Infinity and Beyond

June 27, 2023

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Sets

$\{\}$

$\{$  $\}$

$\{1,2,3,4\}$

$\{890, \text{“foo”}, \pi\}$

Infinite Sets

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$$
$$\mathbb{Z} = \{0, -1, 1, -2, 2, -3, 3, -4, 4, -5, 5, \ldots\}$$
$$\mathbb{Q} = \{0, 1, 2, \tfrac{1}{2}, 3, 4, \tfrac{3}{2}, \tfrac{2}{3}, \tfrac{1}{4}, \tfrac{1}{5}, 5, 6, \tfrac{5}{2}, \ldots\}$$

Infinite Sets

{ every possible book }
{ every possible book that starts with “supercalifragilisticexpialidocious”}
{ every point on a circle }
{ every possible board game }
{ every color }
{ every triangle }

Size

How can we tell if two sets are the same size?

Size

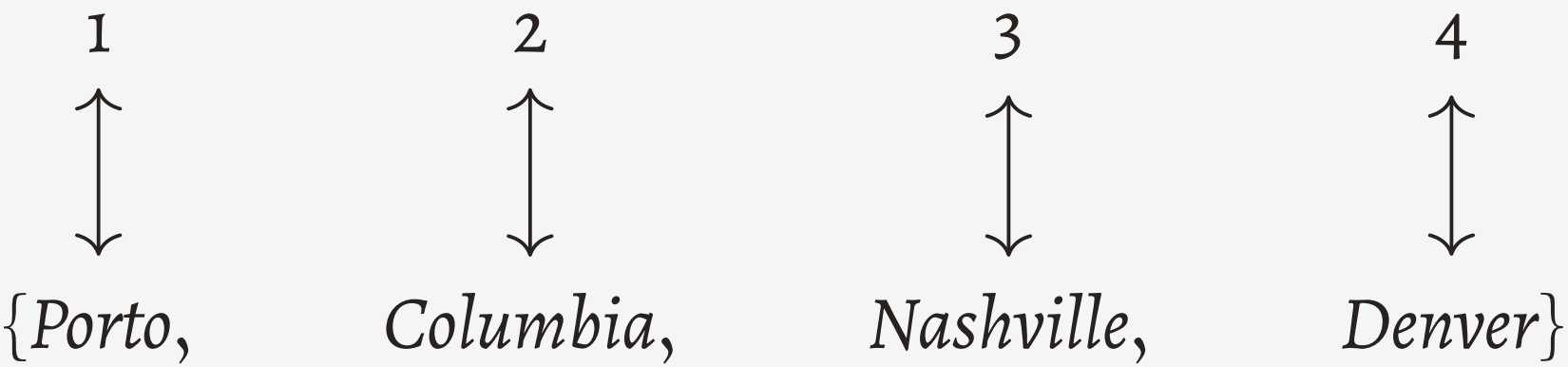
How can we tell if two sets are the same size?

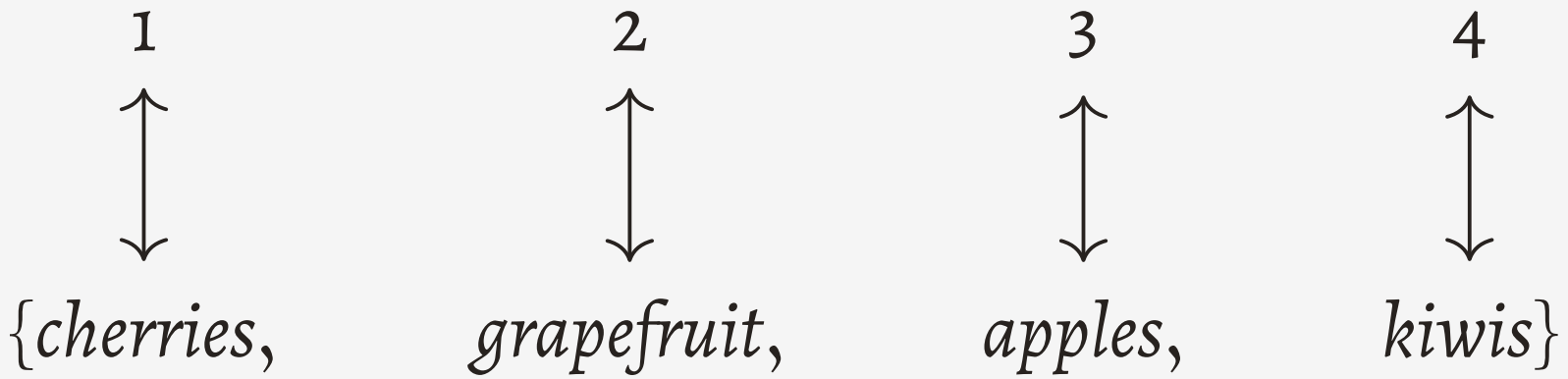
Just count them!

