(In)determinacy in Mathematics

NUS Presidential Conference

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About

The topic of this conference is a cluster of related and hotly debated issues in the philosophy of mathematics involving determinacy and pluralism. Questions of determinacy and indeterminacy concern, roughly, how sharp or precise our mathematical words and concepts are, and whether there is an objective ‘fact of the matter’ corresponding to every mathematical claim. Questions of pluralism concern the issue of whether there are multiple, equally good, legitimate theories of certain mathematical subject-matters, such as arithmetic, geometry, or set theory. This conference will bring together researchers in these fields to present their recent work and discuss new options for development. Possible questions for discussion include the following.

- How should we understand determinacy and indeterminacy in mathematics? Is determinacy fundamentally best understood as linguistic, or conceptual, or metaphysical? How (if at all) is it best formalized?
- Are there different kinds of (in)determinacy in mathematics? What are the relations between different putative kinds of (in)determinacy in mathematics, e.g. indeterminacy of reference vs indeterminacy of truth value?
- Do different mathematical subfields (e.g. geometry, arithmetic, set theory) exhibit different kinds of (in)determinacy?
- How should pluralism about some mathematical domain be best understood?
- Is pluralism the right response to indeterminacy?
- What is the bearing of second-order or higher-order logic on all these debates?

Speakers

Andrew Bacon (USC)
Sharon Berry (Indiana Bloomington)
Tim Button (UCL)
Justin Clarke-Doane (Columbia)
Walter Dean (Warwick)
Joel Hamkins (Notre Dame)
Julien Murzi (Salzburg)
Ethan Russo (NYU)
Brett Topey (Salzburg)

Organizers (all at NUS): Neil Barton, Zachary Goodsell, Lavinia Picollo, Daniel Waxman, and Isaac Wilhelm.

Sponsors: NUS Presidential Young Professorship grant (Isaac Wilhelm).

Schedule

November 20

10:15-10:30am	Welcome coffee
10:30am-12pm	*What Is Carnap's Problem and How Can We Solve It?* Julien Murzi (Salzburg)
12-2pm	Lunch
2—3:30pm	*On Putnam's constructivization argument and reverse philosophy* Walter Dean (Warwick)
3:30–4pm	Coffee break
4–5:30pm	*(In)tolerance and analyticity: problems for Carnapian logicism about arithmetic* Tim Button (UCL)

November 21

10:30am-12pm	*If the omega rule is a solution, what was the problem?* Brett Topey (Salzburg)
12-2pm	Lunch
2—3:30pm	*Absolute Generality as Higher-Order Identity* Ethan Russo (NYU)
3:30–4pm	Coffee break
4–5:30pm	*Could the truths of mathematics have been different?* Andrew Bacon (USC)

November 22

10:30am-12pm	*Determinateness of truth does not come for free from determinateness of objects* Joel Hamkins (Notre Dame)
12-2pm	Lunch
2—3:30pm	*Multiverse Set Theory and Mathematical Explanations for Physical Facts* Sharon Berry (Indiana Bloomington)
3:30–4pm	Coffee break
4–5:30pm	*Intuition and Observation* Justin Clarke-Doane (Columbia)

Abstracts

November 20

What Is Carnap's Problem and How Can We Solve It?

Julien Murzi (Salzburg)

On a plausible approach to logical metasemantics, our dispositions to treat a logical expression's I- and E-rules as valid determine its contribution to the truth conditions of sentences in which it appears. Carnap's so-called Categoricity Problem is that the rules in question don't seem to fix a unique interpretation of our logical vocabulary: there appear to be deviant interpretations of both the connectives and the quantifiers that are compatible with the validity of their rules. And although standard responses are available to Carnap's problem as it applies to propositional logic (by appeal, e.g., to bilateral rules or to a local notion of validity), the case of the quantifiers is more difficult. In this paper, we offer a more precise account of how Carnap-style arguments work than has ever before been given, one that makes clear why certain such arguments succeed while others fail. In so doing, we demonstrate that despite recent criticisms, a particular recent account of the categoricity of the quantifiers isn't threatened by any of the alleged deviant interpretations that have been discussed in the literature: each of these either is incompatible with the validity of the quantifier rules or else results in an illegitimate Carnap-style argument.

On Putnam's constructivization argument and reverse philosophy

Walter Dean (Warwick)

This talk presents a case study in the use of reverse mathematics to assess the strength of the mathematical premises employed in philosophical arguments. I will focus on the role of the Lévy-Shoenfield Absoluteness Lemma and the Barwise Completeness Theorem in variants of the so-called "constructivization argument" originally presented by Putnam in "Models and reality" (1980). When seen in this light, it becomes clear that the conditional form of the argument Putnam likely intended sidesteps many of the concerns framed by Bays (e.g. 2001). But the proof of the theorem underlying the argument also highlights the non-absoluteness of the predicate "x is a constructible real number". I will close by considering how this bears more generally on the significance of (in)determinacy arguments involving definability-theoretic notions.

(In)tolerance and analyticity: problems for Carnapian logicism about arithmetic

Tim Button (UCL)

In Logical Syntax of Language, Carnap suggested two ways to secure the determinacy of arithmetic: use of the ω-rule (Language I) and full higher-order logic (Language II). Several folks (e.g. Beth and Gödel) rightly found such moves suspicious. But internal categoricity promises to yield the determinacy of arithmetic. So in this paper I want to ask: does internal categoricity give new life to LSL-style logicism? Sadly, it does not. Indeed, we get two sorts of conflict with the Principle of Tolerance. First: the Beth/Gödel problem recurs as the problem of how I can even so much as set up frameworks. Second: it becomes impossible for me to move freely between "equally good" frameworks; internal categoricity makes me rather judgy.

