, the research takes place in a very standardized fashion that can become like studying a process in a vacuum. So, even if a process is theoretically causal in a vacuum, it may act differently in the real world when it interacts with other processes.

Correlational designs have greater capacity for external validity than experimental designs because the participants can be observed in their natural environments to evaluate how variables are related in the real world. However, the greater external validity comes at a cost of lower internal validity. Correlational designs are not well-positioned to make causal inferences. Correlational studies can account for potential confounds using *covariates* or for the reverse direction of effect using longitudinal designs, but the researcher has less control over the variables than in an experiment.

As the internal validity of a study's design increases, its external validity tends to decrease. The greater control we have over variables (and, therefore, have greater confidence about causal inferences), the lower the likelihood that the findings reflect what happens in the real world because it is studying things in a metaphorical vacuum. Because no single research design can have both high internal and external validity, scientific inquiry needs a combination of many different research designs so we can be more confident in our inferences—experimental designs for making causal inferences and correlational designs for making inferences that are more likely to reflect the real world.

Case studies, because they have smaller sample sizes and inferences drawn in a subjective, qualitative way, tend to have lower external validity than both experimental and correlational studies. Case studies also tend to have lower internal validity because they have less control over variables, and thus fail to remove the possibility of illusory correlations, potential confounds, or the reverse direction of effect. Thus, case studies are among the weakest forms of evidence. Nevertheless, case studies can still be useful for generating hypotheses that can then be tested empirically with a larger sample in experimental or correlational studies.

6.4.4 Conclusion Validity

Conclusion validity is the extent to which a study's conclusions are reasonable about the association among variables based on the data. That is, were the correct statistical analyses performed, and are the interpretations of the findings from those analyses correct?

6.5 Mediation vs Moderation

Both types of effects involve (at least) three variables:

1. An independent/predictor variable, which will be labeled as `X`.
2. A dependent/outcome variable, which will be labeled as `Y`.
3. The mediator or moderator variable, which will be labeled as `M`.

A mnemonic to help remember the difference between mediation and moderation is in Figure 6.4.

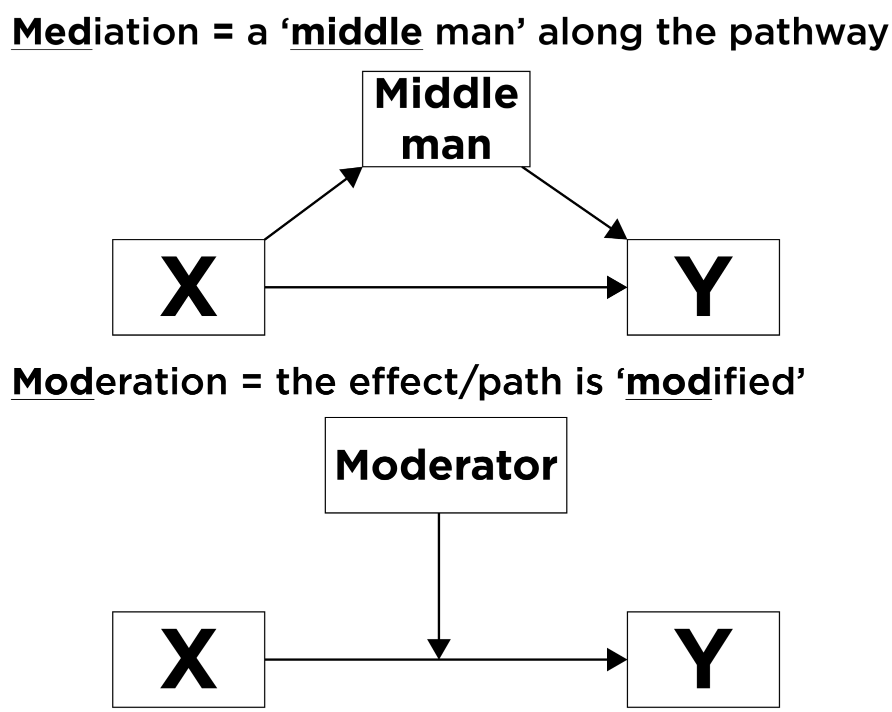


Figure 6.4 Mediation Versus Moderation Mnemonic.

6.5.1 Mediation

6.5.1.1 Overview

Mediation is a causal chain of events, where one variable (a mediator variable) at least partially explains (or accounts for) the association between two other variables (the predictor variable and the outcome variable). In mediation, a predictor (`X`) leads to a mediator (`M`), which leads to an outcome (`Y`). Mediation answers the question of, "**Why (or how)** does `X` influence `Y`? A mediator (`M`) is a variable that helps explain the assocation between two other variables, and it answers the question of why/how `X` influences `Y`. That is, the mediator is the variable that helps explain how/why `X` is related to `Y`. In other words, you can think of the mediator as the mechanism that helps explain why `X` has an impact on `Y`. The association between `X` and `Y` gets smaller when accounting for `M`. Visually this can be written as in Figure 6.5:

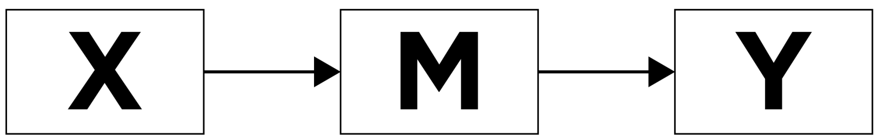


Figure 6.5 Mediation.

where `X` is causing `M`, which in turn is causing `Y`. In other words, `X` leads to `M`, and `M` leads to `Y`.

For instance, if we determine that consuming sports drinks improves player performance, we may want to know how/why. That is, what is the mechanism that leads consumption of sports drinks to improve player performance? We might hypothesize that consumption of sports drink helps increase a player's hydration, which in turn will improve the player's performance. In this case, increased hydration mediates (i.e., helps explain or account for) the effect of the sports drink consumption on improved player performance.

Question: Why/how does sports drink consumpion lead players to perform better?
Answer: increased hydration

As a picture, we can draw this assocation as in Figure 6.6:

6.5.1.2 Types of Mediation

6.5.1.2.1 Full Mediation

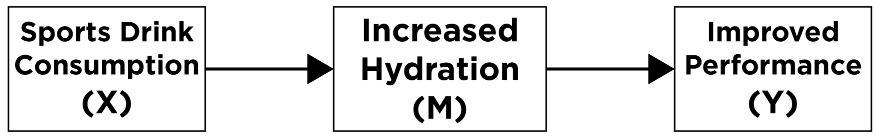


Figure 6.6 Mediation Example.

When one mechanism fully accounts for the effect of the predictor variable on the outcome variable, this is known as **full mediation**, as depicted in Figure 10.15:

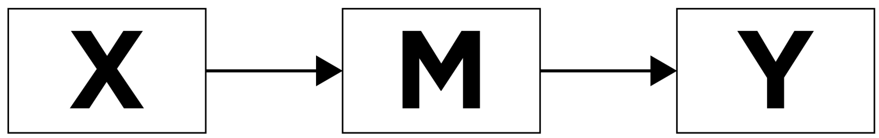


Figure 6.7 Full Mediation.

6.5.1.2.2 Partial Mediation

When a single process partially—but does not fully—accounts for the effect of the predictor variable on the outcome variable; this is known as **partial mediation** and is depicted in Figure 10.16:

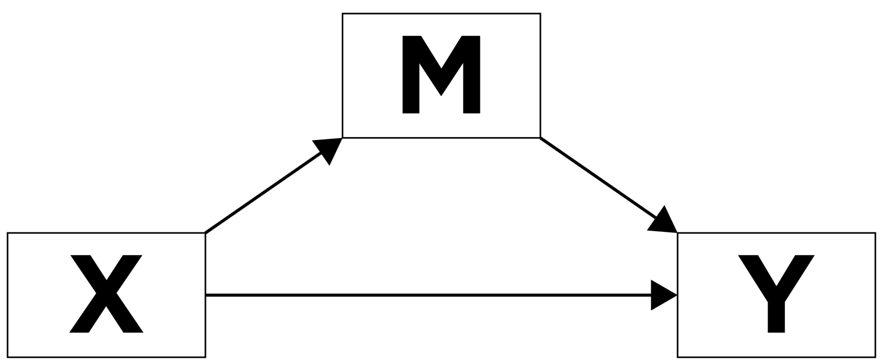


Figure 6.8 Partial Mediation.

6.5.1.2.3 Multiple Mediators

In addition, there can be multiple mediators/mechanisms that account for the effect of a predictor variable on an outcome variable, as depicted in Figure 6.9:

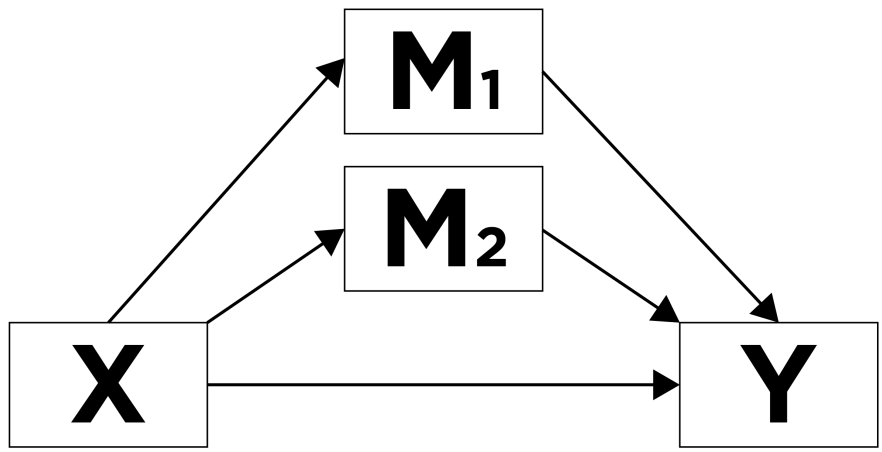


Figure 6.9 Multiple Mediators.

6.5.2 Moderation (i.e., Interaction)

6.5.2.1 Overview

Moderation (sometimes called an "interaction"), on the other hand, occurs when there is a variable or condition (`M`; called a "moderator") that changes the assocation between `X` and `Y`. That is, the effect of the predictor variable on the outcome variable differs at different levels of the moderator variable. In these cases, `X` and `M` work together to have an effect on `Y`; here `X` does not have a direct effect on `M`. Moderation answers the question of, "**For whom** does `X` influence `Y`?" If `X` influences `Y` more strongly for some people or in some circumstances, we would say that there is an interaction such that the effect of `X` on `Y` depends on `M`, as depicted in Figure 6.10:

For example, if the effect of consuming sports drinks on player performance differs for Quarterbacks and Wide Receivers, the interaction could be depicted in Figure 6.11 and Figure 6.12:

An interaction can be identified visually by non-parallel lines at different levels of the moderator. In this example, the player's position moderates the effect consuming sports drinks on player performance. In particular, there is a strong

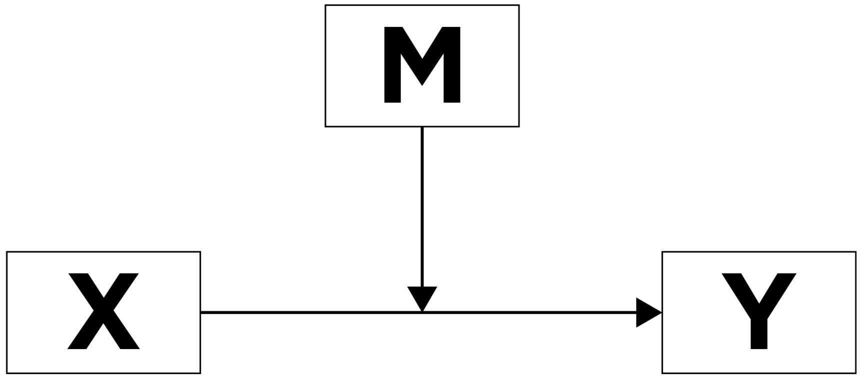


Figure 6.10 Moderation.

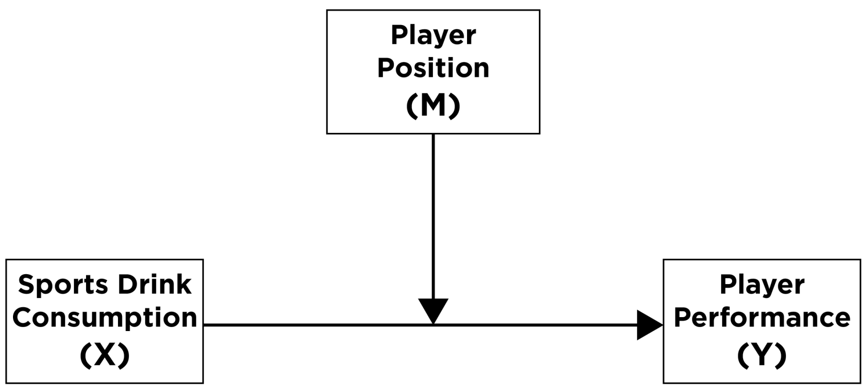


Figure 6.11 Moderation Example: Path Diagram.

positive association between consuming sports drinks and player performance for Wide Receivers (as evidenced by the upward slope of the best-fit regression line), whereas there is no association between consuming sports drinks and player performance for Quarterbacks (as evidenced by the flat line).

6.6 Levels of Measurement

It is important to know the levels of measurement of your data, because the level(s) of measurement of your data constrain the types of comparisons and

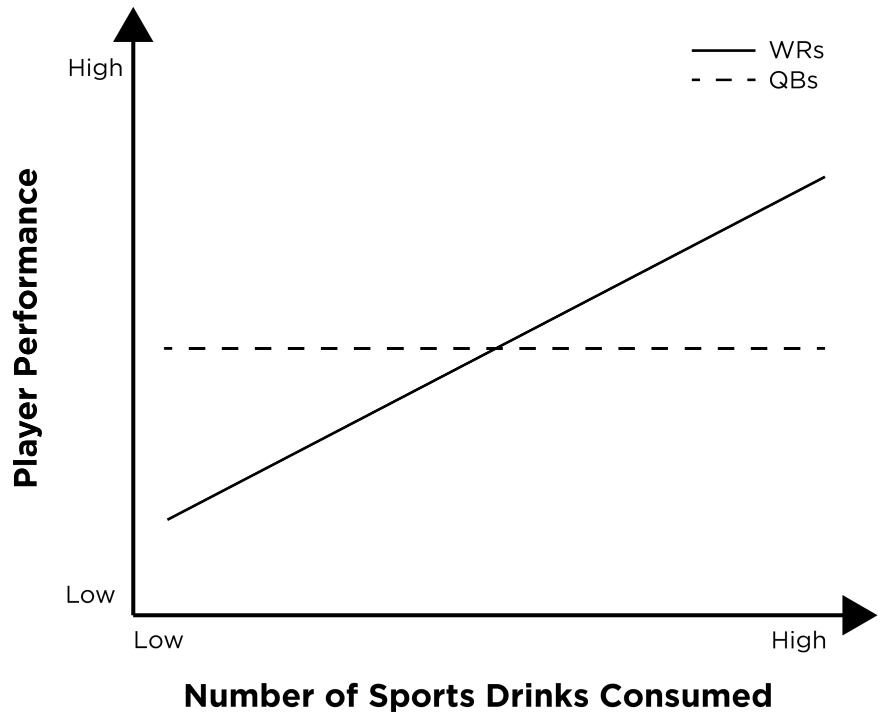


Figure 6.12 Moderation Example: Interaction Graph.

analyses that you can meaningfully perform. There are four levels of measurement that any variable can have:

- nominal
- ordinal
- interval
- ratio

Each is described below:

6.6.1 Nominal

A variable is considered nominal if it is composed of qualitative classifications. You cannot meaningfully evaluate whether one number in the variable is larger than another number in the variable because higher numbers do not reflect higher levels of the concept. Examples of nominal variables include:

- sex (e.g., 1 = male; 2 = female)

- race (e.g., 1 = American Indian; 2 = Asian; 3 = Black; 4 = Pacific Islander; 5 = White)
- ethnicity (e.g., 0 = Non-Hispanic/Latino; 1 = Hispanic/Latino)
- zip code
- jersey number

A football player's jersey number is an example of a nominal variable. A jersey number of 7 is not higher on whatever concept of interest compared to a jersey number of 6.

To examine the central tendency of a nominal variable, you can determine the mode, but you cannot calculate a mean or median.

6.6.2 Ordinal

A variable is considered ordinal if the classifications are ordered. However, ordinal variables do not have equally spaced intervals. Examples of ordinal intervals include:

- likert response scales (e.g., 1 = strongly disagree; 2 = disagree; 3 = neutral; 4 = agree; 5 = strongly agree)
- educational attainment (e.g., 1 = no formal education; 2 = elementary school; 3 = middle school; 4 = high school; 5 = college; 6 = graduate degree)
- academic grades on A–F scale (e.g., 1 = A; 2 = B; 3 = C; 4 = D; 5 = F)
- player rank (1 = 1st; 2 = 2nd; 3 = 3rd, etc.)

A football player's fantasy rank is an example of an ordinal variable. A player with a fantasy rank of 1 has a higher rank than a player with a rank of 2, but it is not known how far apart each player is—i.e., the intervals do not all reflect the same distance. For instance, the distance between the top-ranked player and the 2nd-best player might be 30 points, whereas the distance between the 2nd-best player and the 3rd-best player might be 2 points.

To examine the central tendency of ordinal data, the median and mode are most appropriate; however, the mean may be used (unlike for nominal data).

6.6.3 Interval

A variable is considered interval if the classifications are ordered (similar to ordinal data) and have equally spaced intervals (unlike ordinal data). However, interval variables do not have a meaningful zero that reflects absence. Examples of interval data include:

- temperature on the Fahrenheit or Celsius scale

- time of day

For instance, the temperature difference between 80 and 90 degrees Fahrenheit is the same as the temperature difference between 90 and 100 degrees Fahrenheit. However, 0 degrees Fahrenheit does not reflect absence of temperature/heat.

Interval data can be meaningfully added or subtracted. For instance, if a game starts at 4 pm and ends at 7 pm, you know the game lasted 3 hours ($7 - 4 = 3$). However, interval data cannot be meaningfully multiplied or divided. For instance, 100 degrees Fahrenheit is not twice as hot as 50 degrees Fahrenheit.

To examine the central tendency of interval data, you can compute the mean, median, or mode.

6.6.4 Ratio

A variable is considered ratio if the classifications are ordered (similar to ordinal data), have equally spaced intervals (like interval data), and have an absolute zero point that reflects absence of the concept. Examples of ratio data include:

- temperature on the Kelvin scale
- height
- weight
- age
- distance
- speed
- volume
- time elapsed
- income
- years of formal education
- points in football

For instance, points in football has order, equally spaced intervals, and an absolute zero—a team cannot score less than zero points, and zero points reflects absence of points (though it could be argued to be interval data because zero points does not reflect absence of skill.)

Ratio data can be meaningfully added, subtracted, multiplied, or divided. A player who weighs 350 pounds weighs twice as much as someone who weighs 175 pounds.

To examine the central tendency of ratio data, you can compute the mean, median, or mode.

6.7 Psychometrics

Below, I provide brief discussions of various aspects of measurement reliability and validity. For more information on these and other aspects of psychometrics, see Petersen (2024b) and Petersen (2024c).

6.7.1 Measurement Reliability

The *reliability* of a measure's scores deals with the *consistency* of measurement. This book focuses on the following types of reliability:

- test–retest reliability
- inter-rater reliability
- intra-rater reliability
- internal consistency
- parallel-forms reliability

For more information on these and other aspects of reliability, see https://isaactpetersen.github.io/Principles-Psychological-Assessment/reliability.html (Petersen, 2024b, 2024c).

6.7.1.1 Test–Retest Reliability

Test–retest reliability evaluates the consistency of scores across time. For a construct that is expected to be stable across time (e.g., hand size in adults), we would expect our measurements to be consistent across time. The consistency of scores across time can be examined in terms of relative or absolute test–retest reliability. Relative test–retest reliability—i.e., the consistency of individual differences across time—is commonly evaluated using the coefficient of stability (i.e., the Pearson correlation coefficent). Absolute test–retest reliability—i.e., the absolute consistency of people's scores across time—is commonly evaluated using the coefficient of repeatability.

6.7.1.2 Inter-Rater Reliability

Inter-rater reliability evaluates the consistency of scores across raters. For instance, if we have a strong measure for assessing college players' aptitude to succeed in the NFL, the measure should yield a similar score for a given player regardless of which (trained) rater (e.g., coach or talent scout) uses it to rate the player. The consistency of scores across raters is commonly evaluated using the intraclass correlation coefficient (for continuous variables) and Cohen's kappa (κ; for categorical variables).

6.7.1.3 Intra-Rater Reliability

Intra-rater reliability evaluates the consistency of scores within a given rater. If we have a strong measure for assessing college players' aptitude to succeed in the NFL, the measure should yield a similar score for a given player from the same (trained) rater (e.g., coach or talent scout) each time they rate the same player (assuming the player's aptitude has not changed). The consistency of scores within raters can be evaluated using similar approaches as those evaluating inter-rater reliability.

6.7.1.4 Internal Consistency

Internal consistency evaluates the consistency of scores across items within a measure. If we develop a strong questionnaire measure to assess a college players' aptitude to succeed in the NFL, the scores should be relatively consistent across items. The consistency of scores across items within a measure is commonly evaluated using Cronbach's alpha (α) or McDonald's omega (ω).

6.7.1.5 Parallel-Forms Reliability

Parallel-forms reliability evaluates the consistency of scores across different but equivalent forms of a measure. If we develop two equivalent versions of the Wonderlic Contemporary Cognitive Ability Test (Form A and Form B) so that players sitting next to each other do not receive the same items, we would expect a player's score on Form A would be similar to their score on Form B. Parallel-forms reliability is is commonly evaluated using the coefficient of equivalence (i.e., the Pearson correlation coefficent).

6.7.2 Measurement Validity

The *validity* of a measure's scores deals with the *accuracy* of measurement. This book focuses on the following types of validity:

- face validity
- content validity
- criterion-related validity
  - concurrent (criterion-related) validity
  - predictive (criterion-related) validity
- construct validity

- convergent validity
- discriminant validity
- incremental validity

For more information on these and other aspects of validity, see https://isaactpetersen.github.io/Principles-Psychological-Assessment/validity.html (Petersen, 2024b, 2024c).

6.7.2.1 Face Validity

Face validity evaluates the extent to which a measure "looks like" (on its face) it assesses the construct of interest. For instance, if a measure is developed to assess aptitude of Wide Receivers for the position, it would be considered to have face validity if everyday (lay) people believe that it assesses aptitude for being a successful Wide Receiver.

6.7.2.2 Content Validity

Content validity evaluates the extent to which the measure assesses the full breadth of the content, as determined by context experts. For the measure to have content validity, it should not have gaps (missing content facets) or intrusions (facets of other constructs). For instance, a strong measure for assessing a player's aptitude to succeed in the NFL might need to include a player's speed, strength, size, lateral quickness, etc. If the measure is missing their speed, this would be a content gap. If the measure assesses a construct-irrelevant facet (e.g., their attractiveness), this would be a content intrusion.

6.7.2.3 Criterion-Related Validity

Criterion-related validity evaluates the extent to which the measure's scores are related to meaningful variables of interest. Criterion-related validity is commonly evaluated using a Pearson correlation or some form of regression.

There are two types of criterion-related validity:

- concurrent (criterion-related) validity
- predictive (criterion-related) validity

6.7.2.3.1 Concurrent (Criterion-Related) Validity

Concurrent criterion-related validity evaluates the extent to which the measure's scores are related to meaningful variables of interest assessed at the same point in time. That is, concurrent validity could evaluate whether current player statistics (e.g., passing yards) are associated with their fantasy points.

6.7.2.3.2 Predictive (Criterion-Related) Validity

Predictive criterion-related validity evaluates the extent to which the measure's scores are related to meaningful variables of interest that are assessed at a later point in time. For example, predictive validity could evaluate whether scores on the measure we developed to assess a player's aptitude to succeed in the NFL predicts later performance in the NFL.

6.7.2.4 Construct Validity

Construct validity evaluates the extent to which the measure's scores accurately assess the construct of interest. If we develop a measure with intent to assess aptitude for being a successful Running Back, and it appears to more accurately assess aptitude for being a successful Wide Receiver, then our measure has poor construct validity for assessing aptitude for being a successful Running Back. Construct validity subsumes convergent and discriminant validity, in addition to all of the other forms of measurement validity.

6.7.2.5 Convergent Validity

Convergent validity evaluates the extent to which the measure's scores are related to other measures of the same construct. For instance, if we develop a new measure to assess intelligence, its scores should be related to scores from other measures designed to assess intelligence (e.g., Wonderlic Contemporary Cognitive Ability Test).

6.7.2.6 Discriminant Validity

Discriminant validity evaluates the extent to which the measure's scores are unrelated to measures of the different constructs. For instance, if we develop a new measure to assess intelligence, its scores should be less strongly associated with measures of other constructs (e.g., measures of happiness).

