Section 1

$\boxed{\textbf{Exercise 2.1.i}}$

$\boxed{\textbf{Exercise 2.1.ii}}$ If F is representable then we have a natural isomorphism α and the following commutative square:

$$\begin{tikzcd}
Fx \arrow[r, leftrightarrow, "\alpha_x"] \arrow[d, "Ff"'] & C(c,x) \arrow[d, "f_*"] \\
Fy \arrow[r, leftrightarrow, "\alpha_y"] & C(c,y)
\end{tikzcd}$$

Now suppose f is monic and pick two morphisms $g, h : W \rightrightarrows C(c, x)$ for some set W, such that $f_*g = f_*h$. This is equivalent to saying $\forall w \in W, f_*(g(w)) = f_*(h(w)) \implies f \circ (g(w)) = f \circ (h(w)) \implies g(w) = h(w) \implies g = h$ by the fact that f is monic. Hence f_* is monic.

Now, by the square, we have $\alpha_y Ff = f_* \alpha_x \implies Ff = \alpha_y^{-1} f_* \alpha_x$. Hence Ff is a composition of three injective maps which together are also injective. Therefore F preserves monomorphisms. ///