November 21

If the omega rule is a solution, what was the problem?

Brett Topey (Salzburg)

It's sometimes suggested that we can secure determinacy of truth value for arithmetical sentences by appeal to the claim that the omega rule is one of the rules governing the use of our arithmetical vocabulary, but those who make this suggestion rarely given any argument for the claim that finite beings like us can follow infinitary rules like the omega rule. Jared Warren, though, is an exception: he has recently argued that we can and do follow the omega rule, presenting a case in which it's purportedly clear that we're actually disposed to use that rule. Here I try to establish that this argument fails. Then, taking a step back, I consider just what the determinacy worry is supposed to be, for Warren, and I conclude that, given the shape of his unrestricted inferentialist picture, there can be no such worry after all—it turns out to be trivially easy, on that picture, to show that our arithmetical language is both categorical and truth-theoretically determinate. Finally, I suggest that the sorts of easy determinacy arguments that are available to Warren may turn out not to depend on the specifics of his picture; they may turn out to be available to anyone in need of a determinacy argument.

Absolute Generality as Higher-Order Identity

Ethan Russo (NYU)

The question of Absolute Generality is whether quantifiers are ever as general as can be. Absolutists claim that quantifiers sometimes are absolutely general, while Relativists claim that quantifiers are never absolutely general. Although diverse philosophers have found the Relativist ethos compelling, it has been hard to articulate a consistent thesis which says what the Relativist seems to want to say. In this paper, I offer Relativists a way forward: I argue that what is needed to successfully state Relativism is a way of generalizing that is non-quantificational. After showing how to define such a device of generalization in terms of identity between properties in a higher-order logical language, I use the device to articulate a form of Relativism which I prove to be consistent and which I argue captures the intuitive vision of the Relativist.

Could the truths of mathematics have been different?

Andrew Bacon (USC)

Could the truths of mathematics have been different than they in fact are? If so, which truths could have been different? Do the contingent mathematical facts supervene on physical facts, or are they free floating? I investigate these questions within a framework of higher-order modal logic, drawing sometimes surprising connections between the necessity of arithmetic and analysis and other theses of modal metaphysics: the thesis that possibility in the broadest sense is governed by a logic of S5, that what is possible holds in some maximally specific possibility, and that every property can be rigidified. The investigation will distinguish sharply between platonic contingency—contingency about whether particular abstract "platonic" mathematical objects are arranged in a certain way (e.g. in a natural number or real number structure)—from a deeper variety of structural contingency concerning what holds of objects whenever they are arranged in that way.

November 22

Determinateness of truth does not come for free from determinateness of objects (joint work with Ruizhi Yang)

Joel Hamkins (Notre Dame)

I shall discuss the question whether we may regard determinateness of truth as flowing from determinateness of objects in a mathematical structure. I shall showcase several results in the model theory of set theory and arithmetic that seem to speak against this. For example, there are two models of ZFC set theory that share exactly the same arithmetic structure of the natural numbers $\langle \mathbb{N}, +, *, 0, 1, < \rangle$, what they each view as the standard model of arithmetic, but they disagree about which arithmetic sentences are true in that structure. There are models of ZFC set theory with the same arithmetic structure and the same arithmetic truth, but which disagree on truth-about-truth, or that agree on that, but disagree on higher levels of iterated truth, at any desired level. There are models of set theory with the same natural numbers and real numbers, but which disagree on projective truth. There are models of ZFC that have a rank initial segment $V\theta$ in common, but they disagree about whether it is a model of ZFC. All these examples show senses in which determinateness about objects does not seem to cause determinatess about truth.

Multiverse Set Theory and Mathematical Explanations for Physical Facts

Sharon Berry (Indiana Bloomington)

Width multiverse approaches to set theory reject reference to a notion of all possible ways of choosing that could fix the structure of an intended hierarchy of sets (up to height). Yet many seemingly cogent physical explanatory hypotheses appeal to such a notion. In this talk, I'll argue this presents a prima facia problem for width multiverse theories. I'll then distinguish (and note problems for) three ways of responding to this challenge.

Intuition and Observation (joint work with Avner Ash)

Justin Clarke-Doane (Columbia)

The motivating question of this talk is: 'How are our beliefs in the theorems of mathematics justified?' This is distinguished from the question 'How are our mathematical beliefs reliably true?' We examine an influential answer, outlined by Russell, championed by Gödel, and developed by those searching for new axioms to settle undecidables, that our mathematical beliefs are justified by 'intuitions', as our scientific beliefs are justified by observations. On this view, axioms are analogous to laws of nature. They are postulated to best systematize the data to be explained. We argue that there is a decisive difference between the cases. There is agreement on the data to be systematized in the scientific case that has no analog in the mathematical one. There is virtual consensus over observations, but conspicuous dispute over intuitions. In this respect, mathematics more closely resembles philosophy. We conclude by distinguishing two ideas that have long been associated – realism (the idea that there is an independent reality) and objectivity (the idea that in a disagreement, only one of us can be right). We argue that, while realism is true of mathematics and much of philosophy, these domains fail to be objective. One upshot of the discussion is that even questions of fundamental physics may fail to be objective insofar as the mathematical, logical, and evaluative hypotheses that they presuppose fail to be. Another is pragmatism. Factual questions in mathematics, modality, logic, and evaluative areas go proxy for non-factual practical ones.