6.7.2.7 Incremental Validity

Incremental validity evalutes the extent to which the measure's scores provide an increase in predictive accuracy compared to other information that is easily and cheaply available. That is, in order to be useful, a strong measure should tell us something that we did not already know. For instance, if we develop a strong measure of intelligence, it should result in increased predictive accuracy (for success in the NFL) compared to when just relying on the Wonderlic Contemporary Cognitive Ability Test.

6.7.3 Reliability vs Validity

Reliability and validity are different but related. Reliability refers to the *consistency* of scores, whereas accuracy refers to the *accuracy* of scores. Validity depends on reliability. Reliability is necessary—but insufficient for—validity. That is, consistency is necessary—but insufficient for—accuracy. As depicted in Figure 6.13, a measure can be no more valid than it is reliable. A measure can be consistent but inaccurate; however, a measure cannot be accurate but inconsistent.

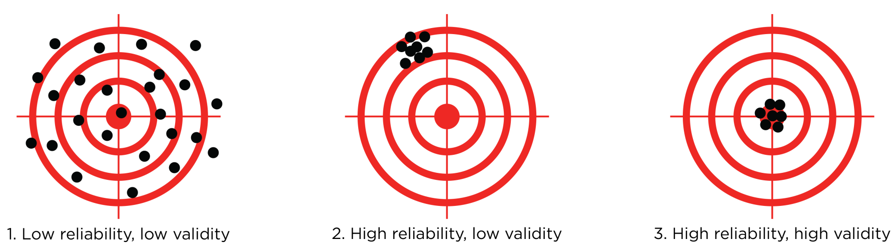


Figure 6.13 Reliability Versus Validity.

7

Basic Statistics

7.1 Getting Started

7.1.1 Load Packages

```
library("petersenlab")
library("pwr")
library("pwrss")
library("WebPower")
library("grid")
library("tidyverse")
```

7.2 Descriptive Statistics

Descriptive statistics are used to describe data. For instance, they may be used to describe the center, spread, or shape of the data. There are various indices of each.

7.2.1 Center

Indices to describe the *center* (central tendency) of a variable's data include:

- mean
- median
- mode

The mean of X (written as: $\bar{X}$) is calculated as:

$$\bar{X} = \frac{\sum X_i}{n} = \frac{X_1 + X_2 + ... + X_n}{n} \tag{7.1}$$

That is, to compute the mean, sum all of the values and divide by the number of values (n).

The median is determined as the value at the 50th percentile (i.e., the value that is higher than 50% of the values and is lower than the other 50% of values).

The mode is the most common/frequent value.

Below is R code to estimate each:

7.2.2 Spread

Indices to describe the *spread* (variability) of a variable's data include:

- standard deviation
- variance
- range
- minimum and maximum
- interquartile range (IQR)

The (sample) variance of X (written as: s^2) is calculated as:

$$s^2 = \frac{\sum (X_i - \bar{X})^2}{n - 1} \tag{7.2}$$

The (sample) standard deviation of X (written as: s) is calculated as:

$$s = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n - 1}} \tag{7.3}$$

The range is calculated of X is calculated as:

$$\text{range} = \text{maximum} - \text{minimum} \tag{7.4}$$

The interquartile range (IQR) is calculated as:

$$\text{IQR} = Q_3 - Q_1 \tag{7.5}$$

where Q_3 is the score at the third quartile (i.e., 75th percentile), and Q_1 is the score at the first quartile (i.e., 25th percentile).

Below is R code to estimate each:

7.2.3 Shape

Indices to describe the *shape* of a variable's data include:

- skewness
- kurtosis

Below is R code to estimate each:

7.2.4 Combination

To estimate multiple indices of center, spread, and shape of the data, you can use the following code:

```
#psych::describe(mydata)

#mydata %>%
#  summarise(across(
#      everything(),
#      .fns = list(
#        n = ~ length(na.omit(.)),
#        missingness = ~ mean(is.na(.)) * 100,
#        M = ~ mean(., na.rm = TRUE),
#        SD = ~ sd(., na.rm = TRUE),
#        min = ~ min(., na.rm = TRUE),
#        max = ~ max(., na.rm = TRUE),
#        range = ~ max(., na.rm = TRUE) - min(., na.rm = TRUE),
#        IQR = ~ IQR(., na.rm = TRUE),
#        median = ~ median(., na.rm = TRUE),
#        mode = ~ petersenlab::Mode(., multipleModes = "mean"),
#        skewness = ~ psych::skew(., na.rm = TRUE),
#        kurtosis = ~ psych::kurtosi(., na.rm = TRUE)),
#      .names = "{.col}.{.fn}")) %>%
#    pivot_longer(
#      cols = everything(),
#      names_to = c("variable","index"),
#      names_sep = "\\.") %>%
#    pivot_wider(
#      names_from = index,
#      values_from = value)
```

7.3 Scores and Scales

There are many different types of scores and scales. This book focuses on raw scores and z-scores. For information on other scores and scales, including percentile ranks, T-scores, standard scores, scaled scores, and stanine scores, see here: https://isaactpetersen.github.io/Principles-Psychological-Assessment/scoresScales.html#scoreTransformation (Petersen, 2024c).

7.3.1 Raw Scores

Raw scores are the original data on the original metric. Thus, raw scores are considered *unstandardized*. For example, raw scores that represent the players' age may range from 20 to 40. Raw scores depend on the construct and unit; thus raw scores may not be comparable across variables.

7.3.2 z Scores

z scores have a mean of zero and a standard deviation of one. z scores are frequently used to render scores across variables more comparable. Thus, z scores are considered a form of a *standardized* score.

z scores are calculated using Equation 7.6:

$$z = \frac{X - \bar{X}}{\sigma} \tag{7.6}$$

where X is the observed score, $\bar{X}$ is the mean observed score, and σ is the standard deviation of the observed scores.

You can easily convert a variable to a z score using the `scale()` function:

```
scale(variable)
```

With a standard normal curve, 68% of scores fall within one standard deviation of the mean. 95% of scores fall within two standard deviations of the mean. 99.7% of scores fall within three standard deviations of the mean.

The area under a normal curve within one standard deviation of the mean is calculated below using the `pnorm()` function, which calculates the cumulative density function for a normal curve.

```
stdDeviations <- 1

pnorm(stdDeviations) - pnorm(stdDeviations * -1)
```

```
[1] 0.6826895
```

The area under a normal curve within one standard deviation of the mean is depicted in Figure 7.1.

```
x <- seq(-4, 4, length = 200)
y <- dnorm(x, mean = 0, sd = 1)
plot(x, y, type = "l",
     xlab = "z Score",
     ylab = "Normal Density")

x <- seq(stdDeviations * -1, stdDeviations, length = 100)
y <- dnorm(x, mean = 0, sd = 1)
polygon(c(stdDeviations * -1, x, stdDeviations),
        c(0, y, 0),
        col = "blue")
```

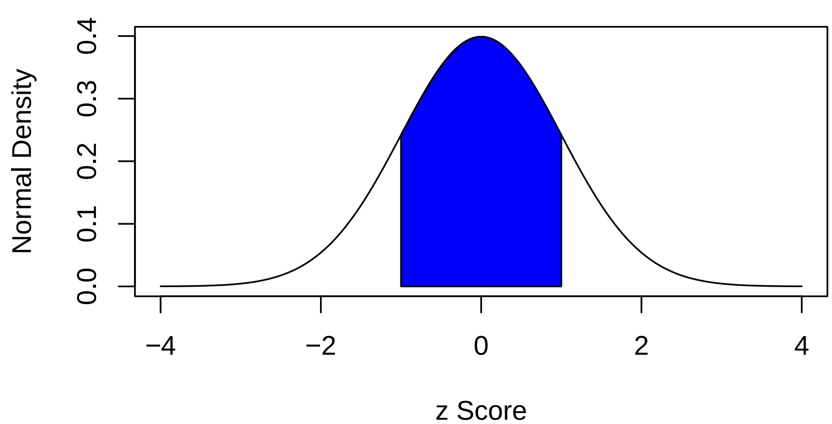


Figure 7.1 Density of Standard Normal Distribution. The blue region represents the area within one standard deviation of the mean.

The area under a normal curve within two standard deviations of the mean is calculated below:

```
stdDeviations <- 2

pnorm(stdDeviations) - pnorm(stdDeviations * -1)
```

```
[1] 0.9544997
```

The area under a normal curve within two standard deviations of the mean is depicted in Figure 7.2.

```
x <- seq(-4, 4, length = 200)
y <- dnorm(x, mean = 0, sd = 1)
plot(x, y, type = "l",
     xlab = "z Score",
     ylab = "Normal Density")

x <- seq(stdDeviations * -1, stdDeviations, length = 100)
y <- dnorm(x, mean = 0, sd = 1)
polygon(c(stdDeviations * -1, x, stdDeviations),
        c(0, y, 0),
        col = "blue")
```

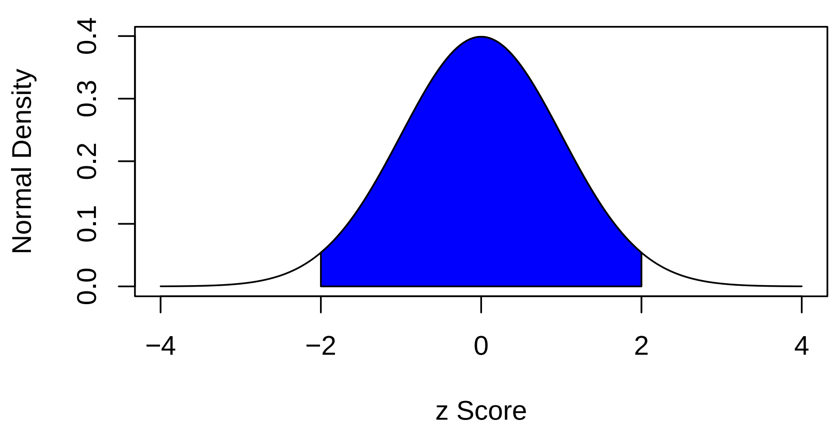


Figure 7.2 Density of Standard Normal Distribution. The blue region represents the area within two standard deviations of the mean.

The area under a normal curve within three standard deviations of the mean is calculated below:

```
stdDeviations <- 3

pnorm(stdDeviations) - pnorm(stdDeviations * -1)
```

```
[1] 0.9973002
```

The area under a normal curve within three standard deviations of the mean is depicted in Figure 7.3.

```
x <- seq(-4, 4, length = 200)
y <- dnorm(x, mean = 0, sd = 1)
plot(x, y, type = "l",
     xlab = "z Score",
     ylab = "Normal Density")

x <- seq(stdDeviations * -1, stdDeviations, length = 100)
y <- dnorm(x, mean = 0, sd = 1)
polygon(c(stdDeviations * -1, x, stdDeviations),
        c(0, y, 0),
        col = "blue")
```

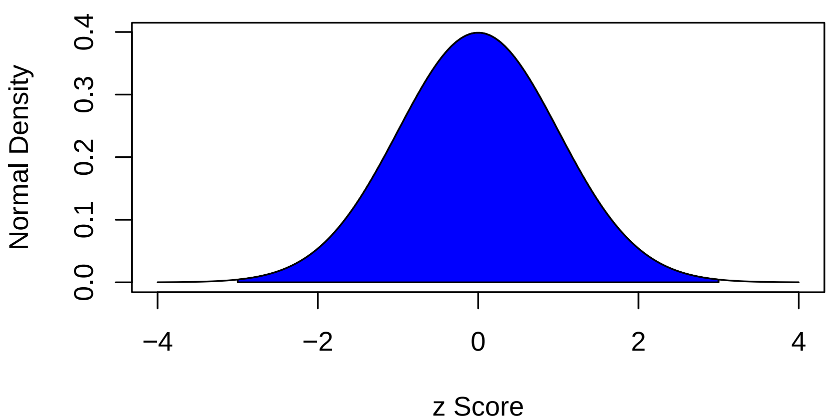


Figure 7.3 Density of Standard Normal Distribution. The blue region represents the area within three standard deviations of the mean.

If you want to determine the z score associated with a particular percentile in a normal distribution, you can use the `qnorm()` function. For instance, the z score associated with the 37th percentile is:

```
qnorm(.37)
```

```
[1] -0.3318533
```

7.4 Inferential Statistics

Inferential statistics are used to draw inferences regarding whether there is (a) a difference in level on variable across groups or (b) an association between variables. For instance, inferential statistics may be used to evaluate whether Quarterbacks tend to have longer careers compared to Running Backs. Or, they could be used to evaluate whether number of carries is associated with injury likelihood. To apply inferential statistics, we make use of the null hypothesis (H_0) and the alternative hypothesis (H_1).

7.4.1 Null Hypothesis Significance Testing

7.4.1.1 Null Hypothesis (H_0)

When testing whether there are differences in level across groups on a variable of interest, the null hypothesis (H_0) is that there is no difference in level across groups. For instance, when testing whether Quarterbacks tend to have longer careers compared to Running Backs, the null hypothesis (H_0) is that Quarterbacks do not systematically differ from Running Backs in the length of their career.

When testing whether there is an association between variables, the null hypothesis (H_0) is that there is no association between the variables. For instance, when testing whether number of carries is associated with injury likelihood, the null hypothesis (H_0) is that there is no association between number of carries and injury likelihood.

7.4.1.2 Alternative Hypothesis (H_1)

The alternative hypothesis (H_1) is the researcher's hypothesis that they want to evaluate. An alternative hypothesis (H_1) might be directional (i.e., one-sided) or non-directional (i.e., two-sided).

Directional hypotheses specify a particular direction, such as which group will have larger scores or which direction (positive or negative) two variables will be associated. Examples of directional hypotheses include:

- Quarterbacks have longer careers compared to Running Backs
- Number of carries is positively associated with injury likelihood

Non-directional hypotheses do not specify a particular direction. For instance, non-directional hypotheses may state that two groups differ but do not specify which group will have larger scores. Or, non-directional hypotheses may state that two variables are associated but do not state what the sign is of the association—i.e., positive or negative. Examples of non-directional hypotheses include:

- Quarterbacks differ in the length of their careers compared to Running Backs
- Number of carries is associated with injury likelihood

7.4.1.3 Statistical Significance

In science, statistical significance is evaluated with the p-value. The p-value does not represent the probability that you observed the result by chance. The p-value represents a conditional probability—it examines the probability of one event given another event. In particular, the p-value evaluates the likelihood that you would detect a result as at least as extreme as the one observed (in terms of the magnitude of the difference or of the association) given that the null hypothesis (H_0) is true.

This can be expressed in conditional probability notation, $Pr(A|B)$, which is the probability (likelihood) of event A occurring given that event B occurred (or given condition B).

The conditional probability notation for a left-tailed directional test (i.e., Quarterbacks have shorter careers than Running Backs; or number of carries is negatively associated with injury likelihood) is in Equation 7.7.

$$p\text{-value} = Pr(T \leq t|H_0) \tag{7.7}$$

where T is the test statistic of interest (e.g., the distribution of t-, $r-$, or F values, depending on the test) and t is the observed test statistic (e.g., t-, $r-$, or F-coefficient, depending on the test).

The conditional probability notation for a right-tailed directional test (i.e., Quarterbacks have longer careers than Running Backs; or number of carries is positively associated with injury likelihood) is in Equation 7.8.

$$p\text{-value} = Pr(T \geq t|H_0) \tag{7.8}$$

The conditional probability notation for a two-tailed non-directional test (i.e., Quarterbacks differ in the length of their careers compared to Running Backs; or number of carries is associated with injury likelihood) is in Equation 7.9.

$$p\text{-value} = 2 \times \min(Pr(T \leq t|H_0), Pr(T \geq t|H_0)) \tag{7.9}$$

where `min(a, b)` is the smaller number of `a` and `b`.

If the distribution of the test statistic is symmetric around zero, the p-value for the two-tailed non-directional test simplifies to Equation 7.10.

$$p\text{-value} = 2 \times Pr(T \geq |t||H_0) \tag{7.10}$$

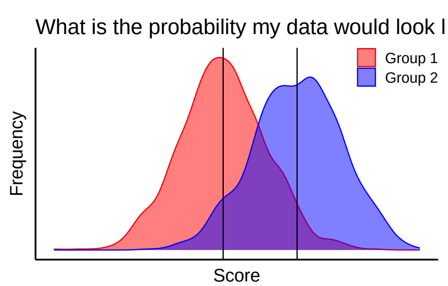
Nevertheless, to be conservative (i.e., to avoid false positive/Type I errors), many researchers use two-tailed p-values regardless whether their hypothesis is one- or two-tailed.

For a test of group differences, the p-value evaluates the likelihood that you would observe a difference as large or larger than the one you observed between the groups if there were no systematic difference between the groups, as depicted in Figure 7.4. For instance, when evaluating whether Quarterbacks have longer careers than Running Backs, and you observed a mean difference of 0.03 years, the p-value evaluates the likelihood that you would observe a difference as larger or larger than 0.03 years between the groups if Quarterbacks do not differ from Running Backs in terms of the length of their career.

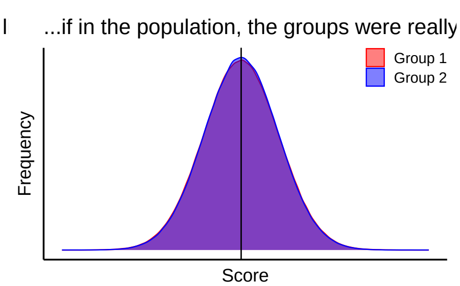
```
set.seed(52242)

nObserved <- 1000
nPopulation <- 1000000

observedGroups <- data.frame(
  score = c(rnorm(nObserved, mean = 47, sd = 3), rnorm(nObserved, mean = 52, sd = 3)),
  group = as.factor(c(rep("Group 1", nObserved), rep("Group 2", nObserved)))
)

populationGroups <- data.frame(
  score = c(rnorm(nPopulation, mean = 50, sd = 3.03), rnorm(nPopulation, mean = 50, sd = 3)),
  group = as.factor(c(rep("Group 1", nPopulation), rep("Group 2", nPopulation)))
)

ggplot2::ggplot(
  data = observedGroups,
  mapping = aes(
    x = score,
    fill = group,
```

```
    color = group
  )
) +
  geom_density(alpha = 0.5) +
  scale_color_manual(values = c("red", "blue")) +
  scale_fill_manual(values = c("red","blue")) +
  geom_vline(xintercept = mean(observedGroups$score[which(observedGroups$group == "Group 1")])) +
  geom_vline(xintercept = mean(observedGroups$score[which(observedGroups$group == "Group 2")])) +
  ggplot2::labs(
    x = "Score",
    y = "Frequency",
    title = "What is the probability my data would look like this..."
  ) +
  ggplot2::theme_classic(
    base_size = 16) +
  ggplot2::theme(
    legend.title = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank(),
    #plot.title.position = "plot"
    legend.position = "inside",
    legend.margin = margin(0, 0, 0, 0),
    legend.justification.top = "left",
    legend.justification.left = "top",
    legend.justification.bottom = "right",
    legend.justification.inside = c(1, 1),
    legend.location = "plot")

ggplot2::ggplot(
  data = populationGroups,
  mapping = aes(
    x = score,
    fill = group,
    color = group
  )
) +
  geom_density(alpha = 0.5) +
  scale_color_manual(values = c("red", "blue")) +
  scale_fill_manual(values = c("red","blue")) +
  geom_vline(xintercept = mean(populationGroups$score[which(populationGroups$group == "Group 1")])
  geom_vline(xintercept = mean(populationGroups$score[which(populationGroups$group == "Group 2")])
  ggplot2::labs(
```

```
    x = "Score",
    y = "Frequency",
    title = "...if in the population, the groups were really this:"
  ) +
  ggplot2::theme_classic(
    base_size = 16) +
  ggplot2::theme(
    legend.title = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank(),
    #plot.title.position = "plot",
    legend.position = "inside",
    legend.margin = margin(0, 0, 0, 0),
    legend.justification.top = "left",
    legend.justification.left = "top",
    legend.justification.bottom = "right",
    legend.justification.inside = c(1, 1),
    legend.location = "plot")
```



(a) What is the probability my data would look like this...



(b) ...if in the population, the groups were really this?

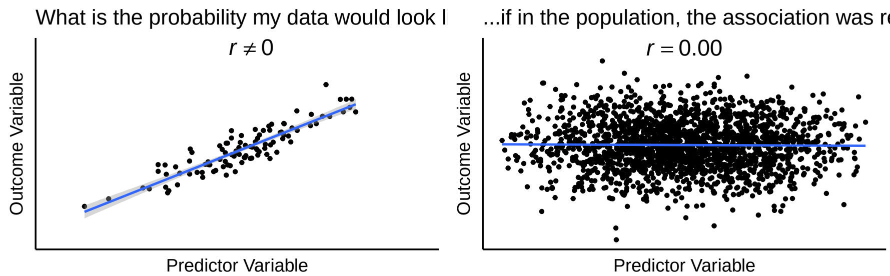
Figure 7.4 Interpretation of p-Values When Examining The Differences Between Groups. The vertical black lines reflect the group means.

For a test of whether two variables are associated, the p-value evaluates the likelihood that you would observe an association as strong or stronger than the one you observed between the groups if there were no association between the variables, as depicted in Figure 7.5. For instance, when evaluating whether number of carries is positively associated with injury likelihood, and you observed a correlation coefficient of $r = .25$ between number of carries and injury likelihood, the p-value evaluates the likelihood that you would observe a correlation as strong or stronger than $r = .25$ between the variables if number of carries is not associated with injury likelihood.

```
set.seed(52242)

observedCorrelation <- 0.9

correlations <- data.frame(criterion = rnorm(2000))
correlations$sample <- NA
correlations$sample[1:100] <- complement(correlations$criterion[1:100], observedCorrelation)
correlations$population <- complement(correlations$criterion, 0)

ggplot2::ggplot(
  data = correlations,
  mapping = aes(
    x = sample,
    y = criterion
  )
) +
  geom_point() +
  geom_smooth(method = "lm") +
  scale_x_continuous(
    limits = c(-3.5,3)
  ) +
  annotate(
    x = 0,
    y = 4,
    label = paste("italic(r) != ", 0, sep = ""),
    parse = TRUE,
    geom = "text",
    size = 7) +
  ggplot2::labs(
    x = "Predictor Variable",
    y = "Outcome Variable",
    title = "What is the probability my data would look like this..."
  ) +
  ggplot2::theme_classic(
    base_size = 16) +
  ggplot2::theme(
    legend.title = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())

ggplot2::ggplot(
```

```
  data = correlations,
  mapping = aes(
    x = population,
    y = criterion
  )
) +
  geom_point() +
  geom_smooth(
    method = "lm",
    se = FALSE) +
  scale_x_continuous(
    limits = c(-2.5,2.5)
  ) +
  annotate(
    x = 0,
    y = 4,
    label = paste("italic(r) == '", "0.00", "'", sep = ""),
    parse = TRUE,
    geom = "text",
    size = 7) +
  ggplot2::labs(
    x = "Predictor Variable",
    y = "Outcome Variable",
    title = "...if in the population, the association was really this:"
  ) +
  ggplot2::theme_classic(
    base_size = 16) +
  ggplot2::theme(
    legend.title = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())
```

Using what is called null-hypothesis significance testing (NHST), we consider an effect to be *statistically significant* if the p-value is less than some threshold, called the *alpha level*. In science, we typically want to be conservative because a false positive (i.e., Type I error) is considered more problematic than a false negative (i.e., Type II error). That is, we would rather say an effect does not exist when it really does than to say an effect does exist when it really does not. Thus, we typically set the alpha level to a low value, commonly .05. Then, we would consider an effect to be *statistically significant* if the p-value is less than .05. That is, there is a small chance (5%; or 1 in 20 times) that we would observe an effect at least as extreme as the effect observed, if the null hypothesis were true. So, you might expect around 5% of tests where the



(a) What is the probability my data would look like this...

(b) ...if in the population, the association was really this?

Figure 7.5 Interpretation of p-Values When Examining The Association Between Variables.

null hypothesis is true to be statistically significant just by chance. We could lower the rate of Type II (i.e., false negative) errors—i.e., we could detect more effects—if we set the alpha level to a higher value (e.g., .10); however, raising the alpha level would raise the possibility of Type I (false positive) errors.

If the p-value is less than .05, we reject the null hypothesis (H_0) that there was no difference or association. Thus, we conclude that there was a statistically significant (non-zero) difference or association. If the p-value is greater than .05, we fail to reject the null hypothesis; the difference/association was not statistically significant. Thus, we do not have confidence that there was a difference or association. However, we do not accept the null hypothesis; it could be there we did not observe an effect because we did not have adequate power to detect the effect—e.g., if the effect size was small, the data were noisy, and the sample size was small and/or unrepresentative.

There are four general possibilities of decision making outcomes when performing null-hypothesis significance testing:

1. We (correctly) reject the null hypothesis when it is in fact false ($1-\beta$). This is a true positive. For instance, we may correctly determine that Quarterbacks have longer careers than Running Backs.
2. We (correctly) fail to reject the null hypothesis when it is in fact true ($1 - \alpha$). This is a true negative. For instance, we may correctly determine that Quarterbacks do not have longer careers than Running Backs.
3. We (incorrectly) reject the null hypothesis when it is in fact true (α). This is a false positive. When performing null hypothesis testing, a false positive is known as a Type I error. For instance, we may incorrectly determine that Quarterbacks have longer careers than

Running Backs when, in fact, Quarterbacks and Running Backs do not differ in their career length.

