

Section 1

Exercise 2.1.i

Exercise 2.1.ii If F is representable then we have a natural isomorphism α and the following commutative square:

$$\begin{array}{ccc} Fx & \xleftarrow{\alpha_x} & C(c, x) \\ Ff \downarrow & & \downarrow f_* \\ Fy & \xleftarrow{\alpha_y} & C(c, y) \end{array}$$

Now suppose f is monic and pick two morphisms $g, h : W \rightrightarrows C(c, x)$ for some set W , such that $f_*g = f_*h$. This is equivalent to saying $\forall w \in W, f_*(g(w)) = f_*(h(w)) \implies f \circ (g(w)) = f \circ (h(w)) \implies g(w) = h(w) \implies g = h$ by the fact that f is monic. Hence f_* is monic.

Now, by the square, we have $\alpha_y Ff = f_* \alpha_x \implies Ff = \alpha_y^{-1} f_* \alpha_x$. Hence Ff is a composition of three injective maps which together are also injective. Therefore F preserves monomorphisms.

As for part two, pick a functor $\mathbf{2} \rightarrow \mathbf{Set}$ that takes the morphism to a non-injective function. By the contrapositive of the above statement, the functor is not representable because it does not preserve monomorphisms. ///