Size

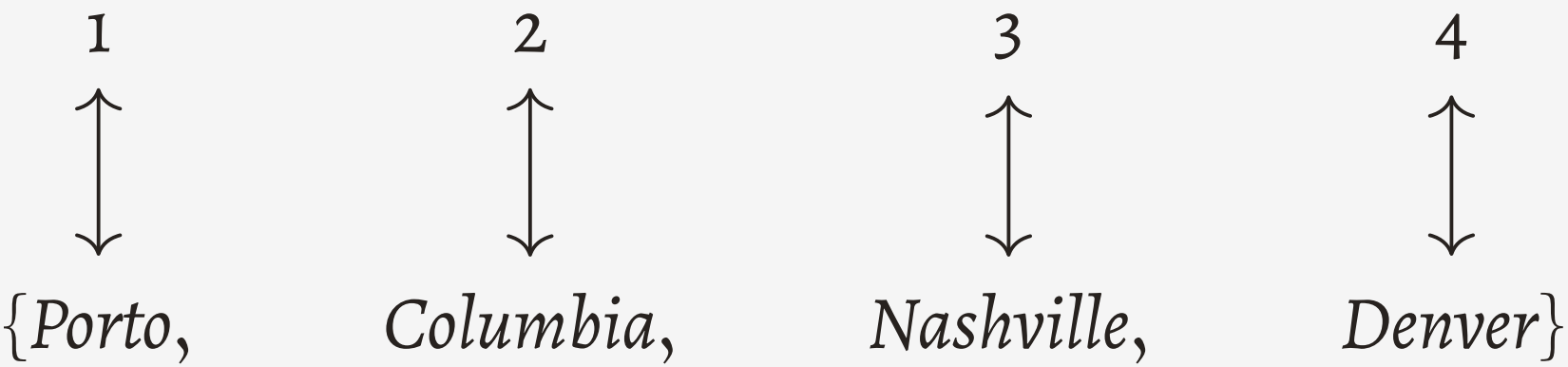
{Porto, Columbia, Nashville, Denver}
{ cherries, grapefruit, apples, kiwis }

Size


1
2
3
4
{Porto,
Columbia,
Nashville,
Denver}


1
2
3
4
{cherries,
grapefruit,
apples,
kiwis}

Size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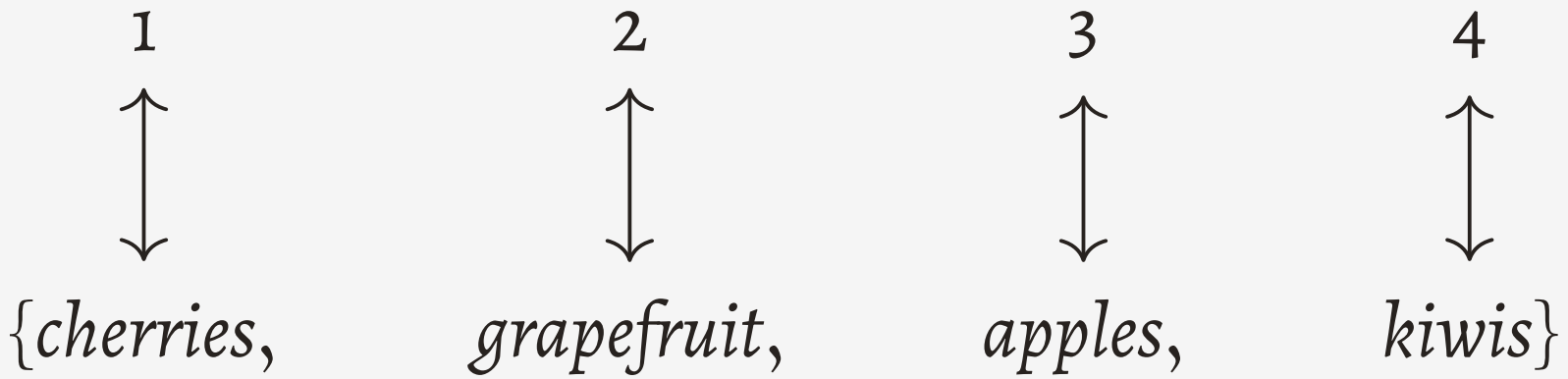

1
2
3
4
{cherries,
grapefruit,
apples,
kiwis}

Size

$$4 = 4$$

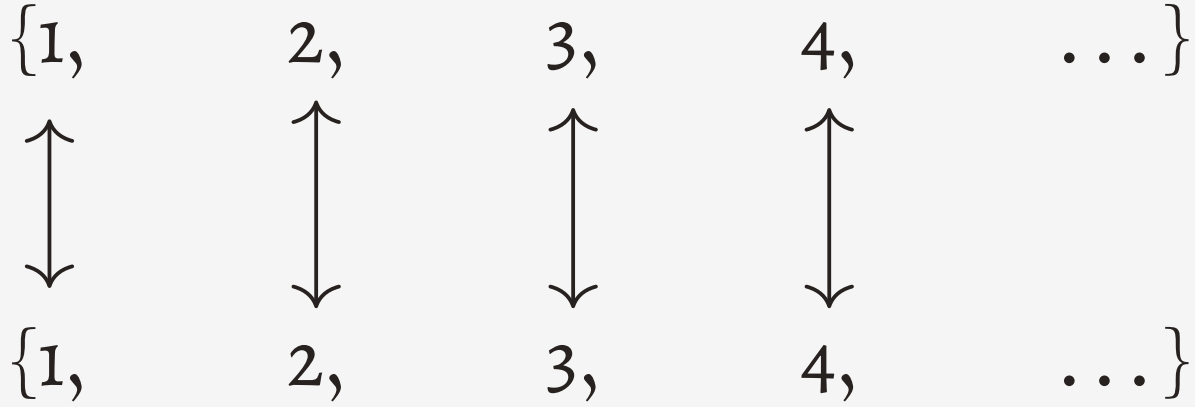
Size

$4 = 4$

$\#\ \text{cities} = \#\ \text{fruits}$

$$\{1, 2, 3, 4, 5, 6, \ldots\}$$

$$\{1, 2, 3, 4, 5, 6, \ldots\}$$


{1, 2, 3, 4, ...}
{1, 2, 3, 4, ...}