4. We (incorrectly) fail to reject the null hypothesis when it is in fact false (β). This is a false negative. When performing null hypothesis testing, a false negative is known as a Type II error. For instance, we may incorrectly determine that Quarterbacks and Running Backs do not differ in their career length when, in fact, Quarterbacks have longer careers than Running Backs.

A two-by-two confusion matrix for null-hypothesis significance testing is in Figure 7.6.

Truth \ **Decision**	Reject H_0	Fail to reject H_0
H_0 false	Correct True Positive $1 - \beta$ ("power")	Type II error False Negative beta (β)
H_0 true	Type I error False Positive alpha (α)	Correct True Negative $1 - \alpha$

Figure 7.6 A Two-by-Two Confusion Matrix for Null-Hypothesis Significance Testing.

In statistics, *power* is the probability of detecting an effect, if, in fact, the effect exists. Otherwise said, power is the probability of rejecting the null hypothesis, if, in fact, the null hypothesis is false. Power is influenced by several variables:

- the sample size (N): the larger the N, the greater the power
  - for group comparisons, the power depends on the sample size of each group
- the effect size: the larger the effect, the greater the power
  - for group comparisons, larger effect sizes reflect:
    * larger between-group variance, and
    * smaller within-group variance (i.e., strong measurement precision, i.e., reliability)
- the alpha level: the researcher specifies the alpha level (though it is typically set at .05); the higher the alpha level, the greater the power; however, the higher we set the alpha level, the higher the likelihood of Type I errors (false positives)

- one- versus two-tailed tests: one-tailed tests have higher power than two-tailed tests
- within-subject versus between-subject comparisons: within-subject designs tend to have greater power than between-subject designs

A plot of statistical power is in Figure 7.7.

```
m1 <- 0  # mu H0
sd1 <- 1.5 # sigma H0
m2 <- 3.5 # mu HA
sd2 <- 1.5 # sigma HA

z_crit <- qnorm(1-(0.05/2), m1, sd1)

# set length of tails
min1 <- m1-sd1*4
max1 <- m1+sd1*4
min2 <- m2-sd2*4
max2 <- m2+sd2*4
# create x sequence
x <- seq(min(min1,min2), max(max1, max2), .01)
# generate normal dist #1
y1 <- dnorm(x, m1, sd1)
# put in data frame
df1 <- data.frame("x" = x, "y" = y1)
# generate normal dist #2
y2 <- dnorm(x, m2, sd2)
# put in data frame
df2 <- data.frame("x" = x, "y" = y2)

# Alpha polygon
y.poly <- pmin(y1,y2)
poly1 <- data.frame(x=x, y=y.poly)
poly1 <- poly1[poly1$x >= z_crit, ]
poly1<-rbind(poly1, c(z_crit, 0))  # add lower-left corner

# Beta polygon
poly2 <- df2
poly2 <- poly2[poly2$x <= z_crit,]
poly2<-rbind(poly2, c(z_crit, 0))  # add lower-left corner

# power polygon; 1-beta
poly3 <- df2
poly3 <- poly3[poly3$x >= z_crit,]
poly3 <-rbind(poly3, c(z_crit, 0))  # add lower-left corner
```

```
# combine polygons.
poly1$id <- 3 # alpha, give it the highest number to make it the top layer
poly2$id <- 2 # beta
poly3$id <- 1 # power; 1 - beta
poly <- rbind(poly1, poly2, poly3)
poly$id <- factor(poly$id,  labels=c("power","beta","alpha"))

# plot with ggplot2
ggplot(poly, aes(x,y, fill=id, group=id)) +
  geom_polygon(show.legend=F, alpha=I(8/10)) +
  # add line for treatment group
  geom_line(data=df1, aes(x,y, color="H0", group=NULL, fill=NULL), linewidth=1.5, show_guide=F) +
  # add line for treatment group. These lines could be combined into one dataframe.
  geom_line(data=df2, aes(color="HA", group=NULL, fill=NULL),linewidth=1.5, show_guide=F) +
  # add vlines for z_crit
  geom_vline(xintercept = z_crit, linewidth=1, linetype="dashed") +
  # change colors
  scale_color_manual("Group",
                     values= c("HA" = "#981e0b","H0" = "black")) +
  scale_fill_manual("test", values= c("alpha" = "#0d6374","beta" = "#be805e","power"="#7cecee")) +
  # beta arrow
  annotate("segment", x=0.1, y=0.045, xend=1.3, yend=0.01, arrow = arrow(length = unit(0.3, "cm"))
  annotate("text", label="beta", x=0, y=0.05, parse=T, size=8) +
  # alpha arrow
  annotate("segment", x=4, y=0.043, xend=3.4, yend=0.01, arrow = arrow(length = unit(0.3, "cm")), 
  annotate("text", label="frac(alpha,2)", x=4.2, y=0.05, parse=T, size=8) +
  # power arrow
  annotate("segment", x=6, y=0.2, xend=4.5, yend=0.15, arrow = arrow(length = unit(0.3, "cm")), li
  annotate("text", label=expression(paste(1-beta, "  (\"power\")")), x=6.1, y=0.21, parse=T, size=
  # H_0 title
  annotate("text", label="H[0]", x=m1, y=0.28, parse=T, size=8) +
  # H_a title
  annotate("text", label="H[1]", x=m2, y=0.28, parse=T, size=8) +
  ggtitle("Statistical Power") +
  # remove some elements
  theme(
    panel.grid.minor = element_blank(),
    panel.grid.major = element_blank(),
    panel.background = element_blank(),
    plot.background = element_rect(fill="white"),
    panel.border = element_blank(),
    axis.line = element_blank(),
    axis.text.x = element_blank(),
    axis.text.y = element_blank(),
```

```
    axis.ticks = element_blank(),
    axis.title.x = element_blank(),
    axis.title.y = element_blank(),
    plot.title = element_text(size=22))
```

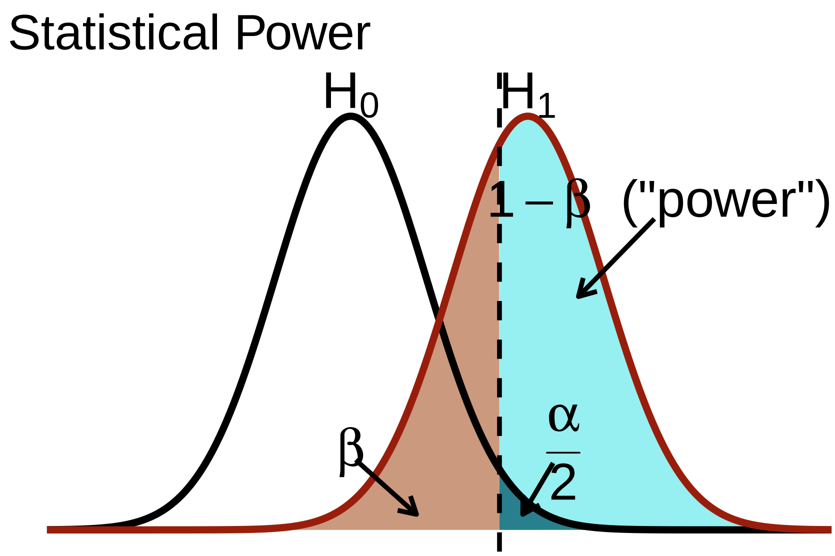


Figure 7.7 Statistical Power (Adapted from Kristoffer Magnusson: https://rpsychologist.com/creating-a-typical-textbook-illustration-of-statistical-power-using-either-ggplot-or-base-graphics; archived at https://perma.cc/FG3J-85L6). The dashed line represents the critical value or threshold.

Interactive visualizations by Kristoffer Magnusson on *p*-values and null-hypothesis significance testing are below:

- https://rpsychologist.com/pvalue/ (archived at https://perma.cc/JP9F-9ZVY)
- https://rpsychologist.com/d3/pdist/ (archived at https://perma.cc/BE96-8LSJ)
- https://rpsychologist.com/d3/nhst/ (archived at https://perma.cc/ZU9A-37F3)

Twelve misconceptions about *p*-values (Goodman, 2008) are in Table 7.1.

Table 7.1 Twelve Misconceptions About p-Values from Goodman (2008). Goodman also provides a discussion about why each statement is false.

Number	Misconception
1	If $p = .05$, the null hypothesis has only a 5% chance of being true.
2	A nonsignificant difference (eg, $p > .05$) means there is no difference between groups.
3	A statistically significant finding is clinically important.
4	Studies with p-values on opposite sides of .05 are conflicting.
5	Studies with the same p-value provide the same evidence against the null hypothesis.
6	$p = .05$ means that we have observed data that would occur only 5% of the time under the null hypothesis.
7	$p = .05$ and $p < .05$ mean the same thing.
8	p-values are properly written as inequalities (e.g., "$p \leq .05$" when $p = .015$).
9	$p = .05$ means that if you reject the null hypothesis, the probability of a Type I error is only 5%.
10	With a $p = .05$ threshold for significance, the chance of a Type I error will be 5%.
11	You should use a one-sided p-value when you don't care about a result in one direction, or a difference in that direction is impossible.
12	A scientific conclusion or treatment policy should be based on whether or not the p-value is significant.

That is, the p-value is not:

- the probability that the effect was due to chance
- the probability that the null hypothesis is true
- the size of the effect
- the importance of the effect
- whether the effect is true, real, or causal

Statistical significance involves the *consistency* of an effect/association/difference; it suggests that the association/difference is reliably non-zero. However, just because something is statistically significant does not mean that it is important. For instance, consider that we discover that players who consume sports drink before a game tend to perform better than players who do not ($p < .05$). However, what if consumption of sports drinks is associated with an average improvement of 0.002 points per game. A small effect such as this might be detectable with a large sample size. This effect would be considered to be reliable/consistent because it is statistically significant. However, such an effect would not be practically important because it has an effect size so small that it results in differences that are not practically important. Thus, in addition to statistical significance, it is also important to consider practical significance.

7.4.2 Practical Significance

Practical significance deals with how large or important the effect/association/difference is. It is based on the magnitude of the effect, called the *effect size*. Effect size can be quantified in various ways including:

- Cohen's d
- Standardized regression coefficient (beta; β)
- Correlation coefficient (r)
- Cohen's ω (omega)
- Cohen's f
- Cohen's f^2
- Coefficient of determination (R^2)
- Eta squared (η^2)
- Partial eta squared (η_p^2)

7.4.2.1 Cohen's d

Cohen's d is calculated as in Equation 7.11:

$$\begin{aligned} d &= \frac{\text{mean difference}}{\text{pooled standard deviation}} \\ &= \frac{\bar{X}_1 - \bar{X}_2}{s} \end{aligned} \tag{7.11}$$

where:

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \tag{7.12}$$

where n_1 and n_2 is the sample size of group 1 and group 2, respectively, and s_1 and s_2 is the standard deviation of group 1 and group 2, respectively.

7.4.2.2 Standardized Regression Coefficient (Beta; β)

The standardized regression coefficient (beta; β) is used in multiple regression, and is calculated as in Equation 7.13:

$$\beta_x = B_x \times \frac{s_x}{s_y} \tag{7.13}$$

where B_x is the unstandardized regression coefficient of the predictor variable x in predicting the outcome variable y, s_x is the standard deviation of x, and s_y is the standard deviation of y.

7.4.2.3 Correlation Coefficient (r)

The formula for the correlation coefficient is in Chapter 8.

7.4.2.4 Cohen's ω

Cohen's ω is used in chi-square tests, and is calculated as in Equation 7.14:

$$\omega = \sqrt{\frac{\chi^2}{N} - \frac{df}{N}} \tag{7.14}$$

where χ^2 is the chi-square statistic from the test, N is the sample size, and df is the degrees of freedom.

7.4.2.5 Cohen's f

Cohen's f is commonly used in ANOVA, and is calculated as in Equation 7.15:

$$\begin{aligned} f &= \sqrt{\frac{R^2}{1-R^2}} \\ &= \sqrt{\frac{\eta^2}{1-\eta^2}} \end{aligned} \tag{7.15}$$

7.4.2.6 Cohen's f^2

Cohen's f^2 is commonly used in regression, and is calculated as in Equation 7.16:

$$\begin{aligned} f^2 &= \frac{R^2}{1-R^2} \\ &= \frac{\eta^2}{1-\eta^2} \end{aligned} \tag{7.16}$$

To calculate the effect size of a particular predictor, you can calculate Δf^2 as in Equation 7.17:

$$\begin{aligned}\Delta f^2 &= \frac{R^2_{\text{model}} - R^2_{\text{reduced}}}{1 - R^2_{\text{model}}} \\ &= \frac{\eta^2_{\text{model}} - \eta^2_{\text{reduced}}}{1 - \eta^2_{\text{model}}}\end{aligned} \tag{7.17}$$

where R^2_{model} is the R^2 of the model with the predictor variable of interest and R^2_{reduced} is the R^2 of the model without the predictor variable of interest.

7.4.2.7 Coefficient of Determination (R^2)

The coefficient of determination (R^2) reflects the proportion of variance in the outcome variable that is explained by the predictor variable(s). R^2 is commonly used in regression, and is calculated as in Equation 7.18:

$$\begin{aligned}R^2 &= 1 - \frac{\sum(Y_i - \hat{Y}_i)^2}{\sum(Y_i - \bar{Y})^2} \\ &= 1 - \frac{SS_{\text{residual}}}{SS_{\text{total}}} \\ &= 1 - \frac{\text{sum of squared residuals}}{\text{total sum of squares}} \\ &= \frac{f^2}{1 + f^2} \\ &= \eta^2 \\ &= \frac{\text{variance explained in } Y}{\text{total variance in } Y}\end{aligned} \tag{7.18}$$

where Y_i is the observed value of the outcome variable for the ith observation, $\hat{Y}_i$ is the model predicted value for the ith observation, $\bar{Y}$ is the mean of the observed values of the outcome variable. The total sum of squares is an index of the total variation in the outcome variable.

7.4.2.8 Eta Squared (η^2) and Partial Eta Squared (η^2_p)

Like R^2, eta squared (η^2) reflects the proportion of variance in the dependent variable that is explained by the independent variable(s). η^2 is commonly used in ANOVA, and is calculated as in Equation 7.19:

$$\begin{aligned}\eta^2 &= \frac{SS_{\text{effect}}}{SS_{\text{total}}} \\ &= 1 - \frac{SS_{\text{residual}}}{SS_{\text{total}}} \\ &= 1 - \frac{\text{sum of squared residuals}}{\text{total sum of squares}} \\ &= \frac{f^2}{1+f^2} \\ &= R^2\end{aligned} \tag{7.19}$$

where SS_{effect} is the sum of squares for the effect of interest and SS_{total} is the total sum of squares.

Partial eta squared (η_p^2) reflects the proportion of variance in the dependent variable that is explained by the independent variable while controlling for the other independent variables. η_p^2 is commonly used in ANOVA, and is calculated as in Equation 7.20:

$$\eta_p^2 = \frac{SS_{\text{effect}}}{SS_{\text{effect}} + SS_{\text{error}}} \tag{7.20}$$

where SS_{effect} is the sum of squares for the effect of interest and SS_{error} is the sum of squares for the residual error term.

7.4.2.9 Effect Size Thresholds

Effect size thresholds (McGrath & Meyer, 2006) for small, medium, and large effect sizes are in Table 7.2.

Table 7.2 Effect Size Thresholds for Small, Medium, and Large Effect Sizes.

Effect Size Index	Small	Medium	Large
Cohen's d	$\geq \lvert .20 \rvert$	$\geq \lvert .50 \rvert$	$\geq \lvert .80 \rvert$
Standardized regression coefficient (beta; β)	$\geq \lvert .10 \rvert$	$\geq \lvert .24 \rvert$	$\geq \lvert .37 \rvert$
Correlation coefficient (r)	$\geq \lvert .10 \rvert$	$\geq \lvert .24 \rvert$	$\geq \lvert .37 \rvert$
Cohen's ω	$\geq .10$	$\geq .30$	$\geq .50$
Cohen's f	$\geq .10$	$\geq .25$	$\geq .40$
Cohen's f^2	$\geq .01$	$\geq .06$	$\geq .16$
Coefficient of determination (R^2)	$\geq .01$	$\geq .06$	$\geq .14$
Eta squared (η^2)	$\geq .01$	$\geq .06$	$\geq .14$
Partial eta squared (η_p^2)	$\geq .01$	$\geq .06$	$\geq .14$

7.5 Statistical Decision Tree

An example statistical decision tree is depicted in Figure 7.8.

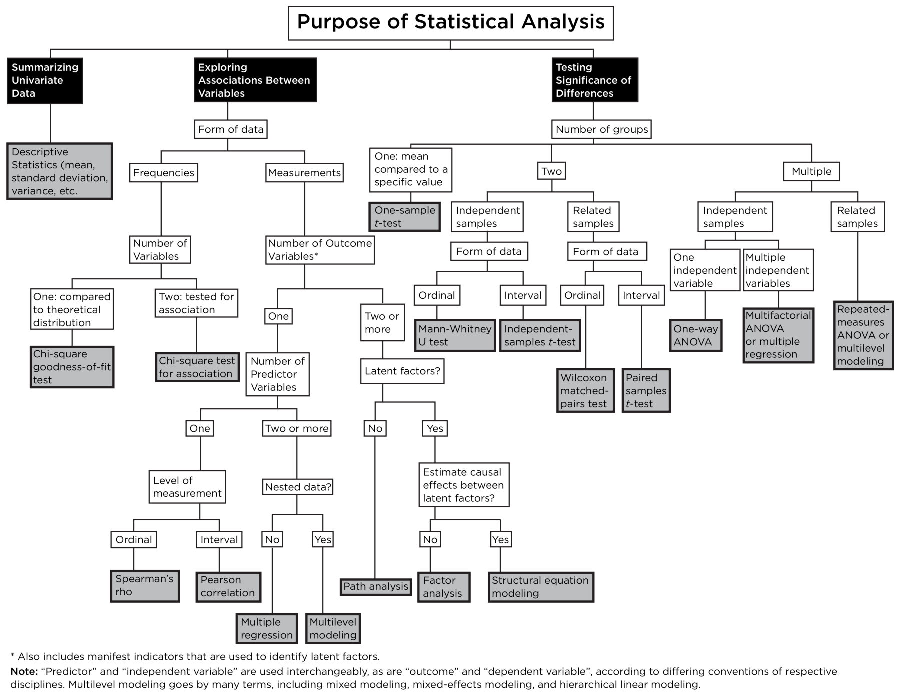


Figure 7.8 A Statistical Decision Tree For Choosing an Appropriate Statistical Procedure. Adapted from: https://commons.wikimedia.org/wiki/File:InferentialStatisticalDecisionMakingTrees.pdf. The original source is: Corston, R. & Colman, A. M. (2000). *A crash course in SPSS for Windows*. Wiley-Blackwell. Changes were made to the original, including the addition of several statistical tests.

However, many statistical tests can be re-formulated in a regression framework, as in Figure 7.9.

For an online, interactive statistical decision tree to help you decide which statistical analysis to use, see here: https://www.statsflowchart.co.uk

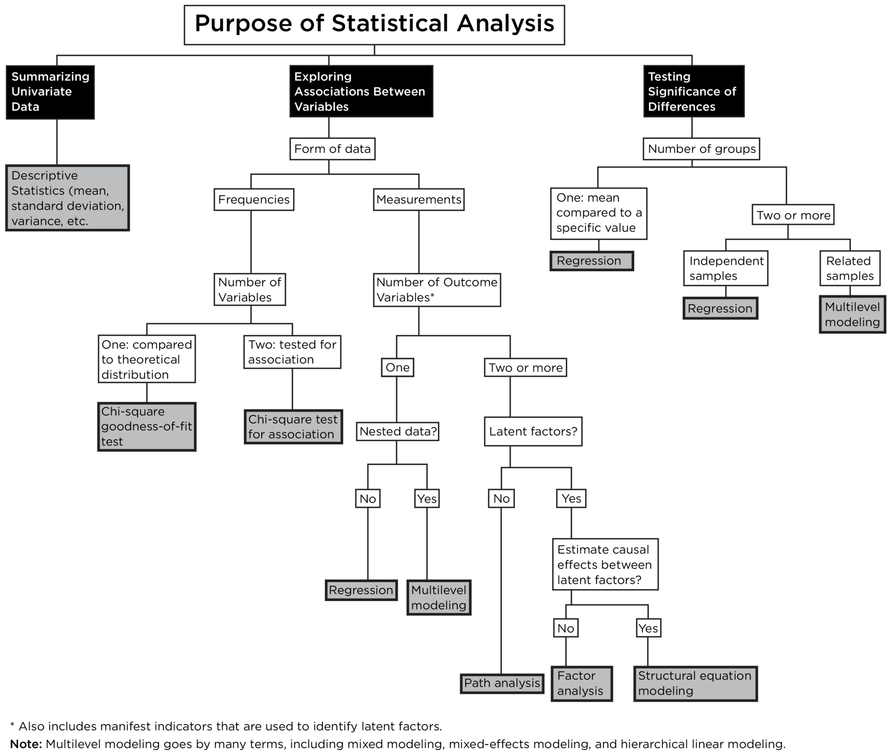


Figure 7.9 A Statistical Decision Tree For Choosing an Appropriate Statistical Procedure, Re-Formulated in a Regression Framework. Adapted from: https://commons.wikimedia.org/wiki/File:InferentialStatisticalDecisionMakingTrees.pdf. The original source is: Corston, R. & Colman, A. M. (2000). *A crash course in SPSS for Windows*. Wiley-Blackwell. Changes were made to the original, including re-formulating the tests in a regression framework.

7.6 Statistical Tests

7.6.1 *t*-Test

There are several *t*-tests:

- one-sample *t*-test
- two-samples *t*-test
    - independent samples *t*-test

- paired samples t-test

A one-sample t-test is used to evaluate whether a sample mean differs systematically from a particular value. The null hypothesis is that the sample mean does not differ systematically from the pre-specified value. The alternative hypothesis is that the sample mean differs systematically from the pre-specified value. For instance, let's say you want to test out a new draft strategy. You could participate in a mock draft and draft players using the new strategy. Then, you could use a one-sample t-test to evaluate whether your new draft strategy yields players with more projected points than the average of players' projected points for other teams.

Two-samples t-tests are used to test for differences between scores of two groups. If the two groups are independent, the independent samples t-test is used. If the two groups involve paired samples, the paired samples t-test is used. The null hypothesis is that the mean of group 1 does not differ systematically from the mean of group 2. The alternative hypothesis is that the mean of group 1 differs systematically from the mean of group 2. For instance, you could use an independent-samples t-test if you want to examine whether Quarterbacks tend to have have longer careers than Running Backs. By contrast, you could use a paired samples t-test if you want to examine whether Quarterbacks tend to score more points in the second year of their contract compared to their rookie year, because the same subjects were assessed twice (i.e., a within-subject design).

7.6.2 Analysis of Variance

Analysis of variance (ANOVA) allows examining whether groups differ systematically as a function of one or more factors. There are multiple variants of ANOVA:

- one-way ANOVA
- factorial ANOVA
- repeated measures ANOVA (RM-ANOVA)
- multivariate ANOVA (MANOVA)

Like two-samples t-tests, ANOVA allows examining whether groups differ as a function of an independent variable. However, unlike a t-test, ANOVA allows examining multiple multiple independent variables and more than two groups. The null hypothesis is that the the groups' mean value does not differ systematically. The alternative hypothesis is that the groups' mean value differs systematically.

A one-way ANOVA examines whether two or more groups differ as a function of an independent variable. For instance, you could use a one-way ANOVA to evaluate if you want to evaluate whether multiple positions differ in their length of career. Factorial ANOVA examines whether two or more groups differ as a function of multiple independent variables. For instance, you could use factorial ANOVA to evaluate whether one's length of career depends on one's position and weight. Repeated measures ANOVA examines whether scores differ across repeated measures (e.g., across time) for the same participants. For instance, you could use repeated-measures ANOVA to evaluate whether rookies score more points as the season progresses. Multivariate ANOVA examines whether multiple dependent variables differ as a function of one or more factor(s). For instance, you could use MANOVA to evaluate whether one's contract length and pay differ as a function of one's position.

7.6.3 Correlation

Correlation examines the association between a predictor and outcome variable. The null hypothesis is that the the two variables are not associated. The alternative hypothesis is that the two variables are associated.

The Pearson correlation coefficient (r) is calculated as in Equation 7.21:

$$r = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum(X_i - \bar{X})^2 \sum(Y_i - \bar{Y})^2}} \tag{7.21}$$

where X is the predictor variable and Y is the outcome variable.

7.6.4 (Multiple) Regression

Regression, like correlation, examines the association between a predictor and outcome variable. However, unlike correlation, regression allows multiple predictor variables.

Regression with a single predictor takes the form in Equation 9.1. A regression line is depicted in Figure 9.4. Multiple regression (i.e., regression with multiple predictors) takes the form in Equation 9.2.

