$\boxed{\textbf{Exercise 1.5.i}}$ Let $\phi : 0 \to 1$ be the single nonidentity morphism in $\mathbf{2}$. Then consider the category $C \times \mathbf{2}$ and pick a morphism $f : x \to y$ in C. We have

$$\begin{tikzcd} (x,0) \arrow[r, "(\mathrm{id}_x,\phi)"] \arrow[d, "(f,\mathrm{id}_0)"'] \arrow[dr, "(f,\phi)"] & (x,1) \arrow[d, "(f,\mathrm{id}_1)"] \\ (y,0) \arrow[r, "(\mathrm{id}_y,\phi)"'] & (y,1) \end{tikzcd}$$

A quick check shows that this diagram commutes. Now apply H to $C \times \mathbf{2}$. Observe that because $F = Hi_0$, we have $H(f, \mathrm{id}_0) = Ff$. Similarly for G, we have $H(f, \mathrm{id}_1) = Gf$. Now because H is itself a single functor, it preserves compositions of morphisms. Hence the above square is lifted to D where it forms a naturality square for f for some natural transformation where $H(\mathrm{id}_x, \phi)$ and $H(\mathrm{id}_y, \phi)$ are the components.

Let $\mathcal{H}$ be the set of all functors $C \times \mathbf{2} \to D$ that restrict along i_0 and i_1 to F and G and $\mathcal{A}$ be the set of natural transformations from F to G. Observe then that applying H defines a function $\mathcal{F} : \mathcal{H} \to \mathcal{A}$.

We may now define $\mathcal{F}^{-1}$. For any $\alpha : F \Rightarrow G$, map the component α_x to (id_x, ϕ). This is enough to define a particular functor in $\mathcal{H}$ as the other mappings are constrained by F and G.

Hence $\mathcal{H} \cong \mathcal{A}$. ///

$\boxed{\textbf{Exercise 1.5.iii}}$ Let the two ismorphisms be $g : a \to a'$ and $h : b \to b'$ and let the leftmost diagram define f'. Then $f' = hfg^{-1}$. Hence the second diagram commutes because $f'g = hf$, the third because $h^{-1}f' = fg^{-1}$, and the fourth because $h^{-1}f'g = f$. ///

$\boxed{\textbf{Exercise 1.5.iv}}$

(i) Because F is fully faithful, $C(y, x) \cong D(Fy, Fx)$. Hence $(Ff)^{-1}$ has a preimage in C, say g. Thus $Ff \circ Fg = \mathrm{id}_y = F(fg) = F\mathrm{id}_y$. Hence $fg = \mathrm{id}_y$. Similar reasoning shows that $gf = \mathrm{id}_x$. Therefore f is an isomorphism. ///

(ii) Because F is fully faithful, we know that for any $x, y \in C$, $C(x, y) \cong D(Fx, Fy)$. There is at least one morphism in $D(Fx, Fy)$, namely the isomorphism Ff. Thus there is a morphism $f \in C(x, y)$. Now apply (i) to see that f is an isomorphism so $x \cong y$. ///

$\boxed{\textbf{Exercise 1.5.ix}}$ Theorem 1.5.9 says that a functor defining an equivalence of categories is fully faithful. Hence, for a category to be equivalent to a locally small category, the hom sets of each category must be isomorphic. Sets are only isomorphic to other sets; therefore the equivalent category is also locally small. ///

$\boxed{\textbf{Exercise 1.5.xi}}$ Consider the inclusion Ab $\to$ Group. It is fully faithful because the hom sets between abelian groups are the same in each category. It is not essentially surjective because nonabelian groups are not isomorphic to abelian groups. ///