The null hypothesis is that the the predictor variable(s) are not associated with the outcome variable. The alternative hypothesis is that the predictor variable(s) are associated with the outcome variable.

7.6.5 Chi-Square Test

There are two primary types of chi-square tests:

- chi-square goodness-of-fit test
- chi-square test for association (aka test of independence)

The chi-square goodness-of-fit test evaluates whether a set of categorical data came from a specified distribution. The null hypothesis is that the data came from the specified distribution. The alternative hypothesis is that the data did not come from the specified distribution.

The chi-square test for association evaluates whether two categorical variables are associated. The null hypothesis is that the two variables are not associated. The alternative hypothesis is that the two variables are associated.

7.6.6 Formulating Statistical Tests in Terms of Partitioned Variance

Many statistical tests can be formulated in terms of partitioned variance.

For instance, the t statistic from the independent-samples t-test and the F statistic from ANOVA can be thought of as the ratio of between-group variance to within-group variance, as in Equation 7.22:

$$t \text{ or } F = \frac{\text{between-group variance}}{\text{within-group variance}} \tag{7.22}$$

The correlation coefficient can be thought of as the ratio of shared variance (i.e., covariance) to total variance, as in Equation 7.23:

$$r = \frac{\text{shared variance}}{\text{total variance}} \tag{7.23}$$

The coefficient of determination (R^2) is the proportion of variance in the outcome variable that is explained by the predictor variables. η^2 is the proportion of variance in the dependent variable that is explained by the independent variables. The coefficient of determination and η^2 can be expressed as the ratio of variance explained in the outcome or dependent variable to the total variance in the outcome or dependent variable, as in Equation 7.24:

$$R^2 \text{ or } \eta^2 = \frac{\text{variance explained in the outcome variable}}{\text{total variance in the outcome variable}} \tag{7.24}$$

7.6.7 Critical Value

The critical value is the test value for a given test, above which the effect is considered to be statistically significant. The critical value for statistical significance for each test can be determined based on the degrees of freedom and alpha level. The degrees of freedom (df) refer to the number of values in the calculation of a test statistic that are free to vary.

```
alpha <- .05
N <- 200
nGroup1 <- 150
nGroup2 <- 150
numGroups <- 4
numLevelsFactorA <- 3
numLevelsFactorB <- 4
numMeasurements <- 4
numPredictors <- 5
numCategories <- 6
numRows <- 5
numColumns <- 2
```

7.6.7.1 One-Sample *t*-Test

For a one-sample t-test, the degrees of freedom is in Equation 7.25:

$$df = N - 1 \tag{7.25}$$

where N is sample size.

```
df_oneSampleTtest <- N - 1
```

One-tailed test:

```
qt(1 - alpha, df_oneSampleTtest)
```

```
[1] 1.652547
```

Two-tailed test:

```
qt(1 - alpha/2, df_oneSampleTtest)
```

```
[1] 1.971957
```

7.6.7.2 Independent-Samples *t*-Test

For an independent-samples t-test, the degrees of freedom is in Equation 7.26:

$$df = n_1 + n_2 - 2 \tag{7.26}$$

where n_1 is the sample size of group 1 and n_2 is the sample size of group 2.

```r
df_independentSamplesTtest <- nGroup1 + nGroup2 - 2
```

One-tailed test:

```r
qt(1 - alpha, df_independentSamplesTtest)
```

```
[1] 1.649983
```

Two-tailed test:

```r
qt(1 - alpha/2, df_independentSamplesTtest)
```

```
[1] 1.967957
```

7.6.7.3 Paired-Samples *t*-Test

For a paired-samples t-test, the degrees of freedom is in Equation 7.27:

$$df = N - 1 \tag{7.27}$$

where N is sample size (i.e., the number of paired observations).

```r
df_pairedSamplesTtest <- N - 1
```

One-tailed test:

```r
qt(1 - alpha, df_pairedSamplesTtest)
```

```
[1] 1.652547
```

Two-tailed test:

```r
qt(1 - alpha/2, df_pairedSamplesTtest)
```

```
[1] 1.971957
```

7.6.7.4 One-Way ANOVA

For a one-way ANOVA, the degrees of freedom is in Equation 7.28:

$$\begin{aligned} df_{\text{between}} &= g - 1 \\ df_{\text{within}} &= N - g \end{aligned} \tag{7.28}$$

where N is sample size and g is the number of groups.

```
df_betweenOneWayANOVA <- numGroups - 1
df_withinOneWayANOVA <- N - numGroups
```

One-tailed test:

```
qf(1 - alpha, df_betweenOneWayANOVA, df_withinOneWayANOVA)
```

```
[1] 2.650677
```

Two-tailed test:

```
qf(1 - alpha/2, df_betweenOneWayANOVA, df_withinOneWayANOVA)
```

```
[1] 3.183378
```

7.6.7.5 Factorial ANOVA

For a factorial two-way ANOVA, the degrees of freedom is in Equation 7.29:

$$\begin{aligned} df_{\text{Factor A}} &= a - 1 \\ df_{\text{Factor B}} &= b - 1 \\ df_{\text{Interaction}} &= (a - 1)(b - 1) \\ df_{\text{error}} &= ab(N - 1) \end{aligned} \tag{7.29}$$

where N is sample size, a is the number of levels for factor A, and b is the number of levels for factor B.

```
df_factorA <- numLevelsFactorA - 1
df_factorB <- numLevelsFactorB - 1
df_interaction <- df_factorA * df_factorB
df_error <- numLevelsFactorA * numLevelsFactorB * (N - 1)
```

Factor A (one-tailed test):

```r
qf(1 - alpha, df_factorA, df_error)
```

```
[1] 2.999494
```

Factor B (one-tailed test):

```r
qf(1 - alpha, df_factorB, df_error)
```

```
[1] 2.608629
```

Interaction (one-tailed test):

```r
qf(1 - alpha, df_interaction, df_error)
```

```
[1] 2.102376
```

Factor A (two-tailed test):

```r
qf(1 - alpha/2, df_factorA, df_error)
```

```
[1] 3.694584
```

Factor B (two-tailed test):

```r
qf(1 - alpha/2, df_factorB, df_error)
```

```
[1] 3.121587
```

Interaction (two-tailed test):

```r
qf(1 - alpha/2, df_interaction, df_error)
```

```
[1] 2.413504
```

7.6.7.6 Repeated Measures ANOVA

For a repeated measures ANOVA, the degrees of freedom is in Equation 7.30:

$$
\begin{aligned}
df_1 &= T - 1 \\
df_2 &= (T - 1)(N - 1)
\end{aligned}
\tag{7.30}
$$

where N is sample size and T is the number of measurements (i.e., the number of levels of the within-person factor: e.g., timepoints or conditions).

```
df1_RMANOVA <- numMeasurements - 1
df2_RMANOVA <- (numMeasurements - 1) * (N - 1)
```

One-tailed test:

```
qf(1 - alpha, df1_RMANOVA, df2_RMANOVA)
```

```
[1] 2.619828
```

Two-tailed test:

```
qf(1 - alpha/2, df1_RMANOVA, df2_RMANOVA)
```

```
[1] 3.138017
```

7.6.7.7 Correlation

For a correlation, the degrees of freedom is in Equation 7.31:

$$df = N - 2 \tag{7.31}$$

where N is sample size.

```
df_correlation <- N - 2
```

One-tailed test:

```
qt(1 - alpha, df_correlation)
```

```
[1] 1.652586
```

Two-tailed test:

```
qt(1 - alpha/2, df_correlation)
```

```
[1] 1.972017
```

7.6.7.8 Multiple Regression

For multiple regression, the degrees of freedom is in Equation 7.32:

$$\begin{aligned} df_1 &= p \\ df_2 &= N - p - 1 \end{aligned} \tag{7.32}$$

where N is sample size and p is the number of predictors.

```
df1_regression <- numPredictors
df2_regression <- N - numPredictors - 1
```

One-tailed test:

```
qf(1 - alpha, df1_regression, df2_regression)
```

```
[1] 2.260647
```

Two-tailed test:

```
qf(1 - alpha/2, df1_regression, df2_regression)
```

```
[1] 2.63243
```

7.6.7.9 Chi-Square Goodness-of-Fit Test

For the chi-square goodness-of-fit test, the degrees of freedom is in Equation 7.33:

$$df = c - 1 \tag{7.33}$$

where c is the number of categories.

```
df_chisquareGOF <- numCategories - 1
```

One-tailed test:

```
qchisq(1 - alpha, df_chisquareGOF)
```

```
[1] 11.0705
```

Two-tailed test:

```
qchisq(1 - alpha/2, df_chisquareGOF)
```

```
[1] 12.8325
```

7.6.7.10 Chi-Square Test for Association

For the chi-square test for association, the degrees of freedom is in Equation 7.34:

$$df = (r - 1) \times (c - 1) \tag{7.34}$$

where r is the number of rows in the contingency table and c is the number of columns in the contingency table.

```
df_chisquareAssociation <- (numRows - 1) * (numColumns - 1)
```

One-tailed test:

```
qchisq(1 - alpha, df_chisquareAssociation)
```

```
[1] 9.487729
```

Two-tailed test:

```
qchisq(1 - alpha/2, df_chisquareAssociation)
```

```
[1] 11.14329
```

7.6.8 Statistical Power

As described above, *statistical power* is the probability of detecting an effect, if, in fact, the effect exists. Statistical power for a given test can be calculated based on three factors:

- effect size
- sample size
- alpha level

Knowing any three of the following, you can calculate the fourth: statistical power, effect size, sample size, and alpha level. Below is `R` code for calculating power for each of various statistical tests (i.e., a *power analysis*). For free point-and-click software for calculating statistical power, see G*Power: https://www.psychologie.hhu.de/arbeitsgruppen/allgemeine-psychologie-und-arbeitspsychologie/gpower.html

```
power <- .8
effectSize_d <- .5
effectSize_r <- .24
effectSize_beta <- .24
effectSize_f <- .25
effectSize_fSquared <- .06
effectSize_omega <- .3
```

7.6.8.1 One-Sample *t*-Test

Solving for statistical power achieved (given effect size, sample size, and alpha level):

```
pwr::pwr.t.test(
  n = N,
  d = effectSize_d,
  sig.level = alpha,
  type = "one.sample",
  alternative = "two.sided")
```

```

     One-sample t test power calculation

              n = 200
              d = 0.5
      sig.level = 0.05
          power = 0.9999998
    alternative = two.sided
```

Solving for sample size needed (given effect size, power, and alpha level):

```
pwr::pwr.t.test(
  power = power,
  d = effectSize_d,
  sig.level = alpha,
  type = "one.sample",
  alternative = "two.sided")
```

```
     One-sample t test power calculation

              n = 33.36713
              d = 0.5
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

Solving for the minimum detectable effect size (given sample size, power, and alpha level):

```
pwr::pwr.t.test(
  power = power,
  n = N,
  sig.level = alpha,
  type = "one.sample",
  alternative = "two.sided")
```

```
     One-sample t test power calculation

              n = 200
              d = 0.1990655
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

7.6.8.2 Independent-Samples *t*-Test

7.6.8.2.1 Balanced Group Sizes

Solving for statistical power achieved (given effect size, sample size per group, and alpha level):

```
pwr::pwr.t.test(
  n = N,
  d = effectSize_d,
  sig.level = alpha,
  type = "two.sample",
  alternative = "two.sided")
```

```
     Two-sample t test power calculation

              n = 200
              d = 0.5
      sig.level = 0.05
          power = 0.9987689
    alternative = two.sided

NOTE: n is number in *each* group
```

Solving for sample size per group needed (given effect size, power, and alpha level):

```
pwr::pwr.t.test(
  power = power,
  d = effectSize_d,
  sig.level = alpha,
  type = "two.sample",
  alternative = "two.sided")
```

```
     Two-sample t test power calculation

              n = 63.76561
              d = 0.5
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

Solving for the minimum detectable effect size (given sample size per group, power, and alpha level):

```
pwr::pwr.t.test(
  power = power,
  n = N,
  sig.level = alpha,
  type = "two.sample",
  alternative = "two.sided")
```

```
     Two-sample t test power calculation
```

```
              n = 200
              d = 0.2808267
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

7.6.8.2.2 Unbalanced Group Sizes

Solving for statistical power achieved (given effect size, sample size per group, and alpha level):

```
pwr::pwr.t2n.test(
  n1 = nGroup1,
  n2 = nGroup2,
  d = effectSize_d,
  sig.level = alpha,
  alternative = "two.sided")
```

```
     t test power calculation

             n1 = 150
             n2 = 150
              d = 0.5
      sig.level = 0.05
          power = 0.9907677
    alternative = two.sided
```

Solving for sample size per group needed (given effect size, power, and alpha level):

```
pwr::pwr.t2n.test(
  power = power,
  n1 = nGroup1,
  d = effectSize_d,
  sig.level = alpha,
  alternative = "two.sided")
```

```
     t test power calculation
```

```
             n1 = 150
             n2 = 40.22483
              d = 0.5
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

Solving for the minimum detectable effect size (given sample size per group, power, and alpha level):

```
pwr::pwr.t2n.test(
  power = power,
  n1 = nGroup1,
  n2 = nGroup2,
  sig.level = alpha,
  alternative = "two.sided")
```

```

     t test power calculation

             n1 = 150
             n2 = 150
              d = 0.3245459
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

7.6.8.3 Paired-Samples *t*-Test

Solving for statistical power achieved (given effect size, sample size per group, and alpha level):

```
pwr::pwr.t.test(
  n = N,
  d = effectSize_d,
  sig.level = alpha,
  type = "paired",
  alternative = "two.sided")
```

```

     Paired t test power calculation
```

```
              n = 200
              d = 0.5
      sig.level = 0.05
          power = 0.9999998
    alternative = two.sided

NOTE: n is number of *pairs*
```

Solving for sample size per group needed (given effect size, power, and alpha level):

```
pwr::pwr.t.test(
  power = power,
  d = effectSize_d,
  sig.level = alpha,
  type = "paired",
  alternative = "two.sided")
```

```

     Paired t test power calculation

              n = 33.36713
              d = 0.5
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number of *pairs*
```

Solving for the minimum detectable effect size (given sample size per group, power, and alpha level):

```
pwr::pwr.t.test(
  power = power,
  n = N,
  sig.level = alpha,
  type = "paired",
  alternative = "two.sided")
```

```

     Paired t test power calculation

              n = 200
              d = 0.1990655
```

```
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number of *pairs*
```

7.6.8.4 One-Way ANOVA

Solving for statistical power achieved (given effect size, sample size per group, and alpha level):

```
pwr::pwr.anova.test(
  n = N,
  f = effectSize_f,
  sig.level = alpha,
  k = numGroups)
```

```

     Balanced one-way analysis of variance power calculation

              k = 4
              n = 200
              f = 0.25
      sig.level = 0.05
          power = 0.9999962

NOTE: n is number in each group
```

Solving for sample size per group needed (given effect size, power, and alpha level):

```
pwr::pwr.anova.test(
  power = power,
  f = effectSize_f,
  sig.level = alpha,
  k = numGroups)
```

```

     Balanced one-way analysis of variance power calculation

              k = 4
              n = 44.59927
              f = 0.25
```

```
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

Solving for the minimum detectable effect size (given sample size per group, power, and alpha level):

```
pwr::pwr.anova.test(
  power = power,
  n = N,
  sig.level = alpha,
  k = numGroups)
```

```

     Balanced one-way analysis of variance power calculation

              k = 4
              n = 200
              f = 0.117038
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

The power analysis code above assumes the groups are of equal size (i.e., a balanced design). If the design is unbalanced (i.e., there are different numbers of participants in each group), it may be necessary to conduct a power analysis via a simulation. Below is an example of evaluating the statistical power for detecting an effect unbalanced designs via simulation:

```
nSim <- 1000 # number of simulations

# Function to generate data and perform ANOVA
simulate_anova <- function(nGroup1, nGroup2, f, alpha) {
  # Means for each group
  mean1 <- 0
  mean2 <- f * sqrt((nGroup1 + nGroup2) / 2)

  # Generate data
  group1 <- rnorm(nGroup1, mean = mean1, sd = 1)
  group2 <- rnorm(nGroup2, mean = mean2, sd = 1)

  # Combine data
```

```
  data <- data.frame(
    value = c(group1, group2),
    group = factor(rep(c("Group1", "Group2"), c(nGroup1, nGroup2)))
  )

  # Perform ANOVA
  aov_result <- aov(value ~ group, data = data)
  p_value <- summary(aov_result)[[1]][["Pr(>F)"]][1]

  # Check if p-value is less than alpha
  return(p_value < alpha)
}

# Run simulations
set.seed(52242) # for reproducibility
powerSimulationOneWayAnova <- replicate(
  nSim,
  simulate_anova(
    nGroup1 = 10,
    nGroup2 = 25,
    f = effectSize_f,
    alpha = alpha))

# Estimate power
mean(powerSimulationOneWayAnova)
```

```
[1] 0.774
```

7.6.8.5 Factorial ANOVA

The power analysis code below assumes the groups are of equal size (i.e., a balanced design). If the design is unbalanced (i.e., there are different numbers of participants in each group), it may be necessary to conduct a power analysis via a simulation. See Section 7.6.8.4 for an example power analysis simulation for one-way ANOVA.

Solving for statistical power achieved (given effect size, sample size per group, and alpha level):

```
pwr::pwr.anova.test(
  n = N,
  f = effectSize_f,
  sig.level = alpha,
  k = numLevelsFactorA)
```

```
     Balanced one-way analysis of variance power calculation

              k = 3
              n = 200
              f = 0.25
      sig.level = 0.05
          power = 0.9999238

NOTE: n is number in each group
```

```
pwr::pwr.anova.test(
  n = N,
  f = effectSize_f,
  sig.level = alpha,
  k = numLevelsFactorB)
```

```
     Balanced one-way analysis of variance power calculation

              k = 4
              n = 200
              f = 0.25
      sig.level = 0.05
          power = 0.9999962

NOTE: n is number in each group
```

```
pwr::pwr.anova.test(
  n = N,
  f = effectSize_f,
  sig.level = alpha,
  k = numLevelsFactorA + numLevelsFactorB)
```

```
     Balanced one-way analysis of variance power calculation

              k = 7
              n = 200
              f = 0.25
      sig.level = 0.05
          power = 1

NOTE: n is number in each group
```

Solving for sample size per group needed (given effect size, power, and alpha level):

```
pwr::pwr.anova.test(
  power = power,
  f = effectSize_f,
  sig.level = alpha,
  k = numLevelsFactorA)
```

```

     Balanced one-way analysis of variance power calculation 

              k = 3
              n = 52.3966
              f = 0.25
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

```
pwr::pwr.anova.test(
  power = power,
  f = effectSize_f,
  sig.level = alpha,
  k = numLevelsFactorB)
```

```

     Balanced one-way analysis of variance power calculation 

              k = 4
              n = 44.59927
              f = 0.25
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

```
pwr::pwr.anova.test(
  power = power,
  f = effectSize_f,
  sig.level = alpha,
  k = numLevelsFactorA + numLevelsFactorB)
```

```
     Balanced one-way analysis of variance power calculation

              k = 7
              n = 32.05196
              f = 0.25
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

Solving for the minimum detectable effect size (given sample size per group, power, and alpha level):

```
pwr::pwr.anova.test(
  power = power,
  n = N,
  sig.level = alpha,
  k = numLevelsFactorA)
```

```
     Balanced one-way analysis of variance power calculation

              k = 3
              n = 200
              f = 0.1270373
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

```
pwr::pwr.anova.test(
  power = power,
  n = N,
  sig.level = alpha,
  k = numLevelsFactorB)
```

```
     Balanced one-way analysis of variance power calculation

              k = 4
              n = 200
              f = 0.117038
      sig.level = 0.05
          power = 0.8
```

```
NOTE: n is number in each group
```

```r
pwr::pwr.anova.test(
  power = power,
  n = N,
  sig.level = alpha,
  k = numLevelsFactorA + numLevelsFactorB)
```

```
     Balanced one-way analysis of variance power calculation 

              k = 7
              n = 200
              f = 0.09889082
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

7.6.8.6 Repeated Measures ANOVA

Solving for statistical power achieved (given effect size, sample size per group, and alpha level):

```r
WebPower::wp.rmanova(
  n = N,
  ng = numGroups,
  nm = numMeasurements,
  f = effectSize_f,
  alpha = alpha,
  type = 0)
```

```
Repeated-measures ANOVA analysis

      n    f ng nm nscor alpha     power
    200 0.25  4  4     1  0.05 0.8484718

NOTE: Power analysis for between-effect test
URL: http://psychstat.org/rmanova
```

```r
WebPower::wp.rmanova(
  n = N,
```

```r
  ng = numGroups,
  nm = numMeasurements,
  f = effectSize_f,
  alpha = alpha,
  type = 1)
```

```
Repeated-measures ANOVA analysis

      n    f ng nm nscor alpha     power
    200 0.25  4  4     1  0.05 0.8536292

NOTE: Power analysis for within-effect test
URL: http://psychstat.org/rmanova
```

```r
WebPower::wp.rmanova(
  n = N,
  ng = numGroups,
  nm = numMeasurements,
  f = effectSize_f,
  alpha = alpha,
  type = 2)
```

```
Repeated-measures ANOVA analysis

      n    f ng nm nscor alpha     power
    200 0.25  4  4     1  0.05 0.6756298

NOTE: Power analysis for interaction-effect test
URL: http://psychstat.org/rmanova
```

Solving for sample size per group needed (given effect size, power, and alpha level):

```r
WebPower::wp.rmanova(
  power = power,
  ng = numGroups,
  nm = numMeasurements,
  f = effectSize_f,
  alpha = alpha,
  type = 0)
```

```
Repeated-measures ANOVA analysis
```

```
          n    f ng nm nscor alpha power
   178.3971 0.25  4  4     1  0.05   0.8

NOTE: Power analysis for between-effect test
URL: http://psychstat.org/rmanova
```

```
WebPower::wp.rmanova(
  power = power,
  ng = numGroups,
  nm = numMeasurements,
  f = effectSize_f,
  alpha = alpha,
  type = 1)
```

```
Repeated-measures ANOVA analysis

          n    f ng nm nscor alpha power
   175.7692 0.25  4  4     1  0.05   0.8

NOTE: Power analysis for within-effect test
URL: http://psychstat.org/rmanova
```

```
WebPower::wp.rmanova(
  power = power,
  ng = numGroups,
  nm = numMeasurements,
  f = effectSize_f,
  alpha = alpha,
  type = 2)
```

```
Repeated-measures ANOVA analysis

          n    f ng nm nscor alpha power
   253.2369 0.25  4  4     1  0.05   0.8

NOTE: Power analysis for interaction-effect test
URL: http://psychstat.org/rmanova
```

Solving for the minimum detectable effect size (given sample size per group, power, and alpha level):

```
WebPower::wp.rmanova(
  power = power,
```

```
  n = N,
  ng = numGroups,
  nm = numMeasurements,
  alpha = alpha,
  type = 0)
```

```
Repeated-measures ANOVA analysis

      n         f ng nm nscor alpha power
    200 0.2358259  4  4     1  0.05   0.8

NOTE: Power analysis for between-effect test
URL: http://psychstat.org/rmanova
```

```
WebPower::wp.rmanova(
  power = power,
  n = N,
  ng = numGroups,
  nm = numMeasurements,
  alpha = alpha,
  type = 1)
```

```
Repeated-measures ANOVA analysis

      n         f ng nm nscor alpha power
    200 0.2342726  4  4     1  0.05   0.8

NOTE: Power analysis for within-effect test
URL: http://psychstat.org/rmanova
```

```
WebPower::wp.rmanova(
  power = power,
  n = N,
  ng = numGroups,
  nm = numMeasurements,
  alpha = alpha,
  type = 2)
```

```
Repeated-measures ANOVA analysis

      n         f ng nm nscor alpha power
    200 0.2817486  4  4     1  0.05   0.8

NOTE: Power analysis for interaction-effect test
URL: http://psychstat.org/rmanova
```

7.6.8.7 Correlation

Solving for statistical power achieved (given effect size, sample size per group, and alpha level):

```
pwr::pwr.r.test(
  n = N,
  r = effectSize_r,
  sig.level = alpha,
  alternative = "two.sided")
```

```

     approximate correlation power calculation (arctangh transformation) 

              n = 200
              r = 0.24
      sig.level = 0.05
          power = 0.9310138
    alternative = two.sided
```

Solving for sample size per group needed (given effect size, power, and alpha level):

```
pwr::pwr.r.test(
  power = power,
  r = effectSize_r,
  sig.level = alpha,
  alternative = "two.sided")
```

```

     approximate correlation power calculation (arctangh transformation) 

              n = 133.1299
              r = 0.24
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

Solving for the minimum detectable effect size (given sample size per group, power, and alpha level):

```
pwr::pwr.r.test(
  power = power,
```

```
  n = N,
  sig.level = alpha,
  alternative = "two.sided")
```

```

     approximate correlation power calculation (arctangh transformation)

              n = 200
              r = 0.1965767
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

7.6.8.8 Multiple Regression

Solving for statistical power achieved (given effect size, sample size, and alpha level):

```
pwr::pwr.f2.test(
  f2 = effectSize_fSquared,
  sig.level = alpha,
  u = numPredictors,
  v = N - numPredictors - 1)
```

```

     Multiple regression power calculation

              u = 5
              v = 194
             f2 = 0.06
      sig.level = 0.05
          power = 0.7548031
```

```
pwrss::pwrss.t.reg(
  n = N,
  beta1 = effectSize_beta,
  k = numPredictors,
  alpha = alpha,
  alternative = "not equal")
```

```
 Linear Regression Coefficient (t Test)
 H0: beta1 = beta0
 HA: beta1 != beta0
```

```
------------------------------
 Statistical power = 0.936
 n = 200
------------------------------
Alternative = "not equal"
Degrees of freedom = 194
Non-centrality parameter = 3.496
Type I error rate = 0.05
Type II error rate = 0.064
```

Solving for sample size needed (given effect size, power, and alpha level)—$v = N - \text{numberOfPredictors} - 1$; thus, $N = v + \text{numberOfPredictors} + 1$:

```
multipleRegressionSampleSizeModel <- pwr::pwr.f2.test(
  power = power,
  f2 = effectSize_fSquared,
  sig.level = alpha,
  u = numPredictors)

multipleRegressionSampleSizeModel
```

```

     Multiple regression power calculation

              u = 5
              v = 213.3947
             f2 = 0.06
      sig.level = 0.05
          power = 0.8
```

```
vNeeded <- multipleRegressionSampleSizeModel$v
sampleSizeNeeded <- vNeeded + numPredictors + 1
sampleSizeNeeded
```

```
[1] 219.3947
```

```
pwrss::pwrss.t.reg(
  power = power,
  beta1 = effectSize_beta,
  k = numPredictors,
  alpha = alpha,
  alternative = "not equal")
```

```
Linear Regression Coefficient (t Test)
H0: beta1 = beta0
HA: beta1 != beta0
------------------------------
 Statistical power = 0.8
 n = 131
------------------------------
Alternative = "not equal"
Degrees of freedom = 124.427
Non-centrality parameter = 2.823
Type I error rate = 0.05
Type II error rate = 0.2
```

Solving for the minimum detectable effect size (given sample size, power, and alpha level):

```
pwr::pwr.f2.test(
  power = power,
  sig.level = alpha,
  u = numPredictors,
  v = N - numPredictors - 1)
```

```
     Multiple regression power calculation

              u = 5
              v = 194
             f2 = 0.06597765
      sig.level = 0.05
          power = 0.8
```

7.6.8.9 Chi-Square Goodness-of-Fit Test

Solving for statistical power achieved (given effect size, sample size, and alpha level):

```
pwr::pwr.chisq.test(
  N = N,
  w = effectSize_omega,
  df = numCategories - 1,
  sig.level = alpha)
```

```
     Chi squared power calculation

              w = 0.3
              N = 200
             df = 5
      sig.level = 0.05
          power = 0.9269225

NOTE: N is the number of observations
```

Solving for sample size needed (given effect size, power, and alpha level):

```
pwr::pwr.chisq.test(
  power = power,
  w = effectSize_omega,
  df = numCategories - 1,
  sig.level = alpha)
```

```
     Chi squared power calculation

              w = 0.3
              N = 142.529
             df = 5
      sig.level = 0.05
          power = 0.8

NOTE: N is the number of observations
```

Solving for the minimum detectable effect size (given sample size, power, and alpha level):

```
pwr::pwr.chisq.test(
  power = power,
  N = N,
  df = numCategories - 1,
  sig.level = alpha)
```

```
     Chi squared power calculation

              w = 0.2532543
              N = 200
             df = 5
```

```
      sig.level = 0.05
          power = 0.8

NOTE: N is the number of observations
```

7.6.8.10 Chi-Square Test for Association

Solving for statistical power achieved (given effect size, sample size, and alpha level):

```
pwr::pwr.chisq.test(
  N = N,
  w = effectSize_omega,
  df = (numRows - 1)*(numColumns - 1),
  sig.level = alpha)
```

```

     Chi squared power calculation

              w = 0.3
              N = 200
             df = 4
      sig.level = 0.05
          power = 0.9431195

NOTE: N is the number of observations
```

Solving for sample size needed (given effect size, power, and alpha level):

```
pwr::pwr.chisq.test(
  power = power,
  w = effectSize_omega,
  df = (numRows - 1)*(numColumns - 1),
  sig.level = alpha)
```

```

     Chi squared power calculation

              w = 0.3
              N = 132.6143
             df = 4
      sig.level = 0.05
          power = 0.8

NOTE: N is the number of observations
```

Solving for the minimum detectable effect size (given sample size, power, and alpha level):

```
pwr::pwr.chisq.test(
  power = power,
  N = N,
  df = (numRows - 1)*(numColumns - 1),
  sig.level = alpha)
```

```

     Chi squared power calculation 

              w = 0.2442875
              N = 200
             df = 4
      sig.level = 0.05
          power = 0.8

NOTE: N is the number of observations
```

7.6.8.11 Multilevel Modeling

Power analysis for multilevel modeling approaches is more complicated than it is for other statistical analyses, such as correlation, multiple regression, *t*-tests, ANOVA[1], etc.

There are free web applications for calculating power in multilevel modeling:

- https://aguinis.shinyapps.io/ml_power/
- https://koumurayama.shinyapps.io/tmethod_mlm/
- https://webpower.psychstat.org/wiki/models/index

7.6.8.12 Path Analysis, Factor Analysis, and Structural Equation Modeling

Power analysis for latent variable modeling approaches like structural equation modeling (SEM) is more complicated than it is for other statistical analyses, such as correlation, multiple regression, *t*-tests, ANOVA[2], etc.

I provide an example of power analysis in SEM using Monte Carlo simulation in R here: https://isaactpetersen.github.io/Principles-Psychological-Assessment/sem.html#monteCarloPowerAnalysis (Petersen, 2024c).

[1] @sec-anova
[2] @sec-anova

There are also free web applications for calculating power in SEM:

- https://sjak.shinyapps.io/power4SEM/
- https://sempower.shinyapps.io/sempower/
- https://yilinandrewang.shinyapps.io/pwrSEM/
- https://webpower.psychstat.org/wiki/models/index

7.6.8.13 Mediation and Moderation

There are free tools for calculating power for tests of mediation and moderation:

- https://schoemanna.shinyapps.io/mc_power_med/
- https://www.causalevaluation.org/power-analysis.html (web application: https://powerupr.shinyapps.io/index/)
- https://webpower.psychstat.org/wiki/models/index

8

Correlation Analysis

8.1 Getting Started

8.1.1 Load Packages

```
library("petersenlab")
library("XICOR")
library("tidyverse")
```

8.2 Overview of Correlation

Correlation is a metric of the association between variables. Covariance is the association between variables and in an unstandardized metric that differs for variables with different scales. By contrast, correlation is in a standarized metric that does not differ for variables with different scales. When examining the association between variables that are interval or ratio levels of measurement, Pearson correlation is used. When examining the association between variables that are ordinal in level of measurement, Spearman correlation is used. Pearson correlation is an index of the *linear* association between variables. If a nonlinear association is present, other indices like xi [ξ; Chatterjee (2021)] and distance correlation coefficients are better suited to detect the association.

8.3 The Correlation Coefficient (r)

The formula for the correlation coefficient is in Equation 7.21.

The correlation coefficient ranges from −1.0 to +1.0. The correlation coefficient (r) tells you two things: (1) the direction (sign) of the association (positive or negative) and (2) the magnitude of the association. If the correlation coefficient is positive, the association is positive. If the correlation coefficient is negative, the association is negative. If the association is positive, as `X` increases, `Y` increases (or conversely, as `X` decreases, `Y` decreases). If the association is negative, as `X` increases, `Y` decreases (or conversely, as `X` decreases, `Y` increases). The smaller the absolute value of the correlation coefficient (i.e., the closer the r value is to zero), the weaker the association and the flatter the slope of the best-fit line in a scatterplot. The larger the absolute value of the correlation coefficient (i.e., the closer the absolute value of the r value is to one), the stronger the association and the steeper the slope of the best-fit line in a scatterplot. See Figure 8.1 for a range of different correlation coefficients and what some example data may look like for each direction and strength of association.

```
set.seed(52242)
correlations <- data.frame(criterion = rnorm(1000))

correlations$v1 <- complement(correlations$criterion, -1)
correlations$v2 <- complement(correlations$criterion, -.9)
correlations$v3 <- complement(correlations$criterion, -.8)
correlations$v4 <- complement(correlations$criterion, -.7)
correlations$v5 <- complement(correlations$criterion, -.6)
correlations$v6 <- complement(correlations$criterion, -.5)
correlations$v7 <- complement(correlations$criterion, -.4)
correlations$v8 <- complement(correlations$criterion, -.3)
correlations$v9 <- complement(correlations$criterion, -.2)
correlations$v10 <-complement(correlations$criterion, -.1)
correlations$v11 <-complement(correlations$criterion, 0)
correlations$v12 <-complement(correlations$criterion, .1)
correlations$v13 <-complement(correlations$criterion, .2)
correlations$v14 <-complement(correlations$criterion, .3)
correlations$v15 <-complement(correlations$criterion, .4)
correlations$v16 <-complement(correlations$criterion, .5)
correlations$v17 <-complement(correlations$criterion, .6)
correlations$v18 <-complement(correlations$criterion, .7)
correlations$v19 <-complement(correlations$criterion, .8)
correlations$v20 <-complement(correlations$criterion, .9)
correlations$v21 <-complement(correlations$criterion, 1)

par(mfrow = c(7,3), mar = c(1, 0, 1, 0))

# -1.0
plot(correlations$criterion, correlations$v1, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
```

```
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v1 ~ criterion, data = correlations), col = "black")

# -.9
plot(correlations$criterion, correlations$v2, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v2 ~ criterion, data = correlations), col = "black")

# -.8
plot(correlations$criterion, correlations$v3, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v3 ~ criterion, data = correlations), col = "black")

# -.7
plot(correlations$criterion, correlations$v4, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v4 ~ criterion, data = correlations), col = "black")

# -.6
plot(correlations$criterion, correlations$v5, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v5 ~ criterion, data = correlations), col = "black")

# -.5
plot(correlations$criterion, correlations$v6, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v6 ~ criterion, data = correlations), col = "black")

# -.4
plot(correlations$criterion, correlations$v7, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v7 ~ criterion, data = correlations), col = "black")

# -.3
plot(correlations$criterion, correlations$v8, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v8 ~ criterion, data = correlations), col = "black")

# -.2
plot(correlations$criterion, correlations$v9, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v9 ~ criterion, data = correlations), col = "black")

# -.1
```

```
plot(correlations$criterion, correlations$v10, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v10 ~ criterion, data = correlations), col = "black")

# 0.0
plot(correlations$criterion, correlations$v11, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v11 ~ criterion, data = correlations), col = "black")

# 0.1
plot(correlations$criterion, correlations$v12, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v12 ~ criterion, data = correlations), col = "black")

# 0.2
plot(correlations$criterion, correlations$v13, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v13 ~ criterion, data = correlations), col = "black")

# 0.3
plot(correlations$criterion, correlations$v14, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v14 ~ criterion, data = correlations), col = "black")

# 0.4
plot(correlations$criterion, correlations$v15, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v15 ~ criterion, data = correlations), col = "black")

# 0.5
plot(correlations$criterion, correlations$v16, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v16 ~ criterion, data = correlations), col = "black")

# 0.6
plot(correlations$criterion, correlations$v17, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v17 ~ criterion, data = correlations), col = "black")

# 0.7
plot(correlations$criterion, correlations$v18, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v18 ~ criterion, data = correlations), col = "black")
```

```
# 0.8
plot(correlations$criterion, correlations$v19, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v19 ~ criterion, data = correlations), col = "black")

# 0.9
plot(correlations$criterion, correlations$v20, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v20 ~ criterion, data = correlations), col = "black")

# 1.0
plot(correlations$criterion, correlations$v21, xaxt = "n", yaxt = "n", xlab = "" , ylab = "",
     main = substitute(paste(italic(r), " = ", x, sep = ""), list(x = round(cor.test(x = correlati
abline(lm(v21 ~ criterion, data = correlations), col = "black")

invisible(dev.off()) #par(mfrow = c(1,1))
```

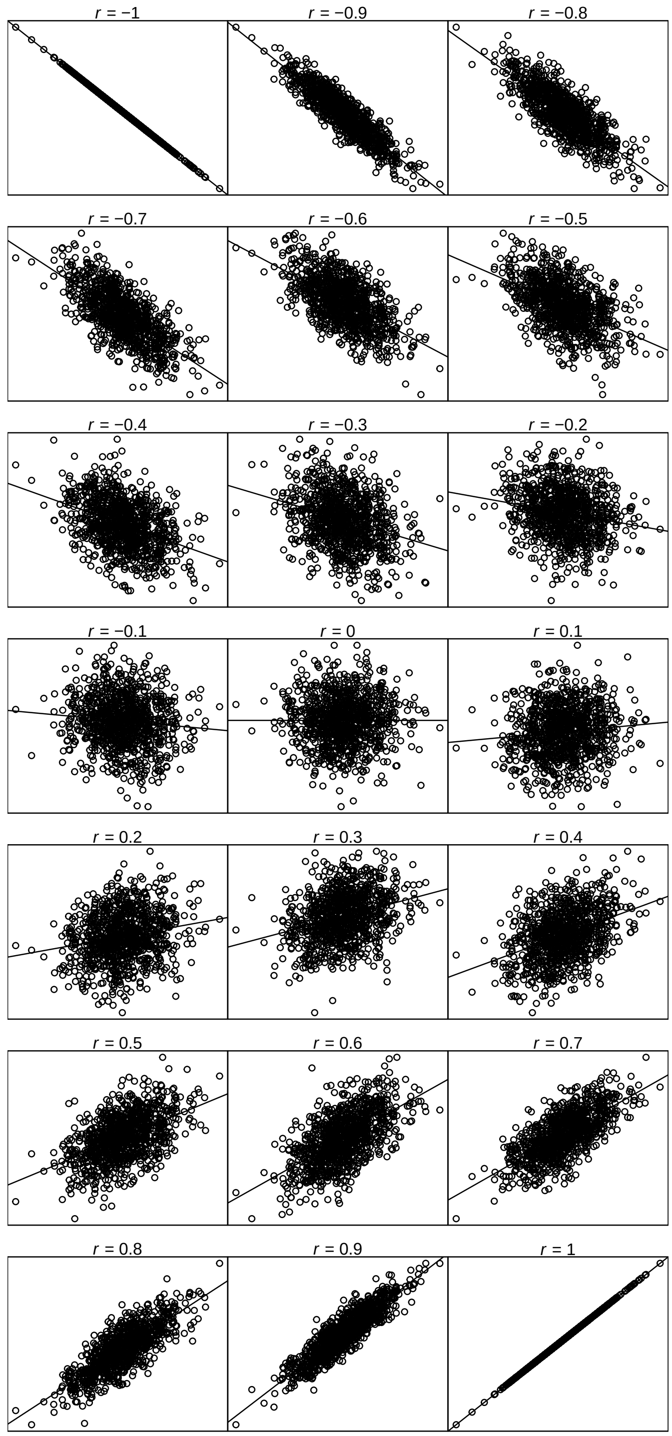


Figure 8.1 Correlation Coefficients.

See Figure 8.2 for the interpretation of the magnitude and direction (sign) of various correlation coefficients.

```
library("patchwork")

set.seed(52242)
correlations2 <- data.frame(criterion = rnorm(15))

correlations2$v1 <- complement(correlations2$criterion, -1)
correlations2$v2 <- complement(correlations2$criterion, -.9)
correlations2$v3 <- complement(correlations2$criterion, -.8)
correlations2$v4 <- complement(correlations2$criterion, -.7)
correlations2$v5 <- complement(correlations2$criterion, -.6)
correlations2$v6 <- complement(correlations2$criterion, -.5)
correlations2$v7 <- complement(correlations2$criterion, -.4)
correlations2$v8 <- complement(correlations2$criterion, -.3)
correlations2$v9 <- complement(correlations2$criterion, -.2)
correlations2$v10 <-complement(correlations2$criterion, -.1)
correlations2$v11 <-complement(correlations2$criterion, 0)
correlations2$v12 <-complement(correlations2$criterion, .1)
correlations2$v13 <-complement(correlations2$criterion, .2)
correlations2$v14 <-complement(correlations2$criterion, .3)
correlations2$v15 <-complement(correlations2$criterion, .4)
correlations2$v16 <-complement(correlations2$criterion, .5)
correlations2$v17 <-complement(correlations2$criterion, .6)
correlations2$v18 <-complement(correlations2$criterion, .7)
correlations2$v19 <-complement(correlations2$criterion, .8)
correlations2$v20 <-complement(correlations2$criterion, .9)
correlations2$v21 <-complement(correlations2$criterion, 1)

# -1.0
p1 <- ggplot(
  data = correlations2,
  mapping = aes(
    x = criterion,
    y = v1
  )
) +
  geom_point() +
  geom_smooth(
    method = "lm",
    se = FALSE) +
  labs(
    title = "Perfect Negative Association",
    subtitle = expression(paste(italic("r"), " = ", "-1.0"))
```

```
  ) +
  theme_classic(
    base_size = 12) +
  theme(
    axis.title.x = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.title.y = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())

# -0.9
p2 <- ggplot(
  data = correlations2,
  mapping = aes(
    x = criterion,
    y = v2
  )
) +
  geom_point() +
  geom_smooth(
    method = "lm",
    se = FALSE) +
  labs(
    title = "Strong Negative Association",
    subtitle = expression(paste(italic("r"), " = ", "-.9"))
  ) +
  theme_classic(
    base_size = 12) +
  theme(
    axis.title.x = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.title.y = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())

# -0.5
p3 <- ggplot(
  data = correlations2,
  mapping = aes(
    x = criterion,
    y = v6
  )
```

```
) +
  geom_point() +
  geom_smooth(
    method = "lm",
    se = FALSE) +
  labs(
    title = "Moderate Negative Association",
    subtitle = expression(paste(italic("r"), " = ", "-.5"))
  ) +
  theme_classic(
    base_size = 12) +
  theme(
    axis.title.x = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.title.y = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())

# -0.2
p4 <- ggplot(
  data = correlations2,
  mapping = aes(
    x = criterion,
    y = v9
  )
) +
  geom_point() +
  geom_smooth(
    method = "lm",
    se = FALSE) +
  labs(
    title = "Weak Negative Association",
    subtitle = expression(paste(italic("r"), " = ", "-.2"))
  ) +
  theme_classic(
    base_size = 12) +
  theme(
    axis.title.x = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.title.y = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())
```

```r
# 0.0
p5 <- ggplot(
  data = correlations2,
  mapping = aes(
    x = criterion,
    y = v11
  )
) +
  geom_point() +
  geom_smooth(
    method = "lm",
    se = FALSE) +
  labs(
    title = "No Association",
    subtitle = expression(paste(italic("r"), " = ", ".0"))
  ) +
  theme_classic(
    base_size = 12) +
  theme(
    axis.title.x = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.title.y = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())

# 0.2
p6 <- ggplot(
  data = correlations2,
  mapping = aes(
    x = criterion,
    y = v13
  )
) +
  geom_point() +
  geom_smooth(
    method = "lm",
    se = FALSE) +
  labs(
    title = "Weak Positive Association",
    subtitle = expression(paste(italic("r"), " = ", ".2"))
  ) +
  theme_classic(
    base_size = 12) +
```

```
  theme(
    axis.title.x = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.title.y = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())

# 0.5
p7 <- ggplot(
  data = correlations2,
  mapping = aes(
    x = criterion,
    y = v16
  )
) +
  geom_point() +
  geom_smooth(
    method = "lm",
    se = FALSE) +
  labs(
    title = "Moderate Positive Association",
    subtitle = expression(paste(italic("r"), " = ", ".5"))
  ) +
  theme_classic(
    base_size = 12) +
  theme(
    axis.title.x = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.title.y = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())

# 0.9
p8 <- ggplot(
  data = correlations2,
  mapping = aes(
    x = criterion,
    y = v20
  )
) +
  geom_point() +
  geom_smooth(
```

```
    method = "lm",
    se = FALSE) +
  labs(
    title = "Strong Positive Association",
    subtitle = expression(paste(italic("r"), " = ", ".9"))
  ) +
  theme_classic(
    base_size = 12) +
  theme(
    axis.title.x = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.title.y = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())

# 1.0
p9 <- ggplot(
  data = correlations2,
  mapping = aes(
    x = criterion,
    y = v21
  )
) +
  geom_point() +
  geom_smooth(
    method = "lm",
    se = FALSE) +
  labs(
    title = "Perfect Positive Association",
    subtitle = expression(paste(italic("r"), " = ", "1.0"))
  ) +
  theme_classic(
    base_size = 12) +
  theme(
    axis.title.x = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.title.y = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())

p1 + p2 + p3 + p4 + p5 + p6 + p7 + p8 + p9 +
  plot_layout(
```

```
  ncol = 3,
  heights = 1,
  widths = 1)
```

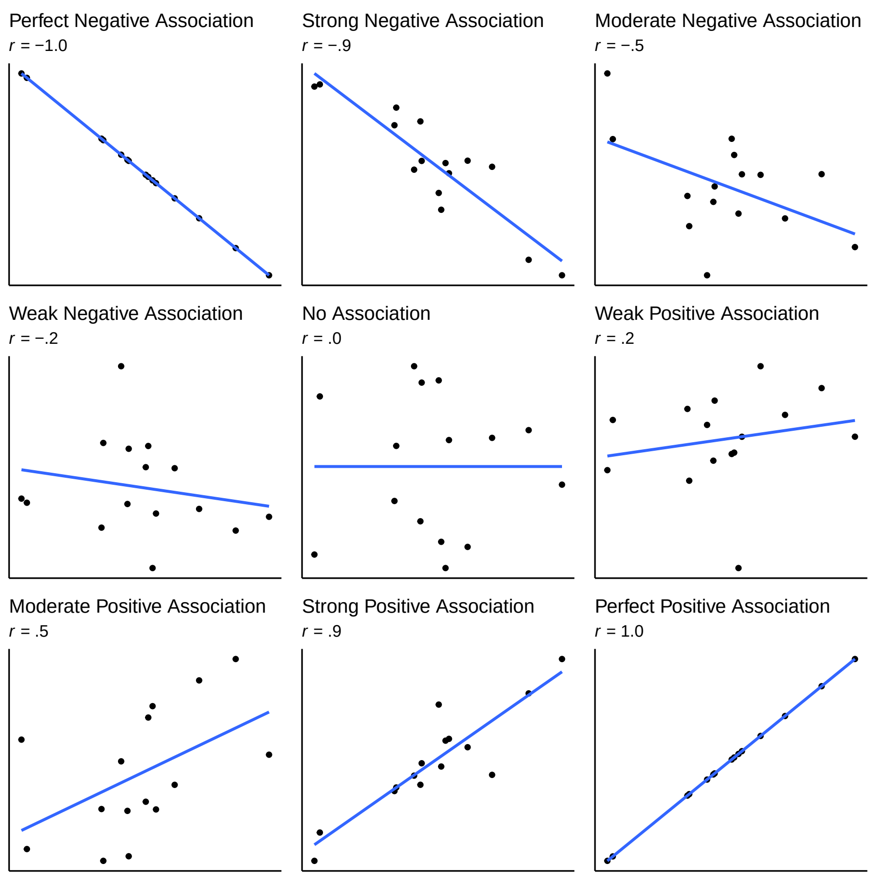


Figure 8.2 Interpretation of the Magnitude and Direction (Sign) of Correlation Coefficients.

Interactive visualizations by Kristoffer Magnusson on p-values and null-hypothesis significance testing are below:

- https://rpsychologist.com/correlation/ (archived at https://perma.cc/G8YR-VCM4)

8.4 Examples

8.4.1 Covariance

8.4.2 Pearson Correlation

8.4.3 Spearman Correlation

8.4.4 Nonlinear Correlation

8.4.5 Correlation Matrix

8.4.6 Correlogram

8.5 Session Info

9

Multiple Regression

9.1 Getting Started

9.1.1 Load Packages

```
library("petersenlab")
library("tidyverse")
library("knitr")
```

9.2 Overview of Multiple Regression

Multiple regression examines the association between multiple predictor variables and one outcome variable. It allows obtaining a more accurate estimate of the unique contribution of a given predictor variable, by controlling for other variables (covariates).

Regression with one predictor variable takes the form of Equation 9.1:

$$y = \beta_0 + \beta_1 x_1 + \epsilon \tag{9.1}$$

where y is the outcome variable, β_0 is the intercept, β_1 is the slope, x_1 is the predictor variable, and ϵ is the error term.

A regression line is depicted in Figure 9.4.

Regression with multiple predictors—i.e., multiple regression—takes the form of Equation 9.2:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_p x_p + \epsilon \tag{9.2}$$

where p is the number of predictor variables.

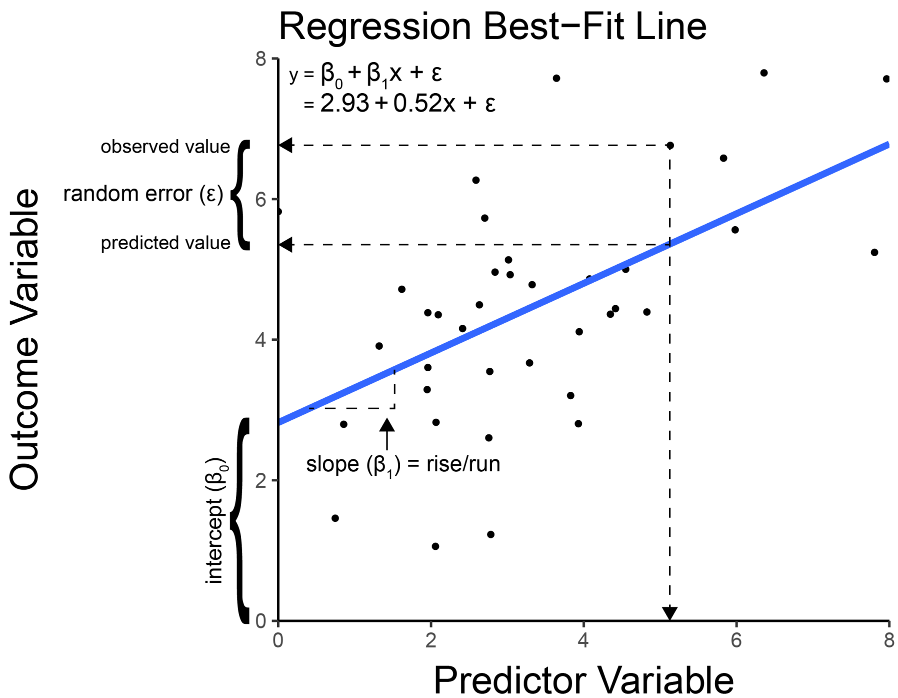


Figure 9.1 A Regression Best-Fit Line.

9.3 Components

- B = unstandardized coefficient: direction and magnitude of the estimate (original scale)
- β (beta) = standardized coefficient: direction and magnitude of the estimate (standard deviation scale)
- SE = standard error: uncertainty of unstandardized estimate

The unstandardized regression coefficient (B) is interpreted such that, for every unit change in the predictor variable, there is a ___ unit change in the outcome variable. For instance, when examining the association between age and fantasy points, if the unstandardized regression coefficient is 2.3, players score on average 2.3 more points for each additional year of age. (In reality, we might expect a nonlinear, inverted-U-shaped association between age and fantasy points such that players tend to reach their peak in the middle of their careers.) Unstandardized regression coefficients are tied to the metric of the raw data. Thus, a large unstandardized regression coefficient for two variables may mean completely different things. Holding the strength of the association constant, you tend to see larger unstandardized regression coefficients for variables with smaller units and smaller unstandardized regression coefficients for variables with larger units.

Standardized regression coefficients can be obtained by standardizing the variables to z-scores so they all have a mean of zero and standard deviation of one. The standardized regression coefficient (β) is interpreted such that, for every standard deviation change in the predictor variable, there is a ___ standard deviation change in the outcome variable. For instance, when examining the association between age and fantasy points, if the standardized regression coefficient is 0.1, players score on average 0.1 standard deviation more points for each additional standard deviation of their year of age. Standardized regression coefficients—though not the case in all instances—tend to fall between $[-1, 1]$. Thus, standardized regression coefficients tend to be more comparable across variables and models compared to unstandardized regression coefficients. In this way, standardized regression coefficients provide a meaningful index of effect size.

9.4 Coefficient of Determination (R^2)

The coefficient of determination (R^2) reflects the proportion of variance in the outcome (dependent) variable that is explained by the model predictions: $R^2 = \frac{\text{variance explained in } Y}{\text{total variance in } Y}$. Various formulas for R^2 are in Equation 7.18. Larger R^2 values indicate greater accuracy. Multiple regression can be conceptualized with overlapping circles (similar to a venn diagram), where the non-overlapping portions of the circles reflect nonshared variance and the overlapping portions of the circles reflect shared variance, as in Figure 9.4.

One issue with R^2 is that it increases as the number of predictors increases, which can lead to overfitting if using R^2 as an index to compare models for purposes of selecting the "best-fitting" model. Consider the following example (adapted from Petersen (2024c)) in which you have one predictor variable and one outcome variable, as shown in Table 9.1.

Table 9.1 Example Data of Predictor (x1) and Outcome (y) Used for Regression Model.

y	x1
7	1
13	2
29	7

Table 9.1 Example Data of Predictor (x1) and Outcome (y) Used for Regression Model.

y	x1
10	2

Using the data, the best fitting regression model is: $y = 3.98 + 3.59 \cdot x_1$. In this example, the R^2 is 0.98. The equation is not a perfect prediction, but with a single predictor variable, it captures the majority of the variance in the outcome.

Now consider the following example where you add a second predictor variable to the data above, as shown in Table 9.2.

Table 9.2 Example Data of Predictors (x1 and x2) and Outcome (y) Used for Regression Model.

y	x1	x2
7	1	3
13	2	5
29	7	1
10	2	2

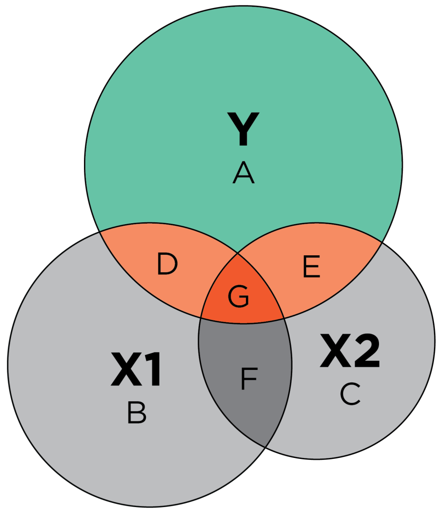
With the second predictor variable, the best fitting regression model is: $y = 0.00 + 4.00 \cdot x_1 + 1.00 \cdot x_2$. In this example, the R^2 is 1.00. The equation with the second predictor variable provides a perfect prediction of the outcome.

Providing perfect prediction with the right set of predictor variables is the dream of multiple regression. So, using multiple regression, we often add predictor variables to incrementally improve prediction. Knowing how much variance would be accounted for by random chance follows Equation 9.3:

$$E(R^2) = \frac{K}{n-1} \tag{9.3}$$

where $E(R^2)$ is the expected value of R^2 (the proportion of variance explained), K is the number of predictor variables, and n is the sample size. The formula demonstrates that the more predictor variables in the regression model, the more variance will be accounted for by chance. With many predictor variables and a small sample, you can account for a large share of the variance merely by chance.

As an example, consider that we have 13 predictor variables to predict fantasy performance for 43 players. Assume that, with 13 predictor variables, we



$$R^2 = \frac{\text{D} + \text{E} + \text{G}}{\text{A} + \text{D} + \text{E} + \text{G}}$$

Figure 9.2 Conceptual Depiction of Proportion of Variance Explained (R^2) in an Outcome Variable (Y) by Multiple Predictors ($X1$ and $X2$) in Multiple Regression. The size of each circle represents the variable's variance. The proportion of variance in Y that is explained by the predictors is depicted by the areas in orange. The dark orange space (G) is where multiple predictors explain overlapping variance in the outcome. Overlapping variance that is explained in the outcome (G) will not be recovered in the regression coefficients when both predictors are included in the regression model. From Petersen (2024b) and Petersen (2024c).

explain 38% of the variance ($R^2 = .38; r = .62$). We explained a lot of the variance in the outcome, but it is important to consider how much variance could have been explained by random chance: $E(R^2) = \frac{K}{n-1} = \frac{13}{43-1} = .31$. We expect to explain 31% of the variance, by chance, in the outcome. So, 82% of the variance explained was likely spurious (i.e., $\frac{.31}{.38} = .82$). As the sample size increases, the spuriousness decreases.

To account for the number of predictor variables in the model, we can use a modified version of R^2 called adjusted R^2 (R^2_{adj}). Adjusted R^2 (R^2_{adj}) accounts for the number of predictor variables in the model, based on how much would be expected to be accounted for by chance to penalize overfitting. Adjusted R^2 (R^2_{adj}) reflects the proportion of variance in the outcome (dependent) variable that is explained by the model predictions over and above what would be expected to be accounted for by chance, given the number of predictor variables in the model. The formula for adjusted R^2 (R^2_{adj}) is in Equation 9.4:

$$R^2_{adj} = 1 - (1 - R^2)\frac{n-1}{n-p-1} \tag{9.4}$$

where p is the number of predictor variables in the model, and n is the sample size.

9.5 Overfitting

Statistical models applied to big data (e.g., data with many predictor variables) can *over-fit* the data, which means that the statistical model accounts for error variance, which will not generalize to future samples. So, even though an over-fitting statistical model appears to be accurate because it is accounting for more variance, it is not actually that accurate—it will predict new data less accurately than how accurately it accounts for the data with which the model was built. In the case of fantasy football analytics, this is especially relevant because there are hundreds if not thousands of variables we could consider for inclusion and many, many players when considering historical data.

Consider an example where you develop an algorithm to predict players' fantasy performance based on 2023 data using hundreds of predictor variables. To some extent, these predictor variables will likely account for true variance (i.e., signal) and error variance (i.e., noise). If we were to apply the same algorithm based on the 2023 prediction model to 2024 data, the prediction model would likely predict less accurately than with 2023 data. The regression coefficients in the

In Figure 9.3, the blue line represents the true distribution of the data, and the red line is an over-fitting model:

9.6 Covariates

Covariates are variables that you include in the statistical model to try to control for them so you can better isolate the unique contribution of the predictor variable(s) in relation to the outcome variable. Use of covariates examines the association between the predictor variable and the outcome variable when

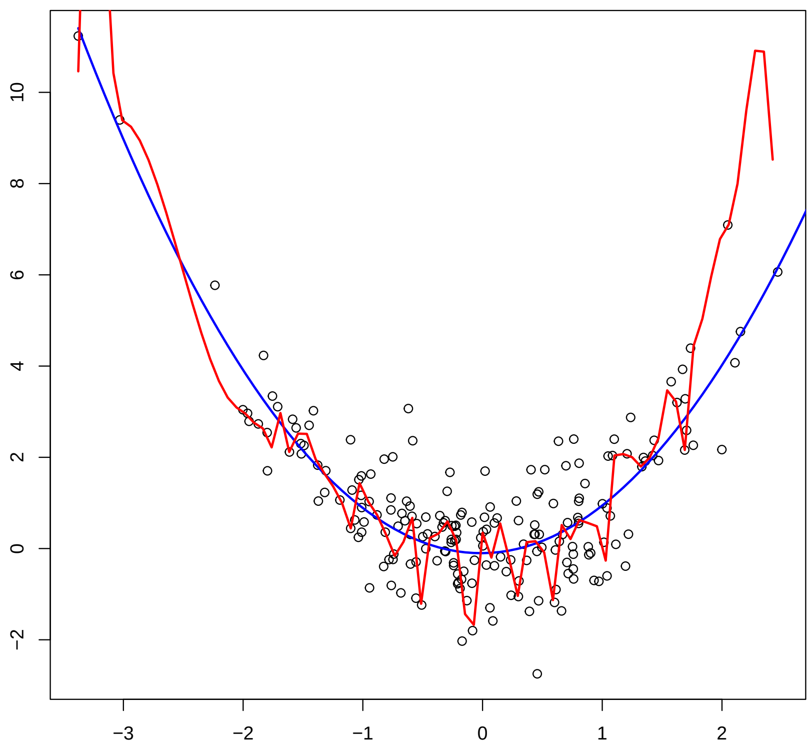


Figure 9.3 Over-fitting Model in Red Relative to the True Distribution of the Data in Blue. From Petersen (2024b) and Petersen (2024c).

holding people's level constant on the covariates. Inclusion of confounds as covariates allows potentially gaining a more accurate estimate of the causal effect of the predictor variable on the outcome variable. Ideally, you want to include any and all confounds as covariates. As described in Section 6.3.2.1, confounds are third variables that influence both the predictor variable and the outcome variable and explain their association. Covariates are potentially (but not necessarily) confounds. For instance, you might include the player's age as a covariate in a model that examines whether a player's 40-yard dash time at the NFL Combine predicts their fantasy points in their rookie year, but it may not be a confound.

9.7 Multicollinearity

Multicollinearity occurs when two or more predictor variables in a regression model are highly correlated. The problem of having multiple predictor variables that are highly correlated is that it makes it challenging to estimate the regression coefficients accurately.

Multicollinearity in multiple regression is depicted conceptually in Figure 9.4.

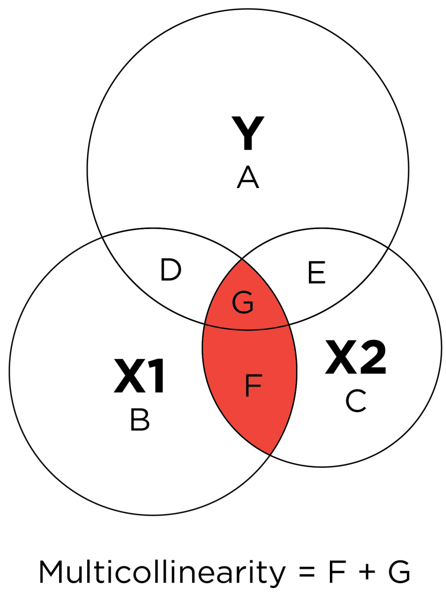


Figure 9.4 Conceptual Depiction of Multicollinearity in Multiple Regression. From Petersen (2024b) and Petersen (2024c).

Consider the following example adapted from Petersen (2024c) where you have two predictor variables and one outcome variable, as shown in Table 9.3.

Table 9.3 Example Data of Predictors (x1 and x2) and Outcome (y) Used for Regression Model.

y	x1	x2
9	2.0	4
11	3.0	6
17	4.0	8
3	1.0	2

Table 9.3 Example Data of Predictors (x1 and x2) and Outcome (y) Used for Regression Model.

y	x1	x2
21	5.0	10
13	3.5	7

The second predictor variable is not very good—it is exactly twice the value of the first predictor variable; thus, the two predictor variables are perfectly correlated (i.e., $r = 1.0$). This means that there are different prediction equation possibilities that are equally good—see Equations in Equation 9.5:

$$\begin{aligned} 2x_2 &= y \\ 0x_1 + 2x_2 &= y \\ 4x_1 &= y \\ 4x_1 + 0x_2 &= y \\ 2x_1 + 1x_2 &= y \\ 5x_1 - 0.5x_2 &= y \\ ... &= y \end{aligned} \tag{9.5}$$

Then, what are the regression coefficients? We do not know what are the correct regression coefficients because each of the possibilities fits the data equally well. Thus, when estimating the regression model, we could obtain arbitrary estimates of the regression coefficients with an enormous standard error around each estimate. In general, multicollinearity increases the uncertainty (i.e., standard errors and confidence intervals) around the parameter estimates. Any predictor variables that have a correlation above $\sim r = .30$ with each other could have an impact on the confidence interval of the regression coefficient. As the correlations among the predictor variables increase, the chance of getting an arbitrary answer increases, sometimes called "bouncing betas." So, it is important to examine a correlation matrix of the predictor variables before putting them in the same regression model. You can also examine indices such as variance inflation factor (VIF).

To address multicollinearity, you can drop a redundant predictor or you can also use principal component analysis or factor analysis of the predictors to reduce the predictors down to a smaller number of meaningful predictors. For a meaningful answer in a regression framework that is precise and confident, you need a low level of intercorrelation among predictors, unless you have a very large sample size.

10

Causal Inference

10.1 Getting Started

10.1.1 Load Packages

```
library("dagitty")
library("ggdag")
```

10.2 Correlation Does Not Imply Causality

As described in Section 6.3.2.1, there are several reasons why two variables, `X` and `Y`, might be correlated:

- `X` causes `Y`
- `Y` causes `X`
- a third variable (i.e., confound), `Z`, influences both `X` and `Y`
- the association between `X` and `Y` is spurious

10.3 Criteria for Causality

How do we know whether two processes are causally related? There are three criteria for establishing causality (Shadish et al., 2002):

1. The cause (e.g., the independent or predictor variable) temporally precedes the effect (i.e., the dependent or outcome variable).

2. The cause is related to (i.e., associated with) the effect.
3. There are no other alternative explanations for the effect apart from the cause.

The first criterion for establishing causality involves temporal precedence. In order for a cause to influence an effect, the cause must occur before the effect. For instance, if sports drink consumption influences player performance, the sports drink consumption (that is presumed to influence performance) must occur prior to the performance improvement. Establishing the first criterion eliminates the possibility that the association between the purported cause and effect reflects reverse causation. Reverse causation occurs when the purported effect is actually the cause of the purported cause, rather than the other way around. For instance, if sports drink consumption occurs only once, and it occurs only before and not after performance, then we have ruled out the possibility of reverse causation (i.e., that better performance causes players to consume sports drink).

The second criterion involves association. The purported cause must be associated with the purported effect. Nevertheless, as the maxim goes, "correlation does not imply causality." Just because two variables are correlated does not necessarily mean that they are causally related. However, correlation is useful because causality requires that the two processes be correlated. That is, correlation is a necessary but insufficient condition for causality. For instance, if sports drink consumption influences player performance, sports drink consumption must be associated with performance improvement.

The third criterion involves ruling out alternative reasons why the purported cause and effect may be related. As noted in Section 10.2, there are four reasons why `X` may be correlated with `Y`. If we meet the first criterion of causality, we have removed the possibility that `Y` causes `X` (i.e., reverse causality). To meet the third criterion of causality, we need to remove the possibility that the association reflects a third variable (confound) that influences both the cause and effect, and we need to remove the possibility that the association is spurious—the possibility that the association between the purported cause and effect is due to random chance. There are multiple approaches to meeting the third criterion of causality, such as by use of experiments, longitudinal designs, control variables, within-subject designs, and genetically informed designs, as described in Section 10.4.

10.4 Approaches for Causal Inference

10.4.1 Experimental Designs

As described in Section 6.3.1, experimental designs are designs in which participants are randomly assigned to one or more levels of the independent variable to observe its effects on the dependent variable. Experimental designs provide the strongest tests of causality because they can rule out reverse causation and third variables. For instance, by manipulating sports drink consumption before the player performs, they can eliminate the possibility that reverse causation explains the effect of the independent variable on the dependent variable. Second, through randomly assigning players to consume or not consume sports drink, this holds everything else constant (so long as the groups are evenly distributed according to other factors, such as their age, weight, etc.) and thus removes the possibility that third variable confounds explain the effect of the independent variable on the dependent variable.

10.4.2 Quasi-Experimental Designs

Although experimental designs provide the strongest tests of causality, many-times they are impossible, unethical, or impractical to conduct. For instance, it would likely not be practical to randomly assign National Football League (NFL) players to either consume or not consume sports drink before their games. Players have their pregame rituals and routines and many would likely not agree to participate in such a study. Thus, we often rely on quasi-experimental designs such as natural experiments and observational/correlational designs.

10.4.2.1 Longitudinal Designs

Research designs can be compared in terms of their internal validity—the extent to which we can be confident about causal inferences. A cross-sectional association is depicted in Figure 10.1:

For instance, we might observe that sports drink consumptions is concurrently associated with better player performance. Among observational/correlational research designs, cross-sectional designs tend to have the weakest internal validity. For the reasons described in Section 10.2, if we observe a cross-sectional association between `X` (e.g., sports drink consumption) and `Y` (e.g., player performance), we have little confidence that `X` causes `Y`. As a result, longitudinal designs can be valuable for more closely approximating causality if an experimental designs is not possible. Consider a lagged association that might be

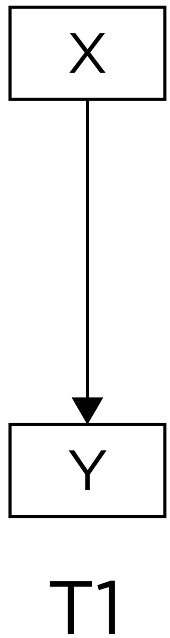


Figure 10.1 Cross-Sectional Association. T1 = Timepoint 1. From Petersen (2024b) and Petersen (2024c).

observed in a longitudinal design, as in Figure 10.2, which is a slightly better approach than relying on cross-sectional associations:

For instance, we might observe that sports drink performance *before* the game is associated with better player performance *during* the game. A lagged association has somewhat better internal validity than a cross-sectional association because we have greater evidence of temporal precedence—that the influence of the predictor *precedes* the outcome because the predictor was assessed before the outcome and it shows a predictive association. However, part of the association between the predictor with later levels of the outcome could be due to prior levels of the outcome that are stable across time. That is, it could be that better player performance leads players to consume more sports drink and that player performance is relatively stable across time. In such a case, it may be observed that sports drink consumption predicts later player performance even though player performance influences sports drink consumption, rather than the other way around Thus, consider an even stronger alternative—a lagged association that controls for prior levels of the outcome, as in Figure 10.3:

For instance, we might observe that sports drink performance *before* the game is associated with better player performance *during* the game, while control-

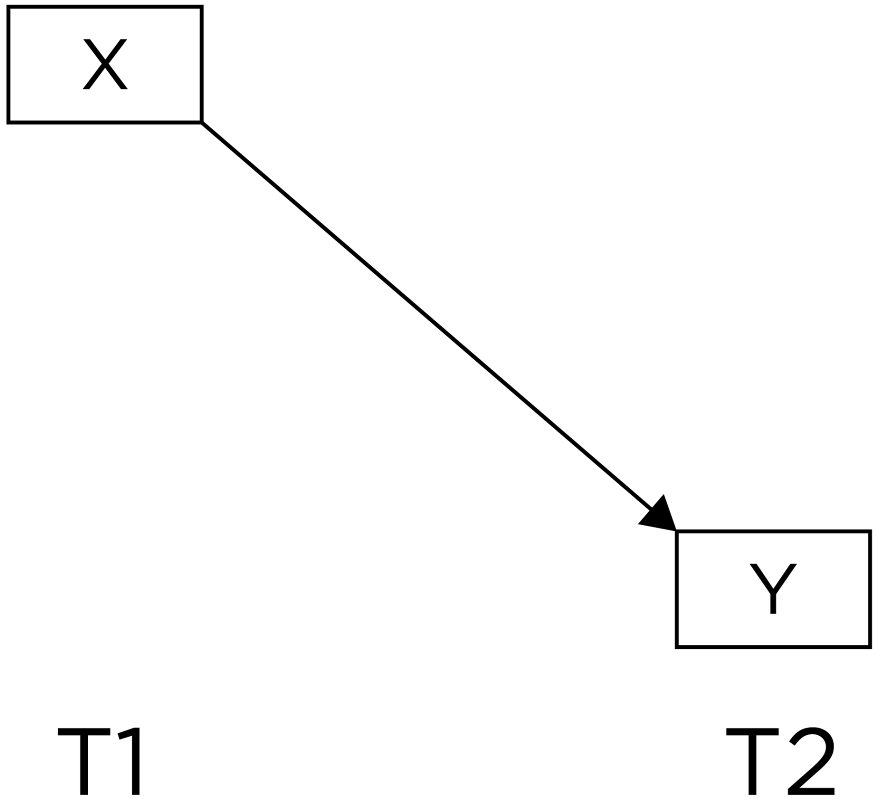


Figure 10.2 Lagged Association. T1 = Timepoint 1. T2 = Timepoint 2. From Petersen (2024b) and Petersen (2024c).

ling for prior player performance. A lagged association controlling for prior levels of the outcome has better internal validity than a lagged association that does not control for prior levels of the outcome. A lagged association that controls for prior levels further reduces the likelihood that the association owes to the reverse direction of effect, because earlier levels of the outcome are controlled. However, consider an even stronger alternative—lagged associations that control for prior levels of the outcome and that simultaneously test each direction of effect, as depicted in Figure 10.4:

Lagged associations that control for prior levels of the outcome and that simultaneously test each direction of effect provide the strongest internal validity among observational/correlational designs. Such a design can help better clarify which among the variables is the chicken and the egg—which variable is more likely to be the cause and which is more likely to be the effect. If there are bidirectional effects, such a design can also help clarify the magnitude of

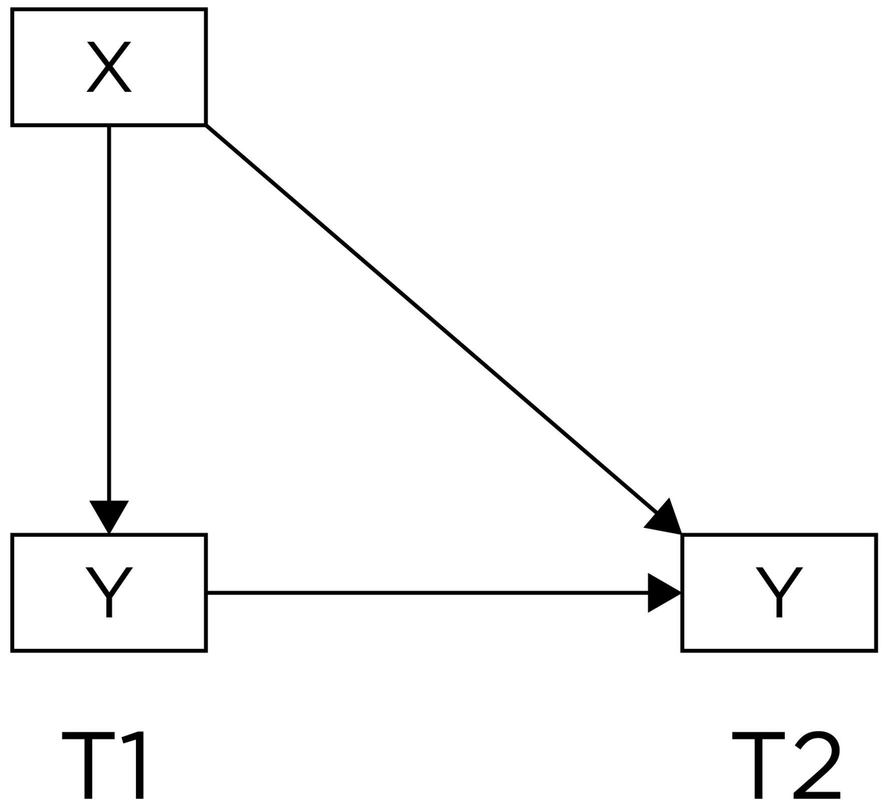


Figure 10.3 Lagged Association, Controlling for Prior Levels of the Outcome. T1 = Timepoint 1. T2 = Timepoint 2. From Petersen (2024b) and Petersen (2024c).

each direction of effect. For instance, we can simultaneously evaluate the extent to which sports drink predicts later player performance (while controlling for prior performance) and the reverse—player performance predicting later sports drink consumption (while contorlling for prior sports drink consumption).

10.4.2.2 Within-Subject Analyses

Another design feature of longitudinal designs that can lead to greater internal validity is the use of within-subject analyses. Between-subject analyses, might examine, for instanc, whether players who consume more sports drink perform better on average compared to players who consume less sports drink. However, there are other between-person differences that could explain any

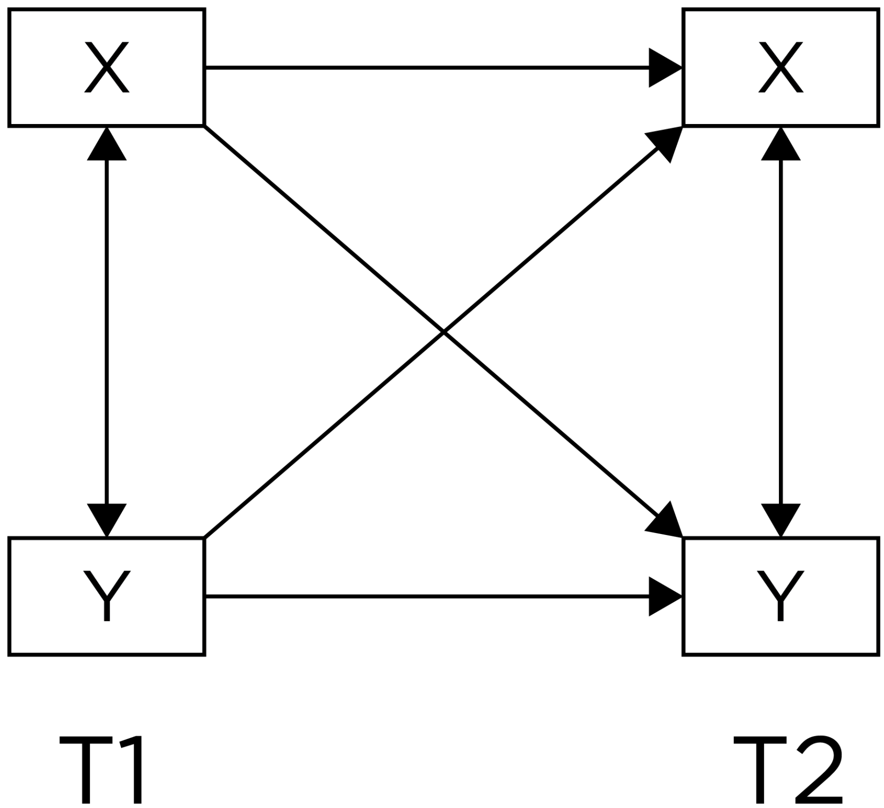


Figure 10.4 Lagged Association, Controlling for Prior Levels of the Outcome, Simultaneously Testing Both Directions Of Effect. T1 = Timepoint 1. T2 = Timepoint 2. From Petersen (2024b) and Petersen (2024c).

observed between-subject associations between sports drink consumption and players performance. Another approach could be to apply within-subject analyses. For instance, you could examining whether, within the same individual, if a player consumes a sports drink, do they perform better compared to games in which they did not consume a sports drink. When we control for prior levels of the outcome in the prediction, we are evaluating whether the predictor is associated with witin-person *change* in the outcome. Predicting within-person change provides stronger evidence consistent with causality because it uses the individual as their own control and controls for many time-invariant confounds (i.e., confounds that do not change across time). However, predicting within-person change does not, by itself, control for time-varying confounds. So, it can also be useful to control for time-varying confounds, such as by use of control variables.

10.4.2.3 Control Variables

One of the plausible alternatives to the inference that `X` causes `Y` is that there are third variable confounds that influence both `X` and `Y`, thus explaining why `X` and `Y` are associated, as depicted in Figure 6.3 and Figure 10.10. Thus, another approach that can help increase internal validity is to include plausible confounds as control variables. For instance, if a third variable such as education level might be a confound that influences both sports drink consumption and player performance, you could include education level as a covariate in the model. Inclusion of a covariate attempts to control for the variable by examining the association between the predictor variable and the outcome variable while holding the covariate variables constant. For instance, such a model would examine whether, when accounting for education level, there is an association between sports drink consumption and player performance.

Failure to control for important third variables can lead to erroneous conclusions, as evidenced by the association depicted in Figure 10.5. In the example, if we did not control for gender, we would infer that there is a positive association between dosage and recovery probability. However, when we examine each men and women separately, we learn that the association between dosage and recovery probability is actually negative within each gender group. Thus, in this case, failure to control for gender would lead to false inferences about the association between dosage and recovery probability.

However, it can be problematic to control for variables indiscriminantly. The use of causal diagrams can inform which variables are important to be included as control variables, and—just as important—which variables not to include as control variables, as described in Section 10.5.

10.4.2.4 Genetically Informed Designs

Another approach to control for variables is to use genetically informed designs. Genetically informed designs allow controlling for potential genetic effects in order to more closely approximate the contributions of various environmental effects. Genetically informed designs exploit differing degrees of genetic relatedness among participants to capture the extent to which genetic factors may contribute to an outcome. The average percent of DNA shared between people of varying relationships is provided in Table 10.1 (https://isogg.org/wiki/Autosomal_DNA_statistics; archived at https://perma.cc/MK3D-DST8):

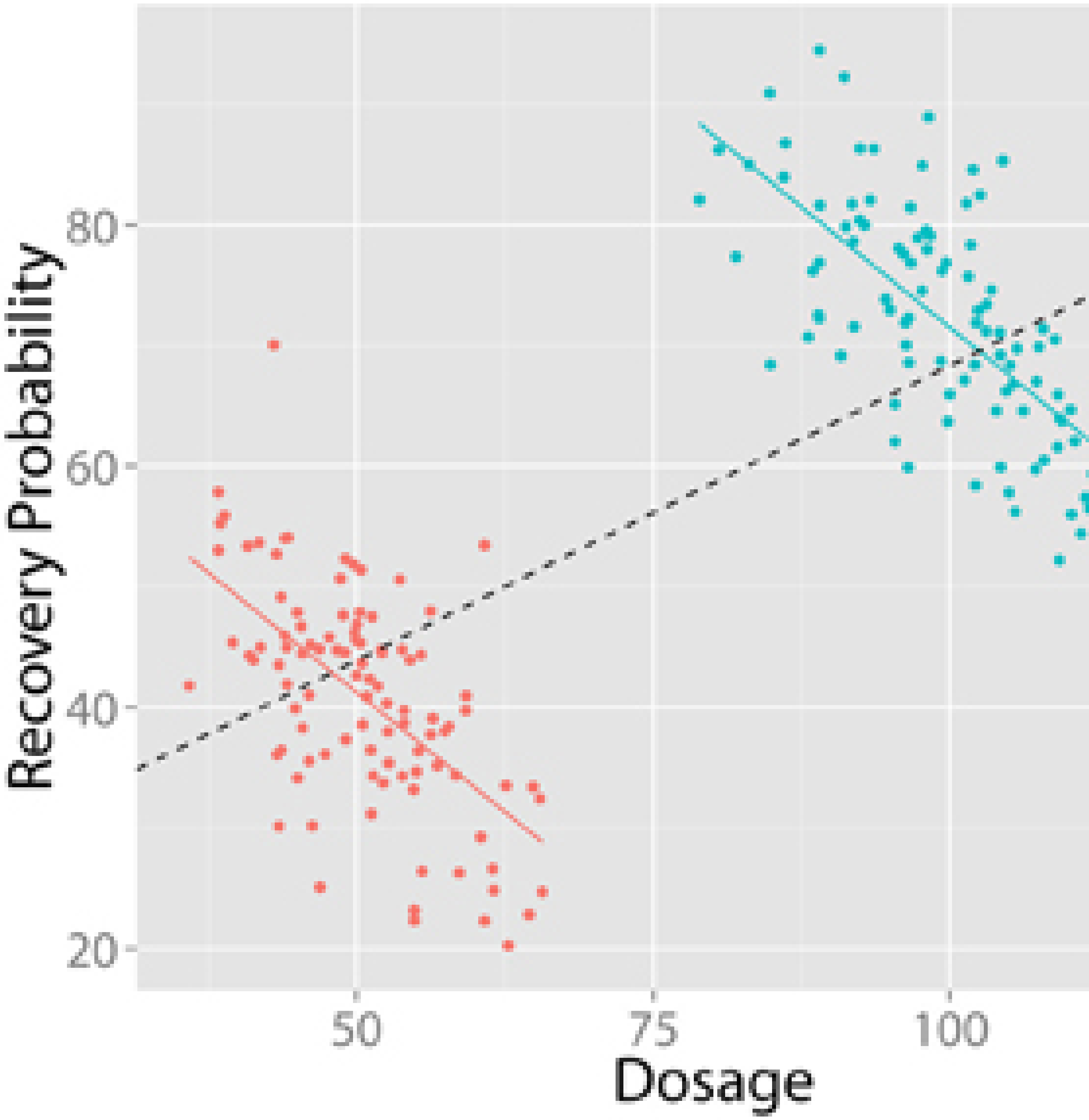


Figure 10.5 Example Where Failing to Control for a Variable (In This Case, Gender) Would Lead to False Inferences. In this example, the association between dosage and recovery probability is positive at the population level, but the association is negative among men and women separately. (Figure reprinted from Kievit et al. (2013), Figure 1, p. 2. Kievit, R., Frankenhuis, W., Waldorp, L., & Borsboom, D. (2013). Simpson's paradox in psychological science: a practical guide. *Frontiers in Psychology*, *4*(513). https://doi.org/10.3389/fpsyg.2013.00513)

Table 10.1 Average Percent of Autosomal DNA Shared by Pairs of Relatives by Relationship Type.

Relationship	Average Percent of Autosomal DNA Shared by Pairs of Relatives
Monozygotic ("identical") twins	100%
Dizygotic ("fraternal") twins	50%
Parent/child	50%
Full siblings	50%
Grandparent/grandchild	25%
Aunt-or-uncle/niece-or-nephew	25%
Half-siblings	25%
First cousin	12.5%
Great-grandparent/great-grandchild	12.5%

For instance, researchers may compare monozygotic twins versus dizygotic twins in some outcome—a so-called "twin study". It is assumed that the trait/outcome is attributable to genetic factors to the extent that the monozygotic twins (who share 100% of their DNA) are more similar in the trait or outcome compared to the dizygotic twins (who share on average 50% of their DNA). Alternatively, researchers could compare full siblings versus half-siblings, or they could compare full siblings versus first cousins.

Genetically informed designs are not as relevant for fantasy football analytics, but they are useful to present as one of various design features that researchers can draw upon to strengthen their ability to make causal inferences.

10.5 Causal Diagrams

10.5.1 Overview

A key tool when describing a research question or hypothesis is to create a conceptual depiction of the hypothesized causal processes. A causal diagram depicts the hypothesized causal processes that link two or more variables. A common form of causal diagrams is the directed acyclic graph (DAG). DAGs provide a helpful tool to communicate about causal questions and help identify how to avoid bias (i.e., over-estimation) in associations between variables due to confounding (i.e., common causes) (Digitale et al., 2022). For instance, from a DAG, it is possible to determine what variables it is important to control for in order to get unbiased estimates of the association between two variables of interest. To create DAGs, you can use the `R` package `dagitty` (Textor et al., 2017) or the associated browser-based extension, DAGitty: https://dagitty.net (archived at https://perma.cc/U9BY-VZE2). Examples of various causal diagrams that could explain why `X` is associated with `Y` are in Figure 10.6, Figure 10.8, and Figure 10.10.

```
XCausesY <- dagitty::dagitty("dag{
  X -> Y
}")

plot(dagitty::graphLayout(XCausesY))

dagitty::impliedConditionalIndependencies(XCausesY)
```

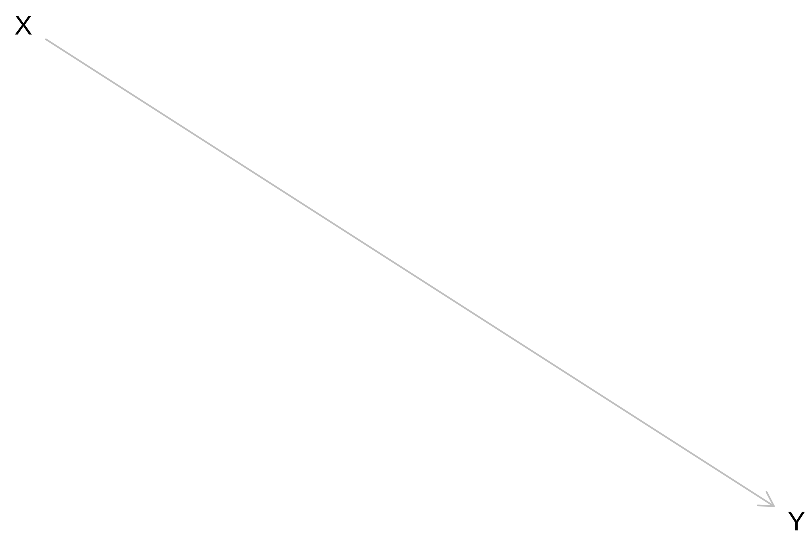


Figure 10.6 Causal Diagram (Directed Acyclic Graph) Depicting `X` Causing `Y`.

Here is an alternative way of specifying the same diagram (more similar to `lavaan` syntax):

```
XCausesY_alt <- ggdag::dagify(
  Y ~ X
)

#plot(XCausesY_alt) # this creates the same plot as above
ggdag::ggdag(XCausesY_alt) + theme_dag_blank()
```

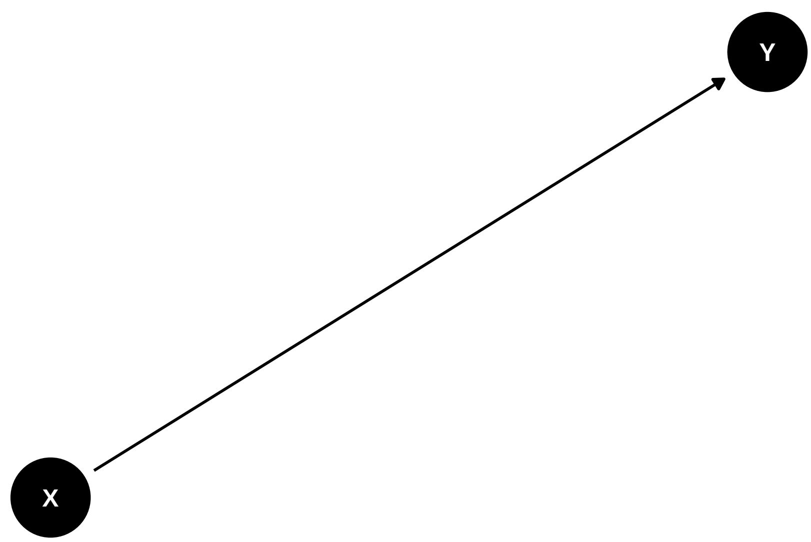


Figure 10.7 Causal Diagram (Directed Acyclic Graph) Depicting `X` Causing `Y`.

```
YCausesX <- dagitty::dagitty("dag{
  Y -> X
}")

plot(dagitty::graphLayout(YCausesX))

dagitty::impliedConditionalIndependencies(YCausesX)
```

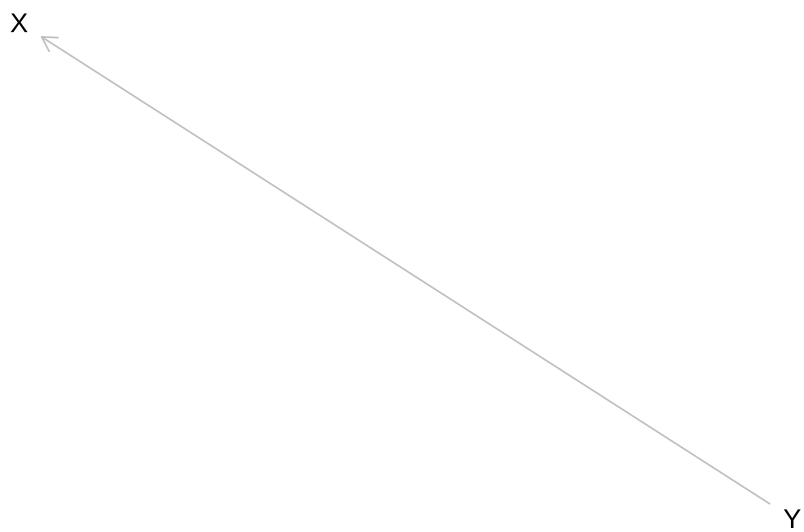


Figure 10.8 Causal Diagram (Directed Acyclic Graph) Depicting Reverse Causation: `Y` Causing `X`.

Here is an alternative way of specifying the same diagram (more similar to `lavaan` syntax):

```
YCausesX_alt <- ggdag::dagify(
  X ~ Y
)

#plot(YCausesX_alt) # this creates the same plot as above
ggdag::ggdag(YCausesX_alt) + theme_dag_blank()
```

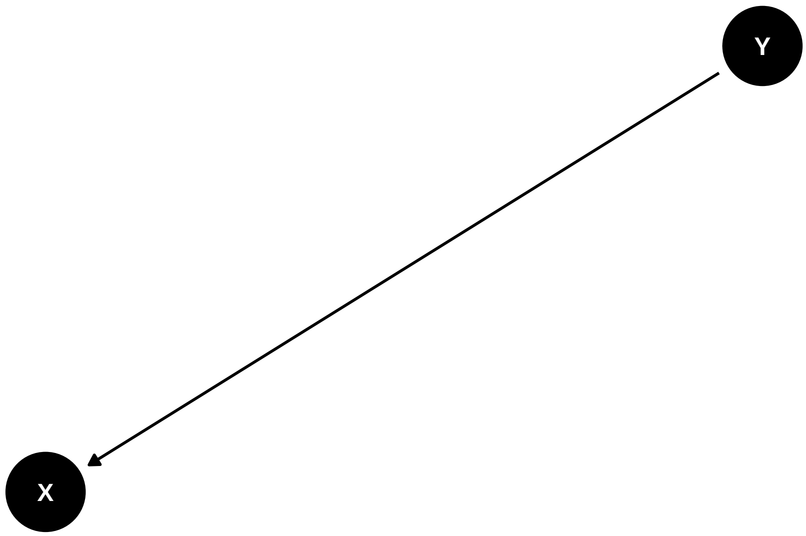


Figure 10.9 Causal Diagram (Directed Acyclic Graph) Depicting Reverse Causation: `Y` Causing `X`.

```
ZCausesXandY <- dagitty::dagitty("dag{
  Z -> Y
  Z -> X
  X <-> Y
}")

plot(dagitty::graphLayout(ZCausesXandY))
```

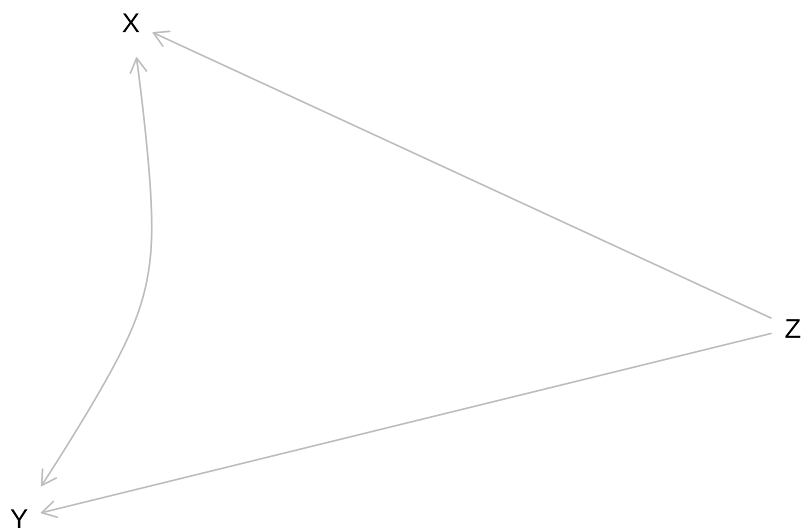


Figure 10.10 Causal Diagram (Directed Acyclic Graph) Depicting a Third Variable Confound, `Z`, Causing `X` and `Y`, Thus Explaining Why `X` and `Y` are associated.

Here is an alternative way of specifying the same diagram (more similar to `lavaan` syntax):

```
ZCausesXandY_alt <- ggdag::dagify(
  X ~ Z,
  Y ~ Z,
  X ~~ Y
)

#plot(ZCausesXandY_alt) # this creates the same plot as above
ggdag::ggdag(ZCausesXandY_alt) + theme_dag_blank()
```

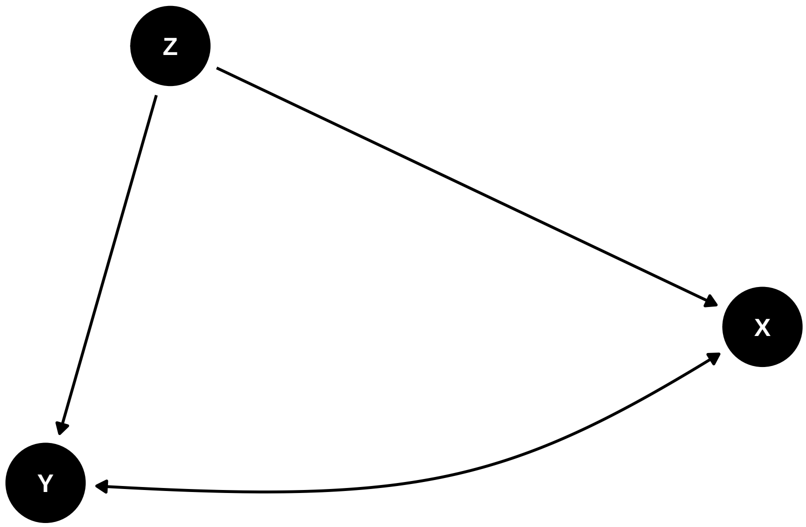


Figure 10.11 Causal Diagram (Directed Acyclic Graph) Depicting a Third Variable Confound, `Z`, Causing `X` and `Y`, Thus Explaining Why `X` and `Y` are associated.

Consider another example in Figure 10.12:

```
mediationDag <- dagitty::dagitty("dag{
  X -> M1
  X -> M2
  M1 -> Y
  M2 -> Y
  M1 <-> M2
}")

plot(dagitty::graphLayout(mediationDag))
```

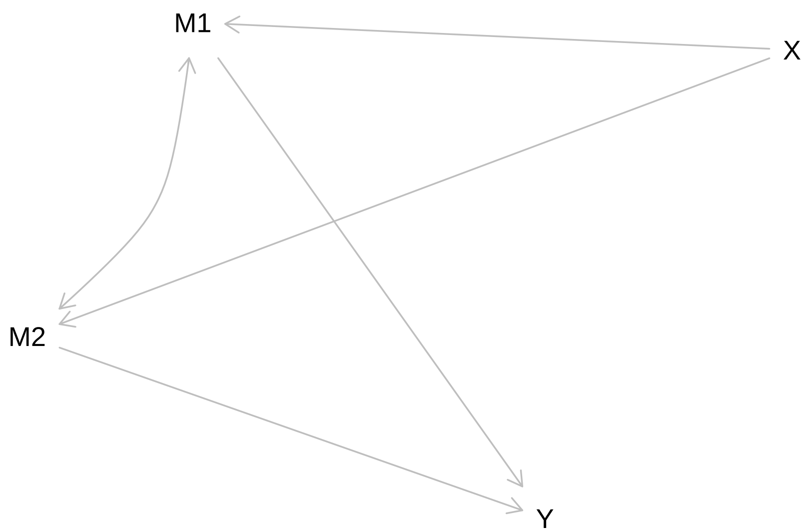


Figure 10.12 Causal Diagram (Directed Acyclic Graph).

```
dagitty::impliedConditionalIndependencies(mediationDag)
```

```
X _||_ Y | M1, M2
```

```
dagitty::adjustmentSets(
  mediationDag,
  exposure = "M1",
  outcome = "Y",
  effect = "total")
```

```
{ M2 }
```

In this example, `X` influences `Y` via `M1` and `M2` (i.e., multiple mediators), and `M1` is also associated with `M2`. The `dagitty::impliedConditionalIndependencies()` function identifies variables in the causal diagram that are conditionally independent (i.e., uncorrelated) after controlling for other variables in the model. For this causal diagram, `X` is conditionally independent with `Y` because `X` is independent with `Y` when controlling for `M1` and `M2`.

The `dagitty::adjustmentSets()` function identifies variables that would be necessary to control for (i.e., to include as covariates) in order to identify an unbiased estimate of the association (whether the total effect, i.e., `effect = "total"`; or the direct effect, i.e., `effect = "direct"`) between two variables (`exposure` and `outcome`). In this case, to identify the unbiased association between `M1` and `Y`, it is important to control for `M2`.

Here is an alternative way of specifying the same diagram (more similar to `lavaan` syntax):

```
mediationDag_alt <- ggdag::dagify(
  M1 ~ X,
  M2 ~ X,
  Y ~ M1,
  Y ~ M2,
  M1 ~~ M2
)

#plot(mediationDag_alt) # this creates the same plot as above
ggdag::ggdag(mediationDag_alt) + theme_dag_blank()
```

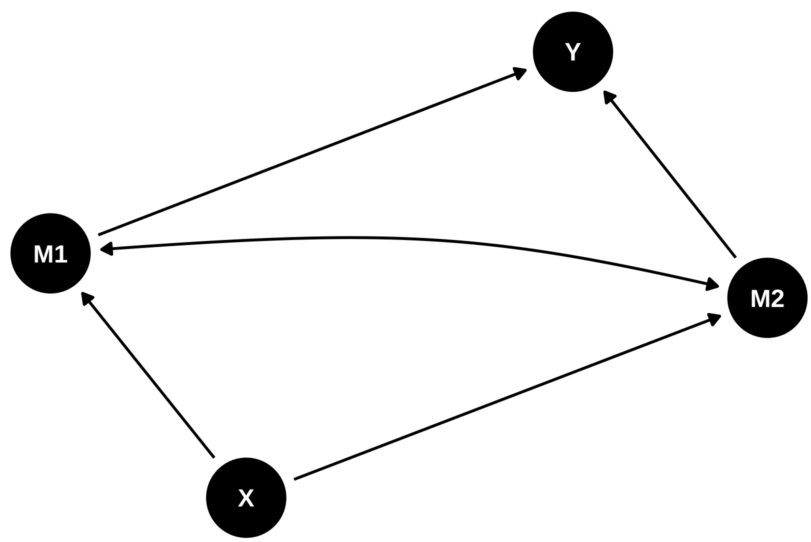


Figure 10.13 Causal Diagram (Directed Acyclic Graph).

10.5.2 Confounding

Confounding occurs when two variables—that are both caused by another variable(s)—have a spurious or noncausal association (D'Onofrio et al., 2020). That is, two variables share a common cause, and the common cause leads the variables to be associated even though they are not causally related. The common cause—i.e., the variable that influences the two variables—is known as a confound (or confounder). An example of confounding is depicted in Figure 10.14:

```
confounding <- ggdag::confounder_triangle(
  x = "Player Endurance",
  y = "Field Goals Made",
  z = "Stadium Altitude")

confounding %>%
  ggdag(
    text = FALSE,
    use_labels = "label") +
  theme_dag_blank()
```

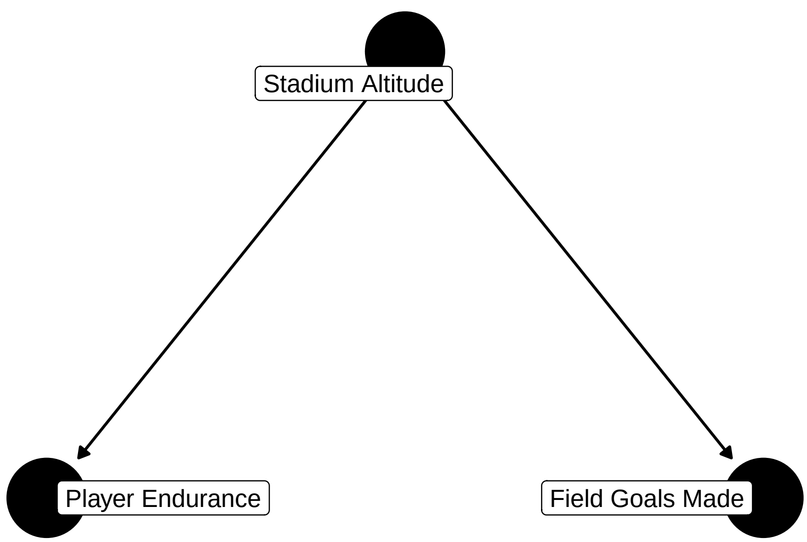


Figure 10.14 Causal Diagram (Directed Acyclic Graph) Example of Confounding.

```
dagitty::impliedConditionalIndependencies(confounding)
```

```
x _||_ y | z
```

The output indicates that player endurance (`X`) and field goals made (`Y`) are conditionally independent when accounting for stadium altitude (`Z`). *Conditional independence* refers to two variables being unassociated when controlling for other variables.

```
dagitty::adjustmentSets(
  confounding,
  exposure = "x",
  outcome = "y",
  effect = "total")
```

```
{ z }
```

The output indicates that, to obtain an unbiased estimate of the causal association between two variables, it is necessary to control for any confounding (Lederer et al., 2019). That is, to obtain an unbiased estimate of the causal association between player endurance (`X`) and field goals made (`Y`), it is necessary to control for stadium altitude (`Z`).

10.5.3 Mediation

Mediation can be divided into two types: full and partial. In full mediation, the mediator(s) fully account for the effect of the predictor variable on the outcome variable. In partial mediation, the mediator(s) partially but do not fully account for the effect of the predictor variable on the outcome variable.

10.5.3.1 Full Mediation

An example of full mediation is depicted in Figure 10.15:

```
full_mediation <- ggdag::mediation_triangle(
  x = "Coaching Quality",
  y = "Fantasy Points",
  m = "Player Preparation")

full_mediation %>%
  ggdag(
    text = FALSE,
    use_labels = "label") +
  theme_dag_blank()
```

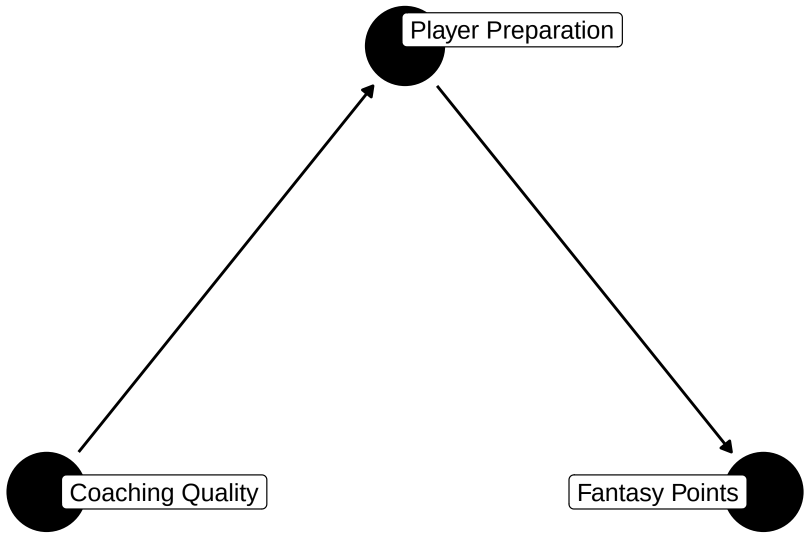


Figure 10.15 Causal Diagram (Directed Acyclic Graph) Example of Full Mediation.

```
dagitty::impliedConditionalIndependencies(full_mediation)
```

```
x _||_ y | m
```

In full mediation, `X` and `Y` are conditionally independent when accounting for the mediator (`Z`). In this case, coaching quality (`X`) and fantasy points (`Y`) are conditionally independent when accounting for player preparation (`M`). In other words, in this example, player preparation is the mechanism that fully (i.e., 100%) accounts for the effect of coaching quality on players' fantasy points.

```
dagitty::adjustmentSets(
  full_mediation,
  exposure = "x",
  outcome = "y",
  effect = "direct")
```

```
{ m }
```

The output indicates that, to obtain an unbiased estimate of the *direct* causal association between coaching quality (`X`) and fantasy points (`Y`) (i.e., the effect that is *not* mediated through intermediate processes), it is necessary to control for player preparation (`M`).

```
dagitty::adjustmentSets(
  full_mediation,
  exposure = "x",
  outcome = "y",
  effect = "total")
```

```
{}
```

However, to obtain an unbiased estimate of the *total* causal association between coaching quality (`X`) and fantasy points (`Y`) (i.e., including the portion of the effect that is mediated through intermediate processes), it is important *not* to control for player preparation (`M`). When the goal is to understand the (total) causal effect of coaching quality (`X`) and fantasy points (`Y`), controlling for the mediator (player preparation; `M`) would be inappropriate because doing so would remove the causal effect, thus artificially reducing the estimate of the association between coaching quality (`X`) and fantasy points (`Y`) (Lederer et al., 2019).

10.5.3.2 Partial Mediation

An example of partial mediation is depicted in Figure 10.16:

```
partial_mediation <- ggdag::mediation_triangle(
  x = "Coaching Quality",
  y = "Fantasy Points",
  m = "Player Preparation",
  x_y_associated = TRUE)

partial_mediation %>%
  ggdag(
    text = FALSE,
    use_labels = "label") +
  theme_dag_blank()
```

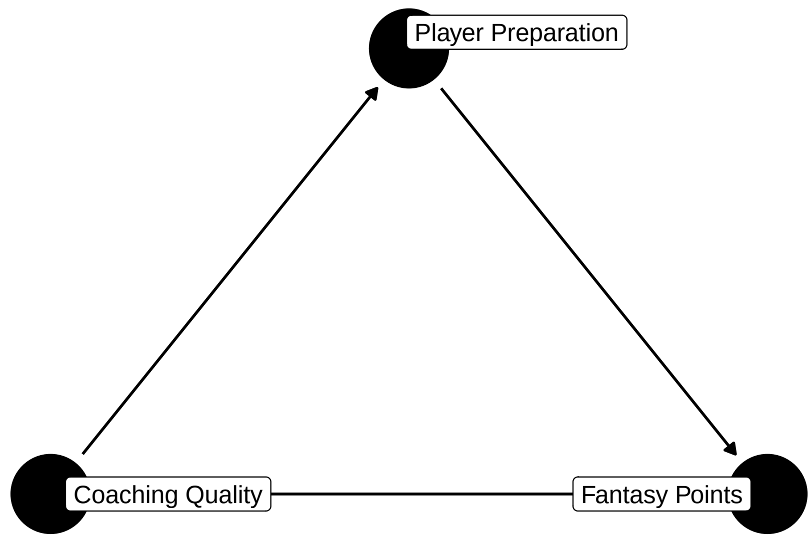


Figure 10.16 Causal Diagram (Directed Acyclic Graph) Example of Partial Mediation.

```
dagitty::impliedConditionalIndependencies(partial_mediation)
```

In partial mediation, `X` and `Y` are *not* conditionally independent when accounting for the mediator (`Z`). In this case, coaching quality (`X`) and fantasy points (`Y`) are still associated when accounting for player preparation (`M`). In other words, in this example, player preparation is a mechanism that partially but does not fully account for the effect of coaching quality on players' fantasy points. That is, there are likely other mechanisms, in addition to player preparation, that collectively account for the effect of coaching quality on fantasy points. For instance, coaching quality could also influence player fantasy points through better play-calling.

```
dagitty::adjustmentSets(
  partial_mediation,
  exposure = "x",
  outcome = "y",
  effect = "direct")
```

```
{ m }
```

As with full mediation, the output indicates that, to obtain an unbiased estimate of the *direct* causal association between coaching quality (`X`) and fantasy points (`Y`) (i.e., the effect that is *not* mediated through intermediate processes), it is necessary to control for player preparation (`M`).

```
dagitty::adjustmentSets(
  partial_mediation,
  exposure = "x",
  outcome = "y",
  effect = "total")
```

```
 {}
```

However, as with full mediation, to obtain an unbiased estimate of the *total* causal association between coaching quality (`X`) and fantasy points (`Y`) (i.e., including the portion of the effect that is mediated through intermediate processes), it is important *not* to control for player preparation (`M`). When the goal is to understand the (total) causal effect of coaching quality (`X`) and fantasy points (`Y`), controlling for a mediator (player preparation; `M`) would be inappropriate because doing so would remove the causal effect, thus artificially reducing the estimate of the association between coaching quality (`X`) and fantasy points (`Y`) (Lederer et al., 2019).

10.5.4 Collider Bias

Collision occurs when two variables influence a third variable, the collider (D'Onofrio et al., 2020). That is, a collider is a variable that is caused by two other variables (i.e., a common effect). An example collision is depicted in Figure 10.17 and Figure 10.18:

```
colliderBias1 <- ggdag::collider_triangle(
  x = "Diet",
  y = "Coaching Strategy",
  m = "Injury Status")

colliderBias1 %>%
  ggdag(
    text = FALSE,
    use_labels = "label") +
  theme_dag_blank()
```

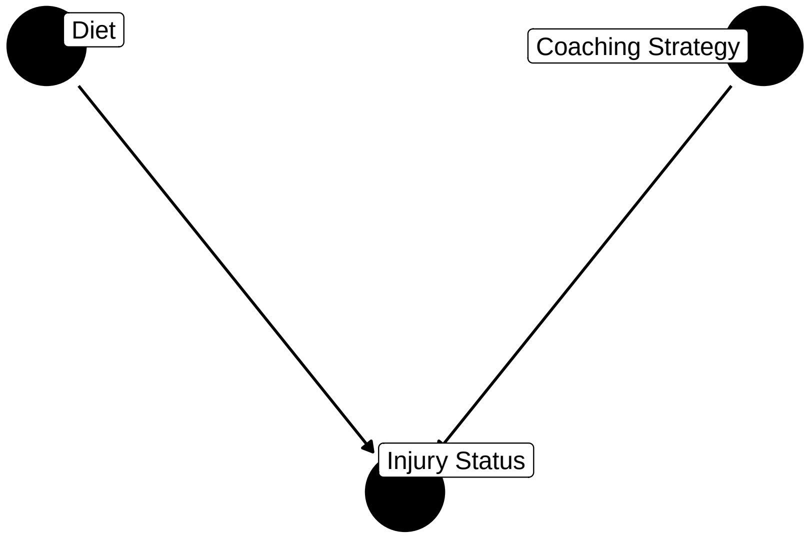


Figure 10.17 Causal Diagram (Directed Acyclic Graph) Example of a Collision with a Collider (Injury Status).

```
dagitty::impliedConditionalIndependencies(colliderBias1)
```

```
x _||_ y
```

In this example collision, diet (`X`) and coaching strategy (`Y`) are independent.

```
dagitty::adjustmentSets(
  colliderBias1,
  exposure = "x",
  outcome = "y",
  effect = "total")
```

```
 {}
```

As the output indicates, we should not control for the collider when examining the association between the two causes of the collider. That is, we should not control for injury status (`M`) when examining the association between diet (`X`) and coaching strategy. Controlling for the collider leads to confounding and can artificially induce an association between the two causes of the collider despite no causal association between them (Lederer et al., 2019). Obtaining a distorted or artificial association between two variables due to inappropriately controlling for a collider is known as *collider bias*.

Consider another example:

```
colliderBias2 <- ggdag::collider_triangle(
  x = "Coaching Quality",
  y = "Player Preparation",
  m = "Fantasy Points",
  x_y_associated = TRUE)

colliderBias2 %>%
  ggdag(
    text = FALSE,
    use_labels = "label") +
  theme_dag_blank()
```

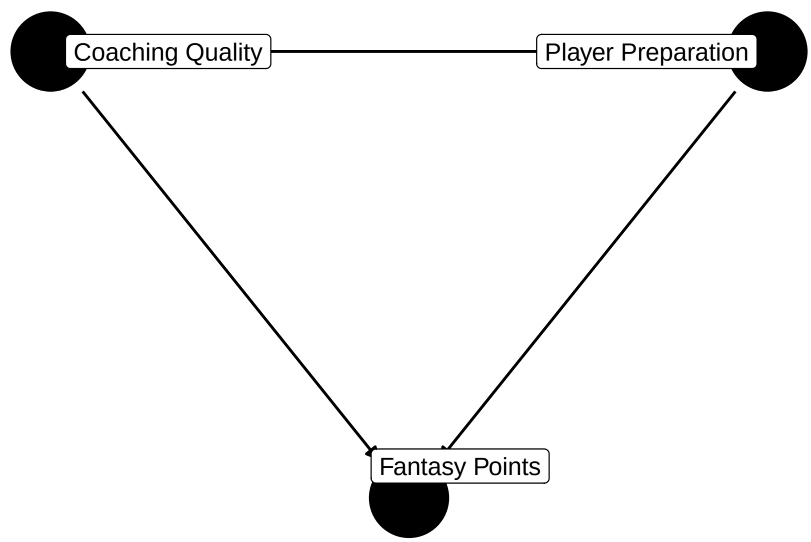


Figure 10.18 Causal Diagram (Directed Acyclic Graph) Example of Collider Bias.

```
dagitty::impliedConditionalIndependencies(colliderBias2)
```

In this example of collider bias, there are no conditional independencies.

```
dagitty::adjustmentSets(
  colliderBias2,
  exposure = "x",
  outcome = "y",
  effect = "total")
```

```
{}
```

Again, it would be important not to control for the collider, fantasy points (`M`), when examining the association between coaching quality (`X`) and player preparation (`Y`). In this case, controlling for the collider would remove some of the causal effect of coaching quality on player preparation and could lead to an artificially smaller estimate of the causal effect between coaching quality and player preparation.

10.5.4.1 M-Bias

Collider bias may also occur when neither variable of interest is a direct cause of the collider (Lederer et al., 2019). M-bias is a form of collider bias that occurs when two variables that are not causally related, `A` and `B`, both influence a collider, `M`, and each (`A` and `B`) also influences a separate variable—e.g., `A` influences `X` and `B` influences `Y`. M-bias is so-named from the M-shape of the DAG. An example of M-bias is depicted in Figure 10.19:

```
mBias <- ggdag::m_bias(
  x = "Number of Media Articles About the Team",
  y = "Fantasy Points",
  a = "Team Record",
  b = "Coaching Quality",
  m = "Fan Attendance at Game")

mBias %>%
  ggdag(
    text = FALSE,
    use_labels = "label") +
  theme_dag_blank()
```

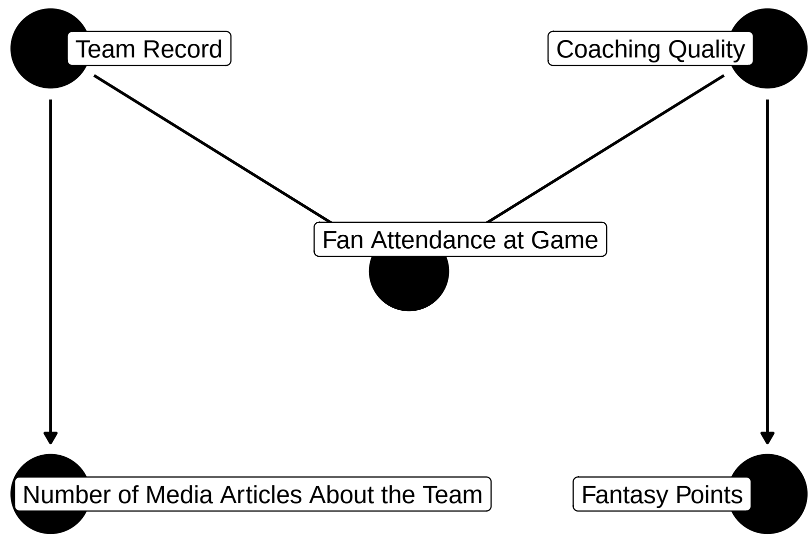


Figure 10.19 Causal Diagram (Directed Acyclic Graph) Example of M-Bias.

In this example, fan attendance is the collider that is influenced separately by the team record and the coaching quality. This is a fictitious example for purposes of demonstration; in reality, coaching quality influences the team's record.

```
dagitty::impliedConditionalIndependencies(mBias)
```

```
a _||_ b
a _||_ y
b _||_ x
m _||_ x | a
m _||_ y | b
x _||_ y
```

As the output indicates, there are several conditional independencies.

```
dagitty::adjustmentSets(
  mBias,
  exposure = "x",
  outcome = "y",
  effect = "total")
```

```
 {}
```

It is important not to control for the collider (fan attendance). If you control for the collider, you can induce an artificial association between team record and coaching quality. Moreover, because doing so induces an artificial association between team record and coaching quality, it can also induce an artifial association between the effects of team record and coaching quality: number of media articles about the team and fantasy points, respectively. That is, controlling for the collider can lead to an artificial association between `X` and `Y` that does not reflect a causal process.

10.5.4.2 Butterfly Bias

Butterfly bias occurs when both confounding and M-bias are present. Butterfly bias (aka bow-tie bias) is so-named from the butterfly shape of the DAG. In butterfly bias, the following criteria are met:

- Two variables (`A` and `B`) influence a collider (`M`).
- The collider influences two variables, `X` and `Y`.
- `A` also influences `X`.
- `B` also influences `Y`.
- `A` and `B` are not causally related
- `X` and `Y` are not causally related

Or, more succinctly:

- `A` influences `M` and `X`.
- `B` influences `M` and `Y`.
- `M` influences `X` and `Y`.

In butterfly bias, the collider (`M`) is also a confound. That is, a variable is both influenced by two variables and influences two variables. An example of butterfly bias is depicted in Figure 10.20:

```
butterflyBias <- ggdag::butterfly_bias(
  x = "Off-field Behavior",
  y = "Fantasy Points",
  a = "Diet",
  b = "Coaching Quality",
  m = "Mental Health")

butterflyBias %>%
  ggdag(
    text = FALSE,
    use_labels = "label") +
  theme_dag_blank()
```

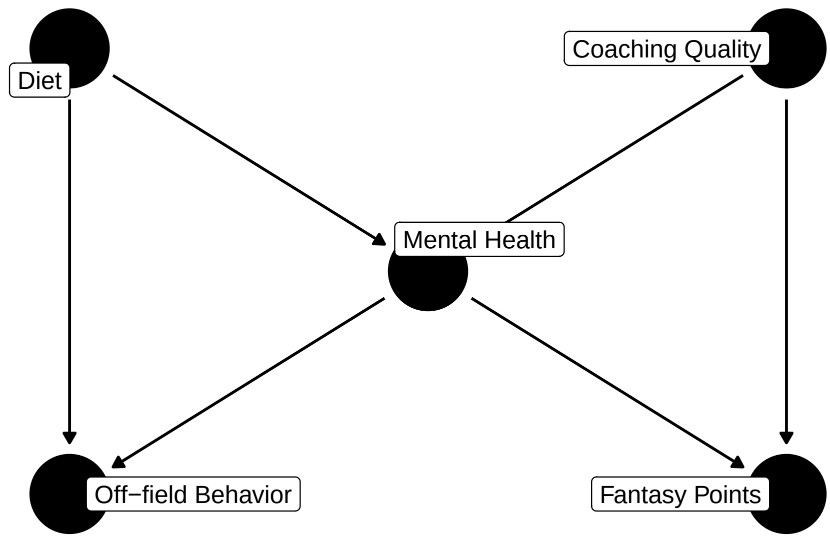


Figure 10.20 Causal Diagram (Directed Acyclic Graph) Example of Butterfly Bias.

In this case, players' mental health is a collider of their diet and the quality of the coaching they receive. In addition, players' mental health is a confound of their off-field behavior and fantasy points.

```
dagitty::impliedConditionalIndependencies(butterflyBias)
```

```
a _||_ b
a _||_ y | b, m
b _||_ x | a, m
x _||_ y | b, m
x _||_ y | a, m
```

As the output indicates, there are several conditional independencies.

```
dagitty::adjustmentSets(
  butterflyBias,
  exposure = "x",
  outcome = "y",
  effect = "total")
```

```
{ b, m }
{ a, m }
```

When dealing with a collider that is also a confound, controlling for either set, `B` and `M` or `A` and `M`, will provide an unbiased estimate of the association between `X` and `Y`. In this case, controlling for either a) coaching quality and mental health or b) diet and mental health—but not both sets—will yield an unbiased estimate of the association between off-field behavior and fantasy points.

10.5.5 Selection Bias

Selection bias occurs when the selection of participants or their inclusion in analyses depends on the variables being studied.

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